Chapter 2

Relative Positioning by Terrestrial Method

Terrestrial Method

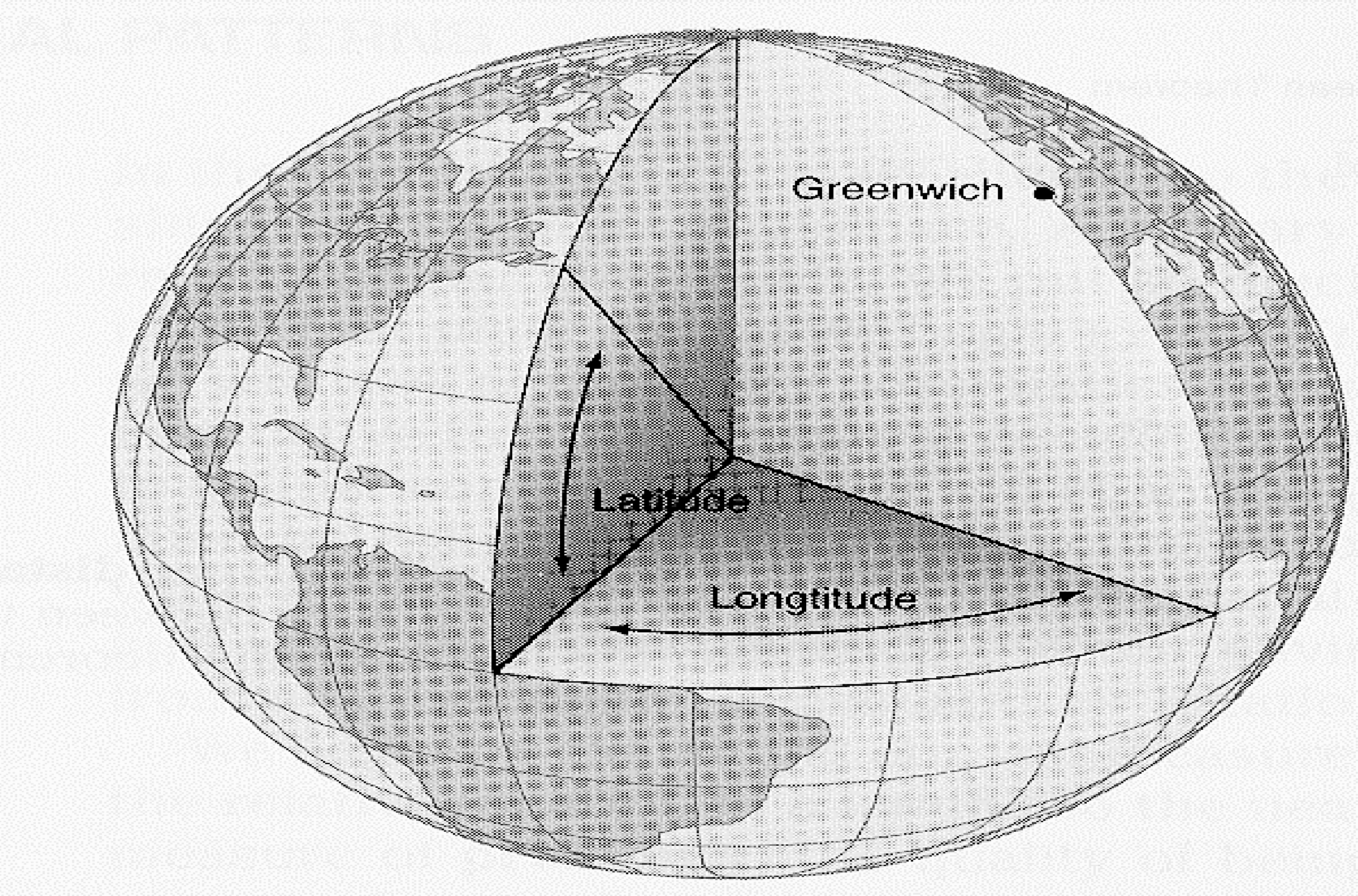
- Instrument is placed on the ground/nearly on the ground.
- Applicable for precise and accurate 2D and 3D position determination.
- Also for the supplementary survey in difficult areas where extraterrestrial method is also impractical (*Overhanging cliff : Automatic targeted Total Stations).*
- Horizontal angles, distance, zenith angles, and height difference obtained from levelling are measured on the earth surface.
- These measurements determine the relative spatial position of the surface points.
- Recently extraterrestrial methods are in practice for the fast and large area coverage at small time.

Three dimensional relative positioning

- For 2D: Easting ,Northing or Latitude or Longitude for the calculation taking as reference of ellipsoidal surface.
- Ellipsoid: local and global
- However, 2D representation is not accurate enough because of Singularity at the poles, Complex behavior near the Poles, Discontinuity at the ±180 degree meridian.

