Positioning by Astronomic Method

Introduction

- > Geodesy is the foundation of all types of surveying and mapping
- > A great role in geodesy is performed by astronomy
- ➤ The branch of astronomy applied to in geodesy is called "Geodetic Astronomy"
- ➤ Main objective of the Geodetic Astronomy are
 - i. Determination of latitude, longitude and azimuth at the origin (datum) and main trigonometrical station (Laplace station)
 - ii. Determination of the deflection of vertical by means of difference between astronomical and geodetic coordinates
 - iii. Determination of polar motions and earth tides
 - iv. Correction of local time

Contd...

- Geometric and physical quantities are observed on the surface of earth and space exterior to it, for the solution of problems of Geodesy.
- Astronomical determination of latitude, longitude and azimuth oriented with respect to direction of plumb line and obtained from measurement of star.
- Measurement of gravity and the components of gravity gradient as well as earth tides measurement.
- Geodetic measurement of horizontal angle, distance, zenith angle and height difference
- Satellite geodesy for relationship between earth and artificial satellite.

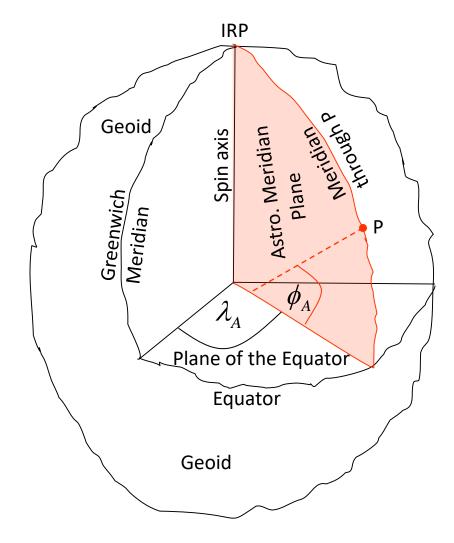
Contd...

- The methods of measurement (data collection) in all these fields depends strongly on <u>technological possibilities</u>.
- development of electronic measuring devices
- > establishment of satellite system
- > application of RADAR, Laser technology etc.

Astronomical Latitude and Longitude

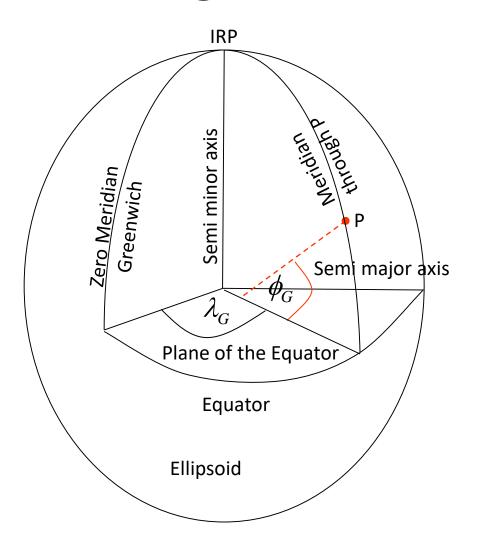
- Astronomical latitude ϕ_A defines the latitude of the vertical (gravity vector) through the point in question (P) to the plane of equator
- Astronomical longitude χ is the angle in the plane of equator between the zero meridian (Greenwich) plane and the meridian plane through P, both of which contain Spin Axis of the Earth

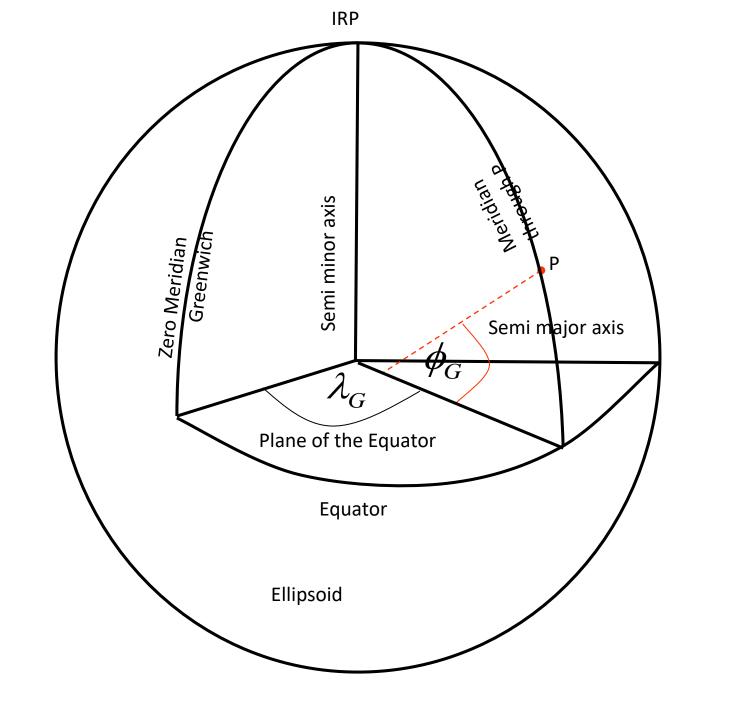
Astronomical Latitude and Longitude



Geodetic Latitude and Longitude

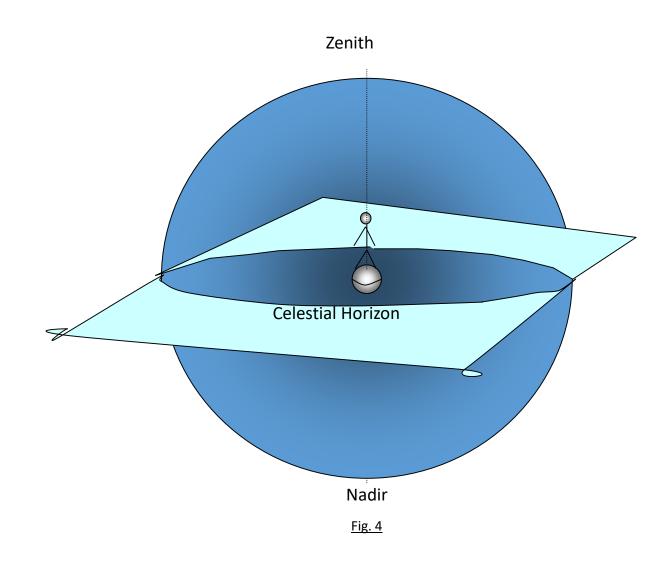
• Considering a point P at height h, measured along the normal through P, above the ellipsoid, the ellipsoidal latitude and longitude will be ϕ_G and λ_G as in the figure





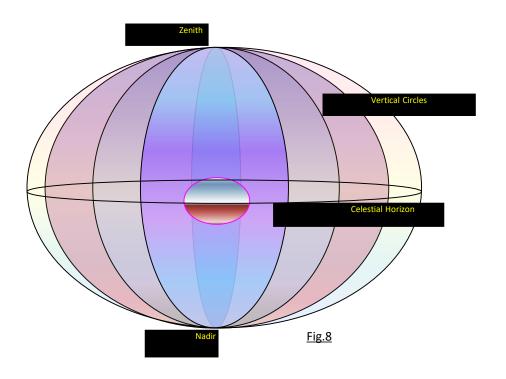
The Celestial Horizon

- The great circle traced upon the celestial sphere by a plane passing through the center of the earth and perpendicular to the zenith-nadir line is called the celestial horizon.
- It is also known as true, rational or geocentric horizon.



