INTRODUCTION 1

Engineering Economics

- Engineering economics deals with the concepts and techniques of analysis useful in evaluating the worth of systems, products, and services in relation to their costs. **It is used to answer many different questions as:**
  - Which engineering projects are worthwhile?
  - Has the mining or petroleum engineer shown that the mineral or oil deposits is worth developing?
  - Which engineering projects should have a higher priority?
  - Has the industrial engineer shown which factory improvement projects should be funded with the available funds?
  - How should the engineering project be designed?
- Engineering economics is the application of economic techniques for the evaluation and design of engineering alternatives. **The role of engineering economics is to:**
  - Assess the appropriateness of a given project.
  - Estimate its value.
  - Justify it from an engineering point of view.
- Engineering economy involves technical analysis, with emphasis on the economic aspects with an objective of assisting in decision making.
- It involves the systematic evaluation of the economic merits of proposed solutions to engineering problems.
- To be economically acceptable, solutions to engineering problems engineering economics must demonstrate a positive balance of long term benefits over long term cost. **It must also:**
  - Promote the wellbeing and survival of an organization.
  - Embed use of creative and innovative technology and ideas.
  - Permit identification and scrutiny of their estimated outcomes.
  - Translate profitability to the lowest tier through a valid and acceptable measure of merit.

1.1 Origin of Engineering Economy

- **Pioneer:** Arthur M. Wellington, civil engineer latter part of nineteenth century; addressed role of economic analysis in engineering projects; area of his interest was railroad building in USA.
- Arthur M. Wellington founder of engineering economics, by profession a civil engineer, wrote a book the economic theory of location of railways in 1887.
- J.C.L. Fish and O.B. Goldman both evaluated engineering structures from the perspective of actuarial mathematics.
- Eugene L. Grant in 1930, in his book principles of engineering economics discussed the importance of judgment factors and short term investment evaluation as well as conventional comparisons of long term investments in capital goods based on compound interest calculations.
- Eugene L. Grant could be called as the father of engineering economics.

1.2 Principles of Engineering Economy

The principles of engineering economic analysis are:

1. Money has a time value.
2. Make investments that are economically justified.
3. Choose the mutually exclusive investment alternative that maximizes economic worth.
4. Two investment alternatives are equivalent if they have the same economic worth.
5. Marginal revenue must exceed marginal cost.
6. Continue to invest as long as each additional increment of investment yields a return that is greater than the investor's time value of money.
7. Consider only differences in cash flows among investment alternatives.
8. Compare investment alternatives over a common period of time.
9. Risks and returns tend to be positively correlated.
10. Past costs are irrelevant in engineering economic analysis, unless they impact future costs.

Engineering Economic Analysis Procedure

- Problem recognition, formulation, and evaluation.
- Development of the feasible alternatives.
- Development of the cash flows for each alternative.
- Selection of a criterion (or criteria).
- Analysis and comparison of the alternatives.
- Selection of the preferred alternative.
- Performance monitoring and post-evaluation results.

1.3 Role of Engineers in Decision Making

Engineers are decision makers, and decision making is what distinguishes engineers from scientists. The decisions that engineers make are often of very high consequence, to the engineer himself or herself, to the engineer's employer, and to society at large. The decisions that an engineer makes often affect his or her job security, income and opportunity for advancement, they impact the profitability or performance of the engineer's employer, and they often impact the environment and our safety. Thus it makes sense that engineers study the mathematics of decision making in order to become better decision makers.

The term engineering economic decision refers to all investment decisions relating to engineering projects. **The main types of engineering economic decisions are:**

- Service improvement.
- Equipment and process selection.
- Equipment replacement.
- New product introduction and product expansion.
- Cost reduction.

Engineering Design Process

- Problem: need definition.
- Problem: need formulation and evaluation.
- Synthesis of possible solutions (alternatives).
- Analysis, optimization, and evaluation.
- Specification of preferred alternative.
- Communication.

1.4 Cash Flow Diagram (CFD)

The costs and benefits of engineering projects over time are summarized on a CFD. Specifically, CFD illustrates the size, sign, and timing of individual cash flows, and forms the basis for engineering economic analysis.

- Engineering projects generally have economic consequences that occur over an extended period of time.
  - For example, if an expensive piece of machinery is installed in a plant were brought on credit, the simple process of paying for it may take several years.
  - The resulting favorable consequences may last as long as the equipment performs its useful function.
- Each project is described as cash receipts or disbursements (expenses) at different points in time.

Categories of Cash Flows

The expenses and receipts due to engineering projects usually fall into one of the following categories:

- **First cost:** Expense to build or to buy and installation.
- **Operations and maintenance (O&M):** Annual expenses, such as electricity, labor, and minor repairs.
- **Salvage value:** Receipt at project termination for sale or transfer of the equipment (can be a salvage cost).
- **Revenues:** Annual receipts due to sale of products or services.
- **Overhaul:** Major capital expenditure that occurs during the asset's life.

Net Cash Flow

a. **Cash Inflows**
   - Receipts from sale of goods and services.
   - Receipts from sale of physical assets.

b. **Cash Outflows**
   - Expenditure on materials, labour and indirect expenses in manufacturing.
   - Selling and administrative.
   - Inventory and taxes, etc.

Drawing a Cash Flow Diagram (CFD)

- In a CFD the end of period n is the same as the beginning of period (n + 1).
- Beginning of period cash flows are: rent, lease, and insurance payments.
- End of period cash flows are: operation and maintenance, salvages, revenues, overhauls.
- The choice of time 0 is arbitrary. It can be when a project is analyzed, when funding is approved, or when construction begins.
- One person's cash outflow (represented as a negative value) is another person's inflow (represented as a positive value).
- A CFD is created by first drawing a segmented time based horizontal line, divided into appropriate time unit.
- Each time when there is a cash flow, a vertical arrow is added, pointing down for costs and up for revenues or benefits.
- The cost flows are drawn to relative scale.

Example

A man borrowed Rs 1, 00,000 from a bank at 8% interest. Two ends of year payments: at the end of the first year, he will repay half of the Rs 1,00,000 principal plus the interest that is due. At the end of the second year, he will repay the remaining half plus the interest for the second year.

Cash flow for this problem is:

End of Year	Cash Flow
0	+ 1,00,000
1	- 58,000 (- 50,000 - 8,000)
2	- 54,000 (- 50,000 - 4,000)

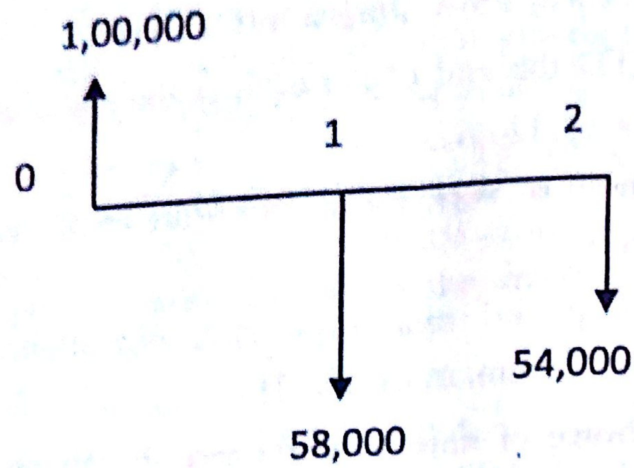


Fig.1.1: *Cash Flow Diagram.*

Questions

1. Engineers play the important role in making the economic decisions. Do you agree with this statement? Discuss.
2. Explain the role of engineers in economic decision analysis.
3. Define engineering economy. Enlist the principle of engineering economy.
4. Describe about the principles of Engineering Economics.
5. Describe uses of engineering economic for engineers.
6. Engineering economics is all about decision making. Explain.
7. What is engineering economics? Why do you think studying this course is important for engineering students?
8. What are the important uses of engineering economics?
9. Define engineering economics? Why it is important to study economics ? Enlist the principles of engineering economics.
10. What are the principles of engineering economics? How does it help to decision making process? Discuss.
11. Write short notes
    a. Origin of engineering economics.
    b. Cash flow diagram.
    c. Role of engineers in decision making.
    d. Principles of engineering economics.

INTEREST AND TIME VALUE OF MONEY

2

2.1 Introduction of Time Value of Money

- Regardless of the value of inflation, money has a time value due to its "earning power".
- Suppose you arrive in a city by airplane and need a car.
  - You can buy a car.
  - You can rent a car.
- Suppose you arrive in a city and need a place to stay.
  - You can buy a house or condominium.
  - You can rent a house, apartment, or hotel room.

Steps to Solve Time Value of Money Problems

- Read problem thoroughly.
- Create a time line.
- Put cash flows and arrows on time line.
- Determine if it is a PV or FV problem.
- Determine if solution involves annuity.
- Solve the problem.

Money Has Value

- Money can be leased or rented.
- The payment is called interest.
- If you put Rs 1,000 in a bank at 9% interest for one year you will receive back your original Rs1,000 plus Rs 90.

