STAGE 4

MATHSCAPE 7

Working Mathematically

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Preface

Mathscape 7 is a comprehensive teaching and learning resource that has been written to address the new Stage 4 Mathematics syllabus in NSW. Our aim was to write a book that would allow students of all abilities to grow in confidence, to improve their understanding of Mathematics and to develop a genuine appreciation of its inherent beauty. Teachers who wish to inspire their students will find this an exciting and yet very practical resource. The text encourages a deeper exploration of mathematical ideas through substantial, well-graded exercises that consolidate students' knowledge, understanding and skills. It also provides opportunities for students to explore the history of Mathematics and to address many practical applications in contexts that are both familiar and relevant.

From a teaching perspective, we sought to produce a book that would adhere as strictly as possible to both the content and spirit of the new syllabus. Together with *Mathscape 8*, this book allows teachers to confidently teach the Stage 4 course knowing that they are covering all of the mandatory outcomes. *Mathscape 7* provides teachers with an extensive number of well-graded exercises designed to cater for students of all abilities.

Content from Stage 3 has been included in each chapter, where appropriate. This will allow teachers to diagnose significant misconceptions and identify any content gaps. For those students who have achieved the relevant Stage 3 outcomes, this material could be used as a review to introduce the Stage 4 topic, or to revise important concepts when they occur. However, for those students who have not achieved these outcomes by the start of Year 7, this material will be new work. All content is clearly listed as either Stage 3 or Stage 4 in the contents section at the front of the book. A detailed syllabus correlation grid has been provided for teachers on the *Mathscape 7 School CD-ROM*.

Mathscape 7 has embedded cross-curriculum content, which will support students in achieving the broad learning outcomes defined by the Board of Studies. The content also addresses the important Key Competencies of the Curriculum Framework, which require students to collect, analyse and organise information; to communicate mathematical ideas; to plan and organise activities; to work with others in groups; to use mathematical ideas and techniques; to solve problems; and to use technology.

A feature of each chapter which teachers will find both challenging and interesting for their students is the ‘Focus on working mathematically’ section. Although the processes of working mathematically are embedded throughout the book, these activities are specifically designed to provoke curiosity and deepen mathematical insight. Most begin with a motivating real-life context, such as sport, newspapers, the sun's rays, or maps, but on occasion they begin with a purely mathematical question. These activities can be used for these purposes.

In our view, there are many legitimate, time-proven ways to teach Mathematics successfully. However, if students are to develop a deep appreciation of the subject, they will need more than traditional methods. We believe that all students should be given the opportunity to appreciate Mathematics as an essential and relevant part of life. They need to be given the opportunity to begin a mathematical exploration from a real-life context that is meaningful to them. To show interest and enjoyment in enquiry and the pursuit of mathematical knowledge, students need activities where they can work with others and listen to their arguments, as well as work individually. To demonstrate confidence in applying their mathematical knowledge and skills to the solution of everyday problems, they will need experience of this in the classroom. If they are to learn to persevere with difficult and challenging problems, they will need to experience these sorts of problems as well. Finally, to recognise that Mathematics has been developed in many cultures in response to human needs, students will need experiences of what other cultures have achieved mathematically.

We have tried to address these values and attitudes in this series of books. Our best wishes to all teachers and students who are part of this great endeavour.

Clive Meyers
Lloyd Dawe
Graham Barnsley
Lindsay Grimison

How to use this book

Mathscape 7 is a practical resource that can be used by teachers to supplement their teaching program. The exercises in this book and the companion text *Mathscape 8* provide a complete and thorough coverage of all content and skills in the Stage 4 course. The great number and variety of questions allows for the effective teaching of students of all abilities. The incorporation of a large amount of Stage 3 material means that this book is also suitable for those students who have not achieved all of the Stage 3 outcomes by the start of Year 7. Each chapter contains:

- a set of chapter outcomes directed to the student
- all relevant theory and explanations, with important definitions and formulae boxed and coloured
- step-by-step instructions for standard questions
- a large number of fully worked examples preceding each exercise
- extensive, thorough and well-graded exercises that cover each concept in detail
- chapter-related, problem-solving activities called 'Try this' integrated throughout
- a language skills section linked to the *Macquarie Learners' Dictionary*
- a mixed problem-solving section
- novel learning activities focussing on the process of working mathematically
- a thorough chapter review.

Explanations and examples

The content and skills required to complete each exercise have been introduced in a manner and at a level that is appropriate to the students in this course. Important definitions and formulae have been boxed and coloured for easy reference. For those techniques that require a number of steps, the steps have been listed in point form, boxed and coloured. Each exercise is preceded by several fully worked examples. This should enable the average students to independently complete the majority of relevant exercises if necessary.

The exercises

The exercises have been carefully graded into three distinct sections:

- Introduction. The questions in this section are designed to introduce students to the most basic concepts and skills associated with the outcome(s) being covered in the exercise. Students need to have mastered these ideas before attempting the questions in the next section.
- Consolidation. This is a major part of the exercise. It allows students to consolidate their understanding of the basic ideas and apply them in a variety of situations. Students may need to use content learned or skills acquired in previous exercises or topics to answer some of these questions. The average students should be able to complete most of the questions in this section, although the last few questions may be a little more difficult.
- Further applications. Some questions presented in this section will be accessible to the average student; however, the majority of questions are difficult. They might require a reverse procedure, the use of algebra, more sophisticated techniques, a proof, or simply time-consuming research. The questions can be open-ended, requiring an answer with a justification. They may also involve extension or off-syllabus material. In some questions, alternative techniques and methods of solution other than the standard method(s) may be introduced, which will only confuse the average student.

Teachers need to be selective in the questions they choose for their students. Some students may not need to complete all of the questions in the Introduction or Consolidations sections of each exercise, while only the most able students should usually be expected to attempt the questions in the Further applications section. Those questions not completed in class might be set as homework at the teacher's discretion. It is not intended that any student would attempt to answer every possible question in each exercise.

Focus on working mathematically

The Working Mathematically strand of the syllabus requires a deeper understanding of Mathematics than do the other strands. As such, it will be the most challenging strand for students to engage with and for teachers to assess. The Working Mathematically outcomes listed in the syllabus have been carefully integrated into each chapter of the book; however, we also decided to include a learning activity in each chapter that will enable teachers to focus sharply on the processes of working mathematically. Each activity begins with a real-life context and the Mathematics emerges naturally. Teachers are advised to work through them before using them in class. Answers have not been provided, but notes for teachers have been included on the *Mathscape 7 School CD-ROM*, with suggested web links. Teachers may wish to select and use the Learning activities in 'Focus on working mathematically' for purposes of assessment. This too is encouraged. The Extension activities will test the brightest students. Suggestions are also provided to assess the outcomes regarding Communication and Reflection.

Problem solving

Each chapter contains a number of small, chapter-related, problem-solving activities called 'Try this'. They may be of some historical significance, or require an area outside the classroom, or require students to conduct research, or involve the use of algebra, while others relate the chapter content to real-life context. A series of 10 mixed problems are also included at the end of each chapter under the heading 'Problem solving'. Again, teachers are advised to work through these exercises before using them in class.

Technology

The use of technology is a clear emphasis in the new syllabus. It is expected that all students will have access to a scientific calculator in Year 7. The clear intention is that it should be used to reinforce mathematical understanding, rather than as an aid in day-to-day computation at this stage. A symbol has therefore been inserted next to those questions where the use of a calculator is required or is deemed appropriate.

Innovative technology for supporting the growth of understanding of mathematical ideas is provided on the *Mathscape 7 School CD-ROM*, which is fully networkable and comes free of charge to schools adopting *Mathscape 7* for student use. Key features of the CD-ROM include:

- Spreadsheet activities
- Dynamic geometry
- Animations
- Original interactive maths programs
- Student worksheets
- Web links.

Language

The consistent use of correct mathematical terms, symbols and conventions is emphasised strongly in this book, while being mindful of the students' average reading age. Students will only learn to use and spell correct mathematical terms if they are required to use them frequently in appropriate contexts. A language section has also been included at the end of each chapter titled 'Language link with Macquarie', where students can demonstrate their understanding of important mathematical terms. This might, for example, include explaining the difference between the mathematical meaning and the everyday meaning of a word. Most chapters include a large number of worded problems. Students are challenged to read and interpret the problems, translate them into mathematical language and symbols, solve the problems, then give the answers in an appropriate context.

Syllabus correlation

A syllabus correlation grid and teaching notes are offered on the *Mathscape 7 School CD-ROM*.

Clive Meyers
Lloyd Dawe
Graham Barnsley
Lindsay Grimison

1 Whole numbers and number systems

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- state the place value of each digit in a number
- write the basic numeral for a number given in words and vice-versa
- add, subtract, multiply and divide numbers
- write a number in the expanded form using powers of 10
- write the basic numeral for a number that has been given in the expanded form
- round off a whole number to the nearest power of 10
- evaluate expressions that contain grouping symbols
- use the order of operations to evaluate expressions
- use mathematical symbols to show the meaning of worded expressions
- find the average of a set of numbers.

Stage 4

After completing this chapter, you should be able to:

- convert between Hindu–Arabic numerals and those of other number systems
- list the strengths and weaknesses of different number systems
- list the advantages of the Hindu–Arabic number system over other number systems
- divide 2- and 3-digit numbers by a 2-digit number using long division
- use a range of mental strategies to aid computation
- use index notation to express powers of numbers
- evaluate expressions that contain numbers given in index form
- evaluate expressions using the associative, commutative and distributive laws
- use the distributive law in reverse to evaluate expressions.

1.1 Early numeration systems

In early human civilisations, people needed only a simple number system for their everyday needs. Before formal number systems were invented, the concept of 'how many' was determined by making notches in bones or pieces of wood, with one notch representing each thing or object. Other practices included tying knots in a piece of rope or moving pebbles from one place to another, one pebble for each object. We call this **one-to-one correspondence**.

Counting with pebbles on a one-to-one basis became difficult when large numbers of objects were involved. Thus a single pebble different in size, shape or colour came to be used in place of a pile of pebbles (ten or a hundred for example). This determined a level of order among the pebbles and therefore among the numbers they represented. It is from these early ideas that we have the concept of a **base**, such as in our number system which is based on the number 10.

As time passed, small groups of people merged, forming larger groups for reasons of security. With greater numbers of people and objects to keep track of, many civilisations found that they needed larger numerals and a consistent way of recording them. In this exercise we will look at the way in which the early Babylonian, Egyptian, Greek and Mayan peoples wrote numbers. Of these, the first three were base 10 systems like ours, however, they did not use place value (at least in the way that we think of it). The Mayan numeration system, however, was a base 20 system that did make use of place value and had a symbol for zero. In larger numbers the Mayans used 360 as the base.

Babylonian numerals

Since they did not have access to papyrus or parchment, the Babylonians pressed their words and numbers into tablets of wet clay with a stylus, forming wedge-shaped or cuneiform characters. These were then baked in the sun or in a kiln, thus creating permanent records.

Babylonian numerals	Hindu–Arabic numerals
▼	1
◀	10
▼▶	100

Egyptian numerals

The Egyptians used everyday objects as numerals. These numerals, or hieroglyphics, were written on papyrus or carved into stone.

Egyptian numerals		Hindu–Arabic numerals
Staff		1
Heel bone		10
Coiled rope		100
Lotus flower		1000
Bent reed		10 000
Fish		100 000
Surprised man		1 000 000

Greek numerals

The Greeks used the first letter of the word of various numbers as numerals. The symbols are used singly and in combination.

Greek numerals	Hindu–Arabic numerals
|	1
Δ	10
H	100
X	1000
M	10 000

Greek numerals	Hindu–Arabic numerals
⌐	5
⌐Δ	50
⌐H	500
⌐X	5000
⌐M	50 000

Mayan numerals

The Mayans of Central America used a system of dots and strokes as well as a symbol for zero to represent their numbers. The lower symbols were units from 1 to 19 while the upper symbols were multiples of 20.

Mayan numerals	Hindu–Arabic numerals
•	1
• •	2
• • •	3
• • • •	4
—	5
—•	6
—• •	7
⋮	⋮
=	10

Mayan numerals	Hindu–Arabic numerals
=•	11
⋮	⋮
⋮	⋮
≡	15
≡•	16
⋮	⋮
⋮	⋮
•⬭	20

Mayan numerals	Hindu–Arabic numerals
••	21
•• •	22
⋮	⋮
⋮	⋮
—•	25
⋮	⋮
⋮	⋮
=•	30

Hindu-Arabic numerals

The Babylonian, Egyptian, Greek (and Roman) numeration systems were cumbersome, requiring many symbols to write even the simplest of numbers. Although they made it possible to count objects and record numbers in a written form, these systems were not user-friendly when it came to arithmetic. Imagine trying to calculate HΔΔΓΠ × HHΔΔΔΓ in Greek numerals! A better numeration system was needed to cope with the growing needs of an increasingly complex society.

Our present numeration system is called the **Hindu–Arabic system**, since it originated in India with the Hindus in approximately 300 BC, and was spread to Europe by the Arabs. The digits in this system underwent constant change as they spread through the Arab world and looked quite different in 300 BC than they appear today. The table to the right shows some of the changes in the digits 0 to 9 as they occurred over the centuries.

AD 2000	300 BC	AD 100	AD 200	AD 400	AD 876	AD 1400
1	—	—	—	—	>	1
2	=	=	=	=	?	2
3	≡	≡	≡	≡	3	3
4	⊤	⊤	⊤	⊤	γ	4
5	M	r	r	⊤	4	5
6	6	6	b	F	6	6
7	7	7	7	⊓	7	7
8	5	5	3	3	8	8
9	9	3	3	3	9	9
0					O	O

The Hindu–Arabic system is a **place value system**, based on the number 10. It is the first numeration system in which it was possible to write any number that one could think of, without having to create new symbols or digits. The system included the use of zero as a place holder, the earliest written evidence of which appears in an inscription in AD 876. In this inscription, the numbers 50 and 270 are both written with zeros.

The greatest advantage of the new system was that it made it possible to perform arithmetic operations with relative ease, unlike the earlier systems.

Example 1

Write each of the following using Babylonian numerals.

a 53

b 264

Solutions

a 53 =

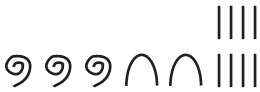
b 264 =

Example 2

Write each of the following using Egyptian numerals.

a 328 **b** 2604 **c** 1 250 013

Solutions

a 328 = 

b 2604 =

c 1 250 013 =

Example 3

Write each of these using Greek numerals.

a 186 **b** 2761 **c** 89 203

Solutions

a 186 = H⌈Δ⌉ΔΔΔ⌈⌉I **b** 2761 = XX⌈H⌉HH⌈Δ⌉ΔI

c 89 203 = ⌈M⌉MMM⌈X⌉XXXXHHIII

Example 4

Write each of these using Mayan numerals.

a 14 **b** 20 **c** 37

d 53 **e** 107 **f** 140

Solutions

a 14 = (2 lots of 5 plus 4)

b 20 = (1 lot of 20 plus 0)

c 37 = (1 lot of 20 plus 17)

d 53 = (2 lots of 20 plus 13)

e 107 = (5 lots of 20 plus 7)

f 140 = (7 lots of 20 plus 0)

Exercise 1.1

1 Write these Babylonian numerals as Hindu–Arabic numerals.

a

b

c

d

e

f

g

h

i

2 Write these Hindu–Arabic numerals as Babylonian numerals.

a 4 **b** 20 **c** 73 **d** 101

e 156 **f** 207 **g** 280 **h** 325

Consolidation

3 Write each of these Egyptian numerals using Hindu–Arabic numerals.

a
b
c
d
e
f
g
h
i

4 Write each of these Hindu–Arabic numerals using Egyptian numerals.

a 45	**b** 427	**c** 560	**d** 1309
e 3012	**f** 10 136	**g** 361 090	**h** 2 450 375

5 Express each of these Greek numerals as Hindu–Arabic numerals.

a IIII	**b** ⌐II	**c** ΔIII
d ΔΔ⌐	**e** ⌈Δ⌉ΔΔΔΔ	**f** H⌈Δ⌉Δ⌐I
g HHHHΔΔΔ	**h** ⌐H⌉HH⌈Δ⌉II	**i** ⌐H⌉HHH⌈Δ⌉ΔΔ⌐III
j XHHΔ⌐IIII	**k** ⌈X⌉XX⌐H⌉⌐	**l** ⌈M⌉MMM⌐H⌈Δ⌉⌐I

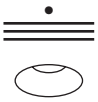
6 Express each of these Hindu–Arabic numerals as Greek numerals.

a 9	**b** 46	**c** 75	**d** 263
e 531	**f** 902	**g** 1380	**h** 5028
i 10 157	**j** 23 090	**k** 50 273	**l** 64 389

Further applications

7 Change these Mayan numerals to Hindu–Arabic numerals. (The numbers increase in value throughout this question to avoid confusion.)

a
b
c
d
e
f
g
h
i
j
k
l
m
n
o
p
q
r
s
t

u **v** **w** **x** 

8 Change these Hindu–Arabic numerals to Mayan numerals.

a 7	**b** 13	**c** 20	**d** 25	**e** 34	**f** 46
g 50	**h** 58	**i** 66	**j** 72	**k** 85	**l** 97
m 111	**n** 120	**o** 188	**p** 259	**q** 298	**r** 300

9 Discuss the relative strengths and weaknesses of:

a the Babylonian number system
b the Egyptian number system
c the Greek number system
d the Mayan number system

1.2 Roman numerals

About two thousand years ago, the Roman Empire had conquered most of the known world including the ancient Egyptians. As the empire spread, so did the use of the Roman numeration system. It was superior to other systems such as the Egyptian system since numbers could be written using many less symbols. It also included the use of the **subtraction principle**, where the placement of a smaller numeral before that of a larger one indicated that the smaller numeral should be subtracted from the larger numeral. This meant that numbers such as 9 could now be written using only two symbols rather than nine symbols as was the case with the Egyptian and Babylonian systems. Although the Roman system included the subtraction principle, it is still considered an additive system, not a place value system, since there are no actual place columns.

Roman numeral	Hindu–Arabic numeral
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

When writing Roman numerals, we note that the symbols I, X, C and M cannot be written more than three at a time and the symbols V, L and D can only be written one at a time. The subtraction principle states that only I, X, C (and M in large numbers) can precede numerals with a greater value, and then only the next two largest numerals. That is, I can only precede V and X, X can only precede L and C, C can only precede D and M. The numerals V, L and D cannot precede a numeral of greater value.

The generally accepted convention for multiplying the value of a Roman numeral by 1000 is to place a bar above it. However, this is not the way that the Romans wrote large numbers, but rather is thought to be a technique adopted by later writers. In fact, large numbers such as 10 000 and 100 000 were written as CCIↃↃ and CCCIↃↃↃ, while 5000 and 50 000 were written as IↃↃ and IↃↃↃ. The Roman numeral for 1000 was at times written as CIↃ. Half of this number, i.e. 500, was also written as IↃ, from which the symbol D most likely evolved.

To write a Hindu–Arabic numeral in Roman numerals:
- **write each of the digits in Roman numerals, one digit at a time**
- **use the subtraction principle to write 4, 40, 400, 9, 90 or 900**
- **place a bar above a numeral to multiply its value by 1000.**

Example 1

Write each of these using Roman numerals.

a 86 **b** 253 **c** 2677

Solutions

a 86 = LXXXVI **b** 253 = CCLIII **c** 2677 = MMDCLXXVII

Example 2

Write each of these using Roman numerals.

a 14 **b** 49 **c** 490 **d** 994

Solutions

a 14 = XIV **b** 49 = XLIX **c** 490 = CDXC **d** 994 = CMXCIV

Example 3

Write each of these using Roman numerals.

a 5000 **b** 9000 **c** 24 605 **d** 369 040

Solutions

a 5000 = $\overline{\text{V}}$ **b** 9000 = M$\overline{\text{X}}$

c 24 605 = $\overline{\text{X}}\,\overline{\text{X}}M\overline{\text{V}}$DCV **d** 369 040 = $\overline{\text{C}}\,\overline{\text{C}}\,\overline{\text{C}}\,\overline{\text{L}}\,\overline{\text{X}}M\overline{\text{X}}$XL

Exercise 1.2

1 Write the Roman numeral for each of the following.

a 1 **b** 5 **c** 10 **d** 50
e 100 **f** 500 **g** 1000

2 Write these numbers in Roman numerals.

a 2 **b** 6 **c** 8 **d** 11 **e** 17
f 20 **g** 25 **h** 28 **i** 31 **j** 33
k 36 **l** 38 **m** 52 **n** 55 **o** 63
p 66 **q** 70 **r** 75 **s** 82 **t** 88

3 Write each of the following in Hindu–Arabic numerals.

a III **b** VII **c** XII **d** XV **e** XVIII
f XXI **g** XXVI **h** XXXII **i** XXXVII **j** LIII

k LVI	**l** LVIII	**m** LX	**n** LXII	**o** LXVII
p LXXIII	**q** LXXVI	**r** LXXX	**s** LXXXV	**t** LXXXVII

Consolidation

4 Change these numerals to Roman numerals.

a 107	**b** 115	**c** 138	**d** 172	**e** 205
f 230	**g** 327	**h** 353	**i** 501	**j** 525
k 552	**l** 611	**m** 638	**n** 666	**o** 687
p 710	**q** 723	**r** 772	**s** 816	**t** 888

5 Express these Roman numerals as Hindu–Arabic numerals.

a CI	**b** CXI	**c** CXXV	**d** CXXXIII
e CLVI	**f** CLXXVII	**g** CC	**h** CCLXV
i CCCXVII	**j** CCCLXXXI	**k** DVIII	**l** DLV
m DLXVIII	**n** DC	**o** DCXXI	**p** DCXXXV
q DCCXXXIII	**r** DCCLI	**s** DCCCXII	**t** DCCCLXVI

6 Write as Roman numerals.

a 1013	**b** 1125	**c** 1266	**d** 1572
e 1705	**f** 2000	**g** 2021	**h** 2678
i 2883	**j** 3250	**k** 3576	**l** 3888

7 Write as Hindu–Arabic numerals.

a MVI	**b** MXXXVII	**c** MC
d MCCX	**e** MDV	**f** MDCCLVIII
g MMLXXII	**h** MMCXXXII	**i** MMDCCCXXV
j MMMXXXI	**k** MMMCCXVI	**l** MMMDCCXXVII

8 Use place value to write each of the following in Roman numerals.

a 4	**b** 40	**c** 400	**d** 9	**e** 90
f 900	**g** 44	**h** 49	**i** 94	**j** 99
k 404	**l** 409	**m** 440	**n** 490	**o** 444
p 449	**q** 494	**r** 499	**s** 904	**t** 909
u 940	**v** 990	**w** 944	**x** 949	**y** 999

9 Write each of these in Roman numerals.

a 14	**b** 19	**c** 24	**d** 39	**e** 43
f 59	**g** 84	**h** 97	**i** 142	**j** 190
k 344	**l** 419	**m** 496	**n** 714	**o** 939
p 1049	**q** 1492	**r** 2744	**s** 2919	**t** 3499

10 Write each of these in Hindu–Arabic numerals.

a XXXIX	**b** LIV	**c** LXXXIX	**d** XCVI
e CXLIV	**f** CCCXC	**g** CDXLV	**h** CDXCII
i DIX	**j** DCCLXXXIV	**k** DCCCXCIX	**l** CMXIX

m CMXLVII	**n** CMXCVIII	**o** MXLVI	**p** MCDXXXIV
q MDCCXL	**r** MMXCIX	**s** MMCMXIV	**t** MMMCDXCI

■ Further applications

11 Express each of the following in Roman numerals.

a 5215	**b** 8924	**c** 4009	**d** 9257	**e** 10 466
f 14 889	**g** 50 412	**h** 75 676	**i** 275 318	**j** 3 549 953

12 Express each of the following in Hindu–Arabic numerals.

a $\overline{\text{V}}$DCC	**b** M$\overline{\text{V}}$DXXIII	**c** $\overline{\text{V}}$MMDLVI
d $\overline{\text{X}}$CCC$\overline{\text{X}}$LI	**e** $\overline{\text{X}}$M$\overline{\text{V}}$CMXLVII	**f** $\overline{\text{L}}\overline{\text{X}}M\overline{\text{M}}$DLIV
g M$\overline{\text{X}}$C$\overline{\text{V}}$CCCXII	**h** M$\overline{\text{C}}\overline{\text{C}}\overline{\text{L}}$MVIII	**i** $\overline{\text{C}}$M$\overline{\text{X}}\overline{\text{L}}\overline{\text{V}}$MXIX

1.3 Place value

The Hindu–Arabic numeration system is a base 10 place value system, with each place column having ten times the value of the column to its immediate right. The place value columns up to one million are:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
1 000 000	100 000	10 000	1000	100	10	1

When writing large numbers, by convention, we leave a space to the left of the hundreds digit and to the left of the hundred thousands digit. However, in typed numbers, the convention is not to leave a space in four-digit numbers.

Example 1

Write down the value of the 9 in each of these.

a 395 **b** 9621 **c** 1 298 310

Solutions

a The 9 is in the tens column, ∴ the value of the 9 is 90.
b The 9 is in the thousands column, ∴ the value of the 9 is 9000.
c The 9 is in the ten thousands column, ∴ the value of the 9 is 90 000.

Example 2

Write these numerals in words.

a 5074
b 92 603
c 2 350 049

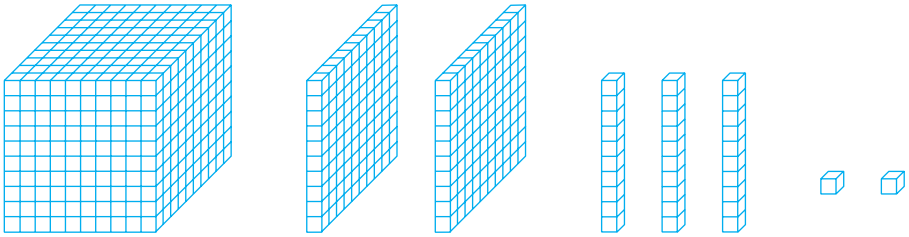
Solutions

a Five thousand and seventy-four.
b Ninety two thousand six hundred and three.
c Two million three hundred and fifty thousand and forty-nine.

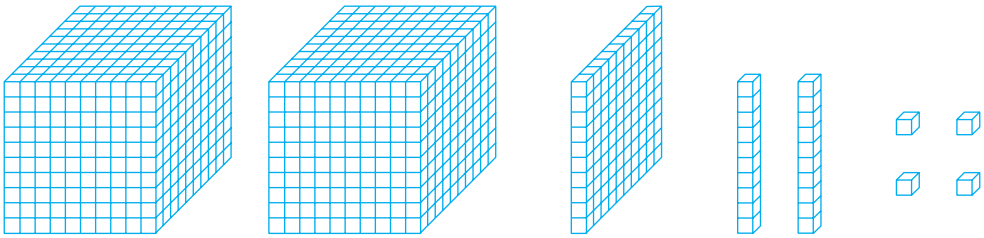
Exercise 1.3

1 Write down the number that is represented by each of the following.

a



b



2 In which place column is the 5 in each of these?

a 315	**b** 452	**c** 5762	**d** 506 217
e 1521	**f** 457 366	**g** 5 200 414	**h** 218 527
i 351 009	**j** 65 277	**k** 1 518 883	**l** 25 786 687

3 Write down the value of the 7 in each numeral.

a 276	**b** 507	**c** 1755	**d** 7243
e 13 749	**f** 78 004	**g** 27 912	**h** 715 089
i 164 076	**j** 287 800	**k** 1 728 441	**l** 37 048 002

4 What is the value of the 4 in each numeral?

a 46	**b** 417	**c** 184	**d** 43 511
e 24 300	**f** 4 381 656	**g** 1 417 899	**h** 6204
i 124 610	**j** 178 452	**k** 47 706	**l** 34 098 555

Consolidation

5 What is the:

a smallest 2-digit number?	**b** largest 2-digit number?
c smallest 3-digit number?	**d** largest 3-digit number?
e smallest 4-digit number?	**f** largest 4-digit number?

6 Find the:

a smallest 2-digit number with a 7 in the tens place
b largest 2-digit number with a 5 in the tens place
c smallest 2-digit number with a 2 in the units place
d largest 2-digit number with a 6 in the units place
e smallest 3-digit number with a 4 in the hundreds place

f largest 3-digit number with an 8 in the hundreds place
g smallest 3-digit number with a 3 in the tens place
h largest 3-digit number with a 1 in the tens place
i smallest 3-digit number with a 9 in the units place
j largest 3-digit number with a 7 in the units place

7 Write the simplest numeral for each of the following.
a 300 + 80 + 6
b 20 000 + 7000 + 500 + 90 + 2
c 900 000 + 40 000 + 6000 + 700 + 10 + 5
d 1 000 000 + 500 000 + 90 000 + 2000 + 400 + 40 + 6
e 5000 + 200 + 90
f 30 000 + 7000 + 10
g 60 + 30 000 + 700 + 8 + 4000
h 300 + 6 000 000 + 2 + 50 000

8 Write the basic numeral for each of the following.
a Three thousand two hundred and sixty-eight
b Forty-two thousand seven hundred and fifty-one
c One hundred and eighty-six thousand five hundred and twenty-two
d Eight hundred and ninety-four thousand one hundred and seventy-three
e Two hundred and thirty-one thousand four hundred and twelve
f Six thousand and five
g Ninety three thousand and seventy
h One hundred and seventeen thousand and nine
i Five hundred and one thousand six hundred
j Three hundred thousand and two
k Four million two hundred and seventy thousand six hundred and eleven
l Eighty nine million fifty three thousand and four

9 Write each of the following numbers in words.
a 5613
b 12 395
c 84 337
d 165 914
e 1 343 572
f 37 411 926
g 8030
h 21 007
i 40 067
j 102 040
k 13 006 090
l 350 100 008

10 Arrange these numbers in ascending order.
a 51 759, 399 812, 216 099, 49 473, 93 686, 8247
b 10 237, 138 834, 20 020, 375 590, 517 933, 20 119
c 365, 1 265 784, 274 998, 1023, 475 662, 98 998
d 17 534 811, 9 938 775, 256 090, 24 489, 800 030, 5 387 751

Further applications

11 How many times greater is the value of the first 7 than the value of the second 7 in each of these?
a 36 797
b 75 671
c 71 338 784
d 743 065 279

12 **a** List all possible 3-digit numbers that could be formed using the digits 2, 5, 9 if none of the digits can be repeated.

b List all possible numbers of length 1, 2 or 3 digits that could be formed using only the digits 3, 6, 7 if repetition of digits is allowed.

c List all possible 3-digit numbers that could be formed using the digits 0, 1, 2 if repetition of digits is allowed.

13 **a** What is the largest 5-digit number that could be formed using any digits without repetition?

b What is the smallest 5-digit number that could be formed using any digits without repetition?

1.4 Addition review

Example

Evaluate:

a 43 + 52

b 1729 + 6587

Solutions

a
$$\begin{array}{r} 43 \\ \underline{52} \; + \\ 95 \end{array}$$

b
$$\begin{array}{r} 1\,7\,2\,9 \\ {}_{1}6_{1}5_{1}8\;7 \; + \\ \hline 8\,3\,1\,6 \end{array}$$

Exercise 1.4

1 Write answers to each of these.

a 7 + 4	**b** 5 + 8	**c** 6 + 6	**d** 9 + 5
e 8 + 7	**f** 4 + 8	**g** 6 + 9	**h** 8 + 8
i 9 + 4	**j** 7 + 7	**k** 9 + 8	**l** 6 + 5
m 6 + 7	**n** 8 + 6	**o** 7 + 5	**p** 9 + 9

2 In a magic square, the sum of the numbers is the same horizontally, vertically and diagonally. Use all of the digits from 1 to 9 and 1 to 16 to complete these magic squares.

a

	7	6
	5	
	3	

b

1	8		12
	11		7
		16	
15	10		6

3 Add the following numbers.

a	**b**	**c**	**d**	**e**	**f**
7	6	8	5	4	9
5	8	8	9	7	6
8	4	5	9	8	6
6	7	7	7	6	9
7	7	6	9	9	6

4 Find the value of:

a 50 + 60	**b** 70 + 80	**c** 90 + 40
d 80 + 90	**e** 800 + 700	**f** 600 + 700
g 900 + 900	**h** 800 + 600	**i** 4000 + 8000
j 7000 + 5000	**k** 6000 + 9000	**l** 7000 + 7000

■ Consolidation

5 Write each addition vertically, then evaluate.

a 25 + 34	**b** 16 + 82	**c** 35 + 28	**d** 16 + 44
e 34 + 59	**f** 49 + 77	**g** 48 + 66	**h** 74 + 97

6 Write each addition vertically, then evaluate.

a 128 + 265 + 417	**b** 292 + 196 + 310
c 508 + 287 + 749	**d** 651 + 824 + 997

7

a 3724	**b** 197	**c** 14	**d** 48
176	19	7253	4239
28	3254	908	57
5 +	817 +	5774 +	695 +

8

a 375 496	**b** 148 481	**c** 402 073
103 219	227 620	720 598
537 628 +	634 947 +	833 646 +

9 Answer each of the following questions by performing an addition and then a subtraction.

a 53 + 19	**b** 78 + 39	**c** 37 + 28
d 84 + 97	**e** 106 + 69	**f** 117 + 48
g 234 + 99	**h** 529 + 98	**i** 463 + 119
j 645 + 128	**k** 314 + 199	**l** 576 + 297

10
- **a** Find the total number of days from the beginning of March to the end of August.
- **b** How many students attend a high school of 457 boys and 519 girls?
- **c** The number of people attending a concert each night were 19 820, 17 455 and 16 998. How many people attended the concert altogether?
- **d** A butcher had four strings of sausages hanging in the window. If there were 14, 17, 18 and 15 sausages on these strings, how many sausages were there?
- **e** An interstate truck driver drove 587 km on the first day, 766 km on the second day and 609 km on the third day. Find the total distance travelled.

■ Further applications

11 Find the missing digits in each of these additions.

a 8 4 □	**b** 6 □ 7	**c** 9 □ 5 8
□ □ 2	3 9 □	□ 6 7 7
5 9 3 +	□ 0 8 +	9 3 4 □ +
□ 2 0 4	1 4 3 0	□ 0 4 8 1

TRY THIS

Ten pin bowling

In bowling, each game has a total of 10 frames.

1	2	3	4	5	6	7	8	9	10
5 2	– 6	8 /	4 2	X	7 1	4 5	8 /	X	3 / 6
7	13	27	33	51	59	?	?	?	?

In each frame you bowl two balls, except if your first ball knocks all the pins down (**strike**).

In the first frame above, the first ball knocked 5 pins down and the second ball knocked 2 pins down for a total of 7.

1
5 2
7

In the second frame, the first ball didn't knock any pins down and the second ball knocked 6 pins down. This gives an overall total for frames 1 and 2 of 13 pins (7 + 6).

In the third frame, the first ball knocked 8 pins down and the second ball knocked the last two pins down. Since all 10 pins were knocked down, we put a slash. This is called a **spare**. If you get a spare, *your score for that frame is 10 plus the number of pins you knock down with your next ball*. Since we hit 4 pins down with the first ball in the next frame, we receive 10 + 4 = 14 pins for the third frame. This gives a total of 27 for the first three frames.

2
– 6
7 + 6 = 13

3
8 /
13 + (10 + 4) = 27

4
4 2
27 + 6 = 33

In the fourth frame, we knocked down a total of 6 pins with our two balls for a total of 33 pins for the first four frames.

In the fifth frame, we knocked down all 10 pins with the first ball—this is called a strike. We put a cross in the first box. If you obtain a strike, *your score for that frame is 10 plus the number of pins you knock down with the next two balls*. Since we knocked down 7 + 1 = 8 pins in the next frame, we receive 10 + 8 = 18 pins for the fifth frame.

5
X
33 + (10 + 8) = 51

This gives a total of 18 + 33 = 51 pins for the first five frames. Now complete the scoring for frames 6, 7, 8, 9 and 10.

Did you obtain the following?

6		
	7	1
59		

51 + (7 + 1) = 59

7		
	4	5
68		

59 + (4 + 5) = 68

8		
	8	╱
88		

68 + (10 + 10) = 88

9		
	╳	
108		

88 + (10 + 10) = 108

10		
3	╱	6
24		

108 + (10 + 6) = 124

Try scoring the following games! You might suggest to your teacher an appropriate excursion!

NAME	1	2	3	4	5	6	7	8	9	10
CLARE	8 –	2 5	3 4	╳	2 ╱	– 4	8 ╱	4 1	6 ╱	5 ╱ 3
BARBARA	2 4	3 ╱	3 1	6 ╱	2 7	– –	8 ╱	6 –	╳	╳ ╳ ╳
MARIE	╳	6 3	5 ╱	2 5	4 3	╳	6 ╱	3 ╱	7 –	4 3
ROSIE	4 ╱	6 ╱	7 2	3 4	5 1	4 2	3 ╱	5 ╱	3 ╱	2 ╱ 8
EDWINA	╳	╳	╳	╳	5 ╱	6 ╱	– ╱	2 7	╳	3 ╱ 5

1.5 Subtraction review

Subtraction is the **inverse** or opposite of addition. We say that subtraction 'undoes' addition. For example, having started with 10 and then added 3, we could get back to 10 by subtracting 3. That is, 10 + **3** – **3** = 10.

When subtracting numbers and faced with having to subtract a larger digit from a smaller digit it is often necessary to take 1 'ten' from the column to the left and add it to the column in question. Remember, it may be that the 'ten' being taken is really ten hundreds or ten thousands, rather than simply ten ones.

Example

Evaluate:

a 58 – 21

b 64 532 – 21 805

Solutions

a

$$\begin{array}{r} 58 \\ \underline{21} \; - \\ 37 \end{array}$$

b

$$\begin{array}{r} 6^{3}\cancel{4}^{1}5^{2}\cancel{3}^{1}2 \\ \underline{2\;1\;8\;0\;5} \; - \\ 4\;2\;7\;2\;7 \end{array}$$

Exercise 1.5

1 Simplify:

a 12 – 7	**b** 14 – 6	**c** 13 – 4	**d** 15 – 7
e 11 – 6	**f** 18 – 9	**g** 12 – 6	**h** 11 – 7
i 12 – 4	**j** 15 – 9	**k** 14 – 7	**l** 16 – 8
m 13 – 7	**n** 14 – 5	**o** 13 – 8	**p** 17 – 8

2 Simplify:

a 6 + 9 + 7	**b** 8 + 5 + 8	**c** 9 + 7 + 4	**d** 7 + 7 + 9
e 25 – 6 – 7	**f** 23 – 8 – 5	**g** 24 – 7 – 8	**h** 21 – 8 – 9
i 8 + 9 – 6	**j** 22 – 8 + 6	**k** 26 – 7 + 9	**l** 18 + 6 – 7

3 Write answers only to each of the following.

a 23 – 8	**b** 34 – 9	**c** 26 – 7	**d** 42 – 5
e 61 – 6	**f** 31 – 4	**g** 76 – 8	**h** 55 – 7
i 92 – 9	**j** 62 – 5	**k** 83 – 6	**l** 46 – 9

■ Consolidation

4 Find answers for each of these.

a 45 – 20	**b** 78 – 37	**c** 57 – 25	**d** 98 – 12

5 Write each subtraction vertically, then evaluate.

a 37 – 19	**b** 42 – 28	**c** 56 – 27	**d** 64 – 17
e 83 – 29	**f** 94 – 56	**g** 77 – 28	**h** 93 – 68

6 Evaluate:

a 325 119 –	**b** 643 257 –	**c** 538 146 –	**d** 317 108 –
e 906 243 –	**f** 860 297 –	**g** 411 354 –	**h** 705 589 –

7 Evaluate:

a 4756 1852 –	**b** 7025 1657 –	**c** 6112 3329 –	**d** 9000 1046 –

8 Find the value of:

a 120 – 40	**b** 110 – 70	**c** 150 – 60	**d** 130 – 80
e 1300 – 600	**f** 1400 – 800	**g** 1800 – 900	**h** 1200 – 500
i 14 000 – 7000	**j** 17 000 – 9000	**k** 15 000 – 8000	**l** 16 000 – 7000

9 **a** If 258 + 879 = 1137, find: **i** 1137 – 879 **ii** 1137 – 258

b If 3746 + 1997 = 5743, find: **i** 5743 – 3746 **ii** 5743 – 1997

10 **a** What number must be added to 46 to give 132?
b In a box of 220 light bulbs, 37 bulbs were faulty. How many still work?
c Pia was driving 835 km from Sydney to Gunnedah. She passed a sign showing that she had driven 450 km. How far did she still have to drive?
d After setting up 3480 dominoes, Carl accidentally knocked down 1253 of them. How many dominoes were still standing?
e A nursery owner bought 650 plants. If she sold 124 of them on Saturday and 155 of them on Sunday, how many plants remain unsold?

■ Further applications

11 Answer each of the following questions by performing a subtraction and then an addition.

a 53 – 19	**b** 85 – 29	**c** 71 – 18	**d** 106 – 47
e 102 – 59	**f** 195 – 78	**g** 230 – 79	**h** 344 – 99
i 495 – 199	**j** 728 – 298	**k** 511 – 397	**l** 967 – 498

12 Find the missing digits in each of these subtractions.

a
$$\begin{array}{r} 5\,\square\,3 \\ \square\,4\,\square\ - \\ \hline 4\ 2\ 7 \end{array}$$

b
$$\begin{array}{r} 8\,\square\,6\,\square \\ \square\,4\,\square\,9\ - \\ \hline 4\ 8\ 7\ 6 \end{array}$$

c
$$\begin{array}{r} 7\,\square\,1\ 4\,\square \\ \square\,3\ 6\,\square\,5\ - \\ \hline 2\ 7\,\square\,0\ 7 \end{array}$$

1.6 Multiplication review

Multiplication is repeated addition. For example, instead of evaluating 4 + 4 + 4 + 4 + 4, it is quicker to work out 5 × 4 (i.e. 5 lots of 4).

To multiply one or more numbers that end in zeros:
- **multiply the non-zero digits**
- **write the same number of zeros in the answer as are present in total in the question.**

To multiply by long multiplication:
- **multiply the top number by the units digit of the bottom number**
- **write a zero in the units place, then multiply the top number by the tens digit of the bottom number**
- **add the numbers that have resulted from these multiplications.**

Example 1

Evaluate:

a 58 × 7
b 25 × 39

Solutions

a
$$\begin{array}{r} 58 \\ {}_{5}7\ \times \\ \hline 406 \end{array}$$

b
$$\begin{array}{r} 25 \\ 39\ \times \\ \hline 225 \\ 750\ + \\ \hline 975 \end{array}$$

Example 2

Evaluate:

a 400×8000 **b** 35×600

Solutions

a To evaluate 400 by 8000, multiply 4×8, then write five zeros in the answer.
$\therefore 400 \times 8000 = 3\,200\,000$.

b To evaluate 35×600, multiply 35×6, then write two *more* zeros in the answer.

$$\begin{array}{r} 35 \\ {}_{3}\underline{6} \times \\ 210 \end{array} \qquad \therefore 35 \times 600 = 21\,000$$

Exercise 1.6

1 Express each of these repeated additions as a multiplication.

a $2+2+2+2$ **b** $5+5+5$ **c** $3+3+3+3+3+3$
d $12+12+12$ **e** $4+4+4+4+4$ **f** $9+9+9+9$

2 Simplify:

a 4×5 **b** 7×3 **c** 6×6 **d** 5×8 **e** 3×9
f 10×6 **g** 12×2 **h** 4×7 **i** 8×6 **j** 9×5
k 11×3 **l** 8×8 **m** 7×9 **n** 4×12 **o** 9×6
p 4×9 **q** 12×5 **r** 10×11 **s** 7×7 **t** 9×9
u 11×11 **v** 12×9 **w** 6×7 **x** 7×8 **y** 12×12

3 Find the missing number in each of these.

a $5 \times \square = 30$ **b** $\square \times 4 = 16$ **c** $7 \times \square = 35$ **d** $\square \times 3 = 24$
e $\square \times 7 = 84$ **f** $9 \times \square = 81$ **g** $\square \times 12 = 96$ **h** $9 \times \square = 72$
i $8 \times \square = 56$ **j** $\square \times 5 = 50$ **k** $10 \times \square = 10$ **l** $\square \times 11 = 132$

4 Evaluate:

a 4×10 **b** 100×9 **c** 3×1000 **d** $10\,000 \times 6$
e 10×25 **f** 32×100 **g** 1000×47 **h** 985×1000

Consolidation

5 Simplify:

a $\begin{array}{r} 15 \\ \underline{5} \times \end{array}$ **b** $\begin{array}{r} 13 \\ \underline{7} \times \end{array}$ **c** $\begin{array}{r} 27 \\ \underline{4} \times \end{array}$ **d** $\begin{array}{r} 46 \\ \underline{9} \times \end{array}$ **e** $\begin{array}{r} 74 \\ \underline{8} \times \end{array}$

f $\begin{array}{r} 126 \\ \underline{4} \times \end{array}$ **g** $\begin{array}{r} 148 \\ \underline{7} \times \end{array}$ **h** $\begin{array}{r} 357 \\ \underline{5} \times \end{array}$ **i** $\begin{array}{r} 509 \\ \underline{8} \times \end{array}$ **j** $\begin{array}{r} 825 \\ \underline{6} \times \end{array}$

6 Evaluate each of the following long multiplications.

a 28×53 **b** 17×84 **c** 75×36 **d** 22×49
e 387×15 **f** 109×47 **g** 324×29 **h** 758×56

7 Find the value of:

a 30×20 **b** 60×40 **c** 50×300 **d** 700×10
e 400×500 **f** 200×1200 **g** 8000×60 **h** 5000×9000
i $20 \times 20 \times 30$ **j** $30 \times 40 \times 50$ **k** $20 \times 60 \times 700$ **l** $50 \times 600 \times 9000$

8 Find answers to each of these.

a 16×30 **b** 41×70 **c** 39×60 **d** 52×40
e 74×700 **f** 65×500 **g** 88×4000 **h** 23×9000

9 **a** Tess is able to sew 118 buttons on garments in an hour. How many buttons can she sew on in 7 hours?
b Joe drives 68 km to and from work each day. How far does he drive in a week if he works from Monday to Saturday?
c Karina is able to stick 40 stamps on each page of her stamp album. How many stamps can she stick into the whole album if it has 32 pages?
d A baker makes 125 loaves of bread each day. How many loaves does she make in a fortnight?
e Find the total weight of a dozen eggs if each egg weighs 16 grams.

10 Outline a simple method by which numbers can be multiplied mentally by:

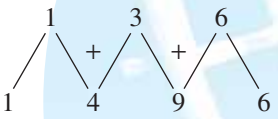
a 4 **b** 20 **c** 15 **d** 300

■ Further applications

11 To quickly multiply a two digit number by 11, we can add the digits together and then write this total between the digits in the number. Extra care must be taken if the sum of the digits is greater than 10, as carrying will then be necessary. For example, to evaluate 25×11, we add the digits $(2 + 5 = 7)$ and write this total between the 2 and the 5. That is, $25 \times 11 = 275$. Use this method to find the value of:

a 23×11 **b** 54×11 **c** 11×31 **d** 60×11
e 11×35 **f** 61×11 **g** 44×11 **h** 11×18
i 48×11 **j** 11×85 **k** 11×69 **l** 98×11

12 The method used in Q11 can be extended to numbers with more than two digits. For example, to evaluate 136×11, add the first two digits and the last two digits to find the middle digits in the answer. Therefore $136 \times 11 = 1496$.



Use this method to evaluate each of the following.

a 243×11 **b** 165×11 **c** 354×11 **d** 852×11

1.7 Division review

Division is the inverse or opposite of multiplication. We say that division 'undoes' multiplication. For example, having started with 5 and then multiplied by 2, we could get back to 5 by dividing by 2. That is, $5 \times \mathbf{2} \div \mathbf{2} = 5$.

In a division such as $18 \div 3 = 6$, the **dividend** is 18, the **divisor** is 3 and the **quotient** is 6. If after performing a division there is a remainder, the remainder is written as a fraction of the divisor. For example, $18 \div 7 = 2 \text{ r } 4$, i.e. $18 \div 7 = 2\frac{4}{7}$.

To divide two numbers that end in zeros:
- **cancel out all zeros in the divisor and the same number of zeros in the dividend**
- **divide the remaining numbers.**

To divide by long division:
- **divide the divisor into the first 2 or 3 digits of the dividend and write the answer at the top**
- **multiply this answer by the divisor, writing the product under the divisor from the left-hand side**
- **subtract this product from the dividend**
- **repeat this procedure until division is no longer possible.**

Example 1

Evaluate:

a $162 \div 6$ **b** $812 \div 4$ **c** $28\,016 \div 4$

Solutions

a
$$\begin{array}{r} 27 \\ 6\overline{)16^{4}2} \end{array}$$

b
$$\begin{array}{r} 203 \\ 4\overline{)812} \end{array}$$

c
$$\begin{array}{r} 7\,004 \\ 4\overline{)28\,016} \end{array}$$

Example 2

Evaluate $374 \div 9$

Solution

$$\begin{array}{r} 41r5 \\ 9\overline{)37^{1}4} \end{array} \qquad \therefore\ 374 \div 9 = 41\tfrac{5}{9}$$

Example 3

Evaluate $24\,000 \div 300$

Solution

$$\begin{aligned} 24\,000 \div 300 &= 24\,0\cancel{00} \div 3\cancel{00} \\ &= 240 \div 3 \\ &= 80 \end{aligned}$$

Example 4

Evaluate $3048 \div 24$

Solution

```
     127
24)3048        ◎ 24 divides into 30 once.
   24↓↓ –      ◎ 1 × 24 = 24
   648         ◎ 24 divides into 64 twice.
   48↓ –       ◎ 2 × 24 = 48
   168         ◎ 24 divides into 168 7 times.
   168 –       ◎ 7 × 24 = 168
     0
```

Exercise 1.7

1 Express these statements as divisions.

a the number of times that 3 can be subtracted from 12
b the number of times that 2 can be subtracted from 16
c the number of times that 7 can be subtracted from 42

2 Simplify:

a $20 \div 4$	**b** $21 \div 3$	**c** $32 \div 8$	**d** $42 \div 7$	**e** $50 \div 10$
f $45 \div 5$	**g** $36 \div 6$	**h** $63 \div 9$	**i** $60 \div 12$	**j** $44 \div 4$
k $80 \div 8$	**l** $54 \div 9$	**m** $48 \div 4$	**n** $49 \div 7$	**o** $24 \div 3$
p $56 \div 8$	**q** $81 \div 9$	**r** $55 \div 11$	**s** $64 \div 8$	**t** $108 \div 9$

3 Evaluate each of the following divisions.

a $2\overline{)284}$	**b** $3\overline{)336}$	**c** $4\overline{)284}$	**d** $5\overline{)455}$
e $3\overline{)609}$	**f** $2\overline{)806}$	**g** $3\overline{)2106}$	**h** $6\overline{)4206}$

Consolidation

4 Evaluate each of these.

a $3\overline{)192}$	**b** $5\overline{)345}$	**c** $6\overline{)174}$	**d** $8\overline{)112}$	**e** $7\overline{)175}$	**f** $9\overline{)288}$
g $4\overline{)152}$	**h** $6\overline{)144}$	**i** $5\overline{)415}$	**j** $7\overline{)469}$	**k** $8\overline{)656}$	**l** $9\overline{)486}$

5 Evaluate each of these short divisions, giving the remainders as fractions.

a $116 \div 3$	**b** $528 \div 7$	**c** $220 \div 9$	**d** $257 \div 6$
e $269 \div 4$	**f** $148 \div 3$	**g** $217 \div 2$	**h** $242 \div 7$

6 Evaluate:

a $216 \div 2$	**b** $621 \div 3$	**c** $545 \div 5$	**d** $654 \div 6$
e $848 \div 8$	**f** $836 \div 4$	**g** $921 \div 3$	**h** $972 \div 9$
i $1216 \div 2$	**j** $2718 \div 3$	**k** $4228 \div 7$	**l** $3025 \div 5$
m $1842 \div 6$	**n** $6336 \div 9$	**o** $6440 \div 8$	**p** $2032 \div 4$

7 Complete each of the following.

a If $18 \times 27 = 486$, find: **i** $486 \div 27$ **ii** $486 \div 18$
b If $134 \times 85 = 11\,390$, find: **i** $11\,390 \div 134$ **ii** $11\,390 \div 85$

8 Write answers to each of these.

a	420 ÷ 10	**b**	670 ÷ 10	**c**	3400 ÷ 100
d	5700 ÷ 100	**e**	13 000 ÷ 1000	**f**	29 000 ÷ 1000
g	500 ÷ 10	**h**	6000 ÷ 10	**i**	38 000 ÷ 100
j	167 000 ÷ 100	**k**	200 000 ÷ 1000	**l**	470 000 ÷ 10 000

9 Write answers to each of these.

a	150 ÷ 30	**b**	240 ÷ 60	**c**	3600 ÷ 400	**d**	21 000 ÷ 7000
e	1800 ÷ 20	**f**	8000 ÷ 200	**g**	45 000 ÷ 50	**h**	72 000 ÷ 1200

10 **a** If 528 football cards are shared equally between 4 boys, how many cards will each boy receive?
b A carpenter cut a 315 cm length of timber into 9 equal pieces. Find the length of each piece.
c Eight people purchased a racehorse for $32 480 and divided the cost equally. How much did each person pay?
d A wine grower planted 1380 plants in 60 rows. How many plants were in each row?
e Find the weekly pay for Sally whose annual salary is $34 840.

11 Outline a simple method by which numbers can be divided mentally by:

a	4	**b**	6	**c**	30	**d**	200

■ Further applications

12 Evaluate each of the following by long division.

a	546 ÷ 21	**b**	576 ÷ 18	**c**	612 ÷ 17	**d**	572 ÷ 22
e	1512 ÷ 24	**f**	1833 ÷ 47	**g**	1377 ÷ 51	**h**	1288 ÷ 28
i	2016 ÷ 42	**j**	1575 ÷ 35	**k**	1248 ÷ 26	**l**	2211 ÷ 33

TRY THIS

3-digit numbers

An investigation 'Why does it work?'

Problem	1 Take any 3-digit number e.g.	735	
	2 Reverse it.	537	
	3 Subtract the smaller number from the larger.	198	
	4 Reverse the answer.	891	+
	5 Add the last two numbers.	1089	Result
Try again e.g. 248		248	
		842	
	Subtract (842 – 248)	. . .	
	Reverse answer	. . .	
	Add	1089	Result

Try again with any 3 digits.
Do you always have the same result?
Answer:

1089 *YES!*

Why does it work? (Big thinking needed here!)

Hints

1 The middle two numbers of the subtraction are always the same (example 1: middle numbers 3) and when you take 1, the middle subtraction is always 9!

2 The outside numbers are always reversed. Example:

$$\begin{array}{r} 75 \\ 57\ - \\ \hline 18 \\ \hline \end{array}$$

Try the subtraction.

What is the total of the digits 1 and 8?
(Is this always true when you reverse numbers and then subtract?)
Look again at all the steps in the example!

1.8 Index notation

In an expression such as 2^5, read as 'two to the fifth' or 'two to the power of five', we say that 2 is the **base** and 5 is the **index** or **power**. The index indicates how many times the base has been multiplied by itself. That is, 2^5 means $2 \times 2 \times 2 \times 2 \times 2$ or 5 factors of 2. Expressions such as 4^2 and 4^3 are read as 'four squared' and 'four cubed'. By convention, we do not usually write an index of 1.

Example 1

Express each of these in index form.

a $7 \times 7 \times 7 \times 7$ **b** $3 \times 3 \times 5 \times 5 \times 5$ **c** $6 \times 6 \times 2 \times 6 \times 2 \times 2 \times 2$

Solutions

a $7 \times 7 \times 7 \times 7$
$= 7^4$

b $3 \times 3 \times 5 \times 5 \times 5$
$= 3^2 \times 5^3$

c $6 \times 6 \times 2 \times 6 \times 2 \times 2 \times 2$
$= 2 \times 2 \times 2 \times 2 \times 6 \times 6 \times 6$
$= 2^4 \times 6^3$

Example 2

Write each of these in index form.

a nine squared
b eight cubed
c seven factors of two
d four to the power of ten

Solutions

a 9^2
b 8^3
c 2^7
d 4^{10}

Example 3

Evaluate each of the following.

a $7^2 - 3^3$ **b** $2^4 + 5^2$ **c** 10×2^3

Solutions

a $7^2 - 3^3$
$= (7 \times 7) - (3 \times 3 \times 3)$
$= 49 - 27$
$= 22$

b $2^4 + 5^2$
$= (2 \times 2 \times 2 \times 2) + (5 \times 5)$
$= 16 + 25$
$= 41$

c 10×2^3
$= 10 \times (2 \times 2 \times 2)$
$= 10 \times 8$
$= 80$

Exercise 1.8

1 Write each of these in index form.

a 5×5 **b** $3 \times 3 \times 3 \times 3$ **c** $6 \times 6 \times 6$
d $4 \times 4 \times 4 \times 4 \times 4$ **e** 7×7 **f** $9 \times 9 \times 9 \times 9$
g $8 \times 8 \times 8$ **h** $2 \times 2 \times 2 \times 2 \times 2 \times 2$ **i** $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

2 Express in index form.

a $7 \times 7 \times 5 \times 5 \times 5$ **b** $2 \times 2 \times 2 \times 9 \times 9$ **c** $6 \times 6 \times 6 \times 6 \times 4$
d $3 \times 5 \times 3 \times 3 \times 5 \times 5$ **e** $2 \times 7 \times 7 \times 2 \times 2 \times 2$ **f** $1 \times 8 \times 1 \times 8 \times 1$

3 Show the meaning of each of these using multiplication signs.

a 6^2 **b** 3^4 **c** 5^3 **d** 7^5
e 2^4 **f** 8^3 **g** 1^6 **h** 9^7

Consolidation

4 Use multiplication signs to show the meaning of each expression.

a $5^2 + 2^3$ **b** $3^5 - 6^2$ **c** $1^4 + 2^3 + 5^2$ **d** $8^2 - 9^3 - 7^2$

5 Write each expression in index form.

a eight squared **b** five cubed
c six to the power of four **d** nine to the power of three
e two factors of eleven **f** twelve factors of five

6 Which one of these expressions means three factors of two? [Write A, B, C or D only.]

A $2 + 2 + 2$ **B** $3 + 3$ **C** $2 \times 2 \times 2$ **D** 3×3

7 Which is larger, five factors of two or two factors of five, and by how much?

8 Evaluate each of the following.

a 3^2 **b** 2^3 **c** 6^2 **d** 1^4 **e** 7^2 **f** 3^3
g 2^4 **h** 4^3 **i** 2^5 **j** 3^4 **k** 5^3 **l** 2^6

9 Express:

a 8 as a power of 2 **b** 9 as a power of 3 **c** 32 as a power of 2

d 6 as a power of 6 **e** 121 as a power of 11 **f** 128 as a power of 2

10 Which one of the following is not equal to 64?
A 8^2 **B** 2^6 **C** 4^3 **D** 3^4

11 Evaluate each of the following powers of 10.
a 10^2 **b** 10^3 **c** 10^5 **d** 10^1 **e** 10^4 **f** 10^6

12 Evaluate:
a 5×10^2 **b** 7×10^3 **c** 2×10^4 **d** 9×10^5
e 4×10^3 **f** 6×10^2 **g** 3×10^6 **h** 8×10^4
i 7×10^5 **j** 5×10^4 **k** 6×10^3 **l** 9×10^6

13 Evaluate:
a $5^2 + 4^2$ **b** $7^2 - 2^3$ **c** $3^3 + 1^5$ **d** $5^2 \times 4$
e $50 - 6^2$ **f** $3^2 \times 2^2$ **g** $10^2 \div 20$ **h** $9^2 \div 9$
i $4^3 + 2^4$ **j** 2×3^3 **k** $2^5 \div 4^2$ **l** $2^3 + 7^2 - 15$
m $10^3 - 12^2$ **n** $7^3 + 2^6$ **o** $2^4 \times 10^4$ **p** $3^4 \times 2^5$

Further applications

14 **a** Evaluate:
i $(2 \times 3)^2$ and $2^2 \times 3^2$ **ii** $(3 \times 4)^2$ and $3^2 \times 4^2$
iii $(5 \times 2)^2$ and $5^2 \times 2^2$ **iv** $(10 \times 5)^2$ and $10^2 \times 5^2$
b Copy and complete this statement.
The square of a _____ is equal to the product of the _____.
c Use this method to evaluate:
i $5^2 \times 8^2$ **ii** $4^2 \times 15^2$ **iii** $20^2 \times 6^2$
d **i** Does $5^2 \times 6^2 = (5 \times 6)^2$? **ii** Hence, evaluate 25×36.

15 Express:
a 24 as the product of a power of 2 and a power of 3 in index form.
b 72 as the product of a power of 2 and a power of 3 in index form.
c 225 as the product of a power of 3 and a power of 5 in index form.

16 Evaluate, without determining the value of the individual terms.
a $2^3 \times 2^7$ **b** $3^2 \times 3^3$ **c** $2^{15} \div 2^8$ **d** $5^{11} \div 5^8$

17 **a** Write down the values of the first 4 powers of 2, 3, 5 in descending order of powers.
b What would be the value of: **i** 2^0? **ii** 3^0? **iii** 5^0?
c What is the value of any positive integer raised to the power of 0?

1.9 Expanded notation

Writing a number in the **expanded form** emphasises the place value of each digit.

Zeros are written as place holders to ensure that each digit is placed in its correct column.

The common powers of 10 in index form are:

$10^1 = 10$ $\quad$ $10^4 = 10\,000$

$10^2 = 100$ $\quad$ $10^5 = 100\,000$

$10^3 = 1000$ $\quad$ $10^6 = 1\,000\,000$

To write a number in the expanded form:

- **express the place value of each digit as the product of the digit and a power of 10**
- **write the number as the sum of these products.**

Example 1

a Write 137 042 in the expanded form.

b Write the basic numeral for $(8 \times 10\,000) + (1 \times 1000) + (2 \times 100) + (7 \times 10) + (9 \times 1)$

Solutions

a $137\,042 = (1 \times 100\,000) + (3 \times 10\,000) + (7 \times 1000) + (0 \times 100) + (4 \times 10) + (2 \times 1)$

b $(8 \times 10\,000) + (1 \times 1000) + (2 \times 100) + (7 \times 10) + (9 \times 1)$

$= 80\,000 + 1000 + 200 + 70 + 9$

$= 81\,279$

Example 2

a Write 25 986 in the expanded form using index notation.

b Write the basic numeral for $(6 \times 10^5) + (4 \times 10^4) + (2 \times 10^3) + (5 \times 10^2) + (0 \times 10^1) + (7 \times 1)$

Solutions

a $25\,986 = (2 \times 10\,000) + (5 \times 1000) + (9 \times 100) + (8 \times 10) + (6 \times 1)$

$= (2 \times 10^4) + (5 \times 10^3) + (9 \times 10^2) + (8 \times 10^1) + (6 \times 1)$

b $(6 \times 10^5) + (4 \times 10^4) + (2 \times 10^3) + (5 \times 10^2) + (0 \times 10^1) + (7 \times 1)$

$= 600\,000 + 40\,000 + 2000 + 500 + 0 + 7$

$= 642\,507$

Exercise 1.9

1 Write the basic numeral for each of the following.

a $(4 \times 100) + (2 \times 10) + (7 \times 1)$

b $(5 \times 1000) + (3 \times 100) + (1 \times 10) + (9 \times 1)$

c $(7 \times 10\,000) + (4 \times 1000) + (6 \times 100) + (5 \times 10) + (4 \times 1)$

d $(2 \times 100\,000) + (3 \times 10\,000) + (2 \times 1000) + (9 \times 100) + (8 \times 10) + (1 \times 1)$

e $(8 \times 10\,000) + (5 \times 100) + (6 \times 10) + (3 \times 1)$

f $(9 \times 1\,000\,000) + (6 \times 100\,000) + (8 \times 10\,000) + (4 \times 1000) + (1 \times 100) + (7 \times 10) + (2 \times 1)$

g $(1 \times 100\,000) + (2 \times 10\,000) + (3 \times 1000) + (2 \times 100) + (1 \times 10) + (5 \times 1)$

h $(6 \times 1\,000\,000) + (9 \times 100\,000) + (1 \times 10\,000) + (7 \times 1000) + (4 \times 100) + (9 \times 10) + (8 \times 1)$

2 Write each of the following numbers in the expanded form.

a 138	**b** 6287	**c** 35 771	**d** 269 145
e 58 892	**f** 7 434 921	**g** 456 179	**h** 5 854 176

Consolidation

3 Write the basic numeral for each of the following.

a $(6 \times 100) + (3 \times 10) + (0 \times 1)$
b $(5 \times 100) + (0 \times 10) + (0 \times 1)$
c $(8 \times 1000) + (0 \times 100) + (1 \times 10) + (7 \times 1)$
d $(4 \times 1000) + (0 \times 100) + (7 \times 10) + (0 \times 1)$
e $(1 \times 10\,000) + (0 \times 1000) + (2 \times 100) + (0 \times 10) + (4 \times 1)$
f $(2 \times 10\,000) + (0 \times 1000) + (0 \times 100) + (8 \times 10) + (0 \times 1)$
g $(9 \times 100\,000) + (6 \times 10\,000) + (0 \times 1000) + (4 \times 100) + (0 \times 10) + (0 \times 1)$
h $(3 \times 100\,000) + (0 \times 10\,000) + (1 \times 1000) + (0 \times 100) + (7 \times 10) + (0 \times 1)$
i $(7 \times 1\,000\,000) + (4 \times 100\,000) + (0 \times 10\,000) + (1 \times 1000) + (0 \times 100) + (0 \times 10) + (6 \times 1)$
j $(5 \times 1\,000\,000) + (0 \times 100\,000) + (0 \times 10\,000) + (6 \times 1000) + (0 \times 100) + (0 \times 10) + (0 \times 1)$

4 Write each of these numbers in the expanded form.

a 409	**b** 350	**c** 5100	**d** 3007
e 80 046	**f** 103 090	**g** 620 008	**h** 4 900 401

5 Write the basic numeral for each of the following.

a $(9 \times 10^2) + (7 \times 10^1) + (3 \times 1)$
b $(8 \times 10^3) + (1 \times 10^2) + (4 \times 10^1) + (5 \times 1)$
c $(4 \times 10^4) + (6 \times 10^3) + (5 \times 10^2) + (2 \times 10^1) + (9 \times 1)$
d $(1 \times 10^6) + (3 \times 10^5) + (8 \times 10^4) + (2 \times 10^3) + (1 \times 10^2) + (9 \times 10^1) + (4 \times 1)$
e $(3 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (7 \times 1)$
f $(7 \times 10^3) + (0 \times 10^2) + (9 \times 10^1) + (0 \times 1)$
g $(6 \times 10^4) + (0 \times 10^3) + (1 \times 10^2) + (0 \times 10^1) + (8 \times 1)$
h $(5 \times 10^6) + (2 \times 10^5) + (0 \times 10^4) + (0 \times 10^3) + (9 \times 10^2) + (3 \times 10^1) + (2 \times 1)$

6 Write each of these numbers in the expanded form using index notation.

a 5618	**b** 45 291	**c** 238 576	**d** 9 136 724
e 20 095	**f** 40 070	**g** 103 409	**h** 3 201 004

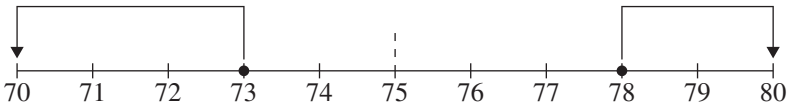
Further applications

7 Write each of these numbers in the expanded form using index notation.

a Five thousand two hundred and sixty-four
b Eighty-nine thousand and twelve
c Three hundred and nine thousand six hundred and one
d Five hundred and ninety thousand and seventy
e Four hundred thousand and fifty-six
f Two million seven hundred and ten thousand nine hundred and thirty
g Six million and twenty-nine thousand one hundred and three
h One hundred and forty million five hundred and fourteen thousand and eight

1.10 Estimation and approximation

To **round off** or **approximate** integers such as 73 or 78 correct to the nearest ten, we need to determine whether each number is closer to 70 or to 80 on a number line.



Now, 73 is closer to 70 and 78 is closer to 80. That is, correct to the nearest ten, we say that $73 \doteqdot 70$ and $78 \doteqdot 80$, where $\doteqdot$ means 'is approximately equal to'.

In general, when rounding off a number to a desired degree of accuracy, it is the size of the digit in the next place that determines whether the number is rounded up or rounded down.

To round off or approximate a number to a desired degree of accuracy:
- **round the number up if the next digit is 5 or greater**
- **round the number down if the next digit is less than 5.**

In order to pick up careless errors in working, it is a good idea to first estimate the answer to a calculation. One method of estimation is called **leading figure estimation**.

To estimate the answer to a calculation using leading figure estimation:
- **round each number to its leading figure**
- **evaluate the resulting expression.**

Example 1

Round off 8537 correct to the nearest:

a ten **b** hundred **c** thousand

Solutions

a To round off 8537 correct to the nearest ten, look at the units digit. Now the units digit is a 7, which is greater than 5, so the number must be rounded up. That is, 8537 is closer to 8540 than it is to 8530.

$\therefore 8537 \doteqdot 8540$.

b To round off 8537 correct to the nearest hundred, look at the tens digit. Now the tens digit is a 3, which is less than 5, so the number must be rounded down. That is, 8537 is closer to 8500 than it is to 8600.

$\therefore 8537 \doteqdot 8500$.

c To round off 8537 correct to the nearest thousand, look at the hundreds digit. Now the hundreds digit is a 5, so the number must be rounded up. That is, 8537 is closer to 9000 than it is to 8000.

$\therefore 8537 \doteqdot 9000$.

Example 2

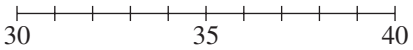
Use leading figure estimation to estimate the value of 284×63.

Solution

Now, $284 \doteqdot 300$ and $63 \doteqdot 60$, so to estimate the value of 284×63, we multiply 300×60.
$\therefore 284 \times 63 \doteqdot 18\,000$.

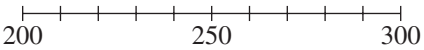
Exercise 1.10

1 Use this number line to round off each number, correct to the nearest ten.



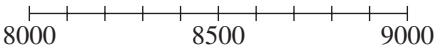
a 31 **b** 37 **c** 35

2 Use this number line to round off each number, correct to the nearest hundred.



a 290 **b** 240 **c** 250

3 Use this number line to round off each number, correct to the nearest thousand.



a 8400 **b** 8800 **c** 8500

Consolidation

4 Round each of these numbers correct to the nearest ten.

a 18	**b** 23	**c** 67	**d** 75
e 7	**f** 72	**g** 15	**h** 49
i 94	**j** 25	**k** 3	**l** 97

5 Round each of these numbers correct to the nearest hundred.

a 360	**b** 237	**c** 850	**d** 485
e 649	**f** 735	**g** 776	**h** 250
i 589	**j** 104	**k** 50	**l** 951

6 Round each of these numbers correct to the nearest thousand.

a 2700	**b** 9430	**c** 7500	**d** 3875
e 1925	**f** 3149	**g** 4334	**h** 1500
i 12 294	**j** 15 937	**k** 31 500	**l** 99 675

7 Round off, correct to the nearest ten:

a 1227	**b** 4355	**c** 3546	**d** 7028
e 6293	**f** 1004	**g** 5275	**h** 7771
i 6005	**j** 9641	**k** 3523	**l** 9998

8 Round off, correct to the nearest hundred.

a 3628	**b** 7162	**c** 1450	**d** 4716
e 2280	**f** 8050	**g** 6345	**h** 5477
i 7624	**j** 5939	**k** 9693	**l** 9950

9 Use leading figure estimation to estimate the answer to each calculation, then use a calculator to find the correct answer.

a $87 + 56$	**b** $755 + 892 + 309$	**c** $4276 + 3920 + 9229$
d $71 - 28$	**e** $664 - 155$	**f** $13\,752 - 7468$
g 48×72	**h** 293×64	**i** 4107×675
j $84 \div 18$	**k** $275 \div 37$	**l** $18\,098 \div 1956$

10 Use leading figure estimation to give an approximate answer to each problem.

a Sergio spent \$92.40 on groceries, paid his electricity bill of \$76.65 and made a monthly car insurance payment of \$85.50. How much did he spend altogether?

b Sonia paid \$34 755 for her new car. Five years later she sold the car for \$14 880. By how much did the value of the car depreciate during this time?

c If there are 18 golf balls in each box, how many balls are there in 150 boxes?

d A small publishing firm received a total income of \$183 656 in their first year. If they published 42 books, how much money was received on the sale of each book, on average?

e The crowds at the first four rugby league matches for the season were 34 458, 29 980, 17 325 and 24 491. How many people attended the first four matches?

f If 427 people can enter a concert hall every minute once the doors open, how many people can enter in $\frac{1}{4}$ of an hour?

g Phillip won \$500 000 in the lottery. He spent \$218 000 on a new house, \$43 250 on a new car, \$16 675 on a holiday to Europe and gave \$27 500 to his brother. How much money does Phillip have left?

h The owner of a mathematics tutoring business wants to print colour brochures and distribute them to advertise for more students. If the owner has \$6240 to spend and each brochure costs \$2.80 to produce and \$0.35 to distribute, how many brochures should she order from the printer?

■ Further applications

11 a The approximate number of students in Year 7 was given as 130, correct to the nearest ten students. Within what range does the exact number of students lie?

b The approximate number of students enrolled in a school was given as 1200, correct to the nearest hundred students. Within what range does the exact number of students lie?

c The crowd at a one-day cricket match was given as 37 000, correct to the nearest thousand. Within what range does the exact number of people at the match lie?

TRY THIS **Add a sign**

For the following problems add the signs required to make the left-hand side equal the right-hand side. Once again, they start easy and finally comes the challenging problem.

1 Add one plus sign.

2 3 4 6 = 69

2 Add one plus and one subtraction sign.

4 6 3 5 = 62

3 Add one plus, one minus and one multiplication sign.

3 2 3 4 5 6 3 5 = 490

1.11 Grouping symbols

Grouping symbols are used to indicate the order in which several operations are to be performed. The most commonly used grouping symbols are:

parentheses ( ) **brackets** [ ] **braces** { }

When an expression contains more than one set of grouping symbols, we evaluate the expression in the innermost grouping symbols first.

Example 1

Evaluate each expression.

a $5 \times (4 + 8)$ **b** $(27 + 13) \div (12 - 7)$ **c** $[48 \div (20 - 12)] + 11$

Solutions

a
$$\begin{aligned} & 5 \times (4 + 8) \\ &= 5 \times 12 \\ &= 60 \end{aligned}$$

b
$$\begin{aligned} & (27 + 13) \div (12 - 7) \\ &= 40 \div 5 \\ &= 8 \end{aligned}$$

c
$$\begin{aligned} & [48 \div (20 - 12)] + 11 \\ &= [48 \div 8] + 11 \\ &= 6 + 11 \\ &= 17 \end{aligned}$$

Example 2

Evaluate $\dfrac{55 - 13}{10 + 5 - 9}$

Solution

The fraction bar acts in the same way as a set of grouping symbols, but indicates division.

$$\begin{aligned} \frac{55 - 13}{10 + 5 - 9} &= \frac{42}{6} \\ &= 7 \end{aligned}$$

Exercise 1.11

1 Evaluate each of the following.

a $(5 + 2) \times 3$ **b** $(8 - 3) \times 4$ **c** $(40 - 10) \div 6$ **d** $15 - (7 - 4)$
e $5 \times (4 + 5)$ **f** $50 \div (7 + 3)$ **g** $25 + (3 \times 5)$ **h** $(29 + 7) \div 9$
i $32 - (3 \times 8)$ **j** $(13 - 7) \times 8$ **k** $7 \times (6 + 6)$ **l** $(15 + 12) \div 3$
m $25 + (7 \times 5)$ **n** $64 \div (12 - 4)$ **o** $35 - (18 - 7)$ **p** $17 + (80 \div 4)$

■ Consolidation

2 Evaluate:

a $10 + (8 \times 3) + 6$ **b** $25 - (3 \times 5) + 7$ **c** $18 + (12 \div 3) - 9$
d $30 - (60 \div 5) - 14$ **e** $36 - (13 - 4) - 15$ **f** $48 + (7 \times 6) + 13$
g $17 - (15 - 8) - 10$ **h** $55 + (45 \div 5) - 12$ **i** $70 - (132 \div 11) + 7$

3 Find the value of each expression.

a $(5 + 6) \times (10 - 3)$ **b** $(9 + 4) \times (15 \div 5)$ **c** $(20 - 11) + (6 \times 3)$
d $(18 - 9) - (24 - 17)$ **e** $(42 \div 6) \times (32 \div 4)$ **f** $(40 - 15) - (12 + 11)$
g $(8 \times 10) - (6 \times 8)$ **h** $(9 \times 8) \div (14 - 8)$ **i** $(60 - 45) \div (27 \div 9)$
j $(56 \div 8) + (10 \times 5)$ **k** $(42 - 12) - (6 \times 5)$ **l** $(21 \div 7) \div (36 \div 12)$

4 Evaluate:

a $[50 - (6 \times 5)] - 14$ **b** $[10 + (28 \div 7)] + 6$
c $9 + [(17 + 18) \div 7]$ **d** $100 - [60 - (5 \times 8)]$
e $30 - [25 - (17 - 4)]$ **f** $60 - [10 \times (8 - 3)]$
g $[(16 - 7) \times 10] \div 30$ **h** $53 + [48 \div (72 \div 12)]$
i $[(8 \times 3) - (66 \div 11)] \div 3$ **j** $[(11 + 9) \div (5 \times 2)] \times 6$
k $12 \times [(7 \times 8) \div (25 - 18)]$ **l** $44 \div [15 - (16 - 5)] + 12$
m $150 - [29 + (14 - 7)] - 10$ **n** $[(8 \times 8) + (7 \times 7) - 13] \div 2$

5 Evaluate each of these fractions.

a $\dfrac{11 + 7}{9 - 3}$ **b** $\dfrac{36 - 8}{16 - 9 + 7}$ **c** $\dfrac{70 - 15 - 7}{3 \times 2 \times 2}$ **d** $\dfrac{[(4 \times 6) \div 3]}{40 - (23 + 13)}$

■ Further applications

6 Insert grouping symbols in each of the following to make a true statement.

a $20 - 4 \times 2 = 12$ **b** $35 - 8 - 6 = 33$ **c** $22 - 10 \times 4 = 48$
d $42 \div 2 \times 3 = 7$ **e** $5 \times 12 \div 10 = 6$ **f** $19 + 9 \times 2 = 37$
g $7 + 5 \times 3 - 4 = 18$ **h** $8 + 5 - 3 + 6 = 4$ **i** $49 \div 7 + 4 \times 3 = 19$
j $25 - 5 - 6 + 5 = 9$ **k** $31 + 35 \div 7 - 8 = 28$ **l** $50 - 7 \times 3 - 5 + 4 = 20$

1.12 The order of operations

The **order of operations** specifies the order in which two or more operations must be performed when they occur within the same question. This is necessary to ensure that everyone solving the question will get the same answer.

The order of operations states that:
- **expressions inside grouping symbols or with indices are worked first, then**
- **multiplications and divisions are worked from left to right, then**
- **additions and subtractions are worked from left to right.**

Example

Evaluate each of these expressions using the order of operations.

a $18 - 10 + 7 - 4$
b $20 + 12 \times 3 \div 9$
c $40 - 30 \div 10 + 6$
d $36 \div 4 + 8 \times 5$

Solutions

a $18 - 10 + 7 - 4$
$= 8 + 7 - 4$
$= 15 - 4$
$= 11$

b $20 + (12 \times 3 \div 9)$
$= 20 + (36 \div 9)$
$= 20 + 4$
$= 24$

c $40 - (30 \div 10) + 6$
$= 40 - 3 + 6$
$= 37 + 6$
$= 43$

d $(36 \div 4) + (8 \times 5)$
$= 9 + 40$
$= 49$

Exercise 1.12

1 Evaluate each expression by working from left to right.

a $12 + 8 + 3$ **b** $25 - 4 - 7$ **c** $18 - 4 + 6$
d $10 - 6 + 5 - 3$ **e** $34 + 5 - 2 - 11$ **f** $22 - 7 - 12 + 9$
g $19 + 10 + 6 - 13$ **h** $39 + 6 - 5 - 7$ **i** $8 - 7 + 14 - 4$
j $20 - 6 - 3 + 8 + 4$ **k** $47 - 4 - 8 - 3 + 9$ **l** $24 - 6 + 10 - 3 - 11$

2 Evaluate each expression by working from left to right.

a $2 \times 2 \times 9$ **b** $8 \times 3 \div 6$ **c** $72 \div 6 \div 4$
d $10 \times 2 \times 3 \div 5$ **e** $33 \div 3 \times 5 \times 2$ **f** $20 \times 2 \div 5 \div 4$
g $21 \div 3 \times 10 \div 2$ **h** $8 \times 9 \div 12 \div 2$ **i** $3 \times 15 \div 5 \times 7$
j $8 \times 2 \times 10 \div 4$ **k** $120 \div 3 \div 5 \div 2$ **l** $18 \times 2 \div 6 \times 12$

3 Evaluate each of the following using the order of operations.

a $30 - 3 \times 7$ **b** $15 + 6 \times 2$ **c** $32 \div 8 - 2$ **d** $5 \times 7 + 6$
e $18 + 12 \div 3$ **f** $10 \times 3 - 5$ **g** $35 - 7 \times 4$ **h** $8 + 36 \div 4$
i $9 \times 7 + 14$ **j** $50 - 13 \times 3$ **k** $16 + 42 \div 6$ **l** $110 \div 10 - 11$

Consolidation

4 Find the value of each expression.

a $9 + 3 \times 4 + 5$ **b** $47 - 7 \times 5 + 6$ **c** $55 - 4 \times 6 - 10$
d $36 + 16 \div 4 - 14$ **e** $15 + 100 \div 10 + 9$ **f** $28 - 9 \times 3 + 15$
g $15 + 2 \times 3 \times 5$ **h** $3 \times 4 \times 8 - 6$ **i** $9 - 18 \div 3 \div 2$
j $40 \div 5 \div 4 - 2$ **k** $30 - 3 \times 2 \times 3$ **l** $17 + 20 \div 2 \div 2$
m $5 + 16 \div 4 \times 3 + 7$ **n** $24 + 5 \times 2 \times 6 - 14$ **o** $84 - 80 \div 10 + 9$

5 Evaluate:

a $5 \times 3 + 2 \times 6$ **b** $8 \times 5 - 7 \times 4$ **c** $12 \div 6 + 39 \div 3$
d $55 \div 5 - 18 \div 2$ **e** $3 \times 6 + 32 \div 4$ **f** $9 \times 4 - 4 \times 6$
g $77 \div 11 + 6 \times 11$ **h** $63 \div 7 - 50 \div 25$ **i** $8 \times 8 + 4 \times 4$

6 Evaluate each of the following expressions.

a $5 \times 9 - 3 \times 7 + 6$ **b** $25 - 36 \div 4 - 6 \times 2$ **c** $50 \div 10 + 13 + 8 \times 3$
d $100 \div 5 - 4 - 5 \times 3$ **e** $13 \times 3 + 8 \times 2 - 11$ **f** $90 - 5 \times 5 - 6 \times 10$
g $8 \times 7 - 2 \times 2 \times 3$ **h** $84 \div 12 + 24 \div 3 \div 2$ **i** $5 \times 4 \times 6 - 10 \times 7$
j $40 + 5 \times 7 - 8 \times 4 + 7$ **k** $35 \div 7 + 15 - 24 \div 4 + 8$ **l** $110 - 12 \times 5 + 10 - 40 \div 8$

Further applications

7 Use the order of operations to evaluate each of these.

a $15 + 5^2 \times 3$ **b** $70 - 2^3 \times 7$ **c** $30 + 6^2 \div 4$
d $7 \times 3^2 + 5 \times 2^2$ **e** $7^2 \times 2 - 3 \times 2^3$ **f** $10^2 \div 5^2 + 2 \times 3^3 \times 10$

1.13 Properties of cardinal numbers

Cardinal numbers are the counting numbers 1, 2, 3, 4, ... Using the **Commutative** and **Associative Laws** will make many calculations involving addition and multiplication of cardinals easier.

The Commutative Laws for addition and multiplication state that the order of the numbers can be *interchanged* without changing the answer. For example,
$3 + 7 = 7 + 3$ and $5 \times 4 = 4 \times 5$.

The Associative Laws for addition and multiplication state that the numbers can be *grouped* in different ways without changing the answer. For example,
$(5 + 2) + 8 = 5 + (2 + 8)$ and $(2 \times 4) \times 3 = 2 \times (4 \times 3)$.

Care must also be taken when evaluating expressions that contain either 0 or 1.

Number property	Example
When 0 is added to a number, the number is unchanged	$5 + 0 = 5$
When 0 is multiplied by a number, the answer is 0	$0 \times 6 = 0$
When a number is multiplied by 1, the number is unchanged	$3 \times 1 = 3$

Related ideas	Example
When 0 is subtracted from a number, the number is unchanged	$7 - 0 = 7$
When 0 is divided by a number, the answer is 0	$0 \div 8 = 0$
When a number is subtracted from itself, the answer is 0	$9 - 9 = 0$
When a number is divided by 1, the number is unchanged	$4 \div 1 = 4$
When a number is divided by itself, the answer is 1	$2 \div 2 = 1$

NOTE: While it is possible to divide zero by a number, it is not possible to divide a number by zero. For example, the expression $3 \div 0$ cannot be evaluated since it is not possible to divide 3 objects into 0 groups or into groups of 0 objects.

Example 1

Write true (T) or false (F) to state which of the following show a correct use of the Associative Laws.

a $2 \times (4 \times 9) = (2 \times 4) \times 9$

b $10 - (5 - 3) = (10 - 5) - 3$

Solutions

a LHS $= 2 \times (4 \times 9)$
$= 2 \times 36$
$= 72$

RHS $= (2 \times 4) \times 9$
$= 8 \times 9$
$= 72$

LHS = RHS, ∴ this is a correct use of the Associative Laws.

b LHS $= 10 - (5 - 3)$
$= 10 - 2$
$= 8$

RHS $= (10 - 5) - 3$
$= 5 - 3$
$= 2$

LHS ≠ RHS, ∴ this is not a correct use of the Associative Laws.

Example 2

Evaluate these expressions using the Commutative Laws.

a $86 + 75 + 14$

b $25 \times 9 \times 4$

Solutions

a $86 + 75 + 14 = 86 + 14 + 75$
$= 100 + 75$
$= 175$

b $25 \times 9 \times 4 = 25 \times 4 \times 9$
$= 100 \times 9$
$= 900$

Exercise 1.13

1 Evaluate:

a 7×0 **b** $15 + 0$ **c** 12×1 **d** $23 + 1$
e $54 + 0$ **f** 36×1 **g** $49 + 1$ **h** 75×0
i 19×1 **j** $11 + 0$ **k** 27×0 **l** $34 + 1$

2 Evaluate:

a $8 - 8$ **b** $3 \div 3$ **c** $4 - 0$ **d** $0 \div 2$
e $5 \div 1$ **f** $25 - 25$ **g** $12 \div 12$ **h** $17 - 0$
i $0 \div 9$ **j** $31 - 31$ **k** $11 \div 1$ **l** $28 \div 28$
m $19 - 19$ **n** $0 \div 1$ **o** $15 \div 15$ **p** $27 - 27$

3 Write true (T) or false (F) to state which of the following show a correct use of the Commutative Laws.

a $9 \times 4 = 4 \times 9$ **b** $11 + 4 = 4 + 11$ **c** $7 - 3 = 3 - 7$
d $8 \div 2 = 2 \div 8$ **e** $23 + 78 = 78 + 23$ **f** $72 \div 12 = 12 \div 72$
g $51 - 14 = 14 - 51$ **h** $18 \times 7 = 7 \times 18$ **i** $88 + 109 = 109 + 88$
j $45 \div 9 = 9 \div 45$ **k** $63 - 27 = 27 - 63$ **l** $26 \times 19 = 19 \times 26$
m $82 - 36 = 36 - 82$ **n** $94 + 187 = 187 + 94$ **o** $91 \div 7 = 7 \div 91$

4 Write true (T) or false (F) to state which of the following show a correct use of the Associative Laws.

a $(5 + 4) + 3 = 5 + (4 + 3)$ **b** $32 \div (8 \div 2) = (32 \div 8) \div 2$
c $20 - (13 - 4) = (20 - 13) - 4$ **d** $(5 \times 2) \times 3 = 5 \times (2 \times 3)$
e $(60 \div 6) \div 2 = 60 \div (6 \div 2)$ **f** $13 + (11 + 7) = (13 + 11) + 7$
g $8 \times (3 \times 2) = (8 \times 3) \times 2$ **h** $12 - (7 + 2) = (12 - 7) + 2$
i $(40 \div 2) \times 5 = 40 \div (2 \times 5)$ **j** $(10 + 9) + 17 = 10 + (9 + 17)$
k $12 \times (5 - 2) = (12 \times 5) - 2$ **l** $(4 \times 3) \times 7 = 4 \times (3 \times 7)$
m $0 + (5 + 9) = (0 + 5) + 9$ **n** $0 \times (13 \times 6) = (0 \times 13) \times 6$

Consolidation

5 Choose a suitable pair of numbers to add first, then evaluate each expression.

a $16 + 77 + 4$ **b** $89 + 66 + 11$ **c** $54 + 27 + 13$
d $35 + 89 + 25$ **e** $29 + 27 + 23$ **f** $132 + 46 + 14$
g $78 + 87 + 122$ **h** $197 + 114 + 103$ **i** $375 + 49 + 25$
j $180 + 375 + 120$ **k** $443 + 101 + 99$ **l** $485 + 148 + 115$

6 Evaluate:

a $83 + 19 + 17 + 11$ **b** $38 + 54 + 6 + 12$ **c** $16 + 75 + 24 + 25$
d $130 + 66 + 24 + 70$ **e** $91 + 19 + 69 + 11$ **f** $133 + 208 + 7 + 92$

7 Choose a suitable pair of numbers to multiply first, then evaluate each expression.

a $5 \times 19 \times 2$ **b** $4 \times 7 \times 3$ **c** $2 \times 8 \times 3$ **d** $4 \times 9 \times 5$
e $13 \times 5 \times 2$ **f** $5 \times 23 \times 4$ **g** $2 \times 70 \times 4$ **h** $19 \times 4 \times 25$
i $4 \times 6 \times 15$ **j** $8 \times 12 \times 5$ **k** $13 \times 2 \times 15$ **l** $5 \times 34 \times 4$

8 Evaluate:

a $19 \times 7 \times 0 \times 12$ **b** $5 \times 17 \times 5 \times 4$ **c** $2 \times 8 \times 5 \times 2$
d $38 \times 5 \times 10 \times 2$ **e** $125 \times 7 \times 2 \times 4$ **f** $9 \times 25 \times 4 \times 6$

Further applications

9 We say that the set of cardinal numbers (i.e. 1, 2, 3, …) is **closed under addition** because the sum of any two cardinal numbers always gives another cardinal number. Is the set of cardinal numbers closed under:

a subtraction? **b** multiplication? **c** division?

10 The product of two numbers can often be found by continually halving one number and doubling the other number until the answer is obvious.

For example,
$$\begin{aligned} 28 \times 15 &= 14 \times 30 \\ &= 7 \times 60 \\ &= 420 \end{aligned}$$

a Explain why this method produces the correct answer.

b Use this method to evaluate each of these products.

i 8×18 **ii** 32×15 **iii** 48×25 **iv** 35×240

11 To multiply a number by 12 we could first double the number then multiply the result by 6. Use this method to evaluate:

a 15×12 **b** 25×12 **c** 12×55

12 Use a similar method to that of Q11 to evaluate:

a 15×14 **b** 35×16 **c** 22×45

13 To divide a number by 20 we could divide the number by 10 then halve the result. Use this method to evaluate:

a $260 \div 20$ **b** $480 \div 20$ **c** $1040 \div 20$

14 Use a similar method to that of Q13 to evaluate:

a $390 \div 30$ **b** $840 \div 40$ **c** $720 \div 60$

15 **a** To divide a number by 4 we could halve the number twice. Find $128 \div 4$.

b To divide a number by 8 we could halve the number three times. Find $216 \div 8$.

c Evaluate $208 \div 16$.

1.14 The Distributive Law

Expressions such as 5 lots of 2 + 3 lots of 2 can be simplified by counting the total number of 'lots of 2'. That is, $(5 \times 2) + (3 \times 2) = (5 + 3) \times 2$ or 8 lots of 2. In reverse, we could say that $2 \times (5 + 3) = (2 \times 5) + (2 \times 3)$. This method of expanding an expression is called the **Distributive Law**, since the number outside the grouping symbols is being *distributed* over each term inside.

It is important that you understand this method before we begin the study of algebra later in the year, where the Distributive Law will prove to be a very important technique for simplifying expressions.

To expand an expression containing grouping symbols using the Distributive Law:
- **multiply the number outside the grouping symbols by each term inside.**

Example 1

Expand each of these expressions using the Distributive Law, then evaluate.

a $3(7 + 4)$ **b** $5(9 - 6)$

Solutions

a $3(7 + 4) = (3 \times 7) + (3 \times 4)$
$= 21 + 12$
$= 33$

b $5(9 - 6) = (5 \times 9) - (5 \times 6)$
$= 45 - 30$
$= 15$

Example 2

Use the Distributive Law to evaluate each expression.

a 23×102 **b** 7×95

Solutions

a $23 \times 102 = 23(100 + 2)$
$= (23 \times 100) + (23 \times 2)$
$= 2300 + 46$
$= 2346$

b $7 \times 95 = 7(100 - 5)$
$= (7 \times 100) - (7 \times 5)$
$= 700 - 35$
$= 665$

Example 3

Complete each of these expressions, then evaluate.

a $5 \times 2 + 5 \times 8 = 5(__ + __)$ **b** $12 \times 9 - 12 \times 4 = 12(__ - __)$

Solutions

a $5 \times 2 + 5 \times 8 = 5(2 + 8)$
$= 5 \times 10$
$= 50$

b $12 \times 9 - 12 \times 4 = 12(9 - 4)$
$= 12 \times 5$
$= 60$

Exercise 1.14

1 Express each of the following in the form '□ lots of Δ', then evaluate.

a 4 lots of 2 + 3 lots of 2
b 9 lots of 5 – 7 lots of 5
c 5 lots of 3 + 4 lots of 3
d 11 lots of 6 – 5 lots of 6
e 12 lots of 13 – 9 lots of 13
f 6 lots of 18 + 4 lots of 18

2 Which one of the following expressions shows the expanded form of $3(2 + 4)$?

A 3×6 **B** $32 + 34$ **C** $3 \times 2 + 4$ **D** $3 \times 2 + 3 \times 4$

3 Write each of these in the expanded form.

a $5(3 + 1)$ **b** $2(7 - 3)$ **c** $4(12 + 5)$ **d** $9(10 - 4)$

Consolidation

4 Evaluate these expressions using the Distributive Law.

a $2(3 + 1)$ **b** $3(7 + 4)$ **c** $5(2 + 6)$ **d** $4(5 + 2)$
e $7(9 - 4)$ **f** $6(8 - 2)$ **g** $10(7 - 3)$ **h** $9(6 - 1)$
i $4(3 + 6)$ **j** $8(11 - 5)$ **k** $7(8 - 4)$ **l** $12(1 + 4)$
m $10(9 + 2)$ **n** $6(4 - 3)$ **o** $11(5 + 6)$ **p** $9(12 - 7)$

5 Use the Distributive Law to evaluate each expression.

a 7×13 **b** 6×14 **c** 8×21 **d** 9×32
e 5×19 **f** 7×29 **g** 8×99 **h** 3×95
i 17×101 **j** 12×92 **k** 16×98 **l** 25×103
m 23×96 **n** 15×90 **o** 13×104 **p** 9×99
q 102×32 **r** 98×26 **s** 91×11 **t** 195×200

Further applications

6 Copy and complete each expression, then evaluate.

a $2 \times 5 + 2 \times 3 = 2 \times (__ + __)$ **b** $3 \times 4 + 3 \times 5 = 3 \times (__ + __)$
c $7 \times 6 + 7 \times 4 = 7 \times (__ + __)$ **d** $8 \times 9 + 8 \times 6 = 8 \times (__ + __)$
e $6 \times 2 + 6 \times 5 = 6 \times (__ + __)$ **f** $5 \times 7 + 5 \times 1 = 5 \times (__ + __)$

7 Copy and complete each expression, then evaluate.

a $4 \times 3 - 4 \times 1 = 4 \times (__ - __)$ **b** $6 \times 12 - 6 \times 3 = 6 \times (__ - __)$
c $2 \times 5 - 2 \times 3 = 2 \times (__ - __)$ **d** $5 \times 11 - 5 \times 6 = 5 \times (__ - __)$
e $7 \times 8 - 7 \times 5 = 7 \times (__ - __)$ **f** $9 \times 10 - 9 \times 2 = 9 \times (__ - __)$

8 Express each of these in the form '$\square \times \Delta$', then evaluate.

a $4 \times 9 + 7 \times 9$ **b** $9 \times 3 - 5 \times 3$ **c** $7 \times 4 + 5 \times 4$
d $16 \times 14 - 6 \times 14$ **e** $18 \times 11 - 13 \times 11$ **f** $14 \times 7 + 6 \times 7$

1.15 Mathematical language and symbols

In worded problems, there are many key words or phrases that indicate the operation to be performed. Some of these key words and phrases are listed below.

- Addition: plus, sum, more than, total, increase, add
- Subtraction: minus, difference, less than, reduce, decrease, take away, subtract

- Multiplication: times, product, multiple, lots of, multiply
- Division: divide, quotient, into, share

There are also a number of symbols that mathematicians use to simplify their working. While '=' means *equals* or *is equal to*, several **inequality signs** are used to show the relationship between two numbers that are not equal in value. These signs are listed in the table.

Symbol	Meaning
$\neq$	is not equal to
$<$	is less than
$>$	is greater than
$\leqslant$	is less than or equal to
$\geqslant$	is greater than or equal to

Some words such as **average** involve the use of more than one operation.

To find the average of a set of numbers:
- **add the numbers**
- **divide this sum by the number of numbers.**

Example 1

Write each of the following statements using numerals and mathematical operations.

a the sum of six and eight
b the product of five and nine
c the quotient of twelve and four
d the difference between fifteen and seven
e three less than eleven
f increase twenty by thirteen

Solutions

a $6 + 8$ **b** 5×9 **c** $12 \div 4$ **d** $15 - 7$ **e** $11 - 3$ **f** $20 + 13$

Example 2

Use inequality signs to show the meaning of:

a 6 is greater than 4
b 3 is less than 10
c 11 is not equal to 16
d 7 is less than or equal to 8
e 24 is greater than or equal to 19

Solutions

a $6 > 4$ **b** $3 < 10$ **c** $11 \neq 16$ **d** $7 \leqslant 8$ **e** $24 \geqslant 19$

Example 3

Find the average of 4, 7, 12, 10, 2.

Solution

i First, we add the numbers, i.e. $4 + 7 + 12 + 10 + 2 = 35$
ii Now there are 5 numbers, so we divide the sum by 5, i.e. $35 \div 5 = 7$
$\therefore$ The average of the numbers is 7.

Exercise 1.15

1 Write each of the following statements using numbers and mathematical operations.

a eight plus four
b nine minus six
c the sum of five and three
d the difference between seven and two
e subtract ten from fifteen
f add nine to eleven
g three more than four
h one less than eight
i from seventeen take twelve
j to twenty add thirty
k increase nineteen by five
l decrease fourteen by ten

2 Write each of the following statements using numbers and mathematical operations.

a five times eight
b twenty divided by four
c three into eighteen
d six lots of seven
e the product of two and eleven
f the quotient of thirty-six and nine
g divide forty-eight by eight
h twelve multiplied by ten

Consolidation

3 Rewrite each of these statements using numerals and mathematical operations, then evaluate.

a 14 minus 6
b add 5 to 7
c 4 lots of 8
d 3 into 21
e 12 less than 25
f divide 42 by 6
g the sum of 30 and 19
h to 23 add 15
i decrease 28 by 17
j the difference between 34 and 5
k the product of 20 and 6
l 8 more than 9
m 75 divided by 25
n increase 18 by 10
o 12 multiplied by 4
p subtract 9 from 19
q the quotient of 16 and 4
r from 40 take 8
s 4 plus 11
t 9 times 7
u reduce 50 by 15
v multiply 13 by 4

4 Write a number sentence that represents each of these statements, then evaluate.

a 3 more than the product of 5 and 2
b 7 less than the product of 6 and 3
c 6 more than the quotient of 12 and 4
d 10 less than the quotient of 60 and 5

5 Find the average of each set of integers.

a 5, 3, 10
b 12, 6, 7, 3
c 10, 4, 9, 8, 14
d 23, 50, 15, 35, 7, 26

6 State the meaning of each inequality sign in words.

a $\neq$ **b** $>$ **c** $<$ **d** $\leqslant$ **e** $\geqslant$

7 Use inequality signs to show the meaning of:

a 5 is less than 9
b 10 is greater than 4
c 2 is less than or equal to 7
d 18 is greater than or equal to 13

e 9 is not equal to 5
f 8 is less than 12
g 3 is greater than or equal to 2
h 15 is not equal to 14
i 10 is greater than 3
j 20 is less than or equal to 25

8 Write true (T) or false (F) for each of the following.

a $4 < 7$ **b** $5 > 8$ **c** $10 \geqslant 6$ **d** $3 \leqslant 3$
e $18 < 13$ **f** $21 \geqslant 24$ **g** $9 < 15$ **h** $17 > 16$
i $35 \leqslant 33$ **j** $28 \neq 29$ **k** $7 \geqslant 7$ **l** $14 > 20$
m $16 \neq 16$ **n** $99 \leqslant 100$ **o** $34 > 36$ **p** $48 \geqslant 48$

9 Write an equivalent statement for each of the following using the opposite inequality sign, then write the meaning of that statement in words.

a $5 > 2$ **b** $4 < 11$ **c** $8 \geqslant 6$ **d** $9 \leqslant 14$

Further applications

10 Which of the inequality signs $<$, $>$, $\leqslant$, $\geqslant$ could be used in place of those below?

a $\ngtr$ **b** $\nleqslant$ **c** $\nless$ **d** $\ngeqslant$

11 What limitation must be placed on $\square$ to make a true statement in each of these?

a $\square + 3 < 8$
b $\square + 5 > 12$
c $\square - 2 \leqslant 6$
d $\square - 4 \geqslant 10$

12 List all of the possible integers that could replace the Δ to make a true statement.

a $0 \leqslant \Delta \leqslant 6$
b $4 < \Delta < 9$
c $13 \leqslant \Delta < 17$
d $19 < \Delta \leqslant 25$

13 The symbol ! (read as 'factorial') written after an integer is used to refer to the product of that integer and every positive integer that is less than or equal to it in value. For example,

$4! = 4 \times 3 \times 2 \times 1$
$= 24$

Evaluate each of the following factorials.

a $3!$ **b** $5!$ **c** $6!$ **d** $7!$
e $\dfrac{9!}{8!}$ **f** $\dfrac{13!}{12!}$ **g** $\dfrac{11!}{9!}$ **h** $\dfrac{8!}{5! \times 3!}$

14 Express each of the following as a single factorial number.

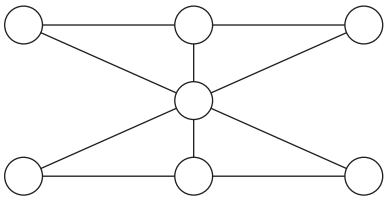
a $10 \times 9!$ **b** $15 \times 14 \times 13!$ **c** $20 \times 3!$ **d** $210 \times 4!$

Problem solving

1 If you were given nine lollies and you ate one every $\frac{1}{2}$ hour, how long would they last?

2 If a twenty-metre-long tree had to be cut into twenty equal pieces, how long would it take if it took 5 minutes to make each cut?

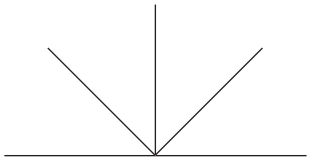
3 Place the numbers 4 to 10 in the seven circles so that every row (vertical, horizontal, and diagonal) adds to 21.



4 Rearrange the following numbers so that the subtraction is correct.

```
 123
  45 –
 ----
 678
```

5 How many angles less than 180° are in the diagram? Be careful!



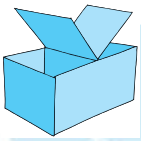
6 Draw two straight lines so that the moon is divided into 6 pieces (not necessarily the same size).

7 What is the least number of moves to put these balls in order of alternating colour?

8 A plane takes one hour and ten minutes to fly from X to Y, and only 70 minutes to fly from Y to X. Can you explain this?

9 Arrange 9 dots so that you have three dots in each of eight rows. (Rows may be vertical, horizontal or diagonal.)

10 A box is to be made to fit two die. It has two lids.
Can you design a pattern or flat layout called a net which could be used to make this box? For more information about nets see page 460.



Focus on Working Mathematically

The Mayan people of Mexico

Introduction

In this chapter we've studied some ways in which people down through the ages have recorded numbers. The Egyptian, Babylonian, Sumerian, Arabic and Hindu cultures all played a role in the development of the base 10 number system we use today.

We will now take a closer look at the Mayan counting system.

The story of the Mayan counting system

The Mayan people of what is now Central America had a wonderful counting system. It was used in organising their everyday lives, for example their calendars, commerce and scientific work, and written down in books. From 2000 BC the Mayan people and their descendants preserved their counting system and developed it to include a symbol for zero.

In 1519 Hernan Cortes, a Spanish soldier and adventurer, sailed to the Yucatan Peninsula. The local Mayan Indians welcomed him, and he soon married a local woman he called Marina. The Mayans suspected something was wrong and called her Malinche, which means traitoress. She acted as Cortes's interpreter and guide and enabled him to conquer the Yucatan Peninsula and the mighty Aztecs of Mexico. The 'New World' then came under strict Spanish control. The cities that held the secrets to the Mayan counting system were largely devastated.

Twenty years later Diego de Landa, a Spanish missionary, visited the Mayan people. They were badly treated by their Spanish masters and Landa did what he could to protect them. He visited the ruins of the great Mayan cities and learnt from the people about their history, their calendars and counting systems. However he saw their religious practices to be 'of the devil' and ordered all Mayan idols and counting books to be destroyed.

Landa was surprised at the great distress his actions caused the Mayan people. Out of regret or perhaps to justify himself, he wrote a book which describes in great detail the culture of the Mayan people. This book was lost for many years but was rediscovered in Madrid in 1869.

A few documents survived Landa's destruction, of which the most important are kept in the libraries of Dresden, Madrid and Paris. As for Malinche, she had a child, and became

symbolically the mother of the first Mexican, the first child of Indian and Spanish blood, the first child who was both European and American.

Learning activities

To get started you will need to study the table of Mayan numerals below. There were just three symbols, a shell to represent zero, a dot to represent 1, and a bar to represent 5.

0	1	2	3	4	5
	•	• •	• • •	• • • •	—

The Mayans counted in 5s and 20s rather than 10s as we do, and they wrote their numbers vertically. The position of the symbols in the vertical line showed their value. In Example 1 below you can see the numeral for 24. There is a gap to separate the 20 from the 4.

However you can see below that the system used by priests and astronomers was not a true base 20 system. Instead of the next place being 20 lots of 20 as we would expect, it is only 18 lots of 20. This is the only system for which we have recorded evidence.

		Example 1	Example 2	Example 3	Example 4
18×20^2	(7200)		•	• • • •	
18×20	(360)		• • —		
20	(20)	•	• —	• • •	• •
Units		• • • •	• =	• • • • —	≡
Hindu–Arabic number		24	9851		

Reading down the second column from the top, we see Example 2 is (1; 7; 6; 11) which represents $(1 \times 7200) + (7 \times 360) + (6 \times 20) + (11 \times 1) = 7200 + 2520 + 120 + 11 = 9851$

1. Convert Examples 3 and 4 of Mayan numerals in the table above into Hindu–Arabic numerals.
2. How might you add numbers in Mayan? Try adding the numbers represented by Examples 1, 2 and 3 in the table.
3. How might you subtract two numbers in Mayan? Try subtracting Example 1 from Example 2, and Example 2 from Example 3 in the table.
4. The Mayans did not have methods to multiply and divide their numbers. Can you write down a reason why it was so difficult?
5. The Mayans did not have numerals for fractions either. Just for fun make up your own Mayan symbols for $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{3}{20}$. Are some fractions easier than others? Why?

Extension activities

1 What happens to a number when you move all the digits one place to the left in our system? For example, 246 to 2460?
2 Work out the Hindu–Arabic equivalent for the Mayan numeral (1; 2; 3) and the Mayan numeral (1; 2; 3; 0).
3 What happens to a Mayan number when you move all the digits one place to the left? Use your results in Q2 to show that the number (1; 2; 3; 0) is not 20 times the number (1; 2; 3). Can you explain why not? Check again the Mayan place value system.

Let's communicate

In pairs, make a poster of what you have learned about Mayan numerals. How does the Mayan system differ from the Hindu–Arabic system we use today? Why do you think they counted in 5s? in 20s? Discuss the strengths and weaknesses of the Mayan number system with your neighbour.

Reflecting

By comparing with other number systems, think over the advantages of the Hindu–Arabic system we use today. List these advantages in your exercise book.

Language link with Macquarie

MACQUARIE

1 *Explain why most* ***civilisations*** *need a* ***numeration*** *system.*
2 *Explain how the* ***subtraction*** *principle works in Roman numerals.*
3 *Explain the difference between* ***division*** *and* ***multiplication****.*
4 *What is meant by the terms* ***base*** *and* ***index****?*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***numeral****:*

numeral *noun* **1.** the word or words for a number: *The numeral for 9 is 'nine'.* **2.** a letter or figure, or a group of letters or figures, which represent a number: *The Roman numeral for 60 is LX.*

How does a numeral differ from a number?

CHAPTER REVIEW

1 Write these Babylonian numerals as Hindu–Arabic numerals.

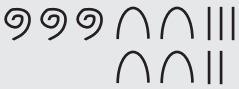
a

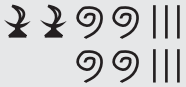
b

c

d

2 Write these Egyptian numerals as Hindu–Arabic numerals.

a 

b 

c

d

3 Write these Greek numerals as Hindu–Arabic numerals.

a ΔΔΔ⌐II
b HH⌈Δ⌉ΔΔΔΔIIII
c XXX⌐H⌈Δ⌉Δ
d ⌈M⌉MM⌈X⌉XXX⌐H

4 Write these Mayan numerals as Hindu–Arabic numerals.

a

b

c

d

5 Write these Roman numerals as Hindu–Arabic numerals.

a LXXVIII **b** CCXXXI
c MMMDCCLXVI
d $\overline{X}\overline{V}$MDII **e** XIX
f XLIV **g** CDXCVII
h M$\overline{V}$XCIX

6 Write each of these using Roman numerals.

a	88	**b**	206	**c**	773
d	2657	**e**	14	**f**	99
g	474	**h**	942	**i**	9489

7 **a** Give one reason why the Roman numeration system was better than that of the ancient Babylonians and Egyptians.
b What features did the Mayan numeration system have that other systems did not?
c Why is the Hindu–Arabic number system superior to all of the ancient number systems?

8 Write down the value of the underlined digit in each of these.

a 2<u>8</u>3 **b** <u>5</u>2 614
c 3<u>9</u> 002 **d** 7 <u>4</u>66 912

9 Write down the:
a smallest 3 digit number with a 6 in the tens place.
b largest 3 digit number with a 4 in the hundreds place.

10 Write the basic numeral for each of these.
a Two thousand four hundred and thirteen
b Fifteen thousand and seventy
c One hundred and forty-eight thousand nine hundred and twenty-six
d Seven million six hundred and eighty thousand five hundred and nine
e Twelve million and thirty-four thousand two hundred and eighty

11 Write each number in words.

a 4010 **b** 162 478
c 390 050 **d** 89 067 211

12 Evaluate:
a 4 + 8 + 9 + 6 + 5
b 67 + 83

c $385 + 965 + 477$
d $32\,062 + 97\,465 + 30\,791$

13 Evaluate:
a $85 - 64$ **b** $74 - 49$
c $600 - 274$ **d** $3072 - 1836$

14 Write answers to each of these.
a 6×7 **b** 9×4
c 8×8 **d** 12×5
e 7×9 **f** 8×12
g 11×11 **h** 9×8

15 Evaluate:
a 27×6 **b** 384×7
c 43×800 **d** 66×39

16 Evaluate:
a $4\overline{)2884}$ **b** $3\overline{)252}$
c $7\overline{)2163}$ **d** $5\overline{)15\,200}$

17 Evaluate $509 \div 6$, giving the remainder as a fraction.

18 Evaluate $24\,000 \div 60$.

19 Evaluate each of these long divisions. (All answers are whole numbers.)
a $414 \div 18$ **b** $962 \div 37$
c $1008 \div 36$

20 Evaluate each of the following.
a 7^2 **b** 2^3
c 10^4 **d** 1^7

21 Which is larger, 3 factors of 4 or 4 factors of 3?

22 Find the value of:
a $6^2 + 3^3$ **b** $5^3 - 2^5$
c 4×10^3 **d** $12^2 - 7^2 - 5^2$

23 **a** Does $5^2 \times 12^2 = (5 \times 12)^2$?
b Hence, evaluate 25×144.

24 Write the basic numeral for each of these.
a $(7 \times 10\,000) + (4 \times 1000) + (6 \times 100) + (0 \times 10) + (2 \times 1)$
b $(3 \times 10^5) + (8 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (9 \times 10^1) + (7 \times 1)$

25 Write 243 586 in the expanded form.

26 Write each of these in the expanded form, using index notation.
a 5219 **b** 38 514
c 129 042 **d** 3 507 068

27 Round off each number correct to the nearest ten.
a 64 **b** 28
c 75 **d** 1844

28 Round off each number correct to the nearest hundred.
a 363 **b** 749
c 150 **d** 23 971

29 Use leading figure estimation to estimate the answer to each calculation.
a $587 + 839$ **b** $22\,289 - 5099$
c 54×375 **d** $815 \div 37$

30 Evaluate:
a $(23 + 19) \div (28 - 7)$
b $60 - [40 - (56 \div 8)] + 12$
c $\dfrac{30 + (7 \times 6)}{13 - 4}$

31 Insert grouping symbols to make a true statement: $5 \times 3 + 11 - 8 = 62$

32 Use the order of operations to evaluate:
a $9 \times 3 - 55 \div 5$ **b** $100 - 2 \times 3 \times 12$
c $30 + 18 \div 3^2 - 7$

33 Write true (T) or false (F) for each of these.
a $17 + 19 = 19 + 17$
b $12 - 8 = 8 - 12$
c $13 \times 4 = 4 \times 13$
d $20 \div 10 = 10 \div 20$

34 Write true (T) or false (F) for each of these.
a $(8 + 5) + 7 = 8 + (5 + 7)$
b $40 - (16 + 3) = (40 - 16) + 3$
c $(5 \times 6) \times 4 = 5 \times (6 \times 4)$

35 Rearrange each expression into a more suitable form, then evaluate.

a $172 + 49 + 28$ **b** $25 \times 19 \times 4$

36 Evaluate each expression using the Distributive Law.

a $4(3 + 5)$ **b** $7(8 - 2)$
c $5(11 + 9)$ **d** $9(12 - 7)$

37 Evaluate 27×99 using the Distributive Law.

38 Copy and complete each expression, then evaluate.

a $5 \times 7 + 5 \times 3 = 5 \times (___ + ___)$
b $12 \times 5 - 12 \times 3 = 12 \times (___ - ___)$

39 Rewrite each of these statements using numerals and mathematical operations.

a 3 lots of 6
b 7 more than 30
c 9 into 36
d the difference between 19 and 6
e the quotient of 40 and 8
f the sum of 13 and 16
g 18 less than 25
h the product of 13 and 3
i decrease 50 by 10

40 Find the average of 14, 7, 11, 8, 20.

41 Use inequality signs to show the meaning of:

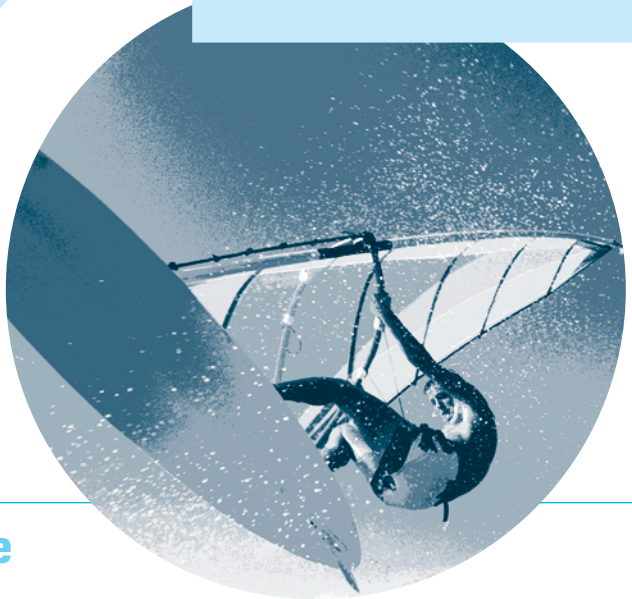
a 6 is greater than 4
b 13 is less than 21
c 27 is not equal to 25
d 15 is less than or equal to 28
e 11 is greater than or equal to 11
f 5 is not less than 3

42 Outline a method by which a number could be:

a increased by 29
b multiplied by 50
c multiplied by 99
d divided by 20

Number theory

2



This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- classify a positive whole number as either odd or even
- list the multiples of a number
- find all factors of a number
- state whether a number is prime, composite or neither
- express a number as the product of its prime factors using a factor tree.

Stage 4

After completing this chapter, you should be able to:

- identify a palindromic number
- list the first 12 square and triangular numbers
- form the Fibonacci number sequence
- use the divisibility tests to determine whether one number is divisible by another
- use the divisibility tests to determine whether a number is prime or composite
- find the lowest common multiple of two numbers
- find the highest common factor of two numbers
- find squares and square roots of numbers
- find cubes and cube roots of numbers
- use prime factors to find the square root or cube root of a number.

2.1 Odd and even numbers

All counting numbers are either odd or even. A number is **even** if it is divisible by 2. For example, 10 is even because $10 \div 2 = 5$, or $2 \times 5 = 10$. A number is **odd** if it leaves a remainder of 1 when divided by 2. For example, 9 is odd because $9 \div 2 = 4$ remainder 1, or $(2 \times 4) + 1 = 9$. The first five even numbers are 2, 4, 6, 8 and 10. The first five odd numbers are 1, 3, 5, 7 and 9. For a large number, we can determine whether it is odd or even by looking at the units digit.

- **A number is even if it ends in 2, 4, 6, 8 or 0.**
- **A number is odd if it ends in 1, 3, 5, 7 or 9.**

EG+S

Example

State whether the following numbers are odd or even.

a 334
b 659
c 5870

Solutions

a The units digit of 334 is 4, which is even, ∴ 334 is even.
b The units digit of 659 is 9, which is odd, ∴ 659 is odd.
c The units digit of 5870 is 0, which means the number is divisible by 10, and hence it is also divisible by 2, ∴ 5870 is even.

Exercise 2.1

1 Complete the following statements.
a All even numbers are divisible by _____ .
b If a number is divisible by 2, then it is an _____ number.

2 List the first 10:
a odd numbers **b** even numbers

3 State whether the following numbers are odd or even.

a 36	**b** 47	**c** 50	**d** 164	**e** 271	**f** 399
g 700	**h** 1003	**i** 4625	**j** 8421	**k** 37 628	**l** 110 101

Consolidation

4 Write down the next:
a odd number after 83 **b** even number after 170 **c** even number after 1098
d odd number before 117 **e** even number before 560 **f** odd number before 2301

5 Write down the:
a smallest 2-digit even number **b** largest 2-digit odd number
c smallest 3-digit odd number **d** largest 3-digit even number
e largest 2-digit even number **f** smallest 2-digit odd number
g largest 3-digit odd number **h** smallest 3-digit even number

6 Write down the:
- **a** smallest 2-digit odd number with a 7 in the tens place.
- **b** largest 2-digit even number with a 4 in the tens place.
- **c** smallest 2-digit even number with a 9 in the tens place.
- **d** smallest 3-digit odd number with a 2 in the tens place.
- **e** largest 3-digit odd number with a 1 in the tens place.
- **f** largest 3-digit even number with a 5 in the tens place.

7 **a** Find the smallest 3-digit odd number with a 4 in the hundreds place and a 7 in the tens place.

b Find the largest 4-digit even number with a 6 in the thousands place and a 3 in the tens place.

Further applications

8 State whether each of the following are odd or even.
- **a** the sum of 2 odd numbers
- **b** the product of 2 odd numbers
- **c** the difference between 2 even numbers
- **d** the sum of an odd number and an even number
- **e** the product of an even number and an odd number
- **f** the difference between an odd number and an even number
- **g** the product of 2 odd numbers and an even number
- **h** the sum of 3 odd numbers

9 How many positive integers from 1 to 100 can be expressed as the product of two even numbers? What else do these numbers have in common?

2.2 Special numbers

Palindromic numbers

A **palindrome** is a word or number that reads the same from left to right or from right to left. For example, madam, toot and deed are palindromic words, while 33, 171 and 52 825 are palindromic numbers.

Square numbers

The pattern of **square numbers** is derived from the following diagrams.

In the diagram, each square has one more star horizontally and one more star vertically than the previous square. The number of stars in each square can be found by multiplying the number of horizontal stars by the number of vertical stars. That is,

$1 \times 1 = 1 \qquad 2 \times 2 = 4 \qquad 3 \times 3 = 9 \qquad 4 \times 4 = 16.$

So the first four square numbers are 1, 4, 9, 16.

When a number has been multiplied by itself, we say that it has been **squared**. If a number can be expressed as the square of a counting number, then it is a square number. For example, $49 = 7 \times 7$, so 49 is a square number. However, $10 = 1 \times 10$ or 2×5 only, so 10 is not a square number.

Triangular numbers

The pattern of **triangular numbers** is derived from the following diagrams.

Each triangle has one more row than the previous triangle and each row has one more star than the row above. The number of stars in each triangle can be found by adding **consecutive** counting numbers. That is,

$1 = 1$
$1 + 2 = 3$
$1 + 2 + 3 = 6$
$1 + 2 + 3 + 4 = 10$

So the first four triangular numbers are 1, 3, 6 and 10.

Example 1

Find the:

a 9th square number
b 16th square number

Solutions

a $9 \times 9 = 81$, so 81 is the 9th square number.
b $16 \times 16 = 256$, so 256 is the 16th square number.

Example 2

Find the 6th triangular number.

Solution

$1 + 2 + 3 + 4 + 5 + 6 = 21$, so the 6th triangular number is 21.

Exercise 2.2

1 Which of these numbers are palindromes?

a 66	**b** 131	**c** 275	**d** 2002
e 3013	**f** 24 342	**g** 478 784	**h** 9 133 319

2 **a** What is the smallest 2-digit palindromic number?
b What is the largest 2-digit palindromic number?

3 Make up four palindromic numbers that have 5 digits.

4 Which digit can never be found in the units place of a palindromic number? Why?

5 **a** Write down the square numbers indicated by the following diagrams.

```
                                    * * * *
                        * * *       * * * *
            * *         * * *       * * * *
    *       * *         * * *       * * * *
```

b Draw the next two squares in the pattern and hence find the next two square numbers.

6 Find the first 12 square numbers.

7 Write down the first:
a 6 odd square numbers **b** 6 even square numbers

8 **a** Write down the triangular numbers indicated by the following diagrams.

```
                                    *
                        *           * *
            *           * *         * * *
    *       * *         * * *       * * * *
```

b Draw the next two triangles in the pattern and hence find the next two triangular numbers.

9 Find the first 12 triangular numbers.

10 Write down the first:
a 6 odd triangular numbers **b** 6 even triangular numbers

■ Consolidation

11 Select all the square numbers from the given list. 25, 35, 9, 48, 36, 99, 18, 64, 1, 44, 96, 121

12 Explain how you could find the 20th square number, then find this number.

13 Find the:
a 13th square number **b** 15th square number **c** 19th square number

14 **a** Evaluate each of the following.
$1 =$
$1 + 3 =$
$1 + 3 + 5 =$
$1 + 3 + 5 + 7 =$
$1 + 3 + 5 + 7 + 9 =$
$1 + 3 + 5 + 7 + 9 + 11 =$

b What type of numbers are the answers in part **a**?
c How could we find the sum of the first 6 odd numbers without doing an addition?

15 Using the method outlined in Q14, write down a multiplication that could be used to evaluate the following sums, then find the answer to each.
a $1 + 3 + 5 + 7 + \ldots + 15$ **b** $1 + 3 + 5 + 7 + \ldots + 19$ **c** $1 + 3 + 5 + 7 + \ldots + 31$

16 How many consecutive odd numbers would have to be added to find the value of 14×14?

17 **a** Find the first two square numbers that are each the sum of two other square numbers.
b Find the first four square numbers that are each the product of two different square numbers other than 1.

18 Select all triangular numbers from the given list. 2, 15, 18, 25, 21, 36, 60, 6, 10, 89, 91, 28

19 Explain how you could find the 20th triangular number. (Do not find the number.)

20 Find the:
a 13th triangular number **b** 14th triangular number **c** 15th triangular number

21 **a** Evaluate each of these sums.
i $1 + 3$ **ii** $3 + 6$ **iii** $6 + 10$ **iv** $10 + 15$ **v** $15 + 21$ **vi** $21 + 28$
b What kind of numbers are the answers in part **a**?

22 Take any three consecutive triangular numbers. Add the numbers, subtract 1, then divide the answer by 3. Repeat this several times. What do you notice?

23 Every even counting number can be expressed as the sum of at most three triangular numbers. Express each of the following as the sum of either 2 or 3 triangular numbers.
a 14 **b** 16 **c** 26 **d** 44

24 **a** Find the first triangular number that is equal to the product of two other triangular numbers.
b Find the first four triangular numbers that are equal to the sum of two other triangular numbers.

25 Two triangular numbers differ by 23. If the smaller of the numbers is 253, what is the other number?

26 **a** Evaluate each of the following.
$[(3 \times 3) - 1] \div 8 =$
$[(5 \times 5) - 1] \div 8 =$
$[(7 \times 7) - 1] \div 8 =$
b Continue this pattern for another three lines. What kind of numbers are the answers?

■ Further applications

27 **a** Write down the square number that lies between 80 and 90.
b Find the first square number that is divisible by 12.
c Find the first odd 2-digit square number.

d Find the two square numbers whose difference is 17.
e Find the first square number whose digits differ by 2.
f Find the first square number that is divisible by 27.
g Find the first 2-digit square number whose digit sum is even.
h Write the first two square numbers that have a 6 in the tens place.
i Find the first two square numbers, each of which is the square of another square number.
j State the first square number with more than one digit that is also a palindrome.
k Find the square number which when doubled and then increased by 2 gives another square number.
l Find the first 4 digit square number.

28 Use the first 14 triangular numbers to find answers to each of these.
a Find the first 2-digit triangular number.
b Write down two triangular numbers that are also square numbers.
c Find two triangular numbers each of which is 1 less than a square number.
d Find two triangular numbers greater than 10 that are also palindromes.
e Write down a triangular number that is divisible by 13.
f State the largest 2-digit triangular number.
g Find the first 2-digit triangular number whose digits are also triangular numbers.
h Write down the first 2-digit triangular number whose digit sum is also a triangular number.
i Find the first 3-digit triangular number.
j Calculate the product of the 7th square number and the 4th triangular number.
k Calculate the quotient of the 9th and 5th triangular numbers.
l Find two triangular numbers whose product is 5 more than one triangular number and 6 less than another.

TRY THIS Palindromes

'Dad' is a word which is the same whether you read it forwards or backwards; so is 'Mum'! 464 is a palindromic number; so is 41314. It is said that every number can be changed to a palindromic number by the following method of reversing digits then adding until it is palindromic, e.g.

1 34:
```
34
43
--
77
```

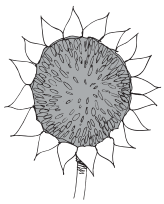
2 124:
```
124
421
---
545
```

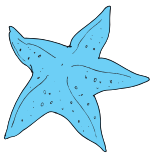
3 96:
```
   96
   69
  ---
  165
  561
  ---
  726
  627
 ----
 1353
 3531
 ----
 4884
```

Can any whole number be changed to a palindromic number in this way? Why?

2.3 Fibonacci numbers

The **Fibonacci numbers** are an important sequence of numbers discovered by the famous Italian mathematician Leonardo Fibonacci (also known as Leonardo of Pisa) in 1202. The sequence of Fibonacci numbers begins with two consecutive 1s and each term thereafter is found by adding the two previous terms.





What makes the Fibonacci numbers interesting is that they occur so often in nature. For example, in the centre of a daisy we see that there are two sets of intersecting spirals that radiate in opposite directions, 34 in one direction and 21 in the other. In the seed cones of fir trees, the ratio of the numbers of spirals is $\frac{13}{8}$ or $\frac{8}{5}$, while the ratio for a pineapple is $\frac{13}{8}$. In ripe sunflower heads the spiralling rows of florets are in the ratio $\frac{21}{13}$, $\frac{34}{21}$, $\frac{55}{34}$ or in a giant sunflower $\frac{144}{89}$. The Fibonacci numbers can also be found in the arrangement of leaves on a stem in many plants. If you count the number of leaves passed before you reach a leaf directly above the first leaf, the number of leaves in most cases is a Fibonacci number. The numbers can also be found in connection with starfish, sea urchins and many other marine animals.

Example

Find the first six Fibonacci numbers.

Solution

The sequence is 1, 1, 1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5, 3 + 5 = 8.
Therefore, the first six Fibonacci numbers are 1, 1, 2, 3, 5, 8.

Exercise 2.3

1 Find the first 10 Fibonacci numbers.

2 Find the first Fibonacci number, other than 1, that is also a square number.

3 Which of the first 10 Fibonacci numbers, other than 1, are also triangular numbers?

Consolidation

4 **a** Is there a pattern with the odd and even Fibonacci numbers? If so, what is it?
b Will the 24th Fibonacci number be odd or even?
c Will the 31st Fibonacci number be odd or even?

5 In which positions are the Fibonacci numbers that are divisible by:
a 3 **b** 5

6 **a** Find the sum of the first 10 Fibonacci numbers. Is it divisible by 11?
b Find the sum of the 10 Fibonacci numbers starting with the second number (i.e. the second 1). Is it divisible by 11?
c Find the sum of the 10 Fibonacci numbers starting with the third number (i.e. the 2). Is it divisible by 11?
d What do these results suggest about Fibonacci numbers?

7 The pattern below highlights an interesting relationship between sets of three consecutive Fibonacci numbers.
For 2, 3, 5: $(3 \times 3) = (2 \times 5) - 1$
For 5, 8, 13: $(8 \times 8) = (5 \times 13) - 1$
a Is this pattern still true for 13, 21, 34?
b How should we adjust the pattern for 3, 5, 8 and 8, 13, 21?

8 Take any four consecutive Fibonacci numbers such as 2, 3, 5, 8.
a Find the product of the inner pair of numbers and find the product of the outer pair of numbers.
b Repeat this for three more sets of four consecutive Fibonacci numbers. What do you notice?

9 **a** If the 13th Fibonacci number is 233, what is the sum of the first 11 Fibonacci numbers?
b If the sum of the first 12 Fibonacci numbers is 376, what is the 14th Fibonacci number?

10 Complete the next three lines of this pattern. What do you notice?
$1^2 + 1^2 = 2 = 1 \times 2$
$1^2 + 1^2 + 2^2 = 6 = 2 \times 3$
$1^2 + 1^2 + 2^2 + 3^2 = 15 = 3 \times 5$

Further applications

In the following questions, the subscript refers to the position of the number in the sequence. For example, F_2 means the second Fibonacci number, and in Q11, L_2 means the second Lucas number.

11 **a** Is 6 divisible by 2? Is F_6 divisible by F_2?
b Is 8 divisible by 4? Is F_8 divisible by F_4?
c In general, one Fibonacci number will be divisible by another if the subscript of the larger number is divisible by the subscript of the smaller number. Which Fibonacci numbers will divide into F_{12}?

12 The Lucas numbers are closely related to the Fibonacci numbers. For Lucas numbers, the first two terms are 1 and 3 and each following term is found by adding the two previous terms, as for Fibonacci numbers.
a Write down the first ten Lucas numbers.
b Show that $F_1 + F_3 = L_2$, $F_2 + F_4 = L_3$, $F_3 + F_5 = L_4$.
c How could you use the Fibonacci numbers to find L_7, the 7th Lucas number?

2.4 Divisibility tests

The following tests can be used to determine whether one number is divisible by another without performing the division.

A number is divisible by:

- **2 if it is even, i.e. if the units digit is 2, 4, 6, 8 or 0**
- **3 if the sum of the digits is divisible by 3**
- **4 if the number formed by the last two digits is divisible by 4**
- **5 if the last digit is either 5 or 0**
- **6 if the number is divisible by both 2 and 3**
- **8 if the number formed by the last three digits is divisible by 8**
- **9 if the sum of the digits is divisible by 9**
- **10 if the units digit is 0**
- **11 if the sum of the digits in the odd positions is equal to the sum of the digits in the even positions, or they differ by a number that is divisible by 11.**

NOTE: A divisibility 'test' for 7 is given in the Further Applications section of Exercise 2.4.

Example

EG+S

Use the divisibility tests to show that:

a 178 is divisible by 2 **b** 768 is divisible by 3 **c** 1524 is divisible by 4
d 635 is divisible by 5 **e** 978 is divisible by 6 **f** 5848 is divisible by 8
g 9576 is divisible by 9 **h** 4320 is divisible by 10 **i** 259 864 is divisible by 11

Solutions

a The number ends in an 8, which is even. Therefore, 178 is divisible by 2.

b The sum of the digits is $7 + 6 + 8 = 21$, which is divisible by 3. Therefore, 768 is divisible by 3.

c The number formed by the last two digits is 24, which is divisible by 4. Therefore, 1524 is divisible by 4.

d The number ends in a 5. Therefore, 635 is divisible by 5.

e The number ends in an 8, so 978 is divisible by 2. The sum of the digits is $9 + 7 + 8 = 24$, so 978 is also divisible by 3. Therefore, 978 is divisible by 6.

f The number formed by the last three digits is 848, which is divisible by 8. Therefore, 5848 is divisible by 8.

g The sum of the digits is $9 + 5 + 7 + 6 = 27$, which is divisible by 9. Therefore, 9576 is divisible by 9.

h The number ends in a zero. Therefore, 4320 is divisible by 10.

i The sum of the digits in the odd positions (i.e. 1st, 3rd, 5th) is $2 + 9 + 6 = 17$. The sum of the digits in the even positions (i.e. 2nd, 4th, 6th) is $5 + 8 + 4 = 17$. The sum of the digits in the even positions is equal to the sum of the digits in the odd positions. Therefore, 259 864 is divisible by 11.

Exercise 2.4

1 Look at the last digit and hence determine which numbers are divisible by 2.
176 297 548 685 900 1374 1469 9999

2 Find the sum of the digits and hence determine which numbers are divisible by 3.
252 284 442 486 2899 9651 20 895 26 237

3 Look at the number formed by the last two digits and hence determine which numbers are divisible by 4.
122 132 328 809 1520 2741 4817 5636

4 Look at the last digit and hence determine which numbers are divisible by 5.
53 95 170 554 862 965 2038 3900

5 Look at the last digit and find the sum of the digits. Hence determine which numbers are divisible by 6.
138 146 234 461 470 693 762 5496

6 Look at the number formed by the last three digits and hence determine which numbers are divisible by 8.
1168 1231 1248 2086 7328 3142 9488 16 583

7 Find the sum of the digits and hence determine which numbers are divisible by 9.
123 135 468 516 3250 5823 9297 14 258

8 Look at the last digit and hence determine which numbers are divisible by 10.
141 220 518 690 744 1001 1500 20 360

9 Look at the sum of the digits in the odd positions and the sum of the digits in the even positions. Hence determine which numbers are divisible by 11.
176 2188 6358 12 346 71 258 82 431 93 523 472 868

■ Consolidation

10 Use the divisibility tests to find a 5-digit number that is divisible by:

a 3 **b** 4 **c** 5 **d** 6
e 8 **f** 9 **g** 10 **h** 11

11 Although not always practical to use, one way to test a number for divisibility by 7 is to multiply by 3 the number formed by all digits other than the units digit, then add the units digit. If the resulting number is divisible by 7, then so is the original number. For example, to test **12**6, we evaluate $(\mathbf{12} \times 3) + 6$. Now, $12 \times 3 + 6 = 42$, which is divisible by 7. Therefore, 126 is also divisible by 7.

Use this method to determine which of the following numbers are divisible by 7.
91 97 98 114 119 125 145 154

12 Use the divisibility tests to find all the numbers from 2 to 11 that will divide into 924.

13 To test a number for divisibility by 6, we test for divisibility by both 2 and 3. How could we test a number for divisibility by:

a 12? **b** 14? **c** 15? **d** 45?

14 **a** Is 795 divisible by 15? **b** Is 518 divisible by 12? **c** Is 126 divisible by 14?

■ Further applications

15 The divisibility 'test' for 7 can be extended as follows. Multiply the left-hand digit by 3 then add the next digit to the right. Rewrite all unused digits at the end. Repeat this procedure as often as necessary until you can clearly determine whether the number is divisible by 7. For example, to determine whether 1526 is divisible by 7 we would write:

$(1 \times 3) + 5 = 8$, so we write 826 (since the last two digits 2 and 6 were not used)
$(8 \times 3) + 2 = 26$, so we write 266 (since the last digit 6 was not used)
$(2 \times 3) + 6 = 12$, so we write 126 (since the last digit 6 was not used)
$(1 \times 3) + 2 = 5$, so we write 56.

Now, 56 is divisible by 7, therefore 1526 is also divisible by 7.

Use the method above to determine which of the following numbers are divisible by 7.

a 196 **b** 1493 **c** 6545 **d** 2012

16 Consider the following statement. 'If two numbers are each divisible by another number, then their sum is also divisible by that number.' Is the statement true or false? Give examples.

2.5 Multiples

The **multiples** of a number are the numbers into which it can divide without remainder. For small numbers, we may think of multiples as the answers to multiplication tables. For example, 12 is a multiple of 3 because $12 \div 3 = 4$, i.e. 3 divides into 12 without remainder.

The first multiple of any number is the number itself, so multiples are all greater than or equal to the given number.

The multiples of a number are the numbers into which it can divide without remainder.

The **lowest common multiple (LCM)** of two or more numbers is the smallest number into which they all divide. For example, the lowest common multiple of 3 and 5 is 15.

To find the lowest common multiple (LCM) of two or more numbers:
- **list the set of multiples of each number**
- **find the smallest number that is common to both sets.**

Example 1

Write the first five multiples of 4.

Solution

$1 \times 4 = 4 \quad 2 \times 4 = 8 \quad 3 \times 4 = 12 \quad 4 \times 4 = 16 \quad 5 \times 4 = 20$

Therefore the first five multiples of 4 are {4, 8, 12, 16, 20}.

Example 2

Find the:

a 9th multiple of 7
b 8th multiple of 5

Solutions

a $9 \times 7 = 63$, so the 9th multiple of 7 is 63.
b $8 \times 5 = 40$, so the 8th multiple of 5 is 40.

Example 3

Find the LCM of 6 and 8.

Solution

The multiples of 6 are: {6, 12, 18, $\boxed{24}$, 30, …}
The multiples of 8 are: {8, 16, $\boxed{24}$, 32, 40, …}
The smallest number common to both sets is 24, so the LCM of 6 and 8 is 24. That is, 24 is the smallest number into which both 6 and 8 will divide.

Exercise 2.5

1 List the first five multiples of:

a 2 **b** 3 **c** 5 **d** 6 **e** 8 **f** 11
g 1 **h** 4 **i** 7 **j** 9 **k** 10 **l** 12

2 Write down the:

a 6th multiple of 4 **b** 4th multiple of 9 **c** 8th multiple of 7
d 9th multiple of 5 **e** 7th multiple of 6 **f** 4th multiple of 8
g 10th multiple of 9 **h** 11th multiple of 12 **i** 12th multiple of 6

Consolidation

3 Which multiple of:

a 3 is 18? **b** 4 is 20? **c** 8 is 24? **d** 5 is 40? **e** 7 is 63?
f 6 is 66? **g** 11 is 110? **h** 12 is 96? **i** 9 is 72? **j** 10 is 110?

4 Write true (T) or false (F) for each of these:

a 15 is a multiple of 3 **b** 3 is a multiple of 15 **c** 7 is a multiple of 7
d 48 is a multiple of 6 **e** 36 is a multiple of 8 **f** 38 is a multiple of 4
g 42 is a multiple of 8 **h** 84 is a multiple of 7 **i** 100 is a multiple of 3

5 Find all multiples of:

a 3 between 20 and 40 **b** 4 between 30 and 55 **c** 5 between 12 and 42
d 7 between 45 and 75 **e** 8 between 25 and 60 **f** 12 between 70 and 125

6 **a** Find the sum of the first three multiples of 6.
b Find the product of the first three multiples of 2.

7 **a** Find the difference between the 8th multiple of 9 and the 5th multiple of 7.
b Find the quotient of the 10th multiple of 6 and the 3rd multiple of 4.

CALCULATOR

8 Use the constant multiplier function on your calculator to find the first five multiples of:
a 13 **b** 15 **c** 24 **d** 29 **e** 35

9 **a** If 48 is a multiple of 4, find the next three multiples of 4.
b If 91 is a multiple of 7, find the next three multiples of 7.
c If 144 is a multiple of 6, find the previous three multiples of 6.

10 Write down any three numbers of which 72 is a multiple.

11 Find the LCM of these numbers.
a 2 and 3 **b** 2 and 5 **c** 3 and 4 **d** 4 and 6 **e** 6 and 8
f 9 and 6 **g** 6 and 10 **h** 8 and 12 **i** 10 and 8 **j** 15 and 10

12 Find two numbers other than those in Q11 whose LCM is:
a 21 **b** 24 **c** 36 **d** 60

CALCULATOR

13 Use a calculator to check that 600 is a common multiple of 25 and 40. Is it the LCM?

Further applications

14 Find the lowest common multiple of:
a 10, 15 and 20 **b** 8, 10 and 12 **c** 3, 15 and 25

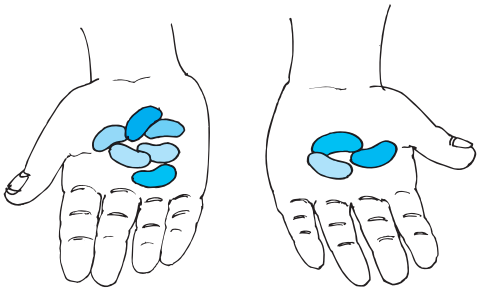
15 **a** Write down the first even multiple of 7.
b Find the first square number that is also a multiple of 20.
c Find the first multiple of 8 that is greater than 30.
d What is the first palindromic 2-digit multiple of 8?
e Find the first multiple of 9 that is also a triangular number.
f Write the largest 2-digit multiple of 6.
g Write down the smallest 3-digit multiple of 7.
h What is the first Fibonacci number that is also a multiple of 11?
i Which even multiple of 7 lies between 40 and 50?
j Find the first multiple of 6 that is also a triangular number.
k Find the smallest multiple of 9 that has a 4 in the tens place.
l What is the first multiple of 7 that is also a Fibonacci number?

16 In how many 2-digit numbers is the sum of the digits a multiple of 7?

17 Find the smallest multiple of 9 that has no odd digits.

TRY THIS **A trick**

Ask a friend to hold an even number of things in one hand and an odd number in the other hand. Tell your friend that if he or she does as you direct, you will be able to tell which is which!



Directions to your friend

1. Multiply the left-hand number by an even number, and the right hand number by an odd number. (Keep the results quiet.)
2. Add these numbers together, and give the result as odd or even.
3. If the result is odd, then the left-hand side is even and the right is odd. But if the result is even, the left is odd and the right is even!

Challenge

Work out how this trick works. Try it on another friend.

2.6 Factors

The **factors** of a number are the numbers that divide into it without remainder. For example, 4 is a factor of 12 because $12 \div 4 = 3$ or $3 \times 4 = 12$, i.e. 4 divides into 12 without remainder. If 4 is a *factor* of 12, then 12 is a *multiple* of 4.

The largest factor of any number is the number itself, so factors are all less than or equal to the given number. Every number has itself and 1 among its factors. If a number is even, then its second largest factor is half of the number.

The factors of a number are the numbers that divide into it without remainder.

The **highest common factor (HCF)** of two or more numbers is the largest number that divides into all of them. For example, the highest common factor of 12 and 15 is 3.

To find the highest common factor (HCF) of two or more numbers:
- **list the set of factors of each number**
- **find the largest number that is common to both sets.**

Example 1

List all of the factors of 30.

Solution

$1 \times 30 = 30$ $\quad 2 \times 15 = 30$ $\quad 3 \times 10 = 30$ $\quad 5 \times 6 = 30$

Therefore, the factors of 30 are {1, 2, 3, 5, 6, 10, 15, 30}.

Example 2

Find the second largest factor of 168.

Solution

168 is even and $\frac{1}{2}$ of 168 is 84. Therefore, the second largest factor of 168 is 84.

Example 3

Find the HCF of 12 and 20.

Solution

The factors of 12 are: {1, 2, 3, $\boxed{4}$, 6, 12}
The factors of 20 are: {1, 2, $\boxed{4}$, 5, 10, 20}

The largest number common to both sets is 4, so the HCF of 12 and 20 is 4. That is, 4 is the largest number that will divide into both 12 and 20.

Exercise 2.6

1 List all factors of each of the following numbers.

a 5	**b** 8	**c** 10	**d** 12	**e** 13	**f** 14
g 15	**h** 18	**i** 20	**j** 21	**k** 22	**l** 24
m 27	**n** 28	**o** 29	**p** 30	**q** 32	**r** 33

2 List all factors of each of these. The number of factors is shown in brackets.

a 35 [4]	**b** 36 [9]	**c** 40 [8]	**d** 42 [8]
e 44 [6]	**f** 45 [6]	**g** 48 [10]	**h** 50 [6]
i 54 [8]	**j** 56 [8]	**k** 60 [12]	**l** 63 [6]
m 66 [8]	**n** 70 [8]	**o** 72 [12]	**p** 75 [6]

Consolidation

3 Write true (T) or false (F) for each of these.

a 9 is a factor of 18	**b** 18 is a factor of 9	**c** 8 is a factor of 12
d 7 is a factor of 49	**e** 21 is a factor of 3	**f** 14 is a factor of 35
g 5 is a factor of 85	**h** 16 is a factor of 4	**i** 9 is a factor of 29

4 Complete these statements by inserting the word 'factor' or 'multiple'.

a 3 is a _____ of 12 **b** 24 is a _____ of 4
c 30 is a _____ of 6 **d** 11 is a _____ of 22

5 **a** Find all factors of 4, 9 and 16.
b Do square numbers have an odd or an even number of factors? Why?

6 Find the second largest factor of each of these.

a 6 **b** 26 **c** 68 **d** 142

7 Which number is a factor of every number?

8 **a** If a number is divisible by 12, then it must be divisible by what other numbers?
b If a number is divisible by 20, then it must be divisible by what other numbers?

9 List all of the factors that are common to:
a 16 and 40 **b** 40 and 60 **c** 28 and 42

10 Which of these numbers are factors of 100? 5, 15, 50, 20, 75, 25, 10, 100, 40, 60, 4, 3

11 Find the HCF of these numbers.
a 8 and 12 **b** 10 and 15 **c** 12 and 18
d 14 and 35 **e** 16 and 24 **f** 15 and 21
g 18 and 45 **h** 24 and 36 **i** 30 and 42

12 Find a pair of 2-digit numbers other than those in Q11 whose HCF is:
a 4 **b** 6 **c** 7 **d** 13

CALCULATOR

13 Use a calculator to check that 17 is a common factor of 85 and 153.

Further applications

14 Find the highest common factor of:
a 24, 28 and 48 **b** 42, 54 and 72

15 Find all factors of each number. The number of factors is shown in brackets.
a 80 [10] **b** 84 [12] **c** 92 [6] **d** 96 [12]

CALCULATOR

16 Find all 24 factors of 420.

17 **a** Which factor of 12 other than 1 is also a square number?
b Which factor of 20 other than 1 is also a triangular number?
c Which factor of 6 is also a multiple of 6?
d Write down the first 2-digit square number that is also a factor of 32.
e State the smallest Fibonacci number that has 17 as one of its factors.
f Find the smallest triangular number that has a square number other than 1 as a factor.
g Find the largest 2-digit number with 8 as one of its factors.
h Write the smallest Fibonacci number with factors of 3 and 7.
i Find the even number whose second largest factor is 19.
j State the largest factor of 16 that is also a factor of 72.
k Find the smallest 3-digit number with a factor of 3.
l Write down the largest odd factor of 36.

2.7 Prime and composite numbers

All counting numbers other than 1 are either prime or composite. A **prime** number has only two factors—one and itself. A **composite** number has more than two factors. For example, 7 is prime because its only factors are 1 and 7 whereas 9 is composite because it has three factors: 1, 3 and 9. Since 1 has only one factor it does not meet either definition. Hence, 1 is neither prime nor composite. The primes are all odd except for 2, which is even. Prime numbers that

differ by 2 are called **twin primes**. For example, 5 and 7 are twin primes because they are both prime and $7 - 5 = 2$.

- **A prime number has only two factors: itself and 1.**
- **A composite number has more than two factors.**

To test whether a number is prime we need only check for divisibility by those primes that are less than the square root of the number. (Square roots will be explained in detail later in this chapter if you are not familiar with them.) Therefore, to test any number that is less than 100, we note that $\sqrt{100} = 10$. The only primes less than 10 are 2, 3, 5 and 7. So we need only test our number for divisibility by 2, 3, 5 or 7.

To test whether a number is prime or composite:
- **test for divisibility by all primes that are less than or equal to the square root of the number**
- **if the number is not divisible by any of these primes, then the number itself is prime.**

Example

Determine whether each of these numbers is prime or composite.

a 15 **b** 17 **c** 97

Solutions

a $15 = 1 \times 15$ and $15 = 3 \times 5$. Therefore the factors of 15 are {1, 3, 5, 15}. Since 15 has more than two factors, it is composite.

b $17 = 1 \times 17$ only. Therefore the factors of 17 are {1, 17}. Since 17 has only two factors, it is prime.

c We use the divisibility tests to test for divisibility by 2, 3, 5 and 7 (since 97 is less than 100).
- 97 ends in a 7 which is odd, so 97 is not divisible by 2.
- $9 + 7 = 16$, which is not divisible by 3, so 97 is not divisible by 3.
- 97 does not end in a 5 or 0, so 97 is not divisible by 5.
- $(3 \times 9) + 7 = 34$, which is not divisible by 7, so 97 is not divisible by 7 (otherwise you will need to do a short division).

Now, 97 is less than 100 and it is not divisible by either 2, 3, 5 or 7. Therefore, 97 is prime.

Exercise 2.7

1 State whether these numbers are prime (P) or composite (C).

a 2	**b** 5	**c** 6	**d** 7	**e** 9	**f** 13
g 15	**h** 17	**i** 18	**j** 21	**k** 23	**l** 25
m 27	**n** 29	**o** 33	**p** 37	**q** 39	**r** 41
s 43	**t** 45	**u** 47	**v** 49	**w** 51	**x** 53

2 Test the following numbers for divisibility by 2, 3, 5 and 7 to determine whether they are prime (P) or composite (C).

a 57 **b** 59 **c** 61 **d** 63 **e** 67 **f** 69
g 73 **h** 79 **i** 81 **j** 85 **k** 87 **l** 91

Consolidation

3 List the numbers from 1 to 100 in straight rows with six numbers in each row. Circle the number 2, then cross out any multiples of 2. Repeat this procedure for 3, 5 and 7. Cross out the 1. Circle all of the remaining numbers. What do you notice about the numbers that you have circled? This is called the Sieve of Eratosthenes.

4 List all pairs of twin primes.

5 List all pairs of primes that differ by 10.

6 Can a square number be prime?

7 Which counting number is neither prime nor composite? Why?

8 What is the only even prime?

9 In 1742, a famous German mathematician Christian Goldbach conjectured that every even number greater than 2 could be expressed as the sum of two primes. As yet, no-one has been able to prove or disprove this conjecture. Express each of these even numbers as the sum of two primes.

a 10 **b** 16 **c** 20 **d** 28
e 30 **f** 38 **g** 50 **h** 84

10 Goldbach also conjectured that every odd number greater than 5 could be expressed as the sum of three primes. Express each of these odd numbers as the sum of three primes.

a 7 **b** 13 **c** 19 **d** 31
e 47 **f** 55 **g** 69 **h** 99

11 Find all pairs of primes whose sum is 100.

12 Express 36 as the sum of four primes.

13 Write down the first prime that can be expressed as the sum of three different primes.

CALCULATOR

14 Use a calculator to determine whether these numbers are prime or composite. (You will need to test for divisibility by all primes up to 13.)

a 109 **b** 133 **c** 117 **d** 153 **e** 187

Further applications

15 **a** If a 2-digit number is a palindrome, can it be prime? Explain.
b With the exception of 11, explain why a number cannot be prime if it is a palindrome and has an even number of digits.

16 Find a three digit palindrome that is:
a prime **b** composite

17 **a** Find the largest 2-digit prime.
b Find the smallest 3-digit prime.
c Find the first prime with a 2 in the tens place.
d Find the largest 2-digit prime that has a 3 in the units place.
e Write down the two prime factors of 44.
f Write the smallest composite number.
g Find a prime that is also a triangular number.
h Find the largest prime that is less than the fourth square number.
i Find the first prime that is greater than the eleventh square number.
j Write down the first composite Fibonacci number.
k Find the first odd composite number.
l Find the smallest triangular number that is the product of a prime number and a square number greater than 1.

18 List all of the 2-digit primes that are still prime when the order of their digits is reversed.

2.8 Prime factors

Every composite number can be expressed as the product of **prime factors**. It is common practice to write any repeated factors in index form. We can find the prime factors of a number by drawing a factor tree. The idea is to express the given number as the product of two smaller numbers and then, if possible, express each of these numbers as the product of two smaller numbers and so on. The process is repeated until all of the remaining numbers are prime. The divisibility tests may be quite useful when working with large numbers.

Highest common factor and lowest common multiple by prime factors

The **highest common factor** and **lowest common multiple** of two or more numbers can be found by the use of their prime factors.

To find the HCF of two numbers by prime factors:
- **find the product of the prime factors that are common to both numbers.**

To find the LCM of two numbers by prime factors:
- **find the product of all of the prime factors of the smaller number and those factors of the larger number that are not contained in the smaller one.**

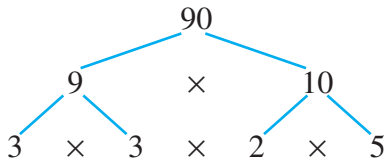
Example 1

Use a factor tree to express each of these numbers as the product of its prime factors.

a 90 **b** 84

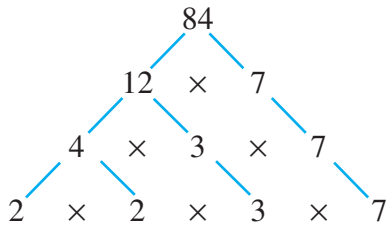
Solutions

a



Therefore, $90 = 2 \times 3^2 \times 5$

b



Therefore, $84 = 2^2 \times 3 \times 7$

Example 2

Find the HCF of 1050 and 7000 by prime factors.

Solution

$1050 = \boxed{2} \times 3 \times \boxed{5} \times \boxed{5} \times \boxed{7}$ and $7000 = \boxed{2} \times 2 \times 2 \times \boxed{5} \times \boxed{5} \times 5 \times \boxed{7}$

The HCF $= 2 \times 5 \times 5 \times 7$

$= 2 \times 5^2 \times 7$

$= 350$ (optional)

Example 3

Find the LCM of 324 and 600 by prime factors.

Solution

$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$ and $600 = 2 \times 2 \times \boxed{2} \times 3 \times \boxed{5} \times \boxed{5}$

The LCM $= [2 \times 2 \times 3 \times 3 \times 3 \times 3] \times [2 \times 5 \times 5]$

$= 2^3 \times 3^4 \times 5^2$

$= 16\,200$ (optional)

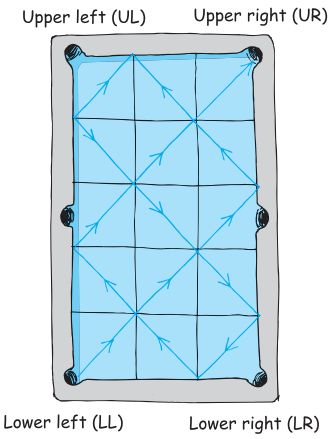
TRY THIS Mathematics of pool tables

Yes, pool players do need to know something about angles! Using angles they can predict the path of the ball.

In this investigation we will always 'hit' our ball from the lower left pocket at an angle of 45° to the side. We will also assume that the pockets are at the corners.

Look at this 3×5 table:

The ball finishes in the upper right pocket.



There are three questions here:

1 If you are given the size of the table (e.g. 3×5), can you predict which pocket the ball will finally enter?

2 Can you predict how many times the ball will touch the sides of the table before entering the pocket?

3 If we multiply both side lengths of the table by the same amount, will the ball then enter the same pocket and touch the sides the same number of times? i.e. Is a 3×5 table the same as a 6×10 table?

Hint

Start by looking at tables with prime number lengths and widths; e.g. 2, 3, 5, 7, 11, 13. Then investigate tables with side lengths that are composite numbers.

Exercise 2.8

1 Draw a factor tree to express each number as the product of its prime factors. Give your answers in index form.

a 20	**b** 24	**c** 28	**d** 30	**e** 36	**f** 40
g 42	**h** 45	**i** 48	**j** 50	**k** 54	**l** 56
m 60	**n** 63	**o** 72	**p** 75	**q** 80	**r** 100

■ Consolidation

2 Draw a factor tree to express each number as the product of its prime factors. Give your answers in index form. You may need to make use of the divisibility tests.

a 240	**b** 540	**c** 792	**d** 882
e 1134	**f** 1225	**g** 1755	**h** 1890

3 Use this list of prime factors to find the HCF of each pair of numbers.

- $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$
- $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
- $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
- $504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7$
- $810 = 2 \times 3 \times 3 \times 3 \times 3 \times 5$
- $900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$
- $990 = 2 \times 3 \times 3 \times 5 \times 11$
- $2058 = 2 \times 3 \times 7 \times 7 \times 7$
- $2450 = 2 \times 5 \times 5 \times 7 \times 7$
- $3267 = 3 \times 3 \times 3 \times 11 \times 11$

a 216 and 2058	**b** 810 and 2450	**c** 900 and 2450
d 504 and 2058	**e** 432 and 3267	**f** 432 and 990
g 2450 and 2058	**h** 990 and 3267	**i** 810 and 900
j 216 and 810	**k** 324 and 504	**l** 216 and 324

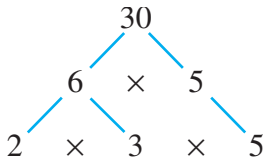
4 Use this list of prime factors to find the LCM of each pair of numbers.

- $24 = 2 \times 2 \times 2 \times 3$
- $36 = 2 \times 2 \times 3 \times 3$
- $54 = 2 \times 3 \times 3 \times 3$
- $56 = 2 \times 2 \times 2 \times 7$
- $200 = 2 \times 2 \times 2 \times 5 \times 5$
- $600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$
- $825 = 3 \times 5 \times 5 \times 11$
- $1188 = 2 \times 2 \times 3 \times 3 \times 3 \times 11$
- $1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7$
- $1400 = 2 \times 2 \times 2 \times 5 \times 5 \times 7$

a 24 and 36 **b** 36 and 54 **c** 24 and 56
d 54 and 56 **e** 36 and 1260 **f** 36 and 600
g 1188 and 1260 **h** 600 and 825 **i** 54 and 1260
j 200 and 1188 **k** 825 and 1188 **l** 1188 and 1400

Further applications

The factors of a number can be determined by finding all possible products of the prime factors one at a time, two at a time, three at a time etc. For example:



Now using the prime factors 2, 3 and 5:

- one at a time: 2, 3, 5
- two at a time: $(2 \times 3) = 6$, $(2 \times 5) = 10$, $(3 \times 5) = 15$
- three at a time: $(2 \times 3 \times 5) = 30$
- 1 is a factor of every number

$\therefore$ the factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

5 Find the prime factors of each number, then use the method outlined above to find all:

a 8 factors of 42 **b** 8 factors of 66
c 8 factors of 70 **d** 8 factors of 105
e 8 factors of 110 **f** 16 factors of 210
g 16 factors of 330 **h** 12 factors of 90

2.9 Square and cube roots

Finding the **square root** ($\sqrt{\ }$) is the opposite of squaring a number, in the same way that subtraction is the opposite of addition and division is the opposite of multiplication. That is, taking the square root 'undoes' the operation of squaring. Similarly, finding the **cube root** ($\sqrt[3]{\ }$) is the opposite of cubing a number.

Example 1

Evaluate each of the following.

a $\sqrt{25}$ **b** $\sqrt[3]{8}$

Solutions

a $\sqrt{25} = 5$
because $5^2 = 25$ (or $5 \times 5 = 25$).

b $\sqrt[3]{8} = 2$
because $2^3 = 8$ (or $2 \times 2 \times 2 = 8$).

Example 2

Evaluate each of the following.

a $\sqrt{441}$ if $441 = 3 \times 3 \times 7 \times 7$

b $\sqrt[3]{2744}$ if $2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$

Solutions

a To find the square root of a number using its prime factors, take one factor from each pair of like factors and multiply them together.

$\therefore \sqrt{441} = 3 \times 7$
$= 21$

b To find the cube root of a number using its prime factors, take one factor from each three like factors and multiply them together.

$\therefore \sqrt[3]{2744} = 2 \times 7$
$= 14$

Example 3

Find:

a $\sqrt{1600}$

b $\sqrt[3]{27\,000\,000}$

Solutions

a $\sqrt{1600} = \sqrt{16} \times \sqrt{100}$
$= 4 \times 10$
$= 40$

b $\sqrt[3]{27\,000\,000} = \sqrt[3]{27} \times \sqrt[3]{1\,000\,000}$
$= 3 \times 100$
$= 300$

Exercise 2.9

1 Find the value of each of these square roots.

a $\sqrt{9}$ **b** $\sqrt{25}$ **c** $\sqrt{4}$ **d** $\sqrt{49}$ **e** $\sqrt{100}$ **f** $\sqrt{1}$
g $\sqrt{16}$ **h** $\sqrt{81}$ **i** $\sqrt{121}$ **j** $\sqrt{36}$ **k** $\sqrt{64}$ **l** $\sqrt{144}$

2 Find the value of each of these cube roots.

a $\sqrt[3]{8}$ **b** $\sqrt[3]{27}$ **c** $\sqrt[3]{1}$ **d** $\sqrt[3]{1000}$ **e** $\sqrt[3]{125}$ **f** $\sqrt[3]{64}$

Consolidation

3 **a** If $13^2 = 169$, find $\sqrt{169}$ **b** If $14^2 = 196$, find $\sqrt{196}$
c If $\sqrt{1225} = 35$, find 35^2 **d** If $\sqrt{1849} = 43$, find 43^2

4 **a** If $7^3 = 343$, find $\sqrt[3]{343}$ **b** If $9^3 = 729$, find $\sqrt[3]{729}$
c If $\sqrt[3]{216} = 6$, find 6^3 **d** If $\sqrt[3]{1331} = 11$, find 11^3

5 **a** Find $\sqrt{4^3}$ **b** Find $\sqrt[3]{8^2}$

6 Does $\sqrt{16} + \sqrt{9} = \sqrt{16 + 9}$?

7 Evaluate each of these.

a $\sqrt{12 + 13}$ **b** $\sqrt[3]{2 + 5^2}$ **c** $\sqrt{5 \times 20}$
d $\sqrt[3]{40 \div 5}$ **e** $\sqrt[3]{10^2 + 5^2}$ **f** $\sqrt{2^2 \times 3^2}$

8 Evaluate each of the following.

a $\sqrt{25} + \sqrt{49}$ **b** $\sqrt{100} - \sqrt{64}$ **c** $\sqrt{121} \times \sqrt{9}$

d $\sqrt{144} \div \sqrt{16}$ **e** $22 - \sqrt{36}$ **f** $13 + \sqrt{4}$

g $\sqrt[3]{8} + \sqrt{100}$ **h** $\sqrt[3]{27} + \sqrt{25}$ **i** $\sqrt{81} - \sqrt[3]{125}$

j $\sqrt[3]{1000} \div \sqrt[3]{125}$ **k** $\sqrt{64} - \sqrt[3]{64}$ **l** $\sqrt{49} + \sqrt[3]{27}$

m $9^2 + \sqrt{9}$ **n** $8^2 - \sqrt[3]{125}$ **o** $\sqrt{121} \times 2^3$

p $2^5 - \sqrt{144}$ **q** $\sqrt[3]{64} \times \sqrt{36} \times \sqrt{9}$ **r** $\sqrt[3]{8} + \sqrt{121} - \sqrt[3]{27}$

9 Use the order of operations to evaluate:

a $5 + \sqrt{9} \times 4$ **b** $\sqrt[3]{8} + 6 \times \sqrt{16}$ **c** $\sqrt{144} \div 6 + 2^3$

d $\sqrt{100} - \sqrt[3]{125} \times 2$ **e** $\sqrt{49} \times \sqrt[3]{1000} \div \sqrt{25}$ **f** $\sqrt{4} \times \sqrt{9} + \sqrt[3]{8} \times \sqrt[3]{125}$

CALCULATOR

10 Estimate the value of each square root, then check your answers using a calculator.

a $\sqrt{169}$ **b** $\sqrt{225}$ **c** $\sqrt{484}$ **d** $\sqrt{625}$ **e** $\sqrt{961}$

CALCULATOR

11 Estimate the value of each cube root, then check your answers using a calculator.

a $\sqrt[3]{216}$ **b** $\sqrt[3]{343}$ **c** $\sqrt[3]{729}$ **d** $\sqrt[3]{512}$ **e** $\sqrt[3]{1331}$

12 State whether or not it would be appropriate to use a calculator to evaluate each of these square roots. If it is appropriate, say why.

a $\sqrt{1369}$ **b** $\sqrt{7}$ **c** $\sqrt{81}$

■ Further applications

13 **a** Evaluate:

i $\sqrt{9 \times 4}$ and $\sqrt{9} \times \sqrt{4}$ **ii** $\sqrt{4 \times 25}$ and $\sqrt{4} \times \sqrt{25}$

iii $\sqrt{16 \times 9}$ and $\sqrt{16} \times \sqrt{9}$ **iv** $\sqrt{4 \times 16}$ and $\sqrt{4} \times \sqrt{16}$

b Copy and complete this statement.
The _____ of a product is equal to the _____ of the square roots.

c Use this method to evaluate:

i $\sqrt{20} \times \sqrt{5}$ **ii** $\sqrt{18} \times \sqrt{2}$ **iii** $\sqrt{24} \times \sqrt{6}$

d Show that $25 \times 49 = 1225$. Hence, find $\sqrt{1225}$.

14 Find:

a $\sqrt{196}$ if $196 = 2 \times 2 \times 7 \times 7$ **b** $\sqrt{225}$ if $225 = 3 \times 3 \times 5 \times 5$

c $\sqrt{1764}$ if $1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$ **d** $\sqrt{324}$ if $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

15 Find:

a $\sqrt{400}$ if $400 = 2^4 \times 5^2$ **b** $\sqrt{576}$ if $576 = 2^6 \times 3^2$

c $\sqrt{1296}$ if $1296 = 2^4 \times 3^4$ **d** $\sqrt{7056}$ if $7056 = 2^4 \times 3^2 \times 7^2$

16 Find:

a $\sqrt[3]{216}$ if $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$ **b** $\sqrt[3]{1000}$ if $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

c $\sqrt[3]{1728}$ if $1728 = 2^6 \times 3^3$ **d** $\sqrt[3]{91\,125}$ if $91\,125 = 3^6 \times 5^3$

17 Evaluate each of these.

a $\sqrt{400}$ **b** $\sqrt{900}$ **c** $\sqrt{490\,000}$

d $\sqrt{250\,000}$ **e** $\sqrt{16\,000\,000}$ **f** $\sqrt{121\,000\,000}$

18 Evaluate each of these.

a $\sqrt[3]{8000}$ **b** $\sqrt[3]{27\,000}$ **c** $\sqrt[3]{125\,000}$

d $\sqrt[3]{1\,000\,000}$ **e** $\sqrt{8\,000\,000}$ **f** $\sqrt[3]{64\,000\,000\,000}$

Problem solving

1 If two different chocolate bars together cost $1.50 and one of them cost 60c more than the other, how much does each cost?

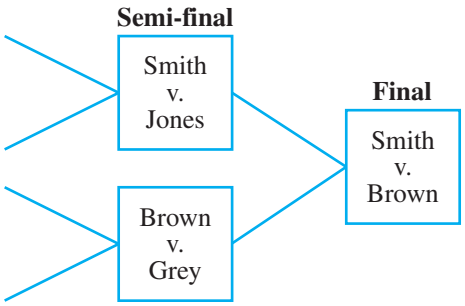
2 Calculate 1 + 3 + 5 + 7 + 9 … + 97 + 99 (i.e. the sum of all the odd numbers less than 100).

3 What would you rather have, a bucket of 50c coins or a half bucket of $1 coins?

4 A man purchased 5 tyres for his car. He knows that he will travel 25 000 km. If he wishes to use the tyres equally, how many kilometres will each tyre travel?

5 In a tennis tournament there are 32 players. How many matches must be played before we have the winner, if all losers drop out and the winners play each other in the next round until only one remains?

Draw a diagram and work backwards until you reach 32 players!



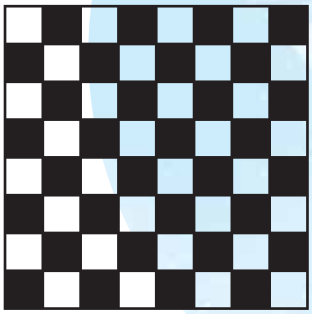
6 A clock loses 10 minutes every hour. If it is 12 midnight on Sunday, will the clock ever show the correct time again? If so, when?

7 In three years' time, Charlotte will be twice as old as Nicholas is now. If Charlotte is 3 years old now, how old is Nicholas now?

8 There are 3 sacks of potatoes. The first and second sacks together weigh 35 kg; the second and third sacks together weigh 45 kg, and the third and first together weigh 40 kg. How much does each sack weigh?

9 At a meeting, everyone shook hands with everyone else. If there were 55 handshakes, how many people were at the meeting?

10 A bishop on a chessboard can only move diagonally. If the piece starts on a black square, it can only visit other black squares. It is not possible to visit all the black squares on the board without retracing your path. How many black squares can you visit without retracing your steps?



Focus on working mathematically

Our beautiful Earth

Introduction

> 'God saw all that he had made, and it was very good. And there was evening, and there was morning—the sixth day. Thus the heavens and the earth were completed in all their vast array.' *Genesis* 1:31

Down through the ages, people of different faiths and cultures have seen the hand of a Creator behind the magnificent world in which we live, and beyond to the stars. The book of *Genesis* in the Bible, quoted above, is one such description. What is so interesting is that in this account God completes his work in exactly six days. However our purpose here is not to debate if it is true, but to draw attention to the number 6 itself. From ancient times people have believed that this number was chosen by God because it was perfect. Saint Augustine (AD 354–430) wrote that God created all things in six days because the number itself is perfect. In the history of number theory, '*perfect numbers*' as they came to be known have held a fascination for Egyptian, Greek, Arab and European mathematicians.

Consider the number 6. It can be written as $6 = 1 + 2 + 3$. You can see that 1, 2 and 3 are the set of divisors of 6. If we exclude 6 itself, 1, 2 and 3 are called 'proper' divisors of 6. The number 6 is called a *perfect* number.

Perfect numbers were very special in ancient times. Pythagoras and his followers (500 BC) thought perfect numbers to have religious and mystical properties.

But the first recorded mathematical result concerning perfect numbers was written by Euclid in his book the *Elements* around 300 BC. Nicomachus of Gerasa (who died about AD 120) suggested, but did not try to prove, that all perfect numbers end in 6 and 8 alternately, and that there are infinitely many perfect numbers.

Perfect numbers are very rare and quickly get very large. Today, only 37 have been discovered. In 1998 the 37th perfect number was discovered using a computer and found to have 1 819 050 digits!

LEARNING ACTIVITIES

Materials needed: Calculator, pencil and paper

1. As you have seen the first perfect number is 6. Let's find the next one. Try various numbers up to 20 checking with your calculator if you wish. Now try up to 30. What is the next perfect number?
2. After this the search becomes time consuming. The next perfect number is 496. Use your calculator to get the complete set of divisors, excluding 496, and add them up. You should get 496.
3. Notice how the gap between the perfect numbers is increasing. The next perfect number is between 8126 and 8136. See if you can find it.
4. In your work you will have found the first four perfect numbers. Write them down in order of size.

EXTENSION ACTIVITY

In 1536 a mathematician called Hudalrichus Regius found the 5th perfect number 33 550 336. Notice that it ends in a 6. Nicomachus' claim that perfect numbers end alternately in 6 and 8 seemed to be true: 6, 28, 496, 8128, 33 550 336, … However as so often happens in mathematics, the sixth perfect number, found by Cataldi in 1603, turned out to be 8 589 869 056 which also ends in a 6 and not 8 as we might have hoped.

As an extra activity you may like to learn how to generate perfect numbers from the series 1, 2, 4, 8, 16, 32, … Look carefully at the pattern in the table below:

You can continue the table by adding the series until you reach the next prime after 31, and multiplying by the last number added in the series. See if you can get 8128.

Sum	Total (A prime number)	Prime × last number added
$1 + 2$	3	$3 \times 2 = 6$
$1 + 2 + 4$	7	$7 \times 4 = 28$
$1 + 2 + 4 + 8 + 16$	31	$31 \times 16 = 496$

Let's communicate

Write a summary of what you have learned about perfect numbers. Don't forget to mention that they are very rare, get very large, and that there have been only 37 discovered so far. We do not know yet if there are infinitely many perfect numbers. Does it surprise you that there are unsolved problems in mathematics? Discuss with your teacher.

Reflecting

Through the ages, a great deal of work went into finding perfect numbers. Although today computers have made a great difference, no-one still knows if the number of perfect numbers is infinite. Nor do we know that *all* perfect numbers are even.

How does this knowledge affect your understanding of what mathematics is?

Finally, did God create the universe in 6 days? In his recent book *Just Six Numbers* (Weidenfeld & Nicolson, London, 1999), Martin Rees demonstrates how it is that just six numbers, imprinted in the 'big bang', determine the essential features of the cosmos. If any one of them varied just a little, there could be no stars and no life. Interesting!

Language link with Macquarie

MACQUARIE

1 *Explain the difference between a* ***multiple*** *and a* ***factor****.*
2 *When a number has been multiplied by itself what do we call it?*
3 *Explain what the expression 2^3 means in your own words.*
4 *Define* ***prime*** *number for a new maths dictionary.*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***number****:*

number *noun* **1.** a sign or symbol used to represent a place in a counting system: *7 is the number which comes after 6 and before 8.* | $\frac{1}{2}$, $\frac{1}{4}$ *and* $\frac{1}{3}$ *are all numbers smaller than 1.* **2.** the sum or total of a collection of things: *What number are you expecting to come?* | *This year's audience was only half the number of last year's.* **3.** a collection or quantity, usually large: *A number of people came.* | *She has a number of good ideas.* **4.** the particular numeral or figure given to something to fix its place in a list or series or to identify it: *Our house number is 67.* | *What is your phone number?* **5.** a telephone number: *I asked her for her number, but she wouldn't give it to me.* | *I could ring my father and ask him, but I don't know his new number.* **6.** a song, especially on a concert program: *Hans will sing the next number.* **7.** *Specialised* in grammar, the number of persons or objects a noun, pronoun or verb refers to.
–*verb* **8.** to mark with a number: *We numbered all the files.* **9.** to amount to a certain number: *The crowd at the cricket match numbered 80 000.*
–*phrase* **10. do a number**, to perform a particular piece, or routine: *Go on, get up on stage and do a number!*

Which jobs need a good understanding of ***numbers*** *in our society?*

CHAPTER REVIEW

1 State whether each number is odd or even.
a 6023 **b** 51 346 **c** 39 470

2 Write down the:
a largest 2-digit even number
b smallest 3-digit odd number
c largest 3-digit odd number with a 7 in the tens place
d smallest 4-digit even number with a 6 in the hundreds place

3 Which of these numbers are palindromes?
24 124 13 231 46 446 7007

4 What is the only digit that cannot be found in the units place of a palindromic number?

5 **a** List the first 10 square numbers.
b Find the 17th square number.

6 **a** Write down the first square number that is greater than 70.
b Give the first two square numbers with a 6 in the units place.
c Which two square numbers have a sum of 52?

7 **a** List the first 10 triangular numbers.
b Find the 14th triangular number.

8 **a** Write down the first triangular number that is greater than 90.
b State the first two triangular numbers that contain the digit 2.
c Find the two triangular numbers whose product is 63.

9 **a** List the first ten Fibonacci numbers.
b Write the number that is 8 less than the 8th Fibonacci number.
c By how much does the sum of the first seven Fibonacci numbers exceed the sum of the first four?

10 Which number in each pair is divisible by the number in the brackets?
a 341 or 670 [2]
b 842 or 4176 [3]
c 124 or 114 [4]
d 465 or 554 [5]
e 243 or 138 [6]
f 281 or 832 [8]
g 809 or 765 [9]
h 105 or 4160 [10]
i 1452 or 1546 [11]

11 How could you test a number for divisibility by 18?

12 Write the first 6 multiples of:
a 4 **b** 6 **c** 7 **d** 9

13 **a** Find the 8th multiple of 5.
b Which multiple of 6 is 42?

14 List all multiples of 7 between 30 and 65.

15 If 1308 is a multiple of 4, find the next 3 multiples of 4.

16 Find the LCM of:
a 4 and 5 **b** 6 and 4 **d** 9 and 12

17 Find all factors of each number.
a 16 **b** 30 **c** 42 **d** 72

18 What is the second largest factor of 418?

19 If a number is divisible by 20, then what other numbers must it also be divisible by?

20 Find the HCF of:
a 15 and 20 **b** 21 and 28
c 32 and 40

21 State whether these numbers are prime or composite.
a 2 **b** 9 **c** 15 **d** 19
e 21 **f** 23 **g** 39 **h** 47

22 Use the divisibility tests to determine whether 87 is prime or composite.

23 Express each even number as the sum of two primes.
a 18 **b** 24 **c** 36 **d** 58

24 **a** What are twin primes?
b List the first three pairs of twin primes.

25 **a** Find the first 3-digit prime with a 6 in the tens place.
b Write down the first composite Fibonacci number.
c What is the next odd composite number after 9?

26 Draw a factor tree and use it to express each number as the product of its prime factors. Give your answers in index form.
a 72 **b** 120 **c** 675 **d** 1936

27 Use the given prime factors to find the HCF of each pair of numbers.
a $441 = 3 \times 3 \times 7 \times 7$ and
$1134 = 2 \times 3 \times 3 \times 3 \times 3 \times 7$
b $3575 = 5 \times 5 \times 11 \times 13$ and
$33\,275 = 5 \times 5 \times 11 \times 11 \times 11$

28 Use the given prime factors to find the LCM of each pair of numbers.
a $18 = 2 \times 3 \times 3$ and $42 = 2 \times 3 \times 7$
b $392 = 2 \times 2 \times 2 \times 7 \times 7$ and
$700 = 2 \times 2 \times 5 \times 5 \times 7$

29 Evaluate:
a $\sqrt{16}$ **b** $\sqrt{25}$ **c** $\sqrt{81}$
d $\sqrt{144}$ **e** $\sqrt[3]{8}$ **f** $\sqrt[3]{27}$
g $\sqrt[3]{125}$ **h** $\sqrt[3]{1000}$

30 Evaluate:
a $\sqrt{36} \times \sqrt{49}$ **b** $\sqrt{64} + \sqrt[3]{64}$
c $\sqrt[3]{2^3 + 19}$ **d** $\sqrt{27^2}$

31 Find:
a $\sqrt{484}$ if $484 = 2 \times 2 \times 11 \times 11$
b $\sqrt{2704}$
if $2704 = 2 \times 2 \times 2 \times 2 \times 13 \times 13$
c $\sqrt[3]{35\,937}$ if $35\,937 = 3 \times 3 \times 3 \times 11 \times 11 \times 11$
d $\sqrt[3]{42\,875}$
if $42\,875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$

32 **a** Show that $36 \times 16 = 576$.
b Hence, find $\sqrt{576}$.

3 Time

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- convert between common units of time
- convert between analog and digital time
- convert between 12-hour and 24-hour time
- solve problems involving the various time zones in Australia
- convert between units of time involving the calendar
- solve problems that involve the calendar.

Stage 4

After completing this chapter, you should be able to:

- add and subtract time mentally
- add and subtract time using the degrees, minutes and seconds key on a calculator
- interpret calculator displays for time calculations
- calculate time differences between major cities
- interpret and use timetables.

3.1 Clocks, watches and time conversions

Units of time

The basic unit of time is the day—the time taken for the Earth to spin on its axis through one **revolution**. This is the time it takes from the sun appearing directly overhead on one day until it reaches the same position on the next day.

