

# University of Michigan 2017 Concrete Canoe Design Paper







The University of Michigan

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#### **Executive Summary**

The University of Michigan has been committed to fostering the Leaders and Best since 1817 as an institution with a longstanding presence within academics, arts, the Ann Arbor community, athletics, and research. This year, the University is celebrating its bicentennial. Much like the University, the Concrete Canoe Team Michigan (MCCT) continuously pushes itself to pioneer into new technology, grow as an organization, and engage an expanding network of alumni both in the past, present, and future. The historic determination surrounding the University inspired the team to select the name VALIANT for the 2017 canoe.

At the 2016 North Central Regional competition hosted by Michigan State University, MCCT's canoe *EXTINCTION* placed third overall. Previously, the 2015 canoe *ALLEGRO* placed second overall while the 2014 canoe *LEGACY* placed third overall.

With a team comprised of many returning members, MCCT decided to focus on project management improvements to assist future teams with annual planning and execution. Frequent checkmarks and improved transparency allowed for more sustainable leadership positions.

This year, more intensive analysis was done on the 2016 canoe, *EXTINCTION*. Due to poor weather conditions, the 2016 competition races were cancelled. As a result, the boat was never tested in a competitive environment. Thus, the team allocated time to test and observe *EXTINCTION* in a nearby body of water. Following these observations, the following adjustments were made to *VALIANT's* hull: height reduction to improve stability, increased beam for a higher prismatic coefficient, as well as a flatter bottom side profile to lower the center of gravity while retaining the center of buoyancy. The team chose to continue the use of a male mold to help achieve higher gunwale quality control.

Pigmented slurry was used for an aesthetic finish of *VALIANT*. The combination of the aesthetic slurry with the maize coloring of the canoe aligned well with our celebration of the University's 200th anniversary. Vinyl letters were chosen again after positive results from last year.

To continue paving the way for future wolverines, the Michigan Concrete Canoe Team presents the 2017 canoe, *VALIANT*.

#### Table 1: VALIANT Specifications

VALIANT										
Weight	265 pounds									
Length	20 1	feet								
Width	31.9 i	nches								
Depth	14.1 i	nches								
Average Hull Thickness	1 inch									
Reinforcement	Fiberglass Mesh									
	Structural Mix	Finishing Mix								
Concrete Colors	Maize	Blue								
Concrete Unit Weight	$57.4 \text{ lb/ft}^3 \text{ (wet)}$ $55.4 \text{ lb/ft}^3 \text{ (dry)}$	$\begin{array}{c} 66.8 \ lb/ft^3 \ (\text{wet}) \\ 65.7 \ lb/ft^3 \ (\text{dry}) \end{array}$								
Compressive Strength	1070 psi	-								
Split Tensile Strength	260 psi	-								
Flexural Strength	240 psi -									
Air Content	5.1%	1.6%								





#### **Project Management**

The goal for MCCT this year was to implement project management changes that would be sustainable for future years. As a result, more extensive records of tasks and events were kept to assist in continuous improvement for future University of Michigan teams.

The 2016-17 project schedule was created based on the schedule created for the 2016 canoe, *EXTINCTION*. Critical path events were laid out at the beginning of the year as follows: mass meeting, *EXTINCTION* hull testing, finalized hull design, finalized mix selection, canoe placement, completion of sanding, and completion of the canoe. Milestone activities were identified for each event which determined the year's work schedule. Buffers were added to for critical path flexibility. Similar to past years, the placement date for *VALIANT* was set for early December. This date allowed ample time for the canoe to cure prior to the finishing process.

The following milestone activities were selected to guide the completion of the critical path events: recruitment, concrete mixing and testing, hull design, mold fabrication, canoe placement, de-molding, sanding and sealing, and creating display pieces. Responsibilities were distributed by the captain.

MCCT's final and initial project schedule aligned well throughout the year. However, modifications were made for some tasks during second semester. Primarily, the team condensed the finishing time to allow for application of an aesthetic slurry. Additionally, the deadline for the construction of a new canoe carrier was shifted back approximately two months due to a lack of available building space.

A greater emphasis on quality control was achieved by designating a specific experienced member to lead and oversee all quality control initiatives. Updated initiatives included canoe thickness gauges, keel angle devices, and thorough training sessions. Safety requirements were met by following ASTM and University of Michigan Guidelines as well as MCCT specific trainings. Required trainings include project area, mixer, and respirator training. Safety procedures were enforced by veteran members.

This year's project was divided into eight main categories; person hours were tracked for each. The breakdown of person hours can be found below in Table 2.

Task	Hours							
Project Management	120							
Hull Design	410							
Structural Analysis	95							
Mix Design Development	350							
Mold Construction	200							
Canoe Construction	575							
Finishing	300							
Academics	295							
Total	2,345							

Table 2: Breakdown of Person-Hours

The budget for VALIANT was \$9600. The majority of the team's budget was allocated to paddling practice, recruitment/meetings, aggregate costs, construction, competition, and display. Funding came from local company donations and sponsorship from school departments and student government. Concrete materials were sourced from both MCCT sponsors and specialty material suppliers. Other materials, such as foam for the mold construction, came from local companies to reduce shipping costs.

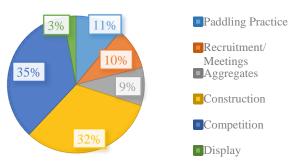


Figure 1: Budget Allocations 2016-2017





#### **Quality Assurance**

A focus of this year's project management was to improve the team's quality assurance. For assessing properties of new materials (i.e. fibers and pigments), the Mix Design Lead and Quality Control (QC) Lead worked together to ensure compliance of any option in consideration. If compliance was not clear, the QC Lead submitted an RFI. The QC Lead also focused attention on RFI's and would alert the leadership team if any RFI's pertained to the team's plans.

Furthermore, because the team purchased a new mixer at the beginning of the year, university equipment was no longer required and thus, the need to have members trained in the university's public lab was removed. However, as a safety consideration, all members who participated in mixing concrete were required by team leadership to attend an instructional safety session hosted by the Team Captain and Mix Design Lead. Additionally, members who would be sanding the canoe were required to be respirator trained and to wear a respirator at all times during the sanding process.

To ensure that calculations were correct, MCCT had several team members review the work of the Mix Design Lead and Hull Design Lead. This review ensured that there were no trivial mistakes in the calculations before beginning the fabrication of the canoe.

