Category theory applications in physics

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What is Category theory?What are it's uses?

Informally, category theory is a general theory of functions. Samuel Eilenberg and Saunders Mac Lane formulated the concepts of categories from 1942–45 in their study of algebraic topology, with the goal of understanding the processes that preserve mathematical structure. It may also be used as an axiomatic foundation for mathematics, as an alternative to set theory.

Applications: As we shall see a certain type of categories called **monoidal categories** are very useful in physics. The category **FdHilb** is useful in Quantum Mechanics and QFT and the category **nCob** is useful in General Relativity.

Introduction

Definition

A category **C** consists of:

- Objects(or nodes): A, B, C, . . . (the class of objects is denoted by $|\mathbf{C}|$ or ob(**C**))
- Morphisms (or arrows or maps): f, g, h, . . .(the set of morphisms from A to B is denoted by **C**(A,B) or $\hom_{\mathbf{C}}(A, B)$)
- For each morphism f, there are given objects called *domain* and *codomain* and denoted by *dom*(f),*cod*(f). If $A = dom(f)$, $B = cod(f)$ then we write $f : A \to B$ or $f \in \mathbf{C}(A, B)$ or $A \xrightarrow{f} B$
- $\forall f : A \to B$ & $g : B \to C$ $\exists g \circ f : A \to C$ called a *composite* of f and g.
- For each object A, there is given an arrow $1_A : A \to A$ called the identity arrow of A.

and it satisfies

- **Associativity**: $h \circ (g \circ f) = (h \circ g) \circ f \; \forall f : A \to B, g : B \to C, h : C \to D.$
- **Unit**: $f \circ 1_A = f = 1_B \circ f \forall f : A \to B$

Examples

The set containing one object and one morphism is a trivial example. Another example is the category **Set**, containing all sets as objects and all functions between them as morphisms. Since functions satisfy associativity and unit properties. Some other examples-

Rel: which have all sets as objects and all relations, which essentially are subsets of the Cartesian product of 2 sets, as morphisms between them.

Mon: which have all so-called monoids, which essentially are groups without inverses, as objects and all monoid morphisms as morphisms between them.

Cat: which have all categories as objects and all so-called functors(defined in next slide), which essentially are maps between categories, as morphisms between them.

Functors

Definition

A functor $F : \mathbf{C} \to \mathbf{D}$ between categories $\mathbf{C}$ and $\mathbf{D}$ is a mapping that of objects to objects and arrows to arrows, in such a way that

- $F(f : A \to B) = F(f) : F(A) \to F(B)\ \forall f \in \hom(\mathbf{C})$
- $F(1_A) = 1_{F(A)}\ \forall A \in |\mathbf{C}|$
- $F(g \circ f) = F(g) \circ F(f)\ \forall f : A \to B$ and $g : B \to C \in \hom(\mathbf{C})$

It can be graphically represented as-

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[dr, "g\circ f"'] & B \arrow[d, "g"] \\ & C \end{tikzcd} \quad \xrightarrow{F} \quad \begin{tikzcd} F(A) \arrow[r, "F(f)"] \arrow[dr, "F(g\circ f)"'] & F(B) \arrow[d, "F(g)"] \\ & F(C) \end{tikzcd}$$

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