QUANTUM PHYSICS ON A QUANTUM COMPUTER*

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MINI-LECTURE SERIES ON QUANTUM COMPUTING AND QUANTUM INFORMATION SCIENCE,

Jefferson Laboratory March 11, 2020

U.S. DEPARTMENT OF ENERGY

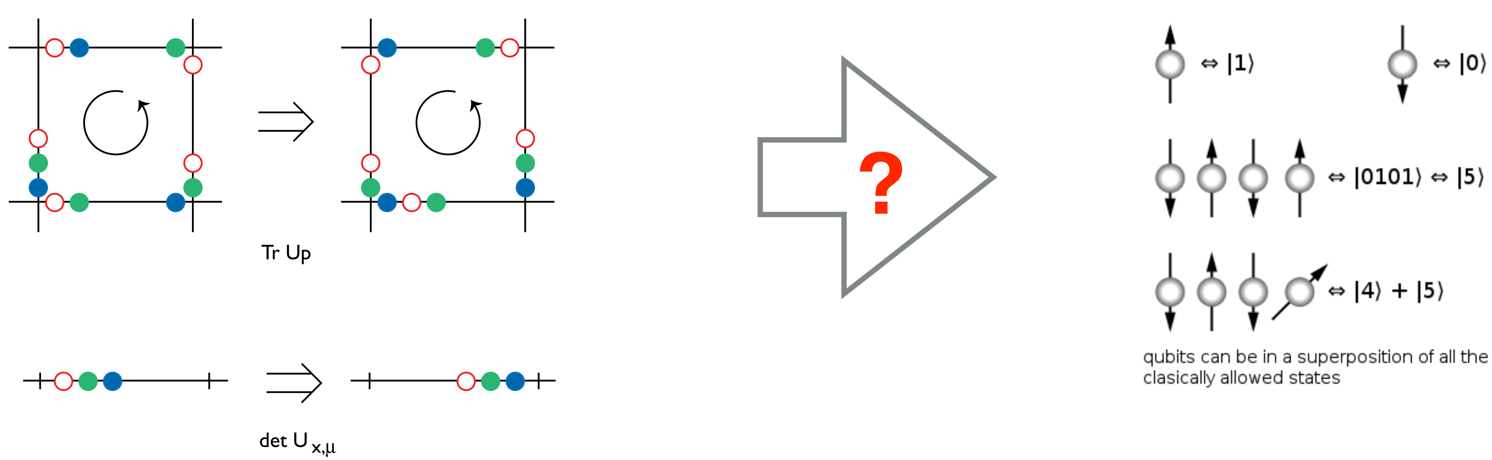
*D-theory: The accidental discovery of a Quantum Algorithm for Lattice Gauge theories: Circa 1998:

R. C. Brower, S. Chandrasekharan, U-J Wise , QCD as quantum link model, Phys. Rev D 60 (1999).

R. C. Brower, S. Chandrasekharan, U-J Wiese, D-theory: Field quantization … discrete variable Nucl. Phys. B (2004)

D-theory :QCD Abacus*

Fermionic Qubit Algorithm ?



Bravyi and Kitaev "Fermionic Quantum Computation" (2002)

R. C. Brower, The QCD Abacus: APCTP-ICPT Conference, Seoul, Korea, May (1997)

Richard Feynman

On quantum physics and computer simulation

"There is plenty of room to make [computers] smaller. . . . nothing that I can see in the physical laws . . . says the computer elements cannot be made enormously smaller than they are now. In fact, there may be certain advantages. —1959"

60 years ago!

"trying to find a computer simulation of physics seems to me to be an excellent program to follow out. . . . the real use of it would be with quantum mechanics. . . . Nature isn't classical . . . and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. —1981"

Feynman's Unfinished Legacy

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

"...The question is, If we wrote a Hamiltonian which *involved only* these [Pauli] operators, locally coupled to corresponding operators on the other space-time points, could we imitate every quantum mechanical *system* which is discrete and has a finite number of degrees of freedom? *I know, almost certainly,* that we could do that for any quantum mechan-*ical system which involves* Bose particles. I'm no sure whether Fermi *particles* could be described by such a system. So, I leave that open..."

Richard P. Feynman

(Simulating Physics with Computers (1982))

International J. of Theoretical Physics Vol 21, now 6./7, 1982

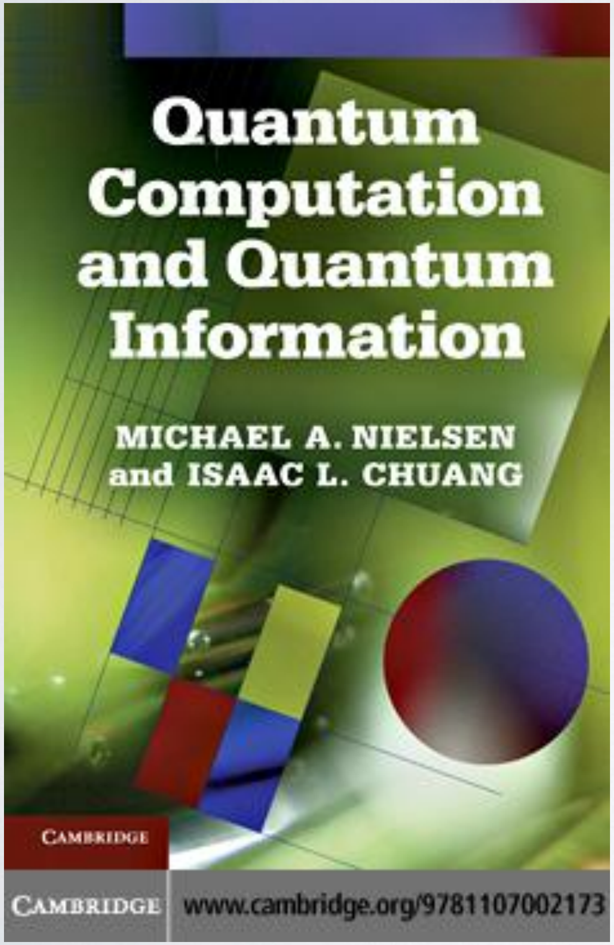
OUTLINE:

Quantum Computing for Quantum Field Theory

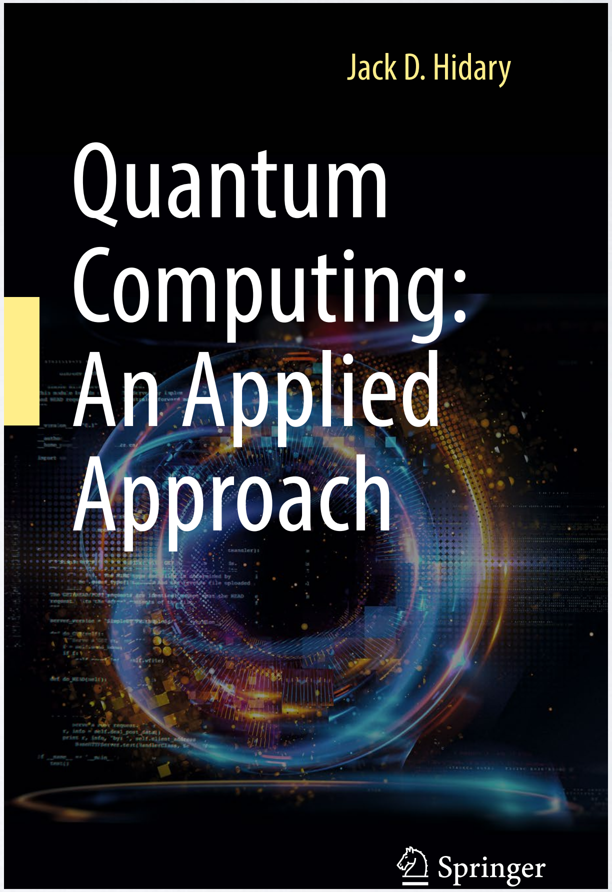
1. What is a Computer?
2. What is Quantum ? (Qubits and all that)
3. Universal vs Efficient/Fault Tol Quantum Computing ? (FFT)
4. What is Quantum Link Lattice Field Theory: Universality?
5. Example for NISQ (Noisy Intermediate Scale Quantum) ?
6. Programming (virtually) the IBM Q ?

GOOD QC REFERENCES

Help you Read this Book!

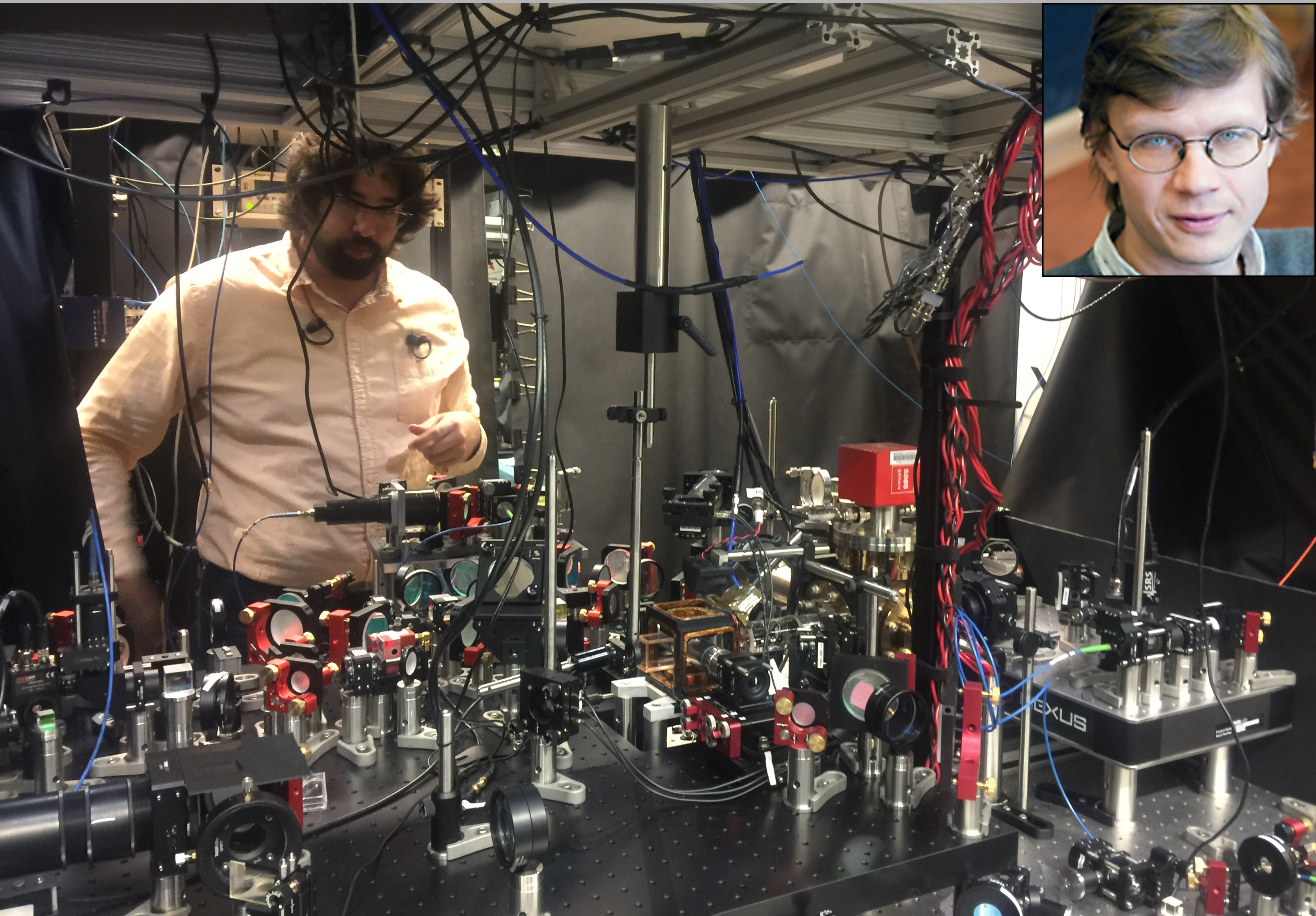


Simple Pedagogical Overview



IBM's new 53-qubit quantum computer is its biggest yet

Mikhail Lukin's Lab Harvard Trapped Ion



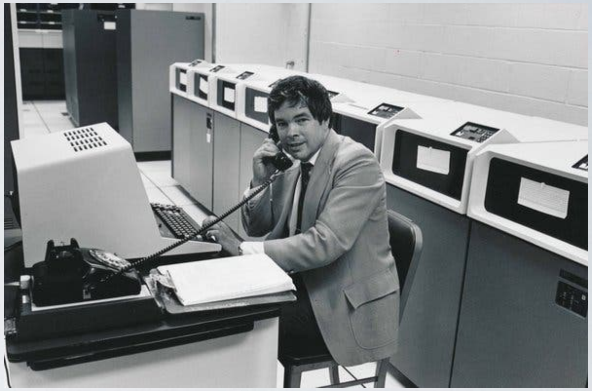
TODAY

Oak Ridge National Laboratory's 200 petaflop supercomputer

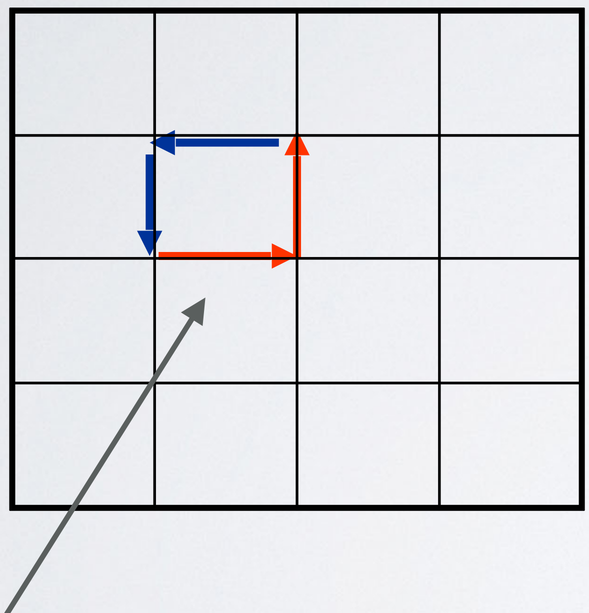
“Lattice Gauge Theory Machine” 200,000,000,000,000,000 Floats/sec
9,216 IBM POWER9 CPUs and 27648 NVIDIA GPUs
Each GPU has 5120 Cores and total of 580,608,000,000,000 transistors

