31/1/22

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ASSIGNMENT ID : Homework #1

1) $S = 50$ (Total number of students)

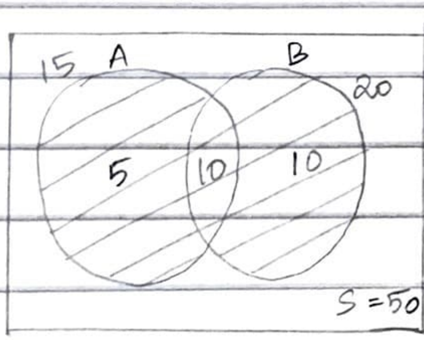
let students who took CSE 5301 be 'A'

let students who took CSE 5305 be 'B'.

let students who took both the classes be $A \cap B$.

1a) Students in either class $(A \cup B)$:

$$A \cup B = |A| + |B| - |A \cap B|$$
$$= 15 + 20 - 10$$
$$= 25.$$



$\therefore$ 25 students are there in either class.

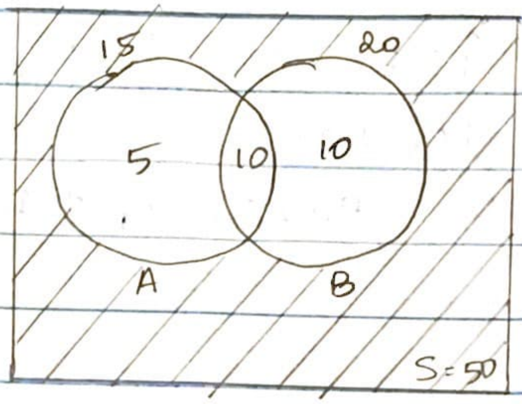
1b) Students in neither class :- $(S - A \cup B)$

Students in neither class = [Total no. of students] - [students in either class]

$= 50 - 25$ = (Sample space - $A \cup B$ (either class))

$= 25$

$\therefore$ Students in neither class are 25 in number

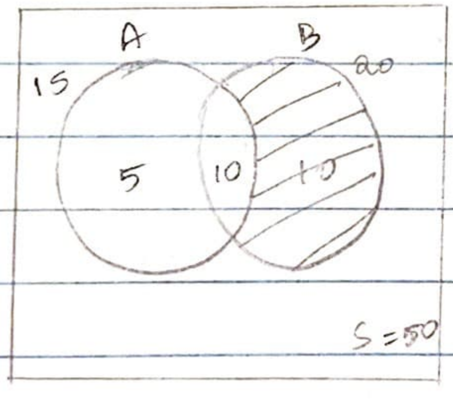


1c)

Probability of taking only CSE 5305

$= \dfrac{\text{No of students taking CSE 5305 only}}{\text{Total no of students}}$

$= \dfrac{10}{50} = 0.20$



2)

Fair coin tossed 5 times

possible outcome $= 2^5 = 32$ outcomes.

2a) Sample space for a fair coin tossed 5 times is =

~~S = {HHHHH, HHHHT, HHHTH, HHTHH,~~
~~HTHHH, THHHH, HHHTT, HHTTH, HTTHH,~~
~~TTHHH, THHHT, THHTH, THTHH,~~

2a) S = { HHHHH, HHHHT, HHHTH, HHHTT,
HHTHH, HHTHT, HHTTH, HHTTT, HTHHH,
HTHHT, HTHTH, HTHTT, HTTHH, HTTHT,
HTTTH, HTTTT, THHHH, THHHT, THHTH,
THHTT, THTHH, THTHT, THTTH, THTTT,
TTHHH, TTHHT, TTHTH, TTHTT, TTTHH,
TTTHT, TTTTH, TTTTT }

2b) Probability of exactly 2 heads & 3 tails

$$= \frac{\text{(No of times we get exactly 2 heads and 3 tails)}}{\text{Total no of outcomes}}$$

$$= \frac{10}{32} = \frac{5}{16} = 0.31$$

2c) Probability of getting atleast 4 heads

$$= \frac{\text{(No of times we get exactly 4 head + No of times we get more than 4 heads)}}{\text{Total no of outcomes}}$$

No of times we get exactly 4 heads = 5
No of times we get more than 4 heads = 1
∴ Probability of getting 4 heads

$$= \frac{6}{32}$$

$$= \frac{3}{16}$$

$$= 0.1875$$

3)
3a) A bag with infinite white and black balls we need to get 2 white balls or 2 black balls in consecutive draw.

