Covid-19:analysis of a modified SEIR model-report

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Abstract

Modelling of infectious diseases is not only an interesting example of nonlinear dynamics, it is also very useful to decide what types of interventions should governments do to contain the disease as early as possible. Among social distancing (SD) and testing-quarantining (TQ) we explain that for same target $R_0^{target} < 1$, TQ is more efficient in controlling the pandemic than lock-downs that only implement SD. This project report is based on the paper arXiv:2005.11511 [q-bio.PE]

1 Introduction and motivation

The COVID-19 pandemic, is an ongoing pandemic caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), first identified in December 2019 in Wuhan, China. As of 1 December 2020, more than 63.2 million cases have been confirmed, with more than 1.46 million deaths attributed to COVID-19. It is very important for governments to decide on the most optimal intervention strategy. Here we analyze intervention strategies in an extended version of the SEIR model.

2 Description of the extended SIER model

The main difference between this model and the normal SIER model is that we take into account the fact that asymptomatic individuals play a significant role in the transmission of Covid-19. We divide the population of size N into eight compartments: 1. S = Susceptible individuals. 2. E = Exposed but not yet contagious individuals. 3. I_a = Asymptomatic, either develop no symptoms or mild symptoms. 4. I_p = Presymptomatic, those who would eventually develop strong symptoms. 5. U_a = Undetected asymptomatic individuals who have recovered. 6. D_a = Asymptomatic individuals who are detected because of directed testing-quarantining, may have mild symptoms, and would have been placed under home isolation (few in India). 7. U_p = Presymptomatic individuals who are detected at a late stage after they develop serious symptoms and report to hospitals. 8. D_p = Presymptomatic individuals who are detected because of directed testing-quarantining. The dynamics are given by the following equations.

$$\dot{S} = -\frac{u(\beta_a I_a + \beta_p I_p)}{N} S (\alpha \text{ :fraction of asymptomatic carriers})$$
$$\dot{E} = \frac{u(\beta_a I_a + \beta_p I_p)}{N} S - \sigma E (\beta_a \text{ : infectivity of asymptomatic carriers})$$
$$\dot{I}_a = \alpha\sigma E - \gamma_a I_a - r\nu_a I_a (\beta_p \text{ :infectivity of presymptomatic carriers})$$
$$\dot{I}_p = (1-\alpha)\sigma E - \gamma_p I_p - r\nu_p I_p (\sigma \text{ :transition rate from exposed to infectious})$$
$$\dot{U}_a = \gamma_a I_a (\gamma_a \text{ :transition rate of asymptomatic carriers to recovery or hospitalization})$$
$$\dot{D}_a = r\nu_a I_a (\gamma_p \text{ :transition rate of presymptomatics to recovery or hospitalization})$$
$$\dot{U}_p = \gamma_p I_p (r\text{:intervention factor due to testing-quarantining})$$
$$\dot{D}_p = r\nu_p I_p (u\text{: intervention factor due to social distancing})$$

$\nu_a; \nu_p$:detection probabilities of asymptomatic & symptomatic carriers. Here we choose $\nu_a = \frac{1}{3}; \nu_p = \frac{1}{2}$ and we have the identity

$$N = S + E + I_a + I_p + U_a + D_a + U_p + D_p$$

From these definitions it follows that the total infectious population size is $I = I_a + I_p$, the cumulative affected population (recovered or dead) is $R = U_a + D_a + U_p + D_p$, the total number of confirmed cases,C and the number of daily recorded new cases D would be $C = D_a + D_p + U_p$,$D = \frac{dC}{dt} r\nu_a I_a + (\gamma_p + r\nu_p) I_p$

3 Discussion of the key results

The total infected population size at any given time is $I = I_a + I_p$, the cumulative affected population(the ones that are dead, in hospital, or have recovered) is $R = U_a + D_a + U_p + D_p$, the reported total confirmed cases is $C = D_a + D_p + U_p$, and the reported new daily cases is $D = dC/dt = r\nu_a I_a + (\gamma_p + r\nu_p) I_p$. The parameters u and r are in general time dependent, u changing from the free value $u = 1$(without interventions) to a target value $u_l < 1$, while r is a rate that changes from 0 to a value $r_l > 0$.