WILSON'S LATTICE QCD*

$$Z_{wilson} = \int_{Haar} dU \, \mathbf{e}^{\frac{6}{g^2} \sum_{\Box} Tr[U_{\Box} + U_{\Box}^{\dagger}]}$$



45 Years Ago



$$U^{ij}(x, x+\mu) = e^{iagA_{\mu}^{ij}(x)}$$

$$i, j = 1, 2, 3$$

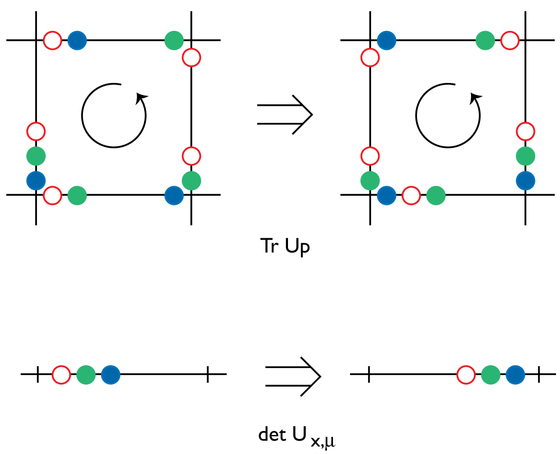
SU(3) Gauge Transport on each link.
Exact per site gauge invariance

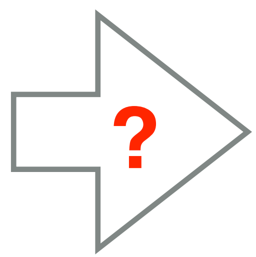
$$U_{\Box_{\mu\nu}}(x) = [U(x, x+\mu)U(x+\mu, x+\mu+\nu)][U(x, x+\nu)U(x+\nu, x+\nu+\mu)]^{\dagger}$$
$$\simeq 1 + a^2 iF_{\mu\nu} - (a^4/2)F_{\mu\nu}^2 + \cdots$$

* K.G. Wilson, Phys. Rev. D 10 (1974), 2445.

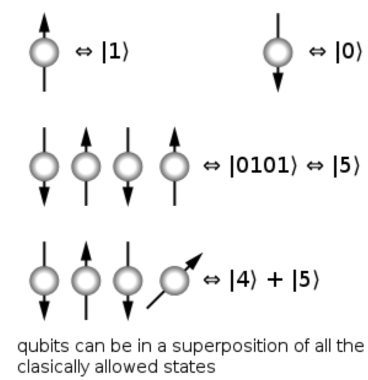
From Bits to Qubits ?

QCD



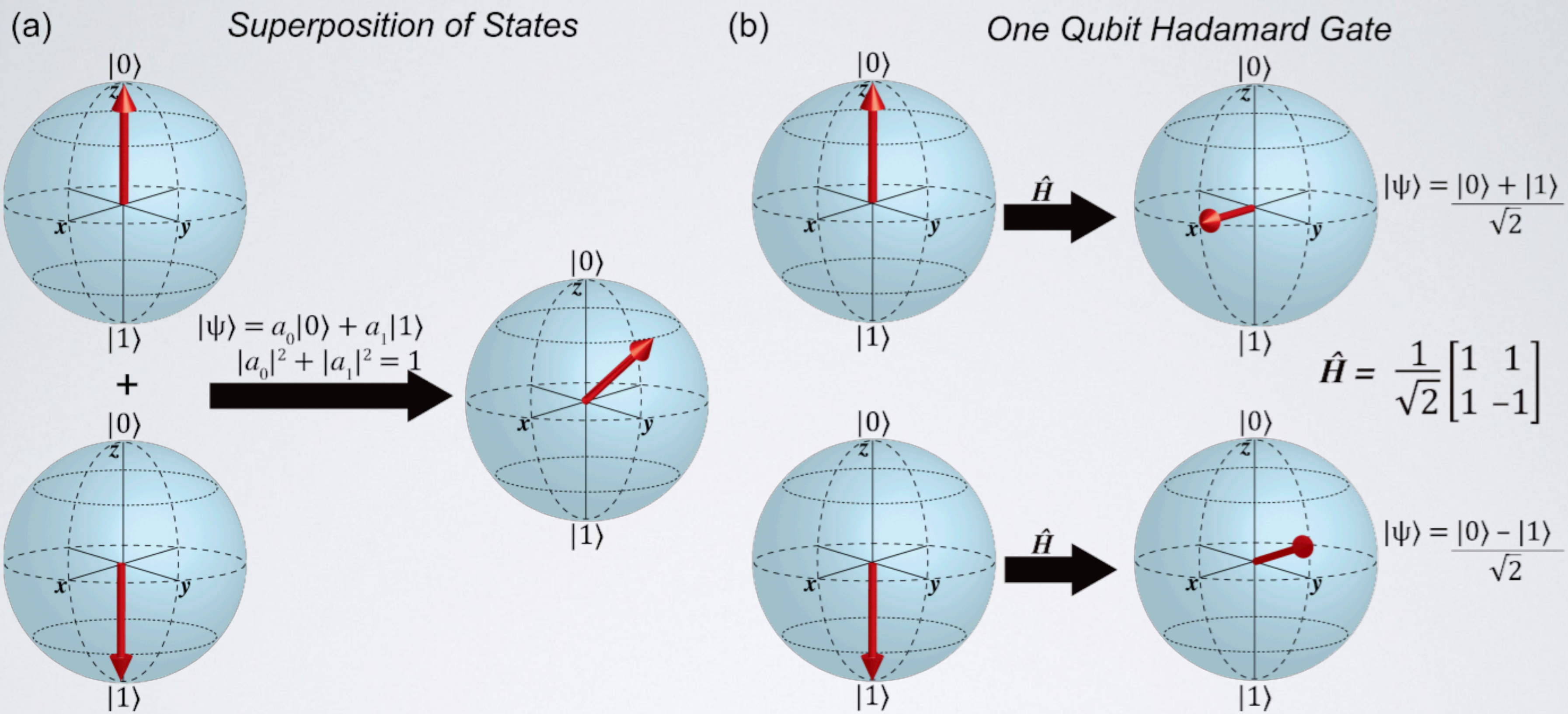


Fermionic Qubit



* See THE QCD ABACUS: A New Formulation for Lattice Gauge Theories R. C. Brower https://arxiv.org/abs/hep-lat/9711027 Lecture at "APCTP-ICTP Joint International Conference '97 on Recent Developments in Non-perturbative Method" May, 1997, Seoul, Korea. MIT Preprint CTP 2693.

WHAT IS A QUBIT



$$|\psi_{block}\rangle = e^{i\gamma}\big(\cos\frac{\theta}{2}|0\rangle + ie^{i\phi}\sin\frac{\theta}{2}|1\rangle\big)$$

$$|\psi\rangle = e^{i(\theta/2)\hat{n}\cdot\vec{\sigma}}|0\rangle = (\cos\theta/2 + i\hat{n}\cdot\vec{\sigma}\sin\theta/2)|0\rangle$$

Math Stuff: U(2) = U(1) x SU(2)^*

$$U = e^{i\phi}e^{i(\theta/2)\hat{n}\cdot\vec{\sigma}} = e^{i\phi}[\cos(\theta/2) + i\,\hat{n}\cdot\vec{\sigma}\,\sin(\theta/2)]$$

$$U = e^{i\phi}\begin{bmatrix}\cos\theta/2 + in_z\sin\theta/2 & i(n_x - in_y)\sin\theta/2 \\ i(n_x + in_y)\sin\theta/2 & \cos\theta/2 - in_z\sin\theta/2\end{bmatrix}$$

THEREFORE:

$$U|0\rangle = e^{i\phi}[\cos(\theta/2) + in_z\sin(\theta/2)]|0\rangle + (in_x + n_y)\sin(\theta/2)|1\rangle$$

$$\alpha\alpha^* + \beta\beta^* = 1 \implies \cos^2(\theta) + \hat{n}\cdot\hat{n}\sin^2(\theta) = 1$$

---

* OR $U = e^{i\phi}[k_0\sigma_0 + i\,\vec{k}\cdot\vec{\sigma}] \implies k_\mu k_\mu = 1 \implies \mathbb{S}^1 \otimes \mathbb{S}^3$

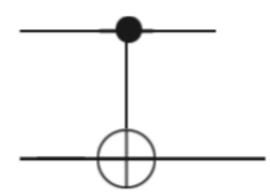
UNIVERSAL GATE SET

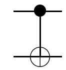
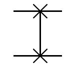
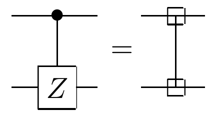
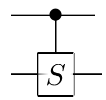
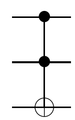
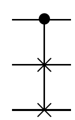
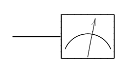
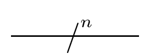
*	Hadamard	H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	
	Pauli-X	X	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	NOT
	Pauli-Y	Y	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	
	Pauli-Z	Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	
	Phase	S	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	
*	$\pi/8$	T	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	

Figure 4.2. Names, symbols, and unitary matrices for the common single qubit gates.

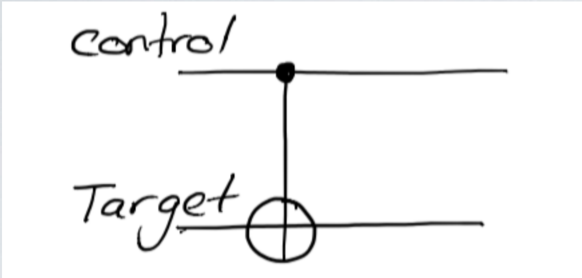
CNOT

* $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$



controlled-NOT		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
swap		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
controlled-Z		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
controlled-phase		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix}$
Toffoli		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$
Fredkin (controlled-swap)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
measurement		Projection onto $\lvert 0\rangle$ and $\lvert 1\rangle$
qubit		wire carrying a single qubit (time goes left to right)
classical bit		wire carrying a single classical bit
n qubits		wire carrying n qubits

WHAT IS CNOT ? UNITARY (GENERALIZED) XOR



c = 1 (true) activates Not t *

$$|c\rangle \otimes |t\rangle \equiv |ct\rangle$$

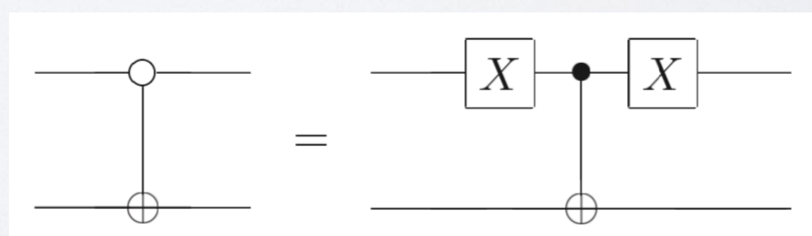
XOR / Mod 2 add

Cnot is $\quad |c\rangle|t\rangle \to |c\rangle|c \oplus t\rangle$

$$\begin{array}{cc} & \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ CNOT = & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = Cnot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

WARNING
NC has a NotCnot with 0



UNIVERSAL QUANTUM COMPUTER

involving only those gates. We now describe three universality constructions for quantum computation. These constructions build upon each other, and culminate in a proof that any unitary operation can be approximated to arbitrary accuracy using Hadamard, phase, CNOT, and $\pi/8$ gates. You may wonder why the phase gate appears in this list, since it can be constructed from two $\pi/8$ gates; it is included because of its natural role in the fault-tolerant constructions described in Chapter 10.

The first construction shows that an arbitrary unitary operator may be expressed *exactly* as a product of unitary operators that each acts non-trivially only on a subspace spanned by two computational basis states. The second construction combines the first construction with the results of the previous section to show that an arbitrary unitary operator may be expressed *exactly* using single qubit and CNOT gates. The third construction combines the second construction with a proof that single qubit operation may be approximated to arbitrary accuracy using the Hadamard, phase, and $\pi/8$ gates. This in turn implies that any unitary operation can be approximated to arbitrary accuracy using Hadamard, phase, CNOT, and $\pi/8$ gates.

Our constructions say little about efficiency – how many (polynomially or exponentially many) gates must be composed in order to create a given unitary transform. In Section 4.5.4 we show that there *exist* unitary transforms which require exponentially many gates to approximate. Of course, the goal of quantum computation is to find interesting families of unitary transformations that *can* be performed efficiently.

Universal Quantum Computing

$$|\Psi(t)\rangle = U(t)|\Psi(t) = U(t)|\Psi(t) = e^{-itH}|\Psi(0)\rangle$$

I. Any dxd unitary U(d) is product of 2x2 U(2) unitary by GAUSSIAN ELIMINATION!

$$2^n(2^n - 1)/2 = 2^{n-1}(2^n - 1) \quad \text{Qubit Rotations}$$

II. General U(2) rotation: Gray coding with *cnot's and one Qubit* rotation

$$n^2 \quad \texttt{cnot} \text{ gates} + \text{one rotation}$$

III. But Solovay-Kitaev theorem approx U(2) on single Qubit can be approximated by Hadamard and pi/8 gates

$$\text{No of gates } \in O(\log^c(1/\epsilon) \quad c \simeq 2$$

Gaussian Elimination: Rotate one Qubit at a time!