Sample Space:
Since there are infinite balls in the bag and we only stop drawing when we draw two consecutive white balls or two consecutive black balls, then sample space will be infinitely large. The sample space will look like below

S = { WW, BB, WBB, BWW, WBBB, BWWW, WBBW, BWWB, BWBW, WBWB, WBBBB, WBWBW, BWBWBW, ... }

The number of draws will keep on increasing as long as we keep drawing alternate coloured balls

(Here W - White balls, B - Black balls)
Sample space will be infinite as the no of drawings increases as long as we keep drawing alternate coloured balls. (2 draw, 3 draw, 4 balls drawing, 5 balls drawing, ... till infinity)

3b) We have a bag containing 6 numbers (1 to 6).
We draw 3 balls from them and save the order.
Let S be sample space.
Drawing 3 balls from 6 (while order matters)
total no of possibilities =

$$P(6,3) = \frac{6!}{3!} = \frac{6\times5\times4\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}} = \underline{120}$$

$\therefore$ No of items in Sample Space = $\underline{120}$.

$\therefore$ Total no of possibilities of choosing 3 balls from total of 6 and save the order is $\underline{120}$.

Sample space (S) = { 123, 124, 125, 126,
132, 134, 135, 136,
142, 143, 145, 146,
152, 153, 154, 156,
162, 163, 164, 165,
.... }

These are the possibilities for picking the balls numbered first and considering other 2 balls. similarly the possibilities for other draws can be made.

4)

Let Probability of bedroom be P(bed)
Let Probability of kitchen be P(kitchen)
Let Probability of living room be P(living)
Let Probability of reading = 52 be P(R=52)

From the readings, it is obtained that

$P(R=52 \mid bed) = 0.08$

$P(R=52 \mid kitchen) = 0.14$

$P(R=52 \mid living) = 0.28$

Using Baye's theorem:

$$P(Bed \mid R=52) = \frac{P(R=52 \mid bed) * P(bed)}{P(R=52)}$$

To get P(R=52) we need to do marginalization.

$$P(R=52) = P(R=52 \mid bed) * P(bed) + P(R=52 \mid kitchen) * P(kitchen) + P(R=52 \mid living) * P(living)$$

~~P(R=52) = 0.08~~

Since we spend about equal time in each room

$P(bed) = P(kitchen) = P(living) = 1/3$.

$\therefore$

$P(R=52) = (0.08 \times 1/3) + (0.14 \times 1/3) + (0.28 \times 1/3)$

$P(R=52) = 0.166$.

$$P(bed \mid R=52) = \frac{P(R=52 \mid bed) * P(bed)}{P(R=52)}$$

$$= \frac{0.08 \times 1/3}{0.166}$$

$$= 0.16$$

$\therefore P(bed \mid R=52) = 0.16$

$$P(kitchen \mid R=52) = \frac{P(R=52 \mid kitchen) * P(kitchen)}{P(R=52)}$$

$$= \frac{0.14 \times 1/3}{0.166}$$

$$= 0.28$$

$\therefore P(kitchen \mid R=52) = 0.28$.

$$P(living \mid R=52) = \frac{P(R=52 \mid living) * P(living)}{P(R=52)}$$

$$= \frac{0.28 \times 1/3}{0.166}$$

$= 0.56$

$\therefore$ P(living | R = 52) = 0.56.

$\therefore$ Probability of each room given a reading of 52 is

P(bed | R = 52) = 0.16

P(kitchen | R = 52) = 0.28

P(living | R = 52) = 0.56.

5)

P(D) = Probability of getting disease = 0.0001

5a) P(K|D) = ~~If~~ Probability of you have disease, it will kill you = 1

P(T|D) = Probability of if you have the disease the test will be positive = 0.95.

