Proyecto Final

Departamento de Ingeniería Eléctrica y Electrónica

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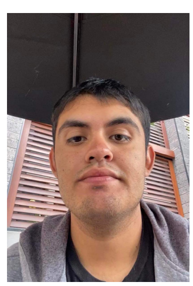

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Carrera: **Ingenieria Biomedica**

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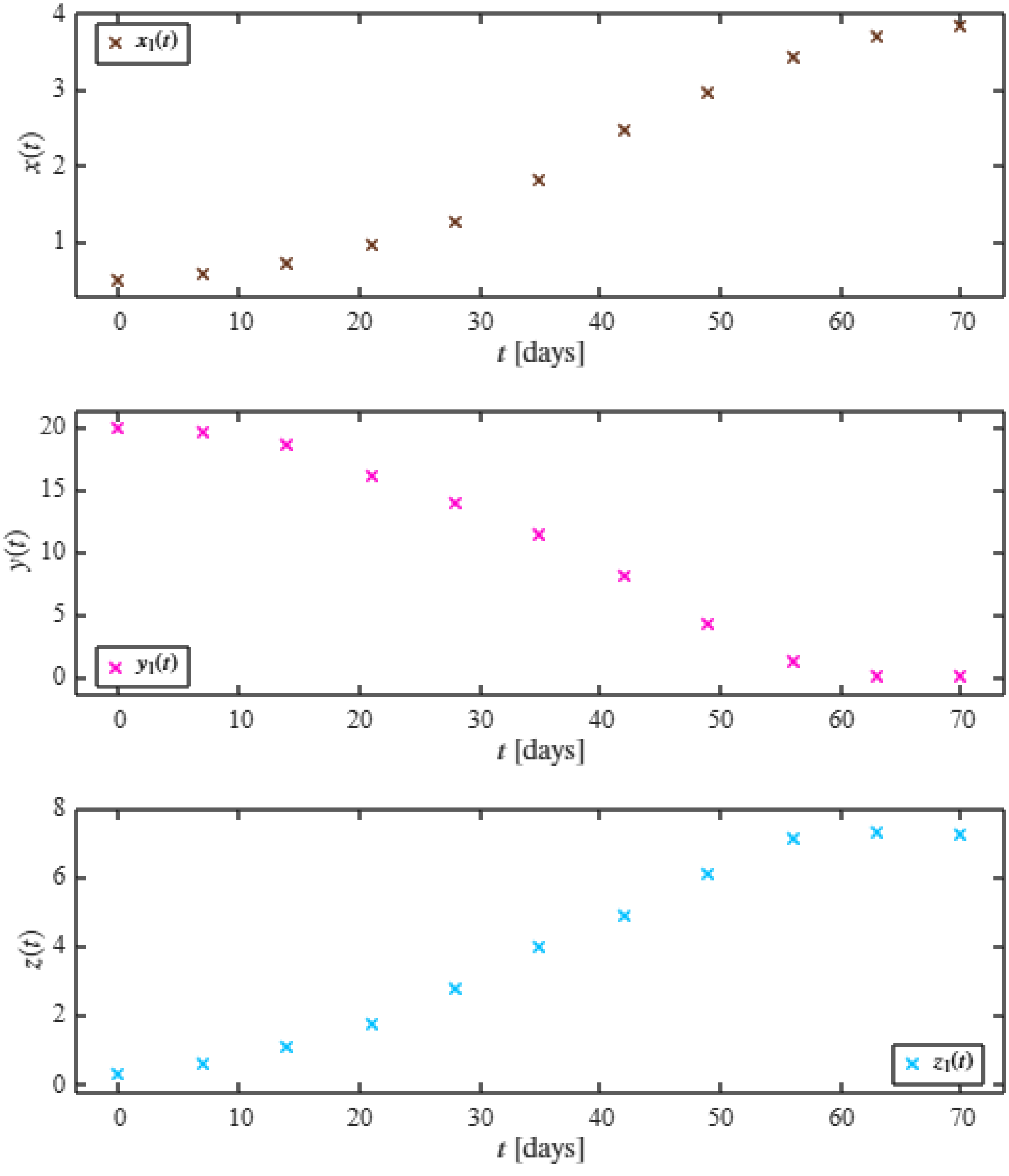
Objetive: To integrate specific competencies related to the modeling, analysis, and control of biological systems, in order to illustrate and predict their dynamics in both the short and long term. The goal is to lay the groundwork for the development of digital twins capable of addressing and solving problems within the field of Systems Biology.

Simulation Data

```
clc; clear; close all; warning('off','all')
filename = 'data2.csv';
sys = readmatrix(filename);
to = sys(:,1);
x1 = sys(:,2);
y1 = sys(:,3);
Z1 = sys(: ,4);
T = array2table([to, x1,y1,Z1], 'VariableNames', {'tiempo', 'x(t)', 'y(t)',
'Z(t)'});
disp(T); plotdata(to, x1, y1, Z1); saveas(gcf, 'RawData.pdf');
```

tiempo	x(t)	y(t)	Z(t)
0	0.517	19.939	0.281
7	0.58	19.578	0.609
14	0.726	18.678	1.063
21	0.965	16.215	1.758
28	1.282	14.005	2.752
35	1.825	11.467	3.975
42	2.473	8.096	4.895
49	2.974	4.327	6.096
56	3.43	1.287	7.152
63	3.717	0.194	7.328
70	3.829	0.056	7.232



Smooth Data

```
clc; clear; close all; warning('off','all');
filename = 'data2.csv';
sys = readmatrix(filename);
```