Label: $|s_n\rangle \otimes \cdots \otimes |s_2\rangle \otimes |s_1\rangle$

$$i = 0, 1, \cdots 2^n \rightarrow 000, 001, 010, 011, 100, 101, 110, 111, \cdots 2^n - 1$$

$$UX = b \implies u_{ij}x_j = b_i$$

$$u_{11}x_1 + u_{12}x_2 + u_{13}x_3 + u_{14}x_4 + \cdots = b_1$$
$$u_{21}x_1 + u_{22}x_2 + u_{23}x_3 + u_{24}x_4 + \cdots = b_2$$
$$u_{31}x_1 + u_{32}x_2 + u_{33}x_3 + u_{34}x_4 + \cdots = b_3$$
$$\cdots$$

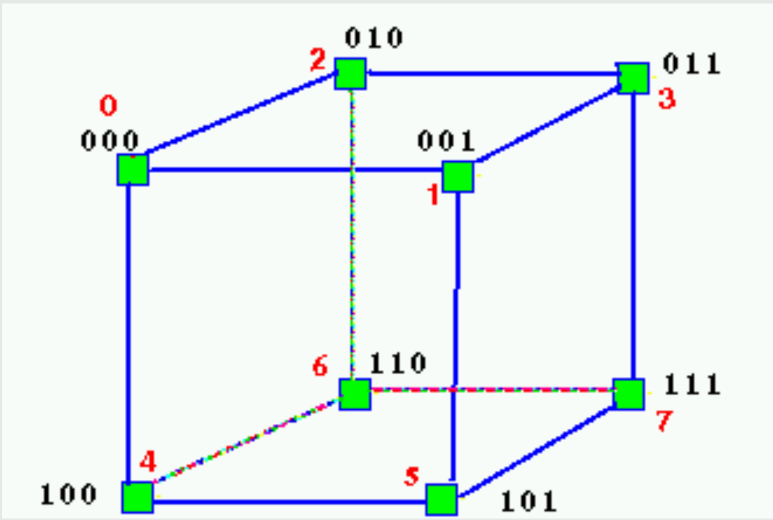
Now multiply $U_1UX = U_1b$ where

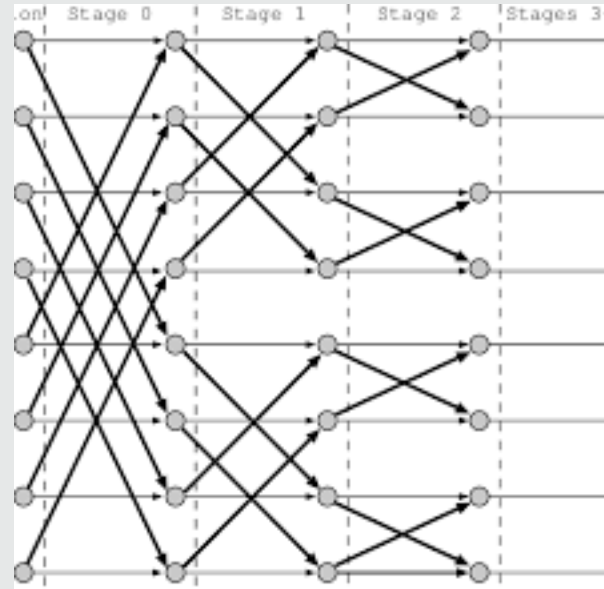
$$U_1 = \begin{bmatrix} \alpha & \beta & 0 & 0 & 0 & 0 & \cdots \\ -\beta^* & \alpha* & 0 & 0 & 0 & \cdots & \\ 0 & 0 & 1 & 0 & 0 & \cdots & \\ 0 & 0 & 0 & 1 & 0 & \cdots & \\ 0 & 0 & 0 & 0 & 1 & \cdots & \\ \cdots & & & & & & \end{bmatrix}$$

$U \rightarrow U_1U$ such that $u_{21}^{(new)} = -\beta^* u_{11} + \alpha^* u_{12} = 0$

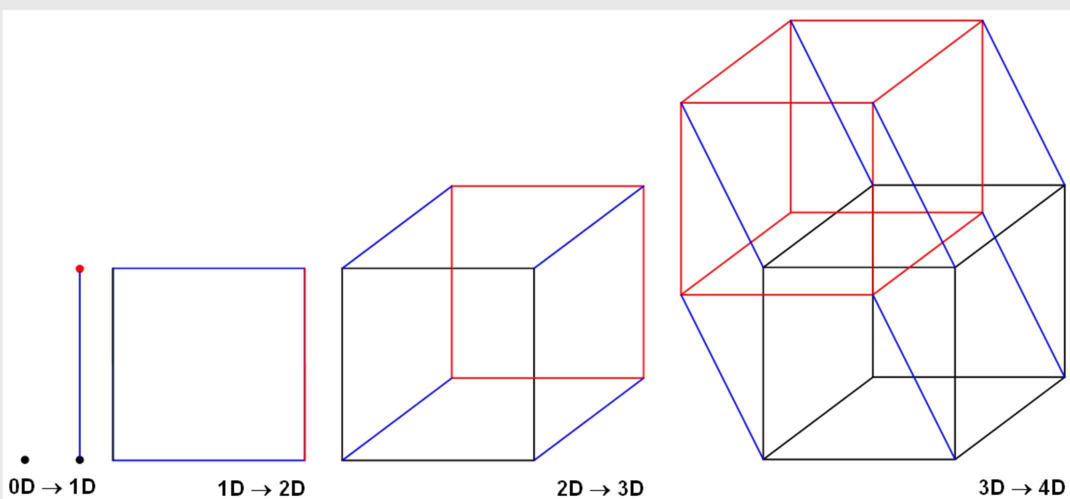
Hypercube - Gray Codes - FFT and all that

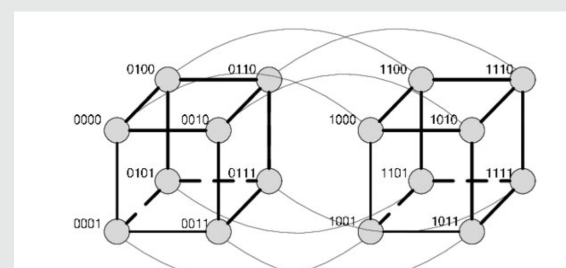
- Gray Coding: Adjacent Vertices Differ by one bit





Texi Cab distance = Hamming Distance.

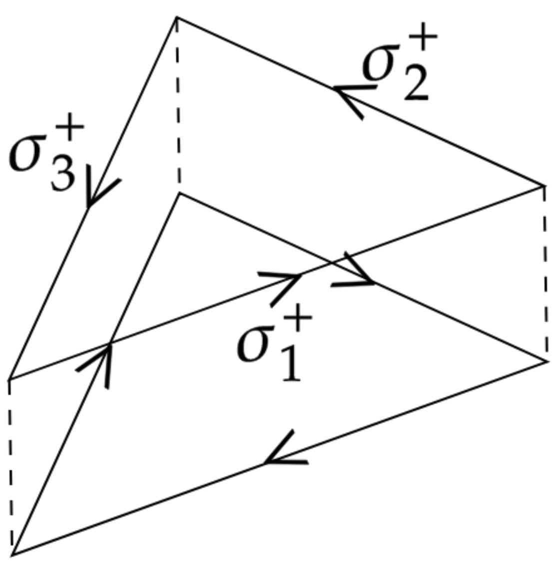




Plaquette term for U(1) is an example

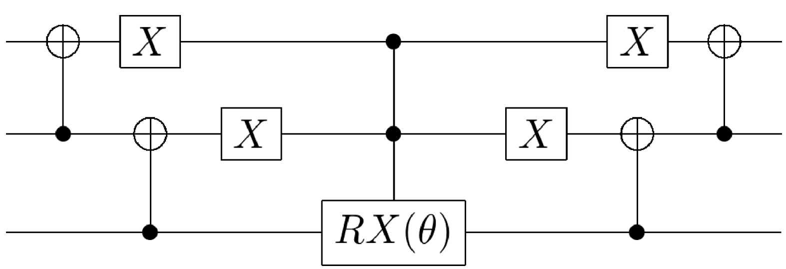
For single triangle:

$$\hat{H} = \sum_s \Big[\quad \frac{g^2}{2} \sum_{j=1}^{3} (\sigma^z_{j,s} + \sigma^z_{j,s+1})^2 + \frac{\alpha}{2g^2} \sum_{j=1}^{3} (\sigma^+_{j,s}\, \sigma^-_{j,s+1} + \sigma^-_{j,s}\, \sigma^+_{j,s+1}) \\ \boxed{- \frac{1}{2g^2} (\sigma^+_{1,s}\, \sigma^+_{2,s}\, \sigma^+_{3,s} + \sigma^-_{1,s}\, \sigma^-_{2,s}\, \sigma^-_{3,s})} \Big]$$



$+h.c.$

Circuit



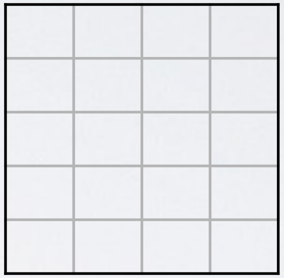
$$\equiv$$

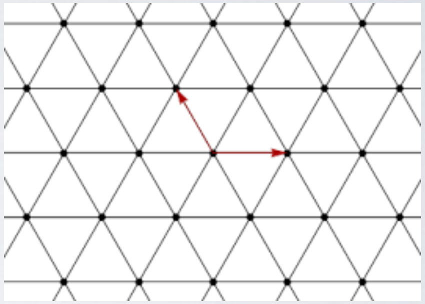
$$e^{-i\hat{H}_B\theta}$$

Universality == Many equivalent LFT

- Different space-time + field discretizations define exactly the same continuum quantum field theory

- *eg. Lattice different lattice give identical c = 1/2 CFT — square, triangle or spherical lattice!*

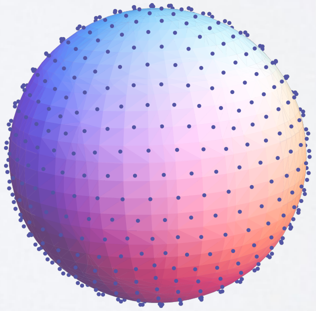


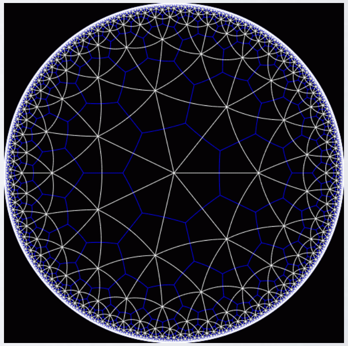


- *Fields: Continuum phi 4th field and single bit Ising fields are equivalent*

$$s \in \pm 1 \iff \phi_x \in \mathbb{R}$$

- *Bosonic Sine Gordon Theory = Fermionic Thirring Theory*





- *N = 4 SUSY is dual AdS Gravity*

WARM UP: O(3) HEISENBERG SPIN MODEL

Classical O(3) MODEL

$$Z = \int dS \, \exp[\frac{1}{g^2} \sum_{\langle x,y \rangle} \vec{S}_x \cdot \vec{S}_y]$$

Quantum antiferromagnet O(3) MODEL

$$\to Tr[e^{-\frac{\beta}{g^2} \sum_{x,\mu} \vec{\sigma}_x \cdot \vec{\sigma}_{x+\mu}}]$$

Warm up with Fermionic Quantum Operator

$$\vec{S}_x \to a_i^\dagger(x) \sigma^{ij} a_j(x) \quad \text{or} \quad \hat{S}^{ij}(x) = a_i^\dagger(x) a_j(x)$$

$$\{a_i(x), a_j^\dagger(x)\} = \delta_{ij}\delta_{xy} \quad , \quad \{a_i^\dagger(x) a_j^\dagger(x)\} = 0 \quad , \quad \{a_i(x), a_j(x)\} = 0$$

$$Z = \mathrm{Tr} \exp(-\beta \hat{H}). \qquad \hat{H} = \sum_{\langle x,y \rangle} Tr[\hat{S}_x \hat{S}_y] \quad , \quad Tr[\hat{S}_x] = 0$$

Global Rotation: $\vec{J} = \sum_x Tr[\vec{\sigma} \hat{S}_x] \implies [\vec{J}, \hat{H}] = 0$

Local Fermion No: $\hat{F}_x = Tr[\hat{S}_x] \implies [\hat{F}_x, \hat{H}] = 0$

B.B. Beard and U-J Wiese

https://arxiv.org/abs/cond-mat/9602164

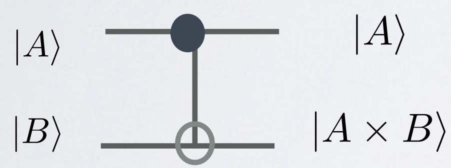
FERMIONIC QUBIT GATES

Each Fermion $a^\dagger a + a a^\dagger = 1$ $aa = a^\dagger a^\dagger = 0$

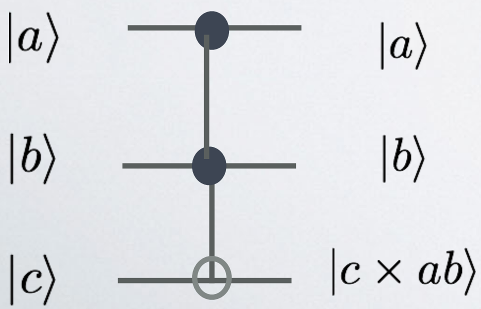
On Each qubit $a^\dagger(\alpha|1\rangle + \beta|0\rangle) = \beta|1\rangle$ $a(\alpha|1\rangle + \beta|0\rangle) = \alpha|0\rangle$

$$a^\dagger + a \implies X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (a + a^\dagger)^2 - 1 \implies H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$(a^\dagger + a)/i \implies Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad (2a^\dagger a - 1) \implies Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



$$(1 - a^\dagger a) + a^\dagger a(b + b^\dagger) \implies \text{ContrNOT}$$



$$(1 - a^\dagger a b^\dagger b) + a^\dagger a b^\dagger b(c^\dagger + c) \implies \text{Toffoli}$$

