Chapter 1.1
Statements, Symbolic Representation, Tautologies

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Logic

- Formal logic
  - **Definition** – foundation for the **organized, careful method of thinking** that characterizes any reasoned activity
    - A criminal investigation, judiciary
    - A scientific experiment
    - A sociological study, personality assessment
  - The study of reasoning – specifically concerned with whether something is **true or false**
  - Formal logic focuses on the **relationship between statements** as opposed to the content of any particular statement
  - **Applications** of formal logic in computer science
    - Prolog – programming language based on logic
    - Circuit logic – logic governing computer science

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Statement

- **Statement (proposition)**
  - A sentence that is **either true or false**, but not both
- **Truth value** of a statement
  - True, T, 1
  - False, F, 0
- Examples:

a. Ten is less than seven. (10 < 7) **False Statement**

b. Austin is the capital of Texas. **True Statement**

c. He is very talented. **Not a Statement**

d. There are life forms on Pluto. **Statement**

Statements and logical connectives

- **Statements letters:**
  - Capital letters like **A, B, C, D,** etc. are used to represent statements
- **Logical connectives:**
  - Symbols like $\wedge$, $\vee$, $\rightarrow$,$\leftrightarrow$, $'$
- **$\wedge$: and**
- **$\vee$: or**
- **$\rightarrow$: implies**
- **$\leftrightarrow$: equivalent**
- **$'$: negation**

Conjunction

- **Connective #1 : Conjunction (∧)**
  - A, B: Statements (or statement variables)
  - A ∧ B: Conjunction of A and B
  - A and B are called *conjuncts of the expression*
  - **A ∧ B is TRUE when both A and B are true**
  - **A ∧ B is FALSE when at least one of A or B is false**
- **Example**
  - A: Ten is less than seven.
  - B: Austin is the capital of Texas.
  - A ∧ B ?

Truth Table

- A table that shows
  - the truth values of a **statement form** (e.g. A ∧ B),
  - which correspond to the **different combinations of truth values for the variable** (A, B)
  - How many combinations of A, B are possible?

A	B	A ∧ B
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

- **Connective #2 : Disjunction (V)**
  - A, B: Statements (or statement variables)
  - A V B: Disjunction of A and B
  - A and B are called *disjuncts of the expression*
  - **A V B is TRUE when at least one of A or B is true**
  - **A V B is FALSE when both A and B are false**
- **Example**
  - A: Ten is less than seven.
  - B: Austin is the capital of Texas.
  - A V B ?

A	B	A V B
T	T	T
T	F	T
F	T	T
F	F	F

Implication

- **Connective #3 : Implication (→)**
  - A and B: Statements (or statement variables)
  - **A → B:** Symbolic form of "If A, then B" or A implies B
  - A : hypothesis or antecedent statement
  - B : conclusion or consequent statement
  - **A → B is FALSE when A is true and B is false**
  - **A → B is TRUE otherwise**

A	B	A→B
T	T	T
T	F	F
F	T	T
F	F	T

Remark: If I pass the exam, I will go for the trip

Equivalence

- **Connective #4: Equivalence (↔)**
  - **A ↔ B** stands for **(A → B) ∧ (B → A)**
  - **TRUE: Both A and B have the *same truth values***
  - **FALSE: A and B have *different truth values***

A	B	A→B	B→A	(A → B) ∧ (B → A) / A ↔ B
T	**T**	T	T	T
T	**F**	F	T	F
F	**T**	T	F	F
F	**F**	T	T	T

Negation

- **Connective #5: Negation (’)**
  - Unary connective
  - AND, OR, Implication, and Equivalence: **Binary connectives**, because they join two expressions to produce the third one
  - The negation of A is “**not A**” and is denoted **A’**
  - It has the *opposite truth value from A*
  - **If A is true, then A’ is false; if A is false, then A’ is true**

A	A’
T	F
F	T

Negation (Examples)

- A: 5 is greater than 2 (5 > 2)
  - A': 5 is less than equal to 2 (5 <= 2)
- B: Jane likes butter
  - B': Jane dislikes butter / hates / doesn't like
- C: John hates butter but (and) likes cream
  - C': John likes butter or hates cream
- **In a negation, AND becomes OR, OR becomes AND**

Another form of implication

- A: You do not do your homework
  - A': You do your homework
- B: You will fail
- A' V B ??
  - You do your homework or you will fail
- If you do not do your homework, then you will fail
  - $A \rightarrow B$
- Therefore, $A \rightarrow B \equiv A' \vee B$

A	B	A→B
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

A	A'	B	A' V B
T	F	T	**T**
T	F	F	**F**
F	T	T	**T**
F	T	F	**T**

Tables

Table 1.5 – Common English words associated with various logical connectives

English Words	Logical Connectives	Logical Expression
And; but; also; in addition; moreover	Conjunction	$A \wedge B$
Or	Disjunction	$A \vee B$
If A, then B. A implies B A, therefore B A only if B B follows from A A is a sufficient condition for B B is necessary condition for A	Implication	$A \rightarrow B$
A if and only if B A is necessary and sufficient for B	Equivalence	$A \leftrightarrow B$
Not A It is false that A … It is not true that A …	Negation	A'

Tables

- Table 1.6 – Examples for the negation of a statement

Statement	Correct Negation	Incorrect Negation
It will rain tomorrow.	It is false that it will rain tomorrow. It will not rain tomorrow.	
Peter is tall and thin.	It is false that Peter is tall and thin. Peter is not tall or he is not thin. Peter is short or fat.	Peter is short **and** fat.
The river is shallow or polluted.	It is false that the river is shallow or polluted. The river is neither shallow nor polluted. The river is deep and unpolluted.	The river is not shallow **or** not polluted.

