E. WRITTEN SOLUTIONS OF LABORATORY

JACOBIAN MATRIX

$$\begin{vmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ W_x \\ W_y \\ W_z \end{vmatrix} = \begin{array}{ccc} R & R & P \\ \multicolumn{3}{c}{\left[\begin{array}{ccc} R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_0^0) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_1^0) & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{array}\right]} \end{array} \begin{bmatrix} \dot{\theta}_1 \\ \theta_2 \\ d_3 \end{bmatrix}$$

$$R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_0^0)$$

$$\underset{R_0^0}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \underset{d_3^0}{\begin{bmatrix} a2\cos\theta_1 + a4\cos\theta_1\cos\theta_2 - a4\sin\theta_1\sin\theta_2 \\ a2\sin\theta_1 + a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2\sin\theta_1 \\ a1 + a3 - a5 - d3 \end{bmatrix}} - \underset{d_0^0}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$$= \begin{bmatrix} a2\cos\theta_1 + a4\cos\theta_1\cos\theta_2 - a4\sin\theta_1\sin\theta_2 \\ a2\sin\theta_1 + a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2\sin\theta_1 \\ a1 + a3 - a5 - d3 \end{bmatrix}$$

$$R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_1^3)$$

$$\underset{R_1^0}{\begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \underset{d_3^0}{\begin{bmatrix} a2\cos\theta_1 + a4\cos\theta_1\cos\theta_2 - a4\sin\theta_1\sin\theta_2 \\ a2\sin\theta_1 + a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2 - \sin\theta_1 \\ a1 + a3 - a5 - d3 \end{bmatrix}} - \underset{d_1^0}{\begin{bmatrix} a2\,c\theta_1 \\ a2\,s\theta_1 \\ d_1 \end{bmatrix}}$$

$$\begin{bmatrix} a4\cos\theta_1\cos\theta_2 - a4\sin\theta_1\sin\theta_2 \\ a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2\sin\theta_1 \\ a3 - a5 - d3 \end{bmatrix}$$

$$R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underset{R_2^0}{\begin{bmatrix} \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 & \cos\theta_2\sin\theta_1 - \cos\theta_1\sin\theta_2 & 0 \\ \cos\theta_2\sin\theta_1 - \cos\theta_1\sin\theta_2 & -\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & 0 \\ 0 & 0 & -1 \end{bmatrix}} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ -1 \end{matrix}$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ W_x\\ W_y\\ W_z\end{bmatrix} = \begin{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix} a2\cos\theta_1 + a4\cos\theta_1\cos\theta_2 - a4\sin\theta_1\sin\theta_2 \\ a2\sin\theta_1 + a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2\sin\theta_1 \\ a1 + a3 - a5 - d3\end{bmatrix} & \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix} a4\cos\theta_1\cos\theta_2 - a4\sin\theta_1\sin\theta_2 \\ a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2\sin\theta_1 \\ a3 - a5 - d3\end{bmatrix} & \begin{matrix}0\\0\\-1\end{matrix} \\ \begin{bmatrix}0\\0\\1\end{bmatrix} & \begin{bmatrix}0\\0\\1\end{bmatrix} & \begin{matrix}0\\0\\0\end{matrix}\end{bmatrix}$$

$$\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix} \times \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix} = \begin{bmatrix} a_2b_3 - a3b_2 \\ a3b_1 - a1b_3 \\ a1b_2 - a2b_1\end{bmatrix}$$

$$\begin{vmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ W_x\\ W_y\\ W_z\end{vmatrix} \doteq \begin{bmatrix} -[a2\sin\theta_1 + a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2\sin\theta_1] & -[a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2\sin\theta_1] & 0 \\ a2\cos\theta_1 + a4\cos\theta_1\cos\theta_2 - a4\sin\theta_1\sin\theta_2 & a4\cos\theta_1\cos\theta_2 - a4\sin\theta_1\sin\theta_2 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{d}_3\end{bmatrix}$$

$\dot{x} = -[a2\sin\theta_1 + a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2\sin\theta_1] \times (\dot{\theta}_1) - [a4\cos\theta_1\sin\theta_2 + a4\cos\theta_2\sin\theta_1] \times (\dot{\theta}_2)$

$\dot{y} = [\dot{\theta}_1] \times [a_2\cos\theta_1 + a_4\cos\theta_1\cos\theta 2 - a_4\sin\theta_2] + (\dot{\theta}_2) \times [a4\cos\theta_1\cos\theta_2 - a4\sin\theta_1\sin\theta_2]$

$\dot{z} = -\dot{d}_3$

$W_x = 0$

$W_y = 0$

$W_z = \dot{\theta}_1 + \dot{\theta}_2$

Alangilan, Batangas City, Philippines
+63 43 425 - 0139 loc. 2121
www.batstate-u.edu.ph
coe.alangilan@g.batstate-u.edu.ph

SINGULARITIES

LET :

$$J = \begin{bmatrix} -[a_2 \sin\theta_1 + a_4\cos\theta_1 \sin\theta_2 + a_4 \cos\theta_2 \sin\theta_1] & -[a_4 \cos\theta_1 \sin\theta_2 + a_4 \cos\theta_2 \sin\theta_1] & 0 \\ a_2 \cos\theta_2 + a_4 \cos\theta_1 \cos\theta_2 - a_4 \sin\theta_1 \sin\theta_2 & a_4 \cos\theta_1 \cos\theta_2 - a_4 \sin\theta_1 \sin\theta_2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

BE REPRESENTED AS ;

$$\begin{bmatrix} a1 & b1 & C1 \\ a2 & b2 & C2 \\ a3 & b3 & C3 \end{bmatrix}$$

$$\text{Det.}(J) = a1\begin{bmatrix} b_2 & C_2 \\ b_3 & C_3 \end{bmatrix} - b1\begin{bmatrix} a_2 & C_2 \\ a_3 & C_3 \end{bmatrix} + C1\begin{bmatrix} a_2 & b2 \\ a_3 & b_3 \end{bmatrix}$$

$$\text{Det.}(J) = a_1[b_2(C_3) - b_3(C_2)] - b_1[a_2(C_3) - a_3(C_2)] + C_1[a_2(b_3) - a_3(b_2)]$$

$a_1 = 10$ $\quad\theta_1 = 90°$
$a_2 = 5$ $\quad\theta_2 = -90°$
$a_3 = 10$ $\quad d_3 = 0$
$a_4 = 5$
$a_5 = 10$

SUBSTITUTING ON THE ORIGINAL MATRIX J ;

$$J = \begin{bmatrix} -5 & 0 & 0 \\ 5 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\text{Det}(J) = a_1[b_2(C_3) - b_3(C_2)] - b_1[a_2(C_3) - a_3(C_2)] + C_1[a_2(b_3) - a_3(b_2)]$$

SINCE b_1 AND C_1 IS ZERO ;

$$\text{Det}(J) = (-5)[5(-1) - 0(0)]$$

$$\text{Det}(J) = 25$$

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