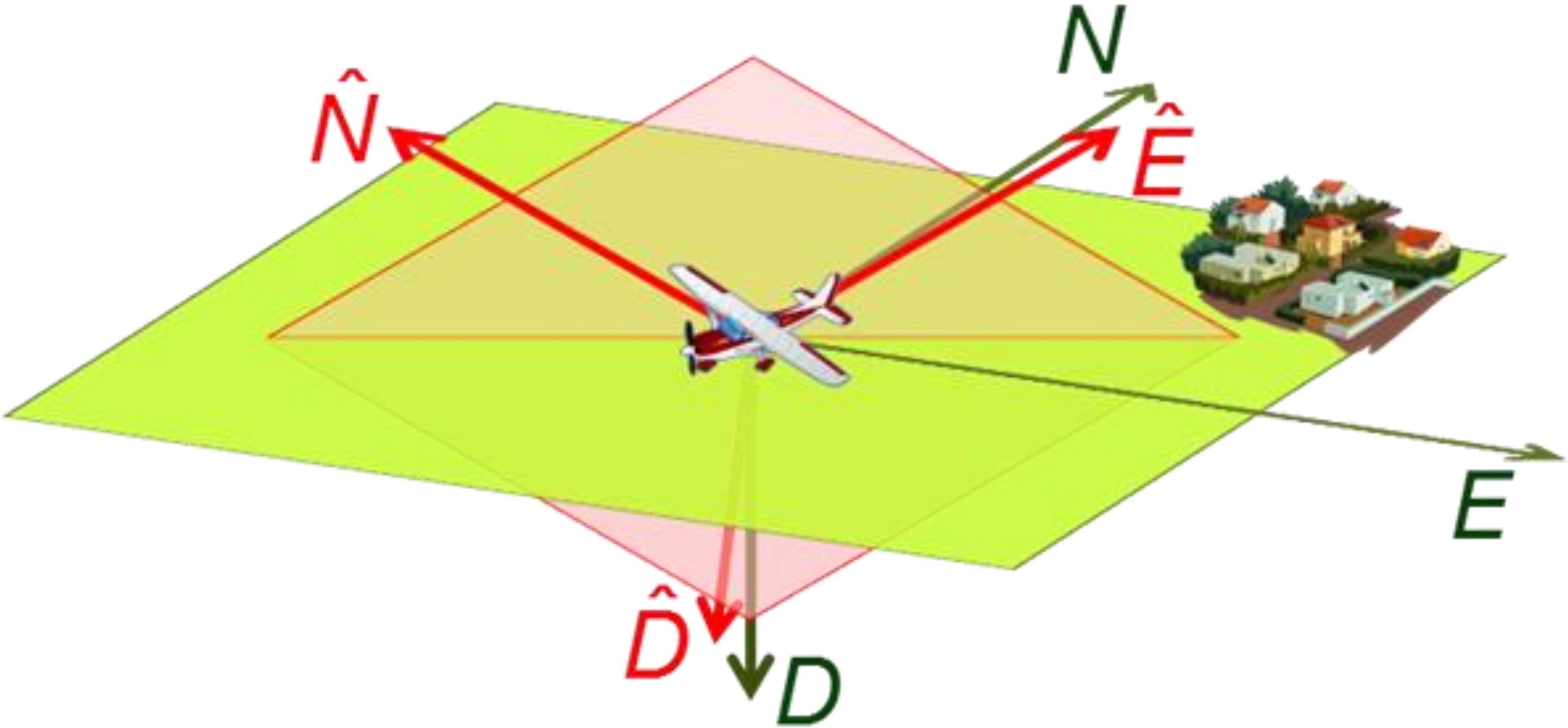
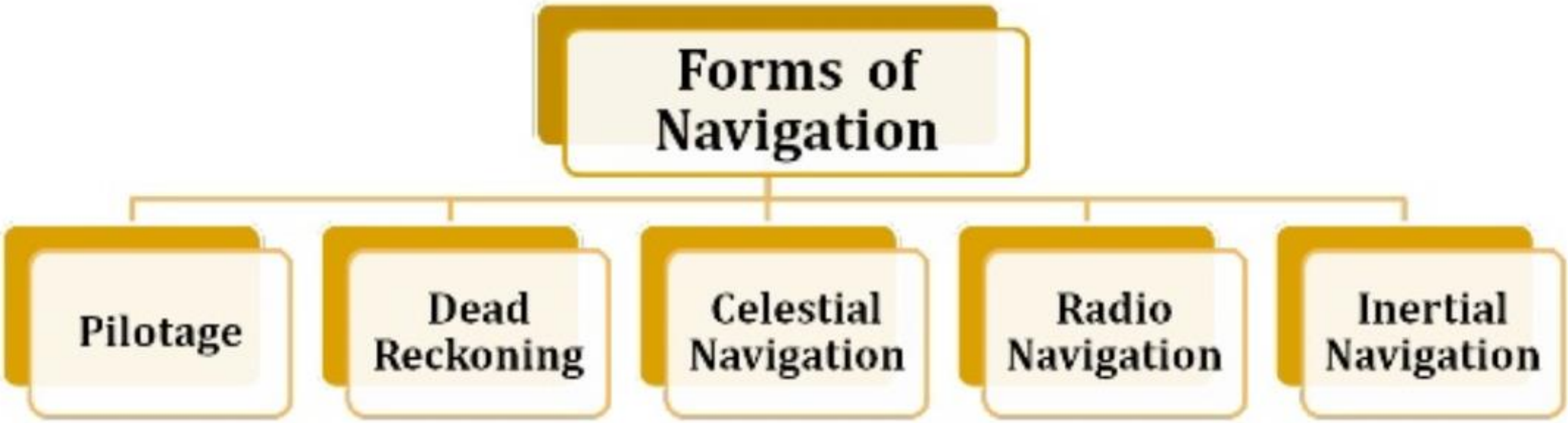
Chapter 5

Positioning by Inertial Navigation System



Basic Form of Navigation



Basic forms of Navigation

1. Pilotage

- Before the advent of satellite navigation pilot had to know how to get from point A to point B, only using limited resource.
- These resources are chart, magnetic compass, clock, landmark below them.
- Even with the advanced navigation system available today pilot are expected to know how to apply pilotage skill.

