Higgs mechanism

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Abstract

This is a brief review of the Higgs mechanism.

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0 Introduction

A spontaneously broken continuous global symmetry generates $n(G)-n(H)$ number of Goldstone bosons, where G is the original symmetry group of the Lagrangian density, H is the symmetry group of the vacuum and $n(G)$ is the number of generators of the group G. These Goldstone bosons are massless. What if the symmetry is not a global symmetry but a gauge symmetry? As we will see, this causes the Goldstone boson to disappear from the spectrum and the gauge boson to become massive through a procedure known as the

Higgs mechanism. This is informally stated as ”gauge fields become massive by the Higgs mechanism, and they “eat” a scalar degree of freedom along the symmetry direction in order to become massive”.

When Yang and Mills discovered the Yang–Mills theory , Pauli criticized their theory because it predicts only massless particles. Pauli himself worked earlier on some non-abelian gauge theory and stopped because of this. Naively adding the mass term will violate gauge invariance. Massless particles should be easy to observe. So the possible answers such that Yang–Mills theory is correct are

1. The Yang-Mills particles acquire mass through some mechanism.
2. the YangMills particles are in fact massless but are not observed because the theory doesn’t allow it to be detected.

The 1st possibility is realised in the electroweak theory through the Higgs mechanism and the 2nd possibility is realised as color confinement in QCD. We will discuss the Higgs mechanism. It is also called ABEGHHK’tH mechanism (for Anderson, Brout, Englert, Guralnik, Hagen, Higgs, Kibble, and ’t Hooft).

1 $U(1)$ gauge theory

The Lagrangian for a complex scalar coupled to a $U(1)$ gauge field is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D^\mu\phi - V(|\phi|),$$

where $D_\mu = \partial_\mu + ieA_\mu$. The gauge transformation is

$$\begin{aligned} \phi(x) &\to e^{-i\alpha(x)}\phi(x), \\ A_\mu(x) &\to A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x). \end{aligned}$$

We choose the most general analytic, gauge-invariant, renormalizable potential (implies powers more than 4 are not contained since they have couplings with negative mass dimension) with a symmetry-breaking term

$$V = -m^2\phi^*\phi + \frac{\lambda}{4}\left(\phi^*\phi\right)^2$$

or we can add a constant and define (which will not contribute as long as we neglect gravity)

$$V = \frac{\lambda}{4}\left(|\phi|^2 - \frac{v^2}{2}\right)^2$$

The wrong-sign mass term for the scalar indicates that the ground state has $|\langle\phi\rangle| = \frac{v}{\sqrt{2}} = \sqrt{\frac{2m^2}{\lambda}}$.

All the vacua are equivalent (by symmetry) so we can pick any convenient parametrization. Let us take $\langle\phi\rangle$ to be real.

Now expand around v by parametrizing $\phi(x)$ in terms of two real fields $\sigma(x)$ and $\pi(x)$ as

$$\phi(x) = \left(\frac{v + \sigma(x)}{\sqrt{2}}\right) e^{i\frac{\pi(x)}{F_\pi}}$$

here F_π a real number used to canonically normalize. Plugging this in, our Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \left(\frac{v+\sigma}{\sqrt{2}}\right)^2 \left[-i\frac{\partial_\mu \pi}{F_\pi} + \frac{\partial_\mu \sigma}{v+\sigma} - ieA_\mu\right]\left[i\frac{\partial_\mu \pi}{F_\pi} + \frac{\partial_\mu \sigma}{v+\sigma} + ieA_\mu\right]$$

$$-\left(-\frac{m^4}{\lambda} + m^2\sigma^2 + \frac{1}{2}\sqrt{\lambda}m\sigma^3 + \frac{1}{16}\lambda\sigma^4\right)$$

Now look at the terms involving only A_μ

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2v^2A_\mu^2 + \cdots$$

comparing with the Proca equation we get

$$m_A = ev$$

similarly

$$m_\sigma = \sqrt{2}m$$

$$m_\pi = 0$$

comparing the coefficient of $(\partial_\mu\pi)^2$ we get

$$F_\pi = v$$

1.1 Gauge invariance

It is frequently stated the Higgs mechanism involves spontaneous breaking of the gauge symmetry. This is, however, wrong. In fact, gauge symmetries cannot be spontaneously broken.