2.2 Simple Interest

Simple interest is charged only on the principal amount. Simple interest is determined by multiplying the interest rate by the principal amount and by the number of periods.

Simple Interest = PiN

Where, P = principal amount.

i = interest rate.

N = duration of the loan.

Example 2.1

Suppose Rs1,000 were invested on January 1, 2015 at 10% simple interest rate for 5 years. Calculate the total simple interest on the amount.

Solution: Given,

Principle	P = Rs 1,000
Interest Rate	i = 10% per year
Time	N = 5 years

Simple Interest (I_s) = P × i × N = 1,000 × 0.1 × 5 = Rs 500 **Ans.**

2.3 Compound Interest

Compound interest is charged on the principal plus any interest accrued till the point of time at which interest is being calculated. **In other words, compound interest system works as follows:**

- Interest for the first period charged on principle amount.
- For the second period, it's charged on the sum of principle amount and interest charged during the first period.
- For the third period, it is charged on the sum of principle amount and interest charged during first and second period, and so on.

It can be proved mathematically, that the interest calculated as per above procedure is given by the following formula:

Compound Interest (I_c) = $P(1+i)^n - P$

Where, P = present sum of money.

i = interest rate.

n = number of periods (years).

Example 2.2

Suppose Rs1,000 were invested on January 1, 2015 at 10% compound interest rate for 5 years. Calculate the total compound interest on the amount.

Solution: Given,

Principle (P) = Rs 1,000

Interest Rate (i) = 10% per year

No of Periods (n) = 5

Compound Interest $(I_c) = 1{,}000\,(1 + 0.1)^5 - 1{,}000$

$= 1{,}000 \times 1.15 - 1{,}000$

$= 1{,}000 \times 1.61051 - 1{,}000$

$= 1{,}610.51 - 1{,}000$

= Rs 610.51 **Ans.**

2.3.1 Nominal Interest Rate

The nominal interest rate is the rate of interest that is reported on loan documents and investment accounts that are not adjusted for inflation. Thus nominal interest rate (r) is an interest rate that does not include any consideration of compounding.

r = interest rate per period × number of periods

A nominal rate may be stated for any period: 1 year, 6 months, weekly, daily.

r = 1.5% per month × 12 months = 18%

Considering 2% per month, all the following are same:

- 2% per month ×12 months = 24% per year.
- 2% per month × 24 months = 48% per 2 years.
- 2% per month × 6 months = 12% semiannually.
- 2% per month × 3 months = 6% quarterly.
- 2% per month × 0.231 months = 0.462 weekly.
- 2% per month × 1/365 months = 0 .005479 daily.

Example 2.3

Given the interest rate18% per year, compounded monthly. Find nominal interest rate per

a. Month

b. 2 month

c. 6 months

d. 2 years

a. Monthly

$$\frac{i}{\text{month}} = \frac{18}{12} = 1.5\%$$

b. **2 months**

$\frac{r}{2}$ Months = 1.5 × 2 = 3%

c. **6 months**

$\frac{r}{6}$ months = 1.5 × 6 = 9%

d. **2 years**

$\frac{r}{2}$ Years = 1.5 × 24 = 36%

2.3.2 Effective Interest Rate

The Effective Annual Rate (EAR) is the annual interest rate that would produce the same answer, with annual compounding, as is obtained with more frequent compounding. **It can be obtained by:**

$$i_{eff.} = (1 + \frac{r}{m})^m - 1$$

Where, r = nominal interest rate per year.

m = number of compounding periods per year.

i = effective interest rate per compounding period (CP) = $\frac{r}{m}$.

$i_{eff.}$ = effective interest rate per year.

Example 2.4

A bank quotes a mortgage rate of 8% (the stated annual rate) but will compute monthly loan payments using standard time value formulas. This implies monthly compounding. What is the effective annual interest rate on the loan?

Solution: Given,

r = 8%, m = 12

$$i_a = (1 + \frac{r}{m})^m - 1 = (1 + \frac{0.08}{12})^{12} - 1 = 0.083 = 8.3\%$$

Therefore, the effective annual interest rate on loan = 8.30% **Ans.**

Example 2.5

A bank quotes a rate of 8% (the stated annual rate), but will compound interest quarterly.

a. *What is the effective annual interest rate on the loan?*
b. *What is the effective semi-annual interest rate?*

Solution: Given,

$r = 8\%$, $m = 4$

a. **For Annual Interest Rate**

$$i_a = (1 + \frac{r}{m})^m - 1 = (1 + \frac{0.08}{4})^4 - 1 = 0.0824$$

Therefore, the effective annual interest rate on loan = 8.24% **Ans.**

b. **For Effective Semi-annual Interest Rate**

For semi-annual, $r = \frac{8}{2} = 4\%$

$$I_{semi\text{-}annual} = (1+\frac{r}{m})^m - 1 = (1+\frac{0.04}{4})^4 - 1 = 0.0404$$

Therefore, the effective semi-annual interest rate on loan = 4.04% **Ans**

Effective Interest Rates for Any Time Period

The payment period (PP) is the frequency of payment or receipts. To evaluate cash flows that occur more frequently than annually, PP < 1 year, the effective interest rate over the PP must be used in the engineering economy relations. **The effective interest rate is given by:**

$$i_{eff} = (1+\frac{r}{m})^m - 1$$

Examples 2.6

Given nominal and effective rates of interest 16% and 16.986 % respectively, what is the compounding period?

Solution :

$$i_{eff} = (1+\frac{r}{m})^m - 1$$

$$16.986 = (1+\frac{0.16}{m})^m - 1$$

Trial and error, m = 4 yields 16.986.

Therefore, compounding period = m = 4 years **Ans.**

Example 2.7

An effective rate of 6.707% per semi-annual period, compounded weekly. What is the equivalent weekly interest rate?

Solution:

m = 26 which is semi-annual, 26 weeks in ½ year.

$$i_{eff.} = (1+\frac{r}{m})^{m} - 1$$

$$0.06707 = (1+\frac{r}{26})^{26} - 1$$

$$\frac{r}{6 \text{ months}} = 6.5\%$$

$$\frac{i}{\text{week}} = \frac{6.5}{26} = 0.25$$

$$\frac{i}{\text{week}} = 0.25\% \text{ per week}$$

The equivalent weekly interest rate= 0.25% **Ans.**

2.3.3 Continuous Compounding

The continuous compounding formula is used to determine the interest earned on an account that is constantly compounded, essentially leading to an infinite amount of compounding periods. Discount factors for continuous compounding are different from those for discrete compounding. The discounting factors can be calculated directly from the nominal interest rate, r, and number of years, n, without having to find the effective interest rate per period.

a. Continuously Compounding and Discrete Cash Flows

It assumes that the cash flows occur at discrete intervals eg. once per year but interest rates are compounded continuously throughout the interval.

Let us consider r be the nominal interest rate per year is compounded m times per year. The effective interest rate will be

$$i = \lim_{m \to \infty} \left\{ \left(1+\frac{r}{m}\right)^{m} - 1 \right\}$$

$$= e^{r} - 1$$

We know,

$F= P(1+i)^{n}$..............................(1)

On substituting the value of i in equation (1)

$F= P(1+e^{r}-1)^{n} = Pe^{r.n}$

Where,

e^{rn} is the continuously compounded amount factor and is denoted by (F/P, i%, n).

$$i_{eff.} = (1+ \frac{r}{m})^{m} - 1$$

$$0.06707 = (1+ \frac{r}{26})^{26} - 1$$

$$\frac{r}{6 \text{ months}} = 6.5\%$$

$$\frac{i}{\text{week}} = \frac{6.5}{26} = 0.25$$

$$\frac{i}{\text{week}} = 0.25\% \text{ per week}$$

The equivalent weekly interest rate= 0.25% **Ans.**

2.3.3 Continuous Compounding

The continuous compounding formula is used to determine the interest earned on an account that is constantly compounded, essentially leading to an infinite amount of compounding periods. Discount factors for continuous compounding are different from those for discrete compounding. The discounting factors can be calculated directly from the nominal interest rate, r, and number of years, n, without having to find the effective interest rate per period.

a. Continuously Compounding and Discrete Cash Flows

It assumes that the cash flows occur at discrete intervals eg. once per year but interest rates are compounded continuously throughout the interval.

Let us consider r be the nominal interest rate per year is compounded m times per year. The effective interest rate will be

$$i = \lim_{m \to \infty} \left\{ \left(1+\frac{r}{m}\right)^{m} - 1 \right\}$$

$$= e^{r} - 1$$

We know,

$F = P(1+i)^{n}$..................................(1)

On substituting the value of i in equation (1)

$F = P(1+e^{r}-1)^{n} = Pe^{r.n}$

Where,

e^{rn} is the continuously compounded amount factor and is denoted by (F/P, i%, n).