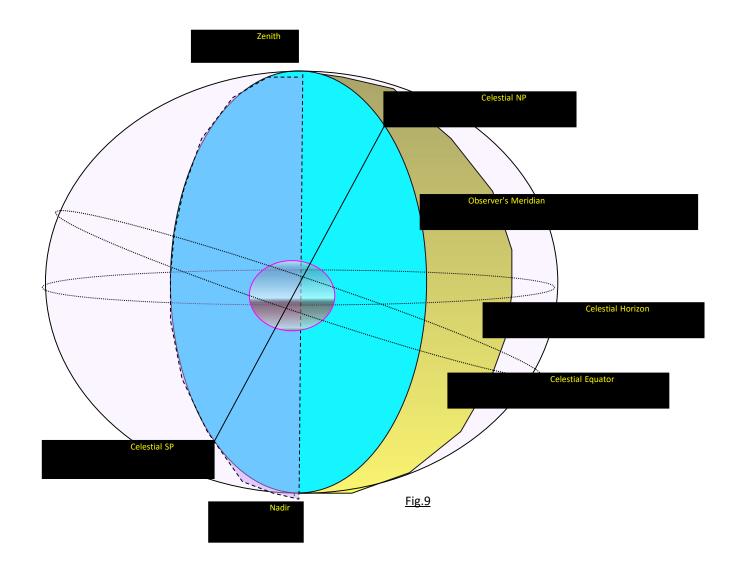
The Vertical Circles

- The great circles of the celestial sphere, which pass through the zenith and nadir points, are called vertical circles.
- They all cut the celestial horizon at right angles



The observer's Meridian

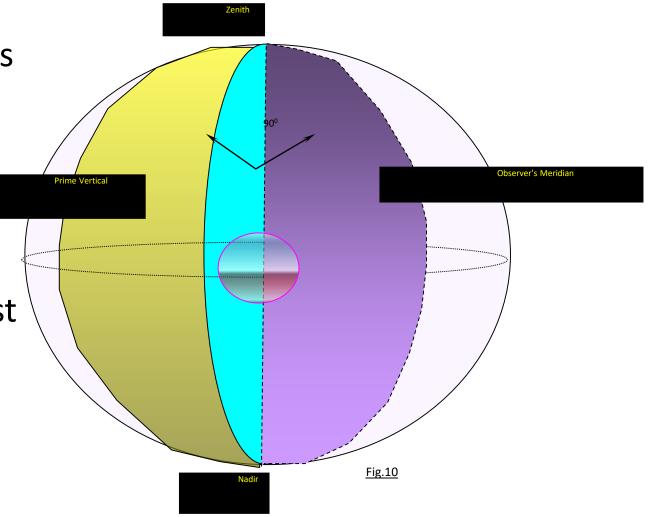
- The great circle of the celestial sphere, which passes through the zenith, nadir and celestial poles, is called the observer's meridian.
- It is the vertical circle passing through the poles or alternatively it is the celestial meridian passing through the zenith and nadir



The Prime vertical

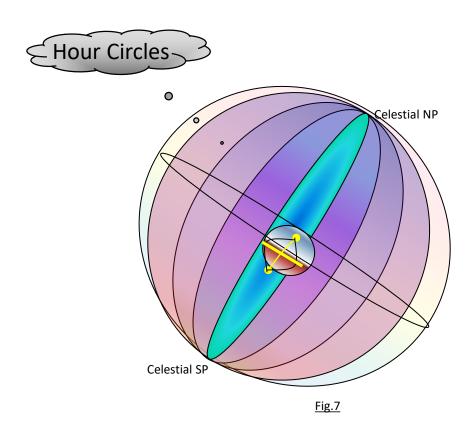
• The vertical circle which is perpendicular to the observer's meridian is termed as the prime vertical.

 the prime vertical passes through the east and west points of the horizon



The Hour Circles

- The hour circles are the great circles passing through the north and south celestial poles.
- these circles are perpendicular to the celestial equator.
- The celestial meridians are actually the hour circles



The Declination Circle and the declination (δ)

- The declination circle of a heavenly body is the great circle passing through the heavenly body and the celestial poles.
- That is it is the celestial meridian or the hour circle through the heavenly body.
- The declination of a celestial body is the angular distance from the plane of equator measured along the declination circle to the celestial body.
- Declination varies from 0-90° and is measured +ve or -ve according as the body is north or south of the equator

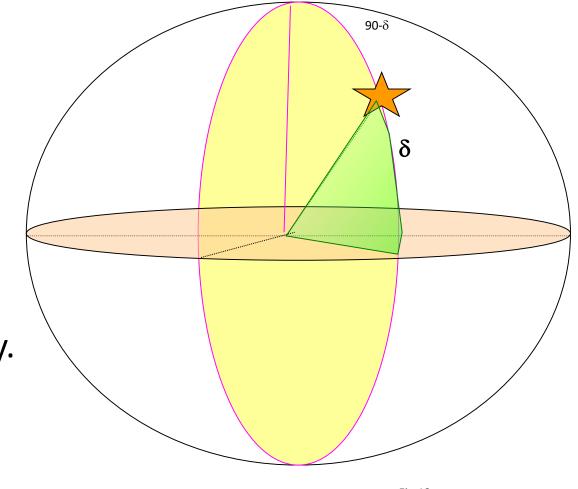
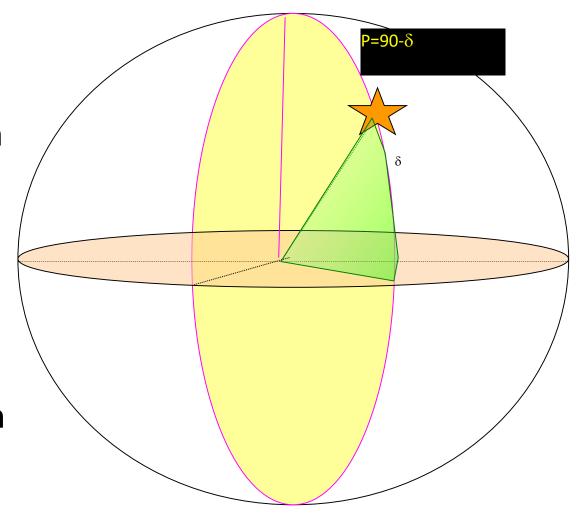


Fig.13

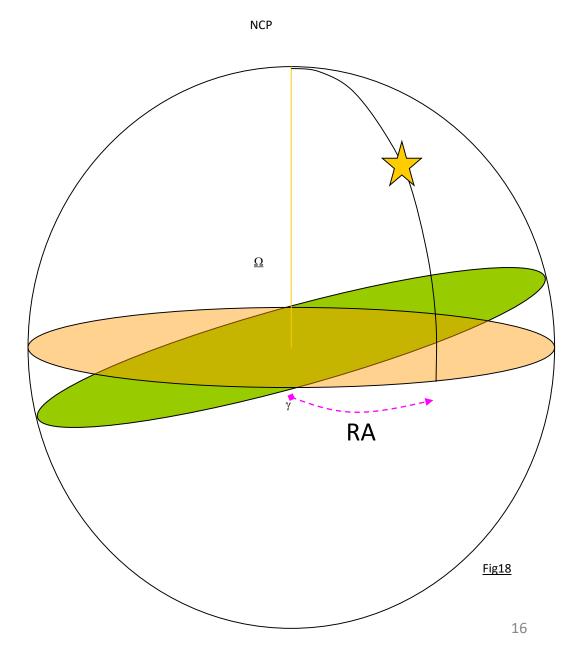
The Co-declination or Polar distance (p)

- It is the angular distance of the heavenly body measured along the declination circle from the near pole.
- It is the complement of the declination (p=90- δ).
- It is always positive,
- for the stars at northern hemisphere the polar distance lies between 0 and 90°.
- However when the declination is south the polar distance is greater than 90°



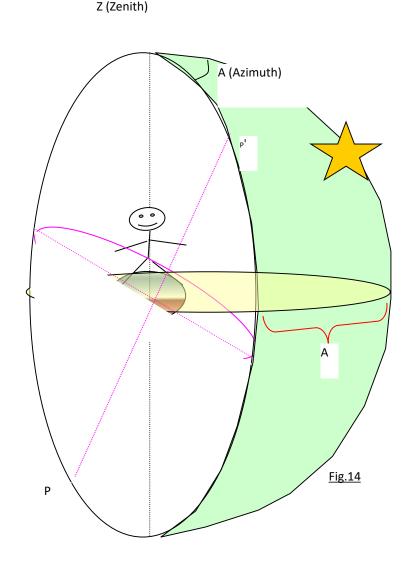
Right Ascension (RA)

- Right ascension of a heavenly body is the arc of the celestial equator intercepted between the first point of Aries and the declination circle through the body.
- Right ascension is measured from the vernal equinox along the equator in anticlockwise direction (while viewing from celestial North Pole) from 00 to 360° or 0 to 24 hrs.
- Actually, Right ascension is a celestial longitude measured in the direction of the Earth's rotation.
- Since that rotation makes a complete circle in 24 hours, the notation adopted for right ascension was in terms of hours and minutes with 24 hours representing the full circle.