A less interesting explanation for this is that gauge symmetries are not actual symmetries, they are just a reflection of a redundancy in our description the system; two states related by a gauge transformation are actually the same physical state. These are merely fake degrees of freedom introduced to simplify calculations. Thus, a gauge symmetry is physically a "do-nothing transformation" and thus it does not make sense for it to be called a symmetry to begin with.

A more satisfying explanation is: even if we interpret gauge symmetries as real symmetries, they can never be spontaneously broken. This result is known as **Elitzur's theorem**. Let us see this in our $U(1)$ example.

The guage transformation

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x), \quad \pi(x) \to \pi(x) - F_\pi \alpha(x)$$

will not change the Lagrangian. So, the gauge symmetry is not broken.

1.2 Unitary and R_ξ gauges

Unitary gauge or **physical gauge**: We can select the gauge transformation such that $\pi(x) = 0$. In this gauge the particle $\pi(x)$ doesn't exist. Since physics should be independent of the particular gauge we chose, we can say that π is not a real particle. It is called as physical gauge because it removes the unphysical Goldstone π particle.

The field σ is called as **Higgs field** and the σ particle is called as **Higgs boson**.

Cartesian representation: Instead of the polar representation that we used earlier we can define the Cartesian representation as

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\zeta(x))$$

$$\begin{aligned}(D^\mu \phi)^\dagger (D_\mu \phi) = \frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) + \frac{1}{2}(\partial^\mu \zeta)(\partial_\mu \zeta) \\ + evA^\mu \partial_\mu \zeta + \frac{1}{2}e^2 v^2 A^\mu A_\mu + \cdots\end{aligned}$$

There is a $A^\mu \partial_\mu \zeta$ term. Notice that it is quadratic in the fields, so it should be part of the free. Lagrangian of the system. However, it contains two different fields, A_μ and ζ involving a derivative, so it is also a interaction term. If written in terms of creation and annihilation operators, this would imply that we can have Feynman diagrams in which an A_μ changes into a ζ, without any other particle interacting with them.

To avoid these problems we add the following gauge fixing term to cancel the problematic term

$$L_{\mathrm{GF}} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi M_A \zeta\right)^2$$

The cross term present here would pair up with the problematic term to make up a total divergence, which can be disregarded in the Lagrangian.

The terms quadratic in the gauge boson field can be summarized as

$$L_0^{(A)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 + \frac{1}{2}M_A^2 A^\mu A_\mu$$

Similarly the quadratic terms involving the ζ-field are given by

$$L_0^{(\zeta)} = \frac{1}{2}\left[\left(\partial^\mu \zeta\right)\left(\partial_\mu \zeta\right) - \xi M_A^2 \zeta^2\right]$$

From these as usual, we look at the terms quadratic in the fields, Fourier transform, and invert. , we can calculate the propagators of both A and ζ, and find

$$iD_{\mu\nu}(k) = \frac{-i}{k^2 - M_A^2 + i\varepsilon}\left[g_{\mu\nu} - \frac{(1-\xi)k_\mu k_\nu}{k^2 - \xi M_A^2}\right]$$

$$i\Delta(k) = \frac{i}{k^2 - \xi M_A^2}$$

Notice that as $\xi \to \infty$ the propagator of ζ becomes 0 that is it becomes unphysical. $\xi \to \infty$ is nothing but the unitary gauge. These gauges are called R_ξ-gauges.

1.3 Renormalizablity

For any finite value of ξ, the propagator of the ζ field does not vanish, and therefore diagrams with internal ζ lines must be taken into account. Hence, in any gauge with finite ξ, the field ζ is called the **unphysical Higgs**. The field is unphysical because it does not represent any physical particle, and therefore cannot appear as external legs of any Feynman diagrams representing a physical process. It is also called **the would-be Goldstone boson**, since this field would have been the Goldstone boson if the symmetry were global rather than local.