Contd…

- Horizontal Datum
  - Reference ellipsoid
  - Horizontal measurements made with respect to this surface



Contd…

- Vertical Datum
  - Normally Geoids/ mean sea level
  - Vertical measurements made with respect to this surface

Relative horizontal positioning on reference ellipsoid

Reference ellipsoid:

- Because the surface of the Earth is so complex, geodesists use simplified mathematical surface model called the ellipsoid to measure the Earth. Because the ellipsoid exists only in theory and not in real life, it can be completely smooth and does not take any irregularities - such as mountains or valleys - - into account.
- The ellipsoid created by rotating an ellipse around its minor axis is called oblate spheroid (ellipsoid)
- The ellipsoid created by rotating an ellipse around its major axis is called prolate spheroid (ellipsoid)

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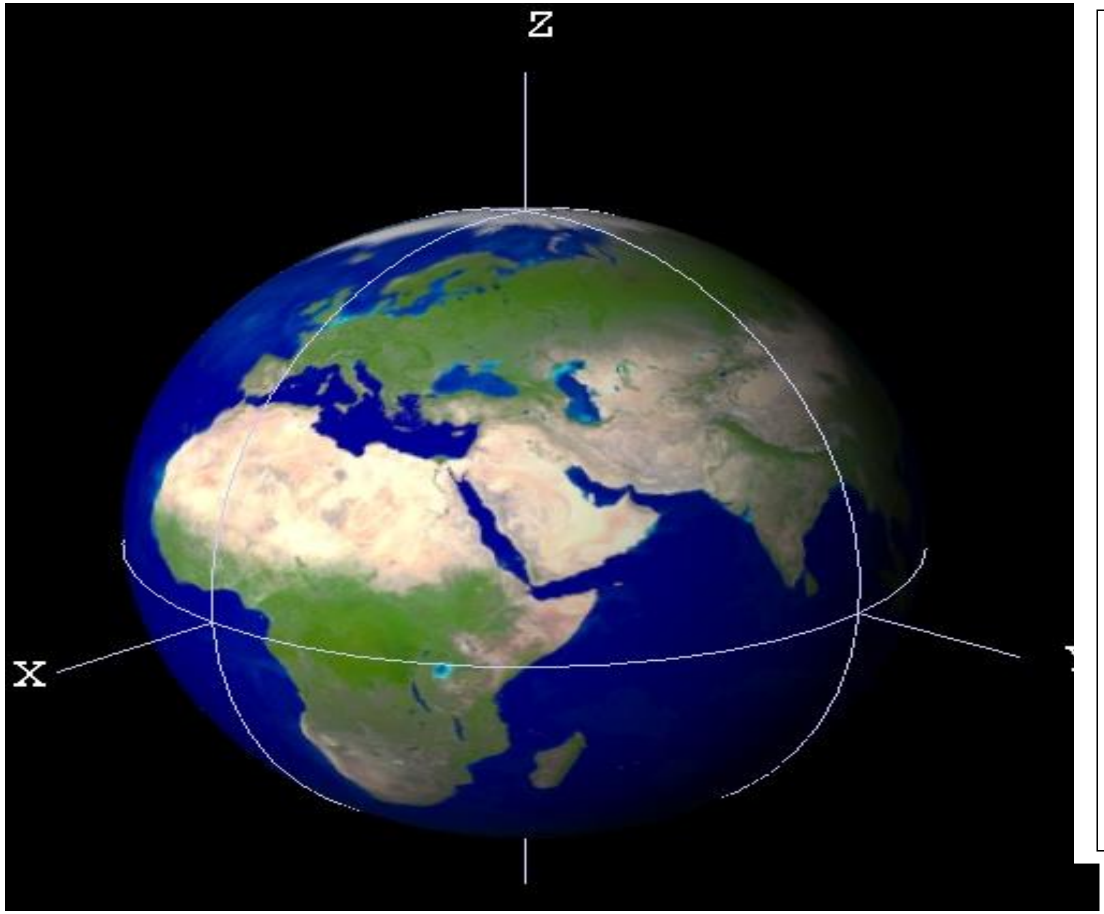
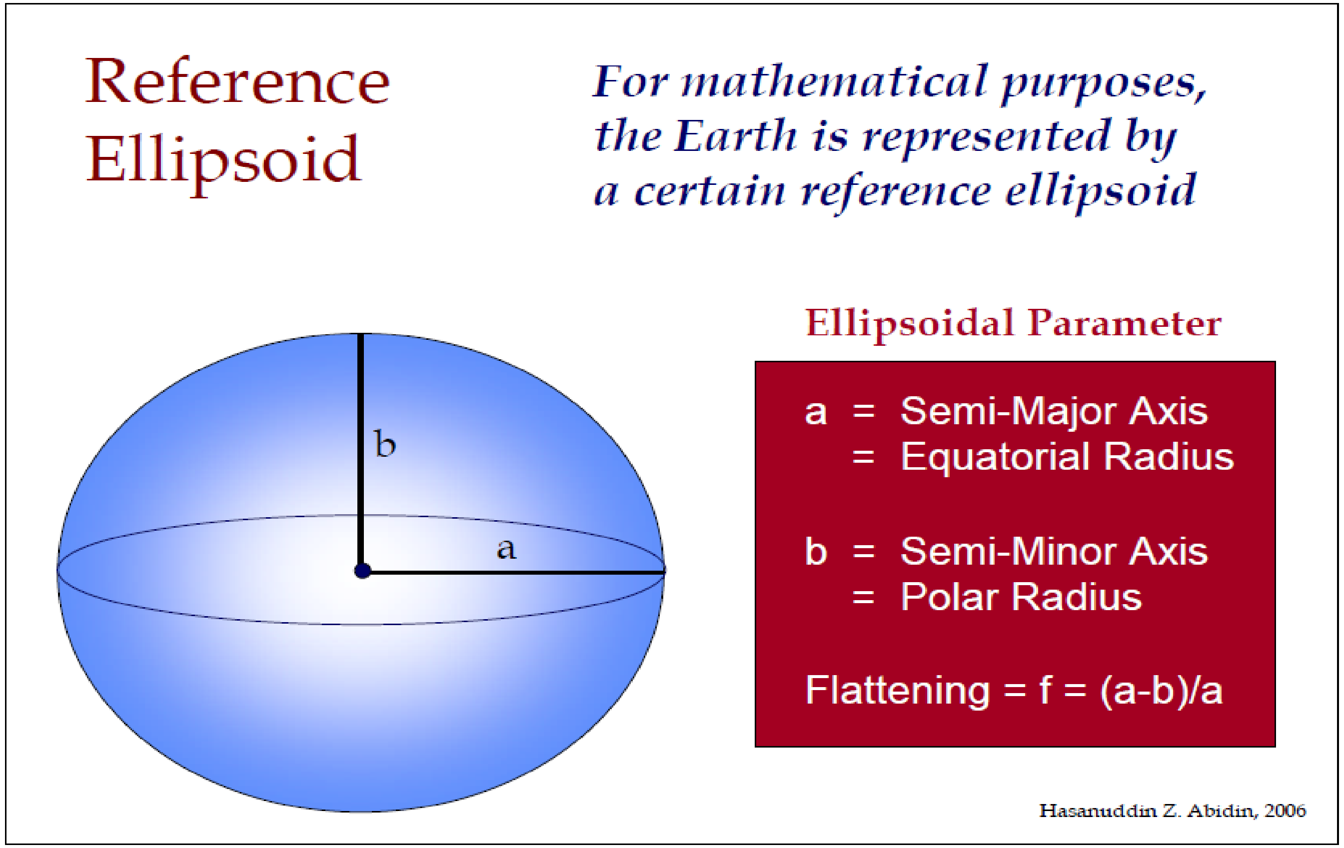
- The oblate spheroid matches the real Earth's shape, because the earth is slightly flattened at the poles and bulges at the equator.





