Comparative Analysis of Alternatives

4

4.1 Comparing Mutually Exclusive Alternatives Having Same Useful Life

4.1.1 Present Worth Analysis

Present worth is one of the ways to compare alternatives. It is most frequently used to determine the present value of future money receipts and disbursements. In the future, income and costs are known, and then using a suitable interest rate, the present worth can be calculated. In present worth analysis, careful consideration must be given to the time period covered by the analysis. Usually, the task to be accomplished has a time period associated with it.

In present worth analysis, the alternative with the maximum present worth (PW) of benefits minus present worth of cost is always selected. This criterion is called the net present worth criterion (NPW). As such,

$$NPW = PW_{\text{of benefits}} - PW_{\text{of costs}}$$

Steps to do Present Worth Analysis

Step 1: Select a desired value of the return on investment (I).

Step 2: Using the compound interest formulas bring all benefits and costs to present worth for each alternative.

Step 3: Select the alternative with the largest net present worth (Present worth of benefits – Present worth of costs).

Cases on Present Worth Analysis

Case I: Useful life of the alternative(s) equals the analysis period.

Case II: Alternatives have useful lives different from the analysis period.

Case III: The analysis period is infinite or long enough to be treated as infinite, n = ά

Present Worth Comparison of Equal Lived Alternatives

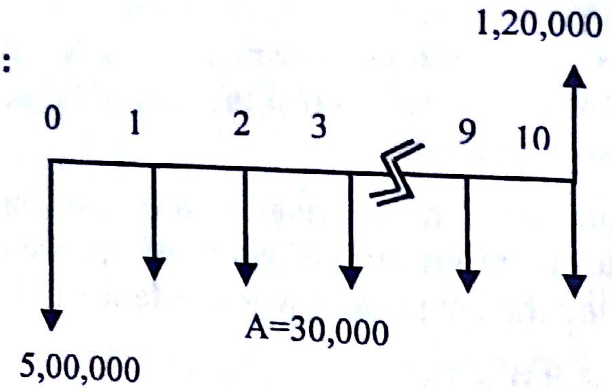
Example 4.1

Two machines are under consideration for a construction company. Machine A will have a first cost of Rs5,00,000 an annual operation and maintenance cost of Rs 30,000, and a Rs1,20,000 salvage value. Machine B will have a first cost of Rs 7,00,000, an annual operation and maintenance cost of Rs20,000, and Rs 1,30,000 salvage value. If both machines are expected to last for ten years, determine which machine should be selected on the basis of PW, FW, AW using an interest rate of 10%.

Solution:

Machines	Machine A	Machine B
Initial Cost	5,00,000	7,00,000
O and M Cost/yr	30,000	20,000
Salvage Value	1,20,000	1,30,000
Useful Life (years)	10	10

Machine A :



Machine B :

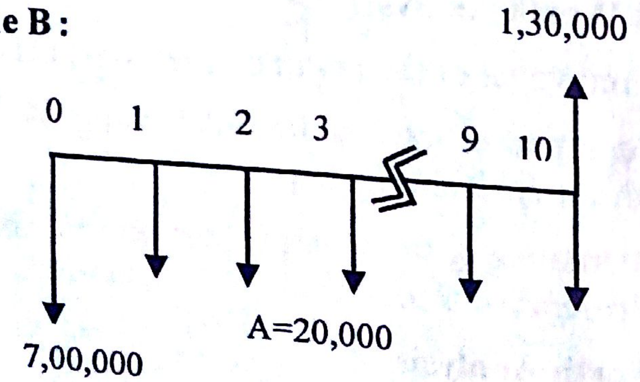


Fig.4.1: *Cash Flow Diagram for Machine A and B.*

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a. **Using Present Worth (PW) Method**

PW_A = –5,00,000-30,000 (P/A, 10%, 10) + 1,20,000 (P/F, 10%, 10)
= –5,00,000-30,000 (6.1446) + 1,20,000 (0.3855)
= – Rs 638,078

PW_B = –7,00,000-20,000 (P/A, 10%, 10) + 1,30,000 (P/F, 10%, 10)
= –7,00,000-20,000 (6.1446) + 1,30,000 (0.3855)
= – Rs7,72,777

Since, both the machines have negative values but $PW_A < PW_B$ ie machine A has less negative value. Therefore, machine A should be selected **Ans.**

b. **Using Future Worth (FW) Method**

FW_A = – 5,00,000(F/P, 10%, 10) – 30,000 (F/A, 10%, 10) +1,20,000
= – 5,00,000(2.5937) – 30,000 (15.9374) +1,20,000
= – Rs 16,54,972

FW_B = – 7,00,000 (F/P, 10%, 10) – 20,000 (/A, 10%, 10) +1,30,000
= – 7,00,000 (2.5937) – 20,000 (15.9374) +1,30,000
= – Rs 20,04,338

Since, both the machines have negative values but $FW_A < FW_B$ ie machine A has less negative value. Therefore, machine A should be selected. **Ans.**

c. **Using Annual Worth (AW) Method**

AW_A = – 5,00,000(A/P, 10%, 10) – 30,000 +1,20,000(A/F, 10%, 10)
= – 5,00,000(0.1627) – 30,000 +1,20,000(0.0627) = – Rs 1,03,826

AW_B = – 7,00,000 (A/P, 10%, 10) – 20,000 +1,30,000 (A/F, 10%, 10)
= – 7,00,000(0.1627) – 20,000 +1,30,000(0.0627) = – Rs1,25,739

Since, both the machines have negative values but $AW_A < AW_B$ ie machine A has less negative value. Therefore, machine A should be selected. **Ans.**

Example 4.2

A firm is considering which of two mechanical devices to install to reduce costs in a particular situation. Both devices cost Rs10,000 and have useful lives of five years and no salvage value. Device A is expected to result in Rs 3,000 savings annually. Device B will provide savings for Rs 4,000 the first year but will decline Rs 500 annually. With interest rate 7%, which device should the firm purchase?

Solution:

Both devices have the same useful lives and both costs Rs10,000. The appropriate decision criterion is to choose the alternative that maximizes the present worth of benefits. The cash flow diagrams are shown below.

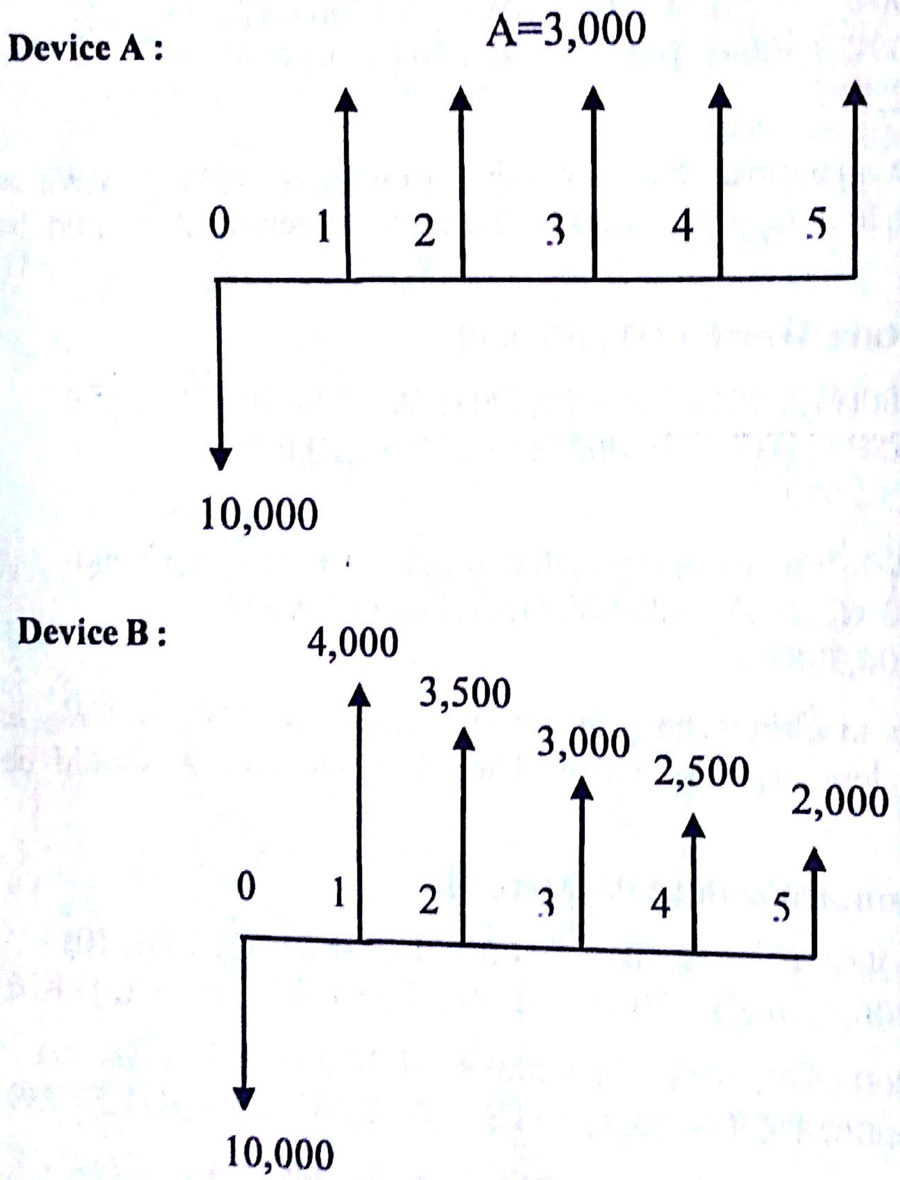


Fig.4.2: *Cash Flow Diagram for Device A and B.*

$PW_A = -10,000 + 3,000(P/A, 7\%, 5)$

$= -10,000 + 3,000\ (4.1002) = \text{Rs } 2,300.6$

$PW_B = -10,000 + 4,000(P/A, 7\%, 5) - 500(P/G, 7\%, 5)$

$= -10,000 + 4,000(4.1002) - 500\ (7.6467) = \text{Rs } 2,577.45$

Here, $PW_B > PW_A$. Therefore, mechanical device B should the firm be purchased **Ans.**

4.1.2 Rate of Return Method and Benefit Cost Ratio Method

1. The Rate of Return (ROR) Method

- Present worth comparisons, Future worth comparisons and Equivalent annual worth comparison methods will tell us which alternative is the best, and whether we are gaining a greater return than our minimum attractive rate of return (MARR).
- The rate of return method is to find the rate of return (i %) for an investment over a specific service life.
- Internal rate of return (IRR) tell us the exact rate of return we are receiving on an individual investment.
- The rate of return method considers all the cash follows that occur during the life cycle of an investment.
- There are two methods to find the rate of return for an investment. **These are:**
  a. The present worth method and
  b. The equivalent uniform annual cost (EUAC) method.
- These calculations must be done through trial and error, or by using commercially available software.

a. ROR Calculation by Present Worth Method

The calculations are done in following steps:

1. Draw a cash flow diagram.
2. Set up the rate of return equation in the following form.

$$0 = \pm P + SV\,(P/F, i\%, n) \pm A\,(P/A, i\%, n)$$

Where,

P = initial cost.
SV = salvage value.
A = annual revenue/expenses.
n = useful life.
i = rate of return.

3. Select values of i by trial and error until the equation is balanced.

b. ROR Calculation by EUAC

The calculations are done in following steps:

1. Draw a cash flow diagram.
2. Set up the rate of return equation in the following form.

$$0 = \pm P\,(A/P, i\%, n) \pm A$$

Where, P = initial cost.
A = annual revenue/expenses.

n = useful life.
i = rate of return.

