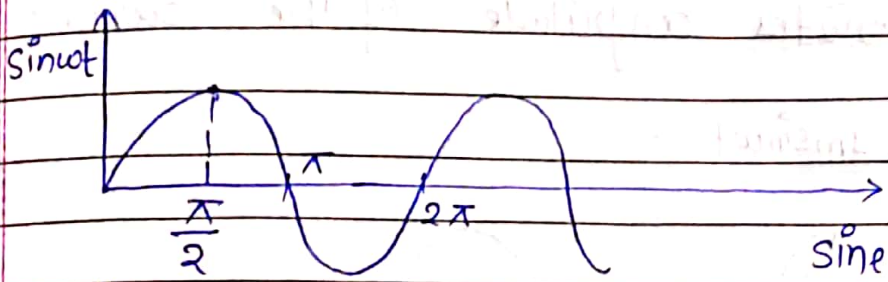
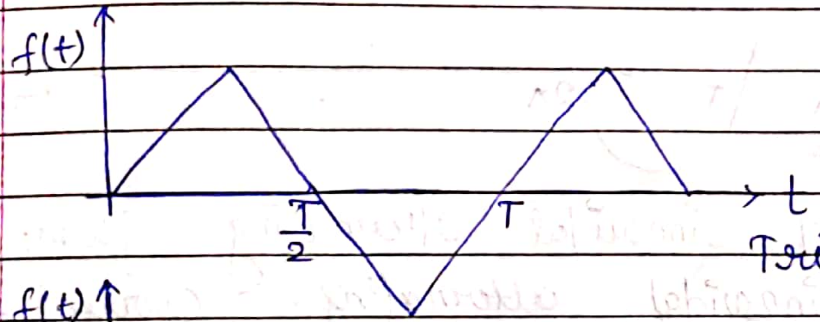
Unit-2

Single phase AC circuit.

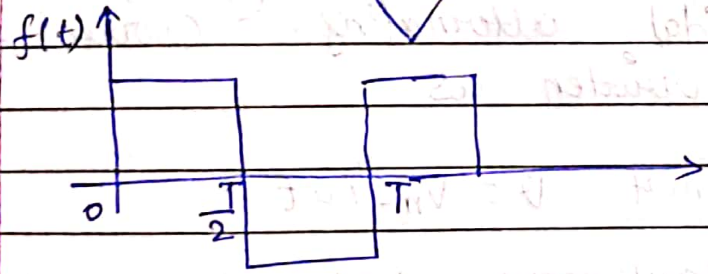
* **Alternating quantity:-**
quantity which varies periodically with time is known as Alternating quantity. It may be voltage or current. some wave forms of Alternating quantities are



Sine wave



Triangular wave.



Square wave.

- **Cycle**
It is the one complete set of positive and negative half cycle of any alternating quantity.

- **Time period:-**
It is the time required in second to complete the one cycle of any alternating quantity.

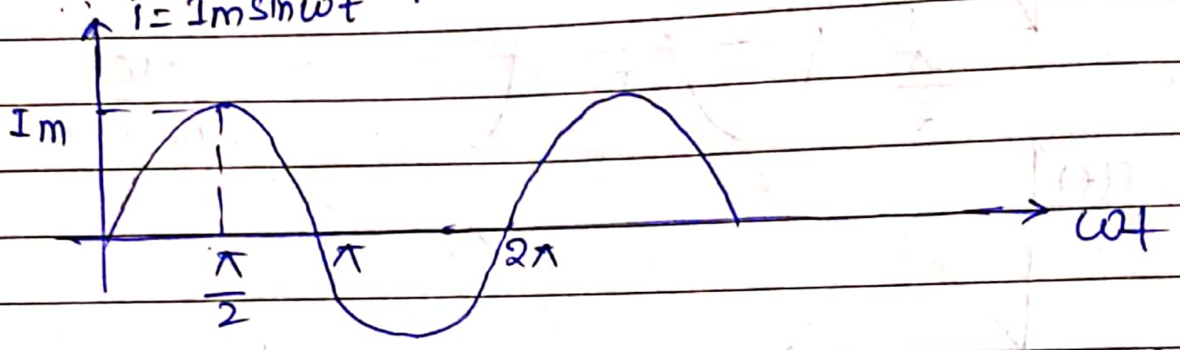
- **frequency:-** $f = NP/120$ (P = poles, N = revolution)
No. of cycles per second is known as the frequency of frequency of alternating quantity.

It is the reciprocal of the time and its unit is cycle/second or Hertz (Hz)

$$\left(f = \frac{1}{T} \text{ in Hertz}\right)$$

- Amplitude:-
It is the peak or maximum value of the alternating quantity in the given fig. I_m denotes amplitude of the current wave.

$i = I_m \sin \omega t$



- Equation of sinosuidal alternating quantity.
The sinosuidal alternating current and voltage is written as.

$$i = I_m \sin \omega t \qquad v = V_m \sin \omega t$$

i, v - instantaneous value of current & voltage

I_m, V_m - Maximum value / peak value.

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad f = \text{frequency in Hz}$$

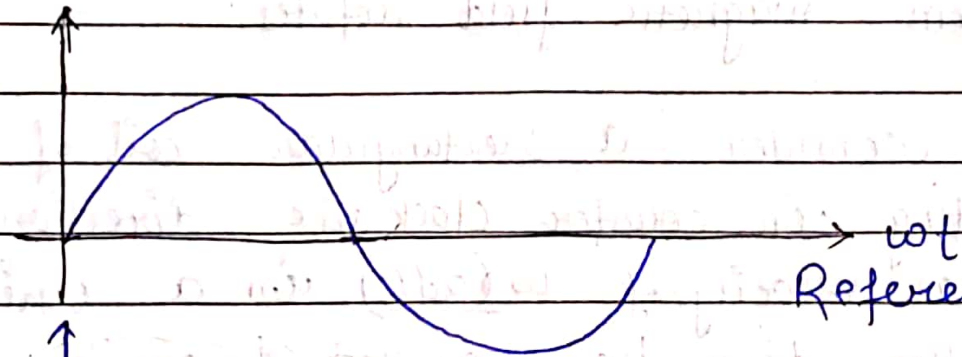
$$\left\{ X_L = \omega L \qquad X_C = \frac{1}{\omega C} \right\}$$

- Phase:-
Phase of any alternating quantity shows the position of wave at any time after it has passed through the zero position of reference.

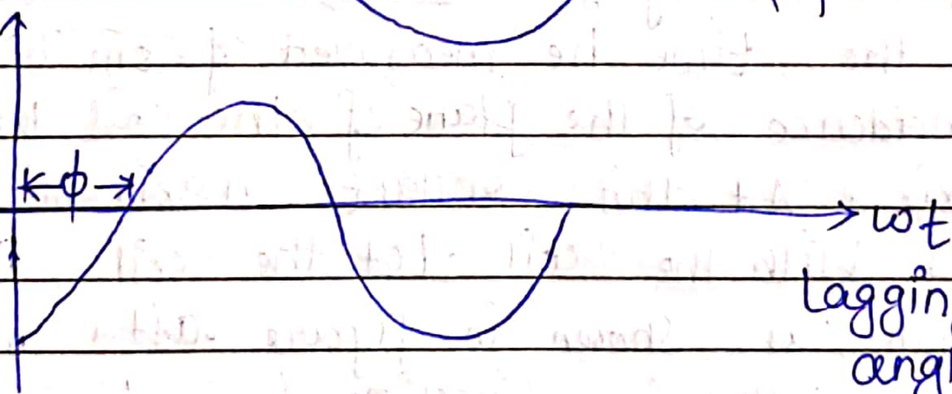
→ Phase angle is the angle of alternating quantity w.r.to reference position.

• Phase difference

When the maxima and minima of two sinosuidal alternating quantities (of same frequency) are not occur at the same instant of time, then these two quantities are said to have phase difference.

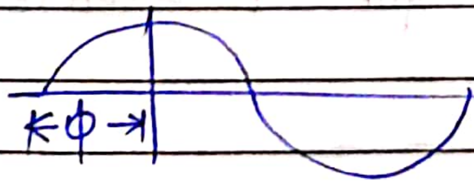


Reference wave.



Lagging wave by angle φ.