Reference Ellipsoid

- An ellipsoid of revolution is uniquely defined by specifying two dimensions.
- Geodesists, by convention, use the semi major axis and flattening. The size is represented by the radius at the equator-the semi major axis-and designated by the letter, a.
- The shape of the ellipsoid is given by the flattening, f, which indicates how closely an ellipsoid approaches a spherical shape. The difference between the ellipsoid of revolution representing the earth and a sphere is very small.

Telluroid, Height anomaly

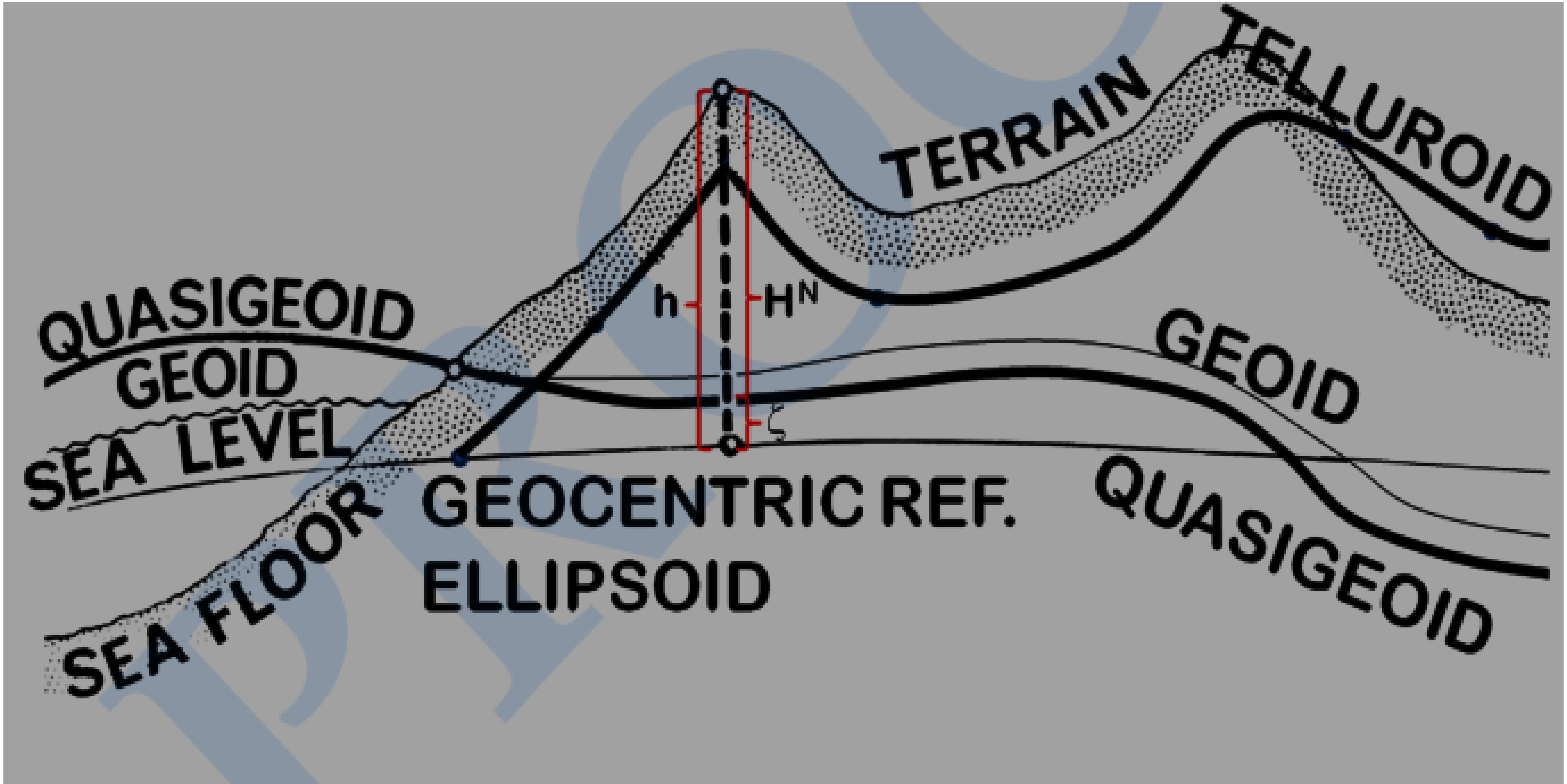


Fig. 6. The relation among the quasigeoid, geoid and reference ellipsoid.

