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(1)

Solution:

Let $y = \sin 3x$

Let Δx & Δy be an small increment corresponding to x & y respectively.

Then,

$$y + \Delta y = \sin 3(x + \Delta x)$$

or, $$\Delta y = \sin 3(x + \Delta x) - y$$

$$= \sin(3x + 3\Delta x) - \sin 3x$$

$$= 2\cos \frac{3x + 3\Delta x + 3x}{2} \cdot \sin \frac{3x + 3\Delta x - 3x}{2}$$

$$= \frac{2\cos(3x + 3\Delta x) \cdot \sin\left(\frac{3\Delta x}{2}\right)}{\frac{3}{2}\Delta x \times \frac{2}{3}} \times \Delta x$$

or, $$\frac{\Delta y}{\Delta x} = \frac{2\cos(3x + 3\Delta x) \cdot \sin\left(\frac{3\Delta x}{2}\right)}{\left(\frac{3\Delta x}{2}\right) \times \frac{2}{3}}$$

Taking $\lim_{\Delta x \to 0}$ on both sides,

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\cos(3x + 3\Delta x) \cdot \sin\left(\frac{3\Delta x}{2}\right)}{\left(\frac{3\Delta x}{2}\right) \times \frac{2}{3}}$$

$$\therefore \frac{dy}{dx} = 3\cos(3x + 3 \times 0)$$

$$= 3\cos 3x$$

Answer

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(2)

(a)

Given,

$f(x) = \sin x$

For all $x \in [0, \pi]$, $f(x)$ has a finite value

so $f(x)$ is ~~differen~~ continous in $[0, \pi]$

Also,

$f'(x) = \cos x$ which exists for all $x \in (0, \pi)$

$\therefore f(x)$ is differentiable in $(0, \pi)$.

Also,

$f(0) = \sin 0 = 0$

$f(\pi) = \sin \pi = 0$

$\therefore f(0) = f(\pi)$

$\therefore$ Hence all condition of Rolle's theorem are satisfied.

So, There exists at least a point $c \in (0, \pi)$ such that

$\therefore f'(c) = 0$

$\cos c = 0$

$c = \pi/_2 \in (0, \pi)$

$\therefore$ Rolle's therom for this function is satisfied.

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(ii) $\lim_{x \to 2} \dfrac{\sqrt{x} - \sqrt{6-x^2}}{x-2}$

Soln:

$$= \lim_{x \to 2} \frac{\sqrt{x} - \sqrt{6-x^2}}{x-2} \times \frac{\sqrt{x} + \sqrt{6-x^2}}{\sqrt{x} + \sqrt{6-x^2}}$$

$$= \lim_{x \to 2} \frac{((\sqrt{x})^2 - (\sqrt{6-x^2})^2)}{(x-2)(\sqrt{x} + \sqrt{6-x^2})}$$

$$= \lim_{x \to 2} \frac{x - 6 + x^2}{(x-2)(\sqrt{x} + \sqrt{6-x^2})}$$

$$= \lim_{x \to 2} \frac{x^2 + x - 6}{(x-2)(\sqrt{x} + \sqrt{6-x^2})}$$

$$= \lim_{x \to 2} \frac{x^2 + 3x - 2x - 6}{(x-2)(\sqrt{x} + \sqrt{6-x^2})}$$

$$= \lim_{x \to 2} \frac{x(x+3) - 2(x+3)}{(x-2)(\sqrt{x} + \sqrt{6-x^2})}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}(\sqrt{x} + \sqrt{6-x^2})}$$

$$= \frac{2+3}{\sqrt{2} + \sqrt{6-2^2}}$$

$$= \frac{5}{\sqrt{2}+\sqrt{2}} = \frac{5}{2\sqrt{2}} \quad \#$$

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③

Solution:

$$f(x) = \begin{cases} 2x+3 & \text{for } x<1 \\ 4 & \text{for } x=1 \\ 6x-1 & \text{for } x>1 \end{cases}$$

Here,

Left hand limit,

$$\lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1^-} 2x+3 = 2\times1+3 = 5$$

Right hand limit,

$$\lim_{x \to 1^+} f(x)$$

$$= \lim_{x \to 1^+} 6x-1 = 6\times1-1 = 5$$

functional value,

$$f(1) = 4$$

$$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \neq f(1)$$

Hence, The above Given function is not Continous at $x=1$. But it can be made Continous by

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redefining the function as follows:

$$f(x) = \begin{cases} 2x+3 & \text{for } x<1 \\ 5 & \text{for } x=1 \\ 6x-1 & \text{for } x>1 \end{cases}$$

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(5) Given,

$P(Q) = 1 - 3Q$

$C(Q) = Q^2 - 2Q$

We have,

$R = P(Q) \times Q$

$= (1 - 3Q)\,Q$

$= Q - 3Q^2$

profit $(\pi) = R - C$

$= Q - 3Q^2 - (Q^2 - 2Q)$

$= 3Q - 4Q^2$

Now,

$$\frac{d\pi}{dQ} = 3 - 8Q$$

$$\frac{d^2\pi}{dQ^2} = -8$$

for minima & maxima,

$$\frac{d\pi}{dQ} = 3 - 8Q = 0$$

or $3 = 8Q$

or $Q = \frac{3}{8}$

So, when $Q = 3/8$, $\frac{d^2\pi}{dQ^2} = -8 > 0$

hence profit is maximum at $Q = 3/8$.