P(~T|D) = Probability of if you have the disease and the test will be negative is = 0.05

P(T|~D) = Probability that you do not have disease and test might come positive = 0.01

P(~T|~D) = Probability you do not have disease and test is negative = 0.99

P(D|T) = ?

Using Bayes' theorem

$$P(D|T) = \frac{P(T|D) * P(D)}{P(T)}$$

$$= \frac{0.95 * 0.0001}{P(T)} \rightarrow (1)$$

using ~~marginalisation~~ marginalisation we get value of P(T)

$$P(T) = P(T|D) * P(D) + P(T|\sim D) * P(\sim D)$$
$$= (0.95 \times 0.0001) + (0.01 \times (1 - P(D)))$$
$$= (0.95 \times 0.0001) + (0.01 \times (1 - 0.0001))$$
$$= 0.000095 + 0.00999$$
$$= 0.010094 \rightarrow (2)$$

Substituting (2) in (1) eqn, we get

$$P(D|T) = \frac{0.95 \times 0.0001}{0.010094}$$

$$P(D|T) = 0.0094$$

The test is not reliable as the value of P(D|T) is very small and closer to 0. Here if the test is positive then probability of having disease is very less (0.0094) so the test is not reliable.

5b) $P(K|C,D) = P(K|C,\sim D) = 0.05$

$P(\sim K|C,D) = 0.95$

$P(K|T,\sim C) = ?$ $\quad P(K|T,C) = ?$ (Need to compare both)

use marginalization

$$P(K|T,\sim C) = P(K|T,\sim C, D) \cdot P(D|T,\sim C) + P(K|T,\sim C,\sim D) \cdot P(\sim D|T,\sim C)$$

value of $P(K|T,\sim C,D) = P(K|\sim C, D)$

Now it is given that $P(K|D) = 1$ which means if you have disease then it will kill with prob 1.

So, $P(K|\sim C, D) = 1$.

Now, $P(K|T,\sim C,\sim D) = P(K|\sim C,\sim D)$

(Here we are ignoring T (test) as K (kill) is not dependant on T)

$$P(K|\sim C,\sim D) = 1 - (P(K|\sim C, D)) = 0.$$

$$\begin{aligned} P(K|T,\sim C) &= P(K|T,\sim C,D) \cdot P(D|T,\sim C) \\ &\quad + P(K|T,\sim C,\sim D) \cdot P(\sim D|T,\sim C) \\ &= P(K|\sim C, D) \cdot P(D|T,\sim C) \\ &\quad + P(K|\sim C,\sim D) \cdot P(\sim D|T,\sim C) \\ &= 1 \cdot P(D|T,\sim C) + 0 \cdot P(\sim D|T,\sim C) \\ &= P(D|T,\sim C) \\ &= P(D|T) \end{aligned}$$

(Here ~C can be ignored as it does not impact on disease)

$= 0.0094$

similarly,

$$P(K|T,C) = P(K|T,C,D) \cdot P(D|T,C) + P(K|T,C,\sim D) \cdot P(\sim D|T,C)$$

(Here T can be ignored as test value has no impact on kill)

Thus,

$$= P(K|C,D) \cdot P(D|T,C) + P(K|C,\sim D) \cdot P(\sim D|T,C)$$

(For the values of $[P(D|T,C)]$ and $[P(\sim D|T,C)]$ we can ignore C as cure has no impact on disease D).

$$= P(K|C,D) \cdot P(D|T) + P(K|C,\sim D) \cdot P(\sim D|T)$$

$$= [0.05 \times 0.0094] + [0.05 \times (1-0.0094)]$$

$$= 0.00047 + 0.04953$$

$$= 0.05$$

$\therefore$ $P(K|T,C) = 0.05$.

As solved previously for $P(K|T,\sim C) = 0.0094$.
Here $P(K|T,\sim C)$ probability value is lesser than $P(K|T,C)$ $[P(K|T,\sim C) < P(K|T,C)]$

So, if test comes positive we should not get cure as probability of kill is more if we get cure as compared to probability of kill if we don't get cure.

In short,

$$P(K|T, C) > P(K|T, \sim C).$$

$\therefore$ One should not take cure when test comes positive.