Classical Reversible Computing Conserving Energy R. Landauer IBM J journal of Research and Development, vol. 5, pp. 183-191, 1961

SYMPLIAL/GAUGE ALGEBRA

$$H = \frac{g^2}{2} \sum_{\langle x, x+\mu \rangle} (Tr[E_L^2(x,\mu)] + Tr[E_R^2(x,\mu)]) - \frac{1}{2g^2} \sum_{\Box} Tr[U_\Box + U_\Box^\dagger]$$

Example: U(1) Abelian P/Q symplectic operators in Q-basis are $$E = i\frac{d}{d\theta} \quad , \quad U = exp[i\theta]$$

U(N) generalization of Gauge Algebra is

$$E^{ij} \equiv \lambda_\alpha^{ij} E^\alpha \qquad \Longrightarrow \qquad [E^\alpha, E^\beta] = 2if^{\alpha\beta\gamma}E^\alpha$$

$$[E_L, U] = -E_L U \quad , \quad [E_R, U] = -UE_R$$

$$E_R = U^\dagger E_L U \quad , \quad [U, U^\dagger] = 0 \qquad UU^\dagger = 1$$

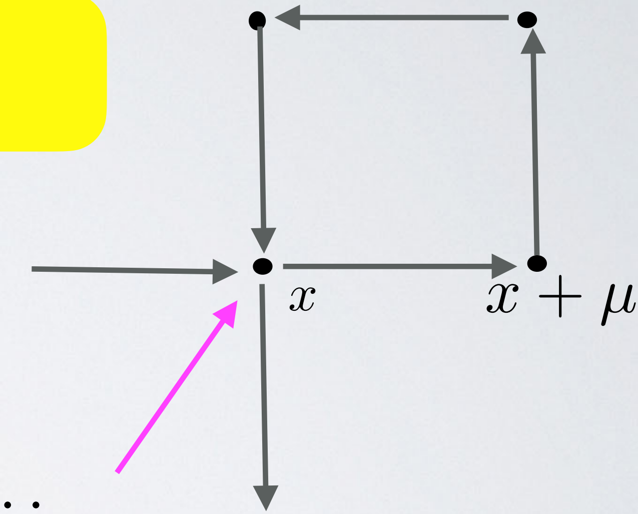
DISCRETE QUANTUM LINK

On each link (x,x + μ) introduce $2N_c$complex fermion

$a_{,}a_i^\dagger$ right(+) moving and $b_j, b_j^\dagger$ left(-) moving fluxon

LINK:

$$a_i \longrightarrow b_j^\dagger$$



$$U_{ij}(x, x+\mu) \quad \to \quad \hat{U}_{ij} = a_i(x)\, b_j^\dagger(x+\mu)$$

$$\{a_i, a_j^\dagger\} = \delta_{ij} \qquad \{b_i, b_j^\dagger\} = \delta_{ij}$$

Local Gaue Operators $\quad \Omega_{ij}(x) = a_i^\dagger(x) a_j(x) + \cdots$

For SU(3) QCD have SU(6) per Link Lie Algebra $\quad \begin{bmatrix} a_i a_j^\dagger & a_i b_j^\dagger \\ b_i a_j^\dagger & b_i b_j^\dagger \end{bmatrix}$

WHAT ABOUT ANTI-SYMMETRIC FERMIONIC FOCK SPACE?

Step #1: PARA-STATISTICS:

D-Theory only require anti-commutator within each a's and b's set

$$[a_i^\dagger a_j , a_p^\dagger b_q] = [a_i^\dagger a_j , a_p^\dagger]\, b_q + a_p^\dagger \cancelto{0}{[a_i^\dagger a_j , b_q]}$$

$$\Longrightarrow \quad [E_L , U] \quad = \quad -E_L\, U$$

same for a —> b

Step #2; Jordan-Wigner:

Apply to Locally to each set of 3 a's and b's.

$$a_1^\dagger = \sigma_1^+ \quad , \quad a_2^\dagger = -\sigma_1^z \sigma_2^+ \quad , \quad a_3^\dagger = \sigma_1^z \sigma_2^z \sigma_3^+$$

same for a —> b

PARA-STATISTICS & JORDAN-WIGNER TO THE RESCUE

Simpler than Bravyi and Kitaev "Fermionic Quantum Computation" (2002)

Math Stuff: “Gamma Matrices are Jordan-Wigner”

$$d = 2: \quad \vec{\gamma}_i^{(2)} \to (\sigma_1, \sigma_2)$$

$$\gamma_{d+1}^{(2)} \to \sigma_3 = -i\sigma_1\sigma_2$$

$$d = 4: \quad \vec{\gamma}_i^{(4)} \to (\sigma_3 \otimes \vec{\gamma}^{(2)}, \sigma_1 \otimes I_2, \sigma_2 \otimes I_2)$$

$$\gamma_{d+1}^{(4)} \to \gamma_1\gamma_2\gamma_3\gamma_4 = \gamma_5$$

$$d \leftarrow d + 2: \quad \vec{\gamma}_i^{(d)} \to (\sigma_3 \otimes \vec{\gamma}^{(d-2)}, \sigma_1 \otimes I_{2^d}, \sigma_2 \otimes I_{2^d})$$

$$\gamma_{d+1}^{(d)} \to \gamma_1\gamma_2\gamma_3 \cdots \gamma_d = \gamma_{d+1}$$

KOGUT SUSKIND HAMILTONIAN

$$H = \frac{g^2}{2} \sum_{\langle x, x+\mu \rangle} (Tr[E_L^2(x,\mu)] + Tr[E_R^2(x,\mu)]) - \frac{1}{2g^2} \sum_{\Box} Tr[U_\Box + U_\Box^\dagger]$$

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$$[E_L, U] = -E_L U \quad , \quad [E_R, U] = -U E_R$$

$$E_R = U^\dagger E_L U \quad , \quad [U, U^\dagger] = 0 \qquad UU^\dagger = 1$$

STARTING TO TEST REAL TIME QUBIT ALGORITHM FOR U(1) QUANTUM LINK GAUGE THEORY

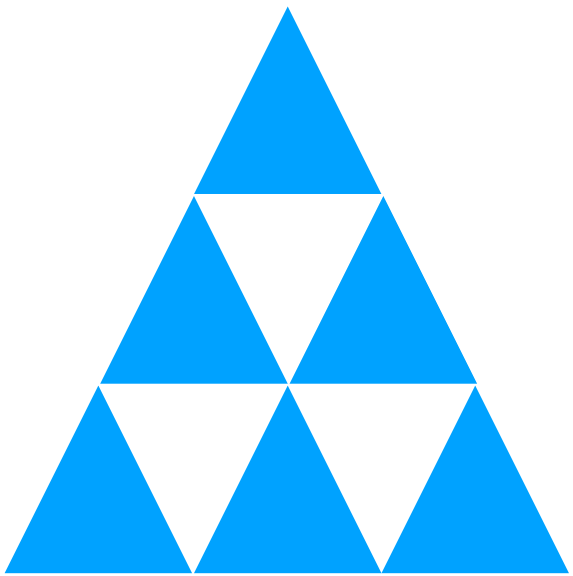
See Gauge Theory for a Quantum Computer

R. C. Brower, D. Berenstein & H. Kawai (Latice 2020)

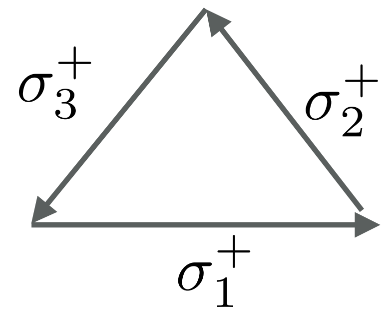
$$\hat{H} = \frac{e^2}{2} \sum_{links,s} (\sigma^z_s + \sigma^z_{s+1})^2 + \alpha \sum_{links,s} [\sigma^+_s \otimes \sigma^-_{s+1} + \sigma^-_s \otimes \sigma^+_{s+1}]$$
$$- \frac{1}{2e^2} \sum_{\triangle,s} [\sigma^+_s \otimes \sigma^+_s \otimes \sigma^+_s + \sigma^-_s \otimes \sigma^-_s \otimes \sigma^-_s]$$

Few very simple kernels in Trotter factorization into
Gauge invariant Unitary operators with very few Qubit width

Choose 2 + 1 on U(1) Hamiltonian on a Triangular spacial lattice



To evaluate lattice: alternate between two coloring of triangles.



+ h.c

Total: about 15-20 consecutive gate operations (coherence time) per qubit per Trotter step

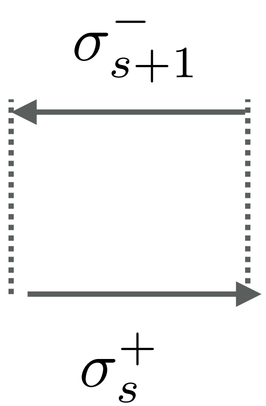
Estimate of current machines: 3 Trotter steps on Lattice

Extra dimension builds local field rep. from XYZ ferromagnetic chain

Using
$$\sigma^+ \otimes \sigma^- + \sigma^- \otimes \sigma^+ = \frac{1}{2}(\sigma^x \otimes \sigma^x + \sigma^y \otimes \sigma^y)$$
$$(\sigma^z \otimes 1 + 1 \otimes \sigma^z)^2 = 2(1 + \sigma^z \otimes \sigma^z)$$

The each links between two triangle are coupled by 2 Qubit ferromagnetic interaction operator to align them.

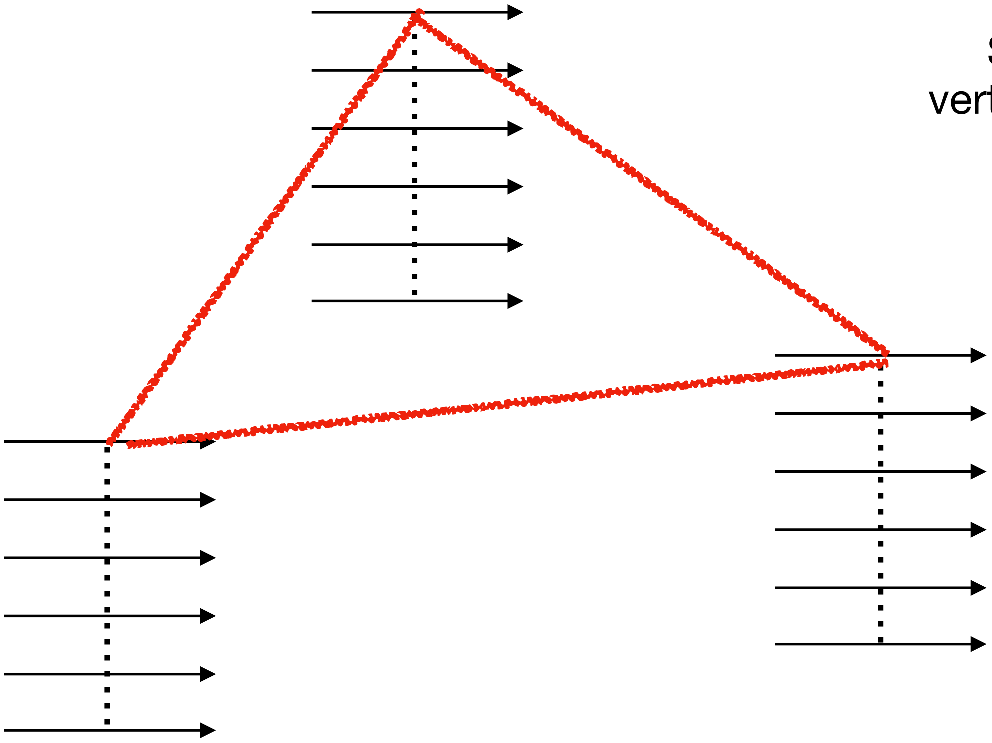
$$H_{align} \simeq -\alpha_{align} \sum (\sigma^x \otimes \sigma^x + \sigma^y \otimes \sigma^y) - \beta_{align} \sum \sigma^z \otimes \sigma^z$$



XY couoling of gauge fixed extra dimension squares!