With a heavy emphasis placed on the quality control for Pour Day, at least one member (in addition to the QC Lead) was required to be the Quality Control Assistant. Quality control during Pour Day consisted of the following: thickness gauges (Figure 2), keel angle markers (Figure 3), and monitoring the timing and fluidity of the mixing process. During Pour Day, it was also important for quality control to look for inconsistencies in the mixture and ensure that fibers were being thoroughly separated in the mixes.



Figure 2: Thickness Gauges

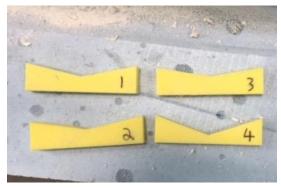


Figure 3: Keel Angle Markers

Technical documents and display plans were revised by the QC Lead for inconsistencies with the ASCE National Rules. This was accomplished by increasing the communication between different portions of the team. MCCT also increased the frequency of meetings dedicated to display planning and technical writing portions of the project. These meetings allowed a greater amount of time to develop ideas that complied with the rules as well as to revise errors that may have otherwise been overlooked.







#### **Organization Chart**







Hull Design Lead Jason Bian

Hull Design Cooper Carpenter Noah Robbins Parker Kurlander Mix Design Lead Eric O'Neill

Mix Design Adrian Berding Payton Piggot

#### Training/Administration

Safety Officer: Alec Distel Public Relations: Olivia Mitchinson Paddling Practice Lead: Michael Kalinowski Faculty Advisor: Will Hansen



Aesthetics and Construction Leads Hannah Bashore, Ashlynn Stanley

Construction Allison Corey Jessica Ma

Display Kristin Lewis Megan Shibley Jacqueline Nisbet



Secretary Mackenzie Horton



Treasurer Mehul Kulkarni



Quality Control Elizabeth Gardner

Name	Year
Gabe Gidley	Sr
Laura Zeiler	Sr
Payton Piggot	Sr
Allison Corey	Jr
Ashlynn Stanley	Jr
Elizabeth Gardner	Jr
Eric O'Neill	Jr
Hannah Bashore	Jr
Mackenzie Horton	Jr
Mehul Kulkarni	Jr
Michael Kalinowski	Jr

Name	Year
Olivia Mitchinson	Jr
Parker Kurlander	Jr
Adrian Berding	So
Alec Distel	So
Cooper Carpenter	So
Jacqueline Nisbet	So
Jason Bian	So
Jessica Ma	So
Kristin Lewis	Fr
Megan Shibley	Fr
Noah Robbins	Fr





#### **Hull Design and Structural Analysis**

Historically, MCCT's hull design has used race results to gauge areas of focus during the design phase. However, during the 2016 competition, safety concerns resulted in race day cancellation. Given the lack of race results from the 2016 canoe, *EXTINCTION*, MCCT adjusted to a data driven approach to hull analysis. Emphasis was placed on engaging new members with design and software. Focal points of design consideration were stability, maneuverability, speed, and ease of paddling.



Figure 4: EXTINCTION Testing

*EXTINCTION* was used to simulate race conditions and gather observations on hull performance. This preliminary testing took place in a nearby pond where tracking and turning were evaluated and ergonomic feedback was gathered from paddlers (Figure 4). The *EXTINCTION* testing results were analyzed alongside national winning designs.

The maximum beam, height, cross section shape, and side profile were modified from *EXTINCTION* to give *VALIANT'S* final hull form.

To improve stability, the team's main objective was lowering the center of gravity and increasing the prismatic coefficient,  $C_p$ , of the new design as compared to *EXTINCTION*. These modifications were implemented and analyzed in Rhinoceros 5.0.



The maximum beam was increased to 31.85 inches to increase the prismatic coefficient. An increase in  $C_p$  from .497 to .568 was achieved with VALIANT'S design. Height was reduced to 14.08 inches to lower VALIANT'S center of gravity for stability. Furthermore, a flatter bottom side profile was opted for in the stern and a reduction in the rate at which the cross section's beam increased from the bow was made. These adjustments lower VALIANT'S center of gravity while retaining the center of buoyancy.

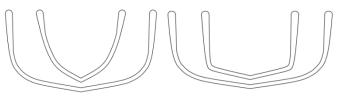


Figure 5: Comparison of *EXTINCTION* and *VALIANT* Cross Sections (1/3 and 2/3 Distance from Bow)

Using Maxsurf simulations, the team used  $GM_T$  and GZ curves to quantify initial stability. Load cases with two male, two female, and 4 co-ed paddlers were used. The simulations showed *VALIANT'S*  $GM_T$  to be 16.132, 16.738, and 11.614 inches for the male, female, and co-ed load cases, respectively. Both the male and female  $GM_T$ 's showed a 20% improvement over *EXTINCTION*.

*VALIANT'S* tipping angles were found from the slope of the GZ curve (Equation 1) and resulted in 25.0, 25.0, and 28.6 for the male, female, and co-ed load cases. The team noted a 6% increase in two paddler races and a 1.4% increase in the co-ed race.

$$Slope \ GZ = \frac{GM \times sin\theta}{\theta} \tag{1}$$

Additional analysis was completed to determine resistance. The Reynold's number is dependent on the kinematic viscosity,  $\nu$ , and forward velocity, V, which can be seen below in Equation 2.



Using the assumption that the hull would be smooth after sanding and sealing the canoe, the frictional resistance coefficient,  $C_F$ , was approximated using the skin friction line developed by the International Towing Tank Conference (ITTC 1978). The skin friction line is defined in Equation 3. Using the resistance coefficient,  $C_F$ , the frictional resistance, R, can be calculated using Equation 4, where  $\rho$  is the density of water and S is the wetted surface area as provided by Maxsurf.

$$Re_S = \frac{VL}{v} \tag{2}$$

$$C_F = \frac{0.075}{(\log_{10}(Re_s) - 2)^2} \tag{3}$$

$$R = C_F \frac{1}{2} \rho S V^2 \tag{4}$$

Using this approximation, the frictional coefficient component was 0.0033 and the total frictional resistance was 1.44 lb.

