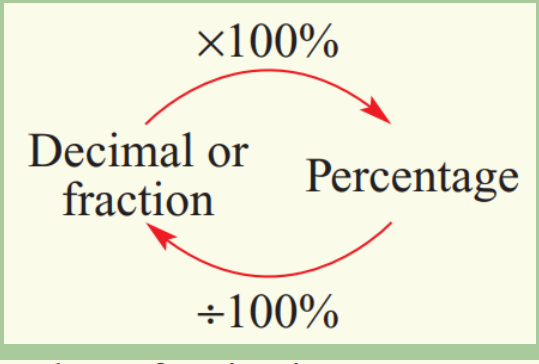
Financial Mathematics

Percentages, fractions and decimals



- To change a decimal or a fraction into a percentage, *multiply* by 100%.

  For example: $\frac{1}{2} \times 100\% = 50\%$ $\quad 0.5 \times 100\% = 50\%$

- To convert a percentage into a fraction, *divide* by 100%, using fraction notation.

  For example: $37\% = \frac{37}{100}$

- To convert a percentage into a decimal, *divide* by 100%.

  For example: $8\% = 8 \div 100$
  $= 0.08$

Percentage composition

- To express one quantity as a percentage of another, write them as a fraction (make sure the units are the same). Then convert this fraction to a percentage by multiplying by 100%.

  For example: 8 grams out of 32 grams $= \frac{8}{32} \times 100\%$
  $= 25\%$

Example 1 Rewriting percentages and fractions

a Write $\frac{12}{25}$ as a percentage.

b Write 7.5% as a simple fraction.

5 Express the following fractions as percentages.

a $\frac{1}{5}$ **b** $\frac{4}{5}$ **c** $\frac{8}{10}$ **d** $\frac{3}{10}$

e $\frac{1}{4}$ **f** $\frac{1}{8}$ **g** $\frac{3}{4}$ **h** $\frac{12}{20}$

i $\frac{14}{25}$ **j** $\frac{7}{20}$ **k** $\frac{9}{100}$ **l** $\frac{3}{40}$

6 Express the following percentages as simplified fractions.

a 19% **b** 23% **c** 99% **d** 5%

e 22% **f** 45% **g** 74% **h** 75%

i 2.5% **j** 17.25% **k** 1% **l** 125%

Example 2 Converting between percentages and decimals

a Write 0.45 as a percentage.

b Write 25% as a decimal.

7 Express the following decimals as percentages.

a 0.78 **b** 0.95 **c** 0.65 **d** 0.48

e 0.75 **f** 1.42 **g** 0.07 **h** 0.3

i 0.03 **j** 1.04 **k** 0.12 **l** 0.1225

8 Express the following percentages as decimals.

a 12% **b** 83% **c** 57% **d** 88%

e 99% **f** 100% **g** 120% **h** 5%

Example 3 Writing a quantity as a percentage

Write 50c out of $2.50 as a percentage.

9 In each of the following cases, express the first quantity as a percentage of the second.

a 5 g out of 200 g **b** 40c out of $4

c 10 km out of 200 km **d** 3 s out of 1 minute

e 200 m out of 1 km **f** 100 mL out of $\frac{1}{2}$ L

g 200c out of $1 **h** 45 marks out of a possible 60 marks

10 Copy and complete the table of the favourite summer sports of Year 9 students.

Sport	Number of students who chose sport	Fraction of the total	Percentage of the total
Swimming	44		
Golf	12		
Volleyball	58		
Cricket	36		
TOTAL			

Answer

5 **a** 20% **b** 80% **c** 80% **d** 30%
e 25% **f** $12\frac{1}{2}\%$ **g** 75% **h** 60%
i 56% **j** 35% **k** 9% **l** $7\frac{1}{2}\%$

6 **a** $\frac{19}{100}$ **b** $\frac{23}{100}$ **c** $\frac{99}{100}$ **d** $\frac{1}{20}$
e $\frac{11}{50}$ **f** $\frac{9}{20}$ **g** $\frac{37}{50}$ **h** $\frac{3}{4}$
i $\frac{1}{40}$ **j** $\frac{69}{400}$ **k** $\frac{1}{100}$ **l** $1\frac{1}{4}$

7 **a** 78% **b** 95% **c** 65% **d** 48%
e 75% **f** 142% **g** 7% **h** 30%
i 3% **j** 104% **k** 12% **l** 12.25%

8 **a** 0.12 **b** 0.83 **c** 0.57 **d** 0.88
e 0.99 **f** 1.0 **g** 1.2 **h** 0.05

