UNIVERSITY OF TEXAS ARLINGTON

# Chapter 4.2
## Counting

**Instructor:** Abhishek Santra

**Email:** abhishek.santra@uta.edu

1

## Counting

- **Combinatorics** is the branch of mathematics that deals with counting
- The idea is to find out **how many members are present in a finite set.**
- Principles of counting answer the following kind of questions:
  - How many one-digit numbers are there?
    **10**
  - How many four-digit numbers can there be if repetition of numbers are allowed and if repetition of numbers is not allowed?
  - If a man has 4 suits, 8 shirts and 5 ties, how many outfits can he put together?



2

## Example: Multiplication Principle

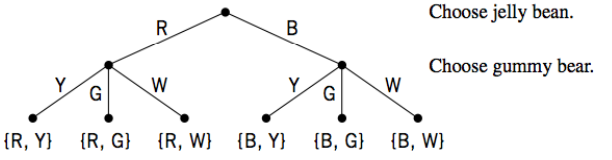
- A child needs to choose
  - one jellybean out of two jellybeans, one red and one black, **AND**
  - one gummy bear out of three gummy bears, yellow, green, and white.
- *How many **different sets of candy** can the child have?*



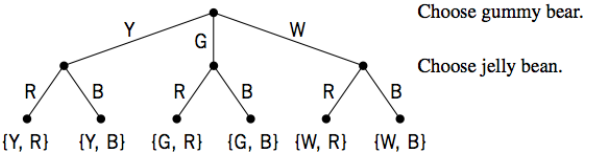
3

## Example: Multiplication Principle

- **2 × 3 = 6** possible outcomes
  - **First Event (Jelly Bean):** 2 Options, AND then in
  - **Second Event (Gummy Bear):** 3 Options




- Or **3×2=6** possible outcomes






4

## Multiplication Principle

- If there are *m* possible outcomes for a first event and *n* possible outcomes for a second event, then there are *m * n* possible outcomes for the **sequence of two events**.
- Hence, from the multiplication principle, it follows that for two sets A and B

$$|A \times B| = |A| \cdot |B|$$



5

## Addition Principle

- If A and B are disjoint events with *m* and *n* outcomes, respectively, then the total number of possible outcomes for event "A or B" is *m + n*.
  - **$|A \cup B| = |A| + |B|$**
- Example: A customer wants to purchase one vehicle. The dealer has 23 cars and 14 trucks in stock. How many options does the customer have?
  - Choose a car: 23 options, **OR**
  - Choose a truck: 14 options
  - Both disjoint events, Total Options: 23 + 14 = 37



6

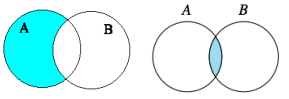
## Addition Principle

- If A and B are disjoint sets, then $|A \cup B| = |A| + |B|$ using the addition principle (A or B).
- [Example 32] Prove that if A and B are finite sets then

$|A-B| = |A| - |A \cap B|$ and $|A-B| = |A| - |B|$ if $B \subseteq A$

$$
\begin{aligned}
(A-B) \cup (A \cap B) &= (A \cap B') \cup (A \cap B) \\
&= A \cap (B' \cup B) && \textit{distributive property} \\
&= A \cap S && \textit{complement property} \\
&= A && \textit{identity property}
\end{aligned}
$$

- Also, **A - B** and **A ∩ B** are *disjoint sets*, therefore using the addition principle




$|A| = |(A-B) \cup (A \cap B)| = |A-B| + |A \cap B|$

- Hence, $|A-B| = |A| - |A \cap B|$
- If $B \subseteq A$, then $A \cap B = B$. Hence, $|A-B| = |A| - |B|$



7

## Example

- How many **three-digit numbers** begin with **7**?
  - Find out the number of options for each digit
    - How many options for the last digit (one's place)? **AND**
    - How many options for the middle digit (ten's place)? **AND**
    - How many options for the first digit (hundred's place)?

Number of options for each digit

| 1 | * | 10 | * | 10 |
|---|---|---|---|---|
| *Options:* ***Only digit 7*** | | *Options:* ***0*** *to 9* | | *Options:* *0 to 9* |

  - **Total: 100**
    - **From 700 to 799**



8

## Class Exercise

- [Example 34] How many four-digit numbers begin with a **4 or a 5**?
  - 4-digit numbers starting with 4: Number of options for each digit: 1 * 10 * 10 * 10
  - 4-digit numbers starting with 5: 1 * 10 * 10 * 10
  - Disjoint Events
  - Total numbers: 1000 + 1000 = 2000
- [Example 36] How many **three-digit integers** (numbers between 100 and 999 inclusive) are even?

Number of options for each digit: 9 * 10 * 5

*Options: **1** to 9* | *Options: **0** to 9* | *Options: 0, 2, 4, 6, 8*

Total = 9*10*5
= **450**



9

## Discussion



10