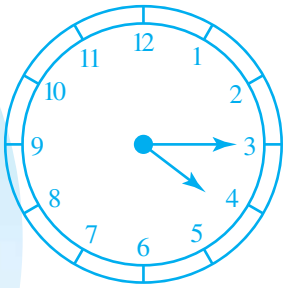
Once we have this unit, we can set smaller and larger units to allow us to measure short or long periods of time. The standard units are **day**, **hour**, **minute**, **second**, **week**, **fortnight**.

1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds
1 week = 7 days
1 fortnight = 2 weeks

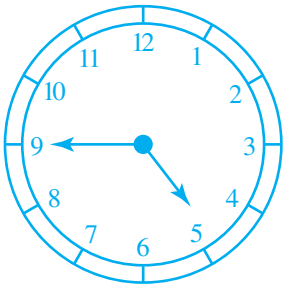
Analog and digital time

Analog time is the system of naming times which relates to clocks and watches of the traditional type with a circular face and pointers (called hands). The terminology developed over centuries may now seem old-fashioned, but it is still very widely used. Some terms are:

noon	—	midday when the sun is directly overhead, i.e. directly over the **meridian** passing through a particular point
midnight	—	12 hours ($\frac{1}{2}$ day) before or after noon
pm	—	post meridian, i.e. after noon
am	—	ante meridian, i.e. before noon
o'clock	—	of the clock, i.e. as shown by the clock
a quarter past	—	a quarter of an hour (15 minutes) after a particular hour
a quarter to	—	a quarter of an hour before a particular hour



a quarter past four



a quarter to five

Digital time is a more recent method of recording times. The time is written with the number of hours and minutes after noon or after midnight.

8:45 am means 8 hours and 45 minutes after midnight
9:23 pm means 9 hours and 23 minutes after noon

Digital time is also written in 24-hour time. The time given is the number of hours and minutes after midnight.

7:42 means 7 hours and 42 minutes after midnight
17:38 means 17 hours and 38 minutes after midnight, which would also be 5 hours and 38 minutes after noon

When more accuracy is required the number of seconds is also included, e.g.

8:34:02 would be 8 hours, 34 minutes and 2 seconds after midnight (in 24-hour time).

Calculations with time

Periods of time can be added and subtracted in much the same way as ordinary numbers, but we must always remember that there are only 60 seconds in 1 minute and 60 minutes in 1 hour, not 100!

Example 1

Convert:

a 3 hours to minutes
b 3 days to hours
c 3 weeks to hours
d $1\frac{1}{2}$ hours to seconds

Solutions

a 3 hours $= 3 \times 60$ minutes
$= 180$ minutes

b 3 days $= 3 \times 24$ hours
$= 72$ hours

c 3 weeks $= 3 \times 7$ days
$= 21 \times 24$ hours
$= 504$ hours

d $1\frac{1}{2}$ hours $= 1\frac{1}{2} \times 60$ minutes
$= 1\frac{1}{2} \times 60 \times 60$ seconds
$= 5400$ seconds

Example 2

Complete the table.

Analog time	Digital time	
	12-hour	24-hour
20 past 3 in the morning	a	b
c	1:04 pm	d
e	f	19:56

Solutions

a 3:20 am
b 3:20
c 4 minutes past 1 in the afternoon
d 13:04
e 4 minutes to 8 in the evening
f 7:56 pm

Example 3

Complete the following calculations.

a
2 hours 32 minutes
4 hours 37 minutes +

b
4 hours 27 minutes 48 seconds
1 hour 57 minutes 23 seconds +

c
5 hours 26 minutes
2 hours 42 minutes –

d
3 hours 4 minutes 0 seconds
1 hour 32 minutes 30 seconds –

Solutions

a
2 hours 32 minutes
4 hours 37 minutes +
7 hours 9 minutes
60 minutes becomes 1 hour carried.

b
4 hours 27 minutes 48 seconds
1 hour 57 minutes 23 seconds +
6 hours 25 minutes 11 seconds
1 minute carried 1 hour carried

c
5 h 26 min = 4 h 86 min
4 hours 86 minutes
2 hours 42 minutes –
2 hours 44 minutes

d
3 h 4 min 0 s = 2 h 63 min 60 s
2 hours 63 minutes 60 seconds
1 hour 32 minutes 30 seconds –
1 hour 31 minutes 30 seconds

Exercise 3.1

1 Convert:

a 1 hour to minutes
b 1 minute to seconds
c 1 day to hours
d 2 hours to minutes
e 5 minutes to seconds
f 2 days to hours
g 7 minutes to seconds
h 5 days to hours
i 4 hours to minutes

2 Convert:

a 120 seconds to minutes
b 180 minutes to hours
c 72 hours to days
d 300 minutes to hours
e 240 hours to days
f 360 seconds to minutes

3 Find the number of minutes in:

a $\frac{1}{2}$ hour
b $\frac{1}{4}$ hour
c $\frac{3}{4}$ hour
d $\frac{1}{3}$ hour
e $\frac{2}{3}$ hour
f $\frac{1}{10}$ hour
g $\frac{1}{5}$ hour
h $\frac{5}{12}$ hour

4 Find the number of seconds in:

a $1\frac{1}{2}$ minutes
b $1\frac{1}{4}$ minutes
c $1\frac{1}{3}$ minutes
d $1\frac{7}{12}$ minutes
e $2\frac{3}{4}$ minutes
f $2\frac{2}{3}$ minutes
g $3\frac{5}{6}$ minutes
h $7\frac{4}{5}$ minutes

5 What fraction of:

a 1 day is 12 hours?
b 1 minute is 20 seconds?
c 1 hour is 15 minutes?
d 1 day is 16 hours?
e 1 minute is 25 seconds?
f 1 hour is 24 minutes?
g 2 days is 8 hours?
h $1\frac{1}{2}$ hours is 20 minutes?

Consolidation

6 **a** How many seconds are there in an hour?
b How many minutes are there in a day?
c How many hours are there in a week?

7 Express each of these analog times as a digital time.

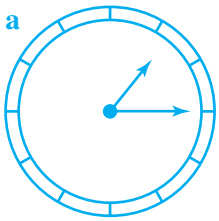
a eight o'clock
b half past seven
c a quarter past three
d a quarter to nine
e ten minutes past eleven
f five minutes to one
g twenty-five minutes past four
h twelve minutes to ten

8 Express each of these digital times as an analog time.

a 7:00 **b** 4:30 **c** 8:15 **d** 1:45
e 10:05 **f** 12:50 **g** 2:25 **h** 11:37

9 Express midnight and midday as digital times.

10 Write down the time showing on each clock in both analog and digital form.

a

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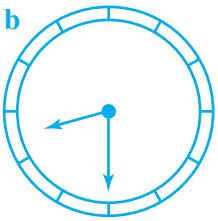
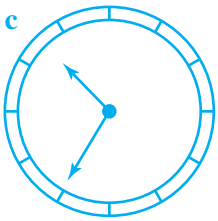
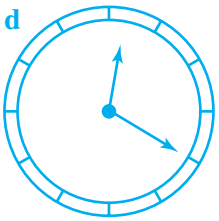

c

d


11 Rewrite each of the following times in 24-hour form.

a 3 am **b** 7 pm **c** 6:30 am **d** 9:15 pm
e 4:55 pm **f** 1:35 am **g** 12:32 pm **h** 2:45 pm
i 3:40 am **j** 8 pm **k** 1:50 pm **l** 12:09 am

12 Rewrite each of the following 24-hour times as 12-hour times.

a 04:00 **b** 11:00 **c** 13:00 **d** 14:30
e 05:10 **f** 17:45 **g** 20:15 **h** 01:20
i 10:47 **j** 16:28 **k** 23:35 **l** 00:15

13 Express in words each of these 24-hour times as analog times.

a 18:45 **b** 07:05 **c** 13:30 **d** 00:10

14 Express each of these analog times in 24-hour form.

a a quarter past eight in the morning
b half past eleven in the evening
c twenty past three in the afternoon
d ten to four in the morning

15 Write answers to each of these:

a 1 h 10 min + 20 min
b 2 h 25 min + 1 h
c 6 h 22 min – 4 h
d 3 h 35 min – 15 min
e 3 h 20 min + 1 h 5 min
f 4 h 30 min – 2 h 10 min
g 7 h 25 min – 1 h 25 min
h 3 h 15 min + 4 h 15 min
i 2 h 17 min + 3 h 9 min
j 5 h 58 min – 2 h 35 min
k 1 h 40 min + 20 min
l 6 h 15 min + 2 h 45 min

16 Write answers to each of these.

a 2 h 50 min + 20 min
b 1 h 45 min + 35 min
c 1 h 40 min + 1 h 25 min
d 3 h 30 min + 2 h 45 min
e 5 h 38 min + 3 h 57 min
f 2 h 27 min + 4 h 48 min

17 Find answers to each of the following.

a 1 h – 10 min
b 1 h – 15 min
c 1 h – 40 min
d 2 h – 30 min
e 2 h – 10 min
f 3 h – 45 min
g 2 h – 1 h 30 min
h 3 h – 2 h 10 min
i 4 h – 1 h 15 min
j 5 h – 2 h 40 min
k 6 h – 4 h 25 min
l 8 h – 3 h 38 min

18 Simplify:

a 1 h 10 min – 20 min
b 1 h 15 min – 35 min
c 2 h 20 min – 45 min
d 2 h 5 min – 1 h 10 min
e 4 h 10 min – 1 h 35 min
f 3 h 25 min – 2 h 40 min
g 5 h 16 min – 2 h 21 min
h 7 h 22 min – 4 h 39 min
i 6 h 34 min – 1 h 55 min
j 4 h 45 min – 3 h 56 min

19 A train left Central Station in Sydney at 2:25 pm and arrived in Wollongong at 3:40 pm. How long was the trip?

20 Virginia wants to set her video recorder to tape a documentary that runs for 1 hour 35 minutes. If the program starts at 2:15 pm, find the starting and finishing times.

21 Hamish went for a run in the evening, leaving home at 5:42 pm. He ran for 35 minutes, rested for 7 minutes, then took a further 41 minutes to run home. At what time did he arrive?

22 Lars boarded a coach at Mittagong and travelled for 3 hours 50 minutes, arriving in Gundagai at 8:15 pm. At what time did the coach leave Mittagong?

23 Erin's clock radio shows the time as Monday 23:55. If the clock is running 15 minutes slow, what is the correct time that should be showing?

24 Aeroplane arrivals and departures are given as 24-hour times. A plane takes off at a quarter to seven in the morning and lands at ten past two in the afternoon.

a State the departure and arrival times in 24-hour form.
b Find the length of the flight in hours and minutes.

25 An aeroplane left Sydney at 8:35 am and landed in Jakarta at 4:20 pm. After a 30-minute stopover, the plane continued on to New Delhi, landing at 12:05 am the next day. Calculate the total flying time. All times are given in Sydney time.

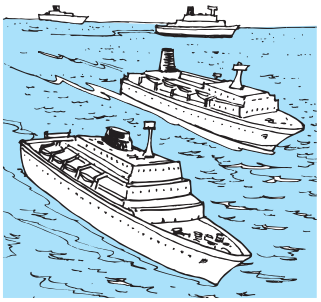
Further applications

26 State the day and time which results in each of the following.

- **a** Monday 10:45 am + 6 hours 10 minutes
- **b** Wednesday 9:20 pm + 4 hours 15 minutes
- **c** Sunday 6:50 pm – 10 hours 35 minutes
- **d** Friday 7:45 am – 15 hours 20 minutes
- **e** Thursday 11:07 pm + 2 days 14 hours 36 minutes
- **f** Saturday 1:00 am – 1 day 22 hours 14 minutes

TRY THIS 'See you in port'

Bruce, Jack, Tracy and Amanda all work on different ships which left Sydney on 1 January 2003. The ships have voyage lengths of 3, 4, 6 and 8 weeks respectively (i.e. after 3 weeks, Bruce will be back in Sydney). When will the four friends again be in port together for another party?



3.2 The calendar

The units of time we used in section 3.1 are relatively small and so they are unsuitable for measuring very long periods of time. For long periods we need a unit which is large, easily observed and unchanging. There is one obvious choice, the year (yr).

The year is the period of time required for the Earth to complete one **orbit** around the sun. The year is slowly changing but these changes are so tiny that they make little difference to a year even over a full human lifetime.

Because both units of time are derived from natural events it is not surprising that the relationship between a year and a day is not simple. One year is between 365 and 366 days, and quite close to, but not quite, $365\frac{1}{4}$ days. By using 365 days for a year we lose a quarter of a day each year, and so after 4 years we are 1 full day behind. To adjust for this we add one day every fourth year (called a leap year). This is still not quite correct so other changes are made at very long intervals.

For the purpose of this chapter:

1 **year** = 365 days
1 **leap year** = 366 days
1 **decade** = 10 years
1 **century** = 100 years
1 **millennium** = 1000 years

12 months = 1 year
3 months = 1 quarter
1 fortnight = 2 weeks

The unit of 1 month is a very unusual one. Seven months have 31 days, four months have 30 days and one has 28 days (or 29 in a leap year). This strange arrangement is summarised by the following rhyme.

30 days hath September
April, June and November.
All the rest have 31 excepting
February which has 28 days clear and 29 in each leap year.

Leap years are the years whose numbers are divisible by 4, e.g. 2004, 2008, 2012, … except where the number is divisible by 100 but not by 400.

So 1900, 2100, 2200, 2300, 2500, … are not leap years, but 2000, 2400, 2800, … are leap years.

These complicated adjustments are needed to allow for the inaccuracy in taking one year as 365 days rather than $365\frac{1}{4}$ days.

Abbreviations These are the accepted abbreviations for the months.

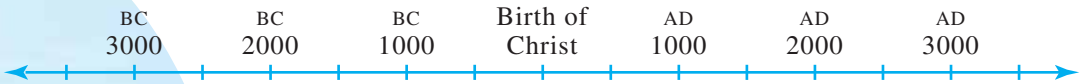
Jan., Feb., Mar., Apr., Aug., Sept., Oct., Nov., Dec.

As well as measuring the duration of longer periods of time, the year is used to locate events. We have numbered the years starting from a particular point in time, so we are able to keep track of events as they occur. Captain James Cook was born in 1728 and Sir Donald Bradman in 1908, so Sir Donald's birth was 180 years after Captain James' birth.

The choice of a beginning point for counting the years is entirely artificial. Any point in time could be chosen and would operate just as well as any other. As a result there are a number of different calendars in use around the world.

The Romans, who originally devised our calendar, were of the Christian religion. They chose the year of the birth of Christ as their zero point. They needed to **distinguish** between the period since His birth and that before, and used the term *anno domini* (meaning in the year of our Lord).

We write AD 1000 meaning the year which is 1000 years after the birth of Christ, and 500 BC for the year which was 500 years before the birth of Christ.



To find the number of years between two dates:
- **subtract the years if both dates are AD or BC.**
- **add the years and subtract 1 if one date is AD and the other is BC.**

NOTE: The year before AD 1 is 1 BC. That is, there was no year 0.

Example 1

a How many days are there from 1 May to 31 July?
b How many days are there from 10 December to 25 January?
c How many days are there from 4 January 2003 to 6 April 2003?

Solutions

a The period will include days from
May—30
June—30
July—31
91 days in total

b The period will include
December—21 days
January—25 days
46 days in total

c The period will include
January—27
February—28 (not a leap year)
March—31
April—6
92 days in total

Example 2

How many years are there from:

a 400 AD to 1761 AD?
b 561 BC to 301 BC?
c 8 BC to 10 AD?
d 27 BC to 64 AD?

Solutions

a $1761 - 400 = 1361$ years
b $561 - 301 = 260$ years
c $8 + 10 - 1 = 17$ years
d $27 + 64 - 1 = 90$ years

Exercise 3.2

1 Write down the number of:
a days in a week
b months in a year
c complete weeks in a year
d days in a year (not a leap year)
e days in a fortnight
f complete fortnights in a year
g quarters in a year
h months in a quarter
i years in a decade
j decades in a century
k years in a century
l centuries in millennium
m years in a millennium
n decades in a millennium

2 Find the number of:
a months in 3 years
b days in 5 weeks
c weeks in 4 fortnights
d completed weeks in 2 years
e months in 2 decades
f decades in 6 centuries
g years in 7 decades
h centuries in 3 millennia
i decades in 25 centuries
j months in 2 centuries

3 Find the number of:
a years in 24 months
b weeks in 21 days
c fortnights in 16 weeks
d years in 156 weeks
e decades in 360 months
f centuries in 40 decades

g centuries in 3000 years
h millennia in 2000 years
i millennia in 1000 decades
j decades in 5000 years

4 **a** What is the number of days in a leap year?
b What date is given to the extra day?

5 **a** Name the months which have 30 days?
b Which months have 31 days?
c How many days has February in a *non-leap* year?

6 During which months do the seasons occur *in Australia*?
a summer **b** autumn **c** winter **d** spring

Consolidation

7 How many days are there in each season in a non-leap year?

8 How many months are there in:
a $\frac{1}{2}$ year? **b** $\frac{1}{3}$ year? **c** $1\frac{1}{2}$ years? **d** $2\frac{1}{4}$ years?

9 How many days after the first date does the second date occur?
a 20 June; 5 July
b 11 March; 23 April
c 27 October; 12 December
d 16 July; 21 September

10 How many days after 17 February does the 6 March fall in:
a a leap year?
b a non-leap year?

11 What will be the date:
a 1 week after 12 January?
b 1 week after 28 May?
c 1 week after 29 December?
d 2 weeks after 17 November?
e 5 weeks after 9 August?
f 10 weeks after 25 June?

12 On 18 August Clive receives a bank statement telling him that the minimum payment on his credit card debt must be paid within 21 days. What is the latest date on which Clive can make this payment?

13 Find in days, the length of the Christmas holidays if the school year ends on 20 December and school resumes again on 28 January.

14 **a** If 2003 begins on a Wednesday, on what day of the week will 2004 begin?
b On what day of the week will 2005 begin?

15 Easter Sunday falls 40 days after Ash Wednesday each year. In the year 2000, Easter Sunday fell on 23 April. What was the date of Ash Wednesday?

16 Ted sees his doctor every 10 days. If his last visit was on 25 May, what will be the dates of his next two appointments?

17 Sasha's brother Lipzi has his birthday 17 days before hers. If her birthday is 5 October, what is Lipzi's birthday?

18 Bradley has decided to start his taper 21 days before the school swimming carnival which is to be held on 8 March. When should Brad begin his taper?

19 Which year followed the year:

a AD 2? **b** 11 BC? **c** 1 BC?

20 How many years are there from:

a 3 AD to 7 AD? **b** 12 BC to 2 BC? **c** 5 BC to 2 AD?
d 6 BC to 18 AD? **e** 22 BC to 15 AD? **f** 50 BC to 43 AD?

21 Twenty years after its construction, a house was destroyed by fire in AD 6. In what year was the house built?

22 How many years separated the births of Pythagoras (580 BC) and Sir Isaac Newton (AD 1642)?

23 In which century (1st, 2nd, 3rd, …) did each of these years fall?

a 1988 **b** 1722 **c** 1463 **d** 586
e 141 **f** 62 **g** 2000 **h** 2001

24 What were the dates of the first and last days of the twentieth century?

■ Further applications

25 State whether or not each year was a leap year.

a 1988 **b** 1922 **c** 1931 **d** 1960 **e** 1964
f 1910 **g** 1500 **h** 1200 **i** 2002 **j** 2000

3.3 Time with a calculator

Most modern calculators have a key which allows us to change hours to minutes, seconds to hours and minutes, etc. The key will be $\boxed{\text{D M S}}$ or $\boxed{\circ\,'\,''}$ depending on the calculator in use.

Example 1

Use your calculator to make the following conversions.

a 367 min = … h … min **b** 5829 s = … h … min … s
c 24.67 min = … min … s **d** 4.126 h = … h … min … s
e 2 h 18 min = … h **f** 26 min 48 s = … min
g 9 min 27 s = … s **h** 3 h 14 min 2 s = … s

Solutions

The final answer is shown followed by the sequence of keys used.

a	6 h 7 min	[0] [°′″] [3] [6] [7] [°′″] [SHIFT] [°′″]
b	1 h 37 min 9 s	[0] [°′″] [0] [°′″] [5] [8] [2] [9] [°′″] [SHIFT] [°′″]
c	24 min 40.2 s	[0] [°′″] [2] [4] [.] [6] [7] [°′″] [SHIFT] [°′″]
d	4 h 7 min 33.6 s	[4] [.] [1] [2] [6] [SHIFT] [°′″]
e	2.3 h	[2] [°′″] [1] [8] [°′″]
f	26.8 min	[2] [6] [°′″] [4] [8] [°′″]
g	567 s	[9] [°′″] [2] [7] [°′″] [×] [6] [0] [=]
h	11 642 s	[3] [°′″] [1] [4] [°′″] [2] [°′″] [×] [6] [0] [×] [6] [0] [=]

Some calculators do not require use of the shift key as shown in these examples. You should check the specific instructions for the calculator you are using.

We can add, subtract, multiply or divide times by converting, as shown in Example 1, and using the [+], [−], [×], [÷], [=] keys as usual.

Example 2

Use your calculator to perform these operations.

a

h	min	s	
	24	26	+
	46	48	

b

h	min	s	
	16	13	+
	15	27	

c

h	min	s	
1	6	25	−
	37	45	

d

h	min	s	
1	6	42	×
		7	

e

h	min	s	
1	6	42	÷
		7	

f

h	min	s	
	20	36	+
1	42		

Solutions

a	1 h 11 min 14 s	[0] [°′″] [2] [4] [°′″] [2] [6] [°′″] [+] [0] [°′″] [4] [6] [°′″] [4] [8] [°′″] [=] [SHIFT] [°′″]
b	31 min 40 s	[1] [6] [°′″] [1] [3] [°′″] [+] [1] [5] [°′″] [2] [7] [°′″] [=] [SHIFT] [°′″]
c	28 min 40 s	[1] [°′″] [6] [°′″] [2] [5] [°′″] [−] [0] [°′″] [3] [7] [°′″] [4] [5] [°′″] [=] [SHIFT] [°′″]
d	7 h 46 min 54 s	[1] [°′″] [6] [°′″] [4] [2] [°′″] [×] [7] [=] [SHIFT] [°′″]
e	9 min 31.71 s	[1] [°′″] [6] [°′″] [4] [2] [°′″] [÷] [7] [=] [SHIFT] [°′″]
f	2 h 2 min 36 s	[0] [°′″] [2] [0] [°′″] [3] [6] [°′″] [+] [1] [°′″] [4] [2] [°′″] [=] [SHIFT] [°′″]

Exercise 3.3

CALCULATOR

Use your calculator for all questions in this exercise.

1 Convert:

a 2 h 36 min to minutes
b 3 h 7 min to minutes
c 7 h 59 min to minutes
d 17 min 41 s to seconds
e 9 min 9 s to seconds
f 53 min 19 s to seconds

2 Convert:

a 5200 s to min and s
b 765 s to min and s
c 7003 s to min and s
d 218 min to h and min
e 1079 min to h and min
f 2123 min to h and min

3 Convert:

a 3 h 14 min 18 s to s
b 17 h 43 min 29 s to s
c 9 h 1 min 47 s to s
d 3670 s to h, min and s
e 7185 s to h, min and s
f 27 963 s to h, min and s

4 Complete the following calculations.

a

h	min	
3	42	+
2	48	

b

h	min	
5	36	+
7	29	

c

min	s	
8	23	+
2	33	

d

min	s	
14	46	+
9	57	

5 Complete the following calculations.

a

h	min	
12	3	–
3	12	

b

h	min	
7	18	–
5	46	

c

min	s	
17	33	–
8	12	

d

min	s	
42	41	–
15	52	

■ Consolidation

6 Convert:

a 5 h 5 min 15 s to seconds
b 11 h 47 s to seconds
c 2 h 26 min to seconds
d 49 min to seconds

7 Convert, giving your answer correct to 2 decimal places:

a 2 h 17 min 3 s to hours
b 3 h 47 min to hours
c 21 min 32 s to hours
d 7 h and 25 s to hours

8 Convert:

a 4 h 17 min 9 s to minutes
b 17 h 39 min to minutes
c 57 min 27 s to minutes
d 23 h 15 s to minutes

9 Convert each of the following to hours, minutes and seconds.

a 2 days
b 14.6 h
c 326 min
d 4000 s
e 0.36 h
f 2.87 h
g 97.5 min
h 1003.7 min
i 723.5 s
j 85.25 s
k 752.75 min
l 52.5 s

10 Complete the following calculations.

a

h	min	s	
1	20	15	+
3	39	53	

b

h	min	s	
8	0	47	+
1	24	41	

c

h	min	s	
5	48	57	+
5	43	27	

d

h	min	s	
	47	18	+
2	53	0	
1	17	39	

e

h	min	s	
1	49	36	+
2	38	18	
3	32	6	

f

h	min	s	
2	26	17	+
4	0	59	
	43	37	

11 Complete the following calculations.

a

h	min	s	
7	41	26	–
3	37	19	

b

h	min	s	
2	18	47	–
1	39	56	

c

h	min	s	
5	27	15	–
	45	47	

d

h	min	s	
6	0	0	–
2	36	36	

e

h	min	s	
5	12	17	–
2	19	8	

f

h	min	s	
3	0	15	–
1	0	16	

12 Complete the following calculations.

a

min	s	
16	21	×
	4	

b

min	s	
30	27	×
	5	

c

h	min	
6	27	×
	3	

d

h	min	
1	28	×
	16	

e

h	min	s	
1	0	41	×
		32	

f

h	min	s	
2	15	10	×
		20	

g

h	min	s	
1	16	44	×
		3	

h

h	min	s	
2	28	46	×
		8	

13 Complete the following calculations.

a 4 h 16 min ÷ 8
b 3 h 35 min ÷ 5
c 26 min 8 s ÷ 7
d 20 min 33 s ÷ 9
e 1 h 26 min 21 s ÷ 11
f 3 h 17 min 14 s ÷ 4
g 4 h 38 min 12 s ÷ 3
h 7 h 28 min 37 s ÷ 5

■ Further applications

14 **a** A marathon begins at 8 am. The winner finishes at 10 min and 5 s after 10 am. What was the winner's race time?

b It takes Grace 43.2 seconds to write her alphabet once. How long will it take her to write it 14 times? Give your answer in minutes and seconds.

c In a long distance relay race the winning team's time is 2 h 14 min 37 s. If the first two runners both took exactly the same time 43 min 17 s, what was the third runner's time? (There are three runners in a team.)

d An Olympic swimmer completes a 1500 m race in 14 min 47 s. What is his average time for each of the 30 laps?

e A woman travels by train and bus from one town to another taking a total of 23 h 5 min 3 s. If it took her 13 min 3 s to change from train to bus, and the bus trip was 57 min 18 s, what was the duration of the train trip?

3.4 Timetables

As we all know our lives are very closely related to the passing of time. We plan our futures taking into account the number of years in **secondary** (high) school, the length of **tertiary** (university) training and the likely stages of our career. We plan an **annual** holiday considering the number of days available, the best season for our holiday, the time taken to reach our destination, and so on. Our daily lives too revolve around the time we wake, the travel time needed to reach school, the timetable of lessons, how long is left in the Science test, etc.

It is clear that 'time' is of constant interest to us.

Very important for our planning are the many timetables we need to use—train, bus, television and school. We need to be efficient users of these if we are to use our time effectively.

Countrylink is the NSW Government transport system for country areas and includes both rail and coach services.

Below is the Countrylink timetable for travel between Sydney and Oberon.

Western region

Sydney—Mt Victoria—Oberon

Operator	CityRail	Corby's coaches	CityRail	Corby's coaches	CityRail	Corby's coaches
Days of operation	Tue/Fri	Tue/Fri	Tue/Fri	Tue/Fri	Sun	Sun
Service No.	507	553	533	555	527	557
SYDNEY arr	08:22		17:16		14:02	
Strathfield	08:33		...		14:13	
Parramatta	08:44		u 17:37		14:24	
Blacktown	...		...		14:36	
Penrith	09:07		...		14:49	
Katoomba	10:12		19:03		15:58	
MT VICTORIA arr	10:31		19:22		16:17	
Connection						
MT VICTORIA dep		10:45		19:35		16:25
Little Hartley		10:54		19:44		16:34
Hartley		10:58		19:48		16:38
Good Forest		11:08		19:58		16:48
Hampton		11:17		20:07		16:57
OBERON arr		11:45		20:35		17:25

Oberon—Mt Victoria—Sydney

Operator	Corby's coaches	CityRail	Corby's coaches	CityRail	Corby's coaches	CityRail	Corby's coaches	CityRail
Days of operation	Tue	Tue	Fri	Fri	Tue/Fri	Tue/Fri	Sun	Sun
Service No.	552	504	554	502	556	538	558	536
OBERON dep	06:55		08:50		17:45		14:55	
Hampton	07:23		09:18		18:13		15:23	
Good Forest	07:32		09:27		18:22		15:32	
Hartley	07:42		09:37		18:32		15:42	
Little Hartley	07:46		09:41		18:36		15:46	
MT VICTORIA arr	07:55		09:50		18:45		15:55	
Connection								
MT VICTORIA dep		08:06		10:06		19:06		16:06
Katoomba		08:25		10:25		19:25		16:25
Penrith		09:32		11:32		20:32		17:31
Blacktown		09:48		11:48		20:48		17:48
Parramatta		09:57		11:57		20:56		17:56
Strathfield		10:09		12:09		21:09		18:09
SYDNEY arr		10:24		12:23		21:23		18:23

Example

Use the timetable above to answer the following questions.

Sylvia, who lives at Strathfield, is to visit her Aunt Ginny in Good Forest. She arrives at Strathfield station at 8:00 am, in time to catch the Tuesday morning train. Sylvia stays with her aunt for five days and returns home on the Sunday afternoon service.

a What major change in her travel occurs at Mt Victoria?
b For how long will she be on the train coming home?
c How long does the train take to reach Penrith on Tuesday morning?
d How long does Sylvia wait at Mt Victoria on her return journey?
e Which takes longer, the outward journey or the return? What causes this difference?
f How could Sylvia have arranged her trip to stay at her aunt's home for a shorter period?

Solutions

a Sylvia changes from the train to a coach for the remainder of her journey.
b The train leaves Mt Victoria at 16:06 and reaches Strathfield at 18:09, so it takes 2 hours and 3 minutes.
c It leaves Strathfield at 8:33 and reaches Penrith at 9:07, which is 34 minutes.
d She arrives at Mt Victoria by coach at 15:55 and departs in the train at 16:06, so waits 11 minutes.
e Outward journey begins at 8:33 and ends at 11:08. It takes 2 hours and 35 minutes. Return journey begins at 15:32 and ends at 18:09. It takes 2 hours and 37 minutes. The return journey takes longer. The train did not stop at Blacktown on the outward trip, saving 2 minutes.
f Sylvia could have gone on Tuesday and returned on Friday, gone on Friday and returned on Sunday, or gone on Sunday and returned on Tuesday. She could even go and return on the same day if she travelled on Tuesday or Friday.

Exercise 3.4

1 Archimedes High—Year 7

Monday	Tuesday	Wednesday	Thursday	Friday
English	Science	Assembly	Lang.	Science
Social Science	Science	Maths	Social Science	Comp.
Lang.	English	Lang.	Maths	Groups
Recess				
Maths	Comp.	Art	English	Lang.
Maths	Comp.	Science	English	Social Science
Lunch				
Science	Social Science	Social Science	Sport	Maths
P.E.	Social Science	Music	Sport	Art
P.E.	Music	English	Sport	Art

Lesson starting times

Use the school timetable to answer the following questions. School finishes at 3:00 pm.

Lesson	M, Tu, F	Wed	Thurs
1	8:40	8:40	8:40
2	9:20	9:05	9:17
3	10:00	9:48	9:54
Recess	10:40	10:30	10:31
4	11:00	10:50	10:49
5	11:40	11:32	11:26
Lunch	12:20	12:14	12:03
6	1:00	12:54	12:40
7	1:40	1:36	
8	2:20	2:18	

a How long is the sport timeslot on Thursday?

b Which subject are the students studying at:
- **i** 10:32 on Monday?
- **ii** 18 minutes to 12 on Wednesday?
- **iii** a quarter past 9 on Thursday?
- **iv** 13:15 on Friday?
- **v** 15:00 on Tuesday?

c How long is:
- **i** the double lesson of Science on Tuesday?
- **ii** the double lesson of English on Thursday?
- **iii** the Music lesson on Wednesday?
- **iv** Assembly?
- **v** the afternoon session on Wednesday?

d On which subject do the students spend most time—Science, Maths or Social Science?

e On which day is lunch shortened?

2 Glenorie Bus Company—Timetable for route 638

ROUTE 638: PENNANT HILLS and CASTLE HILL to GALSTON and BERRILEE															WEEKDAYS			
Showing Route Number	638	638	638	638	638	638	638	638	638	638	638	638	638	638	638	638	638	638
	am	am	am	am	am	am	am	am	pm	pm	pm	pm	pm	pm	pm	pm	pm	pm
	SN	YN		S	SH	YN					H		SH	S	NK	K	H	
Train from City arrives	6.35	6.35	6.56	…	7.39	7.54	8.47	10.36	12.36	…	2.06	…	3.30		4.14	…	5.02	
Train from Hornsby arrives	6.42	6.42	6.57	…	7.40	7.55	8.49	10.19	12.19	…	1.49	…	3.29		4.12	…	4.55	
PENNANT HILLS Station	6.45	6.45	7.00	…	7.45	8.10	8.53	10.40	12.40	…	2.10	…	3.35	…	4.17	…	5.07	…
CASTLE HILL—Castle Towers	…	…	7.28	7.47	…	…	9.20	11.15	1.15	2.15	…	3.25	…	3.55	…	4.15	…	5.15
Old Northern Road & Gilbert Road	…	…	7.34	7.53	…	…	9.26	11.22	1.22	2.22	2.22	3.35	…	4.01	…	4.21	5.21	5.21
Round Corner Dural	…	…	7.38	7.57	8.05	…	9.30	11.26	1.26	2.26	2.26	3.40	4.00	4.05	…	4.25	5.25	5.25
DURAL—Old Northern & Galston Rds	6.58	6.58	7.43	8.02	8.10	8.25	9.35	11.31	1.31	2.31	2.31	3.55	4.10	4.10	4.32	4.32	5.30	5.30
GALSTON—Galston & Arcadia Rds	7.03	7.03	7.48	8.07	8.15	8.30	9.40	11.36	1.36	2.36	2.36	4.10	4.15	4.15	4.37	4.37	5.35	5.35
GALSTON—Rowland Village	…	…	…	…	…	…	…	11.38	1.38	2.38	2.38	…	…	…	…	…	…	…
Old Northern Rd & Mid-Dural Rd	…	…	7.51	…	…	…	9.46	11.42	1.42	…	…	…	…	…	…	…	…	…
ARCADIA School—Bay & Arcadia Rds	7.11	7.11	…	8.28	…	8.38	…	…	…	2.46	2.46	4.18	4.23	4.23	4.55	4.55	5.43	5.43
BERRILEE—Chilcott Road	7.18	…	…	8.35	…	8.45	…	…	…	2.53	2.53	4.25	4.30	4.30	5.02	5.02	5.50	5.50

	WEEKDAYS (CONT.)						
Showing Route Number	638	638	638	638	638	638	638
	pm	pm	pm	pm	pm	pm	pm
	H		H		H		L
Train from City arrives	5.33	…	6.02	…	7.06	…	9.06
Train from Hornsby arrives	5.37	…	6.00	…	6.49	…	8.49
PENNANT HILLS Station	5.40	…	6.07	…	7.10	…	9.10
CASTLE HILL—Castle Towers	…	5.48	…	6.15	…	7.15	9.37
Old Northern Road & Gilbert Road	5.54	5.54	6.21	6.21	7.20	7.20	9.43
Round Corner Dural	5.58	5.58	6.25	6.25	7.22	7.22	9.55
DURAL—Old Northern & Galston Rds	6.03	6.03	6.30	6.30	7.25	7.25	10.00
GALSTON—Galston & Arcadia Rds	6.08	6.08	6.35	6.35	7.30	7.30	10.05
GALSTON—Rowland Village	…	…	…	…	…	…	…
Old Northern Rd & Mid-Dural Rd	…	…	…	…	7.33	7.33	10.08
ARCADIA School—Bay & Arcadia Rds	6.16	6.16	6.43	6.43	…	…	…
BERRILEE—Chilcott Road	6.23	6.23	6.50	6.50	…	…	…

EXPLANATIONS

H – Bus operates via Boundary, New Line, Hastings & Old Northern Roads.

K – Bus operates via Galston Road, Bevans Road, Knights Road and Gribbenmount Road.

L – Bus operates as required to last passenger only.

N – Bus operates via Boundary, New Line and Old Northern Roads.

S – Bus operates during School Term only.

Y – Bus operates during School Holidays only.

Grey shading indicates services with significant variations or diversions—refer to trip code at top of column.

a An elderly resident returns by train from Sydney arriving at Pennant Hills station at 1:49 pm. At what time will he reach home at Rowland Village?

b Katie catches the 3:35 pm bus to her home in Arcadia. At what time does she reach the stop at Arcadia School?

c Kim has a dental appointment in Castle Hill at 10:00 am. He leaves the dentist's rooms at 10:55 am. What is the first bus he can catch to his home in Galston? How long will he have to wait for this bus?

d Anica is to spend a day shopping in Sydney. She will travel by bus to Pennant Hills then catch a train to Sydney. At what time must her train reach Pennant Hills station in the afternoon so that she doesn't miss the last bus home to Berrilee?

e How long does the bus departing Pennant Hills at:

i 12:40 pm take to reach Rowland Village?

ii 2:10 pm take to reach Rowland Village?

What is the explanation for this difference?

f You catch the 5:48 pm bus from Castle Hill. There is heavy traffic and the bus reaches Round Corner Dural at 6:10 pm. How late is the bus?

g Con is travelling by train from Sydney and is due to reach Pennant Hills at 6:02 pm. The guard announces that the train is running 8 minutes late. At what time will Con reach his home at Round Corner Dural by bus?

h Tracy's friend, Michelle, is coming to visit her at her Galston home during the school holidays. Michelle will travel by bus from Pennant Hills. What is the earliest time at which Michelle can reach Galston?

i How many buses travel all the way to Berrilee on a school day?

Consolidation

ABC

6.00 **Rage.** Music videos. 57490398
10.35 **Noise TV** (Final, R). 4332350
11.30 **Heartbreak High** (R). Australian drama series. 8282
12.00 **Stateline** (R, S). 9911
12.30 **New Eden: Cornucopia** (R, S). Six-part series. 8089
1.00 **Indoor Cricket** (Final). National League Series. Grand Final. 31440
2.00 **Australian Story: A Little Black Mare** (R, S). 3534
2.30 **Basketball.** Women's National League. Dandenong v Bulleen. 724195
4.00 **Netball.** International. Third Test. New Zealand v Australia. 912398
6.00 **Equestrian** (S). New six-part series. International Horse Trials. Melbourne Horse Trials. 5319
6.30 **Landline** (S). Rural current affairs program. 8008
7.00 **News** (S). 718
7.30 **SeaChange** (R, S). Laura is tired of being let down by the men in her life and then Warwick Munro, a consultant employed to review the effectiveness of the magistrate's court, arrives. 35602
8.22 **Federal Election 2001: One Nation Talk.** 6071114
8.25 **News** (S). 6078027
8.30 **The Bill** (PG, S). CID are tipped off about two rival gangs of football hooligans planning a pitched battle in Sun Hill. But with no clues about the location there are doubts that they can stop it in time. 79060
9.25 **News** (S). 1801992
9.30 **Movie: Sunday Too Far Away** (M, 75, S). Sheep shearers in the Australian outback become involved in an industrial dispute. Jack Thompson, Max Cullen, Robert Bruning, Jerry Thomas. 70534
11.00 **Noise TV** (Final, M). 22737
11.55 **Rage** (M). Tonight, *Rage* features a Noise Festival Special. 48814973

SBS

6.00 **WorldWatch.** Includes: Cantonese News. 8312534
6.25 **Mandarin News.** 28811992
6.55 **Telegiornale.** 10926114
7.30 **Das Journal.** 6975992
8.00 **Ta Nea Ton Ennea.** 3223027
9.00 **Le Journal.** 1894008
9.30 **Sevodnia.** 4017824
10.10 **Telediario.** 41469718
11.00 **Siaran Berita.** 1874244
11.30 **The Journal.** 1877331
12.00 **Business Report.** 1878060
12.30 **Opera: Alcina** (Germany). 49427805
3.15 **Great Arias** (R). Frank Dunlop's production for the Lyon National Opera (UK). 84382350
3.30 **Soccer.** Futbol Mundial. Highlights. 1452447
4.00 **Soccer.** European Champions League. Highlights. 7477805
5.00 **Soccer.** World Cup. Qualifying match. Czech Republic v Bulgaria. From Prague. 7486553
6.00 **The Movie Show** (R, S). 4441737
6.30 **World News** (S). 4459756
7.00 **World Sport** (S). 8926089
7.30 **National Party Policy Launch.** 9755485
7.45 **As It Happened: Islam – Empire of Faith: The Ottomans** (Final, R, S). Explores the rise of the Ottoman Empire, ending at the reign of Suleiman the Magnificent (US). 4820737
8.45 **Bubblegum Crisis – Tokyo 2040** (S). The Knight Sabers must destroy Galatea before she unleashes a Boomer plague on the world. (Japan). 3811973
9.15 **Stone Forever** (M, R, S). In 1998, 34,000 motorbikes gathered on a Newcastle expressway to replicate the famous funeral ride from the film *Stone*. 39066485
10.20 **Movie: Lips Of Blood** (M, 75, R). Jean-Lou Philippe. (France). 6535094
11.50 **Eat Carpet** (PG). 43166553
12.55 **The Fauves – 15 Minutes To Rock** (M, R). 13994848
1.50 **WeatherWatch.** 51573886

SEVEN PRIME

6.00 **The Powerpuff Girls** (R). 5244
6.30 **Power Rangers Lightspeed Rescue.** 3263
7.00 **Saturday Disney.** 5644737
9.00 **Li'l Horrors.** 9282
9.30 **Junior.** 14466
10.30 **Winnie The Pooh** (R). 5089
11.00 **Timon And Pumbaa.** 6718
11.30 **Pepper Ann.** 9805
12.00 **A*mazing** (R). 5794
12.30 **The Big Arvo** (R). 7669
1.00 **Movie: Trapeze** (56, R). 8584832
3.10 **Movie: The Glass Bottom Boat** (66, R). Doris Day, Rod Taylor. 1454718
5.30 **Sydney Weekender.** 1783
6.00 **News** (S). 1602
6.30 **The World Around Us: The Hill Of Dreams** (S). Broken Hill has changed from a mining centre and a union town to one that is forging a future through art and tourism. It has survived for over a century with no permanent source of water. 32824
7.30 **Movie: Ever After** (PG, 98, S). A modern-day Cinderella story about a feisty orphan who puts her potential prince charming to shame with her bravery and regard for humankind. Drew Barrymore, Angelica Houston. 66182973
10.05 **Movie: From Dusk Till Dawn** (AV, 96, R, S). Two notorious criminal brothers on the run from the police, find themselves and their hostages at a strange bar, unaware that the management and staff are vampires. George Clooney, Quentin Tarantino, Harvey Keitel, Juliette Lewis. 8523282
12.15 **Movie: The Lipstick Camera** (AV, 93). Brian Wimmer (Daylight savings begins at 2am). 51484454
12.15 **PRIME Quantam Global.** 67301063
1.45 **PRIME TV Shopping.** 67301063
3.10 **Victor Paul Shopping.** 4003577
5.10 **USA High** (R). 9848916
5.35 **Sweet Valley High** (R). 7886935

NINE NBN/WIN

6.00 **Thunderbirds** (R). 914176
6.00 **NBN Danoz.** 852737
6.30 **NBN Bimble's Bucket.** 860756
7.00 **Challenger** (R). 861485
7.30 **Goodsports** (R). 831244
8.00 **Today On Saturday** (S). 389466
9.00 **Misery Guts** (R). 210737
9.30 **Now You See It** (R). 213824
10.00 **Postcards: A Long Way To The Top.** 369602
10.00 **NBN Thunderbirds.** 369602
11.00 **Burke's Backyard** (R, S). 747466
11.00 **NBN Waveriders** 290973
11.30 **NBN Our Country** 293060
12.00 **Horse Racing.** Cox Plate Day. From Moonee Valley. 24641319
5.00 **Britain's Worst: Gardeners.** 112756
6.00 **News** (S). 437945
6.30 **Australia's Funniest Home Video Show** (S). 582094
7.00 **The Camera Trap** (S). New series. Elaborate scenarios and hoaxes unfold before your eyes. 311805
7.30 **Movie: Paulie** (98, S). A little bird with a big personality sets out to find the loving girl who raised him. Gena Rowlands. (Includes Lotto). 4922027
9.25 **Movie: The Faculty** (M, 98, S). When the kids at Harrington High discover their teachers are really from another planet and mind-controlling parasites are rapidly spreading from faculty to student bodies, it's up to an unlikely collection of loners, nerds and jocks to save the world from alien domination. Clea Duvall, Josh Hartnett. 3227447
11.34 **Newsbreak** (S). 204756
11.35 **Concert For New York** (PG) (Daylight savings begins at 2am). 54176060
2.30 **Movie: Rasputin – The Mad Monk** (M, 66, R). Christopher Lee. 1558954
4.20 **Rainbow Country.** 9073637
4.50 **Nine Lives.** 4901480
5.00 **Jesse Duplantis.** 3228374
5.30 **Crefio A. Dollar, Jnr.** 3221461

TEN NTHN/CAPITAL

6.00 **Sports Tonight** (R, S). 912718
6.00 **NTHN Meditation Medication.** Religion. 850379
6.30 **NTHN Danoz.** 868398
7.00 **New Adventures Of Ocean Girl** (R). Animated. 869027
7.30 **The Silver Brumby** (R). 862114
8.00 **Totally Wild** (R). 863843
8.30 **Thunderstone** (R). 200350
8.55 **NTHN Victor Paul.** 25286466
9.00 **Video Hits.** 4312350
9.30 **NTHN Video Hits.** 9406350
11.30 **Ocean Girl** (R). 291602
12.00 **Motor Racing.** Indy 300. Qualifying sessions. From Surfers Paradise, Gold Coast. Hosted by Bill Woods and Neil Crompton. 30946701
5.00 **News** (S). 332391
5.30 **Sports Tonight** (S). 687468
6.00 **The Simpsons** (R, S). 237927
6.00 **NTHN Music Country – It's Your Go.** 237927
6.30 **Movie: 3 Ninjas – High Noon At Mega Mountain** (98, S). Three young boys, their neighbour and an ex-TV star must help the owners of an amusement park when an army of ninjas hold the park for ransom. Hulk Hogan. 153391
8.30 **Movie: Casino** (M, 95, R, S). A numbers-obsessed gambler and his boyhood friend are sent to Las Vegas to manage a mob casino. Robert De Niro, Sharon Stone, Joe Pesci. 83348737
11.40 **News** (S). 8977805
12.10 **Sports Tonight** (S). 301886
12.40 **Pepsi Live.** 2342190
1.10 **Inside Sport.** 2363683
1.40 **Video Hits.** 4104428
2.00 **Danoz Shopping.** 3208515 (Daylight savings begins.)
2.00 **CAPITAL Temporary Close.**
3.00 **Victor Paul Shopping.** 3279003
4.00 **Randy Morrison.** 6116867
4.30 **Key Of David** (PG). 3258515
5.00 **Hour Of Power.** 3299867

3 a At what time does the Indoor Cricket begin on Channel ABC?

b If you watch the movie Ever After on Seven how early will you be able to go to bed?

c Doug arrives home from competition tennis at 12:48 pm. How much of the Indy 300 has he missed?

d How long is:

- **i** the opera, Alcina, on SBS?
- **ii** The Bill?
- **iii** Saturday Disney?
- **iv** the Motor Racing on Ten?
- **v** Eat Carpet?
- **vi** Heartbreak High?

e A girl watched the Women's Basketball on ABC then the Third Netball Test against New Zealand. How much TV has she watched?

f Dad was watching the Horse Racing on Channel Nine. When the movie The Glass Bottom Boat came on Mum insisted that it was her turn. How much of the racing did Dad miss?

Monday to Friday Circular Quay to Parramatta

Depart	am								pm										
Circular Quay	–	7.40	7.50	8.10	8.55	9.20	10.00	11.00	12.00	1.00	1.30	2.30	3.10	4.00	4.40	5.20	5.35	6.00	6.45
Milsons Point*	–	–	7.55	–	8.58	9.23	10.03	11.03	12.03	1.03	1.33	2.33	3.13	4.04	4.44	5.24	5.39	6.04	6.48
McMahons Point*	–	–	7.57	–	9.00	9.26	10.06	11.06	12.06	1.06	1.36	2.36	3.16	4.07	4.47	5.27	5.42	6.06	6.50
Drummoyne (Wolseley St)	6.23	–	8.07	–	–	9.36	10.16	11.16	12.16	1.16	1.46	2.46	3.26	4.17	4.57	5.37	5.52	6.16	7.00
Gladesville	6.27	–	–	8.25	–	9.40	10.20	11.20	12.20	1.20	1.50	2.50	3.30	4.22	5.02	5.42	5.57	6.21	7.04
Chiswick	–	–	8.11	–	9.12	9.42	10.22	11.22	12.22	1.22	1.52	2.52	3.32	4.25	5.05	5.45	6.00	6.24	7.06
Abbotsford	6.31	–	8.15	–	–	9.46	10.26	11.26	12.26	1.26	1.56	2.56	3.36	4.30	5.10	5.50	6.05	6.28	7.10
Meadowbank	6.40	8.05	8.25	8.35	9.22	9.55	10.35	11.35	12.35	1.35	2.05	3.05	3.45	4.40	5.20	6.00	6.15	6.38	7.20
Rydalmere	6.55	–	8.35	–	9.32	10.05	10.50	11.50	12.50	1.50	2.20	3.20	3.55	4.52	–	6.12	6.27	6.50	7.30
Arrive Parramatta	7.15	–	8.55	–	9.52	10.25	11.10	12.10	1.10	2.10	2.40	3.40	–	5.10	–	–	6.41	7.05	7.50

*** No set down at Milsons Point and McMahons Point from Circular Quay.**

Monday to Friday Parramatta to Circular Quay

Depart	am										pm											
Parramatta	–	–	–	7.20	–	–	9.05	10.05	10.50	11.25	12.25	1.25	2.25	2.55	–	3.55	5.20	–	–	6.45	7.10	7.55
Rydalmere	–	7.00	–	7.40	–	–	9.25	10.25	11.10	11.45	12.45	1.45	2.45	3.15	4.00	4.15	5.40	–	6.15	7.05	7.25	–
Meadowbank	7.00	7.15	7.30	7.55	8.10	8.40	9.35	10.36	11.25	12.00	1.00	2.00	3.00	3.30	4.10	4.30	5.50	5.25	6.25	7.15	–	–
Abbotsford	7.11	–	7.41	–	8.21	8.50	9.44	10.44	11.35	12.10	1.10	2.10	3.10	3.40	4.19	4.40	5.59	–	6.34	7.24	–	–
Chiswick	–	7.25	–	8.05	–	8.54	9.48	10.48	11.39	12.14	1.14	2.14	3.14	3.44	4.23	4.44	6.03	–	6.38	7.28	–	–
Gladesville	7.16	–	7.46	–	8.26	8.56	9.50	10.50	11.41	12.16	1.16	2.16	3.16	3.46	4.25	4.46	6.05	–	6.40	7.30	–	–
Drummoyne (Wolseley St)	7.21	–	7.51	8.10	8.31	9.00	9.54	10.54	11.45	12.20	1.20	2.20	3.20	3.50	4.29	4.50	6.09	–	6.44	7.34	–	–
McMahons Point	7.32	7.37	8.02	8.19	8.42	9.10	10.04	11.04	11.55	12.30	1.30	2.30	3.30	4.00	4.39	5.00	6.19	–	6.54	7.44	–	–
Milsons Point	7.34	7.39	8.04	8.21	8.44	9.12	10.07	11.07	11.57	12.32	1.32	2.32	3.32	4.02	4.42	5.02	6.22	–	6.57	7.47	–	–
Arrive Circular Quay	7.38	7.43	8.08	8.25	8.48	9.16	10.10	11.10	12.01	12.35	1.35	2.35	3.35	4.05	4.45	5.05	6.25	5.50	7.00	7.50	7.55	8.40

4 a Using the above timetable, at what time does:

- **i** the first ferry leave Parramatta in the morning?
- **ii** the first ferry leave Chiswick in the morning?
- **iii** the first ferry from Meadowbank reach Circular Quay?
- **iv** the last ferry leave Circular Quay for the return trip to Parramatta?
- **v** the 10:25 am from Rydalmere reach McMahons Point?
- **vi** the 5:52 pm from Drummoyne reach Abbotsford?

b How long does each of these journeys take?

- **i** Drummoyne to Meadowbank on the 3:26 pm
- **ii** Circular Quay to Rydalmere on the 5:20 pm
- **iii** Gladesville to Abbotsford on the last ferry of the day
- **iv** Rydalmere to Abbotsford on the 11:45 am

v Rydalmere to Milsons Point on the last ferry of the day
vi Meadowbank to Chiswick on the first morning ferry

Further applications

5 a John catches the 4:04 pm from Milsons Point to Meadowbank. His friend Eli catches the 4:10 pm from Meadowbank to Milsons Point.
   i Who reaches his destination first?
   ii How long does each trip take?
   iii How do you account for the difference in duration of the trips?
b Where does the 12 noon ferry from Circular Quay pass the 11:25 am from Parramatta?
c A couple decide to take a ferry ride from their home jetty at Abbotsford to Circular Quay and then return immediately. How long will their trip take if they catch the ferry at:
   i 8:21 am? ii 9:44 am? iii 1:10 pm? iv 4:40 pm?
d How many Circular Quay to Parramatta ferries does the 9:05 am ferry from Parramatta pass before it reaches Circular Quay?

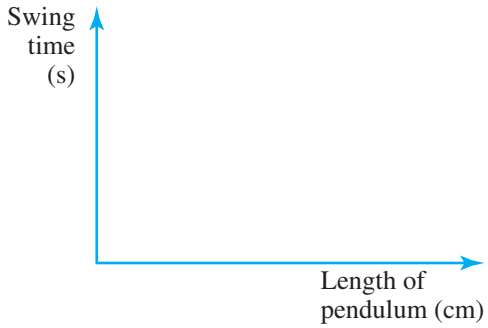
TRY THIS Pendulum clocks

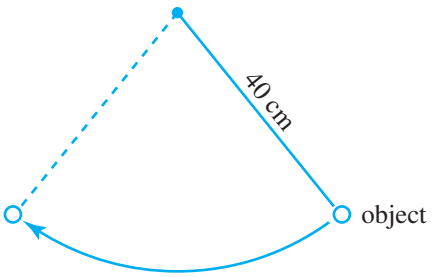
1 The Italian scientist Galileo Galilei made many discoveries (dating from 1588) that led to the invention of the pendulum clock.

Obtain a small weight and attach to it about 2 metres of string. Using the lengths shown in the table, time 20 swings of the weight or pendulum. Then divide your answer by 20 to obtain the time for one swing. If you have a stop watch, then your timing will be more accurate.

Complete the table and draw the graph.

Length of pendulum	Swing time
10 cm	…
20 cm	…
40 cm	…
80 cm	…
160 cm	…
200 cm	…





What length of pendulum is needed for a swing time of 1 second?

2 Have you ever listened closely to the chirps of a cricket? The hotter the temperature, the more chirps a cricket makes. Therefore you could work out the temperature, if you could count the chirps! The formula is:

Temperature (in °C) = (5 × the number of chirps in a minute + 160) ÷ 36.

Work it out for 112 chirps in a minute!

3.5 Time zones

As you are aware we have periods of light and darkness every day because the earth spins on its axis and when the sun is shining on the far side of the earth we are in darkness.

Clearly the time of day will be different in different countries, e.g. when it is 8 am in Sydney it may be midnight in London.

To allow for these differences the world is divided into **time zones**. All the people in a particular time zone use the same daily times for everyday purposes such as TV programs, train timetables, and so on.

In Australia there are three time zones—Eastern Standard Time (EST), Central Standard Time (CST) and Western Standard Time (WST).

- EST is used on the eastern seaboard and applies for most of Queensland, New South Wales, Victoria and Tasmania.
- CST applies for South Australia and the Northern Territory.
- Western Australia uses WST.

Because the earth spins towards the east the sun appears to rise in the east and set in the west. Thus it will be morning in Sydney while it is still dark in Perth, and when we have lunch people in Western Australia may just be starting school.

EST is 30 minutes ahead of CST and 2 hours ahead of WST, e.g. if it is 2 pm in Muswellbrook, NSW, it will be 1:30 pm in Adelaide and only 12 noon in Perth.

Example

Complete the table.

EST	CST	WST
5:30 pm	**i**	**ii**
iii	6:00 am	**iv**
v	**vi**	10:00 pm

Solutions

i 5:00 pm
ii 3:30 pm
iii 6:30 am
iv 4:30 am
v 12:00 midnight
vi 11:30 pm

Some Australian states use Summer Time for daylight saving in late spring, summer and early autumn. Under this system the clocks are set one hour earlier than Standard Time to allow better use of the daylight hours. Since different states start daylight saving at different times, the conversions become rather confusing. We will ignore daylight saving for our questions.

Exercise 3.5

1 Complete this table.

EST	CST	WST
12 noon		
	3:25 pm	
		11:36 am
6:15 pm		
	2:46 am	
		12 midnight

2 **a** I want to ring my friend in Perth. He arrives home from school at 4 pm WST. What is the earliest time (EST) that I can ring him?

b The netball test against New Zealand is being held in Adelaide starting at 3:30 pm (CST). At what time will the telecast begin in NSW?

c Jeff and Narelle leave Melbourne at 6:30 am to drive home to Sydney. If the trip takes 18 hours, including stops, at what time do they arrive home.

Consolidation

3 **a** An aeroplane leaves Brisbane at 11:20 am flying to Adelaide. If the flight takes 1 hour 45 minutes, at what time does it arrive in Adelaide.

b Gemima rings her grandfather at 7:32 pm, Perth time. What time will her grandfather, who lives in Tenterfield, NSW, receive the call?

c Darren turns on his TV just in time to hear that the cricket test against the West Indies will begin the next day at 11 am. The next day, at 11 am, he turns on his TV but the cricket doesn't start until 11:30 am. Can you suggest an explanation?

d A direct flight from Perth to Canberra takes 4 hours and 13 minutes. If the plane lands at 2:32 pm, at what time did it leave Perth.

4 **a** If Los Angeles (USA) is 16 hours behind EST and it is 6 am in Los Angeles, what is the time in Temora, NSW?

b London time is 10 hours behind EST, and a direct flight takes $23\frac{1}{2}$ hours. A plane leaves Sydney at 10:20 am. At what time will it reach London?

c Use the information in part b. If a plane leaves London at 10:20 am, at what time will it land in Sydney?

d Paris time is 9 hours and 30 minutes behind CST. Pierre, who lives in Adelaide, rings his grandfather in Paris at 10 am (CST) one Sunday morning. His grandfather doesn't sound as pleased as he had expected. What might be the reason?

Further applications

5 Ron hears that the cricket test between Australia and South Africa will be televised for 6 hours each day for the 5 days the test is to last. Ron is usually only allowed to watch 10 hours of TV each week, but he asks his father if he can watch the entire test. His father agrees on condition that Ron goes to bed at 9 pm each night and doesn't watch any other programs.
Ron finds that he still only watches 10 hours of TV during that week. Explain why this has occurred.

6 London time is 10 hours behind Sydney time.
New York time is 5 hours behind London time.
Sydney time is 9 hours behind New York time.
Mark leaves Sydney at 11 am Monday on a 23-hour flight to London. He changes planes in London flying straight on to New York on a flight taking 10 hours. He then takes off immediately for Sydney on a 21-hour flight.

a Assuming that the changes take no time, at what time (and day) does Mark arrive in
- **i** London (London time)?
- **ii** New York (New York time)?
- **iii** Sydney (Sydney time)?

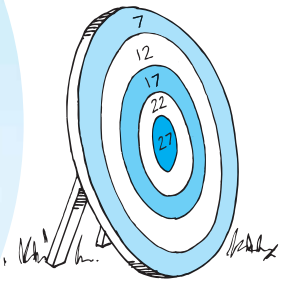
b
- **i** What was the total time of Mark's three flights?
- **ii** Given that he left Sydney at 11 am Monday, when should he arrive back in Sydney?

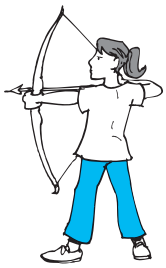
c Did you find a discrepancy in the dates between your answers for parts **a iii** and **b ii**? What is your explanation for this?

Problem solving

1 Using five 9s, write an expression equal to 10. You may use any operations as many times as you need.

2 An archer was firing her shots at the target. With five arrows, her score totalled 100. Where did the arrows land?





3 A snail tries to climb a wall which is 20 metres high. Each day he climbs 6 metres, but each night he slips back 4 metres. How many days will it take him to reach the top of the wall?

4 In MATHSLAND, car number plates have five letters and use only the letters M, A, T, H and S. How many different number plates are possible if each letter can be used only once?

5 This is an old problem! A man looking at a picture in a newspaper said, 'Brothers and sisters have I none, but that man's father is my father's son.' Who is in the picture?

6 A father whose weight was 100 kg and his two sons had to cross a river. If each son weighed 50 kg and the boat could only hold 100 kg, how did they cross the river?

7 Robert was travelling with his parents to Melbourne. At 3.05 they passed the 85 km post, and fifty seconds later went past the 84 km post. How fast were they travelling?

8 What is the next number in this pattern? {4, 9, 25, 49, 121, …}.

9 When Jenny was doing some additions, she asked Donna and Deidre to write down two nine-figure numbers. She then wrote two other numbers. Jenny's trick was that she could immediately write down the answer without adding the numbers.

345 286 468	(Donna)
268 345 325	(Deidre)
654 713 531	(Jenny)
731 654 674	(Jenny)
1 999 999 998	

How do Jenny's numbers relate to Donna's and Deidre's?

10 If 11 + 2 = 1, 6 + 8 = 2, and 9 + 8 = 5, what are we talking about? It should hit you inside a minute!

Focus on working mathematically

The calendars of the Mayan people of Mexico

A Mayan calendar

The civil calendar of 365 days

In chapter 1 we introduced you to the 'base 20' numeration system of the Mayan people who lived in the Yucatan Peninsula in what is now southern Mexico. These people had two calendars, one for everyday civil life and another special one for religious purposes. The everyday calendar called the Haab had 18 months of 20 days, making 360 in all. The Mayan astronomers had calculated 365 days in a full year from their observations of the sun and other stars. So they added a 5-day 'month' called the Wayeb to make 365 days. Any baby born during Wayeb was considered to be doomed to bad luck, and people did not wash, comb their hair or do any hard work during this time.

The religious calendar of 260 days

The Mayan people also had a religious calendar known as the Tzolkin. It was composed of 13 months, named after 13 gods, each of 20 days. In both calendars, the days were numbered from 0 to 19, unlike the way we number days of the month starting from 1. No-one seems to know exactly why they used a 260-day religious calendar. Various theories have been made. However we can say that it was closely linked to their base 20 number system. Their cultural and religious festivals set the ordered pattern of their lives. We know they made careful observations of the movement of the stars, the rising and setting of the sun, the cycles of the moon, and even with very rough instruments were able to calculate the length of the solar year with astonishing accuracy.

Learning activities

1. In this chapter we have learnt about our modern civil calendar. It is called the Gregorian calendar and is based on the day (the time taken for the Earth to turn once on its axis) and the year (the time taken for the Earth to make one complete orbit of the sun).
   Problem: The year is approximately $365\frac{1}{4}$ days, or more accurately 365.242 198 days. What do we do to keep our calendar of 365 days up to date? What about the fact that it is not exactly $365\frac{1}{4}$ days?
2. The Mayans had two calendars, one for civil and one for religious purposes. The Gregorian calendar, named after Pope Gregory XIII, is of Christian origin and can serve both purposes today for many Christians. However many Australians have a separate religious calendar around which their lives are ordered. For example, the Chinese calendar, the Hebrew, Islamic, Indian and so on. If your family follows a religious calendar, see if you can find out about the cycle of seasons and holy days ('holidays'). What special events does your religious calendar commemorate? How are they calculated?
3. The idea of dividing the year into 12 months is another interesting part of the history of the Gregorian calendar. The Mayans named their 13 months after their gods. Where do the names January, February, March and so on of our months come from?

Extension activity

The Mayan people had a third way of measuring time. It was based on a date when they thought the world was created. Time was measured forward from this date. Although scholars disagree on the exact Mayan creation date, one most often taken is 12 August 3113 BC. The number of days which had elapsed since the creation was used by the Mayans for time inscriptions on public buildings, for example, to indicate when the building was completed.
Problem: On a Mayan building an inscription reads 'This building was erected after 1 367 200 days from our creation'. In what year AD was it completed? Take 365.25 days in one year.

Let's communicate

Write a short newspaper story to describe what you have learned about the Mayan calendars. How do they differ from the calendar we use today? What other religious calendars do Australian people use? Can you give examples?

Reflecting

Reflect on the purposes of different cultures and civilisations in creating calendars. Why do you think they were used? Why is the measurement of time important?

Language link with Macquarie

MACQUARIE

1 *What does one* ***revolution*** *mean in terms of the movement of planet Earth?*
2 *Explain the difference between the words* ***century*** *and* ***millennium****.*
3 *Explain the difference between* ***analog*** *and* ***digital*** *methods of counting time.*
4 *In the Christian calendar, what does* ***anno domini*** *mean?*
5 *Read this excerpt from the* Macquarie Learners' Dictionary *entry for* ***time****:*

time *noun* **1.** (*uncount*) the passing of the hours, days, weeks, months and years: *Time goes quickly when you are busy.* **2.** a period or extent of time: *I haven't seen you for a long time.* **3.** (*uncount*) a particular moment shown by a clock: *What is the time?* **4.** an occasion: *The first time I saw you, you were on a bicycle.* | *Take more care next time.* **5.** a particular moment that something is to happen or something takes place: *It is time to go home.* **6.** (*uncount*) the rhythm or tempo of a piece of music: *The tune is in waltz time.* **7. times**, multiplied by: *I know that 5 times 4 is 20.* *–verb* **8.** to measure or record the time or speed of: *They timed her race.* **9.** to choose the moment for: *She timed her arrival perfectly.* *–phrase* **10. at times**, from time to time: *At times I wish I lived in a cold climate.* **11. behind the times**, old-fashioned: *Their ideas are behind the times.* **12. for the time being**, temporarily, or just for the moment: *I'm staying there for the time being.* **13. from time to time**, occasionally: *He visits them from time to time.* **14. In good time**, early, with plenty of time to spare: *We arrived in good time.*

Why is measuring ***time*** *important to the transport industry?*

1 Convert:
- **a** 9 minutes to seconds
- **b** 5400 seconds to hours
- **c** $\frac{5}{6}$ hour to minutes
- **d** 9 days to hours

2 What fraction:
- **a** of one hour is $1\frac{1}{2}$ minutes?
- **b** of 2 days is 6 hours?
- **c** is 40 minutes of 1 hour?

3 Express each analog time as a digital time.
- **a** twenty-five minutes past three
- **b** twelve minutes to seven

4 Express each digital time as an analog time.
a 6:15 **b** 5:30 **c** 4:40

5 Rewrite in 12-hour time.
a 10:47 **b** 23:00 **c** 19:27

6 Rewrite in 24-hour time.
a 4:32 am **b** 12:03 am **c** 5:46 pm

7 Write, in words, as an analog time.
a 23:00 **b** 06:45 **c** 15:20

8 Write in 24-hour time.
- **a** three o'clock in the afternoon
- **b** five to four in the afternoon
- **c** a quarter past twelve in the morning

9 Complete these calculations.
- **a** 2 hours 18 minutes + 4 hours 37 minutes
- **b** 5 hours 42 mins + 6 hours 35 minutes
- **c** 7 hours – 6 hours 41 minutes
- **d** 12 hours 43 minutes – 5 hours 27 minutes
- **e** 8 hours 50 minutes – 1 hour 55 minutes

10 Convert, using a calculator:
- **a** 4 h 13 min 26 s to
  - **i** hours **ii** seconds
- **b** 4721 s to
  - **i** minutes and seconds
  - **ii** hours, minutes and seconds
- **c** 4.72 h to
  - **i** minutes **ii** seconds

11 Complete the following calculations using your calculator.

a

h	min	s	
4	36	25	+
2	15	45	

b

h	min	s	
7	18	37	–
3	46	43	

c

h	min	s	
	33	41	×
		26	

- **d** 4 h 17 min 25 s ÷ 15

12 A driver left Broken Hill at 8:35 am and drove for 3 hours 17 minutes before stopping for a 25-minute rest. At what time did he resume driving?

13 A train left Brisbane for Sydney at 8:46 pm on Monday. If the trip takes 11 hours 36 minutes, when did the train arrive in Sydney?

14 Write down the number of:
- **a** days in April **b** days in spring
- **c** months in a decade
- **d** decades in 3 centuries
- **e** days in 3 fortnights
- **f** hours in half a week

15 How many days are there from:
- **a** 4 May to 19 June?
- **b** 8 Dec to 28 Jan?
- **c** 13 Mar to 1 Nov?
- **d** 28 Jan to 16 Mar (leap year)?

16 How many years are there from:
- **a** 1999 AD to 2010 AD?
- **b** 1427 AD to 2002 AD ?
- **c** 1500 BC to 222 BC?
- **d** 35 BC to 14 AD?

CALCULATOR

CHAPTER REVIEW

17 In which century was:

a 1901? **b** 642? **c** 57?

18 State whether each year will be a leap year.

a 2020 **b** 2100 **c** 2400

Use this timetable for Q19 to 23.

Sydney–Lithgow–Coonabarabran–Baradine

Operator	Dubbo XPT	3F Coach	3F Coach	3F Coach	3F Coach	3F Coach	City Rail	3F Coach	City Rail	3F Coach	City Rail	3F Coach
Days of operation	Daily	Mon	Tue	Wed/ Fri	Thu	Sat	Mon-Fri	Mon-Fri	Sun	Sun	Sun	Sun
Service No.	**427**	**545**	**549**	**547**	**573**	**541**	**537**	**543**	**559**	**539**	**577**	**575**
SYDNEY *dep*	07:10											
Strathfield	u07:21											
Parramatta	u07:33											
Blacktown	c07:44											
Penrith	c07:55											
Katoomba	u08:51											
LITHGOW *arr*	09:31											
Connection												
LITHGOW *dep*		10:00	10:00	10:00	10:00	10:00		19:05		19:05		20:15
Wallerawang		...	...	...	...	...		19:20		19:20		20:30
Portland		...	...	...	...	...		19:30		19:30		20:40
Portland East		...	...	...	...	...		19:33		19:33		20:43
Cullen Bullen		10:25	10:25	10:25	10:25	10:25		19:45		19:45		20:55
Ben Bullen		10:33	10:33	10:33	10:33	10:33		19:53		19:53		21:03
Capertee		10:41	10:41	10:41	10:41	10:41		20:01		20:01		21:11
Running Stream		10:53	10:53	10:53	10:53	10:53		20:13		20:13		21:23
Ilford		11:03	11:03	11:03	11:03	11:03		20:23		20:23		21:33
Clandulla		11:14	...	...	...	11:14		20:34		...		21:44
Charbon		11:17	...	...	...	11:17		20:37		...		21:47
Kandos		11:20	...	...	...	11:20		20:40		...		21:50
Rylstone Hospital		11:26	...	...	...	11:26		20:46		...		21:56
Rylstone		11:28	...	...	...	11:28		20:48		...		21:58
Lue		11:47	...	...	...	11:47		21:07		...		22:17
Mudgee Rail		12:10	11:44	11:44	11:44	12:10		21:30		21:08		22:40
Mudgee		12:15	11:49	11:49	11:49	12:15		21:35		21:13		22:45
GULGONG *arr*		12:39	12:11	12:11	12:11	12:39		21:55		21:35		23:10
Craboon Junction		13:17	12:51	12:51	12:51					22:15		
Dunedoo		13:27	13:01	13:01	13:01					22:25		
Coolah		...	13:36	...	...					...		
Mendooran		14:02	...	13:36	13:36					23:00		
Binnaway		14:45	14:20	14:20	14:20					23:43		
COONABARABRAN *arr*		15:15	14:50	14:50	14:50					00:13		
BARADINE *arr*					15:25							

Baradine–Coonabarabran–Gulgong–Lithgow–Sydney

Operator	3F Coach	City Rail	3F Coach	City Rail	3F Coach	City Rail	3F Coach	3F Coach	Dubbo XPT	3F Coach	City Rail
Days of operation	Sat/ Pub. Hols	Sat/ Pub. Hols	Mon-Fri	Mon-Fri	Fri	Mon-Fri	Mon	Tue/ Wed/ Thu/ Sun	Daily	Sun	Sun
Service No.	**542**	**560**	**544**	**578**	**574**	**582**	**540**	**546**	**428**	**576**	**584**
BARADINE *dep*					09:15						
COONABARABRAN *dep*					09:50		12:35	13:00			
Binnaway					10:20		13:05	13:30			
Mendooran					...		13:45	14:10			
Coolah					11:05		...	...			
Dunedoo					11:42		14:20	14:45			
Craboon Junction					11:52		14:30	14:55			
GULGONG *dep*	06:20		06:30		12:27		15:05	15:30		16:12	
Mudgee	06:45		06:55		12:47		15:25	15:50		16:43	
Mudgee Rail	06:50		07:00		12:52		15:30	15:55		16:37	
Lue	07:20		07:30		...		15:54	...		17:07	
Rylstone	07:35		07:45		...		16:12	...		17:25	
Rylstone Hospital	07:37		07:47		...		16:14	...		17:27	
Kandos	07:43		07:53		...		16:20	...		17:33	
Charbon	07:46		07:56		...		16:23	...		17:36	
Clandulla	07:51		08:01		...		16:28	...		17:41	
Ilford	08:04		08:14		13:38		16:41	16:41		17:54	
Running Stream	08:14		08:24		13:48		16:51	16:51		18:04	
Capertee	08:29		08:39		14:03		17:06	17:06		18:19	
Ben Bullen	08:36		08:46		14:10		17:13	17:13		18:26	
Cullen Bullen	08:44		08:54		14:18		17:21	17:21		18:34	
Portland East	...		...		...		17:32	17:32		18:45	
Portland	...		...		...		17:35	17:35		18:48	
Wallerawang	...		...		...		17:47	17:47		19:00	
LITHGOW *arr*	09:11		09:21		14:47		18:05	18:05		19:18	
Connection											
LITHGOW *dep*											
Katoomba											
Penrith											
Blacktown											
Parramatta											
Strathfield											
SYDNEY *arr*											

19 **a** At what time does the 10 am Monday coach from Lithgow reach Gulgong?

b At what time does service 540 leave Dunedoo on a Monday?

20 How long does the journey take from:

a Parramatta to Gulgong beginning on Saturday morning?

b Lue to Portland East on a Monday?

21 On Tuesday a man catches the 537 service from Sydney to Mudgee. How long does he have to wait at Lithgow?

22 How many services are there from Capertee to Rylstone each week?

23 On which day(s) does the trip from Katoomba to Mendooran take the shortest time?

24 Change these times to allow for the time zones.

a 4:30 am EST = ... WST

b 12 noon CST = ... WST

c 7:19 pm WST = ... CST

d 4:12 pm CST = ... EST

25 **a** If it is 9 am exactly in Brisbane, what is the time in Hobart?

b A flight from Adelaide takes off at 11:05 am. It lands in Hobart at 12:45 pm. How long did the flight take? (All times are local times.)

c Mark leaves Sydney on a 10:55 am flight to New York. The total flight takes 18 hours and 33 minutes. At what time will he land in New York if New York time is 15 hours behind Sydney time?

4 Fractions

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- state the fraction of a figure that has been shaded
- identify the numerator and denominator in a fraction
- express a fraction as a division and vice-versa
- identify proper fractions, improper fractions and mixed numerals
- write down a fraction or mixed numeral that has been placed on a number line
- add and subtract fractions that have a common denominator.

Stage 4

After completing this chapter, you should be able to:

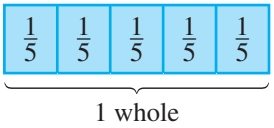
- write a fraction that is equivalent to a given fraction
- convert between improper fractions and mixed numerals
- express a fraction in simplest form
- express one quantity as a fraction of another
- subtract a fraction or mixed numeral from a whole number
- add and subtract fractions that have different denominators
- add and subtract mixed numerals
- multiply fractions and mixed numerals
- state the reciprocal of a fraction
- divide fractions and mixed numerals
- arrange a group of fractions in ascending order by inspection
- arrange a group of fractions in ascending order by expressing them with a common denominator
- find a fraction of a quantity
- use the unitary method to find the whole quantity given a fraction of its value
- solve problems that involve fractions.

4.1 The definition of a fraction

The word **fraction** comes from the Latin word *frangere*, meaning 'to break'. Fractions result when integers (i.e. whole numbers) are 'broken up' or divided into any number of equal parts. That is, a fraction is another way of representing a division. Consider the question:

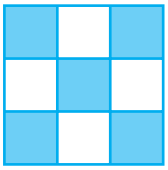
'If a chocolate bar is to be divided equally among five people, what fraction of the chocolate bar would each person receive?'

This can be represented by the division $1 \div 5$ and graphically by the diagram. Each person will receive 'one part out of five' or 'one-fifth' of the chocolate bar.



When writing a fraction, the number above the dividing line is called the **numerator** and the number below the line is called the **denominator**. So, in the fraction $\frac{1}{5}$, 1 is the numerator and 5 is the denominator. The numerator in a fraction can be any whole number, but the denominator cannot be zero. Why? The dividing line in a fraction is called the fraction line or **vinculum**.

In some fractions we may think of the denominator as representing the total number of objects in a set and the numerator as representing a subset or selection of these objects. For example, this figure has been divided into 9 equal parts. Of these, 5 parts have been shaded. We say that 'five-ninths' ($\frac{5}{9}$) of the figure has been shaded.



If in a fraction, the numerator is equal to the denominator, then the fraction has a value of 1 whole. For example, $\frac{3}{3} = 1$, $\frac{8}{8} = 1$.

In any fraction:
- **the number above the fraction line is called the numerator**
- **the number below the fraction line is called the denominator.**

Example 1

What fraction of the square is:

a shaded?
b unshaded?

Solutions

We first divide the figure into 8 equal parts.

a $\frac{5}{8}$ of the square is shaded.

b $\frac{3}{8}$ of the square is unshaded.

Example 2

If $\frac{3}{10}$ of the people in a room are male, what fraction of the people are female?

Solution

If 3 out of every 10 people in a room are male, then $(10 - 3)$ or 7 out of every 10 people are female.

$\therefore \frac{7}{10}$ of the people are female.

Example 3

a Evaluate $2 \div 7$.

b Evaluate $\frac{18}{3}$ by at first expressing it as a division.

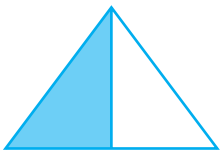
Solutions

a $2 \div 7 = \frac{2}{7}$

b $\frac{18}{3} = 18 \div 3$
$= 6$

Exercise 4.1

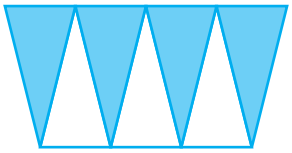
1 What fraction has been shaded in each figure?

a


b

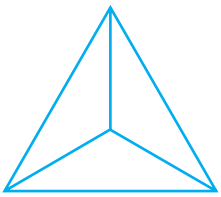
c

d

e


f

2 Shade the given fraction in each of these.

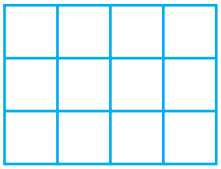
a

$\frac{1}{3}$

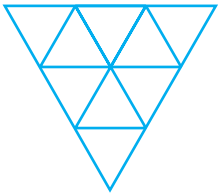
b

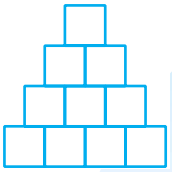
$\frac{3}{4}$

c

$\frac{5}{6}$

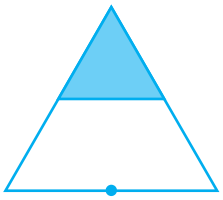
d

$\frac{5}{12}$

e

$\frac{4}{9}$

f

$\frac{7}{10}$

3 What fraction has been shaded in each of these?

a

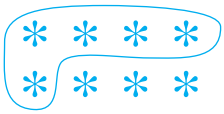
b


c

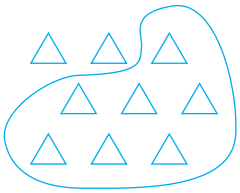
4 What fraction of each group of objects

i has been selected? **ii** has not been selected?

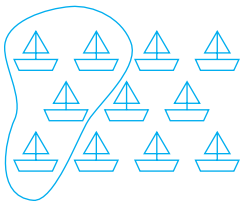
a



b



c



5 For each of these fractions, state the numerator and denominator.

a $\frac{3}{5}$ **b** $\frac{4}{9}$ **c** $\frac{12}{11}$

■ Consolidation

6 Write down the fraction which has a:

a numerator of 6 and denominator of 7 **b** denominator of 13 and numerator of 2

7 Write each of these fractions in words.

a $\frac{1}{4}$ **b** $\frac{2}{3}$ **c** $\frac{8}{9}$ **d** $\frac{3}{7}$ **e** $\frac{5}{12}$ **f** $\frac{11}{15}$

8 Write each of these as a fraction.

a one-half **b** three-quarters **c** seven-tenths
d five-eighths **e** four-fifths **f** two-ninths

9 Complete each of the following statements.

a $\frac{3}{3} = \square$ **b** $\frac{\square}{4} = 1$ **c** $\frac{6}{\square} = 1$

d $\frac{\square}{5} = 1$ **e** $\frac{9}{9} = \square$ **f** $\frac{7}{\square} = 1$

10 **a** Con has completed $\frac{2}{5}$ of a race. What fraction of the race does he still have to run?
b Lisa has read $\frac{3}{4}$ of a book. What fraction of the book does she still have to read?
c Tim has lost $\frac{5}{8}$ of his football swap cards. What fraction of his cards are left?
d Leah gave away $\frac{3}{10}$ of her toys. What fraction of her toys did she keep?
e Toni ate $\frac{7}{15}$ of her lollies. What fraction of her lollies are left?

11 A bag contains 6 red, 3 green and 2 yellow discs. What fraction of the discs are:

a red? **b** green? **c** yellow?

12 A fruit bowl contains 7 oranges, 4 pears and 8 apples. What fraction of the fruit pieces are:

a pears? **b** apples? **c** oranges?
d not apples? **e** not oranges? **f** not pears?

13 In a box of 19 second-hand books there are 7 street directories, 4 cookbooks and the rest are atlases. What fraction of the books are:

a atlases? **b** not atlases? **c** not street directories?

14 Express each of these fractions as a division.

a $\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $\frac{3}{4}$ **d** $\frac{2}{7}$ **e** $\frac{5}{8}$ **f** $\frac{3}{14}$

15 Write each of these divisions as a fraction.

a $1 \div 2$ **b** $2 \div 3$ **c** $7 \div 9$
d $5 \div 8$ **e** $6 \div 13$ **f** $3 \div 10$

16 Evaluate each fraction by treating it as a division.

a $\frac{8}{4}$ **b** $\frac{6}{2}$ **c** $\frac{12}{3}$ **d** $\frac{10}{5}$ **e** $\frac{15}{3}$ **f** $\frac{24}{4}$

g $\frac{12}{4}$ **h** $\frac{16}{2}$ **i** $\frac{21}{3}$ **j** $\frac{30}{5}$ **k** $\frac{14}{2}$ **l** $\frac{36}{4}$

17 Express each of these as a division, then evaluate.

a $\frac{1}{2}$ of 6 **b** $\frac{1}{2}$ of 10 **c** $\frac{1}{2}$ of 22 **d** $\frac{1}{3}$ of 6 **e** $\frac{1}{3}$ of 9 **f** $\frac{1}{3}$ of 15

g $\frac{1}{4}$ of 4 **h** $\frac{1}{4}$ of 8 **i** $\frac{1}{4}$ of 24 **j** $\frac{1}{5}$ of 20 **k** $\frac{1}{6}$ of 18 **l** $\frac{1}{8}$ of 56

18 How many:

a halves are there in 2 wholes? **b** thirds are there in 2 wholes?
c sixths are there in 3 wholes? **d** quarters are there in 3 wholes?
e fifths are there in 2 wholes? **f** eighths are there in 4 wholes?

■ Further applications

19 Toby cut his apple into quarters and then cut each quarter into three equal pieces. If Toby eats seven of the small pieces, what fraction of the apple will remain?

20 Doug bought a pizza and ate half of it. His sister Maree ate two-thirds of the remainder. What fraction of the pizza is left?

21 Pei folded a piece of paper in half four times, then opened it out and ruled a line along each fold line. Into what fractions has the paper been divided?

22 **a** Gary and Debbie each spent half of their pocket money. Did they necessarily spend equal amounts? Explain.

b Linh cut an apple into two pieces and gave one piece to her brother. Did they necessarily receive equal amounts? Explain.

4.2 Proper fractions, improper fractions and mixed numerals

If in a fraction, the numerator is less than the denominator, then the fraction is called a **proper fraction**. It is also referred to as a vulgar or common fraction. Some examples of proper fractions are $\frac{1}{2}$, $\frac{3}{7}$, $\frac{9}{10}$, $\frac{25}{32}$.

If in a fraction, the numerator is greater than or equal to the denominator then the fraction is called an **improper fraction**. Some examples of improper fractions are $\frac{8}{8}$, $\frac{5}{4}$, $\frac{11}{7}$, $\frac{23}{6}$.

If in a fraction:

- the numerator is less than the denominator then the fraction has a value which is less than 1
- the numerator is equal to the denominator then the fraction has a value which is equal to 1
- the numerator is greater than the denominator then the fraction has a value which is greater than 1.

A mixed numeral is a number which is made up of a whole number and a proper fraction. Some examples of mixed numerals are $1\frac{1}{2}$, $2\frac{5}{9}$, $6\frac{7}{10}$, $13\frac{4}{11}$.

To convert a mixed numeral to an improper fraction:

- **multiply the denominator by the whole number then add the numerator.**
- **keep the same denominator.**

To convert an improper fraction to a mixed numeral:

- **divide the numerator by the denominator**
- **write the remainder over the same denominator.**

Example 1

Express $3\frac{4}{7}$ as an improper fraction.

Solution

$$3\frac{4}{7} = \frac{(7 \times 3) + 4}{7}$$

$$= \frac{25}{7}$$

Example 2

Express $\frac{22}{5}$ as a mixed numeral.

Solution

$$\frac{22}{5} = 22 \div 5$$

$$= 4 \text{ remainder } 2$$

$$= 4\frac{2}{5}$$

Exercise 4.2

1 State whether each of these is a proper fraction (PF), an improper fraction (IF) or a mixed numeral (MN).

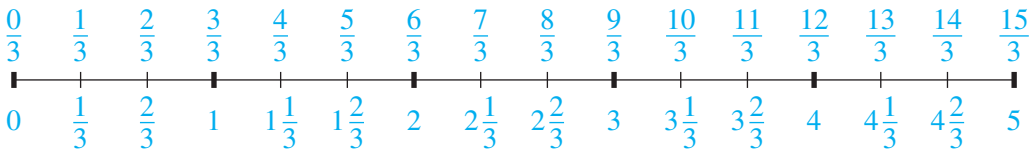
a $\frac{1}{3}$ **b** $\frac{5}{4}$ **c** $1\frac{2}{5}$ **d** $\frac{7}{10}$

e $1\frac{6}{7}$ **f** $\frac{8}{3}$ **g** $2\frac{8}{9}$ **h** $\frac{1}{6}$

i $\frac{11}{12}$ **j** $\frac{15}{11}$ **k** $\frac{21}{8}$ **l** $3\frac{3}{4}$

m $\frac{17}{19}$ **n** $5\frac{9}{13}$ **o** $\frac{23}{20}$ **p** $\frac{9}{16}$

2 Use the number line provided to convert each of the mixed numerals below to an improper fraction.

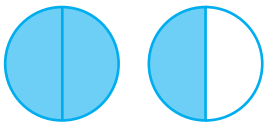


a $1\frac{1}{3}$ **b** $2\frac{2}{3}$ **c** $3\frac{2}{3}$ **d** $4\frac{1}{3}$

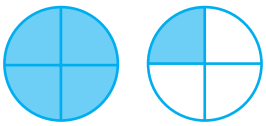
e $1\frac{2}{3}$ **f** $2\frac{1}{3}$ **g** $3\frac{1}{3}$ **h** $4\frac{2}{3}$

3 Write down the mixed numeral and improper fraction suggested by each diagram.

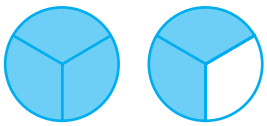
a



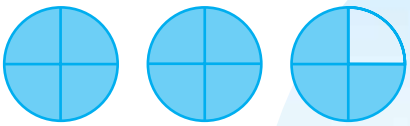
b



c



d



e

f

4 Study this example:

$$2\frac{3}{4} = 2 + \frac{3}{4}$$
$$= 1 + 1 + \frac{3}{4}$$
$$= \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$
$$= \frac{11}{4}$$

Use this method to express each of these mixed numerals as an improper fraction.

a $1\frac{1}{2}$ **b** $1\frac{3}{5}$ **c** $1\frac{1}{3}$

d $2\frac{5}{6}$ **e** $2\frac{1}{8}$ **f** $3\frac{2}{7}$

g $3\frac{7}{9}$ **h** $5\frac{3}{10}$ **i** $4\frac{11}{12}$

5 Study this example:

$$\frac{18}{7} = \frac{7}{7} + \frac{7}{7} + \frac{4}{7}$$
$$= 1 + 1 + \frac{4}{7}$$
$$= 2\frac{4}{7}$$

Use this method to express each of these improper fractions as a mixed numeral.

a $\frac{4}{3}$ **b** $\frac{5}{2}$ **c** $\frac{11}{6}$

d $\frac{9}{4}$ **e** $\frac{16}{7}$ **f** $\frac{19}{5}$

g $\frac{29}{12}$ **h** $\frac{35}{9}$ **i** $\frac{75}{8}$

Consolidation

6 Multiply the whole number by the denominator and add the numerator. This will convert each of these mixed numerals to an improper fraction.

a $1\frac{1}{5}$ **b** $1\frac{3}{4}$ **c** $2\frac{4}{5}$ **d** $3\frac{5}{7}$ **e** $2\frac{7}{12}$

f $5\frac{3}{8}$ **g** $4\frac{6}{11}$ **h** $5\frac{2}{3}$ **i** $4\frac{3}{4}$ **j** $2\frac{9}{10}$

k $6\frac{1}{2}$ **l** $2\frac{2}{7}$ **m** $7\frac{1}{4}$ **n** $9\frac{4}{5}$ **o** $8\frac{5}{6}$

p $10\frac{2}{9}$ **q** $3\frac{7}{11}$ **r** $11\frac{3}{5}$ **s** $5\frac{6}{7}$ **t** $12\frac{3}{8}$

7 Divide the numerator by the denominator. Hence, convert each of these improper fractions to a mixed numeral.

a $\frac{5}{4}$ **b** $\frac{8}{3}$ **c** $\frac{11}{5}$ **d** $\frac{17}{4}$ **e** $\frac{13}{8}$

f $\frac{23}{9}$ **g** $\frac{37}{10}$ **h** $\frac{23}{7}$ **i** $\frac{22}{5}$ **j** $\frac{25}{12}$

k $\frac{38}{7}$ **l** $\frac{34}{5}$ **m** $\frac{45}{4}$ **n** $\frac{23}{3}$ **o** $\frac{35}{11}$

p $\frac{29}{12}$ **q** $\frac{53}{6}$ **r** $\frac{39}{8}$ **s** $\frac{69}{10}$ **t** $\frac{43}{13}$

CALCULATOR

8 Use the fraction key $\boxed{a^b/_c}$ on your calculator to change each mixed numeral to an improper fraction.

a $1\frac{3}{5}$ **b** $2\frac{1}{2}$ **c** $3\frac{9}{10}$ **d** $4\frac{5}{9}$ **e** $7\frac{5}{6}$

CALCULATOR

9 Use the fraction key $\boxed{a^b/_c}$ on your calculator to change each improper fraction to a mixed numeral.

a $\frac{7}{5}$ **b** $\frac{11}{3}$ **c** $\frac{23}{4}$ **d** $\frac{48}{7}$ **e** $\frac{61}{9}$

Further applications

10 Complete the fraction $\frac{\square}{5}$ so that it is greater than $1\frac{2}{5}$ but less than $3\frac{4}{5}$.

11 What values can be placed in the box so that the fraction $\frac{\square}{3}$ is not less than $2\frac{2}{3}$ and not greater than $5\frac{1}{3}$?

4.3 Equivalent fractions

If two or more fractions have the same value, then they are called **equivalent fractions**. For example, the shaded area in this circle could be interpreted as either $\frac{1}{2}$ or $\frac{3}{6}$. Therefore, $\frac{1}{2}$ and $\frac{3}{6}$ are equivalent fractions.

To find a fraction which is equivalent to a given fraction:
- **multiply the numerator and denominator by the same number.**

EG+S

Example

Complete each of these equivalent fractions.

a $\frac{1}{4} = \frac{\square}{12}$ **b** $\frac{2}{3} = \frac{\square}{15}$ **c** $2\frac{3}{5} = \frac{\square}{10}$

Solutions

a $\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$

b $\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$

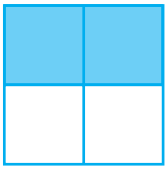
c $2\frac{3}{5} = \frac{13 \times 2}{5 \times 2}$

$= \frac{26}{10}$

Exercise 4.3

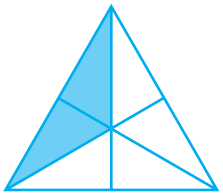
1 Use the diagrams provided to complete the equivalent fractions in each of these.

a



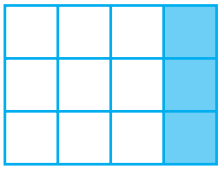
$\frac{1}{2} = \frac{\square}{4}$

b



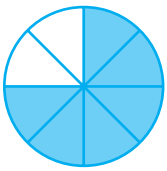
$\frac{1}{3} = \frac{\square}{6}$

c



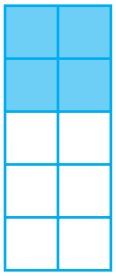
$\frac{1}{4} = \frac{\square}{12}$

d



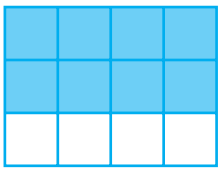
$\frac{3}{4} = \frac{\square}{8}$

e



$\frac{2}{5} = \frac{\square}{10}$

f

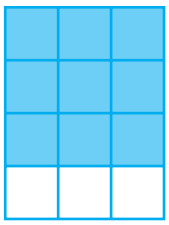


$\frac{2}{3} = \frac{\square}{12}$

g

$\frac{1}{2} = \frac{\square}{8}$

h

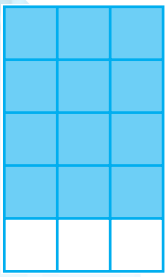


$\frac{3}{4} = \frac{\square}{12}$

i

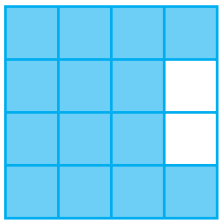
$\frac{2}{3} = \frac{\square}{6}$

j



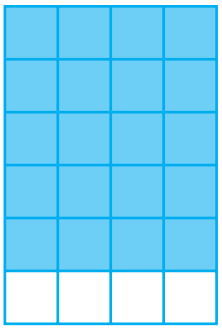
$\frac{4}{5} = \frac{\square}{15}$

k



$\frac{7}{8} = \frac{\square}{16}$

l



$\frac{5}{6} = \frac{\square}{24}$

2 Complete each of these equivalent fractions.

a $\frac{1}{2} = \frac{\square}{4}$ **b** $\frac{1}{3} = \frac{\square}{6}$ **c** $\frac{1}{4} = \frac{\square}{12}$ **d** $\frac{1}{6} = \frac{\square}{30}$ **e** $\frac{1}{5} = \frac{\square}{20}$

f $\frac{1}{7} = \frac{\square}{14}$ **g** $\frac{1}{8} = \frac{\square}{24}$ **h** $\frac{1}{3} = \frac{\square}{12}$ **i** $\frac{1}{6} = \frac{\square}{42}$ **j** $\frac{1}{9} = \frac{\square}{36}$

k $\frac{1}{11} = \frac{\square}{55}$ **l** $\frac{1}{3} = \frac{\square}{27}$ **m** $\frac{1}{2} = \frac{\square}{24}$ **n** $\frac{1}{5} = \frac{\square}{35}$ **o** $\frac{1}{10} = \frac{\square}{60}$

p $\frac{1}{4} = \frac{\square}{20}$ **q** $\frac{1}{7} = \frac{\square}{56}$ **r** $\frac{1}{8} = \frac{\square}{40}$ **s** $\frac{1}{12} = \frac{\square}{96}$ **t** $\frac{1}{11} = \frac{\square}{132}$

■ Consolidation

3 Complete each of these equivalent fractions.

a $\frac{2}{3} = \frac{\square}{9}$ **b** $\frac{3}{4} = \frac{\square}{12}$ **c** $\frac{2}{5} = \frac{\square}{20}$ **d** $\frac{3}{7} = \frac{\square}{14}$ **e** $\frac{2}{11} = \frac{\square}{77}$

f $\frac{5}{6} = \frac{\square}{18}$ **g** $\frac{9}{10} = \frac{\square}{90}$ **h** $\frac{7}{8} = \frac{\square}{32}$ **i** $\frac{6}{7} = \frac{\square}{42}$ **j** $\frac{7}{12} = \frac{\square}{48}$

k $\frac{10}{11} = \frac{\square}{33}$ **l** $\frac{5}{8} = \frac{\square}{48}$ **m** $\frac{3}{4} = \frac{\square}{36}$ **n** $\frac{5}{6} = \frac{\square}{24}$ **o** $\frac{2}{9} = \frac{\square}{72}$

p $\frac{3}{8} = \frac{\square}{40}$ **q** $\frac{5}{9} = \frac{\square}{63}$ **r** $\frac{11}{12} = \frac{\square}{60}$ **s** $\frac{7}{10} = \frac{\square}{40}$ **t** $\frac{2}{3} = \frac{\square}{36}$

4 Complete the following equivalent fractions.

a $\frac{1}{2} = \frac{5}{\square}$ **b** $\frac{\square}{7} = \frac{12}{21}$ **c** $\frac{5}{\square} = \frac{10}{16}$ **d** $\frac{\square}{4} = \frac{18}{24}$ **e** $\frac{7}{\square} = \frac{28}{36}$

f $\frac{9}{10} = \frac{45}{\square}$ **g** $\frac{5}{\square} = \frac{35}{42}$ **h** $\frac{2}{3} = \frac{24}{\square}$ **i** $\frac{3}{\square} = \frac{18}{48}$ **j** $\frac{\square}{9} = \frac{96}{108}$

■ Further applications

5 Express each mixed numeral as an improper fraction, then form an equivalent fraction.

a $1\frac{1}{2} = \frac{\square}{6}$ **b** $1\frac{3}{4} = \frac{\square}{8}$ **c** $2\frac{1}{5} = \frac{\square}{20}$ **d** $3\frac{1}{7} = \frac{\square}{21}$

e $1\frac{5}{6} = \frac{\square}{24}$ **f** $5\frac{1}{4} = \frac{\square}{12}$ **g** $1\frac{3}{7} = \frac{\square}{35}$ **h** $2\frac{7}{8} = \frac{\square}{16}$

i $1\frac{5}{12} = \frac{\square}{24}$ **j** $2\frac{2}{3} = \frac{\square}{24}$ **k** $3\frac{3}{4} = \frac{\square}{12}$ **l** $6\frac{1}{9} = \frac{\square}{18}$

4.4 Simplifying fractions

A fraction is said to be in simplest form when the highest common factor of the numerator and denominator is 1.

To simplify a fraction:
- **find the highest common factor (HCF) of the numerator and denominator**
- **divide both the numerator and denominator by this HCF.**

NOTE: The divisibility tests which were introduced in chapter 2 may prove useful here.

Example 1

Simplify:

a $\frac{5}{20}$

b $\frac{9}{12}$

Solutions

a The HCF of 5 and 20 is 5.

$$\frac{\cancel{5}^{1}}{\cancel{20}_{4}} \begin{matrix}(\div 5)\\(\div 5)\end{matrix}$$

$$= \frac{1}{4}$$

b The HCF of 9 and 12 is 3.

$$\frac{\cancel{9}^{3}}{\cancel{12}_{4}} \begin{matrix}(\div 3)\\(\div 3)\end{matrix}$$

$$= \frac{3}{4}$$

Example 2

Simplify $\frac{60}{400}$.

Solution

The HCF of 60 and 400 is 20, but this may not be obvious. So we simplify the fraction in two stages.

$$\frac{6\cancel{0}}{40\cancel{0}} \begin{matrix}(\div 10)\\(\div 10)\end{matrix}$$

$$= \frac{\cancel{6}^{3}}{\cancel{40}_{20}} \begin{matrix}(\div 2)\\(\div 2)\end{matrix}$$

$$= \frac{3}{20}$$

Exercise 4.4

1 Use the given diagrams to express each fraction in simplest form.

a

$\frac{3}{6} = ___$

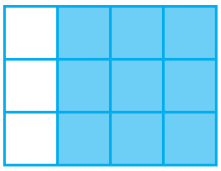
b

$\frac{2}{8} = ___$

c

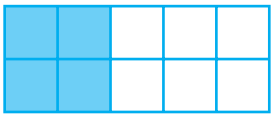
$\frac{6}{9} = ___$

d



$\frac{9}{12} = __$

e



$\frac{4}{10} = __$

f

$\frac{20}{24} = __$

2 Copy and complete each of the following.

a $\frac{2 \div 2}{6 \div 2} = __$ **b** $\frac{3 \div 3}{18 \div 3} = __$ **c** $\frac{5 \div 5}{20 \div 5} = __$

3 Express each fraction in its simplest form.

a $\frac{2}{8}$ **b** $\frac{5}{10}$ **c** $\frac{3}{15}$ **d** $\frac{7}{21}$ **e** $\frac{6}{30}$ **f** $\frac{10}{80}$

g $\frac{4}{28}$ **h** $\frac{8}{32}$ **i** $\frac{2}{22}$ **j** $\frac{9}{27}$ **k** $\frac{7}{35}$ **l** $\frac{3}{36}$

m $\frac{11}{66}$ **n** $\frac{5}{45}$ **o** $\frac{10}{100}$ **p** $\frac{12}{48}$ **q** $\frac{8}{64}$ **r** $\frac{7}{63}$

s $\frac{9}{81}$ **t** $\frac{8}{56}$ **u** $\frac{11}{88}$ **v** $\frac{12}{84}$ **w** $\frac{9}{54}$ **x** $\frac{8}{96}$

■ Consolidation

4 Copy and complete each of the following.

a $\frac{6 \div 2}{10 \div 2} = __$ **b** $\frac{9 \div 3}{15 \div 3} = __$ **c** $\frac{12 \div 4}{20 \div 4} = __$

d $\frac{50 \div 10}{60 \div 10} = __$ **e** $\frac{18 \div 6}{30 \div 6} = __$ **f** $\frac{35 \div 5}{45 \div 5} = __$

g $\frac{14 \div 7}{63 \div 7} = __$ **h** $\frac{36 \div 9}{45 \div 9} = __$ **i** $\frac{60 \div 12}{132 \div 12} = __$

5 Complete these equivalent fractions.

a $\frac{12}{15} = \frac{\square}{5}$ **b** $\frac{20}{24} = \frac{\square}{6}$ **c** $\frac{21}{27} = \frac{\square}{9}$ **d** $\frac{15}{25} = \frac{\square}{5}$ **e** $\frac{32}{36} = \frac{\square}{9}$

6 Express each fraction in its simplest form.

a $\frac{6}{8}$ **b** $\frac{8}{12}$ **c** $\frac{15}{18}$ **d** $\frac{10}{15}$ **e** $\frac{6}{21}$ **f** $\frac{16}{18}$

g $\frac{30}{50}$ **h** $\frac{14}{21}$ **i** $\frac{27}{30}$ **j** $\frac{44}{77}$ **k** $\frac{55}{60}$ **l** $\frac{12}{30}$

m $\frac{14}{22}$ **n** $\frac{10}{45}$ **o** $\frac{12}{44}$ **p** $\frac{40}{48}$ **q** $\frac{18}{21}$ **r** $\frac{28}{35}$

s $\frac{25}{30}$ **t** $\frac{49}{70}$ **u** $\frac{18}{24}$ **v** $\frac{100}{110}$ **w** $\frac{36}{42}$ **x** $\frac{88}{121}$

7 Simplify:

a $\frac{22}{99}$ **b** $\frac{49}{63}$ **c** $\frac{24}{60}$ **d** $\frac{42}{48}$ **e** $\frac{15}{50}$ **f** $\frac{30}{54}$

g $\frac{49}{56}$ **h** $\frac{36}{81}$ **i** $\frac{40}{64}$ **j** $\frac{32}{56}$ **k** $\frac{60}{72}$ **l** $\frac{56}{96}$

m $\frac{45}{108}$ **n** $\frac{72}{84}$ **o** $\frac{35}{63}$ **p** $\frac{42}{49}$ **q** $\frac{70}{80}$ **r** $\frac{121}{132}$

8 Simplify:

a $\frac{10}{300}$ **b** $\frac{30}{500}$ **c** $\frac{20}{150}$ **d** $\frac{90}{400}$

e $\frac{500}{800}$ **f** $\frac{900}{1000}$ **g** $\frac{210}{2500}$ **h** $\frac{300}{20\,000}$

9 Express each of these improper fractions as a mixed numeral, in simplest form.

a $\frac{15}{10}$ **b** $\frac{20}{16}$ **c** $\frac{12}{9}$ **d** $\frac{16}{14}$ **e** $\frac{14}{6}$ **f** $\frac{21}{15}$

g $\frac{28}{10}$ **h** $\frac{27}{12}$ **i** $\frac{24}{9}$ **j** $\frac{55}{15}$ **k** $\frac{39}{24}$ **l** $\frac{63}{14}$

■ Further applications

CALCULATOR

10 Express each fraction in its simplest form. Verify your answers using the fraction key on your calculator.

a $\frac{150}{210}$ **b** $\frac{315}{420}$ **c** $\frac{154}{198}$ **d** $\frac{275}{300}$

e $\frac{320}{480}$ **f** $\frac{240}{288}$ **g** $\frac{168}{240}$ **h** $\frac{252}{315}$

4.5 The relative sizes of fractions

To arrange a set of fractions in ascending order, use one of the following methods.

METHOD 1

- **express the fractions with a common denominator**
- **arrange the fractions in ascending order by ordering the numerators**
- **express the fractions in their original form.**

METHOD 2

- **consider the relative sizes of the fractions by inspection.**

Example 1

Insert $<$ or $>$ to make a true statement in each of these.

a $\frac{4}{5} \square \frac{10}{9}$ **b** $\frac{5}{6} \square \frac{9}{11}$

Solutions

a $\frac{4}{5}$ is a proper fraction, so its value is less than 1, while $\frac{10}{9}$ is an improper fraction, so its value is greater than 1.

$\therefore \frac{4}{5} < \frac{10}{9}$

b Both fractions are proper fractions and it is too difficult to determine which is the larger by inspection. Therefore, we express the fractions with a common denominator, then compare the numerators.

$\frac{5 \times 11}{6 \times 11} = \frac{55}{66}$ and $\frac{9 \times 6}{11 \times 6} = \frac{54}{66}$

Now, $\frac{55}{66} > \frac{54}{66}$, $\therefore \frac{5}{6} > \frac{9}{11}$

Example 2

Arrange these fractions in ascending order by at first expressing them with a common denominator, i.e. using method 1.

$\frac{3}{5}, \frac{1}{2}, \frac{5}{8}, \frac{19}{40}, \frac{9}{20}$

Solution

$\frac{3 \times 8}{5 \times 8} = \frac{24}{40}$, $\frac{1 \times 20}{2 \times 20} = \frac{20}{40}$, $\frac{5 \times 5}{8 \times 5} = \frac{25}{40}$, $\frac{9 \times 2}{20 \times 2} = \frac{18}{40}$

Rearranging these fractions in order gives $\frac{18}{40}, \frac{19}{40}, \frac{20}{40}, \frac{24}{40}, \frac{25}{40}$

i.e. $\frac{9}{20}, \frac{19}{40}, \frac{1}{2}, \frac{3}{5}, \frac{5}{8}$

Example 3

Arrange these fractions in ascending order by inspection, i.e. using method 2.

$\frac{8}{7}, \frac{4}{9}, \frac{2}{15}, \frac{11}{12}$

Solution

i The smallest fraction by inspection is $\frac{2}{15}$.

ii The next largest fraction is $\frac{4}{9}$, which is approximately $\frac{1}{2}$.

iii The next largest fraction is $\frac{11}{12}$, which is slightly less than 1.

iv The largest fraction is $\frac{8}{7}$, which is the only improper fraction and hence it is greater than 1.

$\therefore$ In ascending order, the fractions are $\frac{2}{15}, \frac{4}{9}, \frac{11}{12}, \frac{8}{7}$.

NOTE: Fractions with different denominators cannot be ordered by looking at the numerators.

Exercise 4.5

1 Insert $<$ or $>$ to make a true statement.

a $\frac{3}{4} \square \frac{7}{6}$ **b** $\frac{6}{5} \square \frac{11}{12}$ **c** $\frac{15}{17} \square \frac{9}{8}$

d $\frac{2}{3} \square \frac{10}{9}$ **e** $\frac{16}{13} \square \frac{29}{30}$ **f** $\frac{3}{2} \square \frac{99}{100}$

2 Arrange these fractions in ascending order.

a $1, \frac{8}{7}, 1\frac{2}{7}, \frac{4}{7}$ **b** $\frac{11}{9}, \frac{8}{9}, 1\frac{1}{9}, \frac{5}{9}$ **c** $\frac{10}{11}, 1\frac{3}{11}, \frac{7}{11}, 1$ **d** $\frac{13}{5}, \frac{4}{5}, 1\frac{1}{5}, 2$

3 Express each pair of fractions with a common denominator, then insert $<$ or $>$ to make a true statement.

a $\frac{2}{3} \square \frac{5}{8}$ **b** $\frac{3}{4} \square \frac{4}{5}$ **c** $\frac{5}{6} \square \frac{3}{4}$

d $\frac{5}{12} \square \frac{2}{5}$ **e** $\frac{7}{8} \square \frac{9}{10}$ **f** $\frac{3}{11} \square \frac{1}{3}$

Consolidation

4 Arrange each group of fractions in ascending order, by at first expressing them with a common denominator (i.e. using method 1).

a $\frac{5}{12}, \frac{1}{4}, \frac{1}{6}, \frac{1}{2}, \frac{1}{3}$ **b** $\frac{9}{16}, \frac{19}{32}, \frac{1}{2}, \frac{3}{4}, \frac{5}{8}$ **c** $\frac{7}{10}, \frac{3}{4}, \frac{2}{5}, \frac{9}{20}, \frac{1}{2}$

d $\frac{7}{18}, \frac{4}{9}, \frac{13}{36}, \frac{1}{3}, \frac{5}{12}$ **e** $\frac{4}{9}, \frac{1}{3}, \frac{5}{6}, \frac{13}{18}, \frac{1}{2}$ **f** $\frac{11}{30}, \frac{2}{5}, \frac{1}{3}, \frac{3}{10}, \frac{4}{15}$

g $\frac{11}{24}, \frac{1}{2}, \frac{3}{8}, \frac{5}{12}, \frac{1}{3}$ **h** $\frac{5}{8}, \frac{3}{5}, \frac{11}{20}, \frac{21}{40}, \frac{1}{2}$ **i** $\frac{8}{15}, \frac{3}{10}, \frac{2}{5}, \frac{17}{50}, \frac{1}{2}$

5 Arrange each group of fractions in ascending order by inspection (i.e. using method 2).

a $\frac{7}{6}, \frac{5}{11}, \frac{1}{7}, \frac{11}{12}$ **b** $\frac{3}{7}, \frac{1}{9}, \frac{6}{5}, \frac{12}{13}$ **c** $\frac{7}{8}, \frac{11}{10}, \frac{6}{13}, \frac{1}{14}$

d $\frac{11}{23}, \frac{3}{50}, \frac{8}{7}, \frac{15}{17}$ **e** $\frac{1}{8}, \frac{14}{15}, \frac{10}{19}, \frac{13}{9}$ **f** $\frac{21}{22}, \frac{1}{16}, \frac{8}{5}, \frac{7}{15}$

g $\frac{19}{20}, \frac{1}{2}, \frac{12}{23}, \frac{7}{6}$ **h** $\frac{4}{3}, \frac{8}{15}, \frac{13}{14}, \frac{2}{21}$ **i** $\frac{3}{5}, \frac{1}{18}, \frac{17}{16}, \frac{16}{17}$

Further applications

6 Determine by inspection, which fraction is less than one half and which fraction is greater than one half. Hence, insert $<$ or $>$ to make a true statement.

a $\frac{3}{7} \square \frac{5}{9}$ **b** $\frac{7}{12} \square \frac{4}{9}$ **c** $\frac{3}{8} \square \frac{6}{11}$ **d** $\frac{8}{15} \square \frac{5}{11}$

e $\frac{10}{19} \square \frac{10}{21}$ **f** $\frac{2}{5} \square \frac{7}{13}$ **g** $\frac{9}{20} \square \frac{9}{16}$ **h** $\frac{11}{21} \square \frac{12}{25}$

4.6 Fractions on the number line

To identify fractions on a number line:
- **determine the fraction into which each interval of length 1 unit has been divided**
- **count up or down along the number line by this fraction.**

Example

State the fraction or mixed numeral represented by A, B, C on each number line.

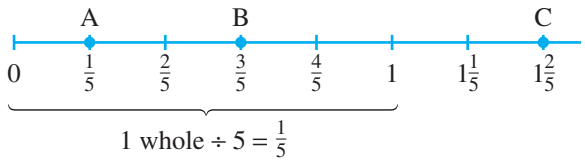
a

b

c

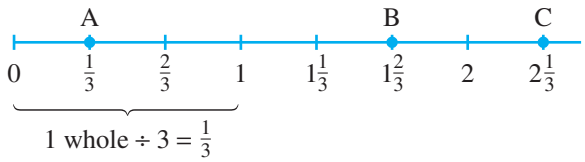
Solutions

a The interval between 0 and 1 is divided into 5 equal units, so each small unit represents the fraction $\frac{1}{5}$.



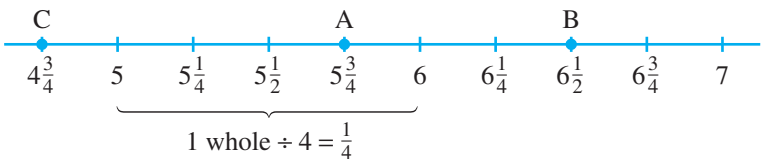
$\therefore A = \frac{1}{5},\ B = \frac{3}{5},\ C = 1\frac{2}{5}$

b The interval between 0 and 1 is divided into 3 equal units, so each small unit represents the fraction $\frac{1}{3}$.



$\therefore A = \frac{1}{3},\ B = 1\frac{2}{3},\ C = 2\frac{1}{3}$

c The interval between 5 and 6 is divided into 4 equal units, so each small unit represents the fraction $\frac{1}{4}$.



$\therefore A = 5\frac{3}{4}$

$B = 6\frac{2}{4}$

$= 6\frac{1}{2}$

$C = 5 - \frac{1}{4}$

$= 4\frac{3}{4}$

Exercise 4.6

1 State the fractions represented by A and B on each number line.

a

b

c

d

e

f

2 Write down the fraction or mixed numeral represented by P, Q and R on each number line.

a

b

c

d

e

f

Consolidation

3 Write down the fraction or mixed numeral represented by X, Y and Z on each number line.

a

b

c

d

4 Write down the fraction, mixed numeral or whole number whose position is marked by E, F, G and H on each number line.

a

b

c

d

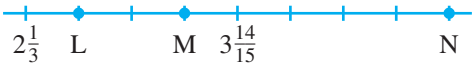
Further applications

CALCULATOR

5 Find the mixed numeral or whole number whose position is marked by L, M and N in each of these. You may need to use a calculator.

a

b



c

4.7 Expressing one quantity as a fraction of another

To express one quantity as a fraction of another:

- **rewrite the question, if necessary, in the form 'What fraction is □ of Δ?'**
- **express the quantities in the same units**
- **write $\dfrac{\text{first quantity}(\square)}{\text{second quantity}(\Delta)}$ and simplify.**

Example 1

EG+S

a What fraction is 5 of 20?

b What fraction is 8 of 12?

Solutions

a $\dfrac{\text{first quantity}}{\text{second quantity}} = \dfrac{\cancel{5}\,1}{\cancel{20}\,4}$

$= \dfrac{1}{4}$

b $\dfrac{\text{first quantity}}{\text{second quantity}} = \dfrac{\cancel{8}\,2}{\cancel{12}\,3}$

$= \dfrac{2}{3}$

Example 2

a What fraction is \$2.50 of \$4?

b What fraction is 50 minutes of $1\frac{1}{2}$ hours?

Solutions

a **i** \$4 = 400c

\$2.50 = 250c

ii $\dfrac{\text{first quantity}}{\text{second quantity}} = \dfrac{25\cancel{0}}{40\cancel{0}}$

$= \dfrac{\cancel{25}\,5}{\cancel{40}\,8}$

$= \dfrac{5}{8}$

b **i** $1\frac{1}{2}$ h $= 1\frac{1}{2} \times 60$ min

$= 90$ min

ii $\dfrac{\text{first quantity}}{\text{second quantity}} = \dfrac{5\cancel{0}\,5}{9\cancel{0}\,9}$

$= \dfrac{5}{9}$

Exercise 4.7

1 What fraction:

a is 5 of 10? **b** is 4 of 12? **c** is 5 of 20? **d** is 10 of 30?
e is 7 of 28? **f** is 8 of 56? **g** is 15 of 20? **h** is 16 of 24?
i is 27 of 33? **j** is 14 of 35? **k** is 25 of 40? **l** is 36 of 84?

2 What fraction:

a is 6 of 3? **b** is 10 of 2? **c** is 12 of 3?
d is 21 of 14? **e** is 15 of 12? **f** is 42 of 36?

Consolidation

3 What fraction:

a is \$0.30 of \$0.60? **b** is \$0.70 of \$1.20? **c** is \$0.50 of \$1.50?
d is \$1 of \$1.60? **e** is \$1.80 of \$2.40? **f** is \$2.50 of \$3.50?
g is \$1.50 of \$5? **h** is \$4 of \$6.40? **i** is \$3.50 of \$2?
j is \$2.10 of \$3.30? **k** is \$5.40 of \$6? **l** is \$2.40 of \$2.70?

4 What fraction:

a is 40 minutes of 1 hour? **b** is 25 minutes of 1 hour?
c is 9 hours of 1 day? **d** is 48 seconds of 1 minute?

e is 30 minutes of 2 hours?
f is 16 hours of 2 days?
g is 10 months of $1\frac{1}{2}$ years?
h is 1 minute of $2\frac{1}{2}$ minutes?
i is 50 seconds of $1\frac{1}{4}$ minutes?
j is 64 hours of 3 days?
k is $1\frac{1}{3}$ hours of 45 minutes?
l is $2\frac{1}{4}$ years of $1\frac{1}{2}$ years?

5 The petrol tank in Lenore's car holds 50 litres when full. If the tank contains 35 litres of petrol at present, what fraction of the tank is full?

6 Pat Rafter served 21 aces from his 280 serves during a tennis match. What fraction of the serves were aces?

7 A baker baked 5 dozen loaves of bread in the early morning. By ten o'clock she had sold 45 loaves. What fraction of the loaves did she sell?

8 Kat earns \$560 p.w. He pays \$210 p.w. in rent. What fraction of his weekly pay does Kat pay in rent?

9 If 120 out of 210 passengers got off the train at Central Station to go to work, what fraction of the passengers remained on the train?

10 Omar bought a new car for \$24 000 in 1999. The value of the car depreciated by \$4500 in 2000 and another \$3500 in 2001. What fraction of the car's original value remains?

11 Lloyd invested \$12 000 in Optel shares, then checked the share price twice during the following year. On the first occasion he found that the shares had increased in value by \$1500, however on the second occasion he found that they had decreased in value by \$3500. What fraction of the original share value was lost overall?

12 A customer paid a deposit of \$48 on a bicycle with a retail price of \$108. What fraction of the cost of the bicycle is still owed?

13 A farmer uses 150 ha of his 450 ha property to grow wheat, 120 ha to grow barley and the remaining area is used to graze cattle. What fraction of the property is used to graze cattle?

14 The ferry fare was increased from \$1.80 to \$2.20 by the state government. By what fraction was the fare increased?

■ Further applications

15 What fraction is one second of one day?

16 By drawing a diagram where necessary, find what fraction:

a $\frac{1}{2}$ is of $1\frac{1}{2}$
b $\frac{1}{4}$ is of $1\frac{3}{4}$
c $\frac{2}{3}$ is of 4
d $1\frac{1}{2}$ is of 6
e $\frac{3}{4}$ is of 2
f $\frac{5}{6}$ is of 3
g $\frac{4}{5}$ is of $1\frac{1}{5}$
h $2\frac{1}{3}$ is of $3\frac{1}{3}$

17 If $1\Omega = 3\Psi$ and $2\Omega = 5\Phi$, what fraction is Ψ of Φ?

4.8 Addition and subtraction of fractions with common denominators

To add or subtract fractions with common denominators:
- **add or subtract any whole numbers**
- **add or subtract the fractions**
- **express any improper fractions as mixed numerals**
- **simplify the fraction if possible.**

Example 1

Evaluate:

a $\frac{4}{11} + \frac{3}{11}$ **b** $\frac{16}{17} - \frac{4}{17}$ **c** $5 + \frac{1}{4}$

d $\frac{3}{8} + \frac{5}{8}$ **e** $4\frac{3}{5} - 1\frac{2}{5}$ **f** $3\frac{5}{9} + 7\frac{4}{9}$

Solutions

a $\frac{4}{11} + \frac{3}{11} = \frac{7}{11}$

b $\frac{16}{17} - \frac{4}{17} = \frac{12}{17}$

c $5 + \frac{1}{4} = 5\frac{1}{4}$

d $\frac{3}{8} + \frac{5}{8} = \frac{8}{8}$
$= 1$

e $4\frac{3}{5} - 1\frac{2}{5} = 3\frac{1}{5}$

f $3\frac{5}{9} + 7\frac{4}{9} = 10\frac{9}{9}$
$= 10 + 1$
$= 11$

Example 2

Evaluate:

a $\frac{9}{10} - \frac{3}{10}$ **b** $2\frac{8}{15} + 1\frac{4}{15}$ **c** $1\frac{6}{7} + 5\frac{4}{7}$

Solutions

a $\frac{9}{10} - \frac{3}{10} = \frac{\cancel{6}^{3}}{\cancel{10}_{5}}$
$= \frac{3}{5}$

b $2\frac{8}{15} + 1\frac{4}{15} = 3\frac{\cancel{12}^{4}}{\cancel{15}_{5}}$
$= 3\frac{4}{5}$

c $1\frac{6}{7} + 5\frac{4}{7} = 6\frac{10}{7}$
$= 6 + 1\frac{3}{7}$
$= 7\frac{3}{7}$

Exercise 4.8

1 Evaluate:

a $\frac{3}{7}+\frac{1}{7}$ **b** $\frac{7}{9}-\frac{2}{9}$ **c** $\frac{1}{3}+\frac{1}{3}$ **d** $\frac{9}{13}-\frac{6}{13}$

e $\frac{8}{11}-\frac{2}{11}$ **f** $\frac{1}{5}+\frac{3}{5}$ **g** $\frac{1}{8}-\frac{1}{8}$ **h** $\frac{2}{3}+\frac{1}{3}$

i $\frac{1}{4}+\frac{3}{4}$ **j** $\frac{3}{10}-\frac{3}{10}$ **k** $\frac{5}{12}-\frac{5}{12}$ **l** $\frac{5}{6}+\frac{1}{6}$

m $5+\frac{1}{2}$ **n** $\frac{2}{3}+4$ **o** $2+\frac{4}{5}$ **p** $\frac{7}{8}+3$

2 Evaluate:

a $3\frac{1}{4}+2$ **b** $7\frac{2}{3}-1$ **c** $4+1\frac{6}{7}$ **d** $6\frac{1}{2}-2$

e $2\frac{1}{7}+\frac{3}{7}$ **f** $9\frac{4}{5}-\frac{3}{5}$ **g** $\frac{2}{9}+1\frac{5}{9}$ **h** $\frac{1}{3}+7\frac{1}{3}$

i $5\frac{10}{11}-\frac{4}{11}$ **j** $12\frac{3}{5}+\frac{1}{5}$ **k** $3\frac{1}{4}-3$ **l** $1\frac{1}{2}-\frac{1}{2}$

m $6\frac{1}{10}-6$ **n** $4\frac{6}{7}-1\frac{1}{7}$ **o** $3\frac{4}{9}+2\frac{4}{9}$ **p** $3\frac{10}{13}+5\frac{2}{13}$

3 Evaluate:

a $1\frac{1}{2}+\frac{1}{2}$ **b** $2\frac{3}{4}+\frac{1}{4}$ **c** $5\frac{1}{3}+\frac{2}{3}$ **d** $4\frac{5}{8}+\frac{3}{8}$

e $7\frac{1}{6}+\frac{5}{6}$ **f** $11\frac{4}{13}+\frac{9}{13}$ **g** $2\frac{1}{2}+1\frac{1}{2}$ **h** $4\frac{2}{3}+1\frac{1}{3}$

i $3\frac{2}{5}+4\frac{3}{5}$ **j** $5\frac{1}{6}+5\frac{5}{6}$ **k** $2\frac{4}{7}+1\frac{3}{7}$ **l** $7\frac{5}{12}+2\frac{7}{12}$

Consolidation

4 Simplify:

a $\frac{3}{8}+\frac{1}{8}$ **b** $\frac{3}{4}-\frac{1}{4}$ **c** $\frac{7}{10}-\frac{3}{10}$ **d** $\frac{1}{12}+\frac{1}{12}$

e $\frac{5}{6}-\frac{1}{6}$ **f** $\frac{5}{14}+\frac{3}{14}$ **g** $\frac{13}{20}+\frac{3}{20}$ **h** $\frac{2}{15}+\frac{7}{15}$

i $\frac{11}{16}-\frac{5}{16}$ **j** $\frac{19}{24}-\frac{1}{24}$ **k** $\frac{17}{18}-\frac{5}{18}$ **l** $\frac{10}{21}+\frac{8}{21}$

5 Simplify:

a $3\frac{1}{9} + 1\frac{2}{9}$ **b** $8\frac{5}{6} - 3\frac{1}{6}$ **c** $2\frac{4}{15} + 1\frac{2}{15}$

d $4\frac{9}{10} - 3\frac{1}{10}$ **e** $1\frac{7}{24} + 5\frac{13}{24}$ **f** $2\frac{10}{21} + 9\frac{4}{21}$

g $7\frac{29}{30} - 1\frac{7}{30}$ **h** $6\frac{17}{18} - 2\frac{5}{18}$ **i** $4\frac{1}{20} + 2\frac{17}{20}$

Further applications

6 Simplify:

a $\frac{9}{10} + \frac{3}{10}$ **b** $\frac{11}{12} + \frac{5}{12}$ **c** $\frac{17}{20} + \frac{7}{20}$

d $\frac{7}{8} + \frac{5}{8}$ **e** $1\frac{5}{6} + 2\frac{5}{6}$ **f** $2\frac{9}{10} + 3\frac{7}{10}$

g $4\frac{9}{14} + 2\frac{11}{14}$ **h** $1\frac{11}{15} + 1\frac{14}{15}$ **i** $6\frac{19}{20} + 8\frac{11}{20}$

4.9 Further subtraction of fractions

In this exercise we will look at two special cases involving the subtraction of fractions.

1 Finding the difference between a whole number and a mixed numeral.

2 Finding the difference between two mixed numerals, where the second fraction is greater than the first.

To subtract a fraction or mixed numeral, where the second fraction is larger than the first:

- **subtract 1 from the first whole number and add it on to the adjacent fraction if there is one**
- **subtract the whole numbers and the fractions**
- **simplify where possible.**

Example 1

Evaluate:

a $1 - \frac{3}{8}$ **b** $6 - \frac{4}{7}$ **c** $4 - 1\frac{5}{12}$

Solutions

a $1 - \frac{3}{8} = \frac{8}{8} - \frac{3}{8}$

$= \frac{5}{8}$

b $6 - \frac{4}{7} = 5\frac{7}{7} - \frac{4}{7}$

$= 5\frac{3}{7}$

c $4 - 1\frac{5}{12} = 3\frac{12}{12} - 1\frac{5}{12}$

$= 2\frac{7}{12}$

Example 2

EG+S

Evaluate:

a $1\frac{4}{9} - \frac{5}{9}$

b $7\frac{2}{5} - 2\frac{4}{5}$

c $5\frac{3}{10} - 3\frac{9}{10}$

Solutions

a $1\frac{4}{9} - \frac{5}{9}$

$= \frac{13}{9} - \frac{5}{9}$

$= \frac{8}{9}$

b $7\frac{2}{5} - 2\frac{4}{5}$

$= \left(6\frac{5}{5} + \frac{2}{5}\right) - 2\frac{4}{5}$

$= 6\frac{7}{5} - 2\frac{4}{5}$

$= 4\frac{3}{5}$

c $5\frac{3}{10} - 3\frac{9}{10}$

$= \left(4\frac{10}{10} + \frac{3}{10}\right) - 3\frac{9}{10}$

$= 4\frac{13}{10} - 3\frac{9}{10}$

$= 1\frac{\cancel{4}^{2}}{\cancel{10}_{5}}$

$= 1\frac{2}{5}$

Exercise 4.9

1 Evaluate:

a $1 - \frac{1}{2}$ **b** $1 - \frac{1}{4}$ **c** $1 - \frac{1}{3}$ **d** $1 - \frac{1}{6}$

e $1 - \frac{3}{10}$ **f** $1 - \frac{5}{7}$ **g** $1 - \frac{2}{5}$ **h** $1 - \frac{7}{12}$

2 Evaluate:

a $2 - \frac{1}{2}$ **b** $3 - \frac{1}{5}$ **c** $5 - \frac{1}{3}$ **d** $7 - \frac{1}{4}$

e $3 - \frac{5}{6}$ **f** $4 - \frac{2}{3}$ **g** $6 - \frac{7}{10}$ **h** $9 - \frac{3}{7}$

i $2 - \frac{8}{11}$ **j** $5 - \frac{3}{13}$ **k** $7 - \frac{3}{4}$ **l** $11 - \frac{2}{9}$

Consolidation

3 Evaluate:

a $2 - 1\frac{1}{2}$ **b** $4 - 1\frac{1}{3}$ **c** $3 - 1\frac{2}{5}$ **d** $6 - 1\frac{7}{8}$

e $7 - 2\frac{1}{3}$ **f** $5 - 3\frac{3}{4}$ **g** $10 - 3\frac{5}{8}$ **h** $4 - 2\frac{7}{12}$

i $6 - 1\frac{4}{9}$ **j** $8 - 4\frac{5}{6}$ **k** $11 - 5\frac{3}{10}$ **l** $15 - 4\frac{13}{16}$

4 Evaluate each of the following.

a $1\frac{1}{3} - \frac{2}{3}$ **b** $1\frac{5}{9} - \frac{7}{9}$ **c** $1\frac{4}{11} - \frac{8}{11}$ **d** $1\frac{2}{5} - \frac{4}{5}$

e $2\frac{1}{13} - \frac{5}{13}$ **f** $3\frac{3}{5} - \frac{4}{5}$ **g** $4\frac{3}{7} - \frac{5}{7}$ **h** $6\frac{1}{3} - \frac{2}{3}$

i $5\frac{7}{15} - \frac{11}{15}$ **j** $7\frac{2}{11} - \frac{8}{11}$ **k** $8\frac{6}{17} - \frac{9}{17}$ **l** $4\frac{13}{19} - \frac{15}{19}$

5 Evaluate each of these.

a $5\frac{1}{7} - 4\frac{2}{7}$ **b** $4\frac{2}{5} - 1\frac{3}{5}$ **c** $6\frac{9}{13} - 1\frac{11}{13}$ **d** $7\frac{1}{3} - 2\frac{2}{3}$

e $6\frac{2}{9} - 2\frac{4}{9}$ **f** $3\frac{4}{15} - 2\frac{11}{15}$ **g** $7\frac{2}{5} - 4\frac{3}{5}$ **h** $10\frac{1}{17} - 3\frac{4}{17}$

Further applications

6 Simplify:

a $2\frac{1}{8} - 1\frac{3}{8}$ **b** $4\frac{3}{10} - 1\frac{9}{10}$ **c** $5\frac{7}{20} - 2\frac{13}{20}$

d $3\frac{1}{6} - 1\frac{5}{6}$ **e** $6\frac{3}{8} - 2\frac{7}{8}$ **f** $5\frac{7}{18} - 1\frac{13}{18}$

g $8\frac{7}{12} - 2\frac{11}{12}$ **h** $4\frac{1}{15} - 3\frac{7}{15}$ **i** $6\frac{7}{20} - 3\frac{13}{20}$

4.10 Addition and subtraction of fractions with different denominators

If two fractions have different denominators, then either one or both fractions must be changed into equivalent fractions with a common denominator, before they can be added or subtracted.

To add or subtract fractions with different denominators:
- **add or subtract any whole numbers**
- **express the fractions with the lowest common denominator (LCD)**
- **add or subtract the fractions**
- **simplify where possible.**

Example 1

Evaluate:

a $\frac{7}{20} + \frac{3}{5}$ **b** $\frac{5}{6} - \frac{2}{9}$ **c** $8\frac{9}{10} - 2\frac{4}{15}$

Solutions

a The LCD of 20 and 5 is 20.

$$\frac{7}{20} + \frac{3 \times 4}{5 \times 4}$$
$$= \frac{7}{20} + \frac{12}{20}$$
$$= \frac{19}{20}$$

b The LCD of 6 and 9 is 18.

$$\frac{5 \times 3}{6 \times 3} - \frac{2 \times 2}{9 \times 2}$$
$$= \frac{15}{18} - \frac{4}{18}$$
$$= \frac{11}{18}$$

c The LCD of 10 and 15 is 30.

$$8\frac{9}{10} - 2\frac{4}{15}$$
$$= 6\frac{9 \times 3}{10 \times 3} - \frac{4 \times 2}{15 \times 2}$$
$$= 6\frac{27}{30} - \frac{8}{30}$$
$$= 6\frac{19}{30}$$

Example 2

Evaluate:

a $\frac{5}{12} + \frac{7}{9}$

b $1\frac{4}{5} + 3\frac{8}{15}$

Solutions

a
$$\frac{5 \times 3}{12 \times 3} + \frac{7 \times 4}{9 \times 4}$$
$$= \frac{15}{36} + \frac{28}{36}$$
$$= \frac{43}{36}$$
$$= 1\frac{7}{36}$$

b
$$1\frac{4}{5} + 3\frac{8}{15}$$
$$= 4\frac{4 \times 3}{5 \times 3} + \frac{8}{15}$$
$$= 4\frac{12}{15} + \frac{8}{15}$$
$$= 4\frac{20}{15}$$
$$= 4 + 1\frac{\cancel{5}^{1}}{\cancel{15}_{3}}$$
$$= 5\frac{1}{3}$$

Exercise 4.10

1 **a** Explain with the use of a diagram why $\frac{1}{2} + \frac{1}{3} \neq \frac{2}{5}$.
b What is the correct answer?

2 Evaluate:

a $\frac{1}{2} + \frac{1}{4}$ **b** $\frac{3}{10} - \frac{1}{5}$ **c** $\frac{7}{8} - \frac{1}{4}$ **d** $\frac{3}{10} + \frac{2}{5}$

e $\frac{5}{6} - \frac{2}{3}$ **f** $\frac{11}{12} - \frac{1}{3}$ **g** $\frac{1}{4} + \frac{9}{16}$ **h** $\frac{19}{20} - \frac{2}{5}$

i $\frac{29}{30} - \frac{3}{5}$ **j** $\frac{5}{18} + \frac{4}{9}$ **k** $\frac{19}{24} - \frac{3}{4}$ **l** $\frac{2}{21} + \frac{2}{3}$

3 Evaluate:

a $\frac{1}{3} + \frac{1}{2}$ **b** $\frac{1}{4} + \frac{1}{3}$ **c** $\frac{1}{5} - \frac{1}{8}$ **d** $\frac{2}{5} - \frac{1}{3}$

e $\frac{1}{2} + \frac{4}{9}$ **f** $\frac{4}{5} - \frac{3}{4}$ **g** $\frac{1}{4} + \frac{1}{6}$ **h** $\frac{3}{4} - \frac{7}{10}$

i $\frac{5}{8} + \frac{1}{6}$ **j** $\frac{5}{6} - \frac{7}{9}$ **k** $\frac{3}{8} + \frac{5}{12}$ **l** $\frac{7}{15} + \frac{3}{10}$

Consolidation

4 Evaluate:

a $2\frac{1}{6} + 1\frac{2}{3}$ **b** $3\frac{11}{12} - 1\frac{5}{6}$ **c** $4\frac{7}{20} + 1\frac{1}{2}$ **d** $1\frac{1}{4} + 2\frac{5}{8}$

e $7\frac{19}{20} - 2\frac{4}{5}$ **f** $5\frac{2}{3} - 1\frac{2}{9}$ **g** $1\frac{2}{9} + 1\frac{1}{2}$ **h** $5\frac{6}{7} - 1\frac{2}{3}$

i $6\frac{3}{4} - 3\frac{2}{5}$ **j** $2\frac{1}{10} + 4\frac{3}{4}$ **k** $3\frac{7}{8} - 2\frac{5}{6}$ **l** $2\frac{5}{12} + 1\frac{4}{9}$

5 Evaluate each of these.

a $\frac{3}{5} + \frac{7}{10}$ **b** $\frac{13}{15} + \frac{2}{5}$ **c** $\frac{2}{3} + \frac{3}{4}$ **d** $\frac{7}{9} + \frac{1}{2}$ **e** $\frac{3}{4} + \frac{9}{10}$

f $\frac{5}{12} + \frac{5}{8}$ **g** $\frac{1}{2} + \frac{8}{11}$ **h** $\frac{5}{8} + \frac{2}{3}$ **i** $\frac{11}{12} + \frac{2}{5}$ **j** $\frac{7}{9} + \frac{5}{6}$

6 Evaluate each of the following.

a $1\frac{5}{12} + 1\frac{2}{3}$ **b** $2\frac{1}{2} + 1\frac{7}{12}$ **c** $1\frac{3}{4} + 1\frac{6}{7}$ **d** $1\frac{4}{5} + 2\frac{5}{6}$

e $2\frac{9}{10} + 4\frac{5}{12}$ **f** $3\frac{5}{6} + 1\frac{7}{8}$ **g** $1\frac{13}{22} + 1\frac{7}{11}$ **h** $2\frac{11}{15} + 1\frac{5}{6}$

i $3\frac{4}{5} + 2\frac{7}{15}$ j $1\frac{2}{3} + 2\frac{3}{4}$ k $3\frac{5}{6} + 1\frac{2}{7}$ l $1\frac{4}{9} + 5\frac{3}{4}$

7 Simplify:

a $\frac{1}{6} + \frac{1}{2}$ b $\frac{1}{3} - \frac{1}{12}$ c $\frac{17}{20} - \frac{3}{5}$ d $\frac{1}{3} + \frac{7}{15}$

e $\frac{1}{11} + \frac{9}{22}$ f $\frac{7}{12} - \frac{1}{3}$ g $\frac{11}{12} + \frac{3}{4}$ h $\frac{5}{6} + \frac{2}{3}$

i $\frac{23}{24} + \frac{3}{8}$ j $\frac{11}{15} + \frac{2}{3}$ k $\frac{13}{24} + \frac{5}{8}$ l $\frac{2}{5} + \frac{17}{20}$

8 Simplify:

a $2\frac{11}{12} - 1\frac{1}{4}$ b $3\frac{17}{24} - 1\frac{3}{8}$ c $4\frac{3}{5} + 2\frac{3}{20}$

d $1\frac{9}{10} + 1\frac{3}{5}$ e $2\frac{17}{18} - 1\frac{1}{6}$ f $5\frac{2}{3} + 3\frac{11}{15}$

Further applications

9 Simplify:

a $3\frac{1}{4} - 1\frac{2}{3}$ b $4\frac{1}{2} - 1\frac{7}{9}$ c $4\frac{5}{6} - 2\frac{8}{9}$ d $3\frac{3}{8} - 2\frac{7}{12}$

e $7\frac{3}{5} - 3\frac{5}{6}$ f $5\frac{5}{8} - 2\frac{2}{3}$ g $6\frac{3}{4} - 1\frac{5}{6}$ h $3\frac{1}{12} - 1\frac{5}{8}$

i $7\frac{3}{10} - 2\frac{5}{8}$ j $3\frac{2}{3} - 1\frac{4}{5}$ k $4\frac{1}{3} - 2\frac{4}{7}$ l $6\frac{5}{12} - 1\frac{9}{10}$

4.11 Multiplication of fractions

To multiply fractions:

- **express any mixed numerals as improper fractions**
- **divide the terms in the numerator and denominator by their HCF, vertically or diagonally**
- **multiply the numerators, then multiply the denominators**
- **express any improper fractions as mixed numerals.**

Example 1

Evaluate:

a $\frac{3}{4} \times \frac{5}{8}$

b $\frac{1}{3} \times 4$

c $3 \times 2\frac{1}{2}$

Solutions

a $\frac{3}{4} \times \frac{5}{8}$

$= \frac{3 \times 5}{4 \times 8}$

$= \frac{15}{32}$

b $\frac{1}{3} \times \frac{4}{1}$

$= \frac{1 \times 4}{3 \times 1}$

$= \frac{4}{3}$

$= 1\frac{1}{3}$

c $3 \times 2\frac{1}{2}$

$= \frac{3}{1} \times \frac{5}{2}$

$= \frac{15}{2}$

$= 7\frac{1}{2}$

Example 2

Evaluate:

a $\frac{3}{10} \times \frac{5}{9}$

b $\frac{15}{16} \times \frac{20}{21}$

c $3\frac{3}{4} \times 1\frac{3}{5}$

Solutions

a $\frac{\cancel{3}^{1}}{\cancel{10}_{2}} \times \frac{\cancel{5}^{1}}{\cancel{9}_{3}}$

$= \frac{1 \times 1}{2 \times 3}$

$= \frac{1}{6}$

b $\frac{\cancel{15}^{5}}{\cancel{16}_{4}} \times \frac{\cancel{20}^{5}}{\cancel{21}_{7}}$

$= \frac{5 \times 5}{4 \times 7}$

$= \frac{25}{28}$

c $3\frac{3}{4} \times 1\frac{3}{5}$

$= \frac{\cancel{15}^{3}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{5}_{1}}$

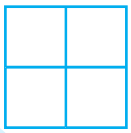
$= \frac{3 \times 2}{1 \times 1}$

$= 6$

Exercise 4.11

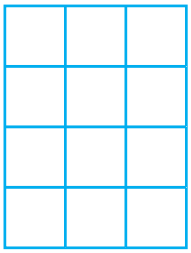
1 Evaluate each of the products below by shading squares. (Each rectangle represents one whole.)

a



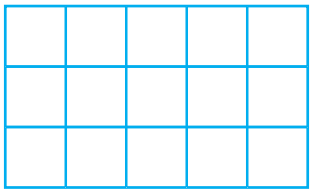
$\frac{1}{2} \times \frac{1}{2}$

b



$\frac{1}{3} \times \frac{1}{4}$

c



$\frac{2}{3} \times \frac{4}{5}$

2 Evaluate:

a $\frac{1}{2} \times \frac{1}{4}$ **b** $\frac{1}{3} \times \frac{1}{2}$ **c** $\frac{2}{5} \times \frac{3}{7}$ **d** $\frac{3}{4} \times \frac{9}{11}$ **e** $\frac{3}{4} \times \frac{7}{10}$

f $\frac{5}{7} \times \frac{3}{8}$ **g** $\frac{1}{6} \times \frac{7}{8}$ **h** $\frac{2}{3} \times \frac{5}{9}$ **i** $\frac{7}{9} \times \frac{4}{5}$ **j** $\frac{11}{12} \times \frac{5}{6}$

3 Evaluate:

a $5 \times \frac{1}{8}$ **b** $\frac{1}{3} \times 2$ **c** $\frac{1}{7} \times 4$ **d** $3 \times \frac{1}{5}$

e $\frac{1}{3} \times 5$ **f** $7 \times \frac{1}{2}$ **g** $\frac{1}{5} \times 6$ **h** $11 \times \frac{1}{4}$

i $\frac{3}{4} \times 7$ **j** $5 \times \frac{4}{9}$ **k** $\frac{2}{3} \times 8$ **l** $12 \times \frac{3}{5}$

4 Evaluate each of the following.

a $3 \times 1\frac{1}{4}$ **b** $5 \times 1\frac{1}{6}$ **c** $2\frac{3}{10} \times 3$ **d** $4\frac{2}{5} \times 2$

e $2 \times 2\frac{3}{5}$ **f** $1\frac{3}{7} \times 8$ **g** $4 \times 1\frac{2}{3}$ **h** $2\frac{1}{4} \times 7$

■ Consolidation

5 Simplify each of the following by cancelling all common factors and then multiplying.

a $\frac{3}{4} \times \frac{5}{6}$ **b** $\frac{11}{12} \times \frac{4}{7}$ **c** $\frac{3}{5} \times \frac{8}{15}$ **d** $\frac{7}{10} \times \frac{5}{14}$

e $\frac{11}{21} \times \frac{7}{22}$ **f** $\frac{5}{12} \times \frac{3}{20}$ **g** $\frac{10}{21} \times \frac{14}{15}$ **h** $\frac{14}{27} \times \frac{18}{35}$

i $\frac{15}{26} \times \frac{13}{20}$ **j** $\frac{42}{55} \times \frac{33}{49}$ **k** $\frac{24}{35} \times \frac{7}{36}$ **l** $\frac{15}{28} \times \frac{16}{21}$

m $\frac{33}{40} \times \frac{35}{44}$ **n** $\frac{21}{44} \times \frac{20}{49}$ **o** $\frac{11}{24} \times \frac{20}{33}$ **p** $\frac{35}{39} \times \frac{13}{60}$

6 Simplify each of these.

a $\frac{3}{5} \times 1\frac{4}{11}$ **b** $\frac{4}{7} \times 1\frac{1}{20}$ **c** $1\frac{7}{25} \times \frac{5}{8}$ **d** $\frac{7}{9} \times 1\frac{7}{20}$

e $5 \times 1\frac{4}{15}$ **f** $2\frac{1}{3} \times 6$ **g** $2\frac{2}{5} \times 1\frac{7}{8}$ **h** $3\frac{3}{4} \times 1\frac{1}{5}$

i $1\frac{1}{6} \times 3\frac{1}{3}$ **j** $2\frac{2}{9} \times 1\frac{1}{5}$ **k** $1\frac{4}{21} \times 1\frac{13}{15}$ **l** $1\frac{17}{28} \times 1\frac{1}{20}$

7 What is the effect of multiplying a number by a fraction that is:

a less than 1? **b** equal to 1? **c** greater than 1?

Further applications

8 Simplify:

a $\frac{3}{5} \times \frac{10}{21} \times \frac{7}{16}$ **b** $\frac{4}{15} \times \frac{5}{16} \times \frac{12}{13}$ **c** $\frac{3}{4} \times \frac{14}{15} \times \frac{20}{21}$

d $\frac{18}{25} \times 1\frac{11}{24} \times \frac{8}{9}$ **e** $\frac{12}{45} \times \frac{25}{36} \times 2\frac{2}{5}$ **f** $1\frac{7}{8} \times \frac{16}{21} \times 1\frac{13}{15}$

4.12 Division of fractions

To divide fractions, we first need to change the question into a multiplication. This will require taking the **reciprocal** of one of the fractions. To find the reciprocal of a fraction, swap the numbers in the numerator and denominator. For example, the reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$. In some simple questions, however, answers can be found by inspection.

To divide fractions:
- **express any mixed numerals as improper fractions**
- **change the division sign to multiplication**
- **take the reciprocal of the fraction which follows the division sign**
- **proceed as for the multiplication of fractions.**

Example 1

Evaluate each of these by inspection.

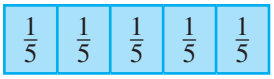
a $1 \div \frac{1}{5}$

b $2 \div \frac{1}{3}$

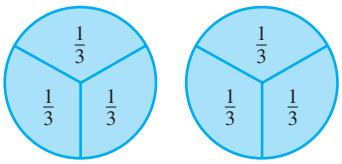
c $1\frac{1}{2} \div \frac{1}{4}$

Solutions

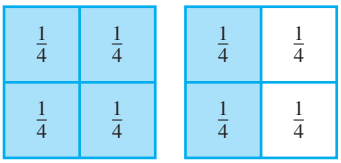
a $1 \div \frac{1}{5}$ can be interpreted as 'how many fifths are there in 1 whole?' Now, there are 5 fifths in 1 whole, $\therefore 1 \div \frac{1}{5} = 5$.



b $2 \div \frac{1}{3}$ can be interpreted as 'how many thirds are there in 2 wholes?' Now, there are 6 thirds in 2 wholes, $\therefore 2 \div \frac{1}{3} = 6$.



c $1\frac{1}{2} \div \frac{1}{4}$ can be interpreted as 'how many quarters are there in 1 whole and a half?' Now, there are 4 quarters in 1 whole and 2 quarters in one half, $\therefore 1\frac{1}{2} \div \frac{1}{4} = 6$.



Example 2

Evaluate:

a $\frac{4}{9} \div \frac{5}{7}$ **b** $\frac{7}{8} \div 3$ **c** $\frac{35}{99} \div \frac{49}{90}$ **d** $1\frac{3}{5} \div \frac{16}{25}$

Solutions

a $\frac{4}{9} \div \frac{5}{7}$

$= \frac{4}{9} \times \frac{7}{5}$

$= \frac{28}{45}$

b $\frac{7}{8} \div \frac{3}{1}$

$= \frac{7}{8} \times \frac{1}{3}$

$= \frac{7}{24}$

c $\frac{35}{99} \div \frac{49}{90}$

$= \frac{\cancel{35}^{5}}{\cancel{99}_{11}} \times \frac{\cancel{90}^{10}}{\cancel{49}_{7}}$

$= \frac{50}{77}$

d $1\frac{3}{5} \div \frac{16}{25}$

$= \frac{8}{5} \div \frac{16}{25}$

$= \frac{\cancel{8}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{16}_{2}}$

$= \frac{5}{2}$

$= 2\frac{1}{2}$

Exercise 4.12

1 Find the reciprocal of each of the following.

a $\frac{3}{4}$ **b** $\frac{5}{11}$ **c** $\frac{2}{3}$ **d** $\frac{4}{9}$ **e** $\frac{7}{12}$

f $\frac{1}{2}$ **g** $\frac{1}{4}$ **h** 3 **i** 5 **j** 9

k $1\frac{1}{2}$ **l** $1\frac{2}{3}$ **m** $2\frac{3}{5}$ **n** $3\frac{1}{7}$ **o** $4\frac{3}{4}$

2 Evaluate each of these by inspection.

a $1 \div \frac{1}{2}$ **b** $1 \div \frac{1}{3}$ **c** $1 \div \frac{1}{5}$ **d** $1 \div \frac{1}{8}$

e $2 \div \frac{1}{2}$ **f** $2 \div \frac{1}{4}$ **g** $3 \div \frac{1}{3}$ **h** $4 \div \frac{1}{5}$

i $5 \div \frac{1}{3}$ **j** $2 \div \frac{1}{6}$ **k** $7 \div \frac{1}{4}$ **l** $6 \div \frac{1}{8}$

m $1\frac{1}{2} \div \frac{1}{2}$ **n** $1\frac{2}{3} \div \frac{1}{3}$ **o** $1\frac{3}{4} \div \frac{1}{4}$ **p** $2\frac{1}{2} \div \frac{1}{4}$

3 a Find:

i $1 \div \frac{1}{4}$ and 1×4 **ii** $2 \div \frac{1}{3}$ and 2×3

iii $3 \div 4$ and $3 \times \frac{1}{4}$ **iv** $5 \div 2$ and $5 \times \frac{1}{2}$

b Is division by a fraction or integer equivalent to multiplication by its reciprocal?

4 Evaluate:

a $\frac{3}{7} \div \frac{4}{5}$ **b** $\frac{2}{13} \div \frac{1}{3}$ **c** $\frac{2}{3} \div \frac{3}{4}$ **d** $\frac{7}{10} \div \frac{8}{9}$ **e** $\frac{5}{9} \div \frac{6}{7}$

f $\frac{3}{14} \div \frac{5}{11}$ **g** $\frac{5}{8} \div \frac{4}{5}$ **h** $\frac{4}{7} \div \frac{5}{6}$ **i** $\frac{2}{7} \div \frac{3}{4}$ **j** $\frac{5}{12} \div \frac{8}{11}$

■ Consolidation

5 Evaluate:

a $\frac{5}{9} \div \frac{2}{5}$ **b** $\frac{10}{11} \div \frac{3}{4}$ **c** $\frac{5}{8} \div \frac{3}{7}$ **d** $\frac{9}{11} \div \frac{5}{7}$ **e** $\frac{5}{12} \div \frac{2}{7}$

6 Evaluate:

a $\frac{1}{3} \div 5$ **b** $\frac{3}{4} \div 2$ **c** $\frac{5}{8} \div 3$ **d** $3 \div \frac{2}{5}$ **e** $4 \div \frac{3}{4}$ **f** $2 \div \frac{5}{6}$

7 Simplify each of the following.

a $\frac{3}{8} \div \frac{9}{16}$ **b** $\frac{27}{50} \div \frac{9}{10}$ **c** $\frac{21}{44} \div \frac{28}{33}$ **d** $\frac{15}{49} \div \frac{25}{42}$

e $\frac{14}{27} \div \frac{49}{66}$ **f** $\frac{40}{99} \div \frac{45}{88}$ **g** $\frac{2}{3} \div \frac{8}{15}$ **h** $\frac{12}{25} \div \frac{2}{5}$

i $\frac{6}{7} \div \frac{36}{77}$ **j** $\frac{16}{21} \div \frac{24}{35}$ **k** $\frac{40}{49} \div \frac{32}{63}$ **l** $\frac{16}{35} \div \frac{24}{55}$

8 Simplify each of these.

a $\frac{5}{6} \div 1\frac{1}{3}$ **b** $\frac{10}{11} \div 1\frac{7}{33}$ **c** $1\frac{1}{14} \div \frac{20}{21}$ **d** $5\frac{3}{5} \div 2\frac{13}{25}$

e $1\frac{13}{35} \div \frac{36}{77}$ **f** $1\frac{1}{7} \div 1\frac{19}{21}$ **g** $1\frac{5}{7} \div 1\frac{11}{49}$ **h** $1\frac{7}{15} \div 4\frac{7}{12}$

■ Further applications

9 Simplify:

a $\frac{3}{5} \times \frac{7}{16} \div \frac{9}{10}$ **b** $\frac{4}{7} \times \frac{15}{25} \div 1\frac{11}{21}$ **c** $\frac{12}{35} \div \frac{18}{25} \div \frac{10}{21}$

d $\frac{25}{28} \times 1\frac{19}{30} \div 1\frac{13}{42}$ **e** $1\frac{7}{33} \div \frac{50}{77} \times \frac{15}{24}$ **f** $1\frac{1}{20} \div 1\frac{1}{26} \div 2\frac{3}{5}$

4.13 Finding a fraction of a quantity

To find a fraction of a quantity:
- **convert the quantity to a smaller unit if necessary**
- **multiply the fraction by the quantity.**

The conversions below will be needed in the following exercise. They will be treated in more depth in later chapters.

10 mm = 1 cm	**1000 mg = 1 g**	**1000 mL = 1 L**
100 cm = 1 m	**1000 g = 1 kg**	**1000 L = 1 kL**
1000 m = 1 km	**1000 kg = 1 t**	
60 s = 1 min	**10 years = 1 decade**	
60 min = 1 hour	**100 years = 1 century**	
24 hours = 1 day		

Example

Find each of the following.

a $\frac{9}{20}$ of \$3 **b** $\frac{7}{25}$ of 5 kg **c** $\frac{3}{5}$ of $1\frac{1}{2}$ hours

Solutions

a $\frac{9}{20}$ of \$3

$= \frac{9}{\cancel{20}_1} \times \cancel{300}^{15}$c

$= 135$c

$= \$1.35$

b $\frac{7}{25}$ of 5 kg

$= \frac{7}{\cancel{25}_1} \times \cancel{5000}^{200}$ g

$= 1400$ g

c $\frac{3}{5}$ of $1\frac{1}{2}$ hours

$= \frac{3}{\cancel{5}_1} \times \cancel{90}^{18}$ min

$= 54$ min

Exercise 4.13

1 Find each of the following.

a $\frac{1}{2}$ of \$1 **b** $\frac{1}{10}$ of \$1 **c** $\frac{1}{4}$ of \$1 **d** $\frac{1}{5}$ of \$2

e $\frac{3}{10}$ of \$5 **f** $\frac{3}{4}$ of \$12 **g** $\frac{7}{20}$ of \$3 **h** $\frac{11}{25}$ of \$4

i $\frac{9}{50}$ of \$7 **j** $\frac{2}{3}$ of \$2.70 **k** $\frac{3}{4}$ of \$3.20 **l** $\frac{4}{5}$ of \$4.50

2 Find each of the following.

a $\frac{1}{3}$ of 1 hour **b** $\frac{1}{2}$ of 1 minute **c** $\frac{1}{4}$ of 1 day **d** $\frac{3}{4}$ of 1 hour

e $\frac{2}{5}$ of 1 decade **f** $\frac{7}{20}$ of 1 century **g** $\frac{2}{3}$ of 2 minutes **h** $\frac{5}{8}$ of 2 days

i $\frac{9}{10}$ of 5 hours **j** $\frac{4}{5}$ of $1\frac{1}{2}$ hours **k** $\frac{2}{3}$ of $2\frac{1}{2}$ days **l** $\frac{3}{25}$ of 2 centuries

3 Find each of the following.

a $\frac{1}{2}$ of 1 cm **b** $\frac{1}{10}$ of 1 m **c** $\frac{1}{4}$ of 1 km **d** $\frac{9}{10}$ of 1 m

e $\frac{3}{5}$ of 1 cm **f** $\frac{7}{20}$ of 1 km **g** $\frac{3}{4}$ of 2 cm **h** $\frac{4}{25}$ of 5 km

i $\frac{5}{6}$ of 3 m **j** $\frac{2}{5}$ of $3\frac{1}{2}$ cm **k** $\frac{7}{25}$ of $1\frac{1}{2}$ m **l** $\frac{3}{4}$ of $2\frac{1}{2}$ km

4 Find each of the following.

a $\frac{1}{10}$ of 1 kg **b** $\frac{1}{5}$ of 1 g **c** $\frac{1}{2}$ of 1 t **d** $\frac{2}{5}$ of 1 L

e $\frac{1}{2}$ of 1 kL **f** $\frac{3}{4}$ of 1 L **g** $\frac{4}{5}$ of 2 kg **h** $\frac{3}{10}$ of 7 g

i $\frac{7}{20}$ of 3 kL **j** $\frac{3}{5}$ of $2\frac{1}{2}$ L **k** $\frac{2}{3}$ of $4\frac{1}{2}$ t **l** $\frac{9}{50}$ of $1\frac{1}{2}$ kL

Consolidation

5 Ariane is a fruit picker. Of the first 112 apples she picked, $\frac{2}{7}$ had gone bad. How many of the apples picked were bad?

6 George spent $\frac{5}{12}$ of the day sleeping. For how many hours was he asleep?

7 Keith got 5 out of every 8 questions correct on his science test. If there were 72 questions on the test, find the number that Keith got correct.

8 Sing-Pey had to drive 900 km from Sydney to Melbourne. She stopped to have lunch after $\frac{7}{10}$ of the journey was completed. How far does she still have to drive?

9 A crowd of 24 000 people turned up to watch a Rugby League match between Brisbane and Parramatta. If $\frac{3}{8}$ of the supporters were Brisbane fans, find the number of Parramatta supporters in the crowd.

10 Alicia spent $1\frac{1}{4}$ hours shopping at the local mall and $\frac{2}{5}$ of this time was spent in the first shop. How much longer did Alicia spend at the mall after leaving the first shop?

11 A carpenter purchased a $1\frac{1}{2}$ metre length of timber. He used $\frac{1}{5}$ of this to make a chair leg and $\frac{3}{8}$ of the remainder to make a picture frame. What length of timber remains unused?

12 The capacity of a backyard swimming pool is 60 kL. If the pool is initially $\frac{3}{4}$ full, and then $\frac{2}{15}$ of the water is lost due to evaporation over several months, find the amount of water remaining in the pool.

■ Further applications

13 An advertising salesman is given a bonus of $\frac{2}{5}$ of his regular weekly pay after successfully managing a major contract. If the salesman's regular monthly pay is $2600, find the amount of his bonus.

14 Josh and David are brothers. Between them, Josh owns $\frac{4}{7}$ of the stamps in their collection and David owns the rest. Two weeks later, Josh has double the number of stamps he owned previously while David has triple the number that he had. If there are now 340 stamps in the collection, find the number of stamps which each boy owns.

4.14 The unitary method

The **unitary method** can be used to find the size of a quantity when only a fraction of its value is known.

To find the size of a quantity given a fraction of its value:
- **find the unit fraction value of the quantity (i.e. divide the quantity by the numerator)**
- **find the entire quantity (i.e. multiply by the denominator).**

A diagram can be very useful for understanding the unitary method.

Example 1

EG+S

Find the number if:

a $\frac{1}{6}$ of the number is 4.

b $\frac{4}{5}$ of the number is 36.

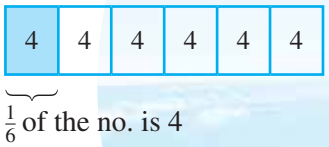
Solutions

a $\frac{1}{6}$ of the number is 4 $\times 6$

$\times 6$

$\frac{6}{6}$ of the number is 24

$\therefore$ the number is 24.



b $\frac{4}{5}$ of the number is 36

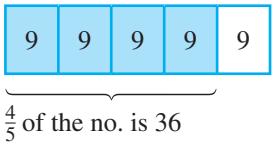
$\div 4$ $\div 4$

$\frac{1}{5}$ of the number is 9

$\times 5$ $\times 5$

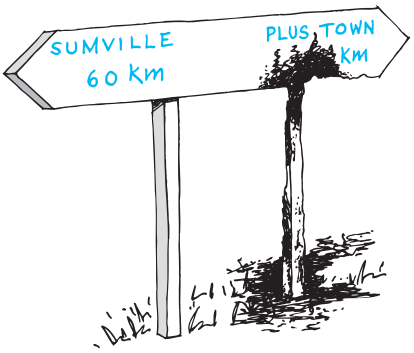
$\frac{5}{5}$ of the number is 45

$\therefore$ the number is 45.



Example 2

After driving $\frac{5}{8}$ of the distance from Sumville to Plus Town, Ray saw this sign which was partially damaged by a recent bushfire.



a How far is it between Sumville and Plus Town?

b How much further does Ray still have to drive?

Solutions

a $\frac{5}{8}$ of the distance between the towns is 60 km

$\frac{5}{8}$ represents 60 km

$\therefore \frac{1}{8}$ represents 12 km ($\div 5$, $\div 5$)

$\therefore \frac{8}{8}$ represents 96 km ($\times 8$, $\times 8$)

$\therefore$ the distance between the towns is 96 km.

b 96 km – 60 km = 36 km

$\therefore$ Ray still has 36 km to drive.

Exercise 4.14

1 In each diagram below, the number is represented by the whole rectangle. Use these diagrams to find the value of the number if:

a

$\frac{1}{2}$ of the number is 5

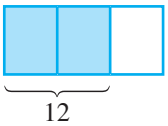
b

$\frac{1}{3}$ of the number is 4

c

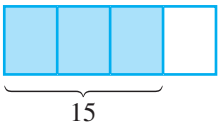
$\frac{1}{4}$ of the number is 7

d



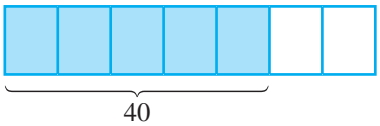
$\frac{2}{3}$ of the number is 12

e



$\frac{3}{4}$ of the number is 15

f



$\frac{5}{7}$ of the number is 40

2 Find the number if:

a $\frac{1}{2}$ of the number is 6 **b** $\frac{1}{3}$ of the number is 5 **c** $\frac{1}{5}$ of the number is 2

d $\frac{1}{4}$ of the number is 9 **e** $\frac{1}{7}$ of the number is 3 **f** $\frac{1}{6}$ of the number is 5

g $\frac{1}{9}$ of the number is 2 **h** $\frac{1}{8}$ of the number is 6 **i** $\frac{1}{11}$ of the number is 4

Consolidation

3 Find the number if:

a $\frac{2}{3}$ of the number is 8 **b** $\frac{3}{4}$ of the number is 18 **c** $\frac{2}{5}$ of the number is 14

d $\frac{7}{8}$ of the number is 28 **e** $\frac{4}{9}$ of the number is 12 **f** $\frac{3}{10}$ of the number is 15

g $\frac{10}{11}$ of the number is 80 **h** $\frac{3}{8}$ of the number is 27 **i** $\frac{5}{6}$ of the number is 25

j $\frac{7}{12}$ of the number is 21 **k** $\frac{4}{11}$ of the number is 48 **l** $\frac{11}{12}$ of the number is 66

m $\frac{9}{10}$ of the number is 45 **n** $\frac{6}{7}$ of the number is 54 **o** $\frac{8}{13}$ of the number is 24

4 **a** If $\frac{3}{7}$ of a number is 12, find $\frac{5}{7}$ of the number.

b If $\frac{2}{9}$ of a number is 14, find $\frac{7}{9}$ of the number.

c If $\frac{4}{11}$ of a number is 32, find $\frac{3}{11}$ of the number.

d If $\frac{7}{12}$ of a number is 42, find $\frac{5}{12}$ of the number.

5 Joel paid \$18 for a book, which represented $\frac{3}{4}$ of the money in his wallet at the time. How much money was in the wallet before the purchase?

6 A car yard contains only Holdens and Nissans. If $\frac{5}{8}$ of the cars in the yard are Holdens and there are 35 Holdens in the yard, find the total number of cars.

7 Jennifer lost $\frac{4}{9}$ of her money at the casino. If she lost \$80, find the amount of money which she had originally.

8 When Kyle had completed $\frac{5}{7}$ of a marathon, he noticed a sign which indicated that the distance to the finish line was 12 km. Over what distance was the marathon run?

9 In Rosie's garden, $\frac{3}{8}$ of the plants are roses and the rest are wildflowers. If there are 45 wildflowers, find the total number of plants in the garden.

10 Todd emptied the coins in his money box onto a table and found that they were all either 20c or 50c coins. If $\frac{9}{13}$ of the coins were 50c coins and the remaining 24 coins were 20c coins, find the total amount of money in the money box.

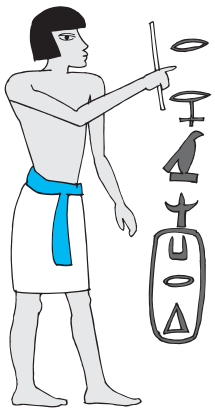
11 Souma and her family live on a farm and raise cattle and sheep. If $\frac{4}{11}$ of animals are cows and the other 35 animals are sheep, find the total number of animals on the farm.

■ Further applications

12 Charles owned $\frac{5}{8}$ of the marbles in a bag and his sister Lorna owned the rest. If Charles gave $\frac{3}{5}$ of his marbles to Lorna, he would then only have 16 marbles. How many marbles did each person have originally?

TRY THIS

Egyptian fractions



The Egyptians wrote all their fractions so that they had a numerator of 1. For example, consider the following fractions and their Egyptian equivalents.

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{2}$$

$$\frac{5}{6} = \frac{1}{3} + \frac{1}{2}$$

$$\frac{7}{12} = \frac{1}{3} + \frac{1}{4}$$

Your task is to investigate Egyptian fractions.

To complete this task you should include the following:

1 Change at least 10 fractions to Egyptian form.

2 Is there more than one way of writing a fraction in Egyptian form? If so, give examples.

3 Can you find a method of changing any fraction to Egyptian form, rather than just using trial and error?

4 How would you add and multiply fractions given that you are to use Egyptian form, e.g. try:

$$\frac{3}{4} + \frac{5}{6}; \quad \frac{3}{4} \times \frac{5}{6}$$

4.15 Solving problems that involve fractions

To solve problems which involve fractions:

- **decide on the operation(s) to be used (i.e. addition, subtraction, multiplication or division)**
- **perform the appropriate calculations**
- **give the solution in the context of the question.**

Example 1

Karen spent $\frac{2}{3}$ of her pocket money on a movie ticket and $\frac{1}{4}$ of her money on a drink. What fraction of her money does Karen have left?

Solution

i We first calculate the fraction of her money which Karen has spent.

$$\frac{2}{3} + \frac{1}{4}$$
$$= \frac{2 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3}$$
$$= \frac{8}{12} + \frac{3}{12}$$
$$= \frac{11}{12}$$

$\therefore$ Karen has spent $\frac{11}{12}$ of her money.

ii To find the fraction of her money which remains, subtract the fraction spent from 1 whole.

$$1 - \frac{11}{12}$$
$$= \frac{12}{12} - \frac{11}{12}$$
$$= \frac{1}{12}$$

$\therefore$ Karen has $\frac{1}{12}$ of her money left.

Example 2

The product of two numbers is $4\frac{1}{2}$. If one of the numbers is $2\frac{5}{8}$, find the other number.

Solution

If $2\frac{5}{8} \times \square = 4\frac{1}{2}$, then $\square = 4\frac{1}{2} \div 2\frac{5}{8}$

Now, $4\frac{1}{2} \div 2\frac{5}{8} = \frac{9}{2} \div \frac{21}{8}$

$$= \frac{\cancel{9}^{3}}{\cancel{2}_{1}} \times \frac{\cancel{8}^{4}}{\cancel{21}_{7}}$$
$$= \frac{12}{7}$$
$$= 1\frac{5}{7}$$

$\therefore$ the other number is $1\frac{5}{7}$.

Exercise 4.15

1 The petrol tank in Mr Wong's car holds 40 litres when full. How much petrol is in the tank when it is $\frac{7}{10}$ full?

2 Anthony spends $\frac{2}{7}$ of his weekly pay on rent. If he earns \$840 p.w., how much does Anthony pay in rent?

3 Bettina is a salesperson who is paid \$300 each fortnight plus a commission of $\frac{3}{11}$ of her fortnightly sales. Find Bettina's total pay for a fortnight when she sells goods worth \$4400.

4 In a class of 30 students, $\frac{2}{5}$ are boys. How many girls are there?

5 Of the 42 employees at a textiles company, 30 belong to the union. What fraction of the employees:
a are in the union? **b** are not in the union?

6 Andrew scored 45 out of 75 on his spelling test in Year 6. If there were 75 words on the test, what fraction of the words did Andrew spell incorrectly?

7 A 2 kg bag of potatoes costs \$3.60. How much should a $2\frac{1}{2}$ kg bag cost at the same rate?

8 The local municipal library has 800 books. After the annual stocktake, the librarian found that 20 of the books were badly damaged and needed to be replaced. What fraction of the books were to be kept?

Consolidation

9 Lionel has \$54. He spends \$25 on a new shirt and \$11 on lunch. What fraction of his money remains?

10 A chef purchased three dozen eggs and used $\frac{5}{6}$ of them in making breakfast orders at his café. How many eggs did the chef use?

11 Greg ate $\frac{1}{3}$ of a pizza and Denise ate $\frac{3}{10}$ of the same pizza.
a Who had the greater share? **b** What fraction of the pizza remains?

12 A bus is full with 65 passengers on board. At the first stop, 4 people get off and 2 people get on. At the next stop, 11 people get off and 3 people get on. What fraction of the bus is full now?

13 The normal retail price of a computer was \$1400. During the January sales period the price tag was marked '$\frac{1}{8}$ off'. Find the new retail price of the computer.

14 Thao is given a pay rise equal to $\frac{1}{6}$ of her regular weekly pay. If she currently earns \$588 p.w., find Thao's new weekly pay.

15 Jane cut a piece of timber into two pieces. If the longer piece is 3 metres long and it exceeds the length of the shorter piece by $1\frac{1}{6}$ metres, find the length of the shorter piece of timber.

16 A boy spent $\frac{1}{4}$ of his pocket money on a new bell for his bicycle and $\frac{1}{6}$ of his money on a bar of chocolate.

a What fraction of his pocket money did the boy spend?

b What fraction of his money remains?

17 A footbridge is being constructed from two opposite sides of a river. If $\frac{1}{3}$ of the bridge has been completed on one side and $\frac{3}{5}$ of the bridge has been completed on the other side, find the fraction of the bridge which remains to be built.

18 The sum of two numbers is $3\frac{9}{10}$. If one of the numbers if $1\frac{2}{5}$, find the other number.

19 The product of two numbers if $4\frac{1}{3}$. If one of the numbers is $1\frac{5}{8}$, find the other number.

20 What must be added to $3\frac{5}{6}$ to give $4\frac{7}{8}$?

21 Five plumbers take $1\frac{1}{4}$ hours to repair a broken water pipe. How long would the job take if only one plumber was available to do the job (assuming they all work at the same rate)?

22 How many times can $\frac{3}{5}$ be subtracted from $14\frac{2}{5}$ to give zero?

23 From a roll of $10\frac{1}{2}$ metres of material, $2\frac{3}{5}$ metres are sold. What length of material remains?

24 An aeroplane was flying at a speed of 20 km/h. How far would the plane fly in $5\frac{3}{4}$ hours?

25 Alex paid a deposit of \$120 towards a stereo. If the deposit is $\frac{1}{4}$ of the purchase price, find:

a the cost of the stereo

b the balance owing on the purchase

26 Seven people shared $\frac{2}{3}$ of a lotto prize. What fraction of the money will each person receive?

■ Further applications

27 After a bad hailstorm, $\frac{3}{8}$ of the cars for sale in a dealership were dented by hailstones and a further $\frac{1}{6}$ of the cars had broken windscreens. If there are 72 cars in the dealership, find the number of cars which can still be sold.

28 If it takes seven men $2\frac{1}{2}$ hours to clean up a street after a New Year's party, how long (in hours and minutes) would it take for three men to do the job?

29 Collect some examples of media statements that involve fractions. Determine the reasonableness of each statement.

TRY THIS Plus equals times!

You know that

$$2 \times 2 = 2 + 2$$

This is a special case where

$$a \times b = a + b$$

Can you find some fractions which give the same result if you add them or multiply them? For example,

$$\frac{7}{3} \times \frac{7}{4} = \frac{49}{12} \quad \text{also} \quad \frac{7}{3} + \frac{7}{4} = \frac{49}{12}$$

Can you find a general method of finding all such fractions?

PROBLEM SOLVING

1 A tank of water started to leak. On the first day, $\frac{1}{4}$ of the water in the tank was lost; on the next day, $\frac{1}{3}$ of what was left was lost; and on the third day, $\frac{1}{2}$ of the remainder was lost. What fraction of the tank now contained water?

2 Write an expression equal to 45 which uses 4 fours and any signs of arithmetic (+, –, ×, ÷) which may be necessary.

3 If we could add all of the numbers $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ … unending, would the total be more than two? Can you explain your answer?

4 Rearrange the following numbers so that the multiplication is correct.

$$\begin{array}{r} 123 \\ \times \quad 4 \\ \hline 678 \end{array}$$

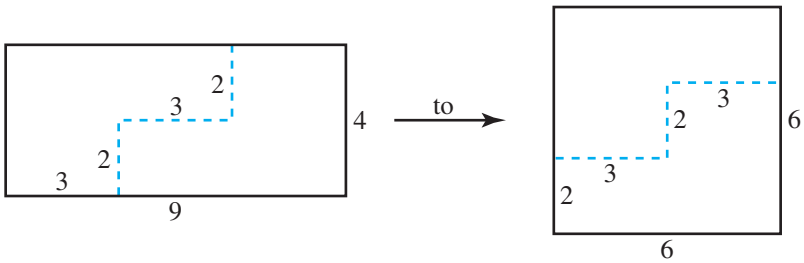
5 Two lines make an angle of 10°. How large will the angle appear if we view it through a magnifying glass which magnifies things five times?

6 A girl has collected some spiders and beetles in a box. If there are five creatures and a total of 32 legs, how many of each type are in the box? Spiders have 8 legs and beetles have 6 legs.

7 Find the missing numbers in this series: 1, 3, 7, …, 31, 63, …, 255.

8 A train is travelling from Melbourne to Sydney at 80 km/h. Another train is travelling from Sydney to Melbourne at 100 km/h. How far apart are they 1 hour before they pass each other?

9 In the diagram a 9×4 rectangle is cut along the dashed line and changed into a 6×6 square.



Now how do you change a 16×9 rectangle into a square?

10 Mrs Higgins bought some apricots and used half of them for a pudding. She then used half of the remainder for a pie and had five left. How many apricots did she buy?

Focus on Working Mathematically

Printing Newspapers

Introduction

Here is a fun activity with newspapers. The idea is to explore the way fractions are used for paper sizes and how newspapers are made.

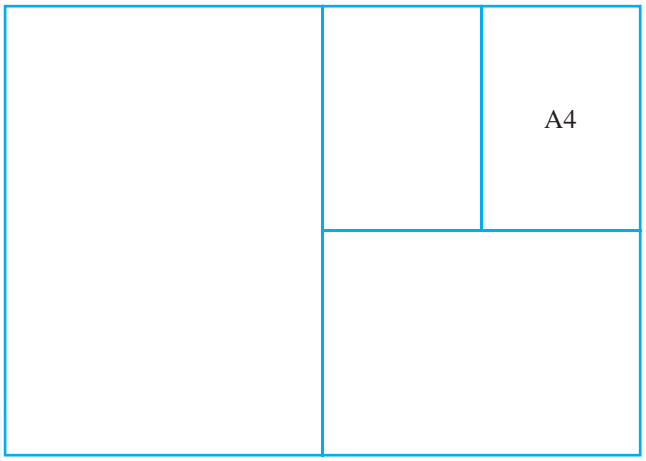
LEARNING ACTIVITIES

Materials needed: A double page of the *Sydney Morning Herald* (SMH) and the *Daily Telegraph* (Tele), 2 sheets of A4 and 3 sheets of A3 paper.

The SMH and the Tele are printed on different sized paper. Printers trim pages during production so the pages are a little less than standard size. The table shows common paper sizes used in the printing industry, showing length and breadth in millimetres.

Size	Length	Breadth
A4	297	210
A3	420	297
A2	594	420
A1	841	594
A0	1189	841
2A0	1682	1189
4A0	2378	1682

1. Take a piece of A4 and a piece of A3 paper. Compare them by placing the A4 on top of the A3. What do you notice? Measure the length and breadth of each sheet and check your answers from the table.
2. What is the approximate size of a single page of the SMH? Of the Tele? Try covering the page of each newspaper with your A4 and/or A3 paper. Check your answers from the table.
3. Now unfold the SMH and the Tele to make a double page. What is the size now? On what size paper is the SMH printed? The Tele?
4. Make a drawing of the double page of the SMH to show the way you covered it with A4 and A3. Copy and complete to show the sizes of all the pieces:



5 You will have found that A4 is $\frac{1}{2}$ of A3. Write down the fraction of A4 to A2, A4 to A1, A4 to A0, A4 to 2A0, and A4 to 4A0. Record your pattern in order, starting with $\frac{1}{2}, \frac{1}{4}, \ldots$ and so on.

6 Suppose this pattern was to continue indefinitely. What do you think might happen if we add them up? That is $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ gets closer and closer to what number?

7 If you can, make a drawing of this sequence, starting with your drawing of the double page of the SMH.

Extension activities

1 Not all sequences of fractions like $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ work like this. Some just keep getting bigger and bigger, and some grow more slowly than others. However the sum of the numbers of this particular sequence do get closer and closer to a number we call a *limit*. This is a good word because no matter how many terms you add, you can never go beyond it.

2 Now look closely at the sequence $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ What is the pattern? Write down the next few terms.

3 Investigate $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ to see if it approaches a limit. Use your calculator if you like.

4 Here is a sequence which does *not* approach a limit: $2 + 4 + 6 + 8 + \ldots$ What do you notice about it?

5 Make up a sequence which you think might have a limit. Check with your teacher.

Let's communicate

1 Make a display of what you have learned about newspaper sizes used in the print media. Include what you now know about the paper sizes used for the SMH and the Tele newspapers.

2 You also learned about a special sequence of fractions, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ Write a summary of what you found out in your exercise book. Include a drawing if you wish.

Reflecting

We now know that the limit of the series of fractions $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ is 1, and consequently, the limit of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ is 2.

However not all patterns of fractions have limits. Remember this! A real surprise is that $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots$ gets bigger and bigger, slowly but surely, as we keep adding more and more terms. Consider this and chat about it with your teacher.