$i = I_m \sin(\omega t + \phi)$



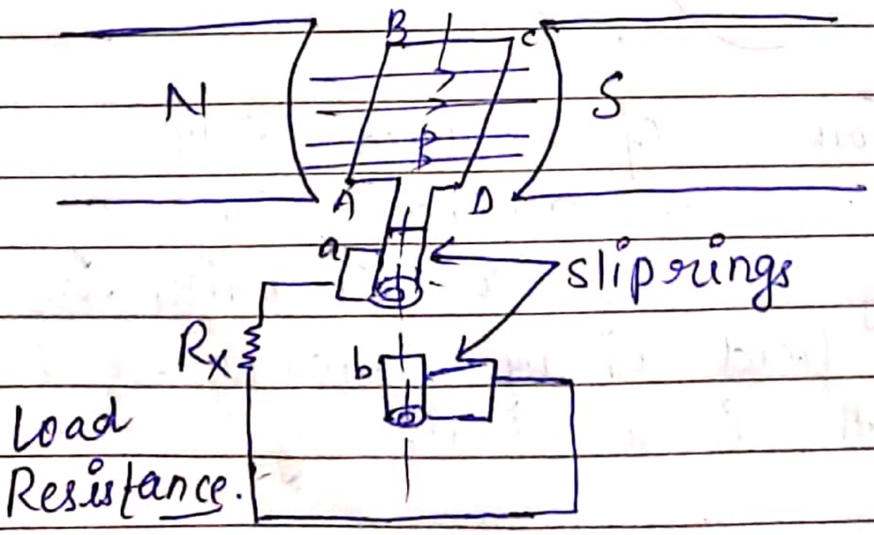
• Generation of Induced emf:-

We know that alternating emf can be generated either by rotating a coil with in a stationary magnetic field or by rotating a magnetic field with in a stationary coil.

The emf generated in either case will be of Sinusoidal wave form. The magnitude of emf generated in the coil ① depends up on the number of turns on the coil ② the strength of Magnetic field and the ③ speed at which the coil or Magnetic field rotates.

Now consider a rectangular coil of N-turns rotating in counter clockwise direction with angular velocity of ω (rad/s) in a uniform magnetic field rotates.

Now consider a rectangular coil of N-turns rotating in counter clockwise direction with angular velocity of ω (rad/s) in a uniform M.f. Let the time be measured from the instant of coincidence of the plane of the coil with the x-axis. At this instant maximum flux (ϕ) links with the coil. Let the coil, assume the position, as shown in figure after moving in counterclock wise direction for 't' seconds



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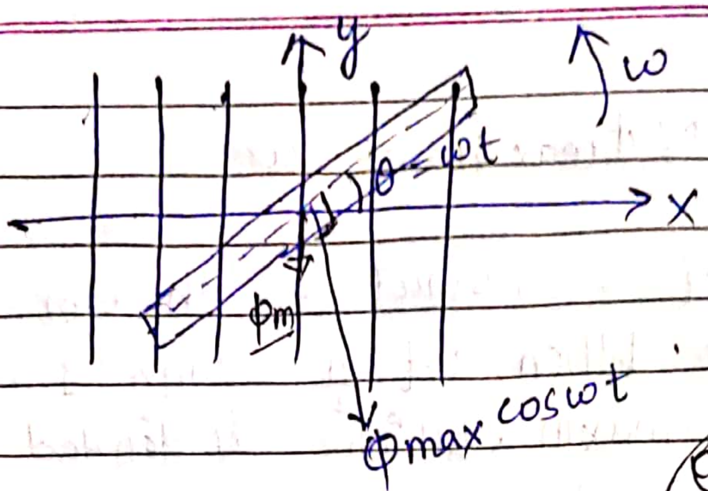

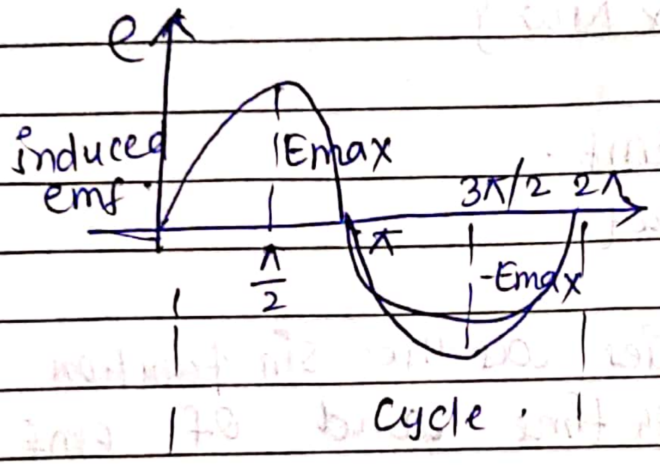

$\theta = \omega t$

$N\phi = N\phi max \cos\omega t$
↳ Total flux

→ The angle θ through which the coil has rotated in "t" seconds = ωt.

→ In this position the component of flux along perpendicular to the plane of coil = $\phi max \cos\omega t$.

→ Hence φ Linkage ~~of~~ the coil at this Instant is equal to {No. of turns on coil x Linking flux}

That is instant flux Linkage = $N \phi max \cos\omega t$.

According to faraday's law of Electromagnetic Induction.

→ Emf Induced in a coil =

Rate of change of flux linkage with −ve sign.

$$e = -\frac{d}{dt}(N \phi max \cos\omega t).$$

$e = +N \phi_{max} \sin\omega t \cdot \omega$

When $\omega t = 0$, $\sin\omega t = 0$ therefor induced emf = 0 When $\omega t = \frac{\pi}{2}$ $\sin\frac{\pi}{2} = 1$. induce emf is maxm. which is denoted by Emax.

$$E_{max} = \phi_{max} N \omega$$

Instantaneous emf.

$$e = E_{max} \sin\omega t.$$

So that emf varies as the sin function of ~~the time~~ varies with time. and of emf induced in plotted against time, A curve of sin wave shape is obtained. such an emf is called sinosuidal emf. The sin wave is completed when the coil rotate through an angle of 2π radians.

- **Reactance:-**

This is the opposition in ohm offered by Inductance and capacitance in the circle

Inductive reactance $X_L = \omega L$ (Ω)

ω = angular frequency in rad/sec.

$= 2\pi f$.

f = frequency in Hz or cycle/second.

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- capacitive reactance (Ω) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$.
- Impedence complex summation of Resistance and reactance. Its unit is Ω.

$$Z = R + jX$$
$$= R + j(X_L - X_C)$$
$$R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$|Z| = \sqrt{R^2 + X^2}$$

- $\angle Z = \phi = \tan^{-1}\left(\frac{X}{R}\right)$

this is also the angle between voltage and current in the circuit and known as power factor angle.

- Different forms of emf equation

$$e = E_m \sin\theta = E_m \sin\omega t = E_m \sin\frac{2\pi}{T} t = E_m \sin 2\pi f t.$$

E_m → maxm ~~emf~~ voltage

$$e = I_m \sqrt{R^2 + 4\omega^2 L^2}.$$

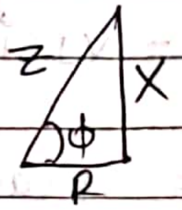
$f = \omega/\pi$

$$\left\{ I_{rms} = \frac{I_{max}}{\sqrt{2}} \right\}$$

- The RMS value of an alternating quantity is given by that steady (d.c) current which when flowing through a given circuit for given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time. Also known as effective or virtual value of AC.

* Impedence triangle.

$$|Z| = \sqrt{R^2 + X^2}$$



Admittance –

$$Y = \frac{1}{Z} = \frac{I}{V}$$

$$Y = G + jB$$

G → Conductance → reciprocal of Resistance.

B → susceptance → reciprocal of reactance.

Power:-

There are three types of power in Alternating circuit

i) Active power –

It is the power which is actually consumed in the circuit or in resistor. then

$$P = VI\cos\phi = I^2R$$

It is also known as real power or true power.

ii) Reactive power:– It is the power taken by the reactance of the circuit. its unit is ~~Volt~~ volt-Amp-reactive (VAr).

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It is denoted by Q.

$$Q = VI\sin\phi = I^2X$$

Where X = Reactance in ohm.

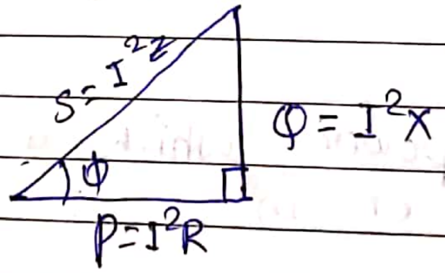
iii) Appearant power :- (S) - denotation

It is given by the product of Rms value of voltage and current. Its unit is volt-amp (VA) and is given by

$$S = VI = I^2Z$$

Z = impedence

- $S = P + jQ$. ⟶ Relation.



$$S = \sqrt{P^2 + Q^2}$$

$$\phi = \tan^{-1}\left(\frac{Q}{P}\right).$$

- Power factor :- It is the cosine of the angle b/w voltage & current.