Azimuth (A):

- The azimuth of a heavenly body is the angle between the observer's meridian and the vertical circle passing through the heavenly body
- Measured on the horizon from 0 to 360 degree from north point towards the east point (sometimes towards west also)
- When the azimuth of a star is 90°E or 90°W, it is on the prime vertical



Altitude (α)

 The altitude of a heavenly body is the angular distance above the horizon measured along the vertical circle passing through the heavenly body

Star α vertical circle

zenith

<u>Fig 15</u>

Co-altitude or Zenith distance (z)

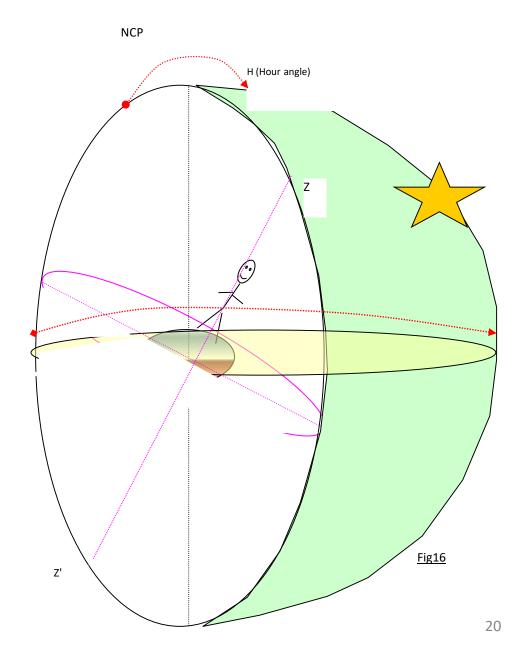
• The co-altitude of a heavenly body is the angular distance of it from zenith measured along the vertical circle passing through it. it is the complement of altitude i.e. $z = 90 - \alpha$

Star vertical circle Fig 15 19

zenithi

Hour angle

- The hour angle of a heavenly body is the angle between the plane of observer's meridian and the plane of the declination circle or hour circle of the heavenly body. it is always measured westward (clockwise when observed from the celestial pole)
- or from 0 to 24 hour, each hour being equivalent to 15°



Culmination

- When a celestial body crosses the observer's meridian, it is said to culminate or transit.
- The instant of meridian passage, when the celestial body is above the pole (altitude is maximum), is known as upper culmination
- When below the pole (altitude is minimum it is known as lower culmination.

• For example, for star S2, L and L1 are upper and lower culmination respectively

Elongation

- When a celestial body appears to attain its maximum distance from the meridian, it is said to be at elongation.
- The elongation can be eastern or western.

Circumpolar star

- The star having polar distance less than the latitude of the place of observation are known as circumpolar star.
- These always remain above the horizon and therefore do not set
- In field astronomy, stars having polar distance less than 10° are only reckoned as circumpolar star

The Celestial Reference System or Astronomical Coordinate Systems

• The system of determination of the location of any celestial (heavenly) body is termed as the Celestial Reference System or the Astronomical Coordinate System. Since all the heavenly bodies are in a continuous motion in the celestial sphere the position of a particular body can be located for a particular moment of time.

The Celestial Reference System or Astronomical Coordinate Systems or Celestial System

- There are the following systems of referencing for the celestial bodies at a particular moment of time
 - Horizontal Coordinate System
 - Equatorial Coordinate System
 - Ecliptical Coordinate System
 - Glactic Coordinate System

Coordinate System

- When observing a fixed star the distance from the point of observation to the center of mass of earth can be neglected in comparison to the distance to the star.
- If we circumscribe celestial sphere about earth the position of the fixed star are determined on the sphere by two direction .
- Of various coordinate system: Equatorial coordinate system

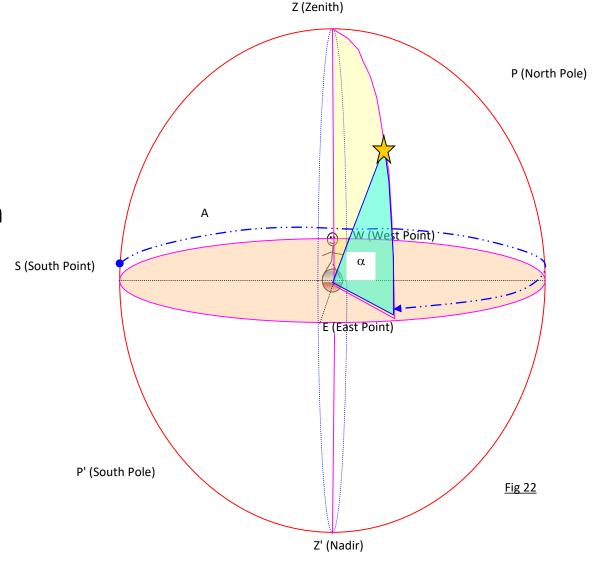
: Horizon coordinate system

are of particular interest in Geodetic Astronomy

Horizontal Coordinate System

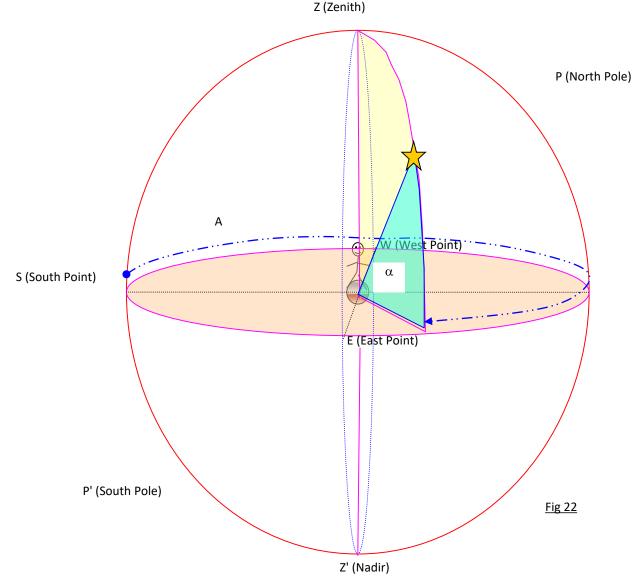
The Horizon System

• The horizon system is defined locally for each observer, or site, on Earth (or another celestial body). Its origin is the observer's location, its reference axis is the local vertical or plumb line and its reference plane is the apparent horizon i.e. simply horizon perpendicular to the zenith nadir line at observer's position.



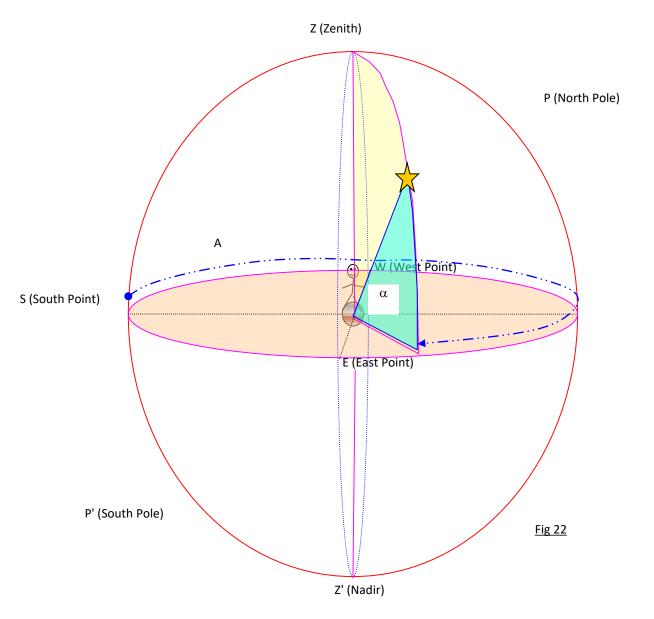
The Horizon System

- Through the position of a star, a unique plane (or great circle) perpendicular to the horizon can be found. Within the plane of its vertical circle, the position under consideration can be characterized by the angle to the horizon, called altitudeα.
- Alternatively and equivalently, one could take the angle between the direction and the zenith, the zenith distance z. All objects above the horizon have positive altitudes (or zenith distances smaller than 90 deg). The horizon itself can be defined, or recovered, as the set of all points for which altitude = 0 deg (or zenith distance = 90 deg).