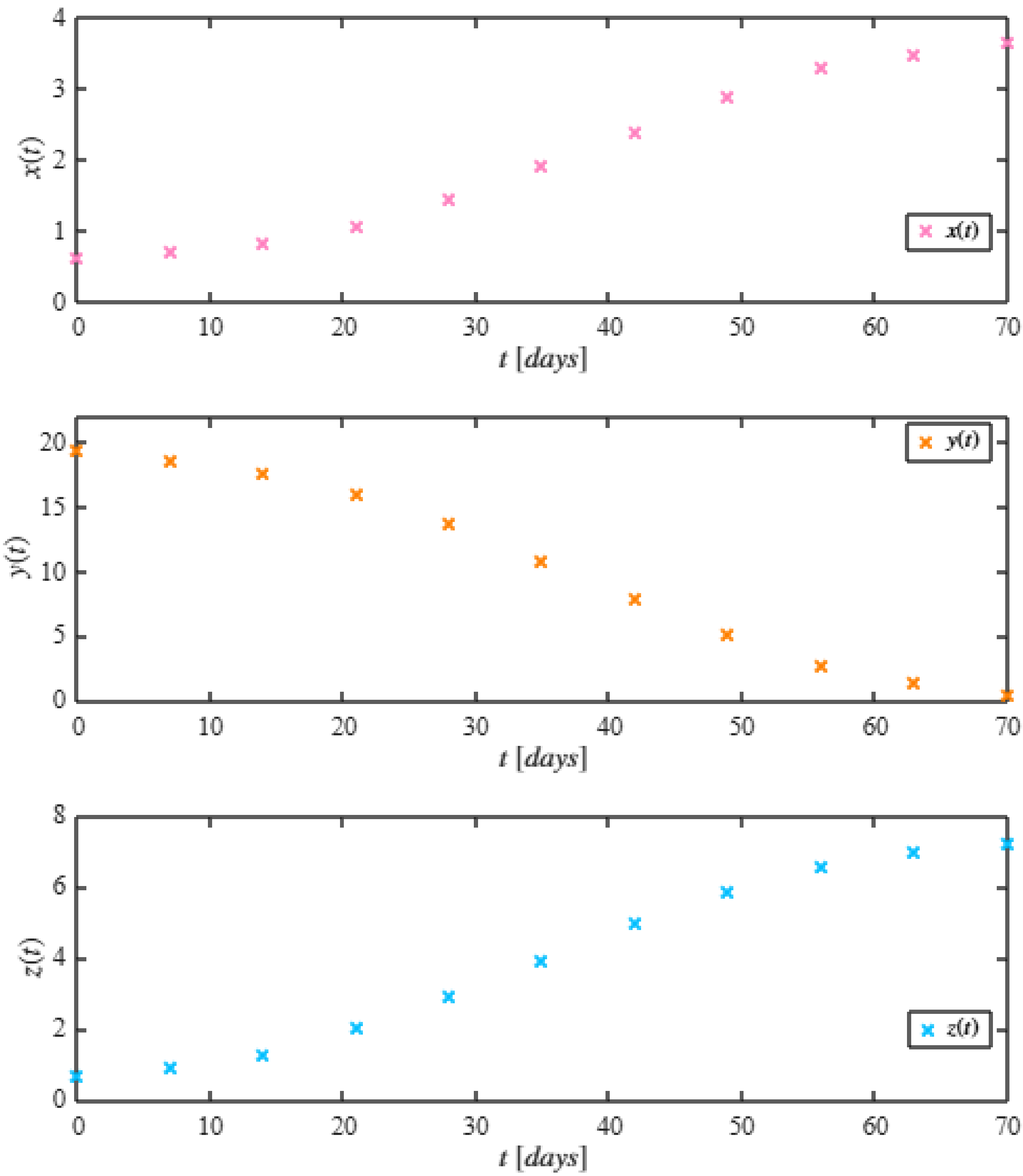
```
to = sys(:,1);
x1 = sys(:,2);
y1 = sys(:,3);
Z1 = sys(:,4);

windowSize = 5;
x1_smooth = movmean(x1, windowSize);
y1_smooth = movmean(y1, windowSize);
Z1_smooth = movmean(Z1, windowSize);

T_smooth = array2table([to, x1_smooth, y1_smooth, Z1_smooth], ...
    'VariableNames', {'tiempo', 'x(t)', 'y(t)', 'z(t)'});
disp(T_smooth);%writetable(T_smooth, 'data2_suavizado.csv');
```

tiempo	x(t)	y(t)	z(t)
0	0.60767	19.398	0.651
7	0.697	18.602	0.92775
14	0.814	17.683	1.2926
21	1.0756	15.989	2.0314
28	1.4542	13.692	2.8886
35	1.9038	10.822	3.8952
42	2.3968	7.8364	4.974
49	2.8838	5.0742	5.8892
56	3.2846	2.792	6.5406
63	3.4875	1.466	6.952
70	3.6587	0.51233	7.2373

```
plotEDOs_fit(to, x1_smooth, y1_smooth, Z1_smooth);saveas(gcf,
'SmoothData.pdf');
```



Nonlinear Regression

The resulting equations from the nonlinear regression, obtained using the Eureqa software, are as follows:

