Image Compression using Principal Component Analysis (PCA)

Submitted By :-

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Course : **MSc Computer Science**

Subject : **Machine Learning**

➢ Introduction

- Principal Component Analysis is a popular dimensionality reduction technique which condenses information from a large set of variables into fewer variables by applying some sort of transformation onto them.

- The transformation is applied in such a way that linearly correlated variables get transformed into uncorrelated variables.

- Correlation tells us that there is a redundancy of information and if this redundancy can be reduced, then information can be compressed.

- The direction of projection is determined using eigenvalues and eigenvectors.

- The first few transformed features termed as Principal Components (PCs) are rich in information (explains most of the variance), whereas the last features contain mostly noise with negligible information in them.

- We retain the first few principal components, thus reducing the number of variables significantly with minimal loss of information.

➢ Data Exploration

- **Original Image**

Gray Scale Image

- **Size of the Image**

Image Shape: (1280, 960)

Loaded the original image (left picture) in python and then converted it into gray scale (right image) to do Principal Component Analysis.

- **Image after retaining 5% of the Principal Components**

```
Image shape:  (1280, 960)
nump_pc:  48
Shape of Z (centered data matrix * W): (1280, 48)
Reduction:  95.0
```

- ✓ After retaining only 5% of the PCs, the image obtained is a bit blurred. Facial features are not so clearly visible and the background is also a bit hazy and not clear. A shadow is observed under the neck.

- **Image after retaining 10% of the Principal Components**

```
Image shape:  (1280, 960)
nump_pc:  96
Shape of Z (centered data matrix * W): (1280, 96)
Reduction:  90.0
```

- ✓ When we retained 10% of the PCs, the image gets somewhat clear as compared to when we retained 5% of the PCs. Some facial features can be seen a bit clearly now, the shadow under the neck has gone. Also the background of the picture is not that hazy now.

- **Image after retaining 20% of the Principal Components**

```
Image shape:  (1280, 960)
nump_pc:  192
Shape of Z (centered data matrix * W): (1280, 192)
Reduction:  80.0
```

  - ✓ When we retained 20% of the PCs, the image gets clearer as compared to when we retained only 10% of the PCs. Facial features are very much clear now and the background of the image can easily be distinguished.

- **Image after retaining 30% of the Principal Components**

```
Image shape:  (1280, 960)
nump_pc:  288
Shape of Z (centered data matrix * W): (1280, 288)
Reduction:  70.0
```

  - ✓ When we retained 30% of the PCs, the image is clearly visible, facial features like eyes, structure of lips can be easily be observed now and the background is not intermixing with the person. This image tries to retain the information (minute details of the image) very well.
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- **Image after retaining 40% of the Principal Components**

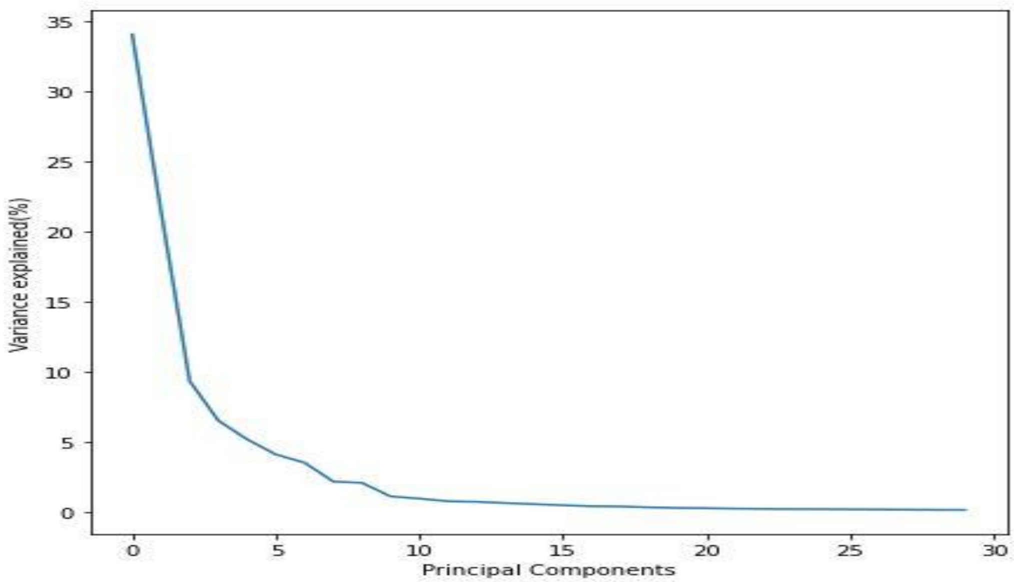
```
Image shape:  (1280, 960)
nump_pc:  384
Shape of Z (centered data matrix * W): (1280, 384)
Reduction:  60.0
```