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- The fact that the topographic density was not known with an adequate accuracy, back in 1960’s Molodenski declare the geoid is impossible to determine to a sufficient accuracy and to introduced an alternative quantity known as Quasi-geoid.
- For determining the quasi-geoid it would not be necessary to know the topo-density and as all the computation are done not on the geoid surface but on the surface called telluroid (i.e. at an almost identical surface to earth surface) but not on the geoid.

Contd…

- The vertical distance between the quasi-geoid and reference ellipsoid is called the Quasi-geoidal height (Height anomaly).
- The telluroid surface is a surface that looks like earth surface expect that it is displace from the earth surface by the quasi-geoidal height.

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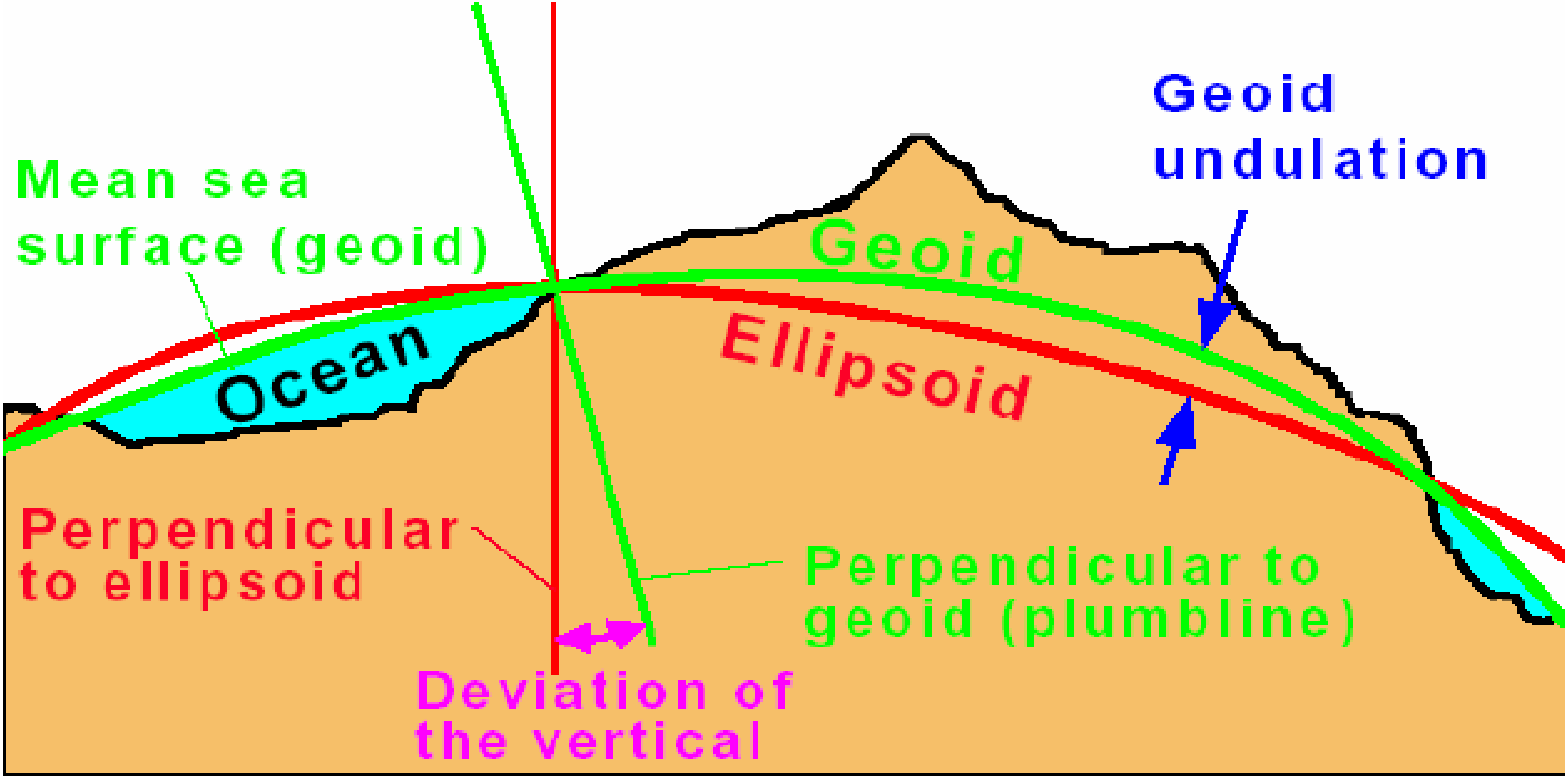


Mean sea
surface (geoid)
Ocean
Geoid
Ellipsoid
Geoid
undulation
Perpendicular
to ellipsoid
Perpendicular to
geoid (plumbline)
Deviation of
the vertical