3. Select values of i by trial and error until the equation is balanced.

Example 4.3

An investment resulted in the following cash flow. Compute the rate of return.

Year	0	1	2	3	4
Cash Flow	– 70,000	+ 10,000	+ 17,500	+ 25,000	+ 32,500

Solution:
Using the equivalent uniform annual cost (EUAC) method.

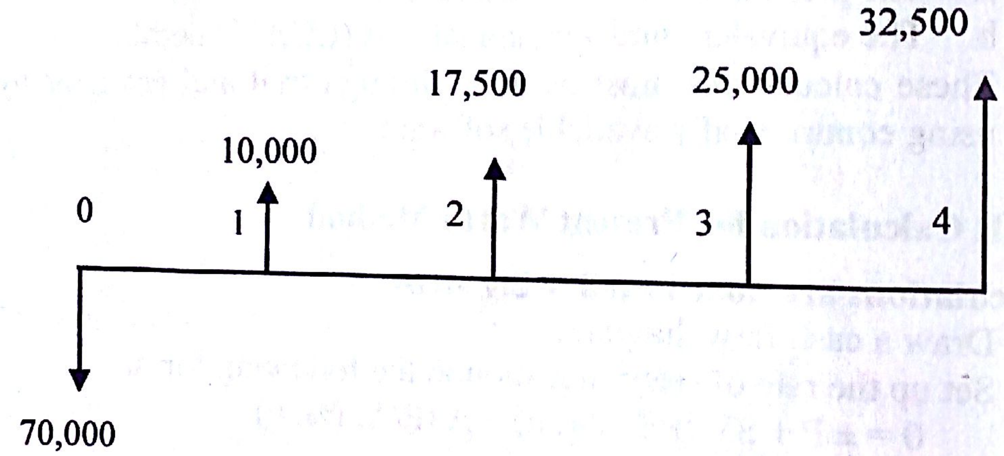


Fig.4.3: *Cash Flow Diagram.*

Equivalent uniform annual benefit (EUAB) - Equivalent uniform annual cost (EUAC) = 0
Or, EUAB – EUAC = 0
Or, 10,000 + 7,500(A/G, i%, 4) – 70,000(A/P, i%, 4) = 0
This equation is solved using trial and error.

@ i = 5%

EUAB – EUAC = 10,000 + 7,500(A/G, 5%, 4) – 70,000(A/P, 5%, 4)
= 10,000 + 7,500(1.4391) – 70,000(0.2820)
= + Rs 1,053.25

@ i = 8%

EUAB – EUAC = 10,000 + 7,500(A/G, 8%, 4) – 70,000(A/P, 8%, 4)
= 10,000 + 7,500(1.4040) – 70,000(0.3019)
= – Rs 603

Example 4.4

Use IRR method to select the best project.

Projects	*A*	*B*	*C*	*D*
Initial Investment	*10,000*	*15,000*	*27,000*	*20,000*
Annual Income	*5,000*	*7,000*	*12,000*	*9,000*
Useful Life	*4*	*4*	*4*	*4*
Salvage Value	*2,500*	*5,000*	*8,000*	*10,000*
MARR		*15%*		

Solution:

For Project A

NPW = 0 at the rate of return

$NPW_A = -10,000 + 5,000(P/A, i\%, 4) + 2,500(P/F, i\%, 4)$

@ i = 35%

$= -10,000 + 5,000\ (1.9969) + 2,500\ (0.3010)$

$= 737$

@ i = 40%

$= -10,000 + 5,000\ (1.8492) + 2,500\ (0.2603)$

$= -103.25$

By interpolating

$IRR_A = 39.38\% > MARR$

For Project B

NPW = 0 at the rate of return

$NPW_B = -15,000 + 7,000(P/A, i\%, 4) + 5,000(P/F, i\%, 4)$

@ i = 35%

$= -15,000 + 7,000\ (1.9969) + 5,000\ (0.3010)$

$= 483.30$

@ i = 40%

$= -15,000 + 7,000\ (1.8492) + 5,000\ (0.2603)$

$= -754.10$

By interpolating

$IRR_B = 36.95\% > MARR$

For Project C

NPW = 0 at the rate of return

$NPW_C = -27,000 + 12,000(P/A, i\%, 4) + 8,000(P/F, i\%, 4)$

@ i = 30%

$= -27,000 + 12,000\ (2.1662) + 8,000\ (0.3501)$

$= 1,795.20$

@ i = 35%

$= -27,000 + 12,000\ (1.9969) + 8,000\ (0.3010)$

$= -629.20$

By interpolating

$IRR_C = 33.70\% > MARR$

For Project D

NPW = 0 at the rate of return

$NPW_D = -20,000 + 9,000\ (P/A, i\%, 4) + 10,000\ (P/F, i\%, 4)$

@ i = 35%

$= -20,000 + 9,000\ (1.9969) + 10,000\ (0.3010)$

$= 982.10$

@ i = 40%

$= -20,000 + 9,000\ (1.8492) + 10,000\ (0.2603)$

$= -754.20$

By interpolating

$IRR_D = 37.82\% > MARR$

Incremental Analysis

Taking alternative A as base alternative having lowest investment.

Projects	A	B	C	D	B-A
Initial Investment	10,000	15,000	27,000	20,000	5,000
Annual Income	5,000	7,000	12,000	9,000	2,000
Salvage Value	2,500	5,000	8,000	10,000	2,500

For Project B-A

NPW = 0 at the rate of return

$NPW = -5,000 + 2,000\ (P/A, i\%, 4) + 2,500\ (F/P, 1\%, 4)$

@ i = 30%

$= -5,000 + 2,000\ (2.1662) + 2,500\ (0.3501)$

$= 207.65$

@ i = 35%

$= -5,000 + 2,000\ (1.9969) + 2,500\ (0.3010)$

$= -253.7$

By interpolating

$IRR_{B-A} = 32.25\% > MARR$

A is eliminated from consideration and taking B as base alternative.

Projects	B	C	D	C-B
Initial Investment	15,000	27,000	20,000	12,000
Annual Income	7,000	12,000	9,000	5,000
Salvage Value	5,000	8,000	10,000	3,000

For Project C – B

NPW = 0 at the rate of return

NPW = −12,000 + 5,000 (P/A, i%, 4) + 3,000 (F/P, 1%, 4)

@ i = 25%

= −12,000 + 5,000 (2.3616) + 3,000 (0.4096)

= 1,036.80

@ i = 30%

= −12,000 + 5,000 (2.1662) + 3,000 (0.3501)

= −118.7

By interpolating

IRR_{C-B} = 29.48% > MARR

B is eliminated from consideration. Then remaining alternatives are C and D.

Projects	**C**	**D**	**C-D**
Initial Investment	27,000	20,000	7,000
Annual Income	12,000	9,000	3,000
Salvage Value	8,000	10,000	−2,000

For Project C-D

NPW = 0 at the rate of return

NPW_{C-D} = −7,000 + 3,000 (P/A, i%, 4) − 2,000 (F/P, 1%, 4)

@ i = 15%

= −7,000 + 3,000 (2.8550) − 2,000 (0.5718)

= 421.40

@ i = 20%

= −7,000 + 3,000 (2.5887) − 2,000 (0.4823)

= −198.50

By interpolating

IRR_{C-D} = 18.39% > MARR

Therefore, alternative C is selected **Ans.**

2. Benefit-cost Analysis

- Benefit-cost analysis method is mainly used for economic evaluation of public projects which are mostly funded by government organizations.
- In addition, this method can also used for economic evaluation of alternatives for private projects.
- The main objective of this method is used to find out desirability of public projects as far as the expected benefits on the capital investment are concerned.

- As the name indicates, this method involves the calculation of ratio of benefits to the costs involved in a project.
- The benefit-cost ratio of projects is determined in different forms namely conventional benefit-cost ratio and modified benefit-cost ratio.
- The benefit-cost ratio is generally designated as B/C ratio.

Example 4.5

There is four mutually exclusive alternatives for a public project. Select the best alternative using incremental B/C ratio analysis if interest rate is 7% per year. The cash flow details of the alternatives are shown in the following table. Each alternative has the useful life of 40 years.

Table 4.1: Cash Flow of Alternatives for the Project

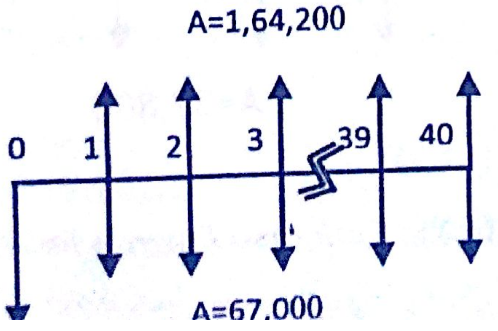
Alternatives / *Cash Flow*	A_1	A_2	A_3	A_4
Initial Investment	10,10,000	11,20,000	14,52,000	12,28,000
O and M Costs/yr (E)	67,000	64,500	57,800	61,350
Annual Benefits (R)	1,64,200	1,72,000	1,91,000	1,79,000
Useful Life	40	40	40	40

Solution:

First the conventional B/C ratio will be used for the incremental benefit-cost analysis for the comparison of above mutually exclusive alternatives. Present worth method will be used for the calculation of equivalent worth of benefits and costs.

Present Worth Formulation

Alternative-1 (A_1)



$$PW_1 = 10{,}10{,}000 + 67{,}000(P/A, i, n)$$
$$= 10{,}10{,}000 + 67{,}000(P/A, 7\%, 40)$$
$$= 10{,}10{,}000 + 67{,}000\ (13.3317)$$
$$= \textbf{Rs 19,03,223.90}$$

Alternative-2 (A_2)

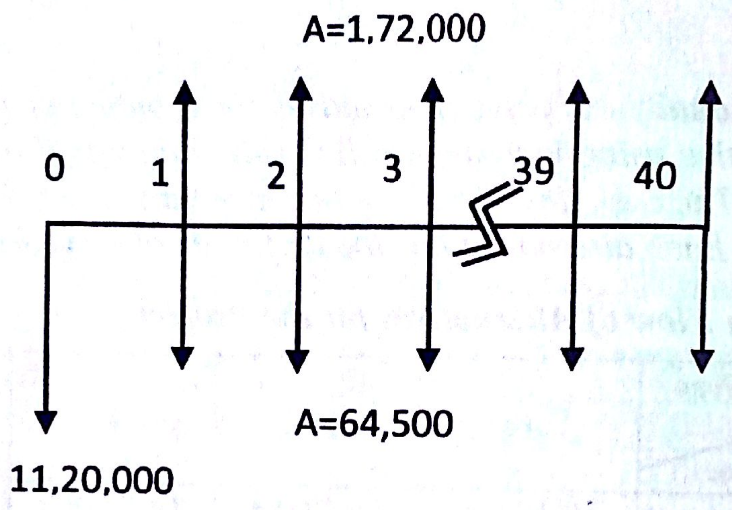


Fig.4.5: *Cash Flow Diagram for Alternative 2.*

$$PW_2 = 11{,}20{,}000 + 64{,}500(P/A, 7\%, 40)$$
$$= 11{,}20{,}000 + 64{,}500\ (13.3317)$$
$$= \textbf{Rs 19,79,894.65}$$

Alternative- 3 (A_3)

