NATIONAL ECONOMICS UNIVERSITY

FACULTY OF MATHEMATICAL ECONOMICS

Business Analysis
Winter 2023

FACIAL RECOGNITION
DSEB 63 - GROUP 04
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Abstract

This study explores the intricacies of face recognition and detection, emphasizing the integration of distance measurement techniques and various machine learning methodologies. Adopting a holistic approach, the research delves into the preprocessing of data across different scales, incorporating well-known datasets such as AT&T, Pins, LFW (Labeled Faces in the Wild), and YaleB. The core objective is to assess the adaptability and efficiency of distinct machine learning strategies in the context of facial recognition tasks, with a special focus on the variability and challenges posed by diverse data scales and qualities. Representing a demo for face recognition with different layer for detection and classification for visualizing the nearest neighborface due to various distance measurement methods.

The methodology section details the implementation of advanced distance measurement algorithms, which are pivotal in enhancing the accuracy of face detection, especially in varying lighting conditions and poses. The study evaluates several machine learning models, including but not limited to, convolutional neural networks (CNNs), support vector machines (SVMs), and deep learning frameworks, to ascertain their efficacy in capturing the nuanced features of facial data.

A significant portion of the research is dedicated to preprocessing techniques, which are crucial for standardizing the datasets and improving model performance. These techniques address challenges such as alignment, normalization, and scaling, ensuring that the machine learning models are trained on high-quality, consistent data.

The experimental results, particularly with the AT&T dataset, provide insightful comparisons between the different machine learning methods, highlighting their strengths and limitations in the context of face recognition. The study concludes with a comprehensive evaluation of the tested methodologies, offering recommendations for their application in practical scenarios and suggesting directions for future research in enhancing the accuracy and reliability of face recognition systems. This exploration contributes to the ongoing advancements in biometric technology, proposing a framework that integrates diverse machine learning techniques and preprocessing methods to optimize facial recognition accuracy.

Key words : Face recognition, dimension reduction, classification model, face detection

Contribution and Work Progress

Role:

Name	Role	Contribution (%)
Nguyen Quy Duong	Group leader	23
Duong Nhat Thanh	Member	23
Nhan Yen Trang	Member	22
Vu Mai Dung	Member	17
Hoang Anh Quan	Member	15

Overall, the team demonstrated effective collaboration and consistently maintained high standards throughout the process. The entire group successfully completed the required tasks, displaying a thorough understanding and comprehensive research. There is room for enhancement in coding practices, specifically in terms of optimization for reuse and better management. The presentation was well-executed, showcasing clear visuals and evident preparation. Each member stayed aligned with the project's progress, significantly influencing and contributing to the overall outcome of the project.

Working progress:

Phase	**Name**	**Work**	**Performance**
1	Nguyen Quy Duong	- Paper summarization PCA, LDA - Distance based implement - Progress tracking - Detect misclassified images in each distances based and visualiztion - Applying distance based to classification problem - Coding review and comment - Baseline coding	100%
	Duong Nhat Thanh	- Paper summarization PCA, modular PCA, LDA, PCA with SVM, PCA for face cognition, eigenfaces,... - Apply PCA, modular PCA, 2d-PCA, kPCA to the AT&T dataset. - Build a demo application.	100%
	Nhan Yen Trang	- Paper summarization - Build a face detection function	100%

		- Build a face recognition system using eigenface, which is derived from PCA - Apply Distance base	
	Hoang Anh Quan	- Paper summarization - Implement PCA	100%
	Vu Mai Dung	- Paper summarization Survey, Distance based - Implement PCA - Distance based, distance measurements:	100%
2	Nguyen Quy Duong	- Using different dimension reduction into machine learning classifications model - Tuning and cross validate - Preprocessing dataset - Progress tracking - Coding review and comment - Baseline coding	100%
	Duong Nhat Thanh	- Apply various classifications models such as Logistic Regression, SVM, KNN together with above PCA extensions to improve the accuracy for face recognition. Insight report and baseline coding	95%
	Nhan Yen Trang	- Using general classifications models such as SVM, KNN, RandomForest, Logistic Regression with PCA and tune to improve the accuracy for AT&T dataset - Using KernelPCA with SVM, tune and cross validate - Visualization the incorrectly predicted images of each model	95%
	Hoang Anh Quan	- Using different dimension reduction into machine learning classifications model - Tuning and cross validate	95%
	Vu Mai Dung	- Method for the dimension reduction - Classifiers - Cross-validation - Printout the true and predicted labels - Models extract face parts	95%

3	Nguyen Quy Duong	- Applying to bigger dataset - Survey dataset - Face detection - Visualization and presentation preparation - Report writing - Tuning for YaleB dataset	95%
	Duong Nhat Thanh	Utilize an expanded dataset for facial recognition purposes. Adjust the image size, convert them to grayscale, and implement the methodology employed in the AT&T dataset. Incorporate supplementary PCA techniques, including LDA, in conjunction with SVM and KNN.	95%
	Nhan Yen Trang	- Find a larger dataset (LFW dataset) - Applying distance base - Using general classifications models such as SVM, KNN, Logistic Regression with PCA - Using Kernel PCA combined with the SVM classification model - Tuning and cross validation - Visualization the incorrectly predicted images of each model	95%
	Hoang Anh Quan	- Find and use bigger dataset - Prepare Presentation	95%
	Vu Mai Dung	- Adjust the image size, grayscale - Visualization - Presentation	95%

Github working environment : [Github - Group 04]

Problem

In today's rapidly evolving digital landscape, the integration of face recognition technology has become indispensable across various sectors, reflecting a significant shift towards enhancing security, convenience, and efficiency. At a macro level, governments worldwide are embracing digitalization by incorporating face recognition for identity verification, enhancing airport and border crossing security, and combating fraud, cyber threats, and trafficking. Notable examples include the implementation of biometrical facial recognition systems in banking and healthcare for advanced diagnosis and fraud prevention. A striking instance of its application was observed in China, where police utilized a smart monitoring system equipped with live facial recognition to apprehend a suspect of economic crime in a crowd of 50,000 at a concert, showcasing the technology's precision and scalability.

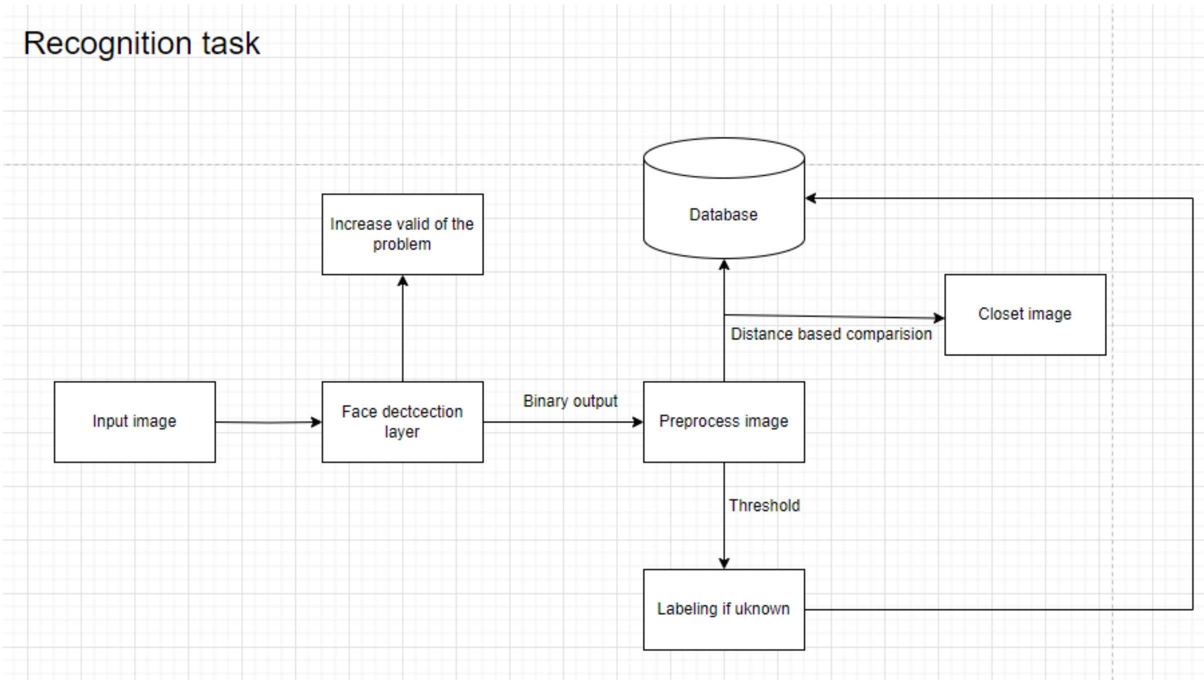
On a micro scale, face recognition technology has permeated everyday life, revolutionizing the way we interact with devices and systems. From attendance monitoring to secure device unlocking, leading technology giants like Apple and Samsung have integrated facial recognition as a key security feature in their devices, underscoring its growing importance and reliability. Further than daily life application, face detection and recognition have also been used as a security system in Crown Casino in Australia for avoiding fraud activities made by gamblers.