The Horizon System

- The second coordinate of a position in the horizon system is defined by the point where the vertical circle of the position cuts the horizon called azimuth A
- Azimuth is defined as the angle between the vertical through the north point and the vertical through the star at S, measured eastwards from the north point along the horizon from 0 to 360°.
- The vertical circle passing through the south and north point (as well as zenith and nadir) is called local meridian (Observer's meridian, the one perpendicular to it through west point, zenith, east point and nadir is called prime vertical

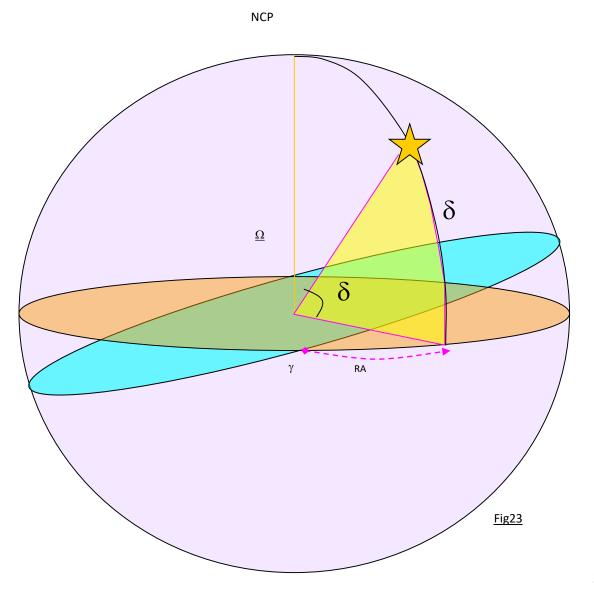


The Equatorial Coordinate System

- The main disadvantage of the horizon system is the steady change of coordinates for a given astronomical object as Earth rotates during the course of the day.
- This can be removed by using a coordinate system which is fixed at the stars (or the celestial sphere).
- The most frequently used such system is the equatorial coordinate system which is still related to planet Earth and thus convenient for observers.
- In principle, the celestial coordinate system can be introduced in the simplest way by projecting Earth's geocentric coordinates to the sky at a certain moment of time.
- Practically, projecting Earth's equator and poles to the celestial sphere by imagining straight half lines from the Earth's center produces the celestial equator as well as the north and the south celestial pole.
- We take the equator and the hour circles as the reference planes for this system
- There are further two systems in this category
 - Right Ascension and Declination System
 - Hour angle and Declination System

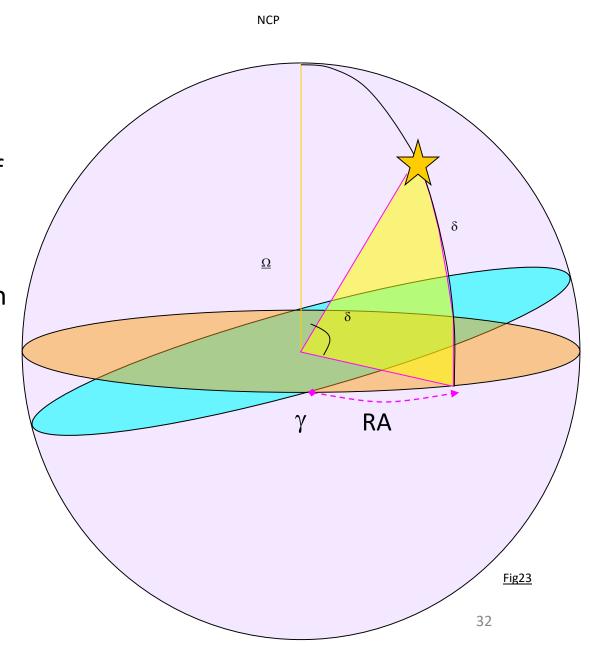
Right Ascension and Declination System

• The first coordinate in the equatorial system, corresponding to the latitude, is called Declination (δ), and is the angle between the position of an object and the celestial equator (measured along the hour circle)



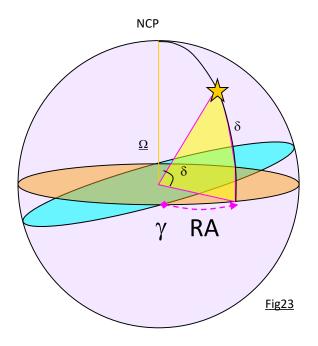
Right Ascension and Declination System

- It remains to fix the zero point of the longitudinal coordinate, called Right Ascension (RA).
- For this, the intersection points of the equatorial plane with Earth's orbital plane, the ecliptic, are taken, more precisely vernal equinox or "First Point of Aries".
- During the year, as Earth moves around the Sun, the Sun appears to move through this point each year around March 21 when spring begins on the Northern hemisphere, and crosses the celestial equator from south to north.
- The opposite point is called the "autumnal equinox", and the Sun passes it around September 23 when it returns to the Southern celestial hemisphere.
- As a longitudinal coordinate, RA can take values between 0 and 360 deg



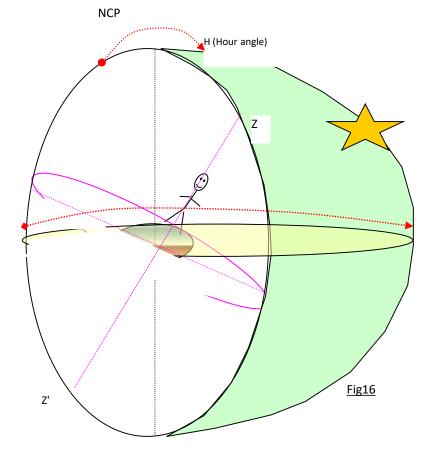
Right Ascension and Declination System

- this coordinate is more often given in time units hours (h), minutes (m), and seconds (s),
- where 24 hours correspond to 360 degrees (so that RA takes values between 0 and 24 h); the correspondence of units is as follows:
 - vernal equinox at RA = 0 h = 0 deg
 - summer solstice at RA = 6 h = 90 deg
 - autumnal equinox at RA = 12 h = 180 deg
 - winter solstice is at RA = 18 h = 270 deg



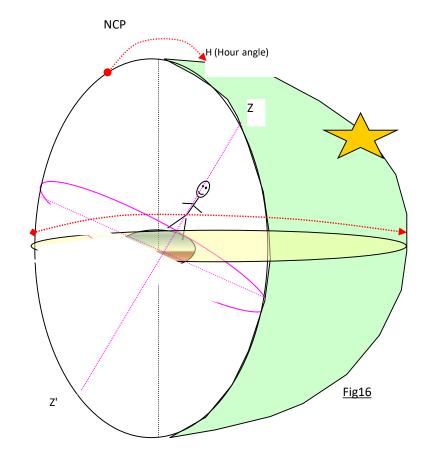
Hour angle and Declination System

- This system has again the celestial equator and poles as reference frames
- declination as latitudinal coordinate,
- longitudinal coordinate is hour angle
- Celestial objects are at constant RA, but change their hour angle as time proceeds
- In this system, a star or other celestial object moves contrary to Earth's rotation along a circle of constant declination during the course of the day



Hour angle and Declination System

- The local meridian is taken as the hour circle for HA=0
- Celestial objects are at constant RA, but change their hour angle as time proceeds
- The standard convention is that HA is measured from east to west so that it increases with time.
- HA will change for the same amount as the elapsed time interval is, as measured in star time (ST),



Summary

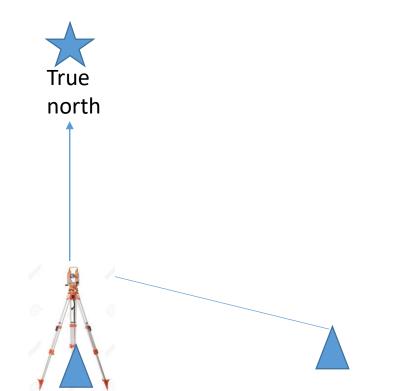
- Horizontal Coordinate System
 - Azimuth and Altitude i. e. (A, α)
- Equatorial Coordinate System
 - Right Ascension and Declination System (RA, δ)
 - Hour angle and Declination System (H, δ)
- Ecliptical Coordinate System
 - Celestial Latitude and Celestial longitude (θ_e , λ_e)
- Glactic Coordinate System
 - Galactic latitude and Galactic longitude (θ_G , λ_G)