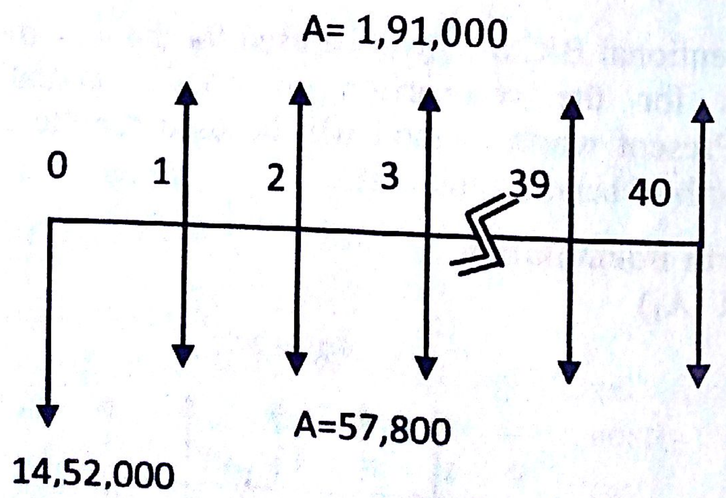


Fig.4.6: *Cash Flow Diagram for Alternative 3.*

$$PW_3 = 14{,}52{,}000 + 57{,}800(P/A, 7\%, 40)$$
$$= 14{,}52{,}000 + 57{,}800\ (13.3317)$$
$$= \textbf{Rs 22,22,572.26}$$

Alternative-4 (A_4)

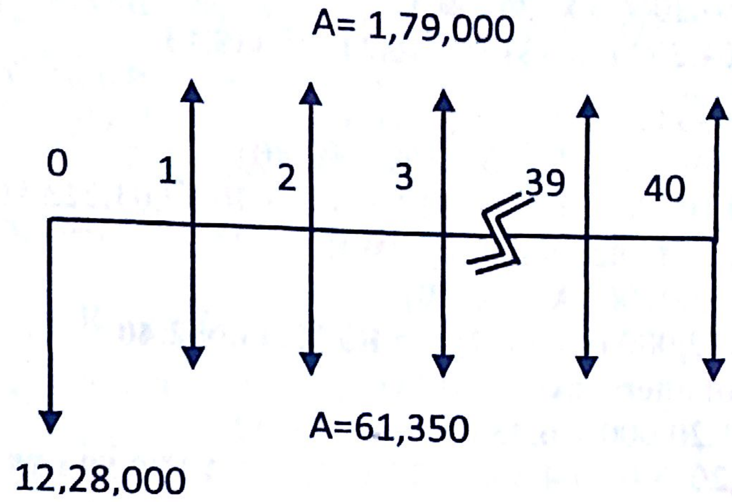


Fig.4.7: *Cash Flow Diagram for Alternative 4.*

$$
\begin{aligned}
PW_4 &= 12{,}28{,}000 + 61{,}350(P/A, 7\%, 40)\\
&= 12{,}28{,}000 + 61{,}350\ (13.3317)\\
&= \textbf{Rs 20,45,899.80}
\end{aligned}
$$

As observed from the above calculations, the order of alternatives from lowest equivalent cost to highest equivalent cost is **A_1, A_2, A_4 and A_3.**

The lowest equivalent cost alternative A_1 is first compared against do-nothing alternative i.e. the B/C ratio of alternative A_1 on its cash flow is calculated.

$$\text{B/C ratio of } A_1 = \frac{\text{PW of benefits}}{\text{Initial investment} + \text{PW of O \& M cost}}$$

$$\text{B/C ratio of } A_1 = \frac{1{,}64{,}200\left(\frac{P}{A}, 7\%, 40\right)}{10{,}10{,}000 + 67{,}000\left(\frac{P}{A}, 7\%, 40\right)}$$

$$\text{B/C ratio of } A_1 = \frac{1{,}64{,}20{,}000\ (13.3317)}{10{,}10{,}000 + 67{,}000\left(\frac{P}{A}, 7\%, 40\right)} = 1.15$$

As the B/C ratio of alternative A_1 is greater than 1.0, A_1 now becomes the base alternative and is compared against the next higher equivalent cost alternative i.e. alternative A_2.

The incremental B/C ratio between alternatives A_2 and A_1 can also be calculated by finding out the ratio of the differences in present worth of benefits of alternatives to that of costs. This calculation is shown below.

PW of benefits of alternative A_1 (PWB_{A1})

$= 1,64,200(P/A, 7\%, 40)$
$= 1,64,200\ (13.3317) =$ **Rs21,89,065.14**

PW of costs of alternative A1 (PWC_{A1})
$= 10,10,000 + 67,000(P/A, 7\%, 40)$
$= 10,10,000 + 67,000\ (13.3317) =$ **Rs19,03,223.90**

PW of benefits of alternative A_2 (PWB_{A2})
$= 1,72,000(P/A, 7\%, 40)$
$= 1,72,000\ (13.3317)$ $=$ **Rs 22,93,052.40**

PW of costs of alternative A2 (PWC_{A2})
$= 11,20,000 + 6,45,00(P/A, 7\%, 40)$
$= 1,20,000 + 64,500\ (13.3317) =$ **Rs19,79,894.65**

Difference in PW of benefit between A_2 and A_1
$= PWB_{A2} - PWB_{A1}$
$= 22,93,052.40 - 21,89,065.14$ $=$ **Rs 1,03,987.26**

Difference in PW of cost between A_2 and A_1
$= PWC_{A2} - PWC_{A1}$
$= 19,79,894.65 - 19,03,223.90 =$ **Rs 76,670.75**

Incremental B/C ration between A_2 and A_1 ($\Delta B/C_{2\text{-}1}$) is calculated as:

$$\Delta B/C_{2\text{-}1} = \frac{PWBA_2 - PWB_1}{PWC_2 - PWC_1} = \frac{1,03,98,726}{76,67,075} = 1.356$$

As the incremental B/C ratio is greater than 1.0, then A_2 becomes the new base alternative and alternative A_1 is removed from further analysis. Alternative A_2 is now compared against the next higher equivalent cost alternative i.e. alternative A_4. The incremental B/C ratio between alternatives A_4 and A_2 is determined in the same manner as that was determined between alternatives A_2 and A_1.

PW of benefits of alternative A_4 (PWB_{A4})

$= 1,79,000\ (P/A, 7\%, 40) = 1,79,000\ (13.3317)$

$=$ **Rs23,86,374.30**

PW of costs of alternative A_4 (PWC_{A4})

$= 12,28,000 + 61,350\ (P/A, 7\%, 40)$
$= 12,28,000 + 61,350\ (13.3317) =$ **Rs20,45,899.79**

Difference in PW of benefit between A_4 and A_2

$= PWB_{A4} - PWB_{A2}$
$= 23,86,374.30 - 22,93,052.40$ $=$ **Rs 93,321.90**

Difference in PW of cost between A_4 and A_2

$= PWC_{A4} - PWC_{A2}$

$= 20,45,899.79 - 19,79,894.65$ $=$ **Rs 66,005.14**

Incremental B/C ratio between A_4 and A_2 ($\Delta B/C_{4\text{-}2}$) is calculated as:

$$\Delta B/C_{4\text{-}2} = \frac{PWBA_4 - PWB_2}{PWC_4 - PWC_2} = \frac{93,32,190}{66,00,514.5} = 1.41 > 1$$

Again, A_4 being base alternative. Incrementas B/C ratio between A_4 and A_3

$$PWB_{A_4} = \text{Rs. } 23,86,374.30$$

$$PWC_{A_4} = \text{Rs. } 20,45,899.79$$

$$PWB_{A_3} = 1,91,000\ (P/A, 7\%, 40)$$

$$= 1,91,000 \times 13.3317 = \text{Rs. } 25,46,354.70$$

$$PWC_{A_3} = \text{Rs. } 22,22,572.26$$

∴ Incremental B/C ratio between A_4 and A_3 is :

$$= \frac{PWB_{A_3} - PWB_{A_4}}{PWC_{A_3} - PWC_{A_4}}$$

$$= \frac{25,46,354.70 - 23,86,374.30}{22,22,572.26 - 20,45,899.79}$$

$$= \frac{1,59,980.40}{1,76,672.47} = 0.906 < 1$$

Here, B/C ratio of $A_3 - A_4$ is <1, hence alternative A_4 is best and is selected.

4.2 Comparing Mutually Exclusive Alternatives Having Different Useful Lives By

Present Worth Comparison of Different-Lived Alternatives

In the previous examples, the useful life of each alternative was equal to the analysis period. But, there will be many situations where the alternatives have useful lives different from the analysis period. This situation will be examined in this section.

Techniques for Dealing with Unequal Useful Lives

4.2.1 Repeatability Assumptions

- We will assume the same costs and benefits and repeat a project all the way to the end of the analysis period.

- Two alternatives having different useful life are changed into projects having same useful life by repeating first cycle of cash flow for successive cycles up to least common year.
- All cash flows will have the same estimated values in every life cycle.
- The study period is equal to the least common multiple of the lives of alternatives.
- For example if useful lives are 3 years and 4 years, then the least common multiple is 12 years.

Example 4.6

A purchasing agent is considering the purchase of some new equipment. Two different manufacturers have provided quotations. An analysis of the quotations indicates the following:

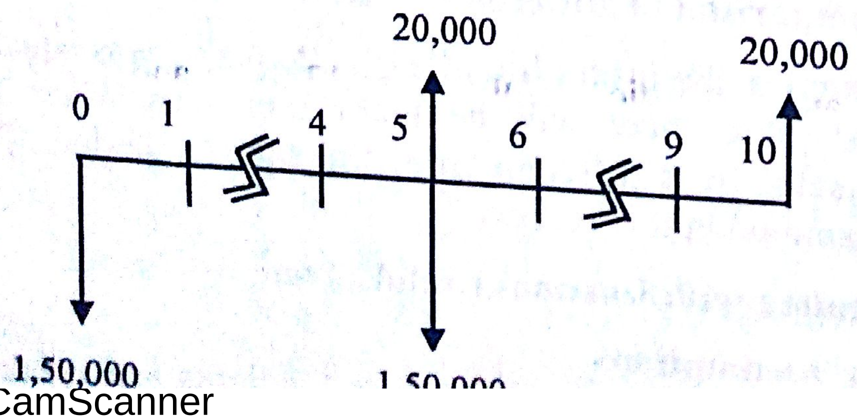
Equipments	*Equipment A*	*Equipment B*
Initial Cost	*1,50,000*	*1,60,000*
Salvage Value	*20,000*	*32,500*
Useful Life (years)	*5*	*10*

Which manufacturer's equipment should be selected? Assume 7% interest rate and equal maintenance cost.

Solution:

As equipment A has a useful life of 5 years and B has useful life of 10 years, 10 year is to select as an analysis period which is the least common multiplier of the lives of both equipments. Thus, we would compare the ten-year life of B against an initial purchase of A plus its replacement with a new one with the same values.. The basis is to compare both alternatives on 10 years period.

