ECO 5201 Professor Pubali Chakraborty

Due: 10.00 AM, September 16, 2021

Assignment 2: Solow Model

1. **[25 points]** Consider a modified version of the Solow growth model, where the aggregate production function takes a Cobb-Douglas form in capital K, and efficiency units of labor, where L denotes labor units and A represents labor augmenting technology, and is given by:

$$Y_t = K_t^{\frac{1}{2}}(A_t L_t)^{\frac{1}{2}} \tag{1}$$

Further, capital depreciates at the rate $\delta \in (0,1)$ and the savings rate is exogenously given at $s \in (0,1)$.

Suppose there is no population growth and no technological progress. Normalize L and A to equal 1.

(a) Does this production function satisfy the Assumptions required in the Solow model? [3 points]

(b) Derive the law of motion of capital. [2 points]

(c) Find the non-trivial steady-state capital stock and steady-state output. [3 point]

(d) Argue that the non-trivial steady state is unique. Argue that it is globally stable. [3 point]

(e) What is the growth rate of output in the long run? [1 point]

(f) What is the golden rule savings rate? [3 points]

Suppose there is population growth at the rate n and technological progress at the rate g.

(a) Derive the law of motion of capital stock per efficiency unit of labor. [3 points]

(b) Find the non-trivial steady-states for capital stock per efficiency unit and output per efficiency unit. [3 points]

(c) In the long run, what is the growth rate of output? What is the growth rate of per capita output level? What is the growth rate of output per efficiency unit? [4 points]

2. **[25 points]** Consider the Solow growth model (no population growth, no technological growth), where output, Y is given by

$$Y_t = K_t^{\alpha} \tag{2}$$

where K is capital stock. Let $s = 0.3, \alpha = 0.5$ and $\delta = 0.1$. Use MATLAB to do the following:

(a) Consider a linearly spaced grid for K_t which has a range of [0,12]. Let the number of gridpoints, N_k, be 20.

i. Calculate K_{t+1} and Y_t for every point on the grid. [4 points]

ii. Plot the phase diagram associated with K_t on a figure. [4 points]

iii. Plot a separate graph with K_t on the x-axis and total savings, total output and total depreciation on the y-axis. Label the graph. Label the graph [7 points]

(b) Find the steady state, K_{ss}, Y_{ss}, C_{ss}. Use the method of convergence described below: [10 points]

- Start with a guess for K_{ss} (for example $K_{ss} = 1$)
- Calculate TK_{ss} by using the capital accumulation equation
- Keep iterating on K_{ss} till $|TK_{ss} - K_{ss}| < 0.00001$.
- Calculate the steady state values, Y_{ss} and C_{ss} using the derived value of K_{ss}

Does the answer change if you had started with a guess of $K_{ss} = 12$?