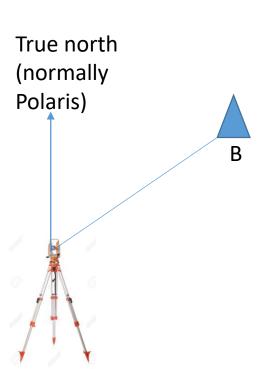
In case of Nepal

- To carry out these astronomical activities the Geodetic Survey Branch established the Astronomical Division in the year 1975.
- Presently, it has various section as
- Latitude Section
- ii. Longitude Section
- iii. Time Service Section
- iv. Computing Section
- v. Administrative Section

Determination of Azimuth

- ☐ Azimuth
- It is the horizontal angle a celestial body makes with the pole
- The observation for azimuth determination of a survey line consist in measuring the horizontal angel between reference mark, (a triangulation station appropriately illuminated and the celestial body

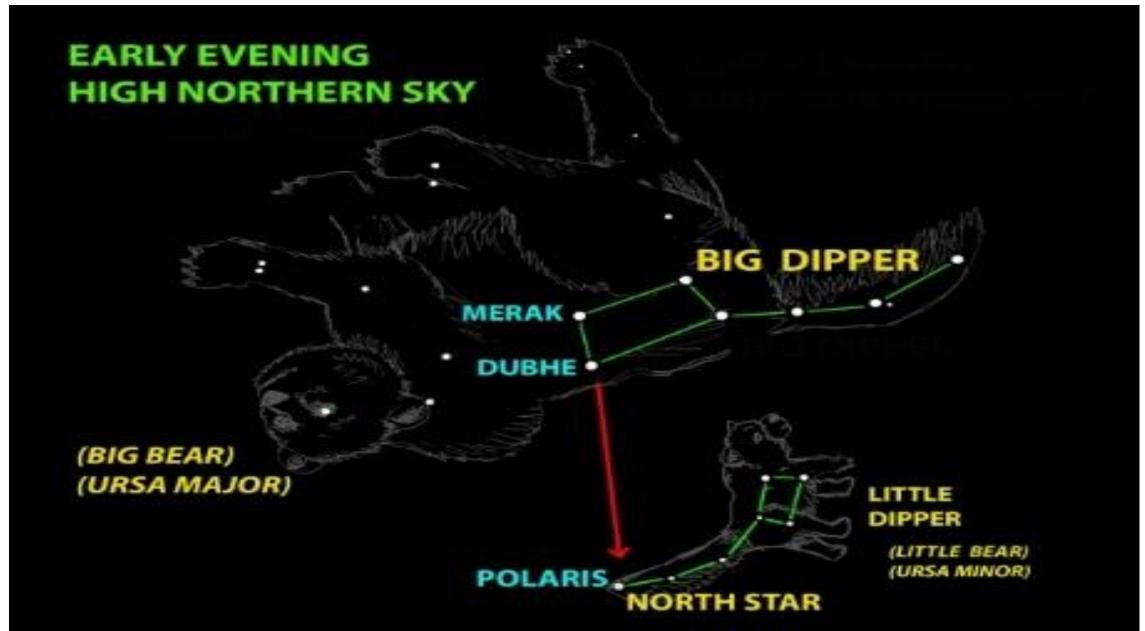


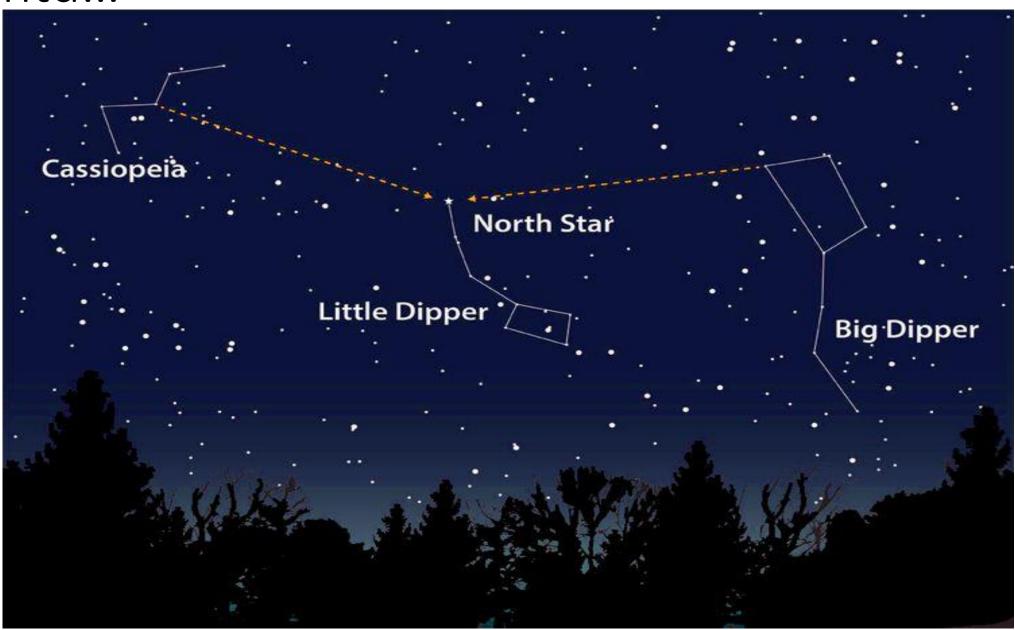


- There are several method prevalent for determining the azimuth but the one permitting both left and right face observation is more prrefered.
- It is advisable to choose a circumpolar star, since it changes very slowly with time in azimuth.
- In northern hemisphere = Polaris is used
- ➤ It is the brightest star of Little bear (Ursa Minor)
- It can be seen with naked eyes and describe a circle of about 1° radius around the pole
- ➤It can readily identified by means of the constellation called Great Bear (Ursa Major) "Sapta rishi" in summer
- This consist of 7 bright star, two of which points directly towards north and are called pointer.

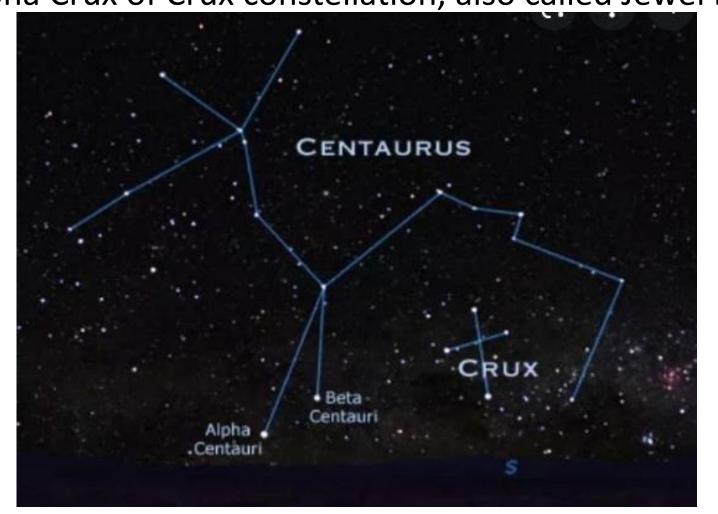
- The line joining these two star passes very closely through Polaris
- In winter Polaris can be identified by constellation Cassiopeia
- It lies at the bisector of large angel of W-shaped Cassiopeia, the two constellation being on the opposite side of Polaris