Table 3: Resistance Calculation Summary

V	4.64 ft/s	$C_F$	0.00332
L	20 ft	ρ	1.94 $slug/ft^3$
v	1.664 * 10 <sup>-5-</sup> ft/s <sup>2</sup>	S	$36.9ft^2$
<b>R</b> e <sub>s</sub>	$5.5769 * 10^{6}$	R	1.44 <i>lb</i>

To analyze the strength of *VALIANT*, the team used two male, two female, and 4 co-ed paddlers as load cases. Moments were calculated for the female races, the male races, and the coed race. The tensile stress in the gunwales was calculated using D, the maximum distance from the neutral axis, I, the moment of inertia, and M, the global bending moment. This can be seen in Equation 5.



$$\sigma = \frac{MD}{I} \tag{5}$$

Distributed weight, buoyancy, and point loads were analyzed to find the global bending moment. Maxsurf Stability Suite was used to analyze the difference between buoyant force and distributed weight to calculate tensile strength along the length of the canoe. The maximum value was found to be 3835.33 lbf-in. Using this value with the stress formula (Equation X), the maximum tensile force in the gunwale of *VALIANT* was calculated to be 26.8 psi. With a concrete tensile strength of 260 psi, the safety factor for this year's design is 9.7.

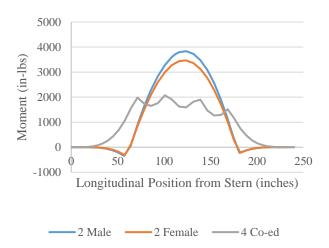


Figure 6: Loading Cases for VALIANT



#### **Development and Testing**

The strength and success of the final mix used for the 2016 canoe *EXTINCTION* allowed MCCT to focus on creating a mix which prevented small shrinkage cracks during the curing process as well as an integrally colored structural mix. To achieve this goal, the Mix Design Team used an iterative design process holding the mix composition constant while altering the amount of pigment, PVA fibers, and polypropylene fibers. MCCT selected this method to understand the effects each type of fiber had on the strength of the concrete as well as how various pigments affected the concrete properties.



Figure 7: EXTINCTION Structural Crack

In 2016, *EXTINCTION* suffered structural cracks during transport to and from competition which were believed to have stemmed from small shrinkage cracks (Figure 7). To protect against shrinkage cracks, the Mix Design Team introduced additional secondary reinforcement of three sizes of PVA fibers in addition to the previously used polypropylene fibers. The Spiderlath Fiberglass Mesh used in previous years was retained for primary reinforcement due to its excellent performance.

Table 4: Mix Results										
6	Baseline	Fiber	Integrally Colored	Final						
Density (lb/ft <sup>3</sup> )	57.8	57.6	58.1	57.4						
Compressive Strength (psi)	720	890	710	1070						
Tensile Strength (psi)	250	230	230	260						
Air Content	5.8%	6.4%	1.3%	5.1%						

To ensure that the causes of any change to the mix was completely clear, the Mix Design Lead used the final mix from 2016 as a baseline. The test results of this baseline mix are available in Table 4. Similar to previous years, three mix sets were created to progress independently from one another: a fiber focused set, an integrally colored set, and a hybrid set which combined the best mixes from the integrally colored and fiber sets with slight modifications.

The fiber set introduced three lengths of PVA fibers (6mm, 8mm, 12mm), in addition to the polypropylene fibers (12mm) used in 2016. Four combinations of fiber lengths and fiber dosing were tested with the mix composition from 2016. It was determined that a combination of all four fiber types produced the greatest strength increase (Table 4). MCCT chose PVA fibers because, according to the manufacturer, they are able to bond with the cement matrix and provide additional strength compared to concrete without PVA fibers. This was tested and confirmed by compressive and tensile strength tests of the fiber mix set ASTM C 109.





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The team found that the fibers were best distributed throughout the concrete mix after first separating fibers that clumped together in packaging. Because of this time consuming process, many team members were designated to be 'separating fibers' during mix sessions and Pour Day as seen in Figure 8.



Figure 8: PVA Fiber Separating

The integrally colored set tested the changes to the mix caused by three colors of iron oxide pigment. Without the ability to stain VALIANT, MCCT sought to create an integrally colored mix to match the bicentennial theme. Maize, blue, and black pigments were tested for their effect on setting time, density, and strength (Figure 9). The specific gravity of the powdered pigment led the Mix Design Team to lower the dosing from the manufacturer's recommendation in order to achieve the desired density and to prevent bleeding once cured. The blue pigment lengthened the setting time of the concrete significantly and negatively influenced both compressive and tensile strength and was therefore not considered for the final mix. The maize pigment performed similarly to the 2016 baseline mix in terms of strength and setting time (Table 4) and was used in the hybrid mixes for optimization.



Figure 9: Pigment Test Samples

Based on compressive and tensile strengths, density, and aesthetics, the best mixes of the fiber and pigment sets were chosen. These were combined into hybrid mixes to accomplish the overall goal of creating an integrally colored mix while preventing small shrinkage cracks. After compiling and receiving the expected stress on VALIANT from the Hull Design Team, the final mix was chosen. The mix is a variant of a hybrid mix with the cementitious material to aggregate ratio lowered slightly to achieve the desired density of 57.4 lb/ft3, which is comparable to the final mix used last year. Additionally, the tensile strength is 260 psi - a 10 psi increase from the baseline mix. The compressive strength is 1070 psi, which is comparable to previous years' final mixes.





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Figure 10: Mixing

To avoid introducing too many variables, the MCCT Mix Design Team used the same amount of the following admixtures from the 2016 final mix in the 2017 final mix design: liquid latex, Glenium 7500 a superplasticizer and AE90, an air entrainer.

One layer of Spiderlath Fiberglass Mesh, placed at a depth of  $\frac{3}{8}$  inches, was used as the primary reinforcement in *VALIANT*. This reinforcement design is identical to both the 2015 and 2016 canoes and was retained due to successful history and ease of application during construction.

The MCCT Mix Design Team utilized two methods to prevent shrinkage cracks during the curing process, particularly the first 24-48 hours after According manufacturer's setting. to the specifications, PVA fibers protect against shrinkage cracking, an issue the team faced extensively in 2016. Also, to compensate for plastic shrinkage cracking, the final mix uses a shrinkage compensating cementitious material, Komponent. Manufacturing specifications for Komponent dictated that it must comprise ~15% of the total cementitious material. This requirement is to ensure the Komponent still minimizes the contraction of concrete while curing, and thus minimizes shrinkage cracking.

The major breakthrough for the MCCT Mix Design Team in 2017 was the use of pigment to create an integrally colored canoe (Figure 11). In addition to the maize pigment used in the structural mix, a finishing slurry mix was created by removing the larger aggregates and increasing the water to cementitious material ratio. This aesthetic slurry testing was added to the team's project schedule during the second semester project management revision meeting. The finishing mix allowed color and graphics to be applied to the canoe while creating a smooth finish.