- ✓ When we retained 40% of the PCs, the image is very much clear and gets a bit brighter. Formation of eyes, nose, and ears is smooth. No intermixing of the pixels is observed.

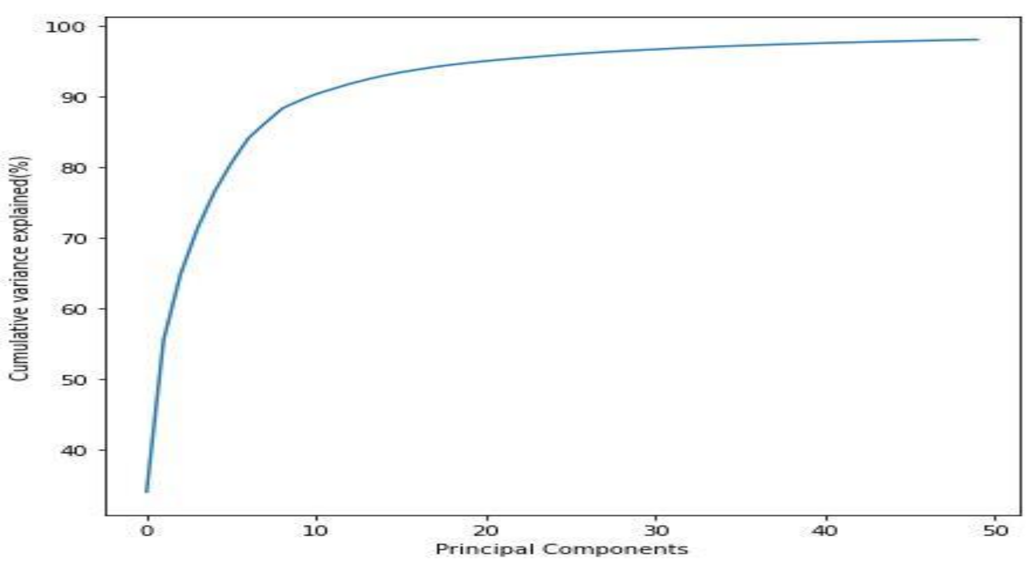
- ✓ After looking at all the above images, it can be clearly observed that retaining 30% of the principal components of the image can be a good choice since it preserves the maximum variance with minimum loss of the information (minimal loss in reduction of the minute details of the image).

❖ Scree plots

1) Plot between Principal Components(x axis) and variance explained (in %) (y axis)



2) Plot between Principal Components(x axis) and cumulative variance explained (in %) (y axis)



➢ Observations

✓ Shape of the original image is (1280, 960).

✓ In the below table :
First column represents the principal components retained whereas the second column represents the size of the Centered data matrix * W [Matrix used for Reconstructing the Image].

Principal Components Retained	Image size (Centered Data Matrix * W)
1) 5%	(1280,48)
2) 10%	(1280,96)
3) 20%	(1280,192)
4) 30%	(1280,288)
5) 40%	(1280,384)

✓ Image obtained after reconstruction in the case when only 5% principal components were retained is blur and the minute details of the image are lost.

✓ Image obtained after reconstruction in the case when 30% principal components were retained defines the features of the picture well (minute details of the image).