This dual utility of face recognition, spanning critical national security measures to personal device security, highlights the technology's versatility and its pivotal role in driving forward the digital age, making it a cornerstone of modern technological advancement and machine learning innovation.

Introduction

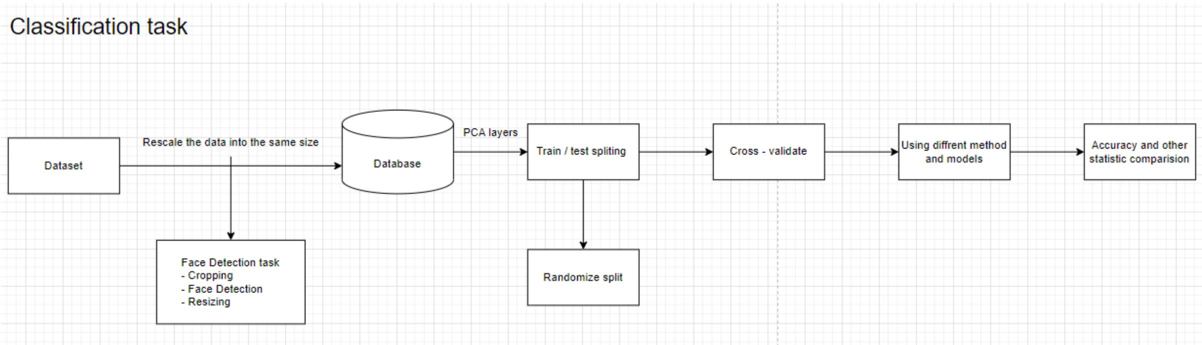
The field of face recognition has witnessed significant advancements over the past few decades, driven by the relentless pursuit of creating systems that can identify or verify a person's identity with high accuracy. This pursuit has led to the exploration of various methodologies that aim to improve the efficiency and reliability of face recognition systems. In this context, our research delves into a comprehensive examination of the face recognition problem through a structured three-phase approach, focusing on dimension reduction techniques, classification models, and enhancement strategies to boost the overall performance of these systems.

The first phase of our study emphasizes the preprocessing of input images, which involves a critical step of face detection followed by the recognition process. This phase lays the foundation by utilizing an array of dimension reduction techniques, including Principal Component Analysis (PCA), Linear Discriminant Analysis (LDA), Kernel PCA, Incremental PCA, Sparse PCA, Modular PCA, and Two-Dimensional PCA (2D PCA). These techniques are pivotal in reducing the computational complexity and enhancing the feature extraction process, making them suitable for handling high-dimensional data inherent in face images. Moreover, we provide a comprehensive overview of different distance measurement metrics and their application in the face recognition task, ensuring the selection of the most similar face images based on the closest distance criterion.



Pipeline in face recognition task using different databases and distance measurement.

In the second phase, our research extends into the domain of classification models. By integrating the dimension reduction techniques mentioned in the first phase, we explore various classification algorithms to discern the most effective approach for the face recognition task. This phase is characterized by rigorous experimentation with different preprocessing methods, aiming to refine the quality of input images and, consequently, improve the accuracy of the recognition system. The use of the AT&T dataset in this phase serves as a benchmark for evaluating the performance of our proposed methods under controlled conditions.



Pipeline in face classification task using different databases and distance measurement.

The third phase of our study seeks to further enhance the model and processing techniques by incorporating larger datasets and deploying advanced methods for feature extraction. This phase is dedicated to fine-tuning the models and applying cross-validation techniques to mitigate the risk of overfitting, ensuring the generalizability of our approach across diverse datasets. The introduction of a bigger dataset in this phase allows for a more robust evaluation of the model's performance, facilitating the exploration of new methods that can lead to a significant increase in accuracy.

Our research does not merely focus on the technical aspects of face recognition but also emphasizes the importance of understanding the entire process, including the intricacies of the data and the interpretation of results. An essential part of our analysis involves the examination of misclassified images, which offers valuable insights into the limitations of the current models and guides future improvements. Through a meticulous process of model tuning and validation, we aim to develop a face recognition system that not only achieves high accuracy but also maintains robustness across various conditions and datasets.

In summary, our study presents a holistic approach to tackling the face recognition problem, employing a systematic investigation into dimension reduction techniques, classification models, and model enhancement strategies. By addressing the challenges at each stage of the process and continuously seeking improvements, we endeavor to push the boundaries of what is possible in the realm of face recognition technology.

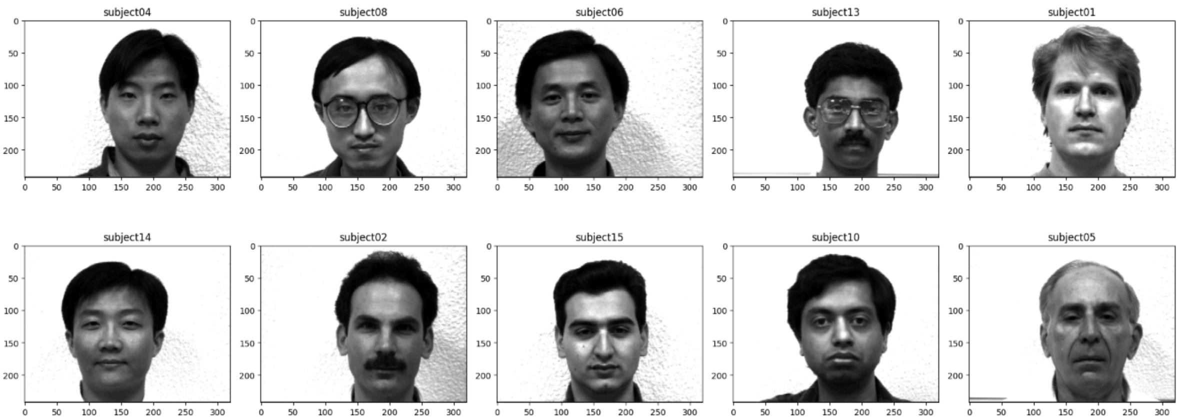
Exploring database analysis

AT&T dataset

The initial dataset utilized in this study is the AT&T dataset, comprising ten unique images for each of 40 distinct subjects. These images encapsulate variations across several aspects; they were captured at different times, incorporating changes in lighting conditions, facial expressions (including eyes open/closed, and smiling/not smiling), and facial attributes (presence or absence of glasses). The photographs were consistently taken against a dark, uniform background, with the subjects positioned upright, facing forward, albeit with a slight allowance for lateral movement. The image files are formatted in PGM, with dimensions of 92x112 pixels, each pixel representing one of 256 gray levels. The dataset is systematically arranged into 40 separate directories, corresponding to the subjects, labeled as sX, where 'X' denotes the subject identifier, ranging from 1 to 40. Within each directory, the ten images of the respective subject are stored, named as Y.pgm, where 'Y' represents the specific image number for that subject, ranging from 1 to 10.