Figure 11: Pigmented Concrete Canoe





#### Construction

MCCT used an iterative approach to select the ideal materials to maximize strength and buoyancy. To accomplish this, MCCT built upon previous years' mix design research to analyze relevant trends. Each mix built upon the previous mix's strengths while minimizing weaknesses. Along the way, the type and quantity of material was altered based on design goals. The final mix was a culmination of months of research and testing.

The form used was constructed of polystyrene, which was modeled in Rhinoceros 5.0 and cut into 80 3-inch cross sections with flat bases. Cross sections were cut using a CNC router (Figure 12). The team chose to use a male mold for multiple reasons such as increasing accuracy of the gunwale shape as well as to reduce slumping. However, this decision was made with the sacrifice of keel integrity.



Figure 12: CNC Router Cutting Sections from Foam Sheets

All cross section pieces were glued together and placed over three reinforcing wooden beams and a polystyrene base to increase stability (Figure 13). In the final step prior to concrete placement, the mold was coated in a thin layer of release agent.



Figure 13: Reinforcement Bars on Mold Base

The team sanded the exterior of the mold. This assisted with removal of the mold once the canoe had cured. In addition, sanding the mold helped give the canoe interior a smoother finish.

During mixing sessions, the Mix Design Lead determined which mix was to be tested. The Mix Design Team then pre-measured all of the necessary materials to increase the efficiency of the process. Concrete was mixed using a Hobart D300 mixer. Canoe construction involved a steady and continuous process of concrete mixing and placing with a focus on reducing the time between finishing mixing and beginning of placement per batch to reduce hardening prior to placement. After placing the first layer of concrete on the mold, a layer of reinforcing fiberglass mesh was placed on the concrete, followed by a second layer of concrete.

Team members who constructed the canoe used the following techniques to ensure complete incorporation of the fiberglass mesh between the two concrete layers and prevent air bubbles from becoming trapped. First, to make sure the mesh was placed at the correct depth, the thickness of the first layer of concrete was tested with <sup>3</sup>/<sub>4</sub>" quality control





devices. Next, while wearing latex gloves, team members sprayed Sika Liquid Latex onto the first layer of concrete. The mesh was cut to form fit the mold and to eliminate any bulges or gaps. Finally, the second layer of concrete was hand placed on top of the mesh and carefully incorporated before being trowelled smooth (Figure 14).



Figure 14: Concrete Placement

This year, an increased focus was placed on quality control. The Team Captain provided stricter guidelines to the Quality Control Lead. Specifically, color coded nails were utilized to test the thickness of the different sections of the canoe - an improvement from previous years' less accurate thickness gauges. Thickness testing was done continuously throughout Pour Day by the same person to ensure uniformity. This attention to quality control paid off as the final product reflects the modeled design well.

Through extensive research and trial and error from previous years, the team determined the ideal curing condition was in a moist environment and the ideal curing time was 28 days. Moist conditions ensure that the concrete does not prematurely dry out while curing which may cause shrinkage cracks. Based on the team's experience from previous years, this setup has been found to yield optimal strength in MCCT's canoes as opposed to other curing conditions.

To remove the canoe from its mold, a two-step process was used. First, the canoe was flipped into a female mold. Then, the male mold was removed from the canoe. Following this step, the team began sanding and finishing the interior of the canoe.

With the continued absence of stains, the team became creative with the aesthetics of *VALIANT*. The structural concrete of *VALIANT* is Maize, a color of the University of Michigan. Together, the Mix Design Lead and Aesthetics Lead collaborated to implement a pigmented concrete slurry as a way to create decorative designs on *VALIANT* which follow MCCT's 2017 theme of Bicentennial. A sample of aesthetic slurry testing can be seen in Figure 15 below.



Figure 15: Aesthetic Slurry Testing

In an effort to be economically and environmentally stable, the team used pieces of concrete from previous canoes as canvases for slurry testing (Figure 15). These slabs provided accurate simulation for tests without making any impact on materials or budget.