For Equipment A



$PW_A = -1,50,000 - (1,50,000\text{-}20,000)\ (P/F, 7\%, 5) + 20,000\ (P/F, 7\%, 10)$
$= -1,50,000 + 1,30,000(0.713) + 20,000(0.5083) = -\text{Rs } 2,32,524$

For Equipment B

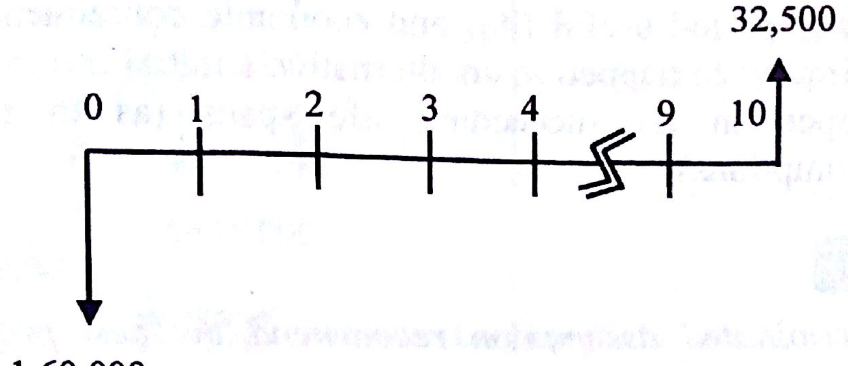


Fig.4.9: *Cash Flow Diagram for Equipment B.*

$PW_B = -1,60,000 + 32,500\ (P/F, 7\%, 10)$
$= -1,60,000 + 32,500(0.5083) = -\text{Rs } 1,43,480.25$

Since $PW_B > PW_A$, Therefore, equipment B should be selected **Ans.**

4.2.2 Co-terminated Assumption

- It uses finite and identical study period for all alternatives.
- This planning horizon combined with appropriate adjustments to the estimated cash flows plus the alternatives on a common and comparable basis.
- The planning horizon chosen could be:
  - o Life of shorter lived alternatives
  - o Life of longer lived alternatives.
  - o Less than the shorter lived alternative.
  - o Greater than the longer lived alternative.
  - o In between the shortest and longest lived alternatives.
- Sometimes the least common multiple method (LCM) creates an unrealistic useful life (eg.13 years and 7 years = LCM of 91 years).
- Instead, pick a terminal year and repeat all projects up until the terminal year.
- Shorten all costs and benefits after the terminal year.

Case I: Study period longer than the useful life.

Following two assumptions are considered:

- Cash flow accumulated at the end of the useful life will be reinvested for the extended periods.
- Replacement/reinvestment is necessary for remaining period (study period-useful life) and economic consequences that are estimated to happen in an alternative's initial life span will also happen in all succeeding life spans (as in repeatability assumptions)

Example 4.7

Using co-terminated assumption recommend the best project taking study period as 8 years.

Projects	*A*	*B*
Initial Investment	3,50,000	5,00,000
Annual Revenue	1,30,000	1,75,000
Annual Costs	15,000	25,000
Salvage Value	35,000	50,000
Useful Life	5 Years	8 Years
MARR	10%	

Solution:

For Project B

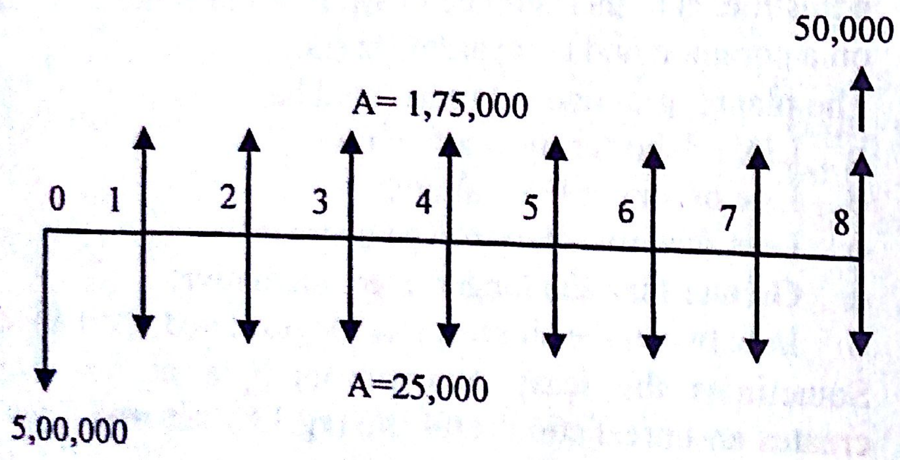


Fig. 4.10: *Cash Flow Diagram for Project B.*

FW_B

$= -5,00,000(F/P, 10\%, 8) + (1,75,000 - 25,000)(F/A\ 10\%\ 8) + 50,000$

$= -5,00,000\ (2.1436) + 1,50,000\ (11.4359) + 50,000$

$=$ Rs 6,93,585

For Project A

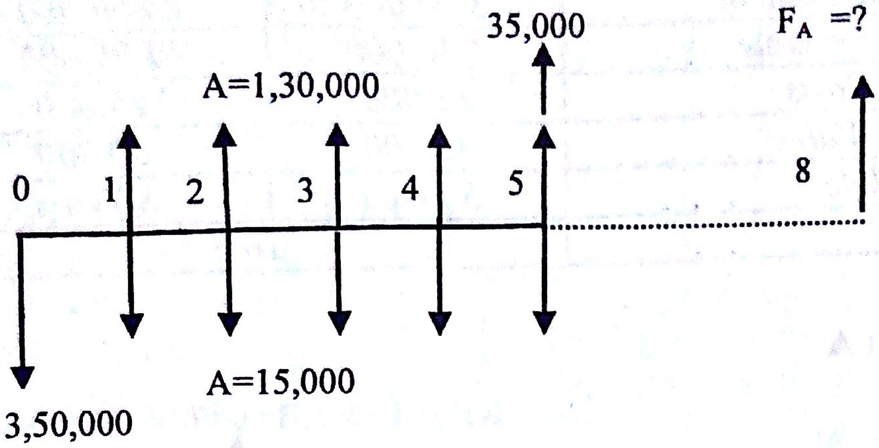


Fig. 4.11: *Cash Flow Diagram for Project A.*

Using FW Method

FW_{A5}

$= -3,50,000(F/P, 10\%, 5) + (1,30,000 - 15,000)\ (F/A\ 10\%\ 5)\ + 35,000$

$= -3,50,000\ (1.6105) + 1,15,000\ (6.1051) + 35,000 = Rs 1,73,411.50$

FW_A

$= FW_{A5}\ (F/P, 10\%, 3)$

$= 1,73,411.5\ (1.3310) = Rs\ 2,30,810.70$

Since, $FV_B > FV_A$. Therefore, project B is recommended **Ans.**

Case II: When Study Period Shorter than the Useful Life

- In this case satisfy the rest of the required service period of the alternative having longest useful life.
- For this shortens the alternative at the end of the study period using the estimated market value.
- Imputed market value is used for this purpose.

Imputed Market Value Calculation

$MV = PW_{CR} + PW_{MV}$

Where, PW_{CR} = present worth of capital recovery.

PW_{MV} = present worth of market value.

MV = market value at study period.

Example 4.8

Using co-terminated assumption recommend the best project taking study period as 5 years.

Projects	A	B
Initial Investment	3,50,000	5,00,000
Annual Revenue	1,30,000	1,75,000
Annual Costs	15,000	25,000
Salvage Value	35,000	50,000
Useful Life	5 Years	8 Years
MARR	10%	

Solution:

For Project A

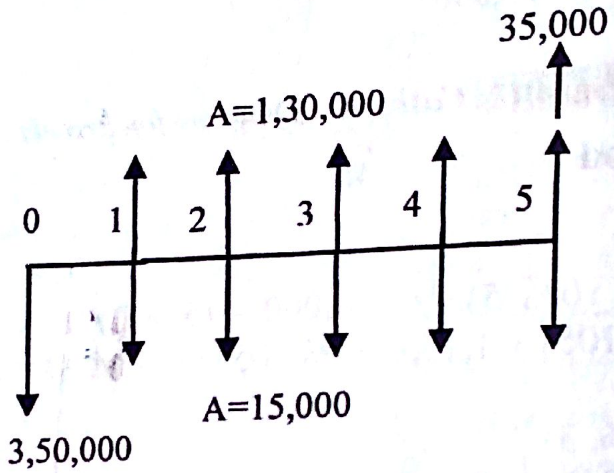


Fig. 4.12: *Cash Flow Diagram for Project A.*

Using FW Method

FW_A

$= -3,50,000(F/P, 10\%, 5) + (1,30,000 - 15,000)\ (F/A\ 10\%\ 5) + 35,000$

$= -3,50,000\ (1.6105) + 1,15,000\ (6.1051) + 35,000$

$=$ Rs1,73,411.50

For Project B

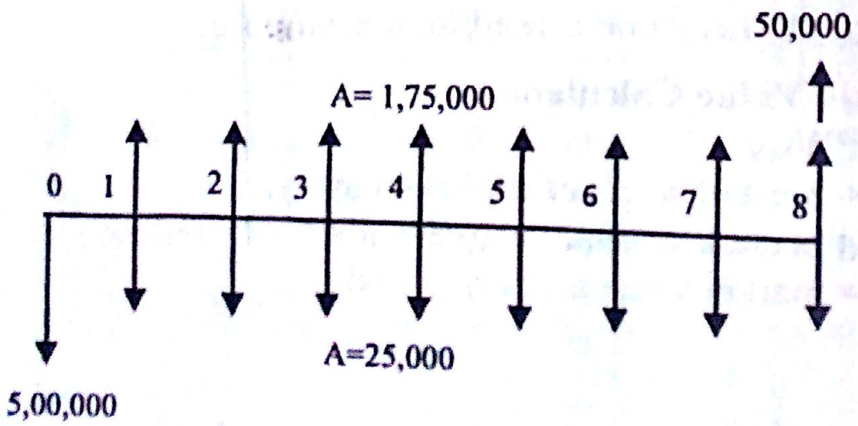


Fig. 4.13: *Cash Flow Diagram for Project B.*

Imputed Market Value Calculation

Capital Recovery (CR) $= 5,00,000\ (A/P, 10\%, 8) - 50,000(A/F, 10\%\ 8)$

$= 5,00,000\ (0.1874) - 50,000\ (0.0874)$

$= \text{Rs } 89{,}330$

PW at Year 5 (n=8-5=3) of CR

$PW_{CR} = 89{,}330\ (P/A, 10\%\ 3)$

$= 89{,}330\ (2.4869)$

$= \text{Rs } 2{,}22{,}154.77$

PW at Year 5 (n=8-5=3) of MV

$PW_{MV} = 50{,}000(P/F, 10\%, 3)$

$= 50{,}000\ (0.7513)$

$= \text{Rs } 37{,}565$

Market Value (MV) at Study Period

$MV_5 = PW_{CR} + PW_{MV}$

$= 2{,}22{,}154.77 + 37{,}565$

$= \text{Rs } 2{,}59{,}719.77$

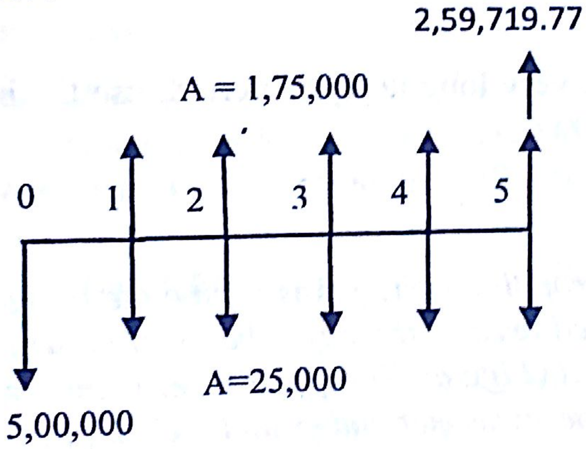


Fig.4.14: *Revised Cash Flow Diagram for Alternative B.*

Using FW Method

$FW_B = -5{,}00{,}000(F/P, 10\%, 5) + (17{,}50{,}000 - 15{,}000)\ (F/A\ 10\%\ 5)$

$+ 2{,}59{,}719.77$

$= -5{,}00{,}000\ (1.6105) + 1{,}50{,}000\ (6.1051) + 2{,}59{,}719.77$

$= \text{Rs } 3{,}70{,}234.77$

Since, $FV_B > FV_A$ .Therefore, project B is recommended **Ans.**

4.2.3 Capitalized Worth Method/ Capitalized Cost Method (For n = infinity)

Capitalized Cost (CC):

It is the present worth of a project which lasts forever. **Examples of such projects are:**

- Public sector projects.
- Bridges.
- Dams.