Table 2.1 **Interest Factors and Symbols**

Factor Name	Converts	Symbol	Formula
Continuous Compounding Amount	To F given P	(F / P, r%, n)	e^{rn}
Continuous Compounding Present Worth	To P given F	(P/F, r%, n)	e^{-rn}
Continuous Compounding Compound Amount	To F given A	(F/A, r%, n)	$\frac{e^{rn}-1}{e^{r}-1}$
Continuously Compounding Present Worth	To P given A	(P/A, r%, n)	$\frac{e^{rn}-1}{e^{rn}(e^{r}-1)}$
Continuous Compounding Sinking Fund	To A given F	(A/ F, r%, n)	$\frac{e^{r}-1}{e^{rn}-1}$
Continuous Compounding Capital Recovery	To A given P	(A/P, r%, n)	$\frac{e^{rn}(e^{r}-1)}{e^{rn}-1}$

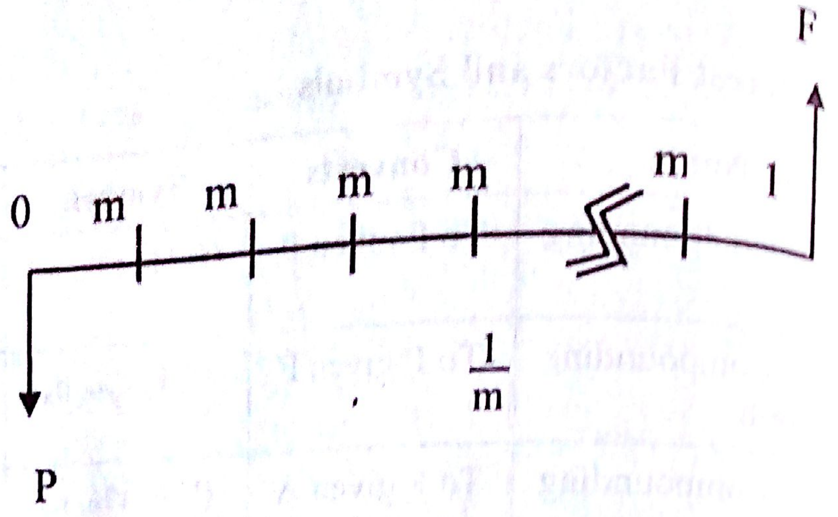
Note:

- *The effective annual interest rate determined on a daily compounding basis will not be significantly different than if continuous compounding is assumed.*
- *To Find P or F equivalent we should multiply factor formula by e^{r} if annuity starts from the beginning of each period.*

b. Continuously Compounding and Continuous Cash Flow

- It is a series of cash flows occurring at infinitely short interval of time.
- This formula could apply to companies having receipts and expenses that occur frequently during each working day. In such case interest is compounded continuously.

Let us consider nominal interest rate per year = r and m be the number of payments per year.



We have,

$$F = A\left\{\frac{(1+i)^n - 1}{i}\right\}$$

For 1 year $n = m$, $i = \frac{r}{m}$ and $A = \frac{1}{m}$

$$F = \frac{1}{m}\left\{\frac{\left(1+\frac{r}{m}\right)^m - 1}{\frac{r}{m}}\right\} = \frac{\left(1+\frac{r}{m}\right)^m - 1}{r}$$

Again,

$$F = P\left(1+\frac{r}{m}\right)^m$$

Or, $$\frac{\left(1+\frac{r}{m}\right)^m - 1}{r} = P\left(1+\frac{r}{m}\right)^m = \frac{\left(1+\frac{r}{m}\right)^m - 1}{r\left(1+\frac{r}{m}\right)m}$$

As, $(m \to \infty)$ $\left(1+\frac{r}{m}\right)^m = e^r$

The present equivalent of continuous compounding for one year i

$$P = \frac{e^r - 1}{r\,e^r}$$

$$(P/A, i\%, n) = \frac{e^{rn} - 1}{re^{rn}}$$

Where, A is the cash flow uniformly and continuously over one year.

Table 2.2 **Interest Factors and Symbols for Continuously Compounding and Continuous Cash Flow.**

Factor Name	Converts	Symbol	Formula
Continuous Compounding Amount	To F given A	(F / A, r%, n)	$\frac{e^{rn}-1}{r}$
Continuous Compounding Present Worth	To P given A	(P/A, r%, n)	$\frac{e^{rn}-1}{re^{rn}}$
Continuous Compounding Sinking Fund	To A given F	(A/ F, r%, n)	$\frac{r}{e^{rn}-1}$
Continuous Compounding Capital Recovery	To A given P	(A/P, r%, n)	$\frac{re^{rn}}{e^{rn}-1}$

Example 2.8

If you invest Rs1,00,000 at an annual interest rate of 5% compounded continuously, calculate the final amount you will have in the account after five years.

Solution: Given,

P = Rs 1,00,000, r = 5%, n = 5 years, F =?

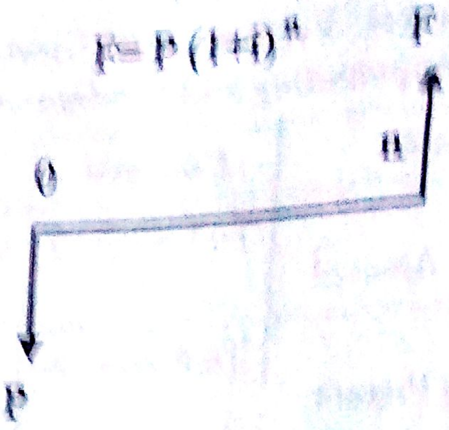
$F = Pe^{r.n} = 1{,}00{,}000\ e^{(0.05\times5)} =$ Rs 1,28,402.54 **Ans.**

2.4 Economic Equivalence

- Economic equivalence exists between cash flows that have the same economic effect and could therefore be traded for one another.
- Even though the amounts and timing of the cash flows may differ, the appropriate interest rate makes them equal in economic sense.

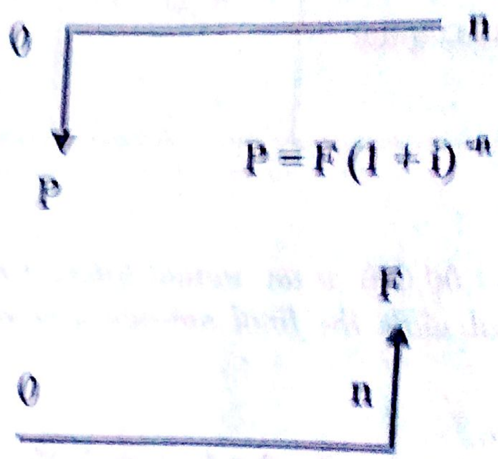
Equivalence from Personal Financing Point of View

If you deposit P dollars today for n periods at i, you will have F dollars at the end of period n.



Alternate Way of Defining Equivalence

- F dollars at the end of period n is equal to a single sum P dollars now, if your earning power is measured in terms of interest rate i.

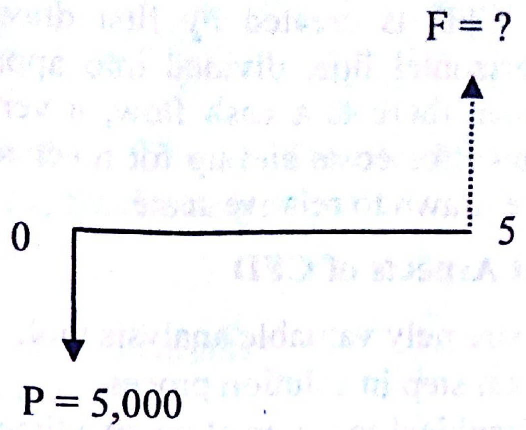


General Principles of Equivalent Calculation

1. Equivalent calculations made to compare alternatives require a common time basis.
2. Equivalence depends on interest rate. Change in interest rate destroys equivalence between the sum values of alternatives.
3. Equivalence calculations require the conversion of multiple payment cash flows to a single cash flow.

Example 2.9

1. *If you deposit Rs 5,000 today in a savings account that pays an 8% interest annually, how much would you have at the end of 5 years?*
2. *At an 8% interest, what is the equivalent worth of Rs 5,000 now in 5 years?*



$$F = P(1+i)^n = 5{,}000(1+0.08)^5 = \text{Rs } 7{,}346.64$$

1. We would have Rs 7,346.64 at the end of 5 years.
2. At an interest rate 8% Rs 5,000 received now is equivalent to Rs 7,346.64 in 5 years.

2.5 Development of Interest Formulas

2.5.1 The Five Types of Cash Flows

Cash Flow Diagram

- Used to describe any investment opportunity.
- Typical investment.

Symbols and Cash Flow Diagrams

- Engineering projects generally have economic consequences that occur over an extended period of time.
- Each project is described as cash receipts or disbursements (expenses) at different points in time.
- For any practical engineering economy problems, the cash flows must be:
  - o Known with certainty.
  - o Estimated.
  - o Range of possible realistic values.
  - o Generated from assumed distribution and simulation.

Cash Flow Diagrams (CFD)

- The costs and benefits of engineering projects over time are summarized on a cash flow diagram (CFD). Specifically, CFD illustrates the size, sign, and timing of individual cash flows, and forms the basis for engineering economic analysis.