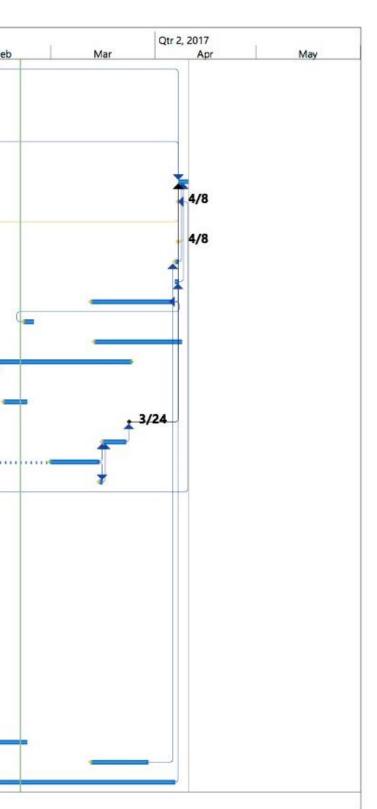


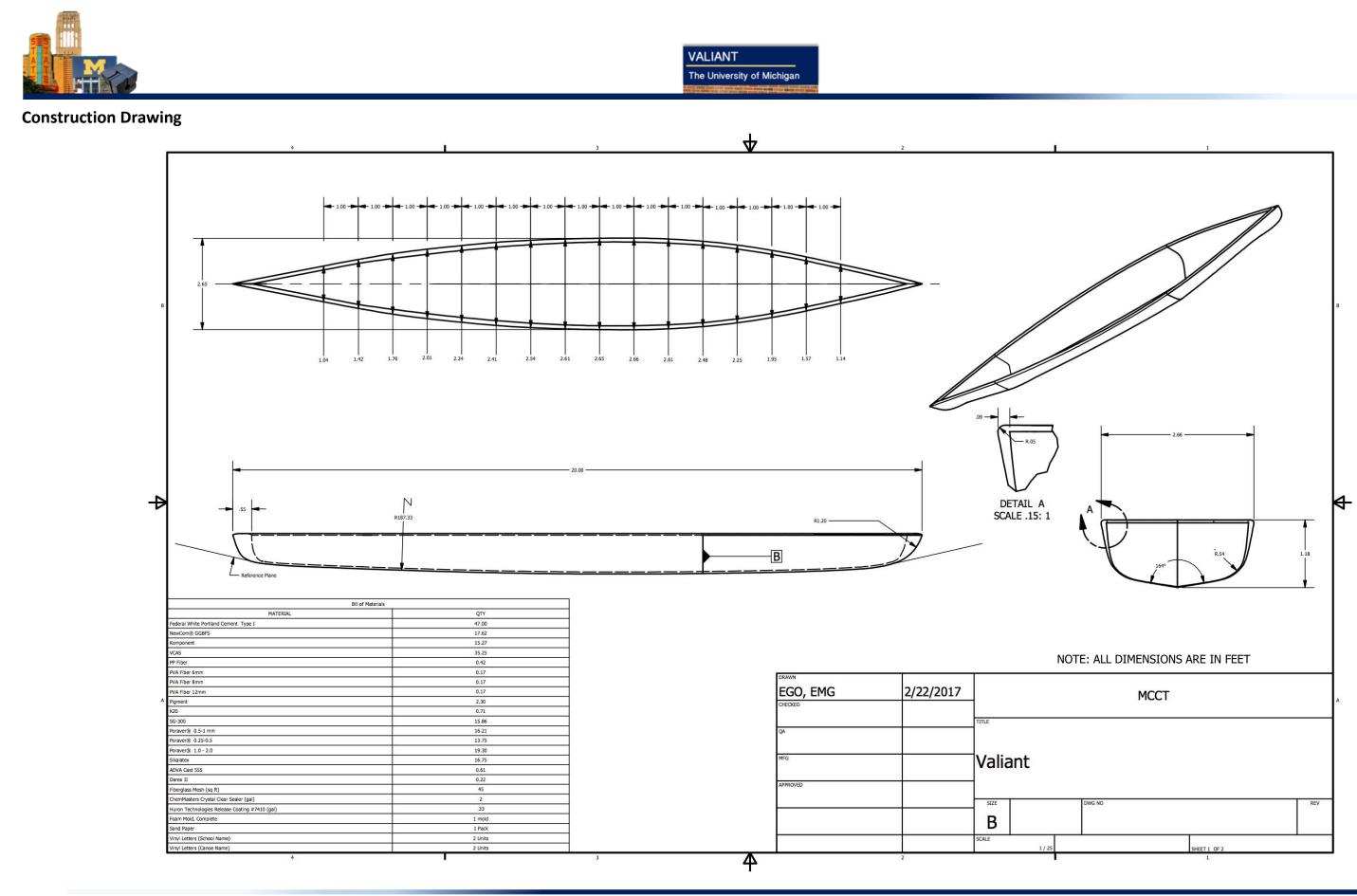
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## Project Schedule

D	Task Name	Duration	Start	Finish	Aur	Free	Qtr 4, 2016	N Lass		Qtr 1, 2017	E.c.
1	New Member Recruitment	14 days?	Thu 9/1/16	Tue 9/20/1	Aug	Sep	Oct	Nov	Dec	Jan	Feb
2	Mass Meeting	1 day	Mon 9/12/16			<b>9/12</b>					
3	Draft & Mail Sponsorship Letters	15 days	Tue 10/11/16	Mon 10/31/16		1.0004.001.01	. 10/31				
4	Fundraising	31 days	Thu 12/1/16	a prior in the second se							
5	Giving Blueday	1 day	Tue 11/29/16	Tue 11/29/					+ 11/29		
6	ASCE Regional Competition	2 days	Sat 4/8/17	Mon 4/10/							
7	Concrete Design	0 days	Sat 4/8/17	Sat 4/8/17							
8	Hull Design	0 days	Tue 1/17/17	Tue 1/17/1						1/17	
9	Technical Paper	0 days	Sat 4/8/17	Sat 4/8/17						T	
10	Technical Presentation	1 day	Fri 4/7/17	Fri 4/7/17							
11	Competition Races	1 day	Fri 4/7/17	Fri 4/7/17							
12	Display/Aesthetics	18 days?	Mon 3/13/17	Wed 4/5/1							
13	Engineer's Notebook (Final Edits)	3 days	Tue 2/21/17	Thu 2/23/1							
14	Design Display Poster	20 days	Tue 3/14/17	Sat 4/8/17							
15	Construct Stands	30 days	Mon 2/13/17	Fri 3/24/17							-
16	Design/Brainstorm	14 days	Tue 1/24/17	Fri 2/10/17						-	J
17	Finish Cross Section	5 days	Wed 2/15/17	Tue 2/21/1							-
18	Finish Canoe	1 day?	Thu 3/23/17	Fri 3/24/17							
19	Seal	5 days	Thu 3/16/17	Thu 3/23/1							
20	Sand and Slurry	19 days	Wed 1/11/17	Wed 3/15/						·	
21	Apply Lettering	1 day	Wed 3/15/17	Thu 3/16/1							
22	Cylinder Testing and Data Analysis	29 days	Tue 10/18/16	Fri 11/25/1			G		•		
23	Mix Testing	29 days	Tue 10/11/16	Fri 11/18/1				<u>a</u> 1	1/18		
24	Organize Testing Space	5 days	Mon 11/14/16	5 Fri 11/18/1				_			
25	Order New Materials	8 days	Thu 9/29/16	Mon 10/10			-				
26	Research New Materials	20 days	Thu 9/1/16	Wed 9/28/			_				
27	Cure Canoe	23 days	Fri 12/16/16	Tue 1/17/1					*		
28	Pour Day	1 day	Sat 12/10/16	Sat 12/10/:					12/10		
29	Construct Foam Mold	1 day	Thu 12/8/16	Thu 12/8/1					×		
30	CNC Cross Section	4 days	Fri 12/2/16	Wed 12/7/					-		
31	Slice Mold	7 days	Wed 11/23/16	Thu 12/1/1				-	_		
32	Structural Calculations	4 days	Wed 11/16/16	Mon 11/21				-			
33	Design	38 days	Fri 9/23/16	Tue 11/15/		-		J			
34	Edit Paper	15 days	Wed 2/1/17	Tue 2/21/1							4
35	Prepare Prezi/PowerPoint	13 days	Mon 3/13/17	Wed 3/29/							
36	NCRB Practice	51 days	Thu 1/26/17	Thu 4/6/17						-	











#### **Appendix A: References**

(1992). "The Reynolds Number: About Rowing and Flying." <a href="http://www.aerodrag.com/Articles/ReynoldsNumber.htm">http://www.aerodrag.com/Articles/ReynoldsNumber.htm</a>> (Oct. 12, 2013).

(2007). "3M Scotchlite Glass Bubbles: K and S series".

ACI. (2008). "ACI 318-08 Building Code Requirements for Structural Concrete." American Concrete Institute. Farmington Hills, Michigan.