Everest Ellipsoid

- Semi major Axis a = 6377276.345 m
- Semi minor Axis b= 6356075.413
- Flattening $f = \frac{(a-b)}{a} \approx 1/300.8017$

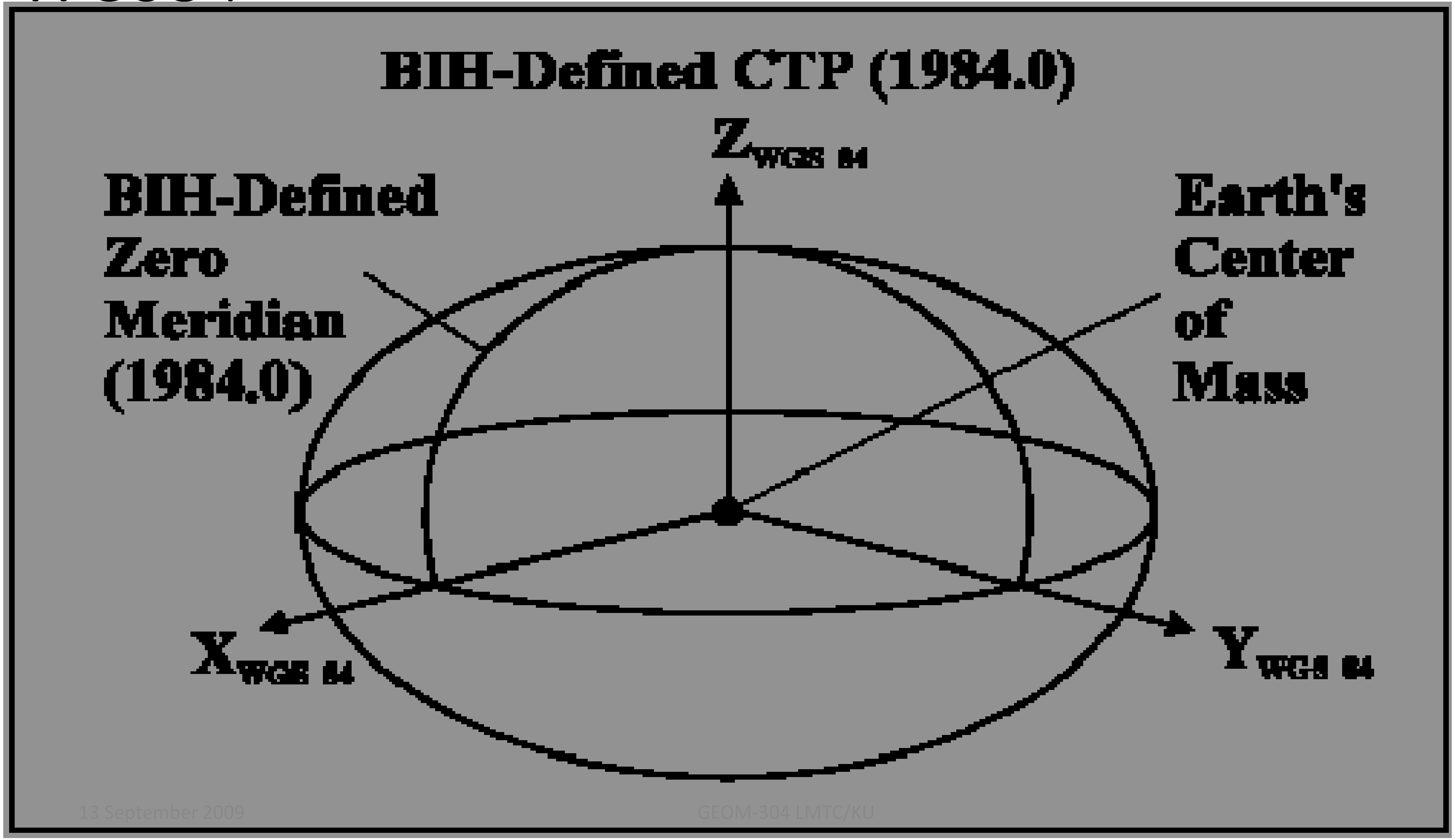
WGS84

- The World Geodetic System - 1984 (WGS 84) coordinate system is a Conventional Terrestrial System (CTS), realized by modifying the Navy Navigation Satellite System (NNSS), or TRANSIT, Doppler Reference Frame in origin and scale, and rotating it to bring its reference meridian into coincidence with the Bureau International de l’Heure (BIH)- defined zero meridian.

Origin and axes of the WGS 84 coordinate system

- **Origin** = Earth's centre of mass
- **Z-Axis** = The direction of the Conventional Terrestrial Pole (CTP) for polar motion, as defined by BIH on the basis of the coordinates adopted for the BIH stations
- **X-Axis** = Intersection of the WGS 84 reference meridian plane and the plane of the CTP's equator, the reference meridian being the zero meridian defined by the BIH on the basis of the coordinates adopted for the BIH stations.
- **Y-Axis** = Completes a right-handed, orthogonal coordinate system, measured in the plane of the CTP equator, 90° East of the x-axis.