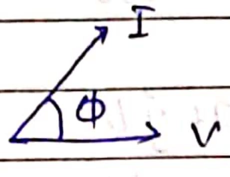
$$P.f = \cos\phi = \frac{R}{Z}$$

$$P.f = \frac{P}{S} = \frac{R}{Z}$$

- Phasor diagram:-

$\vec{V} = |V| \angle 0^\circ$

$\vec{I} = |I| \angle \phi$

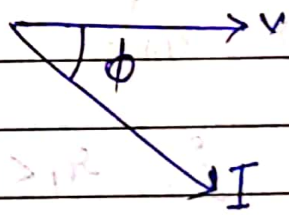


1) → Voltage is reference and current is leading by an angle ϕ

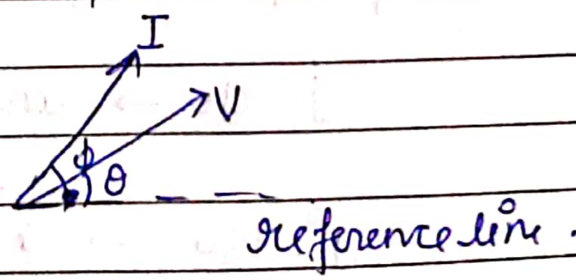
ii) → voltage is reference & current is lagging by an angle ϕ.

$\vec{V} = |V| \angle 0^\circ$

$\vec{I} = |I| \angle -\phi$.



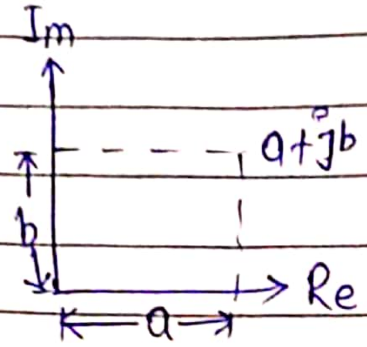
iii) Voltage is not reference but current is leading by an angle ϕ.



$\vec{V} = |V| \angle \theta$

$\vec{I} = |I| \angle \theta + \phi$.

- Complex representation of voltage & current.



Rectangular or Cartesian representation.

$\dot{i} = a + jb$.

$v = c + jd$.

Where $j = \sqrt{-1}$ which gives the rotation through 90° in anticlockwise direction.

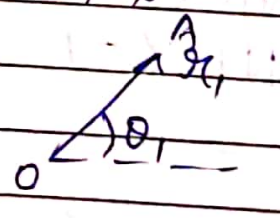
a = Real part   b = img. part

x-axis - real axis

y-axis - Imaginary axis.

$\dot{i} = r_1 \angle \theta_1$

$v = r_2 \angle \theta_2$



- properties of Operator j

$j = 90 \rightarrow$ anticlockwise rotation of a phasor
$= \sqrt{-1}$

$j^2 = 180 \rightarrow$ anticlockwise rotation of a phasor
$= -1$

$j^3 = 270^\circ \rightarrow$ anticlockwise rotation of a phasor
$= -j$.

$$\left\{ I_{rms}^2 = \frac{\int_{t_1}^{t_2} i^2 dt}{\int_{t_1}^{t_2} dt} \right\}$$

$$\frac{1}{j} = -j.$$

* RMS and Avg. value:

RMS / effective value

Root mean square value

$$I_{rms} = \sqrt{\text{average } i^2(t)}$$

Mid ordinate method

$$= \sqrt{\left(\frac{i_1^2 + i_2^2 + i_3^2 + \cdots + i_n^2}{n}\right)}$$

$n \to$ Complete cycle.

Square root of the mean of the square of the instantaneous current over one cycle.

$$I_{av} = \frac{i_1 + i_2 + \cdots i_n}{n}$$

over +ve half cycle

$$i = I_m \cos\omega t = I_m \cos\frac{2\pi}{T} t.$$

$$I_{rms} = \left[\frac{1}{T}\int_0^T i^2 dt\right]^{1/2} \rightarrow \text{Analytical method.}$$

$\Rightarrow$ RMS current.

- Instantaneous power dissipation in a resistance is

$$\{P = i^2 R\}$$

- Avg. power dissipation over one half cycle

$$\left\{P = \frac{1}{T}\int_0^T i^2 R\,dt\right\}$$

$$\boxed{P = I^2 R}$$

Where $I = I_{rms} = \left[\frac{1}{T}\int_0^T i^2 dt\right]^{1/2}$

Rms current.

Consider an AC of sinosuidal current i

$$i = I_m \cos\omega t$$

$$= I_m \cos\frac{2\pi}{T}t$$

$$I_{rms} = \left[\frac{1}{T}\int_0^T I_m^2 \cos^2\frac{2\pi}{T}t\right]^{1/2}$$

$$I_{rms} = \left[\frac{1}{T} I_m^2 \int_0^T \left(\frac{1 + \cos\frac{4\pi}{T}t}{2}\right)dt\right]^{1/2}$$

$$\left[\frac{\frac{1}{T} I_m^2 t + \frac{\sin\frac{4\pi}{T}t}{\frac{4\pi}{T}}}{2}\right]^{1/2}$$

$$\left[\frac{1}{T}\left(\frac{I_m^2 t + \frac{\sin\frac{4\pi}{T}t}{\frac{4\pi}{T}}}{2}\right)_0^T\right]^{1/2}$$

$$I_{rms}^2 = \frac{1}{T}\left[\frac{I_m^2 T}{2} + \left(\frac{\sin\frac{4\pi}{T}\times T}{\frac{4\pi}{T}}\right)\frac{I_m^2}{2} - 0 - 0\right]$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} + \underbrace{\sin 4\pi \times T}_{0 \times T} = \frac{I_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$I_{avg} = \frac{1}{T}\int_0^T I_m \cos\frac{2\pi}{T}t\, dt$$

$$\cancel{I_{avg} = \frac{1}{T} \int_0^T I_m\left(\frac{1+\cos\frac{4\pi}{T}t}{2}\right) dt}$$

$$\cancel{= \frac{I_m}{2}\left(t + \frac{\sin\frac{4\pi}{T}t}{\frac{4\pi}{T}}\right]_0^T}$$

$$\cancel{I_{avg} = \frac{I_m}{2}\left[T + \frac{T}{4\pi}\underbrace{\sin\frac{4\pi T}{T}}_{0}\right]}$$

$$T = \frac{2\pi}{\omega}$$

$$I_{avg} = \frac{1}{T}\int_0^T I_m \cos\omega t\, dt$$

$$I_{avg} = \frac{1}{T}\cdot I_m \cdot\left[\frac{\sin\omega t}{\omega}\right]_0^T$$

$$\frac{1}{T} I_m \cdot \left(\frac{\sin\omega T}{\omega} - \sin 0\right)$$

$$\frac{\omega}{2\pi} \times \frac{I_m}{\omega} \times \sin \omega \times \frac{2\pi}{\omega}$$

$$I_{avg} = \frac{\frac{1}{T}\int_0^{\pi/\omega} I_m \sin\omega t \, dt}{\int_0^{\pi/\omega} dt}$$

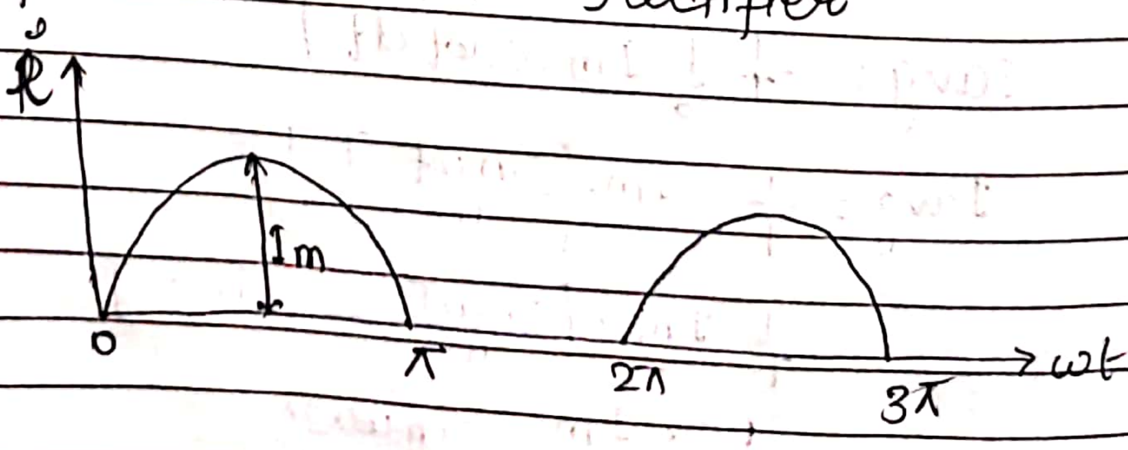
$$I_{avg} = \frac{I_m \left[\frac{\sin\omega t}{\omega}\right]_0^{\pi/\omega}}{\frac{\pi}{\omega}}$$

$$I_{avg} = \frac{I_m}{\cancel{\omega}} \left[\frac{\sin \frac{\pi}{\cancel{\omega}} \times \cancel{\omega}}{\frac{\pi}{\cancel{\omega}}}\right] = \frac{I_m}{\pi} \times 0 = 0$$

$$\left\{ \begin{aligned} I_{av} &= \frac{1}{T}\int_0^T i \, dt \\ I_{rms} &= \left[\frac{1}{T}\int_0^T i^2 \, dt\right]^{1/2} \end{aligned} \right\}$$