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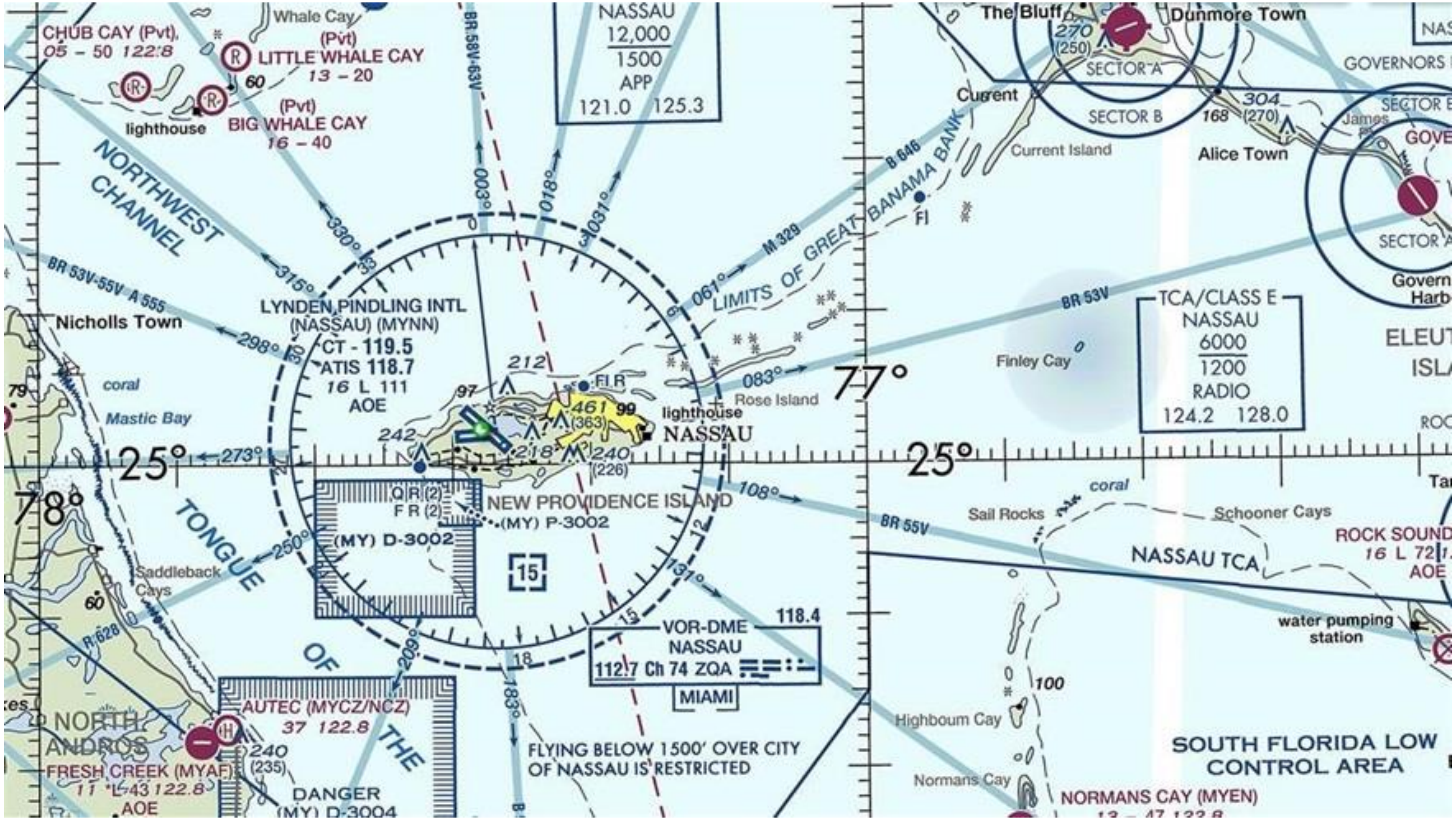

Fig : Aeronautical Chart

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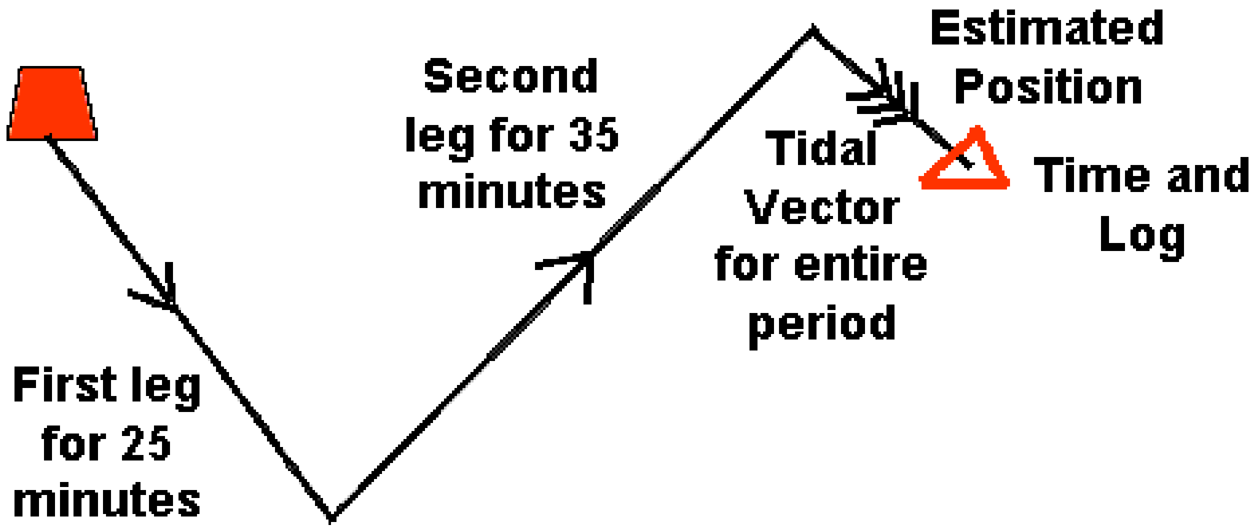
2. Dead reckoning

- It is the process of calculating one's current position by using previously determined position of fix and advancing that position based upon known or estimated speed over elapsed time and course



- Why Dead reckoning

- Sometimes pilot won't be able to clearly see a landmark out the cockpit, this will occur if one fly into clouds, large stretch of water or desert, so by dead reckoning pilot can work out how to reach the next checkpoint.

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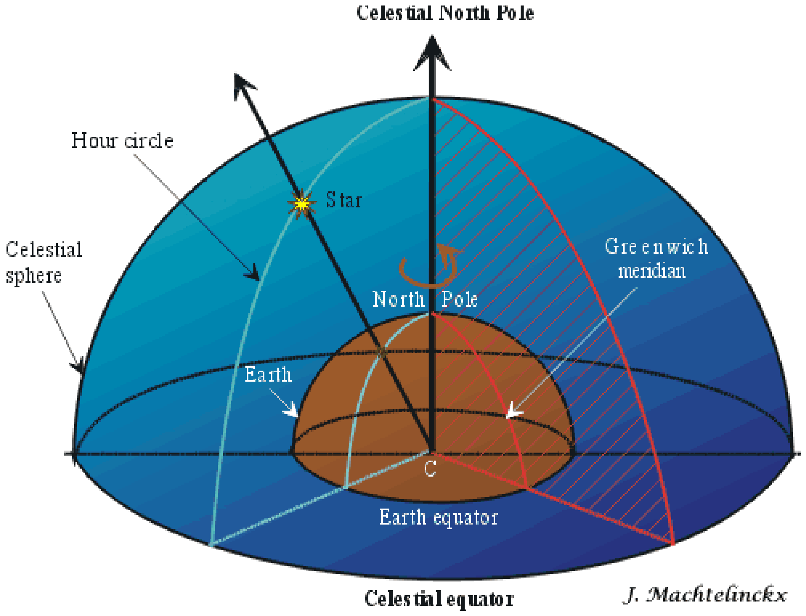
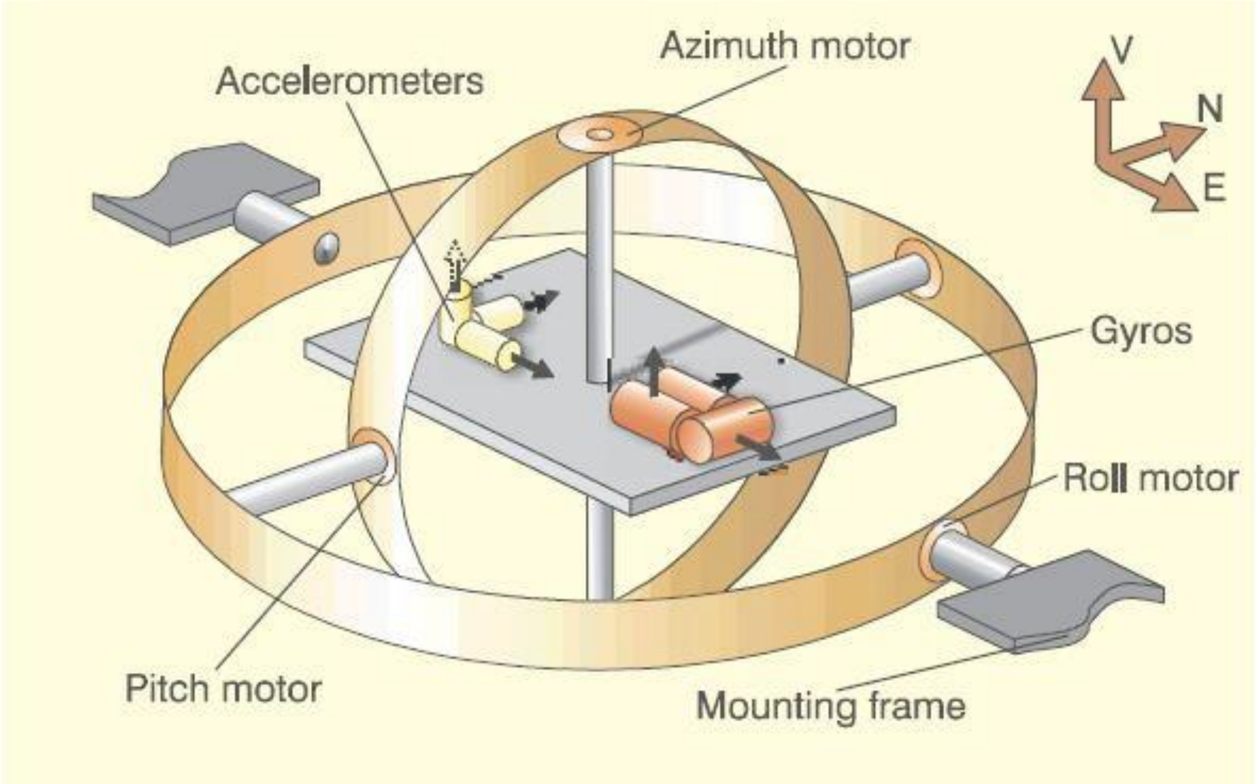
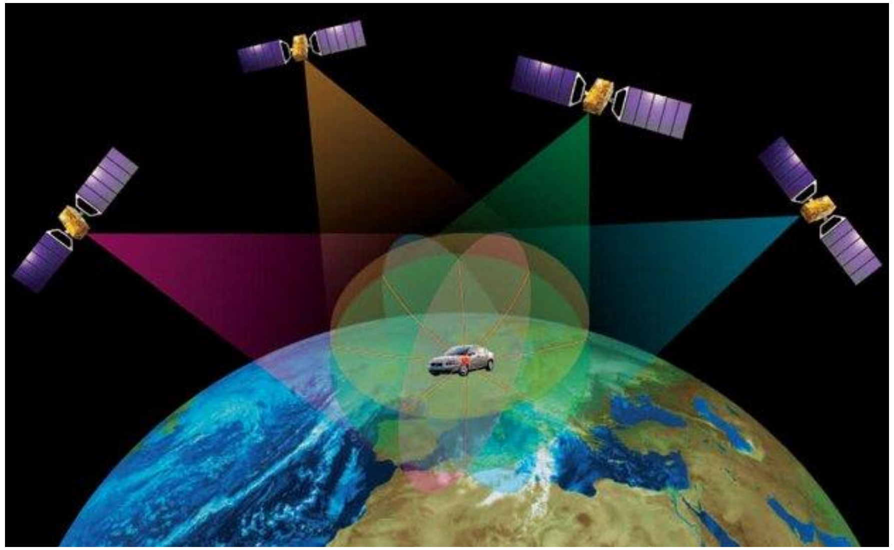