- Irrigation Systems.
- Railroads, etc.

Procedure to Calculate Capitalized Cost (CC)

1. Draw a cash-flow diagram showing all nonrecurring costs and at least two cycles of all recurring (periodic) costs and receipts.
2. Find the present worth of all nonrecurring amounts.
3. Find the equivalent annual worth through one life cycle of all recurring amounts and add this to all other uniform amounts occurring in years 1 through infinity. This results in a total equivalent uniform annual worth (AW).
4. Divide AW obtained in step 3 by the interest rate i to get its capitalized cost.
5. Add the CC values obtained in steps 2 and 4.

Note:

When you have a very long analysis period, use the infinity assumption to simplify problems.

Example 4.9

Pokhara Sub-metropolitan city plans a pipeline to transport water from a distant watershed area to the city. The pipeline will cost Rs 80,00,000 and have an expected life of 70 years. The city anticipates it will need to keep the water line in service indefinitely. Compute the capitalized cost assuming 7% interest.

Solution: We have given,

Initial investment (P) = Rs 80,00,000

Expected life (n) = 70 years

Interest rate (i) = 7%

Capitalized cost (CC) =?

We can find A first based on initial investment.

A = P (A/P, 7%, 70)

= 80,00,000(0.0706)

= Rs 5,64,800

Now, the infinite series payment formula could be applied for n = ∞:

We have the capitalized cost equation, $CC = \frac{A}{i}$

Capitalized cost, $CC = \frac{A}{i} = \frac{564800}{0.07}$ = Rs 80,68,571.43

Therefore, capitalized cost (CC) = Rs 80,68,571.43

4.3 Comparing Mutually Exclusive, Contingent and Independent Project in Combination

Independent Projects

- Independent project is one that may be accepted or rejected without influencing the acceptance or rejection of any other independent project.
- The acceptance or rejection of one does not directly eliminate other projects from consideration or cause the likelihood of their selection.
- For example, the installation of a new air conditioning system and the purchasing of a computer are two independent projects. These two projects are economically independent one another and evaluated separately.

Dependent Projects

- Dependent projects are related to one another in such a way that acceptance or rejection of one project influences the acceptance of others.
- **Dependency among the projects are of following types:**

a. **Contingent Project**

- Contingent project is one if acceptance of one project requires the acceptance of another.
- The acceptance or rejection of one is dependent on the decision to accept or reject one or more other projects.
- Contingent projects may be complementary or substitute. **Example of each would include:**

Complementary

The decision to start stationery may be contingent upon a decision to establish a college in an adjacent building. The cash flows of the stationery will be enhanced by the existence of a nearby college and vice versa.

Substitutes

Two different brands of mobile as two different projects. The success of one project may depend on the non-acceptance of the other project.

b. **Mutually Exclusive Projects**

- When there are several projects to achieve the same objective. In such case the selection of one of these alternatives exclude the choice of any of the others.
- Thus mutually exclusive projects are these when one project is selected all other are excluded in a group.
- For example, a car manufacturing company considering establishing one of its manufacturing complexes can locate it in Kathmadu, Pokhara or Biratnagar. If the company chooses Pokhara, the other two locations are ruled out.

Formation of Mutually Exclusive Combinations of Alternatives

Case 1: Independent Project

Let us consider two independent projects A and B. We can make mutually exclusive combinations are as follows:

Mutually Exclusive Combination	Projects		Remarks
	A	B	
1	-	-	Do nothing
2	A	-	Accept A
3	-	B	Accept B
4	A	B	Accept A and B

Case 2: Mutually Exclusive Projects

Let us consider three mutually exclusive projects A, B and C. We can make combinations are as follows:

Mutually Exclusive Combination	Projects			Remarks
	A	B	C	
1	-	-	-	Do nothing
2	A	-	-	Accept A
3	-	B	-	Accept B
4	-	-	C	Accept C

Case 3: Contingent Projects

Let us consider three projects A, B and C where C is the contingent on acceptance of B and B is the contingent on acceptance on A. We can make mutually exclusive combinations are as follows:

Mutually Exclusive Combination	Projects			Remarks
	A	B	C	
1	-	-	-	Do nothing
2	A	-	-	Accept A
3	A	B	-	Accept A and B
4	A	B	C	Accept All

Additional Solved Examples

Example 4.1

Mr. Kumar has inspected his yearly household expense for the last 10 years. Cost average were steady at Rs 1,00,000 per year for the first 5 years, but has increased consistently by Rs 15,000 per year for each of the last 5 years. Calculate total present worth in year zero. Use gradient formula. MARR=10%. **TU-2069**

Solution:

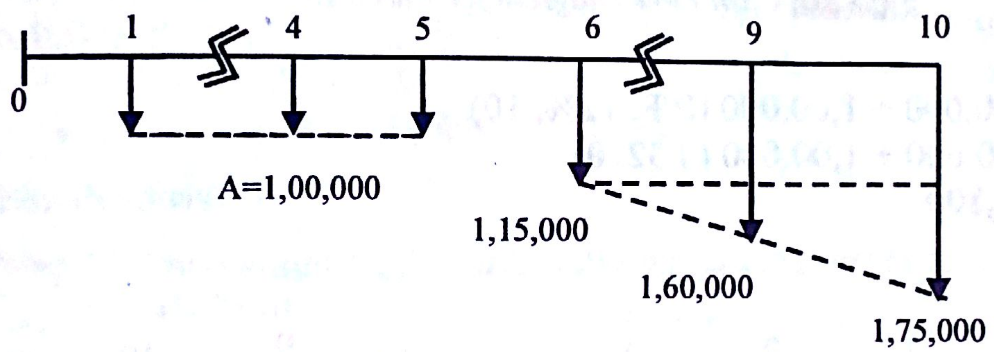


Fig. 4.15: *Cash Flow Diagram.*

P = 1, 00,000 (P/A, 10%, 5) + {1, 15,000 (P/A, 10%, 5)
+15,000 (P/G, 10%, 5)} (P/F, 10%, 5)
=1,00,000(3.7908)+{1,15,000(3.7908) + 15,000 (6.8618)}(0.6209)
= Rs 7,13,663.76

Therefore, total present worth = P = Rs 7,13,663.76 **Ans.**

Example 4.2

A construction company needs equipment which costs Rs 10,00,000 and has a salvage value of Rs 1,00,000 at the end of 10 years. The equipment suppliers are also willing to provide the equipment on hire for Rs 1,25,000 per year for 10 years. What will you do? Purchase or Hire. MARR=12%.

Solution: Given,

I = 10,00,000, SV = 1,00,000, n = 10 Years

Using Present Worth Formulation

For Purchase

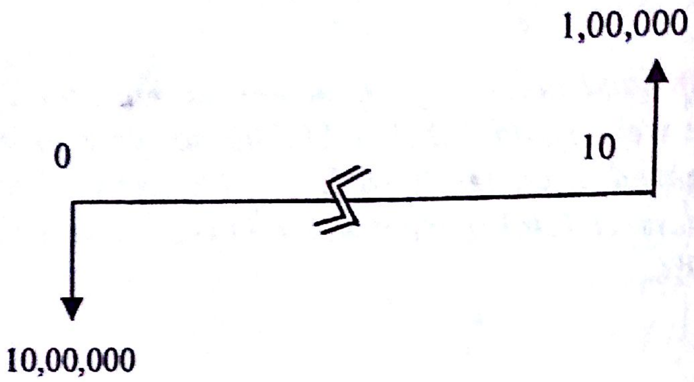


Fig.4.16: *Cash Flow Diagram for Purchase.*

$PW_P = -10,00,000 + 1,00,000\ (P/F, 12\%, 10)$
$= -10,00,000 + 1,00,000\ (0.3220)$
$= -9,67,800$

For Hiring

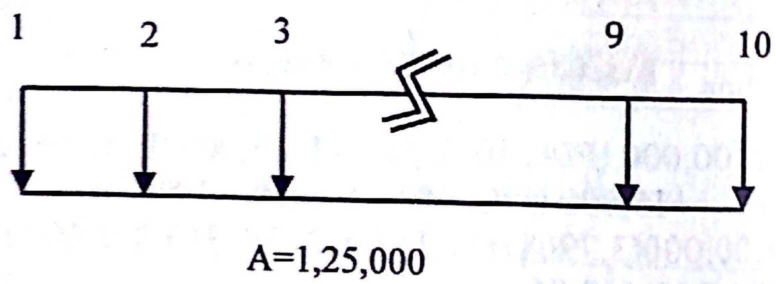


Fig.4.17: *Cash Flow Diagram for Hire.*

$PW_H = -1,25,000\ (P/A, 12\%, 10) = -1,25,000\ (5.6502) = -7,06,275$
Since, $PW_H < PW_P$, Therefore, hire the machine **Ans.**

Example 4.3

Compute the best project using ERR (ε =14%, MARR = 20%) TU-2070

End of Year	Proposal A	Proposal B
0	– 6,400	– 7,500
1	2,620	2,050
2	2,900	4,046
3	3,020	4,000
4	3,100	3,900
5	3,100	3,900
6	2,600	3,400

Solution:

For Proposal A

Step 1: Discounting all the cash outflows to the present at $\epsilon = 14\%$
$= -$ Rs6,400

Step 2: Compounding all the cash inflows to the future at $\epsilon = 14\%$

$= 2,620\ (F/P, 14\%, 5) + 2,900\ (F/P, 14\%, 4) + 3,020(F/P, 14\%, 3) + 3,100(F/P, 14\%, 2) + 3,100(F/P, 14\%, 1) + 2,600$

$= 2,620\ (1.9254) + 2,900\ (1.6890) + 3,020(1.4815) + 3,100(1.2996) + 3,100(1.1400) + 2,600$

$=$ Rs 24,579.54

Step 3: Establishing equivalence between two equations

$6,400\ (F/P, i'\%, 6) = 24,579.54$

$(1 + i')^6 = 3.84$

$i' = 25.14\%$

For Proposal B

Step 1: Discounting all the cash outflows to the present at $\epsilon = 14\%$
$=$ Rs $-$ 7550

Step 2: Compounding all the cash inflows to the future at $\epsilon = 14\%$

$= 2,050\ (F/P, 14\%, 5) + 4,046\ (F/P, 14\%, 4) + 4,000\ (F/P, 14\%, 3) + 3,900\ (F/P, 14\%, 2) + 3,900\ (F/P, 14\%, 1) + 3,400$

$= 2,050\ (1.9254) + 4,046\ (1.6890) + 4,000\ (1.4815) + 3,900\ (1.2996) + 3,900\ (1.1400) + 3,400$

$=$ Rs 29,621.204

Step 3: Establishing equivalence between two equations

$7,500\ (F/P, i'\%, 6) = 29,621.204$

$(1 + i')^6 = 3.95$

$i' = 25.72\%$

Since, ERR of Proposal B> ERR of Proposal A. Therefore, select proposal B **Ans.**

Example 4.4

Select the best project by ERR method. Take MARR =25% and ϵ =14%

EOY	0	1	2	3	4	5	6
A	–64,000	26,200	29,000	30,200	31,000	31000	26,000
B	– 68,000	– 4,000	39,200	38,000	38,000	38,000	38,000
C	– 75,000	20,500	40,460	40,000	39,000	39,000	34,000

Solution:

For Project A

Step 1: Discounting all the cash outflows to the present at $\epsilon = 14\%$

$= -$ Rs 64,000

Step 2: Compounding all the cash inflows to the future at $\epsilon = 14\%$

= 26,200 (F/P, 14%, 5) + 29,000 (F/P, 14%, 4) + 30,200(F/P, 15%, 3) + 31,000 (F/P, 14%, 2) + 31,000 (F/P, 14%, 1) + 26,000

=26,200 (1.9254) + 29,000 (1.6890) + 30,200 (1.4815) + 31,000 (1.2996) + 31,000 (1.1400) + 26,000

= Rs 2,45,795.40

Step 3: Establishing equivalence between two equations

64,000 (F/P, i'%, 6) = 2,45,795.40

$(1 + i')^6 = 3.84$

i' = 25.14% > MARR justified.