Problem

Q. 1 Find rms and avg value of the output of half wave rectifier



$i = I_m \sin\omega t$

$T = 2\pi$

$$I_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{\pi} i^2 \, d(\omega t) + \int_{\pi}^{2\pi} 0 \, d(\omega t)\right]$$

$$I_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{\pi} I_m^2 \sin^2\omega t \, d(\omega t)\right]$$

$$\cancel{\frac{1}{2\pi}\left[-\cos\omega t\right]_0^{\pi}}$$

$$\cancel{\frac{1}{2\pi}\left[-\cos\right]}$$

$$\frac{1}{2\pi}\int_0^{\pi} I_m^2 \left(\frac{1 - \cos 2\omega t}{2}\right) d(\omega t)$$

$$\frac{1}{2\pi}\frac{I_m^2}{2}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}$$

$$\frac{1}{4\pi} I_m^2 \left(\pi - \frac{\sin 2\pi}{2}\right)$$

$$I_{rm}^2 = \frac{I_m^2}{4\cancel{\pi}} \times \cancel{\pi}$$

$$I_{rms} = \frac{I_m}{2} = 0.5\, I_m$$

$$I_{avg} = \frac{1}{2\pi}\int_0^{\pi} i \, d(\omega t)$$

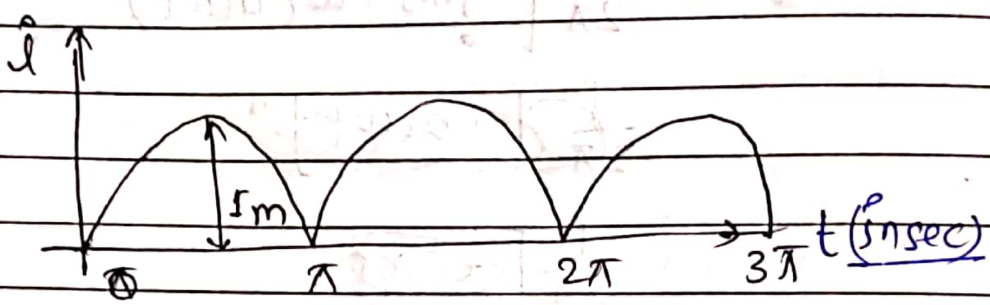
$$\frac{1}{2\pi}\int_0^{\pi} I_m \sin\omega t \, d(\omega t)$$

$$\frac{1}{2\pi} I_m \cdot -(\cos\omega t)_0^{\pi}$$

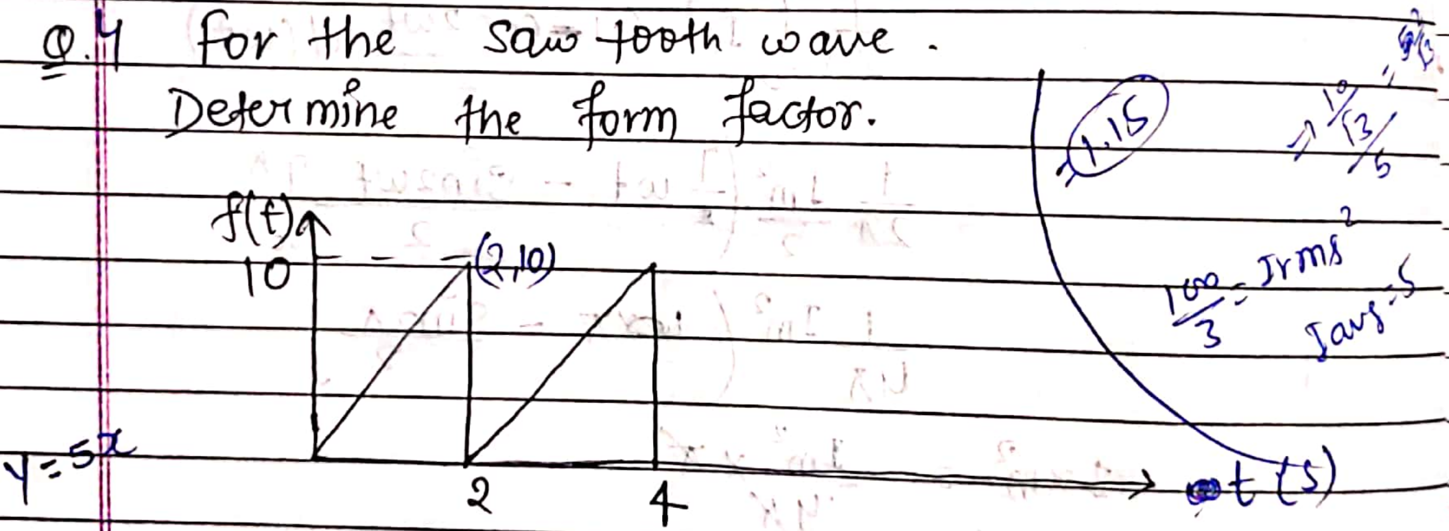
$$(-)\frac{1}{\cancel{2}\pi} \times I_m(-1) - 1) = \frac{I_m}{\pi}$$

Q.2 Find the ~~Poisson~~ factor and peak factor for the above half wave rectifier.

Q.3 For the output of full wave rectifier determine A) RMS value B) Avg value C) form factor d) peak factor.



Q.4 For the saw tooth wave. Determine the form factor.



Q.5 Find the effective or RMS value for the voltage function.

$$v(t) = 20 + 30\sin\omega t + 60\sin(2\omega t - \pi/2)$$

Q.6 Find the RMS value, Avg value, form factor and peak factor for the pulse train shown in figure.

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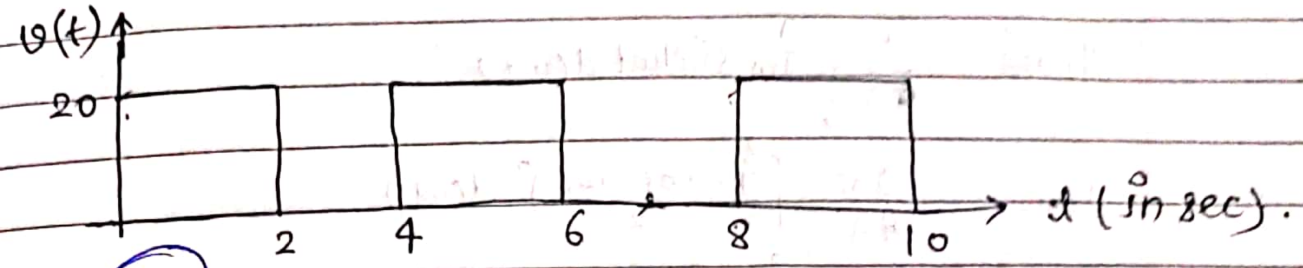


$T=4$

Q 7 An AC has frequency 50Hz and RMS current 25A. ~~Wright~~ equation of and find current at time 0.0025 sec. ~~time at~~
b) Time at which current 14.14 Amp.

$$i = I_m \sin \omega t$$

- form factor $= \frac{I_{rms}}{I_{avg}} = \frac{\pi}{2\sqrt{2}} = 1.11$

power factor $= \frac{I_m}{I_{rms}} = \sqrt{2}$

Soln → (2) form factor $= \frac{0.5 \cancel{I_m} \times \pi}{\cancel{I_m}} = 0.5\pi = 1.57$

~~peak factor~~ $= \frac{I_m}{0.5 I_m} = 2$

Soln → (3). $i = I_m \sin^2 \omega t$

$$I_{avg} = \frac{1}{\pi} \int_0^{\pi} I_m \sin^2 \omega t \, d(\omega t) \qquad \left(I_{avg} = \frac{I_m}{2} \right)$$

$$\frac{I_m}{\pi} \int_0^{\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) d(\omega t)$$

$$\frac{I_m}{2\pi} \times \left[\omega t - \frac{\sin 2\omega t}{2\omega} \right]_0^{\pi} \Rightarrow \frac{I_m}{2\pi} \cdot \pi = \frac{I_m}{2} \quad I_{rms} = \frac{I_m}{2}$$

a) $I = 25\sqrt{2} \sin 100\pi \times \frac{0.0025}{\frac{10000}{4}}$

$I = 25\sqrt{2} \sin \frac{\pi}{4} = 25\sqrt{2} \times \frac{1}{\sqrt{2}} = 25A$

$I_{rms} = 25A$.

$I_m = \sqrt{2} \times 25 = 35.35A$

$i = I_m \sin \omega t$.

$35.355 \sin 2\pi f t$

$i = 35.355 \sin 314t$

b) $14.14 = 25\sqrt{2} \sin 100\pi t$.

$14.14 = 25 \times 1.41 \sin 100\pi t$.

