Instructions: This take home Practice Test is the R portion of Test 1. Work all the problems below in an R notebook, and put your file into **Week 5 Dropbox** in the course website by **noon on Thursday, October 5th**. You are not allowed to discuss these problems with or get help from anyone else other than the instructor. This practice test is intended to give a general idea about the actual test which will be given in class on Friday, October 6th. You should **study all lecture notes and homework exercises** to fully prepare for it. Make sure to **show work** for each problem (both Mathematica and manual solutions) and justify your answer to get full credit. Do well!

Questions:

1. Give precise definitions/statements for each of the following terms.
   (a) $100p$-th quantile of a sample
   (b) Bayes' theorem
   (c) the density $f(x)$ for the geometric random variable
   (d) the geometric summation formula
   (e) Chebyshev's inequality

2. Suppose that we have three boxes identical in form and that one of the boxes contains two gold coins, the other one contains two silver coins, and third contains one of each. The three boxes are mixed up and one box is randomly selected, and a coin is withdrawn from that box at random. If the coin happens to be a gold coin, what is the probability that the remaining coin in the box is also gold? [Use Bayes' formula.]

3. Random integers are selected from among $\{0, 1, 2, \ldots, 8, 9\}$ such that selection of each integer is equally likely as any other. Let X denote the number of trials needed to obtain first zero. Write the sample space for this experiment and compute the density function $f(x)$ for X. What is the probability that it takes more than 3 trials to obtain a zero.

4. Let X be the standard normal random variable whose mean is 0 and standard deviation is 1.
   (a) Find the probability that X is within two standard deviation of its mean.
   (b) Use the Chebyshev's inequality to approximate the same probability. Use this example to explain the strengths and weaknesses of Chebyshev's inequality.

5. Calculate the cumulative distribution function $F(x)$ for the exponential random variable whose density is $f(x) = e^{-x}$ where $x > 0$, and $f(x) = 0$ otherwise.

6. Let the density for X be given by $f(x) = c\,e^{-x}$ for $x = 1, 2, 3, \ldots$. Find the value of c that makes this function a valid density.

7. Consider the function

$$f_{XY}(x, y) = 8xy, \quad 0 < y < x < 1.$$

   (a) Verify that $f_{XY}(x, y)$ is a valid joint density function for the joint (two-dimensional) random variable (X, Y).

   (b) Find the marginal densities $f_X(x)$ and $f_Y(y)$.