3.1 How much testing is required?

Let the no. of tests per day be T and A the typical no. of contacts made by a person before detection over the period of infection. The TQ intervention would be successful only if-

$$T(t) \approx \frac{r(t) A D(t)}{\gamma_p}.$$

Here $r(t)$ is the control rate function which changes from 0 to r_l and should at least be of the order of γ_p, which means $T(t) \approx AD(t)$.

3.2 Comparing different intervention strategies

The targeted reproduction no. is useful in characterizing the system with interventions.

$$R_0^{target} = \frac{\alpha u_l \beta_a}{\gamma_a + r_l \nu_a} + \frac{(1-\alpha) u_l \beta_p}{\gamma_p + r_l \nu_p}$$

A strong intervention is characterized by $R_0^{target} < 1$(suppression of the disease) while weak intervention is characterized by $R_0^{target} \gtrsim 1$(mitigation of the effects of the disease). In the early phase of the pandemic, all populations other than S grow exponentially with time as $\sim e^{\mu t}$ (μ- largest eigenvalue afer linearization). μ becomes negative in case of strong intervention and the disease decays exponentially.

For the purpose of illustrations, we choose the the values of parameters as: $\alpha = 0.67$ and the rates $\beta_a = 0.333$, $\beta_p = 0.5$, $\sigma = 1/3$, $\gamma_a = 1/8$, $\gamma_p = 1/12$ all in units of day^{-1}. With these values, we now compare four different ways in which strong and weak interventions are implemented: (1) 6WLD-NTQ: Six weeks lockdown(strong value of SD parameter) and no testing-quarantining, (2) ELD-NTQ: Extended lockdown and no testing-quarantining,(3) NSD-ETQ: No social distancing and extended testing-quarantining, (4) ESD-ETQ: Extended social distancing and extended testing-quarantining. The case with no social distancing and no testing-quarantining is indicated as NSD-NTQ.

Also suppose, the population $N = 10^7$ and initial conditions $E(0) = 100$, $I_a(0) = I_p(0) = U_a(0) = D_a(0) = U_p(0) = D_p(0) = 0$ and $S(0) = N - E - I_a - I_p - U_a - D_a - U_p - D_p$. We also assume that intervention strategies are switched on when the confirmed cases reach 50 and the full intervention values are attained over a time scale of 5 days.

1. Strong intervention ($R_0^{target} < 1$)
The infection numbers will start decaying exponentially once R_0^{eff} crosses the value 1. Now, linearized theory can be used(since infected people is small compared to the population).
For the purpose of illustrations, the following parameter sets are considered-
Parameter set I [$R_0^{target} = 0.667$]- (i) SD: $u_l = 0.177$, $r_l = 0$, (ii) TQ: $u_l = 1$, $r_l = 1.2$ and (iii) SD-TQ: $u_l = 0.461$, $r_l = 0.4$. This choice corresponds to changing the free value of $R_0 = 3.766$ to a target value $R_0^{target} = 0.667$, for all three strategies.μ changes from the free value $\mu = 0.158$ to the values (i) $\mu = -0.027$, (ii) $\mu = -0.077$, (iii) $\mu = -0.0546$ respectively.