YaleB dataset

The dataset originally from Yale University contains 165 GIF images, featuring 15 different subjects labeled from subject01 to subject15. Each subject is represented with 11 unique images, each corresponding to a distinct facial expression or configuration. These variations include center-light, with glasses, happy, left-light, without glasses, normal, right-light, sad, sleepy, surprised, and wink. However, there is a notable exception in the dataset: the image for subject04 with the 'sad' expression is corrupt. To address this issue, the 'sad' image for subject04 has been replaced with an additional 'normal' expression image. This substitution ensures the dataset maintains a consistent number of images per subject.



Pins face recognition dataset

It is a collection of facial images of various celebrities and public figures, including actors, musicians, politicians, athletes, and social media influencers. The dataset includes a total of 107,818 images of 1,063 different individuals.

The images are provided in JPEG format and are of varying sizes and qualities. Some images are high-quality studio portraits, while others are candid snapshots taken in various settings. The images also vary in terms of lighting, poses, facial expressions, and other factors.

In addition to the images, the dataset also includes a CSV file that contains metadata for each individual, including their name, gender, and the number of images available for that individual.

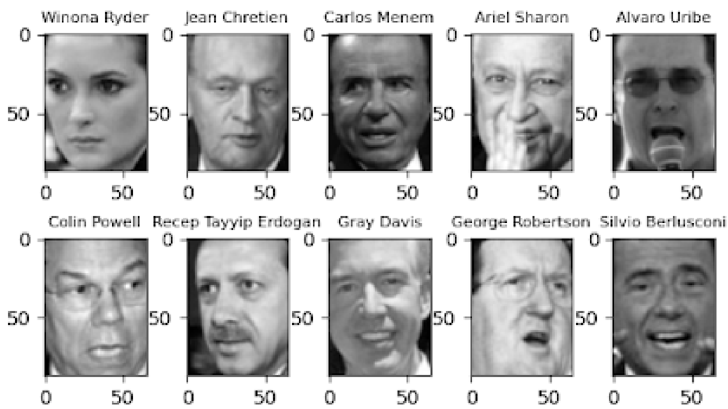
The dataset is intended for use in research related to facial recognition and computer vision, and could be used to train machine learning algorithms for tasks such as face recognition, facial expression analysis, and age estimation.

Label Face in Wild (LFW)

Labeled Faces in the Wild (LFW) is a database of face photographs designed for studying the problem of unconstrained face recognition. This database was created and maintained by researchers at the University of Massachusetts, Amherst (specific references are in the Acknowledgements section).

To access the LFW dataset conveniently, we utilized the `fetch_lfw_people` function provided by the scikit-learn library with specific parameters such as '*min_faces_per_person = 20*' and '*resize = 0.4*'.

After setup, the Dataset includes 3023 face images, each with a height of 50 pixels and a width of 37 pixels, represented by vectors with 1850 features. There are 62 layers, each containing at least 20 images, the distribution between layers varies, some containing up to several hundred images. This variation reflects the diversity of the dataset, ensuring full representation of individual facial variations.



Literature review

Face recognition problem

The evolution of face recognition technology has been a journey of continuous innovation, marked by significant milestones across different periods:

- 1960s-1970s: The inception of automated face recognition featured geometric methods, focusing on manual measurements of facial features. This era laid the groundwork, despite the simplicity and limited robustness of the techniques.

- 1980s: The field witnessed a shift with the introduction of statistical methods, notably Principal Component Analysis (PCA), leading to the development of the eigenfaces technique. This represented a significant advance in capturing facial information efficiently.

- 1990s: The integration of neural networks and machine learning transformed face recognition, improving adaptability and robustness. The use of convolutional neural networks (CNNs) marked a pivotal improvement in feature extraction and recognition accuracy.

- 2000s: The advent of 3D face recognition and model-based approaches addressed some limitations of 2D methods, particularly in handling variations in pose and lighting, enhancing the systems' adaptability and accuracy.

- 2010s-present: The deep learning revolution, with the deployment of deep convolutional neural networks, has dramatically enhanced the performance of face recognition systems, achieving unprecedented accuracy levels. This period is characterized by the use of advanced algorithms that can learn discriminative features from large datasets, pushing the boundaries of accuracy and reliability.

- Challenges and Ethical Considerations : Despite technological advancements, face recognition systems face challenges such as biases in datasets, privacy concerns, and the need for robustness against adversarial attacks. The ethical implications of widespread face recognition, including surveillance and loss of anonymity, have sparked significant debate, leading to calls for regulation and more responsible use of the technology.

The trajectory of face recognition technology showcases a field that has evolved from simple geometric models to sophisticated deep learning frameworks, consistently breaking new ground to improve accuracy, efficiency, and adaptability. The current focus is on enhancing the technology's ethical, privacy, and fairness aspects while exploring new frontiers like multimodal biometrics.

Solution Approach

The advancement of face recognition technology has led to diverse approaches, each with unique strengths and challenges.

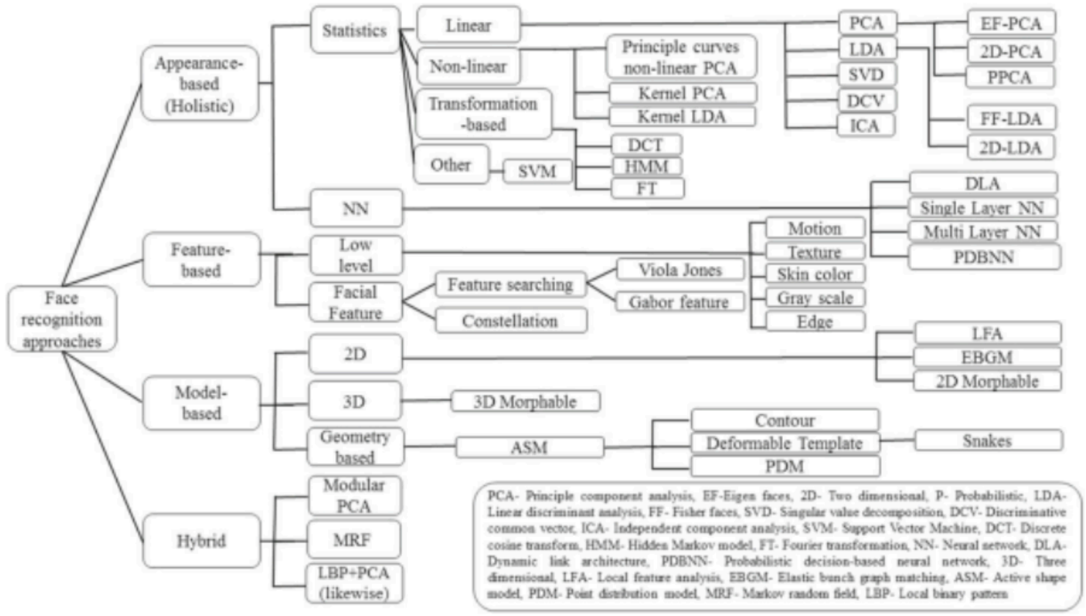


Fig. 3 Classification of face recognition approaches

A comprehensive survey on techniques to handle face identity threats: challenges and opportunities

Feature-based Approaches: Focus on geometric features of the face, offering robustness to variations in position, size, orientation, and lighting. However, they struggle with automatic feature detection and require critical decisions on feature importance, affecting recognition accuracy. An example includes the Histogram of Oriented Gradients (HOGs), known for compact image representation and fast matching.

Holistic Approaches: Utilize the global appearance of the face, achieving higher classification rates with the help of machine learning models like SVM and NN. Despite their success, they are sensitive to lighting conditions, poses, and backgrounds.

Model-Based Approaches: Involve constructing detailed models of the face, either 2D, 3D, or geometrical, to capture facial variations. While offering high accuracy and deep facial pattern understanding, these methods are resource-intensive.

Hybrid Approaches: Combine multiple techniques to improve accuracy and provide comprehensive insights. These methods, however, demand a deep understanding of the domain and face computational complexity challenges.

The choice of approach depends on the application's requirements, dataset size, and available resources. While feature-based and holistic methods suit simple, resource-constrained scenarios, model-based and hybrid approaches are preferred for applications demanding higher accuracy and detailed facial analysis.

