MA 109 Assignment 1

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Q) Evaluate the limit $\lim_{n\to\infty} \frac{an+1}{bn+2}$ and justify your answer using the $\epsilon - N$ definition of the limit.

Sol. Evaluation: $\lim_{n\to\infty} \frac{an+1}{bn+2} = \lim_{n\to\infty} \frac{a+\frac{1}{n}}{b+\frac{2}{n}} = \frac{a}{b}$

Justification: We need to show that $\forall \epsilon > 0\ \exists N_\epsilon \in \mathbb{N}$ such that $n > N_\epsilon \Rightarrow \left|\frac{an+1}{bn+2} - \frac{a}{b}\right| < \epsilon$. For a given ϵ fix N_ϵ such that $N_\epsilon > \frac{|2a-b|}{b^2\epsilon}$. Now $\forall n > N_\epsilon$

$$\frac{|2a-b|}{b^2\epsilon} < N < n$$
$$\Rightarrow \frac{|2a-b|}{b^2 n} < \epsilon$$
$$\Rightarrow \frac{|2a-b|}{b(bn+2)} < \epsilon$$
$$\Rightarrow \left| \frac{an+1}{bn+2} - \frac{a}{b} \right| < \epsilon$$

Hence $\lim_{n\to\infty} \frac{an+1}{bn+2} = \frac{a}{b}$.(One way to fix N_ϵ is $N_\epsilon = \left[\frac{|2a-b|}{b^2\epsilon}\right] + 1$,[.] is G.I.F)