Language link with Macquarie

MACQUARIE

1 *Define the phrase* ***proper fraction****.*
2 *Compare the meaning of* ***numerator*** *with* ***denominator****.*
3 *Consider the words* ***equal*** *and* ***value****. If you put them together, what maths word are you reminded of from this chapter?*
4 *Where does the word* ***fraction*** *come from?*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***fracture****:*

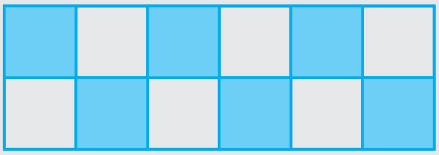
fracture *verb* **1**. to crack or break; *to fall over and fracture your arm | The rock fractured when we hit it.* –*noun* **2**. a break or breaking, especially of a bone: *He was treated for cuts and fractures after the accident.*

What does the word ***fracture*** *have in common with* ***fraction****?*

Chapter Review

1 What fraction has been shaded in each figure?

a



b

c

2 Write the fraction that has a denominator of 11 and a numerator of 7.

3 Write as a fraction:
a one-quarter **b** three-eighths
c thirteen-twelfths

4 If the Hawks basketball team won $\frac{3}{5}$ of their games last season, what fraction of their games did they lose?

5 **a** Express $\frac{6}{11}$ as a division.
b Express $2 \div 5$ as a fraction.

6 How many:
a halves are there in 3 wholes?
b quarters are there in 5 wholes?

7 State whether each of these is a mixed numeral, a proper fraction or an improper fraction.
a $\frac{3}{8}$ **b** $\frac{9}{4}$ **c** $2\frac{6}{7}$

8 Express as improper fractions:
a $1\frac{1}{3}$ **b** $2\frac{3}{4}$ **c** $4\frac{5}{8}$ **d** $7\frac{7}{9}$

9 Express as mixed numerals:
a $\frac{7}{2}$ **b** $\frac{14}{3}$ **c** $\frac{22}{9}$ **d** $\frac{41}{12}$

CHAPTER REVIEW

10 Complete each of these equivalent fractions.

a $\frac{1}{3} = \frac{\square}{12}$ **b** $\frac{2}{5} = \frac{\square}{10}$

c $\frac{4}{7} = \frac{\square}{35}$ **d** $\frac{8}{9} = \frac{\square}{63}$

e $\frac{3}{\square} = \frac{6}{8}$ **f** $\frac{\square}{12} = \frac{15}{36}$

g $\frac{8}{11} = \frac{48}{\square}$ **h** $1\frac{4}{5} = \frac{\square}{25}$

11 Simplify:

a $\frac{5}{20}$ **b** $\frac{3}{18}$ **c** $\frac{10}{12}$ **d** $\frac{15}{18}$

e $\frac{21}{28}$ **f** $\frac{16}{36}$ **g** $\frac{42}{48}$ **h** $\frac{84}{108}$

12 Simplify:

a $\frac{40}{150}$ **b** $\frac{2700}{3300}$ **c** $\frac{240}{150}$

13 Insert $<$ or $>$ to make a true statement:

a $\frac{5}{3} \square \frac{3}{5}$ **b** $\frac{7}{9} \square \frac{5}{6}$

14 Arrange these fractions in ascending order by first expressing them with a common denominator.

a $\frac{11}{18}, \frac{5}{9}, \frac{17}{36}, \frac{3}{4}, \frac{7}{12}$

b $\frac{3}{8}, \frac{1}{2}, \frac{2}{5}, \frac{19}{40}, \frac{9}{20}$

15 Arrange these fractions in ascending order by inspection.

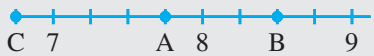
a $\frac{99}{100}, \frac{13}{12}, \frac{1}{22}, \frac{5}{8}$

b $\frac{9}{8}, \frac{2}{23}, \frac{19}{40}, \frac{17}{18}$

16 State in simplest form, the fraction, whole number or mixed numeral represented by A, B and C on each number line.

a

b

c


d

17 In simplest form, what fraction is:

a 7 of 21?
b 12 of 16?
c \$9 of \$15?
d 25 mins of 1 hour?
e 8 months of 2 years?
f 25 mm of $3\frac{1}{2}$ cm?

18 Write answers only for each of these.

a $3 + \frac{2}{5}$ **b** $\frac{2}{7} + \frac{4}{7}$

c $\frac{3}{4} - \frac{3}{4}$ **d** $\frac{5}{9} + \frac{4}{9}$

e $4\frac{3}{10} - 4$ **f** $1\frac{3}{5} + 2\frac{1}{5}$

g $9\frac{2}{5} - 1\frac{2}{5}$ **h** $6\frac{2}{3} + 4\frac{1}{3}$

19 Simplify:

a $\frac{5}{8} + \frac{1}{8}$ **b** $\frac{13}{15} - \frac{4}{15}$

c $5\frac{1}{12} + 2\frac{5}{12}$ **d** $2\frac{23}{24} - 1\frac{7}{24}$

e $\frac{7}{10} + \frac{4}{10}$ **f** $\frac{16}{21} + \frac{8}{21}$

20 Evaluate:

a $1 - \frac{3}{10}$ **b** $7 - \frac{3}{4}$

c $4 - 1\frac{2}{5}$ **d** $8\frac{4}{15} - 3\frac{13}{15}$

21 Evaluate:

a $\frac{1}{2} + \frac{1}{4}$ **b** $\frac{11}{12} - \frac{1}{3}$

c $\frac{2}{7} + \frac{4}{21}$ **d** $\frac{4}{5} - \frac{2}{3}$

e $\frac{3}{4} + \frac{1}{6}$ **f** $\frac{2}{3} + \frac{5}{8}$

g $4\frac{11}{12} - 2\frac{1}{4}$ **h** $3\frac{3}{4} + 2\frac{9}{10}$

22 Evaluate:

a $\frac{1}{6} \times \frac{5}{8}$ **b** $11 \times \frac{1}{3}$

c $1\frac{3}{4} \times 3$ **d** $\frac{15}{16} \times \frac{8}{9}$

e $\frac{25}{49} \times \frac{28}{45}$ **f** $1\frac{1}{14} \times 1\frac{1}{20}$

23 Find the reciprocal of:

a $\frac{5}{7}$ **b** 6 **c** $2\frac{3}{4}$

24 Evaluate:

a $\frac{4}{13} \div \frac{1}{2}$ **b** $\frac{3}{8} \div 5$

c $\frac{2}{9} \div 1\frac{1}{4}$ **d** $\frac{15}{28} \div \frac{20}{21}$

e $\frac{4}{5} \div \frac{8}{25}$ **f** $1\frac{31}{32} \div 1\frac{9}{40}$

25 Find:

a $\frac{1}{4}$ of \$6 **b** $\frac{3}{5}$ of 2 min

c $\frac{7}{20}$ of 3 m **d** $\frac{11}{50}$ of $1\frac{1}{2}$ kg

26 Find the number if:

a $\frac{1}{3}$ of the number is 4

b $\frac{1}{5}$ of the number is 9

c $\frac{3}{4}$ of the number is 15

d $\frac{4}{7}$ of the number is 24

27 **a** In a class of 32 students, $\frac{3}{8}$ are girls. How many boys are there?

b Tina scored 15 goals from her 21 attempts in last week's basketball game. What fraction of her shots were unsuccessful?

c The sum of two numbers is $7\frac{17}{30}$. If one of the numbers is $3\frac{2}{5}$, what is the other number?

d An outdoor table setting was advertised as '$\frac{1}{6}$ off the marked price'. If the marked price was \$378, find the discounted sale price.

5 Number patterns and pronumerals

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- extend a number pattern by finding further terms
- extend a geometric pattern by forming further figures
- write down in words a rule that describes the relationship between two variables
- use a worded rule to complete a table of values
- use a worded rule to find particular terms in a pattern.

Stage 4

After completing this chapter, you should be able to:

- use cups and counters to model and simplify algebraic expressions
- use algebraic conventions to abbreviate expressions
- write down in words, several rules that describe the same number pattern
- write down using pronumerals, several rules that describe the relationship between two variables
- use algebraic rules to complete tables of values
- use algebraic rules to find particular terms in a number pattern
- write down the co-ordinates of a point that has been plotted on a number lattice
- plot points on a number lattice.

5.1 The algebraic symbol system

The need for symbols

In chapter 1 we saw how people from different cultures used symbols to represent numbers. To represent the number *five*, the Romans wrote V, the Egyptians wrote IIIII and the Greeks wrote Γ. When we write 5, we are also using a symbol to represent the number *five*. We use numerals not only to represent our concept of a number but also to write it in as short a way as possible. This is particularly useful when we need to refer to very large numbers. Instead of writing *one million three hundred and forty thousand two hundred and eighty-seven* we can simply write 1 340 287. The use of numerals to represent numbers also makes it much easier to perform arithmetic operations such as addition, subtraction, multiplication and division.

Pronumerals

Letters of the alphabet can be used as symbols to represent numbers. The use of letters in this way is called **algebra** and the letters are then called **pronumerals**. In the two exercises that follow, pronumerals will be used to represent the number of counters in a cup.

For example, we could say here that the number of counters in the cup is represented by the statement $x = 3$, since there are three counters in the cup. In fact, any pronumeral could have been chosen to represent the number of counters. That is, it would have been equally correct to represent the number of counters by the statements $a = 3$, $k = 3$ or $y = 3$.

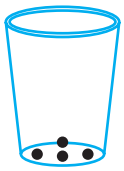
Statements such as $x = 3$ are called **equations**. An equation is a statement which says that two quantities are equal in value. Algebraic statements that do not contain an equals sign are called **expressions**. Some examples of algebraic expressions are $a + 1$, $t - 2$ and $5 \times y$.

Modelling a variable

A pronumeral may also be called a **variable** if it can represent different numbers. In each of the following models, n represents the number of counters in the cup.

$n = 1$

$n = 2$



$n = 4$

The value of n changes according to the number of counters that have been placed in the cup. Therefore, n is a variable.

NOTE: The pronumeral stands for the number of counters, not the counters themselves. Pronumerals can only be used to represent numbers. They cannot be used to represent objects.

Modelling a variable plus a constant

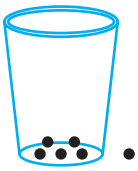
As stated above, a variable is a pronumeral whose value can change. A constant, however, is a number and hence its value cannot change. For example, in the expression $a + 3$, a is a variable and 3 is a constant.

To model an expression that consists of a variable plus a constant:
- **model the value of the variable by the number of counters in the cup**
- **model the constant by the number of counters outside the cup.**

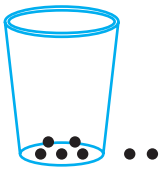
Example 1

Write down the expressions that are being modelled below if $x = 5$ is modelled as

a



b



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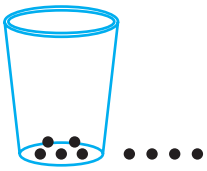
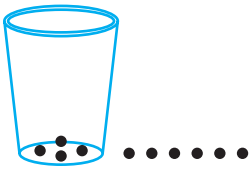


Solutions

a $x + 1$ **b** $x + 2$ **c** $x + 4$

Example 2

If $w + 6$ is modelled as



what is the value of the variable w?

Solution

The value of the variable is equal to the number of counters in the cup, $\therefore\ w = 4$.

Exercise 5.1

1 In each of the following, find the value of x, where x represents the number of counters in the cup.

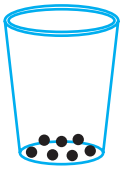
a

$x =$ _____

b

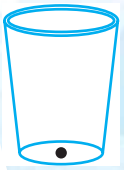
$x =$ _____

c



$x =$ _____

d



$x =$ _____

2 Model each of the following equations using cups and counters, where a represents the number of counters in the cup.

a $a = 1$ **b** $a = 3$ **c** $a = 4$ **d** $a = 6$

3

Which model could represent each equation?

a $m = 2$ **b** $m = 5$ **c** $m = 8$

Consolidation

4 For each of the statements below:

i write an equation to represent the statement using n as the variable

ii model the situation using cups and counters.

a The number of people living in your house.

b The number of letters in your surname.

c The number of books on your desk.

d The number of telephones (including mobile phones) in your house or owned by family members.

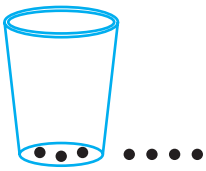
5 In each model below, y stands for the number of counters in the cup.

That is, for , $y = 3$.

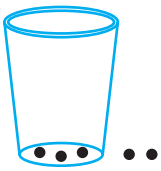
Write down an expression in terms of y for each of these models.

a

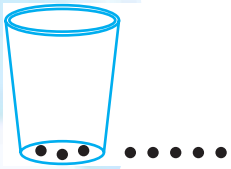
b



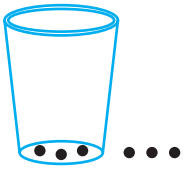
c



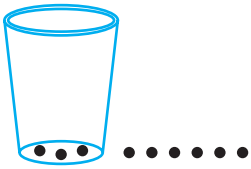
d



e



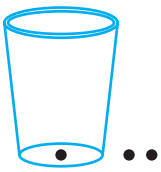
f

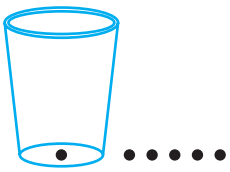


6 Write down an expression for each model in terms of k if:

a $k = 1$ is modelled as

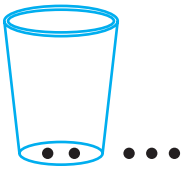
i

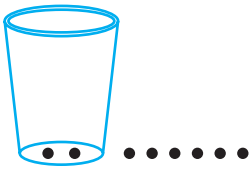
ii 

iii 

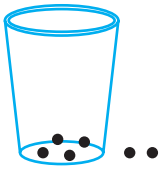
b $k = 2$ is modelled as

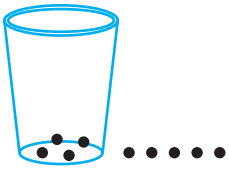
i

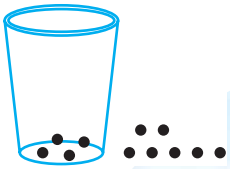
ii 

iii 

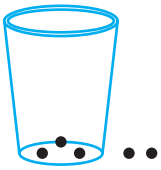
c $k = 4$ is modelled as

i 

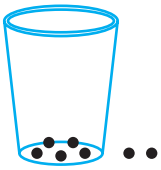
ii 

iii 

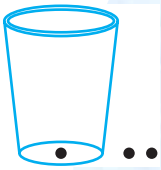
7 Three groups of students were asked to model the expression $p + 2$. Here are the models that they made.



Group 1



Group 2



Group 3

a What is the same in each model?

b How do the models differ?

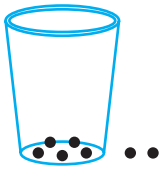
c Did each group correctly model the expression $p + 2$? Explain.

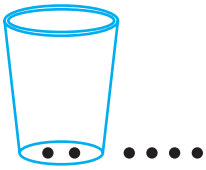
d Draw a different model for this expression.

e Does model $p + 2$? Why or why not?

8 **i** Write down an equation for the number of counters in each cup, using h as the variable.

ii Write down the algebraic expression that is being represented by each model.

a 

b 

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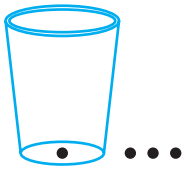
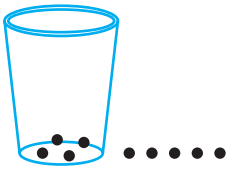
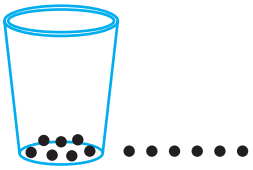
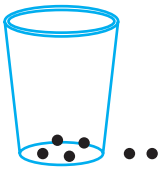
d 

e 

f 

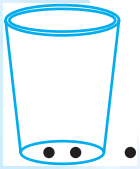
9 Let c represent the number of counters in the cup. Choose a value for c, then use this value to model each of the following expressions using cups and counters.

a 3 **b** c **c** $c + 1$ **d** $c + 3$ **e** $c + 4$

10 If  models the expression $q + 2$, model each of the following expressions.

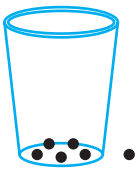
a q **b** $q + 1$ **c** $q + 4$ **d** $q + 7$

11 In each of the following, the expression $b + 1$ is being modelled. Find the value of the variable b in each of these.

a 

b

c

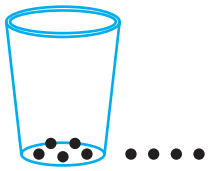
d 

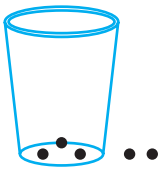
12 If is used to model the expression $u + 4$, what is the value of u?

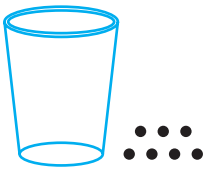
13 Match each of these expressions to one of the models below, then state the value of s in each case.

◎ s ◎ $s + 2$ ◎ $s + 4$ ◎ $s + 7$

a

b 

c 

d 

■ Further applications

14 **a** Choose any value for the pronumeral, then use cups and counters to model $(x + 4) + 2$ and $x + 6$.
b Does $(x + 4) + 2 = x + 6$?

15 Use cups and counters to simplify these expressions.
a $(a + 2) + 1$ **b** $(z + 3) + 2$ **c** $3 + (p + 1)$

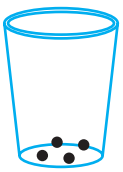
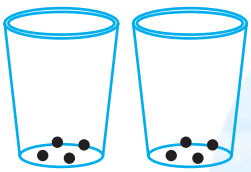
16 Simplify each of the following expressions without the use of cups and counters.
a $(n + 4) + 2$ **b** $(z + 5) + 3$ **c** $6 + (t + 3)$

17 **a** Solve the equation $n + 3 = 7$ using cups and counters.
b In terms of cups and counters, what does the solution represent?

18 Use cups and counters to solve each of these equations.
a $a + 2 = 5$ **b** $t + 1 = 5$ **c** $s + 4 = 9$ **d** $c + 6 = 6$

5.2 Modelling products and sums

When writing algebraic expressions such as $2n$, it is common to leave out the multiplication sign. That is, $2n$ means $2 \times n$. This is called a mathematical **convention**.

If $n = 4$ is modelled by  then $2n$ is modelled by 

How would $3n$ be modelled?

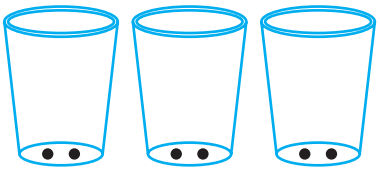
To model the product of a number and a pronumeral:
◎ **model the number of cups equal to the number in the product.**

Example

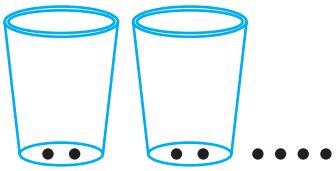
If $d = 2$ is modelled by show the models for: **a** $3d$ **b** $2d + 4$

Solutions

a



b

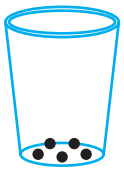


Exercise 5.2

1 If $a = 2$ is represented by , model each of these expressions.

a $2a$ **b** $3a$ **c** $4a$ **d** $5a$

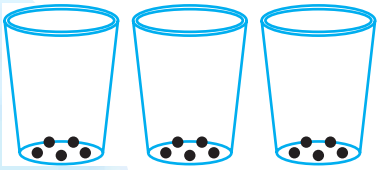
2 Write down as a product, the expressions that are being modelled below if:



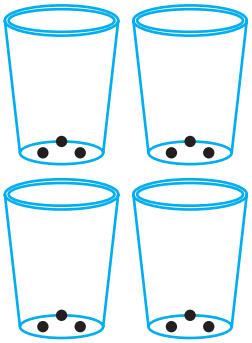
$w = 5$

$k = 3$

a



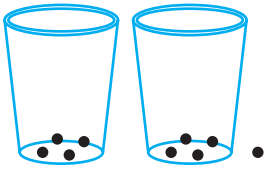
b



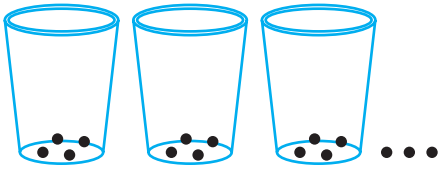
Consolidation

3 In each model below, the variable w stands for the number of counters in each cup. Write down an expression in terms of w for each of these models.

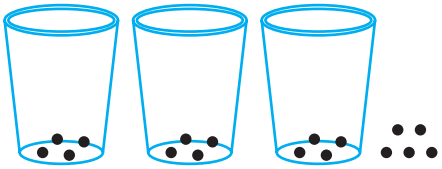
a



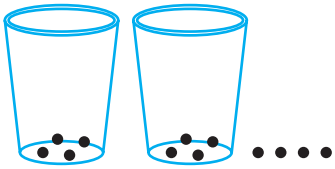
b



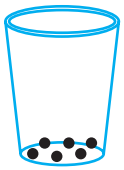
c



d



4 Model the expressions below given that:



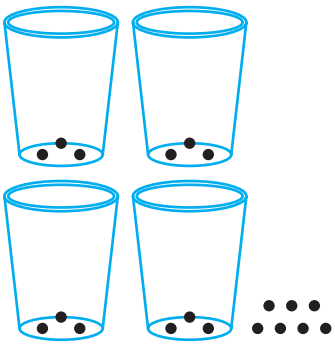
$a = 6$

$b = 2$

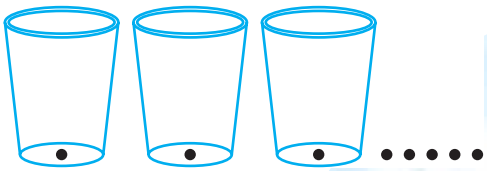
a $2a$ **b** $5a$ **c** $3a + 1$ **d** $4a + 3$
e $3b$ **f** $4b$ **g** $2b + 5$ **h** $5b + 6$

5 **i** Write down an equation for the number of counters in each cup, using j as the variable.
ii Write down the algebraic expression that is being represented by each model.

a



b

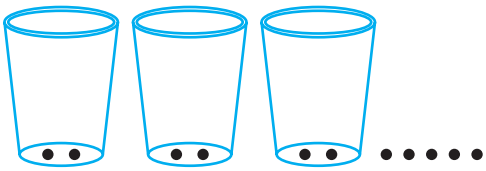


6 Let r represent the number of counters in each cup. Choose a value for r and use it to model each of the following expressions.

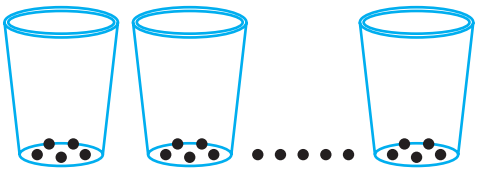
a 5 **b** r **c** $r + 2$ **d** $2r$ **e** $3r + 4$

7 Three groups of students were asked to model the same expression. Here are the models that they made.

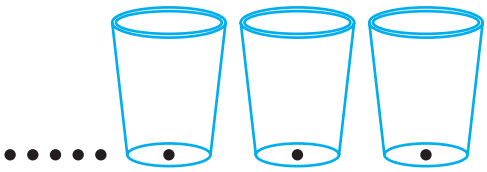
Group 1:



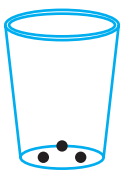
Group 2:



Group 3:



a What is the same in each model?
b How do the models differ?
c What expression do you think the students were modelling? (Let the variable be x.)
d Draw a different model for this expression.

8 How many counters would be needed to build models for these expressions if $v = 3$ is modelled by  ?

a $v + 5$ **b** $v + 8$ **c** $2v$ **d** $7v$
e $4v + 1$ **f** $3v + 5$ **g** $5v + 2$ **h** $10v + 9$

9 **a** Choose any value for the pronumeral, then use cups and counters to model the expressions $a + a$ and $2a$.
b Does $a + a = 2a$? Why?

10 **a** Choose any value for the pronumeral, then use cups and counters to model the expressions $x + x + x$ and $3x$.
b Does $x + x + x = 3x$?

11 Simplify each of these expressions using cups and counters.
a $b + b + b + b$ **b** $j + j + j + j + j + j$ **c** $v + v + v + v + v$

12 **a** Does $2 + 3 = 3 + 2$? Does $5 + 7 = 7 + 5$?
b Choose values for the pronumerals a and b, then model the expressions $a + b$ and $b + a$.

c Does $a + b = b + a$?
d Would the result hold for other values of a and b?
e This result is called the Commutative Law for addition. Why should it be true for pronumerals?

13 **a** Choose a different value for each pronumeral, then use cups and counters to model the expressions $p + p + q + q + q$ and $2p + 3q$.
b Does $p + p + q + q + q = 2p + 3q$?

14 Simplify each of the following expressions using the method outlined in Q13.
a $m + m + m + n$ **b** $g + g + g + g + h + h + h$

15 Simplify these expressions without the use of cups and counters.
a $a + a + b + b + b + b$ **b** $u + v + u + u + v + v$

■ Further applications

16 **a** If $x = 2$, model the expressions $2x + 3$ and $5x$. Does $2x + 3 = 5x$?
b If $d = 3$, model the expressions $4d + 2$ and $6d$. Does $4d + 2 = 6d$?
c Why can't the expressions above be simplified by adding the numbers?

17 Choose any value for the pronumeral, then use cups and counters to show that:

$$\begin{aligned}(a + 3) + (2a + 1) &= (a + 2a) + (3 + 1)\\ &= 3a + 4\end{aligned}$$

18 Simplify each of the following expressions using the method outlined in Q17.
a $(t + 2) + (3t + 1)$ **b** $(3y + 4) + (2y + 5)$

19 Simplify these expressions without the use of cups and counters.
a $(n + 3) + (n + 2)$ **b** $(4f + 2) + (6f + 7)$ **c** $(5r + 7) + (3r + 5)$

20 Choose a value for the variable y.
a Model each of these expressions using cups and counters.
i $2(y + 1)$ **ii** $2y + 1$ **iii** $2y + 2$
b Does $2(y + 1) = 2y + 1$?
c Which expressions are equal? Why?

21 Use cups and counters to simplify each of the following expressions.
a $2(a + 3)$ **b** $3(h + 1)$ **c** $4(r + 2)$

22 Simplify these expressions without the use of cups and counters.
a $3(k + 4)$ **b** $2(d + 5)$ **c** $5(e + 3)$ **d** $7(s + 2)$

23 Show by the use of cups and counters that

$$\begin{aligned}a + a + b + b + b &= 2a + 2b + b\\ &= 2(a + b) + b\end{aligned}$$

5.3 Algebraic conventions

Algebra is used to make long, complicated mathematics shorter and easier to understand. In order to achieve this, mathematicians agree to follow a series of standard algebraic conventions.

A **convention** is a general procedure that everyone agrees to follow. (For example, driving on the left-hand side of the road in Australia is a convention.) The conventions are:

1. **When multiplying expressions, we leave out the multiplication signs. For example, $2 \times n$ is written as $2n$.**
2. **When multiplying a pronumeral by 1, we leave out the 1. For example, $1 \times a$ is written as a, rather than $1a$.**
3. **When dividing one expression by another, we write the division as a fraction. For example, $m \div 3$ is written as $\frac{m}{3}$.**
4. **When a pronumeral is multiplied by itself, we write the product in index form. For example, $n \times n$ is written as n^2, rather than nn.**
5. **When a pronumeral is multiplied by a numeral, the numeral must be written first. For example, $y \times 4$ is written as $4y$, rather than $y4$.**

When a numeral has been multiplied by a pronumeral, we say that the numeral is the **co-efficient** of the pronumeral. For example, in $5k$, 5 is the co-efficient of k. It is not correct to refer to a pronumeral as a co-efficient. That is, in the expression ab, we cannot say that a is the co-efficient of b. In some questions, it will be necessary to use grouping symbols to make clear the meaning of the expression.

Example 1

Write each expression as a product without any multiplication signs.

a $m \times 7$ **b** $p \times 3 \times q$ **c** $1 \times p$

d $5 \times (n - 3)$ **e** $2 \times (3u + 7) \times 4$ **f** $(a - b) \times (3c + 2d)$

Solutions

a $m \times 7$
$= 7m$

b $p \times 3 \times q$
$= 3pq$

c $1 \times p$
$= p$

d $5 \times (n - 3)$
$= 5(n - 3)$

e $2 \times (3u + 7) \times 4$
$= 8(3u + 7)$

f $(a - b) \times (3c + 2d)$
$= (a - b)(3c + 2d)$

Example 2

Insert multiplication signs and grouping symbols where necessary to show the meaning of each expression.

a $9uv$ **b** $3n + 11$ **c** $ab - 8c$

d $2p(q + r)$ **e** $(m + 4)n$ **f** $(y - 1)(y + 1)$

Solutions

a $9uv$
$= 9 \times u \times v$

b $3n + 11$
$= 3 \times n + 11$

c $ab - 8c$
$= a \times b - 8 \times c$

d $2p(q + r)$
$= 2 \times p \times (q + r)$

e $(m + 4)n$
$= (m + 4) \times n$

f $(y - 1)(y + 1)$
$= (y - 1) \times (y + 1)$

Example 3

Write each of these divisions as a fraction.

a $m \div 2$ **b** $1 \div t$ **c** $pq \div rs$
d $(k + 7) \div 6$ **e** $3a \div (2b - c)$ **f** $(w + 4) \div (w - 2)$

Solutions

a $m \div 2$
$= \frac{m}{2}$

b $1 \div t$
$= \frac{1}{t}$

c $pq \div rs$
$= \frac{pq}{rs}$

d $(k + 7) \div 6$
$= \frac{k + 7}{6}$

e $3a \div (2b - c)$
$= \frac{3a}{2b - c}$

f $(w + 4) \div (w - 2)$
$= \frac{w + 4}{w - 2}$

Example 4

Insert multiplication signs, division signs and grouping symbols where necessary to show the meaning of each expression.

a $\frac{k}{3}$ **b** $\frac{1}{c}$ **c** $\frac{3m}{n}$ **d** $\frac{6c}{yz}$ **e** $12 - \frac{a}{5}$ **f** $\frac{m}{3} + \frac{n}{7}$

Solutions

a $\frac{k}{3}$
$= k \div 3$

b $\frac{1}{c}$
$= 1 \div c$

c $\frac{3m}{n}$
$= (3 \times m) \div n$

d $\frac{6c}{yz}$
$= (6 \times c) \div (y \times z)$

e $12 - \frac{a}{5}$
$= 12 - (a \div 5)$

f $\frac{m}{3} + \frac{n}{7}$
$= (m \div 3) + (n \div 7)$

Example 5

Write each product in index form.

a $c \times c \times c \times c$ **b** $5 \times h \times h \times h$ **c** $p \times 3 \times q \times p \times q$

Solutions

a c^4 **b** $5h^3$ **c** $3p^2q^2$

Example 6

Insert multiplication signs to show the meaning of each expression.

a d^3 **b** $7z^5$ **c** r^2s^3

Solutions

a $d \times d \times d$ **b** $7 \times z \times z \times z \times z \times z$ **c** $r \times r \times s \times s \times s$

Exercise 5.3

1 Write each of these expressions in simplest form by leaving out the multiplication signs.

a $3 \times a$ **b** $5 \times m$ **c** $6 \times k$ **d** $n \times 2$ **e** $p \times 7$
f $y \times 8$ **g** $a \times b$ **h** $p \times q$ **i** $m \times n$ **j** $4 \times u \times v$
k $12 \times e \times f$ **l** $c \times d \times 7$ **m** $y \times z \times 6$ **n** $g \times 3 \times h$ **o** $i \times 10 \times j$

2 Write in simplest form.

a $3 \times c + 5$ **b** $k \times 2 - 7$ **c** $m \times 5 + n$
d $6 + 8 \times z$ **e** $10 - 4 \times q$ **f** $t + u \times v$
g $3 \times p + 2 \times q$ **h** $m \times 4 - 2 \times n$ **i** $x \times y + 9 \times z$

3 Express each of these in simplest form by leaving out 1 as a co-efficient.

a $1 \times y$ **b** $1 \times d$ **c** $1 \times q$ **d** $a \times 1$
e $r \times 1$ **f** $v \times 1$ **g** $1 \times g \times h$ **h** $p \times 1 \times q$
i $x \times y \times 1$ **j** $1 \times a + c$ **k** $k + 1 \times n$ **l** $1 \times p - 4$
m $7 + r \times 1$ **n** $1 \times a + 1 \times b$ **o** $s \times 1 - 1 \times t$ **p** $2 + 1 \times e + f$

4 Write each of these products in index form.

a $a \times a$ **b** $p \times p \times p$ **c** $m \times m \times m \times m$
d k **e** $t \times t \times t \times t \times t$ **f** $u \times u \times u$
g $5 \times n \times n$ **h** $2 \times y \times y \times y$ **i** $6 \times c \times c \times c \times c$
j $f \times f \times 4$ **k** $w \times w \times 5 \times w$ **l** $e \times e \times e \times 3 \times e$
m $a \times a \times b$ **n** $m \times n \times n \times n$ **o** $p \times p \times p \times q \times q$
p $3 \times e \times f \times f$ **q** $7 \times x \times x \times y$ **r** $r \times s \times 2 \times s \times r$

5 Write each of these divisions as a fraction.

a $a \div 2$ **b** $k \div 3$ **c** $p \div 7$ **d** $6 \div m$ **e** $14 \div b$
f $1 \div h$ **g** $p \div q$ **h** $x \div y$ **i** $u \div v$ **j** $cd \div 4$
k $6y \div z$ **l** $ef \div 15g$ **m** $8gh \div tv$ **n** $abc \div de$ **o** $2p \div 3rst$

Consolidation

6 Insert multiplication signs to show the meaning of each expression.

a $2c$ **b** $8g$ **c** $9m$ **d** xy **e** cd
f uv **g** $7pq$ **h** $4ef$ **i** $3ijk$ **j** $6b + c$
k $3m - 4$ **l** $mn + 1$ **m** $2 + rs$ **n** $pq + 5r$ **o** $7h - pn$
p $abc + 5$ **q** $6 - 2ghi$ **r** $5 + 3a + b$ **s** $x - yz + w$ **t** $2p + 3q - 6rs$

7 Insert multiplication signs to show the meaning of each expression.

a n^2 **b** k^3 **c** u^5 **d** d^4
e $5p^3$ **f** $7b^2$ **g** $11s^4$ **h** $3g^5$
i xy^2 **j** e^2f **k** mn^3 **l** r^4s^2
m $3p^2q^2$ **n** $5u^3v$ **o** $2a^2b^5$ **p** $11c^4d^3$

8 Insert multiplication and division signs and grouping symbols where necessary to show the meaning of each expression.

a $\frac{w}{4}$ **b** $\frac{y}{9}$ **c** $\frac{11}{s}$ **d** $\frac{1}{a}$ **e** $\frac{g}{h}$
f $\frac{k}{t}$ **g** $\frac{3u}{v}$ **h** $\frac{x}{5y}$ **i** $\frac{2p}{3q}$ **j** $\frac{4}{m} + 2$
k $15 - \frac{t}{7}$ **l** $\frac{1}{c} + 1$ **m** $\frac{x}{2} + \frac{y}{3}$ **n** $\frac{5}{b} - \frac{c}{4}$ **o** $\frac{f}{g} + \frac{7p}{w}$
p $\frac{3x}{y} - \frac{2z}{w}$ **q** $6 + \frac{a}{8} + d$ **r** $13x - \frac{y}{z} + 7$ **s** $\frac{6}{e} - \frac{f}{7} + 1$ **t** $\frac{m}{4} + \frac{n}{2} - \frac{1}{p}$

9 Write each of these products without any multiplication signs.

a $3 \times (a + 2)$ **b** $5 \times (p - q)$ **c** $6 \times (3m - 2n)$
d $(x + 7) \times 4$ **e** $(a - 2) \times 9$ **f** $(2c + 5d) \times 11$
g $2 \times a \times (x + 3)$ **h** $4 \times p \times (y - 1)$ **i** $m \times n \times (7 - t)$
j $6 \times (3 + u) \times k$ **k** $n \times (2w - 1) \times 7$ **l** $5 \times 3 \times (2a + 7b)$
m $(x + 2) \times (y + 5)$ **n** $(p - 1) \times (q + 4)$ **o** $(2a + 3b) \times (c - 6d)$

10 Write each expression as a fraction.

a $(a + 3) \div 2$ **b** $(p - q) \div 8$ **c** $(2m - n) \div 4p$
d $6 \div (p - 2)$ **e** $t \div (u + 5)$ **f** $7f \div (3g + 4h)$
g $(y + 4) \div (z - 4)$ **h** $(1 + u) \div (v + u)$ **i** $(3a + 4b) \div (a + b - c)$

11 Use multiplication signs to show the meaning of each expression.

a $5(a - 7)$ **b** $6(q + 2)$ **c** $e(f - g)$ **d** $(x + y)3$
e $(m - 4)9$ **f** $(5 + c)d$ **g** $2a(w - 4)$ **h** $cd(g + 7)$
i $(3 - n)6s$ **j** $(x + 3)(x + 4)$ **k** $(a + 1)(b - 1)$ **l** $(p - n)(q - m)$

12 Insert only division signs and grouping symbols to show the meaning of:

a $\frac{c + d}{3}$ **b** $\frac{w - 2}{7}$ **c** $\frac{2x + y}{10}$ **d** $\frac{4}{h - 1}$ **e** $\frac{1}{p + 3q}$
f $\frac{9n}{e + f}$ **g** $\frac{n + 2}{y + 7}$ **h** $\frac{g + 1}{g - 1}$ **i** $\frac{10 + 3k}{4m - 13}$ **j** $\frac{2w - 1}{3x + 5y}$

■ Further applications

13 Write each of these expressions without any multiplication or division signs.

a $7 \times (3 \times p + q \times 5) - 4 \times r$
b $5 \times a \times (4 \times b + c - 3 \times d) + 1$

c $x \times (5 \div u + 1 \div v) \times y$
d $(4 \times m + 9 \times n) \div (x \times y \times z - 6 \times v + w)$
e $[(5 + 3 \times e) - f] \div [a \times (6 \times f - 1)]$
f $(5 \times a - b \times 2 + c \times d) \times (t \times 3 - u \times v \times w) \div (a - b \times c + 1)$

5.4 Number patterns

The study of mathematics is basically an attempt to understand the world in which we live. Many things that occur in nature can be explained and understood by studying the various **patterns** and **relationships** that underlie them. Once we have identified a pattern in the behaviour of some object or natural event, the task is then to describe that pattern in some meaningful way. By studying these patterns, we can make *predictions* about the future behaviour of these objects or events. Our study will begin by describing a series of simple number patterns in words. We will then attempt to describe these and other patterns in a more *general* way.

Example 1

Write down in words a rule that describes each number pattern (or sequence), then find the next two numbers.

a 7, 13, 19, 25, … **b** 5, 10, 20, 40, … **c** 4000, 2000, 1000, 500, …

Solutions

a $7 + 6 = 13$
$13 + 6 = 19$
$19 + 6 = 25$

Each term in the pattern is found by adding 6 to the previous term. Now, $25 + 6 = 31$ and $31 + 6 = 37$.
∴ The next two terms are 31 and 37.

b $5 \times 2 = 10$
$10 \times 2 = 20$
$20 \times 2 = 40$

Each term in the pattern is found by multiplying the previous term by 2, or doubling it.
Now, $40 \times 2 = 80$ and $80 \times 2 = 160$.
∴ The next two terms are 80 and 160.

c $4000 \div 2 = 2000$
$2000 \div 2 = 1000$
$1000 \div 2 = 500$

Each term in the pattern is found by dividing the previous term by 2, or halving it.
Now, $500 \div 2 = 250$ and $250 \div 2 = 125$.
∴ The next two terms are 250 and 125.

Example 2

Write down in words the two stage rule which describes the relationship between the terms in the pattern below, then write the next four terms.

2, 5, 4, 7, 6, 9, 8, …

Solution

2 5 4 7 6 9 8 …

+3 −1 +3 −1 +3 −1

The two stage rule for this pattern is to add 3 and then subtract 1.
Now, 8 + 3 = 11, 11 − 1 = 10, 10 + 3 = 13, and 13 − 1 = 12.
∴ The next four terms are 11, 10, 13 and 12.

Exercise 5.4

1 Use the given rule to find the next three terms in each number pattern.

a Add 2; 3, 5, …
b Subtract 3; 28, 25, …
c Multiply by 2; 1, 2, …
d Divide by 2; 400, 200, …
e Add 5; 5, 10, …
f Subtract 4; 30, 26, …
g Double; 3, 6, …
h Halve; 2000, 1000, …
i Divide by 5; 5000, 1000, …
j Add 12; 3, 15, …
k Subtract 11; 99, 88, …
l Divide by 3; 81, 27, …

2 Use the given rule to find the next two terms in each number pattern.

a Add 1 then multiply by 2; 5, 12, …
b Multiply by 3 then subtract 6; 4, 6, …
c Divide by 2 then add 6; 60, 36, …
d Subtract 6 then divide by 3; 105, 33, …
e Divide by 5 then add 10; 450, 100, …
f Multiply by 2 then add 5; 2, 9, …

Consolidation

3 Write down in words a rule for each sequence, then find the next two numbers.

a 11, 23, 35, …
b 62, 51, 40, …
c 7, 14, 28, …
d 20, 35, 50, …
e 80, 40, 20, …
f 60, 47, 34, …
g 4, 8, 16, …
h 19, 28, 37, …
i 10 000, 1000, 100, …

4 In a certain number pattern, each term is found by squaring the previous one. If the first two terms are 2 and 4, find the next two terms in the pattern.

5 Find the next three numbers in each pattern.

a 5, 4, 7, 6, 9, 8, 11, …
b 8, 6, 12, 10, 20, 18, 36, …
c 10, 13, 11, 14, 12, 15, 13, …
d 6, 18, 8, 24, 14, 42, 32, …

6 Write down the next three numbers in each of the following. What kind of numbers are shown in each case?

a 1, 4, 9, 16, 25, …
b 1, 3, 6, 10, 15, …
c 1, 1, 2, 3, 5, 8, …
d 2, 3, 5, 7, 11, 13, …

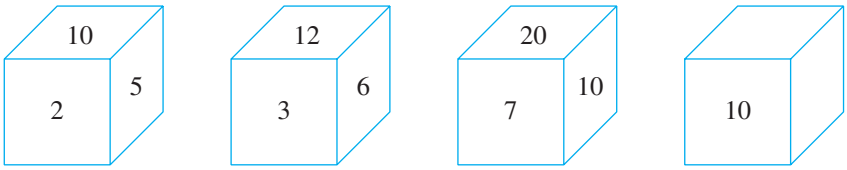
7 This diagram is a very important number pattern in mathematics. It is called Pascal's Triangle, after the famous French mathematician Blaise Pascal (1623–62). Each row begins and ends with the number 1 and every other number in the triangle is found by adding the two numbers above it.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

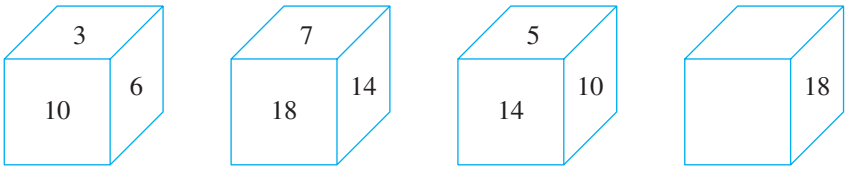
a Locate the counting numbers in this triangle.
b Locate the triangular numbers in this triangle.
c Is the triangle symmetric?
d Write down the next four lines in the triangle.

8 Each set of cubes obeys a certain rule. Write down the rule by referring to the front face, right face and top face, then find the missing numbers on the last cube.

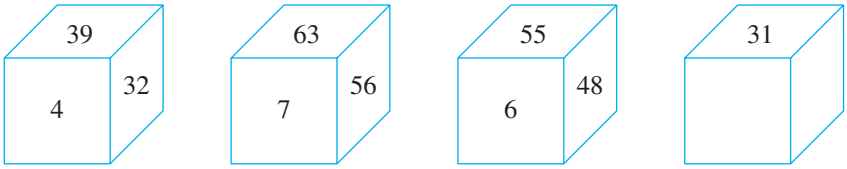
a



b



c



9 Copy each of these patterns, then write down the next 3 lines. Check that each answer is correct.

a $(1 \times 3) + 1 = 2^2 = 4$
$(2 \times 4) + 1 = 3^2 = 9$
$(3 \times 5) + 1 = 4^2 = 16$

b $5^2 - 0^2 = 5 \times 5 = 25$
$6^2 - 1^2 = 7 \times 5 = 35$
$7^2 - 2^2 = 9 \times 5 = 45$

c $8 \times 1 + 1 = 9 = 3^2$
$8 \times (1 + 2) + 1 = 25 = 5^2$
$8 \times (1 + 2 + 3) + 1 = 49 = 7^2$

d $0^2 + 2^2 = 4$
$1^2 + 3^2 = 4 + 6 = 10$
$2^2 + 4^2 = 4 + 6 + 10 = 20$
$3^2 + 5^2 = 4 + 6 + 10 + 14 = 34$

■ Further applications

10 In each of the following, the symbol stands for two operations. For example, if $5 \otimes 3 = 30$, $4 \otimes 5 = 40$ and $7 \otimes 2 = 28$, then $\otimes$ means 'double the first number then multiply the result

by the second number'. Find the meaning of the symbol in each of these, then evaluate the last expression in each line.

a $3 \varnothing 5 = 11,\ 2 \varnothing 8 = 12,\ 7 \varnothing 3 = 17,\ 4 \varnothing 11 = \square$

b $5 \oplus 2 = 2,\ 16 \oplus 3 = 5,\ 21 \oplus 2 = 10,\ 33 \oplus 4 = \square$

c $14 \boxslash 3 = 5,\ 21 \boxslash 1 = 4,\ 35 \boxslash 6 = 11,\ 63 \boxslash 4 = \square$

d $3 \Omega 2 = 7,\ 5 \Omega 6 = 19,\ 6 \Omega 4 = 32,\ 10 \Omega 12 = \square$

11 Further numbers or terms in a number pattern can be found in two different ways:

1 by finding a rule that relates each term to the *previous terms* in the pattern, or

2 by finding a rule that relates each term to its *position* in the pattern.

The questions studied so far in this exercise have all used the first method to find further numbers in a pattern. We will now find further numbers in a pattern by relating the numbers to their position. For example, in the pattern 3, 5, 7, 9, 11, … the first term is 3, the second term is 5, the third term is 7, and so on. We can show this relationship in the form of a table.

Position	1	2	3	4	5
Term	3	5	7	9	11

By observation, we might notice that each term can be found by doubling its position number and then adding 1. For example, the 5th number in the pattern could be found by working out $(2 \times 5) + 1$. That is, the 5th term is 11.

a What would be the 10th number in this pattern?

b What would be the 100th number in this pattern?

c What advantage does this method have over that which uses previous terms?

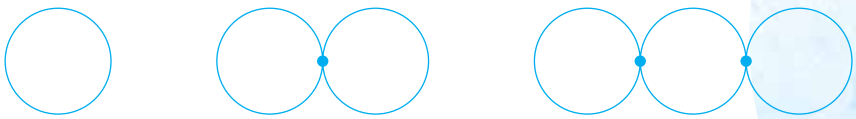
12 Find a rule that relates the terms (T_n) in each pattern to their position number (n), then find the 100th number in each pattern.

a 5, 6, 7, 8, … **b** 9, 10, 11, 12, … **c** 2, 4, 6, 8, … **d** 7, 14, 21, 28, …

e 1, 4, 9, 16, … **f** 5, 7, 9, 11, … **g** 2, 5, 8, 11, … **h** 7, 17, 27, 37, …

5.5 Describing the relationship between two quantities

Finding a relationship between two quantities can often be achieved by drawing up a table of values and looking for a pattern between the numbers. For example, the table below shows the number of points of intersection that would occur when several circles are arranged in a line.



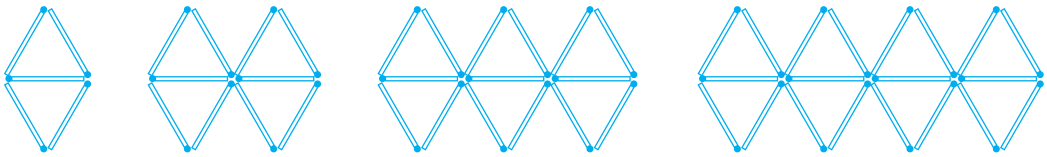
Number of circles	1	2	3	4
Number of points of intersection	0	1	2	3

Clearly, the number of points of intersection will always be 1 less than the number of circles. That is, 'the number of points of intersection = (the number of circles) – 1'.

This rule now defines the relationship between the number of points and the number of circles, when they are arranged in this way. We could then use this rule to determine the number of points of intersection that would arise for *any number* of touching circles. If 20 circles were arranged side by side as above, there would be 20 – 1, or 19 points of intersection.

Example 1

Consider this geometric pattern that has been formed using matchsticks.



a Complete this table of values.

Step number	1	2	3	4	5	6
Number of matches						

b Describe in at least two different ways, the pattern formed by the number of matches.

c Write down a rule that relates the number of matches to the step number in each figure.

d How many matches would be needed to make the figure in step 20?

Solutions

a

Step number	1	2	3	4	5	6
Number of matches	5	10	15	20	25	30

b The pattern formed by the number of matches could be described as:

i increasing by 5, or

ii adding 5 each time, or

iii multiples of 5.

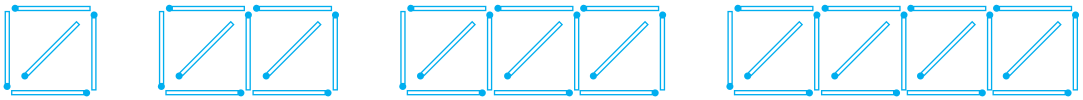
c The number of matches = 5 × the step number.

d In step 20, the number of matches = 5 × 20
= 100

∴ 100 matches would be needed.

Example 2

Consider this geometric pattern which has been formed using matchsticks.



a How many matches need to be added to extend each figure?
b Form a table of values that shows the number of matches in each figure.
c Write down a rule that could be used to determine the number of matches in each step.
d Write down a different rule that could be used to find the number of matches in each step.
e How many matches would be needed to make the figure in step 50?

Solutions

a 4 matches need to be added to extend each figure.

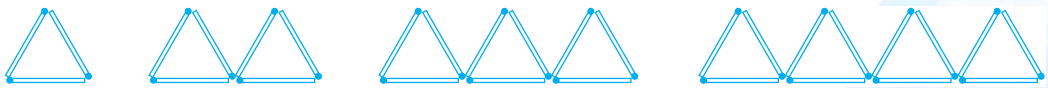
b

Number of squares	1	2	3	4
Number of matches	5 $1 + 4$ $1 + (1 \times 4)$	9 $1 + 4 + 4$ $1 + (2 \times 4)$	13 $1 + 4 + 4 + 4$ $1 + (3 \times 4)$	17 $1 + 4 + 4 + 4 + 4$ $1 + (4 \times 4)$

c The number of matches = 1 + (4 × the step number).
d The number of matches = 5 + (4 × one less than the step number).
e In step 50, the number of matches $= 1 + (4 \times 50)$
$= 1 + 200$
$= 201$
∴ 201 matches would be needed.
Using the alternate rule, the number of matches $= 5 + (4 \times 49)$
$= 5 + 196$
$= 201$

Exercise 5.5

1 Build this geometric pattern using matchsticks and extend it by two more steps.



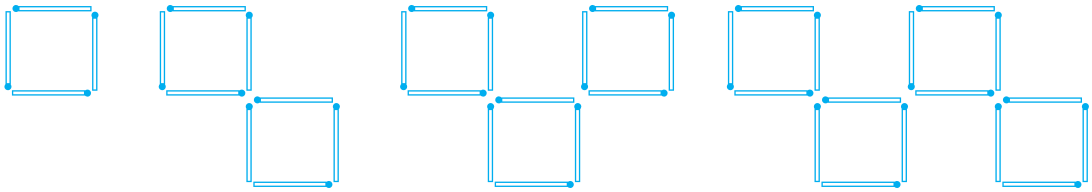
a Complete this table of values.

Step number	1	2	3	4	5	6
Number of matches						

b Describe in at least two different ways, the pattern formed by the number of matches.
c Write down a rule that relates the number of matches to the step number in each figure.

d How many matches would be needed to make the figure in:
i step 8? **ii** step 13? **iii** step 20?

2 Build this geometric pattern using matchsticks and extend it by two more steps.



a Complete this table of values.

Step number	1	2	3	4	5	6
Number of matches						

b Describe in at least two different ways, the pattern formed by the number of matches.
c Write down a rule that relates the number of matches to the step number in each figure.
d How many matches would be needed to make the figure in:
i step 9? **ii** step 12? **iii** step 50?

3 Build this geometric pattern using matchsticks and extend it by two more steps.

a Complete this table of values.

Step number	1	2	3	4	5	6
Number of matches						

b Write down a rule that relates the number of matches to the step number in each figure.
c How many matches would be needed to make the figure in:
i step 10? **ii** step 17? **iii** step 40?

4 Build this geometric pattern using matchsticks and extend it by three more steps.

a Complete this table of values.

Step number	1	2	3	4	5	6
Number of matches						

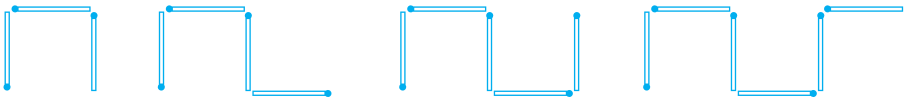
b Write down a rule that relates the number of matches to the step number in each figure.
c How many matches would be needed to make the figure in:
i step 13? **ii** step 35? **iii** step 100?

5 For each of the following matchstick patterns:

i form a table of values showing the number of matches in each of the first 6 steps.
ii write down a rule that links the number of matches to the step number of each figure.
iii find the number of matches in step 20.

a

b



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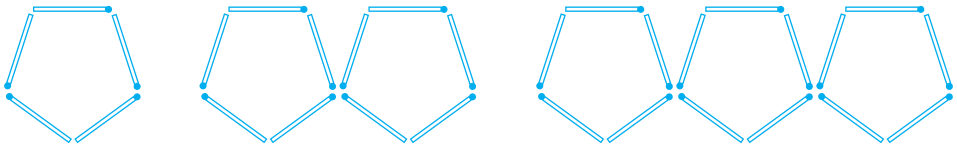

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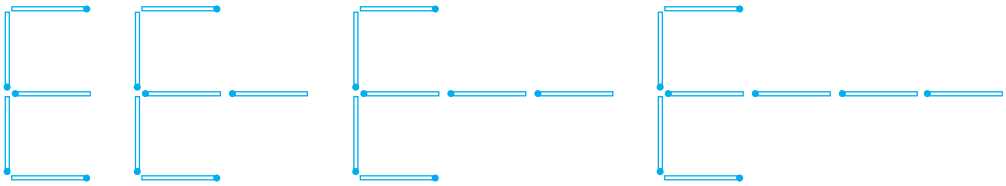
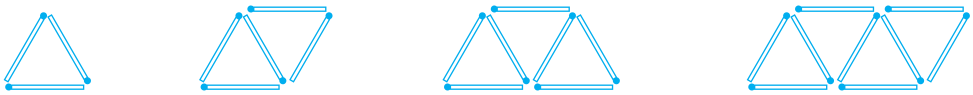


■ Consolidation

6 Build this geometric pattern using matchsticks and extend it by one more step.



a How many matches need to be added to extend each figure?

b Copy and complete this table.

Step number	1	2	3	4
Number of matches	3 1 + 2 1 + (1 × 2)	5 1 + 2 + 2 1 + (2 × 2)	7 1 + 2 + 2 + 2 1 + (3 × 2)	…… …… ……

c Copy and complete this rule which can be used to find the number of matches in each step. 'The number of matches = 1 + (……)'

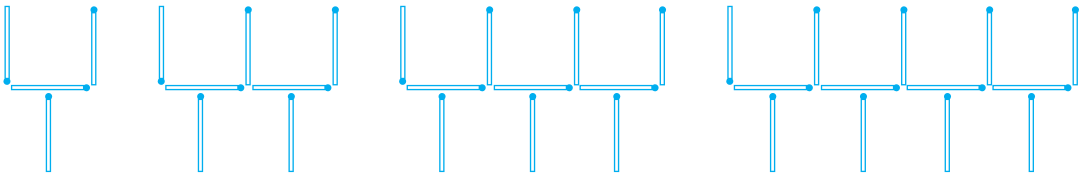
d How many matches would be needed to make the figure in:

i step 7? **ii** step 10? **iii** step 30?

e Write down a different rule that could be used to find the number of matches in each step.

f Which rule do you think is better? Why?

7 Build this geometric pattern using matchsticks and extend it by one more step.



a How many matches need to be added to extend each figure?

b Copy and complete this table.

Step number	1	2	3	4
Number of matches	4 $1 + 3$ $1 + (1 \times 3)$	7 $1 + 3 + 3$ $1 + (2 \times 3)$	…… …… ……	…… …… ……

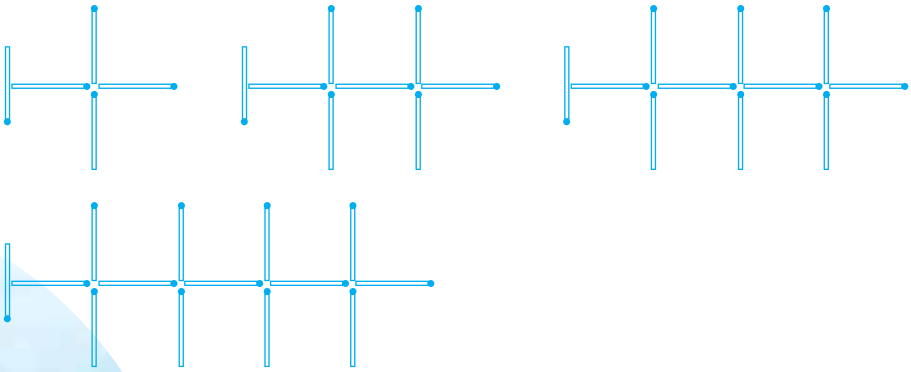
c Copy and complete this rule which can be used to find the number of matches in each step. 'The number of matches = … + (… × 3)'

d How many matches would be needed to make the figure in:

i step 8? **ii** step 12? **iii** step 50?

e Write down a different rule that could be used to find the number of matches in each step.

8 Build this geometric pattern using matchsticks and extend it by one more step.



a How many matches need to be added to extend each figure?

b Copy and complete this table.

Step number	1	2	3	4
Number of matches	5 $2 + 3$ $2 + (1 \times 3)$	8 $2 + 3 + 3$ $2 + (2 \times 3)$	…… …… ……	…… …… ……

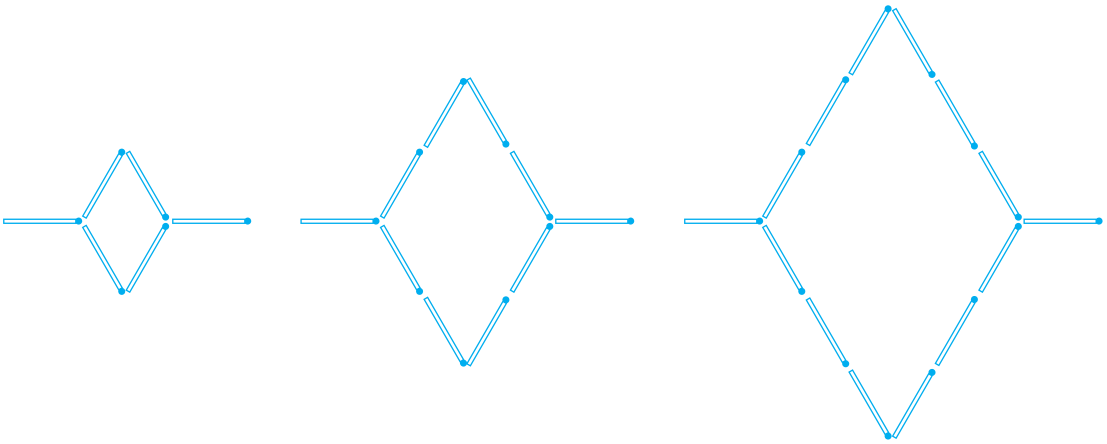
c Copy and complete this rule which can be used to find the number of matches in each step. 'The number of matches = ……'

d How many matches would be needed to make the figure in:

i step 11? **ii** step 20? **iii** step 25?

e Write down a different rule that could be used to find the number of matches in each step.

9 Build this geometric pattern using matchsticks and extend it by one more step.



a How many matches need to be added to extend each figure?

b Copy and complete this table.

Step number	1	2	3	4
Number of matches	6 2 + 4 ……	10 2 + 4 + 4 ……	…… …… ……	…… …… ……

c Copy and complete this rule which can be used to find the number of matches in each step. 'The number of matches = ……'

d How many matches would be needed to make the figure in:

i step 10? **ii** step 17? **iii** step 100?

e Write down a different rule that could be used to find the number of matches in each step.

10 For each of the following matchstick patterns:

i form a table of values showing the number of matches in each of the first 4 steps.

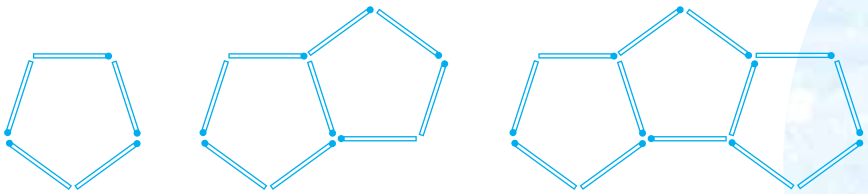
ii predict the number of matches that would be needed to make the figure in step 6.

iii write down a rule that links the number of matches to the step number of each figure.

iv use this rule to check your answer to part ii.

v find the number of matches in step 50.

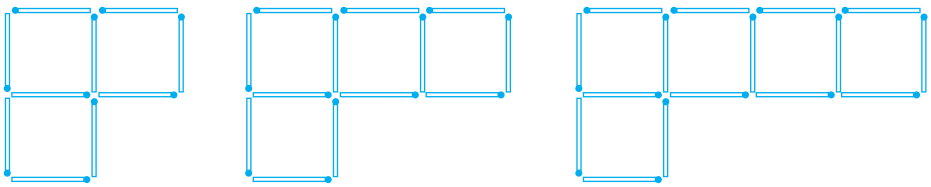
a



b

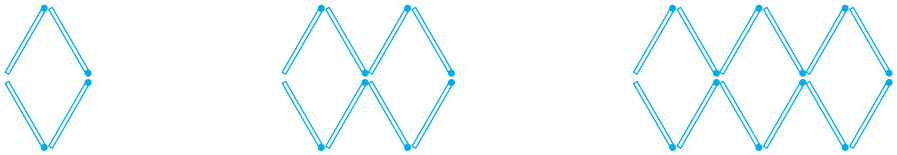
c

d

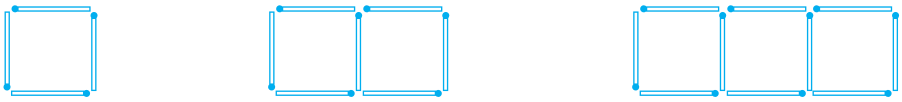


11 In the six matchstick patterns below, three of the patterns follow one rule and the other three follow another rule. Find the two rules linking the number of matches and the step number. State which patterns are described by each rule.

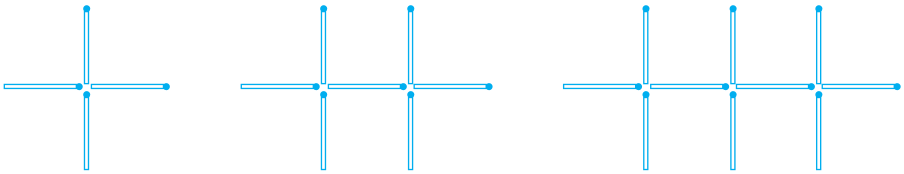
a



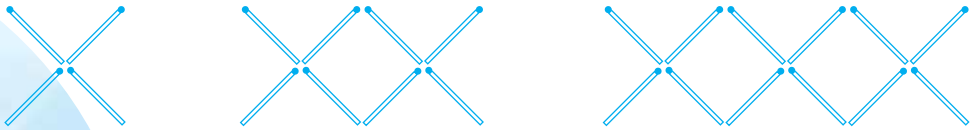
b



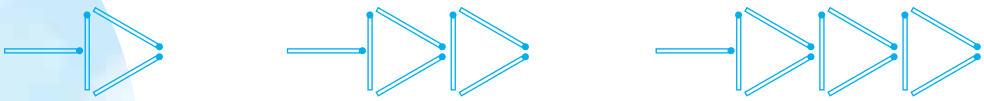
c



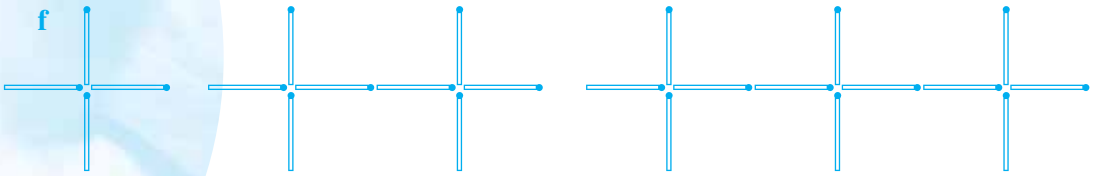
d



e

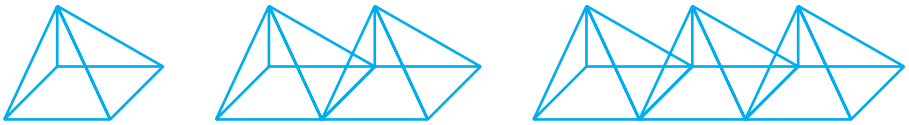


f

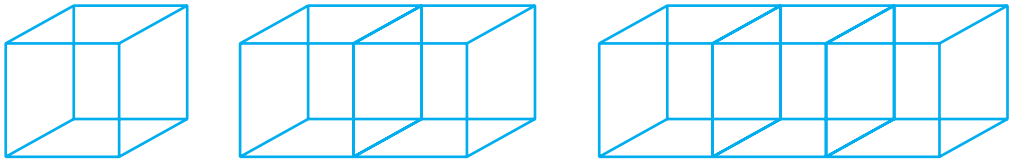


12 The figures below were made using pipe cleaners. Construct a table of values for each, then find a rule that links the number of pipe cleaners to the step number. Use this rule to determine the number of pipe cleaners that would be needed to make the figure in step 12.

a



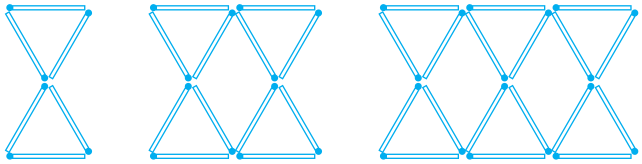
b



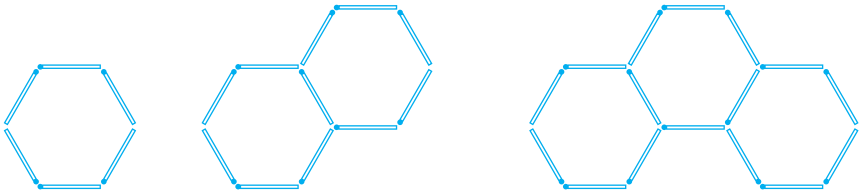
■ Further applications

13 Find a rule for each pattern, linking the number of matches to the step number of each figure. Then find the step number of the figure in which there would be:

a 120 matches



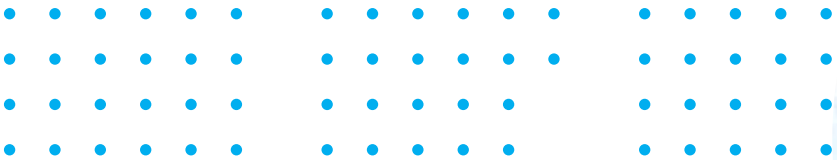
b 46 matches



14 Draw a matchstick pattern (other than those in the exercise) for which:

a the number of matchsticks = five times the step number.
b the number of matchsticks = four more than the step number.
c the number of matchsticks = two more than three times the step number.

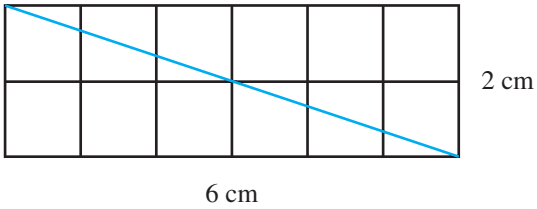
15 Consider this pattern of dots where the number of dots is decreasing in each step.



a Find a rule that links the number of dots in each figure to the step number.
b How many dots would there be in step 7?
c Explain why the number of figures in this pattern is limited. How many are there?

TRY THIS **Diagonals**

The following diagonal line intersects 6 lines and passes through 6 squares of this rectangle.



How many intersections and number of squares passed through are there for rectangles of the following dimensions?

1 3×2 **2** 3×4 **3** 3×5

4 2×3 **5** 2×4 **6** 2×5

Is there a pattern and can you describe it?

Can you tell without a diagram the answers for 6×8 and 11×20?

5.6 Using pronumerals to describe relationships

In exercises 5.1 and 5.2 it was shown that pronumerals could be used to represent numbers whose value may vary or change. In such cases we can also refer to the pronumerals as **variables**. The previous exercise should have led you to understand that the step number is different at each stage of a pattern and that the number of matches, being directly related to the step number, must also vary throughout the pattern.

In this exercise, pronumerals will be used to represent the step number and the number of matches, tiles or cubes in a pattern. This will allow for the writing of shorter and simpler pattern rules. Remember, the pronumerals stand for numbers, not objects.

Example 1

Rewrite each of the following pattern rules using pronumerals, where n represents the number of matches and x represents the step number.

a The number of matches = four times the step number.

b The number of matches = twice the step number, plus five.

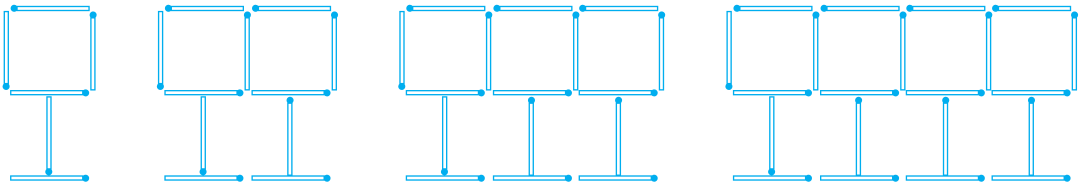
Solutions

a $n = 4x$

b $n = 2x + 5$

Example 2

Consider this geometric pattern that has been formed using matchsticks.



a Form a table of values for this pattern.

b Write down a rule in words for the pattern.

c Rewrite this rule using pronumerals, where n stands for the number of matches and x stands for the step number.

d Use the same pronumerals to write down a different rule for the pattern.

Solutions

a

Step number (x)	1	2	3	4
Number of matches (n)	6	11	16	21

b The number of matches = (5 × the step number) + 1

c $n = 5x + 1$

d The number of matches = 5 × (the step number − 1) + 6. We can use pronumerals to write this rule as $n = 5(x - 1) + 6$.

Exercise 5.6

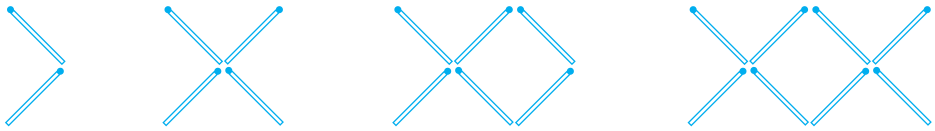
1 Rewrite each of the following pattern rules using pronumerals, where n represents the number of matches and x represents the step number.

a The number of matches = twice the step number.
b The number of matches = five times the step number.
c The number of matches = the step number plus three.
d The number of matches = the step number minus nine.
e The number of matches = three times the step number, plus four.
f The number of matches = six times the step number, minus ten.

2 Rewrite each of the following pattern rules in words, given that a represents the number of matches and b represents the step number.

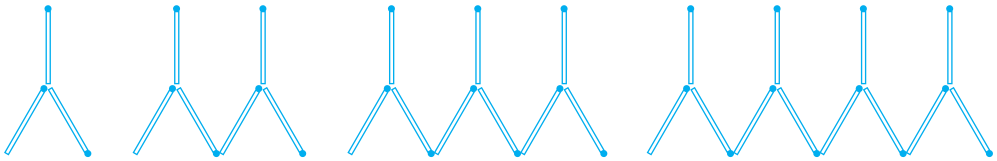
a $a = 3b$ **b** $a = 7b$ **c** $a = 2 + b$ **d** $a = b - 8$
e $a = 2(b + 5)$ **f** $a = 4(b - 1)$ **g** $a = 6(b - 3)$ **h** $a = 9(b + 4)$

3



a Continue this matchstick pattern for one more step.
b Predict the number of matches in step 7.
c Form a table of values up to step 5, where u stands for the number of matches and v stands for the step number.
d Write down a rule in words, that relates the number of matches to the step number.
e Rewrite this rule using pronumerals.

4

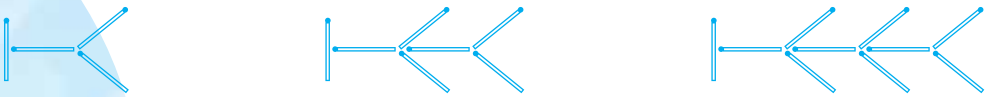


a Continue this matchstick pattern for one more step.
b Predict the number of matches in step 7.
c Form a table of values up to step 5, where a stands for the number of matches and b stands for the step number.
d Write down a rule in words, that relates the number of matches to the step number.
e Rewrite this rule using pronumerals.

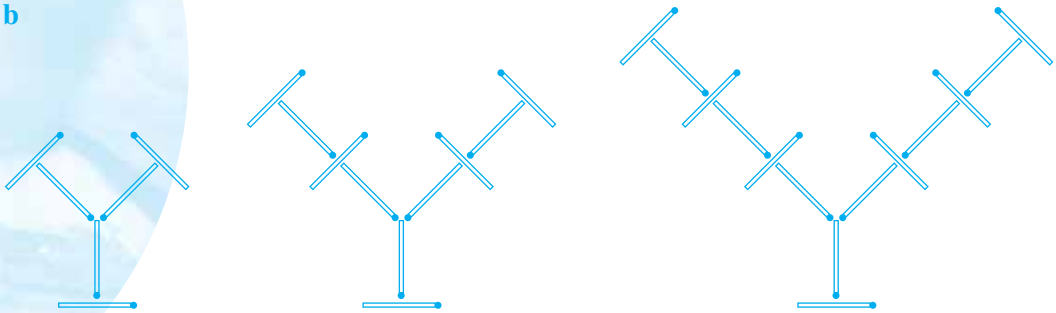
Consolidation

5 For each of the following matchstick patterns:
- **i** continue the pattern for one more step
- **ii** predict the number of matches in step 6
- **iii** form a table of values, where p stands for the number of matches and q stands for the step number
- **iv** write down a rule in words for the pattern
- **v** rewrite this rule using pronumerals
- **vi** use pronumerals to write down a different rule for each pattern.

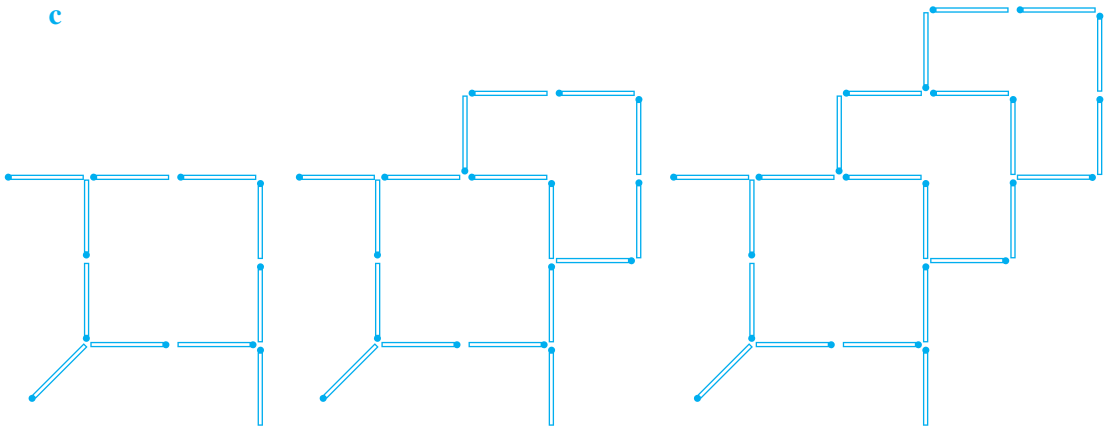
a



b



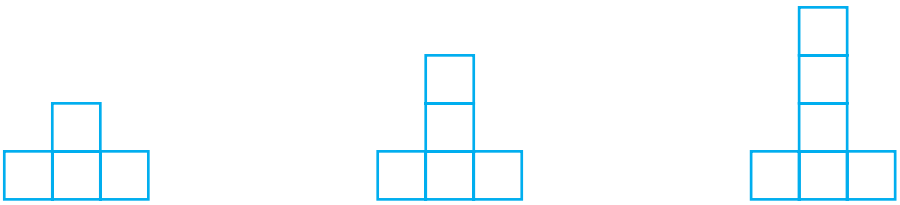
c



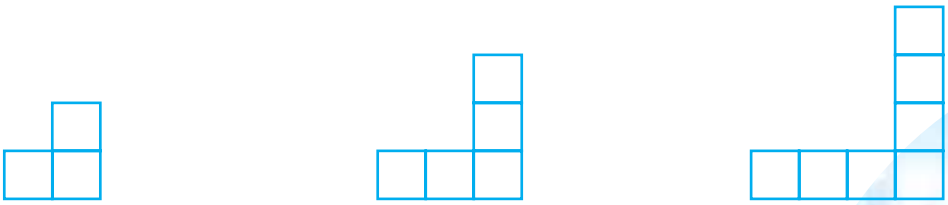
6 Build each of the patterns below using tiles, then:

- **i** continue the pattern for one more step
- **ii** predict the number of tiles in step 5
- **iii** form a table of values, where p stands for the number of tiles and q stands for the step number
- **iv** write down a rule in words for the pattern
- **v** rewrite this rule using pronumerals.

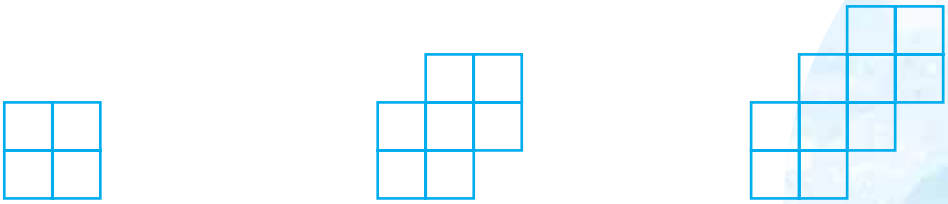
a



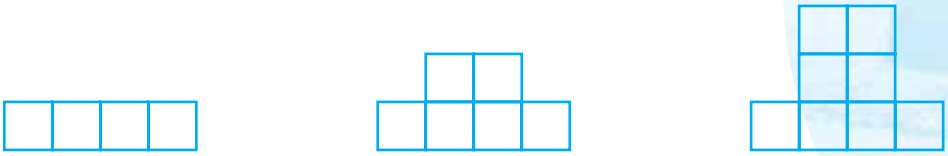
b



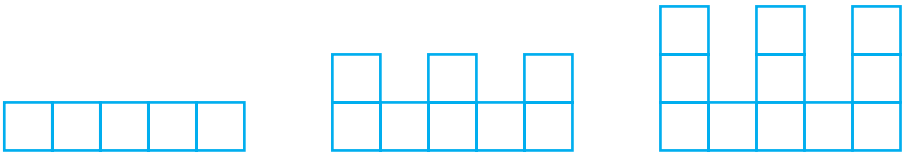
c



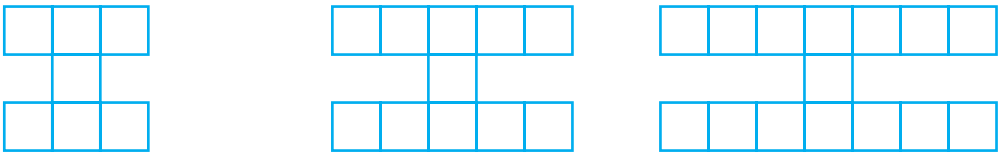
d



e

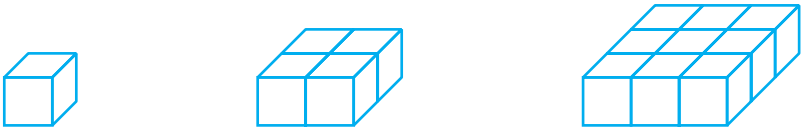


f

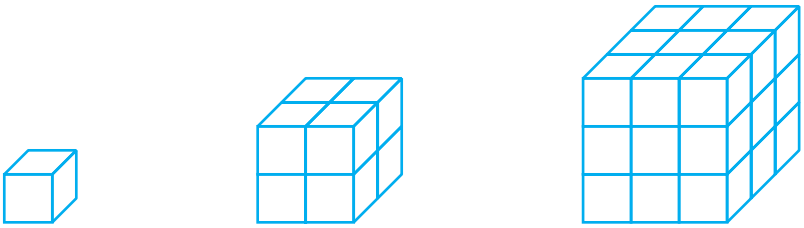


7 Use cubes to build these patterns.

a



b



For each pattern:

- **i** build the next step
- **ii** draw a table of values using two pronumerals of your choice
- **iii** predict the number of cubes in step 6
- **iv** write down a rule in words for the pattern
- **v** rewrite this rule using pronumerals.

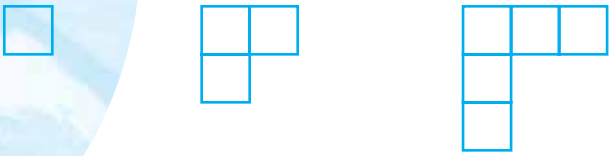
8 Build a geometric pattern with cubes which obeys each of these rules. Let the number of cubes be t and the step number be s.

a $t = 4s$ **b** $t = s + 3$ **c** $t = 2s + 1$ **d** $t = 3s + 1$

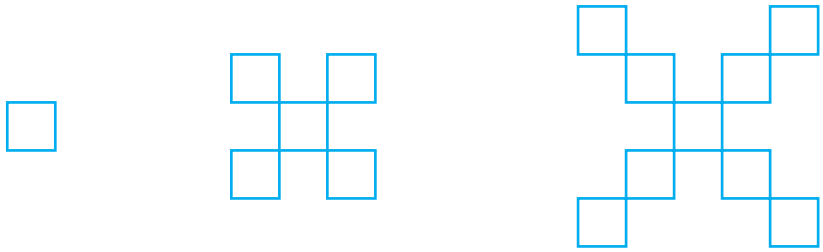
Further applications

9 Find an algebraic rule that links the number of tiles to the step number in each figure. In which step would there be 45 tiles? (Let the step number be x and the number of tiles be n.)

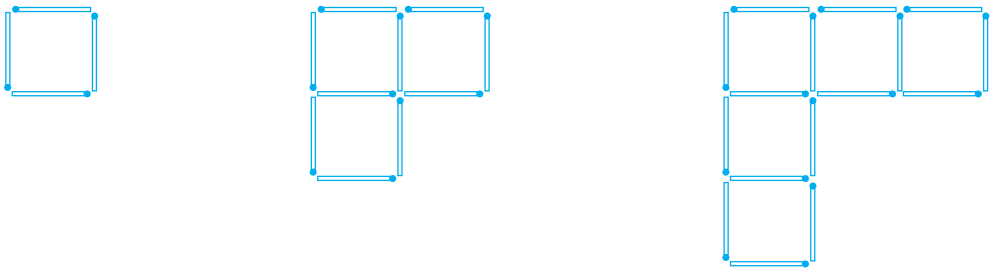
a



b



10



a Draw up the next step in this matchstick pattern.
b Copy and complete this table of values.

Step number	1	2	3	4
Number of squares (s)				
Number of matches (m)				
Perimeter (p)				

c Use pronumerals to write a rule that links:
  i the number of matches to the number of squares
  ii the perimeter to the number of squares.
d Find the number of matches that would be needed to build 9 squares.
e Find the perimeter of a figure that has 9 squares.
f Find the perimeter of a figure that is made up of 40 matches.

5.7 Tables of values

In the previous exercise, algebraic rules were found by investigating patterns formed from matchsticks, tiles and cubes.

In this exercise we will examine further two main problems:
1 using a given algebraic rule to complete a table of values, and
2 finding the rule that links the variables in a table of values.

Example 1

Complete this table of values using the rule $y = 2x + 3$.

x	1	2	3	4
y				

Solution

when $x = 1$:
$y = (2 \times 1) + 3$
$\therefore y = 5$

when $x = 2$:
$y = (2 \times 2) + 3$
$\therefore y = 7$

when $x = 3$:
$y = (2 \times 3) + 3$
$\therefore y = 9$

when $x = 4$:
$y = (2 \times 4) + 3$
$\therefore y = 11$

The completed table of values then is:

x	1	2	3	4
y	5	7	9	11

In each case, the y-value is 3 more than double the corresponding x-value.

Example 2

Find the rule that could be used to generate each table of values.

a

m	0	1	2	3
n	4	5	6	7

b

u	1	2	3	4
v	5	10	15	20

c

p	2	4	6	8
q	1	2	3	4

d

a	1	2	3	4
b	7	9	11	13

Solutions

a $4 = 0 + 4,\ 5 = 1 + 4,\ 6 = 2 + 4,\ 7 = 3 + 4$
$\therefore$ the rule is $n = m + 4$

b $5 = 1 \times 5,\ 10 = 2 \times 5,\ 15 = 3 \times 5,\ 20 = 4 \times 5$
$\therefore$ the rule is $v = u \times 5$, or $v = 5u$

c $1 = 2 \div 2,\ 2 = 4 \div 2,\ 3 = 6 \div 2,\ 4 = 8 \div 2$
$\therefore$ the rule is $q = p \div 2$, or $q = \frac{p}{2}$

d $7 = (1 \times 2) + 5,\ 9 = (2 \times 2) + 5,\ 11 = (3 \times 2) + 5,\ 13 = (4 \times 2) + 5$
$\therefore$ the rule is $b = a \times 2 + 5$, or $b = 2a + 5$

Exercise 5.7

1 Copy and complete each table of values using the given expression.

a

x	1	2	3	4	5	6	10	100
$x + 4$								

b

x	1	2	3	4	5	6	20	100
$5x$								

c

x	7	8	9	10	11	12	30	100
$x - 3$								

d

x	8	10	12	14	16	18	50	100
$\frac{x}{2}$								

2 Complete each table of values using the given rule.

a $y = x + 2$

x	1	2	3	4
y				

b $q = p - 1$

p	5	6	7	8
q				

c $h = 2 \times t$

t	0	1	2	3
h				

d $b = a \div 3$

a	3	6	9	12
b				

e $n = 4 + m$

m	1	3	5	7
n				

f $v = u - 6$

u	8	9	10	11
v				

g $d = c \times c$

c	1	2	3	4
d				

h $w = \frac{24}{k}$

k	1	2	3	4
w				

i $s = 10 - t$

t	0	1	2	3
s				

j $f = \frac{1}{2}e$

e	2	4	6	8
f				

3 Complete each table of values using the given rule.

a $y = 2x + 1$

x	1	2	3	4
y				

b $v = 3u - 2$

u	3	4	5	6
v				

c $q = (p + 3) \times 4$

p	0	1	2	3
q				

d $b = 2 \times (a + 1)$

a	5	6	7	8
b				

$k = 3(c - 2)$

e

c	5	6	7	8
k				

$z = \frac{12}{w} + 2$

f

w	1	2	3	4
z				

$f = \frac{e-1}{2}$

g

e	3	5	7	9
f				

$n = m^2 - 10$

h

m	4	5	6	7
n				

$u + v = 5$

i

u	1	2	3	4
v				

$k = (j - 2) \times (j + 1)$

j

j	3	4	5	6
k				

$h = (g + 1)^2$

k

g	0	1	2	3
h				

$t = (8 - s)(s + 3)$

l

s	0	1	2	3
t				

Consolidation

4 Find a rule that could be used to generate each table of values.

a

x	1	2	3	4
y	6	7	8	9

b

w	9	10	11	12
z	7	8	9	10

c

c	1	2	3	4
d	3	6	9	12

d

t	6	8	10	12
h	3	4	5	6

e

n	10	11	12	13
k	3	4	5	6

f

t	2	4	6	8
d	12	24	36	48

g

g	1	2	3	4
h	5	10	15	20

h

e	1	2	3	4
f	24	12	8	6

i

m	12	13	14	15
n	8	7	6	5

j

p	5	6	7	8
q	13	14	15	16

k

j	11	22	33	44
k	1	2	3	4

l

r	1	2	3	4
s	1	4	9	16

5 Find a rule that could be used to generate each table of values.

a

s	3	4	5	6
t	7	9	11	13

b

p	1	2	3	4
q	5	8	11	14

c

x	1	2	3	4
y	5	7	9	11

d

u	4	5	6	7
v	11	14	17	20

e

c	1	2	3	4
d	1	5	9	13

f

j	0	1	2	3
k	2	7	12	17

g

t	2	3	4	5
v	0	2	4	6

h

m	1	2	3	4
n	0	3	6	9

i

w	2	4	6	8
z	2	3	4	5

j

g	3	6	9	12
h	6	7	8	9

k

a	1	2	3	4
b	2	5	10	17

l

e	3	4	5	6
f	7	14	23	34

6 For each table of values, find a rule that links x and y, then find the missing values.

a

x	1	2	3	5	10
y	4	8		20	

b

x	1	2	5	8	
y	6	7	10		17

c

x	1	3	7	12	
y	3		15	25	51

d

x	2	3		10	11
y	8	11	23		35

e

x	2	4	9		15
y	3	7		25	29

f

x	3	5	6	9	
y		23	28	43	98

■ Further applications

7 Find a rule that could be used to generate each of these decreasing number patterns.

a

n	1	2	3	4
t	6	5	4	3

b

n	1	2	3	4
t	18	16	14	12

c

n	1	2	3	4
t	22	19	16	13

d

n	1	2	3	4
t	23	18	13	8

e

n	1	2	3	4
t	17	13	9	5

f

n	1	2	3	4
t	50	40	30	20

TRY THIS **What's my rule?**

You will need a partner for this activity. Each of you should write down your own table based on a rule, e.g.

Input	5	8	6	12	9
Output	2	5	3	9	6

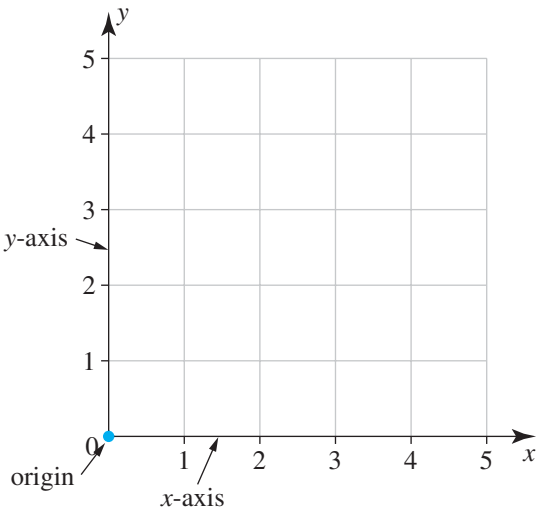
You then swap tables. Your task is to find the rule used by your partner. For example, the rule above is:

Output = Input minus 3

5.8 Representing relationships on a number grid

A **number grid** is a two-dimensional grid drawn on a plane surface. It consists of two perpendicular lines called **axes**. The horizontal axis is usually referred to as the ***x*-axis** and the vertical axis is usually referred to as the ***y*-axis**. The point where the axes meet is called the **origin**.

The position of a point on the number grid is given as an **ordered pair** or **pair of co-ordinates**. In any ordered pair, the horizontal reading (usually the x-value) must be given first and the vertical reading (usually the y-value) must be given second. That is, ordered pairs are given in the form (x, y). The co-ordinates of the origin are (0, 0).

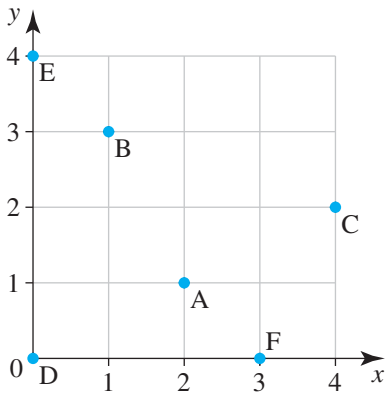


The position of a point on a number grid is given as an ordered pair (x, y), where the x-value is the reading on the horizontal axis and the y-value is the reading on the vertical axis.

Example 1

State the co-ordinates of each of these points.

a A **b** B **c** C
d D **e** E **f** F

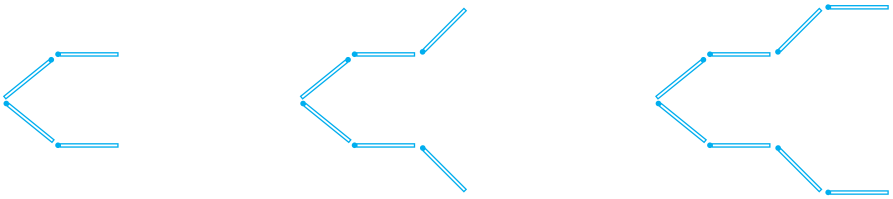


Solutions

a A(2, 1) **b** B(1, 3) **c** C(4, 2) **d** D(0, 0) **e** E(0, 4) **f** F(3, 0)

Example 2

Consider the following matchstick pattern.



a Complete this table of values.

Step number (x)	1	2	3	4	5
Number of matches (y)					

b Plot the ordered pairs from the table on a number grid. Does it make sense to join the points? Explain.

c What do you notice about the points?

d Use pronumerals to write down the rule that links the number of matches to the step number.

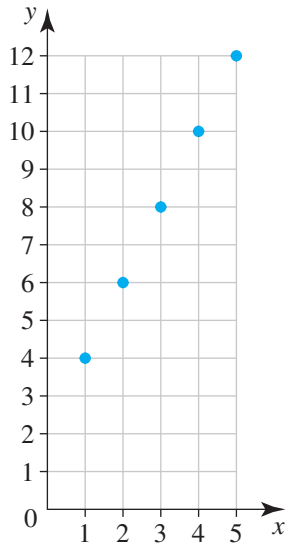
e Would the point (10, 22) lie in the same line as the points on your graph? Explain.

Solutions

a

Step number (x)	1	2	3	4	5
Number of matches (y)	4	6	8	10	12

b



No. We work in whole steps and so should only have values for whole numbers.

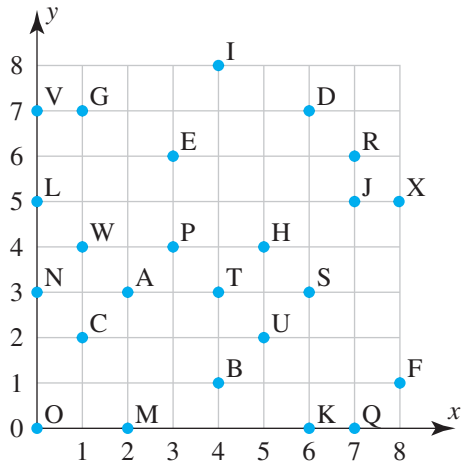
c The points all lie in a straight line. (That is, there is a linear relationship between the values in the table.)

d The number of matches = 2 × (the step number) + 2, i.e. $y = 2 \times x + 2$, or $y = 2x + 2$.

e Yes, the point (10, 22) would lie on the line because when

$$x = 10,\ y = 2 \times 10 + 2$$
$$= 22.$$

Exercise 5.8



1 Write down the letter that marks the position of each point.

a	(1, 2)	**b**	(7, 5)	**c**	(4, 3)
d	(1, 4)	**e**	(8, 1)	**f**	(3, 6)
g	(4, 8)	**h**	(6, 3)	**i**	(6, 7)
j	(8, 5)	**k**	(1, 7)	**l**	(7, 6)
m	(2, 0)	**n**	(0, 3)	**o**	(7, 0)
p	(0, 7)	**q**	(0, 0)	**r**	(4, 1)
s	(5, 2)	**t**	(3, 4)	**u**	(6, 0)
v	(5, 4)	**w**	(2, 3)	**x**	(0, 5)

2 Write down the co-ordinates of each point.

a	A	**b**	B	**c**	C						
d	D	**e**	E	**f**	F						
g	G	**h**	H	**i**	I	**j**	J	**k**	K	**l**	L
m	M	**n**	N	**o**	O	**p**	P	**q**	Q	**r**	R
s	S	**t**	T	**u**	U	**v**	V	**w**	W	**x**	X

3 a The number of matches (y) in each step (x) of a matchstick pattern is shown in this table of values. Plot these points on a number grid. Does it make sense to join the points? Explain.

x	1	2	3	4	5
y	4	5	6	7	8

b What do you notice about the points?

c Write down in words a rule for this number pattern.

d Rewrite this rule using pronumerals.

4 **a** Graph the ordered pairs in this table on a number grid.

x	0	1	2	3	4
y	6	5	4	3	2

b Do the points lie in a straight line?

c Write down the rule for this number pattern using pronumerals.

d How do the points in this graph differ from those in Q3?

Consolidation

5 Graph each set of ordered pairs on a separate number grid.

a

a	0	1	2	3	4
b	1	2	3	4	5

b

p	1	2	3	4	5
q	0	1	2	3	4

c

g	0	1	2	3	4
h	1	3	5	7	9

d

u	2	3	4	5	6
v	0	2	4	6	8

6 Copy and complete each table of values using the given rule, then plot the points on a number grid.

a $y = 2x$

x	0	1	2	3
y				

b $y = x + 2$

x	0	1	2	3
y				

c $y = 4 - x$

x	0	1	2	3
y				

d $y = 2x - 1$

x	1	2	3	4
y				

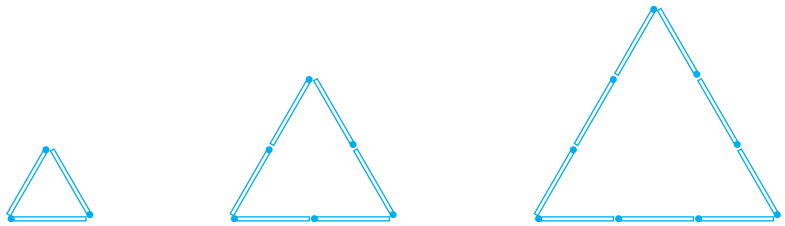
e $y = 3x - 2$

x	1	2	3	4
y				

f $y = 5 - \frac{x}{2}$

x	2	4	6	8
y				

7



a Copy and complete this table of values for the given matchstick pattern.

Step number (n)	1	2	3	4	5
Number of matches (m)					

b Plot the points on a number grid.
c Write down in words a rule that links the number of matches to the number of triangles.
d Rewrite this rule using pronumerals.

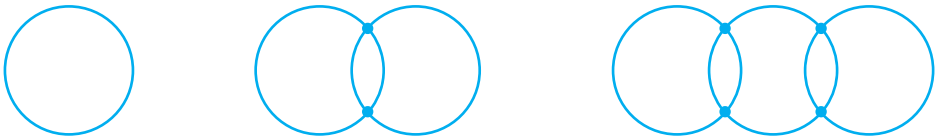
8

a Copy and complete this table of values for the given pattern of triangles.

Number of triangles (q)	1	2	3	4	5
Number of dots (p)					

b Plot the points on a number grid.
c Write down in words a rule that links the number of dots to the number of triangles.
d Rewrite this rule using pronumerals.
e How many dots would there be in a figure made up of 22 triangles?
f Would the point (15, 17) lie in line with the points on your graph? Explain.

9



a Copy and complete this table of values for the given pattern of circles.

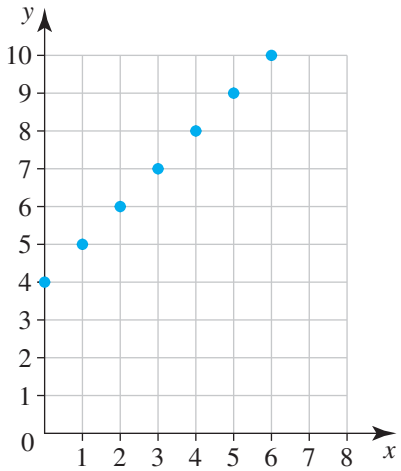
Number of circles (b)	1	2	3	4	5
Number of dots (a)					

b Plot the points on a number grid.
c Use pronumerals to write a rule that links the number of dots to the number of circles.
d How many dots would there be in a figure made up of 15 circles?
e Does it make sense to join the points on your graph? Explain.
f Would the point (9, 20) lie in line with the points on your graph? Explain.
g Write down the co-ordinates of 3 other points that would lie in line with the other points in the table.

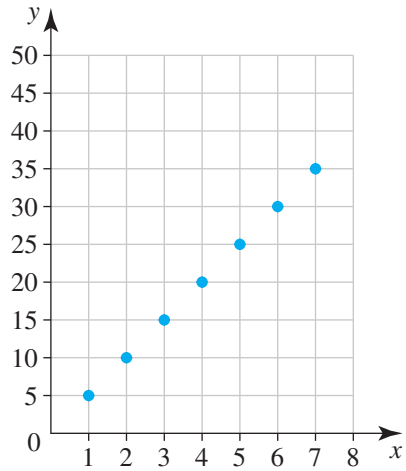
Further applications

10 Complete a table of values for each graph and hence determine the pattern rule.

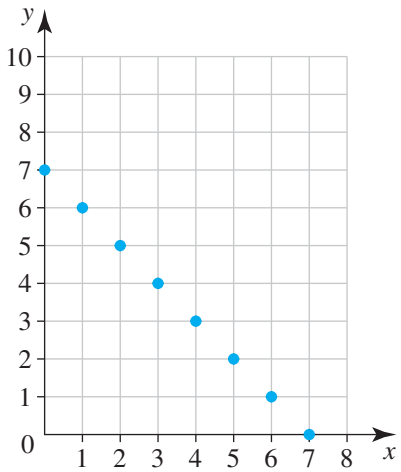
a



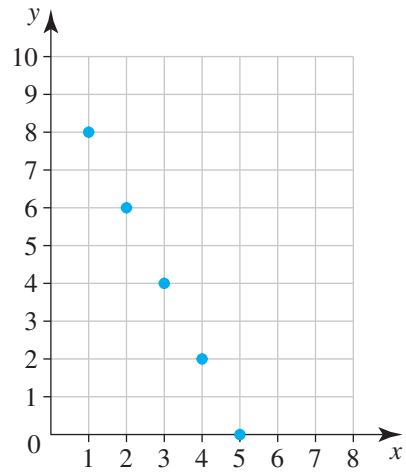
b



c



d



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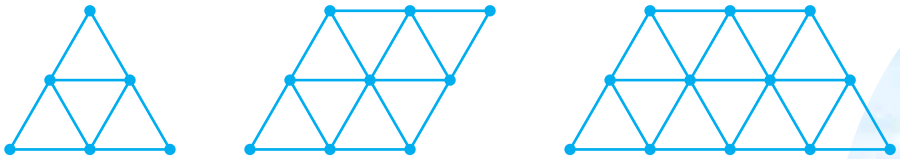


a Copy and complete this table of values for the given pattern.

Number of small triangles (s)	4	8	12	16
Number of dots (r)				

b By choosing suitable scales on the axes, graph the ordered pairs in (s, r) on a number grid.

c Describe this pattern in as many ways as possible.

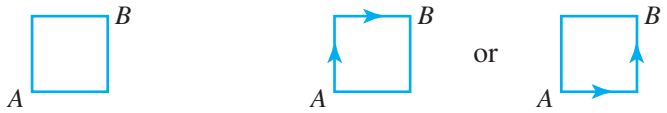
d Use pronumerals to write a rule that links the number of dots to the number of triangles. [HINT: Add 4 to all of the s-values.]

12 What have you learned about number patterns in this chapter? Give examples where necessary.

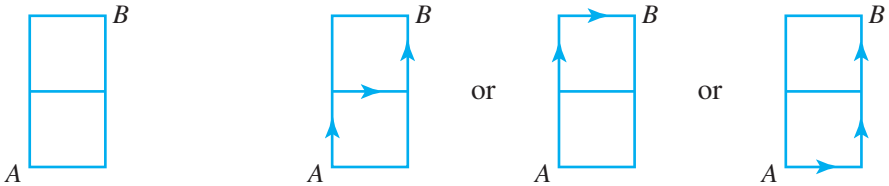
TRY THIS

Trips

How many ways are there to travel from A to B?
You must always travel upwards ↑ or turn to the right →, so there are only two ways.



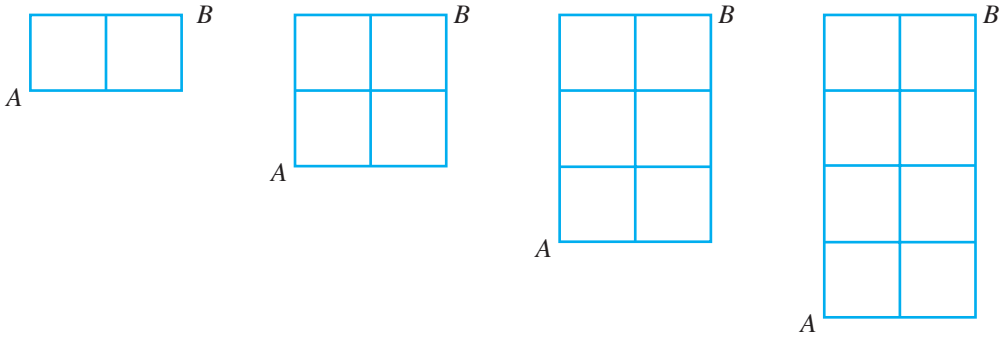
What if we had 2 squares? How many ways are there this time?



Try again with more squares and complete this table.

Number of squares tall (T)	1	2	3	4	5	6
Number of ways (W)	2	3				

What would happen if the area was 2 boxes wide?



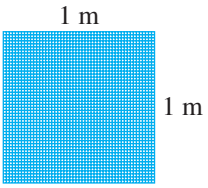
Can you write a rule to describe what is happening?
Complete this table.

Number tall (T)	1	2	3	4	5
Number of ways (W)					

Describe the pattern.

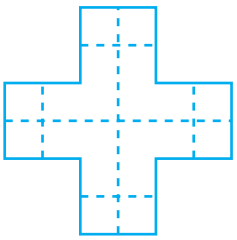
Problem solving

1 Suppose we cut the following piece of material into pieces of area 1 mm^2 or strips of width 1 mm. They are then placed end to end. What would be the total length of these pieces of material?



2 In a 100 metre swimming race, Josh beat Paul by 10 metres. In another 100 metre race, Paul beat Andrew by 10 metres. If Josh raced Andrew, would he win by 20 metres if they swam at their usual speed?

3 Find two ways to cut this cross into four pieces. Each piece must be of the same size and shape.



4 You have just raced against the Olympic 100 metre champion. She beat you by a distance of 50 metres. Suppose you want to reach the finish line at the same time, what advantage should you be given? (It is not the obvious answer.) You might like to try this outside.

5 Draw three straight lines on this letter M in order to form 9 triangles.

6 What are the next two numbers in this sequence: $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \ldots,\right\}$

7 Twenty-seven small cubes have been stuck together to form this shape.
Altogether, how many faces have glue on them?

8 How many faces do not have glue on them?

9 In Q7, how many cubes have glue on all six sides?

10 This sequence of numbers is called the Fibonacci sequence. Write the next two numbers in the sequence. {1, 1, 2, 3, 5, 8, 13, 21, …, …,}

Focus on working mathematically

Seeing is believing

This activity is an exercise in visual thinking to follow up what you have already learned about patterns in mathematics. The idea is to create pictures or diagrams of number patterns to see *why* a particular algebraic statement may be true. Such pictures are often helpful in deciding *how* we might go about *proving* it to be true. As you will see from the history of the subject, Mathematicians are generally very good at making mental pictures of things. This is to help you strengthen your ability to do the same.

Learning activities

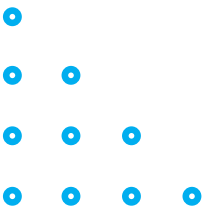
Materials needed: Coloured counters, paper and pen.

You are given the task of adding up all the numbers from 1 to 99:

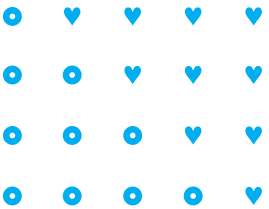
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + \ldots + 97 + 98 + 99$$

You may already know a way to do this but follow the instructions below for a surprising result:

1 Start with the sum $1 + 2 + 3 + 4$ and create a pattern with counters, for example



2 Repeat the pattern with more counters to make the shape of a rectangle so that the sum of the numbers can easily be seen. Use another colour or shape for contrast.



$1 + 2 + 3 + 4$ is half of the rectangular array 4×5

3 Write down a rule in words for adding up $1 + 2 + 3 + 4$ from your shape pattern. Check out your rule by trying it with $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$. Does it work?

4 Extend your pattern so that you can imagine what $1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99$ would look like. Write down a rule in words to find the sum of the numbers $1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99$.

5 Now try to write down your rule in symbols to add up the numbers $1 + 2 + 3 + 4 + \ldots + n$ where n is a whole number and is the last number in the sequence.

6 Check your formula to see if it works for various values of n. For example, when $n = 4$ the formula should give you 10. Try it for $n = 8$ to see if you get 36. Finally, when $n = 99$ you should get 4950.

Extension activities

1 Look at the diagram above of the pattern you made with the counters. Can you see how pairing the numbers works? This is a visual display of pairing the first term with the last term and so on, creating a rectangle whose sum is twice the required sum. You might need your teacher to help you.

2 If you can 'see' that the sum of the numbers $1 + 2 + 3 + 4 + 5 + \ldots + 97 + 98 + 99$ is half of the rectangular pattern of 99 rows with 100 counters in each row, then the total is $\frac{1}{2}$ of (99×100) which gives 4950.

3 You might like to experiment by rewriting the series in reverse and adding up the pairs that way. Write the last beneath the first and so on.

1	2	3	4	5	…	97	98	99
99	98	97	96	95	…	3	2	1
100	100	100	100	100	…	100	100	100

The sum of the first term and the last term will always give you the same total as all the other pairs. The total for one row is half of 99×100.

In general, we get $\frac{1}{2}$ of $n(n + 1)$ where n is the number of terms. We can imagine that this is half of the number of counters in a rectangle with n rows and $n + 1$ columns.

Let's communicate

Write a short letter to a friend who was sick and missed school. Explain what you have learned about adding consecutive whole numbers in sequence.

Reflecting

Would the method you have used to sum consecutive numbers work for a series such as $2 + 4 + 6 + \ldots + 100$? What about $5 + 10 + 15 + 20 + \ldots + 100$?

What sort of series would the method *not* work for do you think? Make a list and check it with your teacher.

Language link with Macquarie

MACQUARIE

1 *Explain what is meant by* ***patterns*** *and* ***relationships*** *between numbers.*
2 *Define the word* ***variable*** *for a new maths dictionary.*
3 *Explain in your own words the meaning of* $y = 2x + 3$.
4 *What can we use to find out how* ***variables*** *relate to each other?*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***convention****:*

convention *noun* **1.** a large meeting: *A science convention was held at the university.* **2.** a rule or principle which is generally accepted and followed: *There are certain conventions about the use of the tennis courts.*
❒ ***Word Family*: conventional** *adjective*

Why are ***conventions*** *important on our roads? Can you think of a mathematical convention you have used in this chapter?*

Chapter Review

1 Write each of these expressions in the abbreviated form.

a $4 \times h$ **b** $b \times 9$
c $c \times d$ **d** $u \times 2 \times v$
e $1 \times p$ **f** $3 \times k + 7$
g $g \times 6 - 5$ **h** $12 - x \times y$
i $d \div 4$ **j** $13 \div m$
k $1 \div t$ **l** $4bc \div 7de$
m $5 \times (c + 8)$ **n** $(s - 3) \times 4$
o $6 \times u \times (2v + 7)$ **p** $(m - n) \times (x + y)$

2 Insert multiplication signs, division signs and grouping symbols where necessary to show the meaning of:

a $2y$ **b** $5ab$
c $8q + 7$ **d** $10t - 3uv$
e $7(k - 5)$ **f** $(d + 2)9$
g $5y(z - 10)$ **h** $(g + h)(p - q)$
i $\frac{r}{2}$ **j** $\frac{1}{z}$
k $\frac{3m}{4n}$ **l** $\frac{a + b}{c}$
m $a + \frac{b}{c}$ **n** $\frac{e}{f - g}$
o $\frac{c}{5} + \frac{9}{d}$ **p** $\frac{3x + 7y}{2z - w}$

3 Use index notation to simplify each of these.

a $m \times m$ **b** $e \times e \times e \times e$
c $c \times d \times d$ **d** $u \times v \times u \times v$

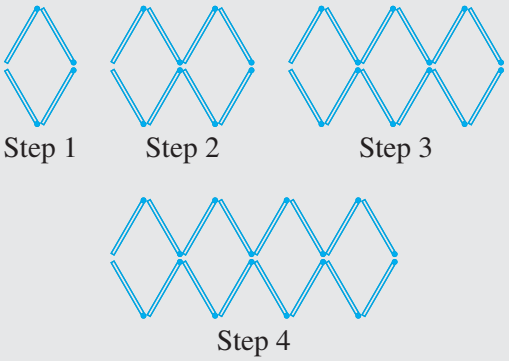
e $4 \times x \times x \times x$
f $p \times p \times 2 \times p \times p \times p$
g $5 \times y \times y \times z \times z \times z$
h $a \times b \times 7 \times a \times a$

4 Insert multiplication signs to show the meaning of:
a a^3 **b** $5k^4$ **c** m^2n^5 **d** $3xy^2$

5 Use the given rule to find the next two numbers in each number pattern.
a Add 3 then multiply by 2; 1, 8, …
b Divide by 2 then add 8; 32, 24, …

6 Describe in words a rule for each sequence, then find the next two numbers.
a 5, 11, 17, 23, …
b 70, 61, 52, 43, …
c 2, 6, 18, 54, …

7 Consider the matchstick pattern below.



a Copy and complete this table of values.

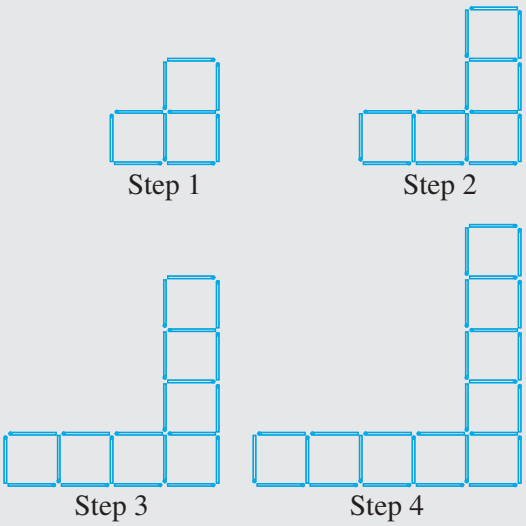
Step number (n)	1	2	3	4	5	6
Number of matches (b)						

b Describe in two different ways the pattern formed by the number of matches.

c Copy and complete this rule that relates the number of matches and the step number.
'The number of matches = _____.'

d Rewrite this rule using the given pronumerals.

8 Consider the matchstick pattern below.



a How many matches must be added to extend each figure?

b Copy and complete this table.

Step number (p)	1	2	3	4
Number of matches (q)				

c Copy and complete this rule that relates the number of matches and the step number.
'The number of matches = ___.'

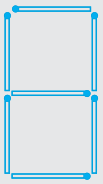
d Rewrite this rule using the given pronumerals.

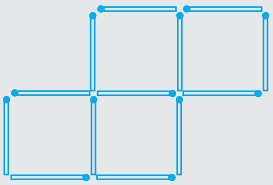
e Write down a different rule for this matchstick pattern using the given pronumerals.

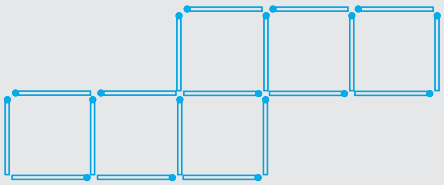
f How many matches would be needed to make the figure in:
- **i** step 7? **ii** step 30?
- **iii** step 100?

9 For each of the following matchstick patterns:
- **i** draw a table of values up to step 5. Let x represent the step number and y represent the number of matches.
- **ii** write down a rule in words, linking the number of matches and the step number.
- **iii** rewrite this rule using pronumerals.

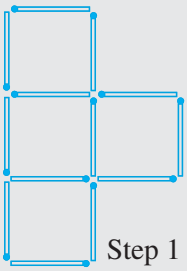
a

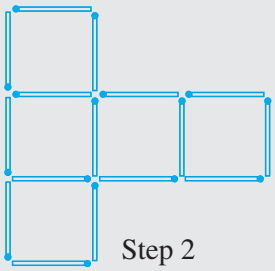

Step 1

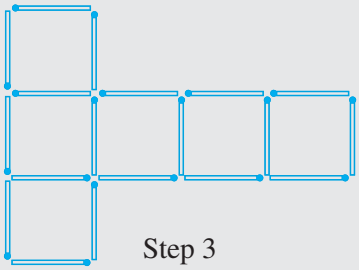

Step 2


Step 3

b

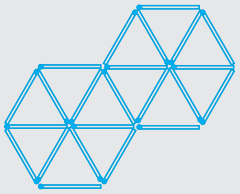

Step 1

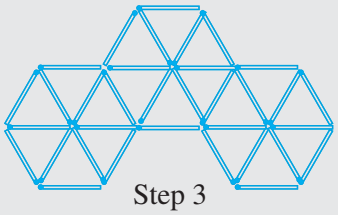

Step 2


Step 3

c

Step 1


Step 2


Step 3

10 Complete each table of values using the given rule.

$y = x + 5$

a

x	1	2	3	4
y				

$n = 4m$

b

m	0	1	2	3
n				

$v = 2u + 7$

c

u	4	5	6	7
v				

$q = 3(p - 2)$

d

p	7	8	9	10
q				

11 Find the algebraic rule that links the top and bottom numbers in each table.

a

m	1	2	3	4
n	5	6	7	8

b

x	0	1	2	3
y	0	7	14	21

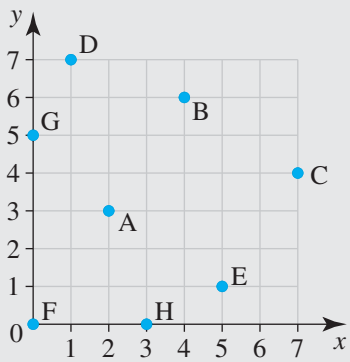
c

p	8	9	10	11
q	5	6	7	8

d

r	1	2	3	4
s	6	8	10	12

12 State the co-ordinates of each point.



a A **b** B **c** C **d** D
e E **f** F **g** G **h** H

13 For each of the tile patterns below:

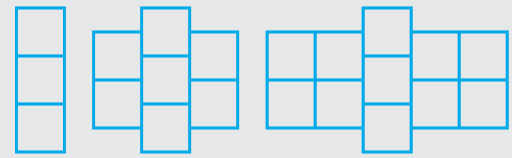
i complete this table of values.

Step number (x)	1	2	3	4	5
Number of tiles (y)					

ii write down a rule linking the number of tiles and the step number, using pronumerals.

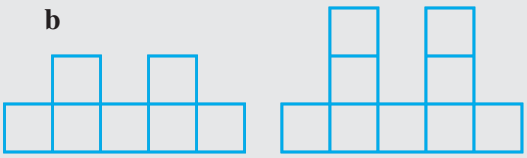
iii plot the ordered pairs from the table on a number grid.

a

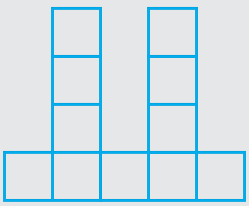


Step 1 Step 2 Step 3

b



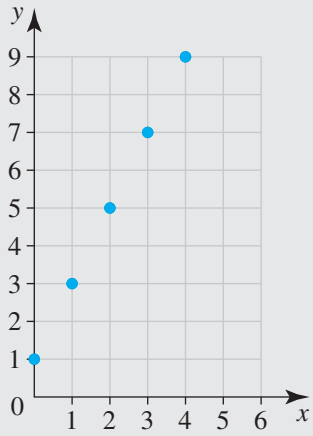
Step 1 Step 2



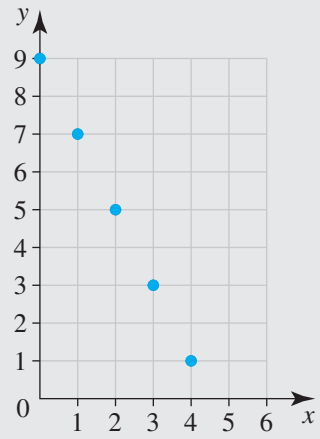
Step 3

14 Find the pattern rule for each set of points.

a



b



6 Decimals

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- convert between decimals and fractions using place value
- state the place value of a digit in a decimal
- find the decimal that lies halfway between two decimals
- identify decimals on a number line.

Stage 4

After completing this chapter, you should be able to:

- arrange a group of decimals in ascending order
- add, subtract, multiply and divide decimals using money concepts
- add, subtract, multiply and divide decimals using fractions
- add, subtract, multiply and divide decimals using formal methods
- find further terms in a decimal number sequence
- multiply and divide decimals by powers of 10
- round off decimals correct to a given number of places
- convert between terminating decimals and fractions
- write recurring decimals using dot notation
- convert fractions and mixed numerals to recurring decimals
- solve problems that involve decimals.

6.1 Decimal place value

We say that our number system is a base 10 or decimal system, since each place column has 10 times the value of the column to its immediate right. The system of decimal place value is an extension of the system of whole number place value, where a decimal point is used to separate the whole number part of a number from the fractional part. In some countries, a comma is used to represent a decimal point; however, by convention, the dot is used in Australia. The table below shows some of the place columns in a decimal system.

Thousands	Hundreds	Tens	Units	•	Tenths	Hundredths	Thousandths
1000	100	10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

It is important to understand decimals in order to cope with the growing use of calculators and computers, as well as the daily interchange of money within society.

Example 1

Write each of these decimals as a fraction with a power of 10 in the denominator.

a 0.9 **b** 0.07 **c** 0.13 **d** 0.003 **e** 1.7 **f** 5.049

Solutions

In each fraction, the number of zeros in the denominator is equal to the number of decimal places in the decimal.

a $0.9 = \frac{9}{10}$ **b** $0.07 = \frac{7}{100}$ **c** $0.13 = \frac{13}{100}$

d $0.003 = \frac{3}{1000}$ **e** $1.7 = 1\frac{7}{10}$ **f** $5.049 = 5\frac{49}{1000}$

Example 2

Write each of these as a single decimal.

a $\frac{3}{10} + \frac{8}{100} + \frac{9}{1000}$ **b** $\frac{7}{10\,000} + 15 + \frac{9}{100}$

Solutions

a $\frac{3}{10} + \frac{8}{100} + \frac{9}{1000}$

$= 0.389$

b $\frac{7}{10\,000} + 15 + \frac{9}{100}$

$= 15 + \frac{9}{100} + \frac{7}{10\,000}$

$= 15.0907$

Example 3

Write 35.709 in the expanded form.

Solution

$35.709 = (3 \times 10) + (5 \times 1) + (7 \times \frac{1}{10}) + (9 \times \frac{1}{1000})$

Example 4

Express each fraction as a decimal.

a $\frac{43}{10}$ **b** $\frac{291}{100}$ **c** $\frac{1075}{1000}$

Solutions

a $\frac{43}{10} = 4\frac{3}{10}$
$= 4.3$

b $\frac{291}{100} = 2\frac{91}{100}$
$= 2.91$

c $\frac{1075}{1000} = 1\frac{75}{1000}$
$= 1.075$

Exercise 6.1

1 Write each of these fractions as a decimal.

a $\frac{7}{10}$ **b** $\frac{3}{100}$ **c** $\frac{9}{1000}$ **d** $\frac{8}{100}$ **e** $\frac{2}{1000}$ **f** $\frac{5}{10}$

g $\frac{1}{100}$ **h** $\frac{4}{10\,000}$ **i** $\frac{6}{1000}$ **j** $\frac{3}{10\,000}$ **k** $\frac{2}{100}$ **l** $\frac{7}{100\,000}$

2 Write each of these decimals as a fraction.

a 0.1 **b** 0.09 **c** 0.003 **d** 0.4
e 0.008 **f** 0.05 **g** 0.2 **h** 0.06
i 0.4 **j** 0.0007 **k** 0.004 **l** 0.000 05

3 Write each of these as a single decimal.

a $\frac{3}{10} + \frac{9}{100}$ **b** $\frac{2}{10} + \frac{7}{100} + \frac{3}{1000}$ **c** $4 + \frac{5}{10} + \frac{1}{100}$

d $2 + \frac{6}{10} + \frac{3}{100} + \frac{1}{1000}$ **e** $\frac{9}{10} + \frac{7}{1000}$ **f** $\frac{4}{100} + \frac{8}{1000}$

g $7 + \frac{2}{10} + \frac{5}{1000}$ **h** $8 + \frac{9}{100} + \frac{6}{1000}$ **i** $5 + \frac{3}{1000}$

j $14 + \frac{3}{10} + \frac{7}{10\,000}$ **k** $22 + \frac{5}{100} + \frac{8}{10\,000}$ **l** $37 + \frac{4}{10} + \frac{2}{1000} + \frac{9}{100\,000}$

Consolidation

4 Write each of these as a single decimal.

a $\frac{2}{100} + \frac{7}{10}$ **b** $\frac{9}{1000} + \frac{5}{100} + \frac{3}{10}$ **c** $\frac{5}{1000} + \frac{6}{10} + \frac{4}{100}$

d $\frac{8}{10} + 6$ **e** $\frac{9}{100} + 7 + \frac{1}{1000} + \frac{4}{10}$ **f** $\frac{8}{1000} + 12 + \frac{3}{10} + \frac{6}{100}$

g $\frac{4}{100} + 8$ **h** $\frac{3}{10} + \frac{2}{1000} + 5$ **i** $\frac{5}{100} + 23 + \frac{7}{1000}$

j $6 + \frac{7}{10\,000} + \frac{8}{100}$ **k** $\frac{4}{1000} + 1 + \frac{9}{10\,000}$ **l** $\frac{2}{10\,000} + 16 + \frac{8}{10}$

5 Express each of these as the sum of integers and fractions.

a 0.57 **b** 0.162 **c** 0.3274 **d** 5.3 **e** 8.09
f 4.103 **g** 11.006 **h** 15.0308 **i** 20.6001 **j** 23.5094

6 Write down the value of the 3 in each of these.

a 0.31 **b** 0.763 **c** 0.0395 **d** 1.4239
e 14.3911 **f** 2.706 38 **g** 6.0395 **h** 9.120 03

7 Write down the value of the underlined digit.

a 0.2<u>7</u>4 **b** 0.<u>9</u>58 **c** 0.46<u>1</u>3 **d** 5.0<u>3</u>2
e 13.62<u>7</u>4 **f** 23.521<u>9</u> **g** 17.6<u>1</u>88 **h** 0.211 4<u>3</u>

8 Express as a single decimal:

a $(3 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + (6 \times \frac{1}{1000})$

b $(4 \times \frac{1}{10}) + (8 \times \frac{1}{1000})$

c $(5 \times 1) + (9 \times \frac{1}{100}) + (4 \times \frac{1}{10\,000})$

d $(3 \times 10) + (2 \times 1) + (7 \times \frac{1}{10}) + (6 \times \frac{1}{100})$

e $(4 \times 100) + (8 \times 10) + (5 \times 1) + (2 \times \frac{1}{10}) + (3 \times \frac{1}{1000})$

f $(7 \times 100) + (8 \times 1) + (4 \times \frac{1}{100}) + (1 \times \frac{1}{1000})$

g $(3 \times 1000) + (6 \times 10) + (5 \times \frac{1}{10}) + (7 \times \frac{1}{10\,000})$

h $(9 \times 1000) + (4 \times 1) + (8 \times \frac{1}{1000}) + (3 \times \frac{1}{10\,000})$

9 Express each fraction as a decimal.

a $\frac{13}{100}$ **b** $\frac{247}{1000}$ **c** $\frac{3596}{10\,000}$ **d** $\frac{57}{100}$ **e** $\frac{198}{1000}$

f $\frac{62}{1000}$ **g** $\frac{425}{10\,000}$ **h** $\frac{67}{10\,000}$ **i** $\frac{13}{100\,000}$ **j** $2\frac{27}{100}$

k $5\frac{134}{1000}$ **l** $9\frac{79}{1000}$ **m** $4\frac{315}{10\,000}$ **n** $12\frac{96}{10\,000}$ **o** $35\frac{102}{100\,000}$

10 Express each decimal as a fraction or mixed numeral.

a 0.73 **b** 0.169 **c** 0.2487 **d** 0.051
e 0.0717 **f** 0.0033 **g** 4.63 **h** 10.067
i 18.0081 **j** 2.0301 **k** 11.099 **l** 24.000 63

11 Write each of these fractions as a decimal.

a $\frac{36}{10}$ **b** $\frac{127}{100}$ **c** $\frac{869}{100}$ **d** $\frac{483}{10}$ **e** $\frac{205}{100}$ **f** $\frac{1354}{1000}$

g $\frac{6708}{100}$ **h** $\frac{3001}{1000}$ **i** $\frac{23\,566}{1000}$ **j** $\frac{1289}{10}$ **k** $\frac{8006}{100}$ **l** $\frac{541\,627}{10\,000}$

12 Express each of these decimals as a fraction or mixed numeral, in simplest form.

a 0.70 **b** 0.030 **c** 0.0010
d 0.900 **e** 0.1000 **f** 5.30
g 8.0900 **h** 12.0070 **i** 15.000 300

13 Choose the two decimals which have the same value in each of these.

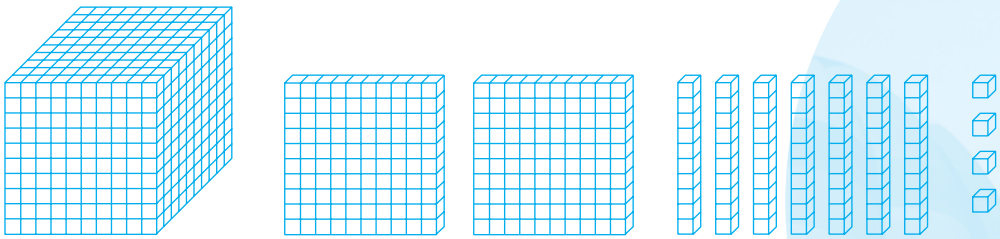
a 0.03, 0.30, 0.300 **b** 0.002, 0.020, 0.02
c 5.070, 5.700, 5.0700 **d** 6, 0.6, 6.0

14 Express each of these with the number of decimal places indicated in the brackets.

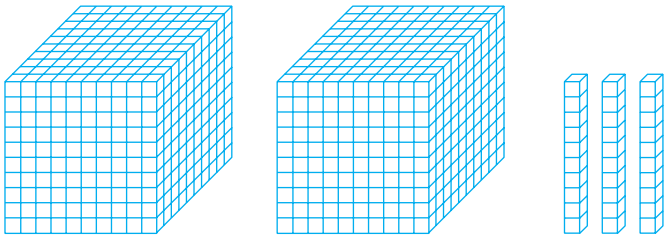
a 5.2 [2] **b** 0.08 [3] **c** 9 [1] **d** 3.6 [3] **e** 12 [2] **f** 5.01 [4]

15 In the diagrams below, the large cube contains 1000 small cubes, the 'flat' contains 100 small cubes and the 'long' contains 10 small cubes. If the large cube represents one whole, write down the decimals that are represented by each of the following.

a



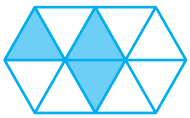
b



16 Write as a decimal, the shaded area in each of these.

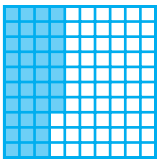
a

b



c

d



e

■ Further applications

17 Evaluate each of these by first converting the decimal to a fraction with a power of 10 in the denominator. Check your answers using a calculator.

a $\sqrt{0.04}$ **b** $\sqrt{0.09}$ **c** $\sqrt{0.25}$ **d** $\sqrt{0.64}$

e $\sqrt{0.0001}$ **f** $\sqrt{0.0016}$ **g** $\sqrt{0.0036}$ **h** $\sqrt{0.0081}$

i $\sqrt{1.21}$ **j** $\sqrt{1.44}$ **k** $\sqrt{2.25}$ **l** $\sqrt{2.89}$

18 If $14^2 = 196$, find:

a $\sqrt{0.0196}$ **b** $\sqrt{1.96}$

19 Can $\sqrt{0.4}$ be worked out exactly? Explain. Check using a calculator.

20 Find each of these cube roots. Check your answers using a calculator.

a $\sqrt[3]{0.001}$ **b** $\sqrt[3]{0.008}$ **c** $\sqrt[3]{0.027}$

TRY THIS

The Dewey decimal system

1 What is the Dewey decimal system and where is it used?

2 Explain how the system is organised.

6.2 The relative sizes of decimals

In section 1.15, we introduced the following inequality signs:
- **$<$ meaning 'less than'**
- **$\leqslant$ meaning 'less than or equal to'**
- **$>$ meaning 'greater than'**
- **$\geqslant$ meaning 'greater than or equal to'**

When comparing the relative sizes of two numbers, ensure that the inequality sign points towards the smaller number.

e.g.
- $2 < 6$ means '2 is less than 6'.
- $7 \geqslant 4$ means '7 is greater than or equal to 4'.

To arrange a series of decimals in ascending order:
- **write each decimal with the same number of decimal places**
- **if the whole numbers are equal, arrange the decimals in order by comparing the decimal parts.**

NOTE: In some simple questions the decimals can be ordered by considering each as a fraction, or as an amount of money.

Example 1

By considering each decimal as an amount of money, insert $<$ or $>$ to make a true statement.

a $0.8 \square 0.52$ **b** $0.94 \square 1.2$

Solutions

a We could think of 0.8 or 0.80 as \$0.80 and 0.52 as \$0.52. Since \$0.80 is greater than \$0.52, then $0.8 > 0.52$.

b We could think of 0.94 as \$0.94 and 1.2 or 1.20 as \$1.20. Since \$0.94 is less than \$1.20, then $0.94 < 1.2$.

NOTE: The length of a decimal is not related to its value. That is, when comparing two decimals, the decimal which has the greater value is not necessarily the longer decimal.

Example 2

Write these decimals in ascending order.

0.303, 0.003, 0.033, 0.33, 0.3

Solution

0.303, 0.003, 0.033, 0.33, 0.3

= 0.303, 0.003, 0.033, 0.330, 0.300

Now, arranging the circled numbers in order, we have 003, 033, 300, 303, 330.

$\therefore$ In ascending order, the decimals are 0.003, 0.033, 0.3, 0.303, 0.33.

Exercise 6.2

1 By considering each decimal as an amount of money, insert < or > to make a true statement.

a 0.6 ☐ 0.4 **b** 0.05 ☐ 0.09 **c** 0.11 ☐ 0.04 **d** 1 ☐ 0.9
e 0.99 ☐ 1 **f** 1.02 ☐ 0.89 **g** 2.387 ☐ 3.2 **h** 4.06 ☐ 1.75
i 0.6 ☐ 0.13 **j** 0.37 ☐ 0.4 **k** 1.2 ☐ 0.98 **l** 7.87 ☐ 8.1

2 Rewrite each pair of decimals with the same number of decimal places by inserting extra zeros, then use < or > to make a true statement.

a 0.47 ☐ 0.3 **b** 0.209 ☐ 0.53 **c** 0.6 ☐ 0.425 **d** 0.08 ☐ 0.065
e 0.11 ☐ 0.078 **f** 0.999 ☐ 0.94 **g** 1.2 ☐ 1.61 **h** 4.675 ☐ 4.68
i 8.106 ☐ 8.09 **j** 12.34 ☐ 12.4 **k** 1 ☐ 0.96 **l** 3 ☐ 2.946

Consolidation

3 Write down the smallest and largest numbers from each group of decimals.

a 0.404, 0.4, 0.004 **b** 0.607, 0.067, 0.76
c 1.22, 1.202, 1.002 **d** 0.099, 0.99, 0.9

4 Arrange these decimals in ascending order.

a 0.87, 1.52, 1.4, 0.9, 0.08 **b** 0.2, 0.18, 0.07, 0.519, 0.64
c 0.91, 1.1, 2, 1.01, 0.6 **d** 3.07, 2.7, 0.099, 3.5, 2.54
e 1.84, 0.3, 0.08, 3.0, 0.65 **f** 0.7, 4.52, 0.95, 1.6, 2.41

5 Arrange these decimals in ascending order.

a 0.4, 0.044, 0.44, 0.404, 0.04 **b** 0.66, 0.066, 0.666, 0.6, 0.606
c 0.707, 7.0, 0.77, 0.007, 0.7 **d** 1.02, 0.21, 2.01, 0.012, 0.102
e 3.8, 0.38, 0.083, 0.803, 0.038 **f** 0.059, 0.95, 0.5, 0.09, 0.905

6 Choose A, B, C or D for each of these. Which number is closest to:

a	0.27?	**A**	0.2	**B**	0.31	**C**	0.24	**D** 26
b	0.6?	**A**	0.7	**B**	0.063	**C**	0.58	**D** 0.059
c	1.43?	**A**	1.5	**B**	1.044	**C**	1.35	**D** 14.2
d	5?	**A**	4.9	**B**	5.3	**C**	6	**D** 5.09
e	0.076?	**A**	0.07	**B**	0.75	**C**	0.77	**D** 0.08
f	0.392?	**A**	0.39	**B**	0.4	**C**	0.3926	**D** 0.3

7 Write down all decimals in the group which are:

a greater than 0.35 0.12, 0.2, 0.4, 0.525, 0.285
b less than 0.85 0.819, 0.5, 0.912, 0.9, 0.095

Further applications

A **recurring decimal** is a decimal in which one or more of the digits are repeated an infinite number of times. To signify a recurring digit, we place a dot above that digit. For example, $0.\dot{7}$ means 0.777 77 …

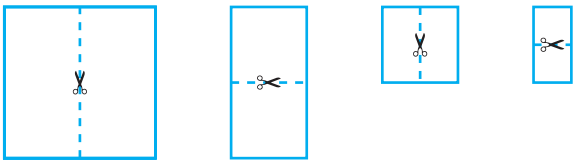
8 Insert < or > to make a true statement.

a $0.\dot{4}$ □ 0.5 **b** $0.\dot{7}$ □ 0.8 **c** $0.\dot{5}$ □ 0.499

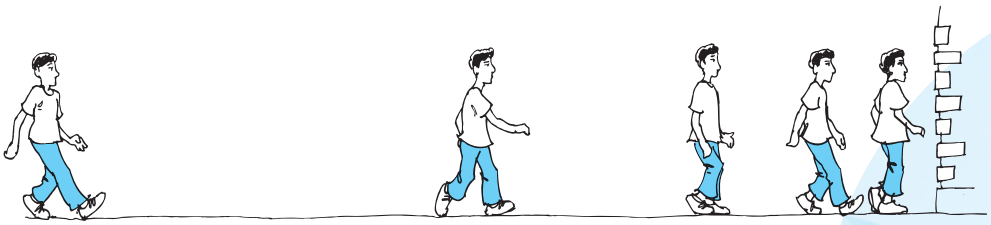
d 0.2 □ $0.\dot{2}$ **e** $0.\dot{4}$ □ 0.45 **f** $0.\dot{8}$ □ 0.8888

6.3 The density of decimals

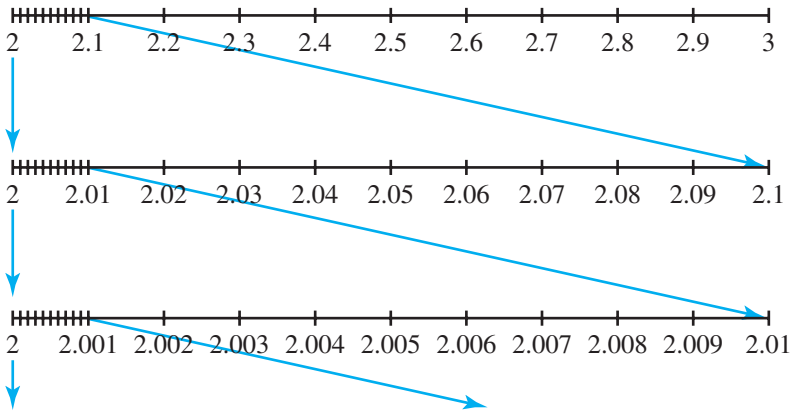
The term **density** in this exercise refers to the study of numbers that lie between other numbers. We begin, therefore, with the question 'How many decimals are there between any two numbers?' To help answer this question, take a sheet of paper and cut it in half. Then cut one of the halves in half, then cut one of the quarters in half, then cut one of the eighths in half, and so on. Keep going until the pieces are too small to cut with a pair of scissors. *In theory*, there is no limit to the number of times that we could cut the paper in half. That is, we could make an **infinite** (or unlimited) number of cuts. Does the paper vanish, or will there always be some small piece left over to cut again?



Further to this idea, imagine that you are walking towards a wall. If you walk halfway to the wall, then walk half of the remaining distance towards the wall, then walk half of this remaining distance, and so on, will you ever reach the wall? *In theory*, you will never reach the wall, because if you only walk half the remaining distance each time then there will always be a small distance separating you and the wall.



So it is with numbers. If we plot two numbers on the number line, we can divide the interval joining the numbers into any number of equal parts, say ten. Each of these parts can then be further divided into ten smaller parts, then each of these can be divided again into ten even smaller parts, and so on. This idea is illustrated on page 223 by continuing to 'zoom in' on the first interval in each number line.



While there are an infinite number of decimals (and hence fractions) between any two numbers, there is only a limited number of integers (whole numbers) between any two numbers.

To find the number that lies halfway between two decimals:
- **add a zero to the end of each decimal**
- **use the last two digits of each decimal to find the number that lies halfway between the given decimals.**

NOTE: In some questions, the number that lies halfway between two decimals can be found quite easily by considering each decimal as an amount of money.

Example

Write the decimal that lies halfway between:

a 6 and 7 **b** 0.2 and 0.3 **c** 0.72 and 0.8 **d** 0.46 and 0.47

Solutions

a If 6 = 6.0 and 7 = 7.0, then the decimal that lies halfway is 6.5.
b If 0.2 = 0.20 and 0.3 = 0.30, then the decimal that lies halfway is 0.25.
c If 0.8 = 0.80, then the decimal that lies halfway between 0.72 and 0.80 is 0.76.
d If 0.46 = 0.460 and 0.47 = 0.470, then the decimal that lies halfway is 0.465.

Exercise 6.3

1 How many decimals lie between 0 and 1?
A 0 **B** 1 **C** 10 **D** an infinite number

2 Which decimal lies halfway between:
a 1 and 2 **b** 7 and 8 **c** 3 and 4
d 14 and 15 **e** 29 and 30 **f** 99 and 100

3 Write the decimal which lies halfway between:

a 2 and 5 **b** 3 and 6 **c** 7 and 10 **d** 0 and 5
e 5 and 10 **f** 2 and 11 **g** 1 and 8 **h** 20 and 17

4 Write the decimal which lies halfway between:

a 0.1 and 0.3 **b** 0.7 and 0.9 **c** 0 and 0.2
d 0.8 and 1 **e** 0.3 and 0.7 **f** 0.2 and 0.8
g 0.9 and 1.1 **h** 0.7 and 1.1 **i** 1.3 and 0.9

Consolidation

5 Find the decimal which lies halfway between:

a 0.1 and 0.2 **b** 0.7 and 0.8 **c** 0.3 and 0.4 **d** 0.6 and 0.7
e 0.9 and 1 **f** 0 and 0.1 **g** 1.4 and 1.5 **h** 3.8 and 3.9
i 7.2 and 7.3 **j** 11.2 and 11.1 **k** 6.1 and 6 **l** 13 and 12.9

6 Find the decimal which lies halfway between:

a 0.02 and 0.03 **b** 0.04 and 0.05 **c** 0.08 and 0.09 **d** 0.31 and 0.32
e 0.73 and 0.74 **f** 0.65 and 0.66 **g** 0.08 and 0.07 **h** 3.12 and 3.11
i 6.24 and 6.23 **j** 10.08 and 10.09 **k** 14.69 and 14.7 **l** 0.09 and 0.1
m 0 and 0.01 **n** 5.99 and 6 **o** 8 and 8.01 **p** 0.113 and 0.114

7 Write down any three decimals which lie between:

a 0.2 and 0.3 **b** 0.17 and 0.18 **c** 0.7 and 0.71

Further applications

8 Find the decimal which lies halfway between:

a 0.01 and 0.04 **b** 0.02 and 0.05 **c** 0.13 and 0.16
d 0.57 and 0.54 **e** 2.07 and 2.1 **f** 6 and 6.03

6.4 Addition and subtraction of decimals (1)

To add or subtract decimals:

- **write the decimals one below the other, with the decimal points and place columns in line**
- **write zeros in empty decimal places**
- **add or subtract as for whole numbers.**

Example

Evaluate:

a 32.6 + 149.743 + 9.28

b 76.5 – 19.38

Solutions

a
```
  32.600
 149.743
   9.280  +
 2 1 1
 191.623
```

b
```
 6 1 4 1
 76.5̸0
 19.38  –
 57.12
```

Exercise 6.4

1 Evaluate:

a
```
  8.2
 5.74
 6.03  +
```

b
```
  5.942
  0.8
 16.37  +
```

c
```
  109.46
   12.9
  234.778  +
```

d
```
   27.455
  742.09
   83.697  +
```

e
```
  3564.4
   908.261
    25.357  +
```

f
```
     72
    156.49
   8565.278  +
```

2 Evaluate:

a
```
 54.76
 13.51  –
```

b
```
 754.3
 204.2  –
```

c
```
 425.16
  38.07  –
```

d
```
 531.2
  57.96  –
```

e
```
 3254.74
  526.821  –
```

f
```
 2581.3
 1494.219  –
```

Consolidation

3 Evaluate each of the following.

a 9.2 + 8.4 + 5.6
b 0.34 + 0.91 + 0.83
c 5.2 + 17.66 + 9.49
d 82.09 + 6.3 + 15.487
e 132.6 + 24.973 + 52.258
f 0.123 + 13.96 + 20.708
g 24 + 181.05 + 246.87
h 312.95 + 60.184 + 290

4 Evaluate each of these.

a 10.4 – 8.7
b 15.23 – 6.41
c 70.8 – 34.72
d 121.06 – 37.5
e 213.1 – 50.768
f 580 – 33.4
g 675 – 357.61
h 974.86 – 90.4
i 500.03 – 167.8
j 1248.56 – 79.81
k 2874.55 – 1067.7
l 6021 – 3504.426

Further applications

5 Find the missing digits in each of these.

a
```
 1 3 . □ 8
 2 □ . 7 □  +
 ----------
 □ 3 . 3 4
```

b
```
 1 2 □ . 5 □
   □ 5 . □ 8  –
 ------------
   7 9 . 3 4
```

c
```
 7 □ 6 □ . 0 4
 5 8 □ 9 . 7 □  –
 ----------------
 □ 5 3 4 . 2 8
```

6.5 Addition and subtraction of decimals (2)

Many simple questions involving the addition or subtraction of decimals can be answered by either considering each decimal as an amount of money or by converting the decimals to fractions.

Example 1

Evaluate each of the following by considering each decimal as an amount of money.

a $0.7 + 0.5$ **b** $1.3 - 0.4$ **c** $0.19 + 0.3$ **d** $2.6 - 1.12$

Solutions

a $\$0.70 + \$0.50 = \$1.20$
$\therefore 0.7 + 0.5 = 1.2$

b $\$1.30 - \$0.40 = \$0.90$
$\therefore 1.3 - 0.4 = 0.9$

c $\$0.19 + \$0.30 = \$0.49$
$\therefore 0.19 + 0.3 = 0.49$

d $\$2.60 - \$1.12 = \$1.48$
$\therefore 2.6 - 1.12 = 1.48$

Example 2

Express each decimal as a fraction and hence evaluate each of these.

a $0.7 + 0.5$
b $0.36 - 0.2$
c $2.9 + 0.15$

Solutions

a
$$0.7 + 0.5 = \frac{7}{10} + \frac{5}{10} = \frac{12}{10} = 1.2$$

b
$$0.36 - 0.2 = \frac{36}{100} - \frac{2}{10} = \frac{36}{100} - \frac{20}{100} = \frac{16}{100} = 0.16$$

c
$$2.9 + 0.15 = \frac{29}{10} + \frac{15}{100} = \frac{290}{100} + \frac{15}{100} = \frac{305}{100} = 3.05$$

Exercise 6.5

1 Evaluate each of the following by considering each decimal as an amount of money.

a $0.1 + 0.2$ **b** $0.4 - 0.3$ **c** $0.2 + 0.8$ **d** $0.7 + 0.4$
e $0.9 + 0.6$ **f** $0.8 + 0.9$ **g** $1.2 - 0.3$ **h** $1.3 - 0.7$
i $1.6 - 0.9$ **j** $2 - 0.3$ **k** $4 - 0.5$ **l** $7 - 0.8$
m $2.3 - 1$ **n** $5.2 + 3$ **o** $8.7 - 8$ **p** $1.6 - 0.6$
q $3.4 - 0.4$ **r** $9.8 - 0.8$ **s** $2.3 - 1.2$ **t** $7 - 6.6$

2 Evaluate each of the following by considering each decimal as an amount of money.

a $0.05 + 0.03$ **b** $0.09 - 0.04$ **c** $0.08 + 0.06$ **d** $0.11 - 0.03$
e $0.16 + 0.05$ **f** $0.24 + 0.07$ **g** $0.19 - 0.06$ **h** $0.47 - 0.13$

i 1 – 0.05	**j** 1 – 0.12	**k** 1 – 0.25	**l** 2 – 0.08
m 5 – 0.04	**n** 10 – 0.15	**o** 3.75 – 1.42	**p** 5.19 – 2.08

3 Explain why $0.5 + 0.03 \neq 0.08$.

Consolidation

4 Evaluate each of the following by considering each decimal as an amount of money.

a 0.1 + 0.06	**b** 0.08 + 0.2	**c** 0.2 – 0.18	**d** 0.37 – 0.1
e 0.5 – 0.09	**f** 0.36 + 0.4	**g** 0.8 + 0.24	**h** 0.15 + 0.9
i 0.65 + 0.7	**j** 1.1 – 0.04	**k** 1.2 – 0.13	**l** 1.08 – 0.1
m 1.04 – 0.2	**n** 3.1 + 0.18	**o** 4.29 – 0.3	**p** 3.09 + 0.41

5 Write the next two decimals in each sequence by considering each decimal as an amount of money.

a 0.2, 0.4, 0.6, 0.8, ____, ____.	**b** 0.3, 0.6, 0.9, ____, ____.
c 0.2, 0.5, 0.8, ____, ____.	**d** 0.7, 1.4, 2.1, 2.8, ____, ____.
e 0.28, 0.26, 0.24, 0.22, ____, ____.	**f** 0.6, 1.2, 1.8, ____, ____.
g 0.9, 0.87, 0.84, ____, ____.	**h** 0.35, 0.3, 0.25, 0.2, ____, ____.
i 1.3, 1.32, 1.34, 1.36, ____, ____.	**j** 0.25, 0.5, 0.75, ____, ____.
k 0.15, 0.3, 0.45, ____, ____.	**l** 1.45, 1.25, 1.05, ____, ____.
m 1.7, 1.4, 1.1, ____, ____.	**n** 0.025, 0.02, 0.015, ____, ____.

6 Express each decimal as a fraction and hence evaluate each of these. Give your answers as decimals or whole numbers.

a 0.3 + 0.5	**b** 0.9 – 0.2	**c** 0.8 + 0.4	**d** 1.3 – 0.7
e 1.5 + 0.7	**f** 2.3 – 0.5	**g** 4.6 – 1.4	**h** 2.6 + 0.9
i 3.7 + 1.3	**j** 0.02 + 0.06	**k** 0.48 – 0.25	**l** 0.77 + 0.59
m 0.185 – 0.042	**n** 0.218 – 0.163	**o** 1.287 + 0.045	**p** 3.496 – 1.256

7 Express each decimal as a fraction and hence evaluate each of these. Give your answers as decimals.

a 0.2 + 0.07	**b** 0.12 + 0.6	**c** 1.3 + 0.08	**d** 0.3 – 0.05
e 0.7 – 0.14	**f** 0.35 – 0.1	**g** 0.42 + 0.5	**h** 0.48 – 0.3
i 1 – 0.2	**j** 1 – 0.06	**k** 2 – 0.2	**l** 5 – 0.15

Further applications

8 Write the next two decimals in these sequences by considering each decimal as an amount of money.

a 0.045, 0.05, 0.055, ____, ____.	**b** 0.025, 0.02, 0.015, ____, ____.
c 0.08, 0.085, 0.09, ____, ____.	**d** 2.63, 2.635, 2.64, ____, ____.
e 1.325, 1.33, 1.335, ____, ____.	**f** 1.28, 1.285, 1.29, ____, ____.

6.6 Multiplying a decimal by a whole number

To multiply a decimal by a whole number:
- **ignore the decimal point and multiply the numbers**
- **insert the decimal point so that the answer has the same number of decimal places as in the question.**

To multiply a decimal by a power of 10:
- **move the decimal point one place to the right for each zero in the power of 10.**

NOTE: In some questions we can multiply a decimal by a whole number by considering the decimal as an amount of money.

Example 1

Evaluate each of the following by considering the decimal as an amount of money.

a 3×0.4 **b** 5×0.06 **c** 4×1.2

Solutions

a $3 \times \$0.40$
$= \$1.20$
$\therefore 3 \times 0.4 = 1.2$

b $5 \times \$0.06$
$= \$0.30$
$\therefore 5 \times 0.06 = 0.3$

c $4 \times \$1.20$
$= \$4.80$
$\therefore 4 \times 1.2 = 4.8$

Example 2

Evaluate 13.62×9

Solution

We first evaluate 1362×9, ignoring the decimal point.

$$\begin{array}{r} 1\,362 \\ \times\ 9 \\ \hline 12\,258 \end{array}$$

Now, there are 2 decimal places in the question, so there must be 2 decimal places in the answer, $\therefore 13.62 \times 9 = 122.58$

Example 3

a 0.87×10 **b** 0.1634×100 **c** 12.3×1000

Solutions

a To multiply by 10, we move the decimal point 1 place to the right, i.e. 0.8.7
$\therefore 0.87 \times 10 = 8.7$

b To multiply by 100, we move the decimal point 2 places to the right, i.e. 0.16.34
$\therefore 0.1634 \times 100 = 16.34$

c To multiply by 1000, we move the decimal point 3 places to the right, filling the extra places with zeros, i.e. 12.300.
$\therefore 12.3 \times 1000 = 12\,300$

Exercise 6.6

1 Evaluate each of these by considering the decimal as an amount of money.

a 5×0.1	**b** 3×0.2	**c** 0.4×2	**d** 2×0.03	**e** 5×0.07
f 0.09×4	**g** 0.06×5	**h** 0.02×10	**i** 5×0.08	**j** 2×0.5
k 5×0.4	**l** 0.9×10	**m** 5×0.3	**n** 0.7×6	**o** 4×0.8
p 4×0.12	**q** 0.11×7	**r** 3×0.15	**s** 2×1.4	**t** 3×1.5
u 2.1×5	**v** 1.04×6	**w** 1.21×4	**x** 3×2.25	**y** 2.12×5

Consolidation

2 Evaluate:

a $\begin{array}{r} 2.1 \\ \underline{6} \times \end{array}$	**b** $\begin{array}{r} 3.4 \\ \underline{7} \times \end{array}$	**c** $\begin{array}{r} 5.2 \\ \underline{4} \times \end{array}$	**d** $\begin{array}{r} 7.6 \\ \underline{9} \times \end{array}$
e $\begin{array}{r} 0.46 \\ \underline{3} \times \end{array}$	**f** $\begin{array}{r} 0.95 \\ \underline{8} \times \end{array}$	**g** $\begin{array}{r} 2.07 \\ \underline{5} \times \end{array}$	**h** $\begin{array}{r} 6.45 \\ \underline{7} \times \end{array}$
i $\begin{array}{r} 5.213 \\ \underline{6} \times \end{array}$	**j** $\begin{array}{r} 13.47 \\ \underline{4} \times \end{array}$	**k** $\begin{array}{r} 7.089 \\ \underline{8} \times \end{array}$	**l** $\begin{array}{r} 23.54 \\ \underline{9} \times \end{array}$

3 Evaluate:

a 4.7×5	**b** 9.6×2	**c** 0.66×3	**d** 1.8×9
e 7.2×6	**f** 12.8×2	**g** 37.48×5	**h** 94.06×8

4 Evaluate:

a 2.17×10	**b** 3.496×100	**c** 7.4623×1000
d 0.75×10	**e** 0.03×10	**f** 0.0316×100
g 0.9×10	**h** 0.62×100	**i** 0.019×1000
j 0.007×100	**k** 1.485×10	**l** 28.743×100
m 0.0063×100	**n** $0.000\,75 \times 10$	**o** $0.001\,23 \times 10\,000$

5 Evaluate each of the following.

a 0.2×100	**b** 0.07×1000	**c** 0.16×1000
d $1.3 \times 10\,000$	**e** 24.83×1000	**f** 5.9×1000
g 9.8×100	**h** 0.04×1000	**i** $0.8 \times 10\,000$
j 512.5×100	**k** $30.61 \times 10\,000$	**l** 700.2×1000
m 62.4×1000	**n** 0.9×100	**o** $67.03 \times 10\,000$

Further applications

6 In each of the following, express the whole number as the product of a power of 10 and a single digit number, then evaluate.

For example, $0.73 \times 200 = 0.73 \times 100 \times 2$
$= 73 \times 2$
$= 146$

a 0.14×200	**b** 0.12×600	**c** 0.13×3000
d 0.16×2000	**e** 0.132×3000	**f** $0.022 \times 40\,000$
g 1.1×70	**h** 1.5×400	**i** 1.25×3000

6.7 Multiplying a decimal by a decimal

To multiply two decimals we could change each decimal to a fraction, multiply the fractions, then convert this answer back to a decimal, as in the following example.

$$0.6 \times 0.04$$
$$= \frac{6}{10} \times \frac{4}{100}$$
$$= \frac{24}{1000}$$
$$= 0.024$$

Now the number of decimal places in the question determines the number of zeros in the individual denominators. However, the number of zeros in the fraction $\frac{24}{1000}$ determines the number of decimal places in the answer. Hence, the number of decimal places in the answer must be equal to the total number of decimal places in the question.

To multiply two or more decimals:
- **ignore the decimal points and multiply the numbers**
- **insert the decimal point so that the number of decimal places in the answer is equal to the total number of decimal places in the question.**

Example

Evaluate:

a 1.7×0.9
b 0.24×0.06
c 0.25×3.4

Solutions

a Ignoring the decimal points, we first evaluate 17×9.

$$\begin{array}{r} 17 \\ \underline{9} \times \\ 153 \end{array}$$

Now there are 2 decimal places in the question, so there must be 2 decimal places in the answer. $\therefore 1.7 \times 0.9 = 1.53$

b Ignoring the decimal points, we first evaluate 24×6.

$$\begin{array}{r} 24 \\ \underline{6} \times \\ 144 \end{array}$$

Now there are 4 decimal places in the question, so there must be 4 decimal places in the answer. $\therefore 0.24 \times 0.06 = 0.0144$

c Ignoring the decimal points, we first evaluate 25×34.

$$\begin{array}{r} 25 \\ \underline{34} \times \\ 100 \\ \underline{750} + \\ 850 \end{array}$$

Now there are 3 decimal places in the question, so there must be 3 decimal places in the answer. $\therefore 0.25 \times 3.4 = 0.850$
$= 0.85$

NOTE: The correct position of the decimal point must be determined before any final zeros are dropped.

Exercise 6.7

1 Express each decimal as a fraction and then evaluate each of these. Give your answers as decimals.

a 0.2×0.4 **b** 0.5×0.7 **c** 0.03×0.6 **d** 0.04×0.03 **e** 0.09×0.5
f 0.6×0.9 **g** 0.7×0.12 **h** 0.6×0.5 **i** 0.08×0.05 **j** 1.2×0.4
k 0.3×1.3 **l** 1.1×0.09 **m** 0.01×1.4 **n** 0.04×1.5 **o** 1.2×1.1
p 1.02×0.7 **q** 1.2×0.9 **r** 1.01×0.05 **s** 1.3×1.4 **t** 2.7×1.6

Consolidation

2 Evaluate each of the following by at first ignoring the decimal points.

a 0.4×0.6 **b** 0.3×0.9 **c** 0.8×0.4
d 0.2×0.06 **e** 0.6×0.07 **f** 0.09×0.2
g 0.03×0.04 **h** 0.07×0.05 **i** 0.06×0.08
j 0.9×0.5 **k** 0.03×0.7 **l** 0.05×0.03
m 0.1×0.02 **n** 0.08×0.04 **o** 0.2×0.6
p $0.3 \times 0.2 \times 0.1$ **q** $0.7 \times 0.3 \times 0.2$ **r** $0.03 \times 0.4 \times 0.6$

3 Evaluate each of these.

a 0.5×0.8 **b** 0.06×0.5 **c** 0.05×0.4 **d** 0.6×0.005
e 0.12×0.5 **f** 0.5×0.02 **g** 0.05×0.16 **h** 0.005×0.6

4 Evaluate:

a 0.13×0.9 **b** 0.18×0.7 **c** 1.4×0.06 **d** 0.04×0.24
e 1.59×0.05 **f** 2.37×0.4 **g** 5.5×0.006 **h** 0.9×28.4
i 37.6×0.08 **j** 16.4×0.7 **k** 162.8×0.003 **l** 729.15×0.005

5 Evaluate each of these.

a 0.3^2 **b** 0.7^2 **c** 0.6^2 **d** 0.04^2
e 0.12^2 **f** 0.1^3 **g** 0.2^3 **h** 0.03^3

6 How many decimal places are there in 1.67^5?

7 What is the effect of multiplying by a decimal that is:

a less than 1? **b** greater than 1?

Further applications

8 Evaluate:

a 0.18×0.19 **b** 0.37×1.5 **c** 0.025×3.9
d 1.04×2.6 **e** 3.42×0.73 **f** 5.91×0.038

9 Evaluate each of these by considering 0.5 as one half and the other decimal as an amount of money.

a 0.5×0.6 **b** 0.5×0.8 **c** 0.9×0.5 **d** 0.7×0.5 **e** 0.5×0.02

f	0.5×0.16	**g**	0.26×0.5	**h**	0.5×1.2	**i**	0.5×2.1	**j**	1.5×0.6
k	1.5×0.04	**l**	1.4×1.5	**m**	2.5×0.6	**n**	2.5×0.08	**o**	3.5×1.2

10 Evaluate each of the following by considering 0.25 as $\frac{1}{4}$, 0.75 as $\frac{3}{4}$ and the other decimal as an amount of money.

a	0.25×0.4	**b**	0.25×0.8	**c**	0.75×0.4	**d**	0.75×0.8
e	0.25×0.04	**f**	0.75×0.08	**g**	0.25×0.12	**h**	0.75×0.16
i	0.25×0.28	**j**	0.25×0.36	**k**	0.75×0.12	**l**	0.75×0.44
m	0.25×1.6	**n**	0.25×3.2	**o**	0.75×2.4	**p**	0.25×0.2
q	0.75×0.6	**r**	0.75×1.8	**s**	0.25×8.4	**t**	0.25×4.12

11 Find the missing number in each of these.

a	$0.3 \times \square = 0.21$	**b**	$\square \times 0.6 = 2.4$	**c**	$5 \times \square = 0.45$
d	$\square \times 0.09 = 0.27$	**e**	$0.4 \times \square = 0.032$	**f**	$0.12 \times \square = 0.072$
g	$\square \times 1.1 = 0.77$	**h**	$\square \times 0.2 = 0.028$	**i**	$0.13 \times \square = 0.0039$
j	$\square \times 8 = 4$	**k**	$0.7 \times \square = 0.007$	**l**	$\square \times 0.05 = 0.6$

6.8 Dividing a decimal by a whole number

To divide a decimal by a whole number:
- **place the decimal point above the decimal point in the question**
- **divide the numbers.**

To divide a decimal by a power of 10:
- **move the decimal point one place to the left for each zero in the power of 10.**

NOTE: In some questions we can divide a decimal by a whole number by considering the decimal as an amount of money.

Example 1

Evaluate each of the following by considering the decimal as an amount of money.

a $0.8 \div 2$ **b** $0.12 \div 4$ **c** $1.5 \div 3$

Solutions

a $\$0.80 \div 2$
$= \$0.40$
$\therefore 0.8 \div 2 = 0.4$

b $\$0.12 \div 4$
$= \$0.03$
$\therefore 0.12 \div 4 = 0.03$

c $\$1.50 \div 3$
$= \$0.50$
$\therefore 1.5 \div 3 = 0.5$

Example 2

Evaluate:

a $42.48 \div 8$
b $0.714 \div 7$
c $1.28 \div 5$

Solutions

a
$$\begin{array}{r} 5.\ 31 \\ 8\overline{)42.^{2}48} \end{array}$$

b
$$\begin{array}{r} 0.102 \\ 7\overline{)0.714} \end{array}$$

c
$$\begin{array}{r} 0.2\ 5\ 6 \\ 5\overline{)1.2^{2}8^{3}0} \end{array}$$

Example 3

Evaluate:

a $14.2 \div 10$ **b** $725.9 \div 100$ **c** $5.6 \div 1000$

Solutions

a To divide by 10, we move the decimal point 1 place to the left, i.e. 1.4.2
$\therefore\ 14.2 \div 10 = 1.42$

b To divide by 100, we move the decimal point 2 places to the left, i.e. 7.25.9
$\therefore\ 725.9 \div 100 = 7.259$

c To divide by 1000, we move the decimal point 3 places to the left, filling the extra places with zeros, i.e. 0.005.6
$\therefore\ 5.6 \div 1000 = 0.0056$

Exercise 6.8

1 Evaluate each of the following by considering the decimal as an amount of money.

a $0.8 \div 2$ **b** $0.9 \div 3$ **c** $0.6 \div 2$ **d** $0.06 \div 3$ **e** $0.15 \div 5$
f $0.24 \div 3$ **g** $0.28 \div 7$ **h** $0.42 \div 6$ **i** $0.72 \div 8$ **j** $0.1 \div 2$
k $0.5 \div 2$ **l** $0.4 \div 5$ **m** $0.6 \div 5$ **n** $0.3 \div 6$ **o** $0.7 \div 10$
p $1.4 \div 2$ **q** $1.2 \div 4$ **r** $2.1 \div 3$ **s** $4.5 \div 9$ **t** $2.5 \div 5$

Consolidation

2 Evaluate each of the following, giving your answers as decimals.

a $6\overline{)33}$ **b** $5\overline{)48}$ **c** $8\overline{)22}$ **d** $8\overline{)67}$

3 Evaluate each of these by short division.

a $4.86 \div 2$ **b** $3.12 \div 3$ **c** $497.4 \div 6$
d $113.88 \div 4$ **e** $372.68 \div 7$ **f** $0.1264 \div 8$
g $12.45 \div 5$ **h** $40.248 \div 9$ **i** $129.477 \div 3$
j $12.076 \div 2$ **k** $102.18 \div 6$ **l** $104.335 \div 5$
m $434.91 \div 7$ **n** $46.592 \div 8$ **o** $46.08 \div 12$
p $83.1 \div 5$ **q** $2.327 \div 4$ **r** $1.725 \div 6$

4 Evaluate:

a $14.5 \div 10$ **b** $186.2 \div 100$ **c** $3504.81 \div 1000$ **d** $3.2 \div 10$
e $169.4 \div 1000$ **f** $15.9 \div 100$ **g** $72 \div 10$ **h** $384 \div 100$

i 52 713 ÷ 1000	**j** 36 ÷ 100	**k** 4 ÷ 10	**l** 498 ÷ 1000
m 30 ÷ 10	**n** 60 ÷ 100	**o** 8100 ÷ 10 000	**p** 5020 ÷ 10 000

5 Evaluate each of these.

a 0.9 ÷ 10	**b** 2.6 ÷ 100	**c** 13.4 ÷ 1000	**d** 5.61 ÷ 100
e 31.84 ÷ 10 000	**f** 375.1 ÷ 1000	**g** 80.2 ÷ 1000	**h** 10.93 ÷ 100 000
i 5.65 ÷ 100 000	**j** 0.8 ÷ 100	**k** 0.02 ÷ 10	**l** 0.43 ÷ 10 000
m 0.007 ÷ 10	**n** 0.014 ÷ 100	**o** 0.06 ÷ 10 000	**p** 0.025 ÷ 100 000

Further applications

6 Find the missing number in each of these.

a 2.4 × □ = 2400	**b** 13.75 × □ = 1375	**c** 560 ÷ □ = 5.6
d 47 290 ÷ □ = 47.29	**e** □ × 10 = 163.4	**f** □ × 1000 = 51 200
g □ ÷ 100 = 2.37	**h** □ ÷ 10 000 = 5.4	**i** 0.09 × □ = 90
j □ × 10 000 = 430	**k** □ ÷ 10 = 0.0028	**l** 3.25 ÷ □ = 0.003 25

6.9 Dividing a decimal by a decimal

To divide two decimals we could rewrite the division in fraction form, then multiply both the numerator and denominator by an appropriate power of 10 so that the denominator becomes a whole number. We now perform a short division as shown in the example.

$$0.615 \div 0.3 = \frac{0.615 \times 10}{0.3 \times 10} = \frac{6.15}{3}$$

$$\begin{array}{r} 2.05 \\ 3\overline{)6.15} \end{array}$$

In practice, however, we need not write the division as a fraction. Rather, we could simply multiply both numbers by the power of 10 which makes the divisor (i.e. the number you are dividing by) a whole number.

To divide two decimals:
- **move the decimal point to the right by the same number of places in each decimal, so that you are dividing by a whole number**
- **divide the first number by the second number.**

Example

Evaluate:

a 0.658 ÷ 0.07

b 5.04 ÷ 0.004

Solutions

a We multiply each number by 100 to change the divisor, 0.07, into a whole number.

$0.65.8 \div 0.07$

$= 65.8 \div 7$

$$\begin{array}{r} 9.\ 4 \\ 7\overline{)65.^{2}8} \end{array}$$

$\therefore 0.658 \div 0.07 = 9.4$

b We multiply each number by 1000 to change the divisor, 0.004, into a whole number.

$5.040 \div 0.004$

$= 5040 \div 4$

$$\begin{array}{r} 1260 \\ 4\overline{)5^{1}0^{2}40} \end{array}$$

$\therefore 5.04 \div 0.004 = 1260$

Exercise 6.9

1 Express each decimal as a fraction and hence evaluate each of these. Give your answers as decimals.

a 0.6 ÷ 0.3 **b** 0.7 ÷ 0.1 **c** 0.08 ÷ 0.02 **d** 0.28 ÷ 0.04
e 0.396 ÷ 0.003 **f** 0.068 ÷ 0.002 **g** 0.24 ÷ 0.4 **h** 0.49 ÷ 0.7
i 0.045 ÷ 0.09 **j** 0.056 ÷ 0.7 **k** 0.096 ÷ 0.8 **l** 1.8 ÷ 0.9
m 4.8 ÷ 0.06 **n** 4.2 ÷ 0.007 **o** 1.6 ÷ 0.04 **p** 6.3 ÷ 0.9
q 7.2 ÷ 0.012 **r** 0.56 ÷ 0.7 **s** 4.68 ÷ 0.9 **t** 0.455 ÷ 0.05

Consolidation

2 Evaluate:

a 0.18 ÷ 0.2 **b** 0.21 ÷ 0.3 **c** 0.063 ÷ 0.7
d 0.064 ÷ 0.04 **e** 1.308 ÷ 0.06 **f** 1.0542 ÷ 0.03
g 8.58 ÷ 0.5 **h** 3.9256 ÷ 0.08 **i** 0.259 ÷ 0.004
j 7.966 ÷ 0.07 **k** 90.96 ÷ 0.8 **l** 0.35668 ÷ 0.002
m 0.0549 ÷ 0.009 **n** 0.1128 ÷ 0.02 **o** 10.3548 ÷ 0.6
p 0.0267 ÷ 0.003 **q** 327.96 ÷ 0.4 **r** 77.36 ÷ 0.05

3 Evaluate:

a 48.8 ÷ 0.08 **b** 21 ÷ 0.3 **c** 69.6 ÷ 0.06
d 8.33 ÷ 0.007 **e** 7.2 ÷ 0.05 **f** 1.2 ÷ 0.004
g 2.745 ÷ 0.0009 **h** 107.8 ÷ 0.011 **i** 6.72 ÷ 0.012

Further applications

4 Evaluate each of the following.

a 315 ÷ 1.5 **b** 265.2 ÷ 1.3 **c** 51.92 ÷ 0.16
d 68.68 ÷ 3.4 **e** 1.197 ÷ 0.21 **f** 29.34 ÷ 1.8
g 2.178 ÷ 0.044 **h** 0.95 ÷ 0.025 **i** 12.75 ÷ 0.051

5 Evaluate each of the following by considering each decimal as an amount of money.

a 0.4 ÷ 0.2 **b** 0.5 ÷ 0.1 **c** 0.9 ÷ 0.3 **d** 0.12 ÷ 0.03
e 0.14 ÷ 0.02 **f** 0.36 ÷ 0.04 **g** 0.35 ÷ 0.05 **h** 0.48 ÷ 0.06
i 0.1 ÷ 0.05 **j** 0.2 ÷ 0.04 **k** 0.3 ÷ 0.05 **l** 0.4 ÷ 0.08
m 0.2 ÷ 0.02 **n** 0.6 ÷ 0.12 **o** 1 ÷ 0.2 **p** 2 ÷ 0.5
q 1 ÷ 0.05 **r** 1.5 ÷ 0.5 **s** 1.8 ÷ 0.3 **t** 2.4 ÷ 1.2

6 Find the missing number in each of these.

a 0.12 ÷ □ = 0.3 **b** 0.28 ÷ □ = 0.4 **c** □ ÷ 0.2 = 0.9
d □ ÷ 0.6 = 0.7 **e** 0.015 ÷ □ = 0.03 **f** 0.024 ÷ □ = 0.6
g □ ÷ 0.4 = 0.09 **h** □ ÷ 0.08 = 0.4 **i** □ ÷ 0.03 = 0.12
j 1.8 ÷ □ = 6 **k** 5.6 ÷ □ = 0.8 **l** □ ÷ 7 = 0.7

TRY THIS

Judging Olympic diving

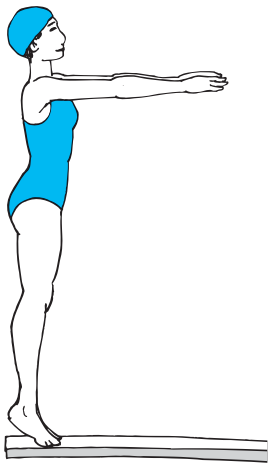
Judging the diving events at the Olympic Games is a most complicated affair. There are usually 7 judges, but the highest and lowest scores are not counted. The scores of the remaining 5 judges are averaged. Since some dives are much harder than others, an allowance is made, called the 'Degree of Difficulty', for the different dives.

Here is how it works:

Donna the diver received the following points out of 10 from 7 judges for doing a forward somersault in the pike position from a three metre board.

7.4, 6.9, 6.4, 7.8, 8.0, 7.5, 7.0

6.4 and 8.0 are eliminated. We average the remaining 5 marks by adding them and dividing the total by 5.



$$\begin{array}{r} 7.4 \\ 6.9 \\ 7.8 \\ 7.5 \\ 7.0 \\ \hline 5\overline{)36.60} \\ \hline 7.32 \end{array}$$

Since scoring in all competitions is based on a 3 judge total, the average score is multiplied by 3.

$$\begin{array}{r} 7.32 \\ 3\ \times \\ \hline 21.96 \end{array}$$

The type of dive that Donna did has a Degree of Difficulty of 1.6, so the total is multiplied by 1.6.

$$\begin{array}{r} 21.96 \\ 1.6\ \times \\ \hline 35.136 \end{array} \leftarrow \text{Donna's score}$$

Problem:

Here are the scores of 5 divers. Do the calculations to find their placings in the event.

Diver	Score	Dives	DD
John	7.2, 7.8, 6.4, 5.8, 6.2, 8.0, 8.2	Reverse somersault	1.9
James	6.8, 7.4, 3.5, 6.7, 8.1, 9.2, 3.4	Triple somersault	2.3
Bill	6.7, 8.3, 9.2, 7.8, 8.6, 9.3, 5.7	Back somersault	1.7
Andy	6.5, 8.4, 9.5, 9.2, 9.6, 7.7, 8.4	Forward pike	1.5
Peter	8.4, 6.7, 4.4, 3.5, 4.7, 5.3, 4.9	Forward $3\frac{1}{2}$ somersault	3.0

6.10 Decimals on the number line

To correctly interpret the information that has been presented in the form of a graph, it is often necessary to read off decimal values from a vertical or horizontal line.

Example

State the decimals represented by A, B and C on each number line.

a

b

c

Solutions

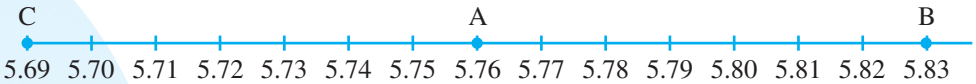
a The interval between 0 and 1 has been divided into 10 smaller intervals. Now, $1 \div 10 = \frac{1}{10} = 0.1$, so each small interval has a length of 0.1 units.

$\therefore$ A = 0.1, B = 0.4, C = 1.2

b The interval between 3 and 4 has been divided into 5 smaller intervals, each of which could be halved, giving 10 even smaller intervals, each of length 0.1 units.

$\therefore$ A = 3.2, B = 3.6, C = 2.8

c The interval between 5.7 and 5.8 has been divided into 10 smaller intervals. Now, $0.1 \div 10 = 0.01$, so each small interval has a length of 0.01 units.



$\therefore$ A = 5.76, B = 5.83, C = 5.69

Exercise 6.10

1 State the decimals represented by P, Q and R on each number line.

a

b

c

d

2 State the decimals represented by X, Y and Z on each number line.

a

b

c

d

Consolidation

3 State the decimals represented by A, B and C in each of these.

a

b

c

d

4 State the decimals represented by F, G and H in each of these.

a

b

c

d

5 State the decimals represented by T, U and V in each of these.

a

b

c

d

6 State the decimals or whole numbers represented by L, M and N in each of these.

a

b

c

d

Further applications

7 State the decimal represented by K in each of these.

a

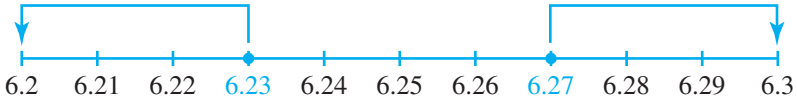
b

c

d

6.11 Rounding off decimals

To **round off**, or **approximate**, decimals such as 6.23 and 6.27 correct to 1 decimal place, we need to determine whether each decimal is closer to 6.2 or to 6.3 on a number line.



Now, 6.23 is closer to 6.2 and 6.27 is closer to 6.3. That is, correct to 1 decimal place we say that $6.23 \doteqdot 6.2$ and $6.27 \doteqdot 6.3$, where $\doteqdot$ means 'is approximately equal to'.

In general, when rounding off a decimal to a certain number of places, it is the size of the digit in the next decimal place which determines whether the decimal is rounded up or down.

To round off, or approximate, a decimal to a certain number of decimal places:
- **remove any digits after the required number of decimal places**
- **add 1 to the last remaining digit if the next digit was 5 or greater.**

Example 1

Round off 7.385 correct to:

a the nearest whole number **b** 1 decimal place **c** 2 decimal places

Solutions

a To round 7.385 correct to the nearest whole number, we note that the next digit is a 3, which is less than 5, so the decimal must be rounded down.
$\therefore$ $7.385 \doteqdot 7$ (to nearest whole number)

b To round 7.385 correct to 1 decimal place, we note that the next digit is an 8, which is greater than 5, so the decimal must be rounded up.
$\therefore$ 7.385 ≒ 7.4

c To round 7.385 correct to 2 decimal places, we note that the next digit is a 5, so the decimal must be rounded up.
$\therefore$ 7.385 ≒ 7.39

Example 2

Round off 2.497 correct to 2 decimal places.