$$\frac{dx}{dt} = P_1 xy + P_2 xyz$$

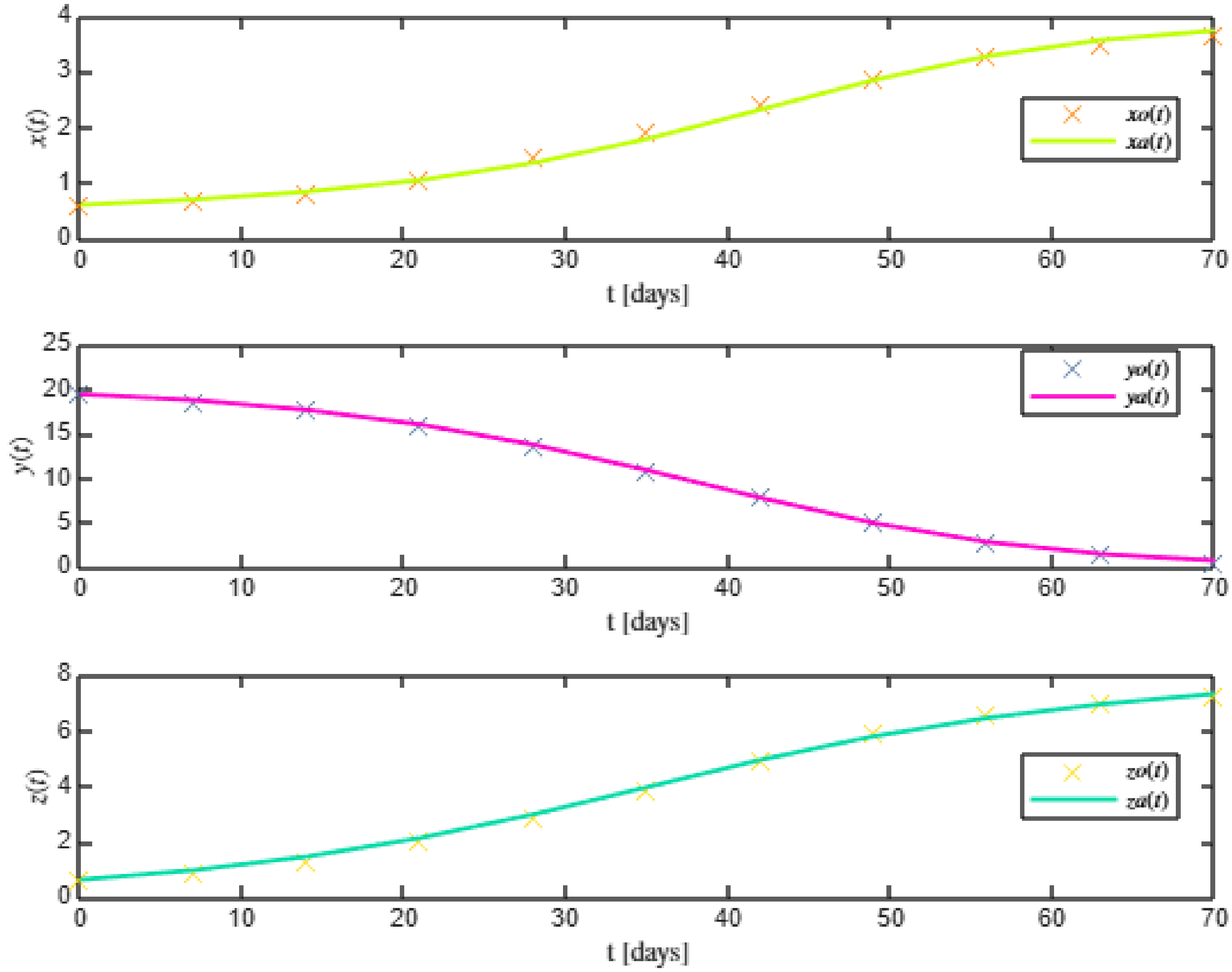
$$\frac{dy}{dt} = P_3y - P_4xy$$

$$\frac{dz}{dt} = P_5z - P_6yz$$

```
warning('off')
sys = readmatrix('data2_suavizado.csv');
to = sys(:,1);
xo = sys(:,2);
yo = sys(:,3);
zo = sys(:,4);
P0 = [
    0.00115591860282494;
    0.000608183514299159;
    0.0139308044869223;
    0.0287876736667008;
    0.00251665147568222;
    0.00321466323165287
];
[mdl, xa, ya, za] = Varied(to, xo, yo, zo, P0); saveas(gcf, 'Nonlinear Regresion.pdf');
```

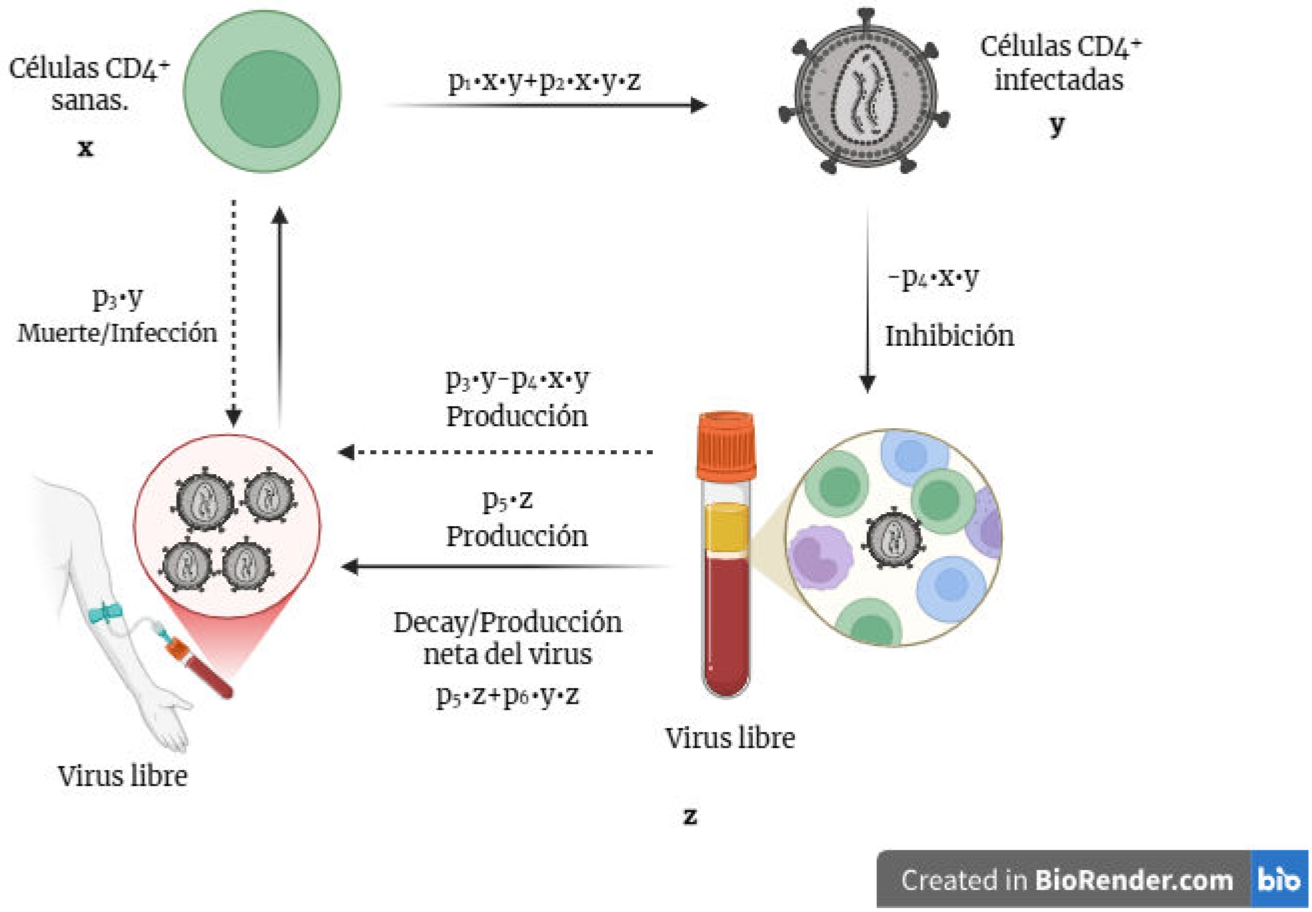
```
Sample size (n): 11
Parameters estimated: 6
Degrees of freedom: 27
R-squared: 0.9998
Corrected AIC: -53.5645
```

Param	Estimate	SE	MoE	CI95		pValue
"p1"	0.00046445	0.00011144	0.00022866	0.00023579	0.00069311	0.00028361
"p2"	0.00078488	4.7378e-05	9.7212e-05	0.00068767	0.0008821	1.1357e-15
"p3"	0.015099	0.00051026	0.001047	0.014052	0.016146	4.1008e-22
"p4"	0.03085	0.00068236	0.0014001	0.02945	0.03225	5.4925e-27
"p5"	0.0042856	0.0007983	0.001638	0.0026476	0.0059236	1.1335e-05
"p6"	0.0029086	6.5984e-05	0.00013539	0.0027732	0.003044	1.0788e-26



Model fitting

The contents include general information, data preprocessing (smoothing and normalization), nonlinear regression methods, model fitting, and system analysis using the Jacobian matrix and equilibrium points. Additional sections cover local stability, prediction algorithms (such as 2t prediction), and corresponding plots for both raw and processed data.