- A CFD is created by first drawing a segmented time-based horizontal line, divided into appropriate time unit. Each time when there is a cash flow, a vertical arrow is added pointing down for costs and up for revenues or benefits. The cash flows are drawn to relative scale.

Important Aspects of CFD

- Extremely valuable analysis tool.
- First step in solution process.
- Graphical representation on a time scale.
- Does not have to be drawn to exact scale.
- Information in one glance.

Cash Flow Diagrams

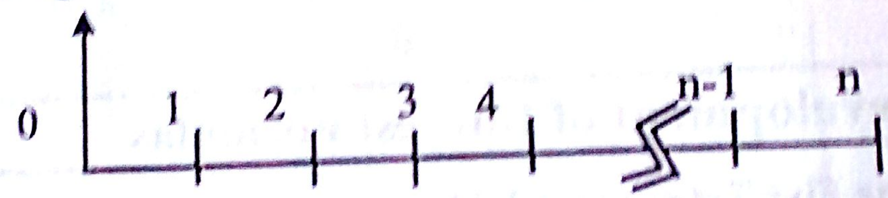


Fig. 2.1: *P-Pattern.*

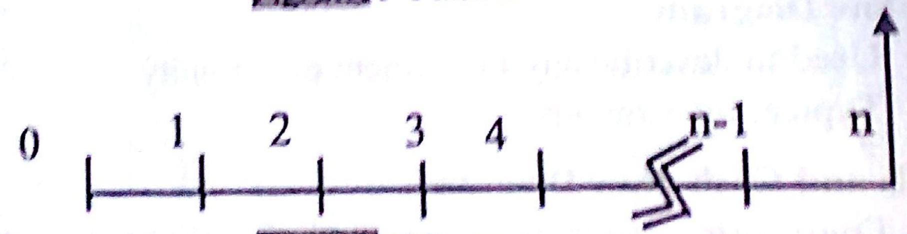


Fig. 2.2: *F-Pattern.*

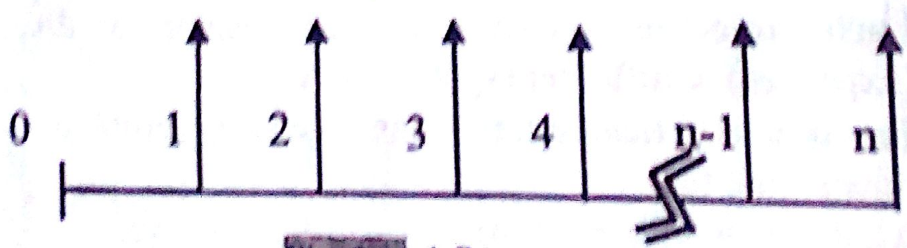


Fig. 2.3: *A-Pattern.*

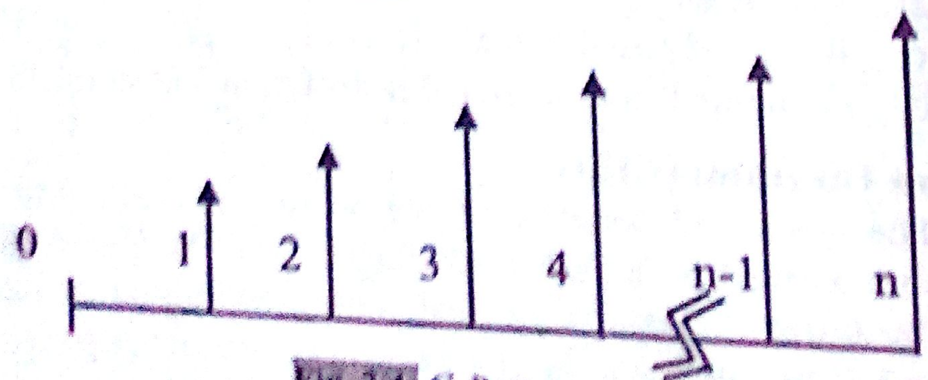


Fig. 2.4: *G-Pattern.*

The Principle of Cash Flow

Principle 1

More Money IN than OUT = Cash flow positive. But high surplus of cash should be avoided in non-interest bearing account.
Inflows = money received from :

- Customers.
- Local and national government grants.
- Sale of property or equipment.
- Loans.

Principle 2

More Money OUT than IN = Cash flow negative. It means shortage of cash to pay bills. **Outflows** = Money spent by the business on:

- Wages and salaries for staff.
- Raw materials or stock.
- Gas, electricity, water and telephone.
- Rent and business rates.
- Interest on loans.
- VAT.
- Equipment purchases.

AIM is to have a positive cash flow or at least a balance.

2.5.2 Single Cash Flow Formulas

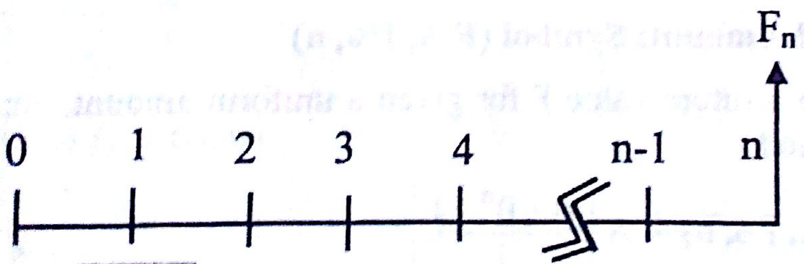


Fig.2.5: *Single Cash Flow Diagram.*

The accumulated value of a present sum invested at a given interest rate after some time can be expressed as:

Future Worth: Symbol (F/P, i%, n)

Converts to a future value F given a present value P.
$F = P(1+i)^n$ .... (1)

Present Worth: Symbol (P/F, i%, n)

Converts to a present value P given a future value F.
$P = F(1+i)^{-n}$.................... (2)

Where, F = accumulated value in the future.
P = principal or present sum invested.
i = interest rate per period.
n = number of interest periods.

The factor $(1 + i)^n$ is known as the single payment compound amount factor.

2.5.3 Uneven Payment Series

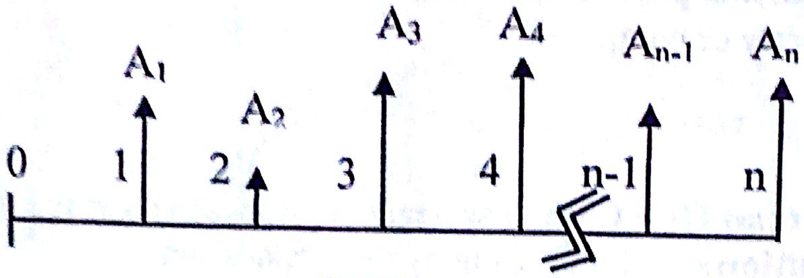


Fig.2.6: *Uneven Payment Series.*

2.5.4 Uniform Series/ Equal Payment Series

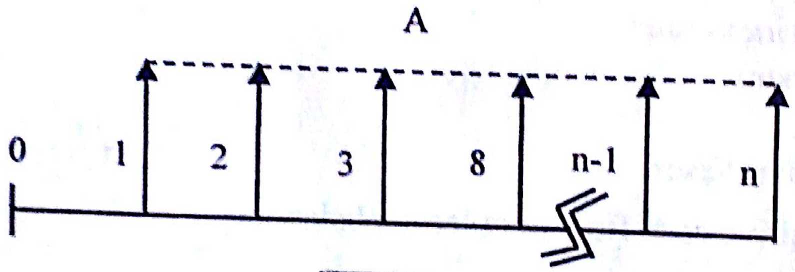


Fig. 2.7: *Equal Payment Series.*

Compound Amount: Symbol (F/A, i%, n)

Converts to a future value F for given a uniform amount, annuity A per interest period.

$$F = A\,(F/A,\, i\%,\, n) = A\left\{\frac{(1+i)^n - 1}{i}\right\}$$

Where, F = future value.
A = uniform amount per period.
i = interest rate.
n = numbers of periods.

2.5.5 Linear Gradient Series

- An arithmetical gradient is cash flow that either increase or decrease by a constant amount.
- The amount increase or decrease is the gradient (G).