Methodology

Metric evaluation

Evaluation Metric Based on Accuracy for Classification and Recognition Tasks in Face Detection and Face Recognition

In the domain of face detection and face recognition, the effectiveness of a system is predominantly evaluated through its accuracy, which serves as a critical metric for both classification and recognition tasks. This section delineates the framework for assessing the accuracy metric, elucidating how it is employed to gauge the performance of algorithms in the context of face detection and recognition.

Accuracy in Face Detection:
Face detection involves identifying and localizing faces within images. The accuracy metric here quantifies the proportion of correctly identified faces over the total number of faces (true positives) plus the number of false positives, where the system incorrectly identifies a face. It is calculated as:

Accuracy in Face Detection

The accuracy for face detection is given by the equation:

$$\text{Accuracy} = \frac{\text{Number of Correctly Detected Faces}}{\text{Total Number of Faces (True Positives + False Positives)}} \tag{1}$$

High accuracy in face detection signifies that the algorithm effectively distinguishes faces from non-faces and accurately localizes them within the image frame.

Accuracy in Face Recognition:
In face recognition, accuracy represents the system's ability to correctly identify or verify a person from a face image. It is defined as the ratio of correct identifications to the total number of identification attempts, encompassing both true positive identifications and false negative identifications, where the system fails to recognize a known face. The formula is expressed as:

Accuracy in Face Recognition

The accuracy for face recognition is defined as:

$$\text{Accuracy} = \frac{\text{Number of Correct Identifications}}{\text{Total Number of Identification Attempts (True Positives + False Negatives)}} \tag{2}$$

For a face recognition system, achieving high accuracy implies that the algorithm not only recognizes the individuals it has been trained on but also maintains this performance consistently across various conditions, including variations in lighting, pose, and facial expressions.

Evaluation Procedure:
The evaluation involves a systematic process where the dataset, typically comprising numerous images with labeled faces, is divided into a training set and a testing set. The model is trained on the training set and then tested on the testing set, where the accuracy metric is computed based on the model's performance. The results are often cross-validated to ensure reliability and generalizability of the model's performance across different data subsets.

Dimension Reduction

Principal Component Analysis - PCA

Assuming we have a collection of human faces, all with the same pixel dimensions (e.g., r×c grayscale images). If we gather M different pictures and vectorize each image into L=r×c pixels, we can represent the entire dataset as an L×M matrix (denoted as matrix A), where each matrix element corresponds to the pixel's grayscale value.

Principal Component Analysis (PCA) is applicable to any matrix, resulting in vectors known as principal components. Each principal component has a length equal to the column length of the matrix. These components are orthogonal, meaning the dot product of any two is zero. Consequently, the various principal components form a vector space, allowing each matrix column to be represented as a linear combination (i.e., weighted sum) of these principal components.

The way it is done is to first take $C = A - a$ where a is the mean vector of the matrix A. So C is the matrix that subtract each column of A with the mean vector a. Then the covariance matrix is

$$S = C \,.\, C^T$$

from which we find its eigenvectors and eigenvalues. The principal components are these eigenvectors in decreasing order of the eigenvalues. Because matrix S is a L×L matrix, we may consider to find the eigenvectors of a M×M matrix $C \,.\, C^T$ instead as the eigenvector v for $C \,.\, C^T$ can be transformed into eigenvector u of $C \,.\, C^T$ by $u = C \,.\, v$. However, it is common to express u as a normalized vector, denoting that its norm is 1.

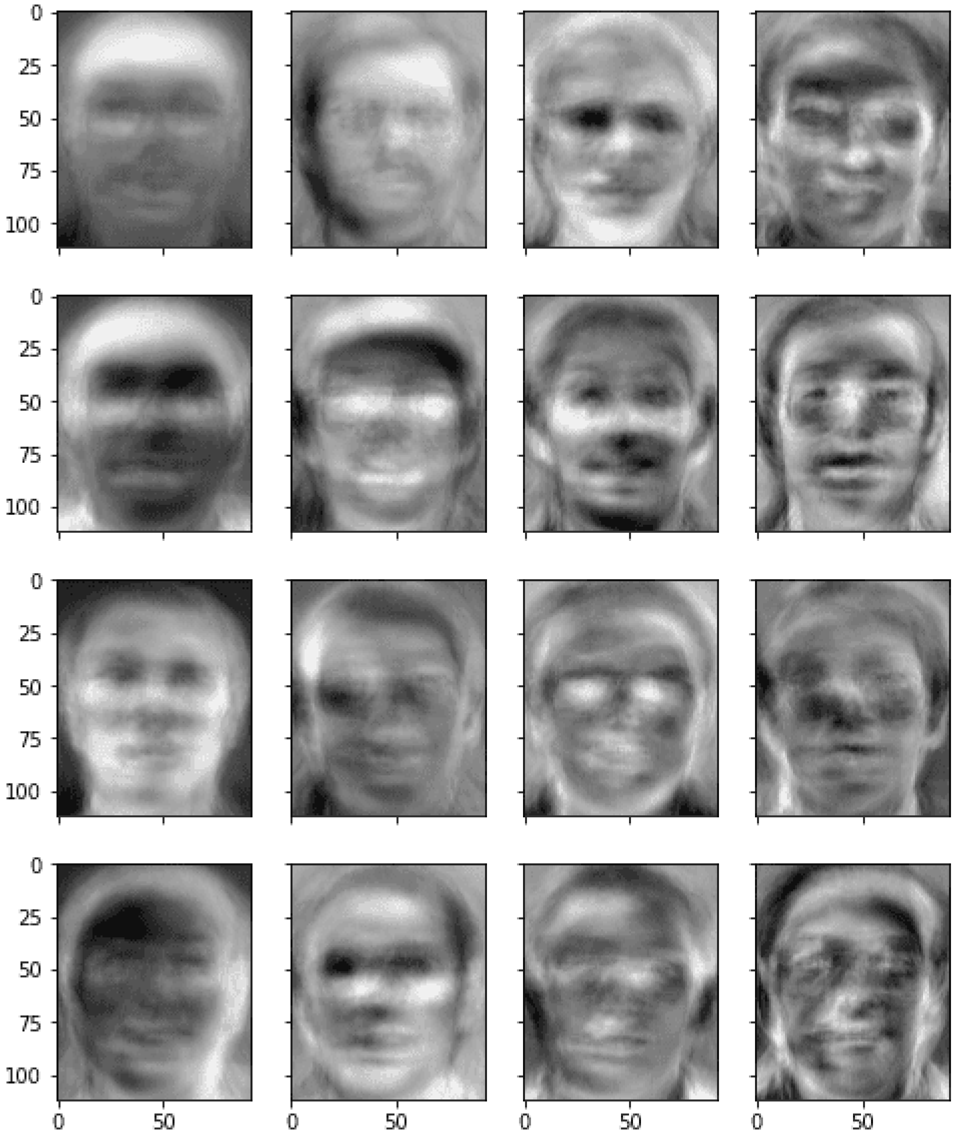
The significance of the principal component vectors of matrix A, or equivalently the eigenvectors of S = C . CT, lies in their role as crucial directions for constructing the columns of matrix A. The importance of each principal component vector can be gauged from its corresponding eigenvalue. A higher eigenvalue indicates a more valuable principal component vector, carrying more information about matrix A. Consequently, we can selectively retain only the first K principal component vectors. In the context of face picture datasets represented by matrix A, these top K principal component vectors are the most significant "face pictures" and are referred to as **eigenface** pictures.

For a given face picture, we can project its mean-subtracted version onto the eigenface pictures using a vector dot-product. The result indicates how closely the face picture is related to the eigenfaces, with zero expected if there is no relation. With K eigenfaces, we can find K dot-products for any given face picture, presenting the results as weights indicating the picture's relation to the eigenfaces. These weights are typically represented as a vector.

Conversely, given a weight vector, we can reconstruct a new face by summing up each eigenface multiplied by its corresponding weight. Denoting the eigenfaces as matrix F (L×K), and the weight vector w as a column vector, the construction of a face picture for any w can be expressed as:

$$z = F . w$$

The resulting vector z, obtained by applying the weight vector w to the eigenfaces, is a column vector with a length of L. Given that only the top K principal component vectors are utilized, the resulting face picture is expected to be distorted while retaining certain facial characteristics.