Solution

To round 2.497 correct to 2 decimal places, we note that the next digit is a 7, which is greater than 5, so the decimal must be rounded up.
$\therefore$ 2.497 ≒ 2.50

Exercise 6.11

1 Estimate the position of each decimal on this number line. Hence, round off each decimal, correct to the nearest whole number.

a 8.2 **b** 8.7 **c** 8.5

2 Estimate the position of each decimal on this number line. Hence, round off each decimal, correct to 1 decimal place.

a 3.68 **b** 3.64 **c** 3.65

3 Estimate the position of each decimal on this number line. Hence, round off each decimal, correct to 2 decimal places.

a 0.423 **b** 0.429 **c** 0.425

Consolidation

4 Round off each of the following decimals, correct to the nearest whole number.

a 12.7	**b** 26.2	**c** 18.6	**d** 41.5	**e** 8.06
f 0.7	**g** 19.378	**h** 36.521	**i** 0.437	**j** 115.16
k 63.499	**l** 17.9	**m** 51.08	**n** 49.099	**o** 199.5

5 Round off each of the following decimals, correct to 1 decimal place.

a 0.38	**b** 0.13	**c** 0.74	**d** 0.65	**e** 17.27
f 46.491	**g** 37.028	**h** 88.25	**i** 6.051	**j** 349.15
k 102.34	**l** 19.949	**m** 550.95	**n** 0.021	**o** 779.98

6 Round off each of the following decimals, correct to 2 decimal places.

a 0.277	**b** 0.614	**c** 0.149	**d** 0.085
e 5.142	**f** 73.0125	**g** 3.7624	**h** 200.398
i 316.455	**j** 697.042	**k** 110.949	**l** 94.4999
m 502.666	**n** 23.8151	**o** 199.4818	**p** 3259.9954

7 Round off each decimal, correct to:

i the nearest tenth **ii** the nearest hundredth

a 0.128 **b** 0.3741 **c** 0.699 **d** 7.0552 **e** 12.506 **f** 26.1459

8 Round off each of these, correct to the number of decimal places indicated in the brackets.

a 15.742 [1]	**b** 83.216 [2]	**c** 1.0409 [3]
d 37.235 [2]	**e** 51.46 [0]	**f** 0.011 28 [3]
g 300.05 [1]	**h** 419.9 [0]	**i** 264.135 [2]
j 6.2842 [3]	**k** 15.99 [1]	**l** 7.4499 [2]

9 Minh rounded off a decimal correct to one decimal place and wrote 3.7.

a Could the decimal have been **i** 3.65? **ii** 3.75?

b Write 3 possible decimals that could have been rounded to 3.7.

10 Sam rounded off a decimal correct to two decimal places and wrote 8.24.

a Could the decimal have been **i** 8.235? **ii** 8.245?

b Write 3 possible decimals that could have been rounded to 8.24.

■ Further applications

11 Round off:

a 2.1746 cm, to the nearest mm	**b** 5.4239 m, to the nearest cm
c 9.0386 km, to the nearest m	**d** 12.76924 m, to the nearest mm
e 0.0376953 km, to the nearest m	**f** 0.1163525 km, to the nearest cm

6.12 Terminating decimals

A **terminating decimal** is a decimal which has a fixed or finite number of decimal places. For example, decimals such as 0.285 and 1.47 are terminating decimals because they have exactly 3 and 2 decimal places respectively. A decimal such as $\sqrt{2} = 1.414\,213\,562\ldots$ has an infinite number of decimal places. It is a non-terminating decimal.

To convert a terminating decimal to a fraction:

- **use place value to express the decimal as a fraction**
- **simplify where possible.**

To convert a fraction to a decimal, use one of the following methods.

METHOD 1

- **multiply the numerator and denominator by a number which will convert the fraction to a fraction with a power of 10 in the denominator**
- **write this fraction as a decimal using place value.**

METHOD 2

- **divide the numerator by the denominator.**

Example 1

Convert each decimal to a fraction in simplest form.

a 0.6 **b** 0.15 **c** 0.028

Solutions

a 0.6

$= \frac{\cancel{6}^{3}}{\cancel{10}_{5}} \begin{matrix}(\div 2)\\(\div 2)\end{matrix}$

$= \frac{3}{5}$

b 0.15

$= \frac{\cancel{15}^{3}}{\cancel{100}_{20}} \begin{matrix}(\div 5)\\(\div 5)\end{matrix}$

$= \frac{3}{20}$

c 0.028

$= \frac{\cancel{28}^{7}}{\cancel{1000}_{250}} \begin{matrix}(\div 4)\\(\div 4)\end{matrix}$

$= \frac{7}{250}$

Example 2

Convert each of these fractions to a decimal using method 1.

a $\frac{2}{5}$ **b** $\frac{9}{25}$ **c** $\frac{11}{200}$

Solutions

a $\frac{2 \times 2}{5 \times 2}$

$= \frac{4}{10}$

$= 0.4$

b $\frac{9 \times 4}{25 \times 4}$

$= \frac{36}{100}$

$= 0.36$

c $\frac{11 \times 5}{200 \times 5}$

$= \frac{55}{1000}$

$= 0.055$

Example 3

Express $\frac{5}{8}$ as a decimal using method 2.

Solution

$\frac{5}{8}$ means $5 \div 8$,

i.e. $\begin{array}{r} 0.625 \\ 8\overline{)5.0^{2}0^{4}0} \end{array}$

$\therefore \frac{5}{8} = 0.625$

Example 4

Express:

a $4\frac{9}{50}$ as a decimal, using method 1

b $2\frac{7}{8}$ as a decimal, using method 2

Solutions

a $4\frac{9}{50}\ \begin{matrix}\times 2\\ \times 2\end{matrix}$

$= 4\frac{18}{100}$

$= 4.18$

b $2\frac{7}{8} = \frac{23}{8}$ as an improper fraction.

$$\begin{array}{r} 2.\ 8\ 7\ 5 \\ 8\overline{)23.^{7}0^{6}0^{4}0} \end{array}$$

$\therefore\ 2\frac{7}{8} = 2.875$

Exercise 6.12

1 Express each of these decimals as a fraction in simplest form.

a 0.2	**b** 0.5	**c** 0.8	**d** 0.4	**e** 0.02
f 0.06	**g** 0.05	**h** 0.14	**i** 0.44	**j** 0.35
k 0.28	**l** 0.25	**m** 0.18	**n** 0.46	**o** 0.75

2 Express each fraction with a power of 10 in the denominator, then convert it to a decimal.

a $\frac{1}{5}$	**b** $\frac{3}{5}$	**c** $\frac{4}{5}$	**d** $\frac{1}{2}$
e $\frac{9}{50}$	**f** $\frac{13}{50}$	**g** $\frac{21}{50}$	**h** $\frac{33}{50}$
i $\frac{3}{25}$	**j** $\frac{9}{25}$	**k** $\frac{16}{25}$	**l** $\frac{24}{25}$
m $\frac{1}{20}$	**n** $\frac{11}{20}$	**o** $\frac{13}{20}$	**p** $\frac{17}{20}$

Consolidation

3 Express each of these decimals as a fraction in simplest form.

a 0.022	**b** 0.046	**c** 0.126	**d** 0.414	**e** 0.036
f 0.164	**g** 0.444	**h** 0.628	**i** 0.055	**j** 0.135
k 0.285	**l** 0.715	**m** 0.008	**n** 0.096	**o** 0.344

4 Express each fraction with a power of 10 in the denominator, then convert it to a decimal.

a $\frac{7}{500}$	**b** $\frac{121}{500}$	**c** $\frac{263}{500}$	**d** $\frac{419}{500}$
e $\frac{1}{250}$	**f** $\frac{27}{250}$	**g** $\frac{108}{250}$	**h** $\frac{213}{250}$

i $\frac{3}{200}$ **j** $\frac{21}{200}$ **k** $\frac{73}{200}$ **l** $\frac{109}{200}$

m $\frac{1}{125}$ **n** $\frac{7}{125}$ **o** $\frac{31}{125}$ **p** $\frac{112}{125}$

5 Express each of these decimals as a mixed numeral in simplest form.

a 1.5 **b** 2.8 **c** 3.45 **d** 6.25
e 7.24 **f** 5.82 **g** 9.75 **h** 10.074
i 11.205 **j** 15.724 **k** 17.104 **l** 20.375

6 Express each of these mixed numerals as a decimal.

a $3\frac{1}{2}$ **b** $5\frac{1}{4}$ **c** $6\frac{3}{4}$ **d** $8\frac{3}{5}$ **e** $9\frac{4}{25}$ **f** $10\frac{13}{20}$

g $12\frac{47}{50}$ **h** $13\frac{53}{500}$ **i** $15\frac{7}{250}$ **j** $19\frac{17}{200}$ **k** $21\frac{9}{125}$ **l** $22\frac{7}{8}$

7 Divide the numerator by the denominator in each of these. Hence, convert each fraction to a decimal.

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{2}{5}$ **e** $\frac{1}{8}$ **f** $\frac{3}{8}$

g $\frac{6}{25}$ **h** $\frac{13}{50}$ **i** $\frac{9}{20}$ **j** $\frac{1}{16}$ **k** $\frac{5}{16}$ **l** $\frac{1}{40}$

■ Further applications

8 Use decimals to evaluate each of the following, then write your answers as fractions or mixed numerals in simplest form.

a $\frac{1}{4} + \frac{1}{2}$ **b** $\frac{1}{5} + \frac{1}{4}$ **c** $\frac{1}{10} + \frac{3}{4}$

d $1\frac{1}{2} + \frac{3}{10}$ **e** $\frac{3}{20} + 1\frac{3}{5}$ **f** $2\frac{9}{20} + 3\frac{3}{4}$

6.13 Recurring decimals

A **recurring decimal** or **repeating decimal** is a decimal in which one or more of the digits are repeated an infinite number of times. To indicate a single recurring digit, we place a dot above that digit. For example, the recurring decimal 0.777 … would be written as $0.\dot{7}$, while 0.125 55 … would be written as $0.12\dot{5}$. Here, the dot indicates that 5 is the only recurring digit.

In some recurring decimals there is a repeating pattern of two or more digits. In these decimals we place a dot above only the first and last digits in the repeating pattern. For example, 0.313 131 … would be written as $0.\dot{3}\dot{1}$, whilst 0.249 249 … would be written as $0.\dot{2}4\dot{9}$.

To convert a fraction to a recurring decimal:
- **express any mixed numerals as improper fractions**
- **divide the numerator by the denominator.**

Example 1

Convert each fraction to a recurring decimal.

a $\frac{4}{9}$ **b** $\frac{7}{11}$ **c** $\frac{5}{6}$

Solutions

a $\frac{4}{9}$ means $4 \div 9$,

i.e. $9\overline{)4.0^{4}0^{4}0}$ $\quad 0.4\ 4\ 4\ \ldots$

$\therefore \frac{4}{9} = 0.\dot{4}$

b $\frac{7}{11}$ means $7 \div 11$,

i.e. $11\overline{)7.0^{4}0^{7}0^{4}0}$ $\quad 0.6\ 3\ 6\ 3\ \ldots$

$\therefore \frac{7}{11} = 0.\dot{6}\dot{3}$

c $\frac{5}{6}$ means $5 \div 6$,

i.e. $6\overline{)5.0^{2}0^{2}0^{2}0}$ $\quad 0.8\ 3\ 3\ 3\ \ldots$

$\therefore \frac{5}{6} = 0.8\dot{3}$

Example 2

Express $1\frac{5}{12}$ as a recurring decimal.

Solution

$1\frac{5}{12} = \frac{17}{12}$ as an improper fraction.

$12\overline{)17.^{5}0^{2}0^{8}0^{8}0^{8}0}$ $\quad 1.\ 4\ 1\ 6\ 6\ 6\ \ldots$

$\therefore 1\frac{5}{12} = 1.41\dot{6}$

Exercise 6.13

1 Write each of the following recurring decimals using the dot notation.

a 0.2222 …	**b** 0.5555 …	**c** 0.377 77 …
d 0.0666 …	**e** 1.3333 …	**f** 4.811 11 …
g 0.494 949 …	**h** 0.121 212 …	**i** 3.757 575 …
j 0.105 105 …	**k** 0.328 328 …	**l** 5.609 609 …
m 0.236 363 6 …	**n** 0.417 817 8 …	**o** 12.034 134 1 …

2 **a** Express $\frac{1}{3}$ as a recurring decimal.

b What would $\frac{2}{3}$ be as a recurring decimal? Verify your answer by division.

3 **a** Convert each of these fractions to a recurring decimal.

i $\frac{1}{9}$ **ii** $\frac{2}{9}$ **iii** $\frac{7}{9}$

b Is there a pattern? Explain.

c What would $\frac{5}{9}$ be as a recurring decimal? Verify your answer by division.

Consolidation

4 Convert each of these fractions to a recurring decimal.

a $\frac{2}{11}$ **b** $\frac{3}{11}$ **c** $\frac{5}{11}$ **d** $\frac{9}{11}$

CALCULATOR

5 Convert each of these fractions to a recurring decimal. Verify your answers using a calculator.

a $\frac{8}{9}$ **b** $\frac{1}{6}$ **c** $\frac{1}{12}$ **d** $\frac{2}{15}$

e $\frac{5}{6}$ **f** $\frac{5}{12}$ **g** $\frac{10}{11}$ **h** $\frac{7}{12}$

6 Express each mixed numeral as an improper fraction, then convert it to a recurring decimal. Check your answers using a calculator.

a $1\frac{1}{3}$ **b** $2\frac{8}{9}$ **c** $4\frac{1}{6}$ **d** $1\frac{11}{12}$

7 **a** Simplify $\frac{1}{3} + \frac{1}{4}$, then express the answer as a decimal.

b Express $\frac{1}{3}$ and $\frac{1}{4}$ as decimals.

c Evaluate $\frac{1}{3} + \frac{1}{4}$ by adding their decimal equivalents. Give your answer as a fraction.

8 **a** Use the fact that $\frac{1}{30} = \frac{1}{10} \times \frac{1}{3}$ to convert $\frac{1}{30}$ to a recurring decimal.

b Hence, express $\frac{1}{300}$ and $\frac{1}{3000}$ as recurring decimals.

Further applications

9 Express each of the following fractions as a recurring decimal. Verify your answers using a calculator.

a $\frac{1}{7}$ **b** $\frac{2}{7}$ **c** $\frac{4}{7}$ **d** $\frac{5}{7}$

10 How should we define $0.\dot{9}$? Explain.

CALCULATOR

11 **a** Convert $\frac{1}{13}$ to a recurring decimal using a calculator.

b Which digit would lie in the:

i 8th decimal place? **ii** 13th decimal place? **iii** 23rd decimal place?

6.14 Solving problems that involve decimals

Example 1

A piece of timber 4.2 m long is cut evenly into lengths of 0.06 m. How many such lengths can be cut?

Solution

To find the number of lengths of timber which can be cut, we divide 4.2 by 0.06.

$4.2 \div 0.06$

$= 420 \div 6$ (Multiplying each number by 100)

$= 70$ $\therefore$ 70 pieces of timber can be cut.

Example 2

A jug holds 0.32 L of water when full. How many litres of water will there be in 25 jugs?

Solution

To find the total quantity of water, we first multiply the numbers, ignoring the decimal points.

$$\begin{array}{rl} 32 & \\ {}_{1}25 & \times \\ \hline 160 & \\ {}_{1}640 & + \\ \hline 800 & \end{array}$$

However, there were 2 decimal places in the question, so there must be 2 decimal places in the answer.

$\therefore 25 \times 0.32 \text{ L} = 8.00 \text{ L}$

$= 8$ L of water

Exercise 6.14

1 During a school athletics carnival, the following times were recorded by eight students in the 100 m sprint.

Alan 15.1 s	Nick 15.08 s	Tran 15.15 s	Pieter 14.9 s
Robert 15 s	Shane 15.32 s	Paolo 14.18 s	Matthew 14.67 s

Write down the finishing order of the runners, from first to last.

2 Theresa bought 1.5 kg of sausages at \$2.10 per kg. How much did she pay?

3 Moira and three of her friends went to a concert and paid \$19.20 for parking. If they shared the cost equally, how much did each person pay?

4 A lottery prize of \$1720.74 was shared equally between 7 people. How much did each person receive?

5 During a national high jump event, Ryan jumped a height of 5.1 m while Nathan jumped 4.85 m. How much higher, in metres, did Ryan jump?

6 A 1.25 L bottle of cola is to be shared between 4 boys, with one boy receiving twice as much as any of the others. Find the larger share.

Consolidation

7 A small water cooler holds 35.6 L. If Maria drinks 0.7 L and Michaela drinks 0.45 L, how much water remains in the cooler?

8 How many 0.25 L glasses of cordial can be *filled* from a container which holds
i 4.5 L? **ii** 5.1 L?

9 A dressmaker bought 3.2 m of ribbon and cut it into strips of length 0.16 m. How many strips can be cut?

10 A scrap metal dealer purchased old car parts weighing 50.3 kg, 28.47 kg, 56.1 kg and 9.384 kg, paying $10 per kilogram. How much did the dealer pay for the parts?

11 Kevin's car holds 40 L of LPG. When the gauge shows that the tank is one-quarter full, Kevin pulls into a garage and fills the tank at 51.4c per litre. Find the cost of the LPG.

12 Find the change from $8 after buying 14 items at 17c each.

13 On the production line at a factory, the average worker can assemble 5.2 gadgets every minute. How many gadgets could be assembled in an hour by 3 workers?

14 Three items weighing 2.1 kg, 1.65 kg and 3.47 kg are to be posted. By how much does the total weight exceed 7.15 kg?

15 During a science experiment, a beaker of liquid is heated so that its temperature rises uniformly from 10.2°C to 32.7°C in 15 minutes.
a By how many degrees does the temperature rise each minute?
b Find the temperature of the liquid 9 minutes after the experiment commenced.

16 When a tourist enquired about the exchange rate between Australian dollars ($A) and English pounds (£), he was told that $1A = £0.45. If $20A is exchanged, how many English pounds would the tourist receive?

17 During an ice-skating competition, the judges awarded Maree the following scores out of 10: 8.2, 8.6, 9.1, 8.8, 7.9 and 7.6. The final score is obtained by removing the highest and lowest scores, then averaging those which remain. Calculate Maree's score.

18 On a scale drawing, the towns of Barker and Devon were drawn 2.7 cm apart. If the scale of the drawing was 1 : 100 000, calculate the real distance between the towns in km.

19 The local council planted 9 new trees along one side of Mason St. The first and last trees are situated 5.6 m from each end of the street and the trees are equally spaced at intervals of 15.3 m. Calculate the length of the street.

20 Karina bought 35 L of petrol at 95.7c per litre. How much did she pay for the petrol?

Further applications

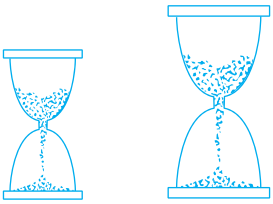
21 The average height of a group of five children is 1.42 m. If four of the children have heights of 1.35 m, 1.4 m, 1.52 m and 1.46 m, find the height of the fifth child.

22 The difference between an integer and its reciprocal is 7.875. Find the integer.

Problem solving

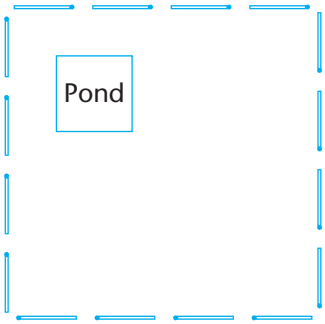
1 This puzzle is quite old, but will keep you thinking!
Three women went to a motel to obtain a room. They decided to share the thirty-dollar bill evenly and so each paid $10. Unfortunately the motel made a mistake, since the actual room cost was $25. When a man was sent to give them back their $5, he felt he would not be able to split this $5 evenly between three people, so he kept $2 and gave each woman $1 change. Hence each woman only paid $9 towards the room.
The room cost was therefore $27, which together with the man's $2 makes $29.
Account for the missing dollar!

2 Here is another sand-timer puzzle:

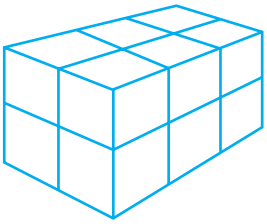


This time, use these timers to measure 15 minutes exactly.

3 Add 10 more matches to divide the area around the pond into 5 equal areas of the same shape.



4 Twelve cubes are placed together to form the box shown. The box is then painted on all sides. How many cubes have 2 sides painted? … 3 sides painted?



5 A magic square!
Arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 in a 3×3 square so that the total in every direction is 15.

6 Janine's parents wish to buy a raffle ticket out of a book of 100 tickets. Seven is their lucky number, but they do not like doubles. How many lucky tickets are there to choose from?

7 A student travels to school at a speed of 2 km/h and returns home at 3 km/h. Altogether, the two trips take 5 hours. How far is it from home to school?

8 What fraction is halfway between $\frac{1}{5}$ and $\frac{1}{7}$?

9 How can Peg use a 5 litre jug and a 3 litre jug to measure exactly 2 litres of water? Also how can she use the two jugs to measure exactly 1 litre of water?

10 Calculate:

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+1}}}$$

Focus on working mathematically

Olympic Decathlon 2000

Introduction

The Olympic Decathlon is a competition held over two consecutive days and tests athletes for all-round excellence in ten events:

Day 1 100 m, long jump, shot put, high jump and 400 m
Day 2 100 m hurdles, discus, pole vault, javelin and 1500 m

The scoring system for each event rewards athletes according to their performance. This activity shows how important decimal fractions of time and distance are for the calculation of total points scored, and subsequent medals.

In 2000 at the Sydney Olympics, the gold medal was won by Erki Nool of Estonia (8641 points). The silver went to Roman Sebrle of the Czech Republic (8606 points), and the bronze to Chris Huffins of the USA (8595 points).

Huffins was the first-day leader on 4554 points. He is a brilliant sprinter and jumper. He was still in first place after nine events with 7907 points. His closest rival, Erki Nool, was close behind on 7893 points and then Roman Sebrle on 7853.

But then came the final event, the 1500 m. This is Huffins' weakness while Nool and Sebrle are much stronger over long distance.

Question: Given the scoring system for the 1500 m, what time would Huffins need to run the 1500 m to take out the gold medal?

Learning activities

The following questions will help you answer the question above. Your teacher may like to provide an opportunity to discuss the information you will need before you start.

1. Nool runs his 1500 m and scores 748 points. Sebrle does even better and collects 753 points. How many points does Huffins need to win the gold medal?
2. What time is required to win these points, given that a time of 4:30.00 (4 min 30.00 s) scores 730 points and a time of 4:20.00 scores 800 points. You may assume that the points are evenly distributed during this interval. Write down your answer to the focus question.
3. Huffins actually runs a time of 4:38.71 (4 min 38.71 s) and scores 688 points. By how many seconds did he miss the gold medal? What was his final score?
4. Draw up and complete the table below.

	Huffins	Nool	Sebrle
After 9 events			
1500 m points			
Total points			
Medal			

Extension activity

Here are the points Huffins scored in each event.

What were his stronger events? His weaker events? What was the average number of points per event? (A world class average is 800.) Make a conclusion about the overall performance of this athlete.

Event	Points	Event	Points
100 m	980	110 m H	986
Long Jump	987	Discus	861
High Jump	806	Pole Vault	819
Shot Put	887	Javelin	687
400 m	894	1500 m	688
		Total	8595

Let's communicate

Discuss with your neighbour how Huffins must have felt as he lined up for the 1500 m. He ran 11 seconds faster than his previous best. What do you think this says about his sportsmanship?

Reflecting

Decimal fractions of time in sport often mean losing or winning at the Olympics. Just a touch in swimming is 0.01 seconds. A time of 9.99 seconds instead of 10.00 seconds for the 100 m track event can mean a medal. However, think how important decimals are in the measurement of daily time, interest rates, scientific research, medical practice, drug prescriptions and so on. Check out the nutrition content on the cereal box at breakfast. You'll see decimals again!

Language link with Macquarie

MACQUARIE

1 *Explain what is meant by a* ***decimal*** *system.*
2 *What is a* ***decimal*** *called when its places are limited?*
3 *Explain the meaning of the following* ***decimal*** *in your own words: 0.541*
4 *What does it mean to* ***convert*** *a fraction to a* ***decimal****.*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***ascend****:*

ascend *verb Rather formal* to climb or go upwards: *I ascended the ladder. | Smoke ascended from the chimney.*
❒ ***Word Family:*** **ascending** *adjective* in order according to an increasing size, importance, etc.: *Please list your requirements in ascending order of importance.*

Opposite: **descend**

If the decimals 0.1, 0.2, 0.3, 0.4 are listed in ***ascending*** *order, what order are 0.4, 0.3, 0.2, 0.1 listed in?*

1 Write each fraction as a decimal.

a $\frac{3}{10}$ **b** $\frac{9}{100}$

c $\frac{7}{1000}$ **d** $\frac{6}{10\,000}$

2 Write each decimal as a fraction.

a 0.7 **b** 0.003

c 0.01 **d** 0.0009

3 Express each of these as a single decimal.

a $2 + \frac{5}{10} + \frac{7}{100} + \frac{4}{1000}$

b $7 + \frac{8}{100} + \frac{2}{10\,000}$

c $\frac{9}{1000} + 14 + \frac{1}{10}$

4 Write down the value of the 7 in each of these.

a 0.375 **b** 7.05

c 6.1078 **d** 0.424 97

5 Express each fraction as a decimal.

a $\frac{21}{100}$ **b** $\frac{249}{1000}$

c $\frac{53}{1000}$ **d** $\frac{871}{100\,000}$

6 Express as a proper fraction or mixed numeral:

a 0.67 **b** 0.051

c 9.23 **d** 4.0309

7 Write each fraction as a decimal.

a $\frac{28}{10}$ **b** $\frac{129}{100}$

c $\frac{485}{10}$ **d** $\frac{43\,778}{1000}$

8 Choose the two decimals that have the same value: 0.90, 0.09, 0.090, 9.00.

9 Express 6 with two decimal places.

10 Insert < or > to make a true statement in each of these.

a 1.05 □ 0.99 **b** 0.08 □ 0.1

c 0.7 □ 0.17 **d** 0.203 □ 0.45

11 Arrange each set of decimals in ascending order.

a 0.73, 1.64, 1.5, 0.9, 0.07

b 0.44, 0.04, 0.404, 0.044, 0.004, 0.4

12 Which decimal is closest to 0.57?

A 0.58 **B** 0.6

C 0.565 **D** 0.578

13 Write the decimal that lies halfway between:

a 0.7 and 0.8 **b** 0.9 and 1

c 0 and 0.1 **d** 5.1 and 5.2

e 0.03 and 0.04 **f** 3.47 and 3.48

g 0.1 and 0.4 **h** 0.2 and 0.18

14 Evaluate vertically:

a 146.95 + 8.487 + 92.664

b 3.861 + 512.99 + 73.065

15 Evaluate vertically:

a 637.4 – 285.6

b 458.7 – 96.08

c 843 – 465.72

16 Evaluate each of the following by considering each decimal as an amount of money.

a 0.7 + 0.5 **b** 1.1 – 0.4

c 1 – 0.6 **d** 3 – 0.8

e 0.05 + 0.09 **f** 0.17 + 0.13

g 1 – 0.15 **h** 2 – 1.75

i 0.2 + 0.09 **j** 0.4 – 0.08

k 0.65 – 0.5 **l** 0.92 + 0.3

17 Evaluate each of these by considering the decimal as an amount of money.

a 2 × 0.7 **b** 0.3 × 4

c 5 × 0.4 **d** 0.2 × 10

e 8 × 0.06 **f** 0.12 × 7

g 2.5 × 3 **h** 9 × 1.08

18 Evaluate vertically:

a 3.7×6 **b** 6.8×5
c 4.32×9 **d** 7.25×8

19 Evaluate:

a 5.67×10 **b** 9.238×100
c 0.0429×1000 **d** 71.44×100
e 0.0015×10 **f** $0.36 \times 100\,000$

20 Evaluate:

a 0.8×0.3 **b** 0.06×0.7
c 0.04×0.09 **d** 0.12×0.5
e 1.3×7 **f** 0.08×2.6
g 54.3×0.05 **h** 0.34×5.7

21 Evaluate each of these by considering the decimal as an amount of money.

a $0.6 \div 3$ **b** $0.14 \div 2$
c $0.35 \div 7$ **d** $0.1 \div 2$
e $0.4 \div 5$ **f** $1.2 \div 3$
g $2.4 \div 6$ **h** $6.3 \div 9$

22 Evaluate each of these by short division.

a $75 \div 8$ **b** $6.15 \div 3$
c $295.2 \div 4$ **d** $580.8 \div 6$

23 Evaluate:

a $27.5 \div 10$ **b** $309.4 \div 100$
c $5626.1 \div 1000$ **d** $10.87 \div 100$
e $68 \div 10$ **f** $722 \div 1000$
g $34.5 \div 10\,000$ **h** $510 \div 100\,000$

24 Evaluate:

a $0.26 \div 0.2$ **b** $0.66 \div 0.3$
c $0.054 \div 0.6$ **d** $0.072 \div 0.04$
e $1.896 \div 0.08$ **f** $9.68 \div 0.5$
g $7.2702 \div 0.003$
h $18.078 \div 0.0006$

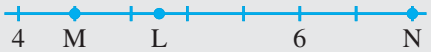
25 State the decimals or whole numbers represented by L, M and N on each number line.

a

b

c

d



26 Round off each decimal, correct to the nearest whole number.

a 37.4 **b** 52.7 **c** 46.5 **d** 199.6

27 Round off each decimal, correct to 1 decimal place.

a 0.47 **b** 0.83 **c** 7.15 **d** 6.099

28 Round off each decimal, correct to 2 decimal places.

a 0.329 **b** 1.694
c 12.085 **d** 25.398

29 Express each decimal as a fraction in simplest form.

a 0.6 **b** 0.25 **c** 0.35
d 0.44 **e** 0.125 **f** 2.5

30 Express each fraction as a decimal.

a $\frac{2}{5}$ **b** $\frac{3}{4}$ **c** $\frac{17}{50}$
d $\frac{9}{20}$ **e** $\frac{107}{250}$ **f** $\frac{11}{200}$

31 **a** Convert $9\frac{21}{25}$ to a decimal.
b Convert 5.406 to a fraction in simplest form.

32 Convert each of these fractions to a decimal by short division.

a $\frac{3}{8}$ **b** $\frac{7}{16}$

33 Write each of these recurring decimals using dot notation.

a 0.444 444 … **b** 0.383 838 …
c 0.722 222 … **d** 0.275 275 …
e 4.917 171 7 … **f** 13.641 341 3 …

34 Convert each of these fractions to a recurring decimal by short division.

a $\frac{5}{9}$ **b** $\frac{1}{6}$ **c** $\frac{8}{11}$ **d** $\frac{11}{12}$

35 **a** If \$1006.60 was shared equally between 7 people, how much would each person receive?

b Glen jumped 5.4 m in the long jump and Matthew jumped 4.87 m. Who jumped further, and by how much?

c Find the cost of 9.4 litres of petrol at 82.3c per litre.

7 Integers

This chapter at a glance

Stage 4

After completing this chapter, you should be able to:

- recognise the direction and magnitude of an integer
- interpret the use of directed numbers in a real world context
- construct a directed number sentence to represent a real situation
- place directed numbers on a number line
- arrange a group of directed numbers in ascending order
- determine whether the + and – signs refer to a direction or an operation, depending on the context
- add and subtract directed numbers
- multiply and divide directed numbers
- simplify expressions that contain adjacent signs
- simplify directed number sentences using a calculator
- find missing terms or operations in incomplete number sentences
- use the order of operations to simplify expressions involving directed numbers
- solve practical problems that involve directed numbers.

7.1 The meaning of directed numbers

Sometimes we are interested not only in the size or **magnitude** of a quantity, but also its **direction**. A **directed number** has both magnitude and direction. For example, think of an elevator that has moved three floors from one level to another. If it went up we would say that it had travelled in the **positive** direction and show this change by the directed number +3 (positive 3). If it went down, we would say that it had travelled in the **negative** direction and represent this change by the directed number -3 (negative 3).

In general, movement upward or to the right is considered to be positive and movement downward or to the left is considered to be negative. With positive numbers, we usually omit the + sign. Thus we would write 3 instead of +3. Zero is neither positive nor negative. We cannot speak of +0 or -0. When the concept of time is involved, we say that a situation such as arriving early is negative while arriving late is said to be positive. Why?

The opposite of a directed number is a number with the same size or magnitude, but which is opposite in sign. For example, the opposite of -7 is +7 (or 7) and the opposite of +4 (or 4) is -4.

An **integer** is a whole number. It may be positive, negative or zero. For example, 3, 7, -4, 0 are all integers. Fractions and decimals are not integers.

- **Directed numbers have both size and direction.**
- **The set of integers is {…, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, …}**

Example 1

Write a directed number to represent each of the following:

a driving 10 km due east
b a debit of $12
c a weight gain of 3 kg
d arriving 5 minutes early
e depositing $20 into an account
f a fall in temperature of 4°C

Solutions

a +10 **b** -12 **c** +3 **d** -5 **e** +20 **f** -4

Exercise 7.1

1 Write a directed number to represent each of these situations.

a 5 m to the right
b 3 cm to the left
c going up 2 floors
d 9 degrees below zero
e walking 17 km south
f 12 points above average
g driving 20 km due east
h descending 6 floors
i a credit of $25
j owing $30

k a test improvement of 10 marks
l going down 7 steps
m depositing $10
n losing 4 kg in weight
o shrinking 3 g in mass
p a price rise of $2
q a fall in temperature of 9°C
r a debit of $18
s sailing 50 km due west
t 45 m below sea level
u withdrawing $28
v arriving 5 mins early
w flying 250 km due north
x arriving 8 mins late

2 Write a statement that shows one possible meaning of each directed number.
a -4 **b** +7 **c** -9 **d** +20 **e** +50 **f** -85

Consolidation

3 Write a directed number to represent each of these statements.
a The temperature fell by 3°C.
b The water level in my swimming pool fell by 2 cm.
c A tree grew 5 cm last year.
d I withdrew $60 from my bank account.
e I received a pay rise of $12 per week.
f The train arrived 3 minutes early at Central Station.
g The number of people visiting a museum fell by 45 visitors per day.
h It took me 15 minutes longer to drive to work today due to heavy traffic.
i The submarine was 100 m below the ocean surface.
j The number of rainy days this summer was 11 fewer than last summer.
k It was 7 minutes before take-off leading up to a launch of the space shuttle *Discovery*.

4 Write a directed number sentence for each statement below.
a Luke lives 3 km west of a major shopping centre. He drove 10 km east and was then 7 km due east of the shopping centre.
b Anne had $20 in her bank account. She paid $35 for a new blouse and her account is now $15 in debit.
c A shark was 2 m below the surface. It went down another 3 m and was then 5 m below the surface.
d The temperature was 8°C below zero. During the day it rose by 5°C and was 3°C below zero at 4 pm.

5 What is the opposite of:
a walking 3 km east?
b arriving 10 minutes early?
c driving 50 km north?
d winning $20 on Lotto?
e moving 8 m to the left?
f descending 4 floors?
g depositing $80?
h a fall in temperature by 5°C?

6 Write the directed number that is the opposite of:
a -4 **b** +7 **c** -9 **d** -20 **e** +50 **f** 85

7 Which of these numbers are integers or can be written as integers?

a -2 **b** +5 **c** $+\frac{1}{2}$ **d** $-1\frac{4}{5}$ **e** 0 **f** $\frac{8}{2}$
g $\frac{2}{8}$ **h** 3.0 **i** 7^2 **j** +0.05 **k** 4.1 **l** $\sqrt{9}$

8 Write true (T) or false (F) for each of these.

a The only integer that does not have a direction is 0.
b All integers (other than 0) are directed numbers.
c All directed numbers are integers.
d 6 means the same as +6.
e -4 means the same as 4.
f The opposite of 9 is -9.
g The opposite of +2 is -2.
h The magnitude of -5 is 5.
i All whole numbers are integers.

Further applications

9 Without performing calculations, state whether each of the following bank accounts will be in credit (+) or debit (–) if the account is:

a $25 in credit, then I withdraw $22
b $18 in credit, then I withdraw $20
c $30 in debit, then I deposit $40
d $50 in debit, then I deposit $45
e $79 in debit, then I withdraw $80
f $99 in credit, then I withdraw $100
g $142 in debit, then I deposit $150
h $220 in debit, then I withdraw $300

7.2 The relative sizes of directed numbers

In the past when you have drawn a number line you started with zero on the left-hand side and then wrote positive numbers increasing in value from left to right.

In reality, however, zero is simply a position in the 'middle' of the number line with the negative numbers continuing indefinitely to the left and the positive numbers continuing indefinitely to the right.

To represent a number on the number line, draw a dot on that number and colour it in.

Example 1

Graph the numbers {-3, -1, 0, 2} on a number line.

Solution

Example 2

Insert < or > to make each statement true.

a 4 ☐ -1 **b** -9 ☐ -2 **c** 0 ☐ -5

Solutions

a 4 > -1 since 4 is further to the right on the number line than -1
b -9 < -2 since -9 is further to the left on the number line than -2
c 0 > -5 since 0 is further to the right on the number line than -5

Exercise 7.2

1 Write down the set of numbers that is shown on each number line.

a -3 -2 -1 0 1 2 3

b -4 -3 -2 -1 0 1 2

c -6 -4 -2 0 2 4 6

d -15 -10 -5 0 5 10 15

e -7 -6 -5 -4 -3 -2 -1

f -5 -4 -3 -2 -1 0 1

g -2 $-1\frac{1}{2}$ -1 $-\frac{1}{2}$ 0 $\frac{1}{2}$ 1

h -0.6 -0.4 -0.2 0 0.2 0.4 0.6

2 Graph each set of numbers on a separate number line.

a {-3, -1, 1, 3} **b** {-4, -1, 0, 2} **c** {-6, -5, -3, -1}
d {-5, -3, -2, 1} **e** $\{-2\frac{1}{2}, -2, -\frac{1}{2}, \frac{1}{2}\}$ **f** {-2, -1.5, 0, 0.5}

3 Which number is further to the left on a number line?

a -2 or 3 **b** 4 or -1 **c** 0 or -3 **d** 5 or 0
e -6 or -5 **f** 7 or -4 **g** -11 or -12 **h** -100 or -99

Consolidation

4 Choose the smallest number from each group.

a 7, -8, -3 **b** -4, -3, -6 **c** 0, -99, -98

5 Choose the largest number from each group.

a -7, -2, -3 **b** -4, 0, -1 **c** -84, -82, -86

6 Insert either < or > to make each statement true.

a 5 ☐ -2 **b** -1 ☐ 4 **c** -3 ☐ 0 **d** -1 ☐ -7
e -21 ☐ 9 **f** -6 ☐ -7 **g** 0 ☐ -14 **h** 17 ☐ -18
i -5 ☐ -1 **j** -12 ☐ -20 **k** -17 ☐ -4 **l** -19 ☐ -18
m -66 ☐ -91 **n** -42 ☐ -38 **o** 0 ☐ -100 **p** -82 ☐ -80
q 15 ☐ -75 **r** -98 ☐ -243 **s** -103 ☐ -102 **t** -150 ☐ 3

7 Arrange these numbers in ascending order.

a -1, 4, 6, 0 **b** 5, -3, -2, 1 **c** -4, -9, -6, 3
d 7, 0, -7, -6 **e** -12, -10, -13, -9 **f** -52, -83, -47, -1
g 3, -67, 0, -92 **h** -74, 76, -73, -75 **i** -1, 29, -113, -114

8 Write down the next two numbers in each number pattern.

a -3, -2, -1, ____, ____ **b** 8, 4, 0, ____, ____
c 9, 5, 1, ____, ____ **d** -5, -7, -9, ____, ____
e -2, 0, 2, ____, ____ **f** -22, -17, -12, ____, ____
g 13, 7, 1, ____, ____ **h** -3, -1, 1, ____, ____
i -21, -14, -7, ____, ____ **j** 32, 22, 12, ____, ____
k -75, -50, -25, ____, ____ **l** -25, -17, -9, ____, ____

9 Write down the integer that lies halfway between:

a 3 and 5 **b** 0 and 2 **c** -1 and 1
d -5 and -3 **e** -2 and 4 **f** -2 and -8
g -5 and 1 **h** -17 and -21 **i** -55 and -49

10 Which of the following transactions would provide the greater bank balance?

a depositing $10 or withdrawing $12 **b** withdrawing $15 or withdrawing $11

11 Which of these is the lower temperature?

a 7°C below zero or 10°C above zero **b** 4°C below zero or 3°C below zero

12 Which position is further east?

a 20 km east or 25 km west **b** 35 km west or 32 km west

13 Which of these is the earlier?

a arriving on time or arriving 7 minutes early
b arriving 12 minutes late or arriving 5 minutes late

14 Which of these would result in the lesser amount?

a shares falling in value by 20c or rising in value by 30c
b shares falling in value by 45c or falling in value by 44c

15 Which account balance is greater?

a a debit of $100 or a credit of $110 **b** a debit of $550 or a debit of $600

■ Further applications

16 Accountants use terms such as 'in the black' and 'in the red'.

a What do these terms mean? **b** Why are they used?
c Which is better, to be $75 in the red or $80 in the black? Why?

17 By considering the integer number line, write down the numbers which are:

a 2 units from 0 **b** 3 units from 1
c 7 units from 2 **d** 6 units from -4
e less than 3 units from -12 **f** less than or equal to 4 units from -38

TRY THIS **Temperature**

What is the temperature approximately at this moment:

1 outside **2** inside **3** at the South Pole
4 in London **5** in Singapore **6** in a refrigerator
7 in the sea **8** in a swimming pool?

7.3 Practical applications of directed numbers

Many everyday situations involve *change*. For example, temperature, elevators, journeys and bank balances involve both a change in size or magnitude and a change in direction.

Example 1

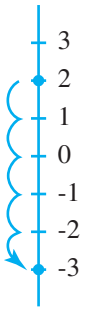
Find the new temperature if initially it was:

a 2°C and then it fell by 5°C

b -3°C and then it rose by 4°C

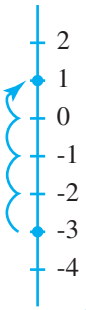
Solutions

a Begin at 2°C and count downward by 5°C



∴ 2°C – 5°C = -3°C.

b Begin at -3°C and count upward by 4°C



∴ -3°C + 4°C = 1°C.

Example 2

a At the casino, Todd won $30 then lost $50. How much better or worse off is Todd?

b A jockey lost 3 kg in weight during December, but gained 7 kg in January. How has his weight changed overall during these two months?

Solutions

a The problem can be thought of as going up 30 places on a number line, then down 50 places. The finishing position would be 20 places below 0. Therefore, Todd has lost $20 overall.

b The problem can be thought of as going down 3 places on a number line, then going up 7 places. The finishing position would be 4 places above 0. Therefore, the jockey has gained 4 kg overall.

Example 3

A train travelled 15 km due west, then 9 km due east.

a Where was the train then in relation to its original position?

b Write a directed number sentence to represent this situation.

Solutions

a After travelling 15 km due west and 9 km due east, the train is then 6 km due west of its original position.

b -15 + 9 = -6

Exercise 7.3

1 a Find the new temperature if initially it was:

- **i** 3°C and then fell by 4°C
- **ii** 2°C and then fell by 6°C
- **iii** 4°C and then fell by 7°C
- **iv** 1°C and then fell by 8°C

b Find the new temperature if initially it was:

- **i** -2°C and then rose by 5°C
- **ii** -4°C and then rose by 6°C
- **iii** -5°C and then rose by 9°C
- **iv** -7°C and then rose by 12°C

c Find the new temperature if initially it was:

- **i** -3°C and then fell by 1°C
- **ii** -5°C and then fell by 3°C
- **iii** -2°C and then fell by 8°C
- **iv** -6°C and then fell by 5°C

d Find the new temperature if initially it was:

- **i** -4°C and then rose by 2°C
- **ii** -6°C and then rose by 5°C
- **iii** -8°C and then rose by 3°C
- **iv** -13°C and then rose by 7°C

Consolidation

2 Find the new temperature if initially it was:

- **a** -3°C and then rose by 4°C
- **b** 4°C and then fell by 6°C
- **c** -1°C and then fell by 7°C
- **d** -9°C and then rose by 5°C
- **e** -7°C and then rose by 2°C
- **f** -8°C and then fell by 3°C
- **g** 10°C and then fell by 12°C
- **h** -2°C and then rose by 2°C

3 In each of the following:

- **i** answer the problem in words
- **ii** write a directed number sentence to represent the situation.

a Mary stepped into the elevator on the second floor below ground level. The elevator went down seven floors and then up three floors. Where did she get out?

b Jeff deposited $10 into his bank account on Monday. He withdrew $12 on Tuesday and then withdrew another $4 on Wednesday. What is the change in his account?

c Linh lost 3 kg in weight during June, gained 5 kg in July and then lost 7 kg in August. How has Linh's weight changed overall since the beginning of June?
d A man drove 15 km due east, 20 km due west and then 4 km due east. Where is he then in relation to his original position?
e A floating buoy rose 2 m with the tide, fell 3 m and then rose 5 m. Where is the buoy then in relation to its original position?
f At the casino, Julianne lost $20 playing blackjack, lost $30 on the roulette wheel and then won $15 playing two-up. How much better or worse off is she?
g A submarine submerged from the surface and went down 10 m. It then went down another 16 m and later rose 12 m. Where is the submarine then in relation to the surface?
h On Friday, the price of petrol rose by 4c per litre. On Saturday it fell by 5c per litre and on Sunday it fell by 8c per litre. How has the price of petrol changed overall since Friday morning?

4 a Paola entered an elevator four floors above ground level and exited two floors below ground level. Write a directed number to represent the movement of the elevator.
b Kwan boarded a train 12 km west of Central station and got off the train 5 km east of Central. Write a directed number to represent the train journey.
c Shona's first test result was 9 marks below the class average and her second test result was 16 marks below the average. Write a directed number to represent the change in her test results.
d Giacomo's temperature was 4 degrees below normal and when measured again two days later, it was back to normal. Write a directed number to describe the change in Giacomo's temperature.

Further applications

5 Write a directed number to represent the original position of a ship if after it has sailed:
a 13 nautical miles due east, the ship is then 5 nautical miles due east of a lighthouse
b 7 nautical miles due west, the ship is then 10 nautical miles due west of a lighthouse
c 6 nautical miles due north, the ship is then 4 nautical miles due south of a lighthouse
d 15 nautical miles due south, the ship is then 6 nautical miles due north of a lighthouse

7.4 Addition and subtraction involving directed numbers

Additions and subtractions that involve directed numbers are best done by either using or visualising a number line. If the number line is vertical, count upward for addition and downward for subtraction. If the number line is horizontal, count to the right for addition and to the left for subtraction. In this exercise we will look at the following four basic types of additions and subtractions that can be performed with negative numbers:

- starting on the negative side and going further away from zero. This is a subtraction.
- starting on the positive side and going through zero to the negative side. This is a subtraction.

- starting on the negative side and going toward zero but not reaching it. This is an addition.
- starting on the negative side and going through zero to the positive side. This is an addition.

To evaluate additions and subtractions that involve directed numbers, count:
- **up or down on a vertical number line, or**
- **left or right on a horizontal number line.**

NOTE: In statements such as -3 – 2, the - is part of the number **negative three**. The second – is an operation. It means **subtract two**.

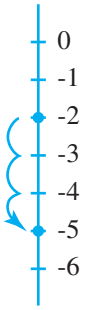
Example 1

Use a vertical number line to evaluate:

a -2 – 3 **b** -6 + 4

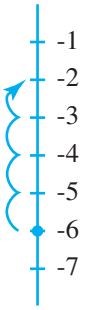
Solutions

a Start at -2 and count down 3 units.



∴ -2 – 3 = -5

b Start at -6 and count up 4 units.



∴ -6 + 4 = -2

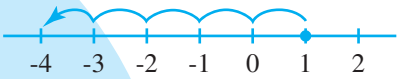
Example 2

Use a horizontal number line to evaluate:

a 1 – 5 **b** -3 + 7

Solutions

a Start at 1 and count 5 units to the left.



∴ 1 – 5 = -4

b Start at -3 and count 7 units to the right.

∴ -3 + 7 = 4

Example 3

Evaluate -2 + 5 – 7

Solution

When there are more than two terms involving addition or subtraction only, the order of operations is to work from left to right.

-2 + 5 – 7
= 3 – 7
= -4

Exercise 7.4

1 Evaluate:

a -1 – 3	**b** -4 – 2	**c** -6 – 1	**d** -2 – 5	**e** -3 – 3
f -2 – 8	**g** -6 – 4	**h** -5 – 3	**i** -2 – 2	**j** -4 – 9
k -8 – 3	**l** -10 – 5	**m** -6 – 7	**n** -9 – 6	**o** -4 – 4
p -11 – 5	**q** -7 – 9	**r** -5 – 8	**s** -4 – 12	**t** -13 – 7

2 Evaluate:

a -5 + 2	**b** -3 + 1	**c** -7 + 3	**d** -8 + 2	**e** -6 + 3
f -4 + 2	**g** -5 + 4	**h** -10 + 6	**i** -6 + 4	**j** -9 + 2
k -11 + 3	**l** -12 + 5	**m** -10 + 1	**n** -13 + 8	**o** -16 + 5
p -11 + 9	**q** -18 + 6	**r** -12 + 7	**s** -19 + 10	**t** -17 + 8

3 Evaluate:

a 2 – 3	**b** 4 – 9	**c** 1 – 4	**d** 5 – 7	**e** 3 – 8
f 2 – 4	**g** 5 – 8	**h** 6 – 10	**i** 4 – 9	**j** 5 – 6
k 1 – 7	**l** 4 – 8	**m** 1 – 10	**n** 6 – 14	**o** 2 – 13
p 0 – 3	**q** 3 – 14	**r** 7 – 16	**s** 10 – 20	**t** 11 – 17

4 Evaluate:

a -1 + 3	**b** -4 + 5	**c** -3 + 6	**d** -1 + 2	**e** -5 + 9
f -2 + 7	**g** -4 + 8	**h** -3 + 10	**i** -2 + 11	**j** -9 + 12
k -4 + 11	**l** -2 + 6	**m** -9 + 10	**n** -8 + 8	**o** -4 + 13
p -8 + 15	**q** -7 + 10	**r** -9 + 17	**s** -16 + 20	**t** -9 + 15

■ Consolidation

5 Evaluate each of the following.

a 1 – 6	**b** -6 + 5	**c** -4 – 6	**d** -3 + 4	**e** -7 + 2
f 2 – 5	**g** -1 + 7	**h** -1 – 2	**i** -3 + 8	**j** -10 + 5
k -9 – 4	**l** 3 – 9	**m** 8 – 12	**n** -7 – 7	**o** -9 + 3
p -2 + 14	**q** -13 + 17	**r** 0 – 4	**s** -1 – 12	**t** -13 + 5

6 Write the number that is:

a 2 less than -7 **b** 8 more than -5
c 12 less than 3 **d** 6 more than -11

7 **a** Increase -15 by 8 **b** Decrease 4 by 16
c Decrease -3 by 11 **d** Increase -4 by 10

8 **a** Take 5 from 1 **b** Add 8 to -2
c Take 7 from -4 **d** Add 9 to -13

9 Find the sum of -6 and 13.

10 Evaluate each of these.

a $1 - 4 + 2$	**b** $4 - 7 - 3$	**c** $-3 + 2 + 6$	**d** $-6 - 4 + 3$
e $-2 + 2 + 4$	**f** $2 - 8 + 3$	**g** $2 - 6 - 1$	**h** $-7 - 2 + 4$
i $-7 + 2 + 9$	**j** $3 - 7 + 1$	**k** $5 - 9 - 7$	**l** $-6 - 6 + 5$
m $0 - 3 + 8$	**n** $3 - 5 - 10$	**o** $5 - 10 + 4$	**p** $-6 + 8 - 11$

■ Further applications

CALCULATOR

11 Evaluate each of the following, then check your answers using a calculator.

a $10 - 30$	**b** $-20 + 50$	**c** $-60 - 40$	**d** $-80 + 70$
e $-35 + 40$	**f** $-25 - 35$	**g** $-90 + 15$	**h** $20 - 65$
i $23 - 32$	**j** $-76 + 45$	**k** $-48 - 53$	**l** $-36 + 78$
m $-89 + 37$	**n** $-103 - 142$	**o** $-16 + 158$	**p** $56 - 260$

7.5 Change involving addition and subtraction

In some questions we are interested in finding the amount of change that has taken place and the direction of that change, rather than the value of an expression.

In directed number sentences, treat:
- **the first term as the *starting position* on a vertical number line**
- **the middle term as a *journey* up or down**
- **the last term as the *finishing position* on a vertical number line.**

EG+S

Example 1

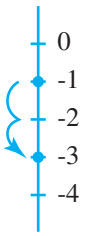
Complete each of the following number sentences.

a $-1 \square = -3$ **b** $-5 \square = -2$

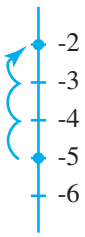
Solutions

In each of these questions we are asked to find the journey up or down given the starting or original position and the finishing position.

a To move from -1 to -3 we must count down 2 units. That is, $-1 \boxed{-2} = -3$.



b To move from -5 to -2 we must count up 3 units. That is, $-5 \boxed{+3} = -2$.



NOTE: The middle term in each of these questions is a mathematical operation (an addition or a subtraction). It is not a directed number. So, in part **a**, $-1 - 2 = -3$ means '*negative* one *minus* two equals *negative* three'.

Example 2

Complete each of the following number sentences.

a $\square + 3 = -1$ **b** $\square - 4 = -2$

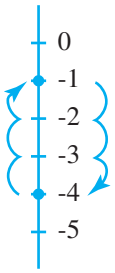
Solutions

In each of these questions we are asked to find the starting or original position given the journey up or down and the finishing position.

a Having counted up 3 units and finished on -1, we need to count *down* 3 units to find the original number.

$\square = -1 - 3$
$= -4$

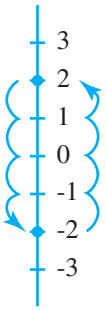
That is, $\boxed{-4} + 3 = -1$.



b Having counted down 4 units and finished on -2, we need to count *up* 4 units to find the original number.

$\square = -2 + 4$
$= 2$

That is, $\boxed{2} - 4 = -2$.



Exercise 7.5

1 Copy and complete each of the following.
a $-2 \square = -6$ **b** $-3 \square = -4$ **c** $-5 \square = -7$ **d** $-4 \square = -9$

2 Copy and complete each of the following.
a $-5 \square = -4$ **b** $-3 \square = -1$ **c** $-6 \square = -3$ **d** $-8 \square = -2$

3 Copy and complete each of the following.
a $3 \square = -1$ **b** $1 \square = -2$ **c** $2 \square = -5$ **d** $4 \square = -2$

4 Copy and complete each of the following.
a $-1 \square = 2$ **b** $-3 \square = 1$ **c** $-6 \square = 4$ **d** $-5 \square = 2$

Consolidation

5 Copy and complete each of the following.

a $-2 \square = -3$	**b** $-10 \square = -5$	**c** $1 \square = -1$	**d** $-3 \square = 5$
e $-9 \square = 0$	**f** $-6 \square = -10$	**g** $-7 \square = -2$	**h** $3 \square = -7$
i $-7 \square = 3$	**j** $-7 \square = -12$	**k** $7 \square = -4$	**l** $-15 \square = -9$
m $-10 \square = -15$	**n** $9 \square = -1$	**o** $-11 \square = 3$	**p** $-8 \square = -3$
q $8 \square = -9$	**r** $-12 \square = 10$	**s** $-13 \square = -4$	**t** $-4 \square = -15$

6 What must be:
a added to -15 to give -4? **b** subtracted from 3 to give -10?
c added to -6 to give 12? **d** subtracted from -5 to give -14?

7 **a** By how much must -1 be increased to give 7?
b By how much should -4 be decreased to give -18?

8 Copy and complete each of the following.
a $\square - 3 = -5$ **b** $\square - 1 = -7$ **c** $\square - 4 = -8$ **d** $\square - 2 = -6$

9 Copy and complete each of the following.
a $\square + 2 = -4$ **b** $\square + 3 = -2$ **c** $\square + 1 = -8$ **d** $\square + 5 = -3$

10 Copy and complete each of the following.
a $\square - 3 = -1$ **b** $\square - 2 = -1$ **c** $\square - 5 = -3$ **d** $\square - 4 = -2$

11 Copy and complete each of the following.
a $\square + 2 = 1$ **b** $\square + 4 = 2$ **c** $\square + 6 = 1$ **d** $\square + 9 = 6$

12 Copy and complete each of the following.
a $\square + 7 = 2$ **b** $\square - 4 = -1$ **c** $\square + 1 = -7$ **d** $\square - 3 = -6$
e $\square + 5 = 3$ **f** $\square + 2 = -11$ **g** $\square - 6 = -2$ **h** $\square - 11 = -12$
i $\square - 8 = -1$ **j** $\square + 10 = 4$ **k** $\square - 12 = -4$ **l** $\square - 9 = -11$
m $\square + 9 = -2$ **n** $\square - 5 = -10$ **o** $\square - 10 = -4$ **p** $\square + 6 = 0$
q $\square - 8 = -5$ **r** $\square + 10 = 6$ **s** $\square + 8 = -6$ **t** $\square - 8 = -15$

13 **a** When 4 is subtracted from a number, the result is -21. What is the number?
b When 7 is added to a number, the result is 5. What is the number?
c When 10 is subtracted from a number, the result is -30. What is the number?
d When 8 is added to a number, the result is -14. What is the number?

14 What number when:
a decreased by 7 gives -8? **b** increased by 2 gives -3?
c decreased by 6 gives -2? **d** increased by 9 gives 5?

Further applications

15 Copy and complete each of the following.
a $3 \square + 6 = 2$ **b** $-5 \square + 3 = -6$ **c** $-7 \square - 2 = -3$
d $-4 \square - 5 = -16$ **e** $\square - 6 - 5 = -7$ **f** $\square + 3 - 4 = -2$
g $\square - 4 + 1 = -5$ **h** $\square + 2 - 9 = -4$ **i** $\square - 3 + 5 = 2$

7.6 Adjacent signs

- **Two like adjacent signs can be replaced with a single addition sign.**

  That is, $\left.\begin{matrix} + & + \\ - & - \end{matrix}\right\}$ *means* $+$

- **Two different adjacent signs can be replaced with a single subtraction sign.**

  That is, $\left.\begin{matrix} + & - \\ - & + \end{matrix}\right\}$ *means* $-$

These results will be justified in Exercise 7.6.

Example 1

Evaluate:

a $6 - -4$ **b** $8 + -3$ **c** $4 + +2$ **d** $5 - +3$ **e** $-7 - -4$ **f** $-2 + -3$

Solutions

a $6 - -4 = 6 + 4$
$= 10$

b $8 + -3 = 8 - 3$
$= 5$

c $4 + +2 = 4 + 2$
$= 6$

d $5 - +3 = 5 - 3$
$= 2$

e $-7 - -4 = -7 + 4$
$= -3$

f $-2 + -3 = -2 - 3$
$= -5$

Example 2

Evaluate $3 - -2 + -8$

Solution

$3 - -2 + -8 = 3 + 2 - 8$
$= 5 - 8$
$= -3$

Exercise 7.6

1 **a** Is (+2) the same as 2?
b Is 7 + (+2) the same as 7 + 2?
c Is 7 – (+2) the same as 7 – 2?

2 Copy and complete the following number patterns.

a
$4 - 4 =$
$4 - 3 =$
$4 - 2 =$
$4 - 1 =$
$4 - 0 =$
$4 - (-1) =$
$4 - (-2) =$
$4 - (-3) =$
$4 - (-4) =$

b
$5 + 4 =$
$5 + 3 =$
$5 + 2 =$
$5 + 1 =$
$5 + 0 =$
$5 + (-1) =$
$5 + (-2) =$
$5 + (-3) =$
$5 + (-4) =$

3 Use your results from Q1 and Q2 to complete each of the following statements.
a We can replace two same adjacent signs (i.e. + + or – –) with a single ____ sign.
b We can replace two different adjacent signs (i.e. + – or – +) with a single ____ sign.

4 Evaluate:
a $5 - (-2)$ **b** $7 - (-4)$ **c** $2 - (-6)$ **d** $1 - (-3)$
e $4 - -6$ **f** $9 - -7$ **g** $6 - -8$ **h** $12 - -9$

5 Evaluate:

a 9 – (+4) **b** 12 + (-5) **c** 13 – (+8) **d** 10 + (-6)
e 13 + -6 **f** 17 – +8 **g** 22 – +7 **h** 25 + -9

Consolidation

6 Evaluate:

a 3 – -4 **b** 9 + -5 **c** 8 – +3 **d** 5 – -7
e 14 – +2 **f** 16 + -9 **g** 10 – +4 **h** 9 – -4
i 7 + (-7) **j** 13 – (+2) **k** 1 – (-10) **l** 15 – (+6)
m 8 + (-4) **n** 9 – (-8) **o** 6 – (-6) **p** 4 + (-4)

7 Evaluate:

a -4 – -3 **b** 5 + -10 **c** -2 – +4 **d** -6 – -9
e -10 – +6 **f** 4 + -8 **g** -12 – +5 **h** -14 – -9
i -3 + (-3) **j** -2 – (-2) **k** -6 – (+7) **l** 5 – (+8)
m -1 – (-7) **n** -14 – (-6) **o** -10 + (-9) **p** -12 – (+10)

8 Find the:

a sum of 3 and -8 **b** difference between 4 and -9
c difference between -2 and -16 **d** sum of -5 and -12

9 Evaluate each of these.

a 5 – -2 + -1 **b** 4 + -5 – -3 **c** -3 + -3 – -3
d -2 – (-6) + (-10) **e** 8 – (+12) – (+7) **f** 1 – (-6) + (-15)

10 **a** By how much does 3 exceed -4? **b** How much less than 6 is -8?
c How much more than -1 is 4? **d** How much less than -7 is -16?

CALCULATOR

11 Evaluate each of the following, then check your answers using a calculator.

a 40 – -70 **b** 50 – +80 **c** 10 + -60 **d** 20 – -40
e -30 + -20 **f** -90 – -60 **g** -40 – +70 **h** -10 – -80
i -15 – (+40) **j** -32 – (-18) **k** 17 + (-50) **l** -24 – (-65)

Further applications

12 Find the missing term in each number sentence.

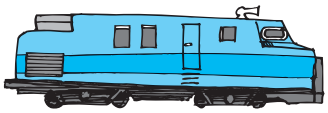
a 3 – □ = 5 **b** 7 + □ = 3 **c** 1 – □ = 8
d 9 + □ = 6 **e** 5 + □ = 4 **f** 0 – □ = 5
g -2 – □ = 4 **h** -9 – □ = -4 **i** -6 + □ = -8
j -12 – □ = -10 **k** -8 – □ = 3 **l** -15 + □ = -23

TRY THIS **Multiplication with directed numbers**

Consider the problem 3×20. This could mean:

Where will a train be in 3 hours if it is travelling forward at 20 km/h?

Do this on a number line with the train starting at zero.



What would -3×20 mean? Clearly:

Where was the train 3 hours ago if it was travelling forward at 20 km/h?

Show this on a number line.

Write a story to explain:

a 3×-20 **b** -3×-20

Show your answers on a number line.

7.7 Multiplication of directed numbers

- **The product of two numbers with like signs is positive. That is,**

$$\left.\begin{array}{l}(+) \times (+) = (+)\\(-) \times (-) = (+)\end{array}\right\}$$

- **The product of two numbers with different signs is negative. That is,**

$$\left.\begin{array}{l}(+) \times (-) = (-)\\(-) \times (+) = (-)\end{array}\right\}$$

In general, when multiplying directed numbers:

- **the product will be negative if there are an odd number of negative numbers**
- **the product will be positive if there are an even number of negative numbers.**

EG+S

These results will be justified in Exercise 7.7.

Example 1

Evaluate:

a -4×-5 **b** -7×2 **c** 3×-6

Solutions

a $-4 \times -5 = 20$ (The product of two negative numbers is positive.)
b $-7 \times 2 = -14$ (The product of a negative number and a positive number is negative.)
c $3 \times -6 = -18$ (The product of a positive number and a negative number is negative.)

Example 2

Evaluate each of these.

a $-4 \times 2 \times -3$ **b** $-5 \times -2 \times -6$

Solutions

a $-4 \times 2 \times -3 = 24$ (The answer is positive because there are an even number of negative numbers.)
b $-5 \times -2 \times -6 = -60$ (The answer is negative because there are an odd number of negative numbers.)

Example 3

Evaluate each of the following:

a $(-7)^2$ **b** $(-2)^3$

Solutions

a $(-7)^2 = -7 \times -7$
$= 49$

b $(-2)^3 = -2 \times -2 \times -2$
$= -8$

Exercise 7.7

1 Copy and complete each of these number patterns.

a
$5 \times 4 =$
$5 \times 3 =$
$5 \times 2 =$
$5 \times 1 =$
$5 \times 0 =$
$5 \times (-1) =$
$5 \times (-2) =$
$5 \times (-3) =$
$5 \times (-4) =$

b
$-3 \times 4 =$
$-3 \times 3 =$
$-3 \times 2 =$
$-3 \times 1 =$
$-3 \times 0 =$
$-3 \times (-1) =$
$-3 \times (-2) =$
$-3 \times (-3) =$
$-3 \times (-4) =$

2 Use your results from Q1 to complete each of the following statements.
a The product of two numbers with like signs is _____.
b The product of two numbers with different signs is _____.

3 Evaluate each of these.

a -4×3	**b** -5×-2	**c** 6×-4	**d** -3×-7	**e** 8×-5
f -6×-6	**g** -12×3	**h** 4×-8	**i** -10×-5	**j** 11×-7
k -6×9	**l** $8 \times (-7)$	**m** $-13 \times (-2)$	**n** $-9 \times (-8)$	**o** $0 \times (-6)$
p $5 \times (-9)$	**q** $-10 \times (-7)$	**r** $-5 \times (-12)$	**s** -7×6	**t** $12 \times (-11)$

Consolidation

4 Evaluate each of the following.

a $5 \times 2 \times -3$ **b** $-4 \times 5 \times -2$ **c** $-6 \times -4 \times -2$ **d** $-1 \times 8 \times -7$
e $-7 \times -6 \times -2$ **f** $5 \times -4 \times 6$ **g** $-4 \times 12 \times 0$ **h** $-9 \times -2 \times 3$
i $3 \times 2 \times (-6)$ **j** $7 \times (-2) \times (-4)$ **k** $5 \times (-3) \times (-3)$ **l** $(-8) \times (-5) \times (-6)$

5 Find:

a double -5 **b** triple -7 **c** quadruple -3
d 3 lots of -8 **e** 5 lots of -12 **f** 9 lots of -6

6 **a** Determine the product of -4 and 7 **b** Determine the product of -9, -2 and -4

7 Evaluate:

a $(-3)^2$ **b** $(-5)^2$ **c** $(-6)^2$ **d** $(-9)^2$ **e** $(-2)^3$ **f** $(-3)^3$
g $(-10)^3$ **h** $(-5)^3$ **i** $(-2)^4$ **j** $(-3)^4$ **k** $(-2)^5$ **l** $(-10)^5$

8 Copy and complete each of these.

a $6 \times \square = -18$ **b** $-5 \times \square = 35$ **c** $\square \times -7 = -21$ **d** $\square \times -4 = -36$
e $\square \times 5 = -30$ **f** $\square \times -3 = 24$ **g** $-9 \times \square = 0$ **h** $\square \times -10 = 20$
i $-8 \times \square = -72$ **j** $\square \times -6 = 36$ **k** $\square \times 12 = -84$ **l** $-11 \times \square = 110$
m $\square \times 13 = -39$ **n** $-9 \times \square = 45$ **o** $-8 \times \square = 96$ **p** $\square \times -9 = -63$

9 The product of two numbers is -70. If one of the numbers is 7, what is the other number?

10 By what must 5 be multiplied to give -55?

11 What number must be multiplied by -8 to give 32?

12 Find the missing number in each of these.

a $-9 \times 4 = 12 \times \square$ **b** $5 \times -8 = \square \times -10$ **c** $\square \times -5 = -10 \times -6$
d $-8 \times \square = -12 \times 4$ **e** $-9 \times 8 = 12 \times \square$ **f** $-7 \times -6 = \square \times -14$
g $\square \times -6 = 18 \times 3$ **h** $10 \times \square = -20 \times 5$ **i** $25 \times \square = -15 \times -5$

CALCULATOR

13 Evaluate each of the following, then check your answers using a calculator.

a $-30 \times (-20)$ **b** $40 \times (-13)$ **c** -15×6 **d** $-25 \times (-12)$
e -20×45 **f** $-6 \times (-35)$ **g** $19 \times (-7)$ **h** $-99 \times (-9)$

Further applications

14 Find two possible values for the missing number in each of these.

a $\square^2 = 16$ **b** $\square^2 = 25$ **c** $\square^2 = 100$

15 If $3 \otimes 7$ means $(3 \times 7) - 2$, evaluate:

a $-4 \otimes 3$ **b** $5 \otimes -6$ **c** $7 \otimes -8$ **d** $-12 \otimes -5$

7.8 Division of directed numbers

The rules for determining whether a quotient is positive or negative are the same as those for multiplication.

- **The quotient of two numbers with like signs is positive. That is,**

$$\left.\begin{array}{l}(+) \div (+) = (+)\\(-) \div (-) = (+)\end{array}\right\}$$

- **The quotient of two numbers with different signs is negative. That is,**

$$\left.\begin{array}{l}(+) \div (-) = (-)\\(-) \div (+) = (-)\end{array}\right\}$$

In general, when dividing directed numbers:
- **the quotient will be negative if there are an odd number of negative numbers**
- **the quotient will be positive if there are an even number of negative numbers.**

These results will be justified in Exercise 7.8.

Example 1

Evaluate:

a $-24 \div -6$

b $21 \div -7$

c $-15 \div 3$

Solutions

a $-24 \div -6 = 4$ (The quotient of two negative numbers is positive.)

b $21 \div -7 = -3$ (The quotient of a positive number and a negative number is negative.)

c $-15 \div 3 = -5$ (The quotient of a negative number and a positive number is negative.)

Example 2

Evaluate:

a $\frac{-30}{-5}$ **b** $\frac{-45}{9}$ **c** $\frac{14}{-7}$

Solutions

a $\frac{-30}{-5} = -30 \div -5$
$= 6$

b $\frac{-45}{9} = -45 \div 9$
$= -5$

c $\frac{14}{-7} = 14 \div -7$
$= -2$

Example 3

Evaluate $-8 \times -5 \div -10$

Solution

When there are more than two terms involving multiplication or division the order of operations is to work from left right.

$-8 \times -5 \div -10 = 40 \div -10$
$= -4$

Exercise 7.8

1 Copy and complete each of the following.

a If $+5 \times (+2) = +10$, then $+10 \div (+2) =$ ____.
b If $+5 \times (-2) = -10$, then $-10 \div (-2) =$ ____.
c If $-5 \times (-2) = +10$, then $+10 \div (-2) =$ ____.
d If $-5 \times (+2) = -10$, then $-10 \div (+2) =$ ____.

2 Use your results from Q1 to complete each of the following statements.

a The quotient of two numbers with like signs is _____.
b The quotient of two numbers with different signs is _____.

3 Evaluate each of these.

a $12 \div -6$	**b** $-18 \div -3$	**c** $-24 \div 8$	**d** $-32 \div -4$
e $49 \div -7$	**f** $-48 \div 4$	**g** $-50 \div -5$	**h** $-27 \div 3$
i $-54 \div 9$	**j** $-21 \div -7$	**k** $64 \div (-8)$	**l** $-63 \div (-9)$
m $70 \div (-7)$	**n** $-25 \div 5$	**o** $-60 \div (-3)$	**p** $-40 \div (-8)$
q $-48 \div (-12)$	**r** $42 \div (-7)$	**s** $110 \div (-10)$	**t** $-96 \div (-8)$

4 Evaluate:

a $\frac{-9}{3}$	**b** $\frac{-33}{-3}$	**c** $\frac{-8}{4}$	**d** $\frac{-28}{7}$
e $\frac{-40}{-4}$	**f** $\frac{-56}{8}$	**g** $\frac{55}{-11}$	**h** $\frac{-90}{-10}$

■ Consolidation

5 Find the value of each expression.

a $60 \div -2 \div -3$	**b** $-54 \div 6 \div 3$	**c** $100 \div -10 \div 5$
d $-72 \div -6 \div -3$	**e** $-90 \div 9 \div -2$	**f** $-80 \div -4 \div -2$

6 Evaluate each of the following.

a $-4 \times 6 \div -3$	**b** $-6 \times -5 \div -3$	**c** $56 \div -7 \times 2$
d $-60 \div (-10) \times (-7)$	**e** $-24 \div 8 \times 0$	**f** $12 \times (-5) \div (-20)$

7 Copy and complete each of the following.

a $35 \div \square = -5$	**b** $-30 \div \square = -6$	**c** $-42 \div \square = 7$	**d** $\square \div 5 = -3$
e $\square \div -3 = -10$	**f** $\square \div -6 = 11$	**g** $80 \div \square = -10$	**h** $\square \div -4 = -9$
i $-84 \div \square = 12$	**j** $-44 \div \square = -4$	**k** $\square \div 9 = -6$	**l** $\square \div -9 = -9$
m $\square \div -12 = 10$	**n** $-49 \div \square = -7$	**o** $108 \div \square = -12$	**p** $-110 \div \square = -10$

8 **a** Find the quotient of -22 and -2 **b** Find the quotient of 32 and -4

9 By what must -24 be divided to give an answer of 8?

10 When a number is divided by -6 the result is -5. What is the number?

11 Evaluate:

a $\dfrac{5 \times -3 \times 2}{-6}$ **b** $\dfrac{-4 \times 3 \times -5}{-12}$ **c** $\dfrac{-10 \times -3 \times -2}{-15}$

CALCULATOR

12 Evaluate each of the following, then check your answers using a calculator.

a $85 \div (-5)$ **b** $-171 \div (-3)$ **c** $-112 \div 4$ **d** $-133 \div (-7)$
e $-144 \div (-6)$ **f** $-207 \div 9$ **g** $280 \div (-8)$ **h** $-336 \div (-12)$

Further applications

13 The quotient of two integers is -4. If one of the numbers is 8, what are the possible values of the other number?

14 The average of three numbers is 2. If two of the numbers are -5 and -3, what is the third number?

15 If 6 Ω 2 means $(6 \div 2) - 10$, evaluate:

a 12 Ω -3 **b** -24 Ω 8 **c** -42 Ω -7 **d** 5 Ω -5

7.9 The order of operations

The **order of operations** applies when there are more than two operations involved in a question.

The order of operations states that:
- **expressions inside grouping symbols or with indices are worked first, then**
- **multiplications and divisions are worked from left to right, then**
- **additions and subtractions are worked from left to right.**

EG+S

Example

Evaluate each of the following.

a $-30 \div -6 - 14 \div 2$ **b** $-6 - 4 \times 2 + 10$

Solutions

a
$$\begin{aligned} -30 \div -6 - 14 \div 2 &= (-30 \div -6) - (14 \div 2) \\ &= 5 - 7 \\ &= -2 \end{aligned}$$

b
$$\begin{aligned} -6 - 4 \times 2 + 10 &= -6 - (4 \times 2) + 10 \\ &= -6 - 8 + 10 \\ &= -14 + 10 \\ &= -4 \end{aligned}$$

Exercise 7.9

1 Use the order of operations to evaluate each of these.

a $5 - 4 \times 2$ **b** $2 - 24 \div 3$ **c** $-4 \times 3 + 5$ **d** $-20 \div -5 - 7$
e $-11 + 5 \times 3$ **f** $-2 + 36 \div 9$ **g** $-6 \times 8 + 50$ **h** $-8 + 21 \div 3$
i $-3 - 28 \div 7$ **j** $-49 \div 7 + 13$ **k** $3 \times 5 - 16$ **l** $4 - 60 \div 5$
m $-10 - 18 \div 3$ **n** $10 - 6 \times 5$ **o** $20 - 7 \times 4$ **p** $-15 - 42 \div 6$

Consolidation

2 Insert grouping symbols and evaluate each of the following.

a $5 \times 2 - 4 \times 3$ **b** $20 \div 5 - 2 \times 6$ **c** $-6 \times 3 + 5 \times 5$
d $-21 \div 3 + 8 \div 2$ **e** $-8 \times 4 - 4 \times 2$ **f** $-5 - 7 \times 2 - 1$
g $2 + 7 \times 3 - 30$ **h** $-12 - 15 \div 5 + 10$ **i** $4 - 3 \times 2 + 5 \times 3$
j $30 \div 3 - 13 + 4 \times 5$ **k** $9 \div 3 - 15 + 2 \times 6$ **l** $-22 \div 11 + 8 - 6 \times 6$

Further applications

3 Use the order of operations and the rules for adjacent signs to evaluate each of these.

a $-7 \times 3 - 2 \times -5$ **b** $2 \times 4 + 6 \times -3$ **c** $-1 - 6 \times -4 - 8$
d $-6 - 54 \div -6 - 7$ **e** $-35 \div 5 - 72 \div -8$ **f** $26 \div -2 + 5 \times -2$
g $-10 - 2 \div -2 - 4$ **h** $15 + 7 \times -2 - 5 \times -4$ **i** $20 \div -4 - 40 \div -8$

Problem solving

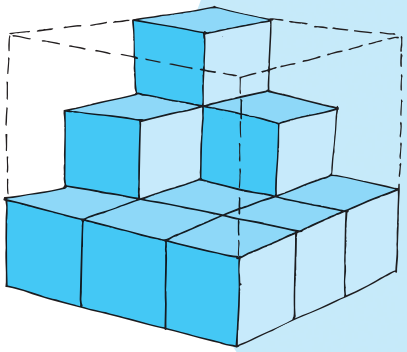
1 A boy had twenty rabbits. All except five escaped from their cage and disappeared. How many does he have now?

2 Complete the magic square so that the rows, columns and diagonals all add to the same total.

		14
7		17
		5

3 Twenty-seven small cubes are put together to form one large cube. The large cube is then painted on all sides. How many small cubes have:

a no sides painted
b one side painted
c two sides painted
d three sides painted
e four sides painted?

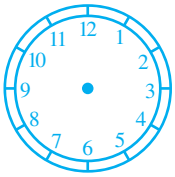


4 Divide 50 by one half, then subtract 50.

5 John and Betty can ride as fast as each other on a bicycle. One day, John rode from the one kilometre post to the ten kilometre post while Betty timed him. Betty then rode from the ten kilometre post to the twenty kilometre post while John timed her. John's time was faster. Why?

6 **a** Twice in twenty-four hours the hands of the clock divide the face into two parts so that the sum of the numbers in each part is the same. When does this happen?

b Draw two straight lines on the face of a clock so that the sum of the numbers in each third is the same.



7 The sportsmaster explained to the headmaster that 3 baseballs and 1 cricket ball weighed the same as 14 tennis balls. He also found that 1 baseball and 6 tennis balls exactly balanced 1 cricket ball. 'That is interesting,' said the headmaster, 'and how many tennis balls weigh the same as 1 cricket ball?' he asked. What was the sportsmaster's answer?

8 At a bus stop 20 students were standing in four groups. The first group contained four students more than the second. The second group contained one student less than the third. The fourth group c ontained twice as many students as the second group. How many students were in each group?

9 In a test, the average score of five students was 7. If Charlotte scored 6, Clare 9, Su-yin 6, and Barbara 5, what was Jenny's score?

10 On the planet Lunar, the Lunies write their numbers in strange ways. For example:

□ □ ▲ = 7
○ ○ ▲ ▲ = 20
○ ○ ○ □ = 30

a What would the number ○ □ ▲ represent?

b How would you write the number 49?

Focus on working mathematically

The loss of the Russian nuclear submarine *Kursk*

Introduction

On 12 August 2000, the Russian nuclear submarine *Kursk* exploded and sank in 107 metres of water in the Barents Sea in the Arctic. Sadly, all 118 sailors aboard perished. After more than a year it was successfully salvaged. The seriously damaged bow, where the explosion occurred, was cut away and left on the seabed. The rest of the sub, including the nuclear reactors that powered it, was lifted to the surface. It was then towed into a Russian naval shipyard.
On 28 October 2001, TV news bulletins showed the devastating effects of the fire inside.

Fortunately the sections containing the nuclear reactors that power the vessel were well protected, averting a potential environmental disaster.

This submarine is a huge vessel, 154 m in length. It took an enormous crane to lift the *Kursk* to the surface.

LEARNING ACTIVITIES

1. A Sydney bus is about 12 m in length. A Boeing 747 Jumbo jet is about 70 m in length. A football field is 100 m in length. The *Kursk* is 154 m in length. Write a mathematical description to show just how big this sub really is.
2. Draw a diagram to show the sub on the seabed. Take sea level as zero. Write a directed number to show the position of the sub on the seabed.
3. If the sub was at a depth of 48 m when the explosion occurred, and it then sank to the bottom (107 m), write in words a possible meaning for the number sentence

$$-48 - (-107) = 59$$

4. Inside the stern of the stricken sub the temperature of the air is 11°C. If the temperature has been falling at a rate of 0.4°C per hour, what was the temperature 3 hours ago? What will the temperature be in 24 hours' time?

EXTENSION ACTIVITIES

1. Using a scale of 10 m = 10 mm, draw a diagram to compare the length of a bus (12 m), a jumbo jet (70 m), a football field (100 m) and the submarine.
2. Would the submarine fit into the Sydney Olympic Stadium? See if you can find out.
3. The height of the Sydney Tower is about 325 m. Imagine the Sydney Tower standing on the seabed of the Barents Sea next to the submarine. Draw a picture of the Tower standing on the seabed to show the depth of the water in which the submarine sank.

Let's communicate

Write a letter to a friend telling about the size of nuclear submarines in comparison to a Sydney bus, a jumbo jet and a football field. Say why negative numbers are used to indicate the depth at which subs travel.

Reflecting

Submarines use special technology to bounce sound waves off the seabed to show a moving profile of the seabed as they travel across it. Fishing vessels use similar technology to show the seabed as they travel across the surface. Reflect on how mathematics has been applied to meet this human need and its close link with science and technology.

Language link with Macquarie

MACQUARIE

1 *Explain what a* ***directed number*** *is in your own words.*
2 *Define the word* ***integer****.*
3 *What is a* ***number sentence****?*
4 *Explain what a* ***number line*** *is in your own words.*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***temperature****:*

temperature *noun* **1.** a measure of the degree of heat or cold of something or someone.
2. an abnormally high amount of heat in the body: *She is running a temperature.*

What does ***temperature*** *have to do with directed numbers?*

Chapter Review

1 Write a directed number to represent each of the following.
- **a** walking 6 km west
- **b** ascending 3 floors
- **c** withdrawing $20
- **d** losing 4 kg in weight
- **e** flying 75 km north
- **f** arriving 10 mins late

2 Write the directed number that is opposite in meaning to 5.

3 Graph the numbers {3, -2, 0, -4} on a number line.

4 Choose the smallest and largest numbers from the set {-6, -34, 0, -99, -1}.

5 Insert < or > to make a true statement.
- **a** 7 ☐ -3
- **b** -5 ☐ 0
- **c** -100 ☐ 2
- **d** -53 ☐ -54

6 Arrange these numbers in ascending order: -48, 17, -12, -13, 0, -81

7 Find the new temperature if it was:
- **a** 2°C and then it fell by 7°C

b -3°C and then it rose by 5°C
c -4°C and then it fell by 8°C
d -11°C and then it rose by 6°C

8 a A woman drove 14 km west, 20 km east then 9 km west. Where is she now in relation to her original position?
b A man stepped into an elevator on the third floor below ground level. The elevator went down two floors and then up four floors. Where is the elevator now in relation to ground level?
c The price of petrol rose by 5c per litre on Tuesday, fell by 8c on Thursday, then fell by another 3c on Saturday. How has the price of petrol changed overall since Tuesday morning?

9 Evaluate each of the following.

a	-7 + 3	**b**	-9 – 5
c	-8 + 6	**d**	2 – 9
e	-6 – 7	**f**	-10 + 9
g	1 – 4	**h**	-5 + 12
i	-15 + 40	**j**	-20 – 60
k	30 – 55	**l**	-50 + 45

10 a Increase -5 by 11
b Decrease -7 by 12
c Take 16 from 3
d Add 14 to -20

11 Evaluate:

a	-7 + 5 + 9	**b**	3 – 4 – 6
c	-12 – 9 + 5	**d**	-8 + 15 – 20

12 Copy and complete each of these.

a	-1 □ = -7	**b**	6 □ = -2
c	-9 □ = 3	**d**	-13 □ = -8
e	0 □ = -3	**f**	-4 □ = 4
g	-7 □ = -16	**h**	-11 □ = -1

13 Copy and complete each of these.

a	□ – 3 = -1	**b**	□ + 6 = 4
c	□ + 15 = -7	**d**	□ – 4 = -18
e	□ + 1 = -5	**f**	□ + 12 = 9
g	□ – 5 = -11	**h**	□ – 13 = -21

14 Copy and complete:
a -4 + 8 □ = -5
b 2 □ + 7 = 3
c □ – 7 + 4 = -13

15 Evaluate:

a	3 – -7	**b**	2 + -8
c	6 – +12	**d**	0 – -5
e	-4 + (-5)	**f**	-9 – (-6)
g	-5 – (+8)	**h**	-1 – (-17)

16 Evaluate 2 – -9 + -6

17 Copy and complete each of these.

a	2 – □ = 5	**b**	6 + □ = -2
c	-7 – □ = -1	**d**	-4 + □ = -8

18 Evaluate:

a	-5 × 3	**b**	-2 × -7
c	6 × -4	**d**	-3 × -9
e	7 × (-5)	**f**	8 × (-8)
g	(-11) × (-6)	**h**	(-12) × (-8)

19 Find the value of:

a	-3 × 2 × 4	**b**	-6 × 2 × -2
c	5 × -8 × -3	**d**	-4 × -3 × -12

20 Evaluate each of these.

a	$(-5)^2$	**b**	$(-3)^3$
c	$(-10)^4$	**d**	$(-2)^5$

21 Copy and complete:
a -7 × □ = 28
b □ × 6 = -54
c -10 × □ = 40
d □ × -8 = -96

22 Evaluate:

a	10 ÷ (-2)	**b**	-18 ÷ (-3)
c	-35 ÷ 7	**d**	-45 ÷ (-9)

e $\frac{-42}{6}$ **f** $\frac{48}{-12}$

g $\frac{-32}{-8}$ **h** $\frac{-121}{11}$

23 Copy and complete:

a $30 \div \square = -5$

b $\square \div (-3) = 7$

c $-24 \div \square = 6$

d $\square \div 9 = -12$

24 Evaluate $\frac{4 \times (-5) \times (-4)}{-10}$

25 Evaluate each of these using the order of operations.

a $10 - 6 \times 4$

b $54 \div 9 - 8 \times 2$

c $-7 - 25 \div 5 + 1$

8 Algebra

This chapter at a glance

Stage 4

After completing this chapter, you should be able to:

- substitute into algebraic expressions
- recognise like terms in algebraic expressions
- add and subtract like terms to simplify algebraic expressions
- simplify algebraic expressions that involve multiplication and division
- determine whether a simplified expression is correct using substitution
- recognise the role of grouping symbols and the different meanings of various expressions
- expand algebraic expressions using the Distributive Law
- translate between everyday language and algebraic language
- interpret statements involving algebraic symbols in spreadsheets.

8.1 Substitution

To **substitute** for a pronumeral is to give the pronumeral the value of a number. The same algebraic expression can have any number of values depending upon the value(s) that are substituted for each pronumeral.

Example 1

Evaluate each expression when $n = 2$.

a $n + 7$ **b** $5n$

c $4n - 1$ **d** $3n^2$

e $\frac{n + 10}{3}$ **f** $7(n + 4)$

Solutions

a
$$\begin{aligned} & n + 7 \\ &= 2 + 7 \\ &= 9 \end{aligned}$$

b
$$\begin{aligned} & 5n \\ &= 5 \times 2 \\ &= 10 \end{aligned}$$

c
$$\begin{aligned} & 4n - 1 \\ &= (4 \times 2) - 1 \\ &= 8 - 1 \\ &= 7 \end{aligned}$$

d
$$\begin{aligned} & 3n^2 \\ &= 3 \times 2^2 \\ &= 3 \times 4 \\ &= 12 \end{aligned}$$

e
$$\begin{aligned} & \frac{n + 10}{3} \\ &= \frac{2 + 10}{3} \\ &= \frac{12}{3} \\ &= 4 \end{aligned}$$

f
$$\begin{aligned} & 7(n + 4) \\ &= 7 \times (2 + 4) \\ &= 7 \times 6 \\ &= 42 \end{aligned}$$

Example 2

Evaluate each expression when $r = 3$ and $s = 5$.

a $7r - 2s$

b $s^2 + 4r$

c $\frac{8r + 6s}{s + 4}$

Solutions

a
$$\begin{aligned} & 7r - 2s \\ &= (7 \times 3) - (2 \times 5) \\ &= 21 - 10 \\ &= 11 \end{aligned}$$

b
$$\begin{aligned} & s^2 + 4r \\ &= 5^2 + (4 \times 3) \\ &= 25 + 12 \\ &= 37 \end{aligned}$$

c
$$\begin{aligned} & \frac{8r + 6s}{s + 4} \\ &= \frac{(8 \times 3) + (6 \times 5)}{5 + 4} \\ &= \frac{54}{9} \\ &= 6 \end{aligned}$$

Example 3

Evaluate each expression when $b = 4$, $c = 7$ and $d = 1$.

a $b - c - d$

b $3d - 2c$

c $b(d - c)$

Solutions

a
$$\begin{aligned} & b - c - d \\ &= 4 - 7 - 1 \\ &= -3 - 1 \\ &= -4 \end{aligned}$$

b
$$\begin{aligned} & 3d - 2c \\ &= (3 \times 1) - (2 \times 7) \\ &= 3 - 14 \\ &= -11 \end{aligned}$$

c
$$\begin{aligned} & b(d - c) \\ &= 4(1 - 7) \\ &= 4 \times -6 \\ &= -24 \end{aligned}$$

Exercise 8.1

1 Evaluate each of these when $a = 3$.

a $a + 4$	**b** $a - 1$	**c** $10 + a$	**d** $20 - a$	**e** $7a$
f $12a$	**g** $6a + 1$	**h** $9a - 3$	**i** $40 - 4a$	**j** $30 - 6a$
k a^2	**l** a^3	**m** $a^2 + 7$	**n** $5a^2$	**o** $2a^3$
p $50 - 3a^2$	**q** $\frac{9}{a}$	**r** $\frac{a}{6}$	**s** $\frac{a+7}{5}$	**t** $\frac{a+9}{3}$

Consolidation

2 Evaluate each of the following when $b = 4$ and $c = 6$.

a $b + c$	**b** $c - b$	**c** $20 - b + c$	**d** $3b + 2c$
e $5c + 2b$	**f** $10c - 5b$	**g** $2bc$	**h** $3bc + 1$
i $25 - bc$	**j** $50 - 2bc$	**k** $80 - 3bc$	**l** $110 - 4bc$
m $\frac{b+c}{5}$	**n** $\frac{c-b}{2}$	**o** $\frac{2b+c}{7}$	**p** $\frac{6b+4c}{bc}$
q $b^2 + c^2$	**r** $c^2 - 2b$	**s** $b^2 + 5c$	**t** $b^2c + bc^2$
u $5(b + c)$	**v** $b(c + 8)$	**w** $c(9 - b)$	**x** $8(c - b)$

3 Evaluate each of the following when $p = 3$, $q = 5$ and $r = 8$.

a $p + q + r$	**b** $r - q + p$	**c** pq	**d** qr
e $2pqr$	**f** $5p + 3q + 2r$	**g** $6r - 5p + q$	**h** $3pq + 2qr$
i $\frac{p+q}{r}$	**j** $\frac{r-2}{p}$	**k** $\frac{r-q}{p}$	**l** $\frac{4p+r}{q}$
m $p^2 + q^2 + r^2$	**n** $3q^2 + r^2$	**o** $r(q - p)$	**p** $9(r - p + q)$

Further applications

4 Evaluate each of the following when $d = 2$, $e = 5$ and $f = 10$.

a $d - e$	**b** $e - f$	**c** $d - f$	**d** $-d + e$	**e** $-e + d$
f $-e - d$	**g** $d - e - f$	**h** $e - f + d$	**i** $-f + d + e$	**j** $-e + f - d$
k $2d - e$	**l** $f - 3e$	**m** $e - 6d$	**n** $2e - 3f$	**o** $3f - 10e$
p $-3d + 4e$	**q** $-6e + 2f$	**r** $d - ef$	**s** $-3d - 5f$	**t** $-7f + 6e$

5 Evaluate each of the following when $t = 7$, $u = -2$ and $w = -5$.

a $t + u + w$	**b** $t - u - w$	**c** $u - t + w$	**d** $w - t - u$
e $-t - u - w$	**f** $3t - 4u + 5w$	**g** $2t + 3w - 5u$	**h** $3tu + 2uw$
i $3u - 4t - 2w$	**j** $-2u + 3t + w$	**k** $t(u + w)$	**l** $t^2u + u^2w$
m $\frac{2u+w}{t}$	**n** $\frac{t+w}{u}$	**o** $\frac{uw}{5t}$	**p** $\frac{2t+2w}{u}$
q $(t + w)(u - w)$	**r** $(t + u)(t - u)$	**s** $u^3t + w^2$	**t** $u^2 - w^2$

8.2 Addition and subtraction of like terms

Algebraic terms with identical pronumerals are called **like terms**. Some examples of:

- like terms are $6k$ and $2k$, $4e$ and $-9e$, ab and ba, $5c^2$ and $7c^2$
- unlike terms are $2p$ and $2q$, uv and vw, $3a^2$ and $10a$.

If there are n apples in a bag, then expressions such as $3n$ and $7n$ could represent the number of apples in 3 bags and 7 bags respectively. Now the number of apples in 3 bags plus the number of apples in 7 bags is equal to the number of apples in 10 bags altogether. In algebra, we could write $3n + 7n = 10n$. Similarly, the number of apples in 7 bags minus the number of apples in 3 bags is equal to the number of apples in 4 bags, i.e. $7n - 3n = 4n$.

From these examples we can see that to add or subtract like terms, we need only add or subtract their *co-efficients*. These co-efficients represent the number of identical items, in this case apples. Remember *the pronumeral represents the number of objects, not the objects themselves.*

To collect the like terms in an algebraic expression:

- **add or subtract the co-efficients**
- **keep the same pronumeral(s).**

Only like terms can be added or subtracted.

Example 1

Simplify each of the following.

a $8m + 5m$ **b** $9t - 5t$ **c** $14p - p$

d $12a^2 + 3a^2$ **e** $11gh - 6gh$ **f** $10cd + 8dc$

Solutions

a $8m + 5m = 13m$ **b** $9t - 5t = 4t$ **c** $14p - 1p = 13p$

d $12a^2 + 3a^2 = 15a^2$ **e** $11gh - 6gh = 5gh$ **f** $10cd + 8dc = 18cd$

Example 2

Simplify each of the following.

a $5a + 2a + 3b + 8b$ **b** $8x + 7 + 4x + 6$

c $11q^2 + 4q + 8q^2 + q$ **d** $10e + 15f - 4e + 5f$

Solutions

a
$$\begin{aligned} & 5a + 2a + 3b + 8b \\ = \; & 7a + 11b \end{aligned}$$

b
$$\begin{aligned} & 8x + 7 + 4x + 6 \\ = \; & 8x + 4x + 7 + 6 \\ = \; & 12x + 13 \end{aligned}$$

c $11q^2 + 4q + 8q^2 + q$
$= 11q^2 + 8q^2 + 4q + q$
$= 19q^2 + 5q$

d $10e + 15f - 4e + 5f$
$= 10e - 4e + 15f + 5f$
$= 6e + 20f$

Exercise 8.2

1 State whether each pair of terms are like terms or unlike terms.

a $5a, 2a$ **b** $3a, 2b$ **c** $6x, x$ **d** $4m, 4n$ **e** $2p, 2$ **f** $7, 5$
g ab, bc **h** xy, yx **i** abc, bcd **j** $7cd, 7d$ **k** a^2, a **l** $2a^3, 5a^3$

2 Select the like terms in each of these.

a $4p, 2q, 9p$ **b** $8a, 8b, 7b$ **c** $15c, 5, 4c, 6$
d $gh, fg, 3fg$ **e** $6ef, 4fe, 9e, 2f$ **f** $2n, 3n^2, 6n, n^2$

3 **a** Are $-5a$ and $-3a$ like terms? **b** Are $-4m$ and $2m$ like terms?

4 **a** Simplify $3m + 2m$.
b Check your answer by substituting several values for m.

5 Simplify:

a $4a + 3a$ **b** $10m - 7m$ **c** $2n + n$ **d** $8k - 6k$ **e** $c + c$
f $7h - 6h$ **g** $10u - 10u$ **h** $4g + 9g$ **i** $15w - 4w$ **j** $7e + 5e$
k $14n - 13n$ **l** $p + 5p$ **m** $11r + 9r$ **n** $17d - 8d$ **o** $10b - 9b$
p $2v + 5v$ **q** $6t + 9t$ **r** $12s + 9s$ **s** $18f - 9f$ **t** $13z - 13z$

Consolidation

6 Simplify each of the following.

a $5m + 3m + 2m$ **b** $7k + 4k + 5k$ **c** $9t + 4t - 3t$
d $10x + x - 6x$ **e** $3c - c + 10c$ **f** $12p - 2p - 3p$
g $8n - n - n$ **h** $6g - g + g$ **i** $j + j + j$
j $5z + 4z + 2z + z$ **k** $7y - 3y + 4y - y$ **l** $11q + 5q - 6q - 10q$
m $14u - 2u - 3u + 7u$ **n** $9a - 6a + 4a + 8a$ **o** $17w + 5w - 4w - 9w$

7 Simplify:

a $4ab + 5ab$ **b** $9pq - 3pq$ **c** $6st + 4st$
d $8uv - 7uv$ **e** $cd + cd$ **f** $11xy - xy$
g $5qr + 2rq$ **h** $13ef - 5fe$ **i** $9gh - 7hg$
j $19yz - zy$ **k** $16jk + 5kj$ **l** $14vw - 14wv$
m $2ab + 5ab + 9ab$ **n** $4pq - 2qp + 6pq$ **o** $8mn - 3mn - nm$
p $7rs + rs - 8rs$

8 Copy and complete each of these as an addition or subtraction.

a $4m$ _____ $= 7m$ **b** $6t$ _____ $= 2t$ **c** $7u$ _____ $= u$
d $3c$ _____ $= 10c$ **e** h _____ $= 2h$ **f** $5p$ _____ $= 0$

g $4a^2$ _____ $= 9a^2$ **h** $8b^2$ _____ $= 10b^2$ **i** $11k^2$ _____ $= 10k^2$
j $13qr$ _____ $= qr$ **k** $5ab$ _____ $= 12ab$ **l** $15pq$ _____ $= 5qp$

9 Find the missing term.
a _____ $+ 4n = 11n$ **b** _____ $+ 3p = 8p$ **c** _____ $- 2x = 7x$
d _____ $- 2a = a$ **e** _____ $+ 6z = 13z$ **f** _____ $- w = 9w$
g _____ $+ 8k^2 = 12k^2$ **h** _____ $- s^2 = 5s^2$ **i** _____ $+ 4y^2 = 18y^2$
j _____ $- 3bc = 0$ **k** _____ $- 7nm = 3mn$ **l** _____ $+ 7rs = 7rs$

10 Simplify these expressions by collecting the like terms.
a $6a + 2a + 3b + 4b$ **b** $7c + 9c + 3k + 2k$
c $4m + 5m + 7n + 3n$ **d** $5x + x + 4y + y$
e $3e + 9e + 8 + 5$ **f** $11 + 7 + 6z + 5z$
g $10p - 3p + 2q + q$ **h** $5r + 6r + 7s - 2s$
i $8x - 7x + 9y - 3y$ **j** $4m - 2m + 13n + 4n$
k $11u - 8u + 12 - 5$ **l** $4 + 5 + 22z - 6z$

11 Simplify by collecting the like terms.
a $2a + 3b + 5a + 2b$ **b** $4p + 6q + 2p + 3q$
c $5m + 2z + 3z + 8m$ **d** $7t + 4n + t + n$
e $8c + 3d + d + 5c$ **f** $e + f + 3e + 4f$
g $9u + 10 + 6u + 1$ **h** $3 + 2k + 4k + 9$
i $6a^2 + 7a + 3a^2 + 5a$ **j** $4 + 7w^2 + 5 + 12w^2$
k $5p^2 + 6p + p + 2p^2$ **l** $9y^2 + 10y + 4y^2 + y$

12 Simplify by collecting the like terms.
a $2p + 8n + 4p - 3n$ **b** $5m + 7y + 4m - 5y$ **c** $6x + 10a - 2x + 4a$
d $7g + 9k - 3g - 6k$ **e** $3n + 2u + n - 2u$ **f** $8c + 2g + g - 7c$
g $11r + 5 - 6r - 3$ **h** $14 + 8t - 9 + 6t$ **i** $4c^2 + 6c - 2c^2 - 3c$
j $8v + 11v^2 - 5v + v^2$ **k** $8c + 2g + g - 7c$ **l** $b^2 + b - b^2 + b$
m $3c + 9 - 7 + 5c$ **n** $20r + 8s - 4r + s$ **o** $6p + 14q - q - 6p$

■ Further applications

13 Simplify these expressions.
a $4x + 3y + 7z + 8x + 9z + 6y$ **b** $5p + 8q + 10r - 2p + 5q - r$
c $7a^2 + 4a + 11 + 3a^2 - a + 5$ **d** $3f + 14g^2 + 9 - f + 5g^2 - 8$
e $bc + c + ca + 7c + 4bc + 6ac$ **f** $8mn + 13m + 6kn - 4m - mn - 6nk$
g $5u + 7vu - u + 6v - 3uv - 4v$ **h** $15yz + 5zx - 6zy + 9 - 2xz - 1$

14 Copy and complete each of these as an addition or subtraction.
a $8t + 3t$ _____ $= 2t$ **b** _____ $+ 3u + 4u = 16u$
c $6b$ _____ $- 2b = 9b$ **d** $2p$ _____ $+ 3q$ _____ $= 5p + 8q$
e $12m$ _____ $+ 10n$ _____ $= 7m + n$ **f** _____ $+ 3b + 4a$ _____ $= 9a + 4b$
g $11j$ _____ $+ 7k$ _____ $= 8j$ **h** $19x$ _____ $+ 15y$ _____ $= x$

8.3 Further addition and subtraction of like terms

The questions in this exercise involve directed number.

Example 1

Simplify each of the following.

a $2p - 6p$ **b** $-7xy - 6xy$ **c** $-5z^2 + 10z^2$
d $4k - 9k - k$ **e** $11a + (-4a)$ **f** $8c - (-9c)$

Solutions

a $2p - 6p = -4p$

b $-7xy - 6xy = -13xy$

c $-5z^2 + 10z^2 = 5z^2$

d
$4k - 9k - k$
$= -5k - k$
$= -6k$

e
$11a + (-4a)$
$= 11a - 4a$
$= 7a$

f
$8c - (-9c)$
$= 8c + 9c$
$= 17c$

Example 2

Simplify each of the following.

a $6p + 3q + p - 5q$ **b** $8s - 12 - 3s + 9$ **c** $2d^2 - 4d + 4d - 3d^2$

Solutions

a
$6p + 3q + p - 5q$
$= 6p + p + 3q - 5q$
$= 7p - 2q$

b
$8s - 12 - 3s + 9$
$= 8s - 3s - 12 + 9$
$= 5s - 3$

c
$2d^2 - 4d + 4d - 3d^2$
$= 2d^2 - 3d^2 - 4d + 4d$
$= -d^2$

Exercise 8.3

1 Simplify each of these.

a $2a - 5a$ **b** $-7m + 3m$ **c** $-8k - 2k$ **d** $-4n + 6n$
e $-5g + 4g$ **f** $-9p - p$ **g** $y - 7y$ **h** $-t + 5t$
i $-u + u$ **j** $e - 2e$ **k** $-7c - 6c$ **l** $-11r + 3r$
m $-6b + 2b$ **n** $-12w - w$ **o** $5s - 10s$ **p** $-7f + 11f$
q $-d - d$ **r** $-9h + 8h$ **s** $-4x - 9x$ **t** $-6k + 6k$

2 Simplify:

a $-2ab - 7ab$ **b** $-4pq + 9pq$ **c** $-12xy + 8xy$
d $3mn - 10mn$ **e** $-8cd + 8cd$ **f** $-3uv + 4uv$
g $-5gh - 4gh$ **h** $2jk - 10jk$ **i** $-9ef + fe$
j $-6wx + 12xw$ **k** $-7bc + 8cb$ **l** $-rs + sr$

Consolidation

3 Simplify:

a $3x - 5x - 4x$ **b** $2u - 7u + 3u$ **c** $-8m + 2m + 4m$ **d** $-4h - h - h$
e $-3w + 8w - 2w$ **f** $n - n - n$ **g** $-q + q - q$ **h** $-6s + 7s - 4s$
i $y - 3y - 8y$ **j** $12b - 7b - 8b$ **k** $10z - 12z + 9z$ **l** $-4v - 12v + 10v$

4 Simplify each of these.

a $4a - (-2a)$ **b** $8u - (-5u)$ **c** $7c - (-c)$ **d** $9w - (-6w)$
e $5k + (-3k)$ **f** $6n + (-n)$ **g** $10t - (+4t)$ **h** $12b - (+7b)$
i $-2p - (-8p)$ **j** $-4q - (+4q)$ **k** $-11h - (-11h)$ **l** $-m + (-8m)$
m $-3xy - (+5xy)$ **n** $2bc + (-4bc)$ **o** $-5tu - (-6tu)$ **p** $7gh + (-7gh)$

5 Copy and complete each of the following as an addition or subtraction.

a $-2w$ _____ $= w$ **b** $-4n$ _____ $= -8n$ **c** $3p$ _____ $= -5p$
d $-5t$ _____ $= 6t$ **e** a _____ $= -a$ **f** $-5g$ _____ $= 0$
g $-7k$ _____ $= -16k$ **h** $2c$ _____ $= -6c$ **i** m _____ $= -12m$
j $-d$ _____ $= 9d$ **k** 0 _____ $= -4s$ **l** $-8e$ _____ $= -17e$

6 Find the missing term in each of these.

a _____ $+ 7u = 2u$ **b** _____ $- 6n = -10n$ **c** _____ $+ f = -3f$
d _____ $- 8r = -5r$ **e** _____ $+ 4w = -w$ **f** _____ $+ 7a = a$
g _____ $- 9d = -15d$ **h** _____ $+ 8s = 0$ **i** _____ $- n = -4n$
j _____ $- 6j = -2j$ **k** _____ $- g = -2g$ **l** _____ $+ 12b = -7b$

7 Simplify each expression by collecting the like terms.

a $7p + 3p + 2q - 5q$ **b** $5m - 4m + 3c - 7c$ **c** $2u - 8u - 4n - 6n$
d $-3b - 5b + x + 4x$ **e** $k - 3k + 5m - 11m$ **f** $-2p + 9p + 8k - 7k$
g $6t - 9t + 1 - 5$ **h** $-3 + 12 - 4y - 8y$ **i** $-14d + 8d - 2 + 3$
j $-7a^2 + 3a^2 + 5a - 11a$ **k** $4p - 13p - 6p^2 - 2p^2$ **l** $n^2 - 5n^2 - 10n + 17n$

8 Simplify by collecting the like terms.

a $7p + 8k - 11p + 2k$ **b** $5u + 3a - 7u - 8a$ **c** $-4d + f - 6f - 4d$
d $-2t - 7e + 6e + 11t$ **e** $-3n - v - 6n - 2v$ **f** $2a + 4q - 2a - 8q$
g $5y - 3b - 10y + 4b$ **h** $-s - 2t + 8t - 12s$ **i** $2k + 5 - 13k - 8$
j $-7 + 3w + 1 + 2w$ **k** $5n - 12 + 3n - 7$ **l** $12a^2 + 3a - 2a^2 - 5a$
m $4p^2 - 7 + 6p^2 - 9$ **n** $u + 6u^2 - 8u^2 - u$ **o** $-4s^2 - 3s - 11s^2 - 10s$
p $5n - 2m^2 + 7m^2 - 6n$ **q** $c + 3c^2 + c - 7c^2$ **r** $-x^2 + y + x^2 - y$

Further applications

9 Copy and complete each of these as an addition or subtraction.

a $2w$ _____ $+ 5w = -3w$ **b** _____ $- 4a - 3a = -13a$
c _____ $- n + 8n = 2n$ **d** $5m$ _____ $+ 6n$ _____ $= 8m - 2n$
e t _____ $- u$ _____ $= -2t + u$ **f** _____ $+ 4a - 2c$ _____ $= 7a - 7c$
g _____ $+ 3x$ _____ $+ y = -3x - y$ **h** $-3a^2$ _____ $+ 4a$ _____ $= -3a$

TRY THIS **Vital capacity**

Have you ever wondered how much air your lungs hold?

Get a balloon! Now take a deep breath and see how much you can blow up your balloon. The amount of air you blow out is called your vital capacity. It is measured in litres. The vital capacity depends on several factors, such as do you smoke? Two important factors are your age and your height. Here is a formula to work out your vital capacity depending on your height and your age.

Men: vital capacity = 0.057H – 0.022A – 4.23
Women: vital capacity = 0.041H – 0.018A – 2.69
H = height in centimetres
A = age in years

For example, if you are a 13-year-old girl of height 130 cm, then your

vital capacity = (0.041 × height) – (0.018 × age) – 2.69
= 0.041 × 130 – 0.018 × 13 – 2.69
= 5.33 – 0.234 – 2.69
= 2.406 litres

Now put in your own height and age, and work out your own vital capacity. You can compare this figure with the one that you get by roughly working out the volume of your breath-filled balloon.

(Volume of the balloon = $\frac{4}{3}\pi r^3 \doteqdot 4r^3$, r is the radius of balloon.)

Say radius = 8 cm. $V \doteqdot 4 \times 8 \times 8 \times 8$ cm^3
= ...? litres

8.4 Multiplication of algebraic terms

Algebraic terms do not have to be like terms in order to be multiplied.

To multiply algebraic terms:
- **multiply the co-efficients**
- **multiply the pronumerals.**

Example

Simplify:

a $h \times 8$ **b** $5a \times 2b$ **c** $\frac{1}{3} \times 15y$

d $4c \times 6c$ **e** $7fg \times 4gh$ **f** $(-8x) \times (-6y)$

Solutions

a $h \times 8 = 8h$ **b** $5a \times 2b = 10ab$ **c** $\frac{1}{3} \times 15y = 5y$

d $4c \times 6c = 24c^2$ **e** $7fg \times 4gh = 28fg^2h$ **f** $(-8x) \times (-6y) = 48xy$

Exercise 8.4

1 Simplify each of the following.

a $3 \times m$ **b** $p \times 7$ **c** $4 \times k$ **d** $g \times 5$
e $a \times b$ **f** $p \times q$ **g** $m \times n$ **h** $j \times k$
i $6 \times u \times v$ **j** $m \times 2 \times n$ **k** $x \times y \times 9$ **l** $r \times 7 \times s$

2 Express in simplest form:

a $a \times 3 \times b \times 2$ **b** $5 \times p \times 3 \times q$ **c** $m \times 7 \times 4 \times n$
d $k \times u \times 6 \times 5$ **e** $c \times 3 \times d \times e$ **f** $2 \times 3 \times w \times 2$

3 Simplify:

a $3 \times 2b$ **b** $5 \times 4n$ **c** $7p \times 2$ **d** $6t \times 3$
e $9 \times 4u$ **f** $5e \times 7$ **g** $8k \times 6$ **h** $7 \times 9w$
i $4a \times 3b$ **j** $6p \times 4q$ **k** $12m \times 4n$ **l** $2k \times 11m$
m $5e \times 8f$ **n** $15c \times 2d$ **o** $10x \times 6y$ **p** $12u \times 7v$

Consolidation

4 Simplify:

a $2c \times 3d \times e$ **b** $5f \times g \times 7h$ **c** $m \times 6n \times 6p$
d $10r \times 4s \times 3t$ **e** $5u \times 9v \times 2w$ **f** $8x \times 3y \times 2z$

5 Simplify the following products.

a $a \times a$ **b** $p \times p$ **c** $c \times c$ **d** $u \times u$
e $3q \times q$ **f** $t \times 2t$ **g** $8g \times g$ **h** $w \times 4w$
i $2y \times 3y$ **j** $5a \times 4a$ **k** $6h \times 2h$ **l** $3v \times 7v$
m $2t \times 5t \times 3$ **n** $4n \times 3 \times 5n$ **o** $6 \times 2s \times 3s$ **p** $8b \times 5 \times 3b$

6 Find each of these products.

a $ab \times ac$ **b** $pq \times qr$ **c** $xy \times yz$ **d** $xy \times 3y$ **e** $2c \times cd$
f $5p \times np$ **g** $4ab \times 3bc$ **h** $5ef \times 2eg$ **i** $2gh \times 7hi$ **j** $4pq \times 8pr$
k $9mp \times 5np$ **l** $8uv \times 7uw$ **m** $11cd \times 5de$ **n** $13uv \times 4tv$ **o** $8rs \times 12rt$

7 Find the missing term in each of these.

a $3m \times$ _____ $= 21m$ **b** _____ $\times 6x = 30x$ **c** _____ $\times 2n = 8n$
d $5c \times$ _____ $= 15cd$ **e** _____ $\times 7q = 42pq$ **f** $3g \times$ _____ $= 27gh$
g $m \times$ _____ $= m^2$ **h** _____ $\times c = c^2$ **i** $6p \times$ _____ $12p^2$
j _____ $\times 4u = 32u^2$ **k** $4f \times$ _____ $48f^2$ **l** _____ $\times 10g = 110g^2$
m $ab \times$ _____ $= a^2bc$ **n** $ij \times$ _____ $= ij^2k$ **o** $4de \times$ _____ $= 20de^2$

8 Simplify each of the following.

a $-3n \times 4$ **b** $-5 \times (-6q)$ **c** $7u \times (-2)$
d $5c \times (-8d)$ **e** $-4r \times (-7s)$ **f** $-9v \times 6w$
g $-10b \times (-7b)$ **h** $8g \times (-8g)$ **i** $-12f \times 10f$
j $3a \times (-2b) \times 7c$ **k** $-2p \times 5q \times (-5r)$ **l** $-4u \times (-3v) \times (-5w)$
m $e \times (-7f) \times (-8g)$ **n** $-6x \times 2y \times (-4z)$ **o** $-5i \times (-3j) \times (-4k)$

Further applications

9 Use the order of operations to simplify:

a $3x + 2x \times 5$ **b** $24a - 3 \times 2a$ **c** $4 \times 5n - n$
d $12p + 6p \times 3 + p$ **e** $10c - 4 \times 2c - 2c$ **f** $4 \times 6k - 3k \times 5$
g $5e + 3e \times 4 + 10$ **h** $20 + 2m \times 7 - 9$ **i** $19u - u \times 7 + 5 \times 3$

8.5 Division of algebraic terms

Algebraic terms do not have to be like terms in order to be divided.

To divide algebraic terms:
- **express the division in fraction form**
- **divide the co-efficients**
- **divide the pronumerals.**

Example 1

Simplify:

a $\frac{10c}{5c}$ **b** $\frac{18ab}{3b}$ **c** $\frac{30n^2}{6n}$ **d** $\frac{-48ef}{4e}$

Solutions

a $\frac{{}^{2}\cancel{10}\cancel{c}}{{}_{1}\cancel{5}\cancel{c}}$
$= 2$

b $\frac{{}^{6}\cancel{18}a\cancel{b}}{{}_{1}\cancel{3}\cancel{b}}$
$= 6a$

c $\frac{{}^{5}\cancel{30}n^{\cancel{2}}}{\cancel{6}\cancel{n}}$
$= 5n$

d $\frac{{}^{12}\cancel{-48}\cancel{e}f}{\cancel{4}\cancel{e}}$
$= -12f$

Example 2

Simplify:

a $40j \div 8j$ **b** $36pqr \div 9pr$ **c** $32b^2 \div 4b$

Solutions

a $40j \div 8j$
$= \frac{{}^{5}\cancel{40}\cancel{j}}{\cancel{8}\cancel{j}}$
$= 5$

b $36pqr \div 9pr$
$= \frac{{}^{4}\cancel{36}\cancel{p}\cancel{q}\cancel{r}}{\cancel{9}\cancel{p}\cancel{r}}$
$= 4q$

c $32b^2 \div 4b$
$= \frac{{}^{8}\cancel{32}b^{\cancel{2}}}{\cancel{4}\cancel{b}}$
$= 8b$

Exercise 8.5

1 Simplify each of these.

a $\frac{2n}{2}$ **b** $\frac{7t}{t}$ **c** $\frac{5m}{5}$ **d** $\frac{ef}{f}$

e $\frac{6ab}{6a}$ **f** $\frac{8xy}{8y}$ **g** $\frac{3cd}{3c}$ **h** $\frac{rst}{rt}$

i $\frac{4pqr}{4q}$ **j** $\frac{10mnp}{mp}$ **k** $\frac{11uvw}{11uw}$ **l** $\frac{wxyz}{wyz}$

2 Simplify each of the following fractions.

a $\frac{6m}{3}$ **b** $\frac{15t}{5}$ **c** $\frac{28u}{7}$ **d** $\frac{40s}{5}$

e $\frac{24r}{4}$ **f** $\frac{33w}{3}$ **g** $\frac{25k}{5}$ **h** $\frac{32e}{8}$

Consolidation

3 Simplify:

a $\frac{12k}{3k}$ **b** $\frac{21c}{7c}$ **c** $\frac{16d}{4d}$ **d** $\frac{5a}{5a}$

e $\frac{42yz}{7y}$ **f** $\frac{26jk}{2k}$ **g** $\frac{50hi}{10h}$ **h** $\frac{36uv}{9v}$

i $\frac{15abc}{3b}$ **j** $\frac{24uvw}{6w}$ **k** $\frac{30mnp}{5m}$ **l** $\frac{49xyz}{7y}$

m $\frac{40efg}{8eg}$ **n** $\frac{66def}{11de}$ **o** $\frac{90bcd}{10bc}$ **p** $\frac{84ijk}{12jk}$

4 Simplify each of the following.

a $\frac{a^2}{a}$ **b** $\frac{k^2}{k}$ **c** $\frac{u^2}{u}$ **d** $\frac{2q^2}{q}$ **e** $\frac{11f^2}{f}$

f $\frac{5c^2}{5c}$ **g** $\frac{8b^2}{8b}$ **h** $\frac{10h^2}{2h}$ **i** $\frac{9b^2}{3b}$ **j** $\frac{18k^2}{9k}$

k $\frac{27z^2}{3z}$ **l** $\frac{42c^2}{6c}$ **m** $\frac{56r^2}{7r}$ **n** $\frac{88s^2}{8s}$ **o** $\frac{45d^2}{5d}$

5 Express each division as a fraction, then simplify.

a $6a \div 6$ **b** $9x \div x$ **c** $mn \div m$ **d** $16e \div 2$

e $40s \div 8$ **f** $21v \div 3$ **g** $5pq \div 5p$ **h** $3ab \div 3b$

i $4jk \div jk$ **j** $12cd \div 3c$ **k** $20uv \div 4v$ **l** $42efg \div 6f$

m $n^2 \div n$ **n** $3e^2 \div e$ **o** $4r^2 \div 2r$ **p** $28y^2 \div 7y$

q $18m^2 \div 3m$ **r** $36c^2 \div 4c$ **s** $15ab^2 \div 3b$ **t** $30p^2q \div 5pq$

6 Simplify:

a $-5g \div 5$ **b** $2m \div (-2)$ **c** $-7c \div (-c)$ **d** $4m \div (-4)$
e $-6d \div (-6)$ **f** $3k \div (-3)$ **g** $-12p \div 6$ **h** $21y \div (-7)$
i $-25w \div (-5)$ **j** $27a \div (-3a)$ **k** $-22j \div 2j$ **l** $-20v \div (-2v)$
m $q^2 \div (-q)$ **n** $-14c^2 \div (-7c)$ **o** $-72r^2 \div 8r$ **p** $-44gh^2 \div (-4gh)$

7 Find the missing term in each of these.

a $3h \div$ _____ $= 3$ **b** $5n \div$ _____ $= 5$ **c** _____ $\div a = 2$
d $15z \div$ _____ $= 3z$ **e** $21b \div$ _____ $= 7$ **f** _____ $\div 3t = 8$
g _____ $\div 9 = 4y$ **h** $33p \div$ _____ $= 11$ **i** _____ $\div 4x = 1$
j $18pq \div$ _____ $= 3q$ **k** _____ $\div 6m = 7n$ **l** $d^2 \div$ _____ $= d$
m _____ $\div e = e$ **n** _____ $\div 4k = k$ **o** $108g^2 \div$ _____ $= 12g$

Further applications

8 Use the order of operations to simplify:

a $16a + 21a \div 3$ **b** $21z - 10z \div 5$ **c** $8p \div 2p + 8$
d $10i + 15i \div 3 - 2i$ **e** $20y - 4y \div 4 - 6y$ **f** $27b \div 9 + 20b \div 4$
g $8f + 54f \div 6 + 7$ **h** $9k - 64k^2 \div 8k + k$ **i** $17 - 36g \div 12g \times 2$

9 Simplify:

a $15n \div 20$ **b** $8a \div 12$ **c** $9x \div 15y$ **d** $18t \div 21u$
e $14u^2 \div 35u$ **f** $21k^2 \div 24k$ **g** $44w^2 \div 55w$ **h** $28y^2 \div 36y$

TRY THIS

Square magic

Here is a magic square.

A	B	C
D	E	F
G	H	I

There is something special about the sums of the rows, columns and diagonals.

Clues:

$A = C + 4$
$B = D^2 + 4$
$4C = 12$
$2D = B - 4$
$E = 2C$
$F = 2(D + C)$
$G = C^2$
$H = 10 - 2C$
$I = 2C - 1$

Most important

Which clue do you start with? Why?

8.6 The Distributive Law

The value of numerical expressions such as $3(5 + 2)$ and $3(5 - 2)$ can be found easily by direct evaluation.

For example, $3(5 + 2) = 3 \times 7 = 21$ or $3(5 - 2) = 3 \times 3 = 9$

In section 1.14 the **Distributive Law** was introduced and used to expand and evaluate expressions such as those above by multiplying the term outside the grouping symbols by each term inside.

For example,

$$\begin{aligned} 3(5 + 2) &= \mathbf{3} \times (5 + 2) \\ &= (\mathbf{3} \times 5) + (\mathbf{3} \times 2) \\ &= 15 + 6 \\ &= 21 \end{aligned}$$

or

$$\begin{aligned} 3(5 - 2) &= \mathbf{3} \times (5 - 2) \\ &= (\mathbf{3} \times 5) - (\mathbf{3} \times 2) \\ &= 15 - 6 \\ &= 9 \end{aligned}$$

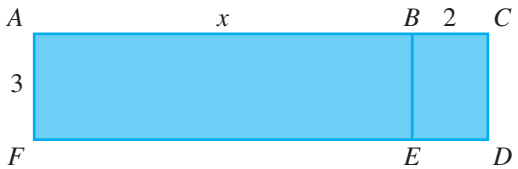
Algebraic expressions such as $3(x + 2)$ and $3(x - 2)$ cannot be simplified directly, since the expressions inside the grouping symbols cannot be simplified. The examples below will justify the use of the Distributive Law to expand and simplify such expressions.

1 $3(x + 2)$ could be interpreted as '3 lots of $(x + 2)$'.

That is,

$$\begin{aligned} 3(x + 2) &= 3 \times (x + 2) \\ &= (x + 2) + (x + 2) + (x + 2) \\ &= x + x + x + 2 + 2 + 2 \\ &= 3x + 6 \end{aligned}$$

This result can also be shown using an area diagram.



Shaded area = area of $ABEF$ + area of $BCDE$

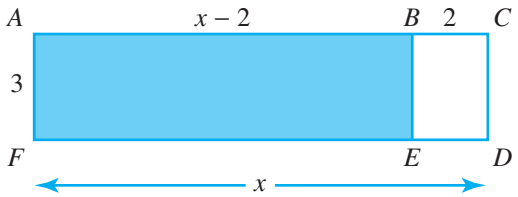
$$\begin{aligned} 3(x + 2) &= (3 \times x) + (3 \times 2) \\ &= 3x + 6 \end{aligned}$$

2 $3(x - 2)$ could be interpreted as '3 lots of $(x - 2)$'.

That is,

$$\begin{aligned} 3(x - 2) &= 3 \times (x - 2) \\ &= (x - 2) + (x - 2) + (x - 2) \\ &= x + x + x - 2 - 2 - 2 \\ &= 3x - 6 \end{aligned}$$

This result can also be shown using an area diagram.



Shaded area = area of $ACDF$ − area of $BCDE$

$$\begin{aligned} 3(x - 2) &= (3 \times x) - (3 \times 2) \\ &= 3x - 6 \end{aligned}$$

The examples above suggest that algebraic expressions containing grouping symbols can be expanded by the use of the Distributive Law.

To expand an expression containing grouping symbols using the Distributive Law:
- **multiply the term outside the grouping symbols by each term inside.**

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

Example 1

Expand:

a $5(t + 6)$ **b** $4(w - 3)$ **c** $2(3c + 8d)$
d $p(q - 4)$ **e** $a(a + 5)$ **f** $3m(7m - 10n)$

Solutions

a $5(t + 6)$
$= (5 \times t) + (5 \times 6)$
$= 5t + 30$

b $4(w - 3)$
$= (4 \times w) - (4 \times 3)$
$= 4w - 12$

c $2(3c + 8d)$
$= (2 \times 3c) + (2 \times 8d)$
$= 6c + 16d$

d $p(q - 4)$
$= (p \times q) - (p \times 4)$
$= pq - 4p$

e $a(a + 5)$
$= (a \times a) + (a \times 5)$
$= a^2 + 5a$

f $3m(7m - 10n)$
$= (3m \times 7m) - (3m \times 10n)$
$= 21m^2 - 30mn$

Example 2

Expand and simplify each of the following.

a $2(x + 12) - 10$ **b** $40 + 5(h - 3)$ **c** $6(j + 8) + 4(j + 3)$

Solutions

a $2(x + 12) - 10$
$= 2x + 24 - 10$
$= 2x + 14$

b $40 + 5(h - 3)$
$= 40 + 5h - 15$
$= 5h + 25$

c $6(j + 8) + 4(j + 3)$
$= 6j + 48 + 4j + 12$
$= 10j + 60$

Exercise 8.6

1 Explain the difference between $2x + 1$ and $2(x + 1)$.

2 Expand:

a $2(x + 3)$ **b** $3(t - 4)$ **c** $5(2 + a)$ **d** $4(y - 2)$
e $7(w + 4)$ **f** $6(n - 5)$ **g** $9(p - 1)$ **h** $8(2 + u)$
i $10(b + c)$ **j** $3(c - d)$ **k** $6(g - h)$ **l** $12(m + n)$

3 Expand:

a $(m + 2)5$ **b** $(n - 4)2$ **c** $(d - 9)3$ **d** $(3 - k)11$
e $(u + 5)7$ **f** $(y + 8)4$ **g** $(6 - c)6$ **h** $(e + 7)12$

4 Expand:

a $a(b + c)$ **b** $p(q + 3)$ **c** $e(f - 4)$ **d** $n(6 - b)$
e $g(y + z)$ **f** $k(z - 1)$ **g** $b(8 + w)$ **h** $c(d - e)$

■ Consolidation

5 Expand each of the following.

a $2(3a + 5)$ **b** $6(2n - 3)$ **c** $5(4 + 3p)$ **d** $4(1 - 2c)$
e $8(5y + 7)$ **f** $9(3 - 8i)$ **g** $3(4u + 3v)$ **h** $6(3a - 5b)$
i $7(7e - 6f)$ **j** $11(t + 9u)$ **k** $10(5g - 4h)$ **l** $12(5d + 9e)$

6 Expand each of these.

a $x(x + 3)$ **b** $a(a - 5)$ **c** $n(2 - n)$ **d** $p(p + q)$
e $u(4 + u)$ **f** $b(a - b)$ **g** $z(2y + z)$ **h** $c(c - 3d)$
i $w(v - 5w)$ **j** $k(2k + 7m)$ **k** $y(3x - 4y)$ **l** $h(5h + 6i)$

7 Expand:

a $3x(y + 4)$ **b** $2p(q - 7)$ **c** $5g(4 + h)$ **d** $7t(3 - u)$
e $4j(k + 8)$ **f** $6y(z - 8)$ **g** $2e(3f + g)$ **h** $5m(3n - 5p)$
i $8i(3j - 2k)$ **j** $10p(2q + 9r)$ **k** $7t(4u + 7w)$ **l** $12c(7d - 11e)$
m $4a(a + b)$ **n** $3n(5 - n)$ **o** $9k(k - 5)$ **p** $2b(3b + 9)$
q $5d(6 - 2d)$ **r** $11f(4f - 3g)$ **s** $7x(3y + 5x)$ **t** $12i(5i - 9j)$

8 Expand and simplify:

a $3(n + 5) + 2$ **b** $4(x - 3) + 2x$ **c** $7(y + 3) - 4$ **d** $5(a - 7) - 4a$
e $5(2c + 3) + 8$ **f** $3(11 - 3w) - 10$ **g** $8(1 + 6m) - 12m$ **h** $6(3d + 7) + 11$
i $10(2 - 3t) - 13$ **j** $9(4k + 8) + 5k$ **k** $11(5u + 6) - 20u$ **l** $7(9 - 2p) - 31$

9 Expand and simplify:

a $5 + 3(n + 4)$ **b** $11 + 2(w - 5)$ **c** $4 + 8(t + 3)$
d $6u + 4(2u + 7)$ **e** $3e + 5(2e - 4)$ **f** $15q + 2(13 - 3q)$
g $4b + 3b(a + 3)$ **h** $12m + 2m(k - 6)$ **i** $18c + 4c(1 - 2d)$

10 Expand and simplify:

a $3(x + 2) + 2(x + 4)$ **b** $7(a + 3) + 4(a + 1)$ **c** $4(m + 9) + 6(m - 1)$
d $8(p + 3) + 3(p - 2)$ **e** $6(u + 5) + 5(2 - u)$ **f** $10(3 + w) + 9(w + 2)$
g $9(n + 10) + 8(n - 5)$ **h** $5(k + 12) + 2(k - 6)$ **i** $7(g + 6) + 4(g + 8)$
j $11(t + 8) + 5(t - 5)$ **k** $4(7 + e) + 9(e - 2)$ **l** $8(6 + t) + 12(t - 4)$

■ Further applications

11 Expand each of the following.

a $-3(n + 5)$ **b** $-4(a - 2)$ **c** $-9(1 + t)$

d $-5(8 - p)$ **e** $-6(b + 7)$ **f** $-2(12 - z)$
g $-2(3m + 7)$ **h** $-5(7y - 4)$ **i** $-7(3 - 10g)$
j $-4(2c + 9)$ **k** $-10(12j + 1)$ **l** $-9(9d - 8)$
m $-11(4 + 6h)$ **n** $-4(8w + 7)$ **o** $-12(7k - 9)$

12 Expand and simplify:
a $2(u + 4) - 9$ **b** $4(m - 3) + 5$ **c** $3(q - 2) + 10$
d $8(y - 6) - 11y$ **e** $5(1 - e) - 4e$ **f** $7(2 - 3j) + 5j$
g $-10 + 3(2x - 3)$ **h** $-14 + 5(3 - 4a)$ **i** $12 + 7(4h - 2)$
j $3(w + 4) + 2(w - 8)$ **k** $6(3 - n) + 2(n - 4)$ **l** $4(y - 10) + 5(3 - y)$

TRY THIS **Number, think and back again**

Try both of these number puzzles by starting with any number.

1 Think of a number, add 6, multiply by 2, subtract 12, divide by 2. What happens every time?

2 Think of a number, add 2, multiply by 4, subtract 8, divide by 4. What happens every time? Can you find out what is happening?

8.7 The meaning of algebraic expressions

Many real-life problems can be solved by expressing them in terms of algebraic statements. We can then use normal mathematical procedures to find solutions. To do this, however, we must first understand the meaning of key words or phrases, so that the correct mathematical expressions can be formed. Some of these key words or phrases are listed below, along with their associated mathematical operations.

- **Addition—sum, increase, add, plus, total, more than**
- **Subtraction—difference, decrease, subtract, take away, reduce, less than**
- **Multiplication—product, multiply, times, double, triple**
- **Division—quotient, divide, share, half, third**

Example 1

Explain in words the meaning of each expression.

a $m + 3$ **b** $x + y$ **c** $k - 4$ **d** $u - v$ **e** pq **f** $\frac{a}{b}$

Solutions

a When a number has been added to a pronumeral, the phrase 'more than' is usually used. So $m + 3$ means '3 more than m'.

b When two or more pronumerals are added, the word 'sum' is usually used. So $x + y$ means 'the sum of x and y'.

c When a number has been subtracted from a pronumeral, the phrase 'less than' is usually used. So $k - 4$ means '4 less than k'.

d When one pronumeral is subtracted from another, the word difference is usually used. So $u - v$ means 'the difference between u and v'.

e When two or more pronumerals are multiplied, the word 'product' is usually used. So pq means 'the product of p and q'.

f When one pronumeral is divided by another, the word 'quotient' is usually used. So $\frac{a}{b}$ means 'the quotient of a and b'.

Example 2

State the meaning of each expression.

a $\frac{x}{2}$

b $\frac{2c}{3}$

c $\frac{3w}{4}$

Solutions

a $\frac{x}{2}$ can be written as $\frac{1}{2}x$. So $\frac{x}{2}$ means 'half of x'.

b $\frac{2c}{3}$ can be written as $\frac{2}{3}c$. So $\frac{2c}{3}$ means 'two-thirds of c'.

c $\frac{3w}{4}$ can be written as $\frac{3}{4}w$. So $\frac{3w}{4}$ means 'three-quarters of w'.

Example 3

Write the meaning of each expression.

a $3m + 4$ **b** $\frac{p}{q} - 13$ **c** $\frac{a-b}{3}$ **d** $\frac{a}{3} - b$ **e** $4a^2$ **f** $5(n + 2)$

Solutions

a $3m + 4$ means '4 more than the product of 3 and m'.

b $\frac{p}{q} - 13$ means '13 less than the quotient of p and q'.

c $\frac{a-b}{3}$ means 'one-third of the difference between a and b'.

d $\frac{a}{3} - b$ means 'the difference between one-third of a and b'.

e $4a^2$ means 'the product of 4 and the square of a'.

f $5(n + 2)$ means 'the product of 5 and 2 more than n'.

Exercise 8.7

1 Write an algebraic expression for each of these.

a three more than k **b** two more than y **c** six more than x
d the sum of p and q **e** the sum of x and y **f** the sum of g and h

2 Write an algebraic expression for each of these.

a seven less than a **b** four less than p
c nine less than w **d** the difference between u and v
e the difference between c and d **f** the difference between r and s

3 Write an algebraic expression for each of the following.

a double h **b** triple n **c** quadruple t
d the product of 8 and f **e** the product of z and 9 **f** the product of j and k

4 Write an algebraic expression for each of the following.

a half of d **b** one-third of m **c** three-quarters of x
d the quotient of a and b **e** the quotient of g and h **f** the quotient of m and n

Consolidation

5 Describe in words each of these expressions.

a $n+6$ **b** $k-1$ **c** $7t$ **d** $\frac{w}{4}$ **e** $g-h$

f $m+n$ **g** $\frac{c}{d}$ **h** ab **i** $z+10$ **j** $\frac{n}{2}$

k $y-8$ **l** $5u$ **m** $\frac{x}{y}$ **n** $j+k$ **o** pq

p $k-m$ **q** $2b$ **r** $\frac{s}{5}$ **s** $v-2$ **t** $f+11$

6 Describe each expression in words.

a $\frac{2a}{3}$ **b** $\frac{3k}{4}$ **c** $\frac{7u}{10}$ **d** $\frac{4c}{5}$

7 Write an algebraic expression for:

a the sum of p, q and r **b** the sum of x, y and 5
c the product of u, v and w **d** the product of 4, s and t

8 Write an algebraic expression for each of these.

a one more than the product of two and x
b four less than the product of k and 3
c five more than the product of a and b
d seven less than the product of g and h

9 Describe each expression in words.

a $5u - 2$
b $cd + 10$
c $yz - 6$
d $4t + 9$

10 Write an algebraic expression for each of the following.

a five more than half of m
b one less than the quotient of r and s
c four less than one-third of n
d nine more than the quotient of a and b
e one-quarter of the sum of p and q
f one-sixth of the number which is five less than p
g one-third of the number which is two more than d
h one-tenth of the difference between b and c

11 Describe each expression in words.

a $\frac{m-n}{3}$
b $\frac{v}{w} + 2$
c $\frac{y}{5} - 8$
d $\frac{f+7}{8}$
e $\frac{k}{9} + 12$
f $\frac{u-1}{4}$
g $\frac{d}{e} - 5$
h $\frac{r+s}{11}$

12 Write an algebraic expression for each of these.

a the square of k
b the cube of j
c y to the power of 9
d v to the seventh power
e twice the square of w
f five times the cube of a
g the product of x and the square of y
h half the cube of e
i one more than the square of m
j ten less than the cube of h
k the sum of c and the square of d
l the difference between the square of v and z

■ Further applications

13 Use grouping symbols to write an algebraic expression for each of the following.

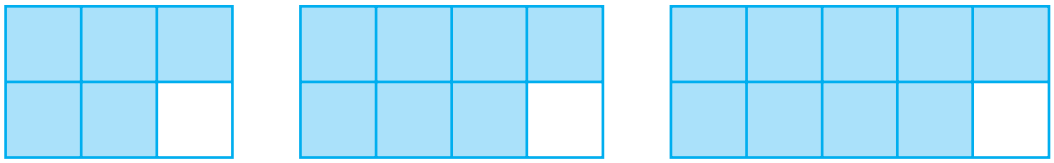
a twice the sum of p and q
b the product of 6 and the difference between u and v
c the product of x and the number which is 7 less than y
d the product of a and the number which is 2 more than c
e five times the number which is 4 less than d
f three times the difference between half of a and four-fifths of b

Problem solving

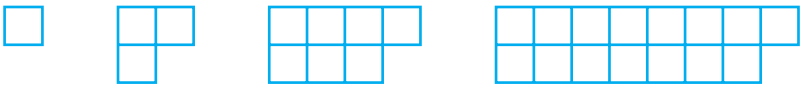
1 When four consecutive numbers are added, the total is 98. What are the numbers?

2 Will you still be alive one billion seconds from now? If so, how old will you be?
NOTE: 1 billion = 1 thousand million = 1 000 000 000.

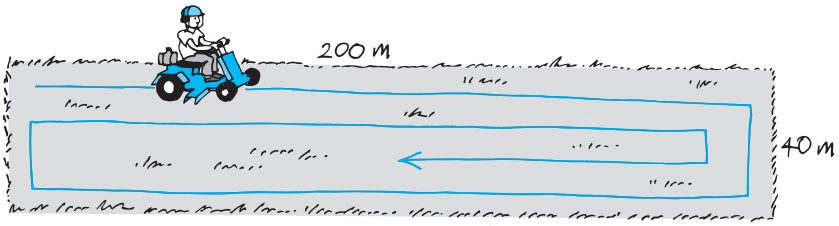
3 Find the next figure in this series.



4 Find the next figure in this series.

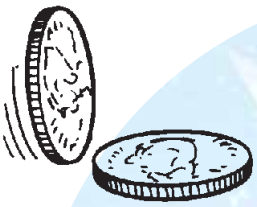


5 A council worker has to mow a rectangular soccer field as shown. If the width of the tractor mower is 3 metres, how many times must the worker go around the field?



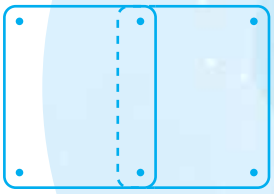
6 In the finals of a basketball competition, a team must lose twice before it can be eliminated. What is the minimum number of games necessary to determine the winner of a 4-team competition?

7 A five-cent coin is rolled around another five-cent coin. Will the Queen's head be facing up or down when it has travelled halfway around the coin?



8 How many ways can you add 6 odd numbers to obtain a total of 30? You may use numbers as many times as you wish?

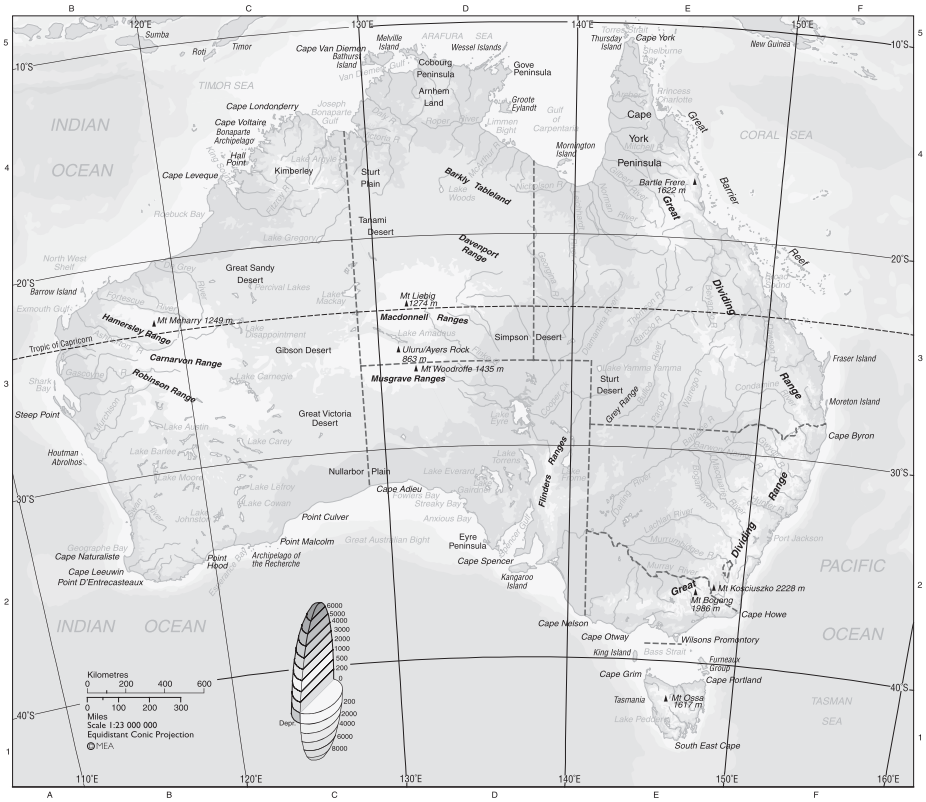
9 How many drawing pins are needed to pin 10 pictures to the wall if every corner of the picture has a drawing pin?



10 A man bought a cow for $100, then sold it for $120. He then bought it back for $140, and sold it for $160. What profit or loss did he make?

Focus on working mathematically

Colouring maps



Introduction

Algebra is a powerful tool for writing down a general rule from specific cases. In this activity we will investigate how the boundaries of a map are related to the number of regions created and the number of corner points.

This discovery was used to solve a problem to do with colouring maps which had not been solved for over 100 years.

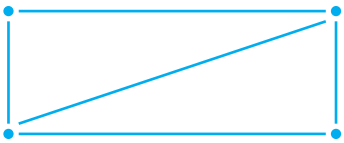
Learning activities

Materials needed: Paper, pen and coloured pencils.

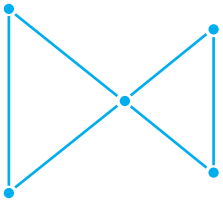
1 Here is a simple map of a piece of land drawn on a flat surface. It has 4 corner posts, and 5 boundary fences. It also divides the plane (the flat surface on which it is drawn) into

3 separate regions. Copy the map and colour the 3 regions, two inside and one outside, with different colours.

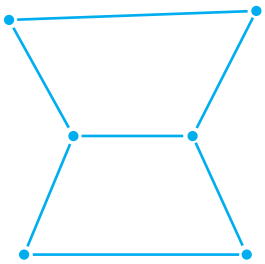
Map 1



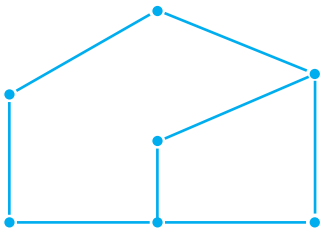
2 Here are three other maps which have just 3 regions. Copy and colour the regions with different colours.



Map 2



Map 3



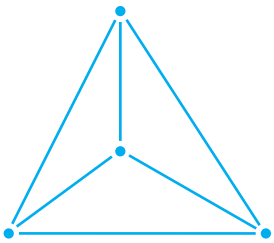
Map 4

3 Let the number of corner posts of any map be *P*. Let the number of boundaries be *B*. Let the number of regions be *R*. Copy and complete this table for the maps above.

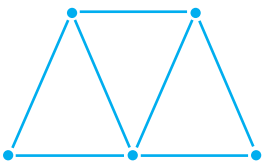
Map	*P*	*B*	*R*
1			
2			
3			
4			

4 Study the pattern of numbers in the table. See if you can write down an equation using the symbols *P*, *B* and *R* to show how they are related.

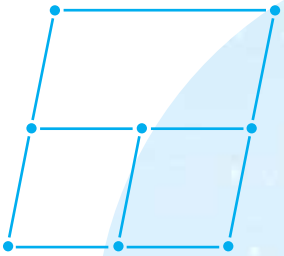
5 Let's try it again but this time using 4 regions. Complete the table for each of the following maps.



Map 5



Map 6



Map 7

6 Check out your equation in part 4 to see if it works for each of the maps with 4 regions.

Map	*P*	*B*	*R*
5			
6			
7			

7 If you investigate these maps, you will find that the equation $P + R - B = 2$ is always true no matter how many regions a map divides the plane into. (Your equation may be equivalent, say $P + R = B + 2$.) It is known as Euler's Formula after the mathematician Leonard Euler who discovered it. Record this in your book.

Extension activity

Try out Euler's Formula to see if it works with three-dimensional solids.

Instead of a post, use a vertex where the edges meet. Instead of a boundary, use an edge of the solid. Instead of a region, use a face of the solid.

For what solids does the formula hold? Try it out with a cube, a rectangular prism, a square pyramid, a cone, a cylinder. If V is the number of vertices, E the number of edges, and F the number of faces, write down Euler's Formula in terms of V, E and F. (See also chapter 12 Solids, p. 457.)

Let's communicate

Make a chart of what Euler's Formula is about. Then write a short account of what you have learned about how useful algebra is in making a general rule from a set of particular cases. Be prepared to read it to the class.

Reflecting

One of the most famous unsolved problems of mathematics was 'the four colour problem'. It was first posed by a mathematician named Francis Guthrie in 1852 and not solved until 1976. Today many proofs have been written.

'Are four colours sufficient to colour every planar (flat) map in such a way that every pair of states or countries with a common border are coloured differently?' The answer is yes. Algebra, in particular Euler's Formula, plays a major role in the proof.

This is a good example of how one branch of mathematics, algebra, can be vital in solving a problem in another branch, in this case geometry. Mathematics is really a complete subject. Different branches relate to each other in special ways. You will discover this as you learn. Think about it now and remember it always when you are solving problems.

Language link with Macquarie

MACQUARIE

1 *In the* ***expression*** $3a + 4b$ *identify the* ***co-efficients****.*
2 *Two skateboarders meet a third skateboarder. Express this situation in* ***algebraic*** *terms.*
3 *In the expression* $2a - 2b$ *what sort of* ***terms*** *are used?*
4 *What do we do with* ***indices*** *when multiplying* ***expressions*** *like:* $2a^2 \times 2a^3$?
5 *Read the* Macquarie Learners' Dictionary *entry for* ***algebra****:*

algebra *noun* (*uncount*) the branch of mathematics which uses letters to stand for numbers
❑ ***Word Family:*** **algebraic** *adjective*

Write your own definition of ***algebra*** *based on what you have learned from this chapter.*

CHAPTER REVIEW

1 Evaluate each of the following expressions given that $a = 3$ and $b = 4$.

a	$a + b$	**b**	$7b$
c	ab	**d**	$8a - b$
e	$5a + 2b$	**f**	$30 - 2ab$
g	$a^2 + 11$	**h**	$b^2 - a^2$
i	$2a^2$	**j**	ab^2
k	$6(a + b)$	**l**	$10(3b + 2a)$
m	$\frac{5b}{2}$	**n**	$\frac{4a + 3b}{8}$
o	$\frac{23 + ab}{b + 1}$	**p**	$\sqrt{a^2 + b^2}$

2 State whether the following are like terms or unlike terms.

a	$7n, 5n$	**b**	$5p, 4q$	**c**	$7y, 7$
d	$3, 8$	**e**	xy, yz	**f**	$5u, u$
g	$4k, -k$	**h**	a^2, a	**i**	pq^2, pq^2

3 Simplify:

a	$5c + 3c$	**b**	$9t - 2t$
c	$8v + v$	**d**	$12p - p$
e	$17f - 16f$	**f**	$10j - 10j$
g	$b + b$	**h**	$4gh + 6gh$
i	$9st - 3ts$	**j**	$4a^2 + 8a^2$
k	$11e^2 - 6e^2$	**l**	$6r - 4r + 9r$

4 Simplify each of the following by collecting the like terms.

a $5a + 3a + 6b + 4b$
b $7u + 5v + 8u + 3v$
c $10c + 4d + 6c + d$
d $5g + 7 + 3g + 9$
e $13r + 12s - 5r + 2s$
f $18x + 7y - 4x - 6y$
g $8w^2 + 9w + 6w^2 - 3w$
h $15ab + 17 - ab - 8$
i $6x + 16x^2 - 5x^2 + 7x$

5 Simplify:

a	$3w - 7w$	**b**	$-6z + 13z$
c	$-9m + 4m$	**d**	$-2d + 11d$

6 Simplify:

a	$5t - (-3t)$	**b**	$12n + (-7n)$
c	$-20k - (+5k)$	**d**	$-4h - (-11h)$

7 Simplify each of the following by collecting the like terms.

a $10m + 2m + 3n - 5n$
b $6e + 3f - 4e - 9f$
c $7x - 4y + 6x + 5y$
d $5u - 1 - 4u - 7$
e $-4g + 6h - 3g - h$
f $2y^2 - 10y - 2y^2 + 3y$

8 Simplify:

a $n \times 6$ **b** $c \times 7 \times d$
c $5 \times 4x$ **d** $3a \times 8b$
e $8p \times 3q \times 2r$ **f** $p \times p$
g $5t \times 9t$ **h** $ab \times bc$
i $6uv \times 7uw$ **j** $-9k \times 4$
k $-12s \times -7s$
l $-3x \times -4y \times -5xyz$

9 Simplify, using the order of operations.

a $5m + 3m \times 7$
b $18e - 4e \times 3 + e$
c $25 - 3c \times 6c + 5 + 2c^2$

10 Simplify each of the following divisions.

a $\frac{4a}{4}$ **b** $\frac{cd}{d}$ **c** $\frac{7abc}{7b}$
d $\frac{45v}{9}$ **e** $\frac{21h}{3h}$ **f** $\frac{40pqr}{5pr}$
g $\frac{m^2}{m}$ **h** $\frac{12x^2}{4x}$ **i** $\frac{ab^2}{ab}$

11 Simplify:

a $24d \div 6$ **b** $35m \div 7m$
c $50ab \div 5a$ **d** $48x^2 \div 4x$
e $-16k \div (-4)$ **f** $-56ef \div 8f$
g $60uv \div (-12u)$ **h** $-72c^2 \div (-9c)$

12 Simplify, using the order of operations.

a $9p + 30p \div 6$
b $20b - 27ab \div 3a + b$
c $5c^2 + 28c^2 \div 4c + 6c - c^2$

13 Expand each of the following.

a $5(a + 3)$ **b** $4(m - 7)$
c $p(q + 8)$ **d** $x(4 - y)$
e $3(6c + 10d)$ **f** $8(3g - 4h)$
g $b(b - 5c)$ **h** $9s(3s + 5t)$

14 Expand and simplify:

a $4(y + 3) + 9$
b $37 + 5(d - 6)$
c $14u^2 + 2u(3u + 5)$
d $3(z + 7) + 6(z - 2)$

15 Write an algebraic expression for each of the following.

a five more than b
b three less than y
c the sum of g and h
d the difference between p and q
e the product of s and seven
f the square of z
g the quotient of c and d
h half of w
i two-thirds of x

16 Describe in words:

a $t - 8$ **b** $r - s$ **c** $u + v$
d $p + 12$ **e** $5m$ **f** $\frac{c}{4}$
g $\frac{5k}{6}$ **h** a^3 **i** $2(m + n)$

17 Write an algebraic expression for:

a three more than the product of five and h
b seven less than the quotient of t and u
c one-third of the difference between e and f
d the difference between e and one-third of f
e twice the square of n
f four more than the cube of c

9 Angles

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- classify angles given either a diagram or the angle size
- draw and measure angles using a protractor.

Stage 4

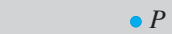
After completing this chapter, you should be able to:

- label and name points, lines and intervals using capital letters
- label the vertex and arms of an angle with capital letters
- label and name angles
- use the common conventions to indicate right angles and equal angles on diagrams
- identify and name adjacent angles, complementary angles and supplementary angles
- find the complement and supplement of an angle
- identify and name vertically opposite angles and angles at a point
- find the unknown angle in a diagram using angle results
- identify and name parallel lines and transversals
- use common symbols for 'is parallel to' and 'is perpendicular to'
- use arrows to indicate parallel lines in diagrams
- identify and name all pairs of alternate, corresponding and co-interior angles in parallel lines
- use angle properties to identify parallel lines
- use angle relationships to find unknown angles in diagrams
- identify parallel and perpendicular lines in the environment
- construct parallel and perpendicular lines using a ruler and a set square
- use dynamic geometry software to investigate angle relationships.

9.1 Points, lines and intervals

Plane or two-dimensional geometrical figures are made up of a series of points, lines and line segments. It is important to ensure that each part of a diagram is labelled correctly so that we can refer to it specifically. A ruler must be used at all times when drawing geometrical figures, as accuracy is essential.

- **A point is a position in space. It has no size or dimension. That is, it has no length, width or height. A dot is used to mark the position of a point, however, the dot is not a point. A capital letter (e.g. *P*) is used to name a point.**



- **A line is one dimensional. It contains an infinite number of points and extends indefinitely in both directions. A single lower case letter (e.g. *l*) can be used to name a line. It can also be named using two points that lie on the line (e.g. *AB*).**

- **An interval is a line segment, or part of a line. It has a definite beginning and a definite ending. An interval consists of its two endpoints and all of the points on the line that lie between them. It is named by stating its endpoints (e.g. *CD*).**

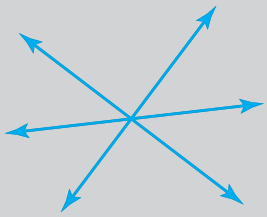
NOTE: Because an interval has a definite beginning and ending, we are able to measure its length. It is not possible to measure the length of a line.

There are a number of other important geometrical terms associated with the study of points, lines and intervals.

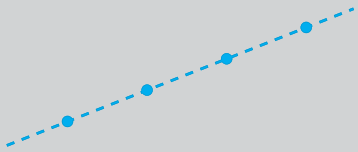
- **The midpoint of an interval is the point that lies halfway between its endpoints.**

$PQ = QR$, $\therefore$ Q is the midpoint of the interval PR.

- **Concurrent lines are three or more lines that intersect at the same point.**



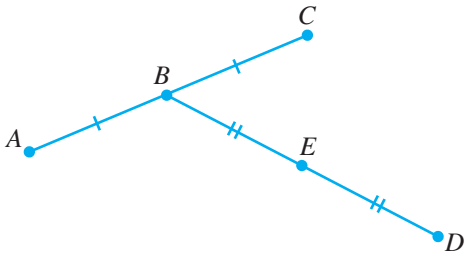
- **Collinear points are three or more points that lie on the same line.**



Example 1

Name the midpoint of the interval:

a *AC* **b** *BD*



Solutions

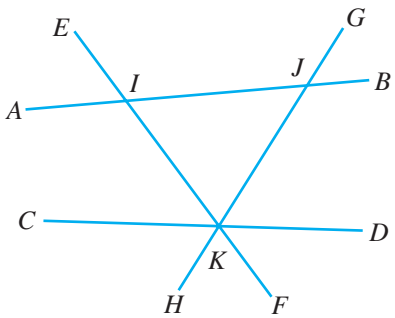
a *B* is the midpoint of *AC* because $AB = BC$.

b *E* is the midpoint of *BD* because $BE = ED$.

NOTE: The point *B* is a midpoint for the interval *AC* and an endpoint for the interval *BD*.

Example 2

Name 3 concurrent lines in the diagram and state their point of intersection.



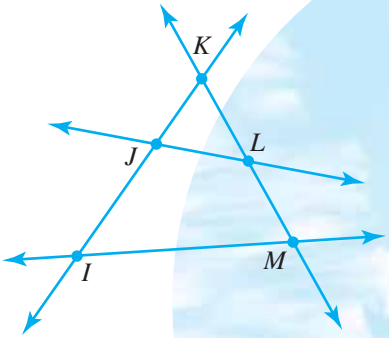
Solution

The lines *CD*, *EF* and *GH* are concurrent and they intersect at *K*.

Example 3

Which point is collinear with both:

a *J* and *K*? **b** *K* and *M*?

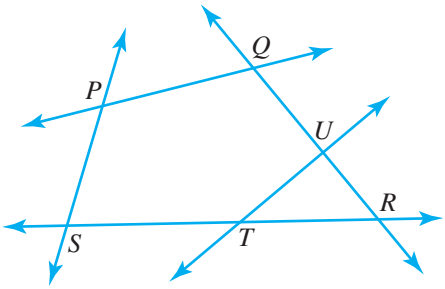


Solutions

a *I* is collinear with the points *J* and *K*. **b** *L* is collinear with the points *K* and *M*.

Exercise 9.1

1 **a** There are 5 lines in this figure. Name them.
b There are 9 intervals in this figure. Name them.



2 **a** Draw any straight line and mark on it the points C and D.
b Name this line in two ways.
c Mark the point E on the line, anywhere between C and D.
d Name all of the intervals that lie on this line.

3 Draw any interval XY and mark on it the point Z, somewhere between X and Y. Name the interval that is equal in length to:
a $XZ + ZY$ **b** $XY - XZ$ **c** $XY - ZY$

4 **a** Draw an interval LN of length 6 cm and mark the point M, such that lies M halfway between L and N. M is called the **midpoint** of LN.
b How far is: **i** M from L? **ii** M from N?
c Does $LM + MN = LN$?

Consolidation

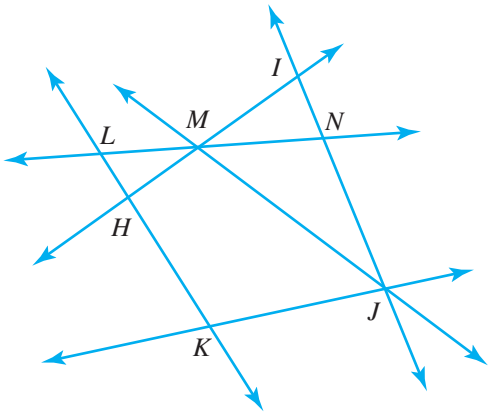
5 **a** Mark two points P and Q and draw the line PQ passing through them.
b Is it possible to draw a different straight line through the points P and Q?
c Copy and complete this statement: 'Any 2 points in a plane lie on a unique _____.'

6 **a** Draw two non-parallel lines TU and VW and label their point of intersection X.
b Can these lines intersect at any point other than X?
c Copy and complete this statement: 'Any 2 lines in a plane intersect at a unique _____, if they intersect at all.'

7 If the same point lies on two different lines, what can you conclude about the lines?

8 If 2 lines drawn in the same plane have no points of intersection, what can you conclude about the lines?

9 Three or more lines are said to be **concurrent** if they all intersect at the same point. Name two sets of 3 concurrent lines on the diagram.



10 Draw any 3 lines that are concurrent at *R*.

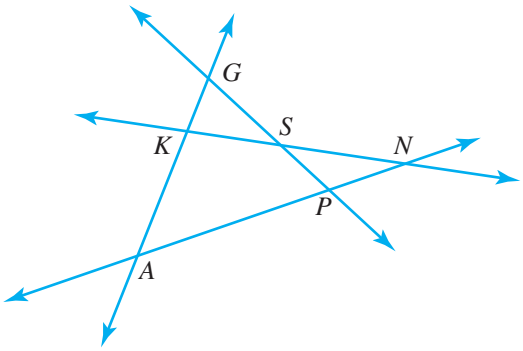
11 Three or more points that lie on the same line are said to be **collinear**.

a Are the points, *A*, *K*, *G* collinear?

b Are the points *A*, *S*, *N* collinear?

c Which point is collinear with both:

i *G* and *S*? **ii** *A* and *N*?



12 'Any 3 points in the same plane can always be joined to form a triangle.' Is this statement true or false? Why?

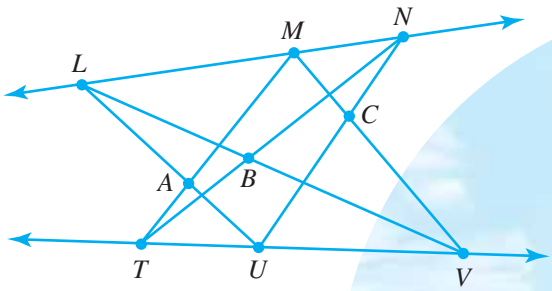
13 Draw a line and mark on it any 3 points *L*, *M* and *N*. Draw a second line (which does not intersect the first) and mark on it any 3 points *T*, *U* and *V*.

a Draw the intervals *LU* and *MT*, then label their point of intersection *A*.

b Draw the intervals *LV* and *NT*, then label their point of intersection *B*.

c Draw the intervals *MV* and *NU*, then label their point of intersection *C*.

d What do you notice about the points *A*, *B* and *C*?



Further applications

14 Draw any triangle *RST*.

a Measure the lengths of the sides and mark *X*, *Y* and *Z*, the midpoints of *RS*, *ST* and *TR* respectively.

b Using a ruler, join each vertex of the triangle to the midpoint of the opposite side. The intervals RY, SZ and TX are called **medians**.

c Are the medians concurrent?

15 Sketch a diagram that shows all of the given information for each of these.

a The intervals AB and CD intersect at E, $AE = EC$ and $DE = BE$.

b Q is the midpoint of the interval PR and R is the midpoint of the interval LM.

c The intervals EF and GH bisect each other at D and $EF > GH$.

d The lines AB, AC and AD intersect at A. A fourth line cuts these lines at P, Q and R repectively.

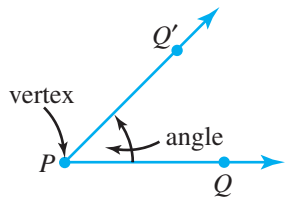
e The interval UV is trisected at C and D by the lines BC and BD.

f $OA = OB = OC$ and $AB = BC = CA$.

g The interval AB is produced to C and BA is produced to D. P is a point not on DC, such that $PD = PC$ and $PA = PB$.

9.2 Naming angles

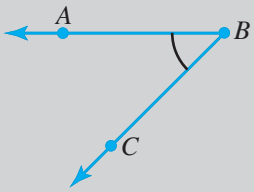
A **ray** is a section of a line. It has a definite beginning but no end point. That is, a ray extends indefinitely in one direction and therefore its length cannot be measured. We draw the ray $\overrightarrow{PQ}$ as P ●——●→ Q. If this ray $\overrightarrow{PQ}$ is rotated about the fixed point P, an **angle** is formed between the rays $\overrightarrow{PQ}$ and $\overrightarrow{PQ'}$. We define an angle as the amount of turning by a ray about a fixed point. The fixed point P is called the **vertex** of the angle.



It is important to understand that the rays $\overrightarrow{PQ}$ and $\overrightarrow{PQ'}$ are not the angle. The angle is the amount of turning that has occurred to rotate the ray $\overrightarrow{PQ}$ to its new position $\overrightarrow{PQ'}$.

There are several ways in which an angle can be named. The angle shown could be named as either:

- **$\angle ABC$ or $\angle CBA$, or**
- **$A\hat{B}C$ or $C\hat{B}A$, or**
- **$\angle B$**



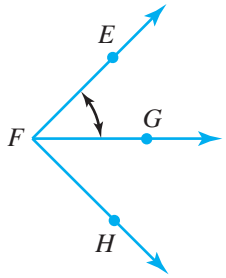
Whenever three letters are used to name an angle, the vertex must be written as the middle letter. It is not correct to name the above angle as $\angle ACB$, since this would imply that C is the vertex rather than B.

NOTE: We can only refer to an angle by its vertex (e.g. $\angle B$) when there are exactly two rays emanating from this point.

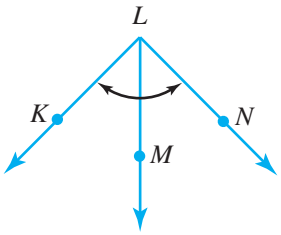
Example

Name the angles indicated in each of the following:

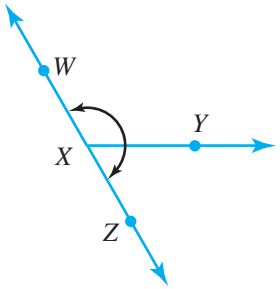
a



b



c



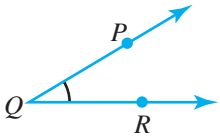
Solutions

a $\angle EFG$ (or $\angle GFE$ or $E\hat{F}G$ or $G\hat{F}E$)
b $\angle KLN$ (or $\angle NLK$ or $K\hat{L}N$ or $N\hat{L}K$)
c $\angle WXZ$ (or $\angle ZXW$ or $W\hat{X}Z$ or $Z\hat{X}W$)

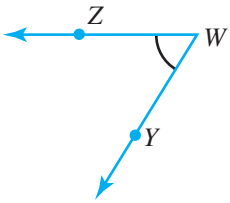
Exercise 9.2

1 In each of the following:
i state the vertex **ii** name the angle

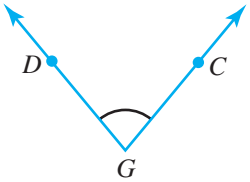
a



b



c



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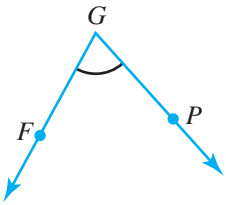

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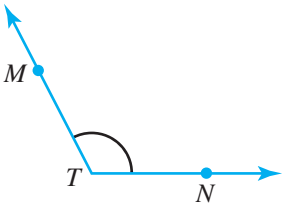

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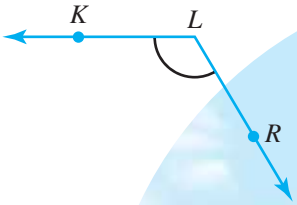

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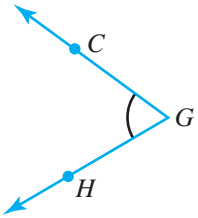

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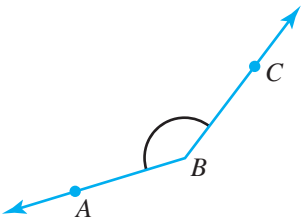
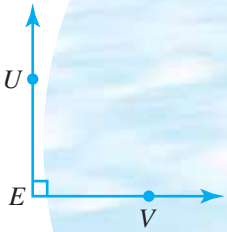


i



2 Name the vertex in each of these angles.

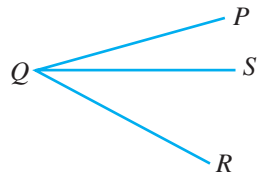
a $\angle MLN$ **b** $\angle PZT$ **c** $\angle KRL$ **d** $\angle UCP$

3 Name the two rays which form these angles.

a $\angle CEF$ **b** $\angle XWY$ **c** $\angle PRN$ **d** $\angle EFW$

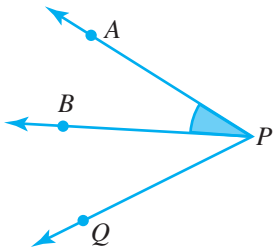
Consolidation

4 Name the three acute angles in this figure.

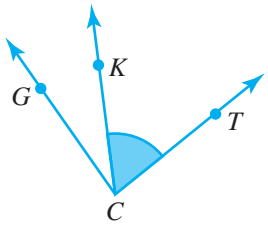


5 Name the shaded angle in each of these.

a



b



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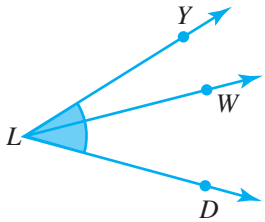

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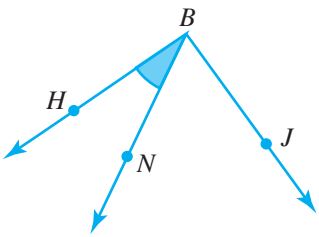

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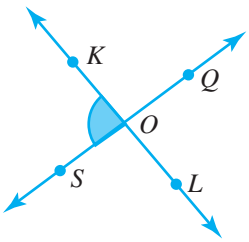

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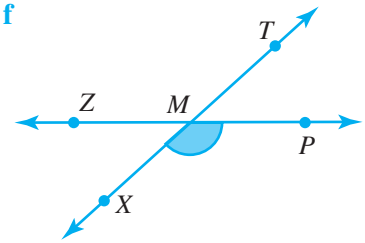

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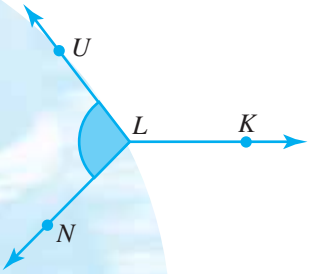

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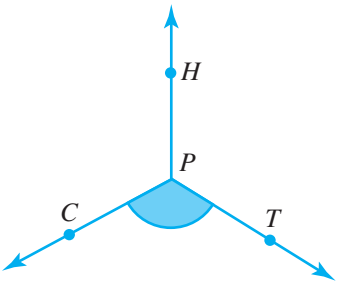

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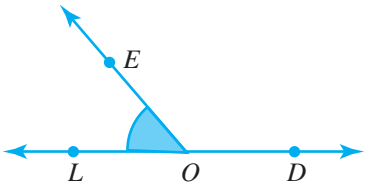

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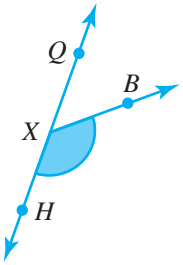

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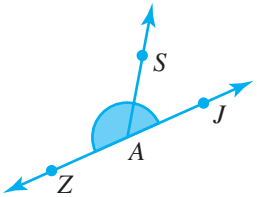
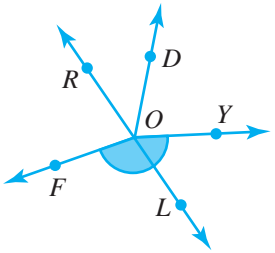
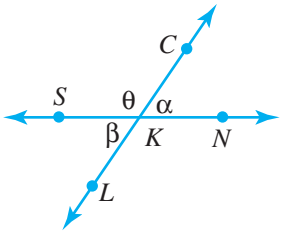


l

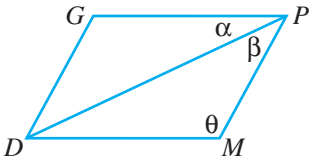


6 Name the angles marked α, β and θ in each of these.

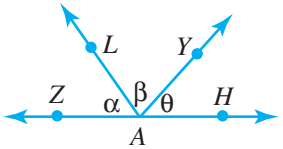
a



b

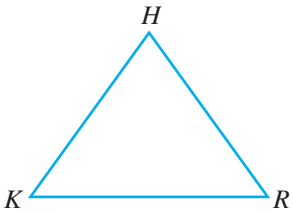


c

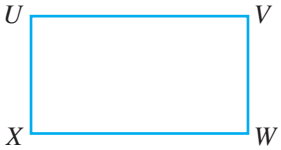


7 Name all of the angles in each figure using three letters.

a



b



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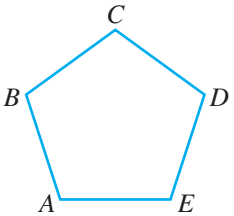
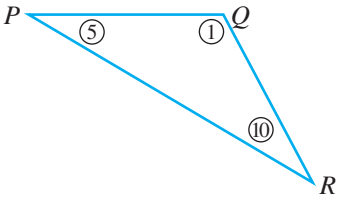
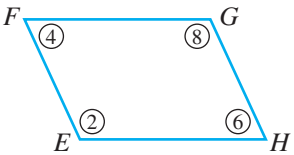
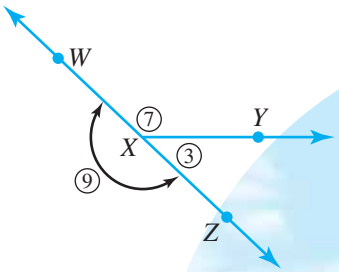


8 Write down the number 1–10 which corresponds to each angle below.



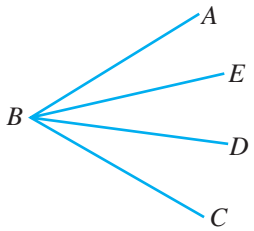




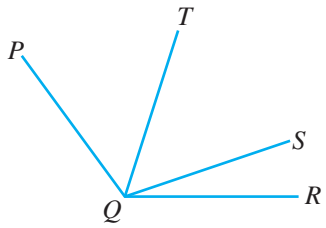
a ∠*GHE*	**b** ∠*YXZ*	**c** ∠*PQR*	**d** ∠*FEH*	**e** ∠*PRQ*
f ∠*WXZ*	**g** ∠*HGF*	**h** ∠*RPQ*	**i** ∠*YXW*	**j** ∠*EFG*

Further applications

9 a In the diagram, $\angle ABE = 15°$, $\angle ABD = 80°$ and $\angle EBC = 75°$. Find the size of $\angle DBC$.



b In the diagram, $\angle PQT = 30°$, $\angle PQS = 75°$ and $\angle PQR = 125°$. Find the size of $\angle TQR$.



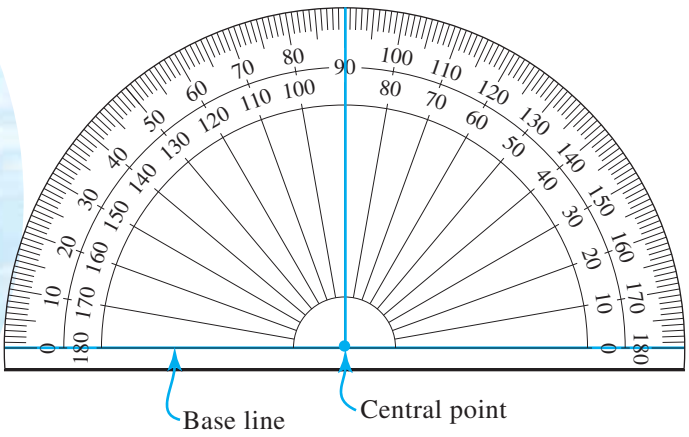
9.3 Measuring and drawing angles

The unit of measurement for angles is the **degree**. The symbol for the degree is a small raised circle. For example, an angle of 60 degrees would be written as 60°. If a ray was rotated through one complete revolution about a fixed point, we would say that the ray had been rotated through an angle of 360°.

Historically, the ancient Babylonians believed that the Earth took 360 days to make one complete revolution about the sun. Hence, they defined the calendar year as 360 days and one day as $\frac{1}{360}$th of a year. It is from this measurement that we define one revolution as 360° and one degree as $\frac{1}{360}$th of a revolution.

The instrument used to measure an angle is called a **protractor**. It has two scales marked on it—an inner scale and an outer scale. To avoid confusion with the different scales it is best to estimate the size of an angle before measuring.

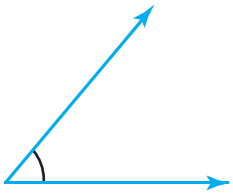
To measure the size of an angle:
- **place the central point of the protractor on the vertex of the angle**
- **line up the base line of the protractor with the lower ray of the angle**
- **measure the angle from the base line using the scale which begins at 0°.**



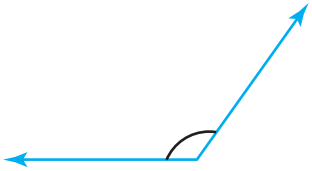
Example

Measure each of these angles.

a

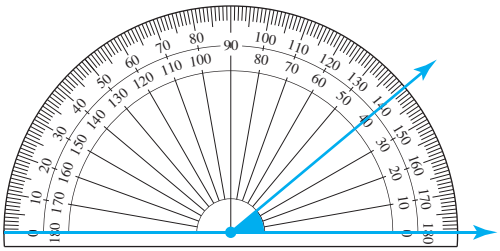


b



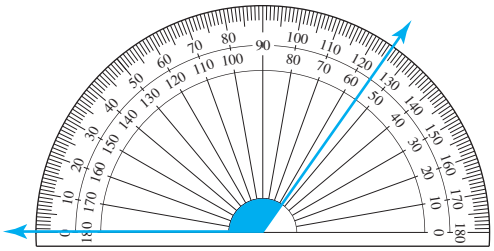
Solutions

a



The angle measures 40° using the inner scale.

b

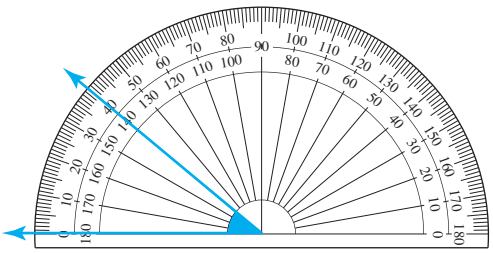


The angle measures 125° using the outer scale.

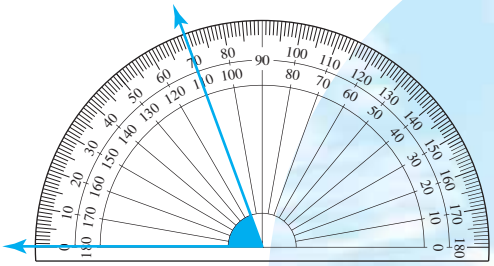
Exercise 9.3

1 Write down the size of the angle shown in each of these.

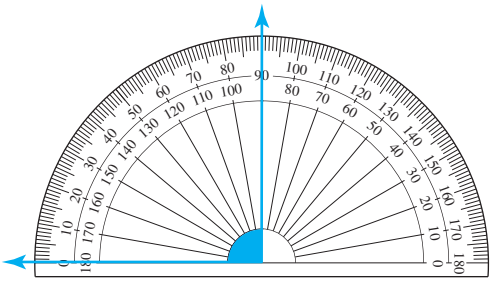
a



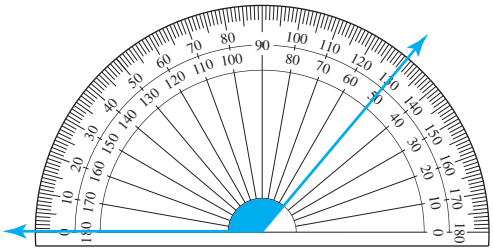
b



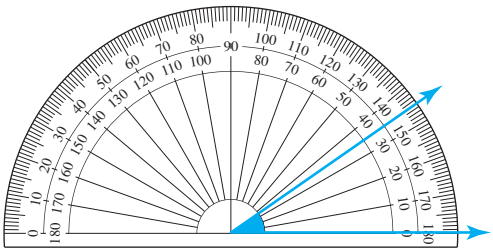
c



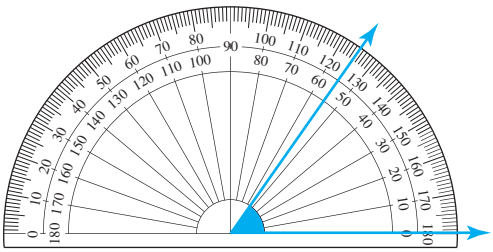
d



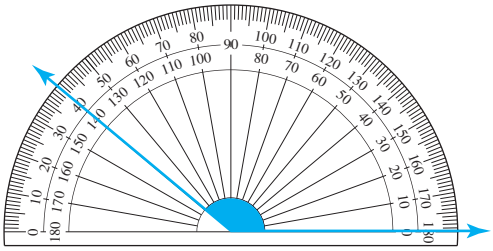
e



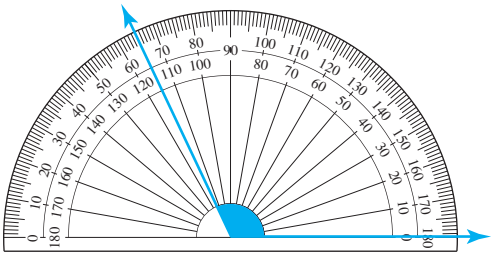
f



g

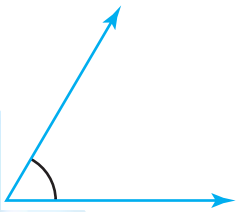


h

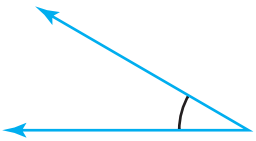


2 Use a protractor to measure the size of each angle. Give your answers correct to the nearest 5°.

a



b



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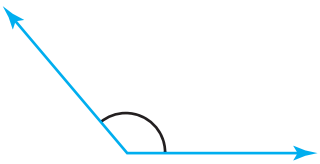
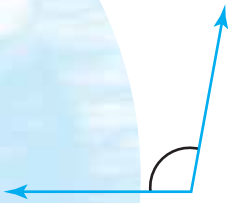
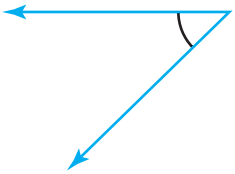


d



e



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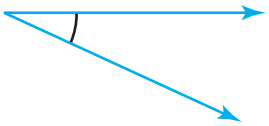

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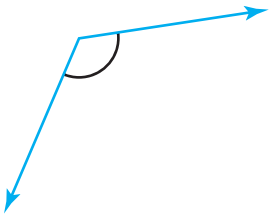

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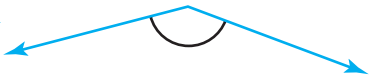
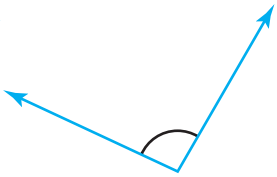


i



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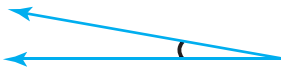
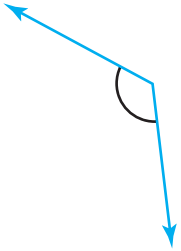


k



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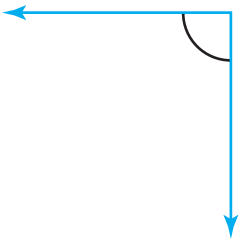


■ Consolidation

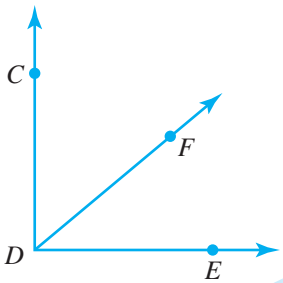
3 Use a protractor to draw an angle whose size is:

a 30° **b** 70° **c** 120° **d** 160°
e 90° **f** 115° **g** 65° **h** 135°

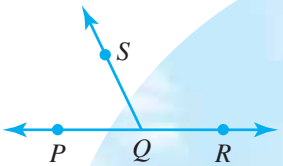
4 Draw an angle whose size is:

a 48° **b** 76° **c** 104° **d** 152°

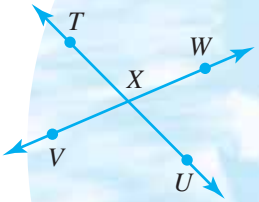
5 a **i** Measure the size of $\angle CDF$ and $\angle FDE$.
ii What is the sum of these angles?