$x = \text{Healthy CD4+ cells}$
$y = \text{Infected CD4+ cells}$
$z = \text{Free virus}$

Equation 1: Healthy cells (x): $P_1xy + P_2xyz$

Interpretation:

- This models the dynamics of healthy CD4+ cells.
- The diagram shows that healthy cells are **lost due to infection** when they interact with infected cells y and free virus z.

Equation 2: Infected cells (y): $P_3y - P_4xy$

Interpretation:

- P_3y: Persistence or reproduction of infected cells.
- $-P_4xy$: Suppression or elimination of infected cells by healthy cells.
- The diagram shows the transition from healthy to infected and the inhibition effect.

Equation 3: Free virus (z): $P_5z - P_6yz$

Interpretation:

- P_5z: Direct replication of free virus.
- $-P_6yz$: Additional virus production driven by infected cells.
- The diagram shows a feedback loop: the virus is produced by infected cells and also amplifies itself.

Jacobian Matrix and Equilibrium Points

```
clc; clear; close all; warning('off','all')

syms x y z
p = sym('p', [1 7]);

  dx = p(1) * x .* y + p(2) * x .* y .* z;
    dy = p(3) * y - p(4) * x .* y;
    dz = p(5) * z + p(6) * y .* z;

J = jacobian([dx, dy, dz], [x, y, z]);
fprintf('Jacobian matrix of the system:\n');disp(J)
```

```
Jacobian matrix of the system:
```

$$\begin{pmatrix} p_1\,y+p_2\,y\,z & p_1\,x+p_2\,x\,z & p_2\,x\,y \\ -p_4\,y & p_3-p_4\,x & 0 \\ 0 & p_6\,z & p_5+p_6\,y \end{pmatrix}$$

```
eq1 = dx == 0;
eq2 = dy == 0;
eq3 = dz == 0;
edos = solve([eq1, eq2, eq3], [x, y, z], 'Real', true);

n = length(edos.x);
fprintf('Equilibrium Points of the system:\n');fprintf('The system has %d
equilibrium point(s).\n\n', n);
```

```
Equilibrium Points of the system:
The system has 2 equilibrium point(s).
```

```
for i = 1:min(3,n)
    X = edos.x(i);
    Y = edos.y(i);
    Z = edos.z(i);
    fprintf('Equilibrium point %d:\n', i);
    disp([X, Y, Z])
end
```

```
Equilibrium point 1:
```

$(0 \;\; 0 \;\; 0)$

```
Equilibrium point 2:
```

$$\begin{pmatrix} \frac{p_3}{p_4} & -\frac{p_5}{p_6} & -\frac{p_1}{p_2} \end{pmatrix}$$

Local estability

```
clc; clear; close all; warning('off','all')
p = [
    0.00115591860282494;
    0.000608183514299159;
    0.0139308044869223;
    0.0287876736667008;
    0.00251665147568222;
    0.00321466323165287
];
syms x y z
dx = p(1)*x*y + p(2)*x*y*z;
dy = p(3)*y - p(4)*x*y;
dz = p(5)*z + p(6)*y*z;
edos = solve([dx == 0, dy == 0, dz == 0], [x, y, z]);
J = [ diff(dx,x), diff(dx,y), diff(dx,z);
      diff(dy,x), diff(dy,y), diff(dy,z);
      diff(dz,x), diff(dz,y), diff(dz,z)];
n_eq = length(edos.x);
for k = 1:n_eq
    x0 = double(edos.x(k));
    y0 = double(edos.y(k));
    z0 = double(edos.z(k));
    J_eval = double(subs(J, {x,y,z}, {x0,y0,z0}));
    L = eig(J_eval);
    L = sort(L,'descend');
    L1 = L(1); L2 = L(2); L3 = L(3);
    var = {sprintf('Equilibrium_%d', k)};
    Lambdas = table(L1, L2, L3, 'RowNames', var);
    disp(Lambdas)
end
```

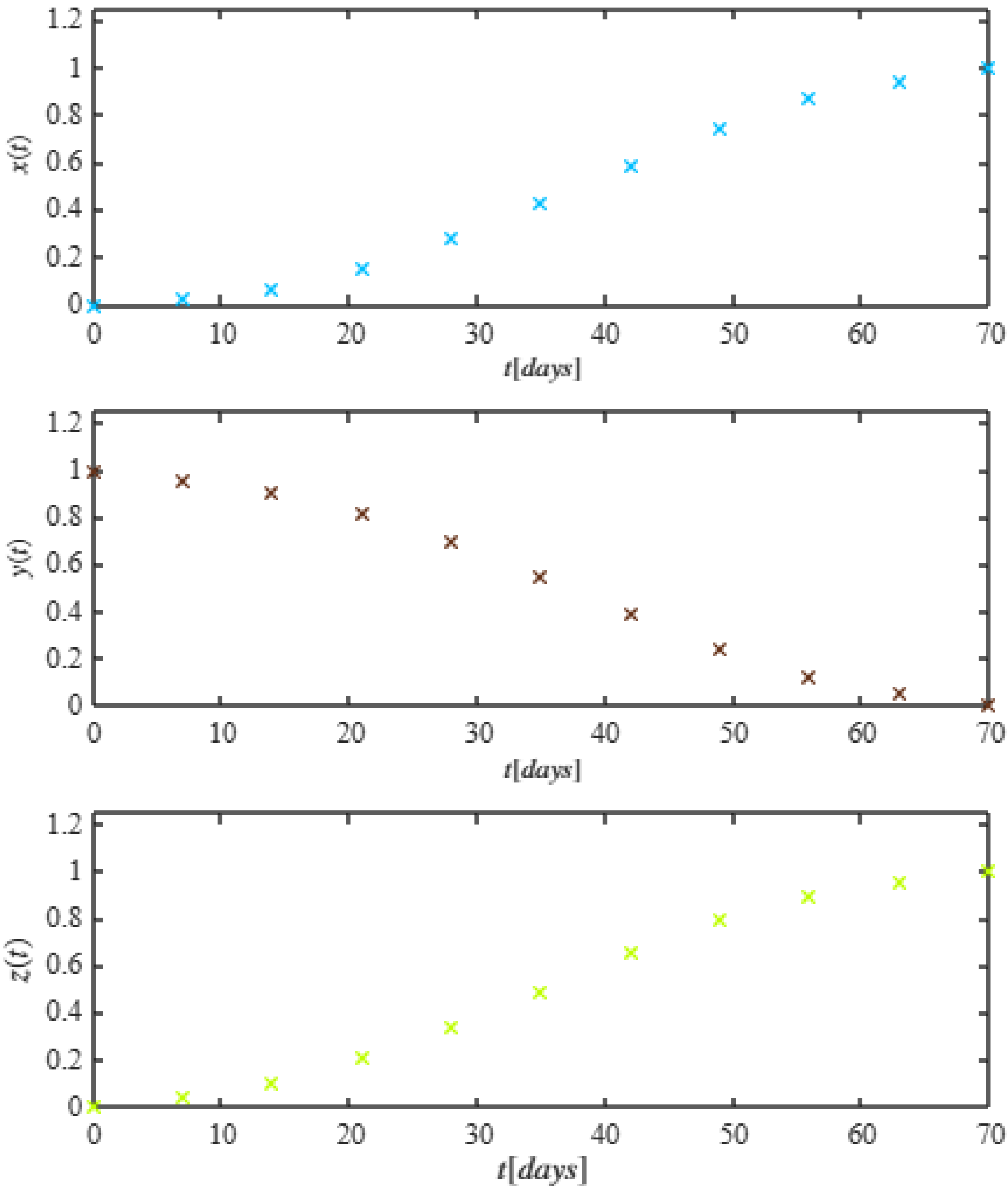
	L1	L2	L3
Equilibrium_1	0.0031657	-0.0015829+0.0027416i	-0.0015829-0.0027416i