Celestial Navigation



Inertial Navigation



Satellite Navigation

Terminology

- Inertia : is the property of bodies to maintain constant translational and rotational velocity, unless disturbed by forces or torques, respectively (Newton's first law of motion).
- Inertial Reference frame :

  is a coordinate frame in which constant velocity exist i.e. acceleration is zero. Inertial reference frames are neither *rotating* nor *accelerating*.

❖Inertial sensors :

  measure *rotation rate* and *acceleration*, both of which are vector valued variables.

- Gyroscopes :

  are sensors for measuring *rotation*: *rate gyroscopes* measure *rotation rate*, and *integrating gyroscopes* (also called *whole-angle gyroscopes*) measure *rotation angle.*

- Accelerometers :

  are sensors for measuring *acceleration*. However, accelerometers *cannot* measure *gravitational acceleration*. That is, an accelerometer in free fall (or in orbit) has no detectable input.

Inertial navigation

- Inertia is the property of a physical body which maintain its velocity (translational or angular) unless disturbed by an external force.
- Inertial sensors measure the translational and rotational motion
- An accelerometer measures acceleration
- A gyroscope measure the angular velocity
- Manufacturers integrates three-axis accelerometer and three-axis gyroscopes into a single device called Inertial Measurement Unit.

Inertial Navigation System

Introduction to INS

- INS is a navigation aid that uses a computer, motion sensors and rotation sensors.
- The motion sensors such as accelerometers.
- The rotational sensors such as gyroscopes.
- It calculates the position, orientation, and velocity of a moving object.
- Inertial navigation is a self-contained navigation technique.
- The INS is initially provided with its position and velocity from another source such as: GPS satellite receiver, etc.
- It is the only form of navigation that does not depend on external references.
- It is used on vehicles such as ships, aircraft, submarines, guided missiles, and spacecraft.

Contd…

- ***An inertial measurement unit (IMU)* or *inertial reference unit (IRU)*** contains a cluster of sensors: *accelerometers* (three or more, but usually three) and ***gyroscopes*** (three or more, but usually three). These sensors are rigidly mounted to a common base to maintain the same relative orientation.
- Inertial measurement units (IMUs) typically contain three orthogonal rate-gyroscopes and three orthogonal accelerometers, measuring angular velocity and linear acceleration respectively.
- Inertial navigation is a self-contained navigation technique in which measurements provided by accelerometers and gyroscopes are used to track the position and orientation of an object relative to a known starting point, orientation and velocity.