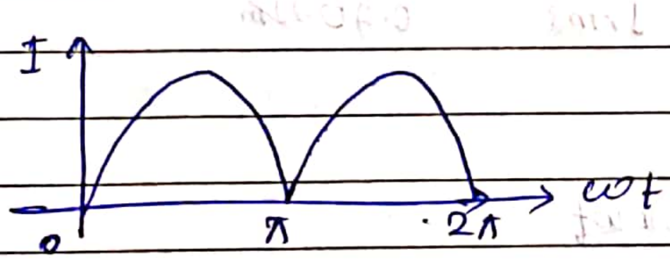
Radian $\frac{\pi}{180} \times \frac{22.6}{100\pi} = t$

$\frac{23.6}{100\cancel{\pi}} \times \frac{\cancel{\pi}}{180} = t$

~~$t = 0.075$ sec~~

$t = 0.0013$ sec.

Soln 3



Time period $= \pi$

$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$

$\sqrt{\frac{1}{\pi} \int_0^\pi I_m^2 \sin^2 \omega t \, d(\omega t)}$

$\sqrt{\frac{I_m^2}{2\pi} \int_0^\pi (1 - \cos 2\omega t) \, d(\omega t)}$

$\frac{I_m}{\sqrt{2}} = 0.707 I_m$

Avg value

$I_{avg} = \frac{1}{T} \int_0^T i \, dt$.

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\omega t \, d(\omega t)$$

$$\frac{I_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$-\frac{I_m}{\pi}\left[-1-1\right] = \frac{2 I_m}{\pi}$$

$$I_{avg} = \underline{0.637 I_m}.$$

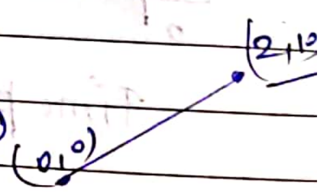
$$\text{Form factor} = \frac{I_{rms}}{I_{avg}} = \frac{0.707 \cancel{I_m}}{0.637 \cancel{I_m}} =$$

$$\text{peak factor} = \frac{I_m}{I_{rms}} = \frac{\cancel{I_m}}{0.707 \cancel{I_m}} =$$

Soln → ⑤ ~~$v(t) = 20 + 30\sin\omega t$~~

Soln → ④

$$I_{rms}^2 = \frac{1}{2}\int_0^2 (5t)^2 dt \qquad f(t) = 5t$$



$$I_{rms}^2 = \frac{1}{2}\left[25\frac{t^3}{3}\right]_0^2 = \qquad I_{rms}^2 = \frac{1}{\cancel{2}}\times 25\left[\frac{\cancel{8}}{3}\right]$$

$$I_{rms}^2 = \frac{50}{3} = \frac{5\sqrt{2}}{\sqrt{3}} \quad \frac{10}{\sqrt{3}}$$

$$I_{avg} = \frac{1}{2}\int_0^2 5t\,dt.$$

$$\frac{1}{2}\left[5\frac{t^2}{2}\right]_0^2 = \frac{1}{\cancel{2}}\left[\frac{5\times 4}{\cancel{2}}\right] = 5$$

$$\text{Form factor} = \frac{I_{rms}}{I_{av}}$$

$$\text{Form factor} = \frac{\frac{5\sqrt{2}}{\sqrt{3}}}{5} \quad \frac{\frac{10}{\sqrt{3}}}{\cancel{5}} = \frac{2}{\sqrt{3}} = 1.1$$

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Soln
(5) $v(t) = 20 + 30\sin\omega t + 60\sin(2\omega t - \frac{\pi}{2})$

$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\, d(t)$

~~$V_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi}$~~

$V_{rms} = \sqrt{(20)^2 + \left(\frac{30}{\sqrt{2}}\right)^2 + \left(\frac{60}{\sqrt{2}}\right)^2}$

$i = I_m \sin\omega t$

$\left(I_{rms} = \frac{I_m}{\sqrt{2}}\right)$

$\sqrt{400 + 450 + 1800}$

$= 51.478\,V.$

(6) $V_{rms}^2 = \frac{1}{T}\int_0^T v^2 dt$

$V_{rms}^2 = \frac{1}{4}\int_0^2 (\cancel{20})^2 dt + \frac{1}{4}\int_2^4 0\, dt$

$V_{rms}^2 = \frac{1}{4}\left[\frac{v^3}{3}\right]_0^2 \Rightarrow \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$

$V_{rms} = \frac{\sqrt{2}}{\sqrt{3}}$

$\left(V_{av} = \frac{1}{4}\int_0^2 v\,dt \quad \frac{1}{4}\times 40 = 10 \quad V_{rms} = \frac{V_m}{\sqrt{2}}\right)$

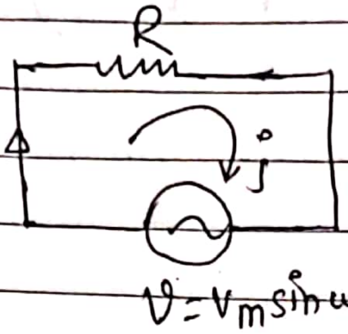
$V_{rms} = \sqrt{200} = 1.41 \times 10 = 14.1\,V$

$V_{rms} = 10\sqrt{2}$

Form factor = 1.414

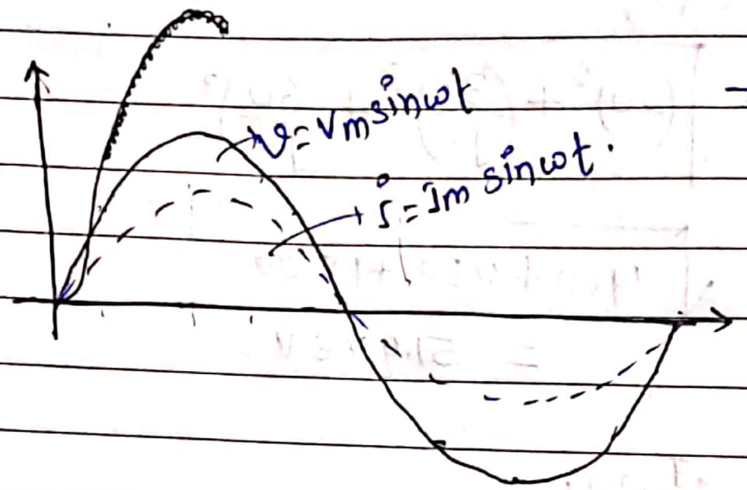
Peak factor = 1.414

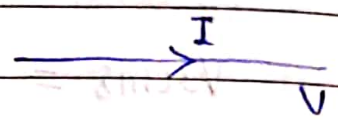
- AC through pure resistive circuit :-



$v = v_m \sin\omega t$

$$\left\{ i = \frac{v}{R} = \frac{v_m \sin\omega t}{R} = I_m \sin\omega t \right\}$$





$p.f = \cos\phi$

$\phi = 0$

$p.f = \cos 0 = 1$

- power. instantaneous power.

$$p = vi$$

$$= v_m \sin\omega t \cdot I_m \sin\omega t$$

$$v_m I_m \sin^2\omega t$$

$$\frac{v_m I_m}{2}(1 - \cos 2\omega t)$$

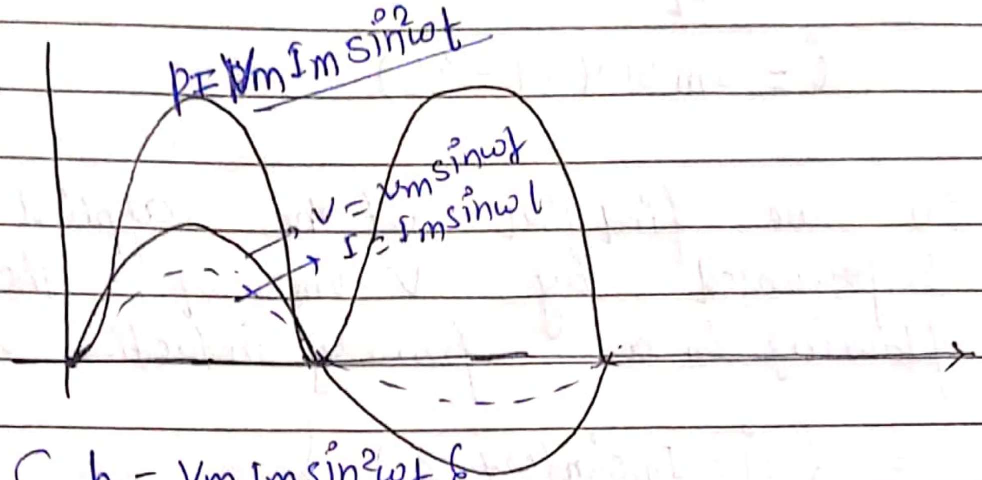
$$p = \frac{v_m I_m}{2} - \frac{v_m I_m}{2}\cos 2\omega t$$

power consists of a constant part $\frac{v_m I_m}{2}$ and a fluctuating part $\frac{v_m I_m \cos 2\omega t}{2}$ of frequency double that of voltage and current wave. for a complete cycle The average value of $\frac{v_m I_m \cos 2\omega t}{2} = 0$

Hence power for the whole cycle is.