For Project B

Step 1: Discounting all the cash outflows to the present at $\epsilon = 14\%$

= – 68,000-4,000(P/F, 14%, 1)

= – 68,000 – 4000 (0.8772)

= – Rs71,508.80

Step 2: Compounding all the cash inflows to the future at $\epsilon = 14\%$

= 39,200 (F/P, 14%, 4) + 38,000 (F/P, 14%, 3) + 38,000 (F/P, 14%, 2) + 38,000 (F/P, 14%, 1) + 38,000

=39,200 (1.6890) + 38,000 (1.4815) + 38,000 (1.2996) + 38,000 (1.1400) + 38,000

= Rs 2,53,210.71

Step 3: Establishing equivalence between two equations

71,508.80 (F/P, i'%, 6) = 2,53,210.71

or, $(1 + i')^6 = 3.5410$

or, i' = 23.46% < MARR [not justified]

For Project C

Step 1: Discounting all the cash outflows to the present at $\epsilon = 15\%$

= Rs – 75,500

Step 2: Compounding all the cash inflows to the future at $\epsilon = 15\%$

= 20,500 (F/P, 15%, 5) + 40,460 (F/P, 15%, 4) + 40,000 (F/P, 15%, 3) + 39,000 (F/P, 15%, 2) + 39,000 (F/P. 15%, 1) + 34,000

$=20,500 (1.9254) + 40,460 (1.6890) + 40,000 (1.4815) + 39,000 (1.2996) + 39,000 (1.1400) + 34,000$

$= \text{Rs } 2,96,212.04$

Step 3: Establishing equivalence between two equations

$75,000 (F/P, i'\%, 6) = 2,96,212.04$

$(1 + i')^6 = 3.95$

$i' = 25.72\% >$ MARR justified.

Since, ERR of Project C > ERR of Project A > ERR of Project B. Therefore, select project C **Ans.**

Example 4.5

From the following information select the best project by using FW method and useful life is 4 years. ***TU-2069***

Projects	*Project A*	*Project B*
Initial Investment	*35,000*	*50,000*
Annual Revenue	*16,450*	*25,000*
Annual Costs	*3,000*	*13,830*
Useful Life (years)	*4*	*8*
Salvage Value	*0*	*0*
MARR	*10%*	*10%*

Solution:

For Project A

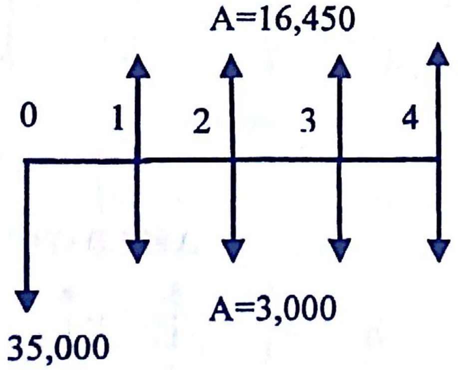


Fig. 4.18: *Cash Flow Diagram for Project A.*

Using FW Method

$FW_A = -35,000 (F/P, 10\%, 4) + (16,450-3,000) (F/A, 10\%, 4)$

$= -35,000 (1.4641) + 13,450 (4.6410)$

$= \textbf{Rs11,178}$

For Project B

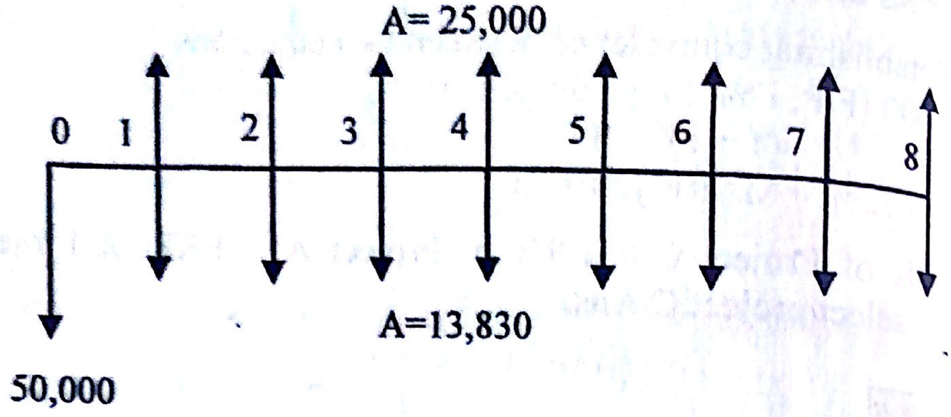


Fig.4.19: *Cash Flow Diagram for Project B.*

Imputed Market Value Calculation

Capital Recovery (CR) = 50,000 (A/P, 10%, 8) – 0(A/F, 10% 8)
= 50,000 (0.1874)
= Rs9,370

PW at Year 4 (n=8-4=4) of CR

PW_{CR} = 9,370 (P/A, 10% 4) = 9,370 (3.1699)
= Rs 29,702

Since, SV = 0

PW_{MV} = Rs 0

Market Value (MV) at Study Period

$MV_5 = PW_{CR} + PW_{MV}$
= 29,702 + 0
= **Rs 29,702**

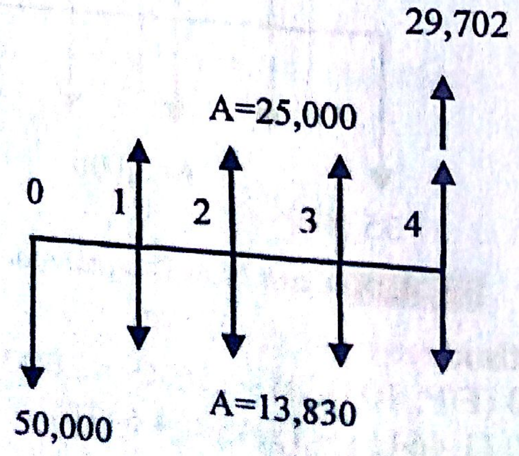


Fig.4.20: *Revised Cash Flow Diagram for Alternative B.*

Using FW Method

FW_B

$= -50,000\ (F/P, 10\%, 4) + (25,000-13,830)\ (F/A\ 10\%\ 4) + 29,702$

$= -50,000\ (1.4641) + 11,170\ (4.6410) + 29,702$

= **Rs 8,337**

Since, $FV_A > FV_B$. Therefore, project A is recommended **Ans.**

Example 4.6

Two types of power converts alpha and beta are under consideration for a specific application. An economic comparison is to be made at an interest rate of 10% and the following cost estimates have been obtained. Select the best option by calculating present worth of both the project if it will be operated for 4 years only. **TU-2069**

Solution:

	Alpha	Beta
Purchase Price	7,50,000	20,00,000
Annual Operating Cost	2,00,000	1,00,000
Salvage Value	0	4,00,000
Estimated Service Life	5 years	9 years

For Alpha

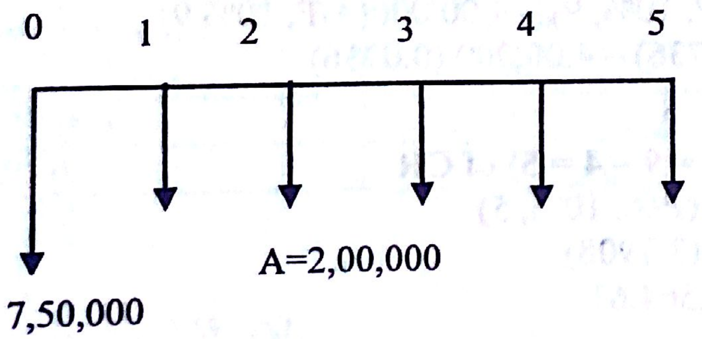


Fig. 4.21: *Cash Flow Diagram for Alpha.*

Imputed Market Value Calculation

Capital Recovery (CR)

$= 7,50,000\ (A/P, 10\%, 5) - 0$

$= 7,50,000\ (0.2638)$

= Rs 1,97,850

PW at Year 4 (n = 5 – 4 = 1) of CR

$PW_{CR} = 1,97,850\ (P/A, 10\%, 1)$

$= 1,97,850\ (0.9091) =$ Rs 1,79,865.43

$PW_{MV} = 0$

Market Value (MV) at Study Period

$MV_4 = PW_{CR} + PW_{MV}$

$= 179865.43 + 0 = \text{Rs } 1,79,865.43$

Using PW Method

$PW_A = -7,50,000 - 2,00,000\ (P/A, 10\%, 4) + 1,79,865.43\ (P/F, 10\%, 4)$

$= -7,50,000 - 2,00,000\ (3.1699) + 1,79,865.43\ (0.6830)$

$= \text{Rs} - 12,61,131.91$

For Beta

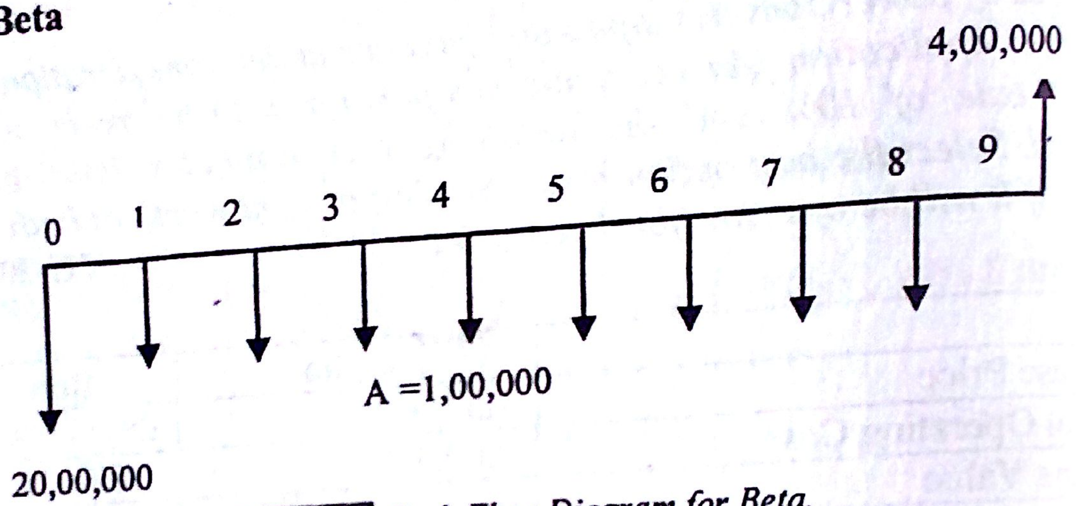


Fig. 4.22: *Cash Flow Diagram for Beta.*

Imputed Market Value Calculation

Capital Recovery (CR)