- ✓ Eigen values give us the order of significance of eigenvectors. Eigenvector with the largest eigenvalue is the most significant PC and so on.
- ✓ After looking at the first plot, we can observe a distinct elbow point at around 3 value in the X axis (Number of Principal Components).
- ✓ Cumulative proportion helps to determine the number of principal components to use. We can see a steep increase in explained variance in the beginning. For 3 Principal Components, almost 70% of the cumulative variance is explained. So, taking 3 Principal components is an excellent choice.

➢ Second Picture

- **Original Image** **Gray Scale Image**

- **Size of the Image**

  Image Shape: (394,446)

Loaded the original image (left picture) in python and then converted it into gray scale (right image) to do Principal Component Analysis.

- **Image after retaining 5% of the Principal Components**

```
Image shape:  (394, 446)
nump_pc:  22
Shape of Z (centered data matrix * W): (394, 22)
Reduction:  95.06726457399103
```

  - ✓ After retaining only 5% of the PCs, the image obtained is completely blurred. No facial features are clearly visible and the background is also very hazy. The image is not at all clear and has lost even its basic features.

- **Image after retaining 10% of the Principal Components**

```
Image shape:  (394, 446)
nump_pc:  44
Shape of Z (centered data matrix * W): (394, 44)
Reduction:  90.13452914798206
```

  - ✓ When we retained 10% of the PCs, the image gets somewhat clear as compared to when we retained 5% of the PCs. Some facial features can be seen although not very clearly. The background of the picture is not that hazy and the sofa can be distinguished now.

- **Image after retaining 20% of the Principal Components**

```
Image shape:  (394, 446)
nump_pc:  89
Shape of Z (centered data matrix * W): (394, 89)
Reduction:  80.04484304932735
```

- ✓ When we retained 20% of the PCs, the image gets clearer as compared to when we retained only 10% of the PCs. Facial features are clearly observable although the right side of the ear is still not that clear. Background can easily be distinguished.

- **Image after retaining 30% of the Principal Components**

```
Image shape:  (394, 446)
nump_pc:  133
Shape of Z (centered data matrix * W): (394, 133)
Reduction:  70.17937219730942
```

- ✓ When we retained 30% of the PCs, the image is almost clearly visible, facial features can be observed now and the background is easily distinguishable. The picture is not that hazy now.

❖ **Image after retaining 40% of the Principal Components**

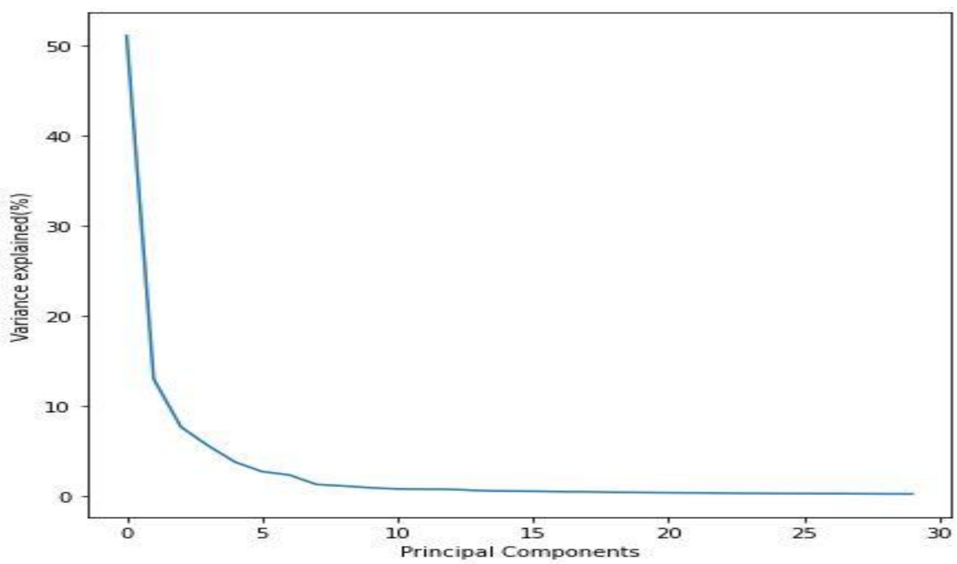
```
Image shape:  (394, 446)
nump_pc:  178
Shape of Z (centered data matrix * W): (394, 178)
Reduction:  60.0896860986547
```

- ✓ When we retained 40% of the PCs, the image is very much clear, facial features like eyes, nose, and ears can easily be observed now. The sofa, the person (along with the facial features and expressions), the wall, all can be clearly seen now and this image is very well explains even the minute details.