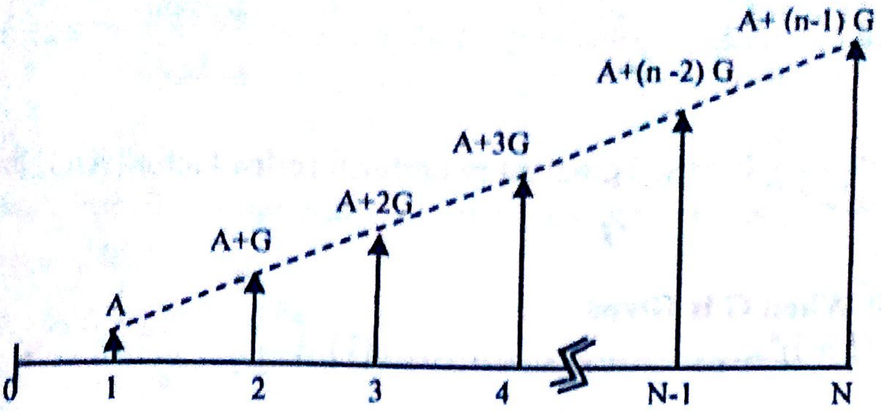


Fig. 2.8: *Increasing Linear Gradient Series.*

1. Find F When G is Given

The future worth of the arithmetic sequence of cash flow as show in Fig. 2.7 is:

F= G (F/A i% n-1) + G (F/A i% n-2) +....................+ G (F/A i% 2) + G (F/A i% 1) + G (F/A i% 0)

$$F = G\left\{\frac{(1+i)^{n-1}-1}{i}+\frac{(1+i)^{n-2}-1}{i}+\ldots\frac{(1+i)^{2}-1}{i}+\frac{(1+i)^{1}-1}{i}+\frac{(1+i)^{0}-1}{i}\right\}$$

$$F = \frac{G}{i}\{(1+i)^{n-1}+(1+i)^{n-2}+\ldots+(1+i)^{2}+(1+i)^{1}+(1+i)^{0}\}-\frac{nG}{i}$$

$$=\frac{G}{i}\{(1+i)^{n-1}+(1+i)^{n-2}+\ldots+(1+i)^{2}+(1+i)^{1}+(1+i)^{0}+1\}-\frac{nG}{i}$$

$$=\frac{G}{i}\left\{\frac{(1+i)^{n}-1}{i}\right\}-\frac{nG}{i}$$

2. Find A When G is Given

$$F = A\left\{\frac{(1+i)^{n}-1}{i}\right\}\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$F = \left[\frac{G}{i}\left\{\frac{(1+i)^{n}-1}{i}\right\}-\frac{nG}{i}\right]$$

On substituting the value of F in equation (1)

$$\left[\frac{G}{i}\left\{\frac{(1+i)^{n}-1}{i}\right\}-\frac{nG}{i}\right] = A\left\{\frac{(1+i)^{n}-1}{i}\right\}$$

$$A = G\left\{\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right\}$$

$\left\{\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right\}$ is called gradient to uniform series factor (A/G, i%, n).

3. Find P When G is Given

$$F = P\,(1+i)^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$F = \left[\frac{G}{i}\left\{\frac{(1+i)^n - 1}{i}\right\} - \frac{nG}{i}\right]$$

On substituting the value of F in equation (1)

$$F = \left[\frac{G}{i}\left\{\frac{(1+i)^n - 1}{i}\right\} - \frac{nG}{i}\right] = P\,(1+i)^n$$

$$P = G\left[\frac{1}{i}\left\{\frac{(1+i)^n - 1}{i\,(1+i)^n}\right\} - \frac{n}{i\,(1+i)^n}\right] = \left\{\frac{(1+i)^n - 1 - ni}{i^2\,(1+i)^n}\right\}$$

The term $\left\{\frac{(1+i)^n - 1 - ni}{i^2\,(1+i)^n}\right\}$ is called gradient to present worth conversion factor and is denoted by (P/G, i%, n).

2.5.6 Geometric Gradient Series

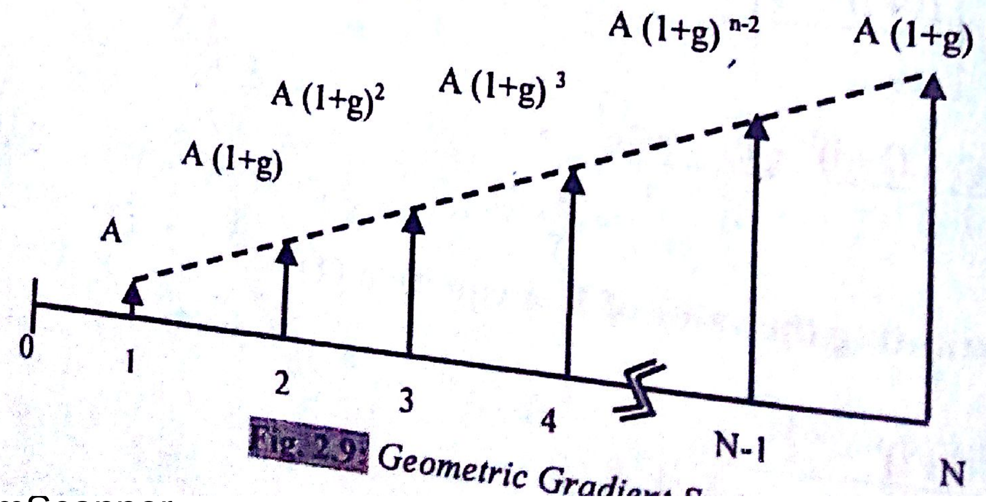


Fig. 2.9: Geometric Gradien...

The magnitude of n^{th}, A_n payment for geometric gradient increasing series is expressed as:

$$A_n = A(1+g)^{n-1}$$

The equation for calculating the present worth of a geometric gradient when $g \neq i$ is:

$$P = \frac{A\left\{1-(1+g)(1+i)^{-n}\right\}}{i-g}$$

Where, P = present worth of all cash flow between 1 and n.
A = cash flow in period 1.
g = rate of change per period.
i = effective interest rate per period.

For a decreasing gradient, change the sign in front of both g's in the present worth equation. When g = i, the present worth of a geometric gradient series is:

$$P = \frac{An}{1+i}$$

Example 2.10

An amount of Rs1,000 is invested at an interest rate 10% per year for 10 years. The accumulated amount can be calculated as:
$F = 1{,}000\,(1+0.1)^{10}$ = Rs 2,594 **Ans.**

Example 2.11

A sum of 1,000 is paid after 10 years. The discount rate is 10% per year. The present value of the future payment can be calculated as:

$$P = \frac{1{,}000}{(1+0.1)^{10}} = \text{Rs } 386 \textbf{ Ans.}$$

Sinking Fund: Symbol (A/F, i%, n)

Converts to a uniform amount, annuity A per interest period for a given specific future value F.

$$A = F\left\{\frac{i}{(1+i)^n-1}\right\}$$

Present Worth: Symbol (P/A, i%, n)

Converts a uniform amount, annuity A per interest period to a present value P.

$$P = A\left\{\frac{(1+i)^n - 1}{i(1+i)^n}\right\}$$

Where, P = present value.

A = amount per interest period.

i = discount rate.

n = discount periods.

Capital Recovery: Symbol (A/P, i%, n)

Converts a present value P to a uniform amount, annuity A per interest period.

$$A = P\left\{\frac{i(1+i)^n}{(1+i)^n - 1}\right\}$$

Present Worth (PW)

$$PW = \frac{F_0}{(1+i)^0} + \frac{F_1}{(1+i)^1} + \frac{F_2}{(1+i)^2} + \ldots + \frac{F_n}{(1+i)^n} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$NPW = \frac{F_0}{(1+i)^0} + \frac{F_1}{(1+i)^1} + \frac{F_2}{(1+i)^2} + \ldots + \frac{F_n}{(1+i)^n}$$

Where, NPW = present worth or value.

F = future cash flow.

i = discount rate.

$(1+i)^n$ is known as the compound amount factor.

Example 2.12

The present worth by investing Rs 1,000 today receiving Rs250 every year in 5 years at an interest or discount rate of 10%, can be calculated as:

$$NPW = -\frac{1,000}{(1+0.1)^0} + \frac{250}{(1+0.1)^1} + \frac{250}{(1+0.1)^2} + \frac{250}{(1+0.1)^3} + \frac{250}{(1+0.1^4} + \frac{250}{(1+0.1)^5}$$

$$= -\text{Rs } 52.3$$

Since, PV is negative and the investment should be avoided **Ans.**

Table 2.3 Equivalence of Cash Flow Patterns.

Factor Name	Converts	Symbol	Formula
Single Payment	To F given P	(F/P, i%, n)	$(1+i)^n$
Single Payment Present Worth	To P given F	(P/F, i%, n)	$(1+i)^{-n}$
Uniform series sinking fund	To A given F	(A/F, i%, n)	$\frac{i}{(1+i)^n-1}$
Capital Recovery	To A given P	(A/P, i%, n)	$\frac{i(1+i)^n}{(1+i)^n-1}$
Uniform Series Compound Worth	To F given A	(F/A, i%, n)	$\frac{(1+i)^n-1}{i}$
Uniform Series Present Worth	To P given A	(P/A, i%, n)	$\frac{(1+i)^n-1}{i(1+i)^n}$
Uniform Gradient Present Worth	To P given G	(P/G, i%, n)	$\left\{\frac{(1+i)^n-1-ni}{i^2(1+i)^n}\right\}$
Uniform Gradient Future Worth	To F given G	(F/G, i%, n)	$\frac{(1+i)^n-1}{i^2}-\frac{n}{i}$
Uniform gradient Uniform Series	To A given G	(A/G, i%, n)	$\frac{1}{i}-\frac{n}{i(1+i)^n-1}$

Additional Solved Examples

Example 2.1

If you deposit Rs 2,000 per month for two years, what will be the amount at the end of five years if bank interest rate is 3% in every six month?