9 **a** 2.5% **b** 10% **c** 5% **d** 5%
e 20% **f** 20% **g** 200% **h** 75%

10

Sport	Number of students who chose sport	Fraction of the total	Percentage of the total
Swimming	44	$\frac{22}{75}$	$29\frac{1}{3}\%$
Golf	12	$\frac{2}{25}$	8%
Volleyball	58	$\frac{29}{75}$	$38\frac{2}{3}\%$
Cricket	36	$\frac{6}{25}$	24%
TOTAL	150	1	100%

Applying percentages

- To find a percentage of an amount, write the percentage as a fraction or a decimal, then multiply by the amount.
  For example: 3% of $200 = \frac{3}{100} \times 200$ or 0.03×200 or 1% of $200 = 2$
  $\therefore$ 3% of $200 = 6$

- To find the original amount when given a percentage, you can work backwards using the unitary method.
  For example: *3% of an amount is 9. What is the original amount?*
  Dividing both numbers by 3 gives $1\% = 3$
  To find 100%, multiply 3 by $100 = 300$

Example 4 Finding a percentage of a quantity

Find 15% of $35.

4 Find the following amounts.

a 10% of 20	**b** 5% of 200	**c** 20% of 40
d 15% of 50	**e** 8% of 720	**f** 5% of 680
g 15% of 8200	**h** 70% of 60	**i** 90% of 500
j 75% of 44	**k** 99% of 200	**l** 3% of 50

5 Find:

a 10% of $360	**b** 50% of $420	**c** 75% of 64 kg
d 12.5% of 240 km	**e** 37.5% of 40 apples	**f** 87.5% of 400 m
g $33\frac{1}{3}\%$ of 750 people	**h** $66\frac{2}{3}\%$ of 300 cars	**i** $8\frac{3}{4}\%$ of $560

Example 5 Finding the original amount

Determine the original amount if 5% of the amount is $45.

6 Determine the original amount if:

a 10% of the amount is $12
b 6% of the amount is $42
c 3% of the amount is $9
d 40% of the amount is $2.80
e 90% of the amount is $0.18
f 6% of the amount is $27
g 12% of the amount is $96
h 15% of the amount is $54

7 Determine the value of x in the following if:

a 10% of x is $54
b 15% of x is $90
c 25% of x is $127
d 18% of x is $225
e 105% of x is $126
f 110% of x is $44

8 Without a calculator, evaluate the following.

a 10% of $58
b 5% of $84
c 1% of $46
d $2\frac{1}{2}\%$ of $20
e $33\frac{1}{3}\%$ of $132
f $66\frac{2}{3}\%$ of $60

9 If $\frac{1}{3}$ of $96 = 32$, what is $66\frac{2}{3}\%$ of 96?

10 If 10% of $800 is $80, explain how you can use this result to find:

a 1% of $800
b 5% of $800
c $2\frac{1}{2}\%$ of $800

Answer

4	**a**	2	**b**	10	**c**	8	**d** 7.5
	e	57.6	**f**	34	**g**	1230	**h** 42
	i	450	**j**	33	**k**	198	**l** 1.5
5	**a**	$36	**b**	$210	**c**	48 kg	
	d	30 km	**e**	15 apples	**f**	350 m	
	g	250 people	**h**	200 cars	**i**	$49	
6	**a**	$120	**b**	$700	**c**	$300	**d** $7
	e	$0.20	**f**	$450	**g**	$800	**h** $360
7	**a**	$540	**b**	$600	**c**	$508	
	d	$1250	**e**	$120	**f**	$40	
8	**a**	$5.80	**b**	$4.20	**c**	$0.46	
	d	$0.50	**e**	$44	**f**	$40	

9 64

10 **a** Divide by 10.

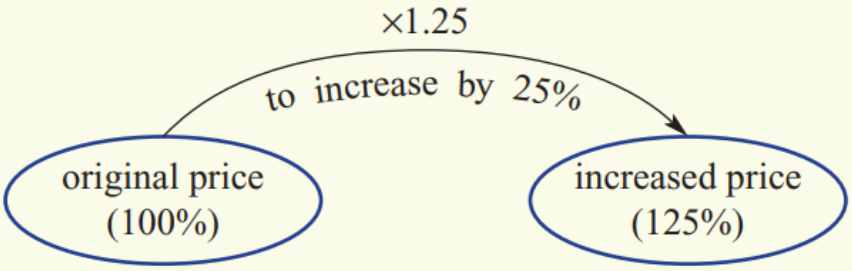
b Divide by 2.

c Divide by 2 and then 2 again (or just 4).