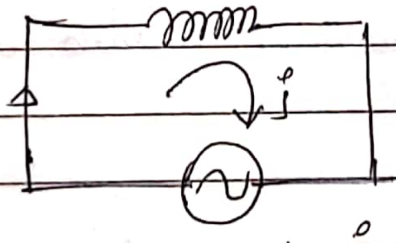
$$\left\{ p = \frac{I_m v_m}{2} = \frac{v_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} \times I_{rms} = V \times I \right\}$$

V = rms value of applied voltage.
I = rms value of the current.



$$\{ p = V_m I_m \sin^2 \omega t \}$$

Ac through pure inductive circuit.



$v = V_m \sin \omega t$

$v = V_m \sin \omega t$.

$v = L \dfrac{di}{dt}$.

$$V_m \sin \omega t = L \frac{di}{dt}.$$

$$\int di = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$i = \frac{V_m}{L} \left[-\frac{\cos \omega t}{\omega} \right]$$

$$i = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\left\{ i = \frac{V_m}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) \right\}$$

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- Maximum value of i is:

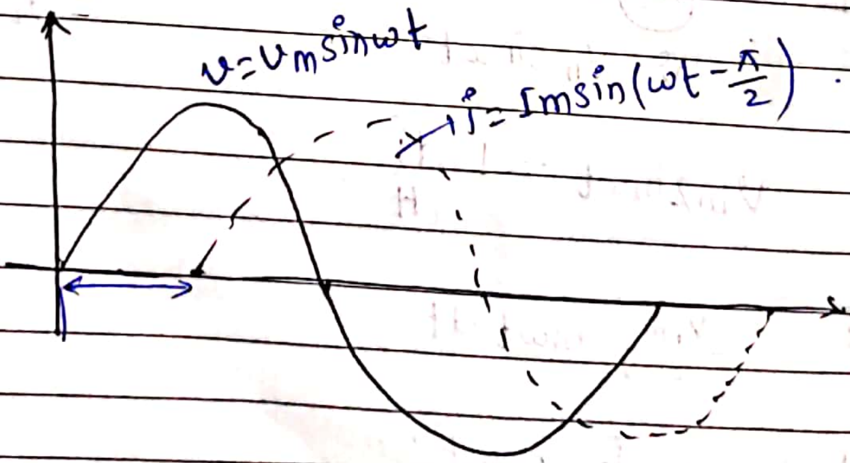
$I_m = \frac{V_m}{X_L}$ when $\sin(\omega t - \frac{\pi}{2})$ is unity.

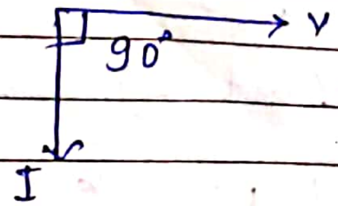
$i = I_m \sin(\omega t - \pi/2)$

So we find that if the applied voltage is represented by $v = V_m \sin\omega t$ then current flowing in a purely inductive circuit

$= \left\{ i = I_m \sin(\omega t - \frac{\pi}{2}) \right\}$

clearly the current lags behind the applied voltage by a quarter cycle or the phase difference between the two is $\frac{\pi}{2}$ (voltage and current) with voltage leading.





phasor diagrams

Instantaneous power.

$$v = V_m \sin\omega t$$
$$i = I_m \sin(\omega t - \frac{\pi}{2})$$ ⊗

$$p = V_m I_m \sin\omega t \cdot - V_m I_m \sin\omega t \cos\omega t$$

$$p = \frac{-V_m I_m}{2} \cdot \sin 2\omega t$$

power ~~factor~~ whole cycle $= -\frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt$

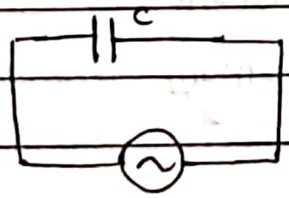
$= 0$.

So the avg. power taken by the inductor for the complete cycle is zero.

$\phi = 90°$ {lagging}

$$\boxed{p.f = \cos\phi = \cos 90° = 0.}$$

- AC ~~circuit~~ through pure Capacitor.



v = potential difference b/w plates at any instant.

q = charge on plates at that instant.

Thus $q = cv$

$q = C V_m \sin\omega t$

$$i = \frac{dq}{dt} = \frac{d}{dt}(V_m \sin\omega t)$$

$$= \omega C V_m \cos\omega t$$

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$$i = \frac{V_m}{1/\omega L} \cos\omega t$$

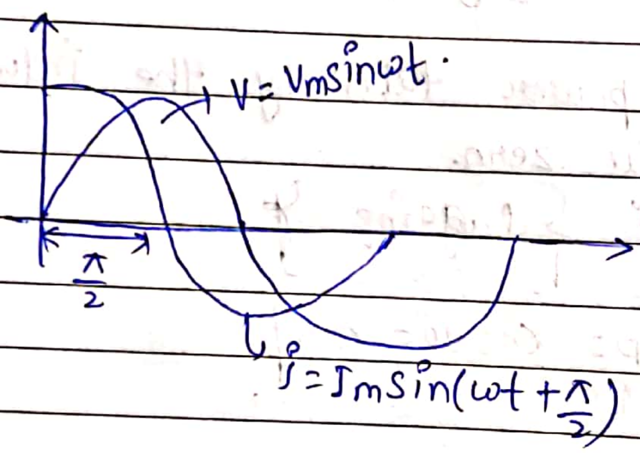
$$i = \frac{V}{1/\omega c} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I_m = \frac{V_m}{c}$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$X_c = \frac{1}{\omega c}$$

= Capacitive Reactance (Ω)



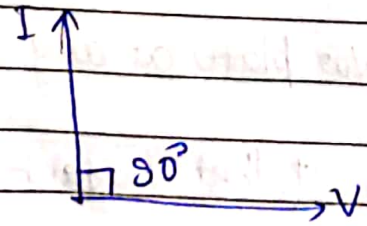
power

$p = vi$

$= V_m \sin\omega t \; I_m \sin(\omega t + \frac{\pi}{2})$

$V_m I_m \sin\omega t \cos\omega t$

$$\left\{ p = \frac{V_m I_m}{2} \sin 2\omega t \right\}$$



power for the whole cycle =

$$\frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

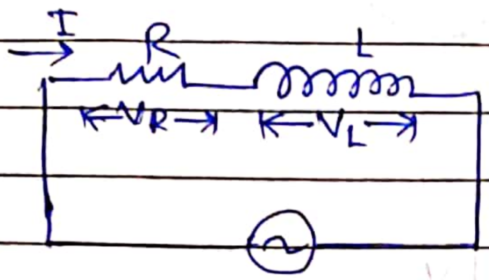
→ power factor = Angle B/w voltage and current

$\phi = 90^\circ$ leading $pf = \cos\phi = \cos 90^\circ = 0$

(leading)

* AC through resistive inductive circuit.

let V = Rms value of the applied voltage.
I = Rms value of the resistive current.



$V_R = IR$ = voltage drop across R.
$V_L = IX_L$ = voltage drop across coil

Total impedence of the circuit.

$$Z = R + jX_L = R + j\omega L$$

$$i = \frac{V}{Z} = \frac{V_m \sin\omega t}{\sqrt{R^2 + X_L^2} \angle \tan^{-1}\left(\frac{X_L}{R}\right)}$$

$$= \frac{V_m}{\sqrt{R^2+X_L^2}} \sin\left(\omega t - \tan^{-1}\left(\frac{X_L}{R}\right)\right)$$

$$= \frac{V_m}{\sqrt{R^2+X_L^2}} \sin(\omega t - \phi)$$

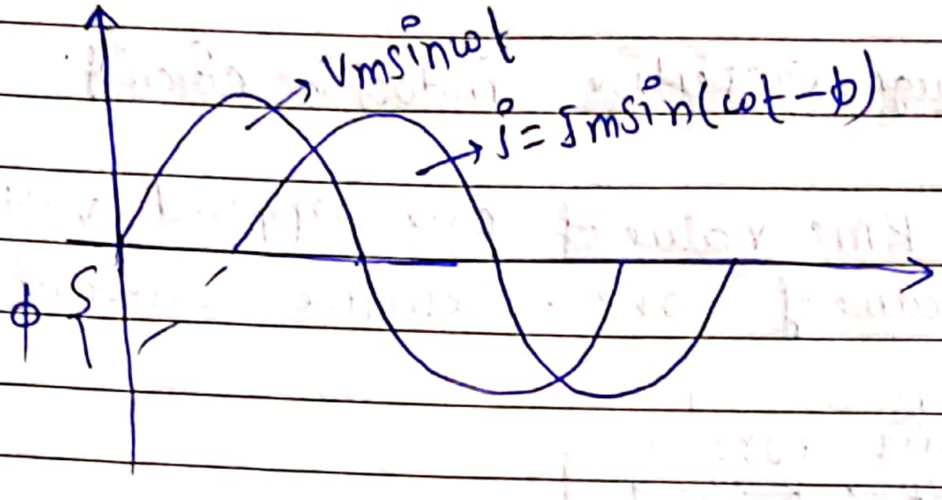
$$i = I_m \sin(\omega t + \phi)$$

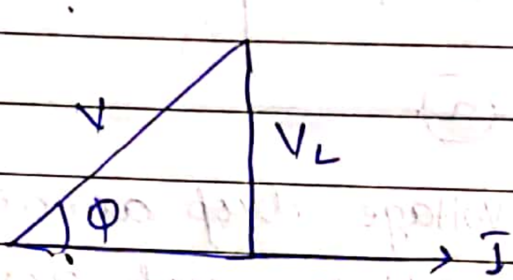
I_m = max value of current.

