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NAME : PREETHI SUBRAMANIAN

STUDENT ID: 1002059233

ASSIGNMENT ID: Homework #2

Q1)a) dataset = {5, 7, 2, 3, 1, 2, 9, 5} ; no of elements = 8.

Mean $E(x) = \sum x_i \cdot P(x_i)$

$= 5(2/8) + 7(1/8) + 2(2/8) + 3(1/8) + 1(1/8) + 9(1/8)$

$= \dfrac{10 + 7 + 4 + 3 + 1 + 9}{8}$

$= \cancel{34/8}\ 34/8$

∴ Mean $E(x) = 4.25$

Median :-

rearranging the dataset in ascending order.

dataset = {1, 2, 2, 3, 5, 5, 7, 9}

since the no of elements in dataset are even in number

It is mean of 2 numbers $= 5+3/2 = 8/2 = 4$.

∴ Median = 4.

Variance:

Variance $= E(x^2) - (E(x))^2$

$E(x^2) = 1/8\,(25 + 49 + 4 + 9 + 1 + 4 + 81 + 25)$

$= \frac{1}{8}(198)$

$= 24.75$

Variance $= 24.75 - (4.25)^2$

$= 24.75 - 18.0625$

Variance $= 6.6875$

1b) dataset = $\{4, 2, 5, 1, 2, 1, 4, 1, 4, 5, 8\}$

no of elements = 11.

Mean $E(x) = \sum x_i \cdot P(x_i)$

$= 4(3/11) + 2(2/11) + 5(2/11) + 1(3/11) + 8(1/11)$

$= \dfrac{12 + 4 + 10 + 3 + 8}{11}$

$= 37/11$

$\therefore$ Mean $E(x) = 3.3636$.

Median :-

rearranging the dataset in ascending order

dataset = $\{1, 1, 1, 2, 2, 4, 4, 4, 5, 5, 8\}$

since the no of elements in the dataset are odd in number, median is the middle value which is 4.

$\therefore$ Median = 4.

Variance :-

Variance $= E(x^2) - (E(x))^2$

$E(x^2) = 1/11\,[16 + 4 + 25 + 1 + 4 + 1 + 16 + 1 + 16 + 25 + 64]$

$= 1/11\,[173]$

$= 15.7272$

variance $= 15.7272 - (3.3636)^2$

$= 15.7272 - 11.3138$

$\therefore$ variance $= 4.4134$

1c) dataset = { 3, 5, -8, 0, 4, 5, 2, 3, -1 }

no of elements = 9.

Mean $E(X) = \sum x_i \cdot P(x_i)$

$= 3(2/9) + 5(2/9) + -8(1/9) + 0(1/9) + 4(1/9) + 2(1/9) + -1(1/9)$

$= \dfrac{6 + 10 + (-8) + 0 + 4 + 2 + (-1)}{9}$

$= 13/9$

$\therefore$ Mean $E(X) = 1.4444$

Median:

Arranging the dataset in ascending / sorted order.

dataset = { -8, -1, 0, 2, 3, 3, 4, 5, 5 }

since the no of elements in the dataset are odd in number, the median is the middle value which is 3

$\therefore$ Median = 3.

Variance :-

variance $= E(x^2) - (E(x))^2$

$E(x^2) = \dfrac{1}{9}[9 + 25 + 64 + 0 + 16 + 25 + 4 + 9 + 1]$

$= 1/9[153]$

$= 17$

Variance $= 17 - (1.4444)^2$

$= 17 - 2.0862$

$\therefore$ Variance $= 14.9138$

Q 2) Effects of some Alzheimer disease and probability of each drug being effective is 15%. which is 0.15.

1) Probability that the first effective drug is the fourth one we examine

since this is geometric distribution problem.