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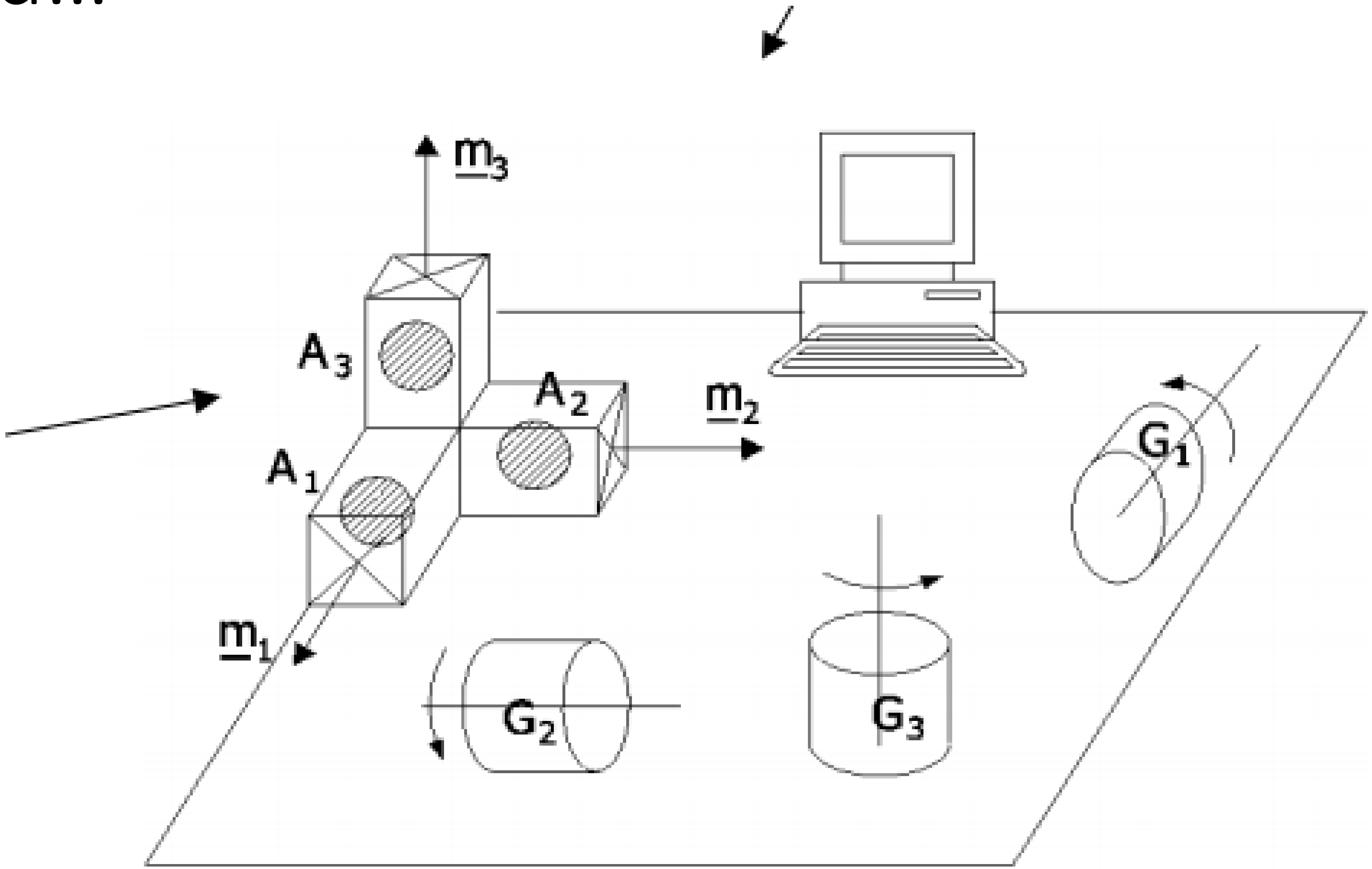
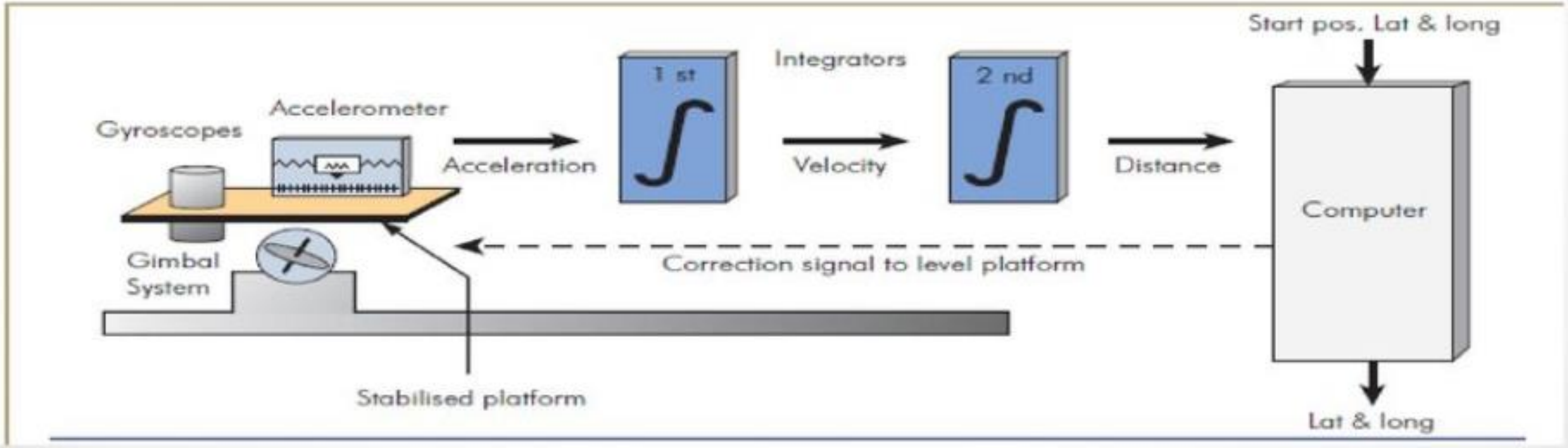


Fig : IMU

Component of INS

Basic components of an INS unit

- A stable platform oriented to maintain the accelerometers horizontal to the Earth and to provide azimuth orientation.
- The accelerometers arranged on the stable platform to supply specific components of acceleration.
- The integrators to receive the output from the accelerometers and to calculate velocity by integrating acceleration once and integrating again to calculate the distance.
- A computer to receive the signals from the integrators and to change to distance travelled into latitude and longitude.



What does an INS consist of?

- An *inertial navigation* uses gyroscopes and accelerometers to maintain an estimate of the position, velocity, and attitude rates of the vehicle in or on which the INS is carried, which could be a land vehicle, aircraft, spacecraft, missile, surface ship, or submarine.
- An INS consists of the following:
  - *An IMU*
  - *Instrument support electronics*
  - *Navigation computers* (one or more) calculate the gravitational

acceleration (not measured by accelerometers) and doubly integrate the net acceleration to maintain an estimate of the position of the host vehicle.

Accelerometers

- The basic principle of accelerometers is to measure the forces acting on a proof mass. (*A proof mass or test mass is a known quantity of mass used in a measuring instrument as a reference for the measurement of an unknown quantity*)
- When the accelerometer experiences an acceleration, the mass is displaced to the point that the spring is able to accelerate the mass at the same rate as the casing. The displacement is then measured to give the acceleration.
- Two types of accelerometers:

  - open loop (e.g. spring based accelerometers) measure the displacement of the proof mass resulting from external forces acting on the sensor.

  - closed loop (e.g. pendulous or electrostatic accelerometers) keep the proof mass in a state of equilibrium by generating a force that is opposite to the applied force.

Cont…

- Consider the 3-axis accelerometer

a_{cc_x} = acc. measurement along x-axis

a_{cc_y} = acc. measurement along y-axis

a_{cc_z} = acc. measurement along z-axis

On Integration,

$\int a_{cc_x} = v_x$

$\int a_{cc_y} = v_y$

$\int a_{cc_z} = v_z$

Again on integration

$\int v_x$ = x(Position along x dir.)