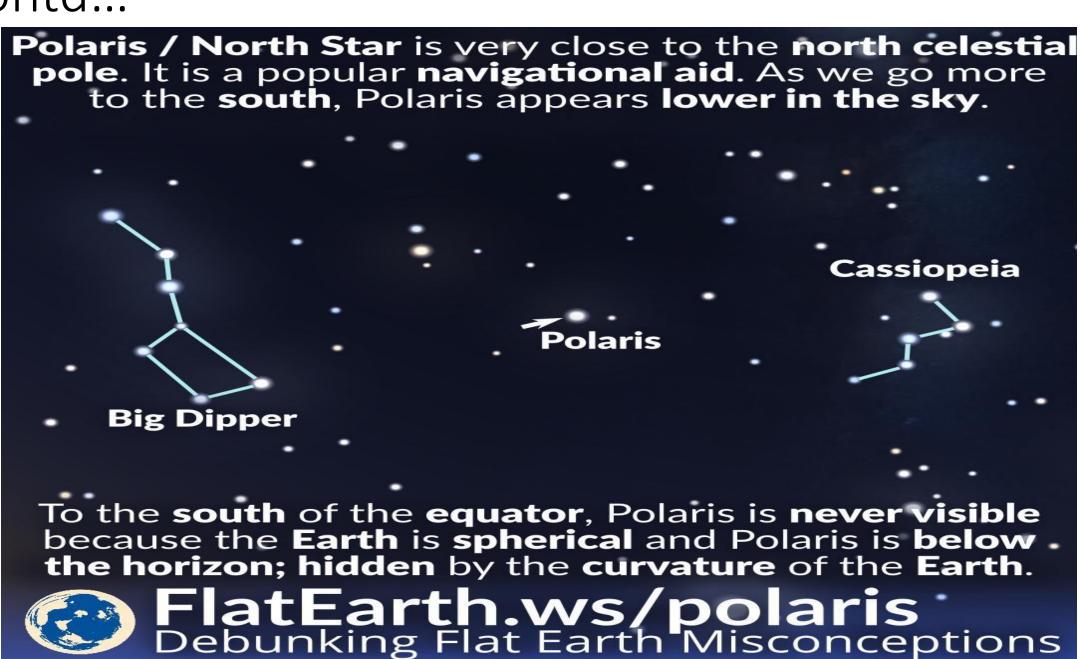
Polar Observation





• In the southern hemisphere, azimuth can be determined with the help of a star Alpha Crux of Crux constellation, also called Jewel Box





Contd....



Observation of Polaris

• In case of Nepal the Polaris is always completely visible from all parts of Nepal, when weather is suitable

• In Astronomical Division the observation of Polaris for determining of azimuth is carried out using Wild T3 Theodolite with graduation of gon

(400g) degrees to gradians conversion scale 80 100 $^{120}_{I}$ 140 160 $^{180}_{I}$ 200 220 $^{240}_{I}$ 260 280 $^{300}_{I}$ 320 340 $^{360}_{I}$ inchcalculator.com

• Before starting the observation the Chronometer should be set or adjusted to radio time signal (Radio time signal stations broadcast the time in both audible and machine-readable time code form that can be used as references for radio clocks and radio-controlled watches)





- Quartz Chronometer is a precision quartz clock which keeps accurate time
- The second hand of chronometer moves twice in a second. Thus possible to read and record the time accurately up to half a second directly
- And any event in between the interval of a half a second can be obtained further with the help of stop watch having the time reading graduation less than a half a second
- The recording of the time when celestial object is bisected by the vertical wire of telescope is done by Chronometer

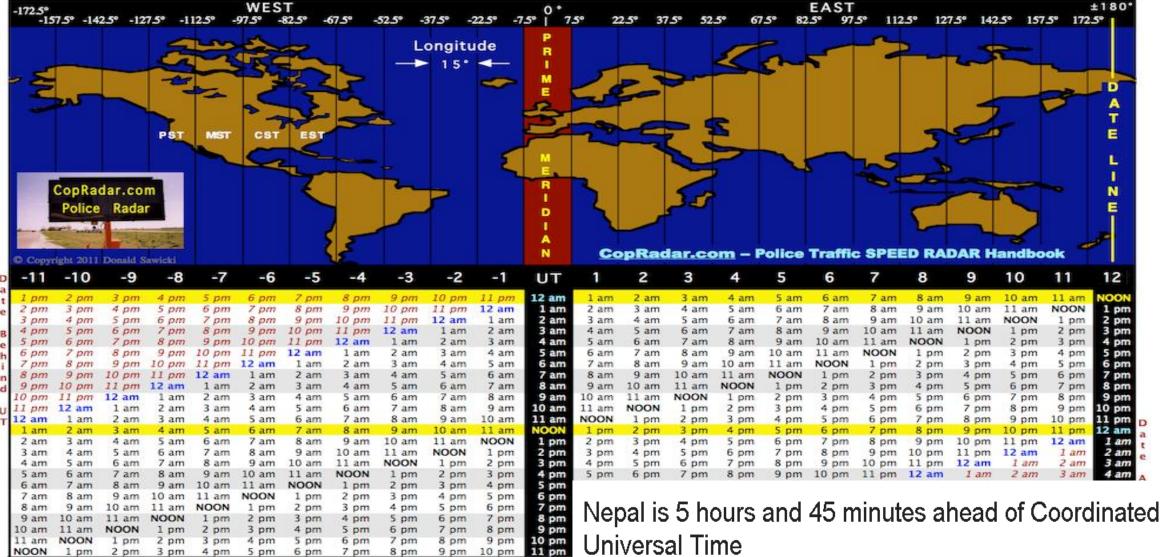
Setting UTC in the Chronometer watch

- Almost every astronomical observation are done in UTC (universal coordinate time)
- So it is necessary to set the time recording instrument to UTC correctly
- UTC is the primary time standard by which world regulates clock and time
- Widely used by military

Universal Time Coordinated

6 pm

NOON



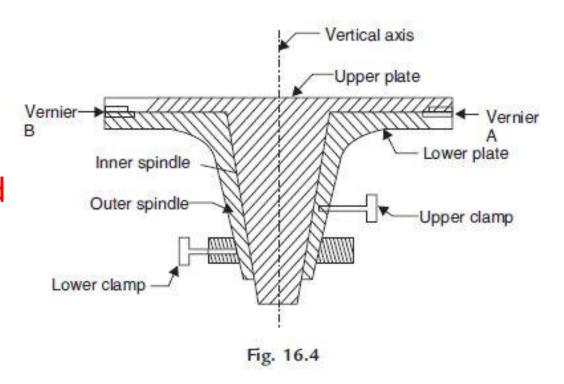
Universal Time

^{2:48} PM Saturday, in Coordinated Universal Time is

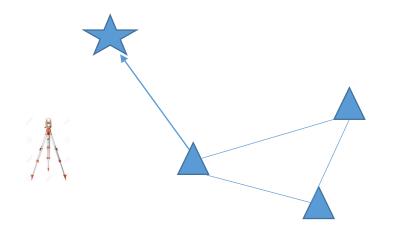
^{8:33} PM Saturday, Nepal (GMT+5:45)

- The wild T3 theodolite is set up at the station and levelled carefully
- The R.O (reference object) to which the azimuth is targeted should be signaled by a light beacon, carefully fixed above the station mark
- The Polaris should be recognized and identified

- The theodolite at first is in its left face then is pointed towards the R.O where light beacon was placed.
- The horizontal circle is set to zero
- Both ends of the plate level should be read and recorded as well as the reading of horizontal and vertical circle
- Than
- The theodolite should be pointed to the Polaris
- Both end of the plate level should be read and recorded



- The observer inform the booker that the time reading is about to occur
- Then the observer follow the Polaris in eyepiece of telescope of theodolite and bisect it by the vertical wire (hair) at the instant he call 'op' (stop)
- The booker instantly read the time (UTC) from chronometer
- After
- Time reading the reading of horizontal and vertical circle should be carried out and recorded.
- Finally again both end of plate level should be read and recorded



- The telescope is then transited for right face
- The procedure for observation in right face is similar to that of left face
- Keeping theodolite in its Right face is pointed to R.O.
- Both end of plate level and horizontal and vertical circle reading should be taken and recorded in the form
- This completes one set of observation
- Their should be at least 9 observation
- But vertical circle reading one set only
- Finally the observed data must be calculated immediately and checked in the field

Other method for determining the azimuth of a survey line are::

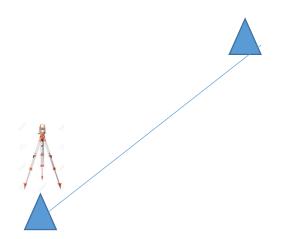
- 1. Extra-meridian observation of sun
- 2. Extra-meridian observation of circumpolar star or of a star near prime vertical
- 3. Azimuth by circumpolar star at elongation
- 4. Azimuth by equal altitude of a circumpolar star
- 5. Azimuth by observation on sun at equal altitude

1. Azimuth by extra-meridian observation of sun

- Is the most common field method
- Where it is done by measuring altitude of the sun and computing the sun's azimuth by sine or tangent formula

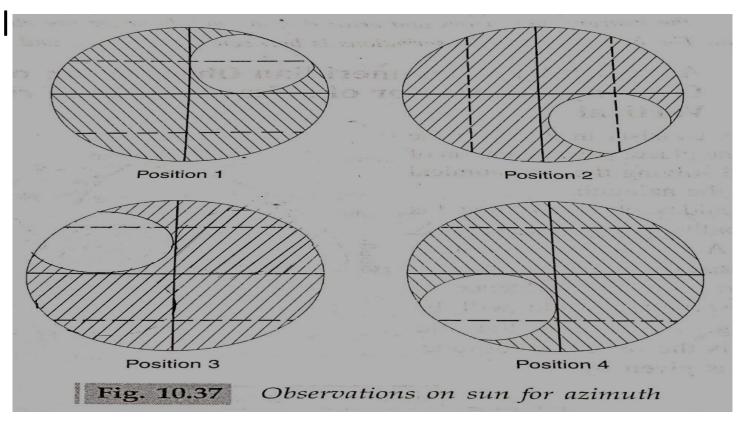
Procedure

i. A theodolite is set up at one end of a line where azimuth is required and a plate Vernier is set to read 0 deg. Bisect the other end, the reference mark of the line with vertical cross-wire



- The coloured glass shade is placed over the eyepiece and the telescope is pointed towards the sun by loosing the upper clamp
- The sun in first observation is in position 1 and 3 and then in position 2 and 4 with changed face.