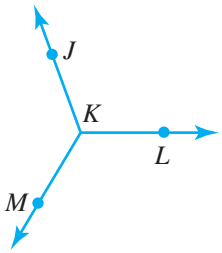
b **i** Measure the size of $\angle PQS$ and $\angle RQS$.
ii What is the sum of these angles?



c **i** Measure the size of $\angle TXW$ and $\angle VXU$.
ii What relationship exists between these angles?

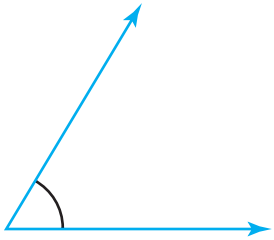


d **i** Measure the size of $\angle JKL$, $\angle LKM$ and $\angle MKJ$.
ii What is the sum of these angles?

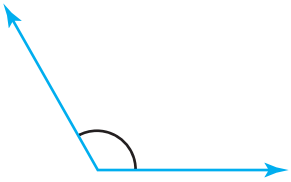


6 Estimate the size of each angle, correct to the nearest 10°. Check your answers by measuring each angle with a protractor.

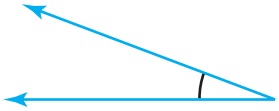
a



b

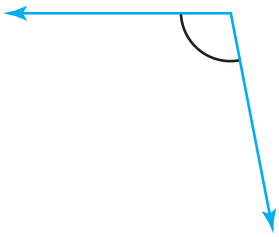


c

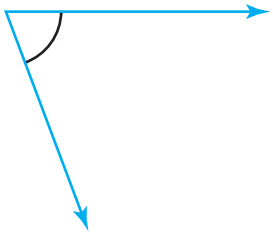


d

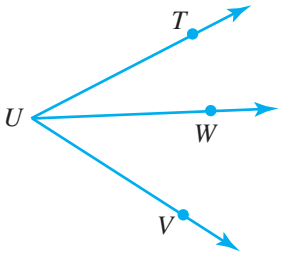
e



f

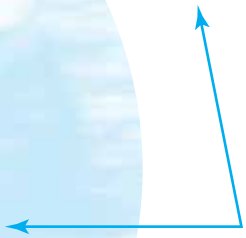


7 Check by direct measurement that $\angle TUW + \angle WUV = \angle TUV$.

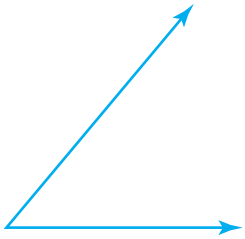


8 Bisect each of these angles using only a ruler and protractor.

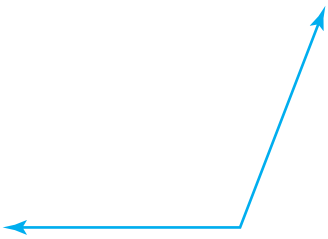
a



b



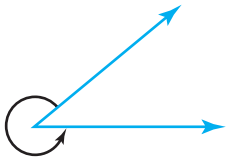
c



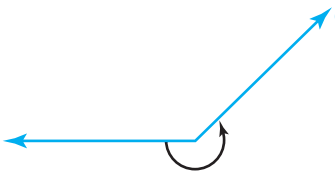
Further applications

9 Find the size of the reflex angle in each of the following. (Hint: Measure the size of the smaller angle first.)

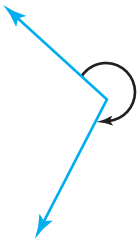
a



b



c

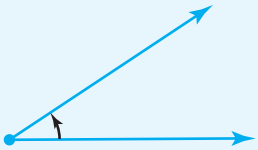
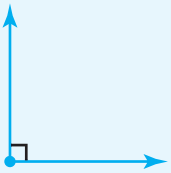
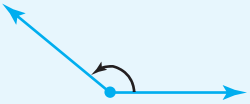
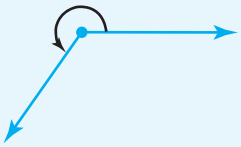


10 Use your protractor to draw an angle of:

a 270° **b** 210° **c** 315° **d** 305°

9.4 Classification of angles

Angles can be classified according to their size. There are six different types of angles.

Acute angle	Right angle	Obtuse angle
 Between 0° and 90°	 Equal to 90°	 Between 90° and 180°
Straight angle	**Reflex angle**	**Revolution**
Equal to 180°	 Between 180° and 360°	Equal to 360°

Example 1

Classify an angle whose size is:

a 130° **b** 360° **c** 55° **d** 90° **e** 217° **f** 180°

Solutions

a Obtuse angle **b** Revolution **c** Acute angle
d Right angle **e** Reflex angle **f** Straight angle

Example 2

Find the number of degrees in:

a $\frac{1}{10}$ of a right angle

b $\frac{3}{5}$ of a revolution

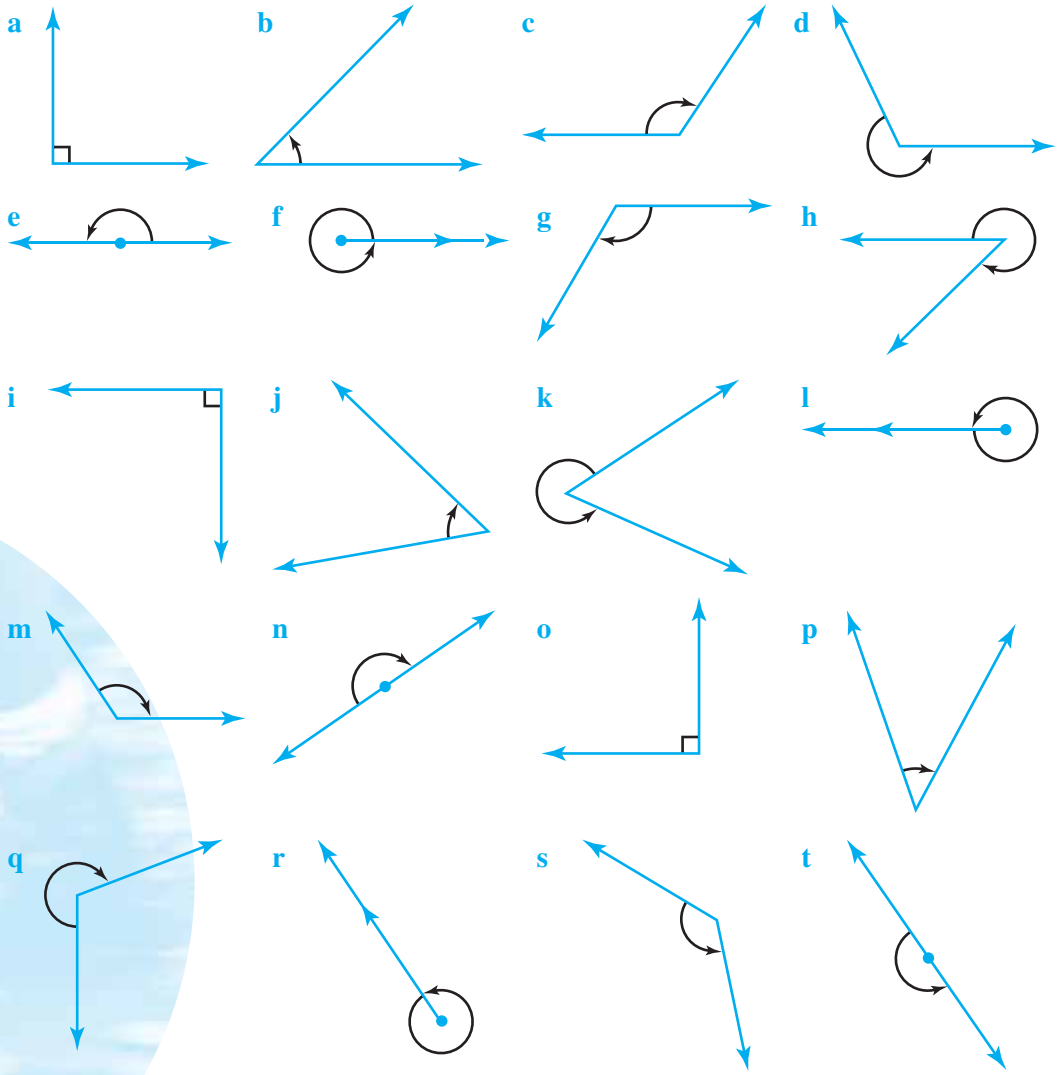
Solutions

a $\frac{1}{\cancel{10}} \times \cancel{90}^{\circ} = 9^{\circ}$

b $\frac{3}{\cancel{5}_{1}} \times \cancel{360}^{72\,\circ} = 216^{\circ}$

Exercise 9.4

1 Classify each of the following angles.



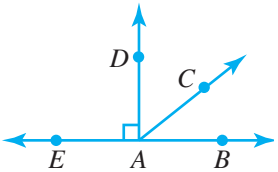
2 Classify these angles according to their size.

a 20°	**b** 160°	**c** 300°	**d** 5°	**e** 90°
f 89°	**g** 270°	**h** 99°	**i** 185°	**j** 180°
k 1°	**l** 179°	**m** 355°	**n** 47°	**o** 360°

■ Consolidation

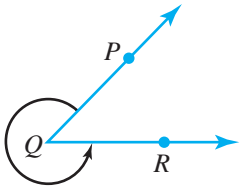
3 What kind of angle is:

a $\angle DAB$? **b** $\angle EAC$? **c** $\angle CAD$?
d $\angle EAB$? **e** $\angle BAC$? **f** $\angle EAD$?

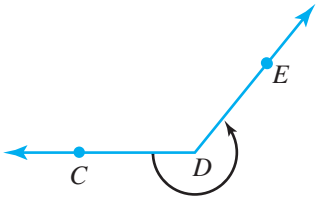


4 Name the reflex angle in each of these.

a



b



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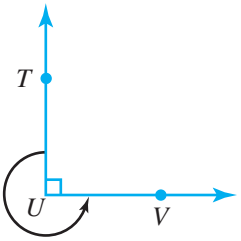


5 Find the number of degrees in:

a $\frac{1}{2}$ of a right angle **b** $\frac{1}{3}$ of a straight angle **c** $\frac{1}{4}$ of a revolution

d $\frac{1}{3}$ of a right angle **e** $\frac{1}{4}$ of a straight angle **f** $\frac{1}{2}$ of a revolution

g $\frac{1}{5}$ of a straight angle **h** $\frac{1}{10}$ of a right angle **i** $\frac{1}{3}$ of a revolution

j $\frac{2}{3}$ of a straight angle **k** $\frac{4}{9}$ of a right angle **l** $\frac{3}{4}$ of a revolution

m $\frac{3}{5}$ of a straight angle **n** $\frac{5}{6}$ of a right angle **o** $\frac{5}{12}$ of a revolution

6 What kind of angle is $\angle ABC$ if:

a $\frac{1}{2}\angle ABC = 55°$? **b** $\frac{1}{4}\angle ABC = 20°$? **c** $\frac{1}{2}\angle ABC = 180°$?

d $\frac{1}{3}\angle ABC = 60°$? **e** $\frac{1}{2}\angle ABC = 45°$? **f** $\frac{1}{5}\angle ABC = 40°$?

■ Further applications

7 What kind of angle(s) could be formed by adding:

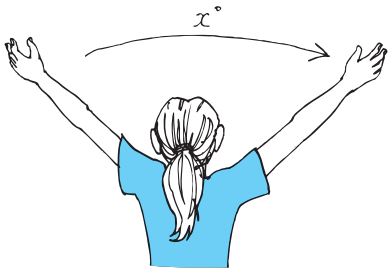
a two acute angles? **b** two obtuse angles?

c an acute angle and an obtuse angle?
d two right angles?
e two straight angles?
f a straight angle and an obtuse angle?

8 What kind of angle(s) could be formed by subtracting:
a a right angle from an obtuse angle?
b an acute angle from an obtuse angle?
c a right angle from a reflex angle?
d a reflex angle from a revolution?

TRY THIS **Angular vision**

What is the widest range of view that you can see when someone is walking towards you? In groups of three, ask your two friends to stand in different positions and use your arms to note the positions where you can just see them. Find this angle ($x°$).

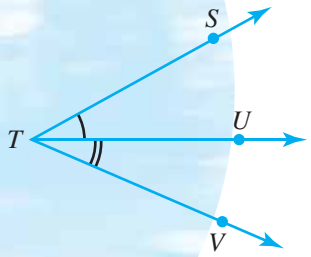


9.5 Pairs of angles

Much of our work in geometry concerns the relationship between a pair of angles. We now define a number of important terms relating to pairs of angles.

Adjacent angles

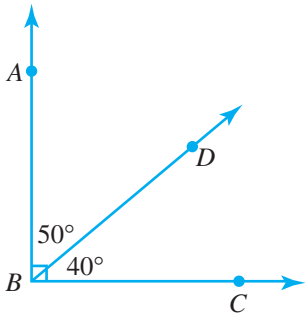
A pair of angles are *adjacent* if they:
- **have a common vertex, and**
- **have a common ray, and**
- **lie on opposite sides of the common ray.**



For example, $\angle STU$ is adjacent to $\angle UTV$ because:
i T is a common vertex, and
ii TU is a common ray, and
iii the angles lie on opposite sides of TU.

Complementary angles

- **A pair of angles that have a sum of 90° are called *complementary angles*.**
- **Each individual angle is the *complement* of the other.**

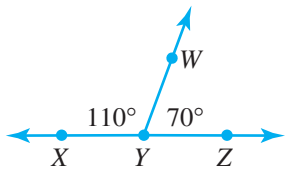


For example, $\angle ABD + \angle DBC = 50° + 40°$
$= 90°$

$\therefore$ $\angle ABD$ and $\angle DBC$ are complementary angles.
We say that 50° is the complement of 40° and vice-versa.

Supplementary angles

- **A pair of angles that have a sum of 180° are called *supplementary angles*.**
- **Each individual angle is the *supplement* of the other.**

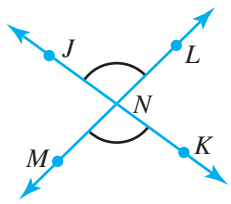


For example, $\angle XYW + \angle WYZ = 110° + 70°$
$= 180°$

$\therefore$ $\angle XYW$ and $\angle WYZ$ are supplementary angles.
We say that 110° is the supplement of 70° and vice-versa.

Vertically opposite angles

- **A pair of *vertically opposite angles* is formed by the intersection of two straight lines.**
- **Vertically opposite angles are equal.**



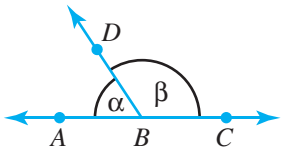
For example, JK and ML are straight lines intersecting at N.

Now, $\angle JNL$ and $\angle MNK$ are vertically opposite angles,
$\therefore$ $\angle JNL = \angle MNK$

Example 1

State whether α and β are adjacent angles in each of the following.

a



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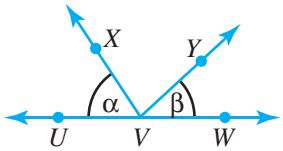

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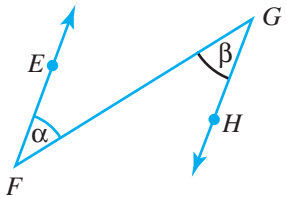


Solutions

a The angles have a common vertex (B), a common ray (BD) and lie on opposite sides of the common ray, ∴ α and β are adjacent angles.

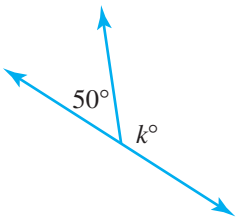
b The angles do not have a common ray, ∴ α and β are not adjacent angles.

c The angles do not have a common vertex, ∴ α and β are not adjacent angles.

Example 2

Classify each pair of angles as complementary, supplementary or vertically opposite, then find the value of the pronumeral.

a



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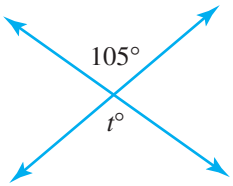

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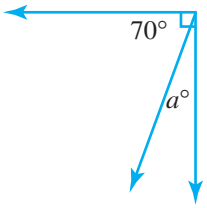


Solutions

a Adjacent angles on a straight line are supplementary.

$\therefore k + 50 = 180$
$k = 180 - 50$
$\therefore k = 130$

b Vertically opposite angles are formed by the intersection of 2 straight lines, $\therefore t = 105$.

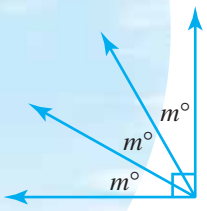
c Adjacent angles in a right angle are complementary.

$\therefore a + 70 = 90$
$a = 90 - 70$
$\therefore a = 20$

Example 3

Find the value of each pronumeral.

a



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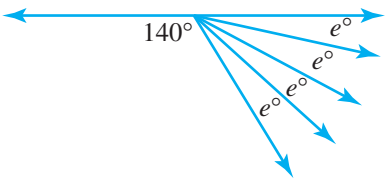

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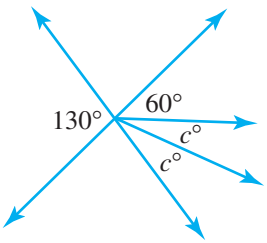


Solutions

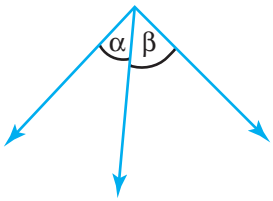
a $m + m + m = 90$
$m = 90 \div 3$
$\therefore m = 30$

b $140 + e + e + e + e = 180$
$e = (180 - 140) \div 4$
$e = 40 \div 4$
$\therefore e = 10$

c $c + c + 60 = 130$
$c = (130 - 60) \div 2$
$c = 70 \div 2$
$\therefore c = 35$

Exercise 9.5

1 For each of the following, state whether the angles α and β are adjacent. If they are not adjacent, give a reason.

a


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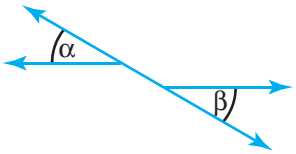

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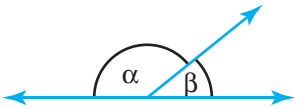

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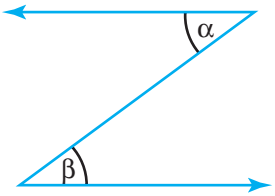

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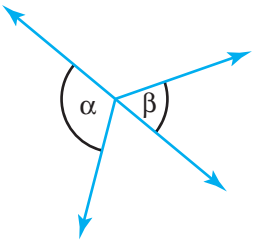

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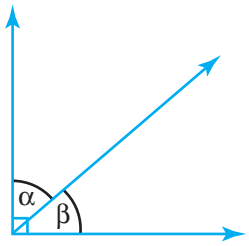

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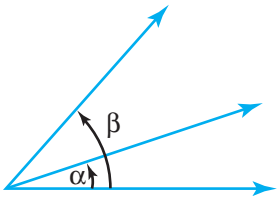

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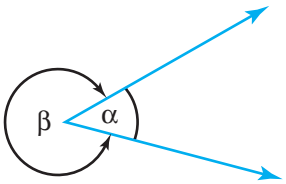

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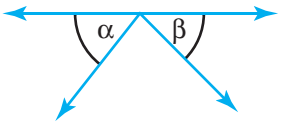

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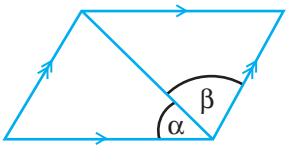

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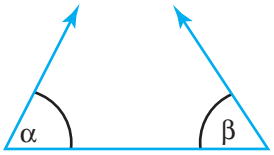

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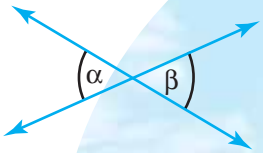

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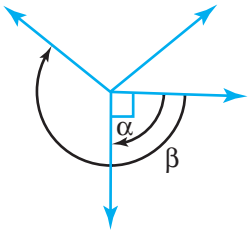

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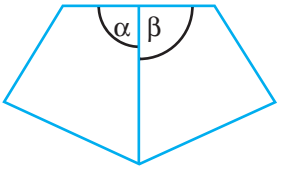

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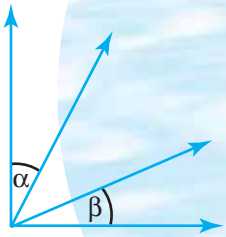
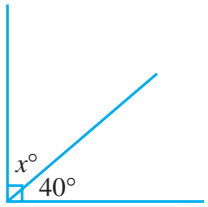


2 Find the value of the pronumeral in each of these.

a



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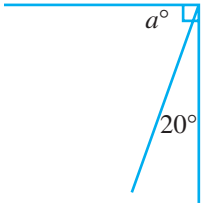

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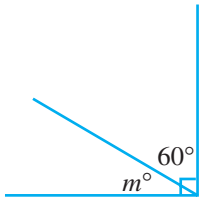

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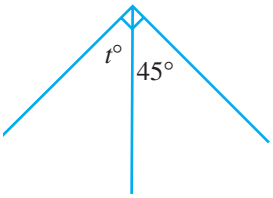

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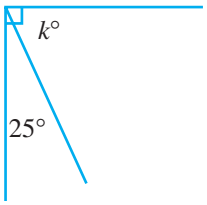

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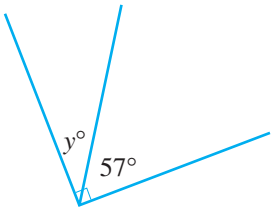
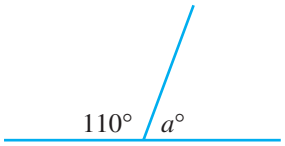


3 Find the value of the pronumeral in each of these.

a



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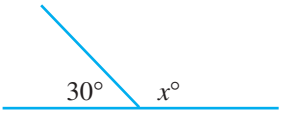

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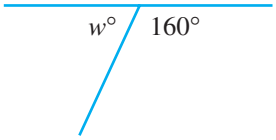

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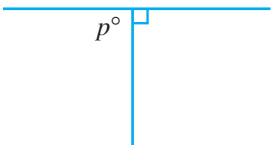

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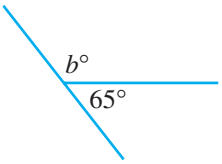

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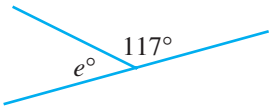
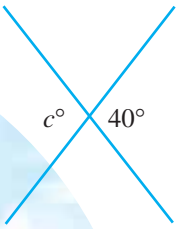


4 Find the value of the pronumeral in each of these.

a



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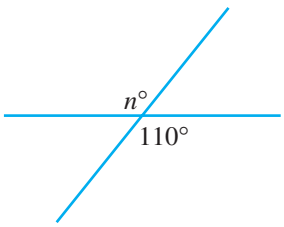

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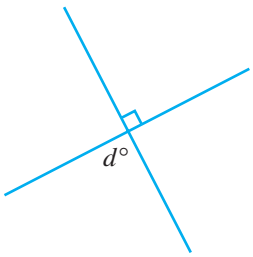

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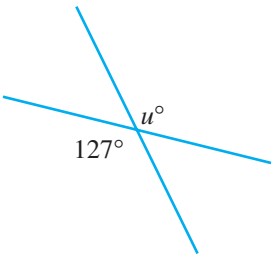
e



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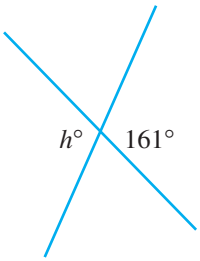
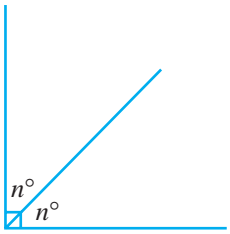


Consolidation

5 Find the value of the pronumeral in each of these.

a



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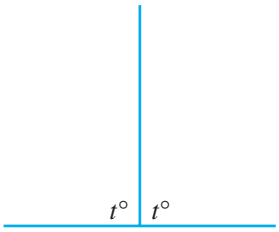

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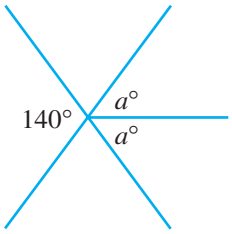

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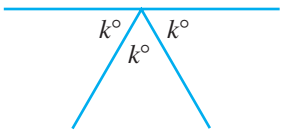

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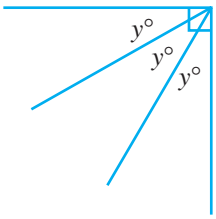

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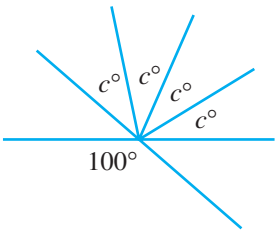

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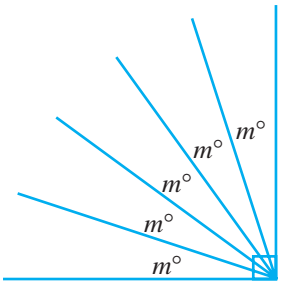

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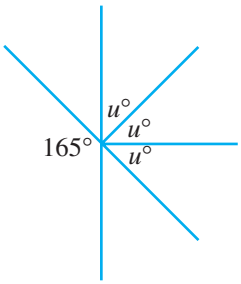

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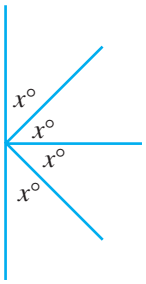


6 State the complement of:

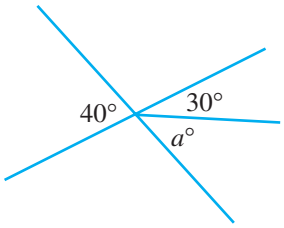
a 70°	**b** 10°	**c** 15°	**d** 35°
e 41°	**f** 16°	**g** 39°	**h** 72°

7 State the supplement of:

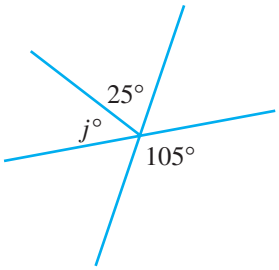
a 50°	**b** 140°	**c** 135°	**d** 75°
e 36°	**f** 102°	**g** 84°	**h** 99°

8 Find the value of the pronumeral in each of these.

a



b



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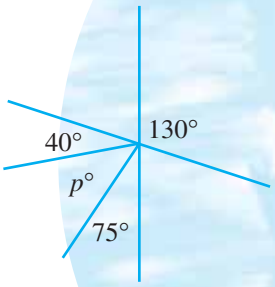
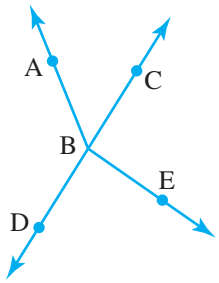


9 Is $\angle ABC$ vertically opposite to $\angle DBE$? Explain.



10 Find a pair of complementary angles such that one angle is:

a twice the size of the other
b 10° more than the other
c 20° less than the other
d one-quarter the size of the other

11 Find a pair of supplementary angles such that one angle is:

a five times as large as the other
b 80° more than the other
c 110° less than the other
d half the size of the other

■ Further applications

12 Find:

a half the complement of 40°
b half the supplement of 50°
c one-third of the complement of 24°
d one-quarter of the supplement of 64°
e the angle which is 12° less than the supplement of 55°
f the angle which is 15° greater than the complement of 80°
g the complement of the complement of 30°
h the supplement of the supplement of 70°
i the complement of the supplement of 100°
j the supplement of the complement of 60°

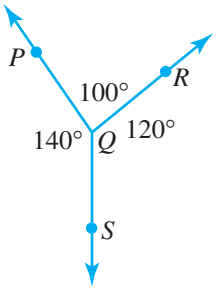
13 Find the size of the angle which is:

a vertically opposite to the complement of 23°
b supplementary to the angle which is vertically opposite to a 58° angle
c vertically opposite to the supplement of 119°
d complementary to the angle which is vertically opposite to a 13° angle

9.6 Angles at a point

The sum of two or more angles with a common vertex which form a revolution is 360°.

For example,
$\angle PQR + \angle RQS + \angle SQP$
$= 100° + 120° + 140°$
$= 360°$

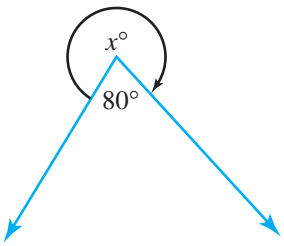


EG + S

Example

Find the value of each pronumeral.

a



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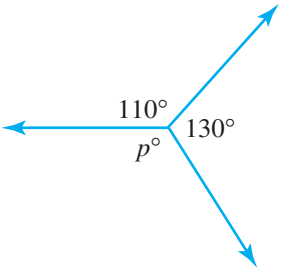

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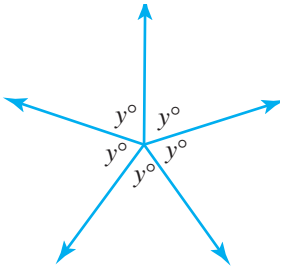


Solutions

a $x + 80 = 360$
$x = 360 - 80$
$\therefore x = 280$

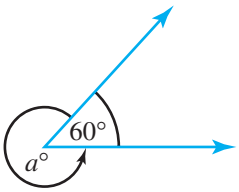
b $p + 110 + 130 = 360$
$p + 240 = 360$
$p = 360 - 240$
$\therefore p = 120$

c $y + y + y + y + y = 360$
$y = 360 \div 5$
$\therefore y = 72$

Exercise 9.6

1 Find the value of the pronumeral in each of these.

a



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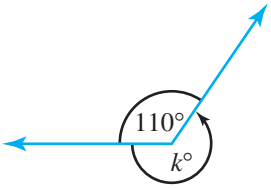

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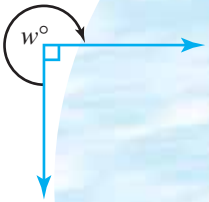

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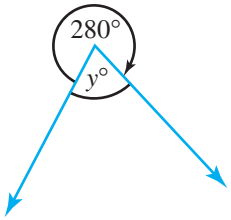

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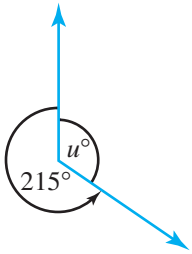

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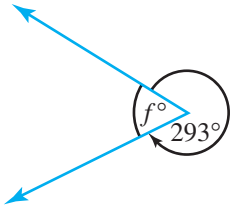
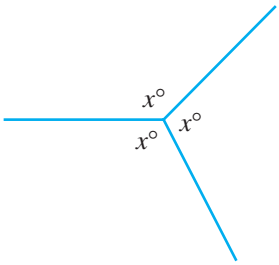


2 Find the value of the pronumeral in each of these.

a



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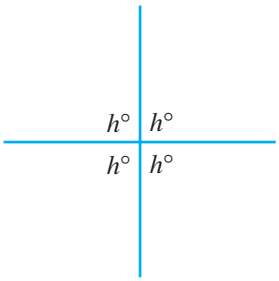

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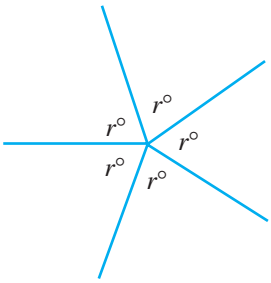
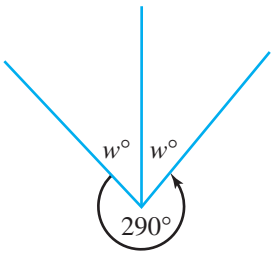


Consolidation

3 Find the value of w in each of these.

a



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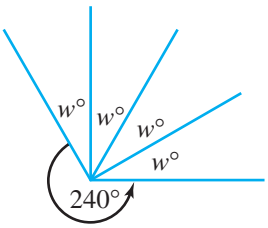

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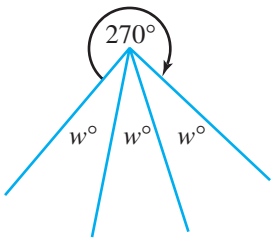
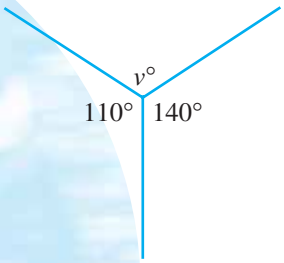


4 Find the value of the pronumeral in each of these.

a



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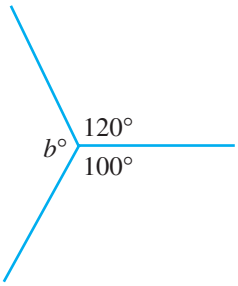

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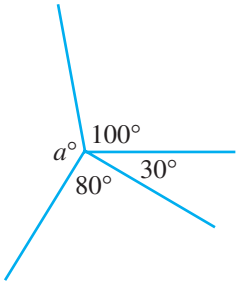

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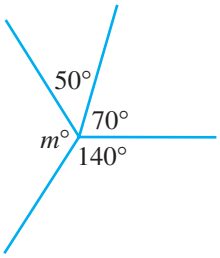

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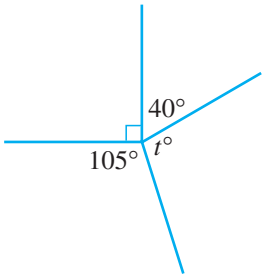

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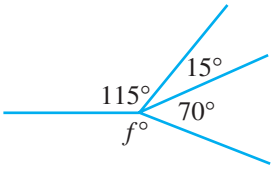

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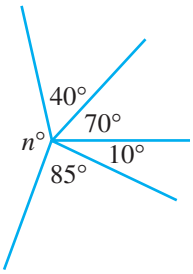

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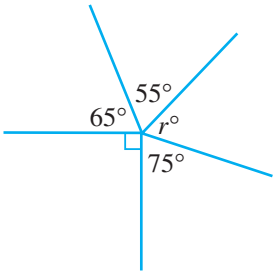

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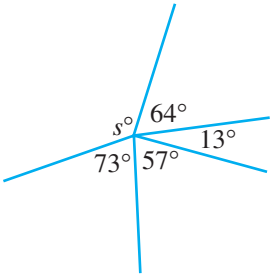
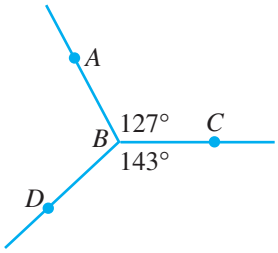
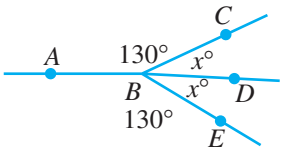


Further applications

5 Prove that AB is perpendicular to BD.



6 Prove that $\angle ABD$ is a straight angle.



7 A pair of angles that have a sum of 360° are called **conjugate angles**. Find:

a the conjugate of 100°
b the conjugate of 215°
c the conjugate of the complement of 40°
d the conjugate of the supplement of 75°
e the complement of the conjugate of 290°
f the supplement of the conjugate of 215°

8 Find the conjugate of the supplement of the complement of 40°.

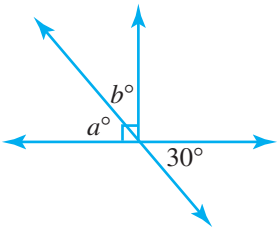
9.7 Miscellaneous questions on angles

EG+S

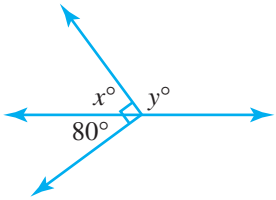
Example

Find the value of each pronumeral.

a



b



Solutions

a **i** Vertically opposite angles are equal, $\therefore a = 30$

ii Adjacent angles in a right angle are complementary,
$a + b = 90$
$30 + b = 90$
$b = 90 - 30$
$\therefore b = 60$

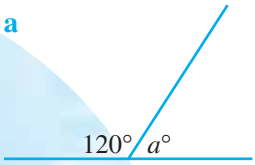
b **i** Adjacent angles in a right angle are complementary,
$x + 80 = 90$
$x = 90 - 80$
$\therefore x = 10$

ii Adjacent angles on a straight line are supplementary,
$x + 10 = 180$
$x = 180 - 10$
$\therefore x = 170$

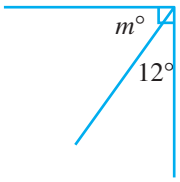
Exercise 9.7

1 Find the value of the pronumeral in each of these.

a



b



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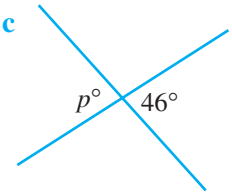

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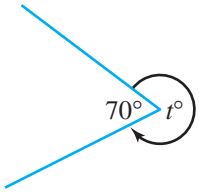

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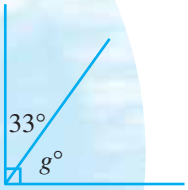

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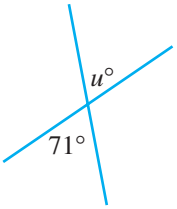

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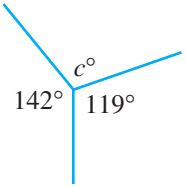

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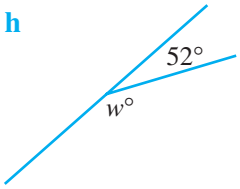

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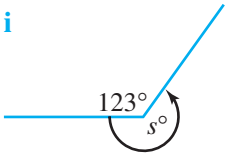

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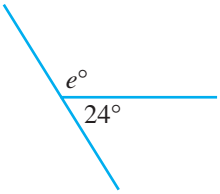

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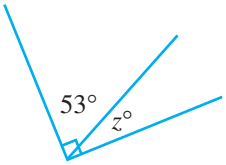

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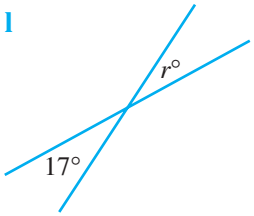

■ Consolidation

2 Find the value of the pronumeral in each of the following.

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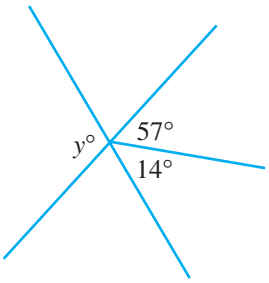

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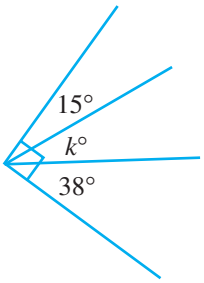

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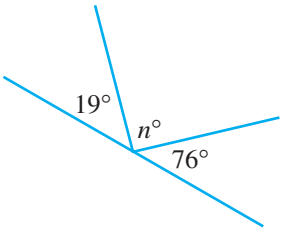

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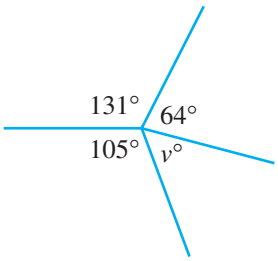

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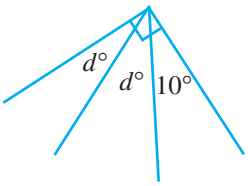

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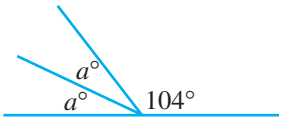

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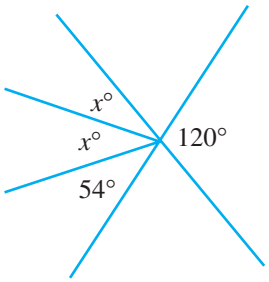

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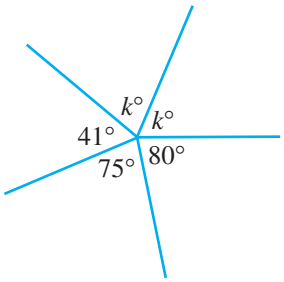

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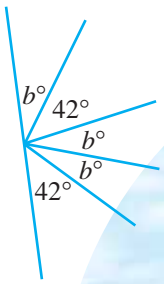

3 Find the value of all pronumerals in each of these.

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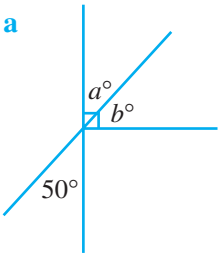

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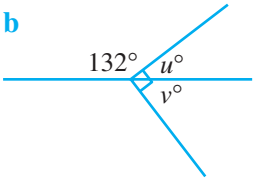

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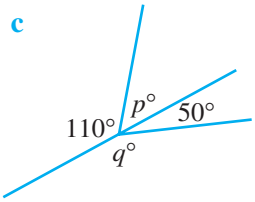

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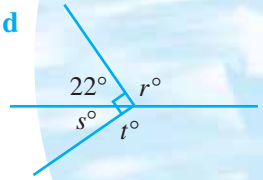

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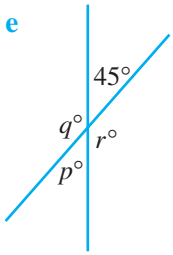

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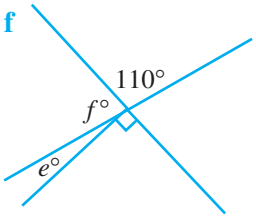

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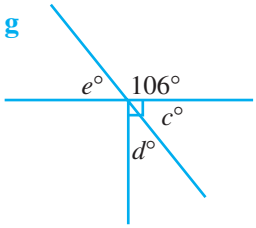

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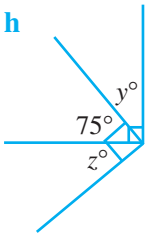

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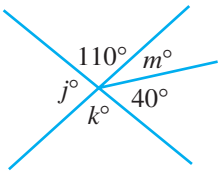

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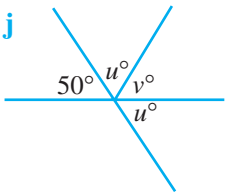

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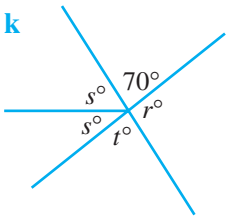

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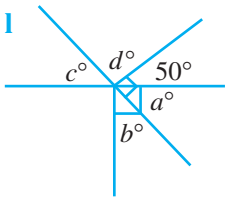

■ Further applications

4 Find the value of all pronumerals in each of the following.

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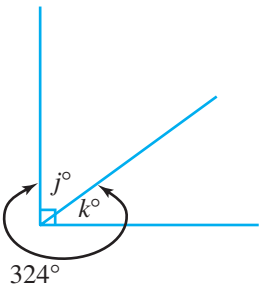

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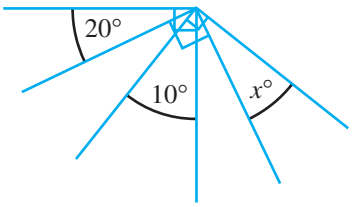

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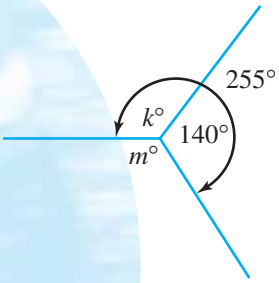

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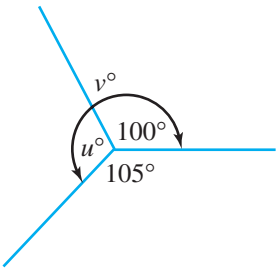


TRY THIS Leaning towers

1 When will the Leaning Tower of Pisa fall over? You will need a milk carton which has been filled with soil or sand. Your task is to find at which angle the carton will fall over.

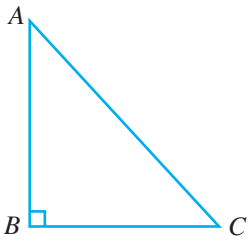
Place an object under the tower to produce the lean necessary.

2 Look up details in your library about the current lean on the Tower of Pisa. Can you predict when it will fall over?

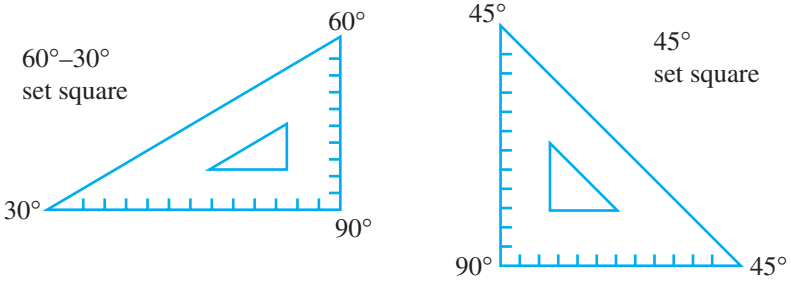
9.8 Perpendicular and parallel lines

Perpendicular lines

Perpendicular lines are lines that intersect at right angles. That is, they meet at an angle of 90°. A small box is drawn near the point of intersection to indicate that the lines are perpendicular. The notation ⊥ means 'is perpendicular to'. For example, in the right-angled triangle *ABC*, $AB \perp BC$ (i.e. *AB* is perpendicular to *BC*).



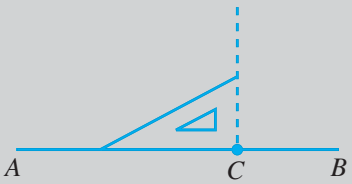
A **set square** can be used to determine whether or not two lines are perpendicular. There are two standard set squares: the 60°–30° set square and the 45° set square.



Set squares can also be used to construct a pair of perpendicular lines.

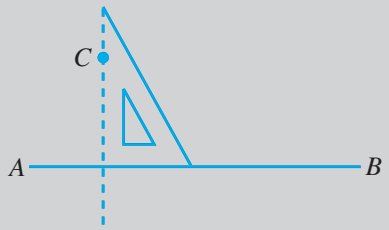
To construct a line perpendicular to *AB* at a point *C* on the line:

- **place one edge of the set square along the line *AB* with the right angle at *C***
- **draw a line along the edge perpendicular to *AB*.**



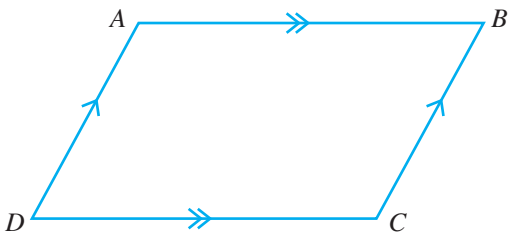
To construct a line perpendicular to *AB* at a point *C* which is not on the line:

- **place one edge of the set square along the line *AB* so that the edge perpendicular to this passes through *C***
- **draw a line through *C* along the edge perpendicular to *AB*.**



Parallel lines

Parallel lines are lines in the same plane that never meet. The distance between the lines remains constant. That is, it does not change. We say that parallel lines are **equidistant**. Arrowheads are drawn on the lines to indicate that they are parallel. The notation ∥ means 'is parallel to'. For example, in the parallelogram *ABCD*, *AB*∥*DC* and *AD*∥*BC* (i.e. *AB* is parallel to *DC* and *AD* is parallel to *BC*).

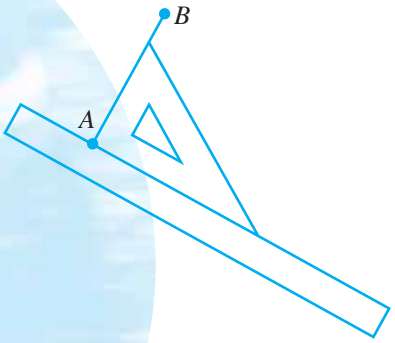


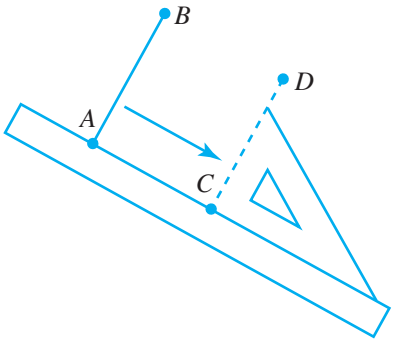
A set square can be used to determine whether or not two lines are parallel. They can also be used to construct a pair of parallel lines.

To construct a line *CD* parallel to *AB*:

- **place one edge of the set square along the edge of a ruler**
- **draw a line *AB* perpendicular to the edge of the ruler**
- **slide the set square along the same edge of the ruler**
- **draw a line *CD* perpendicular to the edge of the ruler.**

NOTE: If two lines are both perpendicular to another line, then they must be parallel to each other.

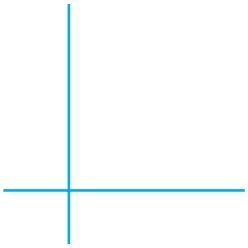




Exercise 9.8

1 Use a set square to check whether each pair of lines is perpendicular.

a



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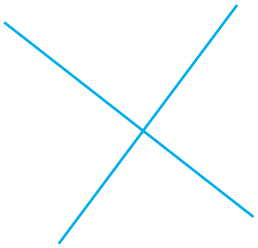

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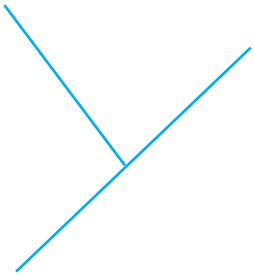

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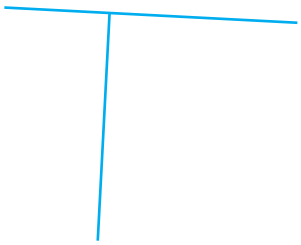
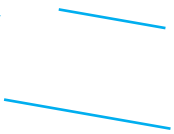


2 Use a ruler and a set square to determine whether each pair of lines is parallel.

a



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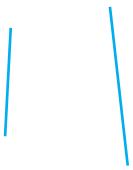
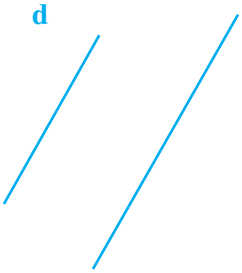
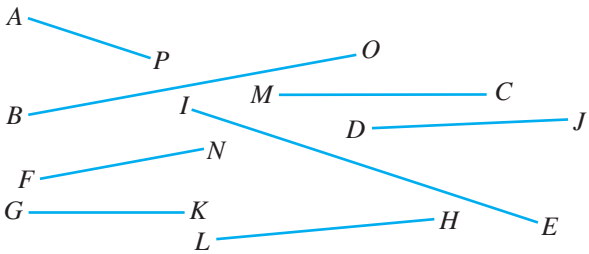


c

d



3 Check this diagram carefully and state which lines are parallel to each other.

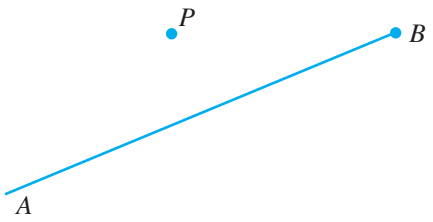


Consolidation

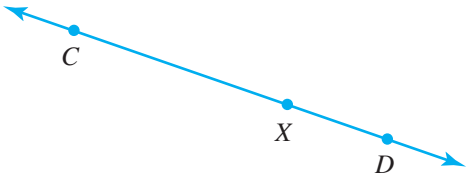
4 Draw a line *AB*. Mark a point *P*, not on this line. Use your ruler and set square to draw another line:

a through *P* parallel to *AB*

b through *P* perpendicular to *AB*.

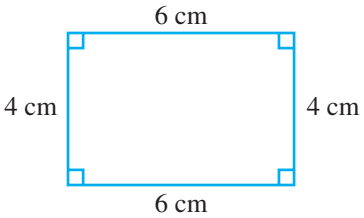


5 Draw a line *CD* and mark a point *X* on this line. Draw another line through *X* perpendicular to *CD*.



6 Use a ruler and set square to draw a square with sides 5 cm long.

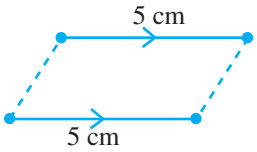
7 Use a ruler and set square to draw a rectangle with sides 6 cm and 4 cm long.



8 **a** Draw a pair of parallel lines 5 cm long.

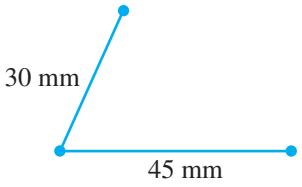
b Join the ends. Are these lines parallel?

c What type of plane figure have you drawn?

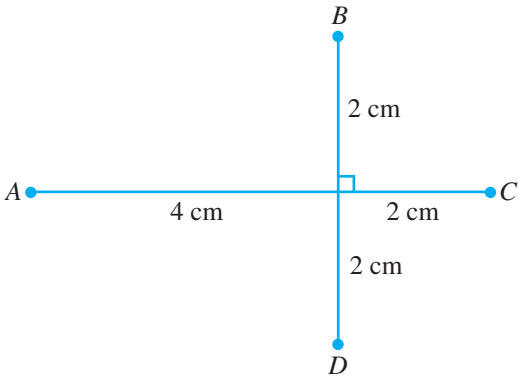


9 **a** Draw 2 lines 45 mm and 30 mm long as shown.

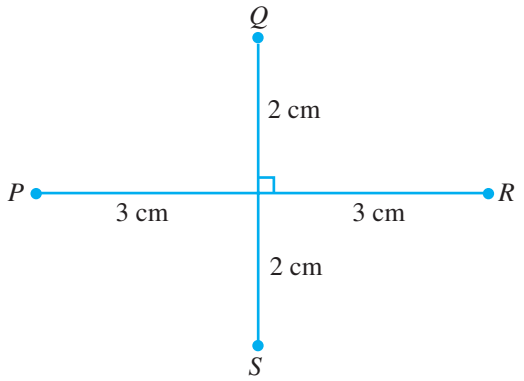
b Draw lines parallel to them to complete a parallelogram.



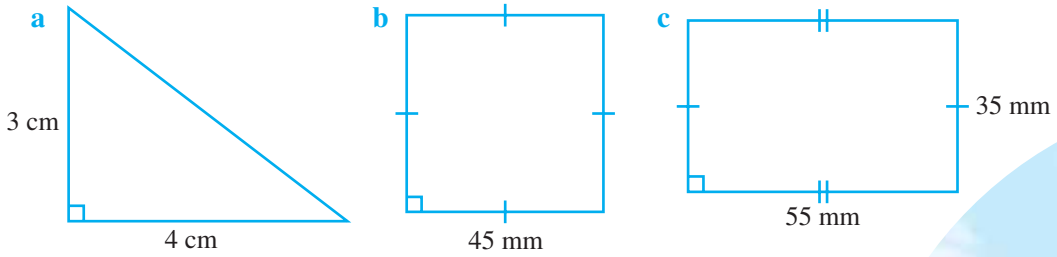
10 **a** Use a ruler and set square to draw this figure.
b Join the points A, B, C and D to form a quadrilateral.
c Does it have an axis of symmetry?
d What type of quadrilateral have you drawn?



11 **a** Use a ruler and set square to draw this figure.
b Join the points P, Q, R and S to form a quadrilateral.
c Measure the lengths of the sides of $PQRS$.
d What type of quadrilateral is $PQRS$?

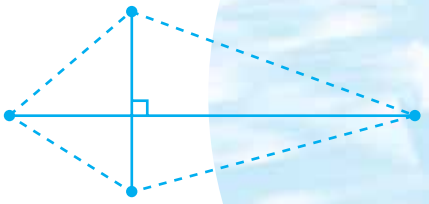


12 Use a ruler and set square to construct these figures to correct size.



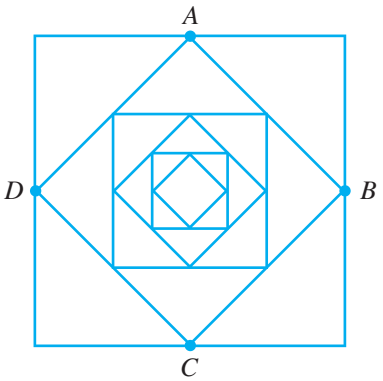
Further applications

13 Use a set square to draw 2 lines that intersect at right angles. Join up the endpoints of the lines to form a quadrilateral.
What type of quadrilateral will you obtain if:
a one of the perpendicular lines is bisected?
b both of the perpendicular lines are bisected?



Consider the cases when the 2 perpendicular lines are:
i the same length **ii** different lengths

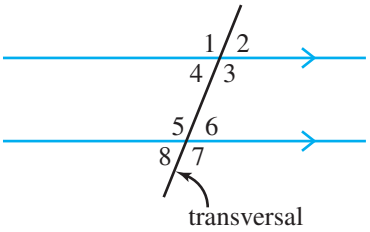
14 Can you copy this construction?
- **a** Start with a square with sides 8 cm long.
- **b** Points A, B, C and D are the midpoints of each side. Join $ABCD$.
- **c** What type of figure is $ABCD$?
- **d** Find the midpoints of the sides of $ABCD$ and join them.
- **e** Keep repeating the process of joining midpoints of sides.



Midpoint means the halfway point.

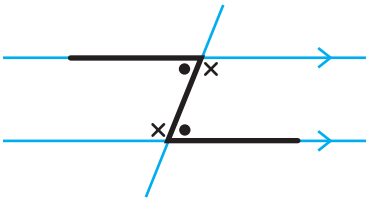
9.9 Angles in parallel lines

A line that cuts two or more parallel lines is called a **transversal**. When a transversal is drawn to cut a pair of parallel lines, eight angles are formed. These angles can be classified into three special pairs of angles: **alternate angles**, **corresponding angles** and **co-interior angles**.



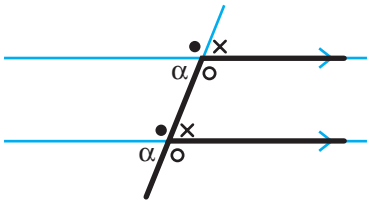
Alternate angles

Alternate angles:
- **lie between the parallel lines and on opposite sides of the transversal**
- **are equal in size**
- **form a Z shape.**



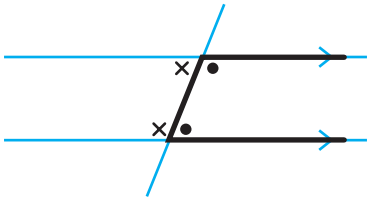
Corresponding angles

Corresponding angles:
- **lie on the same side of the parallel lines and on the same side of the transversal**
- **are equal in size**
- **form a F shape.**



Co-interior angles

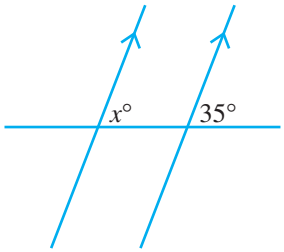
Co-interior angles:
- **lie between the parallel lines and on the same side of the transversal**
- **are supplementary**
- **form a C shape.**



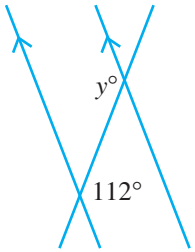
Example 1

State whether the angles shown are alternate, corresponding or co-interior, then find the value of each pronumeral.

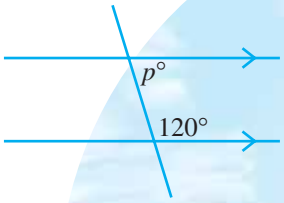
a



b



c



Solutions

a Corresponding angles in parallel lines are equal, $\therefore x = 35$

b Alternate angles in parallel lines are equal, $\therefore y = 112$

c Co-interior angles in parallel lines are supplementary, $\therefore p + 120 = 180$

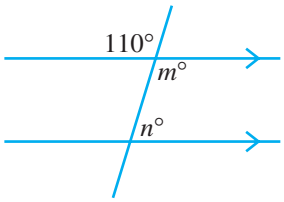
$p = 180 - 120$

$\therefore p = 60$

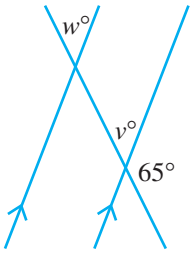
Example 2

Find the value of each pronumeral.

a



b



Solutions

a **i** Vertically opposite angles are equal, $\therefore m = 110$

ii Co-interior angles in parallel lines are supplementary, $\therefore n = 70$

b **i** Adjacent angles on a straight line are supplementary, $\therefore v = 115$

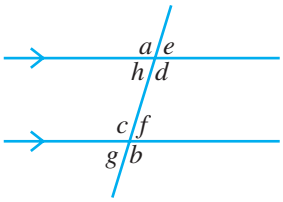
ii Corresponding angles in parallel lines are equal, $\therefore w = 115$

Exercise 9.9

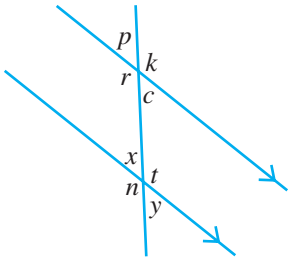
1 In each of the following, write down all pairs of:

i alternate angles **ii** co-interior angles **iii** corresponding angles

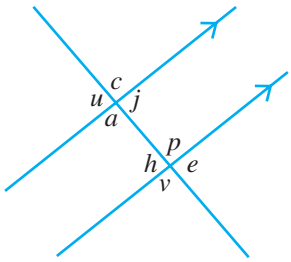
a



b



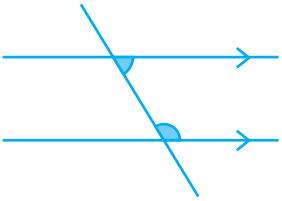
c



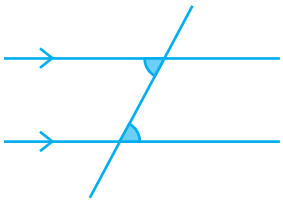
2 State whether the shaded angles are alternate, corresponding or co-interior.

a

b



c



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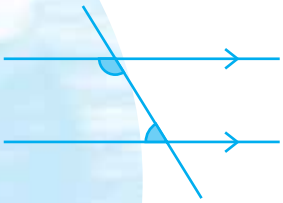

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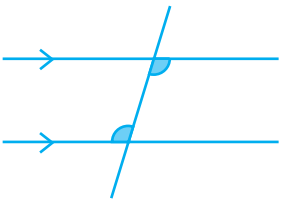

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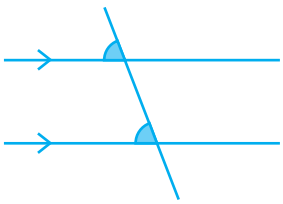

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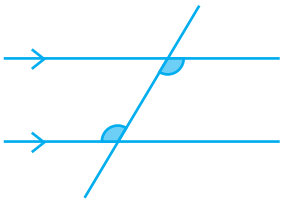

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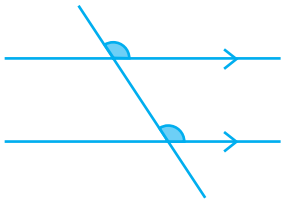

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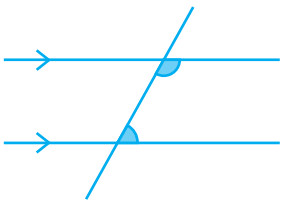

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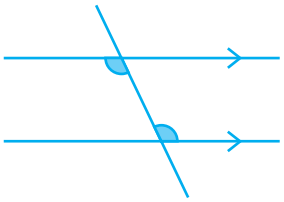

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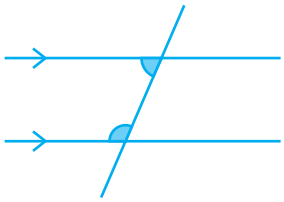

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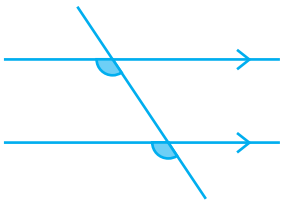
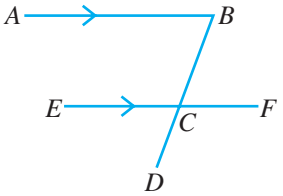


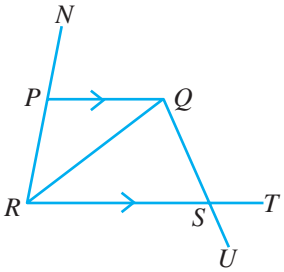
3 Name the angle which is:

a alternate to $\angle BCF$
b corresponding to $\angle ABC$
c co-interior to $\angle ECB$



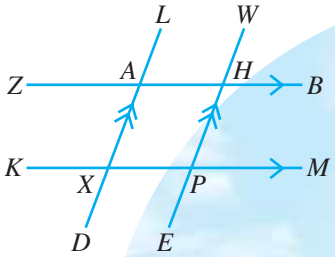
4 Name the angle which is:

a alternate to $\angle QRS$
b co-interior to $\angle RSQ$
c corresponding to $\angle PQS$
d co-interior to $\angle QPR$
e alternate to $\angle PQS$
f corresponding to $\angle PRS$



5 Write down the pair of angles which are:

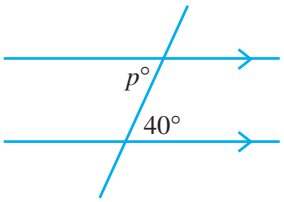
a corresponding to $\angle ZAL$
b co-interior to $\angle HAX$
c alternate to $\angle AHP$
d corresponding to $\angle XPH$
e co-interior to $\angle PHA$
f alternate to $\angle XAH$
g co-interior to $\angle AXP$
h corresponding to $\angle KXA$
i alternate to $\angle XPH$
j corresponding to $\angle LAH$
k alternate to $\angle AXP$
l co-interior to $\angle HPX$



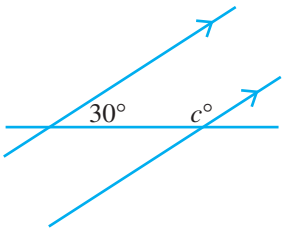
Consolidation

6 Find the value of the pronumeral in each of these.

a



b



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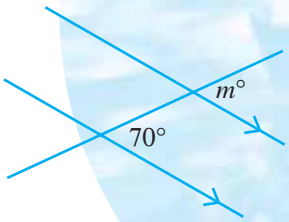

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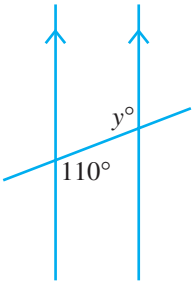

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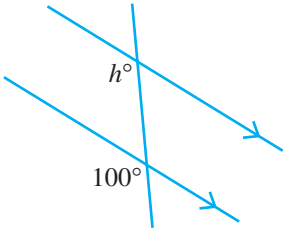

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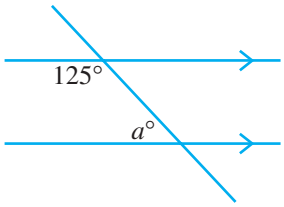

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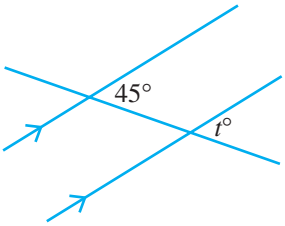

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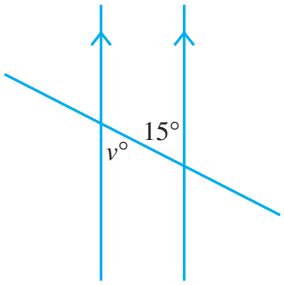

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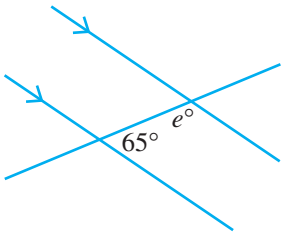

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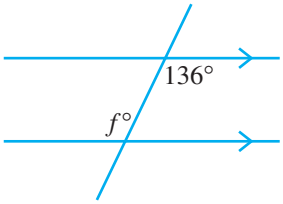

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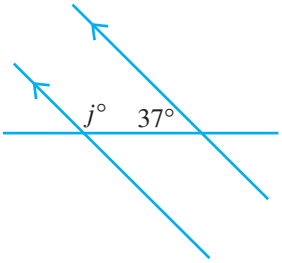

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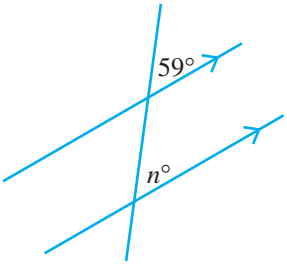

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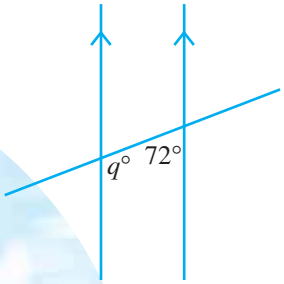

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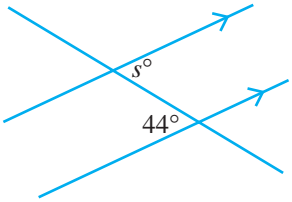

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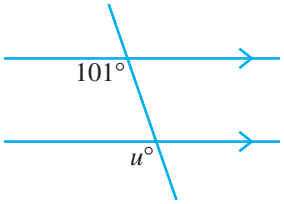

7 Find the value of x in each of these.

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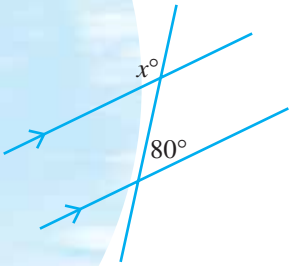

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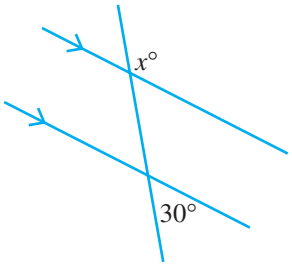

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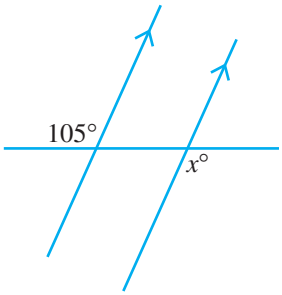

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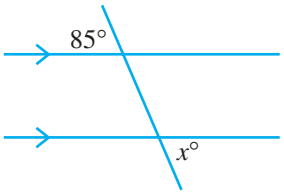

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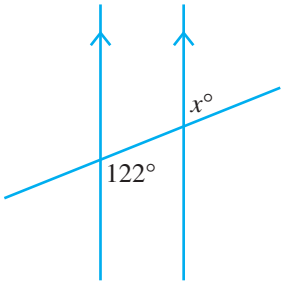

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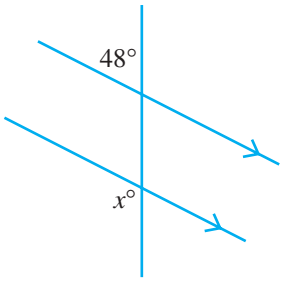

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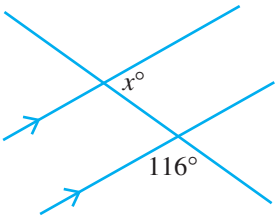

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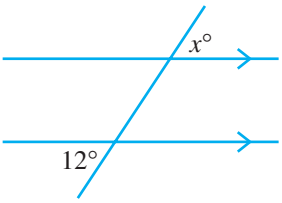

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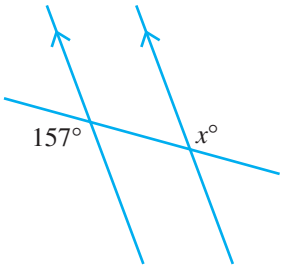

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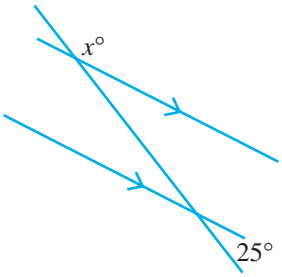

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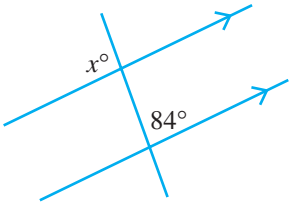

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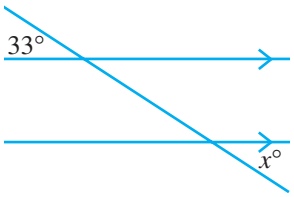

8 Find the value of all pronumerals in each of the following.

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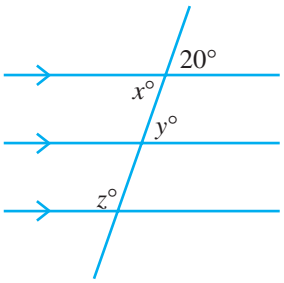

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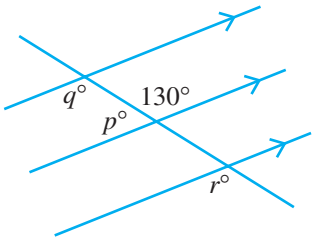

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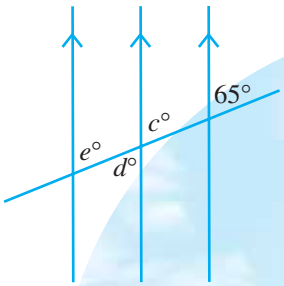

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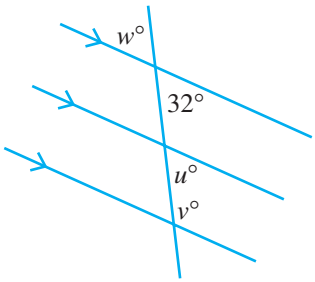

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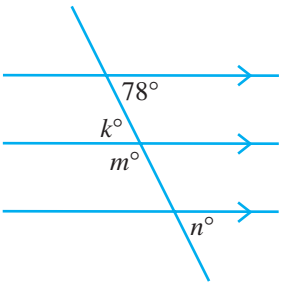

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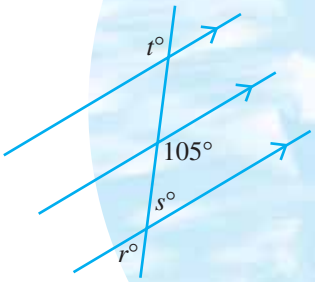

Further applications

9 Find the value of all pronumerals in each of these.

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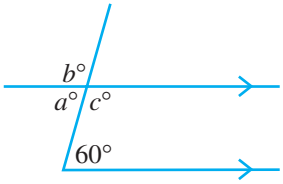

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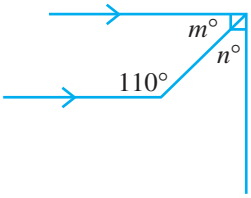

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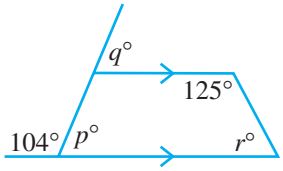

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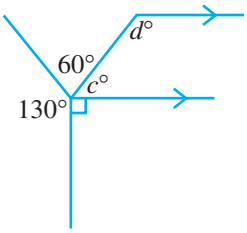

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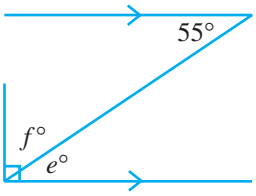

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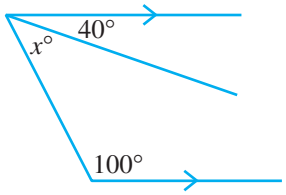

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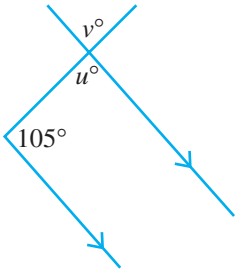

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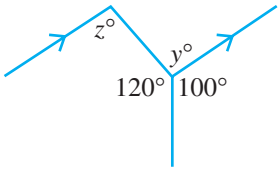

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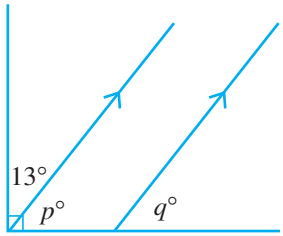

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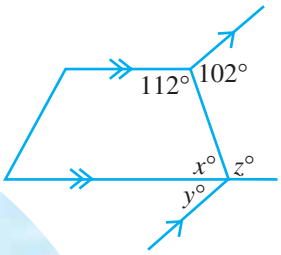

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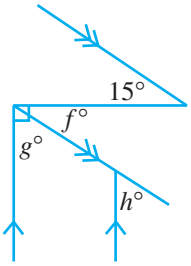

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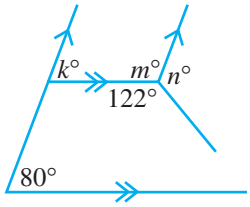
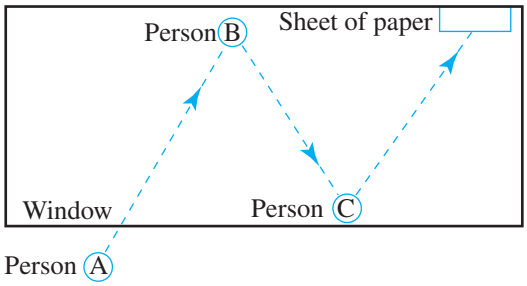


TRY THIS

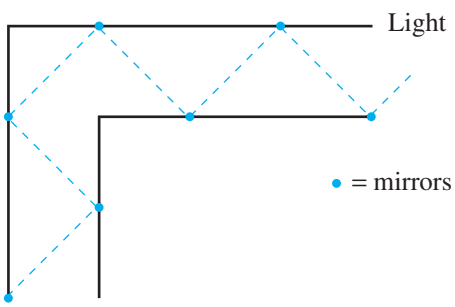
Mirror bounce

The Ancient Egyptians used mirrors to transfer outside light to the inside chambers of the pyramids. In these activities everyone will need a mirror.

1 Place a coloured sheet of paper on a wall opposite the window side. Person A stands outside the room beside a window, Person B stands inside the room at the opposite wall, and Person C stands on the window side as shown.



The aim is to reflect the light from one person to the next until the final reflection is on the sheet of paper. At what angle should you hold the mirrors?

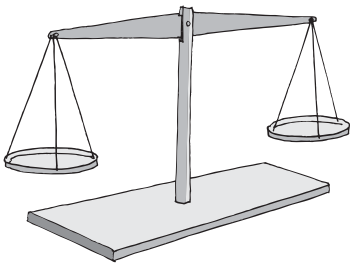


2 Make a box (as shown) to use mirrors to reflect images around corners. What can you conclude about the angle of the mirrors?

Problem solving

1 A kilometre-long train is travelling at 60 km/h. How long will it take for the train to pass through a 2 kilometre tunnel? (60 km/h = 1 km/min.)

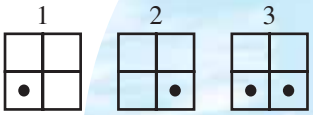
2 Suppose you are given 20 coins, one of which is not genuine and is of slightly lighter weight than the true coins.
By using the balance, what is the least number of weighings that are necessary to find the light coin?



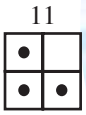
3 2520 is a very special number!
Using indices, can you write it as a product of its prime factors? Can you work out or guess why it is a special number?

4 Jan wrote to 12 of her friends, sending either a letter or postcard. A letter costs 45c and a postcard 30c to send, and she spent $4.20 on postage. How many letters and how many postcards did she send?

5 Dot numbers! Using only one dot per square, we have represented numbers 1, 2 and 3. How can you show 4?

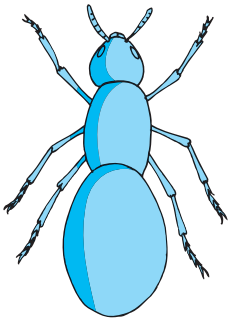


Clues: The largest number you can show is 15, and this is 11. If you can do 4, you should be able to complete all the numbers 1 → 15.

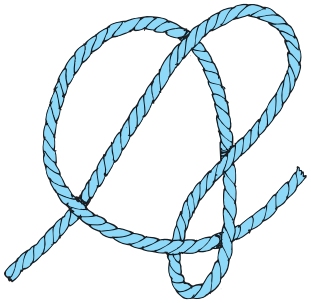


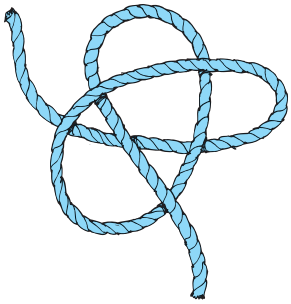
6 Mary is twice as old as her sister Jenny. When Mary turns twenty, their combined ages will total thirty-five. How old are they now?

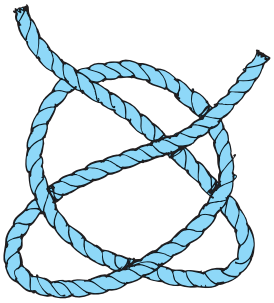
7 An insect has three parts to its body—head, thorax, and abdomen.
If there were only three colours available in the insect world—black, red, and green—and every different arrangement of colour (e.g. red head, green thorax, red abdomen!) was a different species, how many species would be possible?



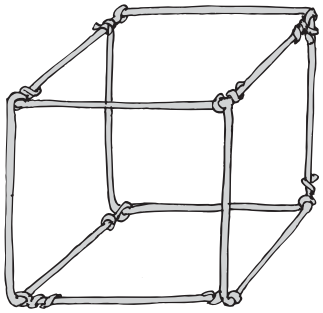
8 Which of these three ropes will tie itself in a knot, if the ends are pulled?



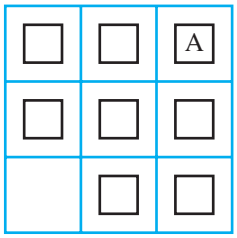




9 This cube is formed by bending and cutting a piece of wire. How many cuts are needed to make this cube? How do you make it?



10 Suppose we have eight squares of paper on a 3×3 board as shown.
Suppose we can only slide these squares one at a time.
What is the minimum number of moves to move piece *A* to the opposite (spare) corner?



Focus on working mathematically

The sun's rays

Introduction

The sun's rays are good examples of parallel lines. You can detect them by observing shadows cast by objects of different height. When the sun is low in the sky the shadows are long. When the sun is high in the sky the shadows are shorter. The highest point of the sun in the sky is called the *zenith*. The angle of the sun's rays can be used to calculate the circumference of the Earth. They can also tell us our latitude, which in New South Wales means the number of degrees of your city or town south of the equator. You will learn much more about this in later years. For this activity we just carry out some simple observations.

Learning activities

Materials needed: metre rule, protractor, set square, string, paper and pencil. You will also need a sunny day! We suggest an outdoor lesson for the whole class, divided into small groups. A large wooden protractor is great for demonstration.

1 Place a number of objects of different height in direct sunlight. Use string to join the top of the object to the tip of its shadow on the ground. Each string can represent a ray of light which just misses the top of the object. Note the time of day, and how high the sun is in the sky.

2 Measure the acute angle made by the string to the ground for each object. A wooden protractor the teacher uses in class may be useful. What do you notice? What will happen if you repeat the measurements in an hour's time?
3 Make a drawing of what you see. Notice that parallel lines in space are not necessarily in the same plane.
4 Draw a right-angled triangle to represent a 1 m rule and its shadow. Mark in the angle you measured which the ray of light makes with the ground. Calculate and mark in the angle the string makes with the vertical 1 m rule.

Extension activities

1 When might the height of the object be exactly equal to the length of its shadow? Make a drawing to show this.
2 The height of the pyramids of Egypt were found in such a way. Make a drawing to show the method.
3 Could you design a way of finding the height of a school building, or other tall object, using this finding? Discuss with your teacher. Try it out if you can.
4 Light rays arrive at Earth from the sun in straight lines. However they are actually bent when they enter the Earth's atmosphere. Would this make a difference to your findings? Check it out. Make a drawing of the Earth and its atmosphere. An atlas may help.
5 How far away is the sun? How fast does light travel? How long would it take a ray of light to reach us? Ask your teacher to help you get the information you need.

Let's communicate

Design a poster to show what you have learned about the sun's rays. Show the sun's rays and the angles they make with the ground at different times of the day. Tell what happens to the path of the sun through the sky during summer and winter, and how this affects the shadows cast by objects such as the pyramids of Egypt.

Reflecting

Geometry is a powerful tool. Think over what you have learned about sunlight, angles and parallel lines in this chapter. Think of how useful the parallel property of the sun's rays is in calculating inaccessible heights like the pyramids. Later you will learn much more about using the sun and the stars to measure distance and location on the surface of the Earth. The early mariners depended on this as they made long journeys across the vast oceans.

Language link with Macquarie

MACQUARIE

1 *How do we define* ***angle*** *in mathematics?*
2 *What word do we use to describe a section of a line?*
3 *What units do we use to measure* ***angles****?*
4 *When are* ***angles*** *described as* ***adjacent****?*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***angle****:*

angle *noun* **1.** the pointed shape made when two straight lines or surfaces meet each other: *The streets met at a sharp angle.* **2.** point of view: *a new angle on the problem.*

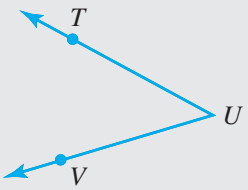
–verb **3.** to bend, move or place at an angle: *He angled the ball away from the fielder.* **4.** to put a bias or slant on: *He angled the question to suit the answer he had prepared.*
–phrase **5. at an angle**, sloping or not at a right angle to: *Straighten the picture because it's at an angle.*

Don't confuse this with **angel**, which is what a messenger of God is called.

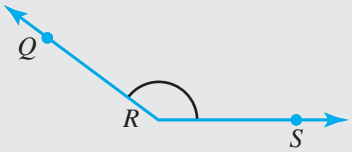
Why is an understanding of ***angles*** *important to a cricketer or softball player?*

CHAPTER REVIEW

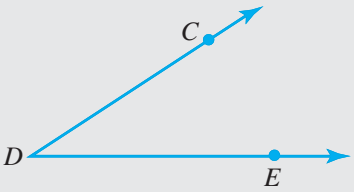
1 State the vertex in this angle.



2 Name this angle.

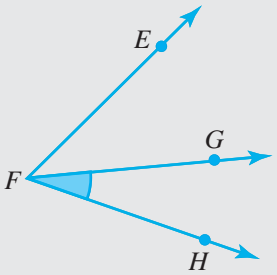


3 Name the two rays that form $\angle CDE$.



4 Name the shaded angle in each of these.

a



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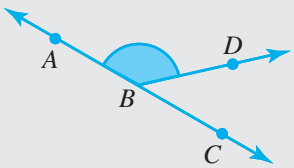

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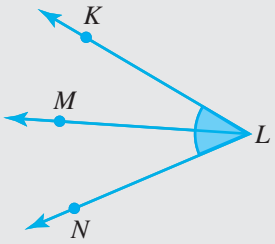
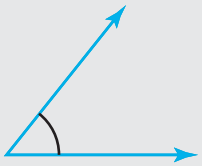


CHAPTER REVIEW

5 Measure each of these angles using a protractor.

a



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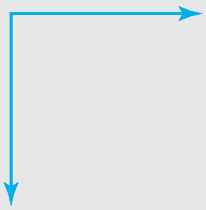
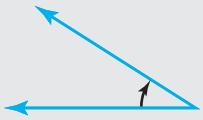


6 Classify each of these angles.

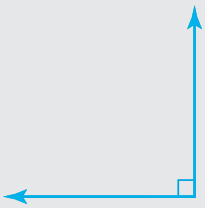
a



b

c

d



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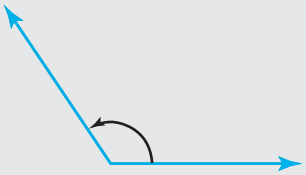

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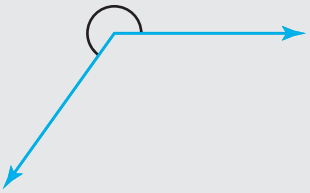


7 What kind of angle measures:

a 110°? **b** 90°? **c** 235°?
d 360°? **e** 50°? **f** 180°?

8 How many degrees are there in:

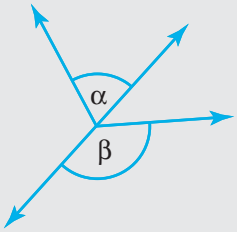
a $\frac{1}{2}$ of a right angle?

b $\frac{1}{3}$ of a straight angle?

c $\frac{2}{5}$ of a revolution?

9 For each of the following, state whether the angles α and β are adjacent.

a



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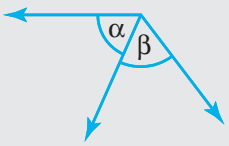

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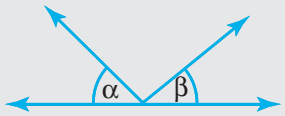
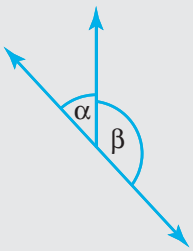
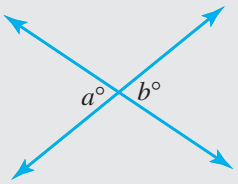


d



10 Name the pairs of angles shown as either complementary, supplementary or vertically opposite angles.

a



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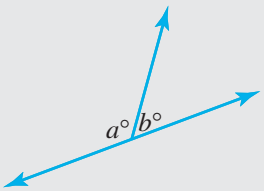

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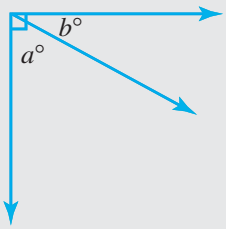
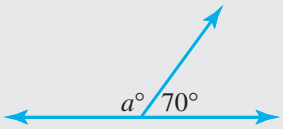


11 Find the value of each pronumeral.

a



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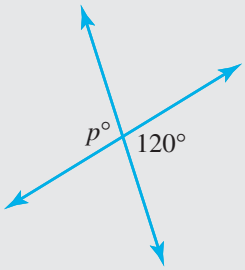

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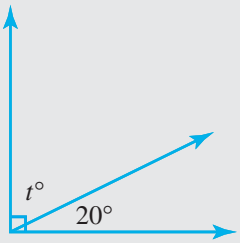

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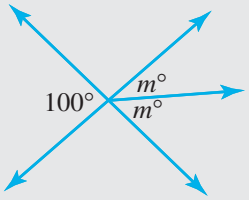

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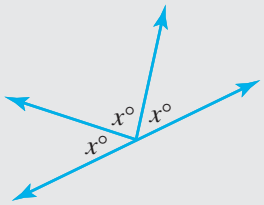

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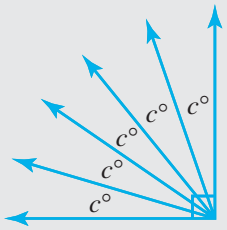
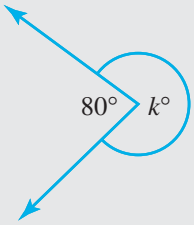


12 **a** Find the complement of 32°.
b Find the supplement of 74°.
c Find the complement of the supplement of 105°.

13 Find the value of each pronumeral.

a



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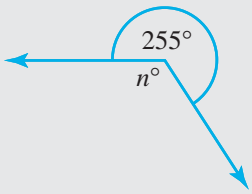

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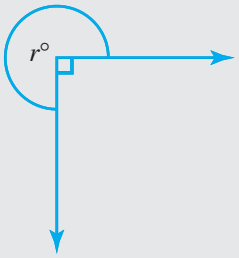

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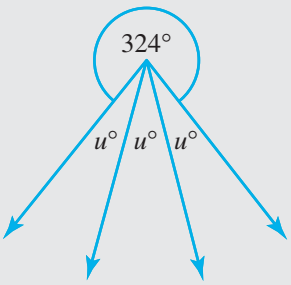

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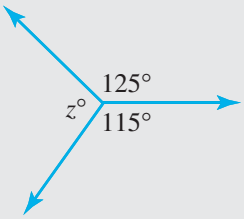

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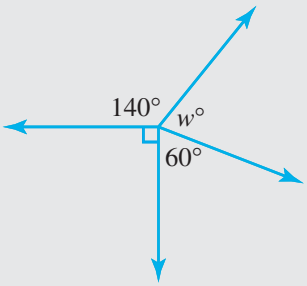
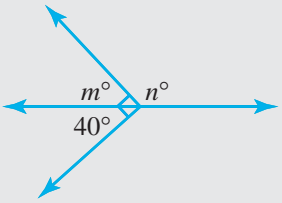
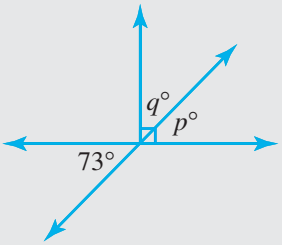


14 Find the value of each pronumeral.

a



b



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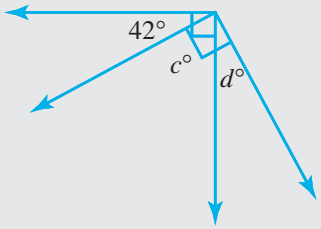
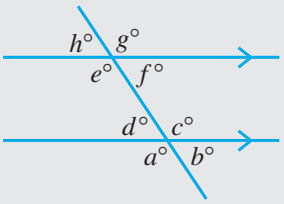


15 Name all pairs of:

a alternate angles

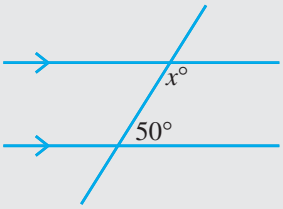
b corresponding angles

c co-interior angles.



16 Find the value of the pronumeral in each of these.

a



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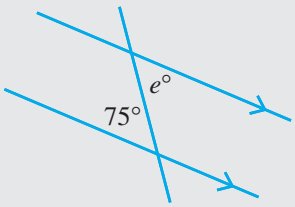

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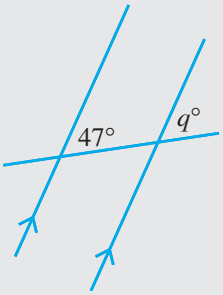

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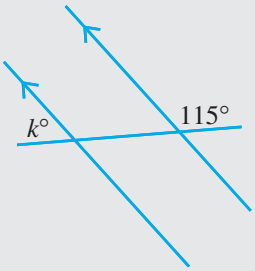

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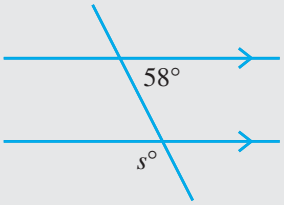

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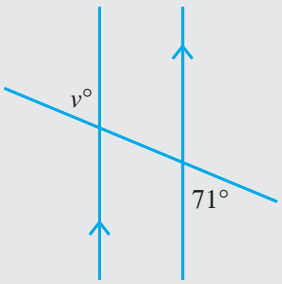
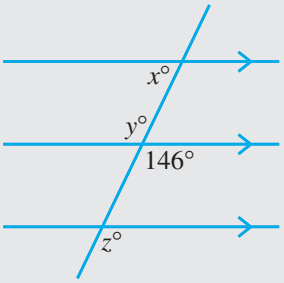
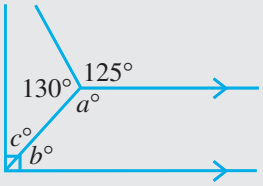


17 Find the value of each pronumeral.

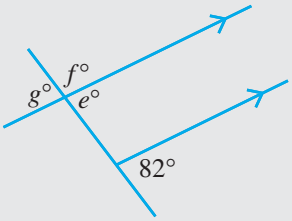


18 Find the values of all pronumerals in each of the following.

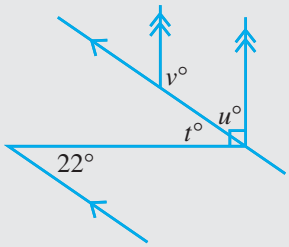
a



b



c

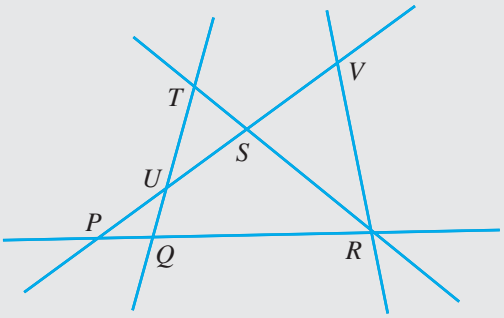


19 Draw a diagram that shows:

a a pair of parallel lines
b a pair of perpendicular lines
c 3 collinear points
d 3 concurrent lines

20 **a** Name the point that is collinear with both U and Q.

b Name the 3 lines that are concurrent.



21 Draw a line GH. Mark a point F, not on this line. Use a ruler and set square to draw another line through F:

a parallel to GH

b perpendicular to GH

22 Draw a line LM and mark on it a point N. Use a ruler and set square to draw another line through N, perpendicular to LM.

10 Properties of geometrical figures

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- identify and name regular and irregular polygons
- identify and draw all axes of symmetry in two-dimensional shapes
- state the order of rotational symmetry for various two-dimensional shapes
- explain the difference between regular and irregular shapes
- identify and draw diagonals in two-dimensional shapes.

Stage 4

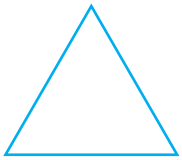
After completing this chapter, you should be able to:

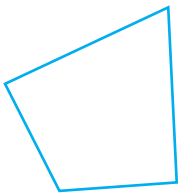
- label and name triangles and quadrilaterals
- use the common conventions to mark equal intervals on diagrams
- recognise and classify types of triangles
- deduce and use the angle sum of a triangle
- establish the exterior angle property of triangles
- recognise the correlation between side length and angle size in triangles
- distinguish between convex and non-convex quadrilaterals
- establish the angle sum of a quadrilateral
- deduce and use the properties of the special quadrilaterals
- apply geometrical facts, properties and relationships to find sides and angles in diagrams
- sketch and label triangles and quadrilaterals from a given verbal description
- indicate all axes of symmetry in various triangles and quadrilaterals
- state the order of rotational symmetry of various triangles and quadrilaterals
- recognise and classify the special quadrilaterals on the basis of their properties
- use dynamic geometry software to investigate the properties of geometrical figures.

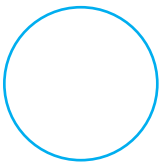
10.1 Recognising plane shapes

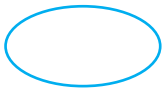
A **plane shape** is a closed, two-dimensional figure which lies in a plane or flat surface. Since it is two-dimensional, a plane shape has an area but not a volume. We can see many different plane shapes in the world around us. What shape are the walls in your classroom or the face of the coins in your wallet?

While the number of possible plane shapes is infinite, we find it easier to make use of just a small number of shapes. In this exercise we will review some of the most common: triangles, quadrilaterals (4-sided figures) and closed curves.


triangle


quadrilateral


circle


ellipse

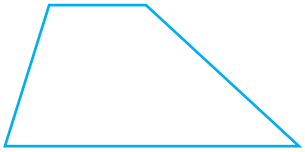
Some of the quadrilaterals have special names. You should already be familiar with them.

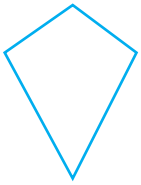
square

rectangle

parallelogram

rhombus

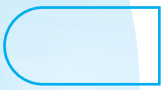

trapezium


kite

Exercise 10.1

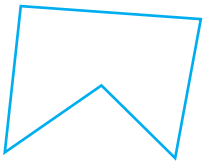
1 State whether or not the following are plane shapes. For those that are not, give a reason.

a



b

c



d one surface of a pane of glass

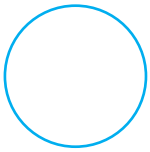
e the surface of a hockey ball

f the surface of a cube

g one surface of a piece of slate

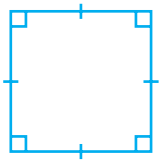
2 Name each of these plane shapes:

a



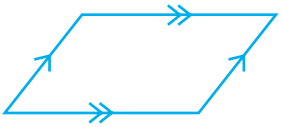
b

c

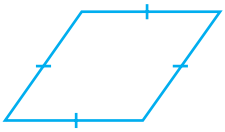


d

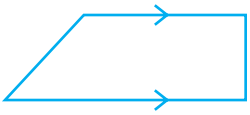
e



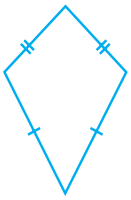
f



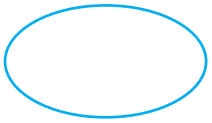
g



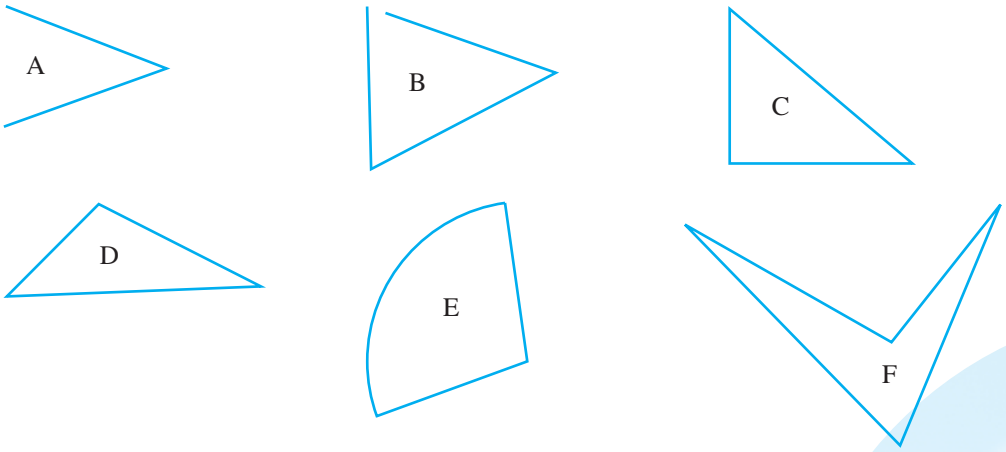
h



i



3 Which of these figures are not triangles?



4 **a** A rectangle is a quadrilateral that has four right angles. Which other quadrilateral is also a rectangle?

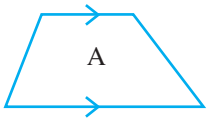
b A parallelogram is a quadrilateral that has two pairs of opposite sides parallel. Which other three quadrilaterals are also parallelograms?

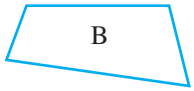
c Name all the quadrilaterals that have at least one pair of opposite sides parallel.

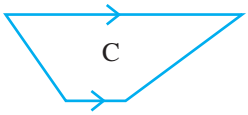
d A rhombus is a quadrilateral that has four equal sides. Which other quadrilateral is also a rhombus?

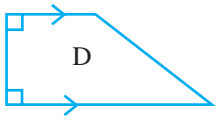
5 Which quadrilateral has two pairs of adjacent sides equal?

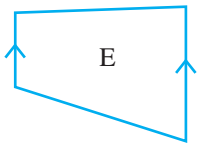
6 Which of the following figures are not trapeziums?

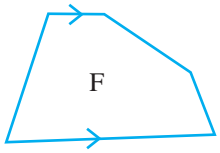
a


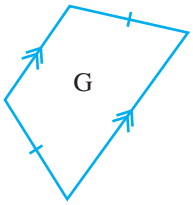
b


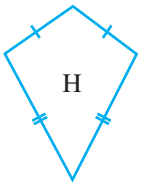
c


d


e


f


g


h


7 **a** Why isn't a trapezium also a parallelogram?
b Why isn't a triangle also a quadrilateral?

Consolidation

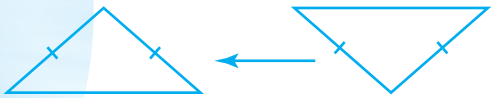
8 Name two plane shapes that do not have any straight sides.

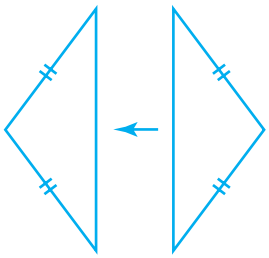
9 **a** Draw a parallelogram and divide it into:
i 2 triangles **ii** 4 triangles **iii** 3 triangles
b Draw a trapezium and divide it into:
i a parallelogram and a triangle **ii** two triangles
iii a rectangle and two triangles **iv** a triangle and a trapezium
v two trapeziums
c If the top of a triangle is 'sliced-off' parallel to the base, what shape is left?
d Draw a parallelogram in such a way that it can be divided into two right-angled triangles.

10 If we think of a parallelogram as a rectangle that has been 'pushed over', how might we think of a rhombus?

11 **a** If two identical squares are placed side by side, what shape is formed?
b If a rhombus is placed alongside another rhombus of the same size, what common shape could be formed?

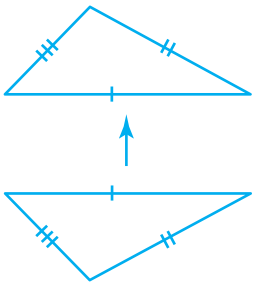
12 **a** An equilateral triangle has three sides of equal length. If two equilateral triangles of the same size are placed side by side, what shape is formed?
b An isosceles triangle has two sides equal. What shape is formed when two isosceles triangles of the same size are placed side by side as shown?

i


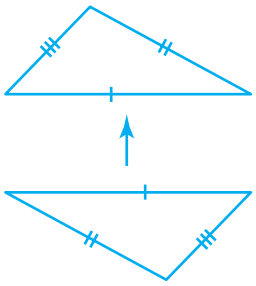
ii


c A scalene triangle has no equal sides. What shape is formed when two scalene triangles of the same size are placed side by side as shown?

i

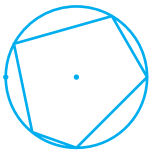


ii



Further applications

13 When a figure is drawn inside another plane shape so as to touch it on all sides, we say that the smaller figure has been **inscribed** in the larger figure. For example, this diagram shows a five-sided plane shape inscribed in a circle.



Draw each of the following:

a a circle inscribed in a square
b a square inscribed in a circle
c a circle inscribed in a triangle
d a triangle inscribed in a circle
e an ellipse inscribed in a rectangle
f a rectangle inscribed in an ellipse

14 Is it possible to inscribe:

a a parallelogram that is not a rectangle in a circle?
b a rectangle that is not a square in a circle?
c a rhombus that is not a square in an ellipse?

10.2 Polygons

Classifying polygons

The name **polygon** is derived from the prefix 'poly' which means *many* and 'gon' which means *side*. Thus, a polygon is a many-sided plane figure. It is a closed figure bounded by straight sides. Therefore, a plane shape with any curved edges, such as a circle, is not a polygon.

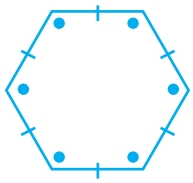
The prefix in the name of each polygon indicates how many sides it has.

The first ten polygons and the number of sides in each are:

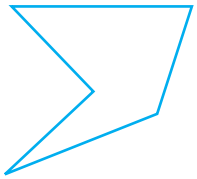
3 sides – **tri**angle
4 sides – **quad**rilateral
5 sides – **penta**gon
6 sides – **hexa**gon
7 sides – **hepta**gon
8 sides – **octa**gon
9 sides – **nona**gon
10 sides – **deca**gon
11 sides – **undeca**gon
12 sides – **dodeca**gon

A polygon is said to be **regular** if all sides are equal and all angles are equal. A polygon that is not regular is said to be **irregular**.

For example,



regular hexagon

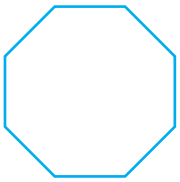


irregular pentagon

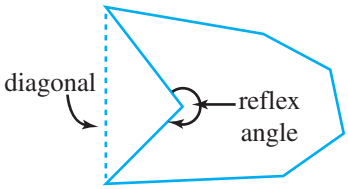
The point where two sides of a polygon meet is called a **vertex** (the plural is vertices). A line joining two vertices is called a **diagonal**.

A polygon is said to be **convex** if all of its diagonals lie entirely within the figure. As a result, all of the interior angles are less than 180°. A polygon is **non-convex** if at least one diagonal does not lie completely inside the figure. A non-convex polygon must have at least one interior reflex angle.

For example,



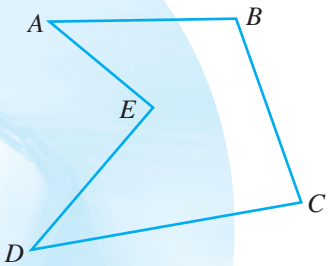
convex octagon



non-convex heptagon

Naming polygons

In many questions it will be necessary to refer to particular polygons in a diagram. In order to avoid confusion, we label the vertices with capital letters and we also use them to name the polygon(s). The letters must be given in order, either clockwise or anti-clockwise.

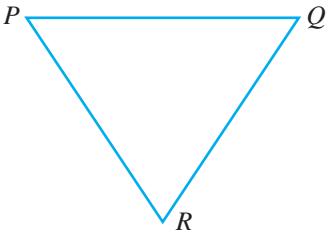


This figure would be referred to as *ABCDE*. Since we could start at any vertex, it could also have other names such as *CDEAB* or *EABCD*. The figure could also have been named in the anti-clockwise direction as *BAEDC* or *DCBAE*, for example.

Since we use triangles and their properties so frequently, it is important that we have a neat, concise way of naming them. The conventional way of doing this is to use a small triangle followed by three capital letters that have been used to name the angles.

The triangle shown would be named ΔPQR.

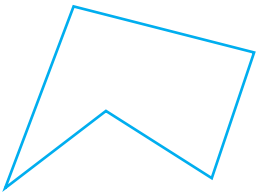
Since it does not matter which letter we begin with, this triangle could also be known as ΔRPQ or ΔRQP, for example.



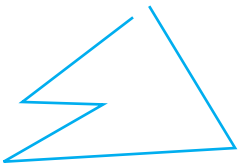
Exercise 10.2

1 State whether or not each figure is a polygon.

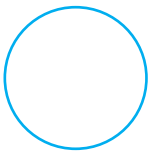
a



b



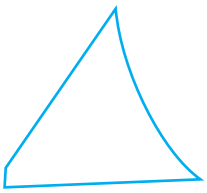
c



d

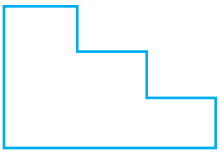
e

f



g

h

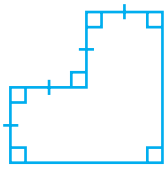


i

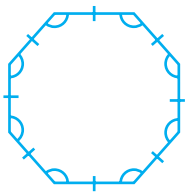
2 For each of the following polygons, state whether the figure is regular or irregular.

a a square **b** a rectangle **c** a rhombus

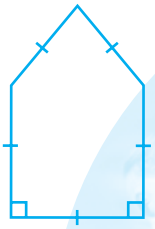
d



e



f



3 Give the names of the polygons which have the following numbers of sides.

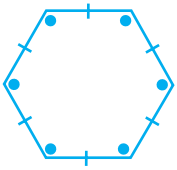
a 3 **b** 4 **c** 5 **d** 6 **e** 7
f 8 **g** 9 **h** 10 **i** 11 **j** 12

4 Without reference to Q3, state the number of sides in each of these polygons.

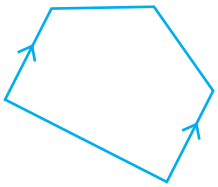
a hexagon **b** decagon **c** triangle

d nonagon **e** dodecagon **f** pentagon
g quadrilateral **h** octagon **i** heptagon
j undecagon

5 **a** Name this polygon.
b Copy the polygon and divide it into:
i two trapeziums
ii two triangles and a rectangle
iii two triangles and a kite

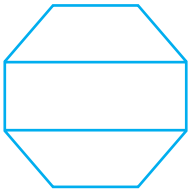


6 **a** Name this polygon.
b Copy the polygon and divide it into:
i a triangle and a trapezium
ii three triangles

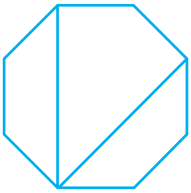


7 A regular octagon has been divided into smaller shapes. What shapes can you see in each figure?

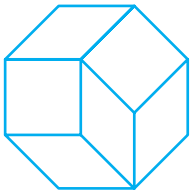
a



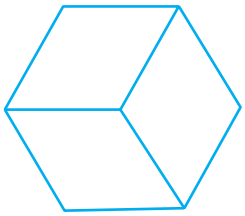
b



c



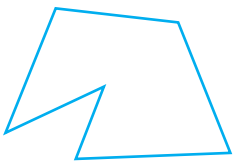
8 This regular hexagon has been divided into three equal shapes. Name the equal shapes.



9 Name the following irregular polygons according to the number of sides.

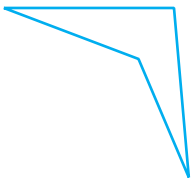
a

b



c

d



e

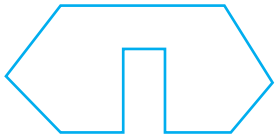
f

g

h

i

j



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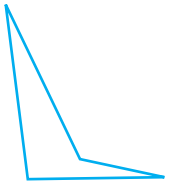


l

Consolidation

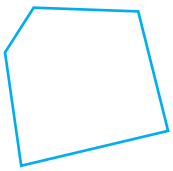
10 **a** Name a quadrilateral that has equal sides but does not have equal angles.
b Name a quadrilateral that has equal angles but does not have equal sides.
c Copy and complete these statements:
i If a polygon which is not a triangle has equal sides, it may or may not have equal _____.
ii If a polygon which is not a triangle has equal angles, it may or may not have equal _____.

11 For each of the special quadrilaterals, state whether it is a regular figure or an irregular figure.

a parallelogram	**b** trapezium	**c** rhombus
d rectangle	**e** square	**f** kite

12 For each of these polygons, state whether it is convex or non-convex.

a



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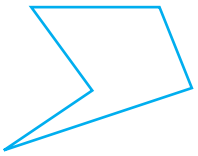
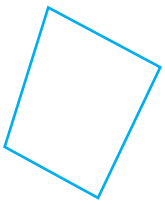


c



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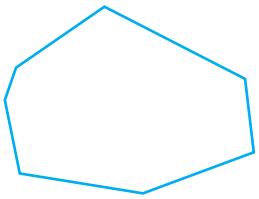
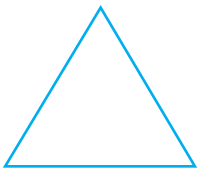
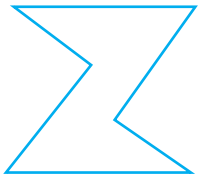


e

f



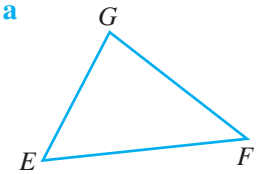
g



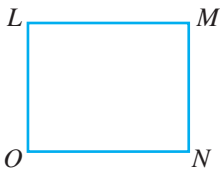
h

13 Name each of these polygons using their vertices.

a



b



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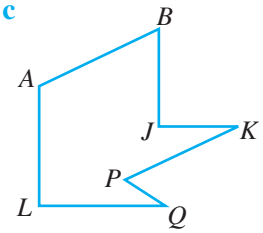

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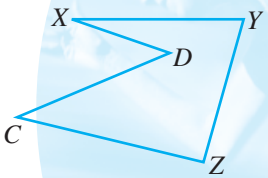
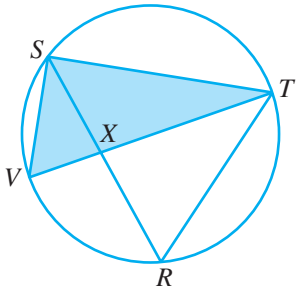
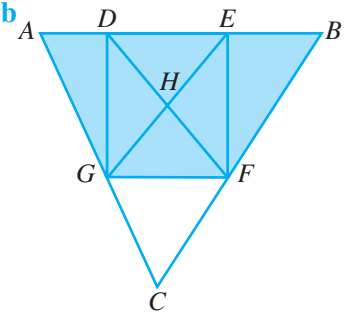


14 Name the shaded figure in each of the following diagrams in two different ways.

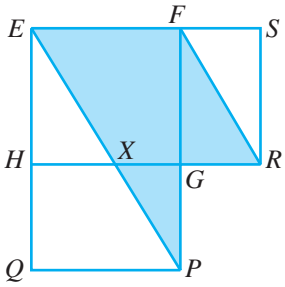
a



b

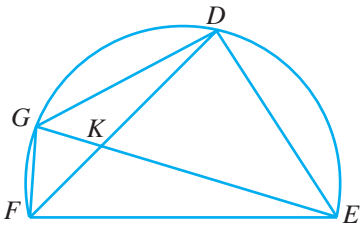


c



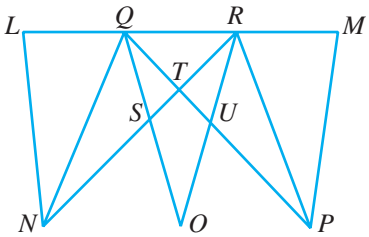
15 For each of the following diagrams sketch a copy of the diagram and shade in the triangle(s) indicated.

a



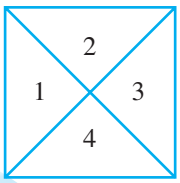
Shade ΔGKD and ΔFKE

b

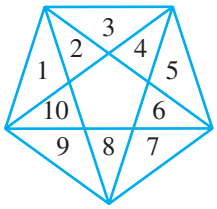


Shade ΔRMP, ΔOSR and ΔQSN

16 The polygons shown below are regular. All their diagonals have been drawn in and the small triangles that share a vertex with the polygon have been counted.

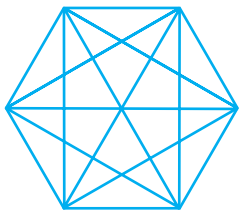


A regular quadrilateral (square)
Diagonals 2
Triangles 4



A regular pentagon
Diagonals 5
Triangles 10

a How many diagonals can be drawn in a regular hexagon?
b How many small triangles share a common vertex with the hexagon?
c Copy and complete this table.



A regular hexagon

	square	pentagon	hexagon
Number of sides (n)	4	5	6
Number of diagonals (d)	2	5	
Number of triangles (t)	4	10	

d Write a sentence that shows the relationship between:
  i the number of triangles and the number of diagonals
  ii the number of triangles and the number of sides

e Write an algebraic formula for each relationship in part **d**.

f Use the second formulae to find the number of triangles in a regular polygon with:
  i 8 sides **ii** 10 sides **iii** 100 sides

g Combine the two formulae in part **d** to find a new formula that could be used to find the number of diagonals in a regular polygon, given its number of sides.

h Use this new formula to find the number of diagonals in a convex:
  i heptagon **ii** dodecagon **iii** 100-sided polygon

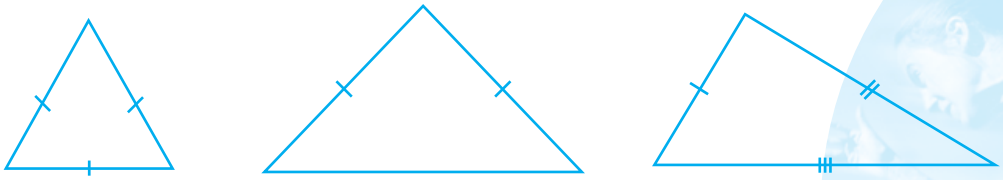
10.3 Classification of triangles and their properties

Triangles have the fewest sides of any polygons. As a result of this they have the least variation in type and they are, in many ways, more useful than other polygons. If we are given the lengths of the three sides of a triangle we have enough information to allow us to calculate everything about that triangle. We can find the sizes of its angles, its area and perimeter, anything we need. This is not true of any other polygon.

The certainty of shape that exists for triangles means that they are very useful in many practical aspects of our lives. The construction engineer building a bridge, surveyor marking out a new subdivision, carpenter making a table, dressmaker cutting out material, landscape gardener planning a garden bed, farmer mending a fence all make use of this special property of triangles.

A triangle can be classified according to the lengths of its sides or the sizes of its angles.

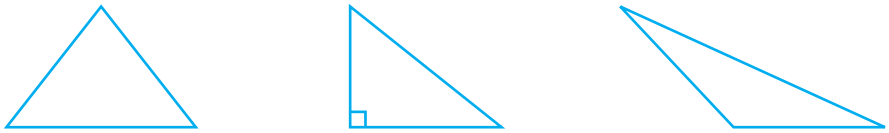
Classification according to sides:
- **An *equilateral triangle* is a triangle in which all three sides are equal in length.**
- **An *isosceles triangle* is a triangle in which two sides are equal in length.**
- **A *scalene triangle* is a triangle in which no sides are equal in length.**



Equilateral Isosceles Scalene

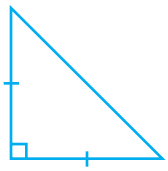
Classification according to angles:

- **An *acute-angled triangle* is a triangle in which all three angles are acute.**
- **A *right-angled triangle* is a triangle in which there is one right angle.**
- **An *obtuse-angled triangle* is a triangle in which there is one obtuse angle.**



Acute-angled Right-angled Obtuse-angled

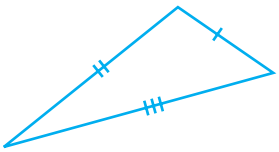
NOTE: A triangle may belong to more than one class. For example, a triangle may be both right-angled and isosceles, as shown.



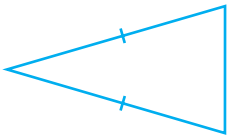
Exercise 10.3

1 Classify these triangles according to the lengths of their sides.

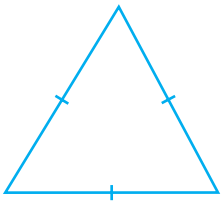
a



b

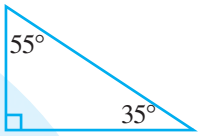


c



2 Classify these triangles according to the sizes of their angles.

a



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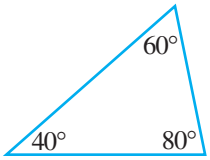
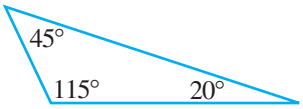


c



3 Draw any triangle. Measure the lengths of the sides and the sizes of the angles.

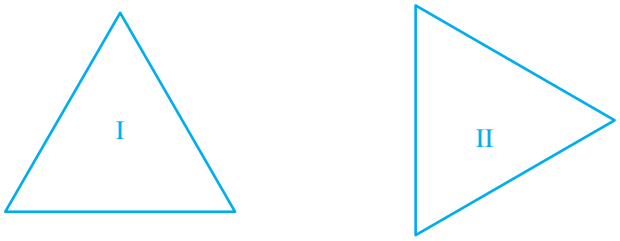
a Is the longest side opposite the largest angle?

b Is the shortest side opposite the smallest angle?

c Copy and complete this statement.

In any triangle, the longest side is _____ the largest angle and the _____ side is opposite the smallest angle.

4



a Measure the sides in each of these triangles. What kind of triangles are they?
b Measure the angles in each triangle. What size are they?
c Copy and complete this statement.
In an _____ triangle, the size of each angle is _____.

5 a Use your ruler to draw:
i an acute-angled isosceles triangle with equal sides of length 3 cm
ii an obtuse-angled isosceles triangle with equal sides of length 4 cm
iii a right-angled isosceles triangle with equal sides of length 3.5 cm
b Measure the angles in each triangle. How many angles are equal?
c Where are the equal angles in relation to the equal sides in each triangle?

6 a Draw a triangle with a ruler, then measure the angles.
b Find the sum of the angles in the triangle.
c Compare your results with those of other students.

7 a Draw a triangle on a piece of paper, using a ruler.
Mark the angles, then tear them off.
b Arrange the angles on the table so that they form a straight angle.
What is the sum of the angles?
c Copy and complete this statement.
The angle sum of any triangle is _____.

8 a Try to draw a triangle with sides of length:
i 2 cm, 3 cm and 7 cm **ii** 3 cm, 3 cm and 6 cm
b What do you notice? Explain why this is the case.
c Copy and complete this statement.
In any triangle, the length of each side must be _____ than the _____ of the lengths of the other two sides.

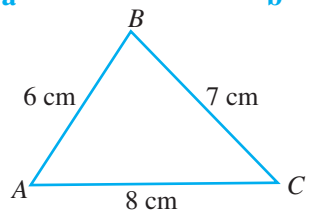
Consolidation

Having completed Q1–8 in this exercise, you should have deduced the following properties of triangles.

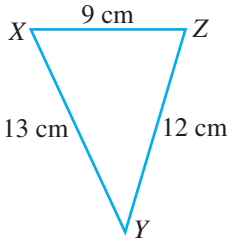
- **In any triangle, the sum of the angles is 180°.**
- **In any triangle, the length of each side must be less than the sum of the lengths of the other two sides.**
- **In any triangle, the longest side lies opposite the largest angle and the shortest side lies opposite the smallest angle.**
- **In an equilateral triangle, all angles are 60°.**
- **In an isosceles triangle, the equal sides lie opposite the equal angles.**

9 Name the smallest and largest angles in each of these triangles.

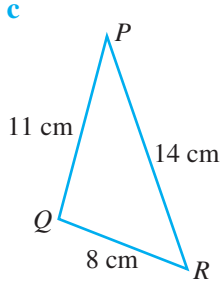
a



b



c



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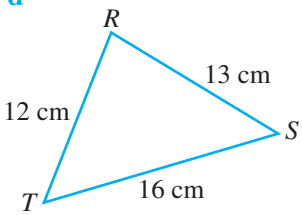
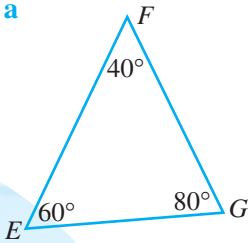


10 Name the shortest and longest sides in each of these triangles.

a



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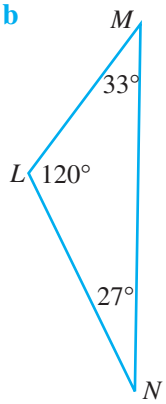

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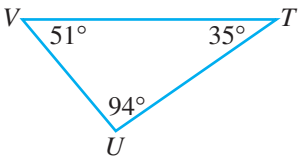

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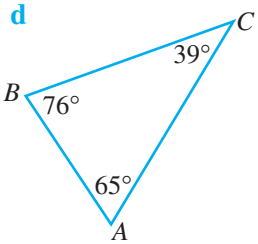
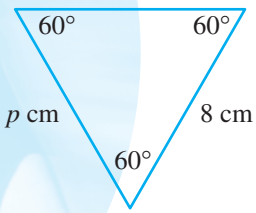
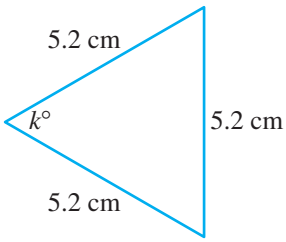


11 Find the value of the pronumeral in each of these.

a

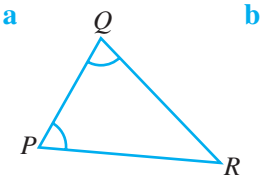


b



12 Name the equal sides in each of these isosceles triangles.

a



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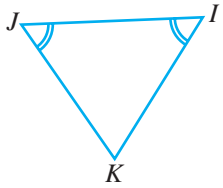
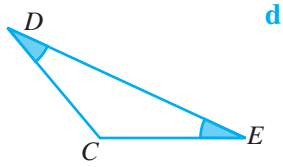


c



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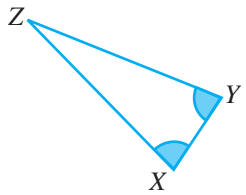
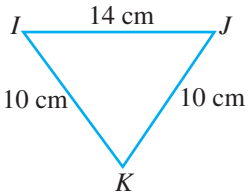


13 Name the equal angles in each of these isosceles triangles.

a



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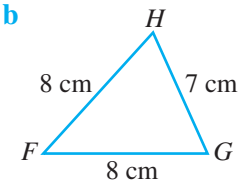

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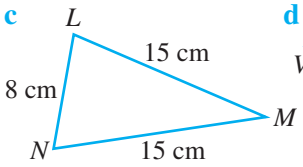

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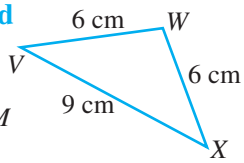
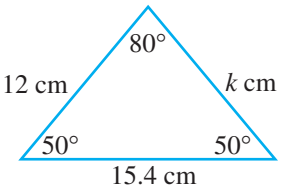
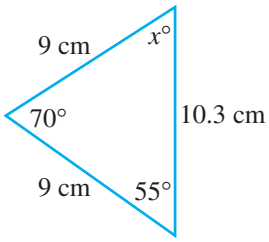


14 Find the value of the pronumeral in each triangle.

a



b



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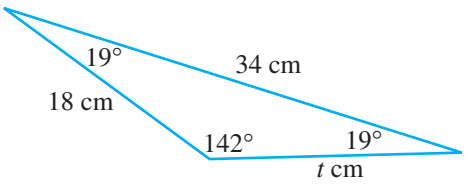

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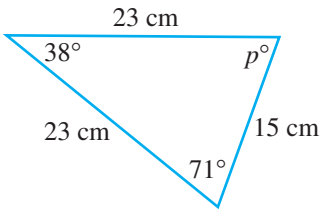
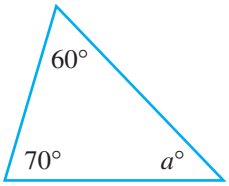
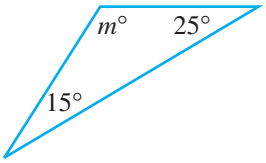


15 Find the value of the pronumeral in each of these.

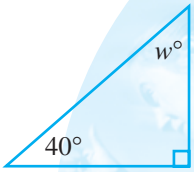
a



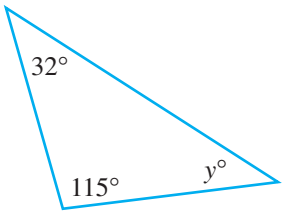
b



c



d



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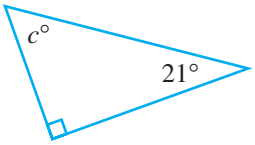

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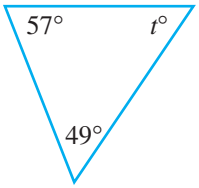
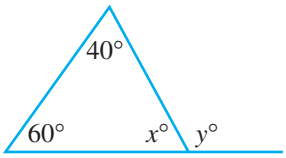
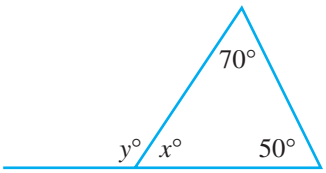


16 a Find values for x and y in each of these.

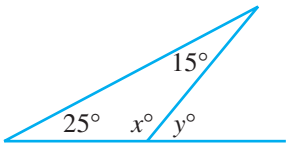
i



ii



iii

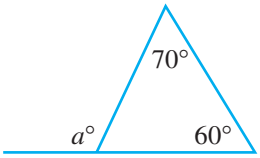


b Is there a way to find y without first finding x?

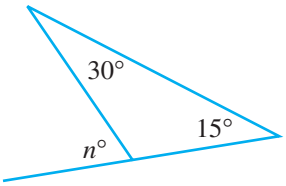
c The angles marked $y°$ are called exterior angles. Copy and complete this statement.
The _____ angle of a triangle is equal to the _____ of the two interior opposite _____.

17 Use the exterior angle property of triangles to find the value of each pronumeral.

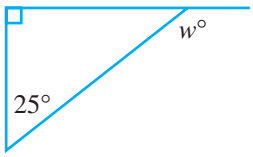
a



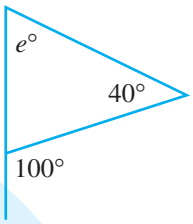
b



c



d



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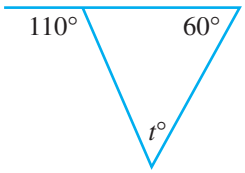

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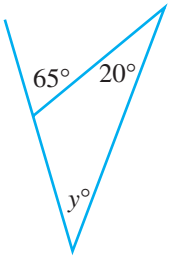

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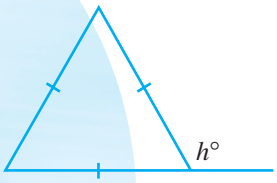

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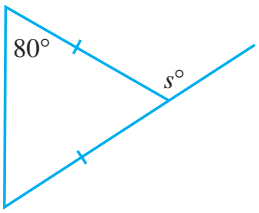
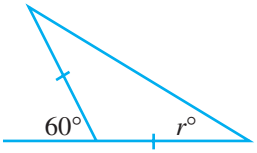


i

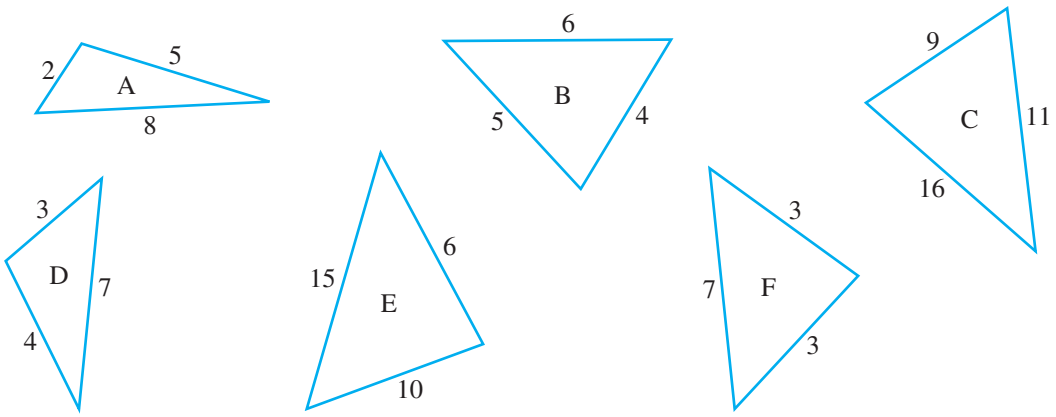


Further applications

18 Can a triangle exist with the following angles?

a 70°, 67°, 43° **b** 55°, 35°, 90° **c** 16°, 27°, 147°
d 93°, 91°, 10° **e** 48°, 31°, 101° **f** 75°, 25°, 100°

19 Which of the following triangles could not possibly exist? (All measurements are in mm.)



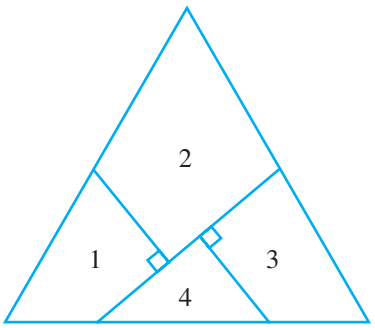
20 Is it possible for:

a a right-angled triangle to have an obtuse angle?
b a triangle to have two right angles?
c a triangle to be both right angled and isosceles?
d an equilateral triangle to contain an obtuse angle?
e a triangle to have two obtuse angles?
f a scalene triangle to be right angled?
g an obtuse-angled triangle to be isosceles?
h a triangle to be both equilateral and right-angled?
i a scalene triangle to have an obtuse angle?

TRY THIS

Triangle trouble

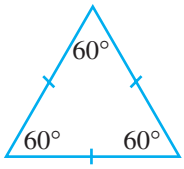
This equilateral triangle has been divided into four pieces. Trace the diagram onto a piece of paper and cut out the four pieces. Can they be rearranged to form a square?



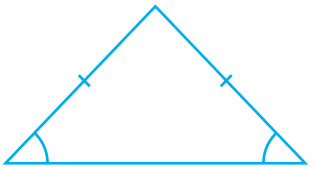
10.4 Angle sum of a triangle

In the previous exercise, we established the following properties for equilateral and isosceles triangles.

- **In an equilateral triangle, the angles are all 60°.**
- **In an isosceles triangle, the equal sides lie opposite the equal angles.**

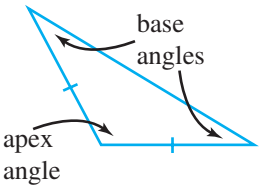


Equilateral triangle



Isosceles triangle

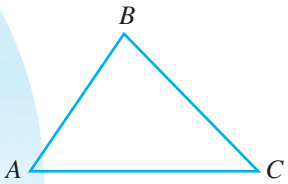
In an isosceles triangle, the equal angles are called **base angles** and the third angle is called the **apex angle**. That is, the apex angle is the angle formed between the equal sides.



NOTE: In an isosceles triangle, the base angles are not necessarily 'at the bottom'. Similarly, the apex angle is not necessarily 'at the top'.

The emphasis in this exercise will be on combining the properties of triangles with other general angle properties to find the size of an angle in a triangle, or the value of a pronumeral. Of particular importance is the result for the angle sum of a triangle.

The angle sum of any triangle is 180°.



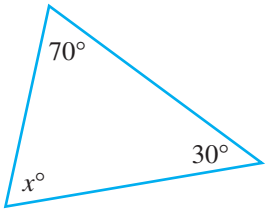
$$\angle A + \angle B + \angle C = 180°$$

In the following examples, reasons for each step of working are given. They have been included for the purpose of explanation only. It is not intended that students give formal reasons at this stage.

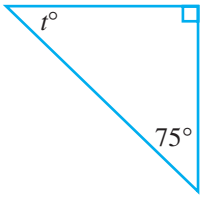
Example 1

Find the value of the pronumeral in each of these.

a



b



Solutions

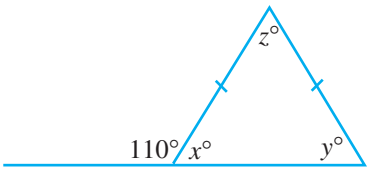
a $x = 180 - (70 + 30)$ (the angle sum of a triangle is 180°)
$= 80$

b $y = 180 - (90 + 75)$ (the angle sum of a triangle is 180°)
$= 15$

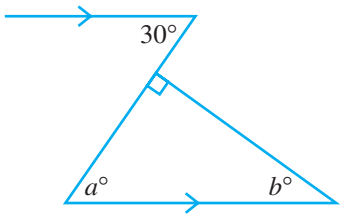
Example 2

Find the value of all pronumerals in each of these.

a



b



Solutions

a **i** $x = 180 - 110$ (adjacent angles on a straight line add to 180°)
$= 70$

ii $y = 70$ (the base angles of an isosceles triangle are equal)

iii $z = 180 - (70 + 70)$ (the angle sum of a triangle is 180°)
$= 40$

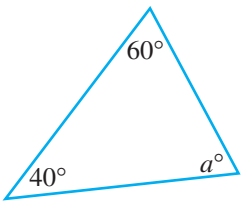
b **i** $a = 30$ (the alternate angles in parallel lines are equal)

ii $b = 180 - (30 + 90)$ (the angle sum of a triangle is 180°)
$= 60$

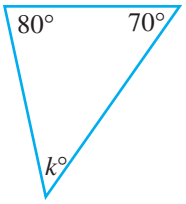
Exercise 10.4

1 Find the value of the pronumerals in each of the triangles:

a



b



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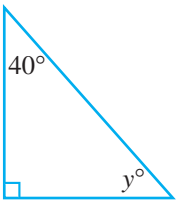

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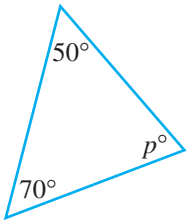
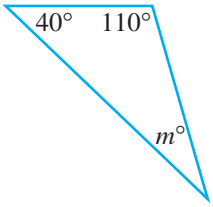
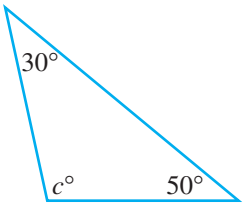


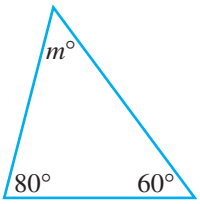
e



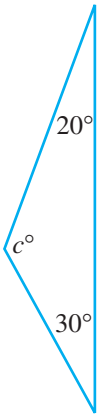
f



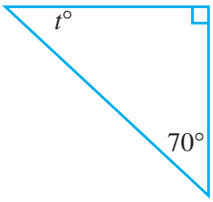
g



h



i



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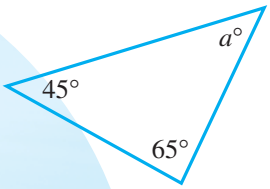

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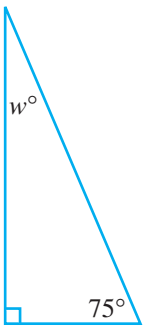

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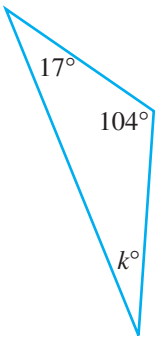
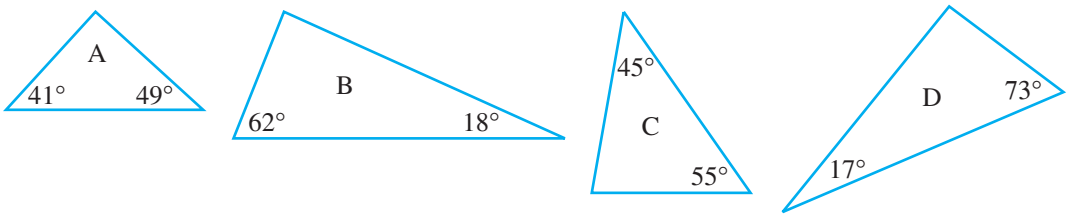
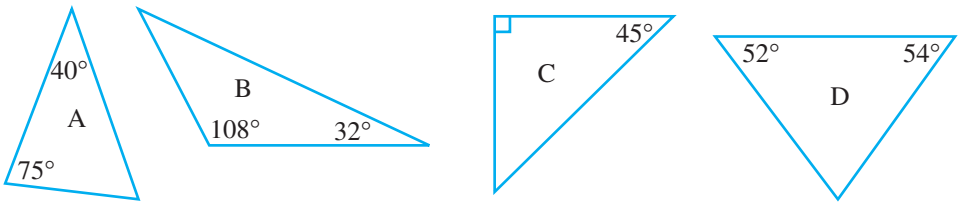


2 Find the sizes of the missing angles and hence determine which triangles are right angled.

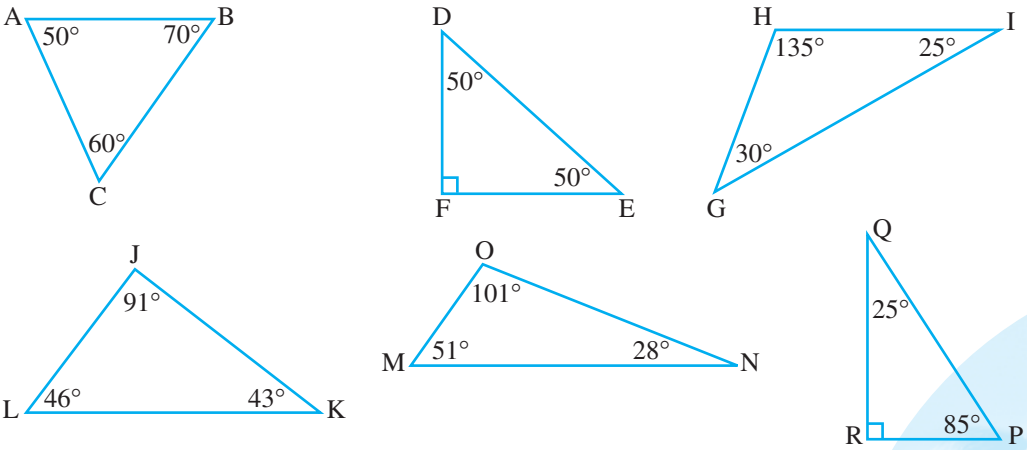


3 Find the sizes of the missing angles and hence determine which triangle(s) are isosceles.



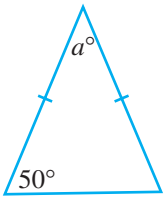
Consolidation

4 Which of the following triangles could not possibly exist?

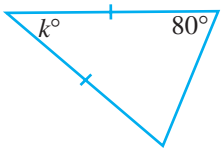


5 Find the value of the pronumeral in each of these isosceles triangles.

a



b



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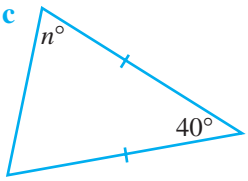

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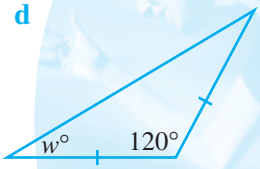

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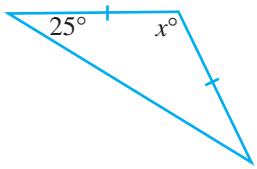

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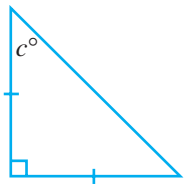

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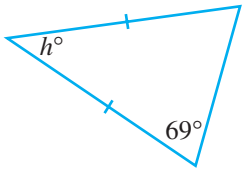

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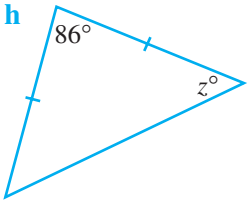
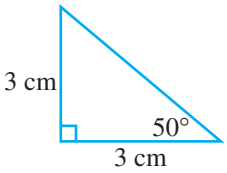


6 Determine whether or not each triangle can exist.

a



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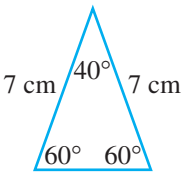

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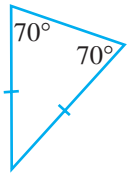

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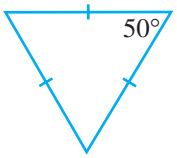

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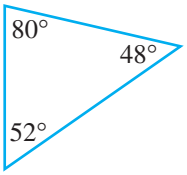

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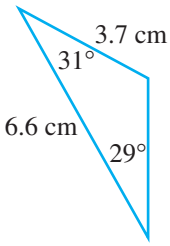
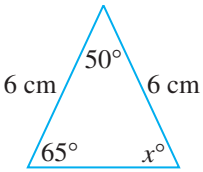


7 Find the value of the pronumeral in each of the following.

a



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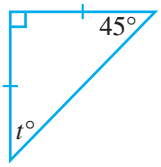

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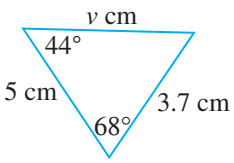
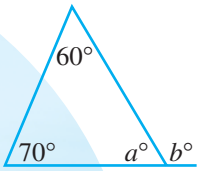


8 Find the values of all pronumerals.

a



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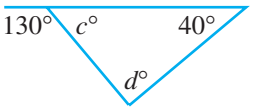

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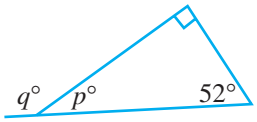
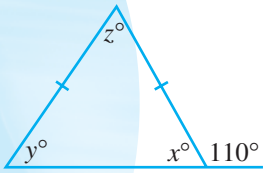


9 Find the values of all pronumerals in the following.

a



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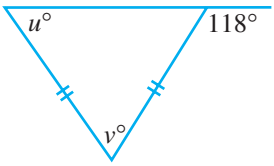

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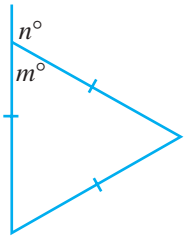
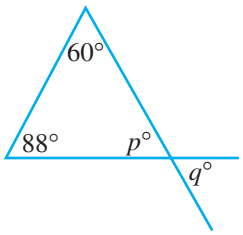


10 Find the values of all pronumerals in the following.

a



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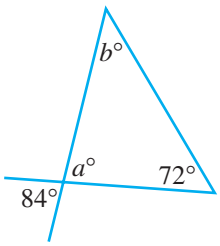

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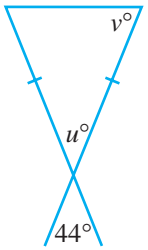
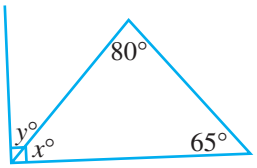


11 Find the values of x and y in each of the following.

a



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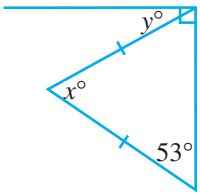

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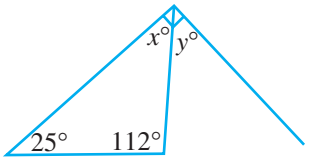
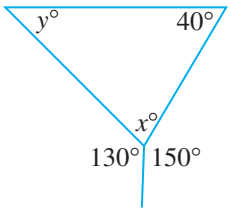


12 Find the values of all pronumerals in each of these.

a



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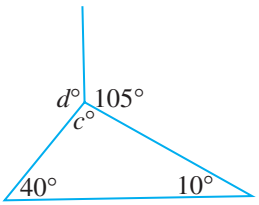

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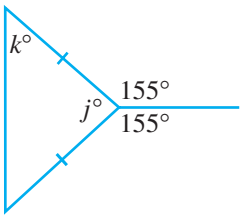
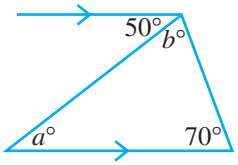


13 Find the value of each pronumeral.

a



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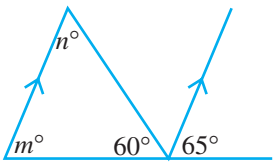

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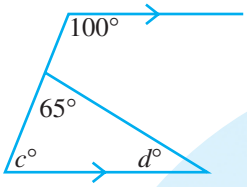

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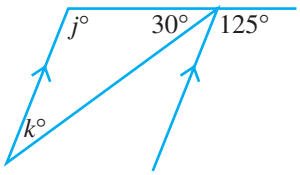

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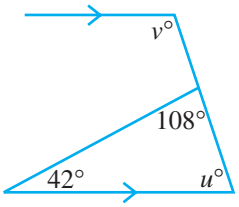

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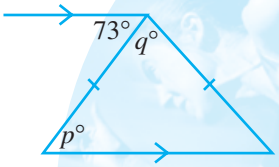

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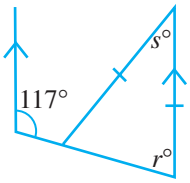

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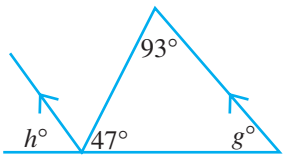

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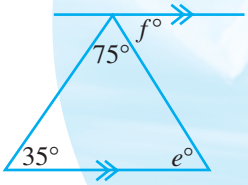

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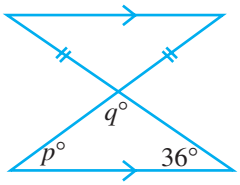

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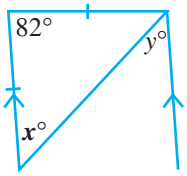

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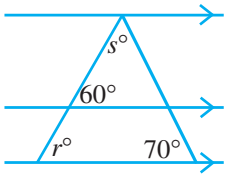
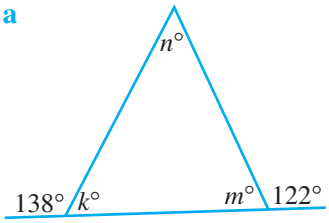


14 Find the values of all pronumerals in each of the following.

a



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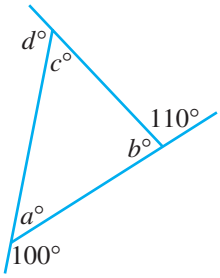
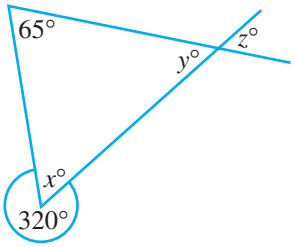


c



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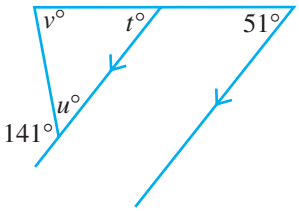

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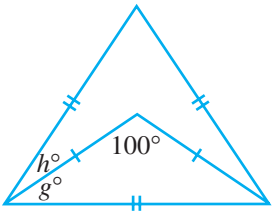

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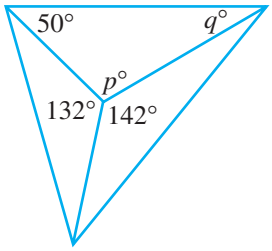
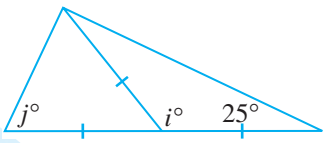


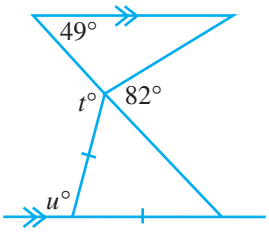
■ Further applications

15 Find the value of the pronumeral(s) in each of the following.

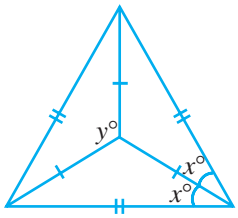
a



b

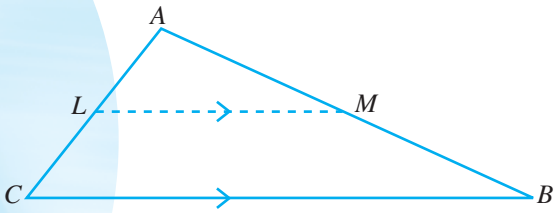


c



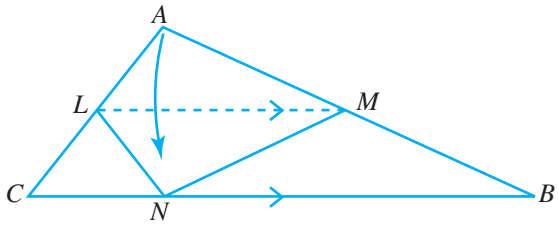
16 **i** Draw any triangle on a piece of paper and cut it out.

ii

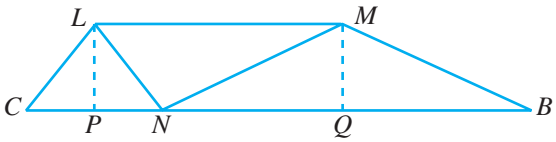


Place the triangle on your desk so that the largest angle ($\angle A$) is at the top as shown in the diagram.

iii Fold along the line LM (which *must* be parallel to CB) so that the point A rests on CB at N.



iv



Now fold along LP so that C rests on N and fold along MQ so that B meets A and C at N.

v Explain how this method proves that the angle sum of a triangle is 180°.

10.5 Properties of quadrilaterals

A quadrilateral is any four-sided polygon. There are six special quadrilaterals whose properties are very important in our study of geometry. They are the square, rectangle, parallelogram, rhombus, trapezium and kite. These figures are closely related and share a number of common properties.

While we think of a parallelogram as a rectangle that has been 'pushed over', its actual definition is 'a quadrilateral with two pairs of opposite sides parallel'. Any quadrilateral that shares this same property is, therefore, also a parallelogram. This means, for example, that a rectangle is also a parallelogram. It has all of the properties of a parallelogram, and more.

In this exercise the emphasis will first be on discovering the properties of the special quadrilaterals. These properties will then be used to solve a number of simple geometry problems.

Exercise 10.5

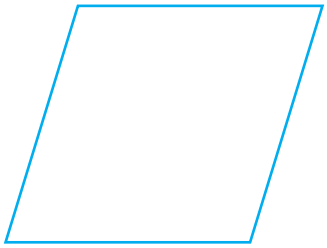
1 Photocopy or trace each of these quadrilaterals, then cut them out. Name each figure.

a

b

c

d



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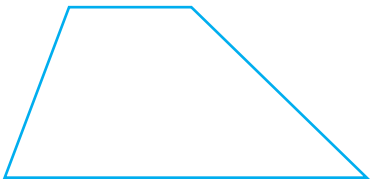

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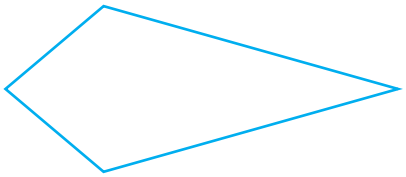


2 Answer the following questions for each of the six special quadrilaterals by folding and measuring the figures.

a Are the opposite sides equal in length?
b Are all sides equal in length?
c How many pairs of opposite sides are parallel, if any?
d Are the opposite angles equal in size?
e Are all angles right angles?
f Are the diagonals equal in length?
g Do the diagonals bisect each other?
h Are the diagonals perpendicular?
i Do the diagonals bisect the angles at the vertices?

3 Copy and complete this table using the results from your investigations.

Property	Square	Rectangle	Parallelogram	Rhombus
all sides are equal				
opposite sides are equal				
opposite sides are parallel				
all angles are right angles				
opposite angles are equal				
diagonals are equal in length				
diagonals intersect at right angles				
diagonals bisect each other				
diagonals bisect angles at vertices				

4 In which quadrilateral is there only one pair of parallel sides?

5 In which quadrilateral are two pairs of adjacent sides equal?

6 Trace or photocopy each of the following trapeziums. For each figure:

a Measure the lengths of the sides and mark those that are equal.
b Measure the sizes of the angles and mark those that are equal or right angles.
c Mark the sides that are parallel.

Right trapezium

Isosceles trapezium

7 **a** Draw a quadrilateral of any size and shape with a ruler, then measure the angles.
b Find the sum of the angles in the quadrilateral.
c Compare your result with that of other students.

8 **a** Draw a quadrilateral of any size and shape on a piece of paper, using a ruler. Mark the angles, then tear them off.
b Arrange the angles on the table so that they form a revolution. What is the sum of the angles?
c Copy and complete this statement.
The angle sum of any quadrilateral is _____.

■ Consolidation

Having completed Q1–8 in this exercise, you should have deduced the following properties of quadrilaterals.

◎ **The angle sum of a quadrilateral is 360°.**

Properties of a parallelogram:
- ◎ **opposite sides are parallel**
- ◎ **opposite sides are equal**
- ◎ **opposite angles are equal**
- ◎ **diagonals bisect each other**

Properties of a rhombus:
- ◎ **all properties of a parallelogram**
- ◎ **all sides are equal**
- ◎ **diagonals are perpendicular**
- ◎ **diagonals bisect the angles at the vertices**

Properties of a rectangle:
- ◎ **all properties of a parallelogram**
- ◎ **all angles are right angles**
- ◎ **diagonals are equal**

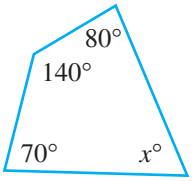
Properties of a square:
- ◎ **all properties of a rectangle**
- ◎ **all sides are equal**
- ◎ **diagonals bisect the angles at the vertices**
- ◎ **diagonals are perpendicular**

Properties of a trapezium:
- ◎ **one pair of opposite sides are parallel**

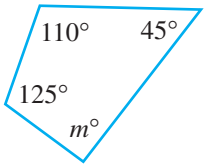
Properties of a kite:
- ◎ **two pairs of adjacent sides are equal**

9 Find the value of the pronumeral in each of these.

a



b



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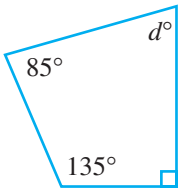

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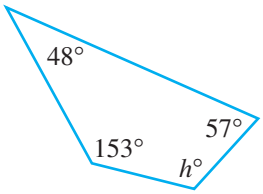

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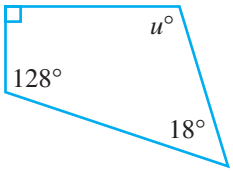

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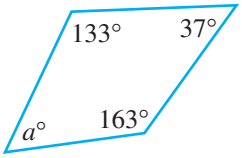
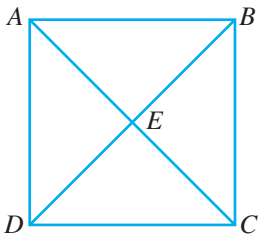
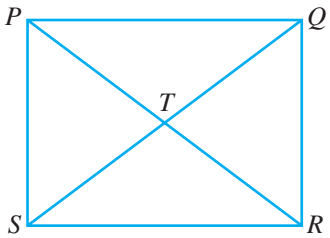


10 *ABCD* is a square.
$AB = 7$ cm, $AC = 10$ cm



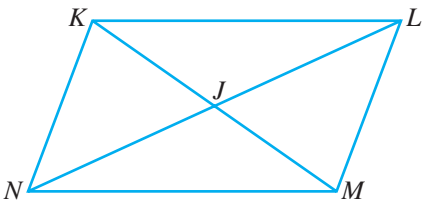
a How long is *BC*?
b What size is $\angle DAB$?
c What length is the diagonal *BD*?
d Which side is parallel to *AD*?
e What size is $\angle BEC$?
f How long is *AE*?
g What size is $\angle DAE$?

11 *PQRS* is a rectangle.
$PQ = 16$ cm, $QR = 12$ cm
$PR = 20$ cm, $\angle PQT = 37°$



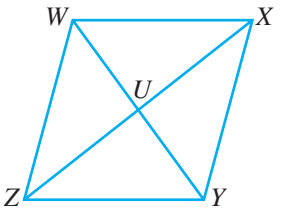
a How long is *SR*?
b What size is $\angle SRQ$?
c What length is the diagonal *QS*?
d Which side is parallel to *SR*?
e Is $PR \perp QS$?
f How long is *QT*?
g What size is $\angle TQR$?

12 *KLMN* is a parallelogram.
$KL = 15$ cm, $KN = 8$ cm
$KM = 13$ cm, $\angle KNM = 60°$



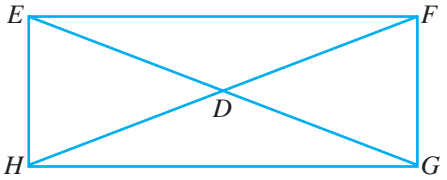
a How long is *LM*?
b What size is:
i $\angle KLM$? **ii** $\angle NKL$?
c Is $LN = KM$?
d Which side is parallel to *MN*?
e Is $KM \perp NL$?
f How long is *JM*?
g Is $\angle LMK = \angle KMN$?

13 *WXYZ* is a rhombus.
$WZ = 13$ cm, $ZX = 24$ cm
$\angle ZWX = 135°$



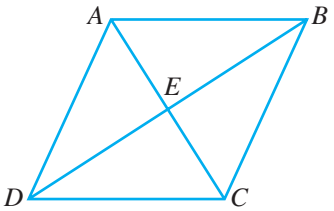
a How long is *ZY*?
b What size is:
  i $\angle XYZ$?
  ii $\angle WZY$?
c Is $WY = ZX$?
d Which side is parallel to *XY*?
e What size is $\angle ZUY$? **f** How long is *UZ*? **g** What size is $\angle XWU$?

14 *EFGH* is a rectangle.
$FH = 14$ cm, $\angle DFG = 70°$



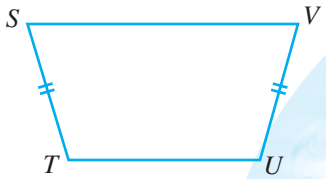
a How long is:
  i *DF*? **ii** *DG*?
  iii *DE*? **iv** *DH*?
b What size is:
  i $\angle DGF$? **ii** $\angle FDG$? **iii** $\angle EDH$? **iv** $\angle DEH$?
  v $\angle DHE$? **vi** $\angle EDF$? **vii** $\angle DEF$? **viii** $\angle DFE$?
  ix $\angle DHG$? **x** $\angle HDG$?

15 *ABCD* is a rhombus.
$\angle EBC = 30°$
What size is:



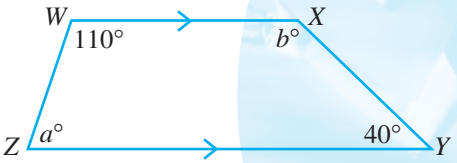
a $\angle BEC$? **b** $\angle AEB$?
c $\angle BCE$? **d** $\angle ABE$?
e $\angle CAB$? **f** $\angle BDC$?
g $\angle ADB$? **h** $\angle DAC$?
i $\angle ACD$? **j** $\angle DAB$?

16 *STUV* is an isosceles trapezium.

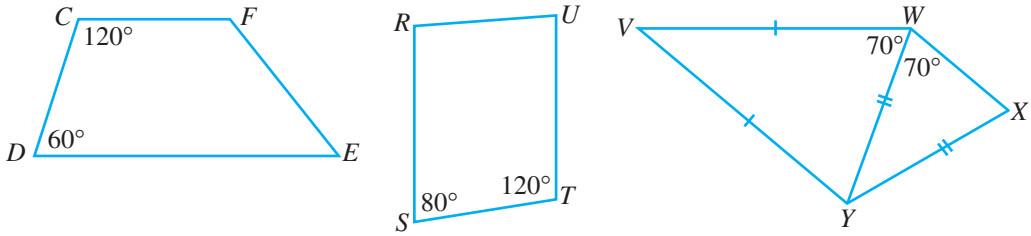


a Which angles must be equal?
b Which sides are parallel?
c If $\angle STU = 140°$, find the sizes of all angles in the trapezium.

17 *WXYZ* is a trapezium.
Find values for *a*, *b*.



18 State whether each quadrilateral is a trapezium. Give reasons for your answers.



19 Draw a kite and show that it can be divided into:

a two isosceles triangles
b two equal scalene triangles
c four right-angled triangles

■ Further applications

20 What kind of quadrilateral (square (S), rectangle (RE), parallelogram (P), rhombus (RH), trapezium (T), kite (K)) could *ABCD* be if:

a the opposite sides are equal?
b the opposite angles are equal?
c the opposite sides are parallel?
d all sides are equal?
e all angles are right angles?
f the diagonals are equal?
g the diagonals bisect each other?
h the diagonals are perpendicular?
i the diagonals bisect the angles at the vertices?

21 What kind of quadrilateral could *KLMN* be (use the abbreviations in Q20) if:

a the opposite sides only are equal and all angles are right angles?
b all sides are equal and all angles are right angles?
c all sides are equal and the opposite sides are parallel?
d the diagonals bisect each other at right angles?
e the diagonals bisect the angles at the vertices and all angles are right angles?
f the diagonals have different lengths and bisect each other?
g the diagonals are equal and do not bisect the angles at the vertices?
h the diagonals are perpendicular and have different lengths?

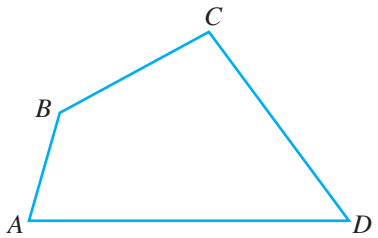
22 Choose the true statement from each pair.

a (A) a rectangle is a parallelogram (B) a parallelogram is a rectangle
b (A) a rhombus is a square (B) a square is a rhombus
c (A) a rhombus is a parallelogram (B) a parallelogram is a rhombus
d (A) a rectangle is a square (B) a square is a rectangle
e (A) a trapezium is a parallelogram (B) a parallelogram is a trapezium

10.6 Angle sum of a quadrilateral

The emphasis in this exercise will be on combining the properties of quadrilaterals with other general angle properties to find the size of an angle in a quadrilateral, or the value of a pronumeral. Of particular importance is the result for the angle sum of a quadrilateral.

The angle sum of a quadrilateral is 360°.

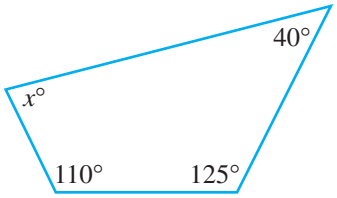


$\angle A + \angle B + \angle C + \angle D = 360°$

In the examples below, reasons for each step of working are given. They have been included for the purpose of explanation only. It is not intended that students give formal reasons in the questions that follow.

Example 1

Find the value of the pronumeral.



Solution

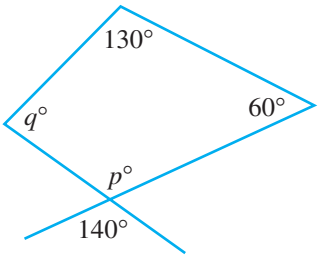
$x = 360 - (110 + 125 + 40)$ (the angle sum of a quadrilateral is 360°)

$= 85$

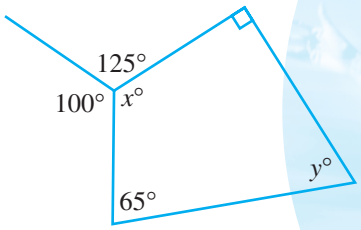
Example 2

Find the values of the pronumerals in each of these.

a



b



Solutions

a **i** $p = 140$ (the vertically opposite angles are equal)

ii $q = 360 - (140 + 60 + 130)$ (the angle sum of a quadrilateral is 360°)
$= 30$

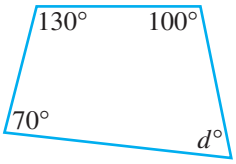
b **i** $x = 360 - (100 + 125)$ (the angles at a point add to 360°)
$= 135$

ii $y = 360 - (65 + 135 + 90)$ (the angle sum of a quadrilateral is 360°)
$= 70$

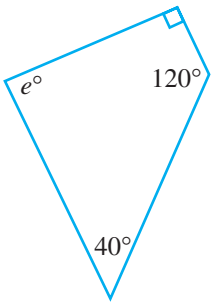
Exercise 10.6

1 Find the value of the pronumeral in each of these.

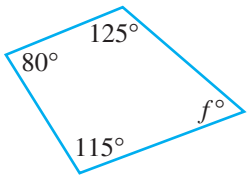
a



b



c



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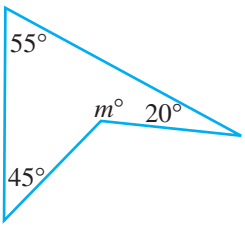
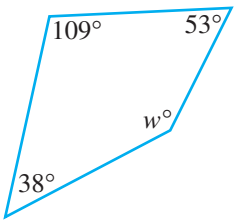


e



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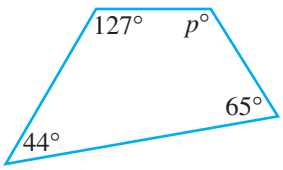
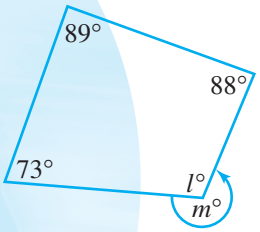


Consolidation

2 Find the values of the pronumerals in each of these.

a



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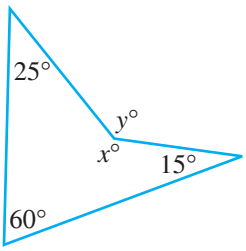

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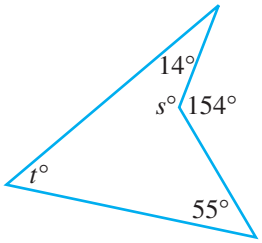
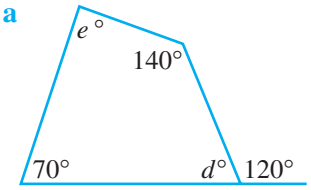


3 Find the values of all pronumerals in the following.

a



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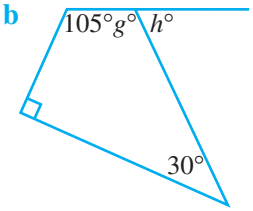

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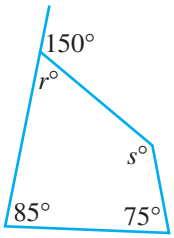
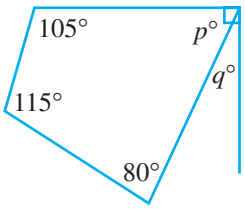


4 Find the values of all pronumerals in each of these.

a



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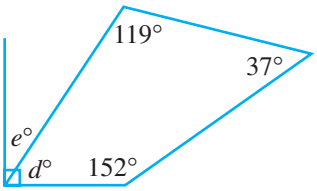

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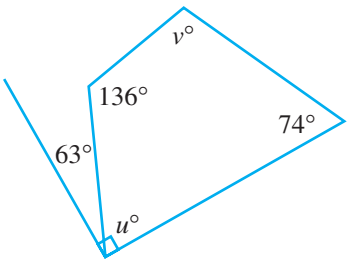
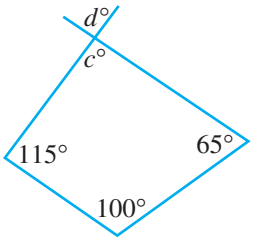


5 Find the values of the pronumerals.

a



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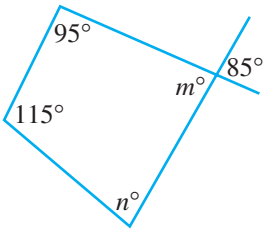

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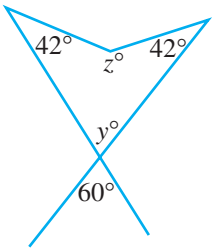
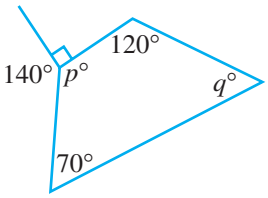


6 Find the values of the pronumerals in each of these.

a



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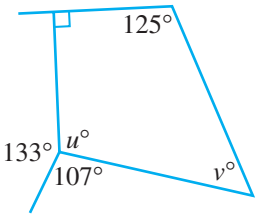

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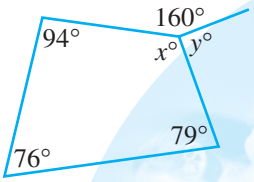
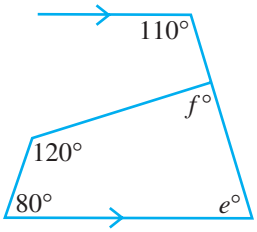
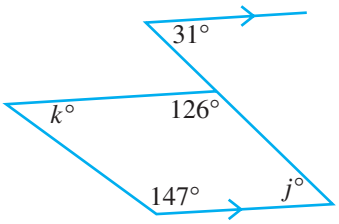


7 Find all pronumerals in each of the following.

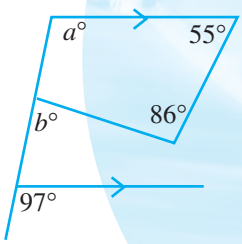
a



b



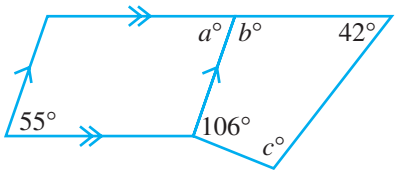
c



Further applications

8 Use the properties of the special quadrilaterals to find all pronumerals in each of these.

a



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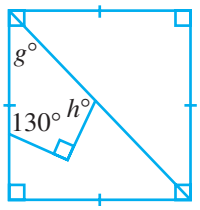

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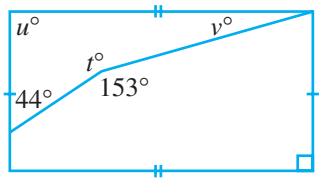
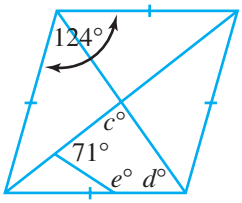


d

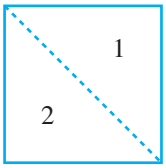


TRY THIS

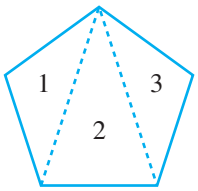
Angle sum of a polygon

Let us investigate and discover if there is a pattern or formula for finding the sum of the angles in any polygon.

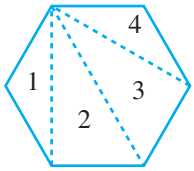
Quadrilateral



Pentagon



Hexagon



Copy and complete this table.

Polygon	Number of sides	Number of triangles	Angle sum
Quadrilateral	4	2	$2 \times 180 = 360°$
Pentagon	5	3	$3 \times 180 = 540°$
Hexagon	6	4	$4 \times 180 =$
Heptagon	7	…	…
Octagon	8	…	…
Nonagon	9	…	…
Decagon	10	…	…
Dodecagon	12	…	…
	50		
	100		
	n		

Hence write the rule which will give you the angle sum, if you know the number of sides of the polygon.

10.7 Symmetry

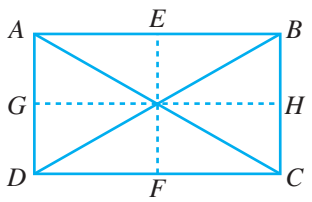
Many of the shapes that we are familiar with from our everyday experiences have **symmetry**. That is, the figure is in some way made up of identical parts, although these parts may be in different orientations. Many artists, sculptors, architects and builders throughout the ages have tried to ensure that their works were symmetric, and thus pleasing to the eye.

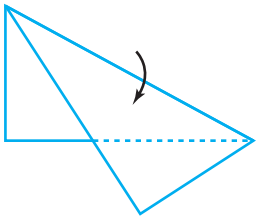
There are many different types of symmetry that a figure may have. In this exercise we will examine **line symmetry** (i.e. symmetry about a line) and **rotational symmetry** (i.e. symmetry about a point).

A figure has line symmetry if a line can be drawn through the figure dividing it into two parts, such that each part is the mirror image of the other.

The dividing line is called a **line of symmetry** or an **axis of symmetry**. Some figures have several axes of symmetry, while others may have none. The axes of symmetry in geometric figures may be thought of as fold lines. That is, lines over which the figure may be folded onto itself.

In this rectangle, the lines *EF* and *GH* are axes of symmetry, however the diagonals *AC* and *BD* are not.

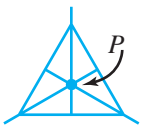




A figure has rotational symmetry if it can be rotated or turned about a fixed point through an angle of 180° or less so as to look identical.

That is, after rotation the figure has the same shape, size, position and orientation. The point about which the figure is rotated is called the **centre of symmetry**. The number of times that the figure can be rotated onto itself in one complete revolution is called the **order of symmetry**.

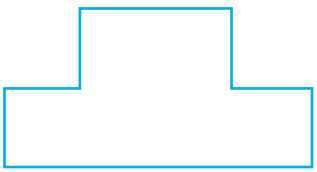
In this diagram, the figure can be rotated onto itself three times in one complete revolution about the point *P*. Therefore, the figure has rotational symmetry of order 3.



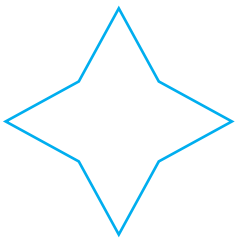
Example 1

Mark in all axes of symmetry in each figure.

a

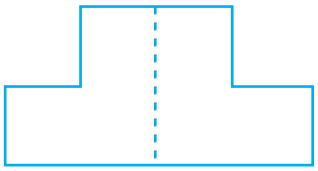


b



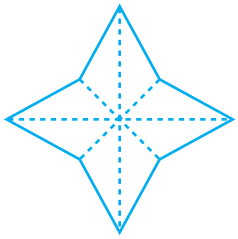
Solutions

a



1 axis of symmetry

b



4 axes of symmetry

Example 2

State whether or not each figure has rotational symmetry. If it does, state the order of symmetry.

a

b

Solutions

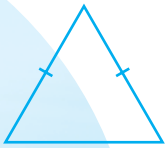
a The figure is based on a regular pentagon. It has rotational symmetry of order 5 since rotations of 72°, 144°, 216°, 288° and 360° all result in a repetition of the same figure.

b The figure does not have rotational symmetry, since there is no rotation (other than 360°) that will result in a repetition of the figure in the same orientation.

Exercise 10.7

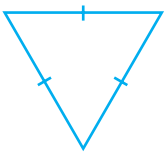
1 Copy each figure and drawn in all axes of symmetry.

a



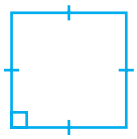
isosceles triangle

b



equilaterial triangle

c

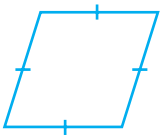


square

d

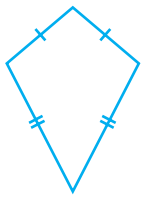
rectangle

e



rhombus

f

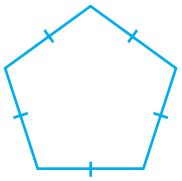


kite

g

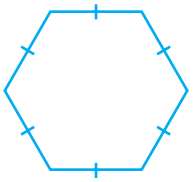
ellipse

h



regular pentagon

i



regular hexagon

2 How many axes of symmetry has a parallelogram?

3 **a** How many axes of symmetry has a trapezium in general?

b Draw a trapezium that has one axis of symmetry. What kind of trapezium have you drawn?

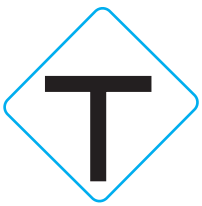
Consolidation

4 **a** If the diagonals of a quadrilateral are axes of symmetry, must they bisect the angles at the vertices?

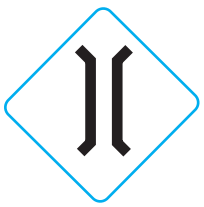
b In which of the special quadrilaterals does this occur?