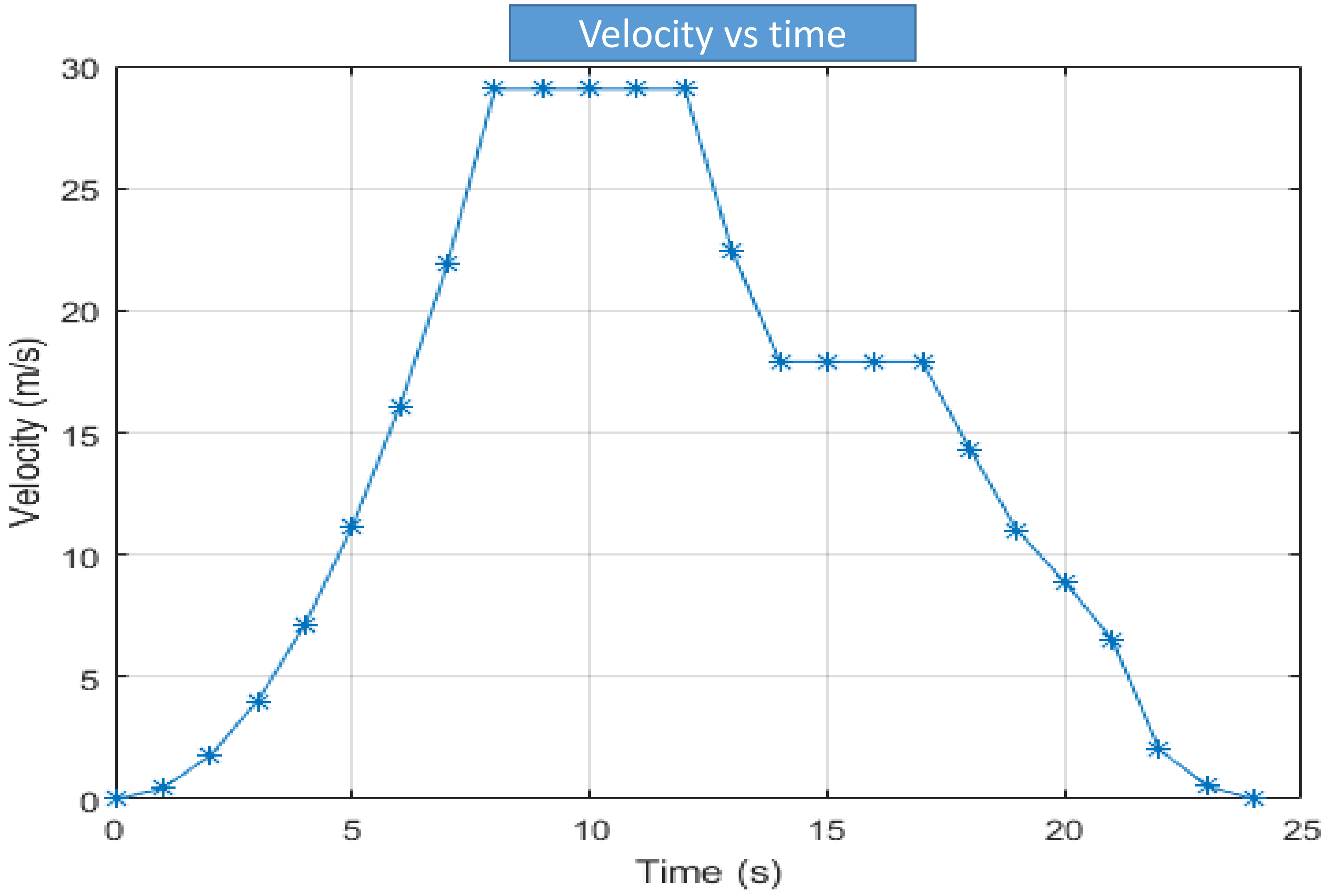
$\int v_y = y$

$\int v_z = z$

Cont…

- Sensors are connected to the Computers 0
- They only take the discrete value.
- Here a complete curve is a true acceleration but the sensor is able to measure the particular points

(Note: we can increase the data by increasing The sampling rate Of the sensor)



Cont…

- So from these discrete data further, calculus is done to find position

$$\frac{dv_x}{dt} = \text{acc}_\text{x}$$

- This derivative form in discrete form can be represented as,

$$\frac{lim}{\Delta t \rightarrow 0} \; \frac{v_x[t+\Delta t]-v[t]}{\Delta t} = \text{acc}_\text{x}$$

where,

Δt = depends on how fast the accelerometer is able to take the sample

$v_x[t + \Delta t]$ = v(t)+ $\text{acc}_{\text{x}*}\, \Delta t \longrightarrow$ it gives the next velocity

Cont…

- Finally let say at any period of time t= k the velocity is given as,

$$\underbrace{v_x(k)}_{\text{Current velocity}}=\underbrace{v_x(k-\Delta t)}_{\text{Previously computed velocity}} + \underbrace{acc_x(k)}_{\text{Current measured acceleration by accelerometer}}*\ \Delta t$$

Cont…

- Similarly for position computation

$$x(k)=x(k-\Delta t) + v_x(k)* \Delta t$$

Current position

Previously computed position

Current velocity

Cont…

- In case of Gyroscope also

  We know that the angular velocity (ω) is the rate of change of angular position with respect to time