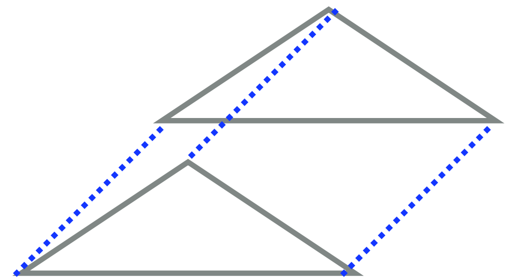
E^2 coupling term

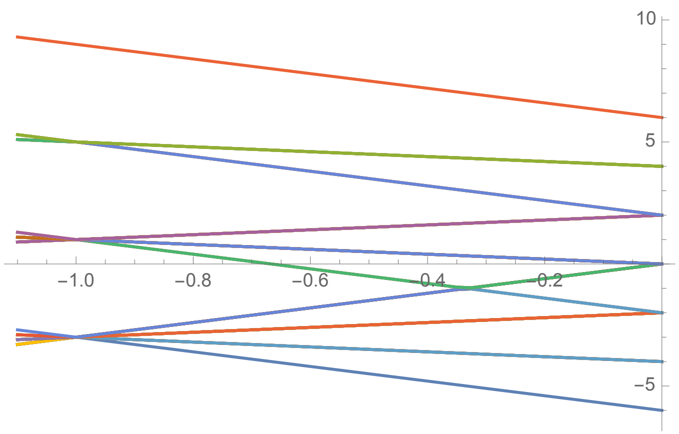
Plaquette



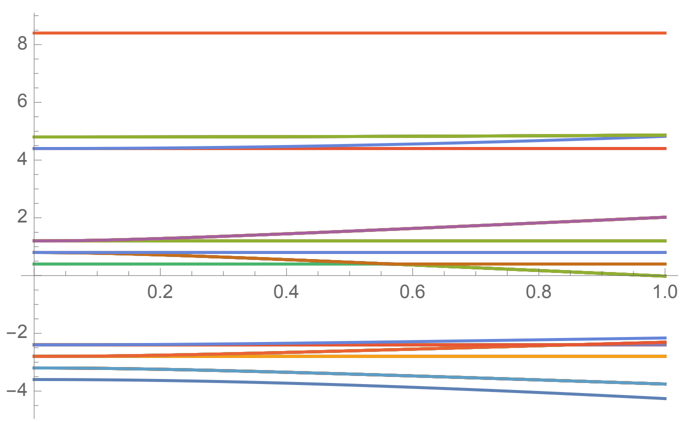
Still gauge invariant if broken
vertically: few operations per qubit.
(Not all to all)

Parameter fitting Two Triangle couple Hamiltonian needing 6 Qubits & eigenvalues of 64x64 matrices





Just XXZ piece: need to avoid level crossings, close to XXX is better



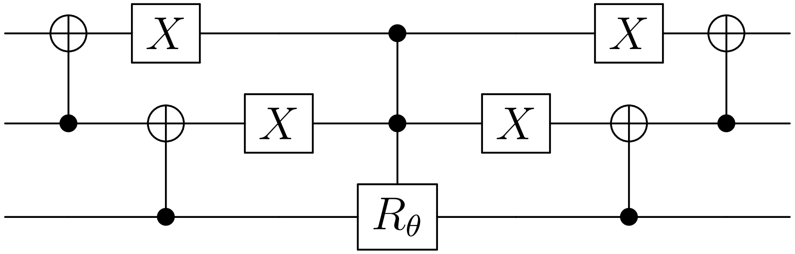
Together with plaquette operator: Gap persists: suggests simulation will not be too polluted by UV

$$\mathcal{H}_\triangle \propto \sigma^+ \otimes \sigma^+ \otimes \sigma^+ + \sigma^- \otimes \sigma^- \otimes \sigma^-$$

$$U_\triangle(t) = \exp(-itH_\triangle)$$

This is a rotation on a 2-plane of 8 dimensional Hilbert space (+++) rotating into (- - -).

Can be written in terms of a double control gate after some bit flips which need to be undone



Depending on details of architecture: it can take anywhere between
5 computing cycles and 20 (depth).

For experts: May be done efficiently with ancillas if Toffoli gates available.

Now we have the spin model

For single triangle:

$$\hat{H} = \sum_s \Bigg[\underbrace{\boxed{\frac{g^2}{2}\sum_{j=1}^{3}(\sigma^z_{j,s} + \sigma^z_{j,s+1})^2}}_{\hat{H}_E} + \underbrace{\boxed{\frac{\alpha}{2g^2}\sum_{j=1}^{3}(\sigma^+_{j,s}\,\sigma^-_{j,s+1} + \sigma^-_{j,s}\,\sigma^+_{j,s+1})}}_{\hat{H}_B}$$

$$\underbrace{\boxed{- \frac{1}{2g^2}(\sigma^+_{1,s}\,\sigma^+_{2,s}\,\sigma^+_{3,s} + \sigma^-_{1,s}\,\sigma^-_{2,s}\,\sigma^-_{3,s})}}_{\hat{H}_{XY}} \Bigg]$$

Now we have the spin model

For single triangle:

$$\hat{H} = \sum_s \Bigg[\quad \frac{g^2}{2} \sum_{j=1}^{3} (\sigma^z_{j,s} + \sigma^z_{j,s+1})^2 + \frac{\alpha}{2g^2} \sum_{j=1}^{3} (\sigma^+_{j,s}\, \sigma^-_{j,s+1} + \sigma^-_{j,s}\, \sigma^+_{j,s+1}) \\ - \frac{1}{2g^2} (\sigma^+_{1,s}\, \sigma^+_{2,s}\, \sigma^+_{3,s} + \sigma^-_{1,s}\, \sigma^-_{2,s}\, \sigma^-_{3,s}) \quad \Bigg]$$

We are interested in time evolution:

$$|\psi(t)\rangle = e^{-i\hat{H}t}\, |\psi(0)\rangle$$

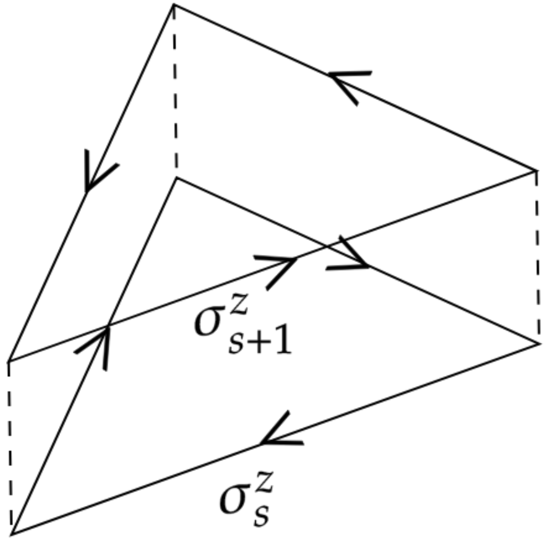
$$\approx \left(e^{-i\hat{H}_E \frac{t}{n}} e^{-i\hat{H}_{XY} \frac{t}{n}} e^{-i\hat{H}_B \frac{t}{n}} \right)^n |\psi(0)\rangle$$

with t/n small enough.

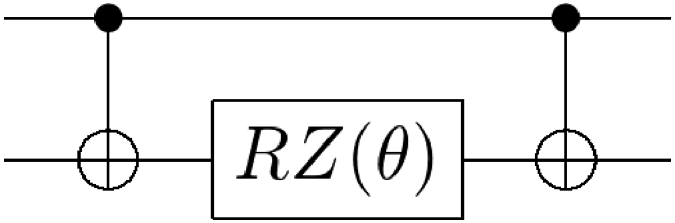
Electric term:

For single triangle:

$$\hat{H} = \sum_s \Big[\boxed{\frac{g^2}{2}\sum_{j=1}^{3} (\sigma^z_{j,s}+\sigma^z_{j,s+1})^2} + \frac{\alpha}{2g^2}\sum_{j=1}^{3}(\sigma^+_{j,s}\,\sigma^-_{j,s+1}+\sigma^-_{j,s}\,\sigma^+_{j,s+1}) - \frac{1}{2g^2}(\sigma^+_{1,s}\,\sigma^+_{2,s}\,\sigma^+_{3,s}+\sigma^-_{1,s}\,\sigma^-_{2,s}\,\sigma^-_{3,s}) \Big]$$



Circuit



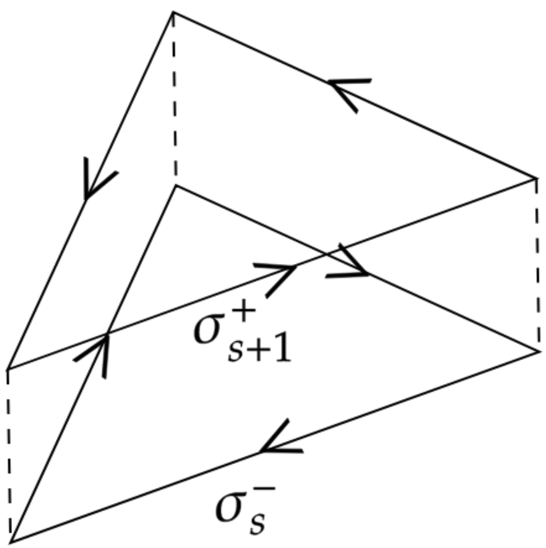
$$\equiv$$

$$e^{-i\hat{H}_E\theta}$$

Coupling term:

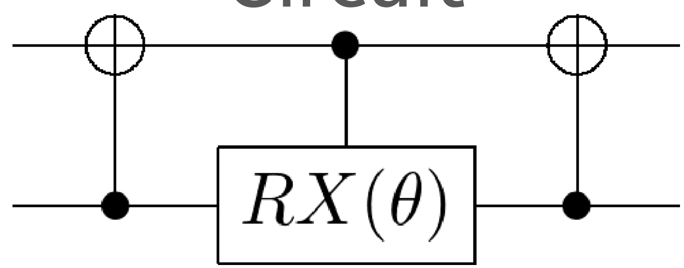
For single triangle:

$$\hat{H} = \sum_s \Bigg[\frac{g^2}{2} \sum_{j=1}^{3} (\sigma^z_{j,s} + \sigma^z_{j,s+1})^2 + \boxed{\frac{\alpha}{2g^2} \sum_{j=1}^{3} (\sigma^+_{j,s}\, \sigma^-_{j,s+1} + \sigma^-_{j,s}\, \sigma^+_{j,s+1})} - \frac{1}{2g^2} (\sigma^+_{1,s}\, \sigma^+_{2,s}\, \sigma^+_{3,s} + \sigma^-_{1,s}\, \sigma^-_{2,s}\, \sigma^-_{3,s}) \Bigg]$$



$+h.c.$

Circuit



$$\equiv$$

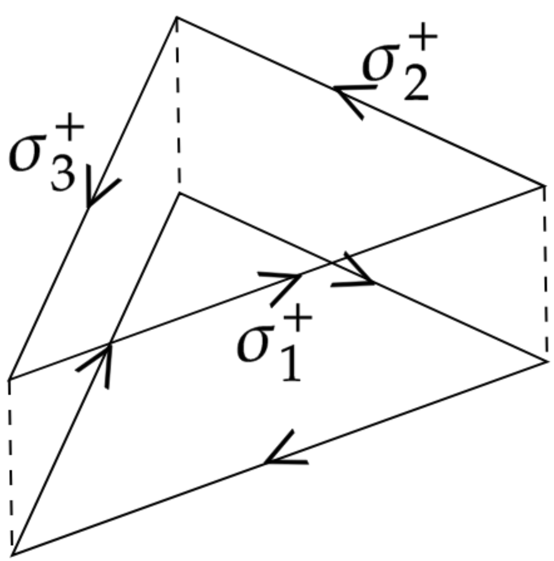
$$e^{-i\hat{H}_{XY}\theta}$$

Plaquette term:

For single triangle:

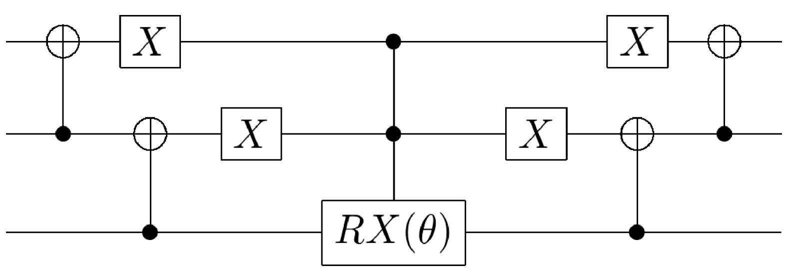
$$\hat{H} = \sum_s \Bigg[\quad \frac{g^2}{2} \sum_{j=1}^{3} (\sigma^z_{j,s} + \sigma^z_{j,s+1})^2 + \frac{\alpha}{2g^2} \sum_{j=1}^{3} (\sigma^+_{j,s}\,\sigma^-_{j,s+1} + \sigma^-_{j,s}\,\sigma^+_{j,s+1})$$

$$\boxed{- \frac{1}{2g^2} (\sigma^+_{1,s}\,\sigma^+_{2,s}\,\sigma^+_{3,s} + \sigma^-_{1,s}\,\sigma^-_{2,s}\,\sigma^-_{3,s})} \Bigg]$$