Eigenfaces in AT&T dataset

Kernel PCA

Kernel PCA is a nonlinear dimensionality reduction technique that enhances face recognition by transforming facial images into a high-dimensional feature space. This transformation allows for the extraction of principal components that capture significant data variance, even in complex, nonlinear image data. The process involves:

1. Preprocessing: Standardizing face images to ensure consistent analysis.
2. Feature Extraction: Using a kernel function (like Gaussian RBF) to perform Kernel PCA, mapping data to a higher-dimensional space to identify key features.
3. Dimensionality Reduction: Selecting the most relevant principal components to reduce the data's dimensionality while retaining crucial information.
4. Classification: Employing classifiers on the transformed data to recognize faces, evaluated using standard performance metrics.

Kernel PCA's ability to uncover complex patterns significantly boosts the accuracy and robustness of face recognition systems, making it a powerful tool for handling the inherent nonlinearities in facial recognition tasks.

Linear Discriminant Analysis

Linear Discriminant Analysis (LDA) is a popular technique used in pattern recognition, including face recognition, primarily for the purpose of dimensionality reduction while preserving as much of the class discriminatory information as possible. In the context of face recognition, LDA seeks to find a projection that maximizes the between-class scatter while minimizing the within-class scatter. Here's how LDA is typically used in face recognition for dimensionality reduction:

1. Preprocessing:

Before applying LDA, faces are usually preprocessed. This might involve normalizing the lighting, cropping to include only the face, and resizing the images to a common dimension.

2. Computing the Mean Faces:

Calculate the mean face for each class (person) and the overall mean face for all the training images. The mean face represents the average face in a particular class or in the entire dataset.

3. Calculating Scatter Matrices:

- Within-class scatter matrix (S_W): This matrix represents the scatter (variance) of the face images within each class. It's calculated by summing up the covariance matrices of each class, considering the difference between the face images and the mean face of their respective class.

- Between-class scatter matrix (S_B): This matrix represents the scatter between different classes. It's calculated based on the differences between the class mean faces and the overall mean face, weighted by the number of samples in each class.

4. Solving the Eigenproblem:

The goal is to find the linear combinations of features (in this context, pixels) that best separate the classes. This is achieved by solving the generalized eigenvalue problem for the matrix $S_W^{-1} S_B$. The eigenvectors corresponding to the largest eigenvalues will form the most discriminative linear subspace.

5. Projecting the Faces:

Once the eigenvectors (also known as Fisherfaces when used in the context of face recognition) are found, the face images are projected onto this new subspace. This transformation from the high-dimensional space to a lower-dimensional space preserves the class-discriminatory information while reducing dimensionality.

Modular PCA

PCA is unable to provide a better recognition rate for large variations in illumination and pose. Under the conditions of varying illumination and pose, some local information of the face does not vary. PCA is unable to take advantage of this. This is shown in the below figure. We can see for the varying illumination case that only one of the sub-images is affected due to illumination variation if we were to cut the image to 4 sub-images. Hence weights of the face regions not affected will closely match with weights of the same individual under normal lighting conditions.

2DPCA

Face Model Construction

We implement a weighted-2DPCA to deal with some practical situations in which some face images in the database are difficult to identify due to their poses (front or profile) or their qualities (noise, blur).

Training data $D = \{(A^{(i)}, w_i), i = 1,..., N\}$

Algorithm 1: Construct proposed face model

Step 1: Compute the mean image

$$\overline{\mathbf{A}} = \frac{\sum_{i=1}^{N} w_i \mathbf{A}^{(i)}}{\sum_{i=1}^{N} w_i} \tag{1}$$

Step 2: Compute matrix

$$\mathbf{G} = \frac{\sum_{i=1}^{N} w_i \left(\mathbf{A}^{(i)} - \overline{\mathbf{A}}\right)^T \left(\mathbf{A}^{(i)} - \overline{\mathbf{A}}\right)}{\sum_{i=1}^{N} w_i} \tag{2}$$

Step 3: Compute eigenvectors $\{\mathbf{\Omega}_1, \mathbf{\Omega}_2, ..., \mathbf{\Omega}_n\}$ and eigenvalues $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ of G.

Feature Extraction

First, a projection point of image A on 2DPCA space is matrix $(X_1, X_2, X_3,..., X_n)$

$$\mathbf{X}_k = \left(\mathbf{A} - \overline{\mathbf{A}}\right)\mathbf{\Omega}_k, k = 1, ..., d \tag{3}$$

Second, the matrix is projected on PCA space to convert matrix to vector and reduce the dimension.

SVM for Face Identification

To apply SVM in face recognition, we use One-Against-All decomposition to transform a multi-class problem to a set of two-class problems.

Training dataset $\{(x^i, y^i);\ x^i \in R^n;\ y^i \in \{-1; 1\};\ i = 1,..., N$ is transformed to series of $D_k = \{(x^i, y^i);\ y^{(i)}{}_k \in \{-1, 1\}\}$

where

$$y_k^{(i)} = \begin{cases} +1 & y^{(i)} = k \\ -1 & y^{(i)} \neq k \end{cases} \tag{11}$$

Algorithm 2 is used to compute the discriminant functions corresponding to D_k.

$$f_k(\mathbf{x}) = \sum_{i=1}^{N} \alpha_i y_k^{(i)} K\left(\mathbf{x}^{(i)}, \mathbf{x}\right) + b \tag{12}$$

In classification phase, we use the following rule to identify the class for input x.

$$k = \arg\max_k \left(f_k(\mathbf{x})\right) \tag{13}$$

Train test split method

In the exploration of machine learning models, particularly in the context of face recognition or any image classification tasks, the method of splitting the dataset into training and testing sets is crucial for evaluating the model's performance. This paper details two distinct approaches to partitioning the dataset for such evaluations:

Method 1: Partitioning into Subsets A, B, C

Initial Setup

The initial dataset consists of an array of data points, represented here as [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The objective is to divide this dataset into three distinct subsets labeled A, B, and C.

Partitioning Process

The dataset undergoes a random partitioning process where a random partition function is employed to allocate the image files into the three subsets. This randomness ensures that the distribution of data across the subsets is unbiased and representative of the overall dataset.

Image File Transfer

Following the partitioning process, image files are systematically transferred from the original dataset to the designated subsets A, B, and C. This transfer is governed by the partitions generated by the random partition function, ensuring each subset receives its respective portion of the dataset.

Experimental Trials

The model undergoes three separate experimental trials to ensure the reliability and consistency of the classification results. Each trial involves using two subsets for training the classifier and the remaining one for evaluation. For instance, if A and B are used for training, C is used for testing, and this rotation continues across trials.

Performance Evaluation
After conducting the three trials, the average performance scores are calculated. This averaging process is instrumental in providing a comprehensive assessment of the classifier's performance across different subsets, offering insights into the model's generalization capabilities and stability.

Method 2: Train-Test Split with Stratification

Stratified Split
This method involves splitting the dataset into training and testing sets with an option to specify the size of the test set. The split is done in a way that maintains the percentage of samples for each class consistent across both training and testing sets—this is known as stratification. Stratification ensures that both training and test sets are representative of the overall dataset, particularly important in datasets with imbalanced class distributions.

Variability in Test Size
The flexibility of this method allows for adjusting the size of the test set, which can be varied to understand its impact on the model's performance. Different test sizes can provide insights into how much data the model needs to train effectively and how it performs with various amounts of unseen data.

Performance Evaluation
The model is trained on the training set and evaluated on the test set. The performance metrics obtained are crucial for understanding the model's effectiveness, generalizability, and robustness. The stratification ensures that the evaluation is fair and representative of the model's ability to generalize from the training data to unseen data.

In summary, both methods provide structured approaches to partitioning data for model evaluation, with the first method focusing on a multi-subset partitioning and trial-based evaluation, and the second emphasizing a stratified split with adjustable test sizes. Each method has its advantages, catering to different experimental needs and providing comprehensive insights into the classifier’s performance.

Distance Measurement

In the realm of face recognition, accurately quantifying the similarity or dissimilarity between facial feature vectors is pivotal. We employ various distance measurements, integral to the algorithm's efficacy in matching and distinguishing facial features. The choice of distance metrics is inspired by established research in the field, and each metric is tailored to capture different aspects of the data's underlying structure.