Now,

$$\text{Max. profit} = 3Q - 4Q^2$$

$$= 3 \times \frac{3}{8} - 4 \times \frac{3^2}{8^2}$$

$$= \frac{9}{8} - \frac{36}{64}$$

$$= \frac{72 - 36}{64}$$

$$= \frac{36}{64}$$

$$= \frac{9}{16}$$

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(6) $\lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x-\theta}$

Soln:

Here, Let $x \to \theta + h$ where $h \to 0$.

Now,

$= \lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x-\theta}$

$= \lim_{h \to 0} \frac{(h+\theta)\cot\theta - \theta\cot(h+\theta)}{h}$

$= \lim_{h \to 0} \frac{h\cot\theta + \theta\cot\theta - \theta\cot(h+\theta)}{h}$

$= \lim_{h \to 0} \frac{\theta(\cot\theta - \cot(h+\theta))}{h} + \lim_{h \to 0} \frac{h\cot\theta}{h}$

$= \theta \cdot \lim_{h \to 0} \frac{\left(\frac{\cos\theta}{\sin\theta} - \frac{\cos(h+\theta)}{\sin(h+\theta)}\right)}{h} + \cot\theta$

$= \theta \cdot \lim_{h \to 0} \frac{\cos\theta \cdot \sin(h+\theta) - \cos(h+\theta) \cdot \sin\theta}{h \cdot \sin\theta \cdot \sin(h+\theta)} + \cot\theta$

$= \theta \cdot \lim_{h \to 0} \frac{\sin(h+\theta-\theta)}{h\sin\theta \cdot \sin(h+\theta)} + \cot\theta$

$= \theta \cdot \lim_{h \to 0} \frac{\sin h}{h} \times \frac{1}{\sin\theta \cdot \sin(h+\theta)} + \cot\theta$

$= \frac{\theta}{\sin\theta \cdot \sin(\theta+\theta)} + \cot\theta = \theta\operatorname{cosec}^2\theta + \cot\theta.$

Answer

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(7)

$$\lim_{x \to 0} \frac{\ln(\tan x)}{\ln x}$$

Soln:

$$= \lim_{x \to 0} \frac{\ln(\tan x)}{\ln x} \quad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{x \to 0} \frac{\frac{1}{\tan x} \times \sec^2 x}{\frac{1}{x}}$$

$$= \lim_{x \to 0} \frac{x \sec^2 x}{\tan x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{x \cdot 2\sec x \cdot \sec x \cdot \tan x + \sec^2 x}{\sec^2 x}$$

$$= \frac{0+1}{1}$$

$$= 1$$

Answer

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Group-B

9 (i)

$y = e^{\cos x}$

Soln:

putting log on both sides.

$\ln y = \ln e^{\cos x}$

or, $\ln y = \cos x \ln e$

or $\ln y = \cos x$ $\quad [\because \ln e = 1]$

Differentiating both sides w.r. to x.

or, $\frac{d \ln y}{dx} = \frac{d \cos x}{dx}$

or $\frac{1}{y} \cdot \frac{dy}{dx} = -\sin x$

or, $\frac{dy}{dx} = -\sin x \times e^{\cos x}$

Answer.

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(ii) $\frac{dy}{dx} + y = e^x$

Soln:

Here,

$\frac{dy}{dx} + y = e^x$ which is in form of

$\frac{dy}{dx} + Py = Q$.

where,

$P = 1$

$Q = e^x$

for Integrating factor (I.F);

$IF = e^{\int 1 dx}$

$= e^{\int dx}$

$= e^x$

Now, Multiplying the above equation by I.F.

$\left(\frac{dy}{dx} + y\right) e^x = e^x \times e^x$

or $e^x \frac{dy}{dx} + e^x y = e^{2x}$

or $d(y.e^x) = e^{2x}$

Integrating on both sides,

$\int d(y.e^x) = \int e^{2x}$

or, $ye^x = \frac{e^{2x}}{2} + C$

Answer

~~Answer~~.

Solution,

Here,

$a=0, b=2, n=4$

$\therefore h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

The five points to be considered are $x_0=0, x_1=0.5, x_2=1$
$x_3=1.5, x_4=2.$

End point	$x_0=0$	$x_1=0.5$	$x_2=1$	$x_3=1.5$	$x_4=2$
$y=2x^2-1$	-1	-0.5	1	3.5	7

Hence, By using trapezoidal rule, we have:

$$\int_0^2 (2x^2-1)\,dx \approx \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right]$$

$$= \frac{0.5}{2}\left[-1 + 2\times(-0.5) + 2\times 1 + 2\times 3.5 + 7\right]$$

$$= 0.25(-1-1+2+7+7) = 3.5$$

Actual value $= \int_0^2 (2x^2-1)\,dx$

$$= \left[\frac{2x^3}{3} - x\right]_0^2 = \frac{2\times 2^3}{3} - 2 = 3.3333$$

$\therefore$ Absolute Error $= |3.3333 - 3.5| = 0.1667$ Ans.