$$\frac{d\Theta_x}{dt} = \omega_x$$

$$\frac{d\Theta_y}{dt} = \omega_y$$

$$\frac{d\Theta_z}{dt} = \omega_z$$

Cont…

- In this way knowing the value of

$v_x(k)$

$v_y(k)$ → 3- Velocity

$v_z(k)$

$x(k)$

$y(k)$ → 3- position

$z(k)$

$\theta_x(k)$

$\theta_y(k)$ → 3- angular orientation and these are called

$\theta_z(k)$

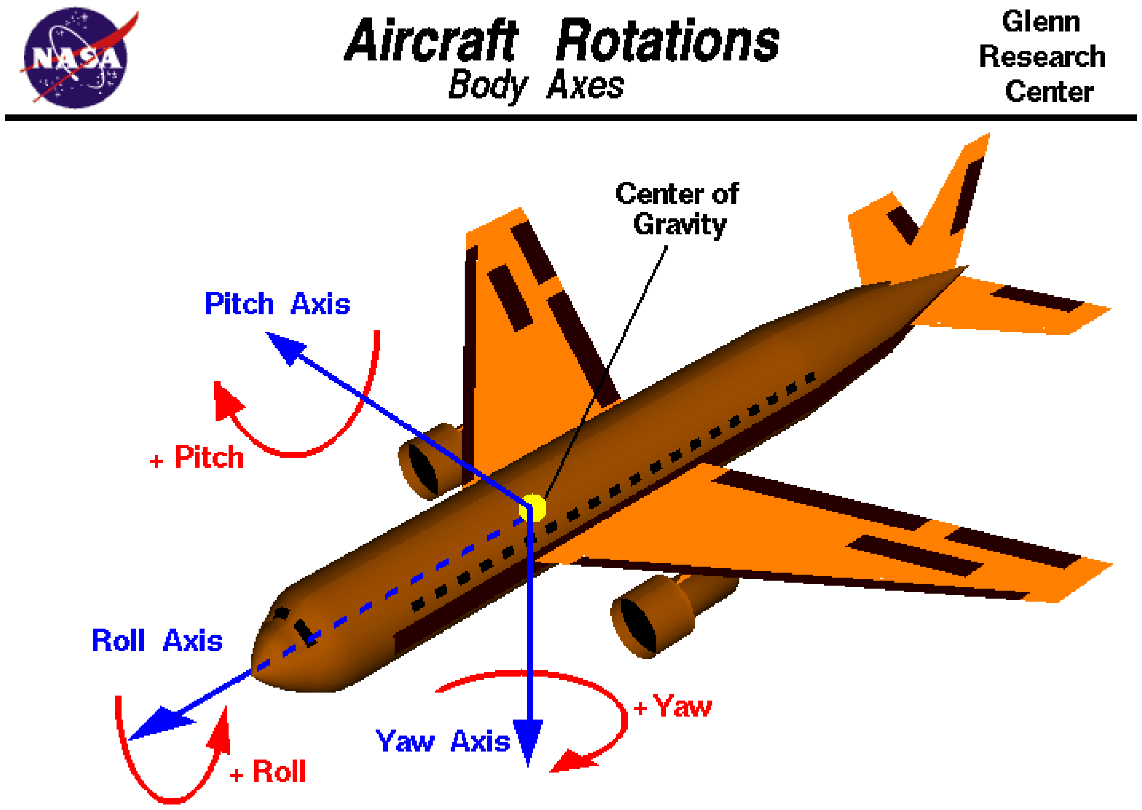
a. Roll

b. Pitch

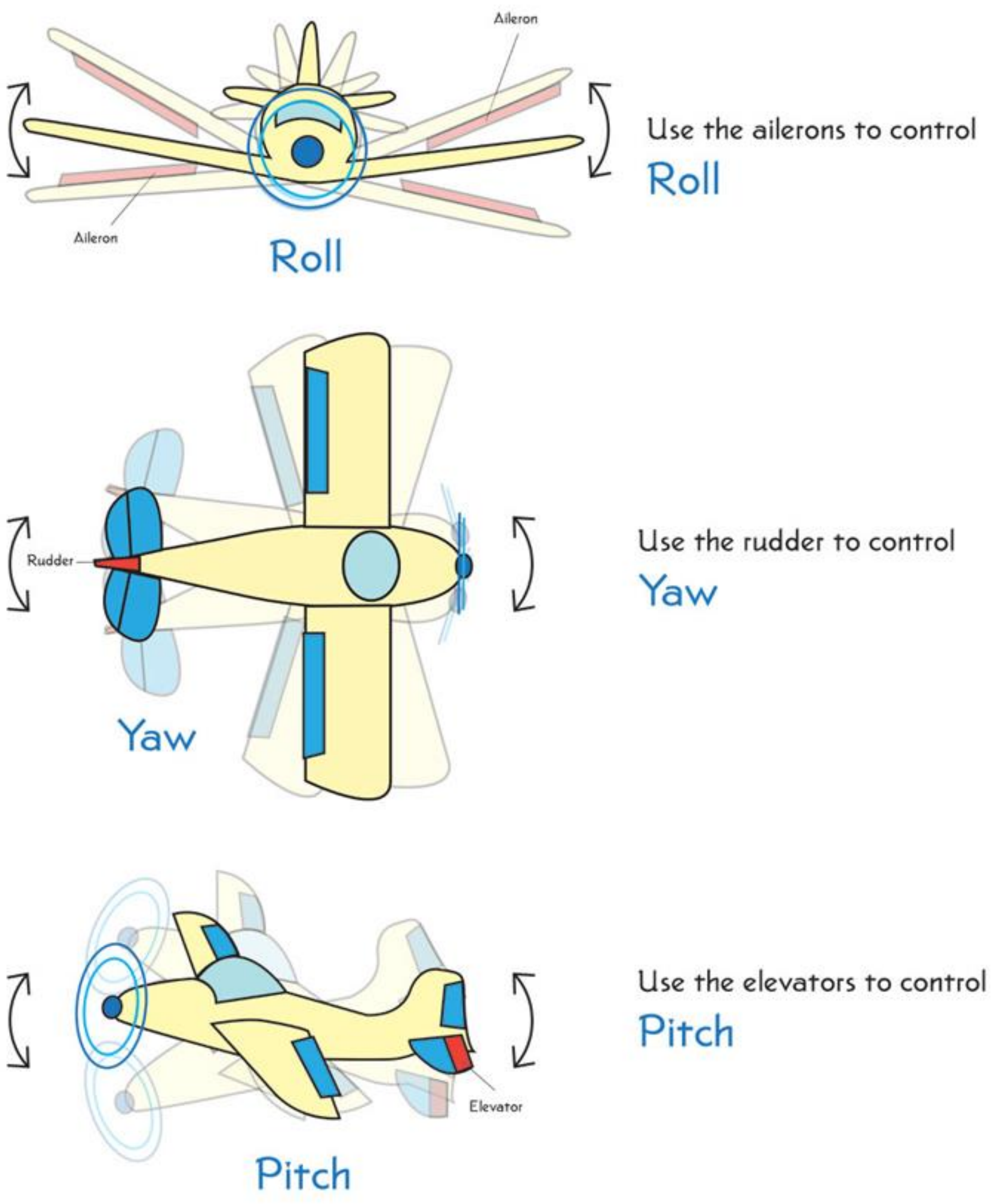
c. Yaw

Attitude

Cont…



Aircraft Rotations
Body Axes
Glenn
Research
Center
Center of
Gravity
Pitch Axis
+ Pitch
Roll Axis
+ Roll
Yaw Axis
+ Yaw



Aileron
Aileron
Roll
Use the ailerons to control
Roll
Rudder
Yaw
Use the rudder to control
Yaw
Elevator
Pitch
Use the elevators to control
Pitch

INS error

- INS ERROR?????
- Error propagate in attitude, position and velocity?????

Attitude

- The attitude of a system is calculated by integrating angular rate (angular velocity) as measured by the gyros (gyroscopes) over a defined time period.

- The equation for a measured angular rate for a single axis can be represented with error sources as follows:

$$w_m = (1+k)\, w_t + b_g + \eta_g$$

where, w_m = measured angular rate

w_t = true angular rate

b_g = time-varying bias

η_g = random Gaussian noise (defined by the angle random walk (ARW) specification)

Contd….

- The error in the attitude calculation is found by integrating the difference between the measured angular rate and the true angular rate

$$\Theta_{err} = \int_0^t (w_m - wt)dt$$

$$\text{or, } \Theta_{err} = \int_0^t ((1+k)\,wt + b_g + \eta_g - w_t)dt$$

$$\text{or, } \quad \Theta_{err} = (\text{ARW})\sqrt{t} + \int_0^t (k\, w_t + bg)dt$$

Q. ARW????

- If a noisy output signal from a sensor is integrated, for example integrating an angular rate signal to determine an angle, the integration will drift over time due to the noise.
- This drift is called random walk, as it will appear that the integration is taking random steps from one sample to the next.
- The two main types of random walk for inertial sensors are referred to as angle random walk (ARW), which is applicable to gyroscopes
- velocity random walk (VRW), which is applicable to accelerometers.
- By multiplying the random walk by the square root of time, the standard deviation of the drift due to noise can be recovered.

Velocity error

- Similar to the measurement equation used to find the angular rate of the gyro, the linear acceleration of the system can be modeled by :

$$a_m = \int_0^t (1+k)a_t + ba + \eta_a + g\sin\Theta_{err}$$

- Assuming attitude error is very small
  - $g\sin\Theta_{err} = g\,\Theta_{err}$

- Now, the velocity error is found by integrating the acceleration and adding that to an initial velocity error at the start of the integration:

$$v_{err} = ver_{r0} + \int_0^t (a_m - at)dt \qquad P_{err} = Pe_{rr0} + \int_0^t V_{err}dt$$

Sensor noise

- Every single measurement by any sensor is corrupted by noise (flaws)
- Since we compute position using these measurement the final computed value also contains flaws

For example

We have for Gyroscope

measured angular velocity = true angular velocity +/* noise

here, *normally noise are additive*

sometimes multiplicative

i.e. $\omega_{meas}(t) = \omega_{true}(t) + \eta\,(t)$

noise parameter

Cont…

- Noise is time dependent variable
- Main factor affecting the noise = constant bias

  For example

  Consider a gyroscope at rest at t= 0 than

  $\theta(0) = 0^\circ$ (initial condition)

  $\theta(1) = \theta(0) + \omega(1) * \Delta t$ *….soon*
- *A*ssume the sampling rate is 1000Hz (i.e at every second 1000 sample *were taken)*

Cont…

- So if the sensor has a constant bias of 0.001 deg/sec

  Initially ,

$$\omega(0)=0, \qquad \Theta(0)=0$$

$$\omega(1) = \omega(0) + bias = 0+0.001=0.001$$

$$\Theta(1) = \Theta(0) + \omega(1)*\Delta t = 0+0.001*1/1000 = 1*10^{-6}$$

$$\omega(2) = \omega(1) + bias = 0.001+0.001=0.002$$

$$\Theta(2) = \Theta(1) + \omega(2)*\Delta t = (1*10^{-6}) + 0.002*1/1000 = 3*10^{-6}$$

Similarly $\Theta(3) = 6*10^{-6}$

$\Theta(4) = 9*10^{-6}$…….so on …..

Note : In this way the constant bias result in measurement other than actual one.

Cont….

