Nathan Robinson

System Modeling and Simulation

Dr. Cooper

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Assignment 3: Bifurcation

Question 1

* Consider the following discrete-time dynamical system:

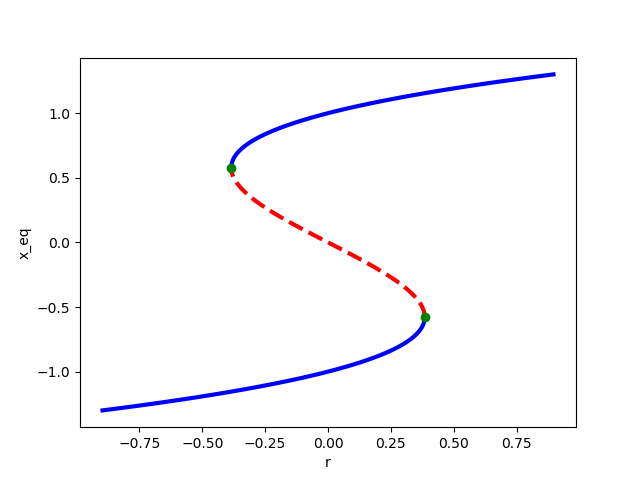
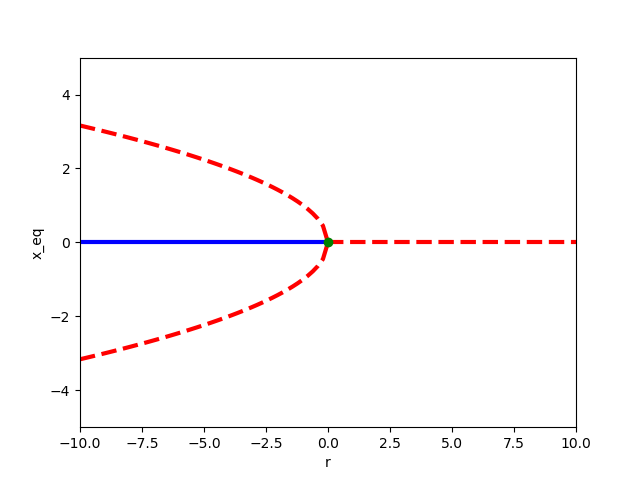
This equation has xeq = 0 as an equilibrium point.  Obtain a value of *a* at which the equilibrium point undergoes a first period-doubling bifurcation.

Given discrete time dynamical system a pitchfork bifurcation is expected. A two period fixed point is a point that repeats itself after two periods:

Equilibrium point xeq = 0 means that the equilibrium point will first double the period at OR and

Substitute and we get and => 1 =>

Therefore *a* = 0 (from ) is where the first period-doubling bifurcation occurs.



Question 2

We first introduced the logistic map earlier in the semester: .

* Conduct a bifurcation analysis of this model to find the critical thresholds of *r* at which bifurcation occurs

Fixed points: x\* = 0 and x\* = (1-r) / r

Bifurcation value: r =1, thus a transcritical bifurcation

To check stability let x` = f(x) so that f `(x\*) = 1 - 2 r x\* - r

f ` (0) = 1- r and f ` ((1-r) / r) = 1 -2 r ((1-r) / r ) = r – 1

x\* = 0 is stable for r > 1 and x\* = (1-r) / r is stable for r < 1

* Study the stability of each equilibrium point in each parameter range and summarize the results in a table

|  |  |  |
| --- | --- | --- |
| Equilibrium point | Parameter range | Stability |
| x\* = 0 | r > 1 | stable |
| x\* = 0 | r = 1 | stable |
| x\* = (1-r) / r | 0 < r < 1 | stable |
| none | r < 0 | unstable |

* Simulate the model with several selected values of *r* to confirm the results of the analysis
* Plot a bifurcation diagram of this model to 0<r<4.