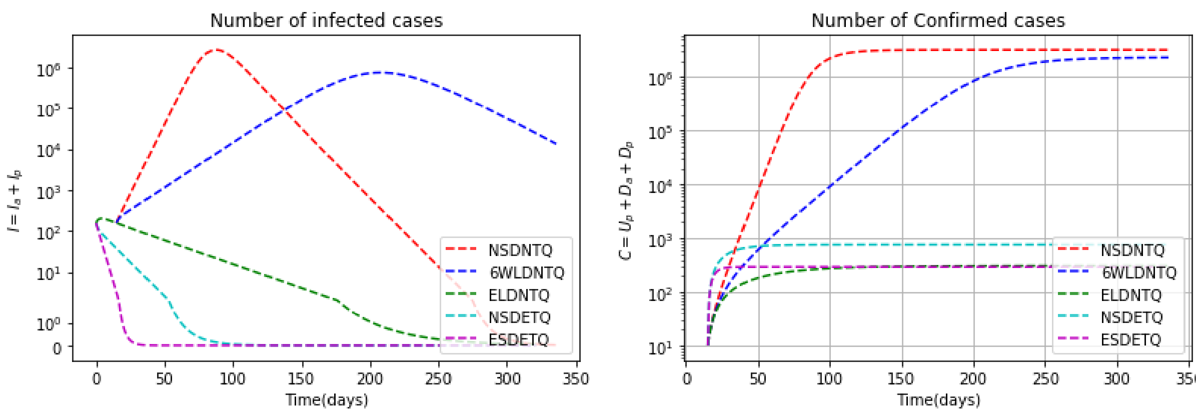

Figure 3.1: For parameter set I

The expected time for the pandemic to die is roughly given by

$$t_{end} \sim \frac{\ln(\text{peak infection number})}{|\mu^{\text{post-intervention}}|}$$

and so it is important that intervention schemes are implemented early and as strongly as possible.

2. Weak intervention ($R_0^{target} \gtrsim 1$)
In this case, a finite fraction of the population is eventually affected, but the intervention succeeds in reducing this from its original free value and in delaying the date at which the infections peak.
The parameter set for this case is-
Parameter set II [$R_0^{target} = 1.205$]- (i) SD: $u_l = 0.32$, $r_l = 0$, (ii) TQ: $u_l = 1$, $r_l = 0.536$ and (iii) SD-TQ: $u_l = 0.634$, $r_l = 0.24$. This choice corresponds to changing the free value of $R_0 = 3.766$ to a fixed target value $R_0^{target} = 1.205$ for all the three different strategies. μ remains positive and changes from the free value $\mu = 0.158$ to the values (i) $\mu = 0.0152$, (ii) $\mu = 0.032$ and (iii) $\mu = 0.0248$ respectively. Assuming that $S(0) \approx N$, the peak value of infections, $I^{(m)}$, (which is proportional to the number of hospitalizations required) and the number of days, $t^{(m)}$, to reach this peak value are approximately given by the following relations

$$I^{(m)} \approx \frac{\sigma}{\gamma_e + \sigma}\left(1 - \frac{1 + \ln R_0}{R_0}\right) N$$

$$t^{(m)} \approx \frac{ln[I^{(m)}/I(0)]}{\mu} \approx \frac{\ln(N/c)}{\mu}$$

$$1 - \bar{x} - e^{-R_0 \bar{x}} = 0$$

where γ_e is an effective recovery rate and c is a constant that depends on initial infected population and other disease parameters and $\bar{x}$ is the fraction of population that is eventually affected. of the equation