	L1	L2	L3
Equilibrium_2	0.013931	0.0025167	0

Data Normalized

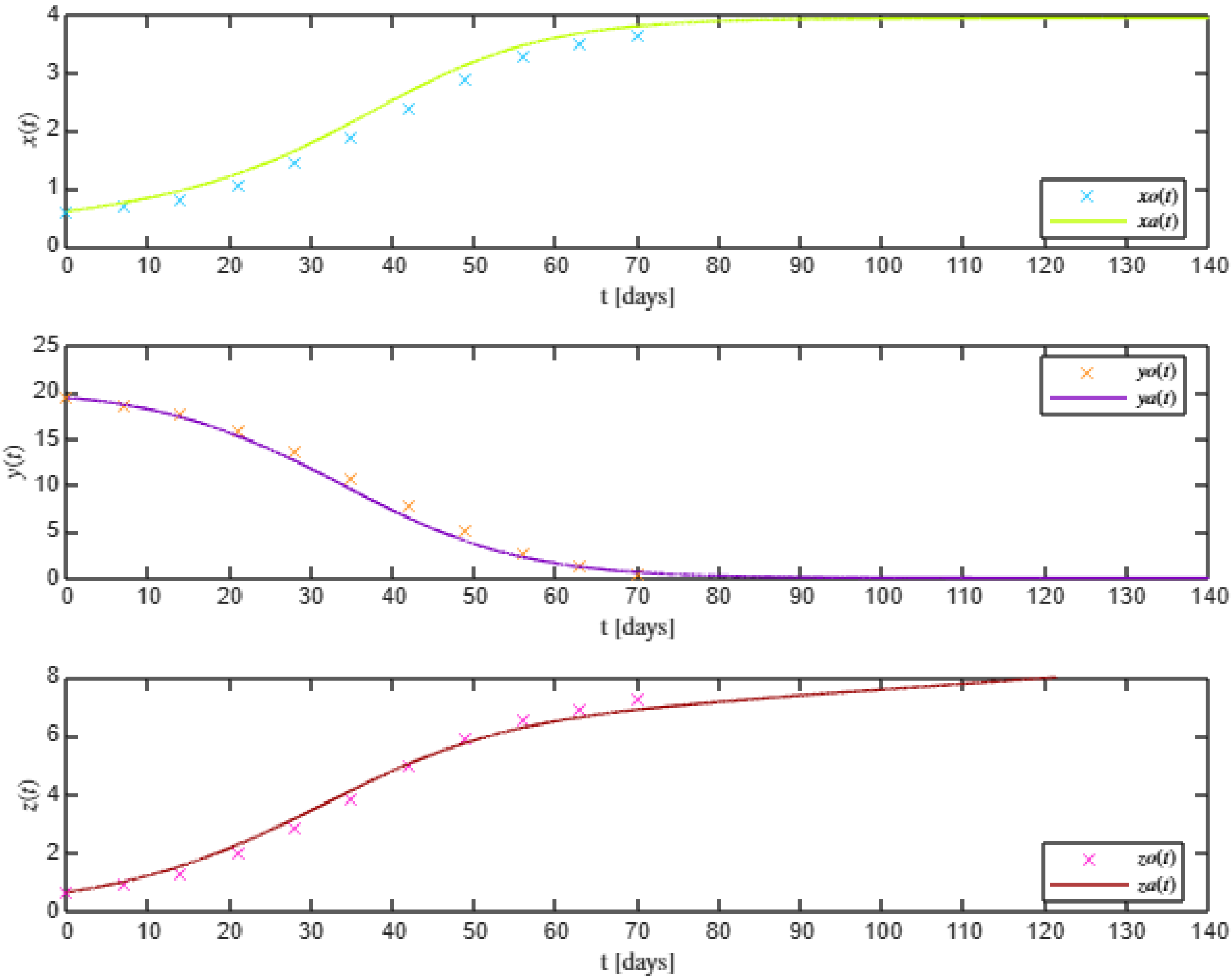
```
sys = readmatrix('data2_suavizado.csv');
to = sys(:,1);
```

```
x1 = sys(:,2);
y1 = sys(:,3);
Z1 = sys(:,4);
x1 = smoothdata(x1);
y1 = smoothdata(y1);
Z1 = smoothdata(Z1);
T = table(to, xo, yo, zo, 'VariableNames', {'Tiempo', 'X_suave', 'Y_suave',
'Z_suave'}); graficarSuavizado(T); saveas(gcf, 'Data Normalized.pdf');
```



2t prediction

```
warning('off')
filename = 'data2_suavizado.csv';
sys = readmatrix(filename);
t = sys(:,1);
x = sys(:,2);
y = sys(:,3);
z = sys(:,4);
P0 = [
    0.00115591860282494;
    0.000608183514299159;
    0.0139308044869223;
    0.0287876736667008;
    0.00251665147568222;
    0.00321466323165287
];
dt = 1E-2;
tend = 140;
[tp, xp, yp, zp] = Predict(x(1), y(1), z(1), dt, tend, P0);
plotXYZPredict(t, x, y, z, tp, xp, yp, zp);saveas(gcf, '2T.pdf');
```



Conclusion

This project has enabled the integration of advanced mathematical tools with real experimental data to construct a dynamic model that accurately describes the behavior of a nonlinear system. Throughout the different stages from data collection to simulation and model validation a deep understanding was developed of both the biological processes involved and the mathematical analysis techniques required for their study.