It seems that the gauges with finite ξ are more complicated because we need to deal with **unphysical internal lines in Feynman diagrams**. But there is a pay-off. Notice that for any finite ξ, the gauge boson propagator indeed falls off as $1/p^2$ for large momenta. So does, in fact, the propagator of the unphysical Higgs. We see that this theory satisfies both conditions needed for renormalizability:

1. The propagator of any bosonic field falls off like $1/p^2$ for large momenta and the propagator of any fermion field falls off like $1/p$.

2. There is no coupling constant with negative mass dimension.

That is why the gauge condition with an arbitrary finite value of ξ is called the renormalizable ξ-gauge or the R_ξ-gauge.

For finite ξ, the theory is renormalizable, although the particle interpretations are somewhat obscure because of the presence of unphysical bosons.

On the other hand, for infinite ξ, the particle spectrum is clear, although renormalizability is not obvious.

But since the physical amplitudes do not depend on ξ, the theory must be both. This means that we have found a renormalizable theory with massive gauge bosons through spontaneous symmetry breaking.

The number of degrees of freedom is same before and after. Massless vector bosons like photons only have 2 independent polarisation degrees, unlike massive vector bosons. For a spin 1 boson with mass μ and with $k^\lambda = (\omega, 0, 0, k)$ the longitudinal mode is given by $\epsilon_\lambda^{(3)} = (k, 0, 0, \omega)/\mu$. In the limit $\mu \to 0$ this longitudinal mode cannot exist and there will only be 2 internal degrees of freedom.

Before the Higgs mechanism massless gauge field has 2 degrees of freedom. After Higgs mechanism massive gauge field has 3 degrees of freedom. The additional degree of freedom came because of "eating" the unphysical Goldstone boson.

2 Non-Abelian gauge theories

We will see an $SO(3)$ gauge theory to understand Higgs mechanism in Non-Abelian gauge theories. We introduce three real scalars ϕ_i and the Lagrangian

$$\mathcal{L} = -\frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{2}\left(\partial_\mu \phi_i - igA^a_\mu \tau^a_{ij}\phi_j\right)^2 + \frac{1}{2}m^2\phi_i^2 - \frac{\lambda}{4!}\left(\phi_i^2\right)^2$$

The potential is minimized for $|\langle\vec{\phi}\rangle| = v = \sqrt{\frac{6m^2}{\lambda}}$. By an SO(3) transformation, we can pick the direction and phase so that $\langle\phi_3\rangle = v$ and $\langle\phi_1\rangle = \langle\phi_2\rangle = 0$. That is, without loss of generality, we take

$$\left\langle \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \right\rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

This vacuum is invariant under $H = \mathrm{SO}(2) \subset G = \mathrm{SO}(3)$, which rotates ϕ_1 and ϕ_2. Since SO(2) has one generator and SO(3) has three (SO(n) has $\frac{n(n-1)}{2}$), there will be two Goldstone bosons that are eaten to form two massive gauge bosons. To see this explicitly, we can expand the Lagrangian in unitary gauge (that is, with $\pi = 0$). We find

$$\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}^a\right)^2 + \frac{g^2}{4} A_\mu^a A_\mu^b \vec{v}^T \left\{\tau^a, \tau^b\right\} \vec{v}$$

where $\vec{v} = \langle\vec{\phi}\rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$. We have symmetrized the $\tau^a\tau^b$ using $\left[A_\mu^a, A_\mu^b\right] = 0$. Plugging in the SO(3) generators:

$$\tau^1 = i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \tau^2 = i\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad \tau^3 = i\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

we see by explicit calculation that $\vec{v}^T \left\{\tau^a, \tau^b\right\} \vec{v}$ is only non-zero for $a = b = 1$ or $a = b = 2$. Thus,

$$\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}^a\right)^2 + \frac{1}{2} m_A^2 \left(A_\mu^1 A_\mu^1 + A_\mu^2 A_\mu^2\right)$$

with $m_A^2 = g^2 v^2$, which describes two massive gauge bosons and one massless one, as expected.

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