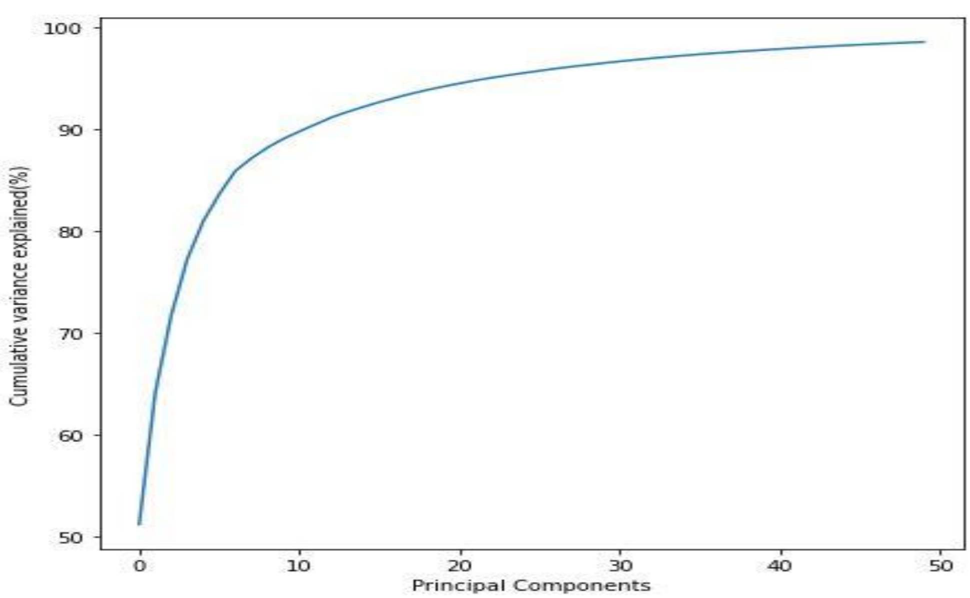
- ✓ After looking at all the above images, it can be clearly observed that retaining 40% of the principal components of the image can be a good choice since it preserves the maximum variance with minimum loss of the information (minimal loss in reduction of the minute details of the image).

❖ **Scree plots**

1) Plot between Principal Components (x axis) and variance explained (in %) (y axis)



2) Plot between Principal Components (x axis) and cumulative variance explained (in %) (y axis)



➢ Observations

✓ Shape of the original image is (394,446).

✓ In the below table :
First column represents the principal components retained whereas the second column represents the size of the Centered data matrix * W [Matrix used for Reconstructing the Image].

Principal Components Retained	**Image size (Centered Data Matrix * W)**
1) 5%	(394,22)
2)10%	(394,44)
3) 20%	(394,89)
4) 30%	(394,133)
5) 40%	(394,178)

✓ Image obtained after reconstruction in the case when only 5% principal components were retained was very blur and the features of the original picture were not at all retained.

✓ Image obtained after reconstruction in the case when 40% principal components define the features of the picture well.

✓ After looking at the first plot, we can observe a distinct elbow point at around 4 value in the X axis (Number of Principal Components).

- ✓ Cumulative proportion helps to determine the number of principal components to use. We can see a steep increase in explained variance in the beginning. For 4 Principal Components, almost 80% of the cumulative variance is explained whereas the first 10 components explain 88% variance. So, taking 4 Principal components will be an excellent choice.

➢ Conclusion

- ✓ It can be concluded that the PCA has done a great job on our Image data in terms of compressibility and information retention.