The use of nonlinear regression facilitated the identification of appropriate parameters, while statistical significance and goodness-of-fit tests ensured the reliability of the results. Additionally, the local and global stability analysis allowed us to explore the qualitative dynamics of the model, providing key insights into its long-term behavior. The simulations carried out during the in silico experimentation stage demonstrated that the model not only fits the available data but also has good predictive potential.

Ultimately, this practice highlights the importance of an interdisciplinary approach in the mathematical modeling of real-world phenomena, where the validity of the model depends not only on its numerical fit but also on its biological consistency and ability to offer useful and meaningful predictions.

Function

Raw data plot

```
function plotdata(t, x, y, z)
    Mecambiareaindustrial = [
        0.60, 0.00, 0.00;
        0.75, 1.00, 0.00;
        0.00, 0.75, 1.00;
        0.50, 0.00, 0.75;
        1.00, 0.50, 0.00;
        0.40, 0.20, 0.10;
        1.00, 0.00, 0.80;
        1.00, 0.50, 0.75;
        0.00, 0.85, 0.65;
        0.30, 0.45, 0.60
    ];

    rng('shuffle');
    indices = randperm(size(Mecambiareaindustrial,1), 3);
    c1 = Mecambiareaindustrial(indices(1), :);
    c2 = Mecambiareaindustrial(indices(2), :);
    c3 = Mecambiareaindustrial(indices(3), :);

    t_min = min(t);
    t_max = max(t);
    t_margin = (t_max - t_min) * 0.05;
    x_axis = [t_min - t_margin, t_max + t_margin];

    %% Crear una sola figura con subplots
    figure('Color', 'w', 'Units', 'Centimeters', 'Position', [2, 2, 18, 20])

    % Gráfica x(t)
    subplot(3,1,1)
    plot(t, x, 'x', 'LineWidth', 1.2, 'Color', c1, 'DisplayName', '$x_1(t)$')
    xlabel('$t$ [days]', 'Interpreter', 'latex', 'FontName', 'Times New 
Roman')
    ylabel('$x(t)$', 'Interpreter', 'latex', 'FontName', 'Times New Roman')
    set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
    xlim(x_axis)
    ylim padded
    grid off
    legend('Interpreter', 'latex', 'Location', 'best')

    % Gráfica y(t)
    subplot(3,1,2)
    plot(t, y, 'x', 'LineWidth', 1.2, 'Color', c2, 'DisplayName', '$y_1(t)$')
```

```
    xlabel('$t$ [days]', 'Interpreter', 'latex', 'FontName', 'Times New
Roman')
    ylabel('$y(t)$', 'Interpreter', 'latex', 'FontName', 'Times New Roman')
    set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
    xlim(x_axis)
    ylim padded
    grid off
    legend('Interpreter', 'latex', 'Location', 'best')

    % Gráfica z(t)
    subplot(3,1,3)
    plot(t, z, 'x', 'LineWidth', 1.2, 'Color', c3, 'DisplayName', '$z_1(t)$')
    xlabel('$t$ [days]', 'Interpreter', 'latex', 'FontName', 'Times New
Roman')
    ylabel('$z(t)$', 'Interpreter', 'latex', 'FontName', 'Times New Roman')
    set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
    xlim(x_axis)
    ylim padded
    grid off
    legend('Interpreter', 'latex', 'Location', 'best')

    set(gcf, 'PaperUnits', 'centimeters', 'PaperPosition', [0 0 18 20]);
    print(gcf, 'datos_txyz.pdf', '-dpdf', '-r300');
end
```

Smooth data plot

```
function plotEDOs_fit(t, x, y, z)
    Mecambiareaindustrial = [
        0.60, 0.00, 0.00;
        0.75, 1.00, 0.00;
        0.00, 0.75, 1.00;
        0.50, 0.00, 0.75;
        1.00, 0.50, 0.00;
        0.40, 0.20, 0.10;
        1.00, 0.00, 0.80;
        1.00, 0.50, 0.75;
        0.00, 0.85, 0.65;
        0.30, 0.45, 0.60
    ];

    rng('shuffle');
    indices = randperm(size(Mecambiareaindustrial,1), 3);
    c1 = Mecambiareaindustrial(indices(1), :);
    c2 = Mecambiareaindustrial(indices(2), :);
    c3 = Mecambiareaindustrial(indices(3), :);

    figure('Color', 'w', 'Units', 'Centimeters', 'Position', [2, 10, 18, 20])
```

```
    % x(t)
    subplot(3,1,1)
    set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
    hold on; box on;
    plot(t, x, 'x', 'LineWidth', 1.5, 'Color', c1, 'DisplayName', '$x(t)$')
    xlabel('$t$ $[days]$', 'Interpreter', 'latex')
    ylabel('$x(t)$', 'Interpreter', 'latex')
    xticks(0:10:70); yticks(0:1:4)
    xlim([0 70]); ylim([0 4])
    legend('Interpreter', 'latex', 'Location', 'best')

    % y(t)
    subplot(3,1,2)
    set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
    hold on; box on;
    plot(t, y, 'x', 'LineWidth', 1.5, 'Color', c2, 'DisplayName', '$y(t)$')
    xlabel('$t$ $[days]$', 'Interpreter', 'latex')
    ylabel('$y(t)$', 'Interpreter', 'latex')
    xlim([min(t), max(t)])
    ylim([0 22])
    legend('Interpreter', 'latex', 'Location', 'best')

    % z(t)
    subplot(3,1,3)
    set(gca, 'FontName', 'Times New Roman', 'FontSize', 10)
    hold on; box on;
    plot(t, z, 'x', 'LineWidth', 1.5, 'Color', c3, 'DisplayName', '$z(t)$')
    xlabel('$t$ $[days]$', 'Interpreter', 'latex')
    ylabel('$z(t)$', 'Interpreter', 'latex')
    xticks(0:10:70); yticks(0:2:8)
    xlim([0 70]); ylim([0 8])
    legend('Interpreter', 'latex', 'Location', 'best')

    % Guardar en PDF si se desea
    % print(gcf, 'EDO_plot.pdf', '-dpdf', '-bestfit')
end
```