$+h.c.$

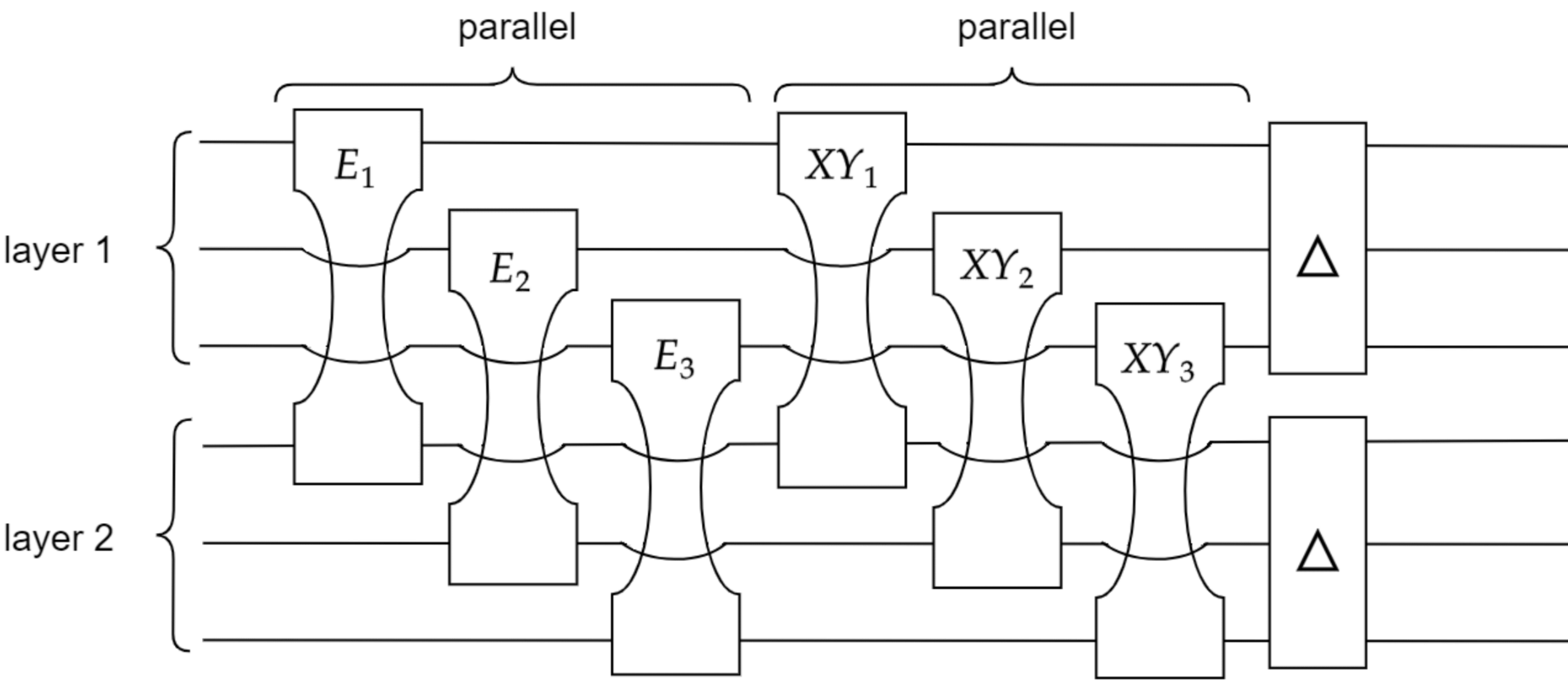
Circuit



$$\parallel$$

$$e^{-i\hat{H}_B\theta}$$

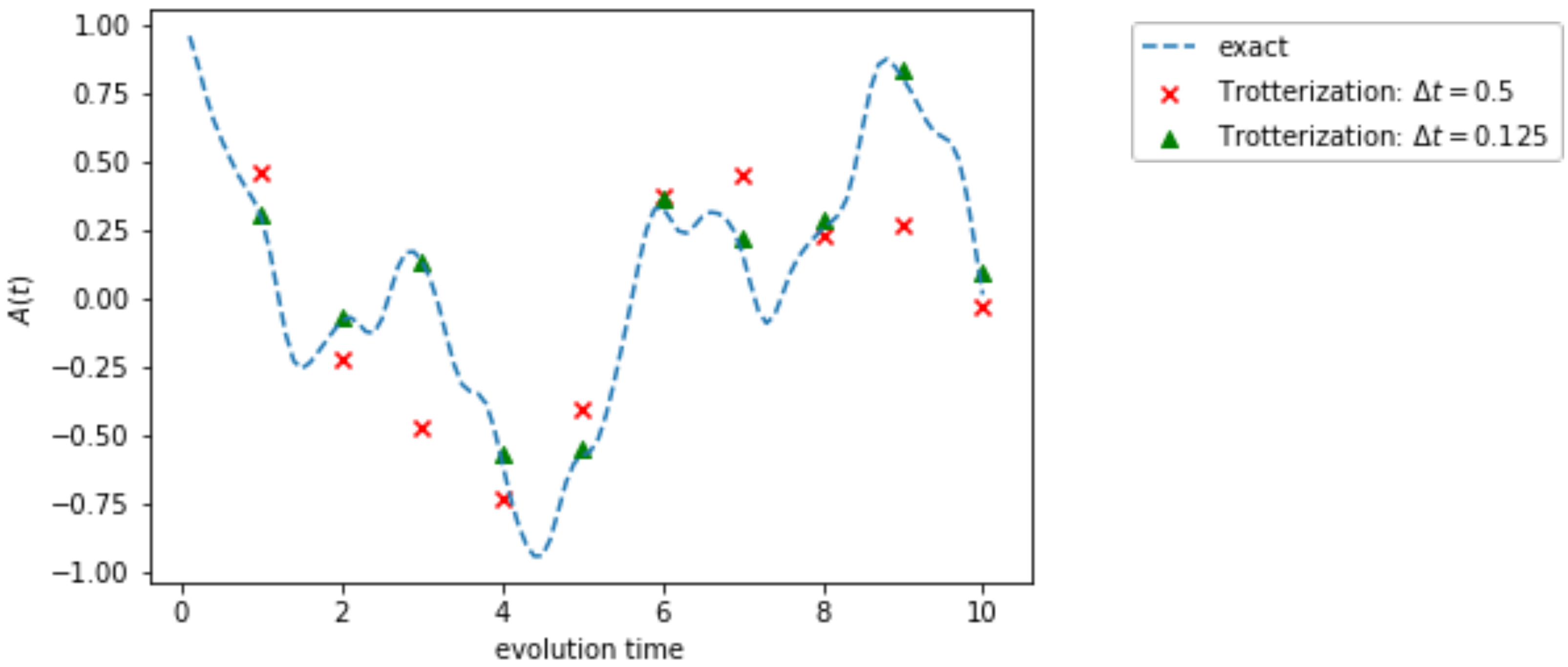
Overall one Trotter step looks like:



50 “physical” qubits per step.

Trotterization

$$U(t) = \left(e^{-i\hat{H}_E \frac{t}{n}} e^{-i\hat{H}_{XY} \frac{t}{n}} e^{-i\hat{H}_B \frac{t}{n}}\right)^n$$



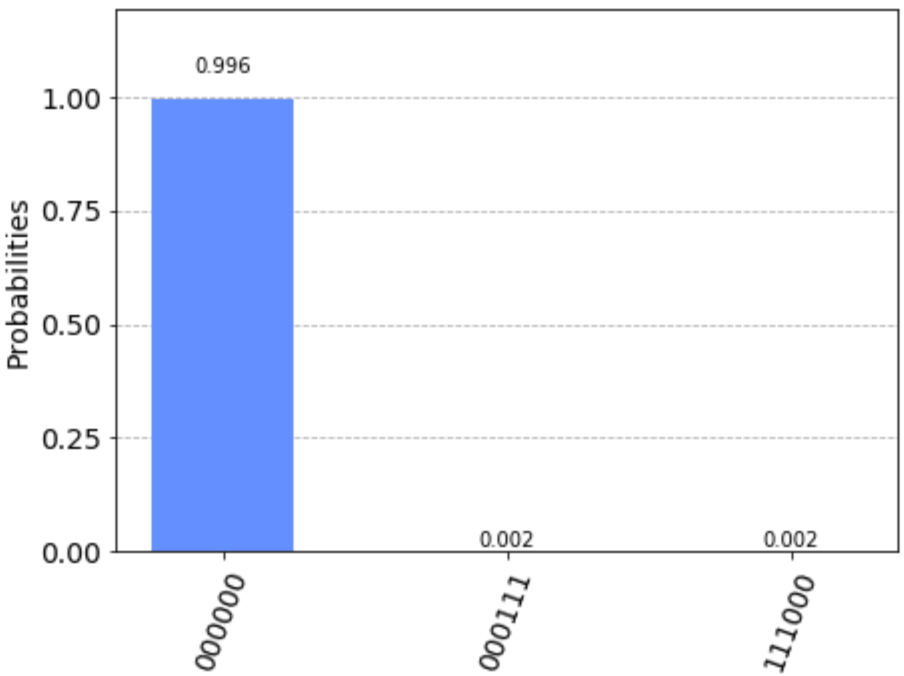
Computation of <0|U(t)|0> (exact).

Around t/n ≃ 0.1 for useful simulation.

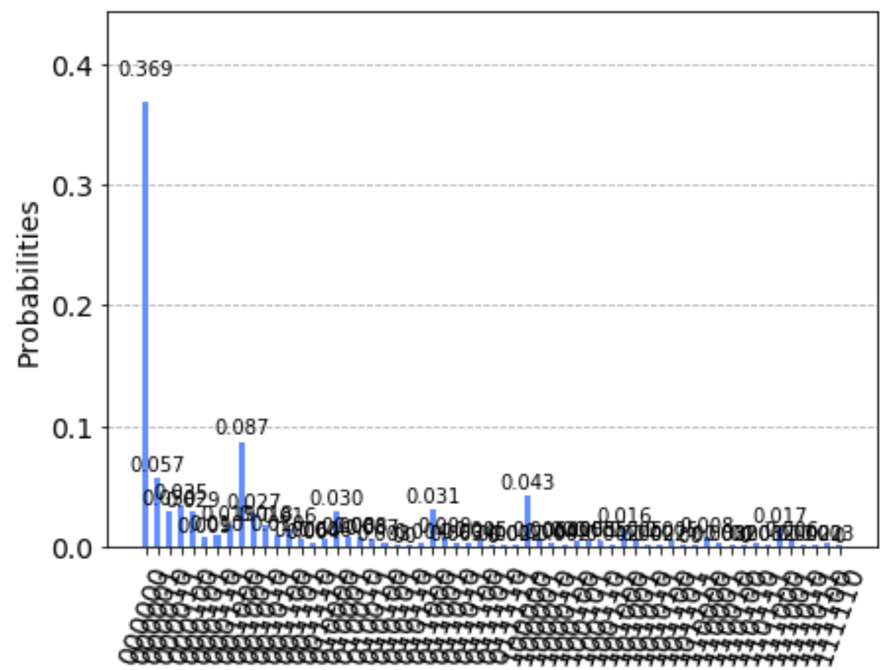
On the actual IBM Q machine

Again, $U(t) = \left(e^{-i\hat{H}_E \frac{t}{n}} e^{-i\hat{H}_{XY} \frac{t}{n}} e^{-i\hat{H}_B \frac{t}{n}} \right)^n$ (t=0.1, n=1)

The distribution $\rho(t) = diag(U(t) \left|0\right\rangle \left\langle 0\right| U(t)^\dagger)$



Ideal

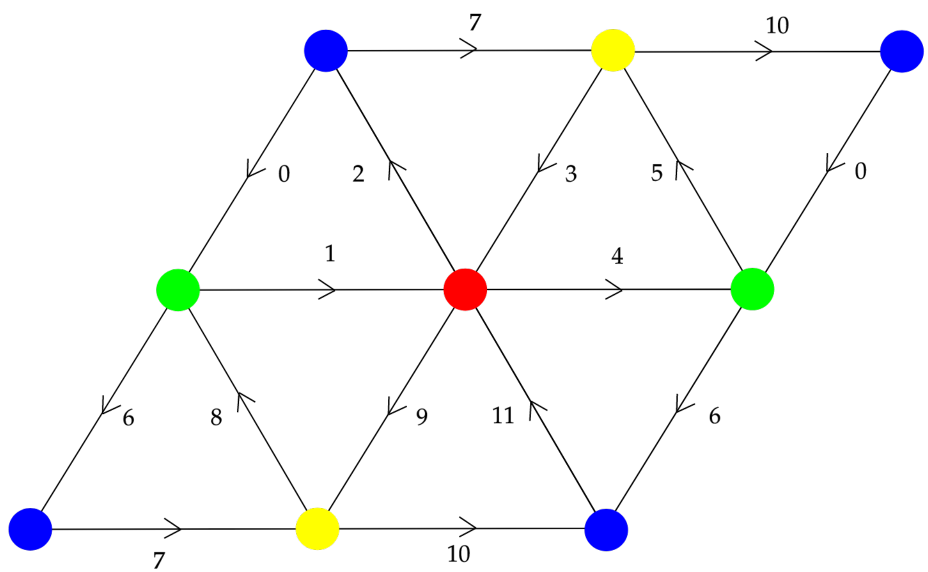


IBM Q

What I have to work on from now

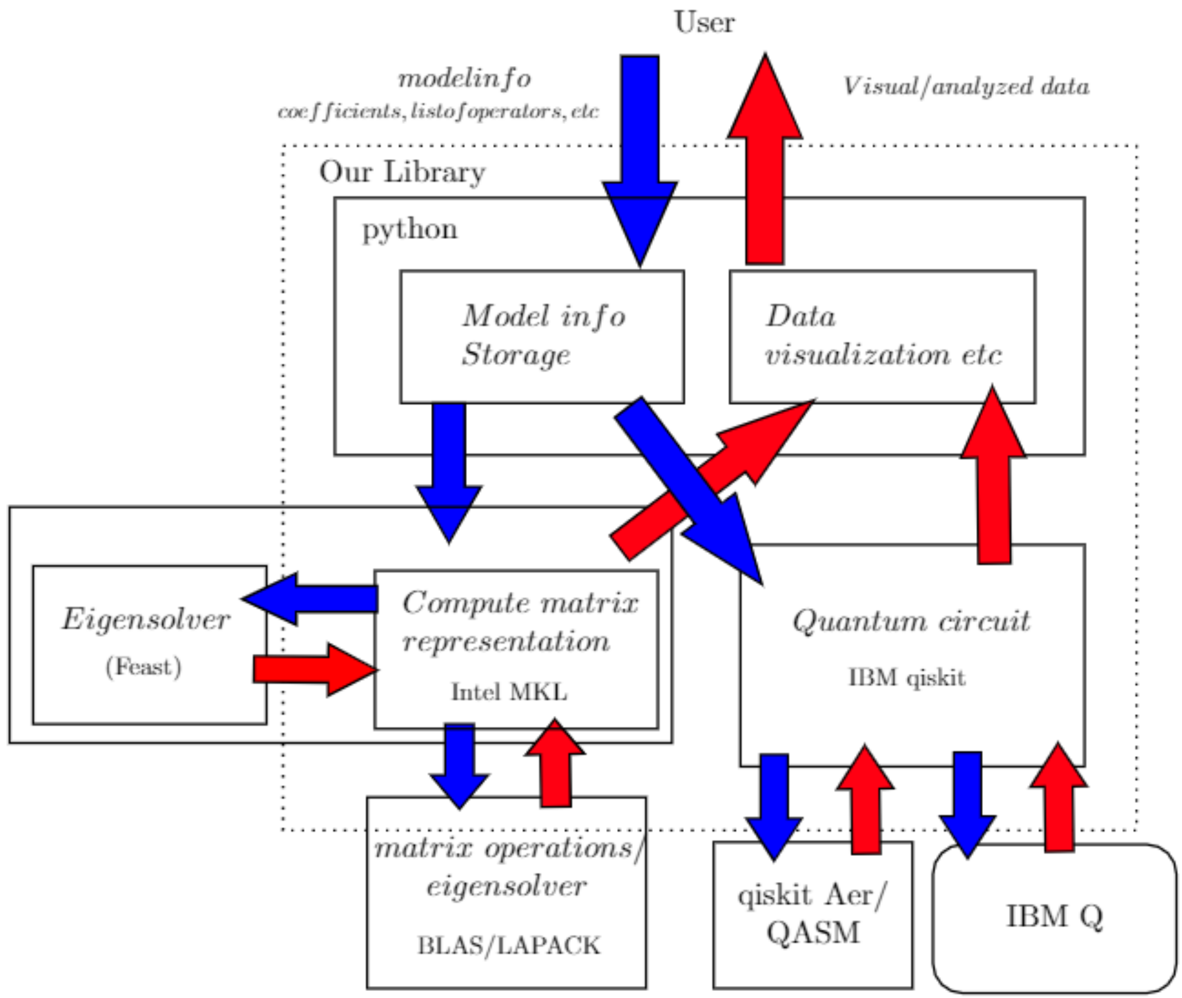
Theory side:

- What if we extend the model to the spacial direction?



