MA 109 Tutorial 4

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Q)2

(a)We know that $L(P) \leq \int_a^b f(x)dx \leq U(P)$,

$$L(P) = \sum_{i=1}^{n} m_i(x_i - x_{i-1})$$
$$\Rightarrow L(P) \geq 0$$
$$\Rightarrow \int_a^b f(x)dx \geq 0$$

Since $m_i \geq 0\ \forall i$. Further, if f is continuous let $F(x)$ be defined by $F(x) = \int_a^x f(t)dt$, then from FTC

$$F'(x) = f(x) \geq 0 \forall x \in [a, b]$$

Now we know that $F'(x) \geq 0$, $F(b) = F(a) = 0 \Rightarrow$
$F(x) = 0 \forall x \in [a, b] \Rightarrow f(x) = 0 \forall x \in [a, b]$

Q)2

(b) Take $f(x) = 0$ if $x \neq \frac{a+b}{2}$, $f(\frac{a+b}{2}) = 1$. Then this function is Riemann integrable and

$$\int_a^b f(x)dx = 0$$

Q)3

(ii) For the function $f(x) = \frac{1}{1+x^2}$, $a = 0, b = 1$ and for the partition $P = \{\frac{1}{n}, \frac{2}{n}, \cdots, \frac{n-1}{n}\}$ this is a Riemann sum. As $n \to \infty$ $||P|| \to 0$. Since $f(x) = \frac{1}{1+x^2}$ is continuous $\Rightarrow$ it is Riemann integrable. So,

$$
\begin{aligned}
lim_{n\to\infty} \sum_{i=1}^{n} \frac{n^2}{i^2+n^2}\frac{1}{n} &= \int_0^1 \frac{1}{1+x^2}dx \\
&= \frac{\pi}{4}
\end{aligned}
$$

(iv) Similar to the above this becomes

$$
\begin{aligned}
lim_{n\to\infty} s_n &= \int_0^1 cos(\pi x)dx \\
&= \frac{sin(\pi) - sin(0)}{\pi} = 0
\end{aligned}
$$

Q)4b)

Let $F(x)=\int_a^x f(t)dt$ then $F'(x)=f(x)$: Now observe that

$$\int_{u(x)}^{v(x)} f(t)dt = \int_a^{v(x)} f(t)dt - \int_a^{u(x)} f(t)dt$$
$$\Rightarrow = F(v(x)) - F(u(x))$$
$$\Rightarrow \frac{d}{dx}\int_{u(x)}^{v(x)} f(t)dt = f(v(x))v'(x) - f(u(x))u'(x)$$

(i)$F'(x)=2cos(4x^2)$
(ii)$F'(x)=2xcos(x^2)$

Q)6

We know that $sin(\lambda(x-t)) = sin(\lambda x)cos(\lambda t) - cos(\lambda x)sin(\lambda t)$. Now in the integrand, take trems in x outside the integral, evaluate $g'(x)$; $g''(x)$, and simplify to show LHS=RHS; from the expressions for $g(x)$ and $g'(x)$ it should be clear that $g(0) = g'(0) = 0$.

$$\begin{aligned} g(x) &= \frac{1}{\lambda}\int_0^x f(t)(sin(\lambda x)cos(\lambda t) - cos(\lambda x)sin(\lambda t))dt \\ &= \frac{1}{\lambda}\left(sin(\lambda x)\int_0^x f(t)cos(\lambda t) - cos(\lambda x)\int_0^x f(t)sin(\lambda t)\right) \end{aligned}$$

You can also do this question using Leibniz integral rule.