Nonlinear regression algorithm

```
function [mdl, xa, ya, za] = Varied(to, xo, yo, zo, p0)
    x0 = xo(1); y0 = yo(1); z0 = zo(1);
    to3 = [to; to; to];
    fo = [xo; yo; zo];

    function fi = model(p, tvec)
        dt = 0.01;
        tmat = reshape(tvec, [], 3);
        t = tmat(:,1);
```

```
        time = (0:dt:max(t))';
        n = numel(time);

        x = zeros(n,1); x(1) = x0;
        y = zeros(n,1); y(1) = y0;
        z = zeros(n,1); z(1) = z0;

        for i = 1:n-1
            [dx, dy, dz] = f(x(i), y(i), z(i), p);
            xp = x(i) + dx*dt;
            yp = y(i) + dy*dt;
            zp = z(i) + dz*dt;

            xp = max(xp, 1e-10);
            yp = max(yp, 1e-10);
            zp = max(zp, 1e-10);

            [dxp, dyp, dzp] = f(xp, yp, zp, p);

            x(i+1) = x(i) + (dx + dxp)*dt/2;
            y(i+1) = y(i) + (dy + dyp)*dt/2;
            z(i+1) = z(i) + (dz + dzp)*dt/2;

            if any(~isfinite([x(i+1), y(i+1), z(i+1)]))
                error('Valor no finito en t=%.4f', time(i+1));
            end
        end

        xi = interp1(time, x, t, 'linear', NaN);
        yi = interp1(time, y, t, 'linear', NaN);
        zi = interp1(time, z, t, 'linear', NaN);

        if any(isnan([xi; yi; zi]))
            error('Interpolación devolvió NaN');
        end

        fi = [xi; yi; zi];
    end

   function [dx, dy, dz] = f(x, y, z, p)
    dx = p(1) * x .* y + p(2) * x .* y .* z;
    dy = p(3) * y - p(4) * x .* y;
    dz = p(5) * z + p(6) * y .* z;
   end
    mdl = fitnlm(to3, fo, @model, p0);

    fa = mdl.Fitted;
    fn = reshape(fa, [], 3);
    xa = fn(:,1);
    ya = fn(:,2);
```

```
    za = fn(:,3);

    coef = mdl.Coefficients;
    Estimate = coef.Estimate;
    SE = coef.SE;
    pValue = coef.pValue;
    CI95 = coefCI(mdl, 0.05);
    dof = mdl.DFE;
    tval = tinv(0.975, dof);
    MoE = SE * tval;

    Params = ["p1"; "p2"; "p3"; "p4"; "p5"; "p6"];
    Results = table(Params, Estimate, SE, MoE, CI95, pValue, ...
        'VariableNames', {'Param','Estimate','SE','MoE','CI95','pValue'});

    fprintf('\nSample size (n): %d', numel(xo));
    fprintf('\nParameters estimated: %d', numel(p0));
    fprintf('\nDegrees of freedom: %d', dof);
    fprintf('\nR-squared: %.4f', mdl.Rsquared.Ordinary);
    fprintf('\nCorrected AIC: %.4f\n\n', mdl.ModelCriterion.AICc);
    disp(Results);

    X_plot = [xo, xa];
    Y_plot = [yo, ya];
    Z_plot = [zo, za];
    plotXYZResults(to, X_plot, Y_plot, Z_plot);
end
```

Nonlinear regression plot

```
function plotXYZResults(t, X, Y, Z)
    Mecambiareaindustrial = [
        0.60, 0.00, 0.00;
        0.75, 1.00, 0.00;
        0.00, 0.75, 1.00;
        0.50, 0.00, 0.75;
        1.00, 0.50, 0.00;
        0.40, 0.20, 0.10;
        1.00, 0.00, 0.80;
        1.00, 0.85, 0.00;
        0.00, 0.85, 0.65;
        0.30, 0.45, 0.60
    ];
    rng('shuffle');
    indices = randperm(size(Mecambiareaindustrial,1), 6);
    c1_obs = Mecambiareaindustrial(indices(1), :);
    c1_mod = Mecambiareaindustrial(indices(2), :);
    c2_obs = Mecambiareaindustrial(indices(3), :);
    c2_mod = Mecambiareaindustrial(indices(4), :);
    c3_obs = Mecambiareaindustrial(indices(5), :);
```

```
    c3_mod = Mecambiareaindustrial(indices(6), :);

    set(figure(),'Color','w')
    set(gcf,'Units','Centimeters','Position',[2,2,20,15])
    set(gca,'FontName','Times New Roman')
    fontsize(12,'points')

    %% Subplot X
    subplot(3,1,1)
    hold on; box on;
    plot(t, X(:,1), 'x', 'Color', c1_obs, 'MarkerSize', 9)
    plot(t, X(:,2), '-', 'Color', c1_mod, 'LineWidth', 1.5)
    xlabel('t [days]','Interpreter','latex')
    ylabel('$x(t)$','Interpreter','latex')
    legend({'$xo{(t)}$','$xa{(t)}$'}, 'Interpreter','latex', 'Location',
'Best')
    xticks(0:10:70); yticks(0:1:4)
    xlim([0 70]); ylim([0 4])

    %% Subplot Y
    subplot(3,1,2)
    hold on; box on;
    plot(t, Y(:,1), 'x', 'Color', c2_obs, 'MarkerSize', 9)
    plot(t, Y(:,2), '-', 'Color', c2_mod, 'LineWidth', 1.5)
    xlabel('t [days]','Interpreter','latex')
    ylabel('$y(t)$','Interpreter','latex')
    legend({'$yo{(t)}$','$ya{(t)}$'}, 'Interpreter','latex', 'Location',
'Best')
    xticks(0:10:70); yticks(0:5:25)
    xlim([0 70]); ylim([0 25])

    %% Subplot Z
    subplot(3,1,3)
    hold on; box on;
    plot(t, Z(:,1), 'x', 'Color', c3_obs, 'MarkerSize', 9)
    plot(t, Z(:,2), '-', 'Color', c3_mod, 'LineWidth', 1.5)
    xlabel('t [days]','Interpreter','latex')
    ylabel('$z(t)$','Interpreter','latex')
    legend({'$zo{(t)}$','$za{(t)}$'}, 'Interpreter','latex', 'Location',
'Best')
    xticks(0:10:70); yticks(0:2:8)
    xlim([0 70]); ylim([0 8])
end
```