5 Many of the street signs that we see every day have line symmetry. How many axes of symmetry do each of these road signs have?

a



b

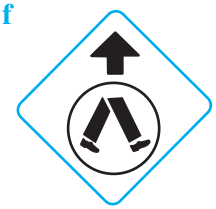


c

d

e

f

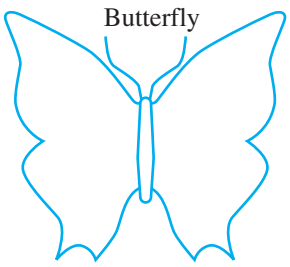


g

h

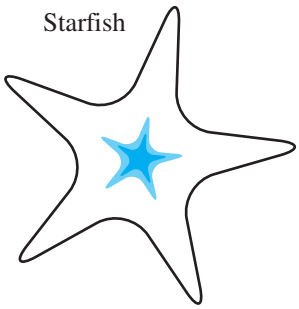
6 Many examples of symmetry occur in nature. How many axes of symmetry can you find in these?

a

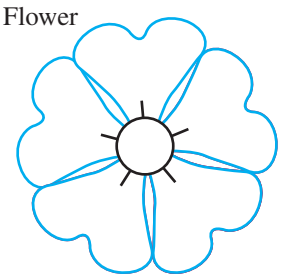


b

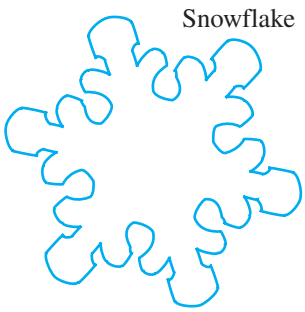
c



d



e



7 Copy these logos, then draw lines to indicate their axes of symmetry.

a

b

c

d

8 Determine whether each figure in Q1 has rotational symmetry. If it does, state the order of symmetry.

9 Does a parallelogram have rotational symmetry? If so, state the order of symmetry.

10 Draw each of the following figures and indicate the centre of rotational symmetry.

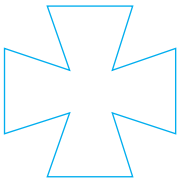
a an equilateral triangle
b a square
c a parallelogram
d a regular octagon

11 What can you deduce about the angles in a quadrilateral that has rotational symmetry of:

a order 2?
b order 4?

12 Many symbols or logos have rotational symmetry. What is the order of rotational symmetry of these figures?

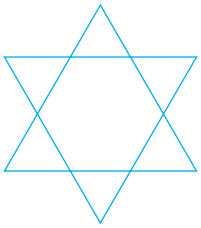
a



b

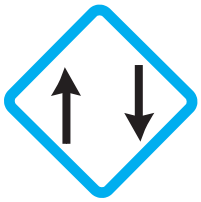
c

d



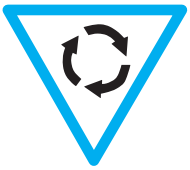
13 What is the order of rotational symmetry of these road signs?

a



b

c



d

14 **a** Copy and complete this table.

Shape	Number of sides	Order of symmetry
equilateral triangle		
square		
regular pentgon		
regular hexagon		

b What relationship can you see between the number of sides and the order of rotational symmetry in a regular polygon?

c What would be the order of symmetry of:

i a regular decagon? **ii** a regular dodecagon?

■ Further applications

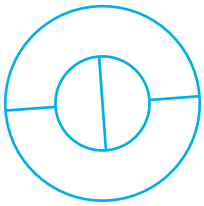
15 A figure has **point symmetry** if it looks exactly the same after a rotation of 180° about a fixed point, i.e. after a half turn. State whether each of these figures has point symmetry.

a

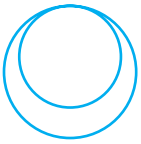
b

c

d



e



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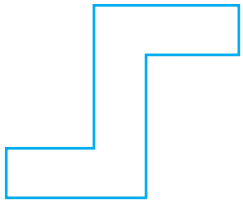
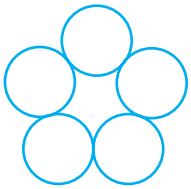


g



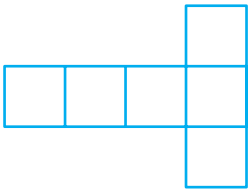
h

i

16 **a** Which of the special quadrilaterals have point symmetry?
b What can you say about the order of rotational symmetry of figures that have point symmetry?

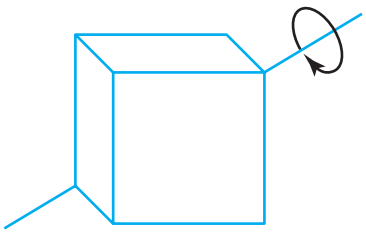
Problem solving

1 There are 11 different nets that can be used to form a model of a cube. One is shown here. Find the other 10.

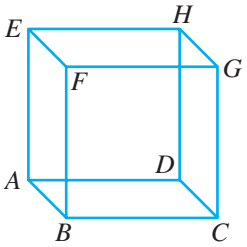


2 Suppose you have 6 red and 6 blue squares of paper, all of the same size. How many different models of a cube can you make using six of the squares?
3 If you spent $100 every minute, how long would it take to spend $1 000 000?
4 You've just received a chain letter with a list of 5 names on it. Your instructions are to send $1 to the person named on the top of the list. You are to then cross her name out and add your name to the bottom. You are then to send 5 copies of this letter to friends with instructions for them to proceed in the same manner. Assume that this process continues and no-one stops the chain. How much money should you finally receive? (Chain letters are illegal in Australia. Can you see what can happen with them?)

5 Describe the shape of the surface formed by spinning a cube around the axis shown.

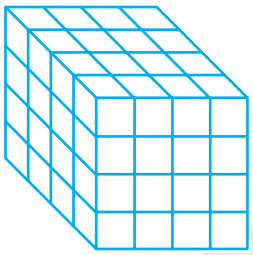


6 List all the paths from *C* to *E*, without backtracking.



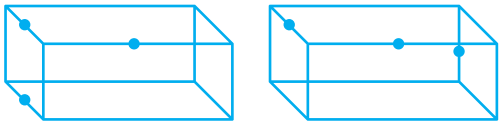
7 In one step take a strip of paper and fold it into a regular pentagon.

8 How many 1×1, 2×2, 3×3 cubes are in this figure?



9 What is the minimum number of colours needed to paint a cuboid so that no two adjacent faces are the same?

10 A cut is made through the midpoints of three sides in the two solid cuboids shown below. Describe the shapes of the exposed surfaces.

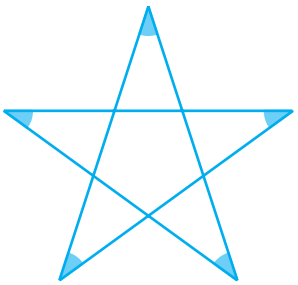


Focus on working mathematically

Stars

Introduction

Star shapes are attractive. They are used widely for flag designs, party decorations, wrapping paper and school badges for example. Humans have used this shape for a long time to depict stars in the sky because stars appear to twinkle. In mathematics they have played an important role in geometrical discoveries such as the pentagram. (You will see this later.)



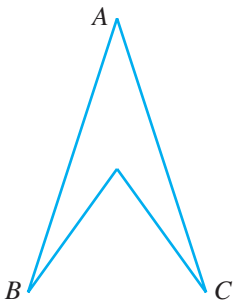
Question: What is the angle sum of a 5-pointed star as shown in the diagram? (It is the sum of the shaded angles that we wish to find.)

Learning activities

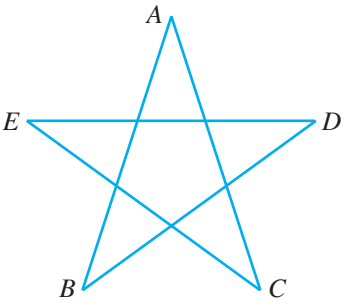
One way to find out is to draw a 5-pointed star and measure the angles.

Materials needed: Drawing paper, set square or ruler and a pencil.

1 Draw this shape on your paper. Call the top angle *A* and the bottom angles *B* and *C*. The angles do not have to be equal.



2 Complete the drawing to make a star shape. Call the angles of the other points of the star *D* and *E*.
3 Measure the angles *A*, *B*, *C*, *D* and *E* of the star with a protractor and write them on your diagram.
4 Find the sum of your angles, $A + B + C + D + E$. Answer the focus question.



5 Draw a 5-pointed star with five 'pointed' angles equal. What will be the size of each angle? This shape is called a *pentagram* and was the secret symbol of the followers of Pythagoras. They drew the symbol on the palms of their hands and showed it to gain entry to secret meetings.
6 Notice the pentagon at the centre of the symbol. It has five equal sides. Join up the diagonals and you will get a surprise. Can you repeat this? What happens?
7 Find out about drawing pentagrams on a computer. Check it out with your teacher.

Extension activities

1 Can you draw a regular 6-pointed star? What will be the size of the angles this time? What shape will be at the centre?
2 What about an 8-pointed star? Investigate.

Let's communicate

Draw a pentagram on a piece of paper without lifting the pen. Now describe what you did in words to your neighbour. Use sufficient detail for them to draw the pentagram from what you describe. Discuss what happens! Take it home for Mum or Dad or a friend to try.

If you can, make some star shapes from coloured cardboard to decorate your classroom and to communicate what you found. A Caltex logo from a local service station could be a help.

Reflecting

Think over what you have learned about the value of practical drawing and measurement to investigate the properties of shapes. What are the advantages and disadvantages of drawing on a computer?

Language link with Macquarie

MACQUARIE

1 *Explain what is meant by a* ***plane*** *shape.*
2 *How is a* ***triangle*** *different from a* ***quadrilateral****?*
3 *When a* ***quadrilateral*** *has one pair of* ***parallel*** *sides what do we call it?*
4 *Define* ***polygon****.*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***shape****:*

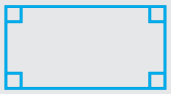
shape *noun* **1.** the outline or form of something or someone. **2.** proper order: *The house is now in shape for our visitors.* **3.** condition: *His business affairs were in bad shape. | He's in pretty good shape considering his age.*
–verb **4.** to give definite form, shape, or character to: *The potter carefully shaped his pots.*
–phrase **5. shape up**, to develop: *The new worker is shaping up well.*

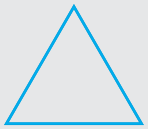
What sorts of jobs need special skills related to ***shape*** *in our society?*

CHAPTER REVIEW

1 Name these plane shapes.

a

b


c


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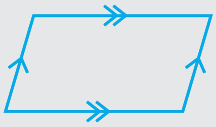

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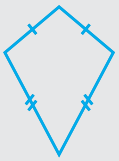
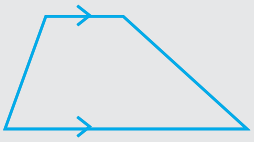
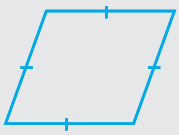
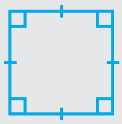


f


g


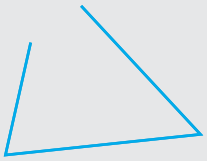
h


i

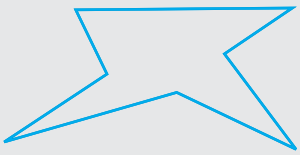
2 Name the polygon that has the given number of sides.

a	4	**b**	3	**c**	6	**d**	10
e	5	**f**	8	**g**	7	**h**	9
i	11	**j**	12				

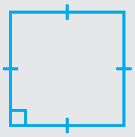
3 State whether or not each figure is a polygon. Give a reason for those that are not polygons.

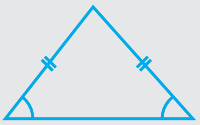
a


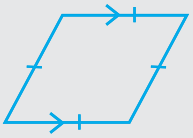
b

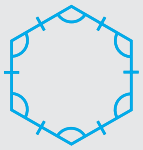
c


4 State whether each figure is regular or not.

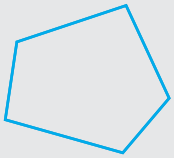
a


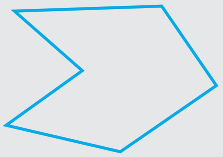
b


c


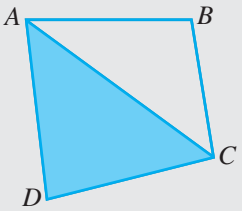
d


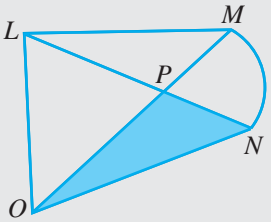
5 State whether each figure is convex or non-convex.

a


b


6 Name the shaded regions using their vertices.

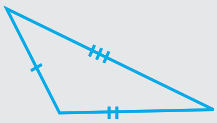
a


b


7 How many diagonals are there in a regular heptagon?

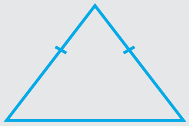
8 Classify these triangles according to their sides.

a



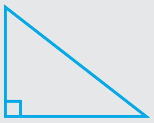
b

c



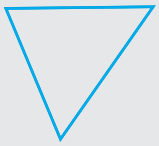
9 Classify these triangles according to their angles.

a



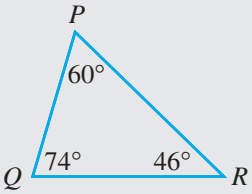
b

c

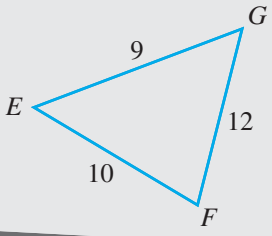


10 What size are the angles in an equilateral triangle?

11 Name the longest and shortest sides.



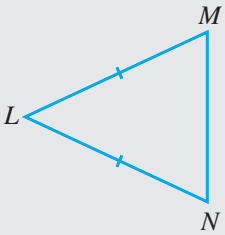
12 Name the largest and smallest angles.



13 Which two of these triangles cannot exist, given the following side lengths. All measurements are in metres.

A 5, 6, 10 **B** 13, 17, 3
C 8, 7, 15 **D** 9, 10, 17

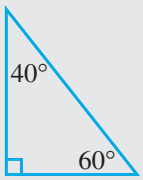
14 **a** Which angles are equal in this triangle?
b Which angle is the apex angle?



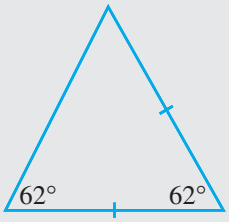
15 Is it possible for:
a an equilateral triangle to be obtuse?
b an isosceles triangle to be right angled?
c a triangle to have both an obtuse angle and a right angle?

16 Which one of these triangles could possibly exist?

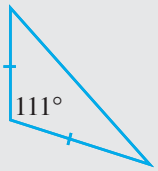
A



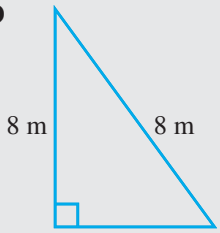
B



C



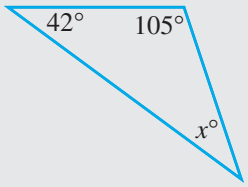
D



CHAPTER REVIEW

17 Find the value of the pronumeral in each triangle.

a



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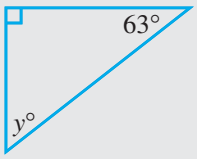

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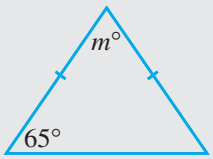

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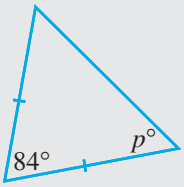


18 In which of the special quadrilaterals:

a are the opposite sides parallel?
b are the opposite angles equal?
c are the diagonals equal?
d do the diagonals intersect at right angles?
e do the diagonals bisect the angles at the vertices?

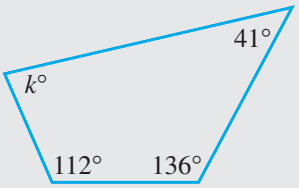
19 In which quadrilateral are two pairs of adjacent sides equal?

20 $PQRS$ is a rectangle and the diagonals intersect at T. If $PR = 22$ cm and $\angle TSR = 36°$, find:

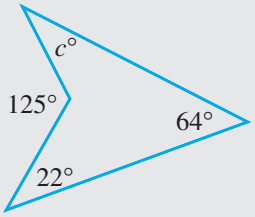
a QS **b** TQ **c** $\angle PRS$
d $\angle PTQ$ **e** $\angle PQT$ **f** $\angle PST$

21 Find the value of the pronumeral in each quadrilateral.

a

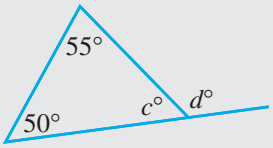


b



22 Find all pronumerals in each of these.

a



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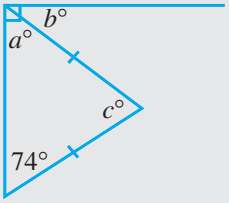

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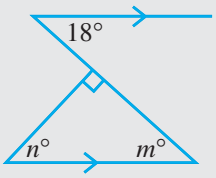

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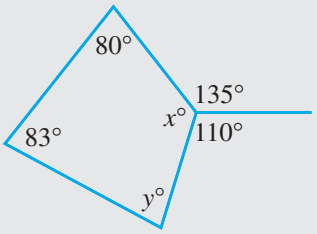


23 How many axes of symmetry are there in:
- **a** an equilateral triangle?
- **b** an isosceles triangle?
- **c** a scalene triangle?

24 **a** Which one of the triangles in Q23 has rotational symmetry?
- **b** What is its order of rotational symmetry?

25 State the number of axes of symmetry in each of the special quadrilaterals.

26 Which of the special quadrilaterals have rotational symmetry? For those that have, state the order of symmetry.

11 Measurement, length and perimeter

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- read the measurements shown on various instruments
- solve problems involving different units of length
- define perimeter
- calculate the perimeter of simple shapes.

Stage 4

After completing this chapter, you should be able to:

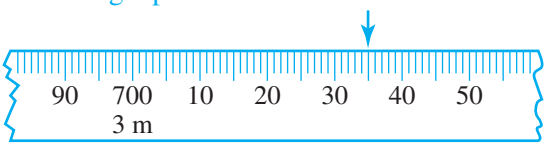
- estimate lengths and distances
- recognise that all measurements are approximate
- choose appropriate units of measurement based on the required degree of accuracy
- select and use appropriate devices to measure lengths and distances
- describe the limits of accuracy of measuring instruments
- interpret the meaning of prefixes in units of measurement
- convert between metric units of length
- find the perimeter of simple composite figures
- solve practical problems that involve length, distance and perimeter.

11.1 Measuring instruments

The ability to take readings from a variety of measuring instruments is an essential skill in mathematics, as well as in other subjects such as science and geography. Reading instruments such as clocks, thermometers, weighing scales, petrol gauges and speedometers plays an important part in our everyday lives.

Example 1

Read off in mm the measurement shown on this measuring tape.



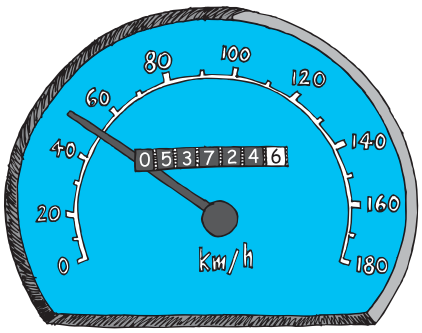
Solution

The reading shown on the measuring tape is 3 m 735 mm, i.e. 3735 mm.

Example 2

a What speed is shown on the speedometer?

b What distance is shown on the odometer?



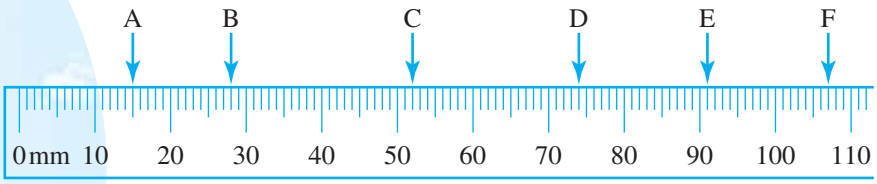
Solutions

a The speedometer shows a speed of 50 km/h.

b The odometer shows a distance of 53 724.6 km. (The last digit measures tenths of a kilometre.)

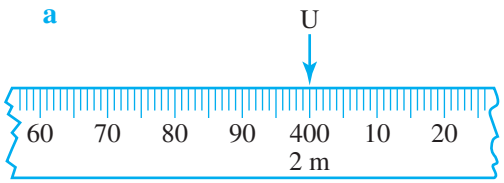
Exercise 11.1

1 Write down each of the measurements shown on this ruler, correct to the nearest mm.

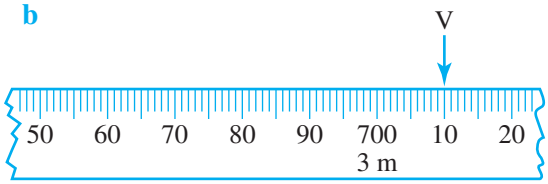


2 Read off in mm each of the measurements shown on these measuring tapes.

a



b



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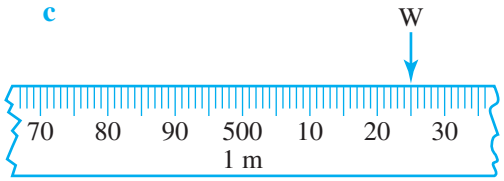

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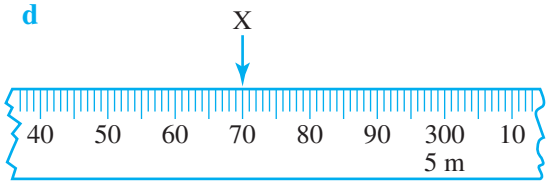

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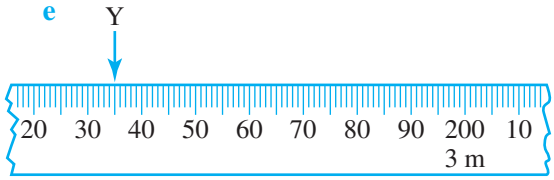
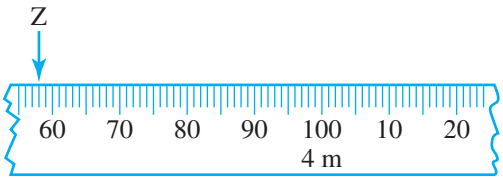
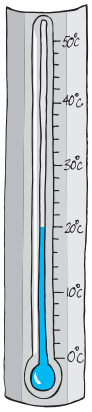


f



3 What temperature is shown on each thermometer?

a



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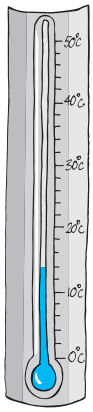

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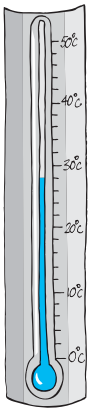

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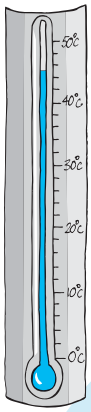
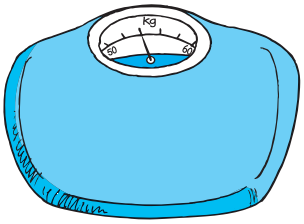


■ Consolidation

4 Find the weight that is shown on each set of scales, correct to the nearest kg.

a



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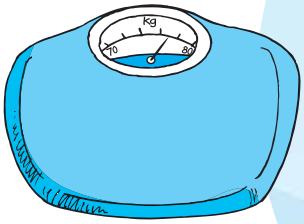
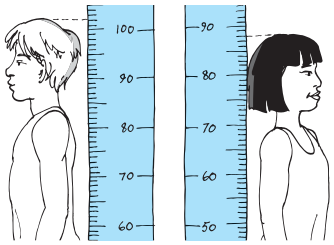
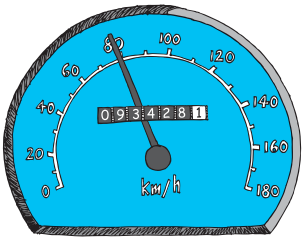


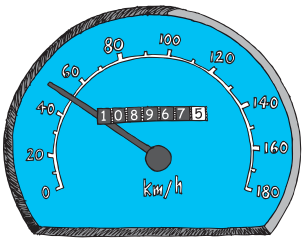
5 As part of their regular medical checkup, Lars and Enikö had their heights measured in cm. How tall is each child?



6 **i**



ii

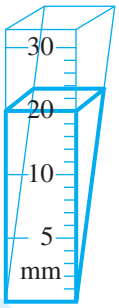


a The speedometer of a car measures the speed at which the car is travelling, in km/h. What speed is shown on each speedometer?

b The odometer measures the total distance travelled by a car, in km, with the last digit measured in tenths of a km. What measurement is shown on each odometer?

7 A rain gauge is used to measure the rainfall, in mm, in a particular area. Write down the measurement that is shown on each rain gauge.

a



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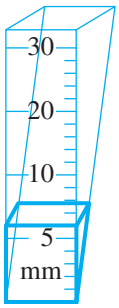

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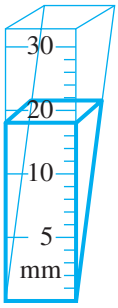
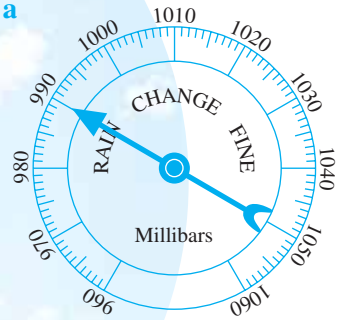


8 Meteorologists measure air pressure in millibars using a barometer. What air pressure is shown on each barometer?

a



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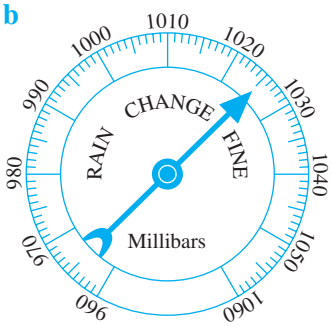

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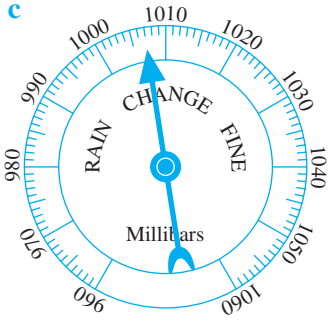
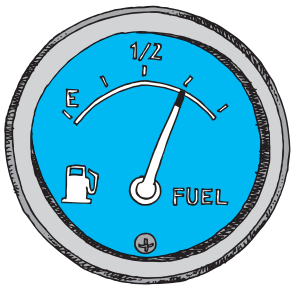


9 Estimate the fraction of the petrol tank that is full in each of these.

a



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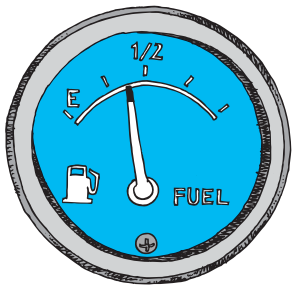

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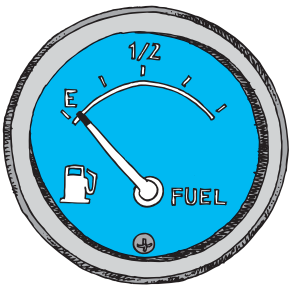
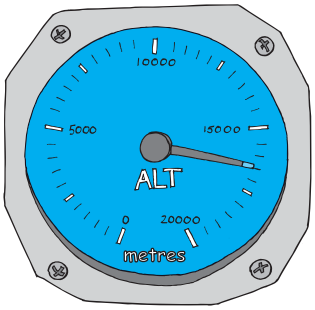
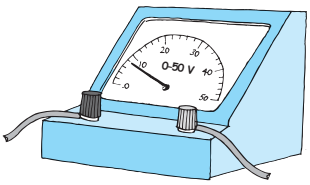


10 The pilot of an aeroplane uses an altimeter to determine the height of the plane, in metres, above sea level. What height is shown on this altimeter?

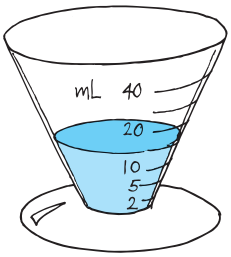


11 Write down the voltage that is shown on this voltmeter.



12 What volume of liquid is shown in each of these.

a



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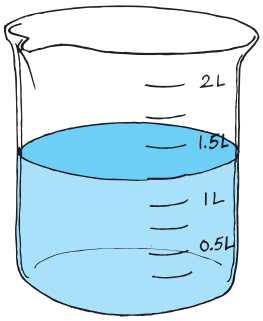

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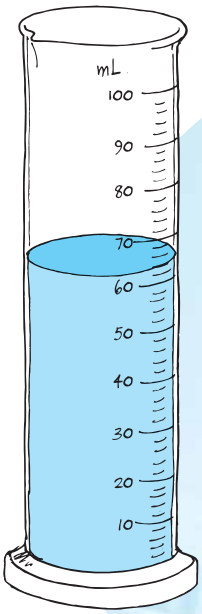
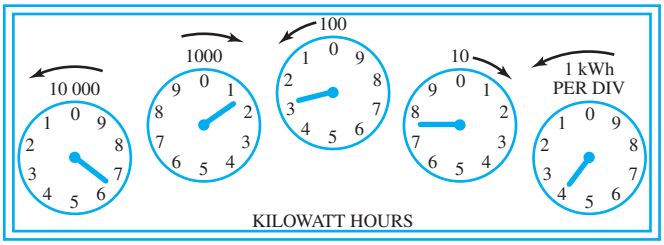


Further applications

13 Write down the reading shown on each of these electricity meters.

a



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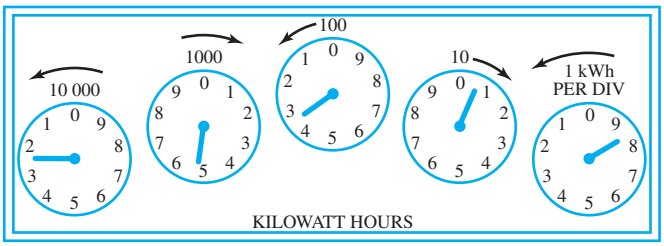

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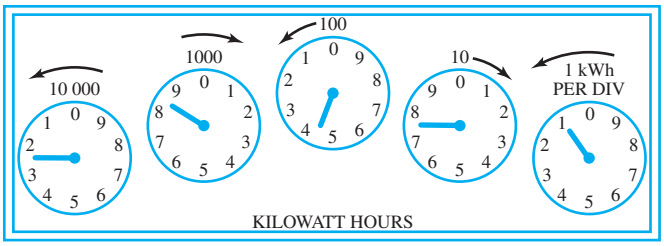
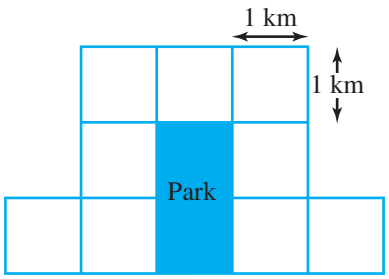


TRY THIS

Police patrol

The city of Mathia has a street plan as shown. Each block is 1 km and, as you can count, the total street length is 28 km. What is the minimum distance the police car must travel to patrol all the streets? Draw the shortest route.



11.2 Converting units of length

Length refers to the distance along an interval and as such is a one-dimensional measurement. Historically, people measured lengths and distances using their body parts. These units are known as natural units.

Some examples of natural units were:
- the cubit—the distance from the elbow to the fingertip
- the span—the length of an outstretched hand
- the pace—the distance travelled in one walking step.

Natural units have an obvious limitation in that everybody's arms and legs are different sizes and so the resulting measurements will vary from person to person.

Eventually, measurements were *standardised* and the **Imperial System** of measurements was formed. This included units such as the inch, foot, yard and mile. The main drawback of the system was that the conversion factors between the units were difficult numbers to deal with mentally. For example, 1 mile = 1760 yards.

The metric or SI system (International System of units) was developed in France in 1789. Today it is used in many countries around the world. The main advantage of the SI system is that the conversion factors between the units are powers of 10, and they are therefore much easier to work with than those in the Imperial System.

In the metric system, the base unit for length is the **metre** (m). The metre was initially intended to be one ten-millionth of the distance from the North Pole to the equator along the line of longitude that passes through Paris, France. This definition was revised in 1960, with the metre then being defined in terms of the wavelength of light. Other units such as the millimetre, centimetre and kilometre are related to the metre according to their prefix. For example, milli means 'one thousandth', centi means 'one hundredth' and kilo means 'one thousand'.

Unit	Fraction or multiple of 1 m
millimetre (mm)	$\frac{1}{1000}$ m
centimetre (cm)	$\frac{1}{100}$ m
metre (m)	1 m
kilometre (km)	1000 m

The common conversions for length are:
- **10 mm = 1 cm (i.e. 10 millimetres = 1 centimetre)**
- **100 cm = 1 m (i.e. 100 centimetres = 1 metre)**
- **1000 m = 1 km (i.e. 1000 metres = 1 kilometre)**

To convert to a smaller unit of length:
- **multiply by the conversion factor.**

To convert to a larger unit of length:
- **divide by the conversion factor.**

Example 1

Convert:

a 5 cm to mm **b** 30 mm to cm **c** 8.2 m to cm
d 740 cm to m **e** 5.91 km to m **f** 3 m to km

Solutions

a 5 cm = (5 × 10) mm
= 50 mm

b 30 mm = (30 ÷ 10) cm
= 3 cm

c 8.2 m = (8.2 × 100) cm
= 820 cm

d 740 cm = (740 ÷ 100) m
= 7.4 m

e 5.91 km = (5.91 × 1000) m
= 5910 m

f 3 m = (3 ÷ 1000) km
= 0.003 km

Example 2

Convert 0.48 km to cm

Solution

0.48 km = (0.48 × 1000) m
= 480 m
= (480 × 100) cm
= 48 000 cm

Exercise 11.2

1 Choose the most appropriate unit (mm, cm, m, km) that could be used to measure the:

a length of an ant **b** height of a child
c length of a bus **d** distance between Sydney and Bathurst
e height of a seat **f** distance run in a horse race
g width of a postage stamp **h** distance between two train stations
i length of a pen **j** length of a cricket pitch
k thickness of a ruler **l** distance between Earth and the moon

2 Convert:

a 2 cm to mm **b** 50 mm to cm **c** 6 cm to mm **d** 110 mm to cm
e 230 mm to cm **f** 30 cm to mm **g** 3.5 cm to mm **h** 47 mm to cm
i 92 mm to cm **j** 8.6 cm to mm **k** 213 mm to cm **l** 10.3 cm to mm

3 Convert:

a 4 m to cm **b** 300 cm to m **c** 9 m to cm **d** 1600 cm to m
e 2200 cm to m **f** 40 m to cm **g** 1.5 m to cm **h** 260 cm to m
i 5.3 m to cm **j** 740 cm to m **k** 189 cm to m **l** 3.75 m to cm

4 Convert:

a 3 km to m **b** 7000 m to km **c** 12 000 m to km **d** 8 km to m
e 20 000 m to km **f** 60 km to m **g** 2500 m to km **h** 4.1 km to m
i 6.35 km to m **j** 1320 m to km **k** 5248 m to km **l** 6.004 km to m

Consolidation

5 Convert:

a 6 m to cm **b** 3 cm to mm **c** 2000 m to km **d** 500 cm to m
e 90 cm to m **f** 4 km to m **g** 2.3 cm to mm **h** 8.2 km to m
i 680 cm to m **j** 4800 m to km **k** 59 mm to cm **l** 9.4 m to cm
m 1.25 km to m **n** 224 cm to m **o** 40 cm to mm **p** 1463 cm to m
q 150 mm to cm **r** 2864 m to km **s** 250 m to cm **t** 0.5 km to m

6 Complete each of these conversions.

a 0.04 m = _____ cm **b** 200 m = _____ km **c** 5 mm = _____ cm
d 0.6 km = _____ m **e** 9 cm = _____ m **f** 0.1 cm = _____ m
g 0.008 m = _____ cm **h** 0.002 km = _____ m **i** 0.3 mm = _____ cm
j 0.15 m = _____ km **k** 0.028 cm = _____ m **l** 1.4 m = _____ km
m 0.04 mm = _____ cm **n** 15.3 km = _____ m **o** 0.7 cm = _____ mm
p 8 cm = _____ m **q** 16 m = _____ km **r** 0.102 m = _____ cm

7 Complete these conversions.

a 3 m = _____ mm **b** 7 km = _____ cm **c** 4000 mm = _____ m
d 2 km = _____ mm **e** 800 000 cm = _____ km **f** 6 000 000 mm = ___ km
g 500 mm = _____ m **h** 1.4 km = _____ cm **i** 3.85 m = _____ mm
j 23 mm = _____ m **k** 5460 cm = _____ km **l** 25 000 mm = _____ km

8 How many:

a mm in $\frac{1}{2}$ cm? **b** m in $1\frac{1}{2}$ km? **c** cm in $\frac{1}{4}$ m?
d m in $\frac{3}{4}$ km? **e** mm in $3\frac{1}{2}$ cm? **f** m in $2\frac{1}{4}$ km?
g cm in $2\frac{1}{10}$ m? **h** mm in $1\frac{1}{5}$ cm? **i** m in $3\frac{7}{10}$ km?

Further applications

This table lists the common units of length as well as some lesser known units. The prefix of each unit shows how it is related to the metre. For example, in the decimetre, the prefix *deci* means *one-tenth of*.

Fraction or multiple of 1 m	Unit
$\frac{1}{1000}$	millimetre (mm)
$\frac{1}{100}$	centimetre cm)
$\frac{1}{10}$	decimetre (dm)
1	metre (m)
10	decametre (dam)
100	hectometre (hm)
1000	kilometre (km)

9 Use the table above to convert:

a 1 m to dm **b** 1 hm to m **c** 1 dam to m **d** 2 km to hm
e 5 dm to mm **f** 4 dam to cm **g** 3 hm to cm **h** 8 km to dm
i 9 hm to mm **j** 1.4 dm to cm **k** 4.9 dam to m **l** 6.2 hm to m

10 Use the table on page 418 to convert:

a 10 cm to dm	**b** 30 hm to km	**c** 80 dm to m
d 500 mm to dm	**e** 4000 cm to dam	**f** 90 000 mm to dm
g 20 m to hm	**h** 670 cm to dam	**i** 4900 dm to km
j 0.7 dam to km	**k** 0.4 dm to hm	**l** 51.9 mm to dam

11.3 Solving problems involving length and distance

Example 1

How many boxes of height 40 cm can be stacked beneath an overhead cupboard of height 1.92 m?

Solution

i 1.92 m = (1.92 × 100) cm
= 192 cm

ii Now, 4 × 40 cm = 160 cm and
5 × 40 cm = 200 cm.
∴ 4 boxes can be stacked beneath the cupboard.

Example 2

From a 3.5 m length of timber, a carpenter uses 1 m 25 cm to make a cabinet door. What length of timber remains? Give your answer in cm.

Solution

i 3.5 m = (3.5 × 100) cm
= 350 cm

ii 1 m 25 cm = 125 cm

iii 350 cm – 125 cm = 225 cm

Exercise 11.3

1 A house plan shows the real length of a lounge room to be 7800 mm. How many metres of carpet will be needed to cover this length?

2 Which is the greater distance, 12.4 m or 11 684 mm, and by how much?

3 How many laps of a 400 m athletics track must be completed by a distance runner competing in the 10 km event?

4 How many matches of length 40 mm can be cut from a 5 m strip of wood?

5 Find, in cm, the total length of timber required to make the four legs of a chair, if the length of each leg is to be 375 mm.

6 What length, in mm, remains from a 14 cm long pencil after 6 cm 8 mm has been used?

7 A new section of highway was opened which bypassed a small town and cut the previous travelling distance of 18 km by 4 km 350 m. Find, in km, the length of the bypass.

8 Will a truck of height 748 cm be able to pass beneath an overpass with a clearance of 8.1 m?

Consolidation

9 How many cartons of height 25 cm can be stacked under an overhead cupboard whose height is 1.95 m?

10 How many books can be covered completely using a 3.5 m roll of plastic, if each book requires a 40 cm length of plastic?

11 The average length of Giann's walking stride is 42 cm. How many metres would Giann walk if she took 600 strides?

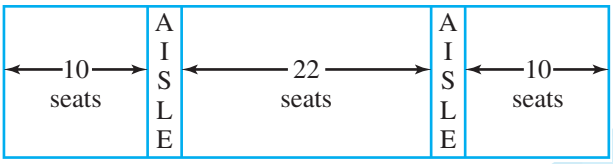
12 Kylie purchased a small tree of height 2.3 m and planted it. Each year the tree grew by 55 cm. Calculate the height of the tree in metres, eight years after the tree was planted.

13 A swimmer completes 45 laps of a 50 m pool each day, five days a week, as part of her training routine. How many km does she swim each week?

14 From a piece of timber of length 2.4 m, a carpenter cuts off three pieces, each of length 278 mm. What length of timber remains? Give your answer in mm.

15 On a roll of toilet paper, each small sheet measures 12 cm × 12 cm. Find, in metres, the total length of toilet paper if each roll contains 500 sheets of paper.

16 The seats in a local movie theatre are arranged as shown. The seats are each 60 cm wide and the width of each aisle is 1.5 m. Find, in metres, the total width of the movie theatre.



Further applications

17 How thick, in cm, is a 540 page book if each page has a thickness of 0.05 mm and the front and back covers each have a thickness of 0.5 mm?

18 The total window length along one side of a railway carriage is 7 m. This area is divided into five smaller windows of equal length, with a gap of 5 cm between each window. Find, in cm, the length of each window.

19 The platform of a suburban railway station is 50 m long. A six-carriage train arrives and just fits along the edge of the platform. If the distance between the carriages is 1.3 m, find the length of each carriage.

20 A builder uses bricks of length 23 cm and height 8 cm to construct a wall. Each level of the wall requires 20 bricks and there are 30 levels, with 7 mm of cement between the bricks. Find, in metres, the length and height of the wall.

TRY THIS **Small thickness!**

What is the thickness of this sheet of paper?

- How would you find this measurement?
- Are all sheets of paper used in different products of the same thickness?

11.4 Accuracy and precision

It is not possible to measure anything exactly. That is, all measurements are approximations. For example, when using a ruler to measure the length of an interval, we commonly ignore the thickness of the strokes marking the intervals and give our answer correct to the nearest cm or mm *as we see it*. The **accuracy** of a measurement refers to how close the reading is to the exact value of the quantity being measured. The degree of accuracy in our measurements, therefore, depends upon the accuracy of the measuring instrument that is being used at the time. Before measuring something, we need to consider the following questions:

- Why are we conducting the measurement?
- How precise does the measurement have to be?
- What measuring instrument would be the most appropriate to use?

The **precision** of a measuring instrument refers to the smallest unit that is marked on it. For example, a typical 30 cm ruler has a precision of 1 mm, since the smallest units that are marked on a ruler are millimetres. It is not possible to be more precise than the smallest unit that is marked on the instrument.

All measurements are accurate to within $\pm\frac{1}{2}$ of the smallest unit marked on the instrument.

Example 1

Between what limits will each of the following measurements lie?

a The height of a child is 84 cm, correct to the nearest cm.

b The length of a driveway is 7 m, correct to the nearest m.

Solutions

a The exact height lies within the range $(84 \pm \frac{1}{2})$ cm, i.e. between 83.5 cm and 84.5 cm.

b The exact length lies within the range $(7 \pm \frac{1}{2})$ m, i.e. between 6.5 m and 7.5 m.

Example 2

The number of people attending a concert is given as 16 300, correct to the nearest hundred people. Within what range does the exact number of people in the crowd lie?

Solution

The exact number of people in the crowd lies within the range ($16\,300 \pm \frac{1}{2} \times 100$), i.e. between 16 250 and 16 350 people.

Example 3

EG+S

State the upper and lower bounds for each measurement.

a 12.8 km **b** 1.47 m

Solutions

a The upper and lower bounds are (12.8 ± 0.05) km. That is, the upper bound is 12.85 km and the lower bound is 12.75 km.

b The upper and lower bounds are (1.47 ± 0.005) m. That is, the upper bound is 1.475 m and the lower bound is 1.465 m.

Exercise 11.4

1 What degree of accuracy is needed when measuring:
- **a** the distance between two cities?
- **b** the length of a fly?
- **c** the height of a cliff?
- **d** the height of a basketball net?

2 Within what limits are each of the following instruments accurate?
- **a** a watch with only hourly markings
- **b** a metre ruler marked in cm
- **c** a set of scales marked in kg
- **d** a protractor with increments of 1°
- **e** a thermometer marked in °C
- **f** a rain gauge marked in mm
- **g** a trundle wheel with a circumference of 1 metre and no other markings
- **h** a measuring tape marked in 10 cm intervals

3 The following measurements are given correct to the nearest metre. Between what limits does the actual length lie?

a 3 m **b** 7 m **c** 12 m **d** 39 m
e 46 m **f** 85 m **g** 100 m **h** 1 km

4 The following measurements are given correct to the nearest 10 cm. Between what limits does the actual length lie?

a 20 cm **b** 30 cm **c** 60 cm **d** 90 cm
e 140 cm **f** 270 cm **g** 1610 cm **h** 1 m

Consolidation

5 The distance between two cities was given as 700 km. Between what limits could the real distance lie, if this measurement was given correct to the nearest:

a kilometre? **b** 10 kilometres? **c** 100 kilometres?

6 After carbon dating the age of an ancient artefact was given as 6300 years, correct to the nearest hundred years. Within what range is the true age of the artefact?

7 The cost of purchasing a lounge suite was quoted as 'about $3400', correct to the nearest hundred dollars. Within what price range is the price of the lounge suite?

8 Between what two limits will each of the following measurements/quantities lie?

a The length of a street is 840 m, correct to the nearest 10 m.
b The number of pens in a box is 270, correct to the nearest 10 pens.
c The time required to drive from Sydney to Melbourne is 9 hours, correct to the nearest hour.
d The length of a table is 144 cm, correct to the nearest cm.
e The height of a police suspect is 180 cm, correct to the nearest 10 cm.
f The number of locusts in a swarm is 13 000, correct to the nearest 1000 locusts.
g The number of pages in a book is 300, correct to the nearest 20 pages.
h The length of a bee is 16 mm, correct to the nearest mm.
i The number of students enrolled in a school is 800, correct to the nearest 100 students.
j The height of a tree is 14 m, correct to the nearest metre.
k The distance between two cities is 6000 km, correct to the nearest 1000 km.
l The weight of a man is 93 kg, correct to the nearest kg.
m The capacity of a child's swimming pool is 120 L, correct to the nearest 10 L.
n The weight of a car is 700 kg, correct to the nearest 100 kg.
o The cost of a CD is $25, correct to the nearest $5.

9 State the lower and upper bounds of each measurement.

a	8 mm	**b**	3 cm	**c**	7 m	**d**	4 km	**e**	7.2 mm
f	8.6 m	**g**	15.3 km	**h**	21.9 cm	**i**	5.24 m	**j**	1.06 km
k	12.75 cm	**l**	8.00 mm	**m**	3.4 cm	**n**	7.83 km	**o**	18.9 mm
p	30.15 m	**q**	5.0 km	**r**	24 m	**s**	36.78 cm	**t**	49.9 mm
u	2182 m	**v**	350.4 mm	**w**	816.17 km	**x**	8.196 cm	**y**	53.742 m

10 The height of a flagpole is given as 12.3 m. What is the shortest possible height of the pole?

11 The length of a piece of timber is given as 2.37 m. What is the shortest possible length of the timber?

12 The length of a workbench was stated as being approximately 3.8 m.

a Could the bench be 3.85 m long? Explain.
b Could its length have been 3.75 m?

■ Further applications

13 The fence wire surrounding a rectangular paddock is to be replaced. The dimensions of the paddock are measured to the nearest metre and recorded as being 110 m × 70 m. What is the greatest amount of fence wire that must be purchased in order to enclose the paddock?

14 The distance between the Earth and the sun is given as 149 000 000 km, correct to the nearest million km. Within what range could the real distance lie?

11.5 Measuring and estimating length

People estimate lengths, distances, times, weights, heights and many other quantities in their everyday lives. It is an essential skill, since although it may often be desirable to actually measure a quantity, this may not always be possible. For example, a group of bushwalkers may not be able to determine the distance across a raging river by direct measurement. In situations such as these we rely on our estimation skills to get some idea of length or distance.

It is also quite common for people to estimate the amount of time that will be required to complete some activity. For example, we need to estimate how long it should take to go from one place to another in order to work out the time that we should leave so as not to be late.

Many of our estimates are to some degree based on our previous experiences. We also make estimates by comparing the quantity in question to another whose value we know. This is particularly so in the case of length, distance or height. The more we practise estimation, the better we become at it.

When we need to measure something, it is essential to be as accurate as the measuring instrument permits.

Example 1

Which instrument (trundle wheel, tape measure or odometer) would be the most appropriate to measure:

a the distance between Parramatta and Campbelltown
b the perimeter of a school playground
c the width of a window

Solutions

a odometer
b trundle wheel
c tape measure

Example 2

Choose the best estimate for:

a the height of a glass
A 5 mm **B** 5 cm **C** 10 cm **D** 20 cm

b the length of a bed
A 0.5 m **B** 1 m **C** 1.5 m **D** 2 m

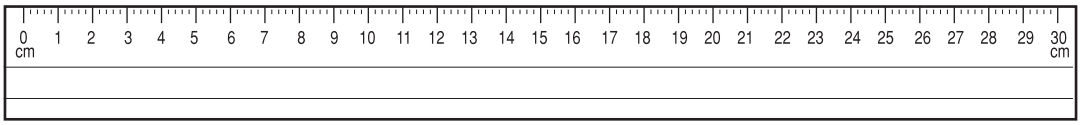
c the distance between Sydney and Melbourne
A 300 km **B** 600 km **C** 900 km **D** 1200 km

Solutions

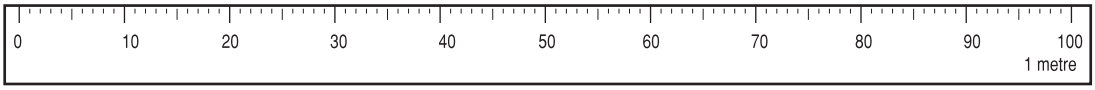
a A drinking glass is about 10 cm tall, ∴ C is the best estimate.
b A bed is about 2 m long, ∴ D is the best estimate.
c The distance between Sydney and Melbourne is about 900 km, ∴ C is the best estimate.

Exercise 11.5

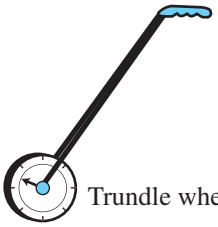
1 Choose the most appropriate instrument from those below to measure each length or distance.



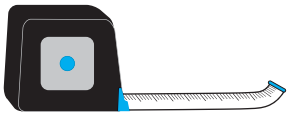
30 cm ruler



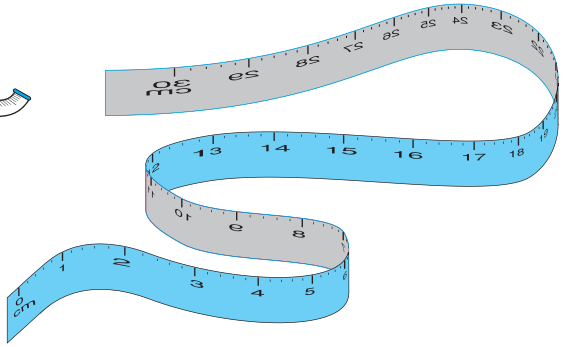
Metre rule



Trundle wheel



Tape measure



Dressmaker's tape

Odometer

a the width of a room
b the length of a football field
c the height of a coffee table
d the distance between two towns
e the length of a calculator
f a person's waist size

2 Which answer is the best estimate for:

a the length of a classroom
A 8 m **B** 12 m **C** 18 m **D** 24 m

b the length of a bee
A 5 mm **B** 12 mm **C** 3 cm **D** 2.5 cm

c the distance between Sydney and Brisbane
A 100 m **B** 1000 m **C** 1000 km **D** 10000 km

d the height of a doorway
A 1 m **B** $1\frac{1}{2}$ m **C** 2 m **D** $2\frac{1}{2}$ m

e the size of a computer screen
A 5 cm **B** 10 cm **C** 15 cm **D** 30 cm

f the arch span of the Sydney Harbour Bridge
A 100 m **B** 500 m **C** 1 km **D** 2 km

g the height of a coffee cup
A 20 cm **B** 15 cm **C** 10 cm **D** 5 cm

h the height of a desk
A 25 cm **B** 75 cm **C** 125 cm **D** 175 cm

i the length of a postage stamp
A 5 mm **B** 10 mm **C** 15 mm **D** 20 mm

Consolidation

3 Measure the length of each interval, correct to the nearest mm:

a _____

b _____

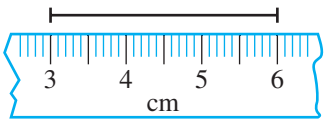
c _____ **d** _____

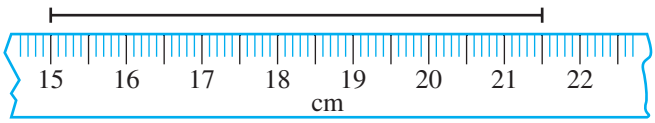
e _____ **f** _____

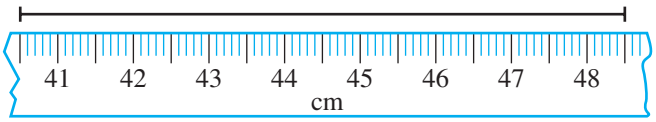
g _____ **h** _____

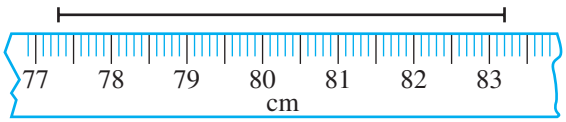
4 Use your ruler to draw intervals of the following lengths. Have another student measure your intervals to check their accuracy.
a 3 cm **b** 25 mm **c** 5.6 cm **d** 7.4 cm

5 Write down, in mm, the length of each interval.

a


b


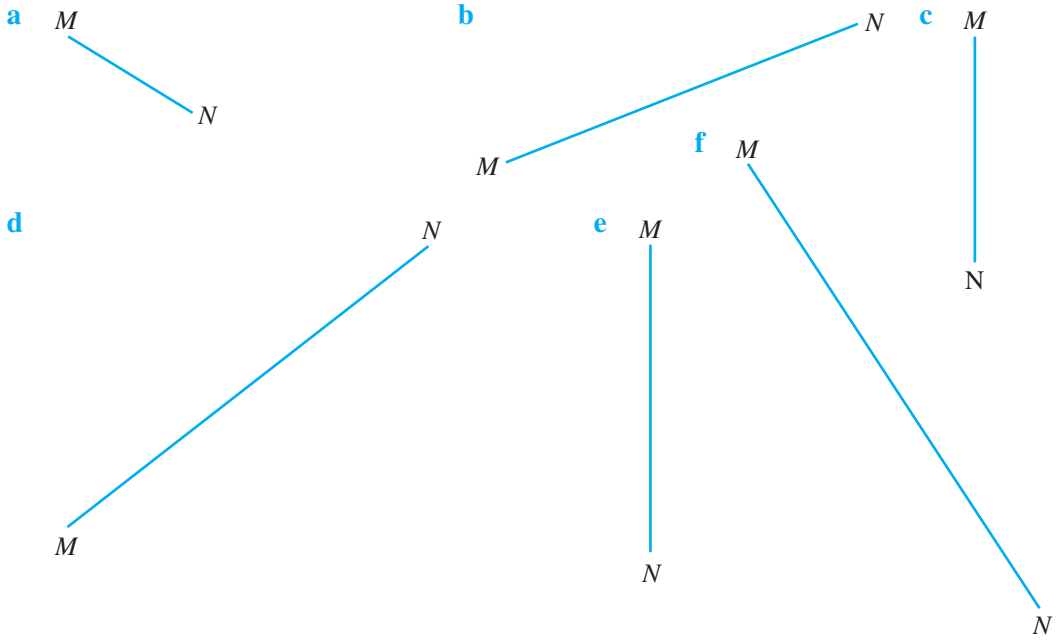
c


d


6

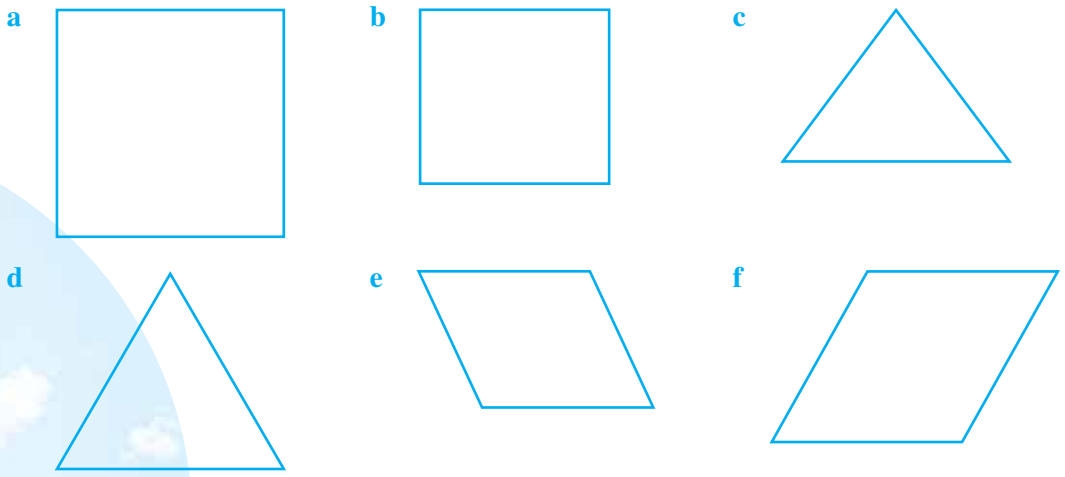
Estimate the length of each interval, to the nearest cm.
a AB **b** BC **c** CD **d** DE

7 Estimate the length of *MN* in each of these. Give your answers correct to the nearest cm.



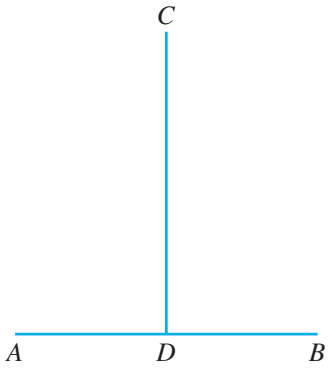
8 Estimate the height of a fellow student, correct to the nearest 10 cm. Verify your estimate using a height chart marked in 1 cm intervals.

9 Measure in mm, the lengths of the sides of each figure, then name the figure.



10 **a** Draw a rectangle with length 4 cm and width 3 cm.
b Estimate the length, in cm, of the diagonals.
c Verify your estimate by direct measurement.

11 Estimate the length of the intervals *AB* and *CD*, then verify your estimates.

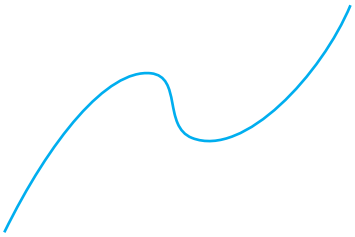


12 Estimate the length of each interval, not including the arrows, then measure the intervals.

■ Further applications

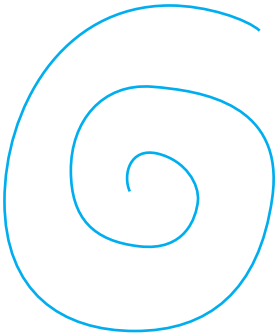
13 Use a piece of thread to find the length of each curve, correct to the nearest cm.

a



b

c

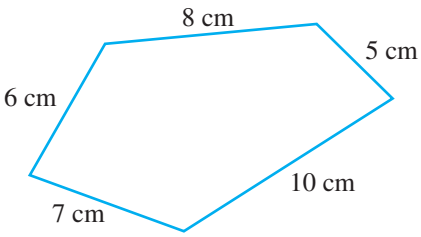


11.6 Perimeter

The **perimeter** of a figure is found by adding the lengths of the sides. Lines drawn inside a figure do not form part of its perimeter. Placing the same mark on two or more sides indicates that these sides are equal in length.

The perimeter of a figure is the total distance around its boundary.

For example:



Perimeter = (6 + 8 + 5 + 10 + 7) cm
= 36 cm

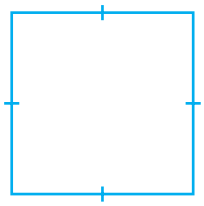
Example 1

Find the perimeter of:

a a square of side 6 cm **b** a rectangle of length 8 mm and breadth 3 mm

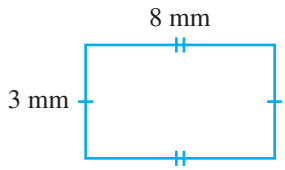
Solutions

a



Perimeter = 4 × 6 cm
= 24 cm

b

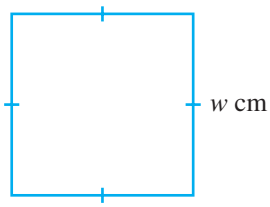


Perimeter = (8 + 3 + 8 + 3) mm
= 22 mm

Example 2

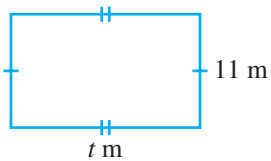
Find the value of the pronumeral in each of these:

a



Perimeter = 48 cm

b



Perimeter = 52 m

Solutions

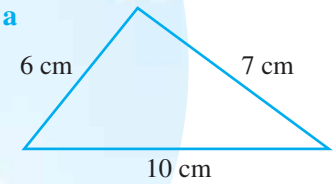
a $w = 48 \div 4$
$= 12$

b $t + t + 11 + 11 = 52$
$2t + 22 = 52$
$\therefore t = (52 - 22) \div 2$
$= 30 \div 2$
$= 15$

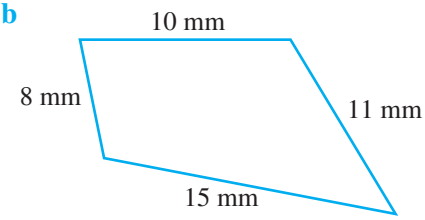
Exercise 11.6

1 Find the perimeter of each figure.

a



b



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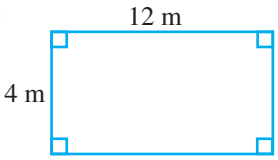

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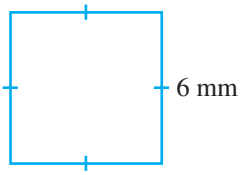

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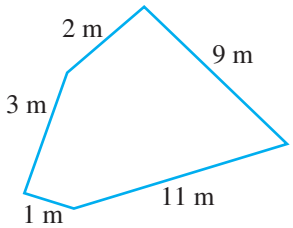

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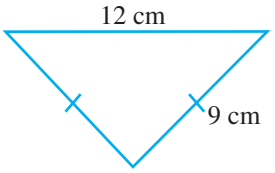
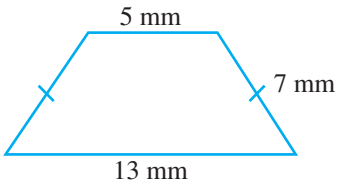


g



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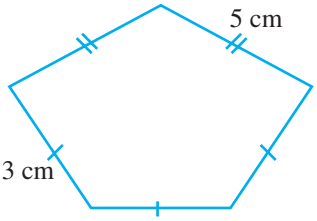
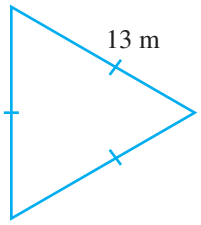


i

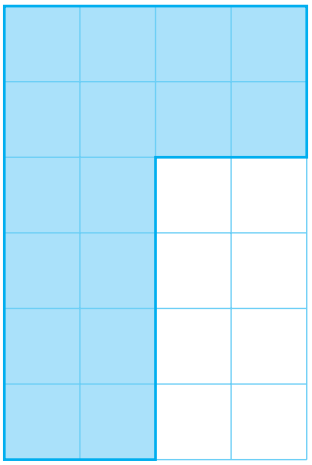


2 Find the perimeter of:

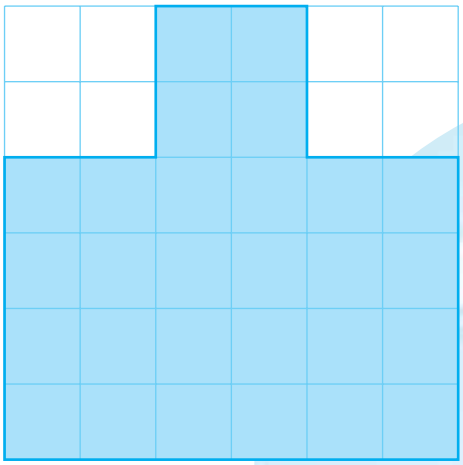
- **a** a square of side 8 mm
- **b** a rectangle whose length is 15 mm and width is 9 mm
- **c** an equilateral triangle whose sides are 25 cm long
- **d** an isosceles triangle with base 17 m and equal sides of length 15 m
- **e** a regular pentagon of side length 9 cm
- **f** a regular octagon of side length 12 mm
- **g** a rhombus whose sides each measure 21 cm
- **h** a kite with adjacent sides of 23 mm and 18 mm

3 The figures below have been drawn on 1 cm grid paper. Find the perimeter of each figure.

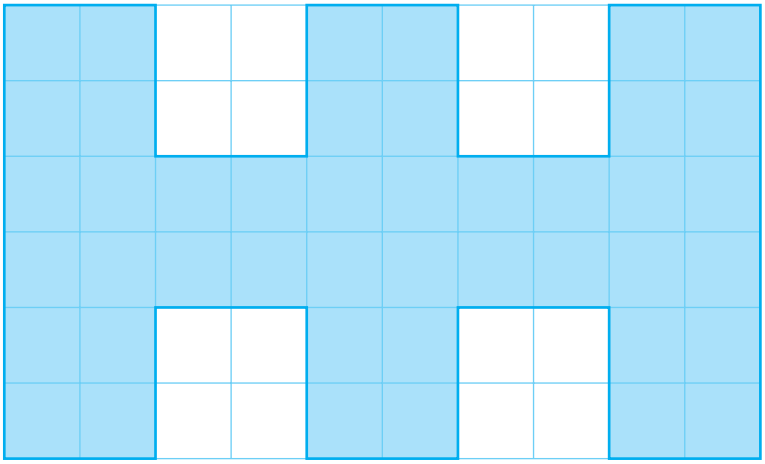
a



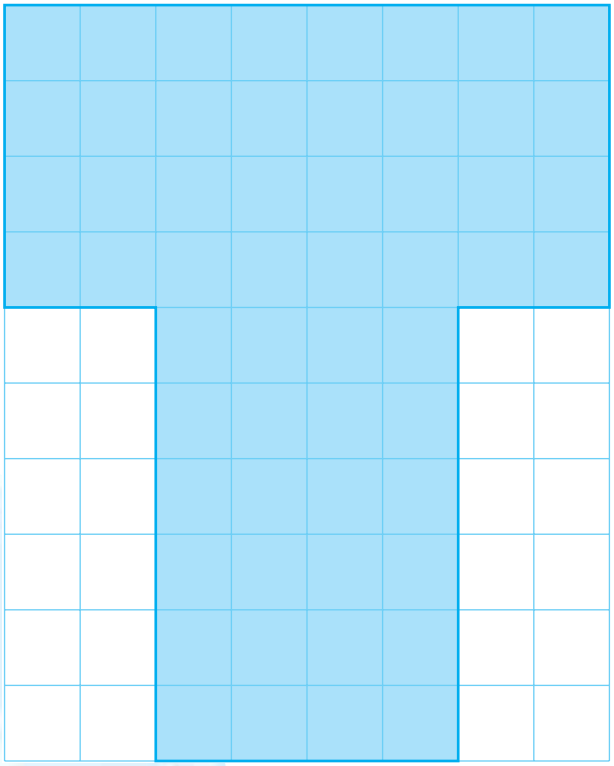
b



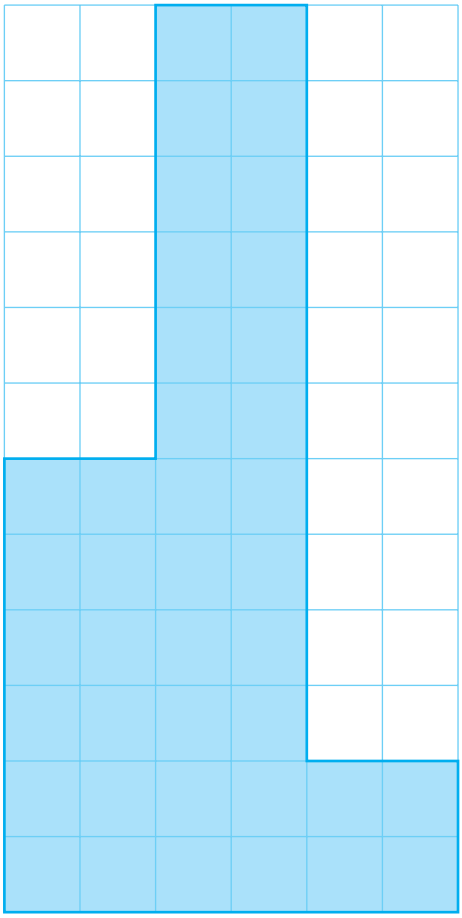
c



d

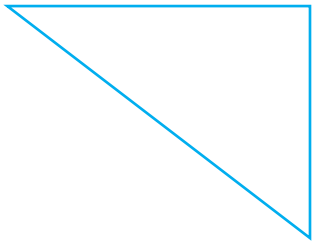


e



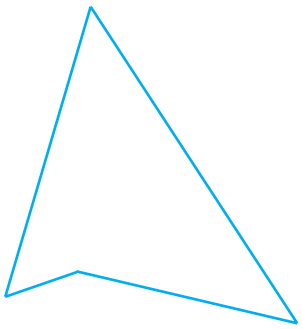
4 Measure the lengths of the sides in each figure, correct to the nearest cm, then calculate the perimeter.

a



b

c



5 The perimeter P of a square with sides of length x is given by the formula $P = 4x$. Use this formula to find the perimeter of a square whose sides are 13 cm.

6 The perimeter P of a rectangle with length L and breadth B is given by the formula $P = 2(L + B)$. Use this formula to calculate the perimeter of a rectangle with:

a length 9 mm and breadth 6 mm
b length 12 cm and breadth 5 cm
c length 10.5 m and breadth 7.5 m
d length 13.4 mm and breadth 8.3 mm

7 Find the side length of a square whose perimeter is:

a 20 cm
b 36 cm
c 68 cm

8 Find the side length of:

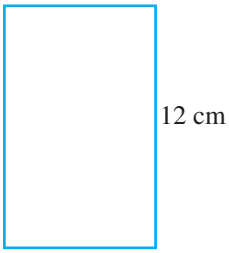
a an equilateral triangle with perimeter 21 cm
b a rhombus with perimeter 32 mm
c a regular hexagon with perimeter 30 m
d a regular dodecagon with perimeter 48 cm

9 Find the value of the pronumeral in each rectangle.

a

Perimeter = 28 cm

b



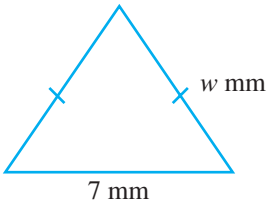
Perimeter = 34 cm

c

Perimeter = 32 cm

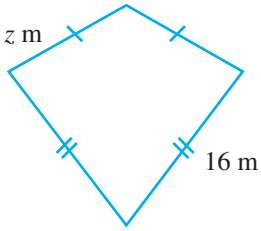
10 Find the value of the pronumeral in each of these.

a



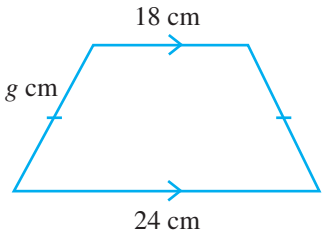
Perimeter = 23 mm

b



Perimeter = 50 m

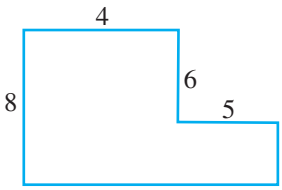
c



Perimeter = 49 cm

11 Find the lengths of any unknown sides, then calculate the total perimeter of each figure. (All angles are right angles and all measurements are in cm.)

a



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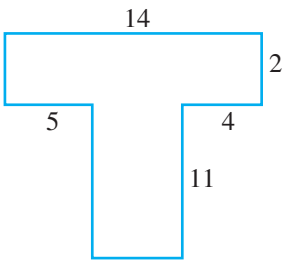

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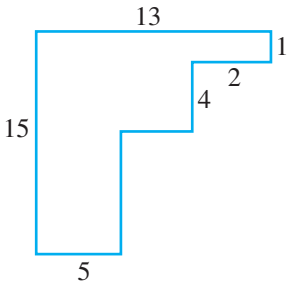

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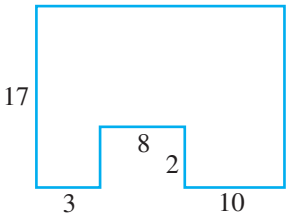
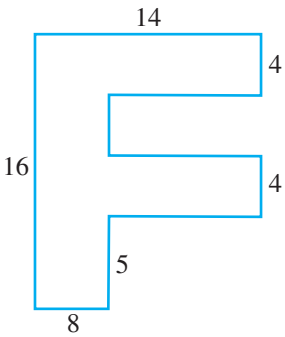


e



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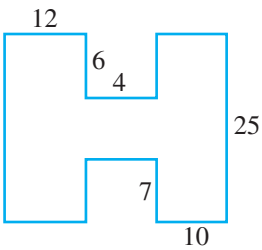

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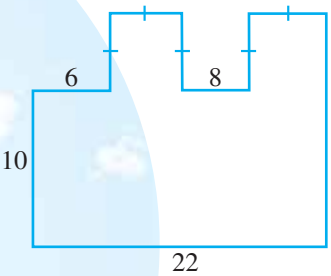
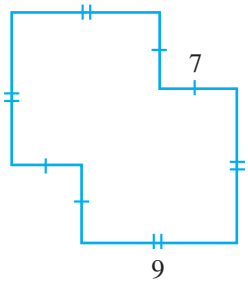


h



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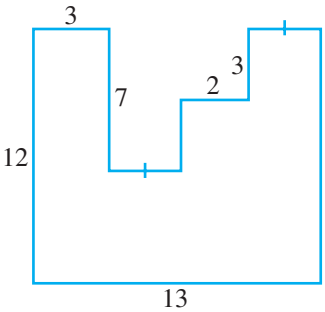

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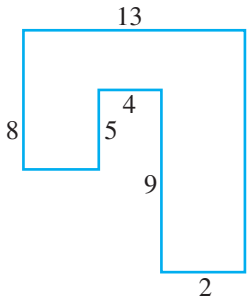

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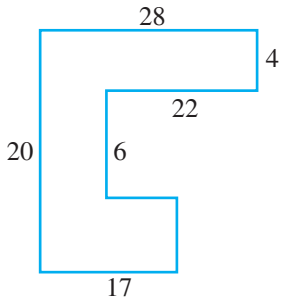

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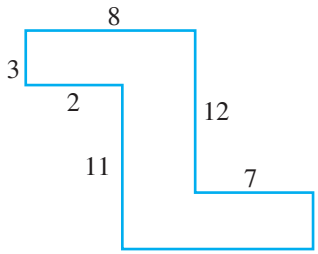
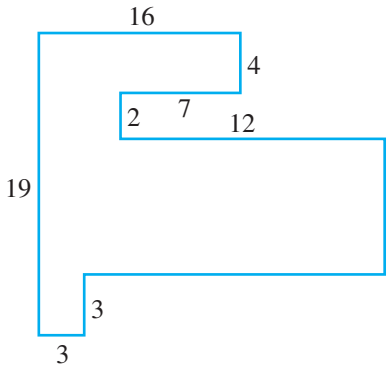


■ Further applications

12 Find the total perimeter of each figure. (All angles are right angles and all measurements are in mm.)

a



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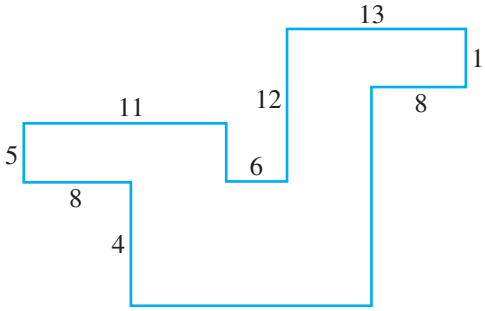

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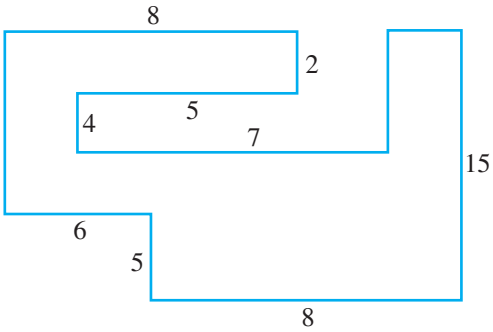

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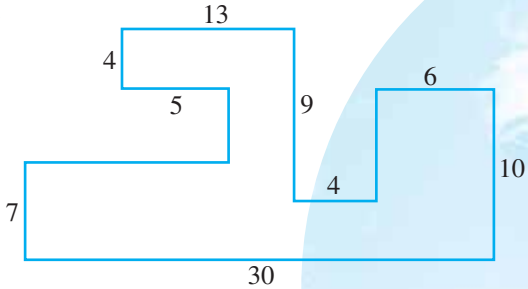
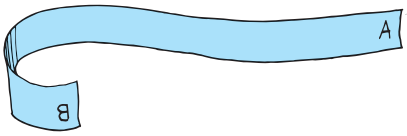


TRY THIS

Möbius strips

Take a piece of paper about 25 cm long and 2 cm wide. (You will need a number of these strips.) At one end, mark it A, and turn it over and mark the other side B.

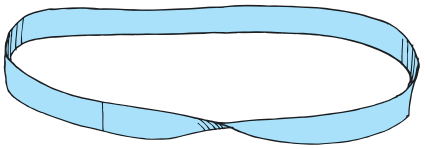


Investigation 1

Give the strip a half twist and join A to B, using tape.

Question:

How many sides has this strip? Two? One? With a pencil, draw a line down its centre until you get back to where you started. Number of sides?



Investigation 2

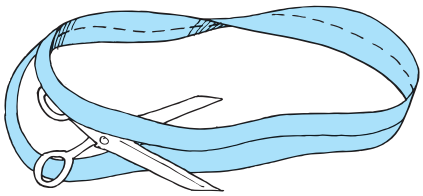
Give another strip two half twists, and then tape.

Question:

How many sides has the strip? Check with a pencil line, as before.

Investigation 3

If you cut the first strip along the pencil line, how many pieces will you get? Cut along the line with scissors, and find out!



Investigation 4

If you cut the second strip down the centre, how many pieces will you get? Try it!

Investigation 5

Use a third strip and give three half twists, and repeat the method of drawing a line down the centre and cutting.

Investigation 6

Take a fourth strip and give one half twist. Draw a line one-third of the way from the edge carefully until you get back to where you started. Cut along the line. Do the same thing with another strip which has two half twists.

Investigation 7

Take a fifth strip and give one half twist. Draw a line one-quarter of the way from the edge around the strip. Cut. Try a band with two half twists.

Have you found out anything of interest?

11.7 Solving perimeter problems

Example 1

A wire fence is to be constructed around the sides of a rectangular chicken coop. The coop measures 3.2 m × 4 m.

a Find the length of fencing required.

b Calculate the cost of the fence wire at $10 per metre.

Solutions

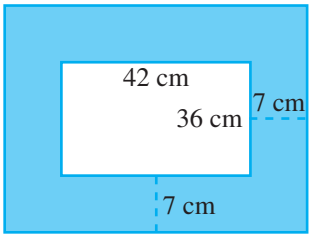
a Perimeter = (3.2 + 4 + 3.2 + 4) m
= 14.4 m
∴ 14.4 m of fence wire is needed.

b 14.4 × 10 = 144
∴ Cost = $144

Example 2

Vincent painted a portrait of the countryside near his home on a canvas measuring 42 cm × 36 cm. The portrait was then set in a wooden frame of width 7 cm. Calculate the perimeter of the framed portrait.

Solution



i We first find the dimensions of the framed portrait.
Length = (42 + 7 + 7) cm
= 56 cm
Width = (36 + 7 + 7) cm
= 50 cm

ii Perimeter = (56 + 50 + 56 + 50) cm
= 212 cm

Exercise 11.7

1 Calculate the perimeter of a rectangular soccer field with dimensions 100 m × 45 m.

2 The Pentagon building in the United States has five outer walls each of length 281 m. Find the perimeter of the building.

3 The perimeter of a rectangular backyard is 44 m. If the yard is 6 m wide, find the length.

4 A square has the same perimeter as an equilateral triangle. If the triangle has sides of length 12 cm, find the side length of the square.

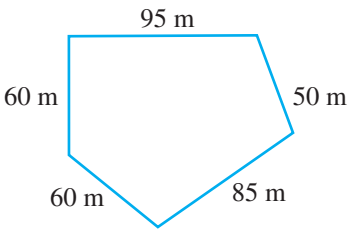
5 The length of a rectangle is twice its width. Find the dimensions of the rectangle if the perimeter is 42 cm.

6 A rose bush is planted every 4 m around the edge of a rectangular flower bed. If there are 30 roses, what is the perimeter of the flower bed?

Consolidation

7 An athletics field is 120 m long and 35 m wide. How far, in km, will an athlete run by completing 20 laps of the track?

8 Christopher runs around the boundary of this irregularly shaped park every morning in order to keep fit. How many laps of the park must he complete in order to run at least 1 km?



9 Lauren runs around the boundary of a local park every morning in order to keep fit. The dimensions of the park are 50 m × 25 m. How many laps of the park must she complete in order to run at least 1 km?

10 A small stone wall is to be built around the boundary of a rectangular park whose length is 150 m and width is 110 m. Two gaps of width 10 m are to be left so that people can access the park.

a Find the length of the stone wall.

b Calculate the cost of the stone at $60 per metre.

11 A frieze design is to be pasted around the walls of a bedroom. The length and width of the room are 8.5 m × 6 m.

a What length of frieze is required?

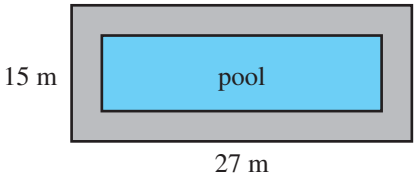
b Find the cost of the frieze if it is sold in 5 m rolls at $7.50 per roll.

12 A print measuring 25 cm by 12 cm is to be surrounded by a wooden frame of width 3 cm. Find the perimeter of the framed print.

13 A swimming pool has a 2 metre wide path around its edge as shown. The outer dimensions of the path are 27 m × 15 m. Find:

a the dimensions of the pool

b the perimeter of the pool



14 A rural primary school is holding its annual 12 km walkathon at a local park which is 250 m long and 150 m wide.

a How many laps of the park must the students walk?

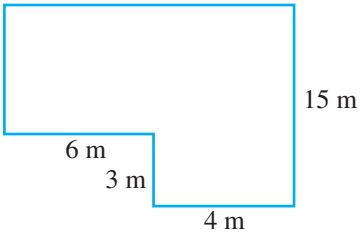
b If one of the students has been promised $7 per lap, how much money will he raise?

15 A mobile phone tower is situated in the middle of a square block of land. The old fence surrounding the property is to be replaced at the cost of $40 per metre. Find the side length of the property if the total replacement cost of the fence is $20 000.

16 Sharon walks at an average speed of 8 km/h while checking the fence around a paddock on her property. The paddock is square in shape and has a side length of 6 km. How long will it take Sharon to check the fence?

17 A small square has side length 5 cm. If 36 of these squares are placed next to each other so that they form a larger square, find the perimeter of the larger square.

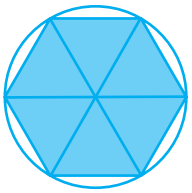
18 The owner of a vegetable garden with the given dimensions wishes to enclose the garden with a wire fence. If the fencing wire costs $30 per metre, find the total cost of enclosing the garden.



Further applications

19 A rectangular fence is supported by posts placed 5 m apart along each side. How many fence posts are needed to support a fence with perimeter 110 m?

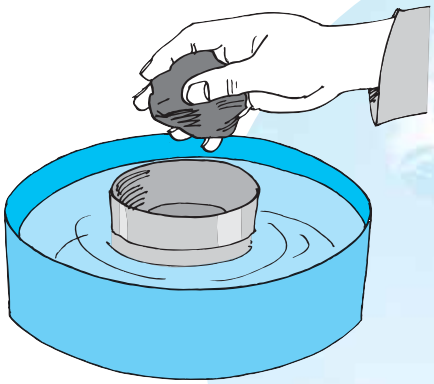
20 A regular hexagon has been inscribed (i.e drawn inside) a circle with diameter 54 mm. Find the perimeter of the hexagon. (HINT: Find the angles in the triangles.)



Problem solving

1 If A, B, C and D are numbers and $B \times D = 48$, $B \times C = 24$, $A \times B = 12$, $C \times D = 32$, find A, B, C and D.

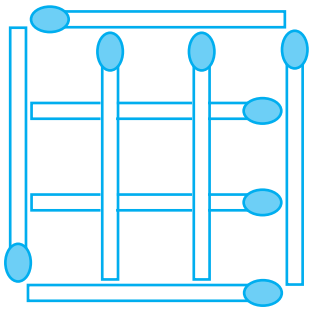
2 Suppose we have a basin of water with a tin floating in it. If we were to place a stone in either the tin or the bottom of the basin, which would result in the greater rise in the water level?



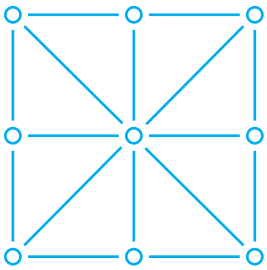
3 If A, B, C and D are numbers and $B + D = 11$, $B + C = 9$, $A + B = 6$, $C + D = 12$, find A, B, C and D.

4 If $\frac{3}{x} = \frac{x}{27}$, find the value of x.

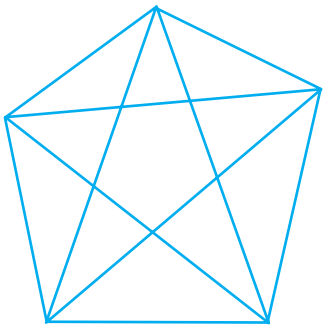
5 This figure made from matches has 14 squares. Count them! Now remove 2 of the matches to leave 3 squares (and some rectangles!).



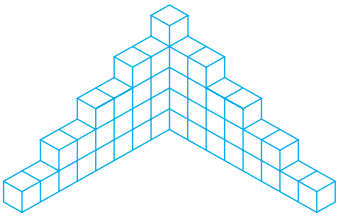
6 At a carnival, an illuminated sign contained 9 light bulbs in 8 rows, columns or diagonals with 3 light bulbs in each row as shown.
Did you check this diagram with the given facts? Can you see the 8 rows? Now can you construct a sign with 10 light bulbs in 5 rows with 4 bulbs in each row? (HINT: STAR.)



7 How many different triangles can you find in this figure?



8 How many cubes are there in this structure?



9 How many cubes would be in a structure 10 cubes high?

10 How many faces would need to be painted? (NOTE: A cube has six faces.)

Focus on working mathematically

Baseball

Focus on working mathematically

Rules for setting out the infield and outfield

Look at the diagram on page 442. The units are in feet and inches as is standard still in the US for measurement of length.

1. The infield is a 90 foot square.
2. The outfield is the area between two foul lines formed by extending two sides of the square.
3. The distance from the pitcher's plate to home base is 60 feet 6 inches.
4. The edge of the grass shall be 90 feet from the pitcher's plate.
5. The distance from home base to the nearest fence, stand or other obstruction on fair territory shall be 320 feet along the foul lines, and 400 feet to centre field.
6. The distance from home base to backstop, and from the base lines to the nearest fence or stand on foul territory shall be 60 feet.

Learning activities

Materials needed: pencil, ruler, pair of compasses, set square, a sheet of A4 paper (preferable because of suggested scale). If students' books are smaller you may wish to change the scale suggested. The length of the diagonal has been calculated for year 7 students.

In this activity we are going to mark out a baseball pitch. Take a piece of A4 paper, and lightly draw a straight line about 6 cm from the bottom of your page. Now mark a point near the middle of the line to represent home base.

Using a scale of 1 cm = 20 feet (1 mm = 2 feet) construct the square as follows.

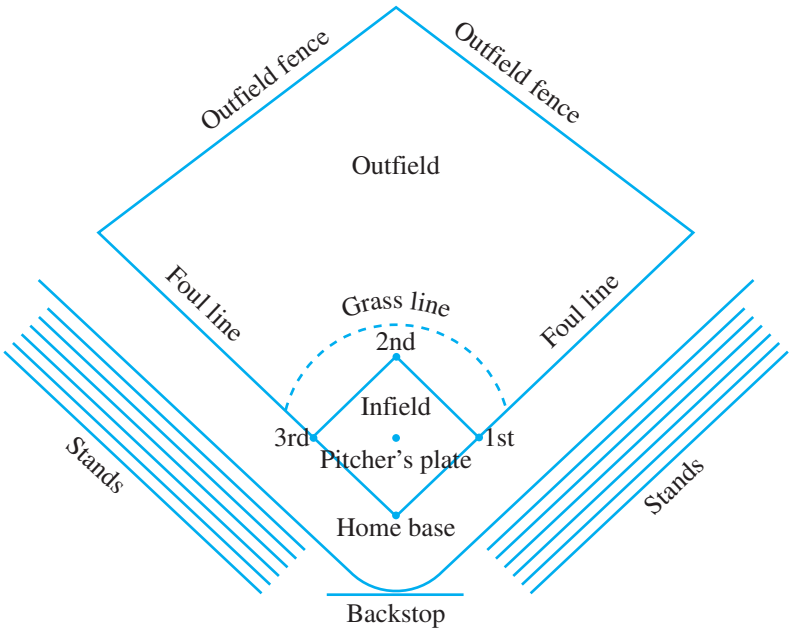
1. Draw a perpendicular to your base line at home base. Measure along this line 6.4 cm to second base.
2. Use your scale to change 90 feet to cm. Then draw two arcs from home and second base to locate first base. Repeat this to locate third base.
3. Use your scale to change 60 feet 6 inches to cm. Locate the pitcher's plate position on the line joining home base to second base.

To complete the whole diagram:

4. Extend the sides of the square to make the foul lines.
5. Use your scale to change 90 feet to cm. With centre on the pitcher's plate, draw an arc of this radius to show the edge of the grass.
6. Use your scale to change 60 feet to cm. With centre home base, draw an arc to show the backstop and continue the fence line 60 feet from the foul line.
7. Use your scale to change 320 feet to cm. Measure along the foul lines from home base to locate the intersection with the far fence. Then change 400 feet to cm, and measure from home base through centre field to the far fence. Join up the points to show the far fence boundary.

Finally, mark your diagram to show the important distances, the bases, the pitcher's plate, the infield and the outfield, foul territory and fair territory, backstop and stands.

Playing area for the game of baseball



Questions

Give the answers in feet. You may use your ruler to measure as appropriate.

1. What is the perimeter of the infield?
2. What is the distance from home to second base?
3. How far does the ball travel from pitcher to batter?
4. What is the distance from home base to the end of each foul line?
5. The pitcher's plate is 10 inches (12 inches = 1 foot) above the home base. But the home base and other bases are level. Draw a diagram to show the slope of the ground from pitcher to home base. It does not have to be to scale.
6. How far is it from the pitcher's base to the edge of the outfield? Why does it vary?

Extension activities

1. If you can, go to the major league baseball web site, www.mlb.com/ and click on baseball basics. Find out where the coach's boxes are. Mark them on your diagram.
2. Find out the fielding positions for the game and show their positions on your diagram. There are 9 players in the team, and the catcher is on home base.

3 If a batter is caught at right midfield, estimate the distance that the batter hit the ball.
4 As a class project you might like to make a three-dimensional model of the baseball field putting in many more details you can find on the web site.

Let's communicate

Write to a friend who does not know anything about baseball. Tell them how the playing field is constructed, providing the dimensions in rough metric units, and giving an outline of the game. Take 1 m to be about 3 feet. Include a diagram.

Reflecting

In different sports, players must run different distances throughout the game. How would baseball compare with football? With hockey? With tennis? Who would have to run further? What effect would this have on training and required fitness? Discuss with your family, friends and teacher.

Language link with Macquarie

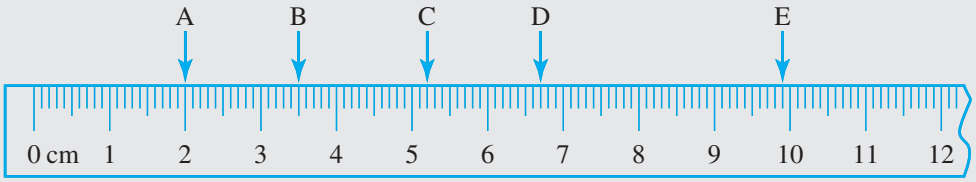
MACQUARIE

1 *What word is used to describe the distance along an* ***interval****?*
2 *What is the* ***base unit*** *of* ***length*** *in the* ***metric*** *system?*
3 *Explain what is meant by the* ***accuracy*** *of a measurement?*
4 *Define* ***perimeter*** *for a new maths dictionary.*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***measure****:*

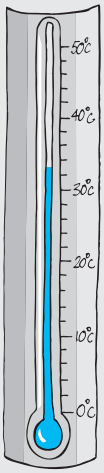
measure *noun* **1.** (*uncount*) a way of seeing how big, extensive, etc., something is: *the measure of storm damage | the measure of her unhappiness* **2.** an agreed unit or standard: *A metre is a measure of length.* **3.** a means to an end: *to take measures to prevent illness*
–*verb* **4.** to decide the size or quantity of, usually by using a special instrument such as a ruler or scales
–*phrase* **5. for good measure**, as an extra precaution: *Take along an umbrella for good measure.* **6. measure up**, to be good enough or suitable: *We tried him out in the new position but he just did not measure up.* **7. measure up to**, to be as good as: *The party did not measure up to expectations.*

Why are ***measuring*** *skills important in the building industry?*

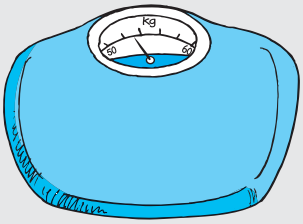
1 Write down each of the measurements shown on this ruler.



2 **a** Read the temperature that is showing on this thermometer.



b Write down the weight that is showing on this set of scales.



3 Choose the most appropriate unit (mm, cm, m, km) that could be used to measure the length of:

a a caravan
b the NSW coastline
c a TV remote control
d an eyelash

4 Convert:

a	3 m to cm	**b**	9 cm to mm
c	2 km to m	**d**	40 mm to cm
e	600 cm to m	**f**	4000 m to km
g	7.5 m to cm	**h**	24 mm to cm
i	8.6 km to m	**j**	910 cm to m
k	80 cm to mm	**l**	0.52 km to m
m	7 cm to m	**n**	16 m to km
o	0.04 m to cm	**p**	0.0005 m to km

5 Convert:

a	6 m to mm	**b**	190 mm to m
c	13 km to cm	**d**	4750 mm to km

6 **a** What length, in cm, remains from a 2.5 m long piece of timber if 1 m 37 cm has been cut off?

b How many toothpicks of length 25 mm can be cut from a 2 m length of wood?

c An athlete training for the 400 m race runs 12 practice races each day, six days a week. What total distance, in km, does this athlete run each day?

7 Between what two limits will each of these measurements lie?

a The length of a bone is 28 cm, correct to the nearest cm.
b The height of a man is 170 cm, correct to the nearest 10 cm.
c The length of a safety pin is 36 mm, correct to the nearest mm.
d The capacity of a dam is 16 400 L, correct to the nearest 100 litres.
e The age of an artefact is 1450 years, correct to the nearest 50 years.

8 State the lower and upper bounds for each of these.

a	4 mm	**b**	6.8 km
c	5.93 m	**d**	12.0 cm
e	31 km	**f**	132.7 m
g	0.46 cm	**h**	2.00 mm

9 Choose the most appropriate instrument from the following list to measure each of the lengths/distances below.

- trundle wheel
- dressmaker's tape
- odometer
- tape measure
- 30 cm ruler
- metre ruler

a the height of a wall
b the width of a rectangle drawn in a book
c the length of a rugby field
d the distance between Newcastle and Dubbo
e the waist size of a customer
f a 70 cm line drawn on a blackboard

10 Which is the best estimate for:

a the distance between Sydney and Canberra?
A 50 km **B** 150 km
C 350 km **D** 750 km

b the height of a snooker table?
A 500 mm **B** 700 mm
C 700 cm **D** 1 m

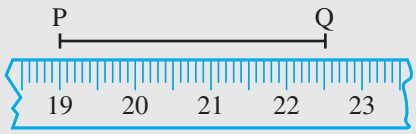
c the length of a CD cover?
A 18 cm **B** 14 cm
C 10 cm **D** 6 cm

11 Measure the length of each interval, correct to the nearest mm.

a _____
b _____
c _____

12 Write down, in mm, the length of each interval *PQ*.

a



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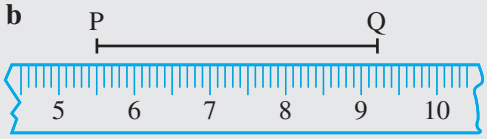
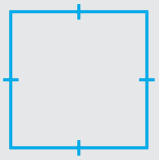


13 Find the perimeter of each figure.

a



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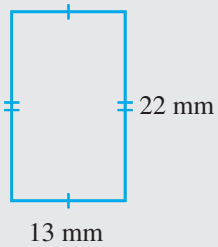

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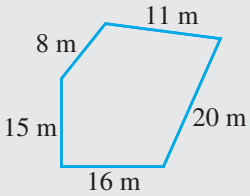

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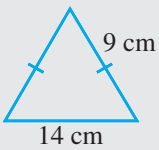


14 Find the perimeter of:

a a rhombus with sides of length 9.4 m
b a kite with adjacent sides of 16 mm and 7 mm
c a regular octagon of side 13 cm
d an isosceles triangle with base 10 m and equal sides of length 12.5 m

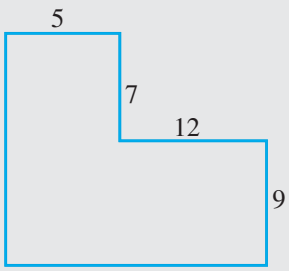
15 **a** Find the side length of a square whose perimeter is 56 cm.
b Find the side length of a regular dodecagon (12-sided polygon) whose perimeter is 84 cm.

16 The perimeter of a rectangle is given by $P = 2(L + B)$. Use this formula to find the perimeter of a rectangle in which:

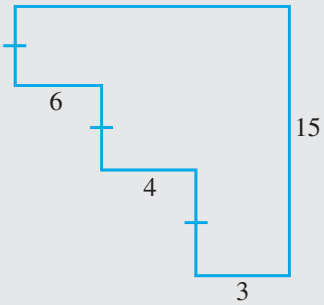
a the length is 17 cm and the breadth is 8 cm
b the length is 13.5 cm and the breadth is 4 cm.

17 Find the lengths of any unknown sides, then calculate the total perimeter of each figure. (All measurements are in mm and all angles are right angles. Diagrams are not to scale.)

a



b



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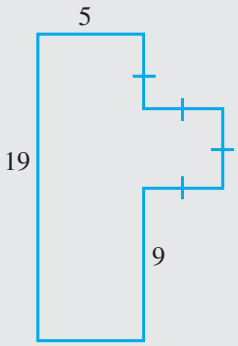

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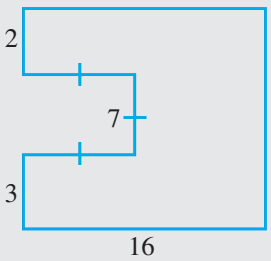

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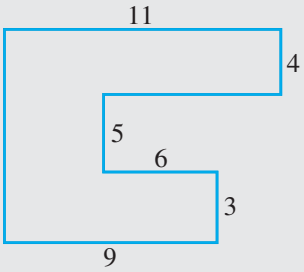

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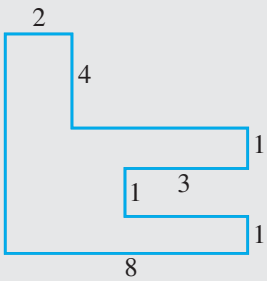
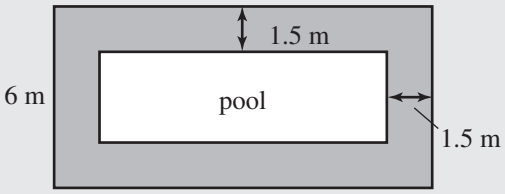


18 The length of a rectangle is three times its width. Find the dimensions of the rectangle if its perimeter is 32 cm.

19 A rectangle with a breadth of 9 cm has the same perimeter as an equilateral triangle of side 16 cm. Find the length of the rectangle.

20 A fence is to be constructed around a rectangular building site measuring 25 m by 32 m. Find the cost of constructing the fence at $23 per metre.

21 A small pool is surrounded by a 1.5 m wide pebble path. The outer perimeter of the path is 28 m and its width is 6 m.



a Find the length of the path.

b Find the dimensions of the pool and hence find its perimeter.

12 Solids

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- name prisms and pyramids according to the shape of their base
- construct three-dimensional models given drawings of different views
- construct three-dimensional models given plans
- visualise and sketch nets for three-dimensional objects.

Stage 4

After completing this chapter, you should be able to:

- describe solids in terms of their geometric properties
- explain why three-dimensional structures in the environment are of a particular shape
- visualise and name a solid given its net
- identify any pairs of parallel faces of a solid
- determine whether two straight edges of a solid are parallel, perpendicular or skew
- identify parallel, perpendicular and skew lines in the environment
- determine whether a solid has a uniform cross-section
- classify solids on the basis of their properties
- identify right prisms and cylinders and oblique prisms and cylinders
- identify right pyramids and cones and oblique pyramids and cones
- sketch solids using oblique projection
- sketch solids using isometric projection
- represent three-dimensional objects in two dimensions from different views
- confirm Euler's formula for various convex polyhedra
- name, recognise and make the Platonic solids.

12.1 Naming and classifying solids

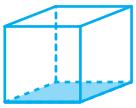
We live in a physical world of three dimensions surrounded by objects that we can touch, feel, lift and see. These objects have many properties, e.g. mass, colour, density, size and chemical composition, but the property that we will study in this chapter is shape.

There are many different solid shapes, however, as with plane shapes, we tend to use a relatively small number of them in our daily lives. Everyday solid objects have a particular shape for practical reasons. For example, a pencil is the shape that it is because it is easy to hold and write with. A tin of tennis balls is a particular shape because the tennis balls fit comfortably inside it. Many regular shapes fit together well, so they are useful for packing objects in.

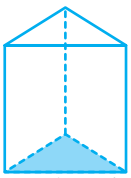
Prisms and pyramids

Most, but not all solids, can be classified into one of two groups: prisms and pyramids.

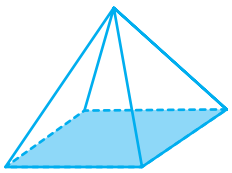
A **prism** is a solid with plane (or flat) surfaces that has a **uniform cross-section**. Each cross-section, or 'slice', is equal in size to a pair of parallel faces called **bases**. In a prism, the bases may be any polygon but all other faces are rectangles. Prisms are named according to the shape of their cross-section. Some common prisms are:

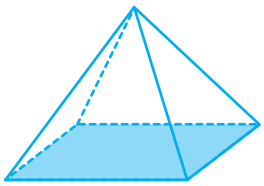

cube

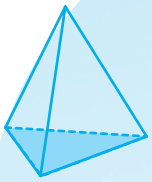
rectangular prism


triangular prism

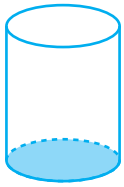
A **pyramid** is a solid with plane (or flat) surfaces in which all vertices lie in the base, except one. The vertex that does not lie in the base is called the **apex**. In a pyramid, the base may be any polygon but all other faces are triangles. Pyramids are named according to the shape of their base. Some common pyramids are:


square pyramid


rectangular pyramid


triangular pyramid

Some solids, such as those that have curved surfaces, are neither prisms nor pyramids. Some common solids with curved surfaces are:


cylinder

cone

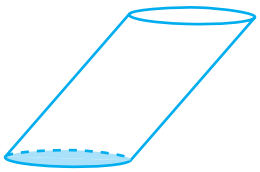
sphere

All points on the surface of a sphere are the same distance from its centre.

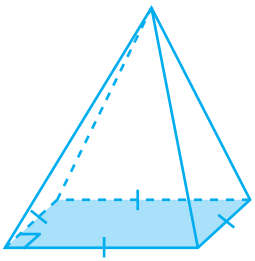
Right and oblique solids

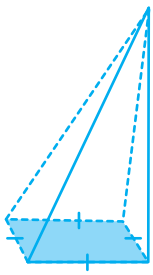
A prism or cylinder is a **right prism** or **right cylinder** if the lines joining corresponding points in the bases are perpendicular to the bases. A prism or cylinder is an **oblique prism** or **oblique cylinder** if the lines joining the corresponding points are not perpendicular to the bases. For example:

right circular cylinder


oblique cylinder

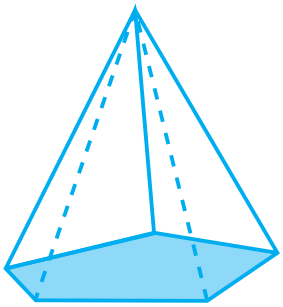
A pyramid or cone is a **right pyramid** or **right cone** if the line joining the apex to the centre of the base is perpendicular to the base. A pyramid or cone is an **oblique pyramid** or **oblique cone** if the line joining the apex to the centre of the base is not perpendicular to the base. For example:


right square pyramid

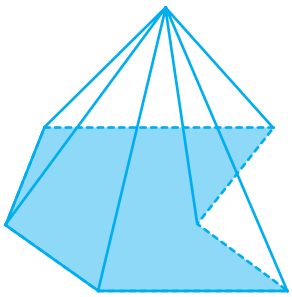

oblique square pyramid

Convex and non-convex solids

A prism or pyramid is convex if its base is convex, that is, if all diagonals of the base lie entirely within the base. A prism or pyramid is non-convex if its base is non-convex, that is, if at least one diagonal of the base does not lie entirely within the base. For example:



convex pyramid



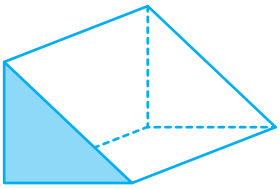
non-convex pyramid

Example 1

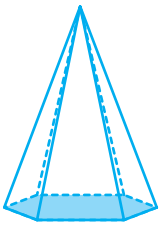
EG+S

Name each of these solids.

a



b

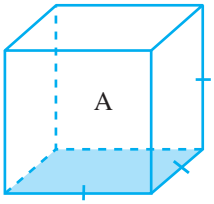


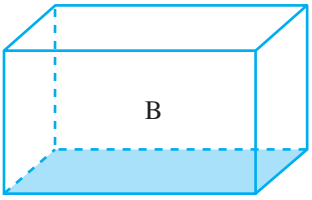
Solutions

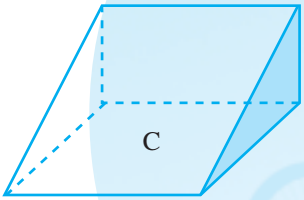
a The solid has a uniform cross-section, so it is a prism. The cross-section is a triangle. Therefore, the solid is a triangular prism.

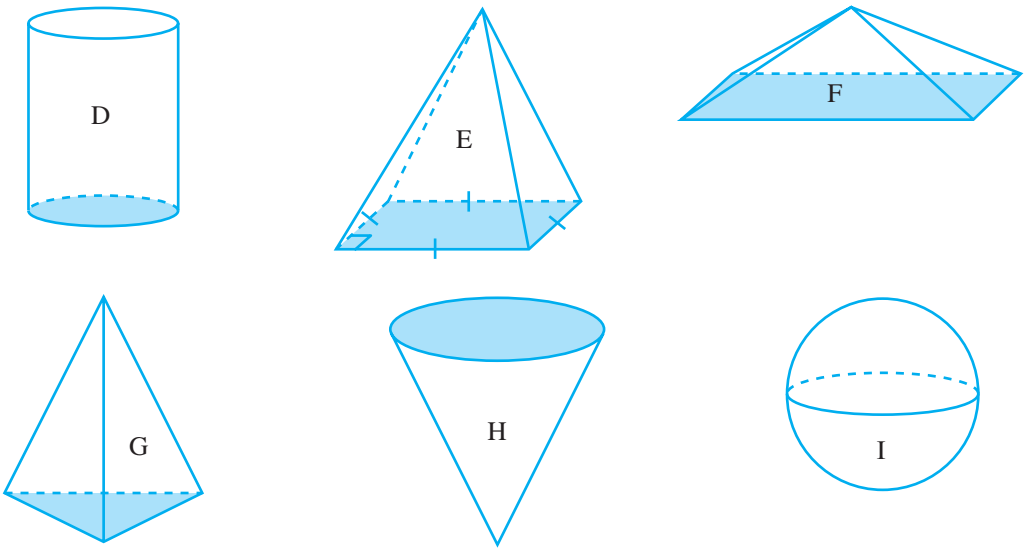
b The solid has exactly one vertex that does not lie in the base, so it is a pyramid. The base is a hexagon. Therefore, the solid is a hexagonal pyramid.

Exercise 12.1









Use the solids above to answer Q1–9. (Write A, B, C, …, I.)

1 What shape is the base in each of these solids?

a A **b** B **c** C **d** D
e E **f** F **g** G **h** H

2 Which of the solids are prisms? Why?

3 Which of the solids are pyramids? Why?

4 Which of the solids are neither prisms nor pyramids?

5 Name each of the solids.

6 Which of the solids can roll?

7 Which solids have a uniform cross-section?

8 Which solids have an apex?

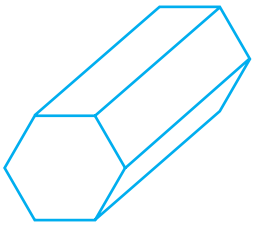
9 What shapes are the faces in each of these solids and how many of each shape are there?

a A **b** C **c** E **d** F

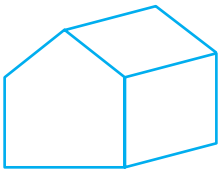
Consolidation

10 Prisms and pyramids are named according to the shape of their base. Name each of these solids.

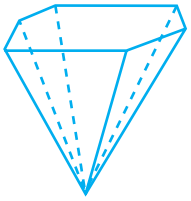
a



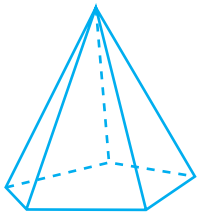
b



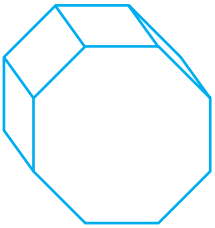
c



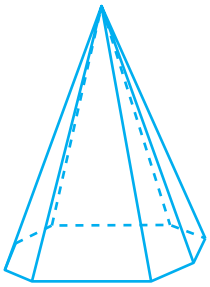
d



e

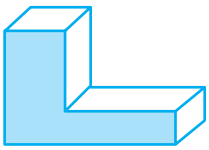


f

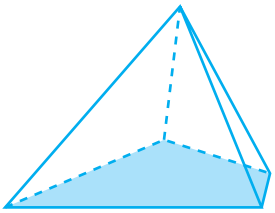


11 State whether these solids are prisms, pyramids or neither.

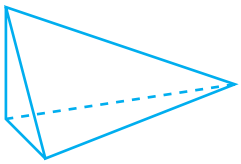
a



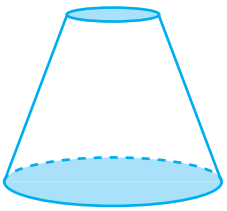
b



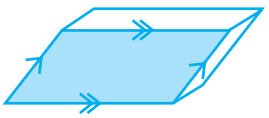
c



d

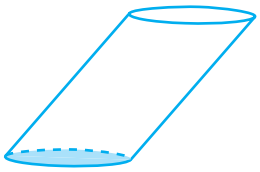


e

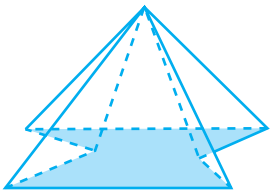


f

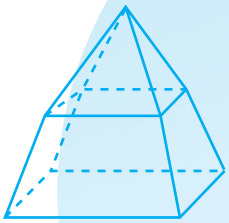
g



h



i

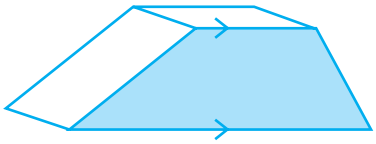


12 Name the solid that most resembles each of these everyday objects.

a a tin of soup
b a tennis ball
c a block of ice
d a stick of chalk
e a housebrick
f a die
g an orange
h a gluestick
i a packet of cornflakes

13 Name each solid and state the number and shape of the faces.

a



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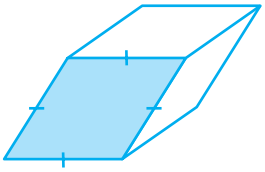
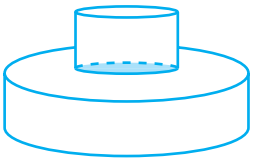


14 Many everyday objects are shaped in a certain way for very practical reasons. Give a reason why most:

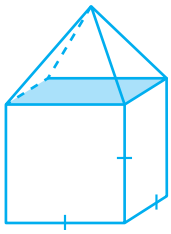
a saucepans are cylindrical
b rooms are rectangular prisms
c balls are spherical
d house roofs are triangular pyramids
e swimming pools are trapezoidal prisms
f cartons are rectangular prisms
g wheels are cylindrical
h dice are cubes
i drinking glasses are cylindrical
j ice-cream cones are cones

15 Name the solids that make up each of these composite shapes.

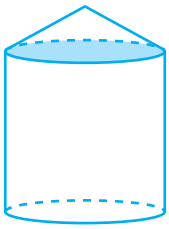
a



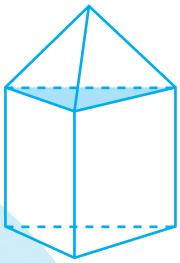
b



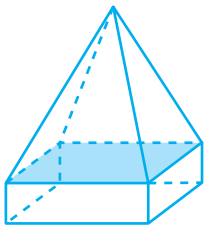
c



d



e



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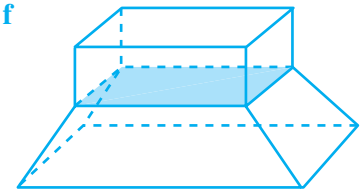
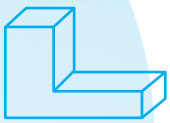


16 State whether each solid is a right prism or cylinder or an oblique prism or cylinder.

a



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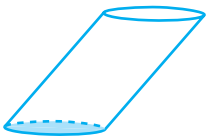

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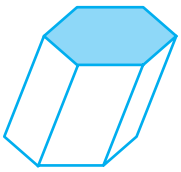
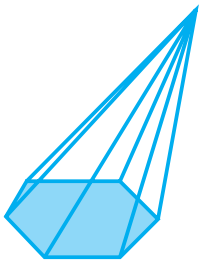


d

17 State whether each solid is a right pyramid or cone or an oblique pyramid or cone.

a



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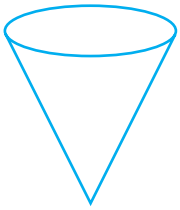
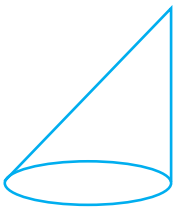


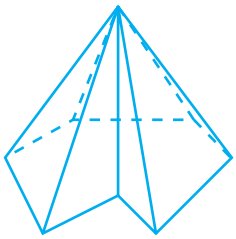
c

d

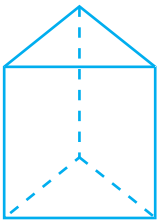


18 State whether each of these solids is convex or non-convex.

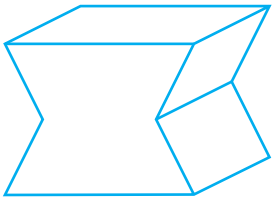
a



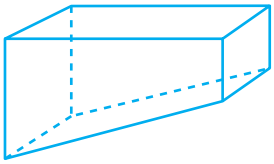
b



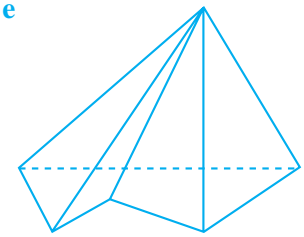
c



d



e



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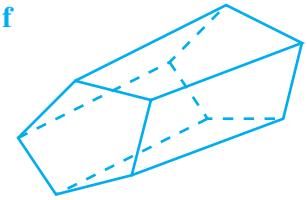
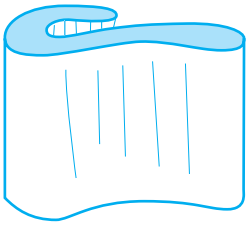


Further applications

19 If the top of a pyramid or cone is cut off by a cross-section parallel to the base, the solid that remains is called a **frustrum**. Draw a frustrum that could have been formed from:

a a cone
b a triangular pyramid
c a rectangular pyramid

20 Solids such as those shown are also referred to as cylinders. A solid is a cylinder if its bases (top and bottom) are identical and the lines joining all pairs of corresponding points on the bases are parallel. Draw two more cylinders of your own design.

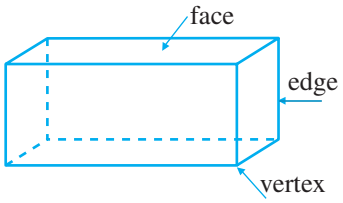


12.2 Faces, vertices and edges in polyhedra

A **polyhedron** is a solid that has only plane (or flat) surfaces. The plural form of polyhedron is polyhedra. Solids that have a curved surface such as the cylinder, cone and sphere are not polyhedra. Polyhedra may be right or oblique, convex or non-convex.

All prisms and pyramids are polyhedra. We may now redefine a prism as a polyhedron with a uniform cross-section. Similarly, we could redefine a pyramid as a polyhedron with one vertex not in the base.

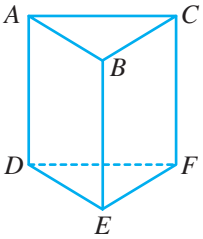
Each plane surface in a polyhedron is called a **face**. In a convex polyhedron, the intersection of exactly two faces is called an **edge**, while the intersection of three faces is called a **vertex**. The plural of vertex is vertices.



A vertex is named using a capital letter, since a vertex is a point. An edge is named using the capital letters at the vertices at each end, since an edge is an interval. A face is named in cyclic order using the capital letters at its vertices, since a face is a plane.

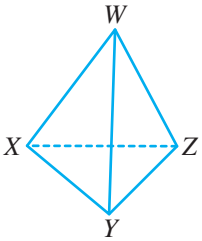
- **Two lines are parallel if the distance between them remains constant (i.e. does not change).**
- **Two lines are perpendicular if they intersect at right angles.**
- **Two lines are skew if they do not lie in the same plane. That is, they neither intersect nor are they parallel.**

These definitions can be used to classify edges and other lines in polyhedra. For example, in this triangular prism the edges *AD* and *CF* are parallel, *AD* and *DE* are perpendicular, *DF* and *BE* are skew.



Example 1

Find the number of faces, vertices and edges in this solid.



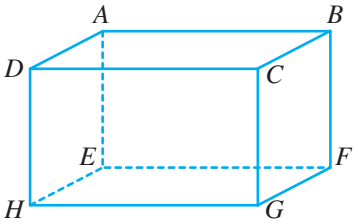
Solution

The solid has 4 faces (*WXY*, *WYZ*, *WXZ*, *XYZ*), 4 vertices (*W*, *X*, *Y*, *Z*) and 6 edges (*WX*, *WY*, *WZ*, *XY*, *YZ*, *ZX*).

Example 2

ABCDEFGH is a rectangular prism.

State whether each pair of edges is parallel, perpendicular or skew.



a *AB* and *BC* **b** *EF* and *DC*

c *EH* and *CG*

Solutions

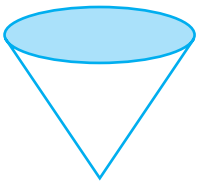
a *AB* and *BC* are perpendicular **b** *EF* and *DC* are parallel

c *EH* and *CG* are skew

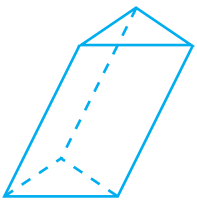
Exercise 12.2

1 State whether or not each solid is a polyhedron.

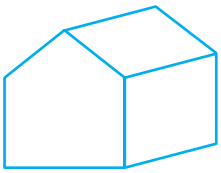
a



b



c



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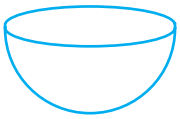

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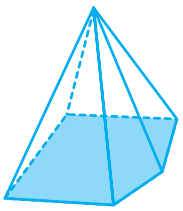

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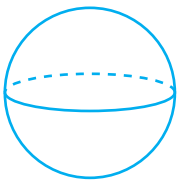
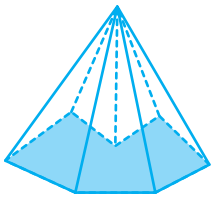
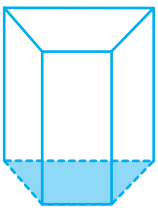


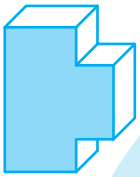
g



h



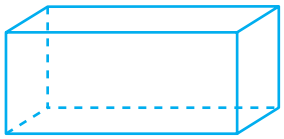
i

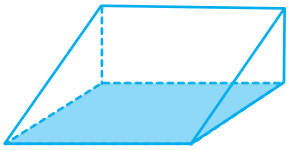


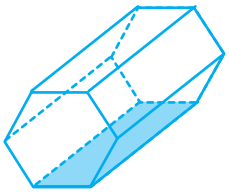
2 A Swiss mathematician named Leonard Euler (1707–83) discovered a relationship between the number of faces, vertices and edges in all convex polyhedra.

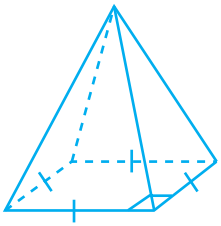
a Copy and complete this table for the solids below.

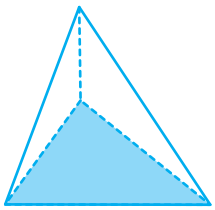
Solid	No. of faces (F)	No. of vertices (V)	No. of edges (E)	$F + V$
Rectangular prism				
Triangular prism				
Hexagonal prism				
Square pyramid				
Triangular pyramid				
Pentagonal pyramid				

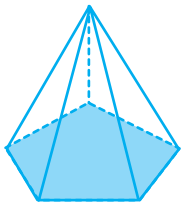

rectangular prism


triangular prism


hexagonal prism


square pyramid


triangular pyramid


pentagonal pyramid

b Use your results from the table to write down Euler's formula for polyhedra, linking the number of faces, vertices and edges of these solids. (Hint: Look at the values in the $F + V$ column and the number of edges.)

3 Euler's formula does not necessarily hold for solids that have curved surfaces. Give an example of a solid that has a curved surface for which the formula:

a does hold **b** does not hold

4 Does Euler's formula necessarily hold for solids that are non-convex?

Consolidation

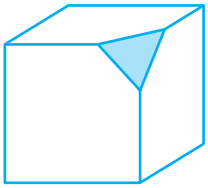
5 **a** Copy and complete this table and use the first five results to generalise an expression for the number of faces, vertices and edges in any n-sided prism.

Solid	No. of sides in base	No. of faces (F)	No. of vertices (V)	No. of edges (E)
Square prism	4			
Pentagonal prism	5			
Hexagonal prism	6			
Heptagonal prism	7			
Octagonal prism	8			
n-sided prism	n			

b Use your results from part **a** to find the number of faces, vertices and edges in a 20-sided prism.

c Verify Euler's formula for the 20-sided prism.

6 Each corner of a cube is removed by cutting it as shown.

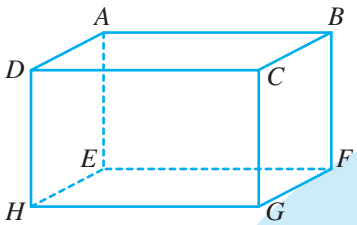


a How many faces, vertices and edges will it have?

b Is the solid a polyhedron?

c Does Euler's formula still hold for this solid?

7 *ABCDEFGH* is a rectangular prism.



a How many:

i edges intersect at each vertex?

ii faces intersect at each edge?

iii faces intersect at each vertex?

b Name the edges that intersect at:

i *C* **ii** *A* **iii** *H*

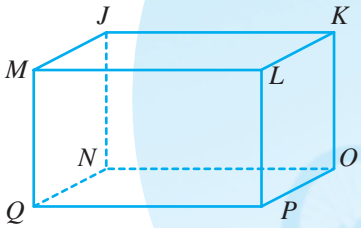
c Name the faces that intersect at each edge:

i *DH* **ii** *AB* **iii** *FG*

d Name the faces that intersect at each vertex:

i *B* **ii** *G* **iii** *E*

8 *JKLMNOPQ* is a rectangular prism.



a Name the face that is parallel to:

i *JKLM* **ii** *MJNQ*

iii *JKON*

b Name the faces that are perpendicular to *MLPQ*.

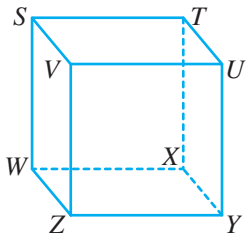
c State whether these edges are parallel or perpendicular.

i *JN* and *KO* **ii** *MQ* and *QP*

iii *ML* and *NO* **iv** *MJ* and *JK*

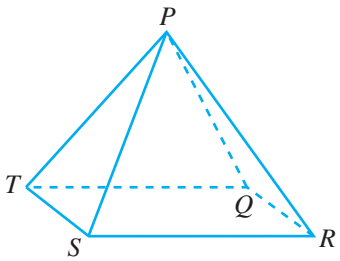
v *LP* and *PO* **vi** *QN* and *LK*

9 *STUVWXYZ* is a cube. State whether or not these edges are skew.



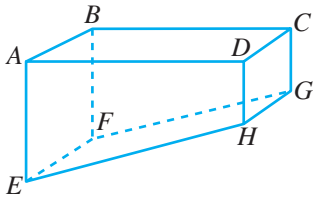
a *ST* and *VZ* **b** *TX* and *WX*
c *VU* and *SW* **d** *TX* and *WZ*
e *SW* and *UY* **f** *ZY* and *VU*
g *XY* and *VZ* **h** *SV* and *SW*
i *UY* and *WZ* **j** *SW* and *ZY*
k *TX* and *SV* **l** *WZ* and *TU*

10 *PQRST* is a rectangular pyramid. For each pair of lines state whether they are parallel, perpendicular or skew.



a *TS* and *QR* **b** *PT* and *QR*
c *PR* and *TS* **d** *TQ* and *ST*
e *SR* and *TQ* **f** *SR* and *QR*

11 *ADHE* is a trapezium and the solid is a trapezoidal prism.



a Name two pairs of parallel lines.
b Name two pairs of perpendicular lines.
c Name two pairs of skew lines.

12 *KLMN* is a triangular pyramid with apex *K*. Which line is skew to:
a *LM*? **b** *MN*? **c** *LN*?

Further applications

13 Identify examples of parallel, perpendicular and skew lines in:
a your classroom **b** the environment

14 Use Euler's formula to find the number of:
a edges in a convex polyhedron with 12 faces and 30 vertices
b vertices in a convex polyhedron with 15 faces and 45 edges
c faces in a convex polyhedron with 55 vertices and 73 edges

15 The Platonic Solids are a special group of solids. Use the internet or your school library to find answers to the following questions.
a What are the Platonic Solids? How many are there? What are their names?
b Who was Plato? Why was he interested in these solids?
c How many faces, vertices and edges are there in each of the Platonic Solids?
d Verify Euler's formula for each of these solids.
e Explain why there are exactly this many Platonic Solids.

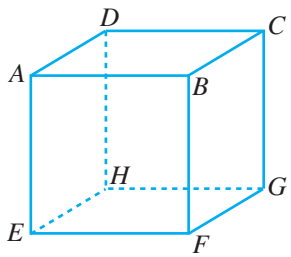
16 Download the nets of the Platonic Solids from the CD-ROM and fold them together.

TRY THIS **Painted cube**

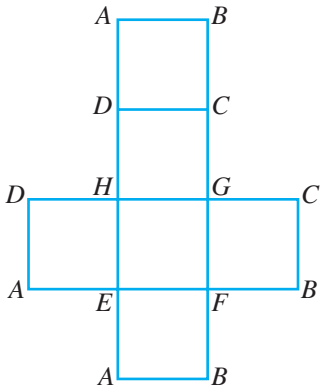
You will need eight cubes. Perhaps you and your friend could each make four cubes. Now put them together to form one large cube of dimensions $2 \times 2 \times 2$. Now paint all sides of the outside of the cube. How many cubes have no sides painted? one side painted? two sides painted? etc. Can you find a pattern?

12.3 Nets of solids

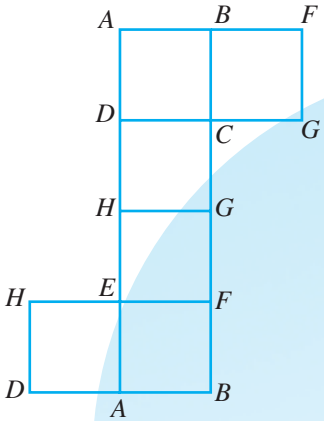
Imagine a cube made of cardboard. If we cut along some edges of the cube we could convert the cube back into a single, plane piece of cardboard.



If we cut along the edges *AB*, *BC*, *AD*, *DH*, *CG*, *BF*, *AE* then the result would be:



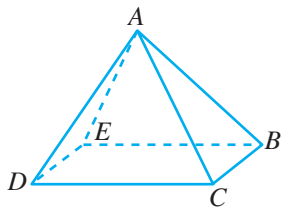
This is just one of a number of shapes that are possible. The shape below could also be folded to produce a cube.



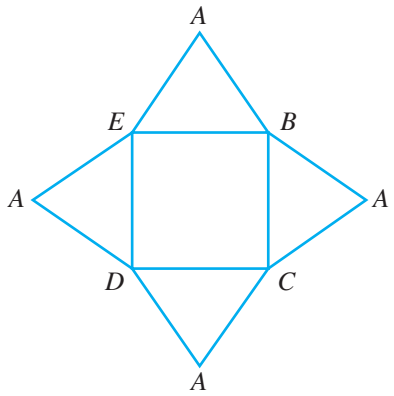
A plane shape which can be folded to produce a solid is called a *net* for the solid.

Example

Draw a net for this pyramid.



Solution

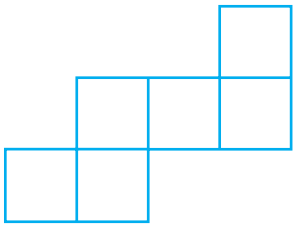


Notice that this net is only one of the possible nets for this solid.

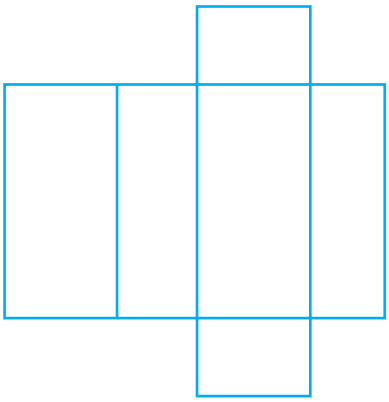
Exercise 12.3

1 Study the nets illustrated below. Which solid do you think that each net could be folded to form? Copy each net onto rectangular grid paper. Cut out each net and fold it to form a solid. Were your answers correct?

a

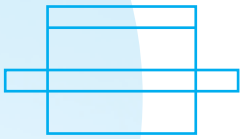


b

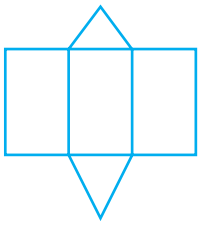


2 Name the solids that have the following nets.

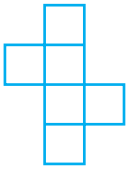
a



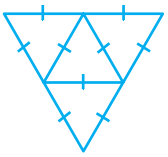
b



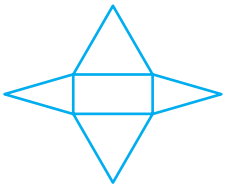
c



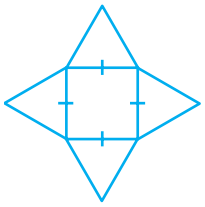
d



e



f

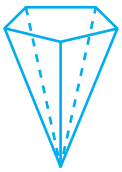


■ Consolidation

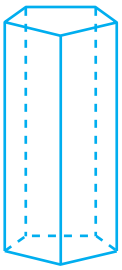
3 Choose the correct net (A, B, C or D) for each solid below, and state its name in each case.

a

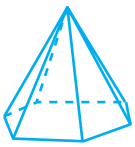
b



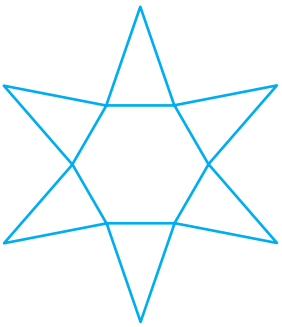
c



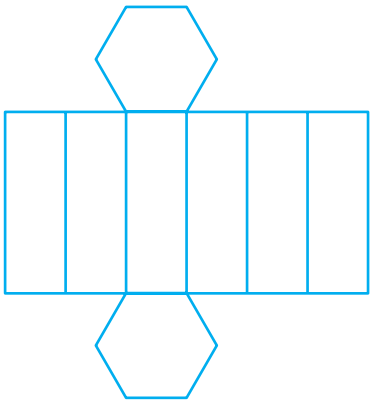
d



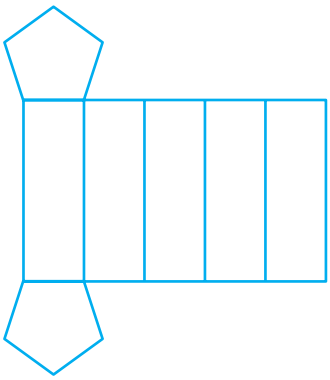
A



B

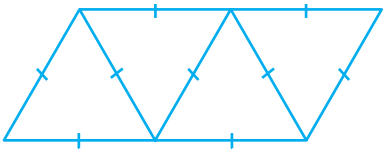


C

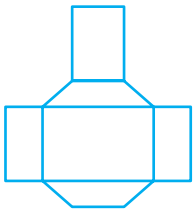


D

4 **a** Name the solid that has this net.
b State the number of faces, vertices and edges of the solid.
c Is the number of intervals in the net equal to the number of edges in the solid? Explain.

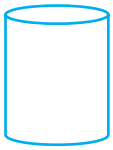


5 **a** Name the solid that has this net.
b State the number of faces, vertices and edges of the solid.

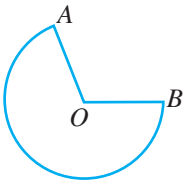


6 **a** Draw three different nets for a cube.
b Draw two different nets for a triangular prism.
c Draw two different nets for a square pyramid.
d Draw one net for an octagonal prism.
e Draw one net for an octagonal pyramid.

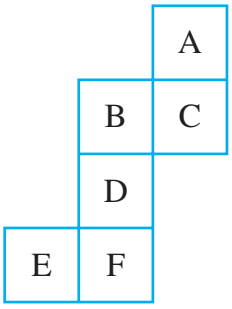
7 Draw the net of a cylinder which is:
a closed **b** open



8 **a** Which *open* shape could be formed by joining the edges *OA* and *OB*?

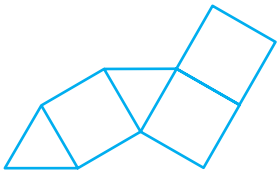


9 This is a net of a cube.
When the sides are folded up and joined, which faces would be opposite each other?

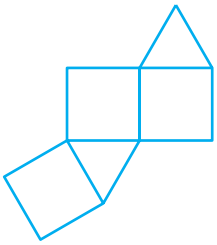


10 Which one of the following could *not* be the net of a triangular prism?

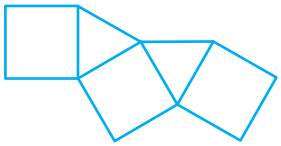
A



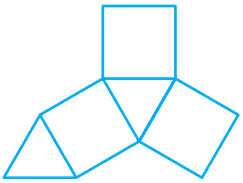
B



C



D



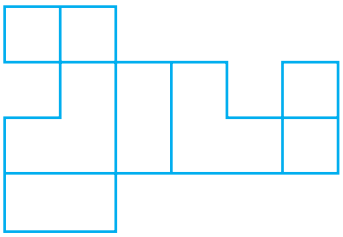
■ Further applications

11 Draw as many different nets as you can for a cube. (There are at least nine!)
NOTE: Rotating or reflecting (turning over) the net does not produce a different net.

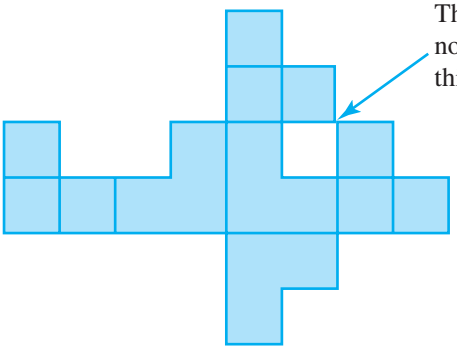
12 Draw as many different nets as you can for a tetrahedron.

13 Copy these nets onto 1 cm grid paper, then cut them out and fold them together to form two solids.

a

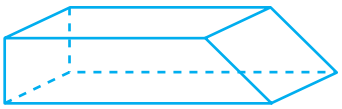


b

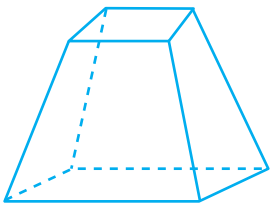


14 Draw nets for each of the following solids.

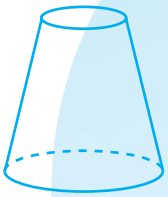
a



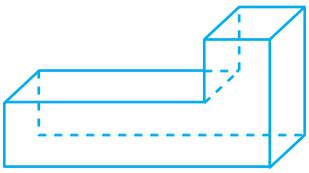
b



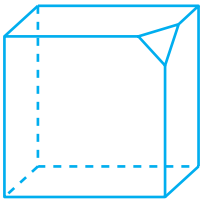
c



d



e



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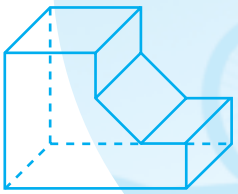
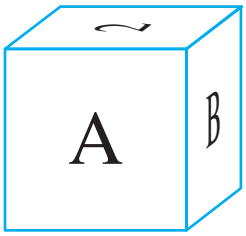


12.4 Sketching solids

Solids are three-dimensional shapes, i.e. they have length, breadth and depth.

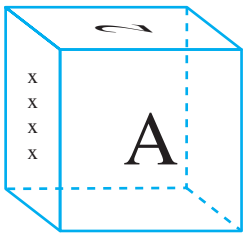
The surface of a sheet of paper is only two dimensional so it is impossible to show a solid as a drawing with complete accuracy and clarity. When we draw a representation of a solid on paper we try to show how the object appears, but we can only show it from one direction.



Looking at this cube we realise that we are viewing it from in front, slightly to the right and slightly above. So we are able to see the front (with the A) very clearly, and we have some view of the right side and the top, but we cannot see the other three sides at all.

One method that is often used to help clarify the illustration is to mark in with dotted lines edges that are out of sight.

This certainly helps, but our view is still far from perfect. Notice how narrow the '2' on top looks!

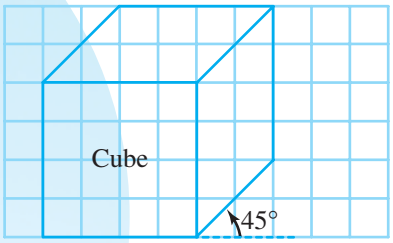


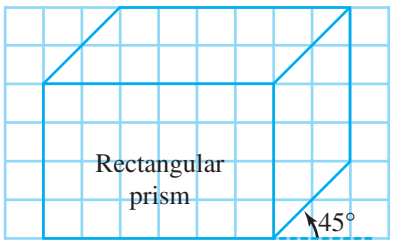
The simplest method of producing diagrams of prisms is to make use of the fact that the base and top of any prism will be identical. We therefore draw the base and then translate it to the top, then join corresponding points.

A solid can be drawn in two different ways—as an **oblique projection** or as an **isometric projection**.

Oblique projection

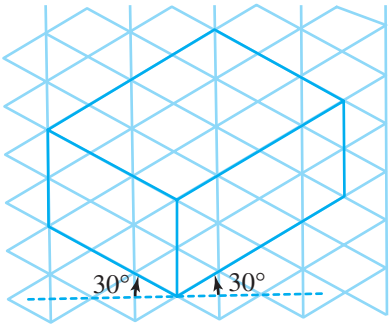
Using **square grid paper**, we draw the front surface correctly. The side faces are drawn at an angle of 45°. The lines going back, indicating depth, are drawn only half the correct length. This way they will look correct.





Isometric projection

When we use **triangular grid or dot paper**, one edge of the solid appears closest to us. The faces go back at an angle of 30°. The sides of squares and rectangles are not drawn at 90° to each other. So the solid looks correct.



Example 1

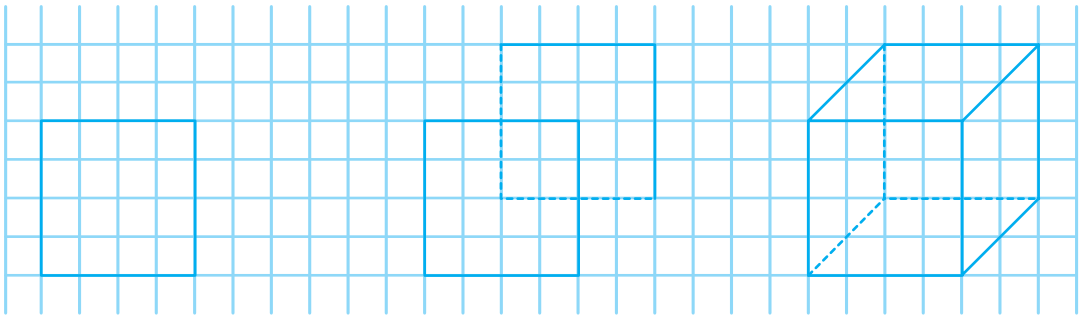
Draw a square prism.

Solution

First draw the base (a square).

Second imagine that the square slides to the right and up, as shown. Edges which cannot be seen from our point of view should be dotted.

Finally join corresponding points.



Example 2

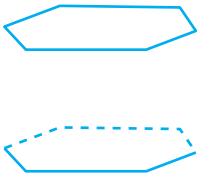
Draw a hexagonal prism oriented vertically.

Solution

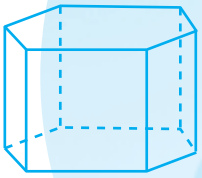
In order to make the diagram appear realistic, we need to show the hexagonal base as seen from the side, i.e. somewhat flattened. The following three diagrams show the process.

Step 1

Step 2



Step 3



Example 3

Draw a triangular pyramid.

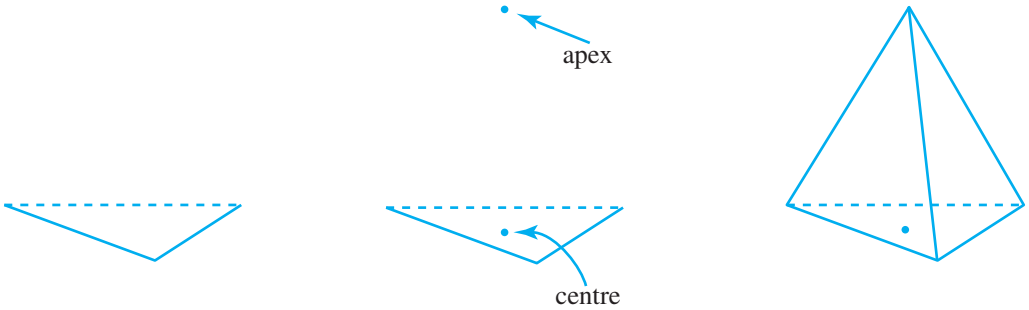
Solution

To produce diagrams of pyramids we simply draw the base, locate the apex (usually vertically above the centre of the base), then join the apex to each vertex of the base.

Step 1: Draw a flattened triangular base.

Step 2: Locate the centre of the base and position the apex.

Step 3: Join the apex to all vertices of the triangle.



Example 4

Use isometric grid paper to sketch:

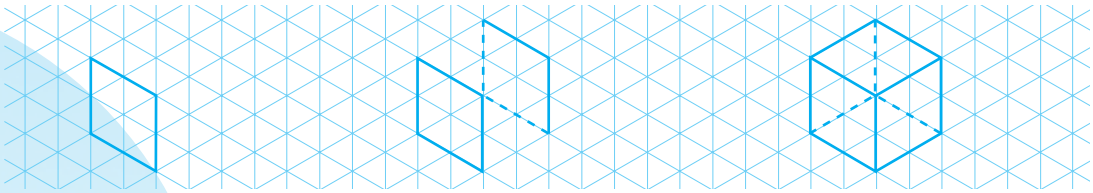
a a cube **b** a rectangular prism

Solutions

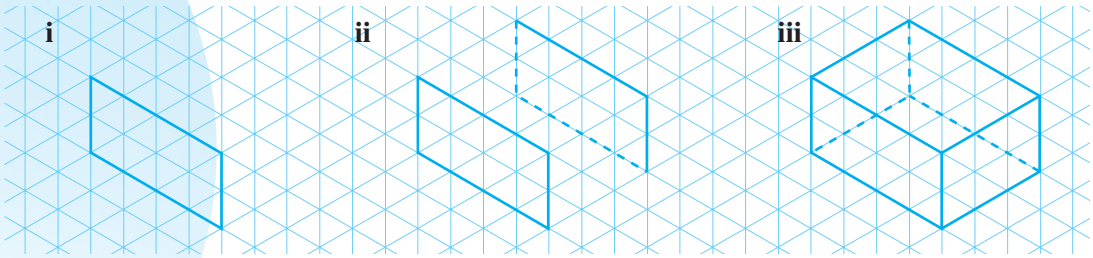
a **i** Draw the front face for the solid.

ii Translate the front face backward along one of the isometric grid lines.

iii Join the corresponding vertices.



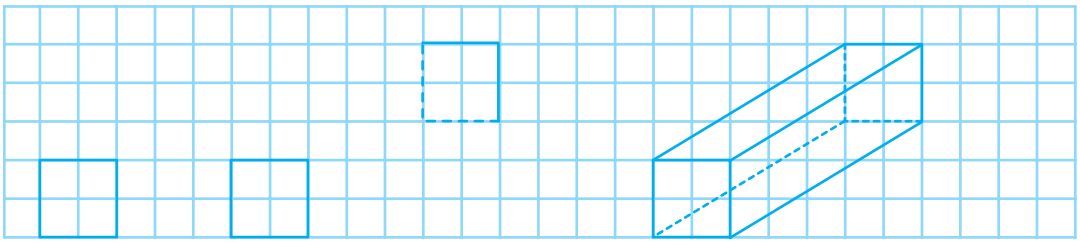
b Follow the steps as in part **a**.



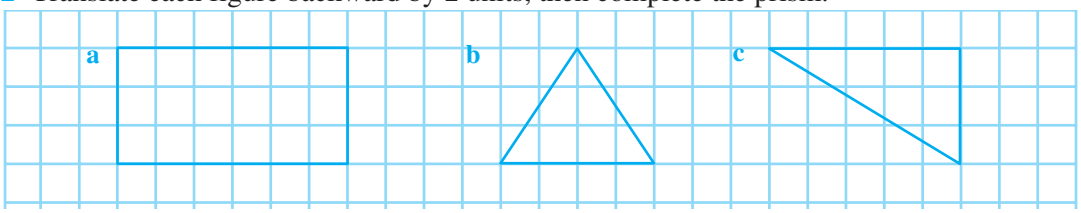
The prism we have drawn has length 4 cm, width 3 cm and height 2 cm.

Exercise 12.4

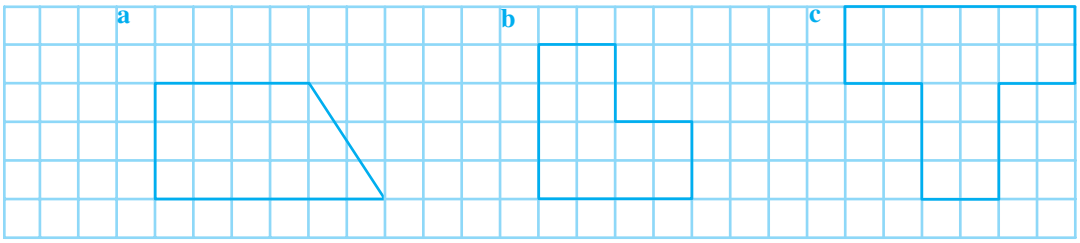
1 Follow these steps to complete the square prism. Is it an oblique projection or an isometric projection?



2 Translate each figure backward by 2 units, then complete the prism.

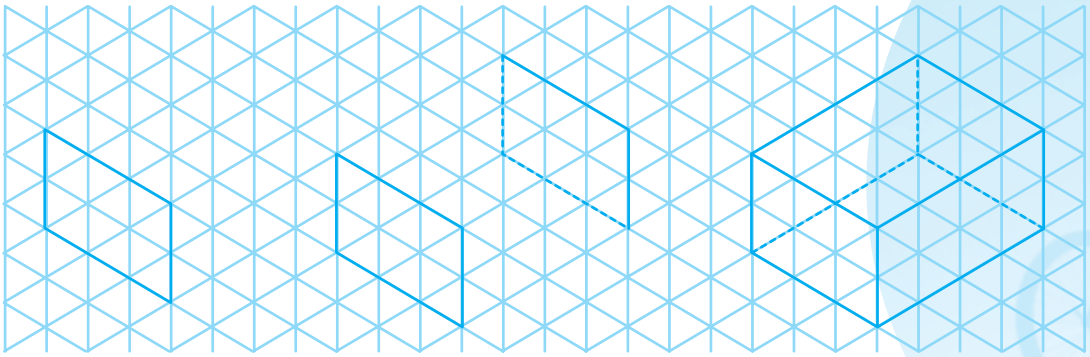


3 Translate each figure backward by 1 unit, then complete the prism.

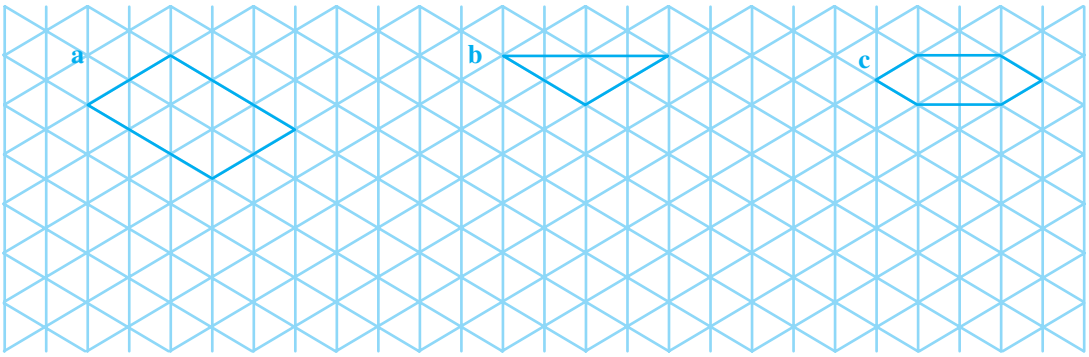


Consolidation

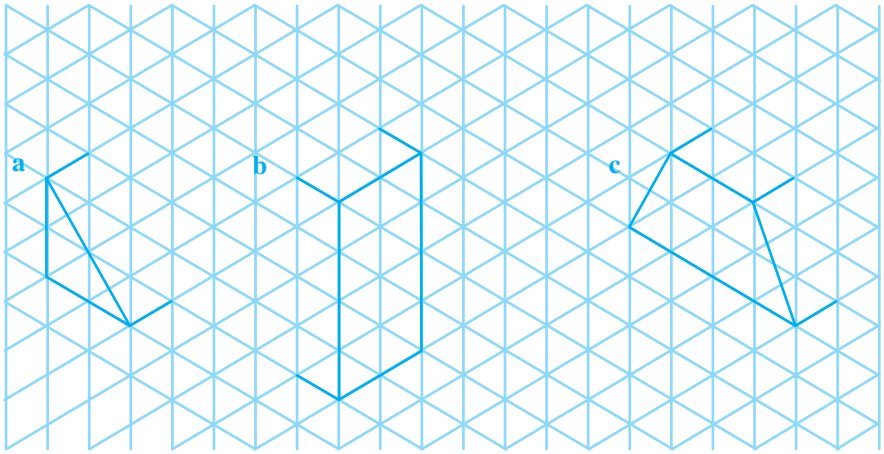
4 Follow these steps to complete the rectangular prism. Is it an oblique or isometric projection?



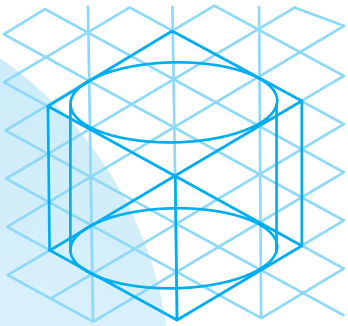
5 Translate each figure downward by 3 units, then complete the prism.



6 Translate each figure 4 units in the direction indicated, then complete the prism.

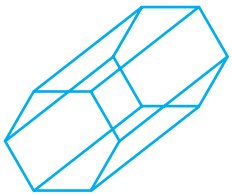


7

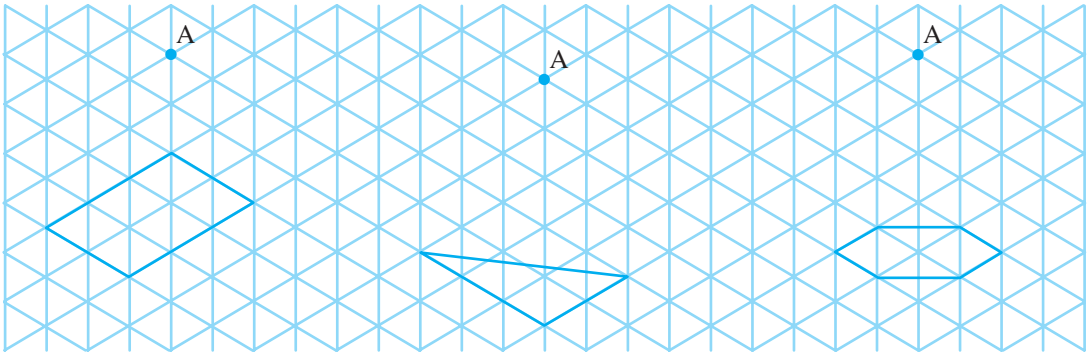


The diagram shows a cylinder drawn inside the isometric projection of a cube. Draw a square prism with a base measuring 2 units × 2 units and a height of 4 units, then draw a cylinder inside it.

8 **a** Draw this solid on square grid paper.
b What is the name of the solid?



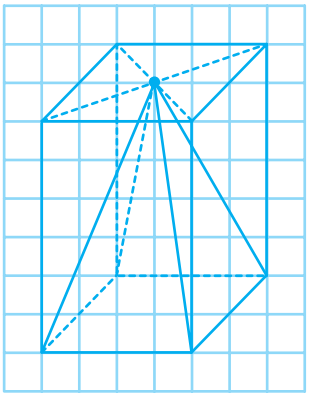
9 Complete these pyramids by joining the apex, A, to each of the vertices in the base.



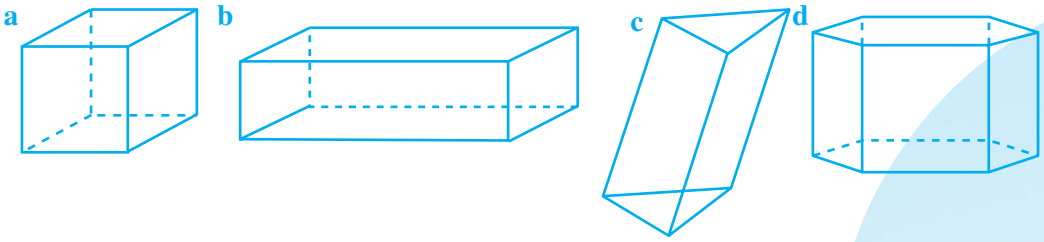
Further applications

Another method of drawing a pyramid is to draw the corresponding prism, mark the apex in the top face, then join it to each of the vertices in the base.

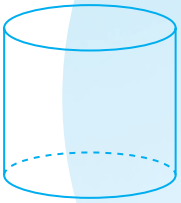
The apex can be found by drawing the diagonals in the top face of the prism and marking their point of intersection.



10 Copy each of these prisms and use them to sketch the corresponding pyramid.



11 Draw any right cylinder and use it to construct a cone that has the same base and height.

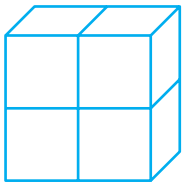


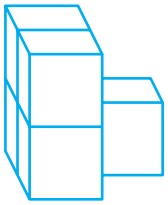
12.5 Constructing and viewing solids

Many people such as architects, draughtsmen and builders need to be able to visualise and draw buildings and other objects from different perspectives. A building, for example, may look quite attractive when seen from the front. However, other less attractive features may become noticeable when the building is seen from a side view or from behind.

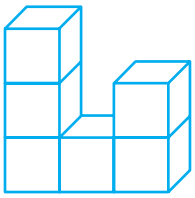
In this section we will construct solids by connecting centicubes. Once a solid has been constructed, you will be able to view the solid from different directions, then draw it on isometric grid paper.

It may not always be possible to tell whether any cubes are 'hidden' behind those at the front or at the side of a solid. One way to overcome this problem is to draw the solid from two (or more) different views, e.g. from the front and from the right.





Another method is to draw a **plan** for the solid. A plan is a diagram that inicates the height of each section of a solid. That is, it tells us the number of cubes that are stacked on top of each other at various points.

The plan for this solid  would be

3	1	2

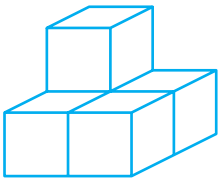
Example 1

Construct this solid using cubes, then remove the shaded cubes. Draw the remaining solid on isometric grid paper.

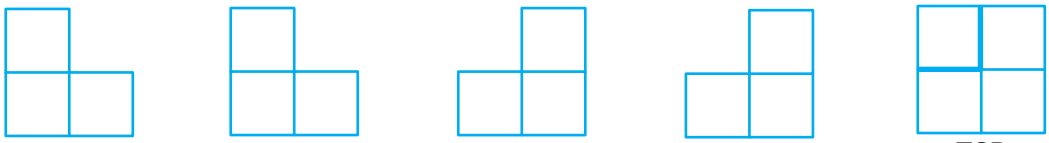
Solution

Example 2

Draw front, left, right, back and top views of this solid.



Solution



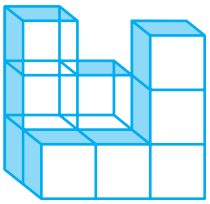
Another method is to draw a top view of the solid and label the heights of each section. This is how maps are often drawn, with land of different heights above sea level in different colours, and features such as mountains labelled with their heights.

Example 3

Sketch the solid represented by this plan.

3	2	
1	1	3

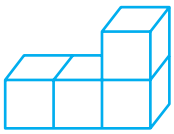
Solution



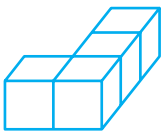
Exercise 12.5

1 Use cubes to build each of these solid shapes.

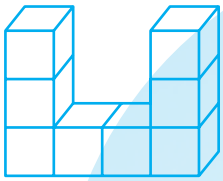
a



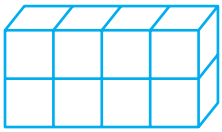
b



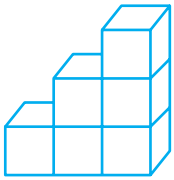
c



d



e



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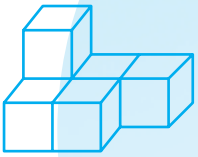
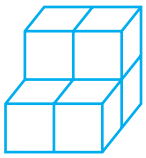
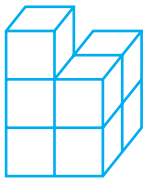


2 Build the following solids using cubes.

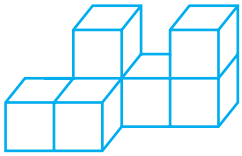
a



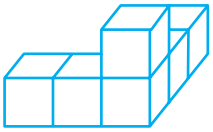
b



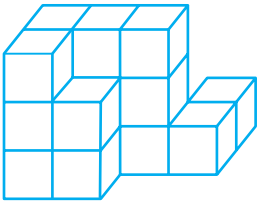
c



d



e



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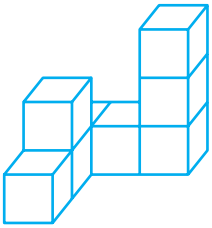
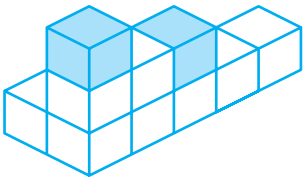


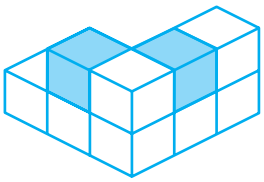
■ Consolidation

3 **i** Construct each of these solids using cubes, then remove the shaded cubes.
ii Sketch the remaining solid on isometric grid paper.

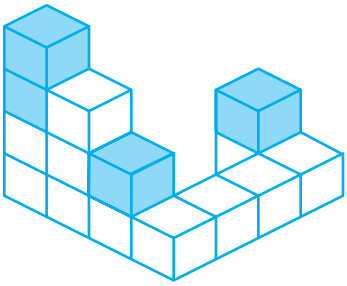
a



b



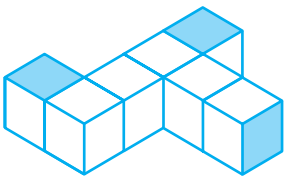
c



4 **i** Construct each of these solids using cubes, then add a cube to each shaded face.
ii Sketch the new solid on isometric grid paper.

a

b

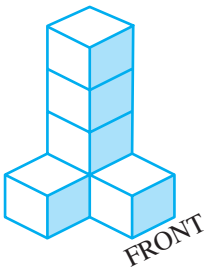


c

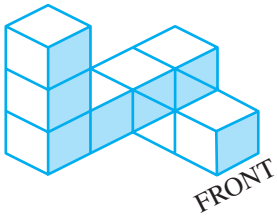
5 Build each of these solids and then sketch the view from the
i front **ii** left **iii** top

a

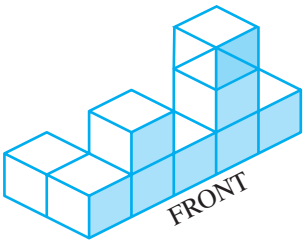
b



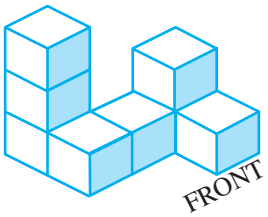
c



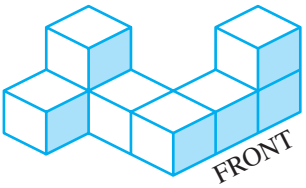
d



e



f



6 Use cubes to construct the solids described by the following plans.

a

3	1	2

b

1	2	2
		1

c

2	3
1	2

d

3	2	3
1		1
3	2	3

e

1	2	3	4
	3	1	
	1		

f

1	3		
1	3	2	4
		1	

7 Draw a plan for each of the solids in Q2, assuming that there are no 'hidden' cubes.

8 Build each solid from the plan, and draw the views from corner *A* and corner *B* on isometric grid paper.

a

2	3	
	1	2

A *B*

b

3	2	2
	1	

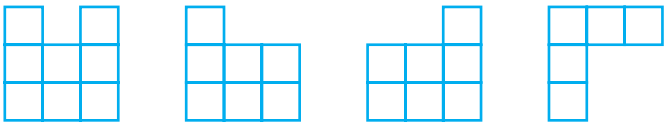
A *B*

9 Build the following solids using cubes, then draw a plan for each solid.

a

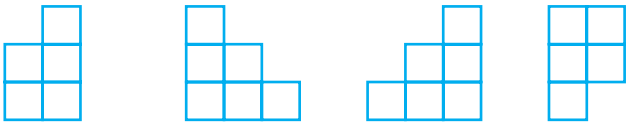
Front Left Right Top

b



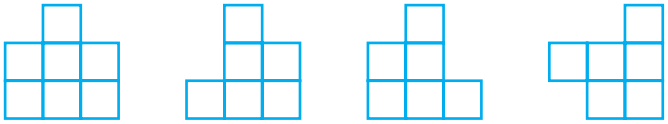
Front Left Right Top

c



Front Left Right Top

d



Front Left Right Top

■ Further applications

10 Sketch on isometric grid paper, the solids represented by these plans.

a

2	1	1

b

2		
2	1	1

c

1	2
1	1

d

1	2	3
		2
		1

e

2		
1	2	2
		1

f

3	3	3	2	3	
3				2	1
2					

TRY THIS

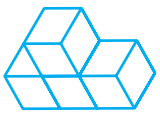
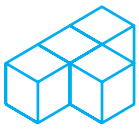
The Soma Puzzle

You will need isometric dot paper to do this activity. If you are given 4 cubes, how many different shapes can be made using

a 1 cube? **b** 2 cubes? **c** 3 cubes? **d** 4 cubes?

Draw these shapes on isometric dot paper. Now complete this table.

Number of cubes used	1	2	3	4
Number of ways	1	1	?	?

Please note that  is the same as 

Rotations of the shape are regarded as being the same shape.

Now how many of these shapes have at least 3 cubes which form a right angle. Circle these shapes on your dot paper.

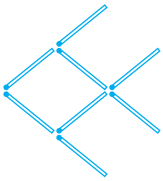
Now make the circled shapes mentioned above. Can you put these 7 shapes together to make one large cube?

Problem solving

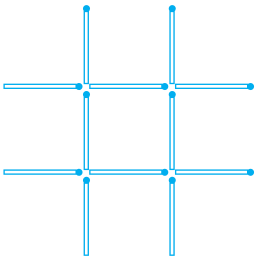
1 If a is a number between 15 and 20 and b is a number between 2 and 5, find the smallest and largest possible values of $\frac{a}{b}$.

2 Find the next pattern in the following series.

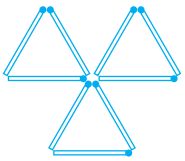
3 These 8 matches look rather like a fish facing to the left. Move 3 matches so that it will face right.



4 Move 3 matches to produce 3 square paddocks that touch each other.



5 Move three matches to form a picture of a cube.

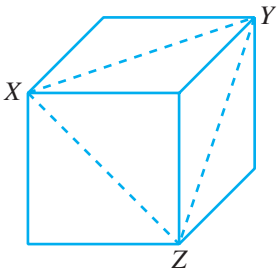


6 John is twice as old as Ann, who is twice as old as Geoff. If the total of their ages is 84, find their ages.

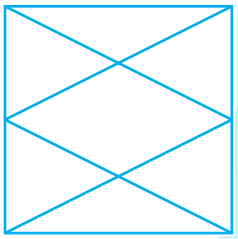
7 In Mathsland, everyone has a driver's licence! The licence number consists of a letter of the alphabet followed by a 2-digit number. What is the largest possible population of Mathsland?

8 Using only two digits, write the number 648 as a product in two different ways. (*Hint:* You may use indices.)

9 If this cube were cut along the dotted lines, what would be the size of angle XYZ?



10 How many triangles have been drawn in this square?



Focus on working mathematically

Shapely thinking: cones and conic sections

Introduction

The imagination plays an important role in the study of geometry. This activity is designed to get you to *visualise* shapes which are formed by cutting a solid, and draw them. In particular, we are going to look at the sections of a cone. They were recorded by the Greek mathematician Appolonius in the second century BC in a book he called *Conics*.

Learning activities

Materials needed: paper and pencil, three-dimensional models if available.

1. Think of a cone. Now draw it on your paper. Try to draw it in three dimensions, not just a front view.
2. Now imagine cutting through the cone and looking at the cross-section formed. What shape would you get if you made a flat cut:
   - **a** parallel to the base?
   - **b** vertically through the apex (top point) of the cone, to the base?
   - **c** vertically through the cone, parallel to the cut in part b?
   - **d** through the side of the cone at an acute angle, but not cutting the base?
   - **e** through the side of the cone, at an acute angle, this time cutting the base?

Draw the shapes you have imagined, check with your teacher, and if possible look at the sections with three-dimensional models.

3. Now draw the curves which form the boundaries of the sections. Appolonius called them 'conics' and they have very important applications in astronomy, rockets and space probes to other planets. See if you can match the name to the sections you have drawn: ellipse, parabola, circle (a special case of an ellipse) and hyperbola.
4. The vertical cut through the vertex does not produce a conic. What did you find?

Extension activities

Your teacher may like to demonstrate parts 1 and 2 if not enough materials are available.

1. Take a piece of string or cotton thread about 30 cm long. Tie the ends to make a loop. Now take a piece of paper and pin to a board. Now place two drawing pins on the paper about 10 cm apart. Loop the string around the pins, and tighten with a pencil. Move the pencil to trace out a curve keeping the string tight at all times.
2. What curve is traced out? Try altering the distance between the pins to see the effect. What happens when there is no distance between the two pins?
3. A ball thrown in the air traces out a conic also. What do you think it might be? What about the curve from a jet of water from a garden hose? Check with your teacher.
4. What curves do the planets in our solar system trace out as they move around the sun?
5. Halley's comet traces a conic path through the stars. Last century it appeared in 1910, and again in 1986. What does this suggest about its path? When will it next be visible from Earth?
6. Finally, the conic known as a hyperbola, unlike the others, has two branches. One branch is identified in part 2c in the *Learning Activities* on the previous page. To get the two branches, visualise two cones joined by the apex. Cut vertically downwards to make a cut through each cone. Draw what you see. Discuss with your teacher.

Let's communicate

1. Make a wall chart to show the conic sections and their occurrence in nature. Check the web for additional information.
2. Unlike the cone, you have also studied solids with flat identical faces. Check out a web link for the 5 Platonic Solids and discuss a project with your teacher.

Reflecting

Think over the differences between solids whose faces are flat surfaces and those that are curved. What is so special about the cone?

Language link with Macquarie

MACQUARIE

1. *Explain the difference between* ***polyhedron*** *and* ***polyhedra****.*
2. *What is a* ***polyhedron*** *with six faces?*
3. *What is an example of a* ***sphere*** *in everyday life?*
4. *Define* ***pyramid*** *for a new maths dictionary.*
5. *Read an excerpt from the* Macquarie Learners' Dictionary *entry for* ***cube****:*

cube *noun* **1.** a solid shape with six equal square sides.

Why is the ***cube*** *important to the building industry in our society?*

1 Name each of these solids.

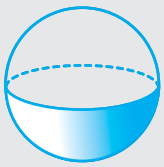
a

b

c

d

e

f


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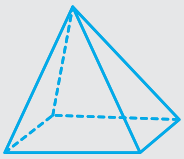

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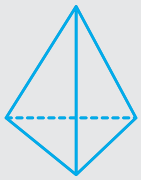
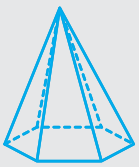


2 Name each of these solids.

a


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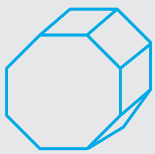
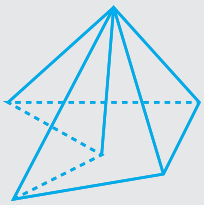


3 Which of the solids in Q1 are:
- **a** prisms?
- **b** pyramids?
- **c** polyhedra?

4 What shape is the cross-section in a:
- **a** cube?
- **b** triangular prism?
- **c** cylinder?
- **d** pentagonal prism?

5 Draw an oblique cylinder and a right circular cylinder.

6 State whether each of these solids is convex or non-convex.

a


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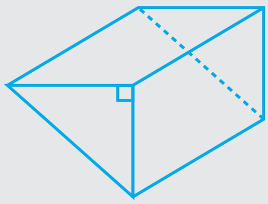


7 What shapes are the faces in a:
- **a** cube?
- **b** rectangular prism?
- **c** triangular prism?
- **d** rectangular pyramid?
- **e** triangular pyramid?
- **f** octagonal pyramid?

8 State the number of faces, vertices and edges in each solid in Q7.

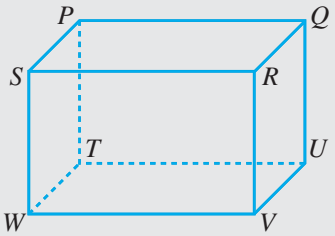
9 **a** Draw a trapezoidal prism and state the number of faces, vertices and edges.
- **b** What shape is the cross-section? Shade this on your diagram.

10 What is a frustrum?

11 Explain why a cylinder is not a polyhedron.

12 What are skew lines?

13 Name 2 edges of this prism that are:



a parallel to *TU*
b perpendicular to *RV*
c skew to *SW*

14 **a** What is Euler's formula for polyhedra?
b Verify the formula for each of these solids.
i hexagonal prism
ii decagonal pyramid

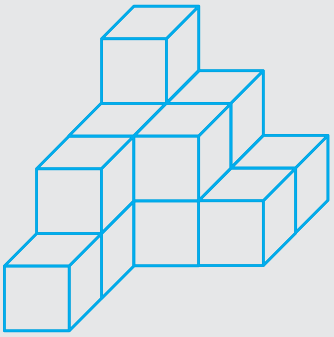
15 Name the five Platonic Solids.

16 Draw a net for each of these solids.
a cube
b rectangular prism
c triangular prism
d square pyramid
e triangular pyramid
f pentagonal pyramid

17 Draw the net for a cylinder.

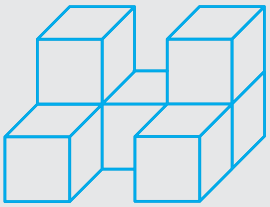
18 Draw each of these solids on isometric grid paper.
a a rectangular prism
b a triangular prism
c a square pyramid
d a hexagonal pyramid

19 **a** Draw the plan for this solid.



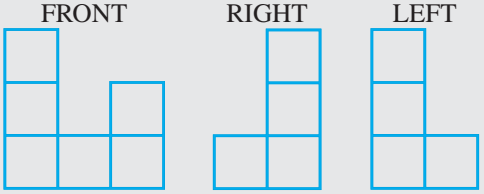
b How many cubes are needed to build it?

20 **a** Build this solid using cubes. How many cubes are needed?
b Draw the plan for this solid.
c Sketch a view of this solid from the:
i top **ii** front **iii** left

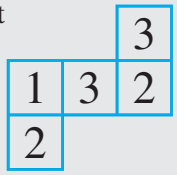


21 Construct the solid that is described by these views. Is there more than one possibility?

FRONT RIGHT LEFT



22 Build the solid with this plan, then sketch the view from the:
a front **b** right



13 Area

This chapter at a glance

Stage 2/3

After completing this chapter, you should be able to:

- use the standard abbreviations for square units
- find areas of shapes that have been drawn on a square centimetre grid
- estimate areas of irregular figures that have been drawn on a square centimetre grid
- select appropriate units to calculate particular areas.

Stage 4

After completing this chapter, you should be able to:

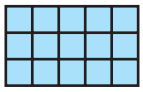
- develop and use formulae for the area of a square and rectangle
- find the side length and perimeter of a square given its area
- find the side length and area of a square given its perimeter
- find the dimensions and perimeter of a rectangle given the area and the length of one side
- find the dimensions and area of a rectangle given the perimeter and length of one side
- develop and use the formula for the area of a triangle
- find one of the perpendicular sides of a right-angled triangle given the other perpendicular side and the area
- find areas of simple composite figures by dissection
- solve practical problems involving area
- convert between metric units of area
- calculate large areas using square kilometres and hectares.

13.1 Definition of area

Area is a measure of the space inside a two-dimensional or plane figure.

We can determine the area of a figure by dividing it into small squares of side length 1 unit, then counting the squares. If each small square has side length 1 unit, then its area is 1 unit^2, read as '1 square unit'.

The rectangle shown has been divided into 15 squares, so its area is 15 square units, i.e. 15 units^2.

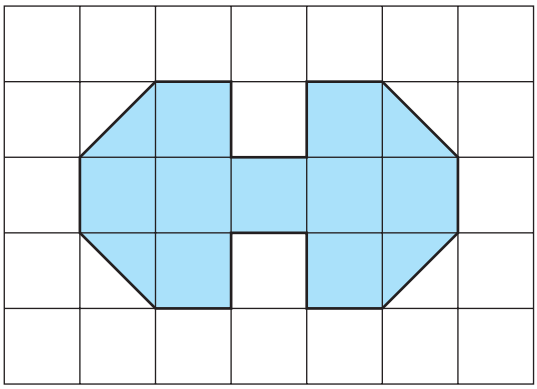


Many questions specify the units of length, such as centimetres. If the figure is divided into small squares of side length 1 cm, then the area is given in square centimetres, i.e. cm^2.

In general, the area of a figure cannot be calculated given only its perimeter, and vice versa.

Example

The figure shown has been drawn on 1 cm grid paper. Find its area.