Z_2xZ_2 lattice equivalent to C_2xC_2 torus with PBC (i.e. quotient mapping)

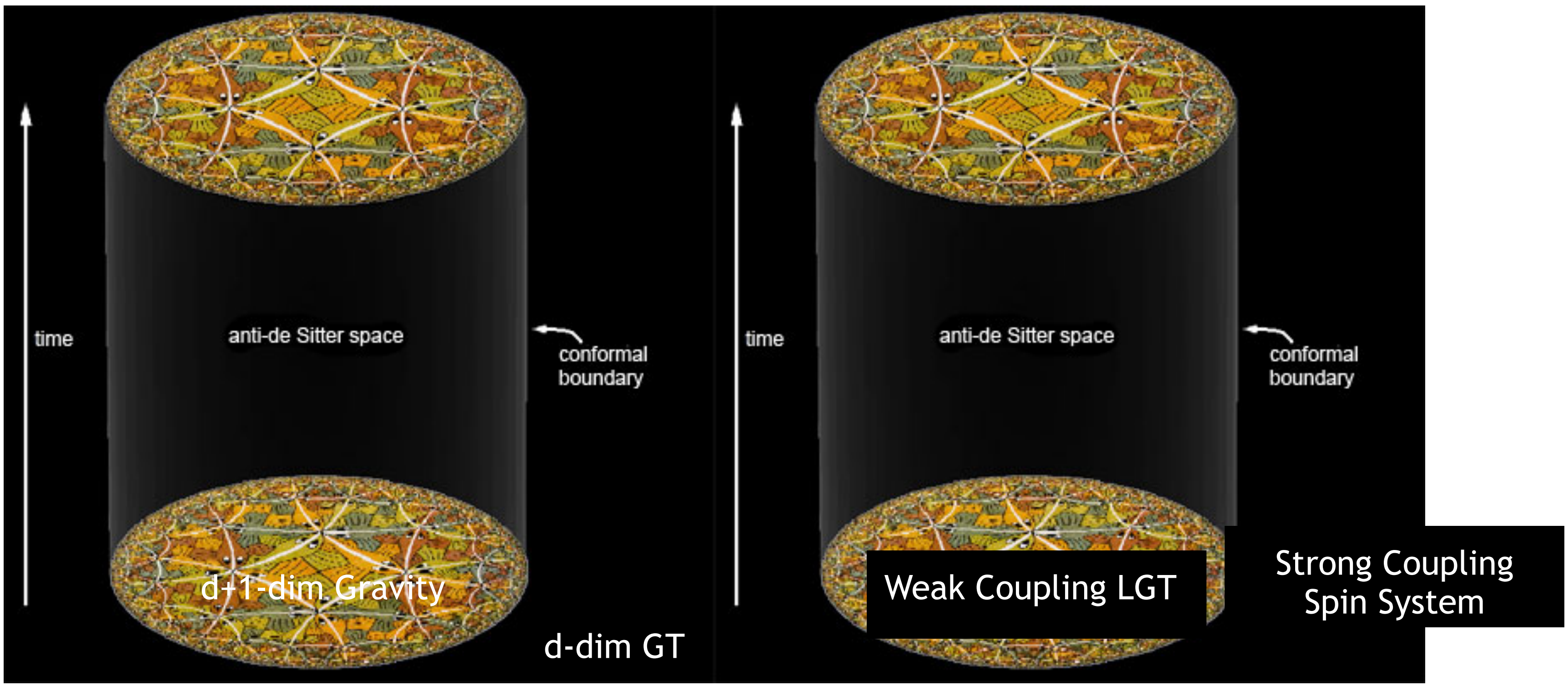
The architecture of the library:



IBM-Q Links for Excellent Python Tutorials, compiler and simulators

https://medium.com/quantum1net/richard-feynman-and-the-birth-of-quantum-computing-6fe4a0f5fcc7

Also Developing Quantum Computing for Hyperbolic Lattice Hamiltonian using triangle group to do strong couponing AdS?CFT or Gravity/Gauge Duality in MInkowski



See Lattice Setup for Quantum Field Theory in AdS2
Richard C. Brower, Cameron V. Cogburn, A. Liam Fitzpatrick, Dean Howarth, Chung-I Tan   https://arxiv.org/abs/1912.07606

Questions?

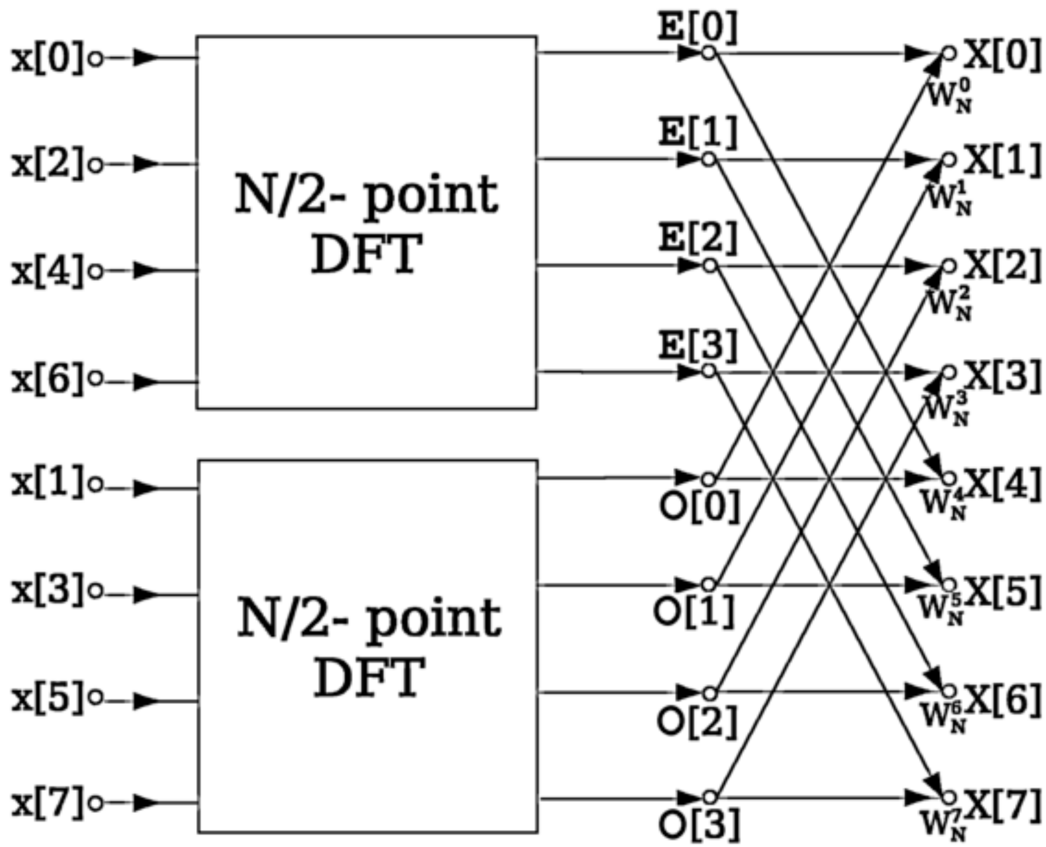
FAST FOURIER TRANSFORM

MAT-VEC O(N^2) —> FFT O(N LOG(N)

Karl Friedrich Gaus (1777-1855)

J. W. Cooley and J.Turkey, 1965

See Wikipedia **https://en.wikipedia.org/wiki/Cooley**



Data flow diagram for *N*=8: a decimation-in-time radix-2 FFT breaks a length-*N* DFT into two length-*N*/2 DFTs followed by a combining stage consisting of many size-2 DFTs called "butterfly" operations (so-called because of the shape of the data-flow diagrams).

FTT: $x_k = \omega_N^k = e^{2\pi i k/N}$

$$y_k \equiv \mathcal{FT}_N[a_n] = \sum_{n=0}^{N-1} (\omega_N^k)^n a_n = \sum_{n=0}^{N-1} e^{i2\pi nk/N} a_n$$

The trick: $n = n_0 + 2\, n_1 + 2^2\, n_2 + \ldots + 2^p\, n_p$

$$\omega_N^n = \omega_N^{n_0}\, \omega_{N/2}^{n_1}\, \omega_{N/4}^{n_2} \cdots\, \omega_2^{n_p}$$

$$\sum_n \omega_N^{nk} = \sum_{n_0=0,1} \omega_N^{n_0 k} \sum_{n_1=0,1} \omega_{N/2}^{n_1 k} \cdots \sum_{n_p=0,1} \omega_2^{n_p k}$$

LOW BIT FIRST

$$y_k \equiv \mathcal{FT}_N[a_n] = \sum_{n=0}^{N-1} (\omega_N^k)^n a_n = \sum_{n=0}^{N-1} e^{i2\pi nk/N} a_n$$

Spit polynomial into even/odd pieces:

$$y_k = \sum_{n=0}^{N/2-1} e^{i2\pi nk/(N/2)} a_{2n} + \omega_N^k \sum_{n=0}^{N/2-1} e^{i2\pi nk/(N/2)} a_{2n+1}$$

➔ One N = Two N/2 Fourier transforms

low k

$$y_k = \sum_{n=0}^{N/2-1} e^{i2\pi nk/(N/2)} [a_n + \omega_N^k a_{n+N/2}]$$

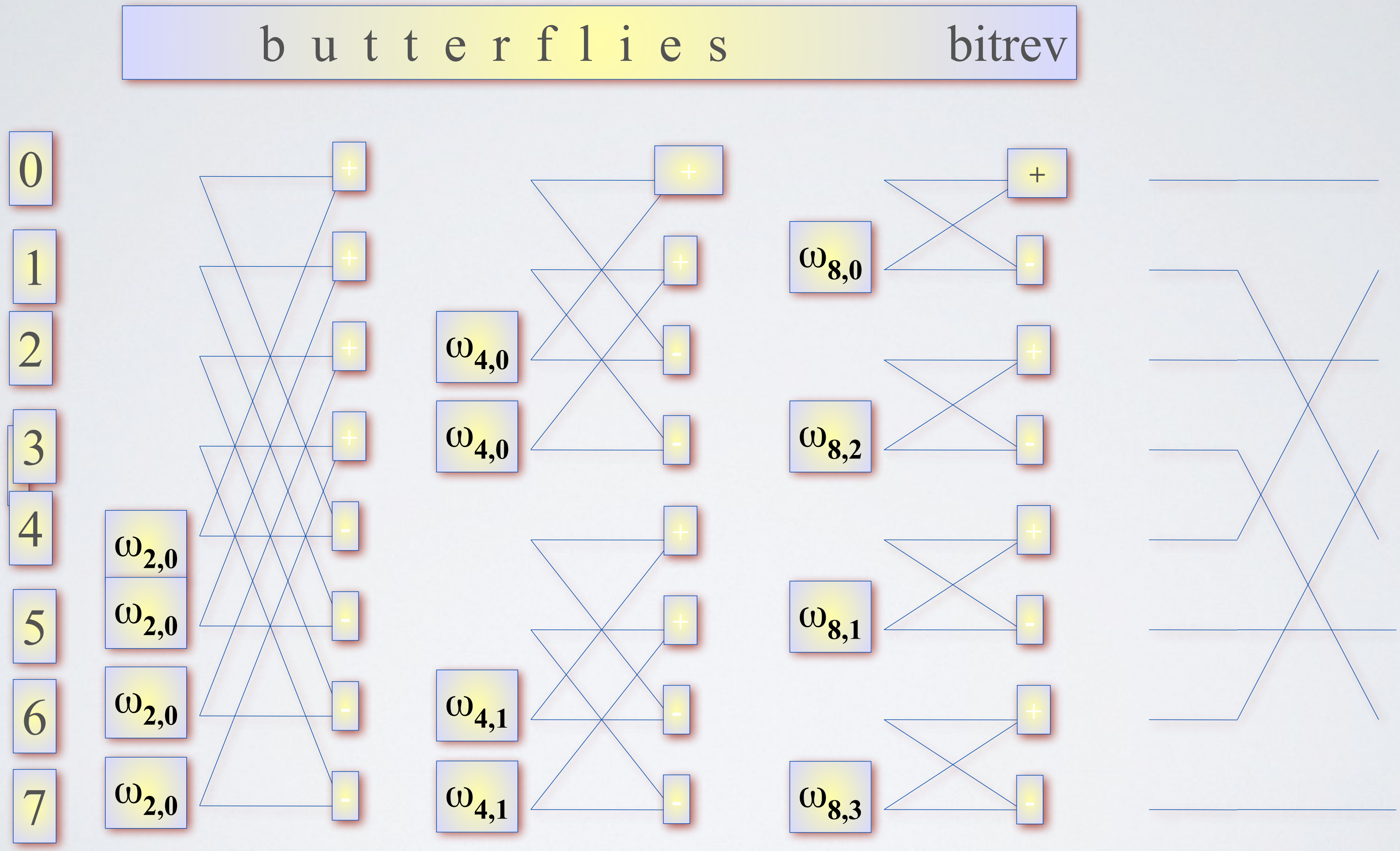
high k

$$y_{k+N/2} = \sum_{n=0}^{N/2-1} e^{i2\pi nk/(N/2)} [a_n - \omega_N^k a_{n+N/2}]$$

BUTTERFLIES NETWORK

$$y_k = \mathcal{FT}_{N/2}[a_{2n} + \omega_N^k a_{2n+1}]$$

$$y_{k+N/2} = \mathcal{FT}_{N/2}[a_{2n} - \omega_N^k a_{2n+1}]$$



Quantum FFT

See Wikipedia **https://en.wikipedia.org/wiki/Quantum_Fourier_transform**

http://demonstrations.wolfram.com/QuantumFourierTransformCircuit/

- Here are slides from David Kaplan @ ICTS Bangaluru Feb 3 2018

Quantum computers to the rescue?