ASCE/NCCC. (2016). "2016 American Society of Civil Engineers National Concrete Canoe Competition. Rules and Regulations."

<http://www.asce.org/uploadedFiles/Membership\_and\_Communities/Student\_Chapters/Concrete\_Canoe/Conte nt\_Pieces/nccc-rules-and-regulations.pdf> (Feb 12 2017).

ASCE/NCCC. (2017). "2017 American Society of Civil Engineers National Concrete Canoe Competition. Rules and Regulations."

<http://www.asce.org/uploadedFiles/Conferences\_and\_Events/Event\_Subpages/Content\_Pieces/nccc-rules-regulations-2017.pdf> (Feb 06 2017).

ASTM. (2004). "Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens." C496/C496M-11, West Conshohocken, Pennsylvania.

ASTM. (2010). "Standard Specification for Air-Entraining Admixtures for Concrete." C260/C260M-10a, West Conshohocken, Pennsylvania.

ASTM. (2010). "Standard Specification for Fiber-Reinforced Concrete." C1116/C1116M-10a, West Conshohocken, Pennsylvania.

ASTM. (2012). "Standard Practice for Making and Curing Concrete Test Specimens in the Field." C31/C31M-12, West Conshohocken, Pennsylvania.

ASTM. (2012). "Standard Specification for Portland Cement." C150/C150M, West Conshohocken, Pennsylvania.

ASTM. (2014). "Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens." C39/C39M-14a, West Conshohocken, Pennsylvania.

ASTM. (2012). "Standard Test Method for Density, Relative Density (Specific Gravity), and Absorption of Coarse Aggregate." C127-12, West Conshohocken, Pennsylvania.

ASTM. (2012). "Standard Test Method for Density, Relative Density (Specific Gravity), and Absorption of Fine Aggregate." C128-12, West Conshohocken, Pennsylvania.





ASTM. (2013). "Standard Specification for Chemical Admixtures for Concrete." C494/C494M-13, West Conshohocken, Pennsylvania.

ASTM. (2013). "Standard Specification for Concrete Aggregates." C33/C33M-13, West Conshohocken, Pennsylvania.

ASTM. (2014). "Standard Specification for Slag Cement for Use in Concrete and Mortars." C989/C989M-14, West Conshohocken, Pennsylvania.

ASTM. (2013). "Standard Test Method for Compressive Strength of Hydraulic Cement Mortars." C109/C109M-13, West Conshohocken, Pennsylvania.

ASTM. (2014). "Standard Test Method for Density (Unit Weight), Yield, and Air Content (Gravimetric) of Concrete." C138/C138M-14, West Conshohocken, Pennsylvania.

Bentley. (2012). Maxsurf Stability Module. (Software).

Fine Wood Water Craft. (2014). "Elements of Hull Design." < http://www.finewoodwatercraft.com/Essentials-of-Hull-Design-1.htm> (15 October 2016)

FormSys. (2007). Maxsurf 13. (Software).

ITTC. (2008). "1978 ITTC Performance Prediction Method." *International Towing Tank Conference*, Society of Naval Architects and Marine Engineers, Alexandria, Virginia, 1-31.

Kinney, F. Skene's Elements of Yacht Design, 5th ed. (p. 284).

Michigan Concrete Canoe Team. (2014). "Legacy." NCCC Design Paper, University of Michigan, Ann Arbor, Michigan.

Michigan Concrete Canoe Team (2015). "Allegro." NCCC Design Paper, University of Michigan, Ann Arbor, Michigan.

Michigan Concrete Canoe Team (2016). "Extinction." NCCC Design Paper, University of Michigan, Ann Arbor, Michigan.

Ramyasree Annam (2015). "Study of Mechanical Properties of PVA Fiber-Reinforced Concrete with Raman Spectroscopic Analysis. *Masters Thesis & Specialist Projects*. Paper 1460. <a href="http://digitalcommons.wku.edu/thesis/1460">http://digitalcommons.wku.edu/thesis/1460</a>>.

Raphael, J. M. (1984). "Tensile Strength of Concrete." < http://www.concrete.org/tempComDocs/1237910-5818/81-17.pdf>. (Nov. 12, 2012).

Rhino 3D. (2016). Rhino 5.0. (Software).







Slade, Stuart. (1998). "Understanding the Prismatic Coefficient". <<u>http://www.navweaps.com/index\_tech/tech-004.htm</u>>. (Oct. 13, 2012).

Vacanti Yacht Design. (2003). Prolines 7. (Software)

VEEM Ltc. (2014). "What is the GM(t) ...And Why it is Important." <a href="http://veemgyro.com/wp-content/uploads/2015/11/Whitepaper-1401-What-is-the-GMT-20140711.pdf">http://veemgyro.com/wp-content/uploads/2015/11/Whitepaper-1401-What-is-the-GMT-20140711.pdf</a>> (Oct. 15, 2016).





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# **Appendix B: Mixture Proportions**