WGS84


BIH-Defined CTP (1984.0)
Z WGS 84
BIH-Defined Zero Meridian (1984.0)
Earth's Center of Mass
X WGS 84
Y WGS 84

WGS-84

- Semi major Axis a = 6378137
- Semi minor Axis b= 6356752.314
- Flattening $f = \frac{(a-b)}{a} \approx 1/298.257223563$

Commonly used Reference Ellipsoids

Name of Spheroid	Semimajor axis (m)	Flattening	Where used
Krassowsky (1940)	6,378,245	1/298.3	Russia
International (1924)	6,378,388	1/297	Europe
Clarke (1880)	6,378,249	1/293.46	France, Africa
Clarke (1866)	6,378,206	1/294.98	North America
Bessel (1841)	6,377,397	1/299.15	Japan
Airy (1830)	6,377,563	1/299.32	Great Britain
Everest (1830)	***6,377,276***	***1/300.80***	***India, Nepal***
WGS 66 (1966)	6,378,145	1/298.25	USA/DoD
GRS 67 (1967)	6,378,160	1/298.25	Australia, South America
WGS 72 (1972)	6,378,135	1/298.26	USA/DoD
WGS 84 (1984)	6,378,137	1/298.2572	USA/DoD
GRS 80 (1979)	6,378,137	1/298.26	

Projected Coordinate System

- a georeferenced Cartesian 2D system with axes of easting and northing.
- The axes may be referred to as for example E and N or in an alternative case as X and Y, and may be given in any prescribed order.
- Projected coordinates result from the conversion of geographic 2D coordinates through a map projection.
- Flat, two-dimensional representation of the Earth.
- Calculations of distance and area are more easily performed.
- Usually expressed in meters as Easting and Northing.

Contd…

- A graticule (lattice of lines of equal latitude and longitude) cannot be projected onto a plane without distortion (in area, shape, direction, distance etc)
  - This is similar to removing an orange peel and trying to force it onto a fat surface - it tears, bends and distorts .
- When one property is preserved, others are distorted.
- The difference in these lengths will vary over the map's extent; distances are true (equal) only along the central meridian.
- Map projection methods have been formulated so that distortion of one or more characteristics (area, shape, direction, distance, etc) is controlled.

Map Projection

Flat Map

Cartesian coordinates: x,y

(Easting & Northing)

Curved Earth

Geographic coordinates: ϕ, λ

(Latitude & Longitude)

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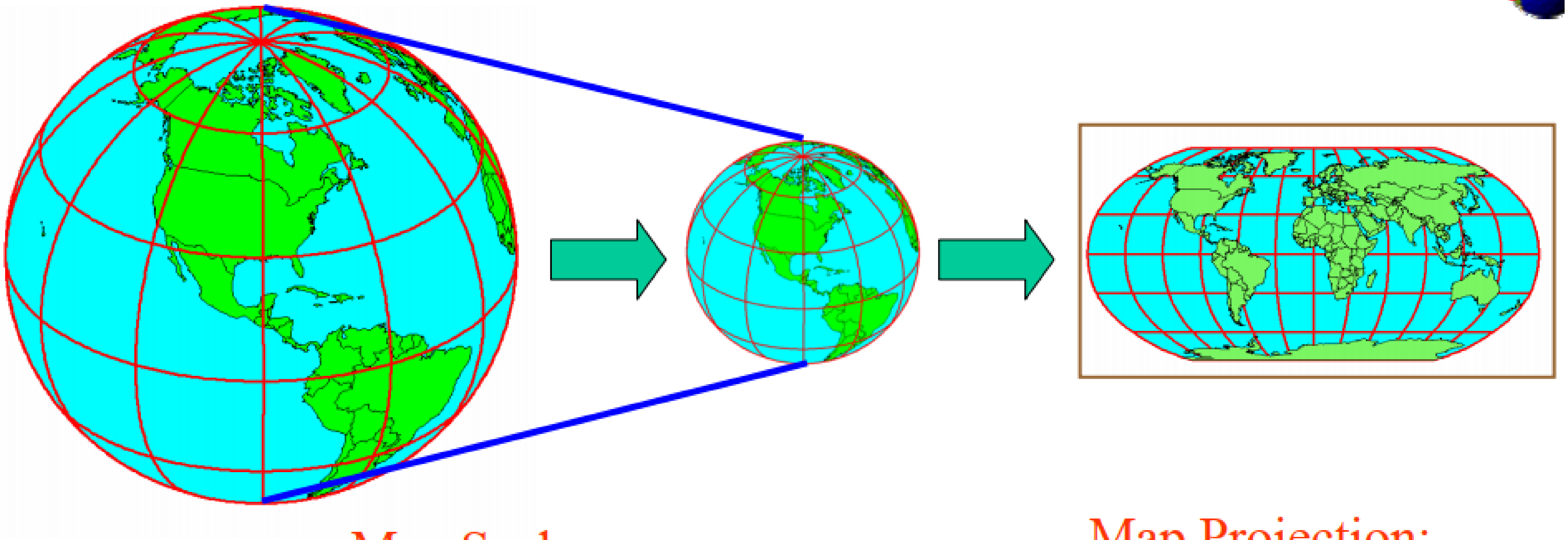

Map Scale:

Representative Fraction

$$= \frac{\text{Globe distance}}{\text{Earth distance}}$$

Map Projection:

Scale Factor

$$= \frac{\text{Map distance}}{\text{Globe distance}}$$

Contd…

- Map projection can be shape preserving (conformal), equivalent (area preserving) and equidistant (distance along certain direction preserving).
- Similarly map projection system can use cylinder, cone or plane surface and respective projections are called cylindrical, conical and azimuthal projection.
- For example Lambert Conic Conformal and Transverse Mercator have this conformal property and use cone and cylinder respectively but in case of both there is distortion in distance and area.
- One meridian, usually at the centre of the mapped area, will be defined to be the longitude of the projection origin.
- It is easily seen that only closely around this meridian the projection is reasonably distortion free.