Solution

Area = 9 whole squares + 4 half squares
= 9 whole squares + 2 whole squares
= 11 whole squares

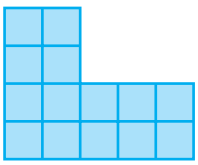
∴ the figure has an area of 11 cm^2

Exercise 13.1

1 Find the area of each figure, in square units.

a

b



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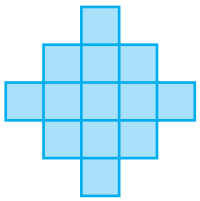

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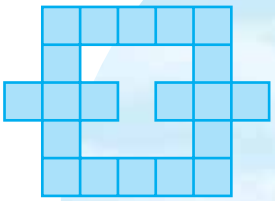

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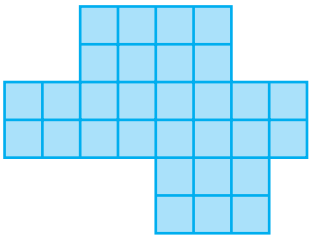

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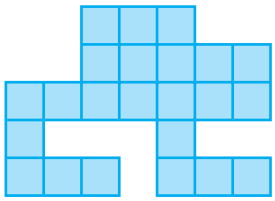

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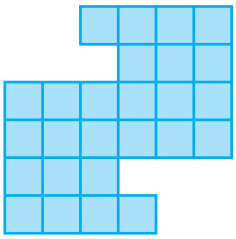

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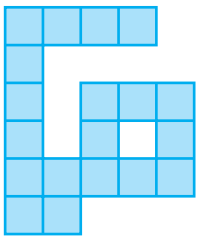
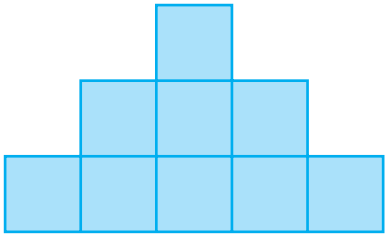


2 Find, in square centimetres, the area of each of these figures.

a



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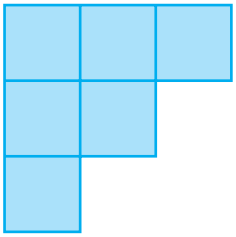

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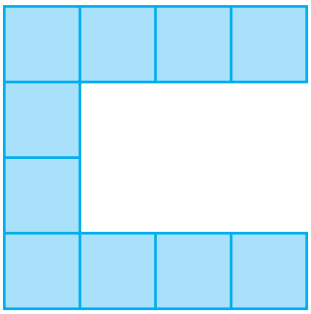

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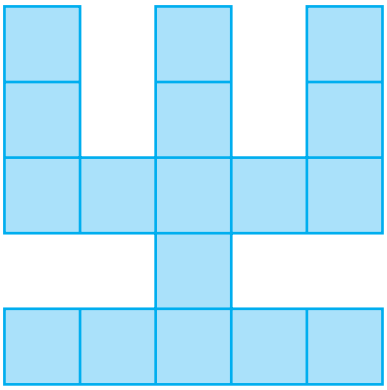

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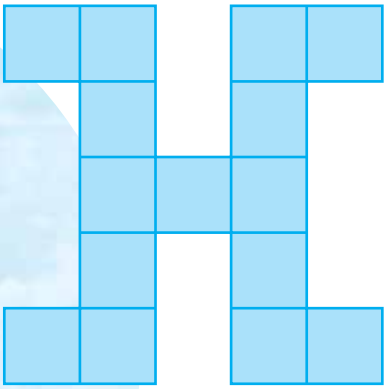

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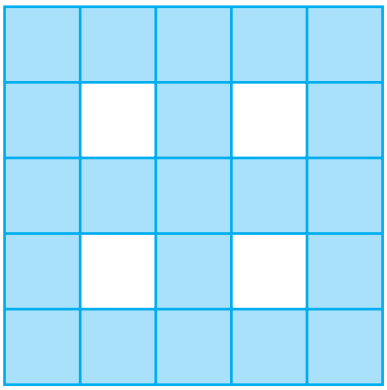
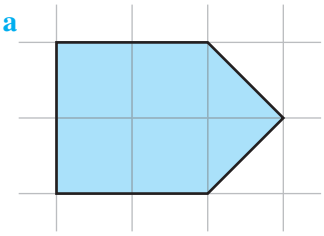


Consolidation

3 Find the area of each figure, in square centimetres.

a



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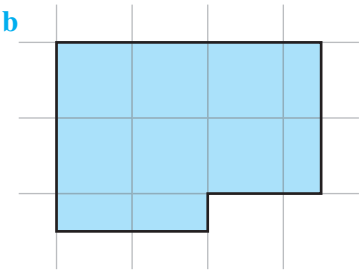

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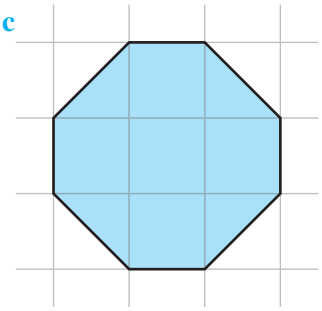

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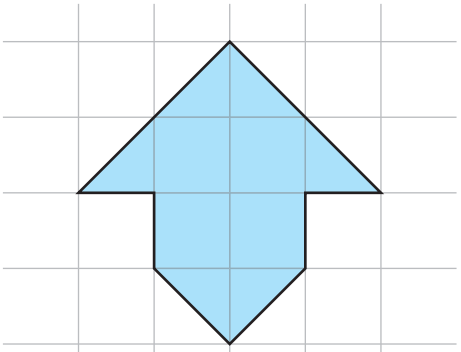

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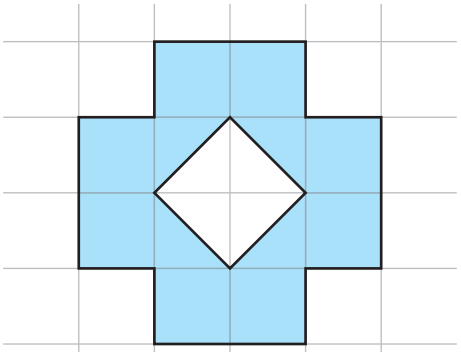

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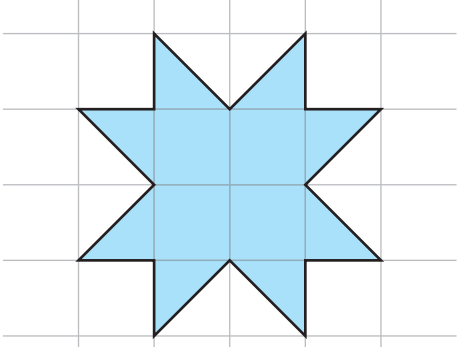

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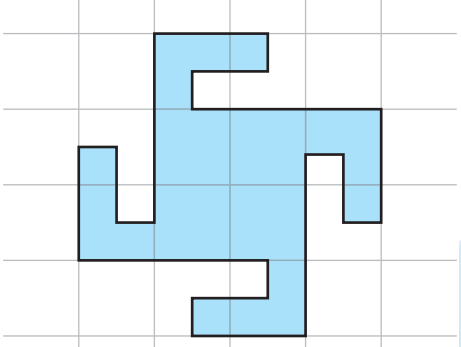

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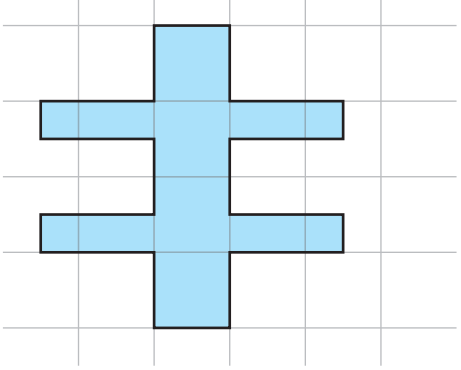
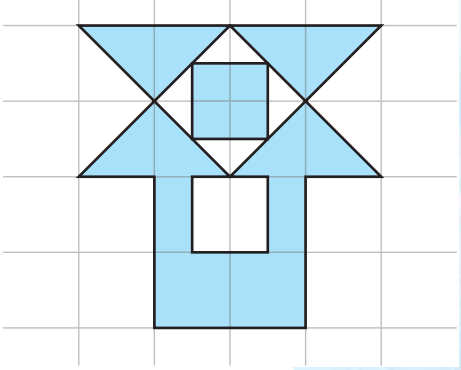


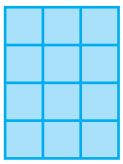
i



4 **i** Find the perimeter and area of each figure.

ii Do figures with the same area always have the same perimeter?

a



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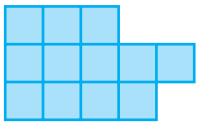

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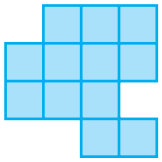
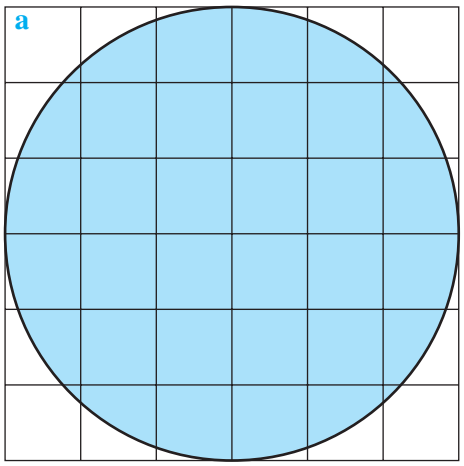
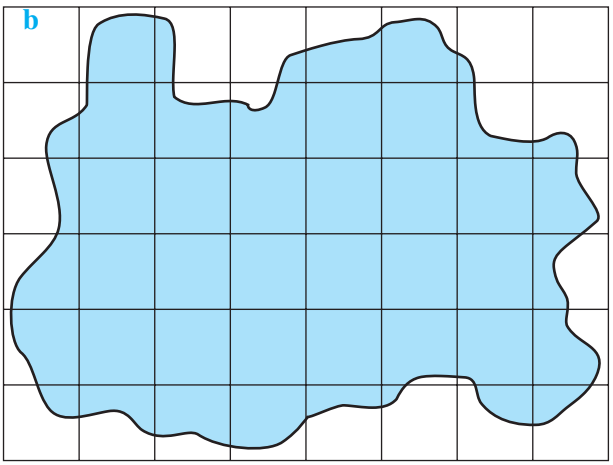


5 The approximate area of a figure can be determined by covering the figure with a 1 cm grid and counting only those squares that are at least half coloured. What is the approximate area of each of the following?

a



b



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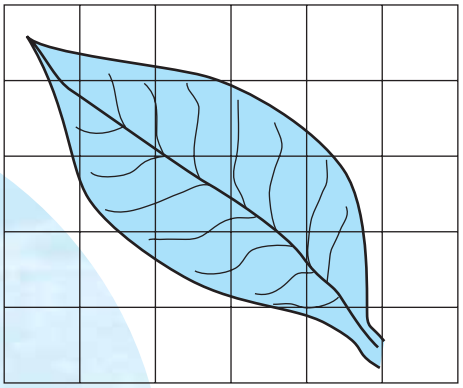

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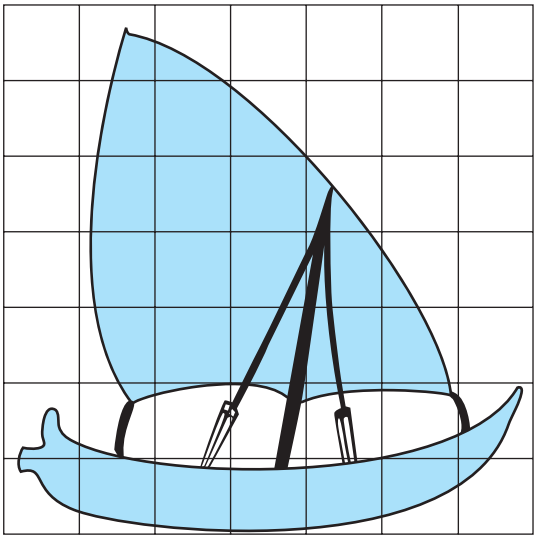


■ Further applications

6 Draw a diagram on 1 cm grid paper that has:

a an area of 15 cm^2 and a perimeter of 16 cm

b an area of 11 cm^2 and a perimeter of 24 cm

7 Trace your hand onto a 1 cm grid sheet and estimate its area.

13.2 Areas of rectangles and squares

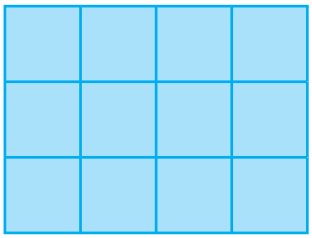
Area of a rectangle

To find the area of a rectangle, we can multiply the number of squares along the length by the number of squares along the breadth.

The area of a rectangle with length l units and breadth b units is given by the formula:

$$A = l \times b$$

For example,



$$\begin{aligned} A &= l \times b \\ &= 4 \times 3 \\ &= 12 \\ \therefore A &= 12 \text{ cm}^2 \end{aligned}$$

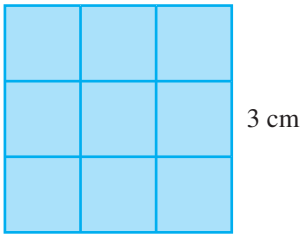
Area of a square

To find the area of a square, we can multiply the number of squares along the length by the number of squares along the breadth. That is, we could *square* the number of squares along one side.

The area of a square of side s units is given by the formula:

$$A = s^2$$

For example,



$$\begin{aligned} A &= s^2 \\ &= 3^2 \\ &= 9 \\ \therefore A &= 9 \text{ cm}^2 \end{aligned}$$

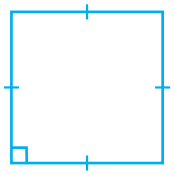
Example 1

Find the value of the pronumeral in each of these.

a

Area = 63 cm^2

b



Area = 64 mm^2

Solutions

a

$A = l \times b$
$63 = 9 \times k$
$k = 63 \div 9$
$\therefore k = 7$

b

$A = s^2$
$64 = p^2$
$p = \sqrt{64}$
$\therefore p = 8$

Example 2

a Find the area of a square whose perimeter is 24 cm.

b Find the perimeter of a square whose area is 49 m^2.

Solutions

a **i**

Side length = perimeter ÷ 4
= 24 ÷ 4
= 6
∴ Side length = 6 cm

ii

Area = s^2
= 6^2
= 36
∴ Area = 36 cm^2

b **i**

Side length = $\sqrt{\text{area}}$
= $\sqrt{49}$
= 7
∴ Side length = 7 m

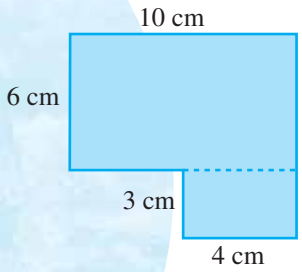
ii

Perimeter = 4 × side length
= 4 × 7
= 28
∴ Perimeter = 28 m

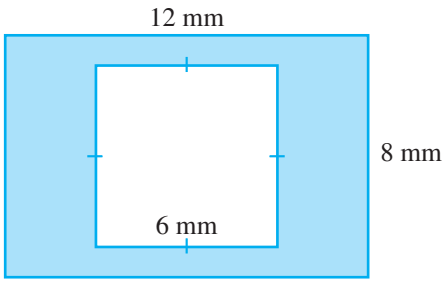
Example 3

Find the shaded area in each of these.

a



b



Solutions

a **i** $A_1 = 6 \times 10$
$= 60$
$\therefore A_1 = 60 \text{ cm}^2$

ii $A_2 = 3 \times 4$
$= 12$
$\therefore A_2 = 12 \text{ cm}^2$

iii $A = A_1 + A_2$
$= 60 + 12$
$\therefore A = 72 \text{ cm}^2$

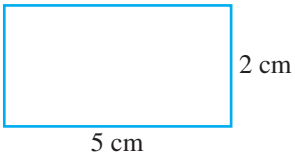
b **i** $A_1 = 12 \times 8$
$= 96$
$\therefore A_1 = 96 \text{ mm}^2$

ii $A_2 = 6^2$
$= 36$
$\therefore A_2 = 36 \text{ mm}^2$

iii $A = A_1 - A_2$
$= 96 - 36$
$\therefore A = 60 \text{ mm}^2$

Exercise 13.2

1 Find the area of each rectangle.

a


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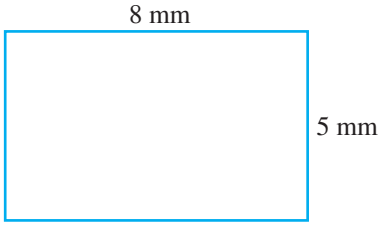
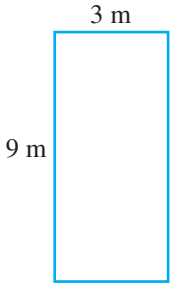
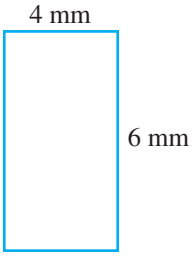
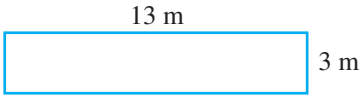


c


d


e


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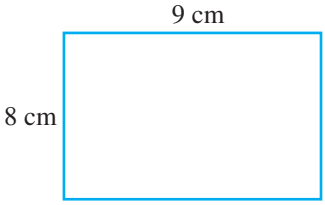


2 Find the area of a square with side length:
a 3 cm **b** 5 mm **c** 11 m

3 Calculate the area of a rectangle whose:
a length is 6 cm and breadth is 7 cm **b** length is 8 m and width is 6 m
c width is 5 mm and length is 12 mm

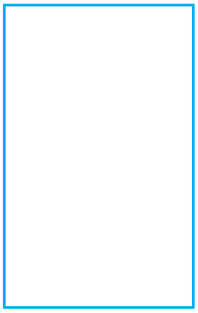
Consolidation

4 Measure the dimensions of each rectangle, then find its area.

a

b

c



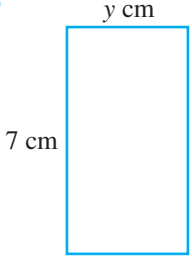
d

5 Find the value of the pronumeral in each of these.

a

Area = 30 cm^2

b



Area = 28 cm^2

c

Area = 96 cm^2

6 **a** Find the length of a rectangle whose width is 5 mm and area is 45 mm^2.
b Find the width of a rectangle whose area is 66 m^2 and length is 11 m.
c Find the breadth of a rectangle whose length is 8 cm and area is 56 cm^2.

7 Find the side length of a square whose area is:
a 9 mm^2 **b** 16 mm^2 **c** 36 mm^2
d 49 mm^2 **e** 81 mm^2 **f** 144 mm^2

8 Find the area of a square whose perimeter is:
a 12 cm **b** 44 cm **c** 52 cm

9 Find the perimeter of a square whose area is:
a 25 m^2 **b** 64 m^2 **c** 100 m^2

10 **a** Find the perimeter and area of a rectangle with dimensions:
i 6 cm × 4 cm **ii** 8 cm × 3 cm **iii** 12 cm × 2 cm
b Do rectangles with the same area necessarily have the same perimeter?

11 **a** Find the perimeter and area of a rectangle with dimensions:
i 11 cm × 4 cm **ii** 9 cm × 6 cm **iii** 8 cm × 7 cm
b Do rectangles with the same perimeter necessarily have the same area?

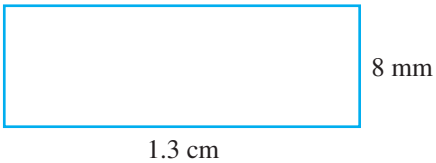
12 Find the area of a rectangle whose:
a perimeter is 18 cm and width is 5 cm **b** perimeter is 28 cm and length is 8 cm
c perimeter is 39 cm and breadth is 7 cm **d** perimeter is 47 cm and width is 3.5 cm

13 Find the perimeter of a rectangle whose:

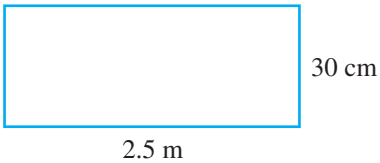
a area is 36 cm^2 and width is 3 cm
b area is 63 cm^2 and length is 9 cm
c area is 84 cm^2 and breadth is 8 cm
d area is 20 cm^2 and width is 2.5 cm

14 Calculate the area of each rectangle.

a

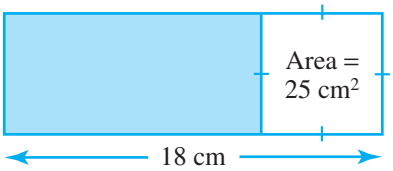


b

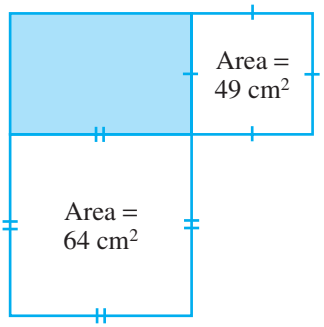


15 Find the shaded area in each figure.

a

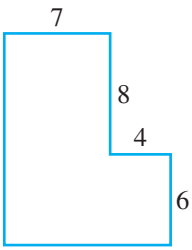


b



16 Find the total area of each figure by addition of areas. (All angles are right angles and all measurements are in metres.)

a



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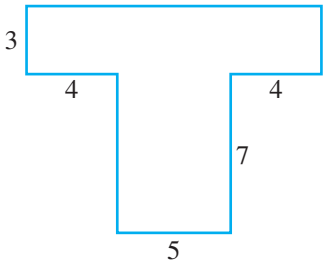
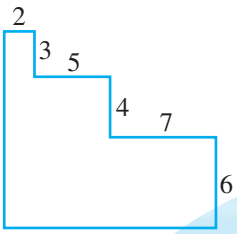


c



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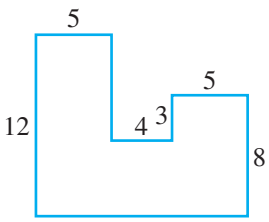
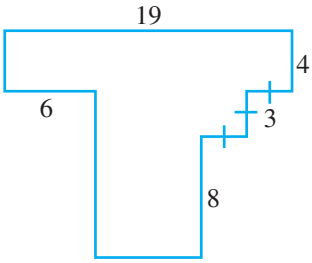


e



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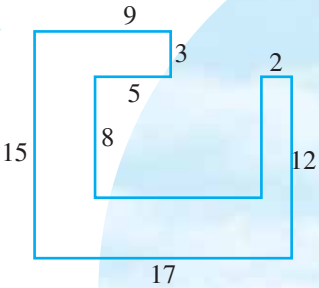
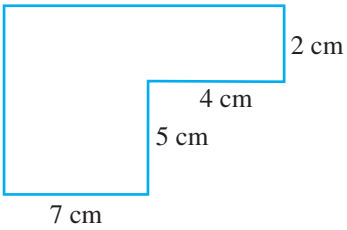
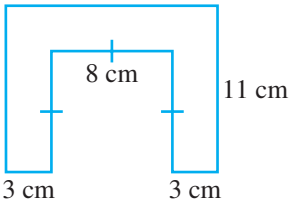


17 Find the area of each figure by subtraction of areas.

a



b



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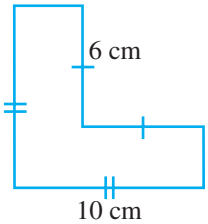
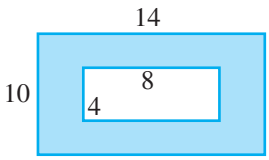
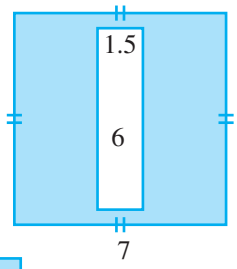


18 Find the shaded area in each figure. (All angles are right angles and all measurements are in mm.)

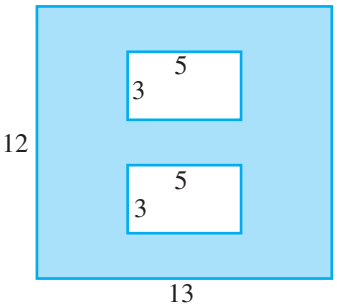
a



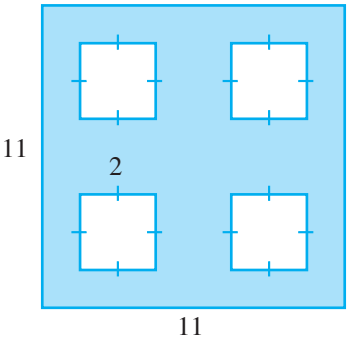
b



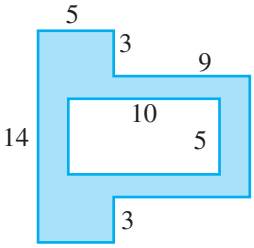
c



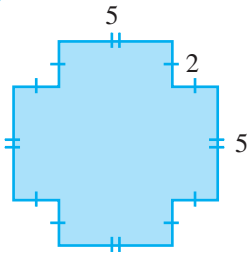
d



e



f



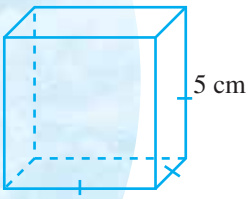
■ Further applications

19 Find, by trial and error, the dimensions of the rectangle whose:

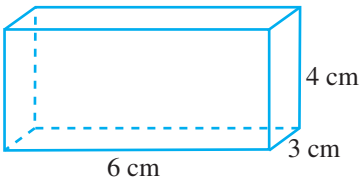
a perimeter is 28 cm and area is 33 cm^2
b perimeter is 26 cm and area is 42 cm^2
c perimeter is 34 cm and area is 72 cm^2
d perimeter is 40 cm and area is 91 cm^2

20 The total area of all faces of a solid is called the **surface area**. Find the total surface area of each solid.

a



b



21 A cube has a surface area of 726 cm^2. How long are the edges of the cube?

TRY THIS

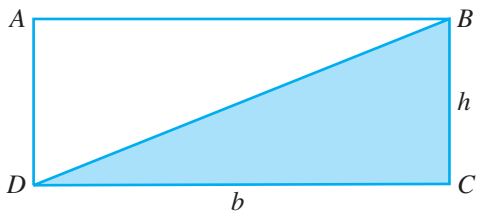
How many people are there in your classroom?

If there wasn't any furniture in your classroom, how many people could stand inside it? Estimate this figure mentally! Now how could you actually calculate an answer?

13.3 Area of a triangle

The area of a triangle is related to the area of a rectangle as shown below.

1 Right-angled triangles



The shaded area is equal to the unshaded area.

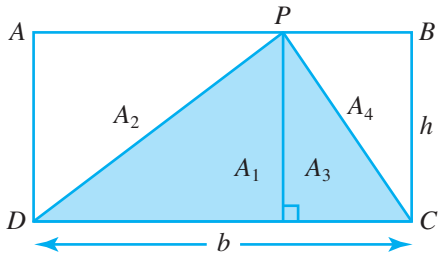
$\therefore$ Area of ΔBCD

$$= \frac{1}{2} \text{ area of rectangle } ABCD$$

$$= \frac{1}{2} \times (b \times h)$$

$$= \frac{1}{2}bh$$

2 Acute-angled triangles



Now $A_1 = A_2$ and $A_3 = A_4$

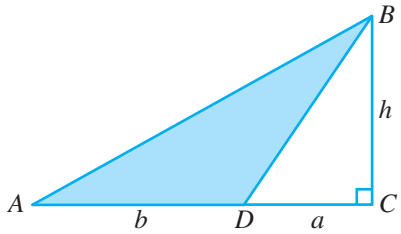
$\therefore A_1 + A_3 = A_2 + A_4$

$\therefore$ Area of ΔDPC

$$= \frac{1}{2} \text{ area of rectangle } ABCD$$

$$= \frac{1}{2}bh$$

3 Obtuse-angled triangles



Area of ΔABD = area of ΔABC – area of ΔDBC

$$= \frac{1}{2}(a+b)h - \frac{1}{2}ah$$

$$= \frac{1}{2}h(a+b) - \frac{1}{2}ah$$

$$= \frac{1}{2}ah + \frac{1}{2}bh - \frac{1}{2}ah$$

$$= \frac{1}{2}bh$$

From these three triangles we can see that the same formula can be used to find the area of a right-angled triangle, an acute-angled triangle or an obtuse-angled triangle. In each case, the area of the triangle is half the area of the surrounding rectangle.

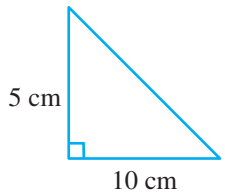
The area of a triangle with base b units and height h units is given by the formula:

$$A = \frac{1}{2}bh$$

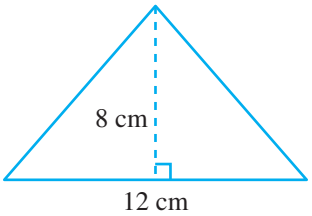
Example 1

Find the area of each triangle.

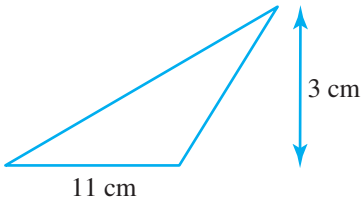
a



b



c



Solutions

a
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 10 \times 5$$
$$= 25$$
$$\therefore A = 25 \text{ cm}^2$$

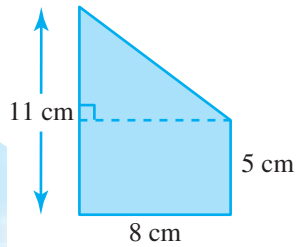
b
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 12 \times 8$$
$$= 48$$
$$\therefore A = 48 \text{ cm}^2$$

c
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 11 \times 3$$
$$= 16.5$$
$$\therefore A = 16.5 \text{ cm}^2$$

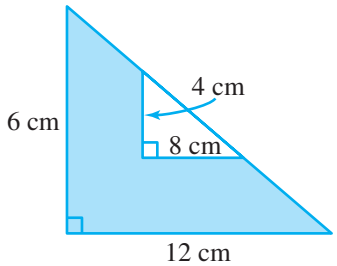
Example 2

Find the shaded area in each of these.

a



b



Solutions

a **i**
$$A_1 = 8 \times 5$$
$$= 40$$
$$\therefore A_1 = 40 \text{ cm}^2$$

ii
$$A_2 = \frac{1}{2} \times 8 \times 6$$
$$= 24$$
$$\therefore A_2 = 24 \text{ cm}^2$$

iii
$$A = A_1 + A_2$$
$$= 40 + 24$$
$$= 64$$
$$\therefore A = 64 \text{ cm}^2$$

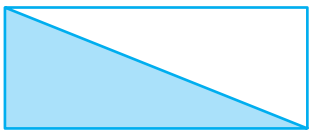
b **i**
$$A_1 = \frac{1}{2} \times 12 \times 6$$
$$= 36$$
$$\therefore A_1 = 36 \text{ cm}^2$$

ii
$$A_2 = \frac{1}{2} \times 8 \times 4$$
$$= 16$$
$$\therefore A_2 = 16 \text{ cm}^2$$

iii
$$A = A_1 - A_2$$
$$= 36 - 16$$
$$= 20$$
$$\therefore A = 20 \text{ cm}^2$$

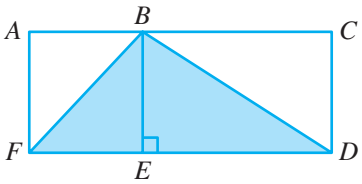
Exercise 13.3

1



a What fraction of this rectangle has been shaded?
b If the area of the rectangle is 12 cm^2, what is the area of the shaded triangle?
c If the area of the shaded triangle is 8 cm^2, what is the area of the rectangle?

2

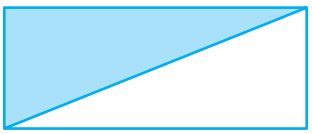


What fraction of each of the following rectangles have been shaded?

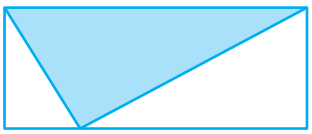
a *ABEF* **b** *BCDE* **c** *ACDF*

3 In which of these figures is the area of the shaded triangle half the area of the rectangle?

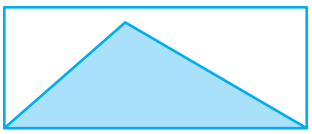
a



b



c



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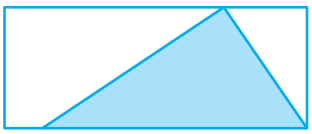
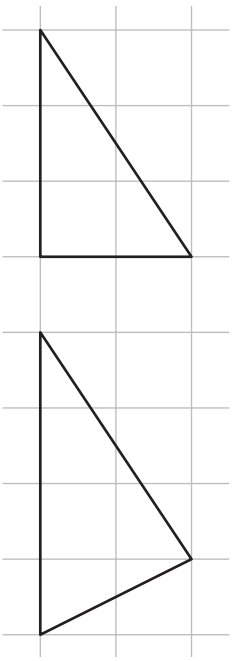


4 Find the area of each of these triangles that have been drawn on 1 cm grid paper.

a



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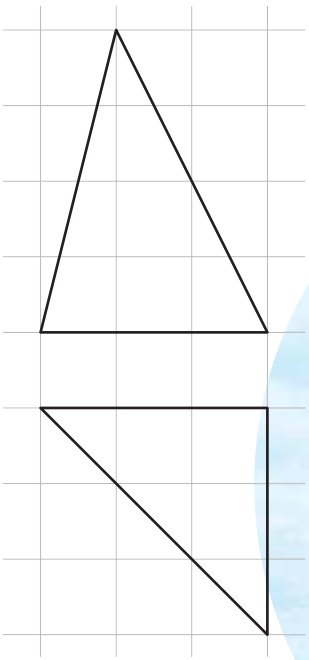
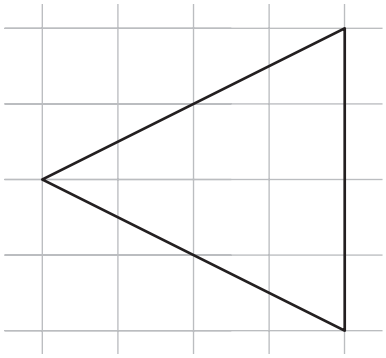


c

d

e



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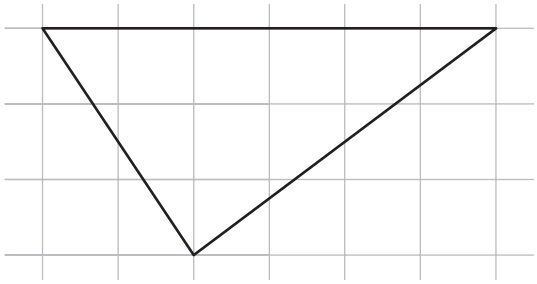
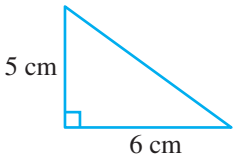


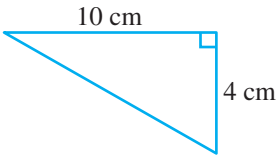
■ Consolidation

5 Find the area of each triangle.

a



b



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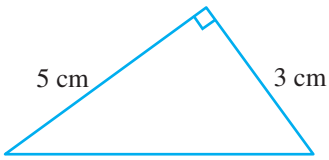

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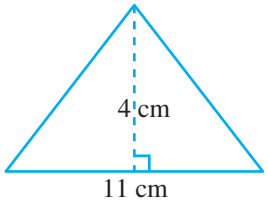

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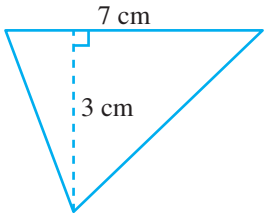

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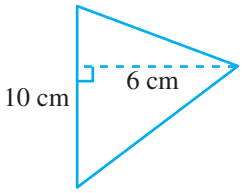

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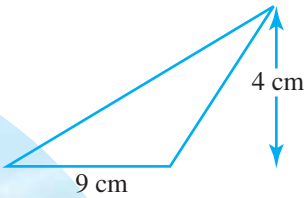

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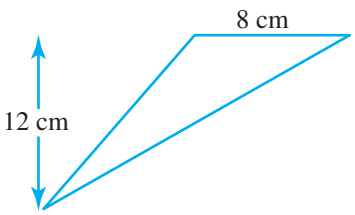
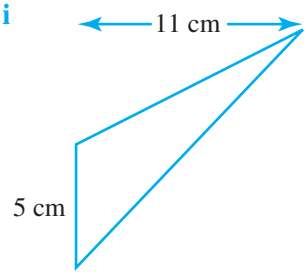


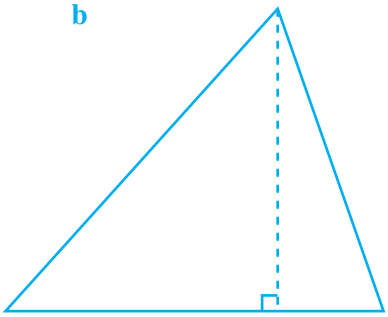
i



6 Measure the length of the base and height of each triangle with a ruler, then find the area.

a

b



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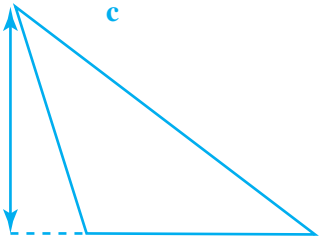


7 Calculate the area of a triangle with:

a a base of 11 m and a height of 16 m

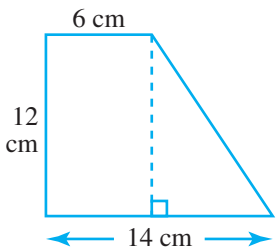
b a base of 9 mm and an altitude of 8 mm

c a base of 15 cm and a height of 7 cm

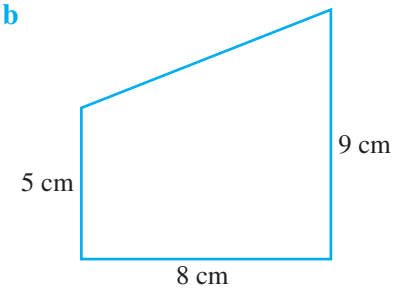
d perpendicular sides of length 13 cm and 5 cm

8 Determine the area of each figure.

a



b



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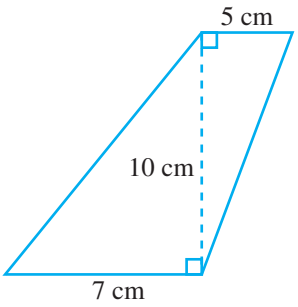

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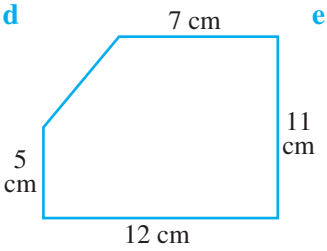

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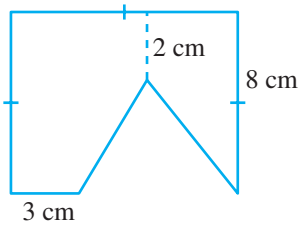

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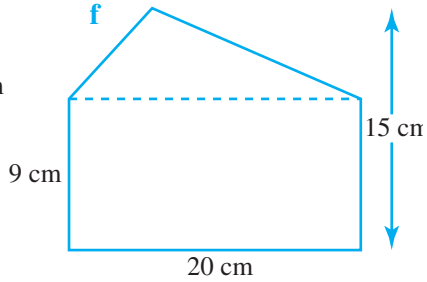

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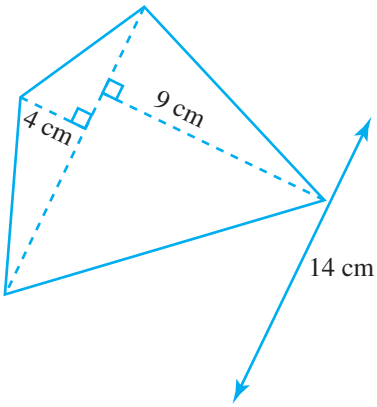

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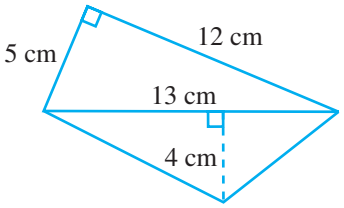

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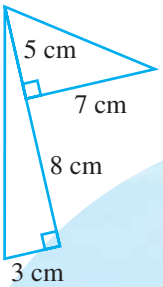
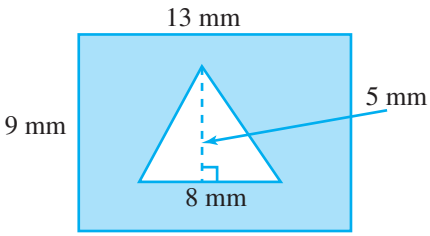
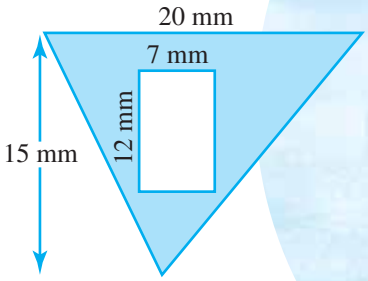


9 Find the shaded area in each of these.

a



b



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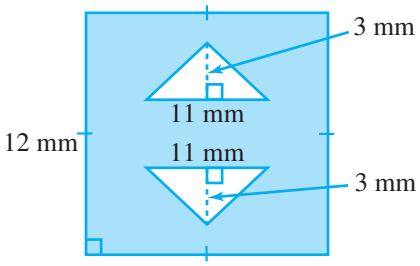

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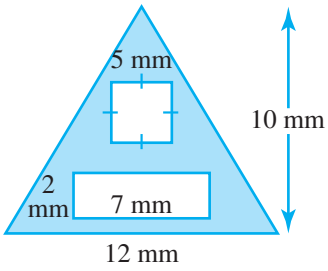
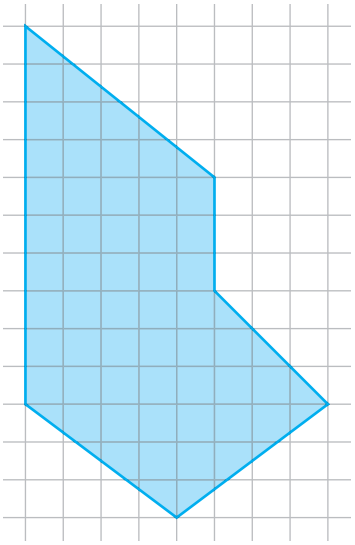
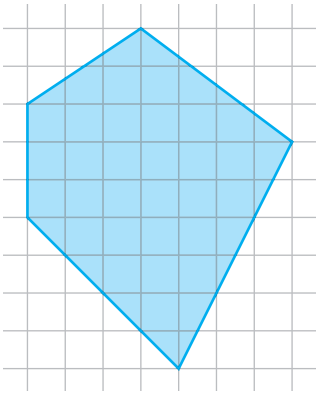


10 Divide these figures into squares, rectangles and triangles, then find the area of each figure in square units.

a



b



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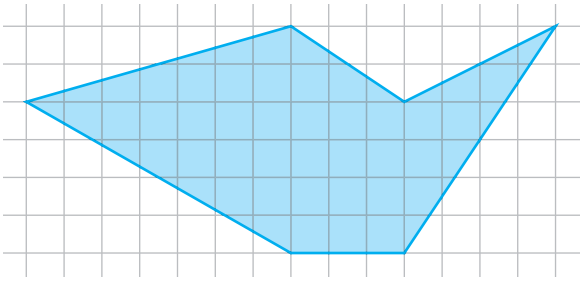
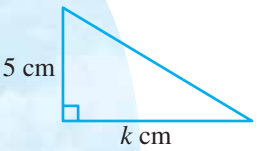


■ Further applications

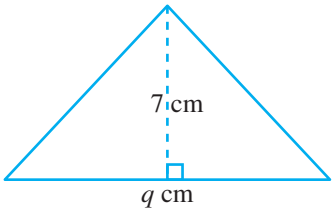
11 Find the value of the pronumeral in each triangle. (HINT: Consider each triangle as half a rectangle.)

a



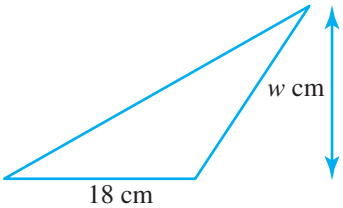
Area = 20 cm^2

b



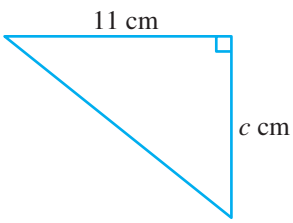
Area = 21 cm^2

c



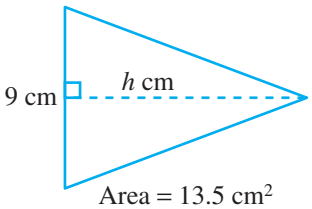
Area = 63 cm^2

d



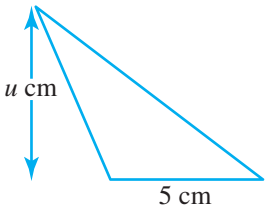
Area = 16.5 cm^2

e



Area = 13.5 cm^2

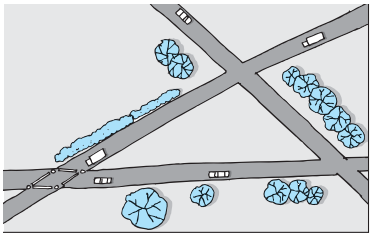
f



Area = 22.5 cm^2

12 Find the area of a rhombus with diagonals of length 16 cm and 12 cm by first dividing the figure into two triangles.

TRY THIS **Building blocks**



Due to poor planning, a block of land is surrounded by three roads.

A land developer decides to divide this land into three blocks. Each block is to have the same land area. How would you do this?

Can you find more than one way of doing it?

13.4 Solving area problems

EG+S

Example

Cork tiles measuring 20 cm × 10 cm are to be laid covering the floor of a kitchen that is 4 m long and 3 m wide.

a Find, in cm^2, the area of the kitchen.

b Find the number of cork tiles that are needed to cover the floor.

c Find the cost of the tiles if they are sold in packets of 200 at \$90 per packet.

Solutions

a Area of the kitchen = 4 m × 3 m
= 400 cm × 300 cm
= 120 000 cm^2

b **i** Area of each tile = 20 cm × 10 cm
= 200 cm^2

ii Number of tiles = area of kitchen ÷ area of each tile
= 120 000 ÷ 200
= 600

∴ 600 tiles are needed to cover the floor.

c **i** Number of packets of tiles needed = 600 ÷ 200
= 3
∴ 3 packets of tiles are needed.

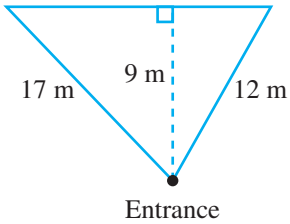
ii Cost = \$90 × 3
= \$270

Exercise 13.4

1 A ceiling measuring 4 m by 7 m is to be repainted. Each tin of paint will cover an area of 14 m^2.
a What is the area of the ceiling? **b** How many tins of paint are required?
c Find the cost of painting the ceiling if paint costs \$8 per tin.

2 Luke wants to wallpaper two walls each measuring 2.6 m high and 5 m wide. One of the walls includes a doorway 1.2 m wide and 2 m high.
a Find the total area that is to be wallpapered.
b Each roll of wallpaper will cover an area of 7 m^2. How many rolls must Luke buy?

3 A small park is triangular in shape and has a perimeter of 51 m. Two sides of the park have lengths 17 m and 12 m respectively, intersecting at the main entrance, which is 9 m from the third side of the park.
a Find the length of the third side.
b Find the area of the park.



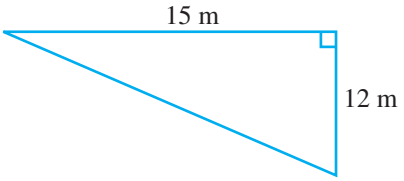
4 A sheep paddock on a farm is in the shape of a right-angled triangle. The perpendicular sides of the paddock measure 40 m and 30 m respectively. Each sheep in the paddock needs a minimum grazing area of 20 m^2.
a Find the area of the paddock.
b What is the maximum number of sheep that the paddock can hold?

Consolidation

5 A square shaped backyard of width 12 m is to be returfed. The grass is sold in rolled rectangular strips measuring 50 cm by 4 m and each strip of turf costs \$10.
a Calculate the area of the yard.
b Find the area of lawn that can be covered by each strip of turf.
c Determine the number of strips of new turf that are required to cover the yard.
d Calculate the cost of returfing the yard.

6 The floor of a laundry 2 m long and 150 cm wide is to be tiled with square tiles of width 10 cm.
a What is the area of the laundry floor in cm^2?
b How many tiles are needed to cover the laundry floor?

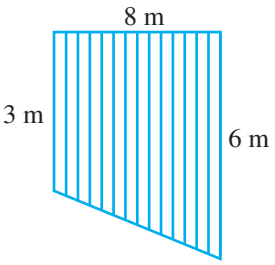
7 Olivia's garden is triangular in shape, with the dimensions shown in the diagram. A lawn mowing service charges \$0.20 per square metre plus a surcharge of \$3.50 on Sundays. How much will Olivia pay to have her lawn mowed on a Sunday?



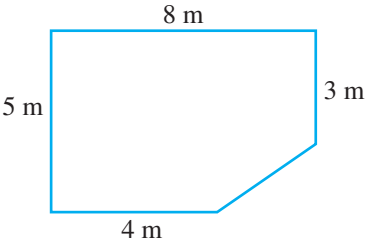
8 Gary wants to build a timber deck in the shape of a trapezium at the back of his house.

a Find the area of the deck.

b Calculate the cost of building the deck if the deck timber costs \$40 per square metre.

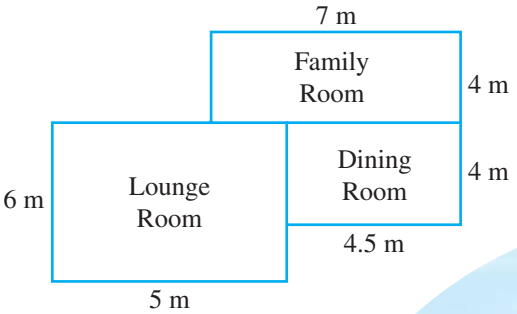


9 The games room shown is to be tiled with slate tiles. Calculate the cost of tiling the room at \$25/m^2.



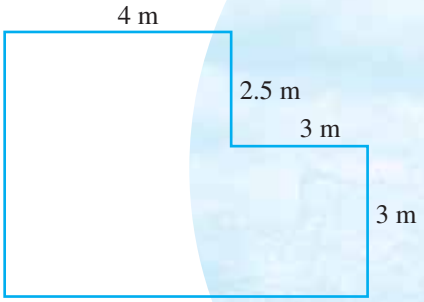
10 The cost of having carpet steam cleaned is advertised as follows:

\$23 per room
or
\$1 per square metre

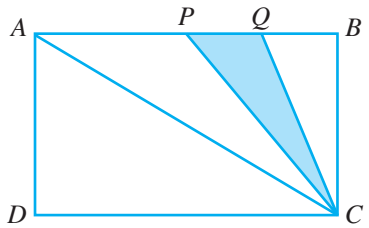


Freda wants to have the carpet cleaned in these three rooms. Which payment option should she choose?

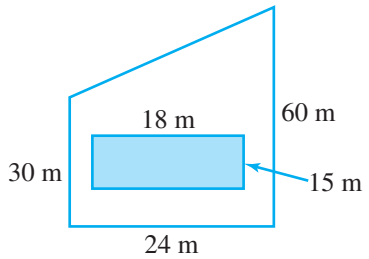
11 The L-shaped lounge/dining room shown is to be carpeted. Perry has to decide between two different carpets. The more expensive carpet costs \$36 per square metre, while the cheaper carpet costs \$21 per square metre. How much would Perry save if he chose the cheaper carpet?



12 What fraction of this rectangle is shaded, given that $AP = PB$ and $PQ = QB$.



13 Julie owns the block of land shown and wants to build a house measuring 18 m by 15 m. The council will not permit a house to be built that occupies more than one-quarter of the area of the block. Will Julie be able to build her house?



14 A stained glass window measuring 180 cm by 350 cm is to be fitted into the wall in a church.

a Find the area of the window in m^2.

b If stained glass costs \$200/m^2 and labour costs amount to \$135, find the total cost of fitting the window.

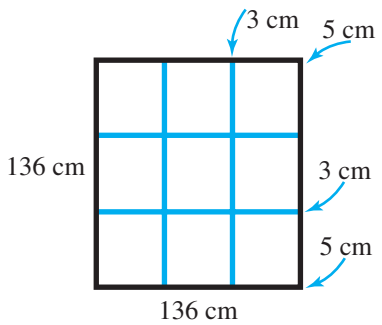
15 A front porch measuring 3 m by 2 m is to be covered with tiles measuring 10 cm by 5 cm. The tiles can be purchased in packets of 100 for \$15.

a Find in cm^2, the area of the porch.

b How many tiles are needed to cover the area?

c Calculate the cost of the tiles.

16 A square window of length 136 cm contains 9 panes of glass as shown. If there is a 3 cm wide aluminium support between each pane and a 5 cm wide aluminium frame surrounding the panes, find the total area of the glass.

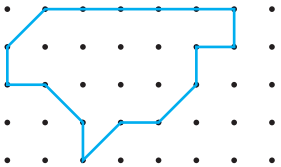


■ Further applications

The area of a polygon that has been drawn in a lattice is given by the formula $A = \frac{1}{2}P + I - 1$, where A is the number of square units in the area, P is the number of lattice points that lie on the perimeter and I is the number of lattice points that lie inside the figure. This formula is known as Pick's rule.

For example, in the diagram shown:

- $P = 16$, since there are 16 points on the perimeter
- $I = 7$, since there are 7 points inside the figure

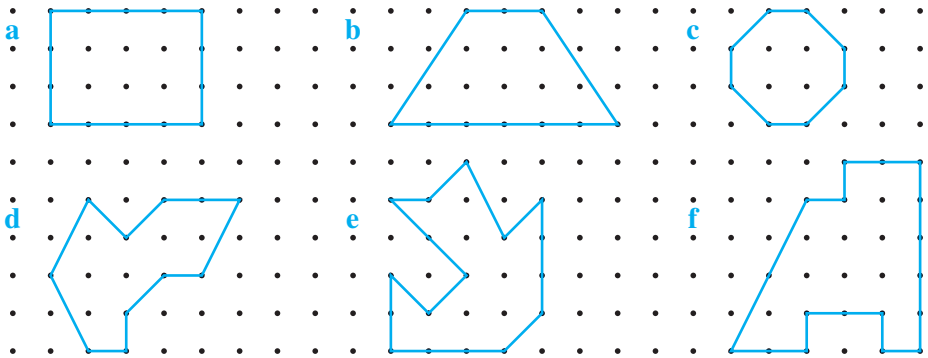


$$A = \frac{1}{2}P + I - 1$$

$$= \left(\frac{1}{2} \times 16\right) + 7 - 1$$

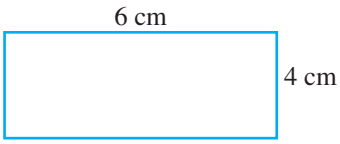
$\therefore A = 14$ units^2

17 Find the area of each figure below using Pick's Rule.



13.5 Large areas and area conversions

Consider a rectangle with length 6 cm and width 4 cm.



Area = 6 cm × 4 cm
= 24 cm^2

or

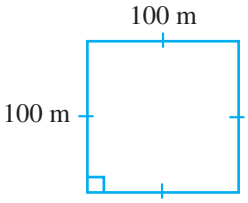
Area = 6 cm × 4 cm
= 6 × 10 mm × 4 × 10 mm
= (6 × 4 × 10^2) mm^2
= 2400 mm^2

From this example, we see that the **conversion factor** between cm and mm (i.e. 10) is a factor in both the length and the width of the rectangle. To convert the area from cm^2 to mm^2 we could have calculated the area in cm^2 then multiplied it by the square of the conversion factor.

That is, 24 cm^2 = (6 × 4 × 10^2) mm^2
= 2400 mm^2

While units such as mm^2, cm^2 and m^2 are sufficient to describe small areas, other units are needed to describe very large areas. In these situations the preferred units of area are the **hectare** (ha) and the **square kilometre** (km^2).

A hectare is an area of 10 000 m^2. To get a proper perspective of the size of a hectare, imagine an area that is 100 m long and 100 m wide. It is approximately equal to the area of two football fields placed side by side. Hectares are used to measure areas such as farms, large urban parks or housing estates.



A square kilometre is an area equal to that of a square 1 km long and 1 km wide. That is, 1 km^2 = 1 000 000 m^2. Square kilometres are used to measure areas such as national parks, states or countries.

To convert to a smaller unit of area:
- **multiply by the square of the conversion factor.**

To convert to a larger unit of area:
- **divide by the square of the conversion factor.**

Example 1

Convert:

a 5 m^2 to cm^2

b 35 000 cm^2 to m^2

Solutions

a $5\ m^2 = (5 \times 100^2)\ cm^2$
$= 50\,000\ cm^2$

b $35\,000\ cm^2 = (35\,000 \div 100^2)\ m^2$
$= 3.5\ m^2$

Example 2

Express:

a 3 ha in m^2

b 42 600 m^2 in ha

Solutions

a $3\ ha = (3 \times 10\,000)\ m^2$
$= 30\,000\ m^2$

b $42\,600\ m^2 = (42\,600 \div 10\,000)\ ha$
$= 4.26\ ha$

Example 3

A national park has an area of 28 km^2. Express this area in hectares.

Solution

i $2.8\ km^2 = (2.8 \times 1000^2)\ m^2$
$= 2\,800\,000\ m^2$

ii $2\,800\,000\ m^2 = (2\,800\,000 \div 10\,000)\ ha$
$= 280\ ha$

Exercise 13.5

1 Convert each of the following.

a 3 cm^2 to mm^2
b 7 m^2 to cm^2
c 9 km^2 to m^2
d 40 000 cm^2 to m^2
e 800 mm^2 to cm^2
f 6 000 000 m^2 to km^2
g 1.8 m^2 to cm^2
h 5.64 km^2 to m^2
i 490 mm^2 to cm^2
j 375 000 m^2 to km^2
k 4670 cm^2 to m^2
l 0.5 cm^2 to mm^2
m 0.9 km^2 to m^2
n 7 mm^2 to cm^2
o 12.95 m^2 to cm^2
p 0.004 cm^2 to mm^2
q 251 000 m^2 to km^2
r 5.3 cm^2 to m^2

2 Complete these area conversions.

a 2 ha = _____ m^2 **b** 60 000 m^2 = _____ ha **c** 1.5 ha = _____ m^2
d 35 000 m^2 = _____ ha **e** 700 000 m^2 = _____ ha **f** 2.75 ha = _____ m^2
g 0.4 ha = _____ m^2 **h** 2000 m^2 = _____ ha **i** 38 m^2 = _____ ha
j 0.025 ha = _____ m^2 **k** 0.0008 ha = _____ m^2 **l** 3 m^2 = _____ ha

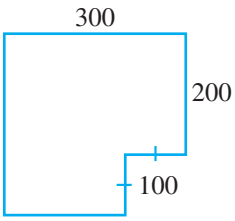
■ Consolidation

3 Choose the most appropriate unit (mm^2, cm^2, m^2, ha, km^2) that could be used to measure:

a a large park **b** a television screen **c** a lounge room
d a fingernail **e** New South Wales **f** a backyard
g an envelope **h** a five cent coin **i** Taronga Park Zoo

4 Find the area of each of these paddocks in hectares. (All angles are right angles and all dimensions are in metres.)

a



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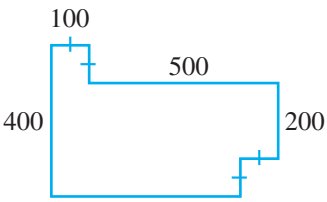

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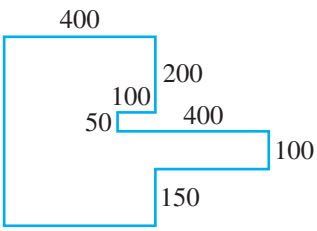


5 A rectangle has a length of 700 cm and a width of 50 cm. Find the area of the rectangle in:

a square centimetres **b** square metres

6 Find in m^2, the area of a square of side 120 cm.

7 One side of a cube has an area of 0.49 m^2. Find the side length of the cube in cm.

8 A racehorse paddock has an area of 3 ha and a length of 500 m. Find the width of the paddock.

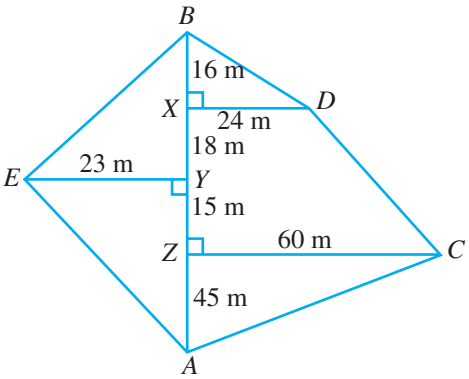
9 A cattle farmer owns 420 ha of land. If he purchases 2.5 km^2 of land from an adjacent farm, how many hectares of land will he own then?

10 A couple purchased a 0.36 ha block of land in order to build their first house. The house is to occupy one-eighth of the area of the property. How many square metres of land will remain after the house is built?

■ Further applications

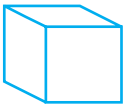
11 **a** How many hectares are equivalent to 1 square kilometre?
b The area of Australia is 768 230 000 ha. Express this area in square kilometres.

12 A surveyor drew the following field sketch for a block of land. If $BX = 16$ m, $XY = 18$ m, $YZ = 15$ m, $ZA = 45$ m, $XD = 24$ m, $EY = 23$ m and $ZC = 60$ m, find the total area of the land in hectares.



Problem solving

1 On a cube, a dot is put on every corner and also the midpoint of every edge. How many dots are there altogether?



2 If $x = 1$, what does $\frac{x}{x+99}$ equal? Now if we make x a very large number, which whole number is closest to $\frac{x}{x+99}$?

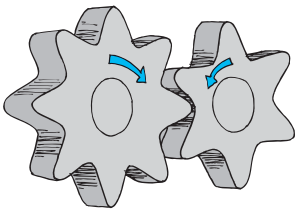
3 Water is poured into a tin so that its volume doubles every minute. If the tin is full at 11 am, at what time was it half full?

4 The council has asked the residents of 500 houses in a street to put in new house numbers. How many of the digits from 1 to 9 would be needed for this job? Tricky!

5 If a new mathematical operation $\otimes$ is given as $a \otimes b = a \times b \times ab$, what does $a \otimes b \otimes c$ equal?

6 At a Chinese restaurant, 30 people decided to have the banquet meal at a cost of \$15 each. As can often happen, 5 people left without paying. How much extra will each of the remaining people pay?

7 A machine has two cogs which intermesh. The back cog has 6 teeth and the front cog has 8 teeth. If the front cog turns 12 times, how many times will the back cog turn?



8 When making concrete, Jill mixes together 4 parts of screenings (small stones), 2 parts of sand, and 1 part of cement. If she uses 8 bags of sand, how many bags of screenings and of cement does she use?

9 Suppose you have 9 coins, one of which weighs less than the others, and a balance scale. How do you find the underweight coin in only 2 weighings?

10 Arrange the numbers 1 to 8 so that consecutive integers are not next to each other, vertically, horizontally or diagonally: i.e. not $\begin{array}{|c|c|}\hline 2 & 3 \\ \hline \end{array}$, $\begin{array}{|c|}\hline 2 \\ \hline 3 \\ \hline \end{array}$, or $\begin{array}{|c|c}\hline 2 & \\ \hline & \multicolumn{1}{|c|}{3} \\ \hline \end{array}$.

Focus on working mathematically

Goal! The World Cup 2002

Introduction

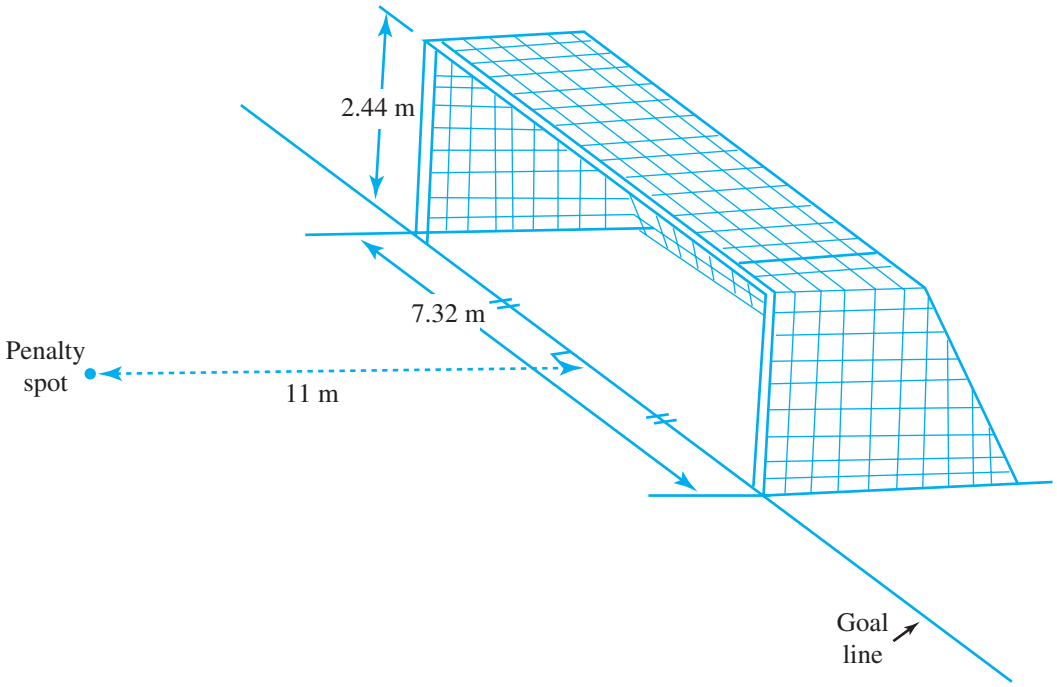
The 2002 World Cup international football competition held in South Korea and Japan attracted a television audience of millions of people. The final, played between Germany and Brazil, was won by Brazil. The game is played on a rectangular field. There is also a very important rectangular area called the penalty area in front of the goal. If a player on the defending team commits a serious offence inside this area, the referee can award a penalty kick to the attacking team. This kick is taken from the penalty spot, marked clearly on the ground in front of the goal. The defending goalie has a tough task to save the goal because the penalty spot is only 11 metres away, directly in front of the goal.

The goals are rectangles too. In this activity we will look at the area of the goal mouth and the angles that players must take into account in shooting for a penalty goal.

LEARNING ACTIVITIES

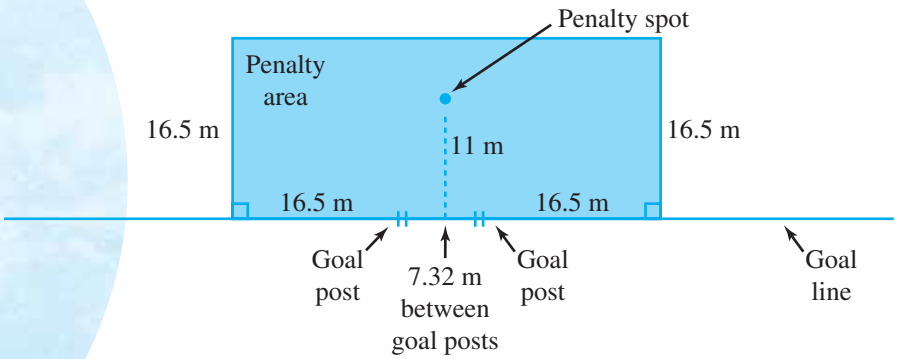
Materials: Paper and pencil, geometrical instruments, calculator, graph paper, and internet access if possible.

1 The goal mouth is a rectangle 7.32 m long and 2.44 m high. Calculate the area of the goal mouth. The diagram (not drawn to scale) also shows the penalty spot, 11 m from the midpoint of the goal mouth and perpendicular to it.



Look around your classroom to see if you can locate an area about the same size. This will give you a 'feel' for the actual size of the goal mouth.

2 In the diagram (not drawn to scale) the rectangular penalty area in front of the goal is shown. Calculate its area.



3 On graph paper, using a scale of 1 cm = 1 m (or choose your own scale to suit your page) draw a diagram to show a side view of the goal and the penalty spot.
When the ball is kicked from the penalty spot, measure the maximum vertical angle for the ball to go into the net.

4 Using the same scale, draw a plan view of the goal and the penalty spot.
When the ball is kicked from the penalty spot, measure the maximum angle to the left or right for the ball to go into the net.

5 Write a single statement regarding the angle at which the ball must be kicked in order that a goal could be scored from the penalty spot, assuming that the goalkeeper misses the ball.

Extension activities

1 The length of a football field is 100 m and breadth 64 m. Write down the area of the field.

2 Use your data for the area of the penalty area (remember there is one at each end of the field) and your calculator to find the fraction of the field covered by the penalty areas.

3 Draw a three-dimensional diagram of the goal, the penalty spot and the angles you measured in Q3 and Q4 above. What is the most difficult kick for a goalie to defend? What is the easiest?

4 On your diagram shade the triangular area between the goal line and the lines drawn from the goal posts to the penalty spot. Find this area. What fraction of the penalty area is it?

5 If you can, find out which four teams made the semifinals of the 2002 World Cup. Which team had the least number of goals scored against them? Who was the goalie? What other information would you need to decide if the goalie on this team was the best in the world? Check out the statistics on the FIFA web site, http://fifaworldcup.yahoo.com

Let's communicate

Make a poster to show the goal size, the penalty spot and the angles within which a player must strike the ball to score a goal. Collect action shots of great saves of the World Cup and the names of the top goalies and their countries. Check the web for additional information and add it to your poster.

Reflecting

Think over the mathematical measures you would need to decide the best goalie in the World Cup for 2002. Look at the statistics collected for the World Cup at the FIFA web site. Who do you think deserves the title?

Language link with Macquarie

MACQUARIE

1 *What does the formula* ***Area = length × breadth*** *refer to?*
2 *Explain the difference between* ***area*** *and* ***perimeter****.*
3 *When do we use* ***hectares*** *to describe area?*
4 *When do we use* ***square kilometres*** *to describe area?*
5 *Read the* Macquarie Learners' Dictionary *entry for* ***area****:*

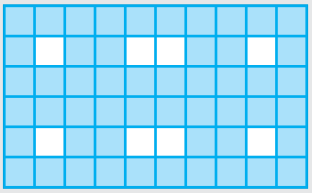
area *noun* **1.** a particular part: *a suburban area | an area of the body* **2.** the size of a flat or curved surface: *The area of the floor of this hall is 100 square metres.*

Why is an understanding of ***area*** *important for an architect or builder?*

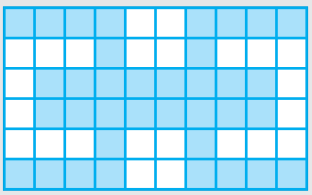
Chapter Review

1 Find the shaded area of each figure, in square units.

a

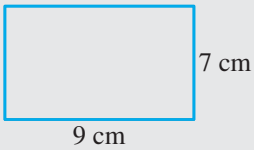


b



2 Find the area of:

a



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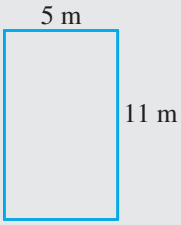

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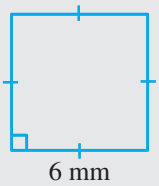

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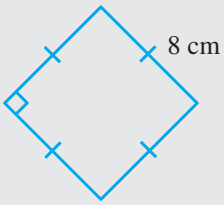
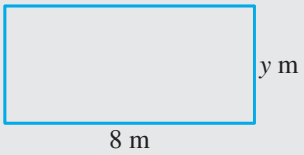


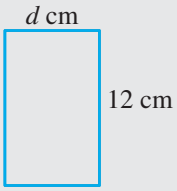
3 Find the value of the pronumeral in each of these.

a



Area = 32 m^2

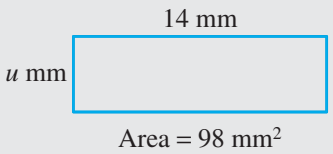
b



Area = 60 cm^2

CHAPTER REVIEW

c



4 Find the side length of a square whose area is 121 cm^2.

5 **a** Find the perimeter of a square whose area is 81 cm^2.

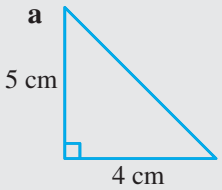
b Find the area of a square whose perimeter is 40 cm.

6 **a** Find the area of a rectangle whose perimeter is 48 cm and width is 7 cm.

b Find the perimeter of a rectangle whose area is 108 cm^2 and length is 12 cm.

7 Calculate the area of each triangle.

a



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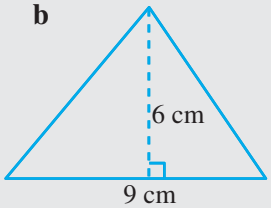

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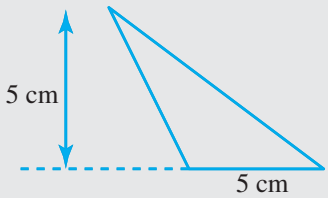


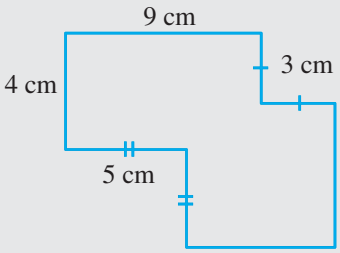
8 Find the area of a triangle with:

a a base of 14 mm and a height of 8 mm

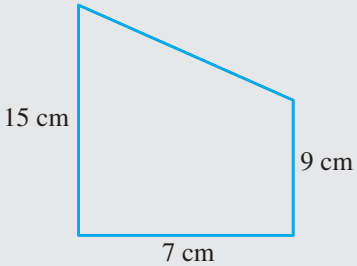
b perpendicular sides of length 16 mm and 9 mm.

9 Find the total area of each figure.

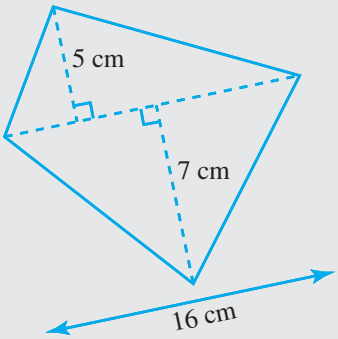
a



b

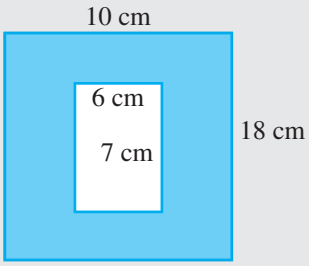


c

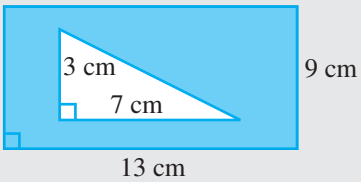


10 Find the shaded area in each of these.

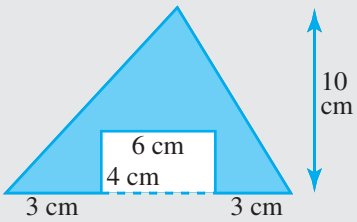
a



b



c



11 Aaliah wants to paint the walls in her bedroom. Two of the walls measure 3 m × 3.5 m and the other two walls measure 4 m × 3.5 m. The doorway is 1 m wide and 2 m high and the window measures 1 m × 1.5 m.

a Find the total area to be painted.

b How many tins of paint will be needed if one tin of paint will cover an area of 9 m^2?

c Calculate the cost of painting the walls if each tin of paint costs $14.50.

12 Trevor intends to lay new tiles on his bathroom floor. The floor is rectangular in shape and its dimensions are 2 m × 2.4 m. One third of the floor area is taken up by the bath, shower and vanity. The tiles are sold in packs of 80, with each tile measuring 5 cm × 4 cm.

a Find in cm^2, the area that is to be tiled.

b How many packets of tiles does Trevor have to buy?

c Find the cost of tiling the floor if each pack of tiles costs $24.

13 Convert:

a 4 cm^2 to mm^2

b 7 m^2 to cm^2

c 0.36 km^2 to m^2

d 65 000 cm^2 to m^2

e 200 mm^2 to cm^2

f 523 000 m^2 to km^2

14 Convert:

a 5 ha to m^2 **b** 80 000 m^2 to ha

c 3.75 ha to m^2 **d** 125 000 m^2 to ha

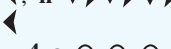
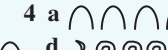
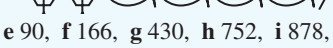
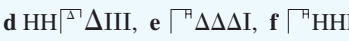
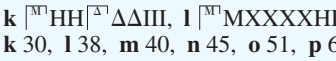
15 Find in hectares, the area of a rectangular paddock measuring 900 m by 1100 m.

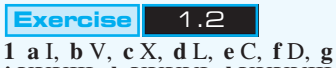
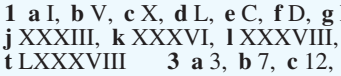
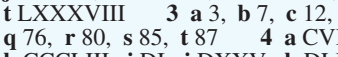
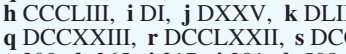
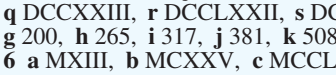
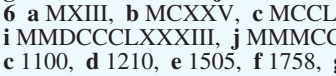
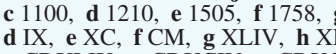
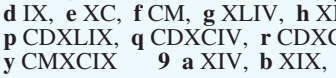
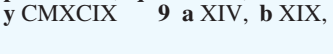
16 A state forest has an area of 140 ha. Express this area in square metres and then in square kilometres.

Answers

1 Whole numbers and number systems

Exercise 1.1

1 **a** 5, **b** 30, **c** 12, **d** 49, **e** 54, **f** 200, **g** 160, **h** 409, **i** 254 **2** **a** , **b** , **c** , **d** , **e**  , **f**  , **g** , **h** **3** **a** 6, **b** 12, **c** 25, **d** 340, **e** 2514, **f** 1247, **g** 53 432, **h** 341 064, **i** 2 400 243 **4** **a**  , **b**  , **c**  , **d** , **e** , **f** , **g** , **h** **5** **a** 4, **b** 7, **c** 13, **d** 25, **e** 90, **f** 166, **g** 430, **h** 752, **i** 878, **j** 1219, **k** 7505, **l** 80 656 **6** **a** ⌐IIII, **b** ΔΔΔΔ⌐I, **c** [Δ]ΔΔ⌐, **d** HH[Δ]ΔIII, **e** ⌐[H]ΔΔΔI, **f** ⌐[H]HHHHII, **g** XHHH[Δ]ΔΔΔ, **h** [X]ΔΔ⌐III, **i** MH[Δ]⌐II, **j** MMXXX[Δ]ΔΔΔΔ, **k** [M]HH[Δ]ΔΔIII, **l** [M]MXXXXHHH[Δ]ΔΔΔ⌐IIII **7** **a** 3, **b** 5, **c** 9, **d** 10, **e** 12, **f** 16, **g** 20, **h** 21, **i** 24, **j** 27, **k** 30, **l** 38, **m** 40, **n** 45, **o** 51, **p** 60, **q** 69, **r** 76, **s** 83, **t** 99, **u** 100, **v** 153, **w** 229, **x** 320

8 **a** , **b**  , **c**  , **d**  , **e**  , **f**  , **g**  , **h**  , **i**  , **j**  , **k** , **l** , **m** , **n** , **o** , **p** , **q** , **r**

Exercise 1.2

1 **a** I, **b** V, **c** X, **d** L, **e** C, **f** D, **g** M **2** **a** II, **b** VI, **c** VIII, **d** XI, **e** XVII, **f** XX, **g** XXV, **h** XXVIII, **i** XXXI, **j** XXXIII, **k** XXXVI, **l** XXXVIII, **m** LII, **n** LV, **o** LXIII, **p** LXVI, **q** LXX, **r** LXXV, **s** LXXXII, **t** LXXXVIII **3** **a** 3, **b** 7, **c** 12, **d** 15, **e** 18, **f** 21, **g** 26, **h** 32, **i** 37, **j** 53, **k** 56, **l** 58, **m** 60, **n** 62, **o** 67, **p** 73, **q** 76, **r** 80, **s** 85, **t** 87 **4** **a** CVII, **b** CXV, **c** CXXXVIII, **d** CLXXII, **e** CCV, **f** CCXXX, **g** CCCXXVII, **h** CCCLIII, **i** DI, **j** DXXV, **k** DLII, **l** DCXI, **m** DCXXXVIII, **n** DCLXVI, **o** DCLXXXVII, **p** DCCX, **q** DCCXXIII, **r** DCCLXXII, **s** DCCCXVI, **t** DCCCLXXXVIII **5** **a** 101, **b** 111, **c** 125, **d** 133, **e** 156, **f** 177, **g** 200, **h** 265, **i** 317, **j** 381, **k** 508, **l** 555, **m** 568, **n** 600, **o** 621, **p** 635, **q** 733, **r** 751, **s** 812, **t** 866 **6** **a** MXIII, **b** MCXXV, **c** MCCLXVI, **d** MDLXXII, **e** MDCCV, **f** MM, **g** MMXXI, **h** MMDCLXXVIII, **i** MMDCCCLXXXIII, **j** MMMCCL, **k** MMMDLXXVI, **l** MMMDCCCLXXXVIII **7** **a** 1006, **b** 1037, **c** 1100, **d** 1210, **e** 1505, **f** 1758, **g** 2072, **h** 2132, **i** 2825, **j** 3031, **k** 3216, **l** 3727 **8** **a** IV, **b** XL, **c** CD, **d** IX, **e** XC, **f** CM, **g** XLIV, **h** XLIX, **i** XCIV, **j** XCIX, **k** CDIV, **l** CDIX, **m** CDXL, **n** CDXC, **o** CDXLIV, **p** CDXLIX, **q** CDXCIV, **r** CDXCIX, **s** CMIV, **t** CMIX, **u** CMXL, **v** CMXC, **w** CMXLIV, **x** CMXLIX, **y** CMXCIX **9** **a** XIV, **b** XIX, **c** XXIV, **d** XXXIX, **e** XLIII, **f** LIX, **g** LXXXIV, **h** XCVII, **i** CXLII, **j** CXC,

k CCCXLIV, **l** CDXIX, **m** CDXCVI, **n** DCCXIV, **o** CMXXXIX, **p** MXLIX, **q** MCDXCII, **r** MMDCCXLIV, **s** MMCMXIX, **t** MMMCDXCIX **10 a** 39, **b** 54, **c** 89, **d** 96, **e** 144, **f** 390, **g** 445, **h** 492, **i** 509, **j** 784, **k** 899, **l** 919, **m** 947, **n** 998, **o** 1046, **p** 1434, **q** 1740, **r** 2099, **s** 2914, **t** 3491 **11 a** $\overline{V}$CCXV, **b** $\overline{V}$MMMCMXXIV, **c** M$\overline{V}$IX, **d** M$\overline{X}$CCLVII, **e** $\overline{X}$CDLXVI, **f** $\overline{X}$M$\overline{V}$DCCCLXXXIX, **g** $\overline{L}$CDXII, **h** $\overline{L}\overline{X}\overline{X}\overline{V}$DCLXXVI, **i** $\overline{C}\overline{C}\overline{L}\overline{X}\overline{X}\overline{V}$CCCXVIII, **j** $\overline{M}\overline{M}\overline{M}\overline{D}\overline{X}\overline{L}M\overline{X}$CMLIII **12 a** 5700, **b** 4523, **c** 7556, **d** 10 341, **e** 14 947, **f** 62 554, **g** 1 095 312, **h** 1 251 008, **i** 946 019

Exercise 1.3

1 a 1232, **b** 2124 **2 a** units, **b** tens, **c** thousands, **d** hundred thousands, **e** hundreds, **f** ten thousands, **g** millions, **h** hundreds, **i** ten thousands, **j** thousands, **k** hundred thousands, **l** millions **3 a** 70, **b** 7, **c** 700, **d** 7000, **e** 700, **f** 70 000, **g** 7000, **h** 700 000, **i** 70, **j** 7000, **k** 700 000, **l** 7 000 000 **4 a** 40, **b** 400, **c** 4, **d** 40 000, **e** 4000, **f** 4 000 000, **g** 400 000, **h** 4, **i** 4000, **j** 400, **k** 40 000, **l** 4 000 000 **5 a** 10, **b** 99, **c** 100, **d** 999, **e** 1000, **f** 9999 **6 a** 70, **b** 59, **c** 12, **d** 96, **e** 400, **f** 899, **g** 130, **h** 919, **i** 109, **j** 997 **7 a** 386, **b** 27 592, **c** 946 715, **d** 1 592 446, **e** 5290, **f** 37 010, **g** 34 768, **h** 6 050 302 **8 a** 3268, **b** 42 751, **c** 186 522, **d** 894 173, **e** 231 412, **f** 6005, **g** 93 070, **h** 117 009, **i** 501 600, **j** 300 002, **k** 4 270 611, **l** 89 053 004
9 a five thousand six hundred and thirteen, **b** twelve thousand three hundred and ninety-five, **c** eighty-four thousand three hundred and thirty-seven, **d** one hundred and sixty-five thousand nine hundred and fourteen, **e** one million three hundred and forty-three thousand five hundred and seventy-two, **f** thirty-seven million four hundred and eleven thousand nine hundred and twenty-six, **g** eight thousand and thirty, **h** twenty-one thousand and seven, **i** forty thousand and sixty-seven, **j** one hundred and two thousand and forty, **k** thirteen million six thousand and ninety, **l** three hundred and fifty million one hundred thousand and eight **10 a** 8247, 49 473, 51 759, 93 686, 216 099, 399 812, **b** 10 237, 20 020, 20 119, 138 834, 375 590, 517 933, **c** 365, 1023, 98 998, 274 998, 475 662, 1 265 784, **d** 24 489, 256 090, 800 030, 5 387 751, 9 938 775, 17 534, 811 **11 a** 100, **b** 1000, **c** 100 000, **d** 10 000 000 **12 a** 259, 295, 529, 592, 925, 952, **b** 3, 6, 7, 33, 36, 37, 63, 66, 67, 73, 76, 77, 333, 336, 337, 363, 366, 367, 373, 376, 377, 633, 636, 637, 663, 666, 667, 673, 676, 677, 733, 736, 737, 763, 766, 767, 773, 776, 777, **c** 100, 101, 102, 110, 111, 112, 120, 121, 122, 200, 201, 202, 210, 211, 212, 220, 221, 222 **13 a** 98 765, **b** 10 234

Exercise 1.4

1 a 11, **b** 13, **c** 12, **d** 14, **e** 15, **f** 12, **g** 15, **h** 16, **i** 13, **j** 14, **k** 17, **l** 11, **m** 13, **n** 14, **o** 12, **p** 18
2 a

2	7	6
9	5	1
4	3	8

, **b**

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

3 a 33, **b** 32, **c** 34, **d** 39, **e** 34 **f** 36 **4 a** 110, **b** 150, **c** 130, **d** 170, **e** 1500, **f** 1300, **g** 1800, **h** 1400, **i** 12 000, **j** 12 000, **k** 15 000, **l** 14 000 **5 a** 59, **b** 98, **c** 63, **d** 60, **e** 93, **f** 126, **g** 114, **h** 171 **6 a** 810, **b** 798, **c** 1544, **d** 2472 **7 a** 3933, **b** 4287, **c** 13 949, **d** 5039 **8 a** 1 016 343, **b** 1 011 048, **c** 1 956 317 **9 a** 72, **b** 117, **c** 65, **d** 181, **e** 175, **f** 165, **g** 333, **h** 627, **i** 582, **j** 773, **k** 513, **l** 873
10 a 184, **b** 976, **c** 54 273, **d** 64, **e** 1962 km
11 a

```
  849
  762
  593 +
 ----
 2204
```

b

```
  627
  395
  408 +
 ----
 1430
```

c

```
 9458
 1677
 9346 +
 -----
20 481
```

Exercise 1.5

1 a 5, **b** 8, **c** 9, **d** 8, **e** 5, **f** 9, **g** 6, **h** 4, **i** 8, **j** 6, **k** 7, **l** 8, **m** 6, **n** 9, **o** 5, **p** 9 **2 a** 22, **b** 21, **c** 20, **d** 23, **e** 12, **f** 10, **g** 9, **h** 4, **i** 11, **j** 20, **k** 28, **l** 17 **3 a** 15, **b** 25, **c** 19, **d** 37, **e** 55, **f** 27, **g** 68, **h** 48, **i** 83, **j** 57, **k** 77, **l** 37 **4 a** 25, **b** 41, **c** 32, **d** 86 **5 a** 18, **b** 14, **c** 29, **d** 47, **e** 54, **f** 38, **g** 49, **h** 25 **6 a** 206, **b** 386, **c** 392, **d** 209, **e** 663, **f** 563, **g** 57, **h** 116 **7 a** 2904, **b** 5368, **c** 2783, **d** 7954 **8 a** 80, **b** 40, **c** 90, **d** 50, **e** 700, **f** 600, **g** 900, **h** 700, **i** 7000, **j** 8000, **k** 7000, **l** 9000 **9 a i** 258, **ii** 879 **b i** 1997 **ii** 3746 **10 a** 86, **b** 183, **c** 385 km, **d** 2227, **e** 371 **11 a** 34, **b** 56, **c** 53, **d** 59, **e** 43, **f** 117, **g** 151, **h** 245, **i** 296, **j** 430, **k** 114, **l** 469
12 a

```
  573
  146 –
 ----
  427
```

b

```
 8365
 3489 –
 -----
 4876
```

c

```
 71 142
 43 635 –
 ------
 27 507
```

Exercise 1.6

1 a 4×2, **b** 3×5, **c** 6×3, **d** 3×12, **e** 5×4, **f** 4×9 **2 a** 20, **b** 21, **c** 36, **d** 40, **e** 27, **f** 60, **g** 24, **h** 28, **i** 48, **j** 45, **k** 33, **l** 64, **m** 63, **n** 48, **o** 54, **p** 36, **q** 60, **r** 110, **s** 49, **t** 81, **u** 121, **v** 108, **w** 42, **x** 56, **y** 144 **3 a** 6,

b 4, **c** 5, **d** 8, **e** 12, **f** 9, **g** 8, **h** 8, **i** 7, **j** 10, **k** 1, **l** 12 **4 a** 40, **b** 900, **c** 3000, **d** 60 000, **e** 250, **f** 3200, **g** 47 000, **h** 985 000 **5 a** 75, **b** 91, **c** 108, **d** 414, **e** 592, **f** 504, **g** 1036, **h** 1785, **i** 4072, **j** 4950 **6 a** 1484, **b** 1428, **c** 2700, **d** 1078, **e** 5805, **f** 5123, **g** 9396, **h** 42 448 **7 a** 600, **b** 2400, **c** 15 000, **d** 7000, **e** 200 000, **f** 240 000, **g** 480 000, **h** 45 000 000, **i** 12 000, **j** 60 000, **k** 840 000, **l** 270 000 000 **8 a** 480, **b** 2870, **c** 2340, **d** 2080, **e** 51 800, **f** 32 500, **g** 352 000, **h** 207 000 **9 a** 826, **b** 408 km, **c** 1280, **d** 1750, **e** 192 grams **11 a** 253, **b** 594, **c** 341, **d** 660, **e** 385, **f** 671, **g** 484, **h** 198, **i** 528, **j** 935, **k** 759, **l** 1078 **12 a** 2673, **b** 1815, **c** 3894, **d** 9372

Exercise 1.7

1 a $12 \div 3$, **b** $16 \div 2$, **c** $42 \div 7$ **2 a** 5, **b** 7, **c** 4, **d** 6, **e** 5, **f** 9, **g** 6, **h** 7, **i** 5, **j** 11, **k** 10, **l** 6, **m** 12, **n** 7, **o** 8, **p** 7, **q** 9, **r** 5, **s** 8, **t** 12 **3 a** 142, **b** 112, **c** 71, **d** 91, **e** 203, **f** 403, **g** 702, **h** 701 **4 a** 64, **b** 69, **c** 29, **d** 14, **e** 25, **f** 32, **g** 38, **h** 24, **i** 83, **j** 67, **k** 82, **l** 54 **5 a** $38\frac{2}{3}$, **b** $75\frac{3}{7}$, **c** $24\frac{4}{9}$, **d** $42\frac{5}{6}$, **e** $67\frac{1}{4}$, **f** $49\frac{1}{3}$, **g** $108\frac{1}{2}$, **h** $34\frac{4}{7}$ **6 a** 108, **b** 207, **c** 109, **d** 109, **e** 106, **f** 209, **g** 307, **h** 108, **i** 608, **j** 906, **k** 604, **l** 605, **m** 307, **n** 704, **o** 805, **p** 508 **7 a i** 18 **ii** 27, **b i** 85 **ii** 134 **8 a** 42, **b** 67, **c** 34, **d** 57, **e** 13, **f** 29, **g** 50, **h** 600, **i** 380, **j** 1670, **k** 200, **l** 47 **9 a** 5, **b** 4, **c** 9, **d** 3, **e** 90, **f** 40, **g** 900, **h** 60 **10 a** 132, **b** 35 cm, **c** \$4060, **d** 23, **e** \$670/week **12 a** 26, **b** 32, **c** 36, **d** 26, **e** 63, **f** 39, **g** 27, **h** 46, **i** 48, **j** 45, **k** 48, **l** 67

Exercise 1.8

1 a 5^2, **b** 3^4, **c** 6^3, **d** 4^5, **e** 7^2, **f** 9^4, **g** 8^3, **h** 2^6, **i** 1^7 **2 a** $7^2 \times 5^3$, **b** $2^3 \times 9^2$, **c** $6^4 \times 4$, **d** $3^3 \times 5^3$, **e** $2^4 \times 7^2$, **f** $1^3 \times 8^2$ **3 a** 6×6, **b** $3 \times 3 \times 3 \times 3$, **c** $5 \times 5 \times 5$, **d** $7 \times 7 \times 7 \times 7 \times 7$, **e** $2 \times 2 \times 2 \times 2$, **f** $8 \times 8 \times 8$, **g** $1 \times 1 \times 1 \times 1 \times 1 \times 1$, **h** $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$ **4 a** $(5 \times 5) + (2 \times 2 \times 2)$, **b** $(3 \times 3 \times 3 \times 3 \times 3) - (6 \times 6)$, **c** $(1 \times 1 \times 1 \times 1) + (2 \times 2 \times 2) + (5 \times 5)$, **d** $(8 \times 8) - (9 \times 9 \times 9) - (7 \times 7)$ **5 a** 8^2, **b** 5^3, **c** 6^4, **d** 9^3, **e** 11^2, **f** 5^{12} **6** C **7** five factors of 2 is larger by 7 **8 a** 9, **b** 8, **c** 36, **d** 1, **e** 49, **f** 27, **g** 16, **h** 64, **i** 32, **j** 81, **k** 125, **l** 64 **9 a** 2^3, **b** 3^2, **c** 2^5, **d** 6^1, **e** 11^2, **f** 2^7 **10** D **11 a** 100, **b** 1000, **c** 100 000, **d** 10, **e** 10 000, **f** 1 000 000 **12 a** 500, **b** 7000, **c** 20 000, **d** 900 000, **e** 4000, **f** 600, **g** 3 000 000, **h** 80 000, **i** 700 000, **j** 50 000, **k** 6000, **l** 9 000 000 **13 a** 41, **b** 41, **c** 28, **d** 100, **e** 14, **f** 36, **g** 5, **h** 9, **i** 80, **j** 54, **k** 2, **l** 42, **m** 856, **n** 407, **o** 160 000, **p** 2592 **14 a i** 36, **ii** 144, **iii** 100, **iv** 2500, **b** product, squares, **c i** 1600, **ii** 3600, **iii** 14 400, **d i** yes, **ii** 900 **15 a** $2^3 \times 3$, **b** $2^3 \times 3^2$, **c** $3^2 \times 5^2$ **16 a** $2^{10} = 1024$, **b** $3^5 = 243$, **c** $2^7 = 128$, **d** $5^3 = 125$

Exercise 1.9

1 a 427, **b** 5319, **c** 74 654, **d** 232 981, **e** 80 563, **f** 9 684 172, **g** 123 215, **h** 6 917 498
2 a $(1 \times 100) + (3 \times 10) + (8 \times 1)$, **b** $(6 \times 1000) + (2 \times 100) + (8 \times 10) + (7 \times 1)$,
c $(3 \times 10\,000) + (5 \times 1000) + (7 \times 100) + (7 \times 10) + (1 \times 1)$,
d $(2 \times 100\,000) + (6 \times 10\,000) + (9 \times 1000) + (1 \times 100) + (4 \times 10) + (5 \times 1)$,
e $(5 \times 10\,000) + (8 \times 1000) + (8 \times 100) + (9 \times 10) + (2 \times 1)$,
f $(7 \times 1\,000\,000) + (4 \times 100\,000) + (3 \times 10\,000) + (4 \times 1000) + (9 \times 100) + (2 \times 10) + (1 \times 1)$,
g $(4 \times 100\,000) + (5 \times 10\,000) + (6 \times 1000) + (1 \times 100) + (7 \times 10) + (9 \times 1)$,
h $(5 \times 1\,000\,000) + (8 \times 100\,000) + (5 \times 10\,000) + (4 \times 1000) + (1 \times 100) + (7 \times 10) + (6 \times 1)$
3 a 630, **b** 500, **c** 8017, **d** 4070, **e** 10 204, **f** 20 080, **g** 960 400, **h** 301 070, **i** 7 401 006, **j** 5 006 000
4 a $(4 \times 100) + (0 \times 10) + (9 \times 1)$, **b** $(3 \times 100) + (5 \times 10) + (0 \times 1)$, **c** $(5 \times 1000) + (1 \times 100) + (0 \times 10) + (0 \times 1)$,
d $(3 \times 1000) + (0 \times 100) + (0 \times 10) + (7 \times 1)$, **e** $8 \times (10\,000) + (0 \times 1000) + (0 \times 100) + (4 \times 10) + (6 \times 1)$,
f $(1 \times 100\,000) + (0 \times 10\,000) + (3 \times 1000) + (0 \times 100) + (9 \times 10) + (0 \times 1)$,
g $(6 \times 100\,000) + (2 \times 10\,000) + (0 \times 1000) + (0 \times 100) + (0 \times 10) + (8 \times 1)$,
h $(4 \times 1\,000\,000) + (9 \times 100\,000) + (0 \times 10\,000) + (0 \times 1000) + (4 \times 100) + (0 \times 10) + (1 \times 1)$ **5 a** 973, **b** 8145, **c** 46 529, **d** 1 382 194, **e** 3007, **f** 7090, **g** 60 108, **h** 5 200 932 **6 a** $(5 \times 10^3) + (6 \times 10^2) + (1 \times 10^1) + (8 \times 1)$,
b $(4 \times 10^4) + (5 \times 10^3) + (2 \times 10^2) + (9 \times 10^1) + (1 \times 1)$,
c $(2 \times 10^5) + (3 \times 10^4) + (8 \times 10^3) + (5 \times 10^2) + (7 \times 10^1) + (6 \times 1)$,
d $(9 \times 10^6) + (1 \times 10^5) + (3 \times 10^4) + (6 \times 10^3) + (7 \times 10^2) + (2 \times 10^1) + (4 \times 1)$,
e $(2 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (9 \times 10^1) + (5 \times 1)$, **f** $(4 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (7 \times 10^1) + (0 \times 1)$,
g $(1 \times 10^5) + (0 \times 10^4) + (3 \times 10^3) + (4 \times 10^2) + (0 \times 10^1) + (9 \times 1)$,
h $(3 \times 10^6) + (2 \times 10^5) + (0 \times 10^4) + (1 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (4 \times 1)$
7 a $(5 \times 10^3) + (2 \times 10^2) + (6 \times 10^1) + (4 \times 1)$, **b** $(8 \times 10^4) + (9 \times 10^3) + (0 \times 10^2) + (1 \times 10^1) + (2 \times 1)$,
c $(3 \times 10^5) + (0 \times 10^4) + (9 \times 10^3) + (6 \times 10^2) + (0 \times 10^1) + (1 \times 1)$,
d $(5 \times 10^5) + (9 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (7 \times 10^1) + (0 \times 1)$,
e $(4 \times 10^5) + (0 \times 10^4) + (0 \times 10^3) + (0 \times 10^2) + (5 \times 10^1) + (6 \times 1)$,
f $(2 \times 10^6) + (7 \times 10^5) + (1 \times 10^4) + (0 \times 10^3) + (9 \times 10^2) + (3 \times 10^1) + (0 \times 1)$,
g $(6 \times 10^6) + (0 \times 10^5) + (2 \times 10^4) + (9 \times 10^3) + (1 \times 10^2) + (0 \times 10^1) + (3 \times 1)$,
h $(1 \times 10^8) + (4 \times 10^7) + (0 \times 10^6) + (5 \times 10^5) + (1 \times 10^4) + (4 \times 10^3) + (0 \times 10^2) + (0 \times 10^1) + (8 \times 1)$

Exercise 1.10

1 **a** 30, **b** 40, **c** 40 **2** **a** 300, **b** 200, **c** 300 **3** **a** 8000, **b** 9000, **c** 9000 **4** **a** 20, **b** 20, **c** 70, **d** 80, **e** 10, **f** 70, **g** 20, **h** 50, **i** 90, **j** 30, **k** 0, **l** 100 **5** **a** 400, **b** 200, **c** 900, **d** 500, **e** 600, **f** 700, **g** 800, **h** 300, **i** 600, **j** 100, **k** 100, **l** 1000 **6** **a** 3000, **b** 9000, **c** 8000, **d** 4000, **e** 2000, **f** 3000, **g** 4000, **h** 2000, **i** 12 000, **j** 16 000, **k** 32 000, **l** 100 000 **7** **a** 1230, **b** 4360, **c** 3550, **d** 7030, **e** 6290, **f** 1000, **g** 5280, **h** 7770, **i** 6010, **j** 9640, **k** 3520, **l** 10 000 **8** **a** 3600, **b** 7200, **c** 1500, **d** 4700, **e** 2300, **f** 8100, **g** 6300, **h** 5500, **i** 7600, **j** 5900, **k** 9700, **l** 10 000 **9** **a** 150, 143, **b** 2000, 1956, **c** 17 000, 17 425, **d** 40, 43, **e** 500, 509, **f** 7000, 6284, **g** 3500, 3456, **h** 18 000, 18 752, **i** 2 800 000, 2 772 225, **j** 4, 4.67, **k** $7\frac{1}{2}$, 7.43, **l** 10, 9.25 **10** **a** \$260, **b** \$20 000, **c** 3000, **d** \$5000, **e** 100 000, **f** 6000, **g** \$210 000, **h** 2000 **11** **a** 125 – 134, **b** 1150 – 1249, **c** 36 500 – 37 499

Exercise 1.11

1 **a** 21, **b** 20, **c** 5, **d** 12, **e** 45, **f** 5, **g** 40, **h** 4, **i** 8, **j** 48, **k** 84, **l** 9, **m** 60, **n** 8, **o** 24, **p** 37 **2** **a** 40, **b** 17, **c** 13, **d** 4, **e** 12, **f** 103, **g** 0, **h** 52, **i** 65 **3** **a** 77, **b** 39, **c** 27, **d** 2, **e** 56, **f** 2, **g** 32, **h** 12, **i** 5, **j** 57, **k** 0, **l** 1 **4** **a** 6, **b** 20, **c** 14, **d** 80, **e** 18, **f** 10, **g** 3, **h** 61, **i** 6, **j** 12, **k** 96, **l** 23, **m** 104, **n** 50 **5** **a** 3, **b** 2, **c** 4, **d** 2
6 **a** $20 - (4 \times 2) = 12$, **b** $35 - (8 - 6) = 33$, **c** $(22 - 10) \times 4 = 48$, **d** $42 \div (2 \times 3) = 7$, **e** $(5 \times 12) \div 10 = 6$, **f** $19 + (9 \times 2) = 37$, **g** $7 + (5 \times 3) - 4 = 18$, **h** $(8 + 5) - (3 + 6) = 4$, **i** $(49 \div 7) + (4 \times 3) = 19$, **j** $(25 - 5) - (6 + 5) = 9$, **k** $31 + (35 \div 7) - 8 = 28$, **l** $50 - (7 \times 3) - (5 + 4) = 20$

Exercise 1.12

1 **a** 23, **b** 14, **c** 20, **d** 6, **e** 26, **f** 12, **g** 22, **h** 33, **i** 11, **j** 23, **k** 41, **l** 14 **2** **a** 36, **b** 4, **c** 3, **d** 12, **e** 110, **f** 2, **g** 35, **h** 3, **i** 63, **j** 40, **k** 4, **l** 72 **3** **a** 9, **b** 27, **c** 2, **d** 41, **e** 22, **f** 25, **g** 7, **h** 17, **i** 77, **j** 11, **k** 23, **l** 0 **4** **a** 26, **b** 18, **c** 21, **d** 26, **e** 34, **f** 16, **g** 45, **h** 90, **i** 6, **j** 0, **k** 12, **l** 22, **m** 24, **n** 70, **o** 85 **5** **a** 27, **b** 12, **c** 15, **d** 2, **e** 26, **f** 12, **g** 73, **h** 7, **i** 80 **6** **a** 30, **b** 4, **c** 42, **d** 1, **e** 44, **f** 5, **g** 44, **h** 11, **i** 50, **j** 50, **k** 22, **l** 55 **7** **a** 90, **b** 14, **c** 39, **d** 83, **e** 74, **f** 544

Exercise 1.13

1 **a** 0, **b** 15, **c** 12, **d** 24, **e** 54, **f** 36, **g** 50, **h** 0, **i** 19, **j** 11, **k** 0, **l** 35 **2** **a** 0, **b** 1, **c** 4, **d** 0, **e** 5, **f** 0, **g** 1, **h** 17, **i** 0, **j** 0, **k** 11, **l** 1, **m** 0, **n** 0, **o** 1, **p** 0 **3** **a** T, **b** T, **c** F, **d** F, **e** T, **f** F, **g** F, **h** T, **i** T, **j** F, **k** F, **l** T, **m** F, **n** T, **o** F **4** **a** T, **b** F, **c** F, **d** T, **e** F, **f** T, **g** T, **h** F, **i** F, **j** T, **k** F, **l** T, **m** T, **n** T **5** **a** 97, **b** 166, **c** 94, **d** 149, **e** 79, **f** 192, **g** 287, **h** 414, **i** 449, **j** 675, **k** 643, **l** 748 **6** **a** 130, **b** 110, **c** 140, **d** 290, **e** 190, **f** 440
7 **a** 190, **b** 84, **c** 48, **d** 180, **e** 130, **f** 460, **g** 560, **h** 1900, **i** 360, **j** 480, **k** 390, **l** 680 **8** **a** 0, **b** 1700, **c** 160, **d** 3800, **e** 7000, **f** 5400 **9** **a** No, **b** Yes, **c** No **10** **b** **i** 144 **ii** 480 **iii** 1200 **iv** 8400 **11** **a** 180, **b** 300, **c** 660 **12** **a** 210, **b** 560, **c** 990 **13** **a** 13, **b** 24, **c** 52 **14** **a** 13, **b** 21, **c** 12 **15** **a** 32, **b** 27, **c** 13

Exercise 1.14

1 **a** $(4 + 3) \times 2 = 14$, **b** $(9 - 7) \times 5 = 10$, **c** $(5 + 4) \times 3 = 27$, **d** $(11 - 5) \times 6 = 36$, **e** $(12 - 9) \times 13 = 39$, **f** $(6 + 4) \times 18 = 180$ **2** **D** **3** **a** $5 \times 3 + 5 \times 1$, **b** $2 \times 7 - 2 \times 3$, **c** $4 \times 12 + 4 \times 5$, **d** $9 \times 10 - 9 \times 4$ **4** **a** 8, **b** 33, **c** 40, **d** 28, **e** 35, **f** 36, **g** 40, **h** 45, **i** 36, **j** 48, **k** 28, **l** 60, **m** 110, **n** 6, **o** 121, **p** 45 **5** **a** 91, **b** 84, **c** 168, **d** 288, **e** 95, **f** 203, **g** 792, **h** 285, **i** 1717, **j** 1104, **k** 1568, **l** 2575, **m** 2208, **n** 1350, **o** 1352, **p** 891, **q** 3264, **r** 2548, **s** 1001, **t** 39 000 **6** **a** 16, **b** 27, **c** 70, **d** 120, **e** 42, **f** 40 **7** **a** 8, **b** 54, **c** 4, **d** 25, **e** 21, **f** 72 **8** **a** $11 \times 9 = 99$, **b** $4 \times 3 = 12$, **c** $12 \times 4 = 48$, **d** $10 \times 14 = 140$, **e** $5 \times 11 = 55$, **f** $20 \times 7 = 140$

Exercise 1.15

1 **a** $8 + 4$, **b** $9 - 6$, **c** $5 + 3$, **d** $7 - 2$, **e** $15 - 10$, **f** $11 + 9$, **g** $4 + 3$, **h** $8 - 1$, **i** $17 - 12$, **j** $20 + 30$, **k** $19 + 5$, **l** $14 - 10$ **2** **a** 5×8, **b** $20 \div 4$, **c** $18 \div 3$, **d** 6×7, **e** 2×11, **f** $36 \div 9$, **g** $48 \div 8$, **h** 12×10 **3** **a** $14 - 6 = 8$, **b** $7 + 5 = 12$, **c** $4 \times 8 = 32$, **d** $21 \div 3 = 7$, **e** $25 - 12 = 13$, **f** $42 \div 6 = 7$, **g** $30 + 19 = 49$, **h** $23 + 15 = 38$, **i** $28 - 17 = 11$, **j** $34 - 5 = 29$, **k** $20 \times 6 = 120$, **l** $9 + 8 = 17$, **m** $75 \div 25 = 3$, **n** $18 + 10 = 28$, **o** $12 \times 4 = 48$, **p** $19 - 9 = 10$, **q** $16 \div 4 = 4$, **r** $40 - 8 = 32$, **s** $4 + 11 = 15$, **t** $9 \times 7 = 63$, **u** $50 - 15 = 35$, **v** $13 \times 4 = 52$
4 **a** $(5 \times 2) + 3 = 13$, **b** $(6 \times 3) - 7 = 11$, **c** $(12 \div 4) + 6 = 9$, **d** $(60 \div 5) - 10 = 2$ **5** **a** 6, **b** 7, **c** 9, **d** 26
6 **a** not equal to, **b** greater than, **c** less than, **d** less than or equal to, **e** greater than or equal to **7** **a** $5 < 9$, **b** $10 > 4$, **c** $2 \leqslant 7$, **d** $18 \geqslant 13$, **e** $9 \neq 5$, **f** $8 < 12$, **g** $3 \geqslant 2$, **h** $15 \neq 14$, **i** $10 > 3$, **j** $20 \leqslant 25$ **8** **a** T, **b** F, **c** T, **d** T, **e** F, **f** F, **g** T, **h** T, **i** F, **j** T, **k** T, **l** F, **m** F, **n** T, **o** F, **p** T **9** **a** $2 < 5$, 2 is less than 5, **b** $11 > 4$, 11 is greater than 4, **c** $6 \leq 8$, 6 is less than or equal to 8, **d** $14 \geq 9$, 14 is greater than or equal to 9

10 **a** $\leqslant$, **b** $>$, **c** $\geqslant$, **d** $<$ **11** **a** $\square < 5$, **b** $\square > 7$, **c** $\square \leqslant 8$, **d** $\square \geqslant 14$ **12** **a** 0, 1, 2, 3, 4, 5, 6, **b** 5, 6, 7, 8, **c** 13, 14, 15, 16, **d** 20, 21, 22, 23, 24, 25 **13** **a** 6, **b** 120, **c** 720, **d** 5040, **e** 9, **f** 13, **g** 110, **h** 56
14 **a** 10!, **b** 15!, **c** 5!, **d** 7!

Chapter 1 Review

1 **a** 3, **b** 40, **c** 62, **d** 211 **2** **a** 345, **b** 2406, **c** 51 023, **d** 1 410 008 **3** **a** 37, **b** 294, **c** 3660, **d** 78 600
4 **a** 13, **b** 40, **c** 76, **d** 229 **5** **a** 78, **b** 231, **c** 3766, **d** 16 502, **e** 19, **f** 44, **g** 497, **h** 4099 **6** **a** LXXXVIII, **b** CCVI, **c** DCCLXXIII, **d** MMDCLVII, **e** XIV, **f** XCIX, **g** CDLXXIV, **h** CMXLII, **i** M$\overline{\text{X}}$CDLXXXIX
8 **a** 80, **b** 50 000, **c** 9000, **d** 400 000 **9** **a** 160, **b** 499 **10** **a** 2413, **b** 15 070, **c** 148 926, **d** 7 680 509, **e** 12 034 280 **11** **a** Four thousand and ten,
b One hundred and sixty-two thousand four hundred and seventy-eight,
c Three hundred and ninety thousand and fifty,
d Eighty-nine million sixty-seven thousand two hundred and eleven **12** **a** 32, **b** 150, **c** 1827, **d** 160 318
13 **a** 21, **b** 25, **c** 326, **d** 1236 **14** **a** 42, **b** 36, **c** 64, **d** 60, **e** 63, **f** 96, **g** 121, **h** 72 **15** **a** 162, **b** 2688, **c** 34 400, **d** 2574 **16** **a** 721, **b** 84, **c** 309, **d** 3040 **17** $84\frac{5}{6}$ **18** 400 **19** **a** 23, **b** 26, **c** 28 **20** **a** 49, **b** 8, **c** 10 000, **d** 1 **21** 4 factors of 3 **22** **a** 63, **b** 93, **c** 4000, **d** 70 **23** **a** Yes, **b** 3600 **24** **a** 74 602, **b** 380 097 **25** $(2 \times 100\,000) + (4 \times 10\,000) + (3 \times 1000) + (5 \times 100) + (8 \times 10) + (6 \times 1)$
26 **a** $(5 \times 10^3) + (2 \times 10^2) + (1 \times 10^1) + (9 \times 1)$, **b** $(3 \times 10^4) + (8 \times 10^3) + (5 \times 10^2) + (1 \times 10^1) + (4 \times 1)$,
c $(1 \times 10^5) + (2 \times 10^4) + (9 \times 10^3) + (0 \times 10^2) + (4 \times 10^1) + (2 \times 1)$,
d $(3 \times 10^6) + (5 \times 10^5) + (0 \times 10^4) + (7 \times 10^3) + (0 \times 10^2) + (6 \times 10^1) + (8 \times 1)$ **27** **a** 60, **b** 30, **c** 80, **d** 1840
28 **a** 400, **b** 700, **c** 200, **d** 24 000 **29** **a** 1400, **b** 15 000, **c** 20 000, **d** 20 **30** **a** 2, **b** 39, **c** 8
31 **a** $5 \times (3 + 11) - 8 = 62$ **32** **a** 16, **b** 28, **c** 25 **33** **a** T, **b** F, **c** T, **d** F **34** **a** T, **b** F, **c** T
35 **a** $(172 + 28) + 49 = 249$, **b** $(25 \times 4) \times 19 = 1900$ **36** **a** 32, **b** 42, **c** 100, **d** 45 **37** 2673
38 **a** $5 \times (7 + 3) = 50$, **b** $12 \times (5 - 3) = 24$ **39** **a** 3×6, **b** $30 + 7$, **c** $36 \div 9$, **d** $19 - 6$, **e** $40 \div 8$, **f** $13 + 16$, **g** $25 - 18$, **h** 13×3, **i** $50 - 10$ **40** 12 **41** **a** $6 > 4$, **b** $13 < 21$, **c** $27 \neq 25$, **d** $15 \leq 28$, **e** $11 \geq 11$, **f** $5 \nless 3$
42 **a** Add 30 then subtract 1, **b** Multiply by 100 then divide by 2, **c** Multiply by 100 then subtract the number, **d** Divide by 10 then divide by 2.

2 Number theory

Exercise 2.1

1 **a** 2, **b** even **2** **a** 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, **b** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 **3** **a** even, **b** odd, **c** even, **d** even, **e** odd, **f** odd, **g** even, **h** odd, **i** odd, **j** odd, **k** even, **l** odd **4** **a** 85, **b** 172, **c** 1100, **d** 115, **e** 558, **f** 2299 **5** **a** 10, **b** 99, **c** 101, **d** 998, **e** 98, **f** 11, **g** 999, **h** 100 **6** **a** 71, **b** 48, **c** 90, **d** 121, **e** 919, **f** 958
7 **a** 471, **b** 6938 **8** **a** even, **b** odd, **c** even, **d** odd, **e** even, **f** odd, **g** even, **h** odd
9 25; they are all divisible by 4.

Exercise 2.2

1 **a** 66, **b** 131, **d** 2002, **f** 24 342, **h** 9 133 319 **2** **a** 11, **b** 99 **4** 0 **5** **a** 1, 4, 9, 16, **b** 25, 36
6 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144 **7** **a** 1, 9, 25, 49, 81, 121, **b** 4, 16, 36, 64, 100, 144
8 **a** 1, 3, 6, 10, **b** 15, 21 **9** 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78 **10** **a** 1, 3, 15, 21, 45, 55, **b** 6, 10, 28, 36, 66, 78 **11** 25, 9, 36, 64, 1, 121 **12** 400 **13** **a** 169, **b** 225, **c** 361
14 **a** 1, 4, 9, 16, 25, 36, **b** square numbers, **c** $6^2 = 36$ **15** **a** 64, **b** 100, **c** 256 **16** 14 **17** **a** 25, 100, **b** 36, 64, 100, 144 **18** 15, 21, 36, 6, 10, 91, 28 **20** **a** 91, **b** 105, **c** 120 **21** **a** **i** 4 **ii** 9 **iii** 16 **iv** 25 **v** 36 **vi** 49 **b** square numbers **23** **a** $1 + 3 + 10$, **b** $15 + 1$ or $6 + 10$, **c** $15 + 10 + 1$, **d** $28 + 10 + 6$ or $1 + 15 + 28$ **24** **a** 36, **b** 6, 21, 36, 55 **25** 276 **26** **a** 1, 3, 6, **b** 10, 15, 21; triangular numbers **27** **a** 81, **b** 36, **c** 25, **d** 64, 81, **e** 64, **f** 81, **g** 64, **h** 64, 169, **i** 16, 81, **j** 121, **k** 1, **l** 1024
28 **a** 10, **b** 1, 36, **c** 3, 15, **d** 55, 66, **e** 78, 91 **f** 91, **g** 36, **h** 10, **i** 105, **j** 490, **k** 3, **l** 6, 10 or 1, 15

Exercise 2.3

1 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 **2** 144 **3** 3, 21, 55 **4** **a** every 3rd number is even, the rest are odd, **b** even, **c** odd **5** **a** every 4th position, **b** every 5th position **6** **a** 143 is divisible by 11, **b** 231 is divisible by 11, **c** 374 is divisible by 11, **d** the sum of any 10 consecutive Fibonacci numbers is divisible by 11 **7** **a** yes, **b** add 1 instead of subtracting 1 **8** **b** The inner product is 1 less than the outer product when you start in an odd

position. The inner product is 1 more when you start in an even position. **9** **a** 232, **b** 377
10 $1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 40 = 5 \times 8$; $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = 104 = 8 \times 13$;
$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 = 273 = 13 \times 21$. The sum of the squares of consecutive Fibonacci numbers is equal to the product of the last Fibonacci number that was squared and the one that follows it. **11** **a** yes, yes, **b** yes, yes, **c** F_1, F_2, F_3, F_4, F_6 **12** **a** 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, **c** $F_6 + F_8$

Exercise 2.4

1 176, 548, 900, 1374 **2** 252, 486, 9651, 20 895 **3** 132, 328, 1520, 5636 **4** 95, 170, 965, 3900
5 138, 234, 762, 5496 **6** 1168, 1248, 7328, 9488 **7** 135, 468, 5823, 9297 **8** 220, 690, 1500, 20 360
9 176, 6358, 71 258, 472 868 **11** 91, 98, 119, 154 **12** 2, 3, 4, 6, 7, 11 **13** **a** test for 3 and 4, **b** test for 2 and 7, **c** test for 3 and 5, **d** test for 5 and 9 **14** **a** yes, **b** no, **c** yes **15** **a** 196, **c** 6545 **16** True

Exercise 2.5

1 **a** 2, 4, 6, 8, 10, **b** 3, 6, 9, 12, 15, **c** 5, 10, 15, 20, 25, **d** 6, 12, 18, 24, 30, **e** 8, 16, 24, 32, 40, **f** 11, 22, 33, 44, 55, **g** 1, 2, 3, 4, 5, **h** 4, 8, 12, 16, 20, **i** 7, 14, 21, 28, 35, **j** 9, 18, 27, 36, 45, **k** 10, 20, 30, 40, 50, **l** 12, 24, 36, 48, 60
2 **a** 24, **b** 36, **c** 56, **d** 45, **e** 42, **f** 32, **g** 90, **h** 132, **i** 72 **3** **a** 6th, **b** 5th, **c** 3rd, **d** 8th, **e** 9th, **f** 11th, **g** 10th, **h** 8th, **i** 8th, **j** 11th **4** **a** T, **b** F, **c** T, **d** T, **e** F, **f** F, **g** F, **h** T, **i** F **5** **a** 21, 24, 27, 30, 33, 36, 39, **b** 32, 36, 40, 44, 48, 52, **c** 15, 20, 25, 30, 35, 40, **d** 49, 56, 63, 70, **e** 32, 40, 48, 56, **f** 72, 84, 96, 108, 120 **6** **a** 36, **b** 48 **7** **a** 37, **b** 5 **8** **a** 13, 26, 39, 52, 65, **b** 15, 30, 45, 60, 75, **c** 24, 48, 72, 96, 120, **d** 29, 58, 87, 116, 145, **e** 35, 70, 105, 140, 175 **9** **a** 52, 56, 60, **b** 98, 105, 112, **c** 126, 132, 138 **11** **a** 6, **b** 10, **c** 12, **d** 12, **e** 24, **f** 18, **g** 30, **h** 24, **i** 40, **j** 30 **13** No (200 is) **14** **a** 60, **b** 120, **c** 75 **15** **a** 14, **b** 100, **c** 32, **d** 88, **e** 36, **f** 96, **g** 105, **h** 55, **i** 42, **j** 6, **k** 45, **l** 21 **16** 12 **17** 288

Exercise 2.6

1 **a** 1, 5, **b** 1, 2, 4, 8, **c** 1, 2, 5, 10, **d** 1, 2, 3, 4, 6, 12, **e** 1, 13, **f** 1, 2, 7, 14, **g** 1, 3, 5, 15, **h** 1, 2, 3, 6, 9, 18, **i** 1, 2, 4, 5, 10, 20, **j** 1, 3, 7, 21, **k** 1, 2, 11, 22, **l** 1, 2, 3, 4, 6, 8, 12, 24, **m** 1, 3, 9, 27, **n** 1, 2, 4, 7, 14, 28, **o** 1, 29, **p** 1, 2, 3, 5, 6, 10, 15, 30, **q** 1, 2, 4, 8, 16, 32, **r** 1, 3, 11, 33 **2** **a** 1, 5, 7, 35, **b** 1, 2, 3, 4, 6, 9, 12, 18, 36, **c** 1, 2, 4, 5, 8, 10, 20, 40, **d** 1, 2, 3, 6, 7, 14, 21, 42, **e** 1, 2, 4, 11, 22, 44, **f** 1, 3, 5, 9, 15, 45, **g** 1, 2, 3, 4, 6, 8, 12, 16, 24, 48, **h** 1, 2, 5, 10, 25, 50, **i** 1, 2, 3, 6, 9, 18, 27, 54, **j** 1, 2, 4, 7, 8, 14, 28, 56, **k** 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60, **l** 1, 3, 7, 9, 21, 63, **m** 1, 2, 3, 6, 11, 22, 33, 66, **n** 1, 2, 5, 7, 10, 14, 35, 70, **o** 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72, **p** 1, 3, 5, 15, 25, 75 **3** **a** T, **b** F, **c** F, **d** T, **e** F, **f** F, **g** T, **h** F, **i** F
4 **a** factor, **b** multiple, **c** multiple, **d** factor **5** **a** (1, 2, 4); (1, 3, 9); (1, 2, 4, 8, 16), **b** odd **6** **a** 3, **b** 13, **c** 34, **d** 71 **7** 1 **8** **a** 1, 2, 3, 4, 6, **b** 1, 2, 4, 5, 10 **9** **a** 1, 2, 4, 8, **b** 1, 2, 4, 5, 10, 20, **c** 1, 2, 7, 14
10 5, 50, 20, 25, 10, 100, 4 **11** **a** 4, **b** 5, **c** 6, **d** 7, **e** 8, **f** 3, **g** 9, **h** 12, **i** 6 **14** **a** 4, **b** 6
15 **a** 1, 2, 4, 5, 8, 10, 16, 20, 40, 80, **b** 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84, **c** 1, 2, 4, 23, 46, 92, **d** 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96 **16** 1, 2, 3, 4, 5, 6, 7, 10, 12, 14, 15, 20, 21, 28, 30, 35, 42, 60, 70, 84, 105, 140, 210, 420 **17** **a** 4, **b** 10, **c** 6, **d** 16, **e** 34, **f** 28, **g** 96, **h** 21, **i** 38, **j** 8, **k** 102, **l** 9

Exercise 2.7

1 **a** P, **b** P, **c** C, **d** P, **e** C, **f** P, **g** C, **h** P, **i** C, **j** C, **k** P, **l** C, **m** C, **n** P, **o** C, **p** P, **q** C, **r** P, **s** P, **t** C, **u** P, **v** C, **w** C, **x** P **2** **a** C, **b** P, **c** P, **d** C, **e** P, **f** C, **g** P, **h** P, **i** C, **j** C, **k** C, **l** C **4** 3, 5; 5, 7; 11, 13; 17, 19; 29, 31; 41, 43; 59, 61; 71, 73 **5** 3, 13; 7, 17; 13, 23; 19, 29; 31, 41; 37, 47; 43, 53; 61, 71; 73, 83; 79, 89 **6** no
7 1 because it has only 1 factor whereas primes have 2 **8** 2 **9** The alternatives are: **a** 7 + 3, 5 + 5, **b** 13 + 3, 5 + 11, **c** 17 + 3, 13 + 7, **d** 17 + 11, 5 + 23, **e** 23 + 7, 19 + 11, 17 + 13, **f** 31 + 7, 19 + 19, **g** 31 + 19, 43 + 7, 47 + 3, 37 + 13, **h** 79 + 5, 71 + 13, 67 + 17, 61 + 23, 47 + 37, 43 + 41, 31 + 53, 73 + 11
10 For example **a** 2 + 2 + 3, **b** 3 + 5 + 5, **c** 5 + 7 + 7, **d** 23 + 5 + 3, **e** 43 + 2 + 2, **f** 11 + 13 + 31, **g** 61 + 5 + 3, **h** 13 + 19 + 67 **11** 97 + 3; 89 + 11; 83 + 17; 71 + 29; 41 + 59; 47 + 53, **12** For example 5 + 7 + 11 + 13
13 19 **14** **a** prime, **b** composite, **c** composite, **d** composite, **e** composite **15** **a** Yes. 11 is a 2 digit prime palindrome. All the others cannot be prime as they are divisible by 11. **b** A palindrome with an even number of digits will have the property that the sum of the even digits equals the sum of the odd digits. This property also means it is divisible by 11, hence it cannot be prime. The number 11 is the exceptional case. **16** For example **a** 101, **b** 121 **17** **a** 97, **b** 101, **c** 23, **d** 83, **e** 2, 11, **f** 4, **g** For example, 3, **h** 13, **i** 127, **j** 8, **k** 9, **l** 28
18 11, 13, 31, 17, 71, 37, 73, 79, 97

Exercise 2.8

1 **a** $2^2 \times 5$, **b** $2^3 \times 3$, **c** $2^2 \times 7$, **d** $2 \times 3 \times 5$, **e** $2^2 \times 3^2$, **f** $2^3 \times 5$, **g** $2 \times 3 \times 7$, **h** $3^2 \times 5$, **i** $2^4 \times 3$, **j** 2×5^2, **k** 2×3^3, **l** $2^3 \times 7$, **m** $2^2 \times 3 \times 5$, **n** $3^2 \times 7$, **o** $2^3 \times 3^2$, **p** 3×5^2, **q** $2^4 \times 5$, **r** $2^2 \times 5^2$ **2** **a** $2^4 \times 3 \times 5$, **b** $2^2 \times 3^3 \times 5$, **c** $2^3 \times 3^2 \times 11$, **d** $2 \times 3^2 \times 7^2$, **e** $2 \times 3^4 \times 7$, **f** $5^2 \times 7^2$, **g** $3^3 \times 5 \times 13$, **h** $2 \times 3^3 \times 5 \times 7$ **3** **a** 6, **b** 10, **c** 50, **d** 42, **e** 27, **f** 18, **g** 98, **h** 99, **i** 90, **j** 54, **k** 36, **l** 108 **4** **a** 72, **b** 108, **c** 168, **d** 1512, **e** 1260, **f** 1800, **g** 41 580, **h** 6600, **i** 3780, **j** 59 400, **k** 29 700, **l** 415 800 **5** **a** 1, 2, 3, 6, 7, 14, 21, 42, **b** 1, 2, 3, 6, 11, 22, 33, 66, **c** 1, 2, 5, 7, 10, 14, 35, 70, **d** 1, 3, 5, 7, 15, 21, 35, 105, **e** 1, 2, 5, 10, 11, 22, 55, 110, **f** 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210, **g** 1, 2, 3, 5, 6, 10, 11, 15, 22, 30, 33, 55, 66, 110, 165, 330, **h** 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

Exercise 2.9

1 **a** 3, **b** 5, **c** 2, **d** 7, **e** 10, **f** 1, **g** 4, **h** 9, **i** 11, **j** 6, **k** 8, **l** 12 **2** **a** 2, **b** 3, **c** 1, **d** 10, **e** 5, **f** 4 **3** **a** 13, **b** 14, **c** 1225, **d** 1849 **4** **a** 7, **b** 9, **c** 216, **d** 1331 **5** **a** 8, **b** 4 **6** No **7** **a** 5, **b** 3, **c** 10, **d** 2, **e** 5, **f** 6 **8** **a** 12, **b** 2, **c** 33, **d** 3, **e** 16, **f** 15, **g** 12, **h** 8, **i** 4, **j** 2, **k** 4, **l** 10, **m** 84, **n** 59, **o** 88, **p** 20, **q** 72, **r** 10 **9** **a** 17, **b** 26, **c** 10, **d** 0, **e** 14, **f** 16 **10** **a** 13, **b** 15, **c** 22, **d** 25, **e** 31 **11** **a** 6, **b** 7, **c** 9, **d** 8, **e** 11 **12** **a** Yes, because it is a large number, **b** Yes, because it has no integer value, **c** No **13** **a** **i** 6, **ii** 10, **iii** 12, **iv** 8 **b** square root; product, **c** **i** 10, **ii** 6, **iii** 12, **d** 35 **14** **a** 14, **b** 15, **c** 42, **d** 18 **15** **a** 20, **b** 24, **c** 36, **d** 84 **16** **a** 6, **b** 10, **c** 12, **d** 45 **17** **a** 20, **b** 30, **c** 700, **d** 500, **e** 4000, **f** 11 000 **18** **a** 20, **b** 30, **c** 50, **d** 100, **e** 200, **f** 4000

Chapter 2 Review

1 **a** odd, **b** even, **c** even **2** **a** 98, **b** 101, **c** 979, **d** 1600 **3** 13 231, 7007 **4** 0 **5** **a** 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, **b** 289 **6** **a** 81, **b** 16, 36, **c** 16, 36 **7** **a** 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, **b** 105 **8** **a** 91, **b** 21, 28, **c** 3, 21 **9** **a** 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, **b** 13, **c** 26 **10** **a** 670, **b** 4176, **c** 124, **d** 465, **e** 138, **f** 832, **g** 765, **h** 4160, **i** 1452 **11** By testing for divisibility by 2 and 9 **12** **a** 4, 8, 12, 16, 20, 24, **b** 6, 12, 18, 24, 30, 36, **c** 7, 14, 21, 28, 35, 42, **d** 9, 18, 27, 36, 45, 54 **13** **a** 40, **b** 7th **14** 35, 42, 49, 56, 63 **15** 1312, 1316, 1320 **16** **a** 20, **b** 12, **c** 36 **17** **a** 1, 2, 4, 8, 16, **b** 1, 2, 3, 5, 6, 10, 15, 30, **c** 1, 2, 3, 6, 7, 14, 21, 42, **d** 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72 **18** 209 **19** 1, 2, 4, 5, 10 **20** **a** 5, **b** 7, **c** 8 **21** **a** prime, **b** composite, **c** composite, **d** prime, **e** composite, **f** prime, **g** composite, **h** prime **22** composite **23** For example **a** 13 + 5, **b** 17 + 7, **c** 31 + 5, **d** 47 + 11 **24** **a** primes that differ by 2, **b** 3, 5; 5, 7; 11, 13 **25** **a** 163, **b** 8, **c** 15 **26** **a** $2^3 \times 3^2$, **b** $2^3 \times 3 \times 5$, **c** $3^3 \times 5^2$, **d** $2^4 \times 11^2$ **27** **a** 63, **b** 275 **28** **a** 126, **b** 9800 **29** **a** 4, **b** 5, **c** 9, **d** 12, **e** 2, **f** 3, **g** 5, **h** 10 **30** **a** 42, **b** 12, **c** 3, **d** 27 **31** **a** 22, **b** 52, **c** 33, **d** 35 **32** **b** 24

3 Time

Exercise 3.1

1 **a** 60 min, **b** 60 s, **c** 24 h, **d** 120 min, **e** 300 s, **f** 48 h, **g** 420 s, **h** 120 h, **i** 240 min **2** **a** 2 min, **b** 3 h, **c** 3 days, **d** 5 h, **e** 10 days, **f** 6 min **3** **a** 30 min, **b** 15 min, **c** 45 min, **d** 20 min, **e** 40 min, **f** 6 min, **g** 12 min, **h** 25 min **4** **a** 90 s, **b** 75 s, **c** 80 s, **d** 95 s, **e** 165 s, **f** 160 s, **g** 230 s, **h** 468 s **5** **a** $\frac{1}{2}$, **b** $\frac{1}{3}$, **c** $\frac{1}{4}$, **d** $\frac{2}{3}$, **e** $\frac{5}{12}$, **f** $\frac{2}{5}$, **g** $\frac{1}{6}$, **h** $\frac{2}{9}$ **6** **a** 3600 s, **b** 1440 min, **c** 168 h **7** **a** 8:00, **b** 7:30, **c** 3:15, **d** 8:45, **e** 11:10, **f** 12:55, **g** 4:25, **h** 9:48 **8** **a** seven o'clock, **b** half past four, **c** a quarter past eight, **d** a quarter to two, **e** five minutes past ten, **f** ten minutes to one, **g** twenty-five minutes past two, **h** twenty-three minutes to twelve **9** midnight 0:00, midday 12:00 **10** **a** a quarter past one, 1:15, **b** half past eight, 8:30, **c** twenty-five minutes to eleven, 10:35, **d** twenty minutes past twelve, 12:20 **11** **a** 03:00, **b** 19:00, **c** 06:30, **d** 21:15, **e** 16:55, **f** 01:35, **g** 12:32, **h** 14:45, **i** 03:40, **j** 20:00, **k** 13:50, **l** 00:09 **12** **a** 4:00 am, **b** 11:00 am, **c** 1:00 pm, **d** 2:30 pm, **e** 5:10 am, **f** 5:45 pm, **g** 8:15 pm, **h** 1:20 am, **i** 10:47 am, **j** 4:28 pm, **k** 11:35 pm, **l** 12:15 am **13** **a** a quarter to seven in the evening, **b** five minutes past seven in the morning, **c** half past one in the afternoon, **d** ten minutes after midnight **14** **a** 08:15, **b** 23:30, **c** 15:20, **d** 03:50 **15** **a** 1 h 30 min, **b** 3 h 25 min, **c** 2 h 22 min, **d** 3 h 20 min, **e** 4 h 25 min, **f** 2 h 20 min, **g** 6 h, **h** 7 h 30 min, **i** 5 h 26 min, **j** 3 h 23 min, **k** 2 h, **l** 9 h **16** **a** 3 h 10 min, **b** 2 h 20 min, **c** 3 h 5 min, **d** 6 h 15 min, **e** 9 h 35 min, **f** 7 h 15 min **17** **a** 50 min, **b** 45 min, **c** 20 min, **d** 1 h 30 min, **e** 1 h 50 min, **f** 2 h 15 min, **g** 30 min, **h** 50 min, **i** 2 h 45 min, **j** 2 h 20 min,

k 1 h 35 min, **l** 4 h 22 min **18** **a** 50 min, **b** 40 min, **c** 1 h 35 min, **d** 55 min, **e** 2 h 35 min, **f** 45 min, **g** 2 h 55 min, **h** 2 h 43 min, **i** 4 h 39 min, **j** 49 min **19** 1 h 15 min **20** Start 14:15, Finish 15:50 **21** 7:05 pm **22** 4:25 pm **23** Tuesday 00:10 **24** **a** Departure 06:45, Arrival 14:10, **b** 7 h 25 min **25** 15 h 30 min **26** **a** Monday 4:55 pm, **b** Thursday 1:35 am, **c** Sunday 8:15 am, **d** Thursday 4:25 pm, **e** Sunday 1:43 pm, **f** Thursday 2:46 am

Exercise 3.2

1 **a** 7, **b** 12, **c** 52, **d** 365, **e** 14, **f** 26, **g** 4, **h** 3, **i** 10, **j** 10, **k** 100, **l** 10, **m** 1000, **n** 100 **2** **a** 36, **b** 35, **c** 8, **d** 104, **e** 240, **f** 60, **g** 70, **h** 30, **i** 250, **j** 2400 **3** **a** 2, **b** 3, **c** 8, **d** 3, **e** 3, **f** 4, **g** 30, **h** 2, **i** 10, **j** 500 **4** **a** 366, **b** February 29 **5** **a** April, June, September, November, **b** January, March, May, July, August, October, December, **c** 28 **6** **a** December, January, February, **b** March, April, May, **c** June, July, August, **d** September, October, November **7** Summer 90 days, Autumn 92 days, Winter 92 days, Spring 91 days **8** **a** 6, **b** 4, **c** 18, **d** 27 **9** **a** 15, **b** 43, **c** 46, **d** 67 **10** **a** 18, **b** 17 **11** **a** 19 January, **b** 4 June, **c** 5 January, **d** 1 December, **e** 13 September, **f** 3 September **12** 8 September **13** 38 days **14** **a** Thursday, **b** Saturday **15** 14 March **16** 4 June, 14 June **17** 18 September **18** 15 February for a non-leap year **19** **a** 3 AD, **b** 10 BC, **c** 1 AD **20** **a** 4, **b** 10, **c** 6, **d** 23, **e** 36, **f** 92 **21** 15 BC **22** 2221 **23** **a** 20th, **b** 18th, **c** 15th, **d** 6th, **e** 2nd, **f** 1st, **g** 20th, **h** 21st **24** 1st January, 1901, 31st December, 2000 **25** **a** Yes, **b** No, **c** No, **d** Yes, **e** Yes, **f** No, **g** No, **h** Yes, **i** No, **j** Yes

Exercise 3.3

1 **a** 156 min, **b** 187 min, **c** 479 min, **d** 1061 s, **e** 549 s, **f** 3199 s **2** **a** 86 min 40 s, **b** 12 min 45 s, **c** 116 min 43 s, **d** 3 h 38 min, **e** 17 h 59 min, **f** 35 h 23 min **3** **a** 11 658 s, **b** 63 809 s, **c** 32 507 s, **d** 1 h 1 min 10 s, **e** 1 h 59 min 45 s, **f** 7 h 46 min 3 s **4** **a** 6 h 30 min, **b** 13 h 5 min, **c** 10 min 56 s, **d** 24 min 43 s **5** **a** 8 h 51 min, **b** 1 h 32 min, **c** 9 min 21 s, **d** 26 min 49 s **6** **a** 18 315 s, **b** 39 647 s, **c** 8760 s, **d** 2940 s **7** **a** 2.28 h, **b** 3.78 h, **c** 0.36 h, **d** 7.01 h **8** **a** 257.15 min, **b** 1059 min, **c** 57.45 min, **d** 1380.25 min **9** **a** 48 h, **b** 14 h 36 min, **c** 5 h 26 min, **d** 1 h 6 min 40 s, **e** 21 min 36 s, **f** 2 h 52 min 12 s, **g** 1 h 37 min 30 s, **h** 16 h 43 min 42 s, **i** 12 min 3.5 s, **j** 1 min 25.25 s, **k** 12 h 32 min 45 s, **l** 52.5 s **10** **a** 5 h 8 s, **b** 9 h 25 min 28 s, **c** 11 h 32 min 24 s, **d** 4 h 57 min 57 s, **e** 8 h, **f** 7 h 10 min 53 s **11** **a** 4 h 4 min 7 s, **b** 38 min 51 s, **c** 4 h 41 min 28 s, **d** 3 h 23 min 24 s, **e** 2 h 53 min 9 s, **f** 1 h 59 min 59 s **12** **a** 1 h 5 min 24 s, **b** 2 h 32 min 15 s, **c** 19 h 21 min, **d** 23 h 28 min, **e** 32 h 21 min 52 s, **f** 45 h 3 min 20 s, **g** 3 h 50 min 12 s, **h** 19 h 50 min 8 s **13** **a** 32 min, **b** 43 min, **c** 3 min 44 s, **d** 2 min 17 s, **e** 7 min 51 s, **f** 49 min 18.5 s, **g** 1 h 32 min 44 s, **h** 1 h 29 min 43.4 s **14** **a** 2 h 10 min 5 s, **b** 10 min 4.8 s, **c** 48 min 3 s, **d** 29.57 s, **e** 21 h 54 min 42 s

Exercise 3.4

1 **a** 2 h 20 min, **b** **i** Language **ii** Science **iii** Language **iv** Maths **v** no subject, **c** **i** 1 h 20 min **ii** 1 h 14 min **iii** 42 min **iv** 25 min **v** 2 h 6 min, **d** Social Science, **e** Thursday **2** **a** 2:38 pm, **b** 4:23 pm, **c** 11:15 am, 20 min, **d** 6:07 pm, **e** **i** 58 min **ii** 28 min, **f** 12 min, **g** 7:22 pm, **h** 7:03 am, **i** 9 **3** **a** 1:00 pm, **b** 10:05 pm, **c** 48 min, **d** **i** 2 h 45 min **ii** 55 min **iii** 2 h **iv** 5 h **v** 1 h 5 min **vi** 30 min, **e** 3 h 30 min, **f** 1 h 50 min **4** **a** **i** 7:20 am **ii** 7:25 am **iii** 7:38 am **iv** 6:45 pm **v** 11:04 am **vi** 6:05 pm, **b** **i** 19 min **ii** 52 min **iii** 6 min **iv** 25 min **v** 42 min **vi** 10 min **5** **a** **i** John **ii** John's trip takes 36 min, Eli's trip takes 32 min **iii** Four minutes extra time allowed for the ferry travelling out of the city since this route will be busier, **b** Between Drummoyne and Gladesville, **c** **i** 1 h 25 min **ii** 1 h 42 min **iii** 1 h 46 min **iv** 1 h 10 min, **d** 3

Exercise 3.5

1

EST	CST	WST
12 noon	11:30 am	10 am
3:55 pm	3:25 pm	1:55 pm
1:36 pm	1:06 pm	11:36 am
6:15 pm	5:45 pm	4:15 pm
3:16 am	2:46 am	1:16 am
2:00 am	1:30 am	12 midnight

2 **a** 6:00 pm, **b** 4:00 pm, **c** 12:30 am the next morning **3** **a** 12:35 pm, **b** 9:32 pm, **d** 8:19 am **4** **a** 10:00 pm, **b** 11:50 pm, **c** 7:50 pm, **d** It is 12:30 am in Paris **6** **a** **i** midnight (12:00 am Tuesday) **ii** 5 am Tuesday, **b** **i** 54 hours **ii** 5 pm Wednesday

Chapter 3 Review

1 **a** 540 s, **b** $1\frac{1}{2}$ h, **c** 50 min, **d** 216 h **2** **a** $\frac{1}{40}$, **b** $\frac{1}{8}$, **c** $\frac{2}{3}$ **3** **a** 3:25, **b** 6:48 **4** **a** a quarter past six, **b** half past five, **c** twenty minutes to five **5** **a** 10:47 am, **b** 11:00 pm, **c** 7:27 pm **6** **a** 04:32, **b** 00:03, **c** 17:46 **7** **a** eleven o'clock in the evening, **b** a quarter to seven in the morning, **c** twenty minutes past three in the afternoon **8** **a** 15:00, **b** 15:55, **c** 00:15 **9** **a** 6 h 55 min, **b** 12 h 17 min, **c** 19 min, **d** 7 h 16 min, **e** 6 h 55 min **10** **a i** 4.22 h (2 decimal places) **ii** 15 206 s, **b i** 78 min 41 s **ii** 1 h 18 min 41 s, **c i** 283.2 min **ii** 16 992 s **11** **a** 6 h 52 min 10 s, **b** 3 h 31 min 54 s, **c** 14 h 35 min 46 s, **d** 17 min 9.67 s **12** 12:17 pm **13** 8:22 am Tuesday **14** **a** 30, **b** 91, **c** 120, **d** 30, **e** 42, **f** 84 **15** **a** 46, **b** 51, **c** 233, **d** 48 **16** **a** 11, **b** 575, **c** 1278, **d** 48 **17** **a** 20th century, **b** 7th century, **c** 1st century **18** **a** Yes, **b** No, **c** Yes **19** **a** 12:39, **b** 14:20 **20** **a** 5 h 6 min, **b** 1 h 38 min **21** 21 min **22** 8 **23** Wednesday, Thursday and Friday **24** **a** 2:30 am, **b** 10:30 am, **c** 8:49 pm, **d** 4:42 pm **25** **a** 9 am, **b** 1 h 10 min, **c** 2:28 pm

4 Fractions

Exercise 4.1

1 **a** $\frac{1}{2}$, **b** $\frac{1}{4}$, **c** $\frac{2}{3}$, **d** $\frac{3}{10}$, **e** $\frac{4}{7}$, **f** $\frac{5}{8}$ **3** **a** $\frac{3}{8}$, **b** $\frac{1}{4}$, **c** $\frac{2}{5}$ **4** **i a** $\frac{5}{8}$, **b** $\frac{7}{9}$, **c** $\frac{4}{11}$, **ii a** $\frac{3}{8}$, **b** $\frac{2}{9}$, **c** $\frac{7}{11}$ **5** **a** numerator = 3, denominator = 5, **b** numerator = 4, denominator = 9, **c** numerator = 12, denominator = 11 **6** **a** $\frac{6}{7}$, **b** $\frac{2}{13}$ **7** **a** one-quarter, **b** two-thirds, **c** eight-ninths, **d** three-sevenths, **e** five-twelfths, **f** eleven-fifteenths **8** **a** $\frac{1}{2}$, **b** $\frac{3}{4}$, **c** $\frac{7}{10}$, **d** $\frac{5}{8}$, **e** $\frac{4}{5}$, **f** $\frac{2}{9}$ **9** **a** 1, **b** $\frac{4}{4}$, **c** $\frac{6}{6}$, **d** $\frac{5}{5}$, **e** 1, **f** $\frac{7}{7}$ **10** **a** $\frac{3}{5}$, **b** $\frac{1}{4}$, **c** $\frac{3}{8}$, **d** $\frac{7}{10}$, **e** $\frac{8}{15}$ **11** **a** $\frac{6}{11}$, **b** $\frac{3}{11}$, **c** $\frac{2}{11}$ **12** **a** $\frac{4}{19}$, **b** $\frac{8}{19}$, **c** $\frac{7}{19}$, **d** $\frac{11}{19}$, **e** $\frac{12}{19}$, **f** $\frac{15}{19}$ **13** **a** $\frac{8}{19}$, **b** $\frac{11}{19}$, **c** $\frac{12}{19}$ **14** **a** $1 \div 3$, **b** $1 \div 5$, **c** $3 \div 4$, **d** $2 \div 7$, **e** $5 \div 8$, **f** $3 \div 14$ **15** **a** $\frac{1}{2}$, **b** $\frac{2}{3}$, **c** $\frac{7}{9}$, **d** $\frac{5}{8}$, **e** $\frac{6}{13}$, **f** $\frac{3}{10}$ **16** **a** 2, **b** 3, **c** 4, **d** 2, **e** 5, **f** 6, **g** 3, **h** 8, **i** 7, **j** 6, **k** 7, **l** 9 **17** **a** 3, **b** 5, **c** 11, **d** 2, **e** 3, **f** 5, **g** 1, **h** 2, **i** 6, **j** 4, **k** 3, **l** 7 **18** **a** 4, **b** 6, **c** 18, **d** 12, **e** 10, **f** 32 **19** $\frac{5}{12}$ **20** $\frac{1}{6}$ **21** sixteenths **22** **a** No, **b** No

Exercise 4.2

1 **a** PF, **b** IF, **c** MN, **d** PF, **e** MN, **f** IF, **g** MN, **h** PF, **i** PF, **j** IF, **k** IF, **l** MN, **m** PF, **n** MN, **o** IF, **p** PF **2** **a** $\frac{4}{3}$, **b** $\frac{8}{3}$, **c** $\frac{11}{3}$, **d** $\frac{13}{3}$, **e** $\frac{5}{3}$, **f** $\frac{7}{3}$, **g** $\frac{10}{3}$, **h** $\frac{14}{3}$ **3** **a** $1\frac{1}{2}$, $\frac{3}{2}$, **b** $1\frac{1}{4}$, $\frac{5}{4}$, **c** $1\frac{2}{3}$, $\frac{5}{3}$, **d** $2\frac{3}{4}$, $\frac{11}{4}$, **e** $2\frac{5}{6}$, $\frac{17}{6}$, **f** $2\frac{3}{5}$, $\frac{13}{5}$ **4** **a** $\frac{3}{2}$, **b** $\frac{8}{5}$, **c** $\frac{4}{3}$, **d** $\frac{17}{6}$, **e** $\frac{17}{8}$, **f** $\frac{23}{7}$, **g** $\frac{34}{9}$, **h** $\frac{53}{10}$, **i** $\frac{59}{12}$ **5** **a** $1\frac{1}{3}$, **b** $2\frac{1}{2}$, **c** $1\frac{5}{6}$, **d** $2\frac{1}{4}$, **e** $2\frac{2}{7}$, **f** $3\frac{4}{5}$, **g** $2\frac{5}{12}$, **h** $3\frac{8}{9}$, **i** $9\frac{3}{8}$ **6** **a** $\frac{6}{5}$, **b** $\frac{7}{4}$, **c** $\frac{14}{5}$, **d** $\frac{26}{7}$, **e** $\frac{31}{12}$, **f** $\frac{43}{8}$, **g** $\frac{50}{11}$, **h** $\frac{17}{3}$, **i** $\frac{19}{4}$, **j** $\frac{29}{10}$, **k** $\frac{13}{2}$, **l** $\frac{16}{7}$, **m** $\frac{29}{4}$, **n** $\frac{49}{5}$, **o** $\frac{53}{6}$, **p** $\frac{92}{9}$, **q** $\frac{40}{11}$, **r** $\frac{58}{5}$, **s** $\frac{41}{7}$, **t** $\frac{99}{8}$ **7** **a** $1\frac{1}{4}$, **b** $2\frac{2}{3}$, **c** $2\frac{1}{5}$, **d** $4\frac{1}{4}$, **e** $1\frac{5}{8}$, **f** $2\frac{5}{9}$, **g** $3\frac{7}{10}$, **h** $3\frac{2}{7}$, **i** $4\frac{2}{5}$, **j** $2\frac{1}{12}$, **k** $5\frac{3}{7}$, **l** $6\frac{4}{5}$, **m** $11\frac{1}{4}$, **n** $7\frac{2}{3}$, **o** $3\frac{2}{11}$, **p** $2\frac{5}{12}$, **q** $8\frac{5}{6}$, **r** $4\frac{7}{8}$, **s** $6\frac{9}{10}$, **t** $3\frac{4}{13}$ **8** **a** $\frac{8}{5}$, **b** $\frac{5}{2}$, **c** $\frac{39}{10}$, **d** $\frac{41}{9}$, **e** $\frac{47}{6}$ **9** **a** $1\frac{2}{5}$, **b** $3\frac{2}{3}$, **c** $5\frac{3}{4}$, **d** $6\frac{6}{7}$, **e** $6\frac{7}{9}$ **10** $\square = 8, 9, 10, 11, 12, \ldots, 18$ **11** $\square = 8, 9, 10, \ldots, 16$

Exercise 4.3

1 **a** $\frac{2}{4}$, **b** $\frac{2}{6}$, **c** $\frac{3}{12}$, **d** $\frac{6}{8}$, **e** $\frac{4}{10}$, **f** $\frac{8}{12}$, **g** $\frac{4}{8}$, **h** $\frac{9}{12}$, **i** $\frac{4}{6}$, **j** $\frac{12}{15}$, **k** $\frac{14}{16}$, **l** $\frac{20}{24}$ **2** **a** $\frac{2}{4}$, **b** $\frac{2}{6}$, **c** $\frac{3}{12}$, **d** $\frac{5}{30}$, **e** $\frac{4}{20}$, **f** $\frac{2}{14}$, **g** $\frac{3}{24}$, **h** $\frac{4}{12}$, **i** $\frac{7}{42}$, **j** $\frac{4}{36}$, **k** $\frac{5}{55}$, **l** $\frac{9}{27}$, **m** $\frac{12}{24}$, **n** $\frac{7}{35}$, **o** $\frac{6}{60}$, **p** $\frac{5}{20}$, **q** $\frac{8}{56}$, **r** $\frac{5}{40}$, **s** $\frac{8}{96}$, **t** $\frac{12}{132}$ **3** **a** $\frac{6}{9}$, **b** $\frac{9}{12}$, **c** $\frac{8}{20}$, **d** $\frac{6}{14}$, **e** $\frac{14}{77}$, **f** $\frac{15}{18}$, **g** $\frac{81}{90}$, **h** $\frac{28}{32}$, **i** $\frac{36}{42}$, **j** $\frac{28}{48}$, **k** $\frac{30}{33}$, **l** $\frac{30}{48}$, **m** $\frac{27}{36}$, **n** $\frac{20}{24}$, **o** $\frac{16}{72}$, **p** $\frac{15}{40}$, **q** $\frac{35}{63}$, **r** $\frac{55}{60}$, **s** $\frac{28}{40}$, **t** $\frac{24}{36}$ **4** **a** $\frac{5}{10}$, **b** $\frac{4}{7}$, **c** $\frac{5}{8}$, **d** $\frac{3}{4}$, **e** $\frac{7}{9}$, **f** $\frac{45}{50}$, **g** $\frac{5}{6}$, **h** $\frac{24}{36}$, **i** $\frac{3}{8}$, **j** $\frac{8}{9}$ **5** **a** $\frac{9}{6}$, **b** $\frac{14}{8}$, **c** $\frac{44}{20}$, **d** $\frac{66}{21}$, **e** $\frac{44}{24}$, **f** $\frac{63}{12}$, **g** $\frac{50}{35}$, **h** $\frac{46}{16}$, **i** $\frac{34}{24}$, **j** $\frac{64}{24}$, **k** $\frac{45}{12}$, **l** $\frac{110}{18}$

Exercise 4.4

1 a $\frac{1}{2}$, **b** $\frac{1}{4}$, **c** $\frac{2}{3}$, **d** $\frac{3}{4}$, **e** $\frac{2}{5}$, **f** $\frac{5}{6}$ **2 a** $\frac{1}{3}$, **b** $\frac{1}{6}$, **c** $\frac{1}{4}$ **3 a** $\frac{1}{4}$, **b** $\frac{1}{2}$, **c** $\frac{1}{5}$, **d** $\frac{1}{3}$, **e** $\frac{1}{5}$, **f** $\frac{1}{8}$, **g** $\frac{1}{7}$, **h** $\frac{1}{4}$, **i** $\frac{1}{11}$, **j** $\frac{1}{3}$, **k** $\frac{1}{5}$, **l** $\frac{1}{12}$, **m** $\frac{1}{6}$, **n** $\frac{1}{9}$, **o** $\frac{1}{10}$, **p** $\frac{1}{4}$, **q** $\frac{1}{8}$, **r** $\frac{1}{9}$, **s** $\frac{1}{9}$, **t** $\frac{1}{7}$, **u** $\frac{1}{8}$, **v** $\frac{1}{7}$, **w** $\frac{1}{6}$, **x** $\frac{1}{12}$ **4 a** $\frac{3}{5}$, **b** $\frac{3}{5}$, **c** $\frac{3}{5}$, **d** $\frac{5}{6}$, **e** $\frac{3}{5}$, **f** $\frac{7}{9}$, **g** $\frac{2}{9}$, **h** $\frac{4}{5}$, **i** $\frac{5}{11}$ **5 a** $\frac{4}{5}$, **b** $\frac{5}{6}$, **c** $\frac{7}{9}$, **d** $\frac{3}{5}$, **e** $\frac{8}{9}$ **6 a** $\frac{3}{4}$, **b** $\frac{2}{3}$, **c** $\frac{5}{6}$, **d** $\frac{2}{3}$, **e** $\frac{2}{7}$, **f** $\frac{8}{9}$, **g** $\frac{3}{5}$, **h** $\frac{2}{3}$, **i** $\frac{9}{10}$, **j** $\frac{4}{7}$, **k** $\frac{11}{12}$, **l** $\frac{2}{5}$, **m** $\frac{7}{11}$, **n** $\frac{2}{9}$, **o** $\frac{3}{11}$, **p** $\frac{5}{6}$, **q** $\frac{6}{7}$, **r** $\frac{4}{5}$, **s** $\frac{5}{6}$, **t** $\frac{7}{10}$, **u** $\frac{3}{4}$, **v** $\frac{10}{11}$, **w** $\frac{6}{7}$, **x** $\frac{8}{11}$ **7 a** $\frac{2}{9}$, **b** $\frac{7}{9}$, **c** $\frac{2}{5}$, **d** $\frac{7}{8}$, **e** $\frac{3}{10}$, **f** $\frac{5}{9}$, **g** $\frac{7}{8}$, **h** $\frac{4}{9}$, **i** $\frac{5}{8}$, **j** $\frac{4}{7}$, **k** $\frac{5}{6}$, **l** $\frac{7}{12}$, **m** $\frac{5}{12}$, **n** $\frac{6}{7}$, **o** $\frac{5}{9}$, **p** $\frac{6}{7}$, **q** $\frac{7}{8}$, **r** $\frac{11}{12}$ **8 a** $\frac{1}{30}$, **b** $\frac{3}{50}$, **c** $\frac{2}{15}$, **d** $\frac{9}{40}$, **e** $\frac{5}{8}$, **f** $\frac{9}{10}$, **g** $\frac{21}{250}$, **h** $\frac{3}{200}$ **9 a** $1\frac{1}{2}$, **b** $1\frac{1}{4}$, **c** $1\frac{1}{3}$, **d** $1\frac{1}{7}$, **e** $2\frac{1}{3}$, **f** $1\frac{2}{5}$, **g** $2\frac{4}{5}$, **h** $2\frac{1}{4}$, **i** $2\frac{2}{3}$, **j** $3\frac{2}{3}$, **k** $1\frac{5}{8}$, **l** $4\frac{1}{2}$ **10 a** $\frac{5}{7}$, **b** $\frac{3}{4}$, **c** $\frac{7}{9}$, **d** $\frac{11}{12}$, **e** $\frac{2}{3}$, **f** $\frac{5}{6}$, **g** $\frac{7}{10}$, **h** $\frac{4}{5}$

Exercise 4.5

1 a <, **b** >, **c** <, **d** <, **e** >, **f** > **2 a** $\frac{4}{7}$, 1, $\frac{8}{7}$, $1\frac{2}{7}$, **b** $\frac{5}{9}$, $\frac{8}{9}$, $1\frac{1}{9}$, $\frac{11}{9}$, **c** $\frac{7}{11}$, $\frac{10}{11}$, 1, $1\frac{3}{11}$, **d** $\frac{4}{5}$, $1\frac{1}{5}$, 2, $\frac{13}{5}$
3 a >, **b** <, **c** >, **d** >, **e** <, **f** < **4 a** $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{5}{12}$, $\frac{1}{2}$, **b** $\frac{1}{2}$, $\frac{9}{16}$, $\frac{19}{32}$, $\frac{5}{8}$, $\frac{3}{4}$, **c** $\frac{2}{5}$, $\frac{9}{20}$, $\frac{1}{2}$, $\frac{7}{10}$, $\frac{3}{4}$, **d** $\frac{1}{3}$, $\frac{13}{36}$, $\frac{7}{18}$, $\frac{5}{12}$, $\frac{4}{9}$, **e** $\frac{1}{3}$, $\frac{4}{9}$, $\frac{1}{2}$, $\frac{13}{18}$, $\frac{5}{6}$, **f** $\frac{4}{15}$, $\frac{3}{10}$, $\frac{1}{3}$, $\frac{11}{30}$, $\frac{2}{5}$, **g** $\frac{1}{3}$, $\frac{3}{8}$, $\frac{5}{12}$, $\frac{11}{24}$, $\frac{1}{2}$, **h** $\frac{1}{2}$, $\frac{21}{40}$, $\frac{11}{20}$, $\frac{3}{5}$, $\frac{5}{8}$, **i** $\frac{3}{10}$, $\frac{17}{50}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{8}{15}$
5 a $\frac{1}{7}$, $\frac{5}{11}$, $\frac{11}{12}$, $\frac{7}{6}$, **b** $\frac{1}{9}$, $\frac{3}{7}$, $\frac{12}{13}$, $\frac{6}{5}$, **c** $\frac{1}{14}$, $\frac{6}{13}$, $\frac{7}{8}$, $\frac{11}{10}$, **d** $\frac{3}{50}$, $\frac{11}{23}$, $\frac{15}{17}$, $\frac{8}{7}$, **e** $\frac{1}{8}$, $\frac{10}{19}$, $\frac{14}{15}$, $\frac{13}{9}$, **f** $\frac{1}{16}$, $\frac{7}{15}$, $\frac{21}{22}$, $\frac{8}{5}$, **g** $\frac{1}{2}$, $\frac{12}{23}$, $\frac{19}{20}$, $\frac{7}{6}$, **h** $\frac{2}{21}$, $\frac{8}{15}$, $\frac{13}{14}$, $\frac{4}{3}$, **i** $\frac{1}{18}$, $\frac{3}{5}$, $\frac{16}{17}$, $\frac{17}{16}$ **6 a** <, **b** >, **c** <, **d** >, **e** >, **f** <, **g** <, **h** >

Exercise 4.6

1 a A = $\frac{1}{3}$ B = $\frac{2}{3}$, **b** A = $\frac{1}{4}$ B = $\frac{3}{4}$, **c** A = $\frac{2}{5}$ B = $\frac{4}{5}$, **d** A = $\frac{1}{6}$ B = $\frac{5}{6}$, **e** A = $\frac{3}{8}$ B = $\frac{5}{8}$, **f** A = $\frac{2}{9}$ B = $\frac{7}{9}$
2 a P = $\frac{1}{2}$ Q = $1\frac{1}{2}$ R = $2\frac{1}{2}$, **b** P = $\frac{2}{3}$ Q = $1\frac{1}{3}$ R = $2\frac{2}{3}$, **c** P = $\frac{1}{4}$ Q = $1\frac{1}{4}$ R = $1\frac{3}{4}$, **d** P = $4\frac{1}{3}$ Q = $4\frac{2}{3}$ R = $5\frac{2}{3}$, **e** P = $5\frac{1}{4}$ Q = $5\frac{1}{2}$ R = $6\frac{3}{4}$, **f** P = $8\frac{2}{5}$ Q = $9\frac{4}{5}$ R = $10\frac{3}{5}$ **3 a** X = $1\frac{2}{3}$ Y = $2\frac{1}{3}$ Z = $\frac{2}{3}$, **b** X = $2\frac{3}{4}$ Y = $4\frac{1}{4}$ Z = $1\frac{3}{4}$, **c** X = $5\frac{4}{5}$ Y = $7\frac{2}{5}$ Z = $4\frac{3}{5}$, **d** X = $9\frac{2}{5}$ Y = $10\frac{1}{5}$ Z = $7\frac{2}{5}$ **4 a** E = 4 F = $3\frac{1}{2}$ G = $5\frac{1}{2}$ H = $2\frac{1}{2}$, **b** E = $6\frac{1}{3}$ F = $7\frac{2}{3}$ G = $8\frac{2}{3}$ H = $5\frac{2}{3}$, **c** E = 7 F = $8\frac{1}{2}$ G = $10\frac{1}{2}$ H = $4\frac{1}{2}$, **d** E = 2 F = $2\frac{2}{3}$ G = $4\frac{1}{3}$ H = $\frac{1}{3}$ **5 a** L = $4\frac{5}{8}$ M = $8\frac{3}{8}$ N = $13\frac{3}{8}$, **b** L = $2\frac{11}{15}$ M = $3\frac{8}{15}$ N = $5\frac{8}{15}$, **c** L = $4\frac{1}{2}$ M = $5\frac{1}{2}$ N = 2

Exercise 4.7

1 a $\frac{1}{2}$, **b** $\frac{1}{3}$, **c** $\frac{1}{4}$, **d** $\frac{1}{3}$, **e** $\frac{1}{4}$, **f** $\frac{1}{7}$, **g** $\frac{3}{4}$, **h** $\frac{2}{3}$, **i** $\frac{9}{11}$, **j** $\frac{2}{5}$, **k** $\frac{5}{8}$, **l** $\frac{3}{7}$ **2 a** 2, **b** 5, **c** 4, **d** $\frac{3}{2}$, **e** $\frac{5}{4}$, **f** $\frac{7}{6}$ **3 a** $\frac{1}{2}$, **b** $\frac{7}{12}$, **c** $\frac{1}{3}$, **d** $\frac{5}{8}$, **e** $\frac{3}{4}$, **f** $\frac{5}{7}$, **g** $\frac{3}{10}$, **h** $\frac{5}{8}$, **i** $\frac{7}{4}$, **j** $\frac{7}{11}$, **k** $\frac{9}{10}$, **l** $\frac{8}{9}$ **4 a** $\frac{2}{3}$, **b** $\frac{5}{12}$, **c** $\frac{3}{8}$, **d** $\frac{4}{5}$, **e** $\frac{1}{4}$, **f** $\frac{1}{3}$, **g** $\frac{5}{9}$, **h** $\frac{2}{5}$, **i** $\frac{2}{3}$, **j** $\frac{8}{9}$, **k** $1\frac{7}{9}$, **l** $1\frac{1}{2}$ **5** $\frac{7}{10}$ **6** $\frac{3}{40}$ **7** $\frac{3}{4}$ **8** $\frac{3}{8}$ **9** $\frac{3}{7}$ **10** $\frac{2}{3}$ **11** $\frac{1}{6}$ **12** $\frac{5}{9}$ **13** $\frac{2}{5}$ **14** $\frac{2}{9}$
15 $\frac{1}{86\,400}$ **16 a** $\frac{1}{3}$, **b** $\frac{1}{7}$, **c** $\frac{1}{6}$, **d** $\frac{1}{4}$, **e** $\frac{3}{8}$, **f** $\frac{5}{18}$, **g** $\frac{2}{3}$, **h** $\frac{7}{10}$ **17** $\frac{5}{6}$

Exercise 4.8

1 a $\frac{4}{7}$, **b** $\frac{5}{9}$, **c** $\frac{2}{3}$, **d** $\frac{3}{13}$, **e** $\frac{6}{11}$, **f** $\frac{4}{5}$, **g** 0, **h** 1, **i** 1, **j** 0, **k** 0, **l** 1, **m** $5\frac{1}{2}$, **n** $4\frac{2}{3}$, **o** $2\frac{4}{5}$, **p** $3\frac{7}{8}$ **2 a** $5\frac{1}{4}$, **b** $6\frac{2}{3}$, **c** $5\frac{6}{7}$, **d** $4\frac{1}{2}$, **e** $2\frac{4}{7}$, **f** $9\frac{1}{5}$, **g** $1\frac{7}{9}$, **h** $7\frac{2}{3}$, **i** $5\frac{6}{11}$, **j** $12\frac{4}{5}$, **k** $\frac{1}{4}$, **l** 1, **m** $\frac{1}{10}$, **n** $3\frac{5}{7}$, **o** $5\frac{8}{9}$, **p** $8\frac{12}{13}$ **3 a** 2, **b** 3, **c** 6, **d** 5, **e** 8, **f** 12, **g** 4, **h** 6, **i** 8, **j** 11, **k** 4, **l** 10 **4 a** $\frac{1}{2}$, **b** $\frac{1}{2}$, **c** $\frac{2}{5}$, **d** $\frac{1}{6}$, **e** $\frac{2}{3}$, **f** $\frac{4}{7}$, **g** $\frac{4}{5}$, **h** $\frac{3}{5}$, **i** $\frac{3}{8}$, **j** $\frac{3}{4}$, **k** $\frac{2}{3}$,

l $\frac{6}{7}$ **5 a** $4\frac{1}{3}$, **b** $5\frac{2}{3}$, **c** $3\frac{2}{5}$, **d** $1\frac{4}{5}$, **e** $6\frac{5}{6}$, **f** $11\frac{2}{3}$, **g** $6\frac{11}{15}$, **h** $4\frac{2}{3}$, **i** $6\frac{9}{10}$ **6 a** $1\frac{1}{5}$, **b** $1\frac{1}{3}$, **c** $1\frac{1}{5}$, **d** $1\frac{1}{2}$, **e** $4\frac{2}{3}$, **f** $6\frac{3}{5}$, **g** $7\frac{3}{7}$, **h** $3\frac{2}{3}$, **i** $15\frac{1}{2}$

Exercise 4.9

1 a $\frac{1}{2}$, **b** $\frac{3}{4}$, **c** $\frac{2}{3}$, **d** $\frac{5}{6}$, **e** $\frac{7}{10}$, **f** $\frac{2}{7}$, **g** $\frac{3}{5}$, **h** $\frac{5}{12}$ **2 a** $1\frac{1}{2}$, **b** $2\frac{4}{5}$, **c** $4\frac{2}{3}$, **d** $6\frac{3}{4}$, **e** $2\frac{1}{6}$, **f** $3\frac{1}{3}$, **g** $5\frac{3}{10}$, **h** $8\frac{4}{7}$, **i** $1\frac{3}{11}$, **j** $4\frac{10}{13}$, **k** $6\frac{1}{4}$, **l** $10\frac{7}{9}$ **3 a** $\frac{1}{2}$, **b** $2\frac{2}{3}$, **c** $1\frac{3}{5}$, **d** $4\frac{1}{8}$, **e** $4\frac{2}{3}$, **f** $1\frac{1}{4}$, **g** $6\frac{3}{8}$, **h** $1\frac{5}{12}$, **i** $4\frac{5}{9}$, **j** $3\frac{1}{6}$, **k** $5\frac{7}{10}$, **l** $10\frac{3}{16}$ **4 a** $\frac{2}{3}$, **b** $\frac{7}{9}$, **c** $\frac{7}{11}$, **d** $\frac{3}{5}$, **e** $1\frac{9}{13}$, **f** $2\frac{4}{5}$, **g** $3\frac{5}{7}$, **h** $5\frac{2}{3}$, **i** $4\frac{11}{15}$, **j** $6\frac{5}{11}$, **k** $7\frac{14}{17}$, **l** $3\frac{17}{19}$ **5 a** $\frac{6}{7}$, **b** $2\frac{4}{5}$, **c** $4\frac{11}{13}$, **d** $4\frac{2}{3}$, **e** $3\frac{7}{9}$, **f** $\frac{8}{15}$, **g** $2\frac{4}{5}$, **h** $6\frac{14}{17}$ **6 a** $\frac{3}{4}$, **b** $2\frac{2}{5}$, **c** $2\frac{7}{10}$, **d** $1\frac{1}{3}$, **e** $3\frac{1}{2}$, **f** $3\frac{2}{3}$, **g** $5\frac{2}{3}$, **h** $\frac{3}{5}$, **i** $2\frac{7}{10}$

Exercise 4.10

1 b $\frac{5}{6}$ **2 a** $\frac{3}{4}$, **b** $\frac{1}{10}$, **c** $\frac{5}{8}$, **d** $\frac{7}{10}$, **e** $\frac{1}{6}$, **f** $\frac{7}{12}$, **g** $\frac{13}{16}$, **h** $\frac{11}{20}$, **i** $\frac{11}{30}$, **j** $\frac{13}{18}$, **k** $\frac{1}{24}$, **l** $\frac{16}{21}$ **3 a** $\frac{5}{6}$, **b** $\frac{7}{12}$, **c** $\frac{3}{40}$, **d** $\frac{1}{15}$, **e** $\frac{17}{18}$, **f** $\frac{1}{20}$, **g** $\frac{5}{12}$, **h** $\frac{1}{20}$, **i** $\frac{19}{24}$, **j** $\frac{1}{18}$, **k** $\frac{19}{24}$, **l** $\frac{23}{30}$ **4 a** $3\frac{5}{6}$, **b** $2\frac{1}{12}$, **c** $5\frac{17}{20}$, **d** $3\frac{7}{8}$, **e** $5\frac{3}{20}$, **f** $4\frac{4}{9}$, **g** $2\frac{13}{18}$, **h** $4\frac{4}{21}$, **i** $3\frac{7}{20}$, **j** $6\frac{17}{20}$, **k** $1\frac{1}{24}$, **l** $3\frac{31}{36}$ **5 a** $1\frac{3}{10}$, **b** $1\frac{4}{15}$, **c** $1\frac{5}{12}$, **d** $1\frac{5}{18}$, **e** $1\frac{13}{20}$, **f** $1\frac{1}{24}$, **g** $1\frac{5}{22}$, **h** $1\frac{7}{24}$, **i** $1\frac{19}{60}$, **j** $1\frac{11}{18}$ **6 a** $3\frac{1}{12}$, **b** $4\frac{1}{12}$, **c** $3\frac{17}{28}$, **d** $4\frac{19}{30}$, **e** $7\frac{19}{60}$, **f** $5\frac{17}{24}$, **g** $3\frac{5}{22}$, **h** $4\frac{17}{30}$, **i** $6\frac{4}{15}$, **j** $4\frac{5}{12}$, **k** $5\frac{5}{42}$, **l** $7\frac{7}{36}$ **7 a** $\frac{2}{3}$, **b** $\frac{1}{4}$, **c** $\frac{1}{4}$, **d** $\frac{4}{5}$, **e** $\frac{1}{2}$, **f** $\frac{1}{4}$, **g** $1\frac{2}{3}$, **h** $1\frac{1}{2}$, **i** $1\frac{1}{3}$, **j** $1\frac{2}{5}$, **k** $1\frac{1}{6}$, **l** $1\frac{1}{4}$ **8 a** $1\frac{2}{3}$, **b** $2\frac{1}{3}$, **c** $6\frac{3}{4}$, **d** $3\frac{1}{2}$, **e** $1\frac{7}{9}$, **f** $9\frac{2}{5}$ **9 a** $1\frac{7}{12}$, **b** $2\frac{13}{18}$, **c** $1\frac{17}{18}$, **d** $\frac{19}{24}$, **e** $3\frac{23}{30}$, **f** $2\frac{23}{24}$, **g** $4\frac{11}{12}$, **h** $1\frac{11}{24}$, **i** $4\frac{27}{40}$, **j** $1\frac{13}{15}$, **k** $1\frac{16}{21}$, **l** $4\frac{31}{60}$

Exercise 4.11

1 a $\frac{1}{4}$, **b** $\frac{1}{12}$, **c** $\frac{8}{15}$ **2 a** $\frac{1}{8}$, **b** $\frac{1}{6}$, **c** $\frac{6}{35}$, **d** $\frac{27}{44}$, **e** $\frac{21}{40}$, **f** $\frac{15}{56}$, **g** $\frac{7}{48}$, **h** $\frac{10}{27}$, **i** $\frac{28}{45}$, **j** $\frac{55}{72}$ **3 a** $\frac{5}{8}$, **b** $\frac{2}{3}$, **c** $\frac{4}{7}$, **d** $\frac{3}{5}$, **e** $1\frac{2}{3}$, **f** $3\frac{1}{2}$, **g** $1\frac{1}{5}$, **h** $2\frac{3}{4}$, **i** $5\frac{1}{4}$, **j** $2\frac{2}{9}$, **k** $5\frac{1}{3}$, **l** $7\frac{1}{5}$ **4 a** $3\frac{3}{4}$, **b** $5\frac{5}{6}$, **c** $6\frac{9}{10}$, **d** $8\frac{4}{5}$, **e** $5\frac{1}{5}$, **f** $11\frac{3}{7}$, **g** $6\frac{2}{3}$, **h** $15\frac{3}{4}$ **5 a** $\frac{5}{8}$, **b** $\frac{11}{21}$, **c** $\frac{8}{25}$, **d** $\frac{1}{4}$, **e** $\frac{1}{6}$, **f** $\frac{1}{16}$, **g** $\frac{4}{9}$, **h** $\frac{4}{15}$, **i** $\frac{3}{8}$, **j** $\frac{18}{35}$, **k** $\frac{2}{15}$, **l** $\frac{20}{49}$, **m** $\frac{21}{32}$, **n** $\frac{15}{77}$, **o** $\frac{5}{18}$, **p** $\frac{7}{36}$ **6 a** $\frac{9}{11}$, **b** $\frac{3}{5}$, **c** $\frac{4}{5}$, **d** $1\frac{1}{20}$, **e** $6\frac{1}{3}$, **f** 14, **g** $4\frac{1}{2}$, **h** $4\frac{1}{2}$, **i** $3\frac{8}{9}$, **j** $2\frac{2}{3}$, **k** $2\frac{2}{9}$, **l** $1\frac{11}{16}$ **7 a** The number will be reduced. **b** The number stays the same. **c** The number will be increased. **8 a** $\frac{1}{8}$, **b** $\frac{1}{13}$, **c** $\frac{2}{3}$, **d** $\frac{14}{15}$, **e** $\frac{4}{9}$, **f** $2\frac{2}{3}$

Exercise 4.12

1 a $\frac{4}{3}$, **b** $\frac{11}{5}$, **c** $\frac{3}{2}$, **d** $\frac{9}{4}$, **e** $\frac{12}{7}$, **f** 2, **g** 4, **h** $\frac{1}{3}$, **i** $\frac{1}{5}$, **j** $\frac{1}{9}$, **k** $\frac{2}{3}$, **l** $\frac{3}{5}$, **m** $\frac{5}{13}$, **n** $\frac{7}{22}$, **o** $\frac{4}{19}$ **2 a** 2, **b** 3, **c** 5, **d** 8, **e** 4, **f** 8, **g** 9, **h** 20, **i** 15, **j** 12, **k** 28, **l** 48, **m** 3, **n** 5, **o** 7, **p** 10 **3 a i** 4, **ii** 6, **iii** $\frac{3}{4}$, **iv** $2\frac{1}{2}$, **b** Yes **4 a** $\frac{15}{28}$, **b** $\frac{6}{13}$, **c** $\frac{8}{9}$, **d** $\frac{63}{80}$, **e** $\frac{35}{54}$, **f** $\frac{33}{70}$, **g** $\frac{25}{32}$, **h** $\frac{24}{35}$, **i** $\frac{8}{21}$, **j** $\frac{55}{96}$ **5 a** $1\frac{7}{18}$, **b** $1\frac{7}{33}$, **c** $1\frac{11}{24}$, **d** $1\frac{8}{55}$, **e** $1\frac{11}{24}$ **6 a** $\frac{1}{15}$, **b** $\frac{3}{8}$, **c** $\frac{5}{24}$, **d** $7\frac{1}{2}$, **e** $5\frac{1}{3}$, **f** $2\frac{2}{5}$ **7 a** $\frac{2}{3}$, **b** $\frac{3}{5}$, **c** $\frac{9}{16}$, **d** $\frac{18}{35}$, **e** $\frac{44}{63}$, **f** $\frac{64}{81}$, **g** $1\frac{1}{4}$, **h** $1\frac{1}{5}$, **i** $1\frac{5}{6}$, **j** $1\frac{1}{9}$, **k** $1\frac{17}{28}$, **l** $1\frac{1}{21}$ **8 a** $\frac{5}{8}$, **b** $\frac{3}{4}$, **c** $1\frac{1}{8}$, **d** $2\frac{2}{9}$, **e** $2\frac{14}{15}$, **f** $\frac{3}{5}$, **g** $1\frac{2}{5}$, **h** $\frac{8}{25}$ **9 a** $\frac{7}{24}$, **b** $\frac{9}{40}$, **c** 1, **d** $1\frac{5}{44}$, **e** $1\frac{1}{6}$, **f** $\frac{7}{18}$

Exercise 4.13

1 a 50c, **b** 10c, **c** 25c, **d** 40c, **e** \$1.50, **f** \$9, **g** \$1.05, **h** \$1.76, **i** \$1.26, **j** \$1.80, **k** \$2.40, **l** \$3.60
2 a 20 min, **b** 30 s, **c** 6 h, **d** 45 min, **e** 4 yr, **f** 35 yr, **g** 80 s, **h** 30 h, **i** 270 min, **j** 72 min, **k** 40 h, **l** 24 yr
3 a 5 mm, **b** 10 cm, **c** 250 m, **d** 90 cm, **e** 6 mm, **f** 350 m, **g** 15 mm, **h** 800 m, **i** 250 cm, **j** 14 mm, **k** 42 cm, **l** 1875 m **4 a** 100 g, **b** 200 mg, **c** 500 kg, **d** 400 mL, **e** 500 L, **f** 750 mL, **g** 1600 g, **h** 2100 mg, **i** 1050 L,

j 1500 mL, **k** 3000 kg, **l** 270 L **5** 32 apples **6** 10 hours **7** 45 **8** 270 km **9** 15 000 **10** 45 mins **11** 75 cm **12** 39 kL **13** \$240 **14** Josh 160, David 180

Exercise 4.14

1 a 10, **b** 12, **c** 28, **d** 18, **e** 20, **f** 56 **2 a** 12, **b** 15, **c** 10, **d** 36, **e** 21, **f** 30, **g** 18, **h** 48, **i** 44 **3 a** 12, **b** 24, **c** 35, **d** 32, **e** 27, **f** 50, **g** 88, **h** 72, **i** 30, **j** 36, **k** 132, **l** 72, **m** 50, **n** 63, **o** 39 **4 a** 20, **b** 49, **c** 24, **d** 30 **5** \$24 **6** 56 **7** \$180 **8** 42 km **9** 72 **10** \$31.80 **11** 55 **12** Charles 40, Lorna 24

Exercise 4.15

1 28 L **2** \$240 **3** \$1500 **4** 18 **5 a** $\frac{5}{7}$, **b** $\frac{2}{7}$ **6** $\frac{2}{5}$ **7** \$4.50 **8** $\frac{39}{40}$ **9** $\frac{1}{3}$ **10** 30 **11 a** Greg, **b** $\frac{11}{30}$ **12** $\frac{11}{13}$ **13** \$1225 **14** \$686 **15** $1\frac{5}{6}$ m **16 a** $\frac{5}{12}$, **b** $\frac{7}{12}$ **17** $\frac{1}{15}$ **18** $2\frac{1}{2}$ **19** $2\frac{2}{3}$ **20** $1\frac{1}{24}$ **21** $6\frac{1}{4}$ h **22** 24 **23** $7\frac{9}{10}$ m **24** 115 km **25 a** \$480, **b** \$360 **26** $\frac{2}{21}$ **27** 33 **28** 5 h 50 min

Chapter 4 Review

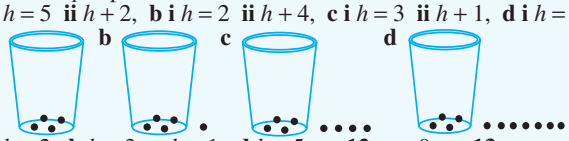
1 a $\frac{1}{2}$, **b** $\frac{5}{8}$, **c** $\frac{3}{4}$ **2** $\frac{7}{11}$ **3 a** $\frac{1}{4}$, **b** $\frac{3}{8}$, **c** $\frac{13}{12}$ **4** $\frac{2}{5}$ **5 a** $6 \div 11$, **b** $\frac{2}{5}$ **6 a** 6, **b** 20 **7 a** PF, **b** IF, **c** MN **8 a** $\frac{4}{3}$, **b** $\frac{11}{4}$, **c** $\frac{37}{8}$, **d** $\frac{70}{9}$ **9 a** $3\frac{1}{2}$, **b** $4\frac{2}{3}$, **c** $2\frac{4}{9}$, **d** $3\frac{5}{12}$ **10 a** $\frac{4}{12}$, **b** $\frac{4}{10}$, **c** $\frac{20}{35}$, **d** $\frac{56}{63}$, **e** $\frac{3}{4}$, **f** $\frac{5}{12}$, **g** $\frac{48}{66}$, **h** $\frac{45}{25}$ **11 a** $\frac{1}{4}$, **b** $\frac{1}{6}$, **c** $\frac{5}{6}$, **d** $\frac{5}{6}$, **e** $\frac{3}{4}$, **f** $\frac{4}{9}$, **g** $\frac{7}{8}$, **h** $\frac{7}{9}$ **12 a** $\frac{4}{15}$, **b** $\frac{9}{11}$, **c** $1\frac{3}{5}$ **13 a** >, **b** < **14 a** $\frac{17}{36}, \frac{5}{9}, \frac{7}{12}, \frac{11}{18}, \frac{3}{4}$, **b** $\frac{3}{8}, \frac{2}{5}, \frac{9}{20}, \frac{19}{40}, \frac{1}{2}$ **15 a** $\frac{1}{22}, \frac{5}{8}, \frac{99}{100}, \frac{13}{12}$, **b** $\frac{2}{23}, \frac{19}{40}, \frac{17}{18}, \frac{9}{8}$ **16 a** $A = \frac{1}{5}$ $B = \frac{4}{5}$ $C = 1\frac{1}{5}$, **b** $A = 3\frac{1}{3}$ $B = 4\frac{2}{3}$ $C = 5\frac{1}{3}$, **c** $A = 7\frac{3}{4}$ $B = 8\frac{1}{2}$ $C = 6\frac{3}{4}$, **d** $A = 2$ $B = 3\frac{1}{2}$ $C = \frac{1}{2}$ **17 a** $\frac{1}{3}$, **b** $\frac{3}{4}$, **c** $\frac{3}{5}$, **d** $\frac{5}{12}$, **e** $\frac{1}{3}$, **f** $\frac{5}{7}$ **18 a** $3\frac{2}{5}$, **b** $\frac{6}{7}$, **c** 0, **d** 1, **e** $\frac{3}{10}$, **f** $3\frac{4}{5}$, **g** 8, **h** 11 **19 a** $\frac{3}{4}$, **b** $\frac{3}{5}$, **c** $7\frac{1}{2}$, **d** $1\frac{2}{3}$, **e** $1\frac{1}{10}$, **f** $1\frac{1}{7}$ **20 a** $\frac{7}{10}$, **b** $6\frac{1}{4}$, **c** $2\frac{3}{5}$, **d** $4\frac{2}{5}$ **21 a** $\frac{3}{4}$, **b** $\frac{7}{12}$, **c** $\frac{10}{21}$, **d** $\frac{2}{15}$, **e** $\frac{11}{12}$, **f** $1\frac{7}{24}$, **g** $2\frac{2}{3}$, **h** $6\frac{13}{20}$ **22 a** $\frac{5}{48}$, **b** $3\frac{2}{3}$, **c** $5\frac{1}{4}$, **d** $\frac{5}{6}$, **e** $\frac{20}{63}$, **f** $1\frac{1}{8}$ **23 a** $\frac{7}{5}$, **b** $\frac{1}{6}$, **c** $\frac{4}{11}$ **24 a** $\frac{8}{13}$, **b** $\frac{3}{40}$, **c** $\frac{8}{45}$, **d** $\frac{9}{16}$, **e** $2\frac{1}{2}$, **f** $1\frac{17}{28}$ **25 a** \$1.50, **b** 72 seconds, **c** 105 cm, **d** 330 g **26 a** 12, **b** 45, **c** 20, **d** 42 **27 a** 20, **b** $\frac{2}{7}$, **c** $4\frac{1}{6}$, **d** \$315

5 Number patterns and pronumerals

Exercise 5.1

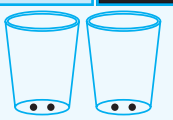
1 a $x = 2$, **b** $x = 5$, **c** $x = 7$, **d** $x = 1$ **3 a** II, **b** III, **c** I **5 a** $y + 1$, **b** $y + 4$, **c** $y + 2$, **d** $y + 5$, **e** $y + 3$, **f** $y + 6$ **6 a i** $k + 1$ **ii** $k + 2$ **iii** $k + 5$, **b i** $k + 1$ **ii** $k + 3$ **iii** $k + 6$, **c i** $k + 2$ **ii** $k + 5$ **iii** $k + 7$ **7 a** The number of counters outside the cup is 2. **b** There is a different number of counters inside each cup. **c** Yes, because they were free to choose what p represented. **e** No, because the two counters outside the cup to represent the +2 are missing. **8 a i** $h = 5$ **ii** $h + 2$, **b i** $h = 2$ **ii** $h + 4$, **c i** $h = 3$ **ii** $h + 1$, **d i** $h = 1$ **ii** $h + 3$, **e i** $h = 4$ **ii** $h + 5$, **f i** $h = 7$ **ii** $h + 6$

10 a **b** **c** **d**



11 a $b = 2$, **b** $b = 3$, **c** $b = 1$, **d** $b = 5$ **12** $u = 0$ **13 a** s, $s = 2$, **b** $s + 4$, $s = 5$, **c** $s + 2$, $s = 3$, **d** $s + 7$, $s = 0$ **14 b** Yes **15 a** $a + 3$, **b** $z + 5$, **c** $p + 4$ **16 a** $n + 6$, **b** $z + 8$, **c** $t + 9$ **17 a** $n = 4$, **b** The number of counters in the cup. **18 a** $a = 3$, **b** $t = 4$, **c** $s = 5$, **d** $c = 0$

Exercise 5.2

1 a

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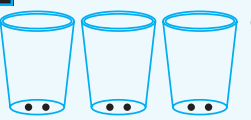
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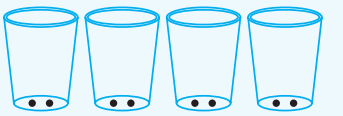
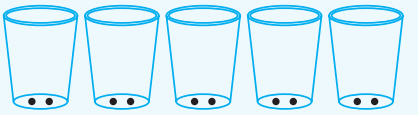
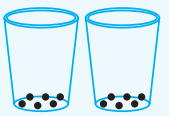

d


2 a $3w$, **b** $4k$ **3 a** $2w+1$, **b** $3w+3$, **c** $3w+5$, **d** $2w+4$

4 a

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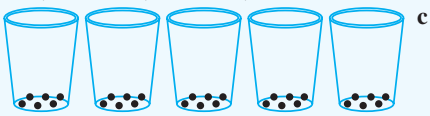
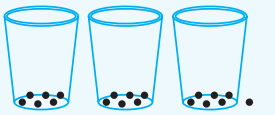

c

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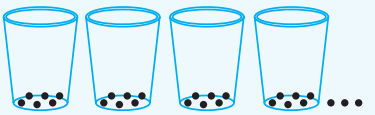
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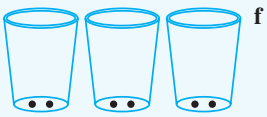
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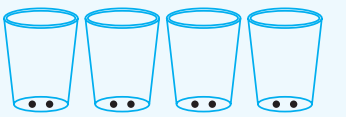
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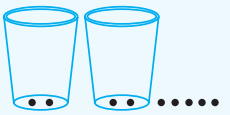
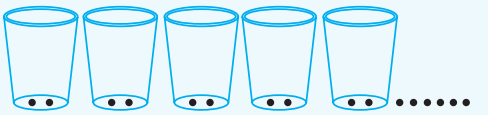

h


5 a i $j=3$ **ii** $4j+7$, **b i** $j=1$ **ii** $3j+5$ **7 a** The number of counters outside the cups. **b** Each group of students chose a different value for the variable—they chose to put a different number of counters in their cups. **c** $3x+5$
8 a 8, **b** 11, **c** 6, **d** 21, **e** 13, **f** 14, **g** 17, **h** 39 **9 b** Yes **10 b** Yes **11 a** $4b$, **b** $6j$, **c** $5v$ **12 a** Yes, **c** Yes, **d** Yes **13 b** Yes **14 a** $3m+n$, **b** $4g+3h$ **15 a** $2a+4b$, **b** $3u+3v$ **16 a** No, **b** No
18 a $4t+3$, **b** $5y+9$ **19 a** $2n+5$, **b** $10f+9$, **c** $8r+12$ **20 b** No, **c** $2(y+1)=2y+2$ **21 a** $2a+6$, **b** $3h+3$, **c** $4r+8$ **22 a** $3k+12$, **b** $2d+10$, **c** $5e+15$, **d** $7s+14$

Exercise 5.3

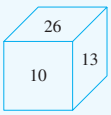
1 a $3a$, **b** $5m$, **c** $6k$, **d** $2n$, **e** $7p$, **f** $8y$, **g** ab, **h** pq, **i** mn, **j** $4uv$, **k** $12ef$, **l** $7cd$, **m** $6yz$, **n** $3gh$, **o** $10ij$
2 a $3c+5$, **b** $2k-7$, **c** $5m+n$, **d** $6+8z$, **e** $10-4q$, **f** $t+uv$, **g** $3p+2q$, **h** $4m-2n$, **i** $xy+9z$ **3 a** y, **b** d, **c** q, **d** a, **e** r, **f** v, **g** gh, **h** pq, **i** xy, **j** $a+c$, **k** $k+n$, **l** $p-4$, **m** $7+r$, **n** $a+b$, **o** $s-t$, **p** $2+e+f$ **4 a** a^2, **b** p^3, **c** m^4, **d** k^1, **e** t^5, **f** u^3, **g** $5n^2$, **h** $2y^3$, **i** $6c^4$, **j** $4f^2$, **k** $5w^3$, **l** $3e^4$, **m** a^2b, **n** mn^3, **o** p^3q^2, **p** $3ef^2$, **q** $7x^2y$, **r** $2r^2s^2$
5 a $\frac{a}{2}$, **b** $\frac{k}{3}$, **c** $\frac{p}{7}$, **d** $\frac{6}{m}$, **e** $\frac{14}{b}$, **f** $\frac{1}{h}$, **g** $\frac{p}{q}$, **h** $\frac{x}{y}$, **i** $\frac{u}{v}$, **j** $\frac{cd}{4}$, **k** $\frac{6y}{z}$, **l** $\frac{ef}{15g}$, **m** $\frac{8gh}{tv}$, **n** $\frac{abc}{de}$, **o** $\frac{2p}{3rst}$ **6 a** $2\times c$, **b** $8\times g$, **c** $9\times m$, **d** $x\times y$, **e** $c\times d$, **f** $u\times v$, **g** $7\times p\times q$, **h** $4\times e\times f$, **i** $3\times i\times j\times k$, **j** $6\times b+c$, **k** $3\times m-4$, **l** $m\times n+1$, **m** $2+r\times s$, **n** $p\times q+5\times r$, **o** $7\times h-p\times n$, **p** $a\times b\times c+5$, **q** $6-2\times g\times h\times i$, **r** $5+3\times a+b$, **s** $x-y\times z+w$, **t** $2\times p+3\times q-6\times r\times s$ **7 a** $n\times n$, **b** $k\times k\times k$, **c** $u\times u\times u\times u\times u$, **d** $d\times d\times d\times d$, **e** $5\times p\times p\times p$, **f** $7\times b\times b$, **g** $11\times s\times s\times s\times s$, **h** $3\times g\times g\times g\times g\times g$, **i** $x\times y\times y$, **j** $e\times e\times f$, **k** $m\times n\times n\times n$, **l** $r\times r\times r\times r\times s\times s$, **m** $3\times p\times p\times q\times q$, **n** $5\times u\times u\times u\times v$, **o** $2\times a\times a\times b\times b\times b\times b\times b$, **p** $11\times c\times c\times c\times c\times d\times d\times d$ **8 a** $w\div 4$, **b** $y\div 9$, **c** $11\div s$, **d** $1\div a$, **e** $g\div h$, **f** $k\div t$, **g** $(3\times u)\div v$, **h** $x\div(5\times y)$, **i** $(2\times p)\div(3\times q)$, **j** $4\div m+2$, **k** $15-t\div 7$, **l** $1\div c+1$, **m** $x\div 2+y\div 3$, **n** $5\div b-c\div 4$, **o** $f\div g+(7\times p)\div w$, **p** $(3\times x)\div y-(2\times z)\div w$, **q** $6+a\div 8+d$, **r** $13\times x-y\div z+7$, **s** $6\div e-f\div 7+1$, **t** $m\div 4+n\div 2-1\div p$
9 a $3(a+2)$, **b** $5(p-q)$, **c** $6(3m-2n)$, **d** $4(x+7)$, **e** $9(a-2)$, **f** $11(2c+5d)$, **g** $2a(x+3)$, **h** $4p(y-1)$, **i** $mn(7-t)$, **j** $6k(3+u)$, **k** $7n(2w-1)$, **l** $15(2a+7b)$, **m** $(x+2)(y+5)$, **n** $(p-1)(q+4)$, **o** $(2a+3b)(c-6d)$
10 a $\frac{a+3}{2}$, **b** $\frac{p-q}{8}$, **c** $\frac{2m-n}{4p}$, **d** $\frac{6}{p-2}$, **e** $\frac{t}{u+5}$, **f** $\frac{7f}{3g+4h}$, **g** $\frac{y+4}{z-4}$, **h** $\frac{1+u}{v+u}$, **i** $\frac{3a+4b}{a+b-c}$ **11 a** $5\times(a-7)$, **b** $6\times(q+2)$, **c** $e\times(f-g)$, **d** $(x+y)\times 3$, **e** $(m-4)\times 9$, **f** $(5+c)\times d$, **g** $2\times a\times(w-4)$, **h** $c\times d\times(g+7)$, **i** $(3-n)\times 6\times s$, **j** $(x+3)\times(x+4)$, **k** $(a+1)\times(b-1)$, **l** $(p-n)\times(q-m)$ **12 a** $(c+d)\div 3$, **b** $(w-2)\div 7$, **c** $(2x+y)\div 10$, **d** $4\div(h-1)$, **e** $1\div(p+3q)$, **f** $9n\div(e+f)$, **g** $(n+2)\div(y+7)$, **h** $(g+1)\div(g-1)$, **i** $(10+3k)\div(4m-13)$, **j** $(2w-1)\div(3x+5y)$ **13 a** $7(3p+5q)-4r$, **b** $5a(4b+c-3d)+1$, **c** $xy(\frac{5}{u}+\frac{1}{v})$, **d** $\frac{4m+9n}{xyz-6v+w}$, **e** $\frac{5+3e-f}{a(6f-1)}$, **f** $\frac{(5a-2b+cd)(3t-uvw)}{a-bc+1}$

Exercise 5.4

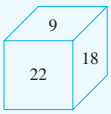
1 a 7, 9, 11, **b** 22, 19, 16, **c** 4, 8, 16, **d** 100, 50, 25, **e** 15, 20, 25, **f** 22, 18, 14, **g** 12, 24, 48, **h** 500, 250, 125, **i** 200, 40, 8, **j** 27, 39, 51, **k** 77, 66, 55, **l** 9, 3, 1 **2 a** 26, 54, **b** 12, 30, **c** 24, 18, **d** 9, 1, **e** 30, 16, **f** 23, 51
3 a Add 12; 47, 59, **b** Subtract 11; 29, 18, **c** Multiply by 2; 56, 112, **d** Add 15; 65, 80, **e** Divide by 2; 10, 5, **f** Subtract 13; 21, 8, **g** Multiply by 2; 32, 64, **h** Add 9; 46, 55, **i** Divide by 10; 10, 1 **4** 16, 256
5 a 10, 13, 12, **b** 34, 68, 66, **c** 16, 14, 17, **d** 96, 86, 258 **6 a** 36, 49, 64; Square numbers, **b** 21, 28, 36; Triangular numbers, **c** 13, 21, 34; Fibonacci numbers, **d** 17, 19, 23; Prime numbers
7 a 1, 2, 3, 4, …, **b** 1, 3, 6, 10, … **c** Yes,

d
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

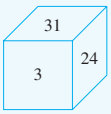
8 a The right face is 3 more than the front face. The top face is double the right face.



b The right face is 4 less than the front face. The top face is half the right face.



c The right face is 8 times the front face. The top face is 7 more than the right face.