		CE	EME	NTITI	ous M	ATER	IALS	5			
Component				ravity	Volum			Amount of	CM (m	ass/volu	me) (lb/vd <sup>3</sup> )
Portland Cement		-1	3.15		1.4			280		Total Amount of	
GGBFS			2.90		0.:	Carlor I	105			cementitious materials	
Komponent		3.10			0.47			91			686 lb/yd <sup>3</sup> c/cm ratio
VCAS-160			2.60	)	1.3	29		210			0.41
				F	IBERS						
Component		Spec	ific G	Fravity	Volum	te (ft <sup>3</sup> )		Amount of F	ibers (r	nass/vo	lume) (lb/yd³)
Grace Fiber (Polypropylene)			1.3		0.0	31		2.5			
NYCON PVA RMS702 6mm			1.3		0.0	12		1		Total.	Amount of Fibers
NYCON PVA RECS15 8mm			1.3		0.0	12		1			5.5 lb/yd <sup>3</sup>
NYCON PVA RECS100 12mm			1.3		0.0	12		1			
				AGG	REGAT	TES					
	AST	M A	bs	MC <sub>stk</sub>		Base	Quan	tity (lb/yd <sup>3</sup> )	Volu	me <sub>SSD</sub> ,	Batch Quantity
Aggregates	C330			(%)	SG <sub>SSD</sub>	0	D	SSD		ft <sup>3</sup> )	(at MC <sub>stk</sub> ) (lb/yd <sup>3</sup> )
Poraver 1.0-2.0		19.	.0	<1	0.40	11	5.0	136.9	4.	.61	115.0
Poraver 0.5-1.0		18.	-	<1	0.50	-	.6	114.0	3.	.10	96.6
Poraver 0.25-0.5		21.	.0	<1	0.70	81	.9	99.1	1.	.88	81.9
SG-300	Č.	1.0	,	<1	0.72	94	.5	94.5	2.	.10	94.5
K20		1.0	)	<1	0.20	53	.0	53.0	4.	.25	53.0
				ADM	IXTUR	RES			1		
Admixture	lb/gal	77.67	Dosag	e	% Nolids Amount of Water in Admixture ()					xture (lb/yd³)	
Sika Liquid Latex Modifier	8.8		200			0	49.99			To	tal Water from
Water Reducer	8.9		8.00		5.0		3.63			Admixtures, $\sum w_{admx}$	
Air Entrainer	8.7		3.00		5.0	0 1.33		<b>54.95</b> <i>lb/yd</i> <sup>3</sup>			
So	LIDS (L	ATEX	K, DY	YES AN	ND POV	VDER	ED AI	MIXTUR	ES)		
Component		Spec	ific (	Fravity	Volun	te (ft <sup>3</sup> )		Amount	(mass/	/volume	) (lb/yd <sup>3</sup> )
Sika Liquid Latex Modifier			1.05		0.7	76 49.80				Total Solids from	
Yellow Pigment			5.24		0.0	0.04		13.72		Admixtures 63.52 lb/yd <sup>3</sup>	
		I		N	ATER						03.32 <i>ibiy</i> u
							/volum	e) (lb/yd³)			Volume (ft <sup>3</sup> )
Water, lb/yd <sup>3</sup>					1111011			314.58			4.40
Total Free Water from All Aggres	gates, lb/y	$d^3$						~0.00			
Total Water from All Admixtures,		2						54.95			
Batch Water, lb/yd <sup>3</sup>								219.45			
	DENSI	TIES.	AIR	CONT	ENT.	RATI	DS AN	D SLUMP			
		cm		1	bers		gates	solids		water	Total
Mass of Concrete, M, (lb)		686.			.50	44	-	63.5		314.6	1510.6
Absolute Volume of Concrete, V	(ft <sup>3</sup> )	3.8	;	0	.07	15	5.9	0.8		4.4	24.97
Theoretical Density, T, $(=\sum M / \sum$		60.:	5	lb/f	t <sup>3</sup>	Air Co	ntent [	= (T-D)/T x	100%	1	5.1 %
Measured Density, D		57.4	4	lb/ft		Slump, Slump flow				$3 \pm 1$ in	
water/cement ratio, w/c:		0.4	0.40				water/cementitious material ratio, w/cm:				0.46

#### MIXTURE DESIGNATION: STRUCTURAL MIX





#### MIXTURE DESIGNATION: FINISHING MIX

			СЕМ	IEN'	TITI	ous M	<b>LATEF</b>	RIALS					
Component			Specific Gravity			Volun	ne (ft <sup>3</sup> )	3) Amount of CM (mass/volume) (lb/yd <sup>3</sup> )					
GGBFS			2.	2.90		6.95			1258		cemer	Total Amount of cementitious materials 1258 lb/yd <sup>3</sup> c/cm ratio	
					F	IBERS						0	
Component		1	Specific	c Gra			ne (ft <sup>3</sup> )	1	Amount of F	ibers	(mass/voi	ume) (lb/yd³)	
			1 5									Amount of Fibers 0 lb/yd <sup>3</sup>	
				F	AGG	REGA	ГES						
		ASTM	Abs	N	1C <sub>stk</sub>		Base	e Quan	tity (lb/yd <sup>3</sup> )	Va	lume <sub>SSD</sub> ,	Batch Quantity	
Aggregates		C330*		1.000	(%)	SG <sub>SSD</sub>	0	D	SSD		(ft <sup>3</sup> )	(at MC <sub>stk</sub> ) (lb/yd <sup>3</sup> )	
Poraver 0.25-0.5mm			21		<1	0.70	17	3.2	209.6		3.97	173.2	
K-20			1		<1	0.20	54	4.70	55.2		4.38	54.7	
							-					-	
					ADM	IXTU	RES						
Admixture	l	b/gal	Dos (fl.oz	age (cwt)		% So	olids Amount of Water in Admixtu					cture (lb/yd³)	
Sika Liquid Latex Modifier		8.8	885			47	7 405.7						
	_				-			-			Adm	tal Water from hixtures, ∑w <sub>admx</sub> 405.7 lb/yd <sup>3</sup>	
	OLI	DS (L)	ATEX.	DYF	ESAN	ND POV	VDER	ED AI	OMIXTURI	ES)	]		
Component			Specifi				ne (ft <sup>3</sup> )			-	ss/volume	) (lb/yd <sup>3</sup> )	
Pigment (Blue or Yellow)				24	0.06 21.0				1	Total Solids from			
Sika Liquid Latex Modifier			1.	05		11	.6		12.2251		Admixtures		
										<b>381.0</b> <i>lb/yd</i> <sup>3</sup>			
					N	ATER	L.						
						Amou	nt (mass	s/volum	e) (lb/yd³)			Volume (ft <sup>3</sup> )	
Water, lb/yd <sup>3</sup>								w:	405.7			6.50	
Total Free Water from All Agg	regate	s, lb/yd	3					$\sum w_{free}$ : ~0.00					
Total Water from All Admixture	es, lb/	vd <sup>3</sup>						$\sum w_{adn}$	<sub>1x</sub> : 405.7				
Batch Water, lb/yd <sup>3</sup>								Wbatch	0.00				
	DI	ENSIT	IES, AI	IR C	CONT	TENT,	RATI	OS AN	D SLUMP				
			сm		fil	bers	aggr	egates	solids		water	Total	
Mass of Concrete, M, (lb)			1258			0	22	7.9	381.0		405.7	∑ <i>M</i> : 2273	
Absolute Volume of Concrete,	-	2	6.95		2162588.8	0	-	.35	11.7		6.50	$\sum V: 33.5$	
Theoretical Density, T, $(=\sum M)$	/∑V)		67.9		lb/f		<i>Air Content</i> [= $(T - D)/T \ge 100\%$ ]				1.6 %		
Measured Density, D			66.8		lb/ft	,		, Slump				10 i	
water/cement ratio, w/c:			-				water/cementitious material ratio, w/cm:				0.32		

\* Indicate if aggregate, other than manufactured glass microspheres and/or cenospheres, is compliant with ASTM C330.