In all position, the horizontal angle, altitude and the time of observation are noted



• During these four observation, the sun changes its position considerably so accurate result cannot be obtained.

For MPV, the altitude and time measured with sun in position 1 and 3 can be averaged and similarly those measured for position 2 and 4 can be averaged

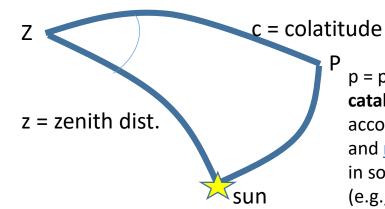
 These two averaged altitude are corrected for index error to give the apparent altitude. (http://www.brainkart.com/article/Sources-of-Error-for-Total-Stations_4653/

The azimuth for these two values of altitude is calculated using tangent

formula.

$$\tan \frac{A}{2} = \sqrt{\frac{\sin(s-z)\sin(s-c)}{\sin s.\sin(s-p)}}$$

$$S = \frac{1}{2}(p+c+z)$$



p = polar distance(from **star catalog**, list of <u>stars</u>, usually according to position and <u>magnitude</u> (brightness) and, in some cases, other properties (e.g., spectral type) as well

- Than, two values of the azimuth obtained are averaged
- The result obtained by observation to the sun are not as accurate as that computed from star observation, but since it is convenient to do the field work at the day time and since only one observation is involved, it is most convenient method of determining the azimuth of a survey line.

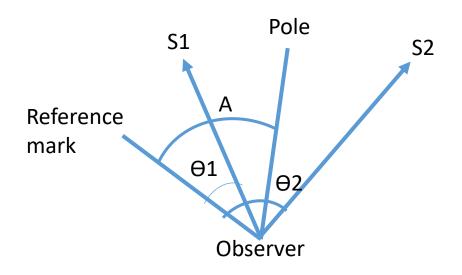
The best period for observation is between 8 to 10 A.M. and 2 to 4 P.M.

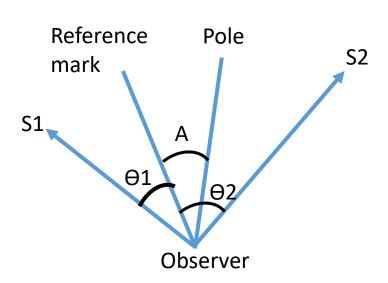
2. Azimuth by equal altitude of a circumpolar star

• This method of azimuth determination is conducted at night

Procedure

 A theodolite is set up and levelled at one end of the line whose azimuth is to be found. At the other end of the line, the reference mark is bisected

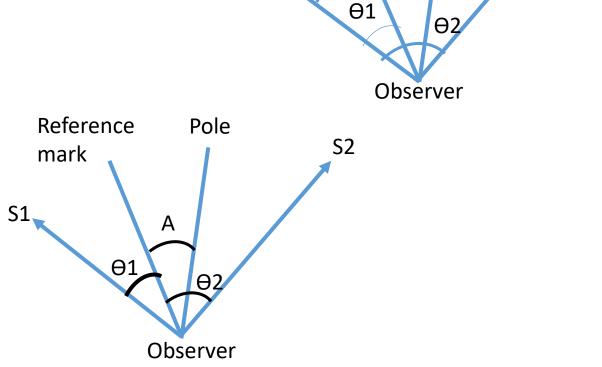




• The reference mark is bisected by vertical hair, with both plates clamped to 0 deg

 The upper clamp is loosened and telescope is revolved clockwise to bisect the star at position S1. The horizontal angle Θ_1 and the altitude α of the star are noted

 The star in its due course reaches the other side of the meridian. The upper clamp is loosened and the star is bisected again when it attains the same altitude.



Reference

mark

S1

Pole

S2

• Since the direction of meridian is midway between the two positions of the star, the azimuth of the line may be determined according to whether both the position of star are to the same side of the Reference mark or on the different side

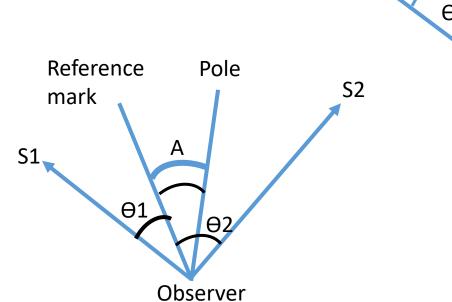
• 1st case: When star on the same side of Reference mark

$$A = \theta 1 + \frac{\theta 2 - \theta 1}{2} = \frac{\theta 1 + \theta 2}{2}$$

• 2nd case: When star on the different side of

Reference mark

$$A = \frac{\theta 1 + \theta 2}{2} - \theta 1 = \frac{\theta 2 - \theta 1}{2}$$



Reference

mark

Pole

 Θ 2

Observer

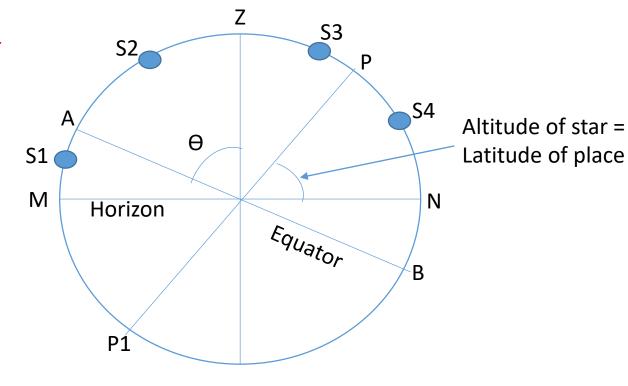
Determination of Latitude

- 1. Latitude by the altitude of a circumpolar star
- 2. Latitude By Meridian altitude of a star
- 3. Latitude by altitude of Sun
- 4. Latitude by extra-meridian altitude of sun or star
- 5. Latitude of extra-meridian observation of Polaris

Latitude by Meridian altitude of Star

- Here the altitude of star is measured, when it crosses the meridian
- And if the declination (hence co-declination) of a star is known, the latitude can be easily computed
- In this case, their arise 4 cases depending upon the position of star
- <u>Case 1</u>: When star S lies between horizon and equator
- AZ = Latitude Θ
- \triangleright MS1 = altitude of star = α 1
- \gt ZS1 = co –altitude (zenith distance) = Z1 = 90 α 1
- \triangleright AS1 = declination of star = δ 1
- > NOW

AZ = ZS1 - AS1
= Z1 -
$$\delta$$
1
:: Θ = (90 - α 1) - δ 1



Case 2: When star S2 lies between equator and zenith

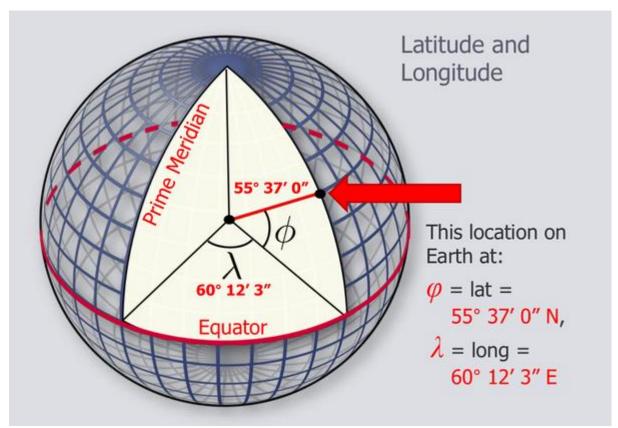
Case 3: When star S3 lies between zenith and pole