Various distance metrics have been developed and are pivotal in the feature space to quantify the differences between facial images effectively. We adopt several distance measurements as described by seminal works in the field (Grudin, 1997; Yambor and Draper, 2002; Phillips et al., 1999, 2000; Cekanavicius and Murauskas, 2002), which are integral to our face recognition framework:

Table 1: Summary of Distance Measurements in Face Recognition

Distance Type	Formula
Minkowski Distance	$d(X,Y) = \left(\sum_{i=1}^{n} \lvert x_i - y_i \rvert^{p}\right)^{1/p}$
Manhattan Distance	$d(X,Y) = \sum_{i=1}^{n} \lvert x_i - y_i \rvert$
Euclidean Distance	$d(X,Y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$
Squared Euclidean Distance	$d(X,Y) = \sum_{i=1}^{n} (x_i - y_i)^2$
Angle-based Distance	$d(X,Y) = 1 - \cos(X,Y)$
Correlation Coefficient-based Distance	$d(X,Y) = 1 - r(X,Y)$
Mahalanobis Distance	$d(X,Y) = \sqrt{\sum_{i=1}^{n} z_i x_i y_i}$
Weighted Manhattan Distance	$d(X,Y) = \sum_{i=1}^{n} z_i \lvert x_i - y_i \rvert$
Weighted SSE Distance	$d(X,Y) = \sum_{i=1}^{n} z_i (x_i - y_i)^2$
Weighted Angle-based Distance	$d(X,Y) = 1 - \sum_{i=1}^{n} z_i x_i y_i$
Chi Square Distance	$d(X,Y) = \sum_{i=1}^{n} \frac{(x_i - y_i)^2}{x_i + y_i}$
Canberra Distance	$d(X,Y) = \sum_{i=1}^{n} \frac{\lvert x_i - y_i \rvert}{\lvert x_i \rvert + \lvert y_i \rvert}$
Modified Manhattan Distance	$d(X,Y) = \frac{\sum_{i=1}^{n} \lvert x_i - y_i \rvert}{\sum_{i=1}^{n} \lvert x_i \rvert \sum_{i=1}^{n} \lvert y_i \rvert}$
Modified SSE-based Distance	$d(X,Y) = \frac{\sum_{i=1}^{n} (x_i - y_i)^2}{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2}$
Weighted Modified Manhattan Distance	$d(X,Y) = \frac{\sum_{i=1}^{n} z_i \lvert x_i - y_i \rvert}{\sum_{i=1}^{n} \lvert x_i \rvert \sum_{i=1}^{n} \lvert y_i \rvert}$
Weighted Modified SSE-based Distance	$d(X,Y) = \frac{\sum_{i=1}^{n} z_i (x_i - y_i)^2}{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2}$

These distance measurements are foundational in defining the algorithm's ability to effectively match and differentiate between facial features, thereby enabling accurate face recognition. Each distance metric has been chosen based on its ability to address specific challenges in the feature space, ensuring robust face recognition performance.

Machine Learning model

In the context of face recognition, the classification task is pivotal for identifying or verifying individuals from their facial images. This section of the methodology outlines the application of various machine learning models that are instrumental in the classification process, each with its unique strengths and suitability for different aspects of the face recognition problem. The models considered include Support Vector Machines (SVM), K-Nearest Neighbors (KNN), Logistic Regression, Decision Trees, Random Forests, and Gaussian Naive Bayes (GNB).

Support Vector Machines (SVM)

SVM is a powerful, widely-used learning algorithm for classification and regression tasks. It operates by finding the hyperplane that best divides a dataset into classes. In face recognition, SVM helps in distinguishing between the feature vectors of different individuals' faces by maximizing the margin between various classes. The effectiveness of SVM in high-dimensional spaces makes it particularly suitable for face recognition, where feature vectors often reside in high-dimensional spaces.

K-Nearest Neighbors (KNN)

KNN is a simple, intuitive, non-parametric method used for classification and regression. For face recognition, it classifies a given face image based on the majority vote of its 'k' nearest neighbors in the feature space. The algorithm assumes that similar instances are close to each other, making KNN a suitable choice for face recognition tasks where facial features of the same individual are expected to cluster together.

Logistic Regression

Though primarily used for binary classification, Logistic Regression can be extended to multiclass classification (via strategies like one-vs-rest), making it applicable for face recognition. It estimates probabilities using a logistic function, which is particularly useful for providing a probabilistic framework for binary classification tasks like verifying whether a given face matches a specific individual in the dataset.

Decision Trees

Decision Trees are a non-parametric supervised learning method used for classification and regression. They are straightforward to understand and interpret, as they mimic human decision-making by splitting data into subsets based on feature values, essentially 'learning' decision rules inferred from the training data. In face recognition, they can efficiently classify faces by learning the decisions based on the facial features extracted.

Random Forests

An ensemble of Decision Trees, Random Forests, combines multiple decision trees to produce a more effective and robust classification. By aggregating the predictions of individual trees, Random Forests reduce the risk of overfitting, making them more reliable for face recognition tasks. They are particularly effective in handling large datasets with higher dimensionality, providing improved accuracy and robustness over single Decision Trees.

Gaussian Naive Bayes (GNB)

GNB is a simple probabilistic classifier based on applying Bayes' theorem with the assumption of independence between the features. In the context of face recognition, GNB works well in cases where the dimensionality of the data is high, which is often the case with facial images. Despite its simplicity, GNB can achieve high accuracy in face recognition, especially when the assumption of feature independence holds reasonably well.

Implementation and Evaluation

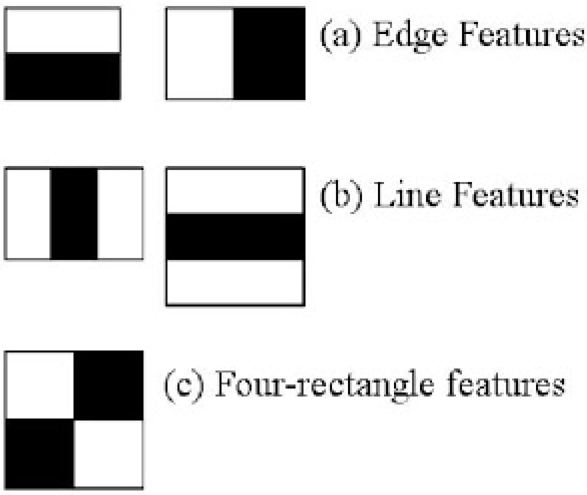
Each of these models will be trained on a dataset consisting of labeled facial images, where the labels correspond to the identities of the individuals in the images. The models will learn to associate the input features (extracted facial features) with these labels. The performance of each model will be rigorously evaluated using standard metrics such as accuracy utilizing a separate test set to ensure the models generalize well to new, unseen data.

In conclusion, the selection of machine learning models in face recognition tasks is crucial for achieving high accuracy and robustness. The chosen models each offer unique advantages and are collectively capable of providing a comprehensive understanding of the dataset's underlying patterns, thereby ensuring effective and efficient face recognition.

Face detection

The Haar feature-based cascade classifier, introduced by Paul Viola and Michael Jones in 2001, is a machine learning technique for object detection, particularly effective in face detection.

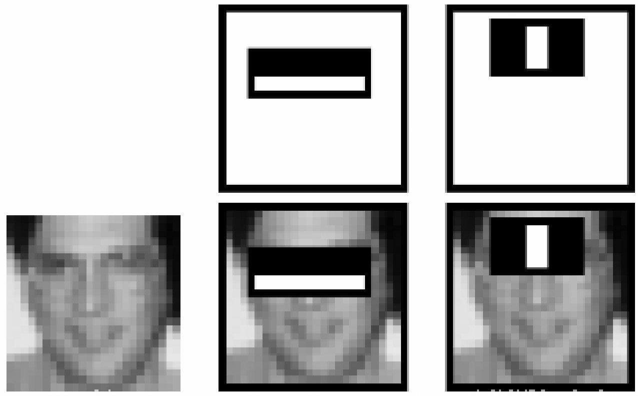
It involves training a classifier with numerous positive (faces) and negative (non-faces) images, followed by feature extraction using Haar features. These features, akin to convolutional kernels, are calculated by subtracting the sum of pixels in black areas from those in white areas within the image.



image

To compute a vast array of features, every potential size and position of the kernel is utilized. This requires calculating the sum of pixels under both the white and black rectangles for each feature. To streamline this process, the concept of integral images was introduced. Integral images simplify the task of summing pixels, regardless of their quantity, reducing it to a procedure that involves just four pixels.