#### **Appendix C: Example Structural Calculation**

The stress calculation was done using the maximum moment for all load cases. The moments were calculated using Maxsurf Stability Suite and can be seen in Figure A1.

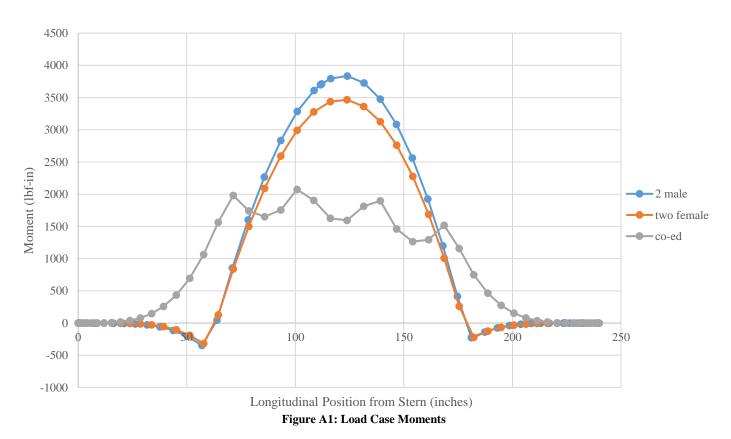


Table 5 below shows the load cases used. The horizontal arm was measured from the bow and the vertical arm was measured from the lowest point on the canoe.

Table 5: Load Cases											
	Horizontal Arm (inches)	Weight (pounds)	Vertical Arm (inches)								
2 Mala	60	152	21.71								
2 Male	180	152	21.71								
2 Famala	60	140	21.71								
2 Female	180	140	21.71								
	70	152	21.71								
	103	140	21.71								
4 Co-ed	137	152	21.71								
	170	140	21.71								





The maximum value is 3835.33 lbf-in. This value occurred 124 inches from the bow during the male load case.

A cross section of the canoe was taken 124 inches from the bow. The moment arm and the area moment of inertia about neutral axis was found using Rhinoceroses 5 moment analysis. The calculations are summarized below:

 $\sigma = stress = \frac{M_{max}y}{I}$ 

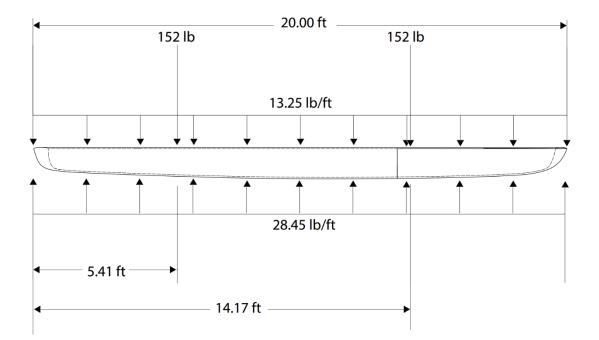
M<sub>max</sub> = max bending moment = 3835.33 lb-in

y = moment arm = 5.00 in

 $I = area moment of inertia about neutral axis = 788.0 in^4$ 

 $\sigma = \frac{3835.33lb - in * 5.00in}{788.0 in^4}$  $\sigma = \frac{21095.1 \, lb - in^2}{788.0 in^4}$ 

$$\sigma = 26.8 \ psi$$







### Appendix D: Hull Thickness/Reinforcement and Percent Open Area

The team used a gradient similar to last year's gradient to strengthen the area below the gunwales. The width and height of the gradient is maintained throughout the entire length of the canoe. The ThicknessAnalysis() function was used in Rhino to find the average thickness between the outer and inner polysurfaces of the canoe design.

The ThicknessAnalysis function failed near the bow and stern of our design due to the rapidly changing curvatures between the inner and our polysurfaces, so the team took two cross sections located 45.16 inches from the bow and 76.35 inches from the bow and stern respectively and used the function to find the average cross sectional thicknesses of all cross sections between the two cross sections selected. The average thickness found was 1.004 inches.

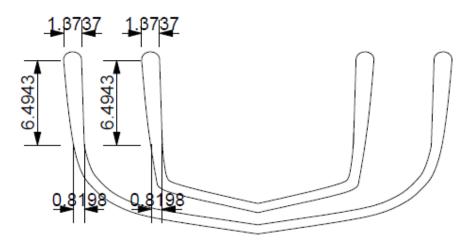


Figure D1: Cross sections chosen

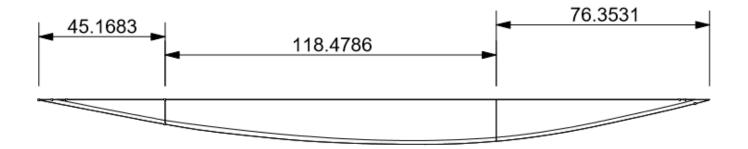


Figure D2: Locations of the cross sections chosen



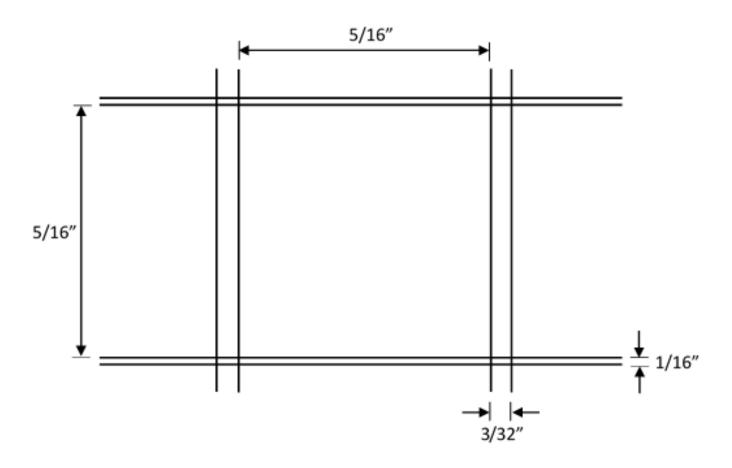




The Spiderlath Fiberglass Mesh that was used in *EXTINCTION* was used in *VALIANT* as well. Percent open area calculations are as follows.

Open Area = 5/16" x 5/16" = 25/256 in<sup>2</sup> Total Area (consider ½ of strand thickness) w = 5/16" + 2\*3/32"\*1/2 = 13/32" h = 5/16" + 2\*1/16"\*1/2 = 6/16" Total Area: 13/32" x 6/16" = 39/256 in<sup>2</sup>

Percent Open Area =  $(25256)/(39256) = (25/39) * 100 \cong 64\%$ 







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