Percentage increase and decrease

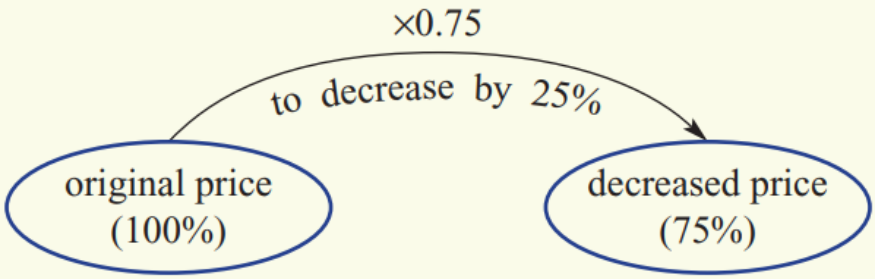
- To increase an amount by a given percentage:
  - add the percentage increase to 100%
  - multiply the amount by this new percentage.

  For example: to increase by 25%, multiply by 100% + 25% = 125% = 1.25



- To decrease an amount by a given percentage:
  - subtract the percentage from 100%
  - multiply the amount by this new percentage.

  For example: to decrease by 25%, multiply by 100% − 25% = 75% = 0.75



- To find a percentage change, use:

$$\text{Percentage change} = \frac{\text{change in price}}{\text{original price}} \times 100\%$$

Example 6 Increasing by a percentage

Increase \$70 by 15%.

5 Complete the following.

a Increase 56 by 10%. **b** Increase 980 by 20%. **c** Increase 100 by 12%.
d Increase 890 by 5%. **e** Increase 180 by 15%. **f** Increase 450 by 20%.
g Increase 8 by 50%. **h** Increase 98 by 100%. **i** Increase 30 by 5%.

Example 7 Decreasing by a percentage

Decrease $5.20 by 40%.

6 Complete the following.

a Decrease 80 by 5%. **b** Decrease 600 by 10%. **c** Decrease 45 by 50%.
d Decrease 700 by 12%. **e** Decrease 8000 by 8%. **f** Decrease 450 by 25%.
g Decrease 68 by 75%. **h** Decrease 9000 by 1%. **i** Decrease 7000 by 100%.

Example 8 Finding a percentage change

a The price of a mobile phone increased from $250 to $280. Find the percentage increase.
b The population of a town decreases from 3220 to 2985. Find the percentage decrease and round to one decimal place.

7 complete the tables, showing percentage change. Round to one decimal place where necessary.

a

Original amount	New amount	Increase	Percentage change
40	60		
12	16		
100	125		
24	30		
88	100		
48	92		
200	250		

b

Original amount	New amount	Decrease	Percentage change
90	81		
100	78		
20	15		
24	18		
150	50		
9	8.3		
3	2.5		

Example 9 Finding the original amount

After rain, the volume of water in a tank increased by 24% to 2200 L. How much water was in the tank before it rained? Round to the nearest litre.

8 Find the original amounts for each of the following if:

- **a** an increase of 10% on the cost of a can of cola drink increased it to $3.30
- **b** an increase of 10% on the cost of a meal increased the cost to $88
- **c** after an increase of 5%, the cost of a pair of running shoes came to $210
- **d** a decrease of 30% made the cost of car insurance $350
- **e** a decrease of 60% brought the price of a used car down to $5000

9 The price of a computer was decreased by 15% in a sale. What is the sale price, if the original price was $2100?

10 Plumbers on a salary of $82 570 were given a $2\frac{1}{2}\%$ pay increase. Find their new annual salary.

11 A car manufacturer intends to increase sales by 14.7% next year. If the company sold 21 390 new cars this year, how many does it expect to sell next year?

12 The length of a bike sprint race is increased from 800 m to 1200 m. Find the percentage increase.

$$\%\,\text{increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$

Answer

5 **a** 61.6 **b** 1176 **c** 112 **d** 934.5 **e** 207
f 540 **g** 12 **h** 196 **i** 31.50

6 **a** 76 **b** 540 **c** 22.5 **d** 616 **e** 7360
f 337.5 **g** 17 **h** 8910 **i** 0

7 **a**

Original amount	New amount	Increase	Percentage change
40	60	20	50%
12	16	4	$33\frac{1}{3}\%$
100	125	25	25%
24	30	6	25%
88	100	12	13.6%
48	92	44	91.7%
200	250	50	25%

b

Original amount	New amount	Decrease	Percentage change
90	81	9	10%
100	78	22	22%
20	15	5	25%
24	18	6	25%
150	50	100	$66\frac{2}{3}\%$
9	8.3	0.7	7.8%
3	2.5	0.5	$16\frac{2}{3}\%$

8 **a** \$3 **b** \$80 **c** \$200 **d** \$500 **e** \$12 500

9 \$1785

10 \$84 634.25

11 24 534 cars

12 50%