$= 20,00,000\ (A/P, 10\%, 9) - 4,00,000 (A/F, 10\% 9)$

$= 20,00,000\ (0.1736) - 4,00,000\ (0.0736)$

$= \text{Rs } 3,17,760$

PW at Year 4 (n = 9 – 4 = 5) of CR

$PW_{CR} = 3,17,760\ (P/A, 10\%, 5)$

$= 3,17,760\ (3.7908)$

$= \text{Rs } 12,04,564.61$

PW at Year 4 (n = 9 – 4 = 5) of MV

$PW_{MV} = 4,00,000 (P/F, 10\%, 5)$

$= 4,00,000\ (0.6209)$

$= \text{Rs } 2,48,360$

Market Value (MV) at Study Period

$MV_5 = PW_{CR} + PW_{MV}$

$= 12,04,564.61 + 2,48,360$

$= \text{Rs } 14,52,924.61$

Using PW Method

$PW_A = -20,00,000 - 1,00,000\ (P/A, 10\%, 4) + 14,52,924.61 (P/F, 10\%, 4)$

$= -20,00,000 - 1,00,000\ (3.1699) + 14,52,924.61\ (0.6830)$
$= \text{Rs} - 13,24,642.49$

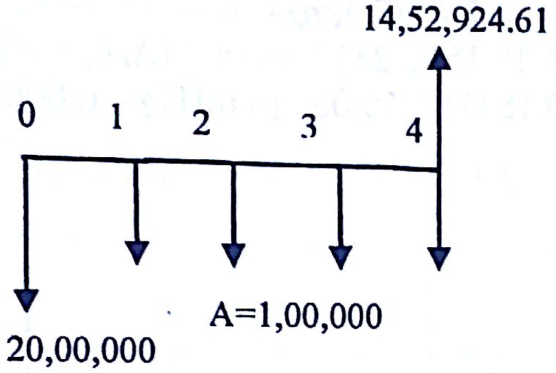


Fig.4.23: *Revised Cash Flow Diagram for Beta.*

Since, both the power converts have negative values but $PW_A < PW_B$ ie power converts Alfa has less negative value. Therefore, Alfa is recommended **Ans.**

Example 4.7

Assume infinite project life; recommend one of the following mutually exclusive projects.

Projects	*A*	*B*
Initial Investment (P)	*50,000*	*1,20,000*
Salvage Value (F)	*10,000*	*10,000*
Annual Cost (A)	*9,000*	*6,000*
Useful Life (n)	*10*	*25*
MARR (i)	*15%*	

Solution:

For Project A

Calculation of Annual Worth

$AW_A = -P\ (A/P, i, n) + F\ (A/F, i, n) - A$
$= -50,000\ (A/P, 15\%, 10) + 10,000\ (A/F, 15\%, 10) - 9,000$
$= -50,000\ (0.1993) + 10,000\ (0.0493) - 9,000$
$= -\text{Rs}\ 18,472$

We have the capitalized cost (CC) equation

$$CC_A = \frac{AW\ A}{i}$$

$$CC_A = \frac{18472}{0.15}$$

$= \text{Rs}\ 1,23,146.66$

For Project B

Calculation of Annual Worth

$AW_B = -P\,(A/P, i, n) + F\,(A/F, i, n) - A$

$= -1{,}20{,}000\,(A/P, 15\%, 25) + 10{,}000\,(A/F, 15\%, 25) - 6{,}000$

$= -1{,}20{,}000\,(0.1547) + 10{,}000\,(0.0047) - 6{,}000$

$= -$ Rs 24,517

$$CC_B = \frac{AW\ B}{i}$$

$$CC_B = \frac{24517}{0.15}$$

= Rs 1,63,446.66

Since, $CC_A < CC_B$. Therefore, select project A **Ans.**

Example 4.8

Calculate the Capitalized Cost of a project which has an initial cost of Rs1,50,000. The annual operating cost is Rs 8,000 for the first 4 years and Rs 5,000 thereafter. There is a recurring Rs15,000 maintenance cost every 15 years. Interest is 15% per year. Take useful life = 30 years.

Solution:

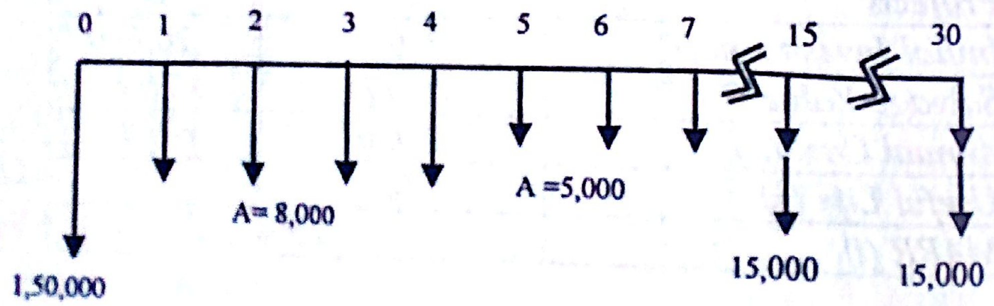


Fig. 4.24: *Cash Flow Diagram.*

Calculation of Annual Worth

1. Consider Rs 3,000 of the Rs 8,000 cost for the first four years to be a onetime cost leaving Rs 5,000 annual operating cost forever.

   $P_0 = -1{,}50{,}000 - 3{,}000\,(P/A, 15\%, 4)$

   $= -1{,}50{,}000 - 3{,}000\,(2.8550)$

   $= -$Rs 1,58,565

2. Recurring annual cost is 5,000 plus the equivalent annual of the 15,000 end of cycle cost.

   $AW = -5{,}000 - 15{,}000\,(A/F, 15\%, 15)$

   $= -5{,}000 - 15{,}000\,(0.0210)$

   $= -$ Rs 5,315

3. Capitalized Cost $= -1{,}58{,}565 + \frac{-5315}{0.15}$

$= -\text{Rs}1{,}93{,}998.33$

Therefore, capitalized cost = – Rs1,93,998.33 **Ans.**

Example 4.9

Compute the following projects by using repeatability assumption when MARR is 12%

Projects	*A*	*B*
Initial Investment	*2,00,000*	*3,00,000*
Annual Revenue	*25,000*	*30,000*
Annual Costs	*7,000*	*9,000*
Useful Life (years)	*6*	*9*
Salvage Value	*10,000*	*20,000*

Solution:

For Project A

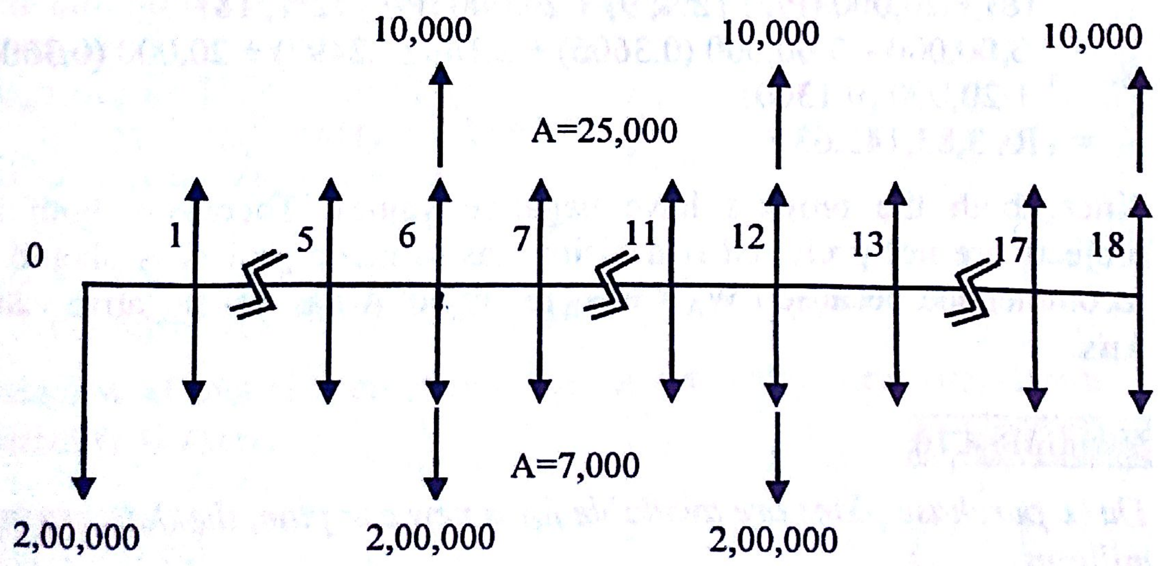


Fig. 4.25: *Cash Flow Diagram for Project A.*

$$PW_A = -2{,}00{,}000 - 2{,}00{,}000\ (P/F, 12\%, 6) - 2{,}00{,}000\ (P/F, 12\%, 12) + (25000 - 7000)\ (P/A, 12\%, 18) + 10{,}000\ (P/F, 12\%, 6) + 10{,}000\ (P/F, 12\%, 12) + 10{,}000\ (P/F, 12\%, 18)$$

$$= -200{,}000 - 200{,}000\ (0.5066) - 200{,}000\ (0.2567) + 18000\ (7.2497) + 10{,}000\ (0.5066) + 10{,}000\ (0.2567) + 10{,}000\ (0.1300)$$

$= \text{Rs } 2{,}13{,}232.40$

For Project B

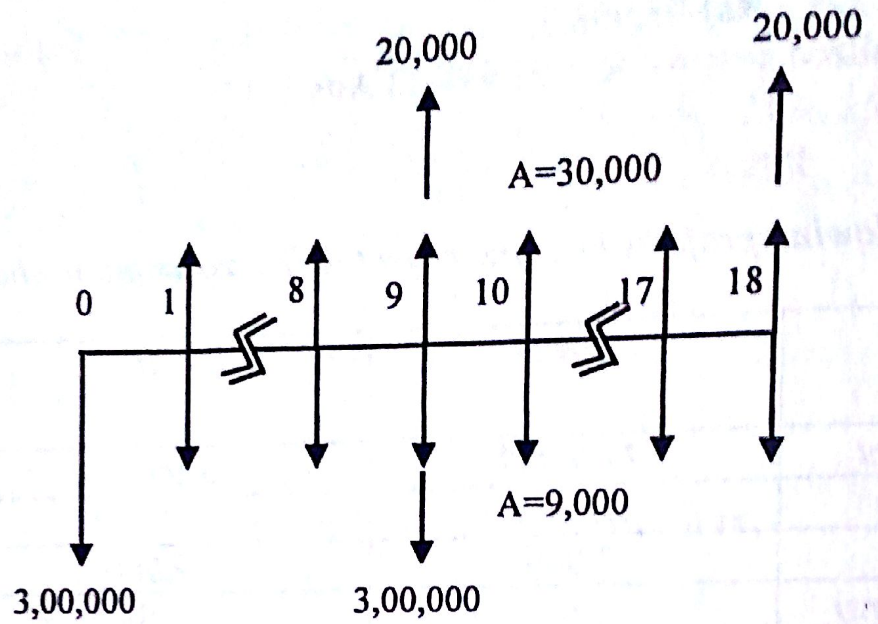


Fig. 4.26: *Cash Flow Diagram for Project B.*

$PW_B = -3{,}00{,}000 - 3{,}00{,}000\ (P/F, 12\%, 9) + (30{,}000\text{-}9{,}000)\ (P/A, 12\%, 18) + 20{,}000\ (P/F, 12\%, 9) + 20{,}000\ (P/F, 12\%, 18)$

$= -3{,}00{,}000 - 3{,}00{,}000\ (0.3606) + 2{,}100\ (7.2497) + 20{,}000\ (0.3606) + 20{,}000\ (0.1300)$

$= -$ Rs 3,83,143.63

Since, both the projects have negative values. Therefore, both the projects are not preferred if decision has to made project A should be recommended because $PW_A < PW_B$ ie project A has less negative value **Ans.**

Example 4.10

Three purchase plans are available for a new car plan, the data are in millions

Plan A: *Rs 5,000 cash immediately*

Plan B: *Rs 1,500 down and 36 monthly payments of Rs 116.25*

Plan C: *Rs 1,000 down and 48 monthly payments of Rs120.50*

If a customer expects to keep the car five years and her cost of money is 18% compounded monthly, which payment plan should she choose?