Well Formed Formula (wff)

- Combining **letters, connectives, and parentheses** can *generate an expression* which is *meaningful*.
- **wff:** An expression that is a *legitimate string*
  - Example 1: (A → B) V (B → A) *(wff)*
  - Example 2: A)) V B (→ C) *(not a wff)*
- To reduce the number of parentheses, an order is stipulated in which the connectives can be applied, called the **order of precedence**
  1. Connectives within innermost parentheses first and then progress outwards
  2. Negation (′)
  3. Conjunction (Λ), Disjunction (V)
  4. Implication (→)
  5. Equivalence (↔)
- Hence, A V B → C is the same as (A V B) → C

Well Formed Formula (wff)

- **Main connective**: The connective to be applied ***last***
  - A Λ (B →C)′
  - Λ is the main connective
- Capital letters, like P,Q,R,S etc. are used to represent **multiple wffs**
  - **[(A V B) Λ C]** → (B V C′) can be represented by **P** → **Q**
  - where,
    - **P** is the wff **[(A V B) Λ C]**
    - **Q** represents **B V C′**

Truth tables for some wffs

- P: A V B′ → (A V B)′
- Main connective (according to the rules of precedence): implication.

A	B	B′	A V B′	A V B	(A V B)′	A V B′ → (A V B)′
T	T	F	T	T	F	F
T	F	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	T	F	T	T

Is every column correct?

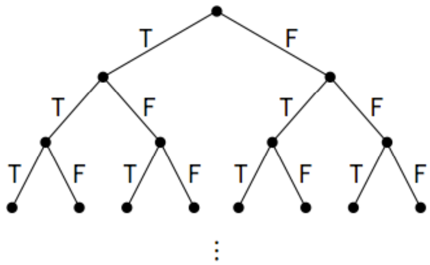
Truth tables for some wffs

- P: A V B′ → (A V B)′
- Main connective (according to the rules of precedence): implication.

A	B	B′	A V B′	A V B	(A V B)′	A V B′ → (A V B)′
T	T	F	T	T	F	F
T	F	T	T	T	F	F
F	T	F	F	T	F	T
F	F	T	T	F	T	T

Wff with n statement letters

- How many rows in truth table?
- The total number of rows in a truth table for n statement letters is 2^n.



Statement letters	Choices
1	$2 = 2^1$ branches
2	$4 = 2^2$ branches
3	$8 = 2^3$ branches
⋮	⋮
n	2^n branches

Tautology and Contradiction

- **Tautology:** A wff that is intrinsically *true*, i.e. no matter what the truth value of the statements that comprise the wff.
  - e.g. It will rain today or it will not rain today (A ∨ A′)
- **Contradiction:** A wff that is intrinsically *false*, i.e. no matter what the truth value of the statements that comprise the wff.
  - e.g. It will rain today and it will not rain today (A ∧ A′)
- Usually, tautology represented by 1 and contradiction by 0

Tautological Equivalences

- Two statement forms are called *logically equivalent* if, and only if, they have **identical truth values** for each possible substitution of statements for their statement variables.
- **Logical equivalence** of statement forms P and Q is denoted by writing **P ⇔ Q or P ≡ Q**. In this case, P and Q are *equivalent wffs*
- Truth table for **(A V B) V C and A V (B V C) ?**

A	B	C	A V B	B V C	(A V B) V C	A V (B V C)
T	T	T	T	T	**T**	**T**
T	T	F	T	T	**T**	**T**
T	F	T	T	T	**T**	**T**
T	F	F	T	F	**T**	**T**
F	T	T	T	T	**T**	**T**
F	T	F	T	T	**T**	**T**
F	F	T	F	T	**T**	**T**
F	F	F	F	F	**F**	**F**

Some common equivalences (Imp Laws)

- Equivalences are listed in *pairs*, called duals of each other.
- One equivalence can be obtained from another by replacing V with Λ and 0 with 1 or vice versa.

Commutative	A V B ⇔ B V A	A Λ B ⇔ B Λ A
Associative	(A V B) V C ⇔ A V (B V C)	(A Λ B) Λ C ⇔ A Λ (B Λ C)
Distributive	A V (B Λ C) ⇔ (A V B) Λ (A V C)	A Λ (B V C) ⇔ (A Λ B) V (A Λ C)
Identity	A V 0 ⇔ A	A Λ 1 ⇔ A
Complement	A V A′ ⇔ 1	A Λ A′ ⇔ 0

De Morgan’s Laws

1. (A V B)′ ⇔ A′ Λ B′
2. (A Λ B)′ ⇔ A′ V B′

A	B	A′	B′	A V B	(A V B)′	A′ Λ B′
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Logical Connectives in the Real World

- **Conditional Statements in programming** use logical connectives with statements.
- Example

if((outflow > inflow) **and** not(pressure < 1000))

do something;

else

do something else;

Algorithm

- How to enroll for a course?
- A **set of instructions** that can be mechanically executed in a **finite amount of time** in order to **solve some problems**
- Algorithms are the **state in between** the verbal form of a problem and the computer program
- Algorithms are usually **represented by pseudocode**
- Pseudocode should be **easy to understand** even if you have no idea of programming

Pseudocode example

j = 1 // initial value
Repeat
    read a value for *k*
    if ((*j* < 5) AND (2**j* < 10) OR (($(3*j)^{1/2}$ > 4)) **then**
        write the value of *j*
    otherwise
        write the value of 4**j*
    end if statement
    increase *j* by 1
Until *j* > 6

Discussion