P(X) being the probability that the K^{th} trial is effective

$$P(X=k) = (1-p)^{k-1} * p.$$

$$P(X=4) = (1-p)^{3} * p \quad \text{where } p = 15\% \text{ or } 0.15$$

$$\therefore P(X=4) = (1-0.15)^{3} * 0.15$$

$$= 0.614125 * 0.15$$

$$= 0.092$$

$\therefore$ Probability that the first effective drug is the 4^{th} one we examine is $P(X=4) = 0.092$

2) Probability that the first effective drug in the first 3 experiments

= P (first trial experiment) + P (second trial experiment) + P (third trial experiment)

$$= P(K=1) + P(K=2) + P(K=3)$$

$$= \left[(1-0.15)^{0} * 0.15\right] + \left[(1-0.15)^{1} * 0.15\right] + \left[(1-0.15)^{2} * 0.15\right]$$

$$= (0.15) + (0.1275) + (0.1008375)$$

$$= 0.385875$$

∴ Probability that the first effective drug in first 3 experiments = 0.385875.

3) This is Geometric distribution.

Q3) In apple store, on average that out of 20 customers, 16 of them wants to buy newest iphone.

$p = 16/20 = 4/5 = 0.8$.

$q = 1-p = 1-0.8 = 0.2$

no of customers randomly chosen $n = 50$.

1) Probability that more than 46 customers buy the newest iphone

$P(k > 46) = P(k=47) + P(k=48) + P(k=49) + P(k=50)$

$P(k) = \binom{n}{k} p^k . q^{n-k}$

where $p = 0.8$, $q = 0.2$, $n = 50$

∴ $P(k=47) = b(47, 50, 0.8)$

$= \binom{50}{47} (0.8)^{47} (0.2)^3$

$= \dfrac{50!}{47! * 3!} * (0.8)^{47} * (0.2)^3$

$P(k=47) = 0.0043$

$P(k=48) = b(48, 50, 0.8)$

$= \dfrac{50!}{48! * 2!} * (0.8)^{48} * (0.2)^2$

$= 0.0010927$

$P(K=49) = b(49, 50, 0.8)$

$= \frac{50!}{49! * 1!} * (0.8)^{49} * (0.2)^{1}$

$= 0.0001784$

$P(K=50) = b(50, 50, 0.8)$

$= \frac{50!}{50! \times 0!} \times (0.8)^{50} \times (0.2)^{0}$

$= 0.00001427$

∴ Probability that more than 46 customers buy the newest iphone is $= 0.0043 + 0.0010927 + 0.0001784 + 0.00001427$

$\underline{P(K>46) = 0.005585}$

2) Probability that exactly 40 customers wants to buy the newest iphone

$P(K=40) = \binom{n}{k} p^k \cdot q^{n-k}$

where $p = 0.8$, $q = 0.2$, $k = 40$, $n = 50$

$P(K=40) = b(40, 50, 0.8)$

$= \frac{50!}{40! * 10!} * (0.8)^{40} * (0.2)^{10}$

$= 0.1398$

3) Variance on the number of purchased iphone

$$\text{Variance} = n \cdot p(1-p)$$
$$= 50 \times 0.8(1-0.8)$$
$$= 8$$

4) This is Binomial distribution

Q4)

Thunderstorm at an average rate of 5 per month during fall. $\boxed{(\lambda)=5}$

~~P(k events in interval)~~

Let x be the no. of thunderstorms per month.

1) Probability that during 2 months we see atmost 6 thunderstorms

$$= 2\lambda = 2\times 5 = 10.$$

P(seeing atmost 6 thunderstorms)

$$\Rightarrow P(X \le 6) = \sum_{x=0}^{6} \frac{e^{-10}\,10^{x}}{x!}$$

$P(X=0) = e^{-10}$

$P(X=1) = 10\times e^{-10}$

$P(X=2) = 50\times e^{-10}$

$P(X=3) = 166.67 \times e^{-10}$

$P(X=4) = 416.67 \times e^{-10}$

$P(X=5) = 833.33 \times e^{-10}$

$P(X=6) = 1388.89 \times e^{-10}$

$\therefore$ P(seeing atmost 6 Thunderstorms) =

$$e^{-10}(1+10+50+166.67+416.67+833.33+1388.89)$$

$$= 2866.56 \times e^{-10}$$

$$= 0.1301$$

2) Probability that during 3 months we see exactly 10 thunderstorms.

$\lambda = 3 \times 5 = 15.$

$$P(X=10) = \frac{\lambda^{10} e^{-15}}{10!}$$

$$= \frac{(15)^{10} e^{-15}}{10!}$$

$$= 0.0486$$

3) Expected value on the number of thunderstorms

$\lambda = 5$ per month.