TU-2072

Solution: Given,

Monthly deposit (A) = Rs 2,000

Semi-annual interest rate (i) = 3%

Compounding period (m) = 2

n = 5 years

F_5=?

Here, yearly interest rate $(i_a) = \{1 + i_{semi-annual}\}^m - 1$

$$= (1 + 0.03)^2 - 1$$
$$= 0.0609$$
$$= 6.09\%$$

Monthly interest rate $(i_m) = (1+i_a)^{\frac{1}{12}} - 1$

$$= (1+0.0609)^{\frac{1}{12}} - 1$$
$$= 0.494\%$$

$F_2 = A\ (F/A, 0.494\%, 24)$

$$= 2{,}000 \left\{ \frac{(1+0.00494)^{24} - 1}{0.00494} \right\}$$

$= \text{Rs } 50{,}828.28$

Future worth at the end of 5 year $= F_5 = 50{,}828.28 \left(\frac{F}{P}, 6.09, 3\right)$

$$= 50{,}828.28\ (1.194)$$

$= \text{Rs } 60{,}691.62$ **Ans.**

Example 2.2

Evaluate FW at the end of 10 years with 8% interest rate compound continuously of a cash flow of Rs 500 at the beginning of each year of first 5 years.

TU-2070

Solution: Given,

Cash flow at the beginning of each year = Rs 500

n = 5 years

r = 8%

$F_{10} = ?$

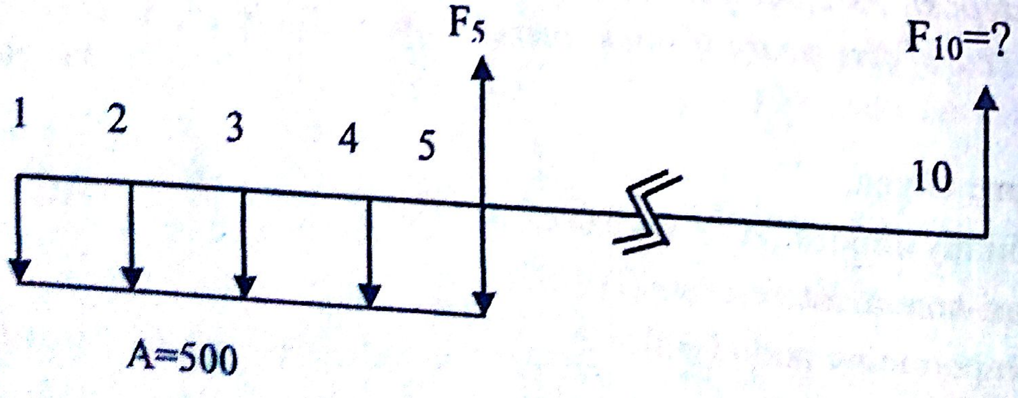


Fig. 2.10: *Cash Flow Diagram.*

Here annuity starts from the beginning of each year so we should multiply the continuous compounding amount factor formula by e^r.

$F = A\,(F/A, r\%, n) \times e^r = 500\,(F/A, 8\%, 5) \times e^r$

$$= 500 \left\{ \frac{(e^{rn} - 1)}{(e^r - 1)} \right\} \times e^r = 500 \left\{ \frac{e^{0.08\times5} - 1}{e^{0.08} - 1} \right\} \times e^{0.08}$$

$= \text{Rs } 3,198.49$

Future Worth at the end of 10 year $= F_{10} = Pe^{rn}$

$= 3,198.47\, e^{0.08\times5}$

$= 4,771.55$ **Ans.**

Example 2.3

How much rupees should you deposit now in a bank account that gives 8% interest per year if you wish to draw Rs 10,000 per month for 10 years. **TU-2070**

Solution: Given,

$P_{10} = ?,\ i = 8\%\ .\ n = 10$ years

Monthly interest rate $(i_m) = (1+i)^{1/m} - 1 = (1+0.08)^{1/12} - 1 = 0.64\%$

$P = A\,(P/A, 0.64\%, 120)$

$$= 10,000 \left\{ \frac{(1+i)^n - 1}{i\,(1+i)^n} \right\} = 10,000 \left\{ \frac{(1+0.0064)^{120} - 1}{0.0064\,(1+0.0064)^{120}} \right\}$$

$= \text{Rs } 8,35,817.6$ **Ans.**

Example 2.4

A man aged 40 years now had borrowed Rs 5,00,000 from bank for his further studies at the age of 20 years. Interest was charged at 11% per year compounded quarterly. He wished to pay loan in semiannual equal installments with the first installment beginning 5 years after receiving the loan. He has just cleared the loan now. What amount did he pay in each installment?

Solution: Given,

$P = 5,00,000$

$A = ?$

Quarterly interest rate $(i_q) = \dfrac{11}{4} = 2.75\%$

Semiannual interest rate $(i_{semi}) = (1+i_q)^n - 1 = (1+0.0275)^2 - 1 = 5.57\%$

We have,

$F = P\,(F/P, 5.57\%, 40) = 5,00,000\,(1.0557)^{40} = 43,71,101.25$

Again,

$$F = A\,(F/A,\ 5.57\%,\ 30) = A\left\{\frac{(1+0.0557)^{30}-1}{0.0557}\right\} = 73.3235 \times A$$

Substituting the value of F in equation 1

$73.3235 \times A = 43{,}71{,}101.25$

Therefore, A = Rs 59,613.93 **Ans.**

Example 2.5

What will be the amount at the end of 10 years if you deposit Rs 5,000 per month for five years continuously if nominal interest rate is 10% compounded quarterly?

Solution: Given

For Effective Interest Rate

$r = 10\%,\ m = 4$

$$i_{eff} = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.1}{4}\right)^4 - 1 = 10.38\%$$

For Monthly Interest Rate

$m = 12$

$$i_m = (1+i_{eff})^{\frac{1}{m}} - 1 = (1+0.1038)^{\frac{1}{12}} - 1 = 0.826\%$$

Here, amount deposited in 5 years = 5 × 12 = 60 months ie = n = 60

FW = 5,000 (F/A, 0.2826%, 60)

$$= 5{,}000\ \frac{\left\{(1+0.00826)^{60}-1\right\}}{0.00826} = \text{Rs } 3{,}86{,}285$$

The future amount in 5 years will be principal for next five years

FW = 3,86,285 (F/P, 10.38%, 5) = Rs 6,32,936 **Ans.**

Example 2.6

Mr. X receives a loan of Rs1,20,000 from a bank at an interest rate of 12% per year. He wishes to repay the load in monthly installment with Rs 3,000 per month. How many installments are necessary to complete his payments?

Solution: Given,

$P = 1{,}20{,}000,\ i_a = 12\%,\ A = 3{,}000$

$i_a = (1 + i_m)^m - 1$

$i_m = (1 + i_a)^{1/m} - 1 = (1 + 0.12)^{1/12} - 1 = 0.95\%$

$P = A\ (P/A,\ 0.95\%,\ n)$

$$\text{Or, } 1{,}20{,}000 = 3{,}000 \left\{\frac{(1+0.0095)^n - 1}{0.0095 \times (1+0.0095)^n}\right\}$$

Or, $0.38 \times 1.0095^n = 1.0095^n - 1$

Or, $1.0095^n = 1.613$

Therefore, $n = 50.56$ Months **Ans.**

Example 2.7

Mr. Ramesh wants to have Rs 10,00,000 for the studies of his daughter after the period of 15 years. How much rupees does he has to deposit each year for 10 continuous years in a saving account that earns 8% interest annually.

Solution:

Discounting the amount Rs 10,00,000 to the year 10

$F = 10{,}00{,}000$

$i = 8\%$

$P = F\ (P/F,\ 8\%\ 5) = 10{,}00{,}000\ (0.6806) = 6{,}80{,}600$

Again,

$A = F\ (A/F,\ 8\%,\ 10) = 6{,}80{,}600(0.0690) = 46{,}961.4$ **Ans.**

Example 2.8

How many deposits of Rs 25,000 each should Dr. Thakur make each month so that the final accumulated amount will be Rs 10,00,000 in the bank interest rate is 12% per year?

Solution: Given,

$A = 25{,}000$ per month

$F = 10{,}00{,}000$

$i = 12\%$ per year

Calculation of Monthly Interest Rate (i_m)

$i = (1 + i_m)^m - 1$

$$\text{Or, } i_m = (1+i)^{\frac{1}{m}} - 1 = (1+0.12)^{\frac{1}{12}} - 1 = 0.94\%$$

Again,

$F = A\ (F/A,\ 0.94\%,\ N)$

$$\text{Or, } 10{,}00{,}000 = 25{,}000 \left\{\frac{(1+i)^n - 1}{i}\right\} = 25{,}000 \left\{\frac{(1+0.0094)^n - 1}{0.0094}\right\}$$

Or, $1.376 = 1.0094^n$

Or, $\log 1.376 = n\log 1.0094 = 34$ Deposits **Ans.**

Example 2.9

Calculate the future worth of the following cash flows deposited at 8% compounded continuously for five years.