Data Normalized Plot

```
function graficarSuavizado(T)
    figure('Color','w')
    set(gcf,'Units','Centimeters','Position',[1,1,15,18])
```

```
    Mecambiareaindustrial = [
        0.60, 0.00, 0.00;
        0.75, 1.00, 0.00;
        0.00, 0.75, 1.00;
        0.50, 0.00, 0.75;
        1.00, 0.50, 0.00;
        0.40, 0.20, 0.10;
        1.00, 0.00, 0.80;
        1.00, 0.85, 0.00;
        0.00, 0.85, 0.65;
        0.30, 0.45, 0.60
    ];

    rng('shuffle');
    indices = randperm(size(Mecambiareaindustrial,1), 4);
    c1 = Mecambiareaindustrial(indices(1), :);
    c2 = Mecambiareaindustrial(indices(2), :);
    c3 = Mecambiareaindustrial(indices(3), :);
    c4 = Mecambiareaindustrial(indices(4), :);

    t = T.Tiempo;
    x = (T.X_suave - min(T.X_suave)) / (max(T.X_suave) - min(T.X_suave));
    y = (T.Y_suave - min(T.Y_suave)) / (max(T.Y_suave) - min(T.Y_suave));
    z = (T.Z_suave - min(T.Z_suave)) / (max(T.Z_suave) - min(T.Z_suave));
    pos1 = [0.13 0.72 0.8 0.22]; pos2 = [0.13 0.42 0.8 0.22]; pos3 = [0.13
0.12 0.8 0.22];

    axes('Position', pos1)
    set(gca,'FontName','Times New Roman'); fontsize(10,'points');
    hold on; box on;
    plot(t, x, 'x', 'LineWidth', 1, 'Color', c1)
    xlabel('$t [days]$', 'Interpreter', 'latex')
    ylabel('$x(t)$', 'Interpreter', 'latex')
    xlim([min(t), max(t)]); ylim([0 1.25]); yticks(0:0.2:1.25)

    axes('Position', pos2)
    set(gca,'FontName','Times New Roman'); fontsize(10,'points');
    hold on; box on;
    plot(t, y, 'x', 'LineWidth', 1, 'Color', c2)
    xlabel('$t [days]$', 'Interpreter', 'latex')
    ylabel('$y(t)$', 'Interpreter', 'latex')
    xlim([min(t), max(t)]); ylim([0 1.25]); yticks(0:0.2:1.25)

    axes('Position', pos3)
    set(gca,'FontName','Times New Roman'); fontsize(10,'points');
    hold on; box on;
    plot(t, z, 'x', 'LineWidth', 1, 'Color', c3)
    xlabel('$t [days]$', 'Interpreter', 'latex')
    ylabel('$z(t)$', 'Interpreter', 'latex')
    xlim([min(t), max(t)]); ylim([0 1.25]); yticks(0:0.2:1.25)
```

```
end
```

2t prediction algorithm

```
function [t, x, y, z] = Predict(x0, y0, z0, dt, tend, p)
    t = (0:dt:tend)';
    N = length(t);
    x = zeros(N, 1); x(1) = x0;
    y = zeros(N, 1); y(1) = y0;
    z = zeros(N, 1); z(1) = z0;

    for i = 1:N-1
        [fx, fy, fz] = f(x(i), y(i), z(i), p);
        xn = x(i) + fx * dt;
        yn = y(i) + fy * dt;
        zn = z(i) + fz * dt;
        [fxn, fyn, fzn] = f(xn, yn, zn, p);

        x(i+1) = x(i) + (fx + fxn) * dt / 2;
        y(i+1) = y(i) + (fy + fyn) * dt / 2;
        z(i+1) = z(i) + (fz + fzn) * dt / 2;
    end
end

 function [dx, dy, dz] = f(x, y, z, p)
    dx = p(1) * x .* y + p(2) * x .* y .* z;
    dy = p(3) * y - p(4) * x .* y;
    dz = p(5) * z + p(6) * y .* z;
 end
```

2t prediction plot

```
function plotXYZPredict(to, xo, yo, zo, tp, xp, yp, zp)
    Mecambiareaindustrial = [
        0.60, 0.00, 0.00;
        0.75, 1.00, 0.00;
        0.00, 0.75, 1.00;
        0.50, 0.00, 0.75;
        1.00, 0.50, 0.00;
        0.40, 0.20, 0.10;
        1.00, 0.00, 0.80;
        1.00, 0.85, 0.00;
        0.00, 0.85, 0.65;
        0.30, 0.45, 0.60
    ];

    rng('shuffle');
    indices = randperm(size(Mecambiareaindustrial,1), 6);
    c1_obs = Mecambiareaindustrial(indices(1), :);
```

```
    c1_mod = Mecambiareaindustrial(indices(2), :);
    c2_obs = Mecambiareaindustrial(indices(3), :);
    c2_mod = Mecambiareaindustrial(indices(4), :);
    c3_obs = Mecambiareaindustrial(indices(5), :);
    c3_mod = Mecambiareaindustrial(indices(6), :);

    figure(); set(gcf,'Color','w')
    set(gcf,'Units','Centimeters','Position',[2,2,20,15])
    set(gca,'FontName','Times New Roman')
    fontsize(12,'points')

    % Subplot X
    subplot(3,1,1); hold on; box on;
    plot(to, xo, 'x', 'Color', c1_obs)
    plot(tp, xp, '-', 'Color', c1_mod)
    xlabel('t [days]','Interpreter','latex')
    ylabel('$x(t)$','Interpreter','latex')
    legend({'$xo{(t)}$','$xa{(t)}$'}, 'Interpreter','latex', 'Location',
'Best')
    xticks(0:10:140); xlim([0 140]);
     ylim([0 4]);yticks(0:1:4);

    % Subplot Y
    subplot(3,1,2); hold on; box on;
    plot(to, yo, 'x', 'Color', c2_obs)
    plot(tp, yp, '-', 'Color', c2_mod)
    xlabel('t [days]','Interpreter','latex')
    ylabel('$y(t)$','Interpreter','latex')
    legend({'$yo{(t)}$','$ya{(t)}$'}, 'Interpreter','latex', 'Location',
'Best')
    xticks(0:10:140); xlim([0 140]);
   ylim([0 25]); yticks(0:5:25);

    % Subplot Z
    subplot(3,1,3); hold on; box on;
    plot(to, zo, 'x', 'Color', c3_obs)
    plot(tp, zp, '-', 'Color', c3_mod)
    xlabel('t [days]','Interpreter','latex')
    ylabel('$z(t)$','Interpreter','latex')
    legend({'$zo{(t)}$','$za{(t)}$'}, 'Interpreter','latex', 'Location',
'Best')
    xticks(0:10:140); xlim([0 140]);
    ylim([0 8]); yticks(0:2:8);
end
```