Expected value is mean value in poisson distribution

Mean $= \mu = \lambda.$

$\therefore \mu = \lambda = 5$ as average rate of 5 per month.

4) This is Poisson Distribution

Q5) 4 sided die and die is not fair

Probability of getting (1) is equal to 3 times of probability of getting other values (2, 3, 4). Probabilities of other possibilities are equal.

1) Expected value of z ?

$P(2) = P(3) = P(4) \rightarrow (1)$

$P(1) = 3P(2) = 3P(3) = 3P(4) \rightarrow (2)$

~~Sub (2) in (1)~~

Total Probability $P(1) + P(2) + P(3) + P(4) = 1 \rightarrow (3)$

Sub 1, 2 in 3

$$3P(2) + P(2) + P(2) + P(2) = 1$$
$$6P(2) = 1$$
$$P(2) = 1/6$$

$\therefore$ $P(3) = P(4) = 1/6$.

$P(1) = 3P(2) = 3(1/6) = 1/2$

$\therefore$ Expected value of z =

$$= 1 \times (1/2) + 2 \times (1/6) + 3(1/6) + 4(1/6)$$
$$= \left(\frac{1}{2}\right) + \left(\frac{2+3+4}{6}\right)$$
$$= \frac{6+18}{12} = \frac{24}{12} = 2.$$

$\therefore$ Expected value of $z = 2$.

2)

Probability mass function for random variable z

$$PMF(z) = \begin{cases} 1/2 & ; z = 1 \\ 1/6 & ; z = 2, 3, 4 \\ 0 & ; \text{otherwise} \end{cases}$$

3) No, it is not a uniform distribution as n values do not have equal probability of $\frac{1}{n}$ values.
Here probability is different for 1 and other values 2, 3, 4. So, it is not a uniform distribution
Here $P(1) = 1/2$, $P(2, 3, 4) = 1/6$.
Probabilities of all n elements are not same.
So, it is not a uniform distribution.

Q6) Bag containing 6 red marbles, 8 blue marbels.
5 marbles are drawn randomly.
1) Probability that 3 of them are blue in color

$$P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

Given :- Total no of marbles = 8+6 = 14 marbles.
no of blue marble (K) = 8
no of blue marble to be drawn (k) = 3
no of red marbles (N-K) = 6
no of red marbles to be drawn (n-k) = 2.

$$= \frac{\binom{8}{3}\binom{6}{2}}{\binom{14}{5}}$$

$$= \frac{\left[\frac{8!}{5!\ 3!}\right] * \left[\frac{6!}{2! \times 4!}\right]}{\left[\frac{14!}{5! \times 9!}\right]} = \frac{\left(\frac{8 \times 7 \times \cancel{6}}{\cancel{6}}\right) * (15)}{2002}$$

$$= 0.41958$$

∴ Probability that 3 of them are blue marbles
$= 0.41958$.

2) Probability that at most 4 of them are blue marbles.

~~P~~ = ~~P~~

$P(k <= 4) = P(k=0) + P(k=1) + P(k=2) + P(k=3)$
$+ P(k=4)$

$P(k=0) = \frac{\binom{8}{0}\binom{6}{5}}{\binom{14}{5}} = 2.99 \times 10^{-3}$

$= 0.00299$

$P(k=1) = \frac{\binom{8}{1}\binom{6}{4}}{\binom{14}{5}} = \frac{8! \times 6! \times 5! \times 9!}{7! \times 2! \times 4! \times 14!}$

$= 0.0599$

$P(k=2) = \frac{\binom{8}{2}\binom{6}{3}}{\binom{14}{5}} = 0.279$

$P(k=3) = \frac{\binom{8}{3}\binom{6}{2}}{\binom{14}{5}} = 0.419$

$P(k=4) = \frac{\binom{8}{4}\binom{6}{1}}{\binom{14}{5}} = 0.209$

$\therefore$ Probability that atmost 4 marbles are blue

$= 0.00299 + 0.0599 + 0.279 + 0.419 + 0.209$

$= 0.96989$ //.

3) This is Hypergeometric distribution.