ML WEEK1

nguyen thi phuong

September 2021

1 Bai 1

H : event that people in group get Hansen
P : event that the test is positive
P(H)=0.05, P(H-)=0.95
P(P/H)=0.98, P(P/H-)=0.03
$\Rightarrow P(P) = 0.05 * 0.98 + 0.95 * 0.03 = 0.0775$
$P(H/P) = (P(P/H).P(H))/P(P) = 0.98 * 0.05/0.0775 = 0.632$

2 Bai 2

A) Proof normalized

- A probability distribution is said to be normalized if sum of all possible outcomes equals to 1.
- The PDF of univariate normal distribution or Gauss distribution is:
f(x) = $\frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$

- we have to proof that :
$\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$ dx =1

- This equals to 1 if and only if A=$\int_{-\infty}^{+\infty} e^{-(x-\mu)^2/2\sigma^2}$ dx=$\sigma\sqrt{2\pi}$
- Let μ=0 then:
A^2=$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-(x^2+y^2)/2\sigma^2} dxdy$

- Transforming Cartesian coordinates (x, y) to polar coordinates (r, θ)by:
Let $x = r\cos\theta$, and $y = r\sin\theta$, cuz $\sin\theta^2 + \cos\theta^2 = 1$, then $x^2 + y^2 = r^2$

- Apply double integral in Polar Coordinates :
A^2=$\int_0^{2\pi} \int_0^{+\infty} e^{-(r)^2/2\sigma^2} r\, dr d\theta$
A^2=$2\pi \int_0^{+\infty} e^{-(r)^2/2\sigma^2} r\, dr$
$A^2 = 2\pi \int_0^{+\infty} \frac{1}{2} e^{-(r)^2/2\sigma^2} d(r^2)$
- After taking integral, we got: $A^2 = \pi[\lim_{r\to\infty} -2\sigma^2 e^{-(r)^2/2\sigma^2} + 2\sigma^2]$

$A^2 = \pi[2\sigma^2.0 + 2\sigma^2]$
A^2=$2\sigma^2\pi \Rightarrow A = \sigma\sqrt{2\pi}$
Therefore , obviously $\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$ =1 or Gauss distribution is normalized

B) Find Expectation and variance

E(x)=$\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} x$
- Let t=$(x-\mu)\Big/\sqrt(2)\sigma \Rightarrow x = t\sqrt{2}\sigma + \mu$ then dx=$\sqrt{2}\sigma d(t)$
E(x)=$\int_{-\infty}^{+\infty} \frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}} e^{-t^2}(t\sqrt{2}\sigma + \mu)dt$

E(x)=$\frac{1}{\sqrt{\pi}}[\sigma\sqrt{2}\int_{-\infty}^{+\infty} te^{-t^2}dt + \mu\int_{-\infty}^{+\infty} e^{-t^2}dt]$

E(x)=$\frac{1}{\sqrt{\pi}}[\sigma\sqrt{2}\int_{-\infty}^{+\infty} \frac{1}{2}e^{-t^2}d(t^2) + \mu\int_{-\infty}^{+\infty} e^{-t^2}dt]$

E(x)=$\frac{1}{\sqrt{\pi}}[\sigma\sqrt{2}\lim_{r\to\infty} \frac{-1}{4}e^{-t^2} + \mu\sqrt{\pi}]$
E(x)=$\frac{1}{\sqrt{\pi}}(\sigma\sqrt{2}.0 + \mu\sqrt{\pi})$
E(x)=$\frac{1}{\sqrt{\pi}}(\mu\sqrt{\pi})$=$\mu$
So, expectation is equal to μ

- Doing similar things to find E(x^2)

E(x^2)=$\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} x^2 e^{-(x-\mu)^2/2\sigma^2}\, dx$
Let t=$(x-\mu)\Big/\sqrt{2}\sigma \Rightarrow x^2 = (t\sqrt{2}\sigma + \mu)^2$ then dx=$\sqrt{2}\sigma dt$
E(x)=$\int_{-\infty}^{+\infty} \frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}} e^{-t^2}(t\sqrt{2}\sigma + \mu)^2 dt$

E(x^2)=$\frac{1}{\sqrt{\pi}}[2\sigma^2\int_{-\infty}^{+\infty} t^2e^{-t^2}dt + 2\sqrt{2}\sigma\mu\int_{-\infty}^{+\infty} te^{-t^2}dt + \mu^2\int_{-\infty}^{+\infty} e^{-t^2}dt]$

E(x^2)=$\frac{1}{\sqrt{\pi}}[2\sigma^2\int_{-\infty}^{+\infty} t^2e^{-t^2}dt + 2\sqrt{2}\sigma\mu.0 + \mu^2\sqrt{\pi}]$

E(x^2)=$\frac{1}{\sqrt{\pi}}[2\sigma^2\int_{-\infty}^{+\infty} t^2e^{-t^2}dt + \mu^2\sqrt{\pi}]$

E(x^2)=$\frac{2\sigma^2}{\sqrt{\pi}}[\lim_{r\to\infty} \frac{-t}{2}e^{-t^2} + \frac{1}{2}\int_{-\infty}^{+\infty} e^{-t^2}dt] + \frac{1}{\sqrt{\pi}}\mu^2\sqrt{\pi}$

E(x^2)=$\frac{2\sigma^2}{\sqrt{\pi}}\frac{1}{2}\int_{-\infty}^{+\infty} e^{-t^2}dt + \mu^2$

E(x^2)=$\frac{2\sigma^2\sqrt{\pi}}{2\sqrt{\pi}} + \mu^2$

E(x^2)=$\sigma^2 + \mu^2$
So, variance of this distribution is Var(x)=E(x^2)-$(E(x))^2$=$\sigma^2 + \mu^2 - \mu^2$=σ^2