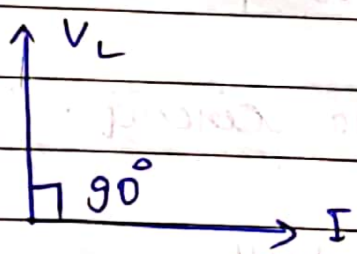
$$\frac{V_m}{\sqrt{R^2+X_L^2}}$$

ϕ = Angle b/w voltage & current.

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\left(\frac{\omega L}{R}\right).$$







power ⇒ Active power

$p = VI\cos\phi$

$V_{rms} I_{rms} \cos\phi$

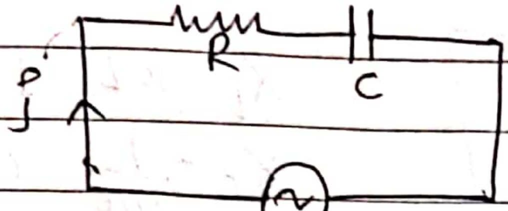
$p = VI\cos\phi$

$VI\frac{R}{2} = \frac{V}{2} I R$

$\{p = I^2 R\}$

power factor = $\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\left(\frac{\omega L}{R}\right)$

- Ac through Resistive and Capacitive circuit.



$V = V_m \sin \omega t$.

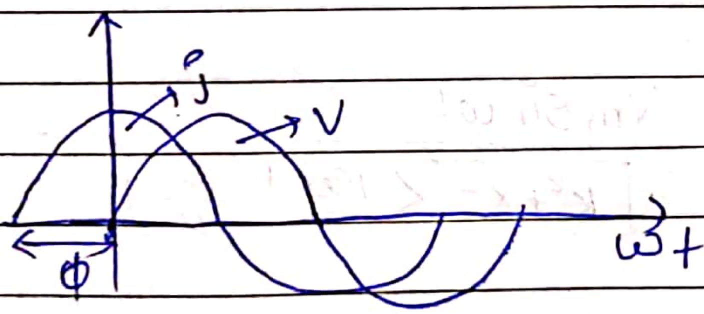
$X_L = \frac{1}{\omega C}$ $Z = R - jX_c = R - \frac{J}{\omega C}$

polar form $\Rightarrow Z = \sqrt{R^2 + X_c^2} < \tan^{-1}\left(\frac{X_c}{R}\right)$.

$$i = \frac{V}{Z} = \frac{V_m \sin \omega t}{\sqrt{R^2 + x_c^2} < \tan^{-1}\left(\frac{x_c}{R}\right)}.$$

$$I_m = \frac{V_m}{\sqrt{R^2 + x_c^2}}.$$

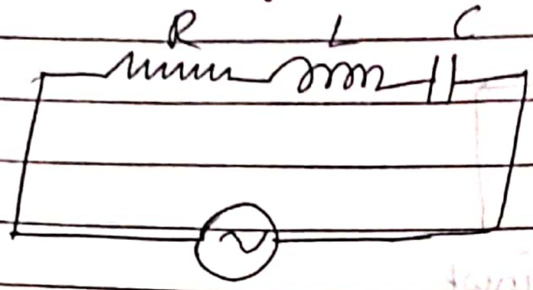
$$\phi = \tan^{-1}\left(\frac{X_c}{R}\right) = \tan^{-1}\left(\frac{1}{\omega C R}\right)$$



Average power is consumed by one zero so the power consume by active power.

power factor angle $\phi = + \tan^{-1}\left(\frac{X_c}{R}\right)$

• A.C through Series R-L-C circuit.



$V = V_m \sin \omega t$

$X_L = \omega L$ Inductive Reactance

$X_C = \frac{1}{\omega C}$ Capacitive Reactance

$$Z = R + j(X_L - X_C)$$
$$R + j\left(\omega L - \frac{1}{\omega C}\right)$$

In polar form:-

$$Z = \sqrt{R^2 + x^2} \angle \tan^{-1} \frac{x}{R}$$
$$X = X_L - X_C$$

The resultant current i in the circuit.

$$i = \frac{v}{Z} = \frac{V_m \sin \omega t}{\sqrt{R^2 + x^2} \angle \tan^{-1}\left(\frac{X}{R}\right)}$$

$$i = \frac{V_m \sin}{\sqrt{R^2 + x^2}} \left(\omega t - \tan^{-1} \frac{X}{R}\right)$$

Now there are two Cases

i) $X_L > X_C$ or $V_L > V_C$

V_L = voltage across the inductor = $i X_L$

V_C = voltage across the Capacitor

$\Rightarrow V = IX_c$.

In this case the circuit is inductive

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$ angle of lag

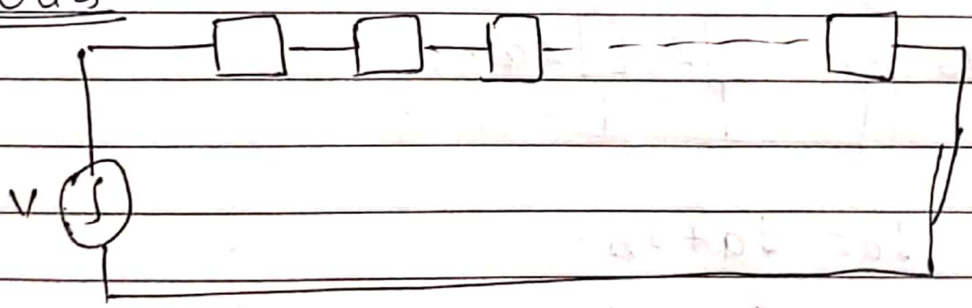
(ii) $X_C > X_L$ or $V_C > V_L$.

In this case nature of circuit is capacitive current i leads the applied voltage v by angle ϕ.

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$ angle of lead.

- **Series and parallel AC circuit**

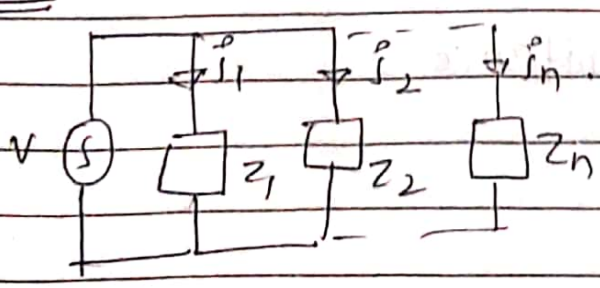
Series



$$V = IZ_1 + IZ_2 + \cdots + IZ_n = IZ_{eq}.$$

$$Z_{eq} = Z_1 + Z_2 + \cdots + Z_n.$$

Parallel



$$I = \frac{V}{Z_1} = VY_1 \quad , I = \frac{V}{Z_2} = VY_2$$

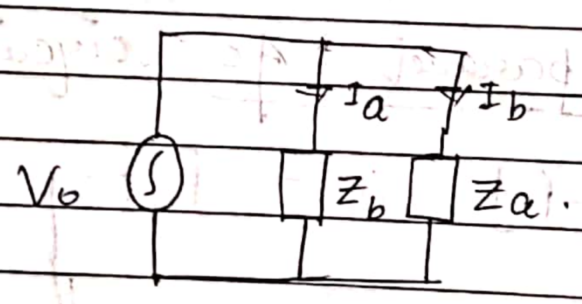
$$I = I_1 + I_2 + \text{-------} + I_n$$

$$VY_{eq} = VY_1 + V_2 + \text{---} + VY_n .$$

$$VY_{eq} = V(Y_1 + Y_2 + \text{---} + Y_n)$$

$$Y_{eq} = Y_1 + Y_2 + \text{---} + Y_n .$$

- Current and voltage division in a.c circuit.



$$I_o = I_a + I_b .$$

$$I_a = \frac{I_o \times Z_b}{Z_a + Z_b} = \frac{Y_a}{Y_a + Y_b} I_o$$

$$I_b = \frac{I_o Z_a}{Z_a + Z_b} = \frac{Y_b}{Y_a + Y_b} Z_o$$

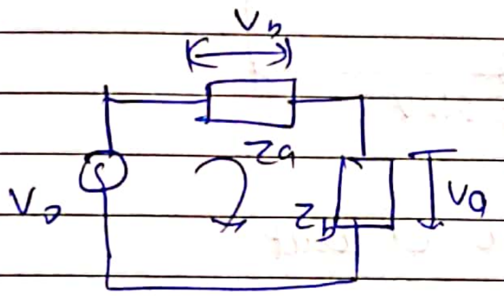
$Y \rightarrow$ Admitance.