Solution:

$i = \frac{18}{12} = 1.5\ \%$

$PW_A = Rs5,000$
$PW_B = 1,500 + 116.25(P/A, 1.5\%, 36)$
$= 1500 + 116.25(27.6607)$
$= Rs4,715.55$
$PW_C = 1,000 + 120.50(P/A, 1.5\%, 48)$
$= 1,000 + 120.50(34.042)$
$= Rs\ 5,102.06$

Since, PW of plan B has lesser value (with respect to payment). Therefore, plan B is the best plan **Ans.**

Example 4.11

Given the following three mutually exclusive alternatives

Alternatives	*A*	*B*	*C*
Initial Cost	*50*	*30*	*40*
Annual Benefits	*15*	*10*	*12*
Useful Life (years)	. *5*	*5*	*5*

What alternative is preferable, if any, assuming i = 10%?

Solution:

$PW_A = -50 + 15(P/A, 10\%, 5)$
$= -50 + 15(3.7908) = -Rs\ 6.87$
$PW_B = -30 + 10(P/A, 10\%, 5)$
$= -30 + 10(3.7908) = -Rs\ 7.91$
$PW_C = -40 + 12(P/A, 10\%, 5)$
$= -40 + 12(3.7908) = -Rs\ 5.49$

Since, PW of cost of alternative C has lowest value. Therefore, choose alternative C **Ans.**

Example 4.12

A farmer must purchase a tractor using a loan of Rs 20, 00,000. The bank has offered the following choice of payment plans each determined by using an interest rate of 8%. If the farmer's minimum attractive rate of return (MARR) is 15%, which plan should he choose?

Plan A: *Rs 5, 01,000 per year for 5 years.*

Plan B: *Rs 2, 95,600 per year for 4 years plus Rs15, 00,000 at end of 5 years*

Plan C: *Nothing for 2 years, then Rs 9, 04800 per year for 3 years*

Solution:

$PW_A = 5,01,000(P/A, 15\%, 5)$
$= 5,01,000 (3.522) = Rs17,64,522$

$PW_B = 2,95,600 (P/A, 15\%, 4) + 15,00,000 (P/F, 15\%, 5)$
$= 2,95,600 (2.8550) + 15,00,000 (0.4972)$
$= Rs15,89,733$

$PW_C = 9,04,800 (P/A, 15\%, 3) (P/F, 15\%, 2)$
$= 9,04,800 (2.2832) (0.7561) = Rs\ 15,61,981.14$

Since, plan C is lowest cost plan. Therefore, choose plan C **Ans.**

Example 4.13

Projects A and B have first costs of Rs 6, 50,000 and Rs 17, 00,000 respectively. Project A has net annual benefits of Rs 2, 00,000 during each year of its 5 year useful life, after which it can be replaced identically. Project B has net annual benefits of Rs 3, 00,000 during each year of its 10 year life. Use present worth analysis, and an interest rate of 10% to determine which project to select.

Solution: We have given,

Projects	A	B
Initial Cost	6,50,000	17,00,000
Annual Benefits/yr	2,00,000	3,00,000
Useful Life (years)	5	10

For Project A

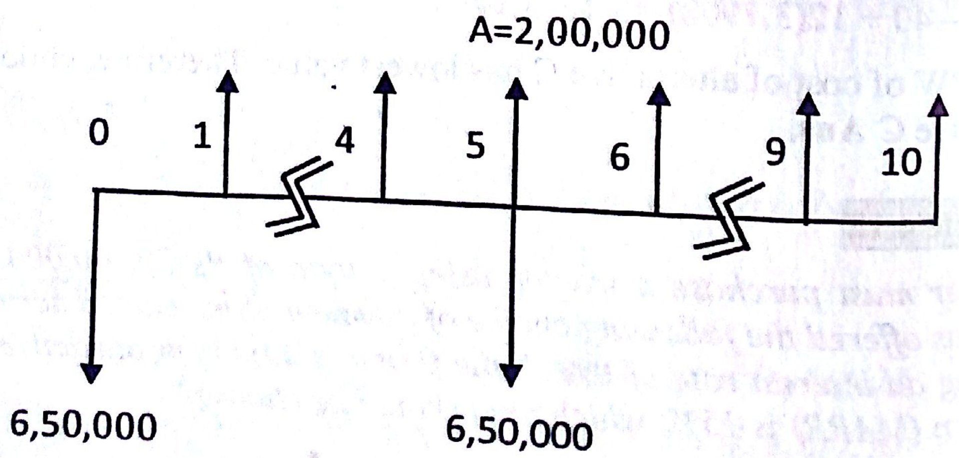


Fig. 4.27: *Cash Flow Diagram for Project A.*

$PW_A = -6,50,000 - 6,50,000(P/F, 10\%, 5) + 2,00,000 (P/A, 10\%, 10)$
$= -6,50,000 - 6,50,000(0.6209) + 2,00,000 (6.1446)$
$= Rs\ 1,75,335$

For Project B

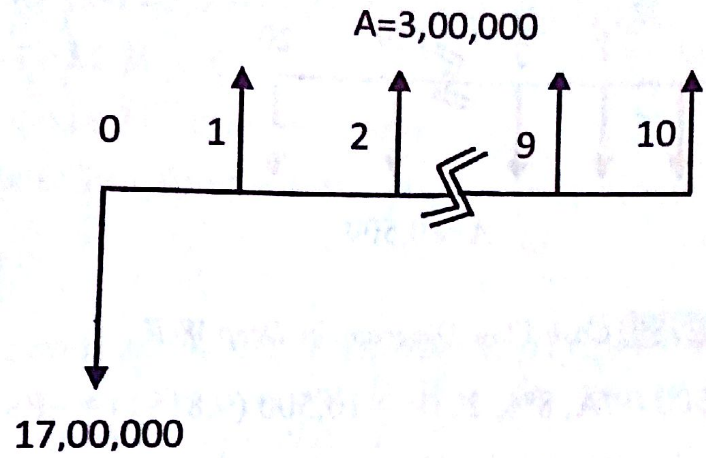


Fig. 4.28: *Cash Flow Diagram for Project B.*

$$PW_B = -17,00,000 + 3,00,000\ (P/A,\ 10\%,\ 10)$$
$$= -17,00,000 + 3,00,000\ (6.1446) = \text{Rs } 1,43,380$$

Since, project A has higher present worth. Therefore, select project A **Ans.**

Example 4.14

The city council wants the municipal engineer to evaluate three alternatives for supplementing the city water supply. The first alternative is to continue deep well pumping at an annual cost of Rs10,500. The second alternative is to install an 18" pipeline from a surface reservoir. First cost is Rs25,000 and annual pumping cost is Rs7,000. The third alternative is to install a 24" pipeline from the reservoir at a first cost of Rs34,000 and annual pumping cost of Rs5,000. Life of all alternatives is 20 years. For the second and third alternatives, salvage value is 10% of first cost. With interest at 8%, which alternative should the engineer recommend? Use present worth analysis.

Solution: We have given,

Cash Flow	**Deep Well**	**18" Pipeline**	**24" Pipeline**
First Cost		25,000	34,000
Annual Cost/yr	10,500	7,000	5,000
Salvage Value	0	10% of 1^{st} cost	10% of 1^{st} cost
Useful Life (years)	20	20	20

Fixed output, therefore minimize cost.

Deep Well:

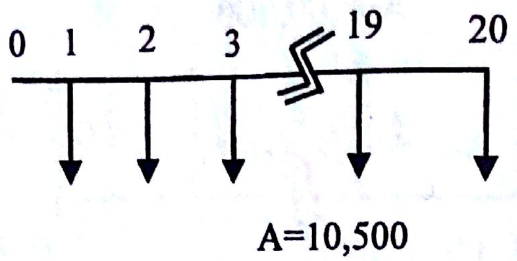


Fig 4.29: *Cash Flow Diagram for Deep Well.*

PW_C = – 10,500 (P/A, 8%, 20) = – 10,500 (9.8181) = –Rs 103,090.

18" Pipeline:

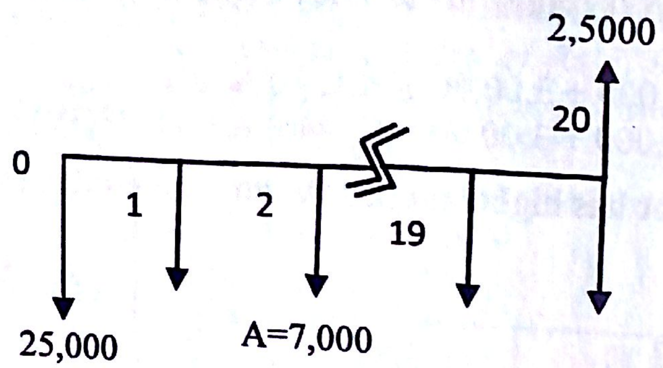


Fig 4.30: *Cash Flow Diagram for 18'' Pipeline.*

PW of Cost = – 25,000 - 7,000 (P/A, 8%, 20) + 2,500 (P/F, 8%, 20)

= – 25,000 - 7,000 (9.8181) + 2,500 (0.2145)

= – Rs 93,190.45

24" Pipeline:

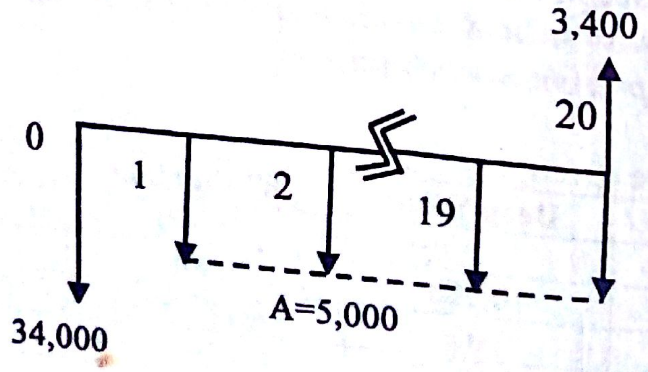


Fig 4.31: *Cash Flow Diagram for 24" Pipeline.*