ML WEEK3

phuong nguyen

September 2021

1 Bai 1

We have a data set of observations $\mathbf{x} = (x1, x2, ..., xN)^t$, representing N observations of the scalar variable x and their corresponding target values $\mathbf{t} = (t1, t2, ..., tN)^T$make predictions for some new value of the input variable x

$t = y(x, w) + N(0, \beta^{-1})so, t = N(y(x, w), \beta^{-1})$

$=> p(t|x, w, \beta) = N(t|y(x, w), \beta^{-1})$

By maximum likelihood we have : $A = p(t|x, w, \beta) = \prod(N(t|y(x, w), \beta^{-1}))$
$=> A = (\frac{1}{\sqrt{2\pi\beta^{-1}}})^N exp^{\frac{-\beta}{2}(t_n - y(x,w))^2}$

$=> lnA = -\frac{N}{2}ln(2\pi) + \frac{N}{2}ln(\beta^{-1}) - \frac{1}{2}\beta\sum(t_n - y(x_n, w))^2$

lnA max when B= $-\frac{1}{2}\beta\sum(t_n - y(x_n, w))^2$ gets max or we'll say -B gets min

Suppose: x is matrix of all observations : X=[1,x] , w is a matrix with entries are w_0 and w_1 then -B=$\|Xw - t\|_2^2$

Apply matrix calculus to find the derivative of -B, we get:
(-B)'=2$X^T(Xw - t)$

Let (B)'=0 => w=$(X^TX)^{-1}X^Tt$

2 Bai 4

Proof A^TA is invertible if and only A has full rank(A's columns are linear independent)

Suppose A is matrix with linear independent columns(a1,a2,a3...an), that means $a1x1 + a2x2 + a3x3 + ...a_nx_n = 0$ and $x1 = x2 = x3 = ...x_n = 0$ or we'll way Ax=0 then nullspace of A is full of vector 0 : $N\{A\} = \{0\}$

We obviously don't know if A is invertible or not but if we can proof that $A^T A$ is invertibale then what we suppose about A is correct.

We know that if a matrix is square and linearly dependent columns then it is invertible
Suppose vector v such that $v \in N\{A^T A\}$

$=> A^T Av = 0 => v^t A^T Av = v^t 0 => (Av)^T.(Av) = 0 => \|Av\|_2^2 = 0$

=> Av=0 then v is also in the nullspace of A, for what we suppose is that A has linear independent columns therefore v has to be zeros vector

or $N\{A^T A\} = N\{A\}$ => the only solution for the equation $A^T Av = 0$ is $v_1 = v_2 = ...v_n = 0$ or we'll say $A^T A$ has linear dependent columns moreover, $A^T A$ is square matrix => $A^T A$ is invertible under the condition that A's columns are linear dependent or A has full rank