- In case of accelerometer

  $a_{meas}(t) = a_{true}(t) + \eta(t)$

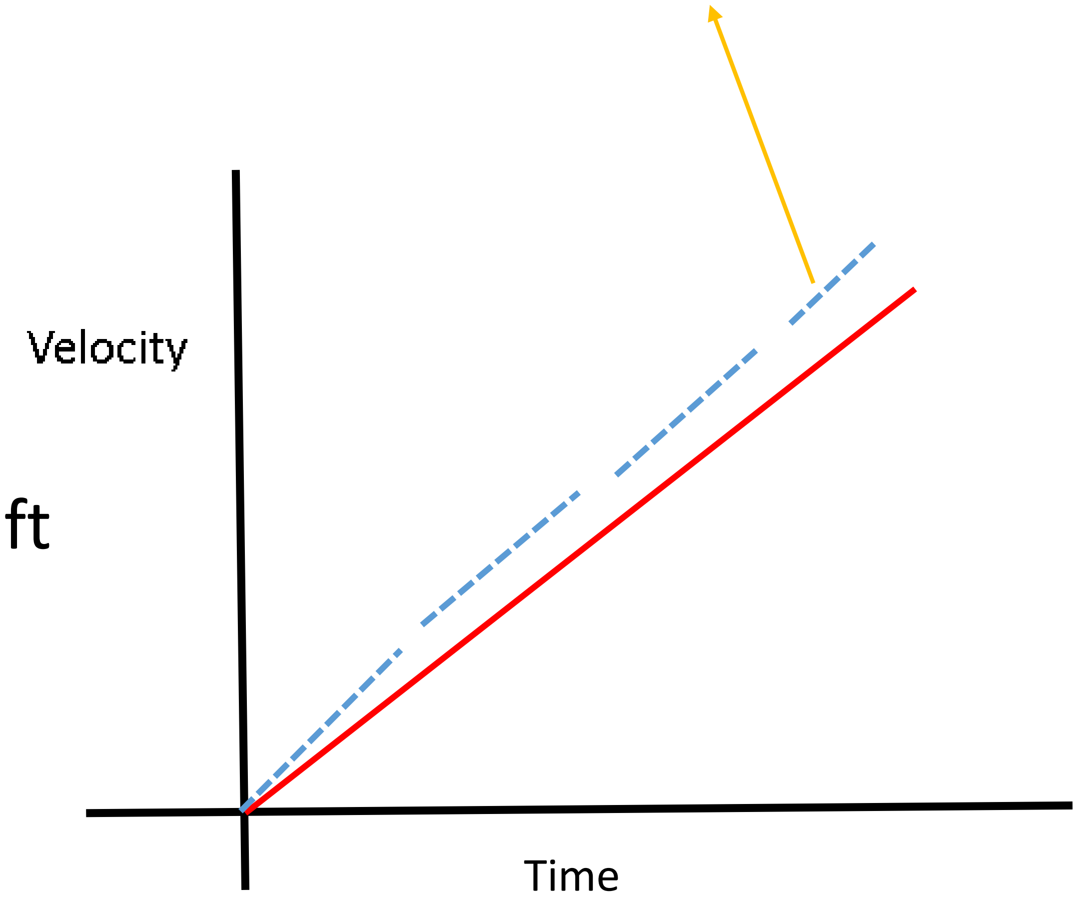
  and

  $\int a_{meas}$ = velocity

  $\int velocity = position$

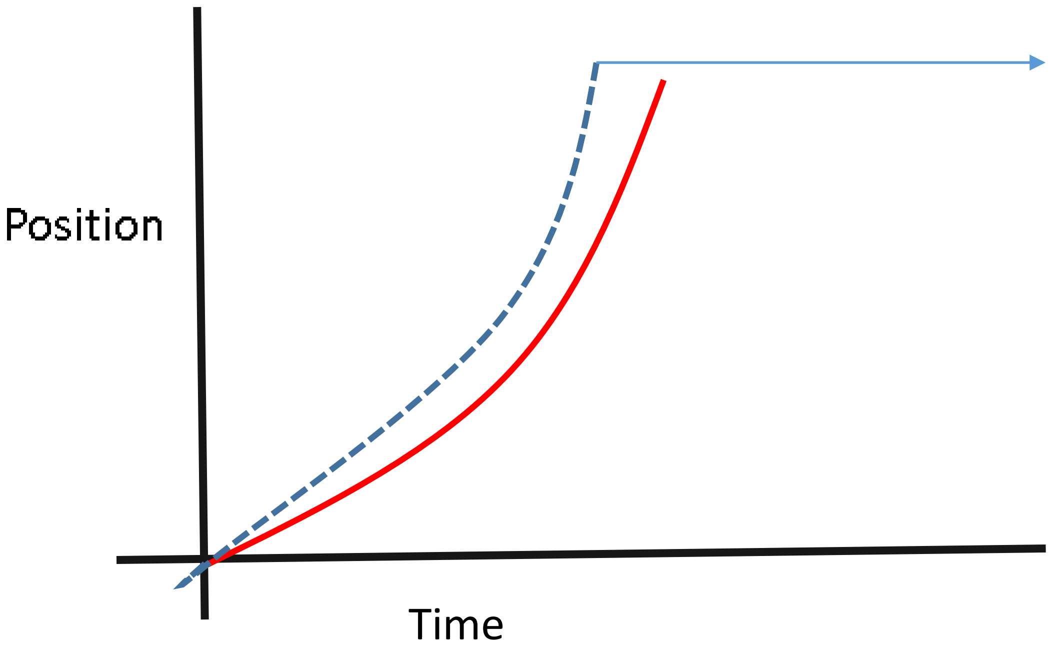
- Single integration result in linear drift

Due to bias measured data will always drift from true one



Cont…

- Double integration results in parabolic drift



Double integration results in a parabolic drift

Cont…

- Beside bias some of the other source error are
- ✓ Scale factor
- ✓ Misalignment
- ✓ Cross-axis response
- ✓ White noise
- ✓ Flicker noise
- ✓ Fixed bias (more dominant)
- ✓ Bias stability

VerctorNAV

Stabilized Platform and Strapdown Technologies

- There are many different designs of INS with different performance characteristics, but they fall generally into two categories:
  - gimbaled or stabilized platform techniques, and
  - strapdown
- The original applications of INS technology used stable platform techniques. In such systems, the inertial sensors are mounted on a stable platform and mechanically isolated from the rotational motion of the vehicle. Platform systems are still in use, particularly for those applications requiring very accurate estimates of navigation data, such as ships and submarines.
- Modern systems have removed most of the mechanical complexity of platform systems by having the sensors attached rigidly, or "strapped down", to the body of the host vehicle. The potential benefits of this approach are lower cost, reduced size, and greater reliability compared with equivalent platform systems. The major disadvantage is a substantial increase in computing complexity.

