Associative memory and Hopfield Neural Network Project -3

Atsushi oba (s1230128), Hitoshi Ikuta (m5221106)
Chowdhury Md Intisar (m5212106), Yunosuke Teshima (m5221151)

May 9, 2018

1 Hopfield Neural Netowork

The hopfield neural network is pattern recognition network which is driven by the auto-associative memory property. It is a energy based model. It is a single layer neural network with feed back.

2 Transition State of the Neuron

The network returns the state of each neuron. The output of each neuron is a binary number -1,1 or 0,1.The output of the network is also called state vector since it contains the state of each neuron in the set. In each run or iteration the state of a certain neuron changes. Thus, only one neuron changes its state at each run which results in the change of the energy state of whole network. At a certain state, the energy function converges to the local or global minima which is called the attractor. Below is a mathematical formulation.

2.1 Mathematical Formulation

Let us assume the current state of the network is $v^k = [v_1^k, v_2^k, ..., v_n^k]$

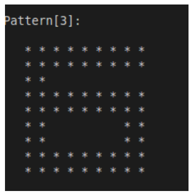
Thus we calculate the **energy function** as follows

$$E = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}v_iv_j + \sum_{i=1}^{n}\theta_i v_i \tag{1}$$

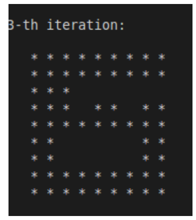
The equation is guranteed to converge to the attractor after certain iteration.

Inorder to save the new pattern we use the following equation,

$$W_{ij} = \sum_{m=1}^{p} s_i^m s_j^m \tag{2}$$

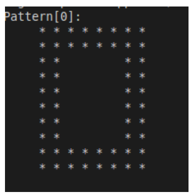


(a) Training pattern for 6



(b) After error rate increased

Figure 1: Output for pattern 6



(a) The pattern 0 for training

(b) After error rate increased

Figure 2: Output for pattern 0

2.2 Program Output

The main advantage of Autoassociative network is that it is able to recover pattern from the memory using just a partial information about the pattern. We can usually recall the pattern in two different ways. One is the synchronous and another is asynchronous. The first method has its limitation. As a result we focus only on the asynchronous method for the recovery.

Since the source code was already given by our Professor we have just changed the parameter noise rate to different value to observed the performance of the network. In doing so we have observed some interesting property and feature of the network.

With noise rate of 0.05 the network restore the learned pattern without any error. Within two iteration the network succesfully restored all the patterns. The learned patterns included 0, 2, 4, 6.The same result we have found for 0.10 and 0.15 noises rate. But increasing the noise rate over 0.20 the network showed some interesting features as depicted in the below image.

From the Figure-1 and Figure-2 we although reconstructed the digit 6 succesfully, the pattern 0 has been overlapped with some other digit.

2.3 Discussion and Conlusion

It is to be noted that, when we store more patterns we get interception between them (its called a crosstalk) and each pattern add some noise to other patterns.

From the given resource and slide we know that the capacity of a network is 0.15N where N is the number of neuron. Saving more than **0.15N** pattern

(a) Two energy minima for two different patterns

(b) Two energy minima overlapped due to error and merged to create single engery minima

Figure 3: A simple example of spurious minima problem

in the neuron will result in confusing the network and saturated. But here the reason of the such near mis-match was the **spurious minima** of the hopfield network. Few points have been presented below to depict the side effects of the network due to increase noise rate.

- **The spurious minima** of the network occurs **when two nearby energy minima combine to make a new minima in wrong place**.
- Increasing the noise rate to a certain extend causes the network to converge to a **new minima**. As a result the network fails to retrieve it's saved pattern. The network gets stucked to the saddle point which is depicted in the Figure 3.
- If the rate of noise is limited to a certain extent the network usually succesfully finds the attractor after few iteration. In this case 0.05 to 0.30 was the limit. Beyond that it gets stucked to new minima.

In the figure 3 the blue curve is the energy minima for one pattern and the green for another. But after error rate has been increased it is possible that during the recall the energy minima of the corrupted image overlaps with the engery minima of completely different pattern. Thus failing to recreate the original intended pattern. So the new energy minima is the **red** curve which is a mixture of two patterns. This kind of problem also occurs when the network is taught a lot of patterns more than its limit i.e. more than 0.15N.

****The image of figure -3 for spurious minima has been taken from the video lecture of Sir Geoffrey Hinton**