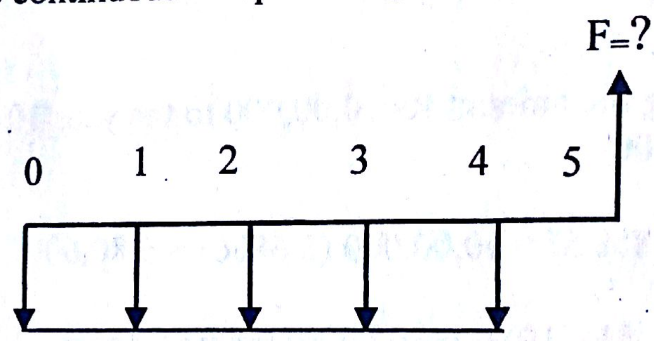
i. Rs 50,000 at the beginning of each year.

ii. Rs 50,000 at the end of each year.

Solution:

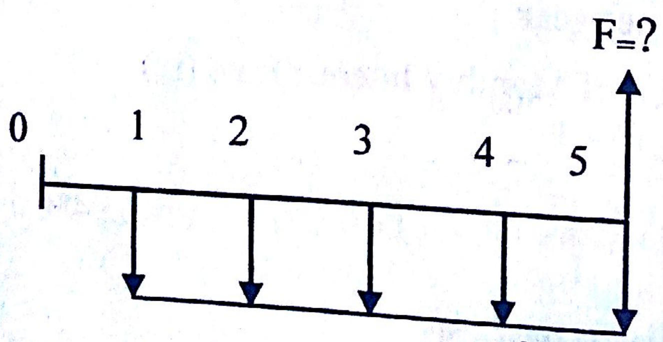
i. Rs 50,000 Deposited at the Beginning of Each Year.

Here annuity starts from the beginning of each year so we should multiply the continuous compounding amount factor formula by e^r.



$$F = A\,(F/A, r\%, n) \times e^r$$
$$= 50{,}000\,(F/A, 8\%, 5) \times e^r$$
$$= 50{,}000 \left\{\frac{(e^{rn} - 1)}{e^r - 1)}\right\} \times e^r = 50{,}000 \left\{\frac{e^{0.08\times5} - 1}{e^{0.08} - 1)}\right\} \times e^{0.08}$$
$$= \text{Rs } 3{,}19{,}849.97 \textbf{ Ans.}$$

ii. Rs 50000 Deposited at the End of Each Year.



$$F = A\,(F/A, r\%, n) = 50{,}000\,(F/A, 8\%, 5)$$

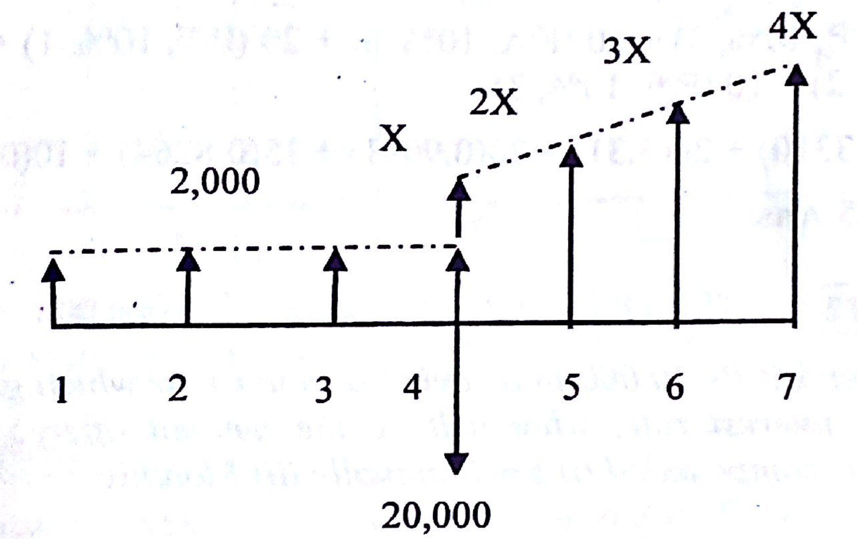
$$= 50{,}000 \left\{\frac{(e^{rn}-1)}{e^{r}-1)}\right\}$$

$$= 50{,}000 \left\{\frac{e^{0.08\times5}-1}{e^{0.08}-1)}\right\}$$

= Rs 2,95,258.74 **Ans.**

Example 2.10

Find the value of X if i =10%. **TU- 2072**



Solution:

FW of Cash Inflow in 4^{th} year

= 2,000 (F/A, 10%, 4) + [X (P/A, 10%, 4) + X (P/G, 10%, 4)] (F/P, 10%, 1)

= 2,000(4.6410) + [X (3.1699) + X (4.3781)] (1.1)

= 9,282 + 8.3028 X.........(1)

FW in 4^{th} year = 20,000................(2)

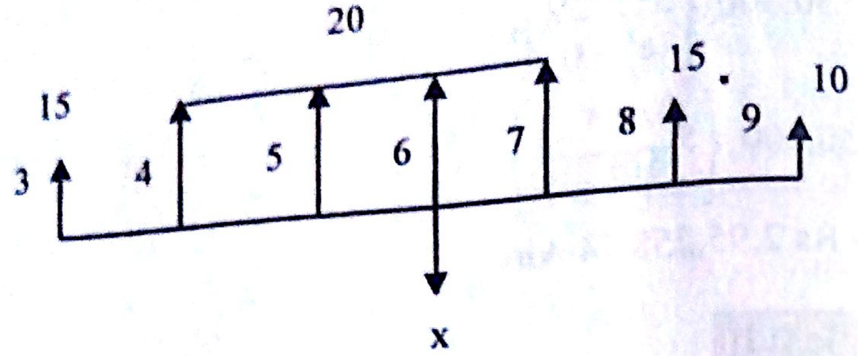
Equating equations (1) and (2)

9,282 + 8.3028 X = 20,000

Therefore, X = 1,290.89 **Ans.**

Example 2.11

Find the value of x from the following figure, i = 10%



Solution:

FW of Cash Inflow in 6th year (x)

$= 15(F/P, 10\%, 3) + 20\ (F/A, 10\%, 3) + 20\ (P/F, 10\%, 1) + 15(P/F, 10\%, 2) + 10\ (P/F, 10\%, 3)$

$= 15(1.3310) + 20(3.31) + 20(0.9091) + 15(0.8264) + 10(0.75130)$

$= 124.25$ **Ans.**

Example 2.12

a) If you deposit Rs 10,000 in a saving account now which gives 10% nominal interest rate, what will be the amount after 5 years if interest is compounded (i) semi-annually (ii) Monthly.

Solution:

a) $F_5 = ?,\ P = 10{,}000,\ r = 10\%,\ n = 5$

(i) $F_5 = ?$ when $m = 2$

$$i_{eff.} = (1 + \frac{r}{m})^m - 1 = (1 + \frac{0.1}{2})^2 - 1 = 10.25\%$$

$F_5 = 10{,}000\ (F/P, 10.25\%, 5) = $ Rs 16,288.94 **Ans.**

Alternatively,

$$i = \frac{10}{2} = 5\%$$

Interest period $= 5 \times 2 = 10$

$F_5 = 10{,}000\ (F/P, 5\%, 10) = $ Rs 16,288.94 **Ans.**

(ii) $F_5 = ?$ when $m = 12$

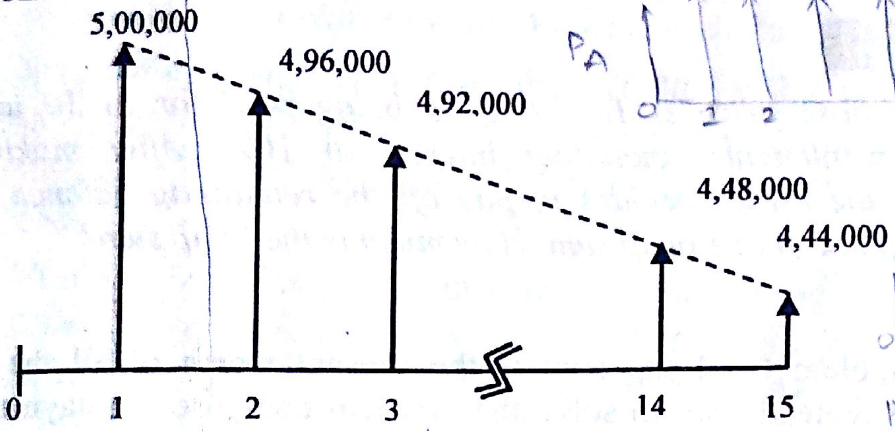
$$i_{eff.} = (1 + \frac{r}{m})^m - 1 = (1 + (1 + \frac{0.1}{12})^{12} - 1 = 10.47\%$$

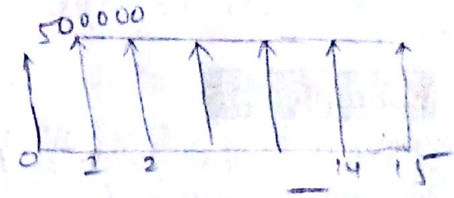
$F_5 = 10{,}000\ (F/P, 10.47\%, 5) = $ Rs 16,452.11 **Ans.**

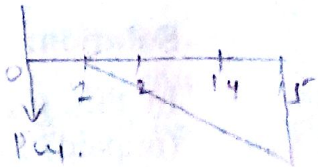
Example 2.13

The annual income of the project starts from Rs 5,00,000 at the end of first year and decreases at the rate of Rs 4,000 per year for 15 years. What is the equivalent present worth when the MARR is 12%.