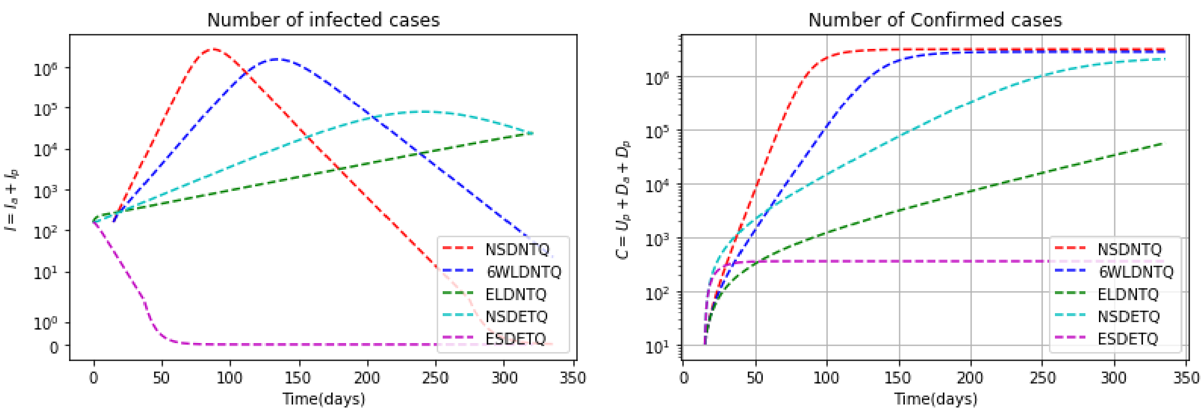

Figure 3.2: For parameter set II

From these figures it is clear that TQ is more efficient in controlling the pandemic than lockdowns that only implement SD. A larger magnitude of μ, corresponding to a faster suppression of the pandemic, is obtained from TQ than that from SD. For TQ to be successful 1). it has to be based on contact-tracing and 2). it is necessary that testing numbers are increased proportional to the number of new detected cases.

3.3 Predictions for India from extended SEIR model

σ	γ	α	R_0 (free)	R_0^{eff}	PDC	Time of peak	Total affected	Total deaths
0.5	0.2	0.67	2.28	1.33	2,456,630	2nd week September	45%	1,936,000
0.5	0.2	0.9	2.16	1.3	686,770	3rd week August	42%	550,700
0.4	0.143	0.67	2.82	1.45	2,956,600	3rd week September	55%	2,363,700
0.4	0.143	0.9	2.65	1.41	832,890	4th week August	52%	676,140

Predictions for India with different choices of parameter values (PDC- peak daily cases)

4 Conclusion and Outlook and open questions, new directions

Summarizing altogether, the modified version of the SEIR model,taking into account the non-clinical interventions namely social distancing(SD) and testing-quarantine(TQ), has successfully helped us to analyze the different intervention protocols to contain the Covid-19 pandemic.The calculations above show that even after a 68 day nationwide lockdown (25/03/2020-31/05/2020),some significant amount of infection(both asymptomatic and pre-symptomatic) were still remaining and the pandemic had continued.To keep the pandemic at bay, only tool remains is that to continue the high rate of testing to those who show positive Covid-19 symptoms,isolating them and contact tracing all contacts of positive patients and quarantining them.Combining this process with that of social distancing, we can keep the pandemic in check.
Keeping in mind that the future is unpredictable,it seems that the only way forward is to take appropriate measures regarding effective contract tracing, testing and isolation and subsequently increasing the awareness and responsibility among the citizens to abide by social/ physical distancing norms. Future works regarding modifying the simulation and analysis maybe as follows:-

- Different newer antigen testing modalities(for example,rapid antigen test,CBNAT and TrueNAT) may be incorporated in a certain population for rapid detection of disease followed by quarantining/isolation.
- We can explore the impact of isolating asymptomatics though testing,where the current policy is to test only symptomatics.
- State-wise sensitivity analysis of various policy levers to help the level of adherence/enforcement of each.
- Rapid antibody testing should also be incorporated in a certain locality to ascertain the percentage of exposed individuals.

At the end, we want to conclude that the number of infected individuals may increase in India if human-to human transmission and poor personal preventive measure continues as the testing status is still very poor in India.However, there still remains some undetermined questions that will help us understand the transmission dynamics of the epidemic. We want to emphasize the uncertainty of accessible authentic data, specially concerning the accurate baseline number of infected individuals which may guide us to inappropriate predictions because nowadays there is a general tendency to suppress the symptoms and testing themselves in fear of being social outcasts.

The code used to generate graphs is at
`https://github.com/iamsreeman/Nonlinear-dynamics/`

References

[1] Arghya Das, Abhishek Dhar, Srashti Goyal and Anupam Kundu. *Covid-19: analysis of a modified SEIR model, a comparison of different intervention strategies and projections for India.* arXiv:2005.11511 [q-bio.PE], 2020.
`https://arxiv.org/abs/2005.11511`