Despite calculating numerous features, many are not useful. For instance, consider an image below where the top row highlights two effective features: one that captures the typically darker area of the eyes compared to the nose and cheeks, and another that notes the darkness of the eyes relative to the nose bridge. However, applying these same features to the cheeks or elsewhere may not be relevant. To sift through the 160,000+ features and identify the most valuable ones, the Adaboost algorithm is employed.



image

Adaboost is pivotal in selecting the most informative Haar features by applying each feature to training images, distinguishing between faces and non-faces. Misclassified images gain higher weight, directing the algorithm's focus towards more challenging cases in subsequent iterations. This process is repeated until the algorithm achieves the desired level of accuracy or gathers the necessary number of features.

These meticulously selected features are structured into a multi-stage cascade within the trained Haar classifier. Each stage in the cascade comprises a subset of these features and operates as a binary classifier. This setup swiftly filters out image regions unlikely to contain the object, optimizing computational efficiency. The cascade processes each stage sequentially, with the outcome of one stage affecting the next, until a final verdict is reached, striking a balance between accuracy and computational demand.

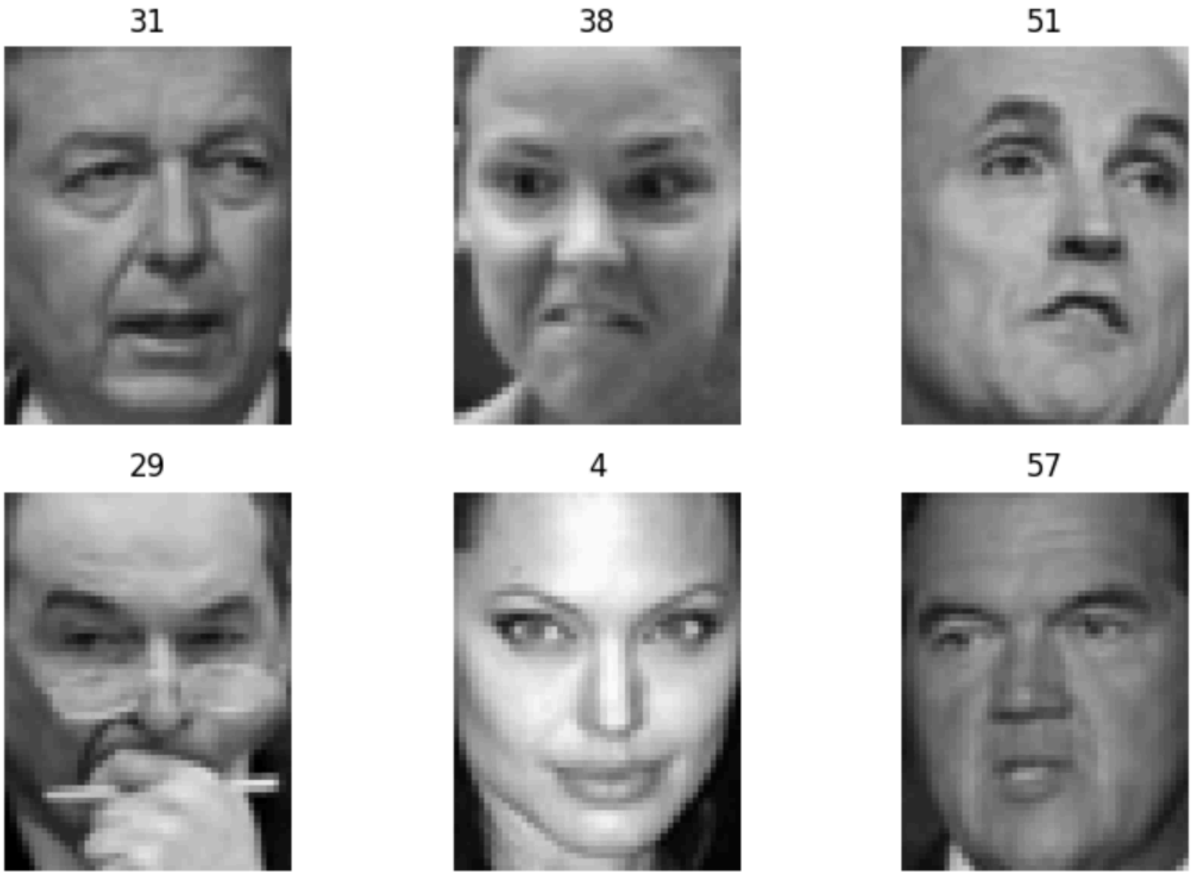
During the detection phase, the sliding window approach is employed, where a window with variable sizes and aspect ratios traverses the image across different regions and scales. At every position, Haar features within the window are calculated using the integral image technique, which the AdaBoost classifier evaluates to determine a match with the object's characteristics. If a match is detected, that window is marked as containing the object, pinpointing its location as a potential detection.

After applying Haar Cascade for face detection in the LFW dataset, there are several reasons why I cannot implement it for the LFW dataset:

The images in the LFW dataset have been pre-processed for face alignment. This might make the face recognition task easier. However, it can decrease the performance of Haar Cascade.

The LFW dataset has a large diversity in angles, expressions, and lighting conditions. Haar Cascade Classifier works best with high-contrast images and faces that are directly facing forward. This diversity can make it challenging for Haar Cascade to accurately detect faces.

Using Haar Cascade Classifier requires careful tuning of parameters such as scaleFactor, minNeighbors, and minSize. In a diverse dataset like LFW, finding a single set of parameters that fits all images can be challenging.



To apply the Haar Cascade technique optimally, we utilized this method in the development phase of our demo application for face recognition, and the results were quite promising.

Result

Phase 1:

Using initial AT&T dataset with baseline PCA and distance measurement based on the stratify dataset and test size = 0.2

	Distance Metric	Precision	Recall	F1-Score	Accuracy
0	Euclidean	0.975	0.973333	80.000	0.9750
1	Manhattan	0.9625	0.953333	80.0000	0.9625
2	Cosine	0.9875	0.986667	80.0000	0.9875
3	Canberra	0.9625	0.953333	80.0000	0.9625
4	Squared Euclidean	0.975	0.973333	80.000	0.9750
5	Correlation Coefficient	0.9875	0.986667	80.0000	0.9875
6	Chi Square	0.0375	0.033333	80.0000	0.0375
7	Minkowski	0.9875	0.986667	80.0000	0.9875

With distance measurement having weighted and modifier estimation

	Distance Metric	Precision	Recall	F1-Score	Accuracy
0	Weighted Manhattan	0.976852	0.970370	80.000	0.975
1	Weighted Squared Euclidean	0.953704	0.943056	80.00	0.950
2	Weighted Angle-Based	0.949074	0.935648	80.00	0.950
3	Modified SSE-Based	0.953704	0.943056	80.00	0.950
4	Modified Manhattan	0.976852	0.970370	80.000	0.975
5	Modified Squared Euclidean	0.953704	0.943056	80.00	0.950
6	Weighted Modified Manhattan	0.976852	0.970370	80.000	0.975
7	Weighted Modified Squared Euclidean	0.953704	0.943056	80.00	0.950

Applying with LDA and stratify method with the same size

	Distance Metric	Precision	Recall	F1-Score	Accuracy
0	Euclidean	0.975	0.973333	80.000	0.9750
1	Manhattan	0.9625	0.953333	80.0000	0.9625
2	Cosine	0.9875	0.986667	80.0000	0.9875
3	Canberra	0.95	0.940000	80.00	0.9500
4	Squared Euclidean	0.975	0.973333	80.000	0.9750
5	Correlation Coefficient	0.9875	0.986667	80.0000	0.9875
6	Chi Square	0.0	0.0	80.0	0.0000
7	New chisq	0.0	0.0	80.0	0.0000
8	Minkowski	0.9875	0.986667	80.0000	0.9875