Solution:





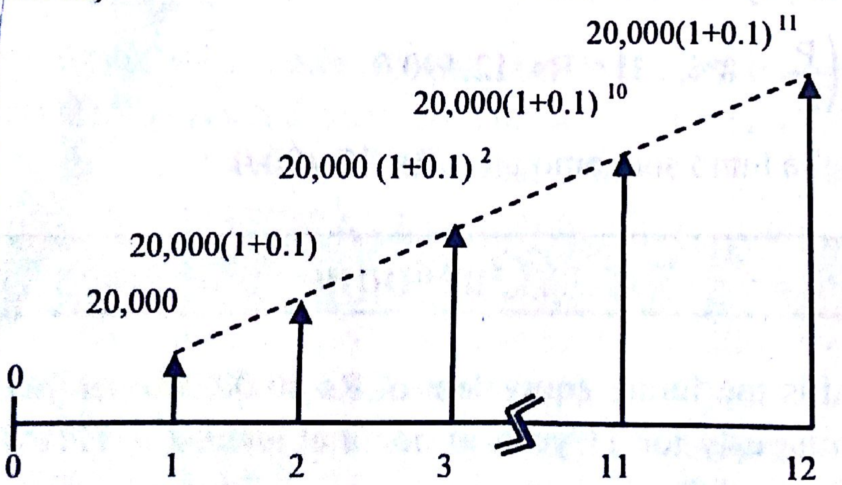


$$\begin{aligned} PW &= 5{,}00{,}000\,(P/A,\ 12\%,\ 15) - 4{,}000\,(P/G,\ 12\%,\ 15) \\ &= 5{,}00{,}000\,(6.8109) - 4{,}000\,(33.9202) = \text{Rs } 32{,}69{,}769.20 \text{ Ans.} \end{aligned}$$

Example 2.14

The revenue produces by a company at the end of 1st year is Rs 20,000 and increasing by 10% per year? Interest rate is 8% per year. What is the equivalent present worth?

Solution: Given,



A = 20,000
g = 10%
i = 8%
n = 12 years
P = ?

$$P = A\left[\frac{1-\left(\frac{1+g}{1+i}\right)^n}{i-g}\right] = 20{,}000\left[\frac{1-\left(\frac{1+0.1}{1+0.08}\right)^{12}}{0.08-0.1}\right]$$

$= 2{,}46{,}313.08$ **Ans.**

Example 2.15

A truck, whose price is Rs18,600, is being paid for in 36 uniform monthly installments, including interest at 10%. After making 13 payments, the owner decides to pay off the remaining balance of the purchase price in one lump sum. How much is the lump sum?

Solution:

In this problem the lump sum is the present worth of all the future (unpaid) payments. So, to solve the problem compute the payment and then compute the PW of the unpaid payments at the stated interest rate.

$$i_m = \left(1+\frac{10}{100}\right)^{\frac{1}{12}} - 1 = 0.80\%$$

$$A = 18{,}600\left(\frac{A}{P}, 0.8\%, 36\right) = 18{,}600\ (0.03208) = \text{Rs. } 596.70$$

After 13 months :

36 – 13 = 23 payments remain

$$P = 596.7\left(\frac{P}{A}, 0.8\%, 23\right) = \text{Rs. } 12{,}490.0$$

Therefore, the lump sum amount = Rs. 12,490.0

Questions

1. What is the future equivalent of Rs 40,00,000 per year that flows continuously for 11 years if nominal interest is 12% compounded continuously?
2. Ram invested Rs 15,000 in a high yield account. At the end of 30 years, he closed the account and received Rs. 5,39,250. Compute the effective interest rate he received on the account.

3. Find the compound amount, if the investment is done Rs 5,000 with the interest rate 12% per year and compounded weekly for 2 years.

4. Hari wants to deposit an amount p now such that he can withdraw an equal amount of Rs 2,000 each year the first 5 years and then Rs 3000 for the following 3 years. Calculate what P is if he interest earned is 6% per year. Draw the cash flow diagram.

5. Suppose one has a bank loan of Rs 10,000, which is to be repaid in equal end of month installments for 5 years with a nominal interest rate of 12% compounded monthly. What is the amount of each payment?

6. You deposited Rs 20,000 in your bank account at 31 December, 2006, if the bank pays 7% simple interest, how much will you accumulate in your account at 3 December 2016? What will be this amount if the bank pays compound interest?

7. A person invests a sum of Rs 50,000 in a bank at a nominal interest rate of 16% for 15 years. The compounding is monthly. Find the maturity amount of the deposited sum after 15 years.

8. Sabina wants to deposit an amount P now such that she can withdraw an equal amount of Rs 2,000 each year for the first 5 years and then Rs 3,000 for the following 3 years. Calculate what is the value of P is if the interest earned is 6% per year? Draw the cash flow diagram.

9. Suppose that certain end of year cash flows are expected to e Rs 10,000 for the second year, Rs 20,000 for the third year and Rs 30,000 for the fourth year and that I interest is 15% per year, calculate the equivalent preset worth at the beginning of the first year using gradient formula.

10. If a farmer wants to save money in a financial company for engineering education of his daughter who is four year old. How much money he needs to save per month if she needs Rs 3,00,000 when her age will be 20 years. The money is compounded semiannually and interest rate is 15%

11. A person wishes to have a sum of Rs 1,00,000 for his son's education after 10 years from now. That is single payment that he should deposit now so that he gets the desired amount after 10

years? The bank gives 15% interest rate compounded annually. If the saving bank pays 1.5% interest every three month, what are the nominal an effective interest rates per year?

12. Ram deposited Rs 2,000 in a saving account when his son was born. The nominal interest rate was 8% per year compounded continuously. On the son's 18^{th} birthday accumulated sum is withdrawn from the account. How much would his accumulate amount be?

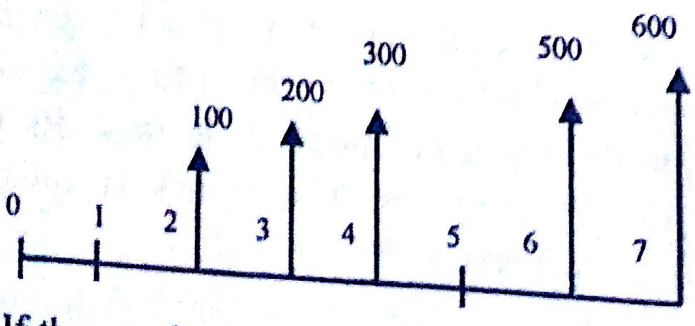
13. Suppose that certain end of year cash flows are expected to be Rs 10,000 for the second year Rs 20,000 for the third year and Rs 30,000 for the fourth year that if interest is 15% per year. Calculate the equivalent present worth at the beginning of the first year using gradient formula.

14. A software company decided to invest Rs 5,50,000 per month how much profit will company gain in six year if yearly maintenance cost is 5,00,000. Suppose that maintenance cost will be spending at the starting of year and effective interest rate is 10%.

15. What is the future equivalent of Rs 10,000 per year that follows continuously for 15.5 years if the nominal interest rate is 10% compounded continuously.

16. a. A bank gives a loan to a company to purchase an equipment worth Rs 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay the bank.

b. Solve for P_0 in the cash flow diagram shown below by using only two interest factors. The interest rate is 11% per year.



c. If the nominal interest rate is 16.5%. Find effective interest rate compound daily, monthly and quarterly

17. Ram wants to have Rs 1,50,000 for saving plan after the period of 10 years. How much rupees he deposit each year in the saving accounts that earn 10% interest annually? Also makes its cash flow diagram.
18. If a farmer want to save money monthly in a financial company for the engineering education of his daughter of 2 years old. How much money he needs to save per month if she needs 4,00,000 when her age will be 18 years old. The company compounded the money monthly and interest rate is 12%.
19. A man aged 40 years now had borrower Rs 5,00,000 from a bank for his further studies at the age of 20 years. Interest was charged at 11% per year compounded quarterly. He wished to pay loan in semiannual equal installments with the first installment beginning 5 years after receiving the loan. He has just cleared the loan now. What amount did he pay in each installment?
20. Define nominal and effective interest rate.
21. What is difference between nominal and effective interest rate?
22. Explain the principle of cash flow.
23. State the important expect of cash flow diagram.
24. Difference between simple and compound interest.
25. Write short note
    a. Cash flow diagram.
    b. Economic equivalence.
    c. Time value of money.