9 a $(4 \times 6) + 1 = 5^2 = 25$; $(5 \times 7) + 1 = 6^2 = 36$; $(6 \times 8) + 1 = 7^2 = 49$, **b** $8^2 - 3^2 = 11 \times 5 = 55$; $9^2 - 4^2 = 13 \times 5 = 65$; $10^2 - 5^2 = 15 \times 5 = 75$, **c** $8 \times (1 + 2 + 3 + 4) + 1 = 81 = 9^2$; $8 \times (1 + 2 + 3 + 4 + 5) + 1 = 121 = 11^2$; $8 \times (1 + 2 + 3 + 4 + 5 + 6) + 1 = 169 = 13^2$, **d** $4^2 + 6^2 = 4 + 6 + 10 + 14 + 18 = 52$; $5^2 + 7^2 = 4 + 6 + 10 + 14 + 18 + 22 = 74$; $6^2 + 8^2 = 4 + 6 + 10 + 14 + 18 + 22 + 26 = 100$
10 a ∅ = double the first number then add the second number; 19, **b** ⊕ = subtract 1 from the first number then divide by the second number; 8, **c** ⧄ = divide the first number by 7 then add the second number; 13, **d** Ω = square the first number then subtract the second number; 88 **11 a** 21, **b** 201 **12 a** $T_n = n + 4$, 104 **b** $T_n = n + 8$, 108 **c** $T_n = 2n$, 200 **d** $T_n = 7n$, 700 **e** $T_n = n^2$, 10 000 **f** $T_n = 2n + 3$, 203 **g** $T_n = 3n - 1$, 299 **h** $T_n = 10n - 3$, 977

Exercise 5.5

1 a

Step number	1	2	3	4	5	6
Number of matches	3	6	9	12	15	18

b Adding 3. Multiples of 3.
c The number of matches = 3 × the step number.
d i 24 **ii** 39 **iii** 60

2 a

Step number	1	2	3	4	5	6
Number of matches	4	8	12	16	20	24

b Adding 4. Multiples of 4.
c The number of matches = 4 × the step number.
d i 36 **ii** 48 **iii** 200

3 a

Step number	1	2	3	4	5	6
Number of matches	2	3	4	5	6	7

b The number of matches = the step number + 1.
c i 11 **ii** 18 **iii** 41

4 a

Step number	1	2	3	4	5	6
Number of matches	4	5	6	7	8	9

b The number of matches = the step number + 3.
c i 16 **ii** 38 **iii** 103

5 a i

Step number	1	2	3	4	5	6
Number of matches	2	4	6	8	10	12

ii The number of matches = 2 × the step number.
iii 40

b i

Step number	1	2	3	4	5	6
Number of matches	3	4	5	6	7	8

ii The number of matches = the step number + 2.
iii 22

c i

Step number	1	2	3	4	5	6
Number of matches	5	10	15	20	25	30

ii The number of matches = 5 × the step number.
iii 100

d i

Step number	1	2	3	4	5	6
Number of matches	5	6	7	8	9	10

ii The number of matches = the step number + 4.
iii 24

6 a 2, **b**

Step number	4
Number of matches	9 1 + 2 + 2 + 2 + 2 1 + (4 × 2)

c The number of matches = 1 + (2 × the step number). **d i** 15 **ii** 21 **iii** 61

7 a 3, **b**

Step number	3	4
Number of matches	10 1 + 3 + 3 + 3 1 + (3 × 3)	13 1 + 3 + 3 + 3 + 3 1 + (4 × 3)

c The number of matches = 1 + (3 × the step number). **d i** 25 **ii** 37 **iii** 151

8 a 3, **b**

Step number	3	4
Number of matches	11 2 + 3 + 3 + 3 2 + (3 × 3)	14 2 + 3 + 3 + 3 + 3 2 + (4 × 3)

c The number of matches = 2 + (3 × the step number). **d i** 35 **ii** 62 **iii** 77

9 a 4, **b**

Step number	1	2	3	4
Number of matches	6 2 + 4 2 + (1 × 4)	10 2 + 4 + 4 2 + (2 × 4)	14 2 + 4 + 4 + 4 2 + (3 × 4)	18 2 + 4 + 4 + 4 + 4 2 + (4 × 4)

c The number of matches = 2 + (4 × the step number). **d i** 42 **ii** 70 **iii** 402

10 a i

Step number	1	2	3	4
Number of matches	5 1 + 4 1 + (1 × 4)	9 1 + 4 + 4 1 + (2 × 4)	13 1 + 4 + 4 + 4 1 + (3 × 4)	17 1 + 4 + 4 + 4 + 4 1 + (4 × 4)

ii 25 **iii** The number of matches = 1 + (4 × the step number) **v** 201

b i

Step number	1	2	3	4
Number of matches	5 3 + 2 3 + (1 × 2)	7 3 + 2 + 2 3 + (2 × 2)	9 3 + 2 + 2 + 2 3 + (3 × 2)	11 3 + 2 + 2 + 2 + 2 3 + (4 × 2)

ii 15 **iii** The number of matches = 3 + (2 × the step number). **v** 103

c i

Step number	1	2	3	4
Number of matches	7 3 + 4 3 + (1 × 4)	11 3 + 4 + 4 3 + (2 × 4)	15 3 + 4 + 4 + 4 3 + (3 × 4)	19 3 + 4 + 4 + 4 + 4 3 + (4 × 4)

ii 27 **iii** The number of matches = 3 + (4 × the step number). **v** 203

d i

Step number	1	2	3	4
Number of matches	10 7 + 3 7 + (1 × 3)	13 7 + 3 + 3 7 + (2 × 3)	16 7 + 3 + 3 + 3 7 + (3 × 3)	19 7 + 3 + 3 + 3 + 3 7 + (4 × 3)

ii 25 **iii** The number of matches = 7 + (3 × the step number). **v** 157 **11** Patterns a, d and f are described by the rule: 'The number of matches = 4 × the step number.' Patterns b, c and e are described by the rule: 'The number of matches = 1 + (3 × the step number).'

12 a

Step number	1	2	3	4
Number of pipe cleaners	8 1 + 7 1 + (1 × 7)	15 1 + 7 + 7 1 + (2 × 7)	22 1 + 7 + 7 + 7 1 + (3 × 7)	29 1 + 7 + 7 + 7 + 7 1 + (4 × 7)

The number of pipe cleaners = 1 + (7 × the step number). 85 pipe cleaners,

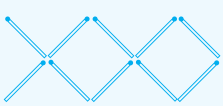
b

Step number	1	2	3	4
Number of pipe cleaners	12 4 + 8 4 + (1 × 8)	20 4 + 8 + 8 4 + (2 × 8)	28 4 + 8 + 8 + 8 4 + (3 × 8)	36 4 + 8 + 8 + 8 + 8 4 + (4 × 8)

The number of pipe cleaners = 4 + (8 × the step number). 100 pipe cleaners **13 a** The number of matches = 6 × the step number. Step 20, **b** The number of matches = 1 + (5 × the step number). Step 9.
15 a The number of dots = 26 – (2 × the step number). **b** 12, **c** The number of figures is limited because there will be no dots left. This will take 13 steps.

Exercise 5.6

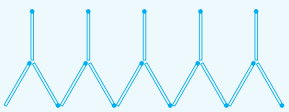
1 a $n = 2x$, **b** $n = 5x$, **c** $n = x + 3$, **d** $n = x - 9$, **e** $n = 3x + 4$, **f** $n = 6x - 10$
2 a Number of matches = 3 × the step number. **b** Number of matches = 7 × the step number.
c Number of matches = 2 + the step number. **d** Number of matches = the step number – 8.
e Number of matches = 2 × (the step number + 5). **f** Number of matches = 4 × (the step number – 1).
g Number of matches = 6 × (the step number – 3). **h** Number of matches = 9 × (the step number + 4).
3 a



b 14 **c**

Step number (v)	1	2	3	4	5
Number of matches (u)	2	4	6	8	10

d The number of matches = 2 × the step number. **e** $u = 2v$ **4 a**



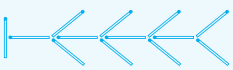
b 21

c

Step number (b)	1	2	3	4	5
Number of matches (a)	3	6	9	12	15

d The number of matches = 3 × the step number.
e $a = 3b$

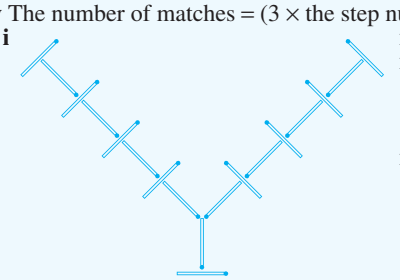
5 a i



ii 19 **iii**

Step number (q)	1	2	3	4	5
Number of matches (p)	4	7	10	13	16

iv The number of matches = (3 × the step number) + 1. **v** $p = 3q + 1$
b i

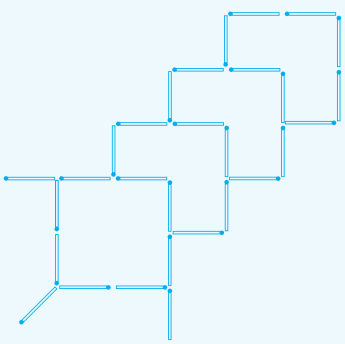


ii 26
iii

Step number (q)	1	2	3	4	5
Number of matches (p)	6	10	14	18	22

iv The number of matches = (4 × the step number) + 2 **v** $p = 4q + 2$

c i

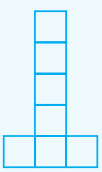


ii 41
iii

Step number (q)	1	2	3	4	5
Number of matches (p)	11	17	23	29	35

iv The number of matches = (6 × the step number) + 5 **v** $p = 6q + 5$

6 a i



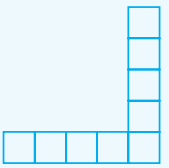
ii 8 **iii**

Step number (q)	1	2	3	4	5
Number of tiles (p)	4	5	6	7	8

iv The number of tiles = the step number + 3
v $p = q + 3$

ANSWERS

b i

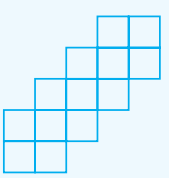


ii 11 **iii**

Step number (q)	1	2	3	4	5
Number of tiles (p)	3	5	7	9	11

iv The number of tiles = (2 × the step number) + 1
v $p = 2q + 1$

c i

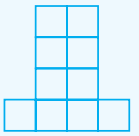


ii 16 **iii**

Step number (q)	1	2	3	4	5
Number of tiles (p)	4	7	10	13	16

iv The number of tiles = (3 × the step number) + 1
v $p = 3q + 1$

d i

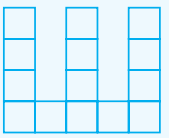


ii 12 **iii**

Step number (q)	1	2	3	4	5
Number of tiles (p)	4	6	8	10	12

iv The number of tiles = (2 × the step number) + 2
v $p = 2q + 2$

e i

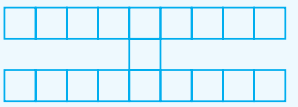


ii 17 **iii**

Step number (q)	1	2	3	4	5
Number of tiles (p)	5	8	11	14	17

iv The number of tiles = (3 × the step number) + 2
v $p = 3q + 2$

f i

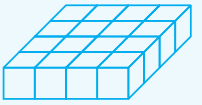


ii 23 **iii**

Step number (q)	1	2	3	4	5
Number of tiles (p)	7	11	15	19	23

iv The number of tiles = (4 × the step number) + 3 **v** $p = 4q + 3$

7a i

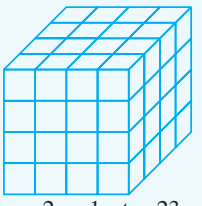


ii

Step number (x)	1	2	3	4	5
Number of cubes (n)	1	4	9	16	25

iii 36
iv The number of cubes = the square of the step number.
v $n = x^2$

b i



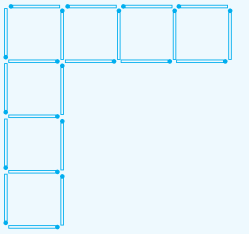
ii

Step number (x)	1	2	3	4	5
Number of cubes (n)	1	8	27	64	125

iii 216
iv The number of cubes = the cube of the step number.
v $n = x^3$

9 a $n = 2x - 1$; step 23,
b $n = 4x - 3$; step 12

10 a



b

Step number	1	2	3	4
Number of squares (s)	1	3	5	7
Number of matches (m)	4	10	16	22
Perimeter (p)	4	8	12	16

c i $m = 3s + 1$ **ii** $p = 2s + 2$,
d 28, **e** 20 units, **f** 28 units

Exercise 5.7

1 a

x	1	2	3	4	5	6	10	100
$x + 4$	5	6	7	8	9	10	14	104

b

x	1	2	3	4	5	6	20	100
$5x$	5	10	15	20	25	30	100	500

c

x	7	8	9	10	11	12	30	100
$x-3$	4	5	6	7	8	9	27	97

d

x	8	10	12	14	16	18	50	100
$\frac{x}{2}$	4	5	6	7	8	9	25	50

2 a

x	1	2	3	4
y	3	4	5	6

b

p	5	6	7	8
q	4	5	6	7

c

t	0	1	2	3
h	0	2	4	6

d

a	3	6	9	12
b	1	2	3	4

e

m	1	3	5	7
n	5	7	9	11

f

u	8	9	10	11
v	2	3	4	5

g

c	1	2	3	4
d	1	4	9	16

h

k	1	2	3	4
w	24	12	8	6

i

t	0	1	2	3
s	10	9	8	7

j

e	2	4	6	8
f	1	2	3	4

3 a

x	1	2	3	4
y	3	5	7	9

b

u	3	4	5	6
v	7	10	13	16

c

p	0	1	2	3
q	12	16	20	24

d

a	5	6	7	8
b	12	14	16	18

e

c	5	6	7	8
k	9	12	15	18

f

w	1	2	3	4
z	14	8	6	5

g

e	3	5	7	9
f	1	2	3	4

h

m	4	5	6	7
n	6	15	26	39

i

u	1	2	3	4
v	4	3	2	1

j

j	3	4	5	6
k	4	10	18	28

k

g	0	1	2	3
h	1	4	9	16

l

s	0	1	2	3
t	24	28	30	30

4 a $y = x + 5$, **b** $z = w - 2$, **c** $d = 3c$, **d** $h = \frac{t}{2}$, **e** $k = n - 7$, **f** $d = 6t$, **g** $h = 5g$, **h** $f = \frac{24}{e}$, **i** $n = 20 - m$, **j** $q = p + 8$, **k** $k = \frac{j}{11}$, **l** $s = r^2$ **5 a** $t = 2s + 1$, **b** $q = 3p + 2$, **c** $y = 2x + 3$, **d** $v = 3u - 1$, **e** $d = 4c - 3$, **f** $k = 5j + 2$, **g** $v = 2t - 4$, **h** $n = 3m - 3$, **i** $z = \frac{w}{2} + 1$, **j** $h = \frac{g}{3} + 5$, **k** $b = a^2 + 1$, **l** $f = e^2 - 2$

6 a $y = 4x$

x	1	2	3	5	10
y	4	8	12	20	40

b $y = x + 5$

x	1	2	5	8	12
y	6	7	10	13	17

c $y = 2x + 1$

x	1	3	7	12	25
y	3	7	15	25	51

d $y = 3x + 2$

x	2	3	7	10	11
y	8	11	23	32	35

e $y = 2x - 1$

x	2	4	9	13	15
y	3	7	17	25	29

f $y = 5x - 2$

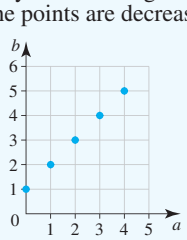
x	3	5	6	9	20
y	13	23	28	43	98

7 a $t = 7 - n$, **b** $t = 20 - 2n$, **c** $t = 25 - 3n$, **d** $t = 28 - 5n$, **e** $t = 21 - 4n$, **f** $t = 60 - 10n$

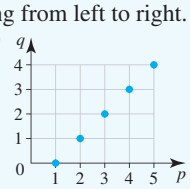
Exercise 5.8

1 a C, **b** J, **c** T, **d** W, **e** F, **f** E, **g** I, **h** S, **i** D, **j** X, **k** G, **l** R, **m** M, **n** N, **o** Q, **p** V, **q** O, **r** B, **s** U, **t** P, **u** K, **v** H, **w** A, **x** L **2 a** (2, 3), **b** (4, 1), **c** (1, 2), **d** (6, 7), **e** (3, 6), **f** (8, 1), **g** (1, 7), **h** (5, 4), **i** (4, 8), **j** (7, 5), **k** (6, 0), **l** (0, 5), **m** (2, 0), **n** (0, 3), **o** (0, 0), **p** (3, 4), **q** (7, 0), **r** (7, 6), **s** (6, 3), **t** (4, 3), **u** (5, 2), **v** (0, 7), **w** (1, 4), **x** (8, 5) **3 a** No, both the step number and the number of matches must be whole numbers. **b** They lie in a straight line. **c** The number of matches = the step number + 3. **d** $y = x + 3$ **4 b** Yes, **c** $y = 6 - x$, **d** The points are decreasing from left to right.

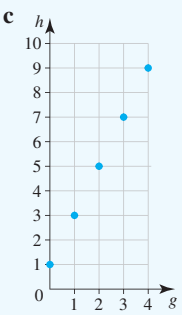
5 a



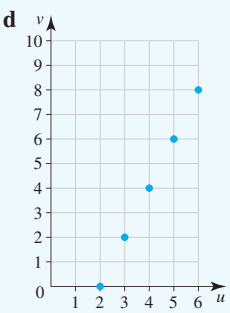
b



c

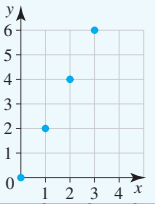


d



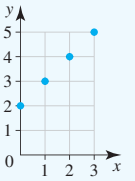
6 a

x	0	1	2	3
y	0	2	4	6



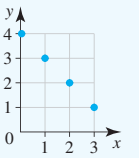
b

x	0	1	2	3
y	2	3	4	5



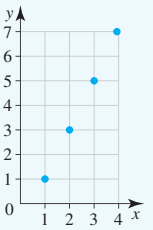
c

x	0	1	2	3
y	4	3	2	1



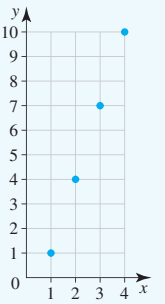
d

x	1	2	3	4
y	1	3	5	7



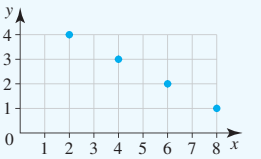
e

x	1	2	3	4
y	1	4	7	10



f

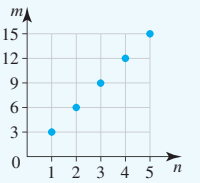
x	2	4	6	8
y	4	3	2	1



7 a

Step number (n)	1	2	3	4	5
Number of matches (m)	3	6	9	12	15

b

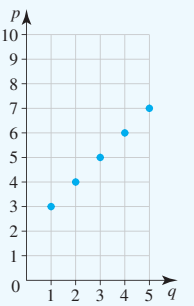


c The number of matches = $3 \times$ the step number.
d $m = 3n$

8 a

Number of triangles (q)	1	2	3	4	5
Number of dots (p)	3	4	5	6	7

b

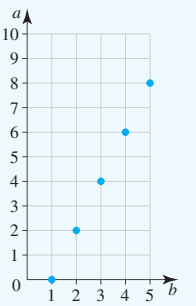


c The number of dots = the number of triangles + 2.
d $p = q + 2$, **e** 24,
f Yes. The number of dots (17) equals 2 plus the number of triangles (15).

9 a

Number of circles (b)	1	2	3	4	5
Number of dots (a)	0	2	4	6	8

b



c $a = 2b - 2$, **d** 28,
e No. You may only have a whole number of circles.
f No. $20 \neq (2 \times 9) - 2$

10 a $y = x + 4$

x	0	1	2	3	4	5	6	7
y	4	5	6	7	8	9	10	11

b $y = 5x$

x	1	2	3	4	5	6	7
y	5	10	15	20	25	30	35

c $y = 7 - x$

x	0	1	2	3	4	5	6	7
y	7	6	5	4	3	2	1	0

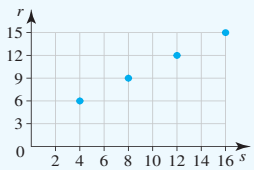
d $y = 10 - 2x$

x	1	2	3	4	5
y	8	6	4	2	0

11 a

Number of triangles (s)	4	8	12	16
Number of dots (r)	6	9	12	15

b



c Increasing by 3, or adding 3 each time, or multiples of 3.
d $4r - 3s = 12$ (or $r = \frac{3}{4}s + 3$)

Chapter 5 Review

1 a $4h$, **b** $9b$, **c** cd, **d** $2uv$, **e** p, **f** $3k + 7$, **g** $6g - 5$, **h** $12 - xy$, **i** $\frac{d}{4}$, **j** $\frac{13}{m}$, **k** $\frac{1}{t}$, **l** $\frac{4bc}{7de}$, **m** $5(c + 8)$, **n** $4(s - 3)$, **o** $6u(2v + 7)$, **p** $(m - n)(x + y)$ **2 a** $2 \times y$, **b** $5 \times a \times b$, **c** $8 \times q + 7$, **d** $10 \times t - 3 \times u \times v$, **e** $7 \times (k - 5)$, **f** $(d + 2) \times 9$, **g** $5 \times y \times (z - 10)$, **h** $(g + h) \times (p - q)$, **i** $r \div 2$, **j** $1 \div z$, **k** $(3 \times m) \div (4 \times n)$, **l** $(a + b) \div c$, **m** $a + (b \div c)$, **n** $e \div (f - g)$, **o** $c \div 5 + 9 \div d$, **p** $(3 \times x + 7 \times y) \div (2 \times z - w)$ **3 a** m^2, **b** e^4, **c** cd^2, **d** u^2v^2, **e** $4x^3$, **f** $2p^5$, **g** $5y^2z^3$, **h** $7a^3b$ **4 a** $a \times a \times a$, **b** $5 \times k \times k \times k \times k$, **c** $m \times m \times n \times n \times n \times n \times n$, **d** $3 \times x \times y \times y$
5 a 22, 50, **b** 20, 18 **6 a** Add six 29, 35, **b** Subtract nine 34, 25, **c** Multiply by three 162, 486

7 a

Step number (n)	1	2	3	4	5	6
Number of matches (b)	4	8	12	16	20	24

b Increasing by 4, or adding 4 each time, or multiples of 4, **c** The number of matches equals four times the step number. **d** $b = 4n$

8 a 6, **b**

Step number (p)	1	2	3	4
Number of matches (q)	10	16	22	28

c The number of matches = 6 × the step number + 4.
d $q = 6p + 4$, **f i** 46 **ii** 184 **iii** 604

9 a i

Step number (x)	1	2	3	4	5
Number of matches (y)	7	13	19	25	31

ii The number of matches equals six times the step number plus one. **iii** $y = 6x + 1$

b i

Step number (x)	1	2	3	4	5
Number of matches (y)	13	16	19	22	25

ii The number of matches equals three times the step number plus 10. **iii** $y = 3x + 10$

c i

Step number (x)	1	2	3	4	5
Number of matches (y)	12	23	34	45	56

ii The number of matches equals eleven times the step number plus one. **iii** $y = 11x + 1$

10 a

x	1	2	3	4
y	6	7	8	9

b

m	0	1	2	3
n	0	4	8	12

c

u	4	5	6	7
v	15	17	19	21

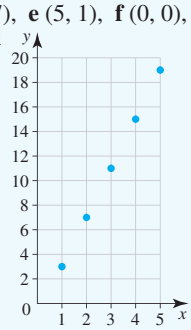
d

p	7	8	9	10
q	15	18	21	24

11 a $n = m + 4$, **b** $y = 7x$, **c** $q = p - 3$, **d** $s = 2r + 4$ **12 a** (2, 3), **b** (4, 6), **c** (7, 4), **d** (1, 7), **e** (5, 1), **f** (0, 0), **g** (0, 5), **h** (3, 0) **13 a i**

Step number (x)	1	2	3	4	5
Number of tiles (y)	3	7	11	15	19

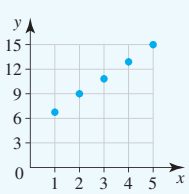
ii $y = 4x - 1$ **iii**



b i

Step number (x)	1	2	3	4	5
Number of tiles (y)	7	9	11	13	15

ii $y = 2x + 5$ **iii**



14 a $y = 2x + 1$, **b** $y = 9 - 2x$

6 Decimals

Exercise 6.1

1 **a** 0.7, **b** 0.03, **c** 0.009, **d** 0.08, **e** 0.002, **f** 0.5, **g** 0.01, **h** 0.0004, **i** 0.006, **j** 0.0003, **k** 0.02, **l** 0.00007
2 **a** $\frac{1}{10}$, **b** $\frac{9}{100}$, **c** $\frac{3}{1000}$, **d** $\frac{4}{10}$, **e** $\frac{8}{1000}$, **f** $\frac{5}{100}$, **g** $\frac{2}{10}$, **h** $\frac{6}{100}$, **i** $\frac{4}{10}$, **j** $\frac{7}{10\,000}$, **k** $\frac{4}{1000}$, **l** $\frac{5}{100\,000}$ **3** **a** 0.39, **b** 0.273, **c** 4.51, **d** 2.631, **e** 0.907, **f** 0.048, **g** 7.205, **h** 8.096, **i** 5.003, **j** 14.3007, **k** 22.0508, **l** 37.40209 **4** **a** 0.72, **b** 0.359, **c** 0.645, **d** 6.8, **e** 7.491, **f** 12.368, **g** 8.04, **h** 5.302, **i** 23.057, **j** 6.0807, **k** 1.0049, **l** 16.8002
5 **a** $\frac{5}{10} + \frac{7}{100}$, **b** $\frac{1}{10} + \frac{6}{100} + \frac{2}{1000}$, **c** $\frac{3}{10} + \frac{2}{100} + \frac{7}{1000} + \frac{4}{10\,000}$, **d** $5 + \frac{3}{10}$, **e** $8 + \frac{9}{100}$, **f** $4 + \frac{1}{10} + \frac{3}{1000}$, **g** $11 + \frac{6}{1000}$, **h** $15 + \frac{3}{100} + \frac{8}{10\,000}$, **i** $20 + \frac{6}{10} + \frac{1}{10\,000}$, **j** $23 + \frac{5}{10} + \frac{9}{1000} + \frac{4}{10\,000}$ **6** **a** $\frac{3}{10}$ **b** $\frac{3}{1000}$ **c** $\frac{3}{100}$ **d** $\frac{3}{1000}$ **e** $\frac{3}{10}$ **f** $\frac{3}{10\,000}$ **g** $\frac{3}{100}$ **h** $\frac{3}{100\,000}$ **7** **a** $\frac{7}{100}$ **b** $\frac{9}{10}$ **c** $\frac{1}{1000}$ **d** $\frac{3}{100}$ **e** $\frac{7}{1000}$ **f** $\frac{9}{10\,000}$ **g** $\frac{1}{100}$ **h** $\frac{3}{100\,000}$ **8** **a** 0.356, **b** 0.408, **c** 5.0904, **d** 32.76, **e** 485.203, **f** 708.041, **g** 3060.5007, **h** 9004.0083 **9** **a** 0.13, **b** 0.247, **c** 0.3596, **d** 0.57, **e** 0.198, **f** 0.062, **g** 0.0425, **h** 0.0067, **i** 0.00013, **j** 2.27, **k** 5.134, **l** 9.079, **m** 4.0315, **n** 12.0096, **o** 35.00102
10 **a** $\frac{73}{100}$, **b** $\frac{169}{1000}$, **c** $\frac{2487}{10\,000}$, **d** $\frac{51}{1000}$, **e** $\frac{717}{10\,000}$, **f** $\frac{33}{10\,000}$, **g** $4\frac{63}{100}$, **h** $10\frac{67}{1000}$, **i** $18\frac{81}{10\,000}$, **j** $2\frac{301}{10\,000}$, **k** $11\frac{99}{1000}$, **l** $24\frac{63}{100\,000}$ **11** **a** 3.6, **b** 1.27, **c** 8.69, **d** 48.3, **e** 2.05, **f** 1.354, **g** 67.08, **h** 3.001, **i** 23.566, **j** 128.9, **k** 80.06, **l** 54.1627 **12** **a** $\frac{7}{10}$, **b** $\frac{3}{100}$, **c** $\frac{1}{1000}$, **d** $\frac{9}{10}$, **e** $\frac{1}{10}$, **f** $5\frac{3}{10}$, **g** $8\frac{9}{100}$, **h** $12\frac{7}{1000}$, **i** $15\frac{3}{10\,000}$ **13** **a** 0.30, 0.300, **b** 0.020, 0.02, **c** 5.070, 5.0700, **d** 6, 6.0 **14** **a** 5.20, **b** 0.080, **c** 9.0, **d** 3.600, **e** 12.00 **f** 5.0100 **15** **a** 1.274, **b** 2.03 **16** **a** 0.7, **b** 0.3, **c** 0.4, **d** 0.37, **e** 0.26 **17** **a** 0.2, **b** 0.3, **c** 0.5, **d** 0.8, **e** 0.01, **f** 0.04, **g** 0.06, **h** 0.09, **i** 1.1, **j** 1.2, **k** 1.5, **l** 1.7 **18** **a** 0.14, **b** 1.4 **19** No **20** **a** 0.1, **b** 0.2, **c** 0.3

Exercise 6.2

1 **a** >, **b** <, **c** >, **d** >, **e** <, **f** >, **g** <, **h** >, **i** >, **j** <, **k** >, **l** < **2** **a** >, **b** <, **c** >, **d** >, **e** >, **f** >, **g** <, **h** <, **i** >, **j** <, **k** >, **l** > **3** **a** 0.004, 0.404, **b** 0.067, 0.76, **c** 1.002, 1.22, **d** 0.099, 0.99 **4** **a** 0.08, 0.87, 0.9, 1.4, 1.52, **b** 0.07, 0.18, 0,2, 0.519, 0.64, **c** 0.6, 0.91, 1.01, 1.1, 2, **d** 0.099, 2.54, 2.7, 3.07, 3.5, **e** 0.08, 0.3, 0.65, 1.84, 3.0, **f** 0.7, 0.95, 1.6, 2.41, 4.52 **5** **a** 0.04, 0.044, 0.4, 0.404, 0.44, **b** 0.066, 0.6, 0.606, 0.66, 0.666, **c** 0.007, 0.7, 0.707, 0.77, 7.0, **d** 0.012, 0.102, 0.21, 1.02, 2.01, **e** 0.038, 0.083, 0.38, 0.803, 3.8, **f** 0.059, 0.09, 0.5, 0.905, 0.95 **6** **a** C, **b** C, **c** A, **d** D, **e** D, **f** C **7** **a** 0.4, 0.525, **b** 0.819, 0.5, 0.095 **8** **a** <, **b** <, **c** >, **d** <, **e** <, **f** >

Exercise 6.3

1 D **2** **a** 1.5, **b** 7.5, **c** 3.5, **d** 14.5, **e** 29.5, **f** 99.5 **3** **a** 3.5, **b** 4.5, **c** 8.5, **d** 2.5, **e** 7.5, **f** 6.5, **g** 4.5, **h** 18.5 **4** **a** 0.2, **b** 0.8, **c** 0.1, **d** 0.9, **e** 0.5, **f** 0.5, **g** 1.0, **h** 0.9, **i** 1.1 **5** **a** 0.15, **b** 0.75, **c** 0.35, **d** 0.65, **e** 0.95, **f** 0.05, **g** 1.45, **h** 3.85, **i** 7.25, **j** 11.15, **k** 6.05, **l** 12.95 **6** **a** 0.025, **b** 0.045, **c** 0.085, **d** 0.315, **e** 0.735, **f** 0.655, **g** 0.075, **h** 3.115, **i** 6.235, **j** 10.085, **k** 14.695, **l** 0.095, **m** 0.005, **n** 5.995, **o** 8.005, **p** 0.1135 **8** **a** 0.025, **b** 0.035, **c** 0.145, **d** 0.555, **e** 2.085, **f** 6.015

Exercise 6.4

1 **a** 19.97, **b** 23.112, **c** 357.138, **d** 853.242, **e** 4498.018, **f** 8793.768 **2** **a** 41.25, **b** 550.1, **c** 387.09, **d** 473.24, **e** 2727.919, **f** 1087.081 **3** **a** 23.2, **b** 2.08, **c** 32.35, **d** 103.877, **e** 209.831, **f** 34.791, **g** 451.92, **h** 663.134 **4** **a** 1.7, **b** 8.82, **c** 36.08, **d** 83.56, **e** 162.332, **f** 546.6, **g** 317.39, **h** 884.46, **i** 332.23, **j** 1168.75, **k** 1806.85, **l** 2516.574 **5** **a** 13.58 + 29.76 = 43.34, **b** 124.52 – 45.18 = 79.34, **c** 7364.04 – 5829.76 = 1534.28

Exercise 6.5

1 **a** 0.3, **b** 0.1, **c** 1, **d** 1.1, **e** 1.5, **f** 1.7, **g** 0.9, **h** 0.6, **i** 0.7, **j** 1.7, **k** 3.5, **l** 6.2, **m** 1.3, **n** 8.2, **o** 0.7, **p** 1, **q** 3, **r** 9, **s** 1.1, **t** 0.4 **2** **a** 0.08, **b** 0.5, **c** 0.14, **d** 0.08, **e** 0.21, **f** 0.31, **g** 0.13, **h** 0.34, **i** 0.95, **j** 0.88, **k** 0.75, **l** 1.92, **m** 4.96, **n** 9.85, **o** 2.33, **p** 3.11 **3** Because 0.5 is five-tenths, not five-hundredths. **4** **a** 0.16, **b** 0.28, **c** 0.02, **d** 0.27, **e** 0.41, **f** 0.76, **g** 1.04, **h** 1.05, **i** 1.35, **j** 1.06, **k** 1.07, **l** 0.98, **m** 0.84, **n** 3.28, **o** 3.99, **p** 3.5
5 **a** 1, 1.2, **b** 1.2, 1.5, **c** 1.1, 1.4, **d** 3.5, 4.2, **e** 0.2, 0.18, **f** 2.4, 3, **g** 0.81, 0.78, **h** 0.15, 0.1, **i** 1.38, 1.4, **j** 1, 1.25, **k** 0.6, 0.75, **l** 0.85, 0.65, **m** 0.8, 0.5, **n** 0.01, 0.005 **6** **a** 0.8, **b** 0.7, **c** 1.2, **d** 0.6, **e** 2.2, **f** 1.8, **g** 3.2, **h** 3.5, **i** 5, **j** 0.08, **k** 0.23, **l** 1.36, **m** 0.143, **n** 0.055, **o** 1.332, **p** 2.24 **7** **a** 0.27, **b** 0.72, **c** 1.38, **d** 0.25, **e** 0.56, **f** 0.25, **g** 0.92, **h** 0.18, **i** 0.8, **j** 0.94, **k** 1.8, **l** 4.85 **8** **a** 0.06, 0.065, **b** 0.01, 0.005, **c** 0.095, 0.1, **d** 2.645, 2.65, **e** 1.34, 1.345 **f** 1.295, 1.3

Exercise 6.6

1 **a** 0.5, **b** 0.6, **c** 0.8, **d** 0.06, **e** 0.35, **f** 0.36, **g** 0.3, **h** 0.2, **i** 0.4, **j** 1, **k** 2, **l** 9, **m** 1.5, **n** 4.2, **o** 3.2, **p** 0.48, **q** 0.77, **r** 0.45, **s** 2.8, **t** 4.5, **u** 10.5, **v** 6.24, **w** 4.84, **x** 6.75 **y** 10.6 **2** **a** 12.6, **b** 23.8, **c** 20.8, **d** 68.4, **e** 1.38, **f** 7.6, **g** 10.35, **h** 45.15, **i** 31.278, **j** 53.88, **k** 56.712, **l** 211.86 **3** **a** 23.5, **b** 19.2, **c** 1.98, **d** 16.2, **e** 43.2, **f** 25.6, **g** 187.4, **h** 752.48 **4** **a** 21.7, **b** 349.6, **c** 7462.3, **d** 7.5, **e** 0.3, **f** 3.16, **g** 9, **h** 62, **i** 19, **j** 0.7, **k** 14.85, **l** 2874.3, **m** 0.63, **n** 0.0075, **o** 12.3 **5** **a** 20, **b** 70, **c** 160, **d** 13 000, **e** 24 830, **f** 5900, **g** 980, **h** 40, **i** 8000, **j** 51 250, **k** 306 100, **l** 700 200, **m** 62 400, **n** 90, **o** 670 300 **6** **a** 28, **b** 72, **c** 390, **d** 320, **e** 396, **f** 880, **g** 77, **h** 600, **i** 3750

Exercise 6.7

1 **a** 0.08, **b** 0.35, **c** 0.018, **d** 0.0012, **e** 0.045, **f** 0.54, **g** 0.084, **h** 0.30, **i** 0.004, **j** 0.48, **k** 0.39, **l** 0.099, **m** 0.014, **n** 0.060, **o** 1.32, **p** 0.714, **q** 1.08, **r** 0.0505, **s** 1.82, **t** 4.32 **2** **a** 0.24, **b** 0.27, **c** 0.32, **d** 0.012, **e** 0.042, **f** 0.018, **g** 0.0012, **h** 0.0035, **i** 0.0048, **j** 0.45, **k** 0.021, **l** 0.0015, **m** 0.002, **n** 0.0032, **o** 0.12, **p** 0.006, **q** 0.042, **r** 0.0072 **3** **a** 0.4, **b** 0.03, **c** 0.02, **d** 0.003, **e** 0.06, **f** 0.01, **g** 0.008, **h** 0.003 **4** **a** 0.117, **b** 0.126, **c** 0.084, **d** 0.0096, **e** 0.0795, **f** 0.948, **g** 0.033, **h** 25.56, **i** 3.008, **j** 11.48, **k** 0.4884, **l** 3.64575 **5** **a** 0.09, **b** 0.49, **c** 0.36, **d** 0.0016, **e** 0.0144, **f** 0.001, **g** 0.008, **h** 0.000027 **6** 10 **7** **a** The number will decrease.
b The number will increase. **8** **a** 0.0342, **b** 0.555, **c** 0.0975, **d** 2.704, **e** 2.4966, **f** 0.22458 **9** **a** 0.3, **b** 0.4, **c** 0.45, **d** 0.35, **e** 0.01, **f** 0.08, **g** 0.13, **h** 0.6, **i** 1.05, **j** 0.9, **k** 0.06, **l** 2.1, **m** 1.5, **n** 0.2, **o** 4.2 **10** **a** 0.1, **b** 0.2, **c** 0.3, **d** 0.6, **e** 0.01, **f** 0.06, **g** 0.03, **h** 0.12, **i** 0.07, **j** 0.09, **k** 0.09, **l** 0.33, **m** 0.4, **n** 0.8, **o** 1.8, **p** 0.05, **q** 0.45, **r** 1.35, **s** 2.1, **t** 1.03 **11** **a** 0.7, **b** 4, **c** 0.9, **d** 3, **e** 0.08, **f** 0.6, **g** 0.7, **h** 0.14, **i** 0.03, **j** 0.5, **k** 0.01, **l** 12

Exercise 6.8

1 **a** 0.4, **b** 0.3, **c** 0.3, **d** 0.02, **e** 0.03, **f** 0.08, **g** 0.04, **h** 0.07, **i** 0.09, **j** 0.05, **k** 0.25, **l** 0.08, **m** 0.12, **n** 0.05, **o** 0.07, **p** 0.7, **q** 0.3, **r** 0.7, **s** 0.5, **t** 0.5 **2** **a** 5.5, **b** 9.6, **c** 2.75, **d** 8.375 **3** **a** 2.43, **b** 1.04, **c** 82.9, **d** 28.47, **e** 53.24, **f** 0.0158, **g** 2.49, **h** 4.472, **i** 43.159, **j** 6.038, **k** 17.03, **l** 20.867, **m** 62.13, **n** 5.824, **o** 3.84, **p** 16.62, **q** 0.58175, **r** 0.2875 **4** **a** 1.45, **b** 1.862, **c** 3.50481, **d** 0.32, **e** 0.1694, **f** 0.159, **g** 7.2, **h** 3.84, **i** 52.713, **j** 0.36, **k** 0.4, **l** 0.498, **m** 3, **n** 0.6, **o** 0.81, **p** 0.502 **5** **a** 0.09, **b** 0.026, **c** 0.0134, **d** 0.0561, **e** 0.003184, **f** 0.3751, **g** 0.0802, **h** 0.0001093, **i** 0.0000565, **j** 0.008, **k** 0.002, **l** 0.000043, **m** 0.0007, **n** 0.00014, **o** 0.000006, **p** 0.00000025 **6** **a** 1000, **b** 100, **c** 100, **d** 1000, **e** 16.34, **f** 51.2, **g** 237, **h** 54 000, **i** 1000, **j** 0.043, **k** 0.028, **l** 1000

Exercise 6.9

1 **a** 2, **b** 7, **c** 4, **d** 7, **e** 132, **f** 34, **g** 0.6, **h** 0.7, **i** 0.5, **j** 0.08, **k** 0.12, **l** 2, **m** 80, **n** 600, **o** 40, **p** 7, **q** 600, **r** 0.8, **s** 5.2, **t** 9.1 **2** **a** 0.9, **b** 0.7, **c** 0.09, **d** 1.6, **e** 21.8, **f** 35.14, **g** 17.16, **h** 49.07, **i** 64.75, **j** 113.8, **k** 113.7, **l** 178.34, **m** 6.1, **n** 5.64, **o** 17.258, **p** 8.9, **q** 819.9, **r** 1547.2 **3** **a** 610, **b** 70, **c** 1160, **d** 1190, **e** 144, **f** 300, **g** 3050, **h** 9800, **i** 560 **4** **a** 210, **b** 204, **c** 324.5, **d** 20.2, **e** 5.7, **f** 16.3, **g** 49.5, **h** 38, **i** 250 **5** **a** 2, **b** 5, **c** 3, **d** 4, **e** 7, **f** 9, **g** 7, **h** 8, **i** 2, **j** 5, **k** 6, **l** 5, **m** 10, **n** 5, **o** 5, **p** 4, **q** 20, **r** 3, **s** 6, **t** 2 **6** **a** 0.4, **b** 0.7, **c** 0.18, **d** 0.42, **e** 0.5, **f** 0.04, **g** 0.036, **h** 0.032, **i** 0.0036, **j** 0.3, **k** 7, **l** 4.9

Exercise 6.10

1 **a** P = 0.1, Q = 0.3, R = 0.7, **b** P = 3.2, Q = 3.5, R = 3.9, **c** P = 6.3, Q = 6.6, R = 6.8, **d** P = 11.1, Q = 11.4, R = 11.7 **2** **a** X = 1.7, Y = 2.1, Z = 0.9 **b** X = 7.2, Y = 8.1, Z = 6.9 **c** X = 2.9, Y = 3.2, Z = 1.8 **d** X = 10.5, Y = 11.3, Z = 9.7 **3** **a** A = 0.2, B = 0.4, C = 0.8, **b** A = 4.2, B = 4.6, C = 4.8, **c** A = 2.6, B = 3.2, C = 1.8, **d** A = 9.4, B = 10.2, C = 8.6 **4** **a** F = 2.31, G = 2.34, H = 2.38, **b** F = 5.72, G = 5.75, H = 5.79, **c** F = 7.13, G = 7.21, H = 7.09, **d** F = 9.96, G = 10.02, H = 9.88 **5** **a** T = 0.05, U = 0.35, V = 0.6, **b** T = 6.15, U = 6.5, V = 6.95, **c** T = 8.25, U = 9.05, V = 7.95, **d** T = 3.2, U = 4.15, V = 2.8 **6** **a** L = 1, M = 1.4, N = 0.5, **b** L = 5.4, M = 6.1, N = 6.9, **c** L = 13.1, M = 15.2, N = 12.9, **d** L = 19.5, M = 20.3, N = 17.8 **7** **a** 2.6, **b** 7.5, **c** 5.86, **d** 6.45

Exercise 6.11

1 **a** 8, **b** 9, **c** 9 **2** **a** 3.7, **b** 3.6, **c** 3.7 **3** **a** 0.42, **b** 0.43, **c** 0.43 **4** **a** 13, **b** 26, **c** 19, **d** 42, **e** 8, **f** 1, **g** 19, **h** 37, **i** 0, **j** 115, **k** 63, **l** 18, **m** 51, **n** 49, **o** 200 **5** **a** 0.4, **b** 0.1, **c** 0.7, **d** 0.7, **e** 17.3, **f** 46.5, **g** 37.0, **h** 88.3, **i** 6.1, **j** 349.2, **k** 102.3, **l** 19.9, **m** 551.0, **n** 0.0, **o** 780.0 **6** **a** 0.28, **b** 0.61, **c** 0.15, **d** 0.09, **e** 5.14, **f** 73.01, **g** 3.76, **h** 200.40, **i** 316.46, **j** 697.04, **k** 110.95, **l** 94.50, **m** 502.67, **n** 23.82, **o** 199.48, **p** 3260.00
7 **a** **i** 0.1, **ii** 0.13, **b** **i** 0.4, **ii** 0.37, **c** **i** 0.7, **ii** 0.70 **d** **i** 7.1, **ii** 7.06, **e** **i** 12.5, **ii** 12.51 **f** **i** 26.1, **ii** 26.15
8 **a** 15.7, **b** 83.22, **c** 1.041, **d** 37.24, **e** 51, **f** 0.011, **g** 300.1, **h** 420, **i** 264.14, **j** 6.284, **k** 16.0, **l** 7.45
9 **a** **i** yes, **ii** no **10** **a** **i** yes, **ii** no **11** **a** 2.2 cm, **b** 5.42 m, **c** 9.039 km, **d** 12.769 m, **e** 0.038 km, **f** 0.11635 km

Exercise 6.12

1 **a** $\frac{1}{5}$, **b** $\frac{1}{2}$, **c** $\frac{4}{5}$, **d** $\frac{2}{5}$, **e** $\frac{1}{50}$, **f** $\frac{3}{50}$, **g** $\frac{1}{20}$, **h** $\frac{7}{50}$, **i** $\frac{11}{25}$, **j** $\frac{7}{20}$, **k** $\frac{7}{25}$, **l** $\frac{1}{4}$, **m** $\frac{9}{50}$, **n** $\frac{23}{50}$, **o** $\frac{3}{4}$ **2** **a** 0.2, **b** 0.6, **c** 0.8, **d** 0.5, **e** 0.18, **f** 0.26, **g** 0.42, **h** 0.66, **i** 0.12, **j** 0.36, **k** 0.64, **l** 0.96, **m** 0.05, **n** 0.55, **o** 0.65, **p** 0.85 **3** **a** $\frac{11}{500}$, **b** $\frac{23}{500}$, **c** $\frac{63}{500}$, **d** $\frac{207}{500}$, **e** $\frac{9}{250}$, **f** $\frac{41}{250}$, **g** $\frac{111}{250}$, **h** $\frac{157}{250}$, **i** $\frac{11}{200}$, **j** $\frac{27}{200}$, **k** $\frac{57}{200}$, **l** $\frac{143}{200}$, **m** $\frac{1}{125}$, **n** $\frac{12}{125}$, **o** $\frac{43}{125}$ **4** **a** 0.014, **b** 0.242, **c** 0.526, **d** 0.838, **e** 0.004, **f** 0.108, **g** 0.432, **h** 0.852, **i** 0.015, **j** 0.105, **k** 0.365, **l** 0.545, **m** 0.008, **n** 0.056, **o** 0.248, **p** 0.896 **5** **a** $1\frac{1}{2}$, **b** $2\frac{4}{5}$, **c** $3\frac{9}{20}$, **d** $6\frac{1}{4}$, **e** $7\frac{6}{25}$, **f** $5\frac{41}{50}$, **g** $9\frac{3}{4}$, **h** $10\frac{37}{500}$, **i** $11\frac{41}{200}$, **j** $15\frac{181}{250}$, **k** $17\frac{13}{125}$, **l** $20\frac{3}{8}$ **6** **a** 3.5, **b** 5.25, **c** 6.75, **d** 8.6, **e** 9.16, **f** 10.65, **g** 12.94, **h** 13.106, **i** 15.028, **j** 19.085, **k** 21.072, **l** 22.875 **7** **a** 0.5, **b** 0.25, **c** 0.75, **d** 0.4, **e** 0.125, **f** 0.375, **g** 0.24, **h** 0.26, **i** 0.45, **j** 0.0625, **k** 0.3125, **l** 0.025 **8** **a** $\frac{3}{4}$, **b** $\frac{9}{20}$, **c** $\frac{17}{20}$, **d** $1\frac{4}{5}$, **e** $1\frac{3}{4}$, **f** $6\frac{1}{5}$

Exercise 6.13

1 **a** $0.\dot{2}$, **b** $0.\dot{5}$, **c** $0.3\dot{7}$, **d** $0.0\dot{6}$, **e** $1.\dot{3}$, **f** $4.8\dot{1}$, **g** $0.\dot{4}\dot{9}$, **h** $0.\dot{1}\dot{2}$, **i** $3.\dot{7}\dot{5}$, **j** $0.\dot{1}0\dot{5}$, **k** $0.\dot{3}2\dot{8}$, **l** $5.\dot{6}0\dot{9}$, **m** $0.2\dot{3}\dot{6}$, **n** $0.4\dot{1}7\dot{8}$, **o** $12.0\dot{3}4\dot{1}$ **2** **a** $0.\dot{3}$, **b** $0.\dot{6}$ **3** **a** **i** $0.\dot{1}$ **ii** $0.\dot{2}$ **iii** $0.\dot{7}$, **c** $0.\dot{5}$ **4** **a** $0.\dot{1}\dot{8}$, **b** $0.\dot{2}\dot{7}$, **c** $0.\dot{4}\dot{5}$, **d** $0.\dot{8}\dot{1}$ **5** **a** $0.\dot{8}$, **b** $0.1\dot{6}$, **c** $0.08\dot{3}$, **d** $0.1\dot{3}$, **e** $0.8\dot{3}$, **f** $0.41\dot{6}$, **g** $0.\dot{9}\dot{0}$, **h** $0.58\dot{3}$ **6** **a** $1.\dot{3}$, **b** $2.\dot{8}$, **c** $4.1\dot{6}$, **d** $1.91\dot{6}$ **7** **a** $\frac{7}{12} = 0.58\dot{3}$, **b** $\frac{1}{3} = 0.\dot{3}$, $\frac{1}{4} = 0.25$, **c** $\frac{7}{12}$ **8** **a** $0.0\dot{3}$, **b** $0.00\dot{3}$, $0.000\dot{3}$ **9** **a** $0.\dot{1}4285\dot{7}$, **b** $0.\dot{2}8571\dot{4}$, **c** $0.\dot{5}7142\dot{8}$, **d** $0.\dot{7}1428\dot{5}$ **10** $0.\dot{9} = \frac{9}{9} = 1$ **11** **a** $0.\dot{0}7692\dot{3}$, **b** **i** 7 **ii** 0 **iii** 2

Exercise 6.14

1 Paolo, Matthew, Pieter, Robert, Nick, Alan, Tran, Shane **2** \$3.15 **3** \$4.80 **4** \$245.82 **5** 0.25 m **6** 0.5 L **7** 34.45 L **8** **i** 18 glasses **ii** 20 glasses **9** 20 strips **10** \$1442.54 **11** \$15.42 **12** \$5.62 **13** 936 gadgets **14** 0.07 kg **15** **a** 1.5°C, **b** 23.7°C **16** £9 **17** 8.375 **18** 2.7 km **19** 133.6 m **20** \$33.50 **21** 1.37 m **22** 8

Chapter 6 Review

1 **a** 0.3, **b** 0.09, **c** 0.007, **d** 0.0006 **2** **a** $\frac{7}{10}$, **b** $\frac{3}{1000}$, **c** $\frac{1}{100}$, **d** $\frac{9}{10\,000}$ **3** **a** 2.574, **b** 7.0802, **c** 14.109 **4** **a** $\frac{7}{100}$, **b** 7, **c** $\frac{7}{1000}$, **d** $\frac{7}{100\,000}$ **5** **a** 0.21, **b** 0.249, **c** 0.053, **d** 0.00871 **6** **a** $\frac{67}{100}$, **b** $\frac{51}{1000}$, **c** $9\frac{23}{100}$, **d** $4\frac{309}{10\,000}$ **7** **a** 2.8, **b** 1.29, **c** 48.5, **d** 43.778 **8** 0.09, 0.090 **9** 6.00 **10** **a** >, **b** <, **c** >, **d** < **11** **a** 0.07, 0.73, 0.9, 1.5, 1.64, **b** 0.004, 0.04, 0.044, 0.4, 0.404, 0.44 **12** C **13** **a** 0.75, **b** 0.95, **c** 0.05, **d** 5.15, **e** 0.035, **f** 3.475, **g** 0.25, **h** 0.19 **14** **a** 248.101, **b** 589.916 **15** **a** 351.8, **b** 362.62, **c** 377.28 **16** **a** 1.2, **b** 0.7, **c** 0.4, **d** 2.2, **e** 0.14, **f** 0.3, **g** 0.85, **h** 0.25, **i** 0.29, **j** 0.32, **k** 0.15, **l** 1.22 **17** **a** 1.4, **b** 1.2, **c** 2, **d** 2, **e** 0.48, **f** 0.84, **g** 7.5, **h** 9.72 **18** **a** 22.2, **b** 34, **c** 38.88, **d** 58 **19** **a** 56.7, **b** 923.8, **c** 42.9, **d** 7144, **e** 0.015, **f** 36 000 **20** **a** 0.24, **b** 0.042, **c** 0.0036, **d** 0.06, **e** 9.1, **f** 0.208, **g** 2.715, **h** 1.938 **21** **a** 0.2, **b** 0.07, **c** 0.05, **d** 0.05, **e** 0.08, **f** 0.4, **g** 0.4, **h** 0.7 **22** **a** 9.375, **b** 2.05, **c** 73.8, **d** 96.8 **23** **a** 2.75, **b** 3.094, **c** 5.6261, **d** 0.1087, **e** 6.8, **f** 0.722, **g** 0.00345, **h** 0.0051 **24** **a** 1.3, **b** 2.2, **c** 0.09, **d** 1.8, **e** 23.7, **f** 19.36, **g** 2423.4, **h** 30 130 **25** **a** L = 2.3, M = 3.1, N = 1.8, **b** L = 7.4, M = 8.2, N = 6.8, **c** L = 1.35, M = 2.15, N = 0.95, **d** L = 5, M = 4.4, N = 6.8 **26** **a** 37, **b** 53, **c** 47, **d** 200 **27** **a** 0.5, **b** 0.8, **c** 7.2, **d** 6.1 **28** **a** 0.33, **b** 1.69, **c** 12.09, **d** 25.40 **29** **a** $\frac{3}{5}$, **b** $\frac{1}{4}$, **c** $\frac{7}{20}$, **d** $\frac{11}{25}$, **e** $\frac{1}{8}$, **f** $2\frac{1}{2}$ **30** **a** 0.4, **b** 0.75, **c** 0.34, **d** 0.45, **e** 0.428, **f** 0.055 **31** **a** 9.84, **b** $5\frac{203}{500}$ **32** **a** 0.375, **b** 0.4375 **33** **a** $0.\dot{4}$, **b** $0.\dot{3}\dot{8}$, **c** $0.7\dot{2}$, **d** $0.\dot{2}7\dot{5}$, **e** $4.9\dot{1}\dot{7}$, **f** $13.6\dot{4}1\dot{3}$ **34** **a** $0.\dot{5}$, **b** $0.1\dot{6}$, **c** $0.\dot{7}\dot{2}$, **d** $0.91\dot{6}$ **35** **a** \$143.80, **b** Glen by 0.53 m, **c** \$7.74

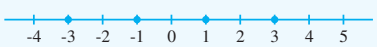
7 Integers

Exercise 7.1

1 **a** +5, **b** -3, **c** +2, **d** -9, **e** -17, **f** +12, **g** +20, **h** -6, **i** +25, **j** -30, **k** +10, **l** -7, **m** +10, **n** -4, **o** -3, **p** +2, **q** -9, **r** -18, **s** -50, **t** -45, **u** -28, **v** -5, **w** +250, **x** +8 **3** **a** -3, **b** -2, **c** +5, **d** -60, **e** +12, **f** -3, **g** -45, **h** +15, **i** -100, **j** -11, **k** -7 **4** **a** $-3 + 10 = +7$, **b** $+20 - 35 = -15$, **c** $-2 - 3 = -5$, **d** $-8 + 5 = -3$ **5** **a** walking 3 km west, **b** arriving 10 minutes late, **c** driving 50 km south, **d** losing \$20, **e** moving 8 m to the right, **f** ascending 4 floors, **g** withdrawing \$80, **h** an increase in temperature by 5°C **6** **a** +4, **b** -7, **c** +9, **d** +20, **e** -50, **f** -85 **7** a, b, e, f, h, i, l **8** **a** T, **b** T, **c** F, **d** T, **e** F, **f** T, **g** T, **h** T, **i** T **9** **a** in credit, **b** in debit, **c** in credit, **d** in debit, **e** in debit, **f** in debit, **g** in credit, **h** in debit

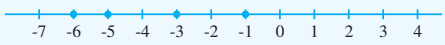
Exercise 7.2

1 **a** -2, -1, 1, 2, **b** -4, -2, 0, 2, **c** -6, -4, 0, 2, **d** -10, -5, 0, 10, **e** -7, -5, -4, -1, **f** -5, -4, -3, 1, **g** $-1\frac{1}{2}$, $-\frac{1}{2}$, 0, 1, **h** -0.6, -0.2, 0.2, 0.4 **2** **a**

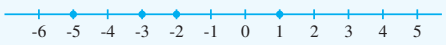


b

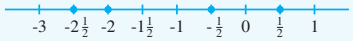
c



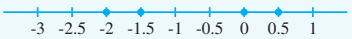
d



e



f



3 **a** -2, **b** -1, **c** -3, **d** 0, **e** -6, **f** -4, **g** -12, **h** -100 **4** **a** -8, **b** -6, **c** -99 **5** **a** -2, **b** 0, **c** -82 **6** **a** >, **b** <, **c** <, **d** >, **e** <, **f** >, **g** >, **h** >, **i** <, **j** >, **k** <, **l** <, **m** >, **n** <, **o** >, **p** <, **q** >, **r** >, **s** <, **t** < **7** **a** -1, 0, 4, 6, **b** -3, -2, 1, 5, **c** -9, -6, -4, 3, **d** -7, -6, 0, 7, **e** -13, -12, -10, -9, **f** -83, -52, -47, -1, **g** -92, -67, 0, 3, **h** -75, -74, -73, 76, **i** -114, -113, -1, 29 **8** **a** 0, 1, **b** -4, -8, **c** -3, -7, **d** -11, -13, **e** 4, 6, **f** -7, -2, **g** -5, -11, **h** 3, 5, **i** 0, 7, **j** 2, -8, **k** 0, 25, **l** -1, 7 **9** **a** 4, **b** 1, **c** 0, **d** -4, **e** 1, **f** -5, **g** -2, **h** -19, **i** -52 **10** **a** depositing \$10, **b** withdrawing \$11 **11** **a** 7°C below zero, **b** 4°C below zero **12** **a** 20 km east, **b** 32 km west **13** **a** arriving 7 minutes early, **b** arriving 5 minutes late **14** **a** falling in value by 20c, **b** falling in value by 45c **15** **a** a credit of \$110, **b** a debit of \$550 **16** **a** in the black means in credit, in the red means in debit, **b** credits are entered in black ink and debits are entered in red ink, **c** \$80 in the black, because it is a credit **17** **a** +2, -2, **b** -2, 4, **c** -5, 9, **d** -10, 2, **e** -14, -13, -12, -11, -10, **f** -42, -41, -40, -39, -38, -37, -36, -35, -34

Exercise 7.3

1 **a** **i** -1°C **ii** -4°C **iii** -3°C **iv** -7°C, **b** **i** 3°C **ii** 2°C **iii** 4°C **iv** 5°C, **c** **i** -4°C **ii** -8°C **iii** -10°C **iv** -11°C, **d** **i** -2°C **ii** -1°C **iii** -5°C **iv** -6°C **2** **a** 1°C, **b** -2°C, **c** -8°C, **d** -4°C, **e** -5°C, **f** -11°C, **g** -2°C, **h** 0°C **3** **a** 6 floors below ground, $-2 - 7 + 3 = -6$. **b** Debit of \$6, $10 - 12 - 4 = -6$. **c** Linh lost 5 kgs, $-3 + 5 - 7 = -5$. **d** The man is 1 km west of his original position, $15 - 20 + 4 = -1$. **e** The buoy is 4 metres above its original position, $2 - 3 + 5 = 4$. **f** Julianne is \$35 worse off, $-20 - 30 + 15 = -35$. **g** The submarine is 14 metres below the surface, $-10 - 16 + 12 = -14$. **h** The price fell 9 cents per litre, $4 - 5 - 8 = -9$. **4** **a** -6, **b** +17, **c** -7, **d** +4 **5** **a** -8, **b** -3, **c** -10, **d** +21

Exercise 7.4

1 **a** -4, **b** -6, **c** -7, **d** -7, **e** -6, **f** -10, **g** -10, **h** -8, **i** -4, **j** -13, **k** -11, **l** -15, **m** -13, **n** -15, **o** -8, **p** -16, **q** -16, **r** -13, **s** -16, **t** -20 **2** **a** -3, **b** -2, **c** -4, **d** -6, **e** -3, **f** -2, **g** -1, **h** -4, **i** -2, **j** -7, **k** -8, **l** -7, **m** -9, **n** -5, **o** -11, **p** -2, **q** -12, **r** -5, **s** -9, **t** -9 **3** **a** -1, **b** -5, **c** -3, **d** -2, **e** -5, **f** -2, **g** -3, **h** -4, **i** -5, **j** -1, **k** -6, **l** -4, **m** -9, **n** -8, **o** -11, **p** -3, **q** -11, **r** -9, **s** -10, **t** -6 **4** **a** 2, **b** 1, **c** 3, **d** 1, **e** 4, **f** 5, **g** 4, **h** 7, **i** 9, **j** 3, **k** 7, **l** 4, **m** 1, **n** 0, **o** 9, **p** 7, **q** 3, **r** 8, **s** 4, **t** 6 **5** **a** -5, **b** -1, **c** -10, **d** 1, **e** -5, **f** -3, **g** 6, **h** -3, **i** 5, **j** -5, **k** -13, **l** -6, **m** -4, **n** -14, **o** -6, **p** 12, **q** 4, **r** -4, **s** -13, **t** -8 **6** **a** -9, **b** 3, **c** -9, **d** -5 **7** **a** -7, **b** -12, **c** -14, **d** 6 **8** **a** -4, **b** 6, **c** -11, **d** -4 **9** 7 **10** **a** -1, **b** -6, **c** 5, **d** -7, **e** 4, **f** -3, **g** -5, **h** -5, **i** +4, **j** -3, **k** -11, **l** -7, **m** 5, **n** -12, **o** -1, **p** -9 **11** **a** -20, **b** 30, **c** -100, **d** -10, **e** 5, **f** -60, **g** -75, **h** -45, **i** -9, **j** -31, **k** -101, **l** 42, **m** -52, **n** -245, **o** 142, **p** -204

Exercise 7.5

1 **a** – 4, **b** – 1, **c** – 2, **d** – 5 **2** **a** + 1, **b** + 2, **c** + 3, **d** + 6 **3** **a** – 4, **b** – 3, **c** – 7, **d** – 6 **4** **a** + 3, **b** + 4, **c** + 10, **d** + 7 **5** **a** – 1, **b** + 5, **c** – 2, **d** + 8, **e** + 9, **f** – 4, **g** + 5, **h** – 10, **i** + 10, **j** – 5, **k** – 11, **l** + 6, **m** – 5, **n** – 10 , **o** + 14, **p** + 5, **q** – 17, **r** + 22, **s** + 9, **t** – 11 **6** **a** 11, **b** 13, **c** 18, **d** 9 **7** **a** 8, **b** 14 **8** **a** -2, **b** -6 , **c** -4, **d** -4 **9** **a** -6, **b** -5, **c** -9, **d** -8 **10** **a** 2, **b** 1, **c** 2, **d** 2, **11** **a** -1, **b** -2, **c** -5, **d** -3 **12** **a** -5, **b** 3, **c** -8, **d** -3, **e** -2, **f** -13, **g** 4, **h** -1, **i** 7, **j** -6 , **k** 8, **l** -2, **m** -11, **n** -5, **o** 6, **p** -6, **q** 3, **r** -4, **s** -14, **t** -7 **13** **a** -17, **b** -2, **c** -20, **d** -22 **14** **a** -1, **b** -5, **c** 4, **d** -4 **15** **a** – 7, **b** – 4, **c** + 6, **d** – 7, **e** 4, **f** -1, **g** -2, **h** 3, **i** 0

Exercise 7.6

1 **a** Yes, **b** Yes, **c** Yes **2** **a** 4 – 4 = 0, 4 – 3 = 1, 4 – 2 = 2, 4 – 1 = 3, 4 – 0 = 4, 4 – (-1) = 5, 4 – (-2) = 6, 4 – (-3) = 7, 4 – (-4) = 8 **b** 5 + 4 = 9, 5 + 3 = 8, 5 + 2 = 7, 5 + 1 = 6, 5 + 0 = 5, 5 + (-1) = 4, 5 + (-2) = 3, 5 + (-3) = 2, 5 + (-4) = 1 **3** **a** +, **b** – **4** **a** 7, **b** 11, **c** 8, **d** 4, **e** 10, **f** 16, **g** 14, **h** 21 **5** **a** 5, **b** 7, **c** 5, **d** 4, **e** 7, **f** 9, **g** 15, **h** 16 **6** **a** 7, **b** 4, **c** 5, **d** 12, **e** 12, **f** 7, **g** 6, **h** 13, **i** 0, **j** 11, **k** 11, **l** 9, **m** 4, **n** 17, **o** 12, **p** 0 **7** **a** -1, **b** -5, **c** -6, **d** 3, **e** -16, **f** -4, **g** -17, **h** -5, **i** -6, **j** 0, **k** -13, **l** -3, **m** 6, **n** -8, **o** -19, **p** -22 **8** **a** -5, **b** 13, **c** 14, **d** -17 **9** **a** 6, **b** 2, **c** -3, **d** -6, **e** -11, **f** -8 **10** **a** 7, **b** 14, **c** 5, **d** 9 **11** **a** 110, **b** -30, **c** -50, **d** 60, **e** -50, **f** -30, **g** -110, **h** 70, **i** -55, **j** -14, **k** -33, **l** 41 **12** **a** -2, **b** -4, **c** -7, **d** -3, **e** -1, **f** -5, **g** -6, **h** -5, **i** -2, **j** -2, **k** -11, **l** -8

Exercise 7.7

1 **a** 5 × 4 = 20, 5 × 3 = 15, 5 × 2 = 10, 5 × 1 = 5, 5 × 0 = 0, 5 × (-1) = -5, 5 × (-2) = -10, 5 × (-3) = -15, 5 × (-4) = –20, **b** -3 × 4 = -12, -3 × 3 = -9, -3 × 2 = -6, -3 × 1 = -3, -3 × 0 = 0, -3 × (-1) = 3, -3 × (-2) = 6, -3 × (-3) = 9, -3 × (-4) = 12 **2** **a** positive, **b** negative **3** **a** -12, **b** 10, **c** -24, **d** 21, **e** -40, **f** 36, **g** -36, **h** -32, **i** 50, **j** -77, **k** -54, **l** -56, **m** 26, **n** 72, **o** 0, **p** -45, **q** 70, **r** 60, **s** -42, **t** -132 **4** **a** -30, **b** 40, **c** -48, **d** 56, **e** -84, **f** -120, **g** 0, **h** 54, **i** -36, **j** 56, **k** 45, **l** -240 **5** **a** -10, **b** -21, **c** -12, **d** -24, **e** -60, **f** -54 **6** **a** -28, **b** -72 **7** **a** 9, **b** 25, **c** 36, **d** 81, **e** -8, **f** -27, **g** -1000, **h** -125, **i** 16, **j** 81, **k** -32, **l** -100 000 **8** **a** -3, **b** -7, **c** 3, **d** 9, **e** -6, **f** -8, **g** 0, **h** -2, **i** 9, **j** -6, **k** -7, **l** -10, **m** -3, **n** -5, **o** -12, **p** 7 **9** -10 **10** -11 **11** -4 **12** **a** -3, **b** 4, **c** -12, **d** 6, **e** -6, **f** -3, **g** -9, **h** -10, **i** 3 **13** **a** 600, **b** -520, **c** -90, **d** 300, **e** -900, **f** 210, **g** -133, **h** 891 **14** **a** 4, -4, **b** 5, -5, **c** 10, -10 **15** **a** -14, **b** -32, **c** -58, **d** 58

Exercise 7.8

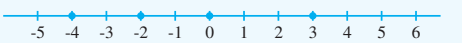
1 **a** 5, **b** 5, **c** -5, **d** -5 **2** **a** positive, **b** negative **3** **a** -2, **b** 6, **c** -3, **d** 8, **e** -7, **f** -12, **g** 10, **h** -9, **i** -6, **j** 3, **k** -8, **l** 7, **m** -10, **n** -5, **o** 20, **p** 5, **q** 4, **r** -6, **s** -11, **t** 12 **4** **a** -3, **b** 11, **c** -2, **d** -4, **e** 10, **f** -7, **g** -5, **h** 9 **5** **a** 10, **b** -3, **c** -2, **d** -4, **e** 5, **f** -10 **6** **a** 8, **b** -10, **c** -16, **d** -42, **e** 0, **f** 3 **7** **a** -7, **b** 5, **c** -6, **d** -15, **e** 30, **f** -66, **g** -8, **h** 36, **i** -7, **j** 11, **k** -54, **l** 81, **m** -120, **n** 7, **o** -9, **p** 11 **8** **a** 11, **b** -8 **9** -3 **10** 30 **11** **a** 5, **b** -5, **c** 4 **12** **a** -17, **b** 57, **c** -28, **d** 19, **e** 24, **f** -23, **g** -35, **h** 28 **13** -32, -2 **14** 14 **15** **a** -14, **b** -13, **c** -4, **d** -11

Exercise 7.9

1 **a** -3, **b** -6, **c** -7, **d** -3, **e** 4, **f** 2, **g** 2, **h** -1, **i** -7, **j** 6, **k** -1, **l** -8, **m** -16, **n** -20, **o** -8, **p** -22
2 **a** -2, **b** -8, **c** 7, **d** -3, **e** -40, **f** -20, **g** -7, **h** -5, **i** 13, **j** 17, **k** 0, **l** -30
3 **a** -11, **b** -10, **c** 15, **d** -4, **e** 2, **f** -23, **g** -13, **h** 21, **i** 0

Chapter 7 Review

1 **a** -6, **b** +3, **c** -20, **d** -4, **e** +75, **f** +10 **2** -5 **3**



4 smallest: -99, largest: 0 **5** **a** >, **b** <, **c** <, **d** > **6** -81, -48, -13, -12, 0, 17 **7** **a** -5°C, **b** 2°C, **c** -12°C, **d** -5°C **8** **a** 3 km west, **b** one floor below ground, **c** decreased by 6 cents **9** **a** -4, **b** -14, **c** -2, **d** -7, **e** -13, **f** -1, **g** -3, **h** +7, **i** 25, **j** -80, **k** -25, **l** -5 **10** **a** 6, **b** -19, **c** -13, **d** -6 **11** **a** 7, **b** -7, **c** -16, **d** -13 **12** **a** – 6, **b** – 8, **c** + 12, **d** + 5, **e** – 3, **f** + 8, **g** – 9, **h** + 10 **13** **a** 2, **b** -2, **c** -22, **d** -14, **e** -6, **f** -3, **g** -6, **h** -8 **14** **a** – 9, **b** – 6, **c** -10 **15** **a** 10, **b** -6, **c** -6, **d** 5, **e** -9, **f** -3, **g** -13, **h** 16 **16** 5 **17** **a** -3, **b** -8, **c** -6, **d** -4 **18** **a** -15, **b** 14, **c** -24, **d** 27, **e** -35, **f** -64, **g** 66, **h** 96 **19** **a** -24, **b** 24, **c** 120, **d** -144 **20** **a** 25, **b** -27, **c** 10 000, **d** -32 **21** **a** -4, **b** -9, **c** -4, **d** 12 **22** **a** -5, **b** 6, **c** -5, **d** 5, **e** -7, **f** -4, **g** 4, **h** -11 **23** **a** -6, **b** -21, **c** -4, **d** -108 **24** -8 **25** **a** -14, **b** -10, **c** -11

8 Algebra

Exercise 8.1

1 **a** 7, **b** 2, **c** 13, **d** 17, **e** 21, **f** 36, **g** 19, **h** 24, **i** 28, **j** 12, **k** 9, **l** 27, **m** 16, **n** 45, **o** 54, **p** 23, **q** 3, **r** $\frac{1}{2}$, **s** 2, **t** 4 **2** **a** 10, **b** 2, **c** 22, **d** 24, **e** 38, **f** 40, **g** 48, **h** 73, **i** 1, **j** 2, **k** 8, **l** 14, **m** 2, **n** 1, **o** 2, **p** 2, **q** 52, **r** 28, **s** 46, **t** 240, **u** 50, **v** 56, **w** 30, **x** 16 **3** **a** 16, **b** 6, **c** 15, **d** 40, **e** 240, **f** 46, **g** 38, **h** 125, **i** 1, **j** 2, **k** 1, **l** 4, **m** 98, **n** 139, **o** 16, **p** 90 **4** **a** -3, **b** -5, **c** -8, **d** 3, **e** -3, **f** -7, **g** -13, **h** -3, **i** -3, **j** 3, **k** -1, **l** -5, **m** -7, **n** -20, **o** -20, **p** 14, **q** -10, **r** -48, **s** -56, **t** -40 **5** **a** 0, **b** 14, **c** -14, **d** -10, **e** 0, **f** 4, **g** 9, **h** -22, **i** -24, **j** 20, **k** -49, **l** -118, **m** $-\frac{9}{7}$, **n** -1, **o** $\frac{2}{7}$, **p** -2, **q** 6, **r** 45, **s** -31, **t** -21

Exercise 8.2

1 **a** like terms, **b** unlike terms, **c** like terms, **d** unlike terms, **e** unlike terms, **f** like terms, **g** unlike terms, **h** like terms, **i** unlike terms, **j** unlike terms, **k** unlike terms, **l** like terms **2** **a** $4p$, $9p$, **b** $8b$, $7b$, **c** $15c$ and $4c$; 5 and 6, **d** fg, $3fg$, **e** $6ef$, $4fe$, **f** $2n$ and $6n$; $3n^2$ and n^2 **3** **a** Yes, **b** Yes **5** **a** $7a$, **b** $3m$, **c** $3n$, **d** $2k$, **e** $2c$, **f** h, **g** 0, **h** $13g$, **i** $11w$, **j** $12e$, **k** n, **l** $6p$, **m** $20r$, **n** $9d$, **o** b, **p** $7v$, **q** $15t$, **r** $21s$, **s** $9f$, **t** 0 **6** **a** $10m$, **b** $16k$, **c** $10t$, **d** $5x$, **e** $12c$, **f** $7p$, **g** $6n$, **h** $6g$, **i** $3j$, **j** $12z$, **k** $7y$, **l** 0, **m** $16u$, **n** $15a$, **o** $9w$ **7** **a** $9ab$, **b** $6pq$, **c** $10st$, **d** uv, **e** $2cd$, **f** $10xy$, **g** $7qr$, **h** $8ef$, **i** $2gh$, **j** $18yz$, **k** $21jk$, **l** 0, **m** $16ab$, **n** $8pq$, **o** $4mn$, **p** 0 **8** **a** $+3m$, **b** $-4t$, **c** $-6u$, **d** $+7c$, **e** $+h$, **f** $-5p$, **g** $+5a^2$, **h** $+2b^2$, **i** $-k^2$, **j** $-12qr$, **k** $+7ab$, **l** $-10pq$ **9** **a** $7n$, **b** $5p$, **c** $9x$, **d** $3a$, **e** $7z$, **f** $10w$, **g** $4k^2$, **h** $6s^2$, **i** $14y^2$, **j** $3bc$, **k** $10mn$, **l** 0 **10** **a** $8a + 7b$, **b** $16c + 5k$, **c** $9m + 10n$, **d** $6x + 5y$, **e** $12e + 13$, **f** $18 + 11z$, **g** $7p + 3q$, **h** $11r + 5s$, **i** $x + 6y$, **j** $2m + 17n$, **k** $3u + 7$, **l** $9 + 16z$ **11** **a** $7a + 5b$, **b** $6p + 9q$, **c** $13m + 5z$, **d** $8t + 5n$, **e** $13c + 4d$, **f** $4e + 5f$, **g** $15u + 11$, **h** $6k + 12$, **i** $9a^2 + 12a$, **j** $9 + 19w^2$, **k** $7p^2 + 7p$, **l** $13y^2 + 11y$ **12** **a** $6p + 5n$, **b** $9m + 2y$, **c** $4x + 14a$, **d** $4g + 3k$, **e** $4n$, **f** $c + 3g$, **g** $5r + 2$, **h** $14t + 5$, **i** $2c^2 + 3c$, **j** $3v + 12v^2$, **k** $c + 3g$, **l** $2b$, **m** $8c + 2$, **n** $16r + 9s$, **o** $13q$ **13** **a** $12x + 9y + 16z$, **b** $3p + 13q + 9r$, **c** $10a^2 + 3a + 16$, **d** $2f + 19g^2 + 1$, **e** $5bc + 8c + 7ac$, **f** $7mn + 9m$, **g** $4u + 4uv + 2v$, **h** $9yz + 3xz + 8$ **14** **a** $-9t$, **b** $9u$, **c** $+5b$, **d** $+3p$, $+5q$, **e** $-5m$, $-9n$, **f** b, $+5a$, **g** $-3j$, $-7k$, **h** $-18x$, $-15y$

Exercise 8.3

1 **a** $-3a$, **b** $-4m$, **c** $-10k$, **d** $2n$, **e** $-g$, **f** $-10p$, **g** $-6y$, **h** $4t$, **i** 0, **j** $-e$, **k** $-13c$, **l** $-8r$, **m** $-4b$, **n** $-13w$, **o** $-5s$, **p** $4f$, **q** $-2d$, **r** $-h$, **s** $-13x$, **t** 0 **2** **a** $-9ab$, **b** $5pq$, **c** $-4xy$, **d** $-7mn$, **e** 0, **f** uv, **g** $-9gh$, **h** $-8jk$, **i** $-8ef$, **j** $6wx$, **k** bc, **l** 0 **3** **a** $-6x$, **b** $-2u$, **c** $-2m$, **d** $-6h$, **e** $3w$, **f** $-n$, **g** $-q$, **h** $-3s$, **i** $-10y$, **j** $-3b$, **k** $7z$, **l** $-6v$ **4** **a** $6a$, **b** $13u$, **c** $8c$, **d** $15w$, **e** $2k$, **f** $5n$, **g** $6t$, **h** $5b$, **i** $6p$, **j** $-8q$, **k** 0, **l** $-9m$, **m** $-8xy$, **n** $-2bc$, **o** tu, **p** 0 **5** **a** $+3w$, **b** $-4n$, **c** $-8p$, **d** $+11t$, **e** $-2a$, **f** $+5g$, **g** $-9k$, **h** $-8c$, **i** $-13m$, **j** $+10d$, **k** $-4s$, **l** $-9e$ **6** **a** $-5u$, **b** $-4n$, **c** $-4f$, **d** $3r$, **e** $-5w$, **f** $-6a$, **g** $-6d$, **h** $-8s$, **i** $-3n$, **j** $4j$, **k** $-g$, **l** $-19b$ **7** **a** $10p - 3q$, **b** $m - 4c$, **c** $-6u - 10n$, **d** $-8b + 5x$, **e** $-2k - 6m$, **f** $7p + k$, **g** $-3t - 4$, **h** $9 - 12y$, **i** $-6d + 1$, **j** $-4a^2 - 6a$, **k** $-9p - 8p^2$, **l** $-4n^2 + 7n$ **8** **a** $-4p + 10k$, **b** $-2u - 5a$, **c** $-8d - 5f$, **d** $9t - e$, **e** $-9n - 3v$, **f** $-4q$, **g** $-5y + b$, **h** $-13s + 6t$, **i** $-11k - 3$, **j** $-6 + 5w$, **k** $8n - 19$, **l** $10a^2 - 2a$, **m** $10p^2 - 16$, **n** $-2u^2$, **o** $-15s^2 - 13s$, **p** $-n + 5m^2$, **q** $2c - 4c^2$, **r** 0 **9** **a** $-10w$, **b** $-6a$, **c** $-5n$, **d** $+3m$, $-8n$, **e** $-3t$, $+2u$, **f** $3a$, $-5c$, **g** $-6x$, $-2y$, **h** $+3a^2$, $-7a$

Exercise 8.4

1 **a** $3m$, **b** $7p$, **c** $4k$, **d** $5g$, **e** ab, **f** pq, **g** mn, **h** jk, **i** $6uv$, **j** $2mn$, **k** $9xy$, **l** $7rs$ **2** **a** $6ab$, **b** $15pq$, **c** $28mn$, **d** $30ku$, **e** $3cde$, **f** $12w$ **3** **a** $6b$, **b** $20n$, **c** $14p$, **d** $18t$, **e** $36u$, **f** $35e$, **g** $48k$, **h** $63w$, **i** $12ab$, **j** $24pq$, **k** $48mn$, **l** $22km$, **m** $40ef$, **n** $30cd$, **o** $60xy$, **p** $84uv$ **4** **a** $6cde$, **b** $35fgh$, **c** $36mnp$, **d** $120rst$, **e** $90uvw$, **f** $48xyz$ **5** **a** a^2, **b** p^2, **c** c^2, **d** u^2, **e** $3q^2$, **f** $2t^2$, **g** $8g^2$, **h** $4w^2$, **i** $6y^2$, **j** $20a^2$, **k** $12h^2$, **l** $21v^2$, **m** $30t^2$, **n** $60n^2$, **o** $36s^2$, **p** $120b^2$ **6** **a** a^2bc, **b** pq^2r, **c** xy^2z, **d** $3xy^2$, **e** $2c^2d$, **f** $5np^2$, **g** $12ab^2c$, **h** $10e^2fg$, **i** $14gh^2i$, **j** $32p^2qr$, **k** $45mnp^2$, **l** $56u^2vw$, **m** $55cd^2e$, **n** $52tuv^2$, **o** $96r^2st$ **7** **a** 7, **b** 5, **c** 4, **d** $3d$, **e** $6p$, **f** $9h$, **g** m, **h** c, **i** $2p$, **j** $8u$, **k** $12f$, **l** $11g$, **m** ac, **n** jk, **o** $5e$ **8** **a** $-12n$, **b** $30q$, **c** $-14u$, **d** $-40cd$, **e** $28rs$, **f** $-54vw$, **g** $70b^2$, **h** $-64g^2$, **i** $-120f^2$, **j** $-42abc$, **k** $50pqr$, **l** $-60uvw$, **m** $56efg$, **n** $48xyz$, **o** $-60ijk$ **9** **a** $13x$, **b** $18a$, **c** $19n$, **d** $31p$, **e** 0, **f** $9k$, **g** $17e + 10$, **h** $14m + 11$, **i** $12u + 15$

ANSWERS

Exercise 8.5

1 **a** n, **b** 7, **c** m, **d** e, **e** b, **f** x, **g** d, **h** s, **i** pr, **j** $10n$, **k** v, **l** x **2** **a** $2m$, **b** $3t$, **c** $4u$, **d** $8s$, **e** $6r$, **f** $11w$, **g** $5k$, **h** $4e$ **3** **a** 4, **b** 3, **c** 4, **d** 1, **e** $6z$, **f** $13j$, **g** $5i$, **h** $4u$, **i** $5ac$, **j** $4uv$, **k** $6np$, **l** $7xz$, **m** $5f$, **n** $6f$, **o** $9d$, **p** $7i$ **4** **a** a, **b** k, **c** u, **d** $2q$, **e** $11f$, **f** c, **g** b, **h** $5h$, **i** $3b$, **j** $2k$, **k** $9z$, **l** $7c$, **m** $8r$, **n** $11s$, **o** $9d$ **5** **a** a, **b** 9, **c** n, **d** $8e$, **e** $5s$, **f** $7v$, **g** q, **h** a, **i** 4, **j** $4d$, **k** $5u$, **l** $7eg$, **m** n, **n** $3e$, **o** $2r$, **p** $4y$, **q** $6m$, **r** $9c$, **s** $5ab$, **t** $6p$ **6** **a** $-g$, **b** $-m$, **c** 7, **d** $-m$, **e** d, **f** $-k$, **g** $-2p$, **h** $-3y$, **i** $5w$, **j** -9, **k** -11, **l** 10, **m** $-q$, **n** $2c$, **o** $-9r$, **p** $11h$ **7** **a** h, **b** n, **c** $2a$, **d** 5, **e** $3b$, **f** $24t$, **g** $36y$, **h** $3p$, **i** $4x$, **j** $6p$, **k** $42mn$, **l** d, **m** e^2, **n** $4k^2$, **o** $9g$ **8** **a** $23a$, **b** $19z$, **c** 12, **d** $13i$, **e** $13y$, **f** $8b$, **g** $17f + 7$, **h** $2k$, **i** 11 **9** **a** $\frac{3n}{4}$, **b** $\frac{2a}{3}$, **c** $\frac{3x}{5y}$, **d** $\frac{6t}{7u}$, **e** $\frac{2u}{5}$, **f** $\frac{7k}{8}$, **g** $\frac{4w}{5}$, **h** $\frac{7y}{9}$

Exercise 8.6

1 $2(x + 1) = 2x + 2$, not $2x + 1$ **2** **a** $2x + 6$, **b** $3t - 12$, **c** $10 + 5a$, **d** $4y - 8$, **e** $7w + 28$, **f** $6n - 30$, **g** $9p - 9$, **h** $16 + 8u$, **i** $10b + 10c$, **j** $3c - 3d$, **k** $6g - 6h$, **l** $12m + 12n$ **3** **a** $5m + 10$, **b** $2n - 8$, **c** $3d - 27$, **d** $33 - 11k$, **e** $7u + 35$, **f** $4y + 32$, **g** $36 - 6c$, **h** $12e + 84$ **4** **a** $ab + ac$, **b** $pq + 3p$, **c** $ef - 4e$, **d** $6n - nb$, **e** $gy + gz$, **f** $kz - k$, **g** $8b + bw$, **h** $cd - ce$ **5** **a** $6a + 10$, **b** $12n - 18$, **c** $20 + 15p$, **d** $4 - 8c$, **e** $40y + 56$, **f** $27 - 72i$, **g** $12u + 9v$, **h** $18a - 30b$, **i** $49e - 42f$, **j** $11t + 99u$, **k** $50g - 40h$, **l** $60d + 108e$ **6** **a** $x^2 + 3x$, **b** $a^2 - 5a$, **c** $2n - n^2$, **d** $p^2 + pq$, **e** $4u + u^2$, **f** $ab - b^2$, **g** $2yz + z^2$, **h** $c^2 - 3cd$, **i** $vw - 5w^2$, **j** $2k^2 + 7km$, **k** $3xy - 4y^2$, **l** $5h^2 + 6hi$ **7** **a** $3xy + 12x$, **b** $2pq - 14p$, **c** $20g + 5gh$, **d** $21t - 7tu$, **e** $4jk + 32j$, **f** $6yz - 48y$, **g** $6ef + 2eg$, **h** $15mn - 25mp$, **i** $24ij - 16ik$, **j** $20pq + 90pr$, **k** $28tu + 49tw$, **l** $84cd - 132ce$, **m** $4a^2 + 4ab$, **n** $15n - 3n^2$, **o** $9k^2 - 45k$, **p** $6b^2 + 18b$, **q** $30d - 10d^2$, **r** $44f^2 - 33fg$, **s** $21xy + 35x^2$, **t** $60i^2 - 108ij$ **8** **a** $3n + 17$, **b** $6x - 12$, **c** $7y + 17$, **d** $a - 35$, **e** $10c + 23$, **f** $23 - 9w$, **g** $8 + 36m$, **h** $18d + 53$, **i** $7 - 30t$, **j** $41k + 72$, **k** $35u + 66$, **l** $32 - 14p$ **9** **a** $3n + 17$, **b** $2w + 1$, **c** $8t + 28$, **d** $14u + 28$, **e** $13e - 20$, **f** $9q + 26$, **g** $13b + 3ab$, **h** $2mk$, **i** $22c - 8cd$ **10** **a** $5x + 14$, **b** $11a + 25$, **c** $10m + 30$, **d** $11p + 18$, **e** $u + 40$, **f** $19w + 48$, **g** $17n + 50$, **h** $7k + 48$, **i** $11g + 74$, **j** $16t + 63$, **k** $13e + 10$, **l** $20t$ **11** **a** $-3n - 15$, **b** $-4a + 8$, **c** $-9 - 9t$, **d** $-40 + 5p$, **e** $-6b - 42$, **f** $-24 + 2z$, **g** $-6m - 14$, **h** $-35y + 20$, **i** $-21 + 70g$, **j** $-8c - 36$, **k** $-120j - 10$, **l** $-81d + 72$, **m** $-44 - 66h$, **n** $-32w - 28$, **o** $-84k + 108$ **12** **a** $2u - 1$, **b** $4m - 7$, **c** $3q + 4$, **d** $-3y - 48$, **e** $5 - 9e$, **f** $14 - 16j$, **g** $6x - 19$, **h** $1 - 20a$, **i** $28h - 2$, **j** $5w - 4$, **k** $10 - 4n$, **l** $-y - 25$

Exercise 8.7

1 **a** $k + 3$, **b** $y + 2$, **c** $x + 6$, **d** $p + q$, **e** $x + y$, **f** $g + h$ **2** **a** $a - 7$, **b** $p - 4$, **c** $w - 9$, **d** $u - v$, **e** $c - d$, **f** $r - s$ **3** **a** $2h$, **b** $3n$, **c** $4t$, **d** $8f$, **e** $9z$, **f** jk **4** **a** $\frac{d}{2}$, **b** $\frac{m}{3}$, **c** $\frac{3x}{4}$, **d** $\frac{a}{b}$, **e** $\frac{g}{h}$, **f** $\frac{m}{n}$ **5** **a** 6 more than n, **b** 1 less than k, **c** the product of 7 and t, **d** one-quarter of w, **e** the difference between g and h, **f** the sum of m and n, **g** the quotient of c and d, **h** the product of a and b, **i** 10 more than z, **j** half of n, **k** 8 less than y, **l** the product of 5 and u, **m** the quotient of x and y, **n** the sum of j and k, **o** the product of p and q, **p** the difference between k and m, **q** the product of 2 and b, **r** one-fifth of s, **s** 2 less than v, **t** 11 more than f **6** **a** two-thirds of a, **b** three-quarters of k, **c** seven-tenths of u, **d** four-fifths of c **7** **a** $p + q + r$, **b** $x + y + 5$, **c** uvw, **d** $4st$ **8** **a** $2x + 1$, **b** $3k - 4$, **c** $ab + 5$, **d** $gh - 7$ **9** **a** 2 less than the product of 5 and u, **b** 10 more than the product of c and d, **c** 6 less than the product of y and z, **d** 9 more than the product of 4 and t **10** **a** $\frac{m}{2} + 5$, **b** $\frac{r}{s} - 1$, **c** $\frac{n}{3} - 4$, **d** $\frac{a}{b} + 9$, **e** $\frac{p+q}{4}$, **f** $\frac{p-5}{6}$, **g** $\frac{d+2}{3}$, **h** $\frac{b-c}{10}$ **11** **a** one-third of the difference between m and n, **b** two more than the quotient of v and w, **c** eight less than one-fifth of y, **d** one-eighth of the number which is seven more than f, **e** twelve more than one-ninth of k, **f** one-quarter of the number which is one less than u, **g** five less than the quotient of d and e, **h** one-eleventh of the sum of r and s **12** **a** k^2, **b** j^3, **c** y^9, **d** v^7, **e** $2w^2$, **f** $5a^3$, **g** xy^2, **h** $\frac{e^3}{2}$, **i** $m^2 + 1$, **j** $h^3 - 10$, **k** $c + d^2$, **l** $v^2 - z$ **13** **a** $2(p + q)$, **b** $6(u - v)$, **c** $x(y - 7)$, **d** $a(c + 2)$, **e** $5(d - 4)$, **f** $3(\frac{a}{2} - \frac{4b}{5})$

Chapter 8 Review

1 **a** 7, **b** 28, **c** 12, **d** 20, **e** 23, **f** 6, **g** 20, **h** 7, **i** 18, **j** 48, **k** 42, **l** 180, **m** 10, **n** 3, **o** 7, **p** 5 **2** **a** like terms, **b** unlike terms, **c** unlike terms, **d** like terms, **e** unlike terms, **f** like terms, **g** like terms, **h** unlike terms, **i** unlike terms **3** **a** $8c$, **b** $7t$, **c** $9v$, **d** $11p$, **e** f, **f** 0, **g** $2b$, **h** $10gh$, **i** $6st$, **j** $12a^2$, **k** $5e^2$, **l** $11r$ **4** **a** $8a + 10b$, **b** $15u + 8v$, **c** $16c + 5d$, **d** $8g + 16$, **e** $8r + 14s$, **f** $14x + y$, **g** $14w^2 + 6w$, **h** $14ab + 9$, **i** $13x + 11x^2$ **5** **a** $-4w$, **b** $7z$, **c** $-5m$, **d** $9d$ **6** **a** $8t$, **b** $5n$, **c** $-25k$, **d** $7h$ **7** **a** $12m - 2n$, **b** $2e - 6f$, **c** $13x + y$, **d** $u - 8$, **e** $-7g + 5h$, **f** $-7y$ **8** **a** $6n$, **b** $7cd$, **c** $20x$, **d** $24ab$, **e** $48pqr$, **f** p^2, **g** $45t^2$, **h** ab^2c, **i** $42u^2vw$, **j** $-36k$, **k** $84s^2$, **l** $-60x^2y^2z$ **9** **a** $26m$, **b** $7e$, **c** $30 - 16c^2$ **10** **a** a, **b** c, **c** ac, **d** $5v$, **e** 7, **f** $8q$, **g** m, **h** $3x$, **i** b **11** **a** $4d$, **b** 5, **c** $10b$, **d** $12x$, **e** $4k$, **f** $-7e$, **g** $-5v$, **h** $8c$ **12** **a** $14p$, **b** $12b$, **c** $4c^2 + 13c$ **13** **a** $5a + 15$, **b** $4m - 28$, **c** $pq + 8p$, **d** $4x - xy$,

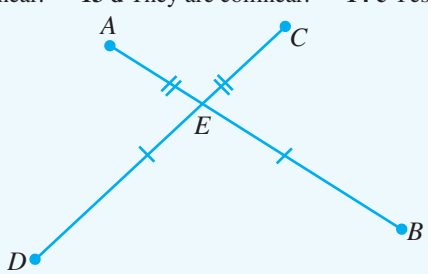
e $18c + 30d$, **f** $24g - 32h$, **g** $b^2 - 5bc$, **h** $27s^2 + 45st$ **14 a** $4y + 21$, **b** $5d + 7$, **c** $20u^2 + 10u$, **d** $9z + 9$ **15 a** $b + 5$, **b** $y - 3$, **c** $g + h$, **d** $p - q$, **e** $7s$, **f** z^2, **g** $\frac{c}{d}$, **h** $\frac{w}{2}$, **i** $\frac{2x}{3}$ **16 a** 8 less than t, **b** the difference between r and s, **c** the sum of u and v, **d** 12 more than p, **e** the product of 5 and m, **f** one-quarter of c, **g** five-sixths of k, **h** the cube of a, **i** twice the sum of m and n **17 a** $5h + 3$, **b** $\frac{t}{u} - 7$, **c** $\frac{e-f}{3}$, **d** $e - \frac{f}{3}$, **e** $2n^2$, **f** $c^3 + 4$

9 Angles

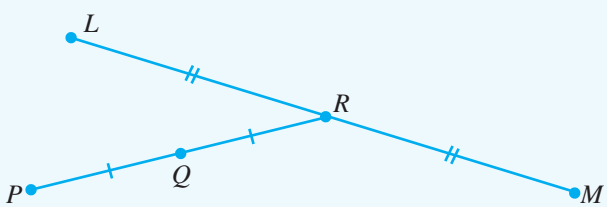
Exercise 9.1

1 a PQ, QR, RS, SP, TU, **b** $PQ, QU, UR, RT, TS, SP, UT, QR, RS$ **2 b** CD, DC, **d** CE, ED, CD **3 a** XY, **b** ZY, **c** XZ **4 b i** 3 cm, **ii** 3 cm, **c** Yes **5 b** No, **c** straight line **6 b** No, **c** point **7** The lines intersect at the point. **8** They are parallel. **9** $LN, MJ, HI; MJ, KJ, NJ$ **11 a** Yes, **b** No, **c i** P, **ii** P **12** False. The points may be collinear. **13 d** They are collinear. **14 c** Yes.

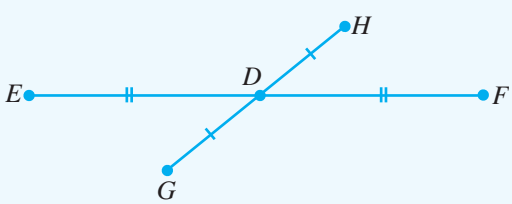
15 a



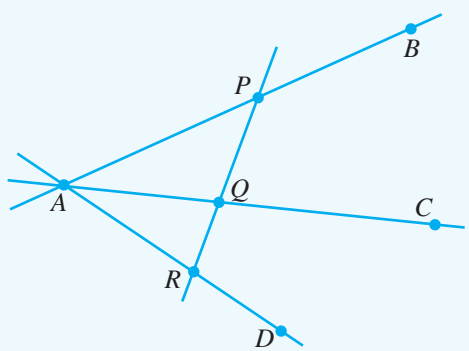
b



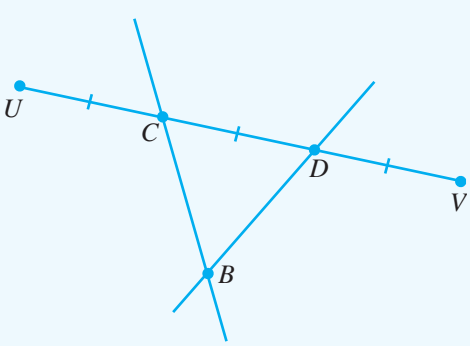
c



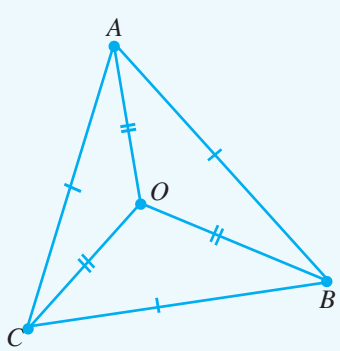
d



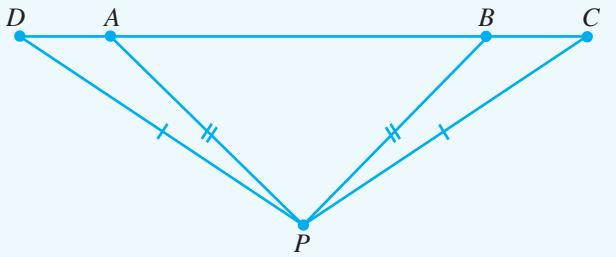
e



f



g



Exercise 9.2

1 **a i** Q **ii** $\angle PQR$, **b i** W **ii** $\angle ZWY$, **c i** G **ii** $\angle CGD$, **d i** G **ii** $\angle FGP$, **e i** T **ii** $\angle MTN$, **f i** L **ii** $\angle KLR$, **g i** G **ii** $\angle CGH$, **h i** B **ii** $\angle ABC$, **i i** E **ii** $\angle UEV$ **2** **a** L, **b** Z, **c** R, **d** C **3** **a** $\overrightarrow{EC}$, $\overrightarrow{EF}$, **b** $\overrightarrow{WX}$, $\overrightarrow{WY}$, **c** $\overrightarrow{RP}$, $\overrightarrow{RN}$, **d** $\overrightarrow{FE}$, $\overrightarrow{FW}$ **4** $\angle PQS$, $\angle SQR$, $\angle PQR$ **5** **a** $\angle APB$, **b** $\angle KCT$, **c** $\angle DLY$, **d** $\angle NBH$, **e** $\angle KOS$, **f** $\angle PMX$, **g** $\angle ULN$, **h** $\angle CPT$, **i** $\angle LOE$, **j** $\angle BXH$, **k** $\angle JAZ$, **l** $\angle FOY$ **6** **a** α: $\angle CKN$, β: $\angle SKL$, θ: $\angle CKS$, **b** α: $\angle DPG$, β: $\angle DPM$, θ: $\angle DMP$, **c** α: $\angle ZAL$, β: $\angle LAY$, θ: $\angle YAH$ **7** **a** $\angle KHR$, $\angle HRK$, $\angle RKH$, **b** $\angle UXW$, $\angle XWV$, $\angle WVU$, $\angle VUX$, **c** $\angle ABC$, $\angle BCD$, $\angle CDE$, $\angle DEA$, $\angle EAB$ **8** **a** 6, **b** 3, **c** 1, **d** 2, **e** 10, **f** 9, **g** 8, **h** 5, **i** 7, **j** 4 **9** **a** 10°, **b** 95°

Exercise 9.3

1 **a** 40°, **b** 70°, **c** 90°, **d** 130°, **e** 35°, **f** 55°, **g** 140°, **h** 115° **2** **a** 60°, **b** 30°, **c** 130°, **d** 100°, **e** 45°, **f** 25°, **g** 120°, **h** 145°, **i** 95°, **j** 10°, **k** 125°, **l** 90° **5** **a i** $\angle CDF = 50°$ $\angle FDE = 40°$, **ii** 90°, **b i** $\angle PQS = 65°$ $\angle RQS = 115°$, **ii** 180°, **c i** $\angle TXW = 110°$ $\angle VXU = 110°$, **ii** The angles are equal. **d i** $\angle JKL = 110°$ $\angle LKM = 120°$, $\angle LMKJ = 130°$, **ii** 360° **6** **a** 60°, **b** 120°, **c** 20°, **d** 140°, **e** 100°, **f** 70° **7** $\angle TUW = 25°$, $\angle TUV = 60°$, $\angle WUV = 35°$, $\angle TUW + \angle WUV = \angle TUV$ **9** **a** 320°, **b** 225°, **c** 255°

Exercise 9.4

1 **a** right, **b** acute, **c** obtuse, **d** reflex, **e** straight, **f** revolution, **g** obtuse, **h** reflex, **i** right, **j** acute, **k** reflex, **l** revolution, **m** obtuse, **n** straight, **o** right, **p** acute, **q** reflex, **r** revolution, **s** obtuse, **t** straight **2** **a** acute, **b** obtuse, **c** reflex, **d** acute, **e** right, **f** acute, **g** reflex, **h** obtuse, **i** reflex, **j** straight, **k** acute, **l** obtuse, **m** reflex, **n** acute, **o** revolution **3** **a** right, **b** obtuse, **c** acute, **d** straight, **e** acute, **f** right **4** **a** $\angle PQR$ (reflex), **b** $\angle CDE$ (reflex), **c** $\angle TUV$ (reflex) **5** **a** 45°, **b** 60°, **c** 90°, **d** 30°, **e** 45°, **f** 180°, **g** 36°, **h** 9°, **i** 120°, **j** 120°, **k** 40°, **l** 270°, **m** 108°, **n** 75°, **o** 150° **6** **a** obtuse, **b** acute, **c** revolution, **d** straight, **e** right, **f** reflex **7** **a** acute, obtuse, right, **b** reflex, **c** obtuse, straight, reflex, **d** straight, **e** revolution, **f** reflex **8** **a** acute, **b** right, obtuse, acute, **c** reflex, straight, obtuse, **d** acute, obtuse, right

Exercise 9.5

1 **a** adjacent, **b** not adjacent – no common vertex, **c** adjacent, **d** not adjacent – no common vertex, **e** not adjacent – no common ray, **f** adjacent, **g** not adjacent – do not lie on opposite sides of the common ray, **h** adjacent, **i** not adjacent – no common ray, **j** adjacent, **k** not adjacent – no common vertex, do not share a common ray, **l** not adjacent – no common ray, **m** not adjacent – do not lie on the opposite sides of the common ray, **n** adjacent, **o** not adjacent – no common ray **2** **a** $x = 50$, **b** $a = 70$, **c** $m = 30$, **d** $t = 45$, **e** $k = 65$, **f** $y = 33$ **3** **a** $a = 70$, **b** $x = 150$, **c** $w = 20$, **d** $p = 90$, **e** $b = 115$, **f** $e = 63$ **4** **a** $c = 40$, **b** $n = 110$, **c** $d = 90$, **d** $e = 45$, **e** $u = 127$, **f** $h = 161$ **5** **a** $n = 45$, **b** $t = 90$, **c** $a = 70$, **d** $k = 60$, **e** $y = 30$, **f** $c = 25$, **g** $m = 18$, **h** $u = 55$, **i** $x = 45$ **6** **a** 20°, **b** 80°, **c** 75°, **d** 55°, **e** 49°, **f** 74°, **g** 51°, **h** 18° **7** **a** 130°, **b** 40°, **c** 45°, **d** 105°, **e** 144°, **f** 78°, **g** 96°, **h** 81° **8** **a** $a = 10$, **b** $j = 80$, **c** $p = 15$ **9** No. AE is not a straight line. **10** **a** 60°, 30°, **b** 50°, 40°, **c** 35°, 55°, **d** 18°, 72° **11** **a** 150°, 30°, **b** 130°, 50°, **c** 35°, 145°, **d** 60°, 120° **12** **a** 25°, **b** 65°, **c** 22°, **d** 29°, **e** 113°, **f** 25°, **g** 30°, **h** 70°, **i** 10°, **j** 150° **13** **a** 67°, **b** 122°, **c** 61°, **d** 77°

Exercise 9.6

1 a $a = 300$, **b** $k = 250$, **c** $w = 270$, **d** $y = 80$, **e** $u = 145$, **f** $f = 67$ **2 a** $x = 120$, **b** $h = 90$, **c** $r = 72$ **3 a** $w = 35$, **b** $w = 30$, **c** $w = 30$ **4 a** $v = 110$, **b** $b = 140$, **c** $a = 150$, **d** $m = 100$, **e** $t = 125$, **f** $f = 160$, **g** $n = 155$, **h** $r = 75$, **i** $s = 153$ **7 a** 260°, **b** 145°, **c** 310°, **d** 255°, **e** 20°, **f** 35° **8** 230°

Exercise 9.7

1 a $a = 60$, **b** $m = 78$, **c** $p = 46$, **d** $t = 290$, **e** $g = 57$, **f** $u = 71$, **g** $c = 99$, **h** $w = 128$, **i** $s = 237$, **j** $e = 156$, **k** $z = 37$, **l** $r = 17$ **2 a** $y = 71$, **b** $k = 37$, **c** $n = 85$, **d** $v = 60$, **e** $d = 40$, **f** $a = 38$, **g** $x = 33$, **h** $k = 82$, **i** $b = 32$ **3 a** $a = 50$, $b = 40$, **b** $u = 48$, $v = 42$, **c** $p = 70$, $q = 130$, **d** $r = 158$, $s = 68$, $t = 112$, **e** $p = 45$, $q = 135$, $r = 135$, **f** $e = 20$, $f = 70$, **g** $c = 74$, $d = 16$, $e = 74$, **h** $y = 15$, $z = 15$, **i** $j = 70$, $k = 110$, $m = 30$, **j** $u = 50$, $v = 80$, **k** $r = 110$, $s = 55$, $t = 70$, **l** $a = 40$, $b = 50$, $c = 40$, $d = 90$ **4 a** $j = 36$, $k = 54$, **b** $x = 60$, **c** $k = 115$, $m = 105$, **d** $u = 155$, $v = 255$

Exercise 9.8

1 a Yes, **b** Yes, **c** No, **d** Yes **2 a** Yes, **b** No, **c** No, **d** Yes **3** $AP \parallel IE$, $GK \parallel MC$, $BO \parallel FN$ **8 b** Yes, **c** parallelogram **10 c** Yes, **d** kite **11 c** all 3.6 cm, **d** rhombus **13 a** kite, **b i** square, **ii** rhombus **14 c** square

Exercise 9.9

1 a i $c, d; h, f$ **ii** $h, c; d, f$ **iii** $a, c; h, g; e, f; d, b$, **b i** $c, x; r, t$ **ii** $r, x; c, t$ **iii** $p, x; k, t; n, r; c, y$, **c i** $a, p; j, h$ **ii** $a, h; j, p$ **iii** $u, h; a, v; c, p; j, e$ **2 a** corresponding, **b** co-interior, **c** alternate, **d** co-interior, **e** alternate, **f** corresponding, **g** alternate, **h** corresponding, **i** co-interior, **j** alternate, **k** co-interior, **l** corresponding **3 a** $\angle ABC$, **b** $\angle ECD$, **c** $\angle ABC$ **4 a** $\angle PQR$, **b** $\angle PQS$, **c** $\angle RSU$, **d** $\angle PRS$, **e** $\angle QST$, **f** $\angle NPQ$ **5 a** $\angle AHW$, $\angle KXA$, **b** $\angle AXP$, $\angle AHP$, **c** $\angle HPM$, $\angle LAH$, **d** $\angle AHW$, $\angle KXA$, **e** $\angle XAH$, $\angle XPH$, **f** $\angle KXA$, $\angle AHW$, **g** $\angle XAH$, $\angle XPH$, **h** $\angle ZAL$, $\angle XPH$, **i** $\angle DXP$, $\angle PHB$, **j** $\angle AXP$, $\angle WHB$, **k** $\angle ZAX$, $\angle XPE$, **l** $\angle AXP$, $\angle AHP$ **6 a** $p = 40$, **b** $c = 150$, **c** $m = 70$, **d** $y = 110$, **e** $h = 100$, **f** $a = 55$, **g** $t = 45$, **h** $v = 15$, **i** $e = 115$, **j** $f = 136$, **k** $j = 143$, **l** $n = 59$, **m** $q = 108$, **n** $s = 44$, **o** $u = 101$ **7 a** $x = 100$, **b** $x = 150$, **c** $x = 105$, **d** $x = 85$, **e** $x = 58$, **f** $x = 132$, **g** $x = 64$, **h** $x = 12$, **i** $x = 157$, **j** $x = 155$, **k** $x = 96$, **l** $x = 33$ **8 a** $x = 20$, $y = 20$, $z = 160$, **b** $p = 50$, $q = 130$, $r = 130$, **c** $c = 65$, $d = 65$, $e = 65$, **d** $u = 32$, $v = 148$, $w = 32$, **e** $k = 78$, $m = 102$, $n = 78$, **f** $r = 75$, $s = 75$, $t = 105$ **9 a** $a = 60$, $b = 120$, $c = 120$, **b** $m = 70$, $n = 20$, **c** $p = 76$, $q = 76$, $r = 55$, **d** $c = 80$, $d = 100$, **e** $e = 55$, $f = 35$, **f** $x = 40$, **g** $u = 75$, $v = 75$, **h** $y = 140$, $z = 140$, **i** $p = 77$, $q = 77$, **j** $x = 68$, $y = 34$, $z = 112$, **k** $f = 15$, $g = 75$, $h = 75$, **l** $k = 80$, $m = 100$, $n = 138$

Chapter 9 Review

1 U **2** $\angle QRS$ **3** $\overrightarrow{DC}$, $\overrightarrow{DE}$ **4 a** $\angle GFH$, **b** $\angle ABD$, **c** $\angle KLN$ **5 a** 52°, **b** 124°, **c** 90° **6 a** acute, **b** straight, **c** revolution, **d** right, **e** obtuse, **f** reflex **7 a** obtuse, **b** right, **c** reflex, **d** revolution, **e** acute, **f** straight **8 a** 45°, **b** 60°, **c** 144° **9 a** No, **b** Yes, **c** No, **d** Yes **10 a** vertically opposite, **b** supplementary, **c** complementary **11 a** $a = 110$, **b** $p = 120$, **c** $t = 70$, **d** $m = 50$, **e** $x = 60$, **f** $c = 18$ **12 a** 58°, **b** 106°, **c** 15° **13 a** $k = 280$, **b** $n = 105$, **c** $r = 270$, **d** $u = 12$, **e** $z = 120$, **f** $w = 70$ **14 a** $m = 50$, $n = 130$, **b** $p = 73$, $q = 17$, **c** $c = 48$, $d = 42$ **15 a** e and c, d and f, **b** h and d, g and c, e and a, f and b, **c** e and d, f and c **16 a** $x = 130$, **b** $e = 75$, **c** $q = 47$, **d** $k = 65$, **e** $s = 122$, **f** $v = 71$ **17** $x = 34$, $y = 146$, $z = 146$ **18 a** $a = 105$, $b = 75$, $c = 15$, **b** $e = 82$, $f = 98$, $g = 82$, **c** $t = 22$, $u = 68$, $v = 112$ **20 a** T, **b** PR, TR, VR

10 Properties of geometrical figures

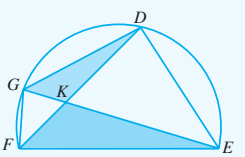
Exercise 10.1

1 a Yes, **b** No, the figure is not closed, **c** Yes, **d** Yes, **e** No, a hockey ball is not flat. **f** No, a cube is not flat. **g** Yes **2 a** circle, **b** triangle, **c** square, **d** rectangle, **e** parallelogram, **f** rhombus, **g** trapezium, **h** kite, **i** ellipse **3** A, B, E, F **4 a** square, **b** rectangle, square, rhombus, **c** rectangle, parallelogram, square, trapezium, rhombus, **d** square **5** kite **6** B, F, H **7 a** A trapezium has only one pair of opposite sides parallel. **b** A triangle has only three sides. **8** circle, ellipse **9 c** trapezium **10** A square that has been 'pushed over'. **11 a** rectangle, **b** parallelogram **12 a** rhombus, **b i** parallelogram, **ii** rhombus, **c i** kite, **ii** parallelogram **14 a** No, **b** Yes, **c** Yes

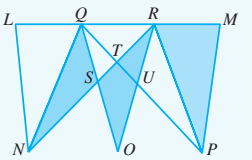
Exercise 10.2

1 a Yes, **b** No, **c** No, **d** Yes, **e** No, **f** No, **g** Yes, **h** Yes, **i** No **2 a** regular, **b** irregular, **c** irregular, **d** irregular, **e** regular, **f** irregular **3 a** triangle, **b** quadrilateral, **c** pentagon, **d** hexagon, **e** heptagon, **f** octagon, **g** nonagon, **h** decagon, **i** undecagon, **j** dodecagon **4 a** 6, **b** 10, **c** 3, **d** 9, **e** 12, **f** 5, **g** 4, **h** 8, **i** 7, **j** 11 **5 a** regular hexagon **6 a** irregular pentagon **7 a** trapezium (2), rectangle, **b** trapezium (2), kite, **c** rhombus (4), square (2) **8** rhombus (3) **9 a** pentagon, **b** hexagon, **c** nonagon, **d** quadrilateral, **e** heptagon, **f** octagon, **g** heptagon, **h** nonagon, **i** dodecagon, **j** decagon, **k** quadrilateral, **l** undecagon **10 a** rhombus, **b** rectangle, **c i** angles, **ii** sides **11 a** irregular, **b** irregular, **c** irregular, **d** irregular, **e** regular, **f** irregular **12 a** convex, **b** non-convex, **c** convex, **d** convex, **e** non-convex, **f** convex, **g** non-convex, **h** non-convex **13** There are many possible answers. **a** ΔEFG, **b** $LMNO$, **c** $ABJKPQL$ **d** $CDXYZ$
14 There are many possible answers.

15 a



b



16 a 9, **b** 18

	Square	Pentagon	Hexagon
sides (n)	4	5	6
diagonals (d)	2	5	9
triangles (t)	4	10	18

d i Number of triangles is equal to twice the number of diagonals. **ii** Number of triangles is equal to the number of sides × (3 less than the number of sides). **e i** $t = 2d$, **ii** $t = n(n - 3)$ **f i** 40, **ii** 70, **iii** 9700 **g** $d = \frac{n(n-3)}{2}$, **h i** 14, **ii** 54, **iii** 4850

Exercise 10.3

1 a scalene, **b** isosceles, **c** equilateral **2 a** right-angled, **b** acute-angled, **c** obtuse-angled **3 a** Yes, **b** Yes, **c** opposite; shortest **4 a** equilateral, **b** 60°, **c** equilateral; 60° **5 b** 2, **c** The equal angles lie opposite the equal sides. **6 b** 180° **7 b** 180° **c** 180° **8 b** The triangles cannot be drawn. **c** less; sum **9 a** smallest $= \angle C$, largest $= \angle B$, **b** smallest $= \angle Y$, largest $= \angle Z$, **c** smallest $= \angle P$, largest $= \angle Q$, **d** smallest $= \angle S$, largest $= \angle R$ **10 a** shortest $= EG$, longest $= EF$, **b** shortest $= LM$, longest $= MN$, **c** shortest $= VU$, longest $= VT$, **d** shortest $= AB$, longest $= AC$ **11 a** $p = 8$, **b** $k = 60$ **12 a** PR and QR, **b** IK and JK, **c** CD and CE, **d** XZ and YZ **13 a** $\angle I$ and $\angle J$, **b** $\angle G$ and $\angle H$, **c** $\angle L$ and $\angle N$, **d** $\angle V$ and $\angle X$ **14 a** $k = 12$, **b** $x = 55$, **c** $t = 18$, **d** $p = 71$ **15 a** $a = 50$, **b** $m = 140$, **c** $w = 50$, **d** $y = 33$, **e** $c = 69$, **f** $t = 74$ **16 a i** $x = 80$, $y = 100$, **ii** $x = 60$, $y = 120$, **iii** $x = 140$, $y = 40$, **b** Add the interior opposite angles. **c** exterior; sum; angles **17 a** $a = 130$, **b** $n = 45$, **c** $w = 115$, **d** $e = 60$, **e** $t = 50$, **f** $y = 45$, **g** $h = 120$, **h** $s = 160$, **i** $r = 30$ **18 a** Yes, **b** Yes, **c** No, **d** No, **e** Yes, **f** No **19** A, D, F **20 a** No, **b** No, **c** Yes, **d** No, **e** No, **f** Yes, **g** Yes, **h** No, **i** Yes

Exercise 10.4

1 a $a = 80$, **b** $k = 30$, **c** $y = 50$, **d** $p = 60$, **e** $m = 30$, **f** $c = 100$, **g** $m = 40$, **h** $c = 130$, **i** $t = 20$, **j** $a = 70$, **k** $w = 15$, **l** $k = 59$ **2** A and D **3** C **4** ΔDEF, ΔGHI, ΔPQR **5 a** $a = 80$, **b** $k = 20$, **c** $n = 70$, **d** $w = 30$, **e** $x = 130$, **f** $c = 45$, **g** $h = 42$, **h** $z = 47$ **6 a** No, **b** No, **c** Yes, **d** No, **e** Yes, **f** Yes **7 a** $x = 65$, **b** $t = 45$, **c** $v = 5$ **8 a** $a = 50$, $b = 130$, **b** $c = 50$, $d = 90$, **c** $p = 38$, $q = 142$ **9 a** $x = 70$, $y = 70$, $z = 40$, **b** $u = 62$, $v = 56$, **c** $m = 60$, $n = 120$ **10 a** $p = 32$, $q = 32$, **b** $a = 84$, $b = 24$ **c** $u = 44$, $v = 68$ **11 a** $x = 35$, $y = 55$, **b** $x = 74$, $y = 37$, **c** $x = 43$, $y = 47$ **12 a** $x = 80$, $y = 60$, **b** $c = 130$, $d = 125$, **c** $j = 50$, $k = 65$ **13 a** $a = 50$, $b = 60$, **b** $m = 65$, $n = 55$, **c** $c = 80$, $d = 35$, **d** $j = 125$, $k = 25$, **e** $u = 30$, $v = 150$, **f** $p = 73$, $q = 34$, **g** $r = 63$, $s = 54$, **h** $g = 40$, $h = 40$, **i** $e = 70$, $f = 70$, **j** $p = 36$, $q = 108$, **k** $x = 49$, $y = 49$, **l** $r = 60$, $s = 50$ **14 a** $k = 42$, $m = 58$, $n = 80$, **b** $a = 80$, $b = 70$, $c = 30$, $d = 150$, **c** $x = 40$, $y = 75$, $z = 75$, **d** $t = 51$, $u = 39$, $v = 90$, **e** $g = 40$, $h = 20$, **f** $p = 86$, $q = 44$ **15 a** $i = 130$, $j = 65$, **b** $t = 131$, $u = 98$, **c** $x = 30$, $y = 120$

Exercise 10.5

1 a square, **b** rectangle, **c** parallelogram, **d** rhombus, **e** trapezium, **f** kite **2** square: **a** Yes, **b** Yes, **c** 2, **d** Yes, **e** Yes, **f** Yes, **g** Yes, **h** Yes, **i** Yes; rectangle: **a** Yes, **b** No, **c** 2, **d** Yes, **e** Yes, **f** Yes, **g** Yes, **h** No, **i** No; parallelogram: **a** Yes, **b** No, **c** 2, **d** Yes, **e** No, **f** No, **g** Yes, **h** No, **i** No rhombus: **a** Yes, **b** Yes, **c** 2, **d** Yes, **e** No, **f** No, **g** Yes, **h** Yes, **i** Yes; trapezium: **a** No, **b** No, **c** 1, **d** No, **e** No, **f** No, **g** No, **h** No, **i** No; kite: **a** No, **b** No, **c** 0, **d** No, **e** No, **f** No, **g** No, **h** Yes, **i** No

3

Property	Square	Rectangle	Parallelogram	Rhombus
All sides are equal	✔	✗	✗	✔
Opposite sides are equal	✔	✔	✔	✔
Opposite sides are parallel	✔	✔	✔	✔
All angles are right angles	✔	✔	✗	✗
Opposite angles are equal	✔	✔	✔	✔
Diagonals are equal in length	✔	✔	✗	✗
Diagonals intersect at right angles	✔	✗	✗	✔
Diagonals bisect each other	✔	✔	✔	✔
Diagonals bisect angles at vertices	✔	✗	✗	✔

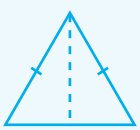
4 trapezium **5** kite **7** **b** 360° **8** **b** 360°, **c** 360° **9** **a** $x = 70$, **b** $m = 80$, **c** $d = 50$, **d** $h = 102$, **e** $u = 124$, **f** $a = 27$ **10** **a** 7 cm, **b** 90°, **c** 10 cm, **d** BC, **e** 90°, **f** 5 cm, **g** 45° **11** **a** 16 cm, **b** 90°, **c** 20 cm, **d** PQ, **e** No, **f** 10 cm, **g** 53° **12** **a** 8 cm, **b** **i** 60°, **ii** 120°, **c** No, **d** KL, **e** No, **f** 6.5 cm, **g** No **13** **a** 13 cm, **b** **i** 135°, **ii** 45°, **c** No, **d** WZ, **e** 90°, **f** 12 cm, **g** 67.5° **14** **a** **i** 7 cm, **ii** 7 cm, **iii** 7 cm, **iv** 7 cm, **b** **i** 70°, **ii** 40°, **iii** 40°, **iv** 70°, **v** 70°, **vi** 140°, **vii** 20°, **viii** 20°, **ix** 20°, **x** 140° **15** **a** 90°, **b** 90°, **c** 60°, **d** 30°, **e** 60°, **f** 30°, **g** 30°, **h** 60°, **i** 60°, **j** 120° **16** **a** $\angle S = \angle V$, $\angle T = \angle U$, **b** SV and TU, **c** $\angle S = 40°$, $\angle V = 40°$, $\angle U = 140°$, $\angle T = 140°$ **17** $a = 70$, $b = 140$ **18** **a** Yes. Co-interior angles are supplementary. $\therefore$ $CF \parallel DE$. **b** No. No parallel sides. **c** Yes. $\angle VYW = 70°$. $\therefore$ $WX \parallel VY$ (alternate angles are equal). **20** **a** S, RE, P, RH, **b** S, RE, P, RH, **c** S, RE, P, RH, **d** S, RH, **e** S, RE, **f** S, RE, **g** S, RE, P, RH, **h** S, RH, K **i** S, RH **21** **a** RE, **b** S, **c** S, RH **d** S, RH, **e** S, **f** P, RH, **g** RE, **h** RH **22** **a** A, **b** B, **c** A, **d** B, **e** B

Exercise 10.6

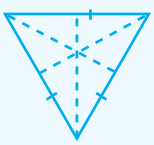
1 **a** $d = 60$, **b** $e = 110$, **c** $f = 40$, **d** $m = 240$, **e** $w = 160$, **f** $p = 124$ **2** **a** $l = 110$, $m = 250$, **b** $x = 260$, $y = 100$, **c** $s = 206$, $t = 85$ **3** **a** $d = 60$, $e = 90$, **b** $g = 135$, $h = 45$, **c** $r = 30$, $s = 170$ **4** **a** $p = 60$, $q = 30$, **b** $d = 52$, $e = 38$, **c** $u = 27$, $v = 123$ **5** **a** $c = 80$, $d = 80$, **b** $m = 85$, $n = 65$, **c** $y = 60$, $z = 216$ **6** **a** $p = 130$, $q = 40$, **b** $u = 120$, $v = 25$, **c** $x = 111$, $y = 89$ **7** **a** $e = 70$, $f = 90$, **b** $j = 31$, $k = 56$, **c** $a = 97$, $b = 58$ **8** **a** $a = 55$, $b = 125$, $c = 87$, **b** $g = 45$, $h = 95$, **c** $t = 207$, $u = 90$, $v = 19$, **d** $c = 90$, $d = 62$, $e = 137$

Exercise 10.7

1 **a**

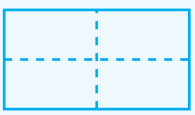


b

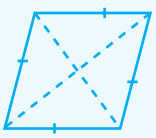


c

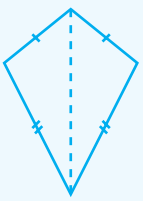
d



e



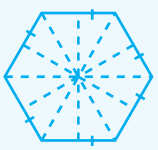
f



g

h

i

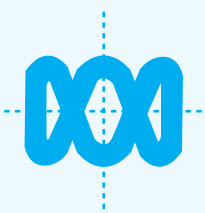


2 0 **3** **a** 0, **b**

Isosceles trapezium

4 **a** Yes, **b** Square, rhombus **5** **a** 1, **b** 2, **c** 1, **d** 4, **e** 1, **f** 0, **g** 1, **h** 0 **6** **a** 1, **b** 1, **c** 5, **d** 5, **e** 6

7 **a**



b

c

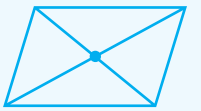
d

8 **a** No, **b** Yes, 3, **c** Yes, 4, **d** Yes, 2, **e** Yes, 2, **f** No, **g** Yes, 2, **h** Yes, 5, **i** Yes, 6 **9** Yes, 2

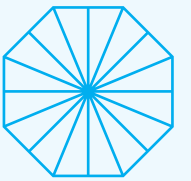
10 **a**

b

c



d



11 **a** The opposite angles are equal. **b** All angles are equal (and 90°). **12** **a** 4, **b** 3, **c** 2, **d** 6 **13** **a** 2, **b** 4, **c** 3, **d** 0 **14** **a**

Shape	Number of sides	Order of symmetry
Equil. Δ	3	3
Square	4	4
Reg. pentagon	5	5
Reg. hexagon	6	6

b The order of rotational symmetry is equal to the number of sides in a regular polygon. **c** **i** 10, **ii** 12 **15** **a** Yes, **b** Yes, **c** No, **d** Yes, **e** No, **f** Yes, **g** No, **h** Yes, **i** Yes
16 **a** square, rectangle, parallelogram, rhombus. **b** The order of rotational symmetry is even and at least 2.

Chapter 10 Review

1 **a** circle, **b** rectangle, **c** triangle, **d** parallelogram, **e** kite, **f** trapezium, **g** rhombus, **h** square, **i** ellipse
2 **a** quadrilateral, **b** triangle, **c** hexagon, **d** decagon, **e** pentagon, **f** octagon, **g** heptagon, **h** nonagon, **i** undecagon, **j** dodecagon **3** **a** No, it is not closed, **b** No, it has a curved side, **c** Yes **4** **a** Yes, **b** No, **c** No, **d** Yes **5** **a** convex, **b** non-convex **6** **a** ΔACD, **b** ΔNOP **7** 14 **8** **a** scalene, **b** equilateral, **c** isosceles
9 **a** right-angled, **b** obtuse-angled, **c** acute-angled **10** 60° **11** longest = PR, shortest = PQ
12 largest = $\angle E$, smallest = $\angle F$. **13** B and C **14** **a** $\angle M$ and $\angle N$, **b** $\angle L$ **15** **a** No, **b** Yes, **c** No
16 C **17** **a** $x = 33$, **b** $y = 27$, **c** $m = 50$, **d** $p = 48$ **18** **a** square, rectangle, parallelogram, rhombus. **b** square, rectangle, parallaleogram, rhombus. **c** square, rectangle. **d** square, rhombus, kite. **e** square, rhombus.
19 kite **20** **a** 22 cm, **b** 11 cm, **c** 36°, **d** 108°, **e** 36°, **f** 54° **21** $k = 71$, **b** $c = 39$ **22** **a** $c = 75$, $d = 105$, **b** $a = 74$, $b = 16$, $c = 32$ **c** $m = 18$, $n = 72$, **d** $x = 115$, $y = 82$ **23** **a** 3, **b** 1, **c** 0 **24** **a** equilateral triangle, **b** 3 **25** square – 4, rectangle – 2, parallogram – 0, rhombus – 2, trapezium – 0, kite – 1. **26** square – 4, rectangle – 2, parallelogram – 2, rhombus – 2.

11 Measurement, length and perimeter

Exercise 11.1

1 **A** 15 mm, **B** 28 mm, **C** 52 mm, **D** 74 mm, **E** 91 mm, **F** 107 mm 2 **2** **a** 2400 mm, **b** 3710 mm, **c** 1525 mm, **d** 5270 mm, **e** 3135 mm, **f** 4058 mm **3** **a** 20°C, **b** 14°C, **c** 28°C, **d** 45°C **4** **a** 54 kg, **b** 77 kg
5 Lars is 102 cm tall, Enikö is 88 cm tall **6** **a** **i** 80 km/h **ii** 50 km/h **b** **i** 93428.1 km **ii** 108 967.5 km
7 **a** 20 mm, **b** 6 mm, **c** 18 mm **8** **a** 990 millibars, **b** 1025 millibars, **c** 1007 millibars **9** **a** $\frac{3}{4}$, **b** $\frac{3}{8}$, **c** $\frac{1}{16}$
10 16 500 m **11** 8 V **12** **a** 15 mL, **b** 1.25 L, **c** 64 mL **13** **a** 61 274 kWh, **b** 25 308 kWh, **c** 28 471 kWh

Exercise 11.2

1 **a** mm, **b** cm, **c** m, **d** km, **e** cm, **f** m, **g** mm, **h** km, **i** cm, **j** m, **k** mm, **l** km **2** **a** 20 mm, **b** 5 cm, **c** 60 mm, **d** 11 cm, **e** 23 cm, **f** 300 mm, **g** 35 mm, **h** 4.7 cm, **i** 9.2 cm, **j** 86 mm, **k** 21.3 cm, **l** 103 mm **3** **a** 400 cm, **b** 3 m, **c** 900 cm, **d** 16 m, **e** 22 m, **f** 4000 cm, **g** 150 cm, **h** 2.6 m, **i** 530 cm, **j** 7.4 m, **k** 1.89 m, **l** 375 cm
4 **a** 3000 m, **b** 7 km, **c** 12 km, **d** 8000 m, **e** 20 km, **f** 60 000 m, **g** 2.5 km, **h** 4100 m, **i** 6350 m, **j** 1.32 km, **k** 5.248 km, **l** 6004 m **5** **a** 600 cm, **b** 30 mm, **c** 2 km, **d** 5 m, **e** 0.9 m, **f** 4000 m, **g** 23 mm, **h** 8200 m, **i** 6.8 m, **j** 4.8 km, **k** 5.9 cm, **l** 940 cm, **m** 1250 m, **n** 2.24 m, **o** 400 mm, **p** 14.63 m, **q** 15 cm, **r** 2.864 km, **s** 25 000 cm, **t** 500 m **6** **a** 4, **b** 0.2, **c** 0.5, **d** 600, **e** 0.09, **f** 0.001, **g** 0.8, **h** 2, **i** 0.03, **j** 0.00015, **k** 0.00028, **l** 0.0014, **m** 0.004, **n** 15 300, **o** 7, **p** 0.08, **q** 0.016, **r** 10.2 **7** **a** 3000, **b** 700 000, **c** 4, **d** 2 000 000, **e** 8, **f** 6, **g** 0.5, **h** 140 000, **i** 3850, **j** 0.023, **k** 0.0546, **l** 0.025 **8** **a** 5, **b** 1500, **c** 25, **d** 750, **e** 35, **f** 2250, **g** 210, **h** 12,

i 3700 **9 a** 10 dm, **b** 100 m, **c** 10 m, **d** 20 hm, **e** 500 mm, **f** 4000 cm, **g** 30 000 cm, **h** 80 000 dm, **i** 900 000 mm, **j** 14 cm, **k** 49 m, **l** 620 m **10 a** 1 dm, **b** 3 km, **c** 8 m, **d** 5 dm, **e** 4 dam, **f** 900 dm, **g** 0.2 hm, **h** 0.67 dam, **i** 0.49 km, **j** 0.007 km, **k** 0.0004 hm, **l** 0.00519 dam

Exercise 11.3

1 7.8m **2** 12.4 m is greater by 716 mm **3** 25 laps **4** 125 matches **5** 150 cm **6** 72 mm **7** 13.65 km **8** Yes **9** 7 cartons **10** 8 books **11** 252 m **12** 6.7 m **13** 11.25 km **14** 1566 mm **15** 60 m **16** 28.2 m **17** 2.8 cm **18** 136 cm **19** 7.25 m **20** length of wall is 4.733 m, height of wall is 2.603 m

Exercise 11.4

1 a nearest km, **b** nearest mm, **c** nearest m, **d** nearest cm **2 a** ± 30 min, **b** ± 5 mm, **c** ± 500 g, **d** ± 0.5°, **e** ± 0.5°C, **f** ± 0.5 mm, **g** ± 0.5 m, **h** ± 5 cm **3 a** 2.5–3.5 m, **b** 6.5–7.5 m, **c** 11.5–12.5 m, **d** 38.5–39.5 m, **e** 45.5–46.5 m, **f** 84.5–85.5 m, **g** 99.5–100.5 m, **h** 999.5–1000.5 m **4 a** 15–25 cm, **b** 25–35 cm, **c** 55–65 cm, **d** 85–95 cm, **e** 135–145 cm, **f** 265–275 cm, **g** 1605–1615 cm, **h** 95–105 cm **5 a** 699.5–700.5 km, **b** 695–705 km, **c** 650–750 km **6** Between 6250 and 6350 years **7** Between $3350 and $3450 **8 a** 835–845 m, **b** 265–275 pens, **c** $8\frac{1}{2}$–$9\frac{1}{2}$ hours, **d** 143.5–144.5 cm, **e** 175–185 cm, **f** 12 500–13 500 locusts, **g** 290–310 pages, **h** 15.5–16.5 mm, **i** 750–850 students, **j** 13.5–14.5 m, **k** 5500–6500 km, **l** 92.5–93.5 kg, **m** 115–125 L, **n** 650–750 kg, **o** $22.50–$27.50 **9 a** 7.5 mm, 8.5 mm, **b** 2.5 cm, 3.5 cm, **c** 6.5 m, 7.5 m, **d** 3.5 km, 4.5 km, **e** 7.15 mm, 7.25 mm, **f** 8.55 m, 8.65 m, **g** 15.25 km, 15.35 km, **h** 21.85 cm, 21.95 cm, **i** 5.235 m, 5.245 m, **j** 1.055 km, 1.065 km, **k** 12.745 cm, 12.755 cm, **l** 7.995 mm, 8.005 mm, **m** 3.35 cm, 3.45 cm, **n** 7.825 km, 7.835 km, **o** 18.85 mm, 18.95 mm, **p** 30.145 m, 30.155m, **q** 4.95 km, 5.05 km, **r** 23.5 m, 24.5m, **s** 36.775 cm, 36.785 cm, **t** 49.85 mm, 49.95 mm, **u** 2181.5 m, 2182.5 m, **v** 350.35 mm, 350.45 mm, **w** 816.165 km, 816.175 km, **x** 8.1955 cm, 8.1965 cm, **y** 53.7415 m, 53.7425 m **10** 12.25 m **11** 2.365 m **12 a** No, **b** Yes **13** 362 m **14** 148 500 000 km, 149 500 000 km

Exercise 11.5

1 a tape measure, **b** trundle wheel, **c** metre rule, **d** odometer, **e** 30 cm ruler, **f** dressmaker's tape **2 a** A, **b** B, **c** C, **d** C, **e** D, **f** B, **g** C, **h** B, **i** D **3 a** 40 mm, **b** 70 mm, **c** 35 mm, **d** 63 mm, **e** 26 mm, **f** 34 mm, **g** 57 mm, **h** 44 mm **5 a** 30 mm, **b** 65 mm, **c** 80 mm, **d** 59 mm **6 a** 3 cm, **b** 5 cm, **c** 2 cm, **d** 4 cm **7 a** 2 cm, **b** 5 cm, **c** 3 cm, **d** 6 cm, **e** 4 cm, **f** 7 cm **9 a** square, **b** rectangle, **c** isosceles triangle, **d** equilateral triangle, **e** parallelogram, **f** rhombus **10 b** 5 cm **11** $AB = CD = 4$ cm **12** Both 5 cm

Exercise 11.6

1 a 23 cm, **b** 44 mm, **c** 32 m, **d** 24 mm, **e** 26 m, **f** 30 cm, **g** 32 mm, **h** 19 cm, **i** 39 m **2 a** 32 mm, **b** 48 mm, **c** 75 cm, **d** 47 m, **e** 45 cm, **f** 96 mm, **g** 84 cm, **h** 82 mm **3 a** 20 cm, **b** 24 cm, **c** 48 cm, **d** 36 cm, **e** 36 cm **4 a** 12 cm, **b** 14 cm, **c** 13 cm **5** 52 cm **6 a** 30 mm, **b** 34 cm, **c** 36 m, **d** 43.4 mm **7 a** 5 cm, **b** 9 cm, **c** 17 cm **8 a** 7 cm, **b** 8 mm, **c** 5 m, **d** 4 cm **9 a** $x = 6$, **b** $k = 5$, **c** $y = 7$ **10 a** $w = 8$, **b** $z = 9$, **c** $g = 3.5$ **11 a** 34 cm, **b** 54 cm, **c** 56 cm, **d** 80 cm, **e** 72 cm, **f** 128 cm, **g** 80 cm, **h** 64 cm, **i** 64 cm, **j** 60 cm, **k** 118 cm, **l** 58 cm **12 a** 94 mm, **b** 102 mm, **c** 80 mm, **d** 108 mm

Exercise 11.7

1 290 m **2** 1405 m **3** 16 m **4** 9 cm **5** width = 7 cm, length = 14 cm **6** 120 m **7** 6.2 km **8** 3 laps **9** 7 laps **10 a** 500 m, **b** $30 000 **11 a** 29 m, **b** $45 **12** 98 cm **13 a** 23 m × 11 m, **b** 68 m **14 a** 15 laps, **b** $105 **15** 125 m **16** 3 hours **17** 120 cm **18** $1500 **19** 22 posts **20** 162 mm

Chapter 11 Review

1 **A** = 2 cm, **B** = 3.5 cm, **C** = 5.2 cm, **D** = 6.7 cm, **E** = 9.9 cm **2 a** 33°C, **b** 53 kg **3 a** m, **b** km, **c** cm, **d** mm **4 a** 300 cm, **b** 90 mm, **c** 2000 m, **d** 4 cm, **e** 6 m, **f** 4 km, **g** 750 cm, **h** 2.4 cm, **i** 8600 m, **j** 9.1 m, **k** 800 mm, **l** 520 m, **m** 0.07 m, **n** 0.016 km, **o** 4 cm, **p** 0.0000005 m **5 a** 6000 mm, **b** 0.190 m, **c** 1 300 000 cm, **d** 0.00475 km **6 a** 113 cm, **b** 80, **c** 28.8 km **7 a** 27.5–28.5 cm, **b** 165–175 cm, **c** 35.5–36.5 mm, **d** 16 350–16 450 L, **e** 1425–1475 years **8 a** 3.5–4.5 mm, **b** 6.75–6.85 km, **c** 5.925–5.935 m,

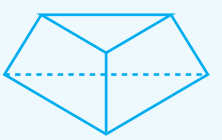
d 11.95–12.05 cm, **e** 30.5–31.5 km, **f** 132.65–132.75 m, **g** 0.455–0.465 cm, **h** 1.995–2.005 mm
9 **a** tape measure, **b** 30 cm ruler, **c** trundle wheel, **d** odometer, **e** dressmaker's tape, **f** metre ruler
10 **a** C, **b** D, **c** B **11** **a** 30 mm, **b** 45 mm, **c** 33 mm **12** **a** 35 mm, **b** 37 mm **13** **a** 72 cm, **b** 70 mm, **c** 70 m, **d** 32 cm **14** **a** 37.6 m, **b** 46 mm, **c** 104 cm, **d** 35 m **15** **a** 14 cm, **b** 7 cm **16** **a** 50 cm, **b** 35 cm
17 **a** 66 mm, **b** 56 mm, **c** 58 mm, **d** 70 mm **e** 58 mm, **f** 36 mm **18** 4 cm × 12 cm **19** 15 cm **20** $2622
21 **a** 8 m, **b** 5 m × 3 m, 16 m

12 Solids

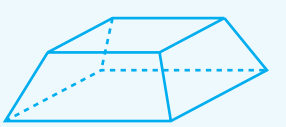
Exercise 12.1

1 **a** square, **b** rectangle, **c** triangle, **d** circle, **e** square, **f** rectangle, **g** triangle, **h** circle **2** *A*, *B*, *C* **3** *E*, *F*, *G* **4** *D*, *H*, *I* **5** **a** cube, **b** rectangular prism, **c** triangular prism, **d** cylinder, **e** square pyramid, **f** rectangular pyramid, **g** triangular pyramid, **h** cone, **i** sphere **6** *D*, *H*, *I* **7** *A*, *B*, *C*, *D* **8** *E*, *F*, *G*, *H* **9** **a** 6 squares, **b** 2 triangles and 3 rectangles, **c** 4 triangles and 1 square, **d** 4 triangles and 1 rectangle **10** **a** hexagonal prism, **b** pentagonal prism, **c** hexagonal pyramid, **d** pentagonal pyramid, **e** octagonal prism, **f** octagonal pyramid
11 **a** prism, **b** pyramid, **c** pyramid, **d** neither, **e** prism, **f** prism, **g** neither, **h** pyramid, **i** neither
12 **a** cylinder, **b** sphere, **c** cube or rectangular prism, **d** cylinder, **e** rectangular prism, **f** cube, **g** sphere, **h** cylinder, **i** rectangular prism **13** **a** trapezoidal prism; 2 trapeziums and 4 rectangles. **b** rhombic prism, 2 rhombuses and 4 rectangles. **15** **a** 2 cylinders **b** a cube and a square pyramid, **c** a cone and a cylinder, **d** a triangular prism and a triangular pyramid, **e** a rectangular prism and a rectangular pyramid, **f** a rectangular prism and a trapezoidal prism **16** **a** right prism, **b** oblique cylinder, **c** oblique prism, **d** right cylinder **17** **a** oblique pyramid, **b** right cone, **c** right pyramid, **d** oblique cone **18** **a** non-convex, **b** convex, **c** non-convex, **d** convex, **e** non-convex, **f** convex
19 **a**

b



c



Exercise 12.2

1 **a** No, **b** Yes, **c** Yes, **d** No, **e** Yes, **f** No, **g** Yes, **h** Yes, **i** Yes
2 **a**

	F	*V*	*E*	*F* + *V*
Rectangular prism	6	8	12	14
Triangular prism	5	6	9	11
Hexagonal prism	8	12	18	20
Square pyramid	5	5	8	10
Triangular pyramid	4	4	6	8
Pentagonal pyramid	6	6	10	12

b $F + V - E = 2$ (or $F + V = E + 2$)
3 **a** cone, **b** sphere or cylinder **4** No

5 **a**

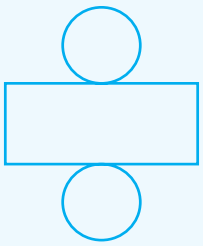
	No. of sides in base	*F*	*V*	*E*
Square prism	4	6	8	12
Pentagonal prism	5	7	10	15
Hexagonal prism	6	8	12	18
Heptagonal prism	7	9	14	21
Octagonal prism	8	10	16	24
n-sided prism	*n*	*n* + 2	2*n*	3*n*

b 22 faces, 40 vertices, 60 edeges,
c 22 + 40 – 60 = 2
6 **a** 14 faces, 24 vertices, 36 edges **b** Yes, **c** Yes **7** **a** **i** 3, **ii** 2, **iii** 3 **b** **i** *CD*, *CB*, *CG*, **ii** *AB*, *AD*, *AE*, **iii** *HD*, *HE*, *HG*, **c** **i** *DAEH*, *DCGH*, **ii** *ABCD*, *ABFE*, **iii** *FGCB*, *FGHE* **d** **i** *ABFE*, *BFGC*, *ABCD*, **ii** *GCBF*, *EFGH*, *GHDC*, **iii** *AEHD*, *ABFE*, *EFGH* **8** **a** **i** *NOPQ*, **ii** *LKOP*, **iii** *MLPQ*, **b** *MJNQ*, *LKOP*, *JKLM*, *NOPQ* **c** **i** parallel, **ii** perpendicular, **iii** parallel, **iv** perpendicular, **v** perpendicular, **vi** parallel
9 **a** Yes, **b** No, **c** Yes, **d** Yes, **e** No, **f** No, **g** Yes, **h** No, **i** Yes, **j** Yes, **k** Yes, **l** No **10** **a** parallel, **b** skew, **c** skew, **d** perpendicular, **e** parallel, **f** perpendicular **12** **a** *KN*, **b** *KL*, **c** *KM* **14** **a** 40, **b** 32, **c** 20

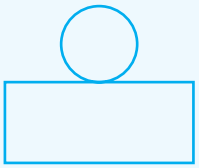
Exercise 12.3

1 **a** cube, **b** rectangular prism **2** **a** rectangular prism, **b** triangular prism, **c** cube, **d** tetrahedron, **e** rectangular pyramid, **f** square pyramid **3** **a** *B*, hexagonal prism, **b** *D*, pentagonal pyramid, **c** *C*, pentagonal prism, **d** *A*, hexagonal pyramid **4** **a** tetrahedron, **b** 4 faces, 4 vertices, 6 edges, **c** No, because some intervals are joined to form the solid. **5** **a** trapezoidal prism, **b** 6 faces, 8 vertices, 12 edges

7 **a**



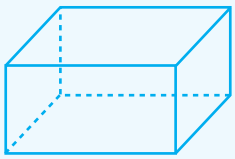
, **b**



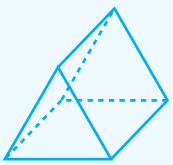
8 cone **9** *B* and *F*, *C* and *E*, *A* and *D* **10** *C*

Exercise 12.4

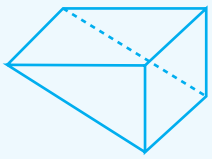
1 Oblique projection **2** **a**



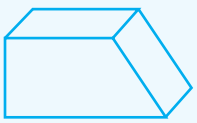
b



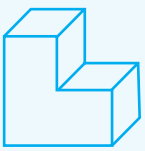
c



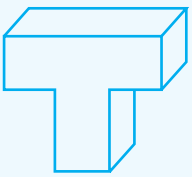
3 **a**



b

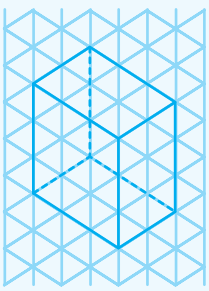


c

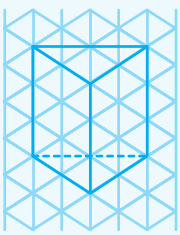


4 Isometric projection

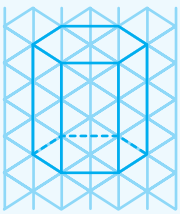
5 **a**



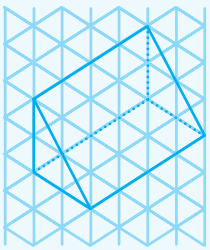
b



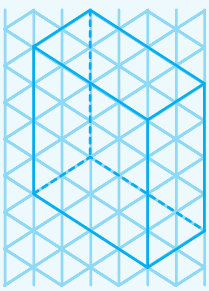
c



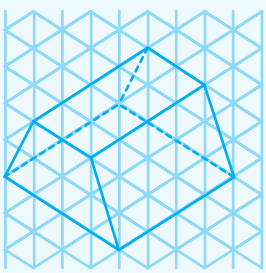
6 **a**



b



c



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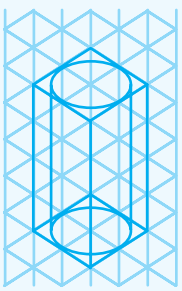
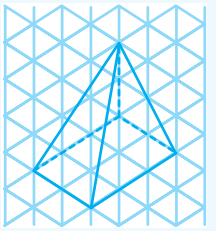
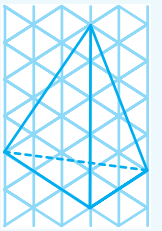
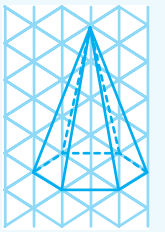
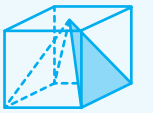


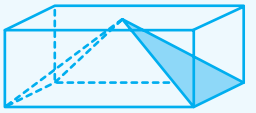
8 **b** Hexagonal prism

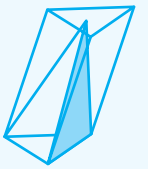
9 a 

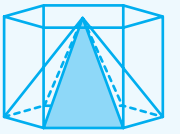
b 

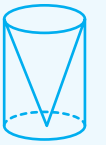
c 

10 a 

b 

c 

d 

11 

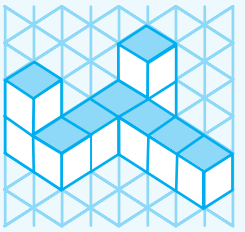
Exercise 12.5

3 a

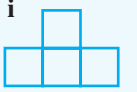
b

c

4 a

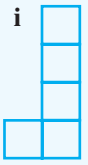
b 

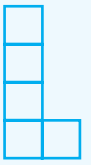
c

5 a i 

ii

iii

b i 

ii 

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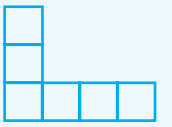
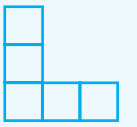
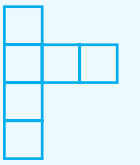
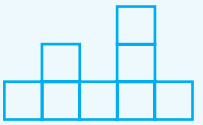
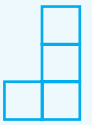
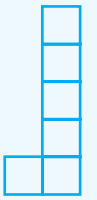
c i 

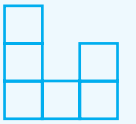
ii 

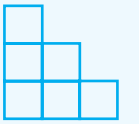
iii 

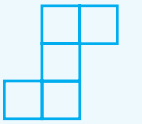
d i 

ii 

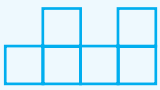
iii 

e i 

ii 

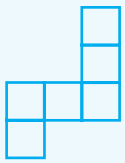
iii 

f **i**



ii

iii



7 a

2	2
1	1

b

	2
3	2

c

	2	1	2
1	1		

d

		1
		1
1	1	2

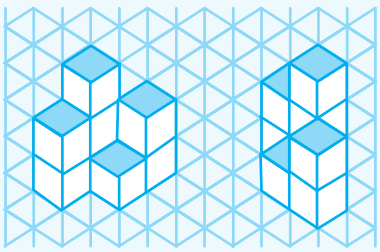
e

			1
3	3	3	1
3	2		

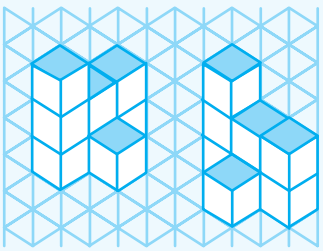
f

1	1	3
2		
1		

8 a



b



9 a

2	1	2
1	1	

b

3	2	3
2		
2		

c

2	3
2	2
1	

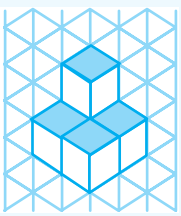
d

		1
2	3	1
	1	2

10 a

b

c



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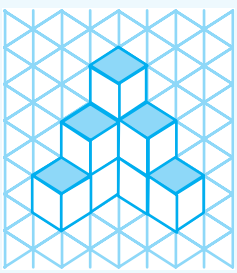

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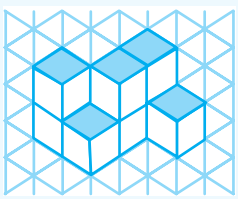
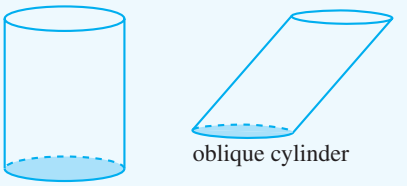


f

Chapter 12 Review

1 **a** cube, **b** rectangular prism, **c** triangular prism, **d** cylinder, **e** cone, **f** sphere, **g** rectangular pyramid, **h** triangular pyramid **2** **a** hexagonal pyramid, **b** octagonal prism **3** **a** cube, rectangular prism, triangular prism, **b** rectangular pyramid, triangular pyramid, **c** cube, rectangular prism, triangular prism, rectangular pyramid, triangular pyramid **4** **a** square, **b** triangle, **c** circle, **d** pentagon

5

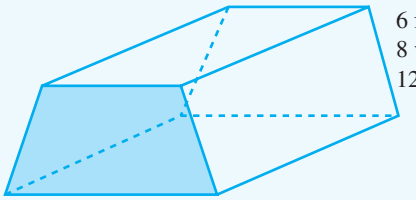


6 **a** non-convex, **b** convex **7** **a** squares, **b** rectangles, **c** triangles, rectangles, **d** rectangle, triangles, **e** triangles, **f** octagon, triangles

8

Solid	*F*	*V*	*E*
cube	6	8	12
rectangular prism	6	8	12
triangular prism	5	6	9
rectangular pyramid	5	5	8
triangular pyramid	4	4	6
octagonal pyramid	9	9	16

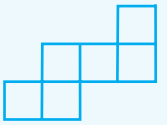
9 **a**



6 faces
8 vertices
12 edges

b trapezium **10** If the top of a pyramid or cone is cut off by a cross-section parallel to the base, the solid that remains is called a frustrum. **11** A cylinder has a curved surface. **12** Two lines are skew if they are neither parallel nor intersect. That is, they do not lie in the same plane. **13** **a** PQ, WV, **b** WV, UV, **c** PQ, TU
14 **a** $F + V = E + 2$, **b** **i** $F = 8$, $V = 12$, $E = 18$, **ii** $F = 11$, $V = 11$, $E = 20$ **15** tetrahedron, icosahedron, octahedron, cube, dodecahedron

16 **a**



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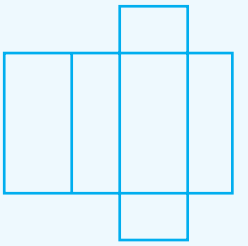

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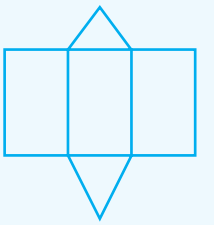

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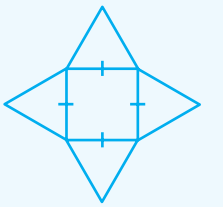

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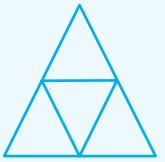

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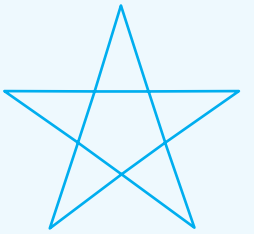

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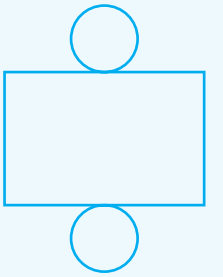
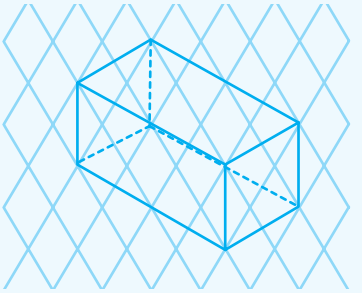
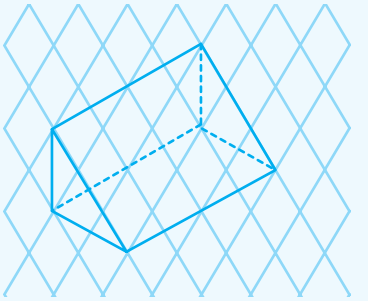


18 **a**



b



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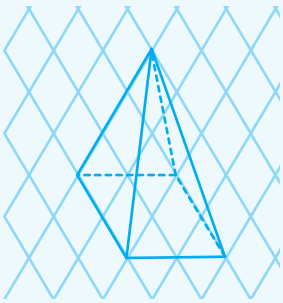
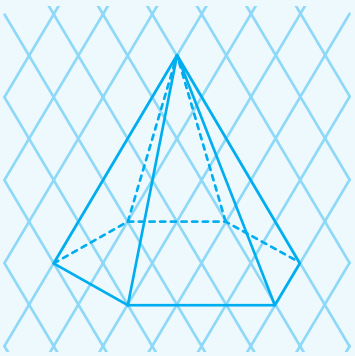


d



19 a

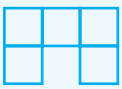
3	2	1
2	2	1
2		
1		

b 14

20 a 7, **b**

2	1	2
1		1

c i



Top

ii

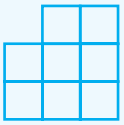
Front

iii

Left

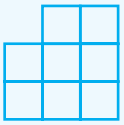
21 Yes

22 a



Front

b



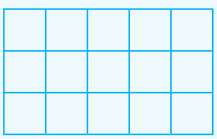
Right

13 Area

Exercise 13.1

1 a 12, **b** 14, **c** 13, **d** 20, **e** 30, **f** 23, **g** 26, **h** 19 **2 a** 9, **b** 6, **c** 10, **d** 17, **e** 15, **f** 21 **3 a** 5, **b** 8, **c** 7, **d** 7, **e** 10, **f** 8, **g** 8, **h** 7, **i** 8 **4 i a** perimeter 14, area 12, **b** perimeter 16, area 12, **c** perimeter 18, area 12 **ii** No **5 a** 32 cm^2, **b** 32 cm^2, **c** 10 cm^2, **d** 19 cm^2

6 a



area = 15, perimeter = 16,

b

area = 11, perimeter = 24

Exercise 13.2

1 a 10 cm^2, **b** 40 mm^2, **c** 27 m^2, **d** 24 mm^2, **e** 39 m^2, **f** 72 cm^2 **2 a** 9 cm^2, **b** 25 mm^2, **c** 121 m^2 **3 a** 42 cm^2, **b** 48 m^2, **c** 60 mm^2 **4 a** 12 cm^2, **b** 20 cm^2, **c** 10 cm^2, **d** 10.5 cm^2 **5 a** $a = 5$, **b** $y = 4$, **c** $p = 8$, **6 a** 9 mm, **b** 6 m, **c** 7 cm **7 a** 3 mm, **b** 4 mm, **c** 6 mm, **d** 7 mm, **e** 9 mm, **f** 12 mm **8 a** 9 cm^2, **b** 121 cm^2, **c** 169 cm^2 **9 a** 20 m, **b** 32 m, **c** 40 m **10 a i** perimeter = 20 cm, area = 24 cm^2, **ii** perimeter = 22 cm, area = 24 cm^2, **iii** perimeter = 28 cm, area = 24 cm^2, **b** No **11 a i** perimeter = 30 cm, area = 44 cm^2, **ii** perimeter = 30 cm, area = 54 cm^2, **iii** perimeter = 30 cm, area = 56 cm^2, **b** No **12 a** 20 cm^2, **b** 48 cm^2, **c** 87.5 cm^2, **d** 70 cm^2 **13 a** 30 cm, **b** 32 cm, **c** 37 cm, **d** 21 cm **14 a** 104 mm^2 or 1.04 cm^2, **b** 7500 cm^2 or 0.75 m^2 **15 a** 65 cm^2, **b** 56 cm^2 **16 a** 122 m^2, **b** 74 m^2, **c** 118 m^2, **d** 120 m^2, **e** 162 m^2, **f** 143 m^2 **17 a** 57 cm^2, **b** 90 cm^2, **c** 64 cm^2 **18 a** 108 mm^2, **b** 40 mm^2, **c** 126 mm^2, **d** 105 mm^2, **e** 92 mm^2, **f** 65 mm^2 **19 a** 3 cm × 11 cm, **b** 6 cm × 7 cm, **c** 8 cm × 9 cm, **d** 13 cm × 7 cm **20 a** 150 cm^2, **b** 108 cm^2 **21** 11 cm

Exercise 13.3

1 **a** $\frac{1}{2}$, **b** 6 cm^2, **c** 16 cm^2 **2** **a** $\frac{1}{2}$, **b** $\frac{1}{2}$, **c** $\frac{1}{2}$ **3** a and b **4** **a** 3 cm^2, **b** 6 cm^2, **c** 4 cm^2, **d** 4.5 cm^2, **e** 8 cm^2, **f** 9 cm^2 **5** **a** 15 cm^2, **b** 20 cm^2, **c** 7.5 cm^2, **d** 22 cm^2, **e** 10.5 cm^2, **f** 30 cm^2, **g** 18 cm^2, **h** 48 cm^2, **i** 27.5 cm^2 **6** **a** 4 cm^2, **b** 10 cm^2, **c** 4.5 cm^2 **7** **a** 88 m^2, **b** 36 mm^2, **c** 52.5 cm^2, **d** 32.5 cm^2 **8** **a** 120 cm^2, **b** 56 cm^2, **c** 60 cm^2, **d** 117 cm^2, **e** 49 cm^2, **f** 240 cm^2, **g** 91 cm^2, **h** 56 cm^2, **i** 37 cm^2 **9** **a** 97 mm^2, **b** 66 mm^2, **c** 111 mm^2, **d** 21 mm^2 **10** **a** 56.5, **b** 37, **c** 44 **11** **a** $k = 8$, **b** $q = 6$, **c** $w = 7$, **d** $c = 3$, **e** $h = 3$, **f** $u = 9$ **12** 96 cm^2

Exercise 13.4

1 **a** 28 m^2, **b** 2, **c** \$16 **2** **a** 23.6 m^2, **b** 4 **3** **a** 22 m, **b** 99 m^2 **4** **a** 600 m^2, **b** 30 **5** **a** 144 m^2, **b** 2 m^2, **c** 72, **d** \$720 **6** **a** 30 000 cm^2, **b** 300 **7** \$21.50 **8** **a** 36 m^2, **b** \$1440 **9** \$900 **10** \$23 per room **11** \$465 **12** $\frac{1}{8}$ **13** Yes **14** **a** 6.3 m^2, **b** \$1395 **15** **a** 60 000 cm^2, **b** 1200, **c** \$180 **16** 14 400 cm^2 or 1.44 m^2 **17** **a** 12, **b** 12, **c** 7, **d** 10.5, **e** 12.5, **f** 16

Exercise 13.5

1 **a** 300 mm^2, **b** 70 000 cm^2, **c** 9 000 000 m^2, **d** 4 m^2, **e** 8 cm^2, **f** 6 km^2, **g** 18 000 cm^2, **h** 5 640 000 m^2, **i** 4.9 cm^2, **j** 0.375 km^2, **k** 0.467 m^2, **l** 50 mm^2, **m** 900 000 m^2, **n** 0.07 cm^2, **o** 129 500 cm^2, **p** 0.4 mm^2, **q** 0.251 km^2, **r** 0.00053 m^2 **2** **a** 20 000, **b** 6, **c** 15 000, **d** 3.5, **e** 70, **f** 27 500, **g** 4000, **h** 0.2, **i** 0.0038, **j** 250, **k** 8, **l** 0.0003 **3** **a** ha, **b** cm^2, **c** m^2, **d** mm^2, **e** km^2, **f** m^2, **g** cm^2, **h** mm^2, **i** ha **4** **a** 8 ha, **b** 18 ha, **c** 23.5 ha **5** **a** 35 000 cm^2, **b** 3.5 m^2 **6** 1.44 m^2 **7** 70 cm **8** 60 m **9** 670 ha **10** 3150 m^2 **11** **a** 100, **b** 7 682 300 km^2 **12** 0.4009 ha

Chapter 13 Review

1 **a** 52 units^2, **b** 36 units^2 **2** **a** 63 cm^2, **b** 55 m^2, **c** 36 mm^2, **d** 64 cm^2 **3** **a** $y = 4$, **b** $d = 5$, **c** $u = 7$ **4** 11 cm **5** **a** 36 cm, **b** 100 cm^2 **6** **a** 119 cm^2, **b** 42 cm **7** **a** 10 cm^2, **b** 27 cm^2, **c** 12.5 cm^2 **8** **a** 56 mm^2, **b** 72 mm^2 **9** **a** 74 cm^2, **b** 84 cm^2, **c** 96 cm^2 **10** **a** 138 cm^2, **b** 106.5 cm^2, **c** 36 cm^2 **11** **a** 45.5 m^2, **b** 6, **c** \$87 **12** **a** 32 000 cm^2, **b** 20, **c** \$480 **13** **a** 400 mm^2, **b** 70 000 cm^2, **c** 360 000 m^2, **d** 6.5 m^2, **e** 2 cm^2, **f** 0.523 km^2 **14** **a** 50 000 m^2, **b** 8 ha, **c** 37 500 m^2, **d** 12.5 ha **15** 99 ha **16** 1 400 000 m^2 = 1.4 km^2