Certain algorithms on a quantum computer can do in polynomial time wh
takes exponential time on a classical computer.

Example: discrete Fourier transform

Classical Fourier transform on a discrete function with N values

$$\{x_0,\ldots x_{N-1}\} \mapsto \{y_0,\ldots y_{N-1}\}$$

$$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j \omega^{jk} \qquad \omega = e^{\frac{2\pi i}{N}}$$

Computational cost = $O(N \log N)$.

When $N = 2^n$, cost (# gate operations) is $O(n\, 2^n)$.

On a quantum computer cost is $O(n^2)$

Fourier transform on a quantum computer

Start with n=2 qubits $|x\rangle = |x_0,x_1\rangle$ where $x_i = 0,1 \ldots$
So $N = 2^2 = 4$ and $\omega = e^{2\pi i/4}$

The Fourier transform is then the unitary transformation on these states

$$\begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix} \to U \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix}$$

$$U = \frac{1}{\sqrt{2^2}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{pmatrix} = \frac{1}{\sqrt{2^2}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & 1 & \omega^2 \\ 1 & \omega^3 & \omega^2 & \omega \end{pmatrix}$$

Then

$$|\psi\rangle = \sum_k x_k |k\rangle \to U\psi = \sum_{j,k} x_j U_{jk} |k\rangle \equiv \sum_k y_k |k\rangle \ , \qquad \text{so } y_k = \sum_j x_j \omega^{jk}$$

The coefficients of the qubits in the final state will be the Fourier transform of the coefficients of the qubits in the initial state

Then

$$|\psi\rangle = \sum_k x_k|k\rangle \to U\psi = \sum_{j,k} x_j U_{jk}|k\rangle \equiv \sum_k y_k|k\rangle \ , \qquad \text{so } y_k = \sum_j x_j \omega^{jk}$$

The coefficients of the qubits in the final state will be the Fourier transform of the coefficients of the qubits in the initial state

In the basis:

$$|x_1 x_2\rangle = \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix} \qquad U = \frac{1}{\sqrt{2^2}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{pmatrix}$$

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$$|\psi\rangle = \sum_k x_k |k\rangle \to U\psi = \sum_{j,k} x_j U_{jk} |k\rangle \equiv \sum_k y_k |k\rangle \ , \qquad \text{so } y_k = \sum_j x_j \omega^{jk}$$

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$\omega^{x_2+2x_1}$

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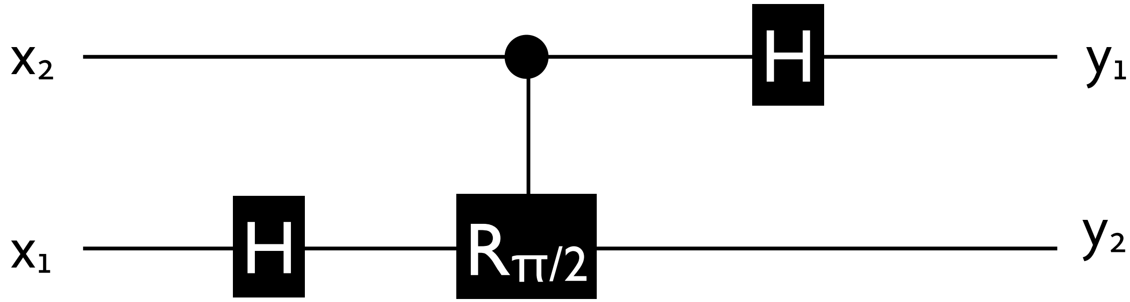
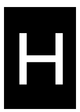
$$\boxed{|y\rangle = U|x\rangle = \frac{1}{2}\left(|0\rangle + \omega^{2x_2}|1\rangle\right)\left(|0\rangle + \omega^{2x_1+x_2}|1\rangle\right)}$$

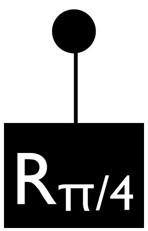
$\omega^4 = 1$

This can be effected (up to overall phase) with 3 basic gates:





= Hadamard gate: $|0\rangle \to \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}\,, \quad |1\rangle \to \dfrac{|0\rangle - |1\rangle}{\sqrt{2}}$

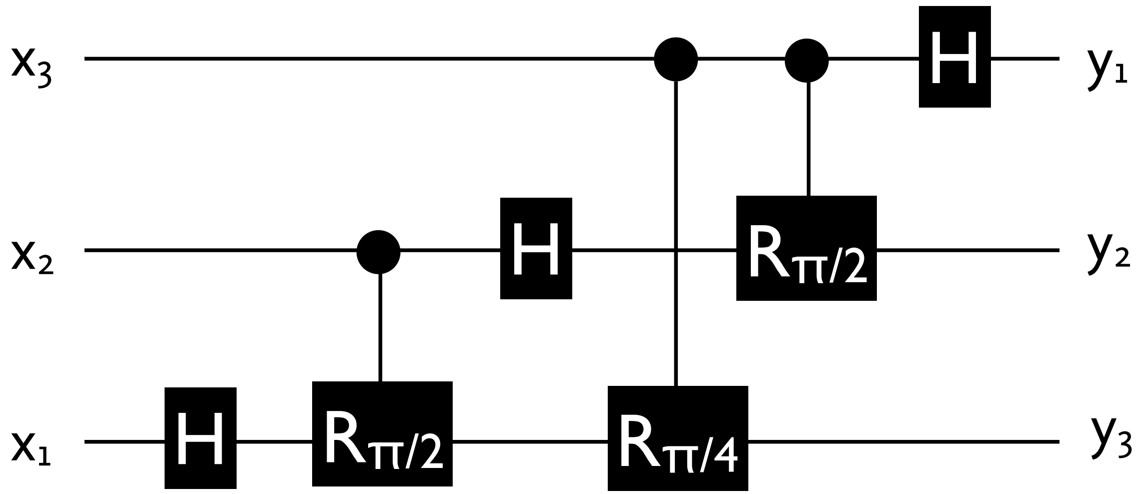


= Controlled Phase Rotation: $|x_1\rangle \to \omega^{x_1}|x_1\rangle \quad \text{iff } x_2 = 1$

$$|x_1\, x_2\rangle \xrightarrow{H} \left(\frac{|0\rangle + \omega^{2x_1}|1\rangle}{\sqrt{2}}\right)|x_2\rangle \xrightarrow{R_{\pi/2}} \left(\frac{|0\rangle + \omega^{2x_1+x_2}|1\rangle}{\sqrt{2}}\right)|x_2\rangle \xrightarrow{H} \left(\frac{|0\rangle + \omega^{2x_1+x_2}|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle + \omega^{2x_2}|1\rangle}{\sqrt{2}}\right)$$

$$|y_1 y_2\rangle = \left(\frac{|0\rangle + \omega^{2x_2}|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle + \omega^{2x_1+x_2}|1\rangle}{\sqrt{2}}\right)$$

The “score” for the n=3 Fourier transform:



3 gates for the n=2 case; 6 gates for n=3. Scales like n^2 for large n

- n H-gates
- n(n+1)/2 R-gates

Same discrete FT scales like $(n\ 2^n)$ on a classical computer.

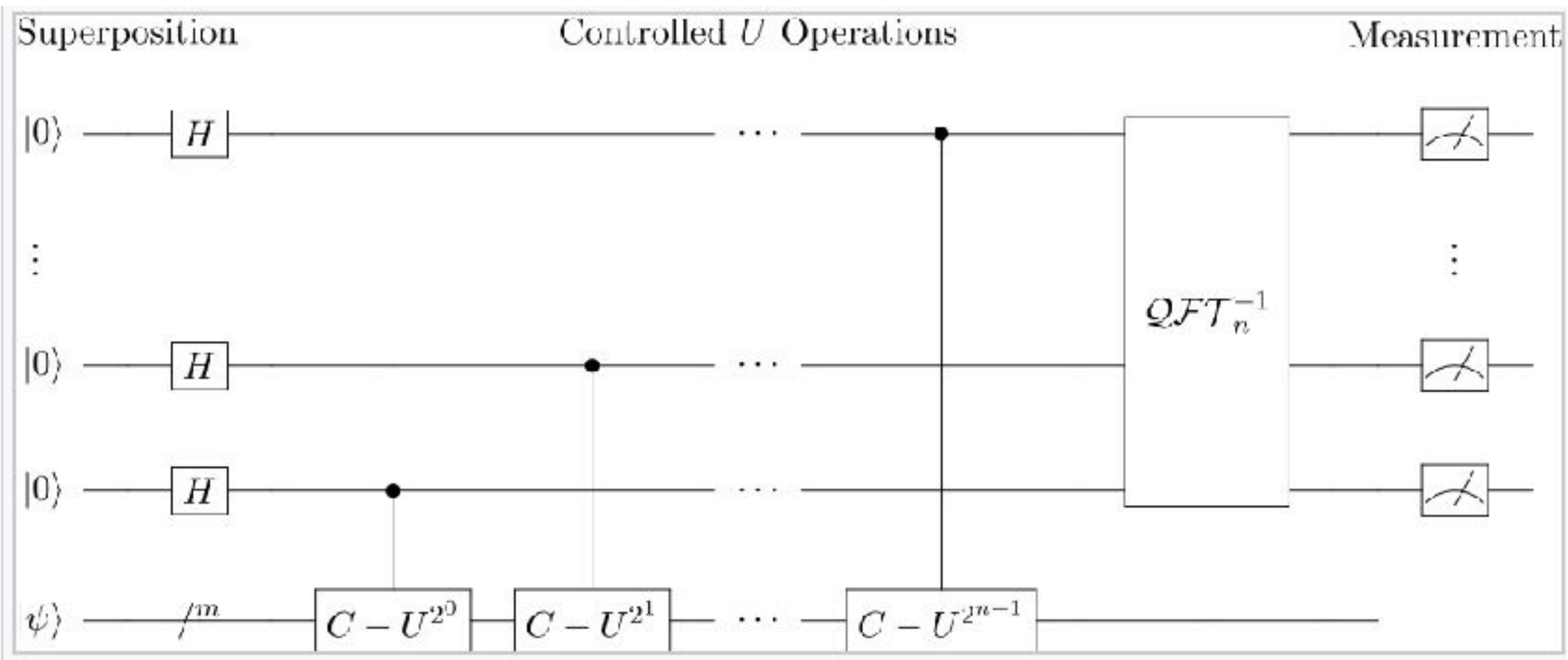
How can you use this for physics?

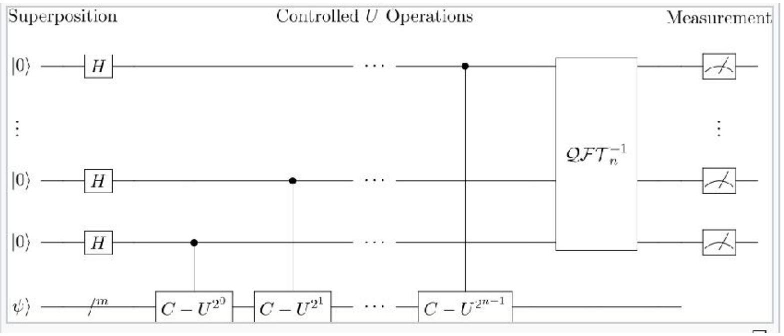
Example: phase estimation algorithm

Suppose |ψ> is the eigenvector of a unitary operator U (= e^{-iHt}), represented by m qubits:

$$U\,|\psi\rangle = e^{2\pi i\theta}\,|\psi\rangle$$

and you want to determine θ to accuracy $1{:}2^{-n}$





Hadamard gates give you the state: $2^{-n/2}(|0\rangle + |1\rangle)^{\otimes n}|\psi\rangle$

Controlled phase rotations by U then give you the state

$$\frac{1}{2^{\frac{n}{2}}}\underbrace{\left(|0\rangle + e^{2\pi i 2^{n-1}\theta}|1\rangle\right)}_{1^{st}\ qubit} \otimes \cdots \otimes \underbrace{\left(|0\rangle + e^{2\pi i 2^{1}\theta}|1\rangle\right)}_{n-1^{th}\ qubit} \otimes \underbrace{\left(|0\rangle + e^{2\pi i 2^{0}\theta}|1\rangle\right)}_{n^{th}\ qubit} = \frac{1}{2^{\frac{n}{2}}}\sum_{k=0}^{2^n-1} e^{2\pi i\theta k}|k\rangle.$$

If $\theta = a\, 2^{-n}$ for integer a, then the inverse Fourier Transform will yield an eigenstate of spin for each of the final qubits $|y\rangle$

Measuring $|y\rangle$ yields the exact answer for a:

$a = 2^0 y_0 + 2^1 y_1 + 2^2 y_2 + \ldots + 2^{n-1} y_{n-1}$,

all y_i measured to be 0 or 1

SOME REFERENCES ON EUCLIDEAN QUANTUM LINKS (AKA D-THEORY)

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GOOD QC REFERENCE