$$V_b = I_o Z_b = \frac{Z_b \times V_o}{Z_a + Z_b} = \frac{Y_a}{Y_a + Y_b} V_o$$

$$V_a = I_o Z_a = \frac{Z_a V_o}{Z_a + Z_b}$$

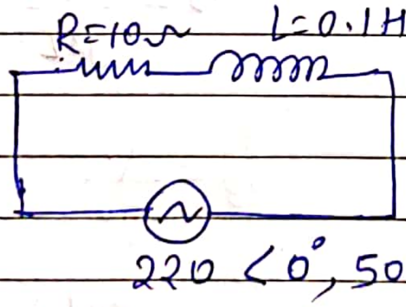
$$= \frac{Y_b V_o}{Y_a + Y_b}$$

$$I_o = \frac{V_o}{Z_a + Z_b}.$$

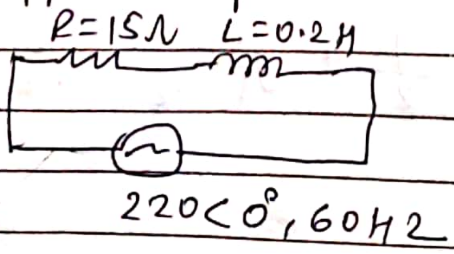


$\left\{ \text{quality factor} = \frac{1}{\cos\phi} \right\}$

Q.1 In series R-L circuit shown in figure find a) impendence b) Resultant current c) power factor, nature d) quality factor

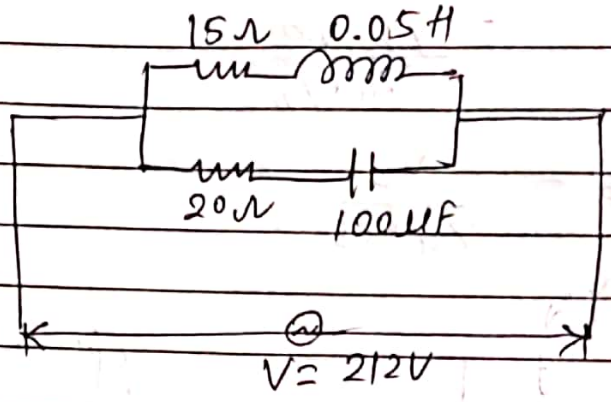


In the given circuit find Active reactive and appearant power.

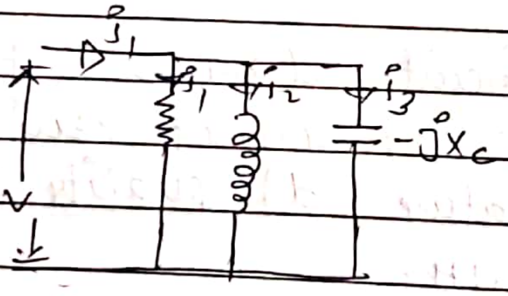


J.b Gupta

Q. 2 Determine the R.m.s value of current in each branch and total current of the circuit shown in figure. Draw the phrasal diagram.



* parallel R-L-C circuit.



$$i_1 = \frac{V}{R}$$

$$i_2 = \frac{V}{jX_L} = \frac{V}{X_L}\angle -90^\circ$$

$$i_3 = \frac{V}{-jX_c} = \frac{V}{X_c}\angle 90^\circ$$

$$i = i_1 + i_2 + i_3 \quad (\text{KCl})$$

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_c}.$$

$$\frac{1}{R} + j\left\{\frac{1}{X_c} - \frac{1}{X_L}\right\}$$

Admittance

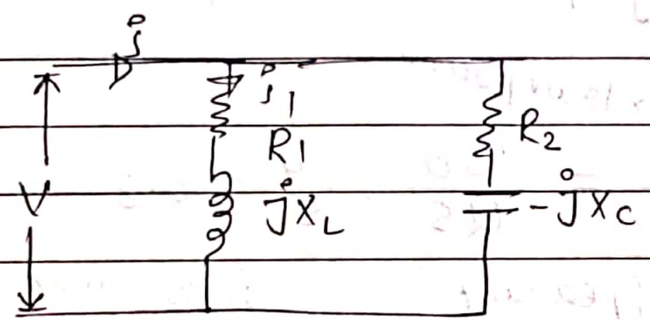
$$Y = \frac{1}{R} + j\left\{\frac{1}{X_C} - \frac{1}{X_L}\right) = \frac{1}{Z}.$$

$$G + jB$$

$$G \rightarrow \frac{1}{R} \qquad B \rightarrow \frac{1}{X_C} - \frac{1}{X_L}$$

B → susceptance.

- parallel Combination of Inductive & Capacitive Impedence.



$Z_1 = R_1 + jX_L$ = impendence of inductive brance.

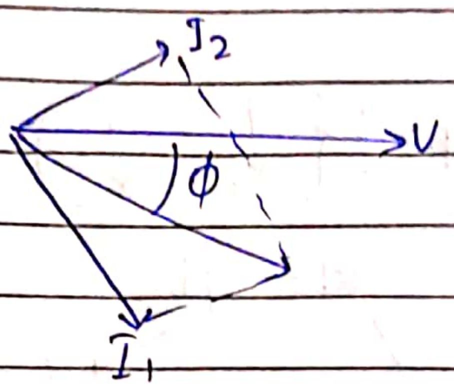
$Z_2 = R_2 - jX_C$ = impedence of capacitive branc

$$\left\{ i_1 = \frac{V}{Z_1} = \frac{V}{R_1 + jX_L} \right\}$$

$$i_2 = \frac{V}{Z_2} = \frac{V}{R_2 - jX_C}$$

$$Y = Y_1 + Y_2$$

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{R_1 + jX_L} + \frac{1}{R_2 - jX_C}$$



Importance of power factor.

A steal stamping operation runs at 100 KW (working power / true power) & the appearant

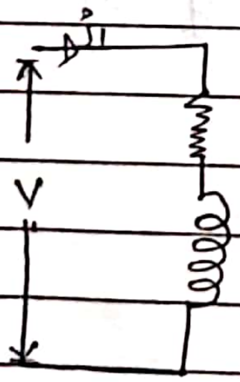
$$\cos\phi = \frac{T_p}{A_p} = \frac{100}{125} = \frac{4}{5} = 0.8$$

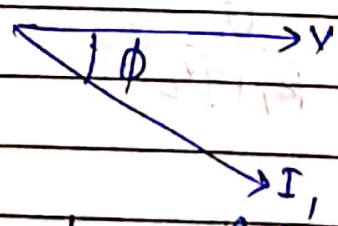
T_p → True power

A_p → appearant power.

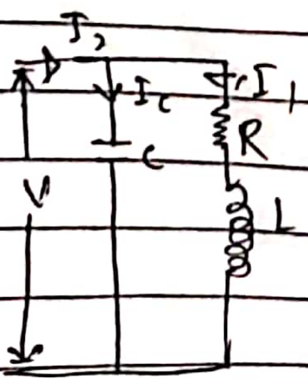
in % = 80%.

- power factor correction by static capacitor.

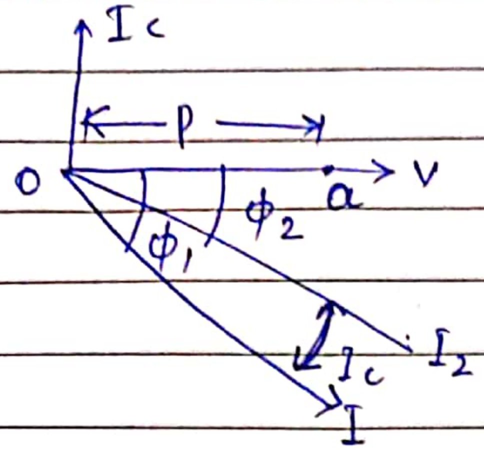




ϕ_1 = original phase angle.



$$I_2 = I_1 + I_2$$



$OA = I_1\cos\phi_1 = I_2\cos\phi_2$ $\qquad$ $\cos\phi_2 > \cos\phi_1$

$I_2 < I_1$

It means that the current I_2 drawn from the supply is less than the load current I_1 there by reducing system loss & Improve efficiency as well From this since

$$\boxed{VI_2\cos\phi_2 = VI_1\cos\phi_1}$$

real power taken from the supply remains the same.

$I_c = I_1\sin\phi_1 - I_2\sin\phi_2$

$V_c = VI_1\sin\phi_1 - VI_2\sin\phi_2$

$P = VI_1\cos\phi = VI_1\cos\phi_2$

we can write

$Q_c = P(\tan\phi_1 - \tan\phi_2)$

$Q_c = VI_c$