Convergence of infinite positive term series –

Comparision Test :

If $\sum u_n$ & $\sum v_n$ be two +ve term series and

① $\sum v_n$ is convergent then $\sum u_n$ is also convergent

if $\lim_{n \to \infty} \frac{u_n}{v_n} = K$ where 'K' is a finite +ve no.

② $\sum v_n$ is divergent then $\sum u_n$ is also divergent

if $\lim_{n \to \infty} \frac{u_n}{v_n} = N'$ where N' is positive (finite) number.

~~To compo~~ To prove the convergenty of series 'u_n' we make a sub series from u_n which we called as v_n and $(u_n \geq v_n)$ then the series is ~~ab~~ divergent.

and when $(u_n \leq v_n)$ then the series is convergent.

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the convergency of v_n we use

series test.

$$\frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots + \frac{1}{n^p} + \cdots$$

① convergent if $p > 1$

② divergent if $p \leq 1$

for eg. # $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

is a divergent series

as $p = 1$ in this case

using p-series test.

Prove the convergency of the series —

$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{2n-1} + \cdots$

$u_n = \frac{1}{2n-1}$; $\Sigma u_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots$

~~Decide v_n the degree of~~

Decide v_n according to the degree of 'n' in Numerator and denominator

for eg: You have

n^{th} term of $u_n = \frac{n}{n^2+1}$ then $v_n = \frac{n}{n^2} = \frac{1}{n}$

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Coming to the question —

We have series

$$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots \frac{1}{2n-1} + \cdots$$

then $u_n = \frac{1}{2n-1}$

Let $v_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{2n-1} \times \frac{n}{1}$$

$$= \lim_{n\to\infty} \frac{n}{2n-1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\left[2 - \frac{1}{n}\right]}$$

now, as $n \to \infty$

$$= \frac{1}{2-0} = \frac{1}{2}$$

$$\boxed{\begin{array}{l} \text{Now, by p-series test we know that the} \\ \text{series } \sum v_n \text{ is convergent if } p>1 \text{ and} \\ \sum v_n \text{ is divergent if } p \le 1 . \end{array}}$$

$\because$ as $n \to \infty$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2} \le 1$$

$\therefore$ the given series is divergent by p-series ~~test~~. Then by comparision test $\sum u_n$ is also ~~divergent~~ divergent.

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Pt.s to Remember

* Comparision test is not applicable on oscillatory series i.e, When the series ~~has~~ ~~c~~ is either convergent or divergent.

* Using p-series test one can find the convergency of v_n. (i.e v_n is either convergent or divergent.)

* If v_n is convergent, u_n is also convergent and when v_n divergent, u_n is also divergent.

* Find v_n according to the degree of u_n in ~~the~~ Numerator or denominator. (as we have discussed in the #1.

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2: $\frac{1}{1\cdot2\cdot3} + \frac{3}{2\cdot3\cdot4} + \frac{5}{3\cdot4\cdot5} + \cdots$

Sol^n:
n^{th} term of the series

$$u_n = \frac{2n-1}{n(n+1)(n+2)}$$

when $n=1$

$$\frac{2-1}{1\cdot(1+1)(1+2)} = \frac{1}{1\cdot2\cdot3}$$

when $n=2$

$$\frac{2(2)-1}{2(2+1)(2+2)} = \frac{3}{2(3)(4)}$$

and so on...

for v_n, we have to take the highest term out of u_n from numerator and denominator.

$$v_n = \frac{n}{n\,n\,n} = \frac{n}{n^3} = \frac{1}{n^2}$$

Now, $$\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{(2n-1)}{n(n+1)(n+2)} \times n^2$$

$$= \frac{2n^3-n^2}{n^3\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)} = \frac{\cancel{n^3}\left(2-\frac{1}{n}\right)}{\cancel{n^3}\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}$$

now as $n\to\infty$

$= 2$ (which is a finite +ve no.)

By p-series test — $\sum v_n = \sum \frac{1}{n^2}$ $(p=2>1)$ is convergent.

$\therefore$ by comparison test
$\sum u_n$ is also convergent.

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D'Alemberts Ratio Test :

Let Σu_n be a series of positive terms with infinite terms then

the series is said to be

(1) Convergent if $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} < 1$

(2) Divergent if $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} > 1$

and the test fails if

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = 1.$$

Remark : This test is only applicable when the series is of +ve terms. ~~and~~

~~Steps~~ :
Remark : $\lim_{n\to\infty} u_n = 0$ (convergent)

$\lim_{n\to\infty} u_n > 0$ (Divergent).

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1 # Test the Convergence of $1^2 + 2^2x^2 + 3^2x^2 + \ldots$ where x is +ve.

Sol.ⁿ

$$u_n = n^2 x^{n-1} \quad \text{—— (1)} \quad \text{where } n = 1, 2, 3, \ldots$$

$$\& \quad u_{n+1} = (n+1)^2 x^{(n+1-1)} = (n+1)^2 x^n \quad \text{—— (2)}$$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{(n+1)^2 x^n}{n^2 x^{n-1}} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(1+\frac{1}{n}\right)^2 x^n}{\cancel{n^2}\left(x^{n-1}\right)}$$

now, as $n \to \infty$

$$= 1\,(x)$$

Now, By D'Alembert's Ratio test ——

$\sum u_n$ is convergent if $x < 1$ & divergent if $x > 1$. if $\underline{x = 1}$ in (1)

$$u_n = n^2$$

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} n^2 = \infty$$

at $n = 1$ u_n is divergent.

$\therefore$ $\sum u_n$ is convergent if $\underline{x < 1}$ and divergent if $\underline{x \geq 1}$.

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Test the convergence : (8)

(2) $\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \frac{x^4}{4\cdot 5} + \dots \quad x > 0.$

Sol^n: First we find u_n, the ~~first~~ n^{th} term is —

$$u_n = \frac{x^n}{n(n+1)} \; ; \quad u_{n+1} = \frac{x^{n+1}}{(n+1)(n+2)}$$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{(\cancel{n+1})(n+2)} \times \frac{n(\cancel{n+1})}{x^n}$$

$$= \lim_{n\to\infty} x \frac{n}{(n+2)} = \frac{\cancel{n} x}{\cancel{n}\left(1+\frac{2}{n}\right)}$$

now as $\lim n \to \infty$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = x.$$

Limit's depends upon x.

Next, by D'Alembert test, the series ~~is~~ and Σu_n is convergent if $x < 1$ and Σu_n is divergent if $x \geq 1$ and test fails when $x = 1$.

$\therefore \quad u_n = \frac{1}{n(n+1)}$ and $v_n = \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{n(n+1)} n^2 = \lim_{n\to\infty} \frac{\cancel{n^2}}{\cancel{n^2}\left(1+\frac{1}{n}\right)} = 1$$

(finite value)

$\Sigma v_n = \Sigma \frac{1}{n^2}$ $\therefore$ by p-series test $\boxed{p = 2}$

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and $\because$ $p = 2 > 1$ $\therefore$ the series v_n 9

is convergent ~~and taken unless clear~~

~~convergates~~ using p-series test.

and by comparison test u_n is also

convergent.

— x —