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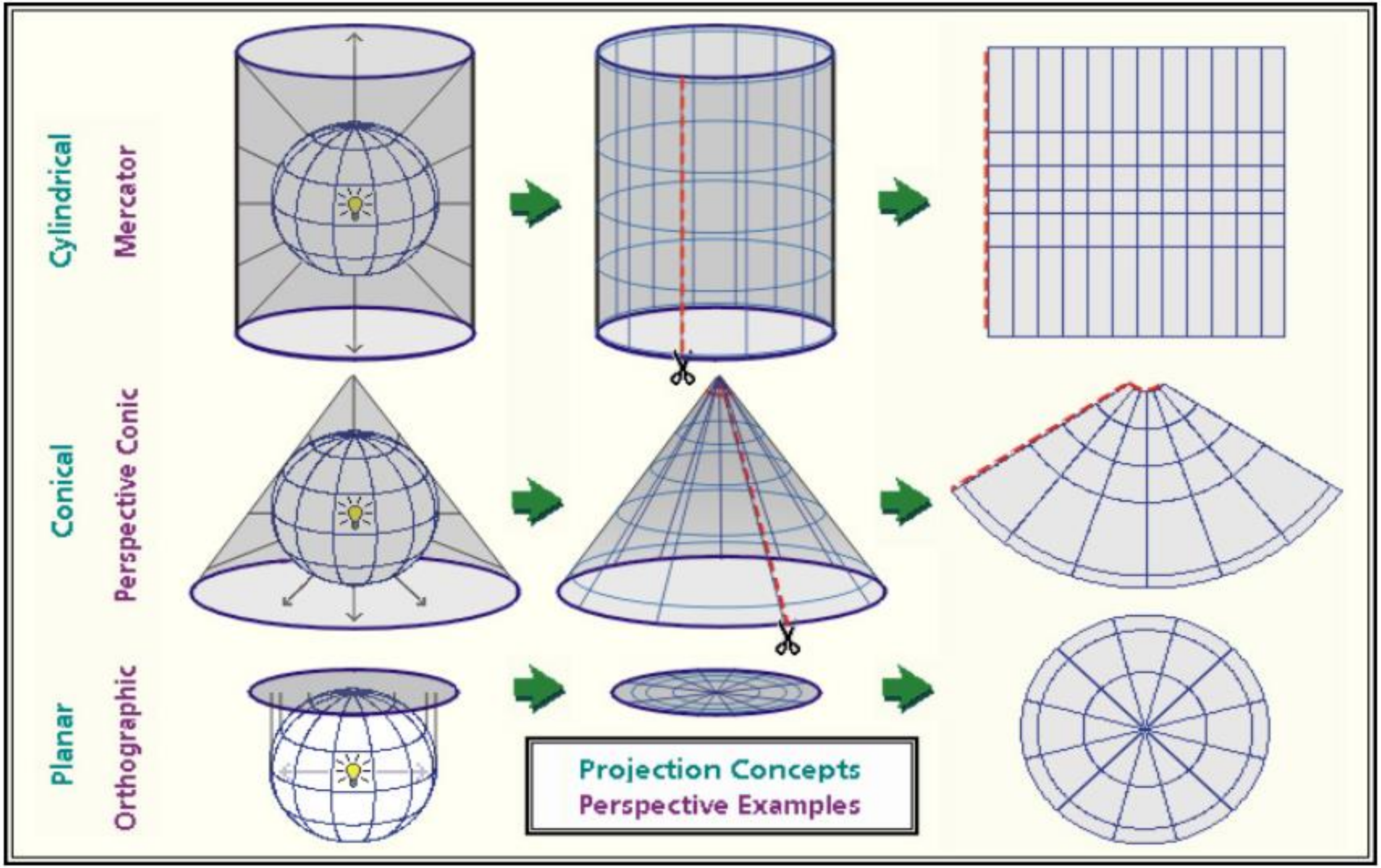

Cylindrical
Mercator
Conical
Perspective Conic
Planar
Orthographic
Projection Concepts
Perspective Examples

Relative horizontal positioning on conformal mapping plane

- Is idle map projection possible?
- Equivalent, Equidistant, Conformal…
- Conformal: Shape preserving
- Conformal maps preserve both angles and the shapes of infinitesimally small figures, but not necessarily their size or curvature.