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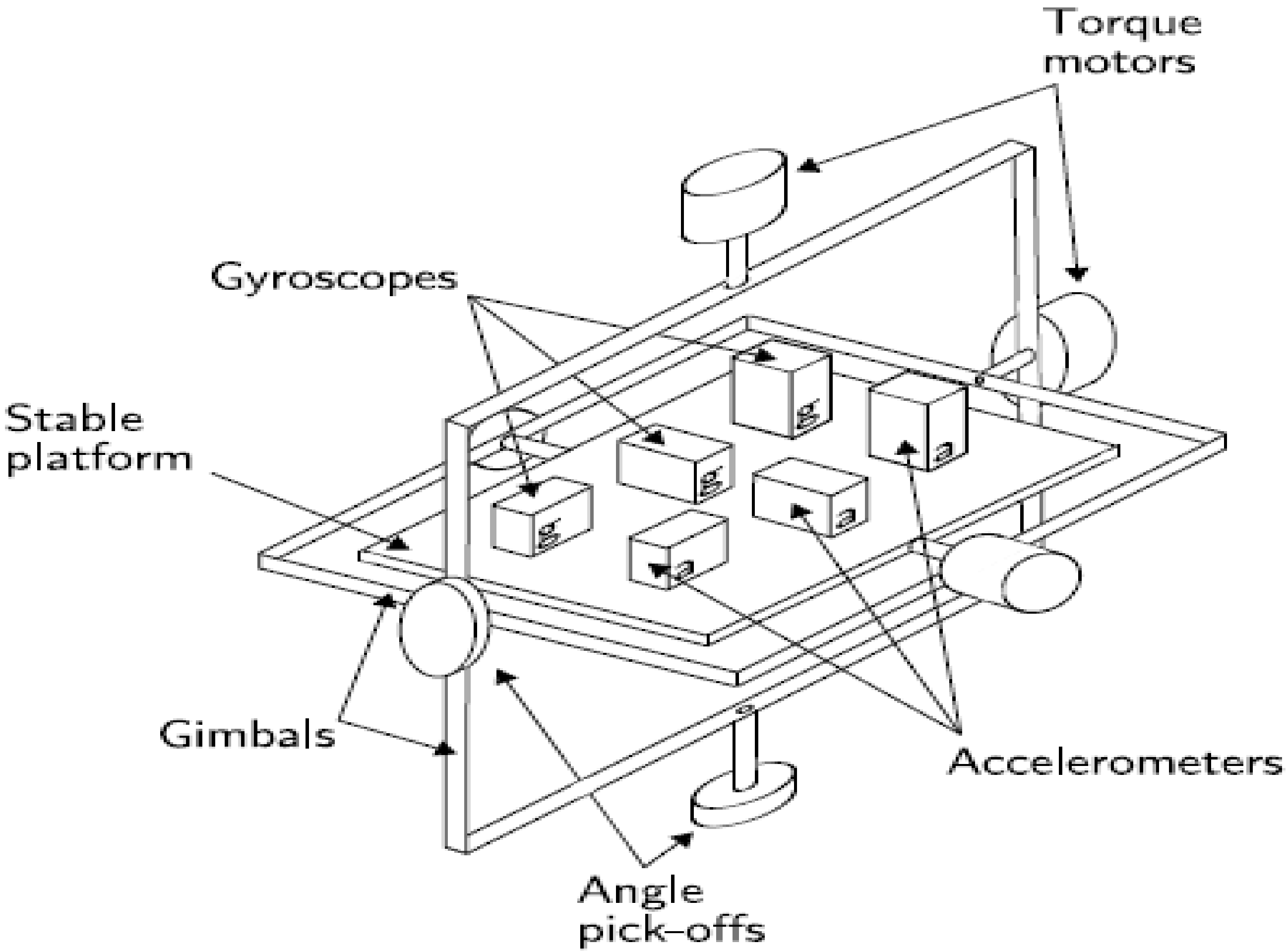
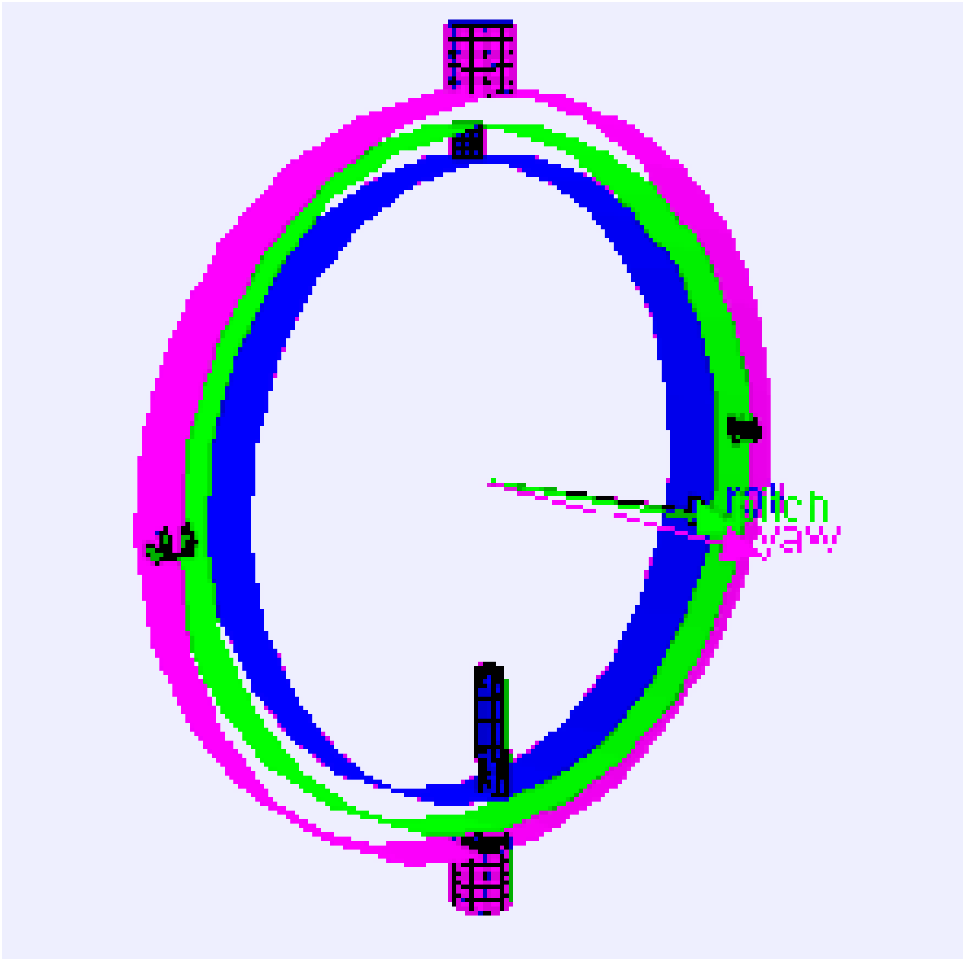


Figure 2: A stable platform IMU.



Gimbled system

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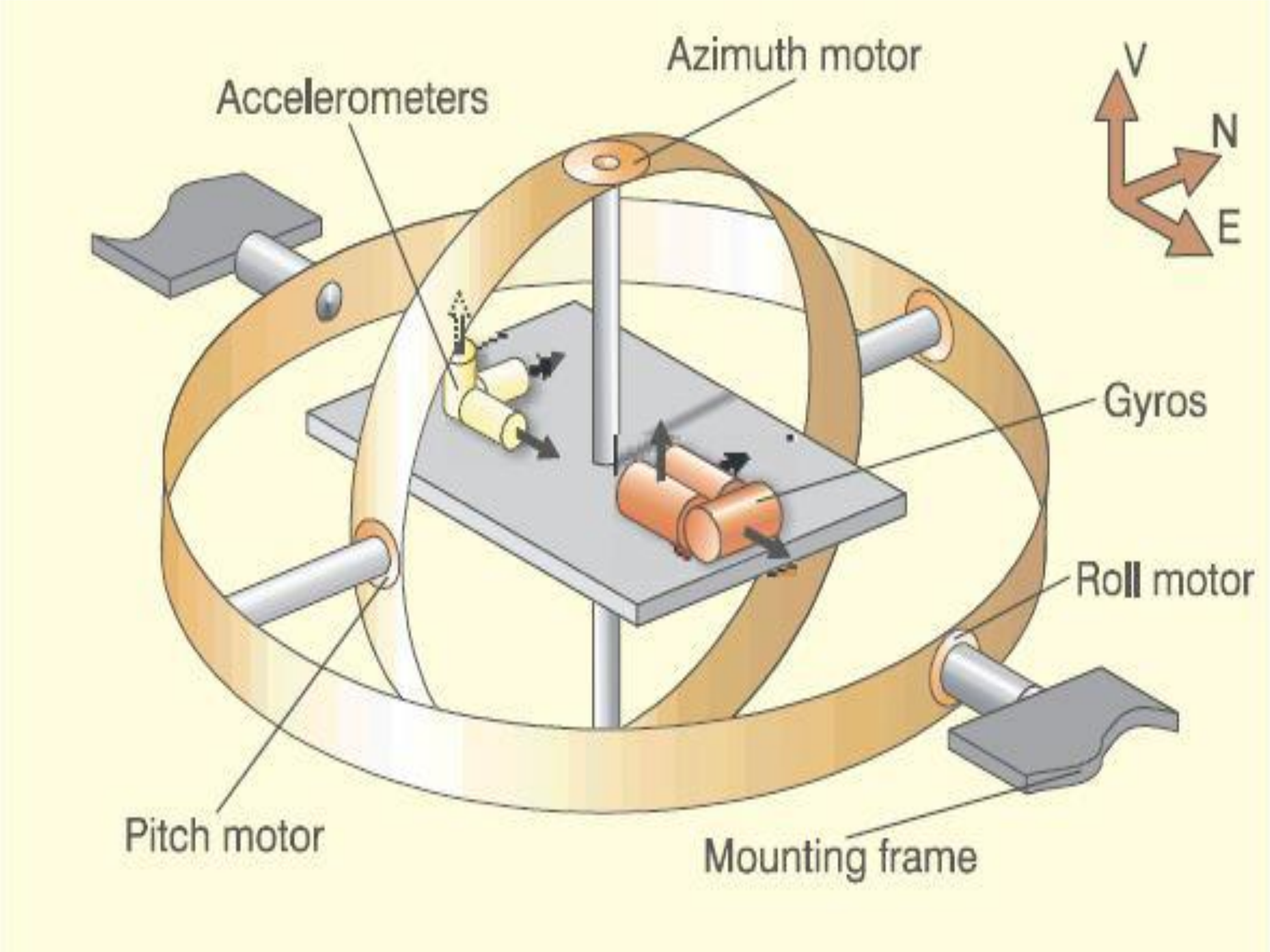
Azimuth motor
V
Accelerometers
N
E
Gyros
Roll motor
Pitch motor
Mounting frame