Phase 2:

Baseline running with strafiy method

Classifier	Accuracy Mean	Accuracy STD
KNN	0.853	0.03
Logistic Regression	0.975	0.02
Random Forest	0.934	0.03
SVC	0.94	0.02
Decision Tree	0.59	0.05
GaussianNB	0.87	0.04

Tunning

Dimension Reduction		Components	Best Param	Accuracy	Tune method
	KNN	50	clf__n_neighbors': 3, 'clf__weights': 'distance'	96,25%	
	Logistic	50	clf__C': 100.0, 'clf__solver': 'lbfgs'	100%	

PCA

GridSeac h - 5 fold

	Random Forest	50	clf__max_depth': None, 'clf__min_samples_split': 2, 'clf__n_estimators': 200	98,75%	
	SVC	50	clf__C': 0.001, 'clf__gamma': 'scale', 'clf__kernel': 'linear'	98,75%	
	Decision Tree	50	clf__max_depth': None, 'clf__min_samples_leaf': 1, 'clf__min_samples_split': 2	58,75%	
	GNB	50		92,50%	
	Logistic		kpca__kernel': 'linear', 'kpca__gamma': 0.1, 'clf__solver': 'lbfgs', 'clf__C': 2.154434690031882	98,75%	
	KNN		kpca__kernel': 'linear', 'kpca__gamma': 0.021544346900318832, 'clf__weights': 'distance', 'clf__n_neighbors': 3	96,25% - 100%	
	Random Forest		kpca__kernel': 'poly', 'kpca__gamma': 0.004641588833612777, 'clf__n_estimators': 200, 'clf__min_samples_split': 2, 'clf__max_depth': None	92,5% -100%	
	SVC		kpca__kernel': 'poly', 'kpca__gamma': 0.46415888336127775, 'clf__kernel': 'linear', 'clf__gamma': 'auto', 'clf__C'	98,75%	
	Decision Tree		kpca__kernel': 'linear', 'kpca__gamma': 10.0, 'clf__min_samples_split': 6, 'clf__min_samples_leaf': 1, 'clf__max_depth': None	51,25% - 66,25%	
KPCA	GNB		kpca__kernel': 'linear', 'kpca__gamma': 0.001	92,50%	Randomized Search - 5 fold

Phase 3:

Label Face in Wild (LFW)

Dimension Reduction	Model Classification	Components	Param	Accuracy	Tune method
KPCA	SVC	150	'kpca__gamma': 0.1, 'kpca__kernel': 'linear'	60.39%	GridSeach - 3 fold
	Logistic		'C': 0.001, 'max_iter': 100	61%	GridSeach - 3 fold
PCA	Logistic	150	'C': 1, 'max_iter': 800	57%	GridSeach - 3 fold
	SVM		'C': 100, 'gamma': 0.0001, 'kernel': 'rbf'	63%	GridSeach - 5 fold
PCA	SVM	150	'svc__C': 1, 'svc__gamma': 0.005	67%	

YaleB dataset

	Classifier	Training Accuracy	Test Accuracy	Mean CV Accuracy	Std CV Accuracy
0	SVM	1.000000	0.939394	0.863533	0.057656
1	Logistic Regression	1.000000	1.000000	0.901709	0.029976
2	KNN	0.787879	0.878788	0.666952	0.047531
3	Decision Tree	1.000000	0.848485	0.705413	0.060861
4	Random Forest	1.000000	0.969697	0.856410	0.043277
5	Naive Bayes	1.000000	0.848485	0.727350	0.050215

Pins Face Recognition

Classifier	Test size	Dimnesion Reduction	Accuracy (%)
PCA + Logistic	80/20	PCA	11.79
PCA + k-NN	80/20	PCA	8.25
PCA + SVM	80/20	PCA	12.85
2DPCA + k-NN	80/20	2DPCA	7.81
2DPCA + SVM	80/20	2DPCA	14.56
KernelPCA + k-NN	80/20	KernelPCA	7.31

KernelPCA + SVM	80/20	KernelPCA	9.80
LDA + k-NN	80/20	LDA	1.77
LDA + SVM	80/20	LDA	1.38
2DPCA + Logistic	80/21	2DPCA	11.63

The comprehensive evaluation of the face recognition system, employing machine learning models alongside dimensionality reduction techniques, particularly PCA, LDA, Kernel PCA, modular PCA, 2D PCA has yielded significant insights. The utilization of the eigenface system provided a basic yet informative exploration into PCA, demonstrating satisfactory face recognition results under controlled conditions. However, the reliance solely on eigenfaces revealed limitations, especially for practical, real-world applications where the environmental variables are unpredictable.

In testing simpler machine learning models like SVM and Logistic Regression, it was observed that these models performed well with datasets characterized by minimal noise and well-aligned facial images. The accuracy and predictive quality of these models were notably influenced by the precision in image preprocessing, including cropping and alignment, underscoring the importance of high-quality input data.

Despite the success in scenarios with single or a small number of facial images, challenges were encountered when dealing with images containing multiple faces or when operated under less controlled conditions. The study emphasized the necessity for enhanced robustness in the face recognition system and suggested the integration of additional discriminant features to improve performance and reliability.

In conclusion, the project highlighted the effectiveness of machine learning models in conjunction with preprocessing techniques and dimensionality reduction for face recognition. Nonetheless, for deployment in real-world scenarios, the system requires further refinements to address the limitations identified, ensuring it can robustly handle a broader range of face recognition challenges.

Conclusion and Future work

The exploration of face recognition systems has laid a solid foundation for advancing the technology further. Considering the insights gained and the current technological trends, the future work will focus on several key enhancements and innovations to elevate the face recognition capabilities. These future directions include:

Integration of Advanced Face Detection and Layered Approaches

Future developments will aim to integrate a more sophisticated face detection system that can seamlessly connect with an advanced layered architecture, forming a full pipeline demonstration. This will involve refining the detection algorithms to enhance accuracy and efficiency, particularly in varying lighting conditions, orientations, and backgrounds. The advanced layered approach will focus on modularizing the system, where each layer specializes in a specific task, leading to a more robust and flexible face recognition pipeline.

Hybrid Models and Advanced Algorithms for Improved Classification

The classification stage will be improved by employing a hybrid approach that combines the strengths of various machine learning models and deep learning architectures. By leveraging more advanced algorithms such as convolutional neural networks (CNNs), generative adversarial networks (GANs), and recurrent neural networks (RNNs), the system will enhance its capability to classify faces accurately, even in challenging scenarios. This approach aims to improve the generalization of the model, making it more adaptable to different faces and conditions.

Enhanced Preprocessing with Face Alignment and Scaling

Preprocessing techniques will be further developed, focusing on face alignment and scaling methods. These enhancements are crucial for standardizing the input data, ensuring that the face recognition system is not biased by variations in scale, pose, or alignment. Improved preprocessing will contribute to the overall accuracy and reliability of the system, enabling it to perform consistently across diverse datasets.

Incorporation of Age and Gender Detection

Adding an age and gender detection layer will broaden the system's applicability, allowing it to provide more detailed analyses. This feature will be particularly beneficial for real-time applications, offering enriched data that can be used for personalized experiences, security enhancements, and demographic analyses. The integration of these additional layers will require the system to be adept at multitasking while maintaining high performance and speed.

Handling Sequential and Complex Structured Data

Future iterations of the face recognition system will focus on effectively handling sequential and complex structured data. This is essential for applications involving dynamic scenes, multiple faces, and changing environments. Enhancements will include the ability to track faces over time, recognize faces in video sequences, and manage data from 3D imaging sources. Improving the system's ability to process and analyze this complex data will significantly enhance its applicability in real-world scenarios, from security systems to user interaction platforms.

By addressing these areas, the next phase of development will aim to create a more sophisticated, accurate, and versatile face recognition system that is well-suited for real-world applications and capable of providing real-time, detailed analyses of facial data.

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