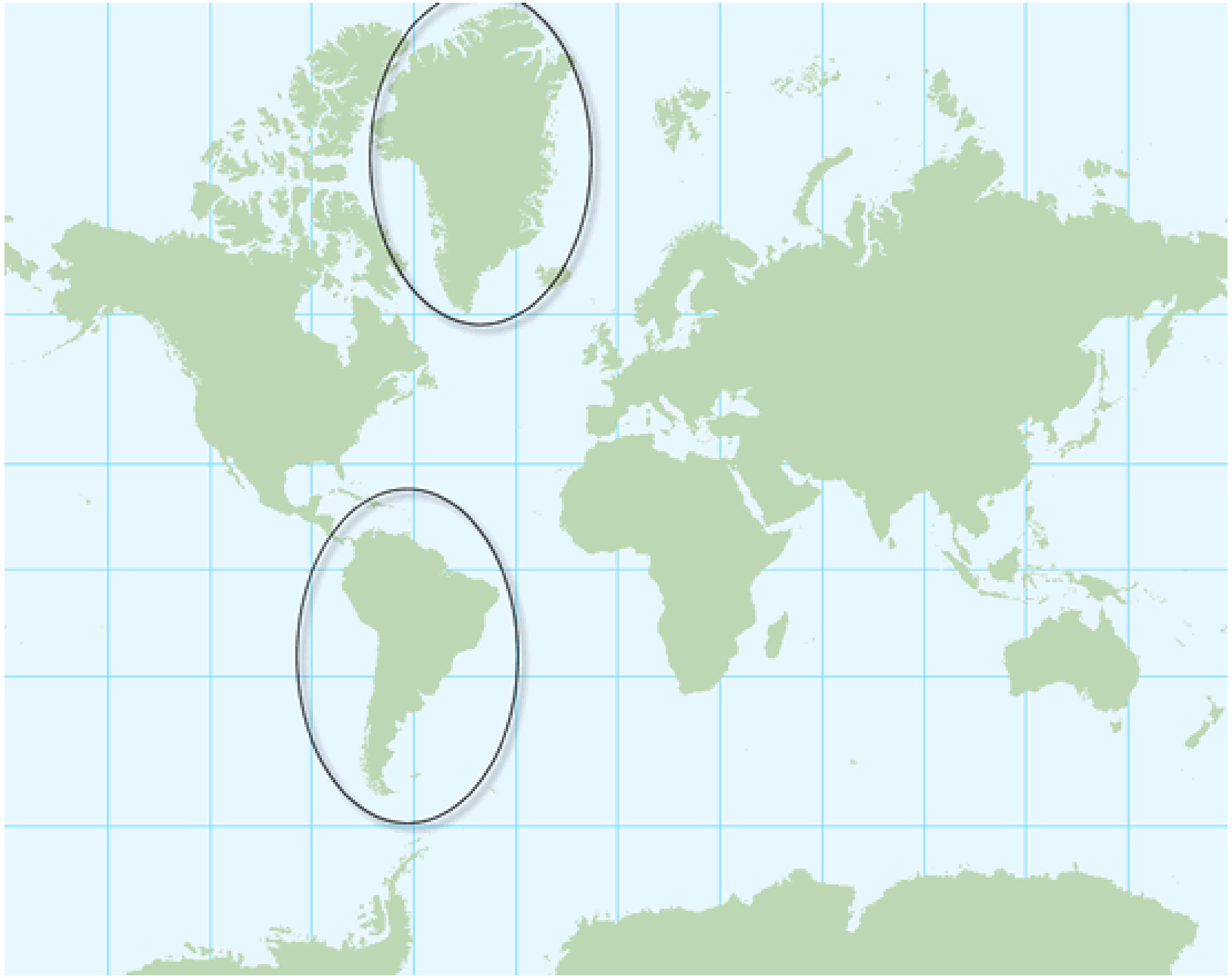
Contd…

- Conformal map projections are frequently employed in large-scale applications, and seldom used for continental or world maps.
- Since no conformal map can be equal-area — most in fact grossly distort dimensions far from the center of the map — conformal projections are almost never applied to thematic and statistical mapping, where comparisons based on size are common.

Contd…

- An example is the Mercator projection. Although Greenland is only one-eighth the size of South America, Greenland appears to be larger Maps used for the measurement of angles (e.g. aeronautical charts, topographic maps) often make use of a conformal map projection.

Contd…



The Mercator projection is a cylindrical map projection with a conformal property. The area distortions are significant towards the polar regions. An example, Greenland appears to be larger but is only one-eighth the size of South America.

Vertical Positioning

- Reference surface for height, two reference surfaces which are commonly used as a basis for height values
  - Mean sea-level surface (or geoid) and
  - the spheroid.
- In most parts of the world, distance above sea-level has been the traditional mechanism for measuring height. This has been due to:
  - a preference for a physically identifiable surface as a reference,
  - the importance of sea level to economic activity (for example shipping),
  - the linkage between sea level, the earth's gravity field, and the conventional
  - instrumentation used to measure height differences,
  - the importance of gravity-related heights to water-flow problems.

Contd….

- Spheroidal heights (heights relative to the spheroid) are now becoming increasingly popular because satellite surveying and satellite navigation receivers producing Spheroidal heights as part of their output.
- The distance between the geoid and the spheroid is referred to as the geoid-spheroid separation or geoidal undulation , If it is known, a sea-level height can be converted to a Spheroidal height and vice-versa.

Instruments for vertical positioning

- Dumpy level
- Y-level
- Reversible level
- Auto level

Methods of determining vertical positions

- **Differential levelling**: The optical instrument used for leveling contains a bubble tube to adjust it in a position parallel to the geoid. When properly "set up" at a point, the telescope is locked in a perfectly horizontal (level) position so that it will rotate through a 360 arc. The exact elevation of at least one point in a leveling line must be known and the rest computed from it.
- **Trigonometric leveling** involves measuring a vertical angle from a known distance with a theodolite and computing the elevation of the point. With this method, vertical measurements can be made at the same time horizontal angles are measured for triangulation.

Contd….

- It is, therefore, a somewhat more economical method but less accurate than differential leveling. It is often the only practical method of establishing accurate elevation control in mountainous areas.
- In **barometric leveling**, differences in height are determined by measuring the difference in atmospheric pressure at various elevations. Air pressure is measured by mercurial or aneroid barometers, or a boiling point thermometer. Although the degree of accuracy possible with this method is not as great as either of the other two, it is a method which obtains relative heights