Appendix A

Derivation Rules for Propositional and Predicate Logic

EQUIVALENCE RULES		
Expression	**Equivalent to**	**Name/Abbreviation for Rule**
$P \vee Q$ $P \wedge Q$	$Q \vee P$ $Q \wedge P$	Commutative—comm
$(P \vee Q) \vee R$ $(P \wedge Q) \wedge R$	$P \vee (Q \vee R)$ $P \wedge (Q \wedge R)$	Associative—ass
$(P \vee Q)'$ $(P \wedge Q)'$	$P' \wedge Q'$ $P' \vee Q'$	De Morgan's laws—De Morgan
$P \rightarrow Q$	$P' \vee Q$	Implication—imp
P	$(P')'$	Double negation—dn
$[(\exists x)A(x)]'$	$(\forall x)[A(x)]'$	Negation—neg

INFERENCE RULES		
From	**Can Derive**	**Name/Abbreviation for Rule**
$P, P \rightarrow Q$	Q	Modus ponens—mp
$P \rightarrow Q, Q'$	P'	Modus tollens—mt
P, Q	$P \wedge Q$	Conjunction—con
$P \wedge Q$	P, Q	Simplification—sim
P	$P \vee Q$	Addition—add
$P \rightarrow Q, Q \rightarrow R$	$P \rightarrow R$	Hypothetical syllogism—hs
$P \vee Q, P'$	Q	Disjunctive syllogism—ds
$P \rightarrow Q$	$Q' \rightarrow P'$	Contraposition—cont
$Q' \rightarrow P'$	$P \rightarrow Q$	Contraposition—cont
P	$P \wedge P$	Self-reference—self
$P \vee P$	P	Self-reference—self
$(P \wedge Q) \rightarrow R$	$P \rightarrow (Q \rightarrow R)$	Exportation—exp
P, P'	Q	Inconsistency—inc
$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$	Distributive—dist
$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$	Distribution—dist

INFERENCE RULES (*CONTINUED*)			
From	**Can Derive**	**Name/Abbreviation for Rule**	**Restrictions on Use**
$(\forall x)P(x)$	$P(t)$, where t is a variable or constant symbol	Universal instantiation—ui	If t is a variable, it must not fall within the scope of a quantifier for t.
$(\exists x)P(x)$	$P(a)$ where a is a constant symbol not previously used in proof sequence	Existential instantiation—ei	Must be the first rule used that introduces a.
$P(x)$	$(\forall x)P(x)$	Universal generalization—ug	$P(x)$ has not been deduced from any hypotheses in which x is a free variable nor has $P(x)$ been deduced by ei from any wff in which x is a free variable.
$P(x)$ or $P(a)$ where a is a constant symbol	$(\exists x)P(x)$	Existential generalization—eg	To go from $P(a)$ to $(\exists x)P(x)$, x must not appear in $P(a)$.