Case 4: When star S4 lies between Pole and horizon

Assignment

Determination of Longitude

- Generally the following method are used for the determination of Longitude
- 1. By triangulation
- 2. By transportation of Chronometer
- 3. By wireless signal
- 4. By observation of moon and star which culminate at the same time
- 5. By lunar distance
- 6. By celestial signal



Astronomical Correction

- The observed altitude of a celestial body is not its true altitude
- Before using this altitude in the solution, it has to be corrected via:
- Correction for Parallax
- ii. Correction for refraction
- iii. Correction for semi-diameter
- iv. Correction for dip

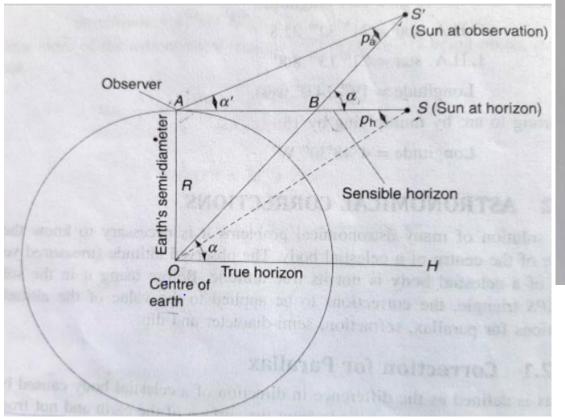
Correction for Parallax

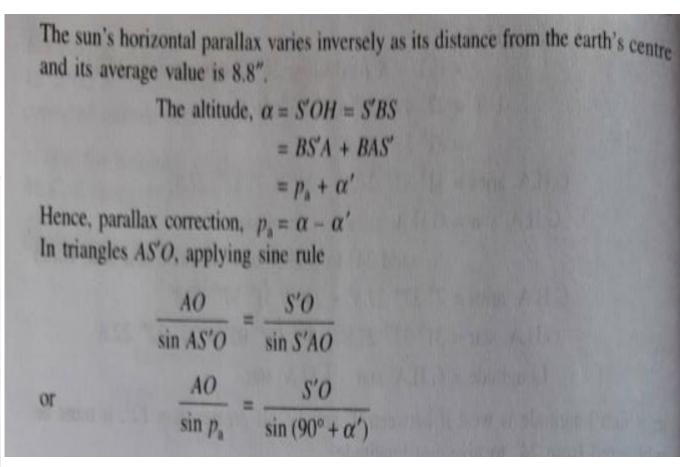
- Parallax as, observation made from the surface of the earth and not from the center of the earth
- This correction is applied when celestial object is at finite distance form the earth
- For infinite distance object this correction tends to be zero
- The parallax correction is always additive and is very small
- For the sun the value = 8.8"

Moon
$$= 1^{\circ}$$

- In fig, angle S'AS(α') = observed altitude , S'OH = α = true altitude
- P_a = parallax correction
- P_h = sun's horizontal parallax

From triangle SAO,
$$\sin p_h = \frac{R}{SO}$$





$$\sin p_{a} = \frac{AO}{S'O} \sin(90^{\circ} + \alpha')$$

$$= \frac{AO}{SO} \cos \alpha' \qquad (\text{since } S'O = SO)$$

$$= \sin p_{h} \cos \alpha' = 8.8'' \cos \alpha'$$

The correction is always positive and is maximum when the sun is at the horizon.

The sun's horizontal parallax varies inversely as its distance from the earth's centre and its average value is 8.8".

The altitude,
$$\alpha = S'OH = S'BS$$

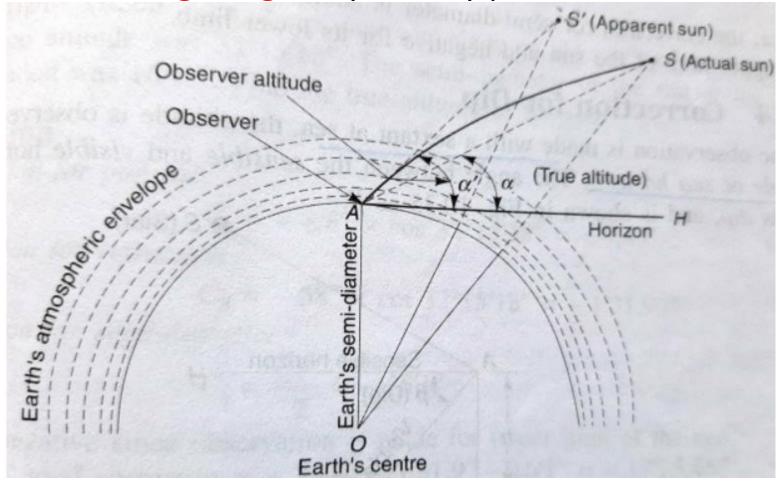
= $BS'A + BAS'$
= $p_a + \alpha'$

Hence, parallax correction, $p_a = \alpha - \alpha'$ In triangles AS'O, applying sine rule

$$\frac{AO}{\sin AS'O} = \frac{S'O}{\sin S'AO}$$
or
$$\frac{AO}{\sin p_a} = \frac{S'O}{\sin (90^\circ + \alpha')}$$

Correction for Refraction

- The layer around earth are dense near surface and get thinner and thinner with distance from the surface of the earth
- This cause bending of light ray as they passes



- In fig:
- Observation is made to a star S
- The apparent direction AS' of sun from earth's surface being tangent to curve AS of refracted ray from the position of observer 'A' result in observation altitude S'AH whereas correct altitude of the body is SAH
- Thus the refraction correction is angle SAS' and is always subtractive
- At a pressure of 76 cm of mercury and a temperature of 10 deg celcius the correction for refraction is

$$C_R = 58'' \cos \alpha'$$

where, α' = apparent altitude of celestial body

• The combined correction for parallax and refraction is subtractive

Correction for semidiameter

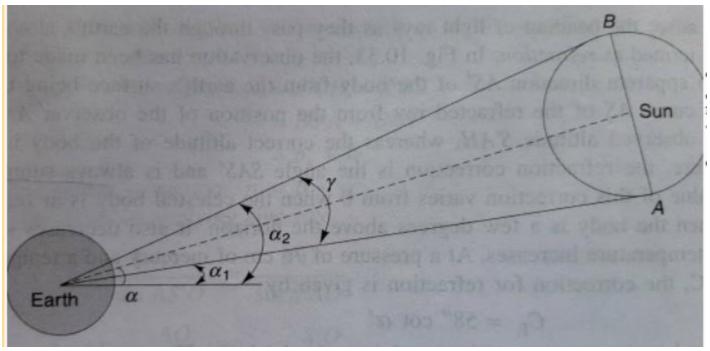
- When measuring the altitude of large object like sun, moon, it is required to obtain the altitude by measuring vertical angles to either upper or lower limb.
- The observed value so obtained must be corrected for the semi-diameter of celestial body
- The semi-diameter of a celestial body is half the angle subtended at the center of the earth by the diameter of the body
- Since the distance of sun from earth varied throughout the year the correction is not constant and varies from 15'46" to 16'18"

Mean =16'1.18"

• If α = correct altitude then

$$\alpha = \alpha \mathbf{1} + \frac{\gamma}{2}$$

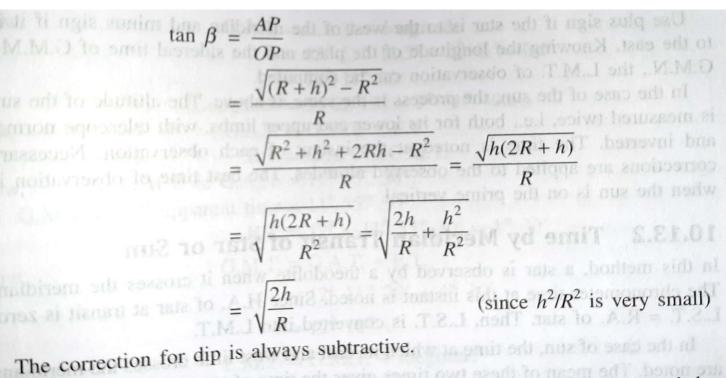
$$\alpha = \alpha 2 - \frac{\gamma}{2}$$

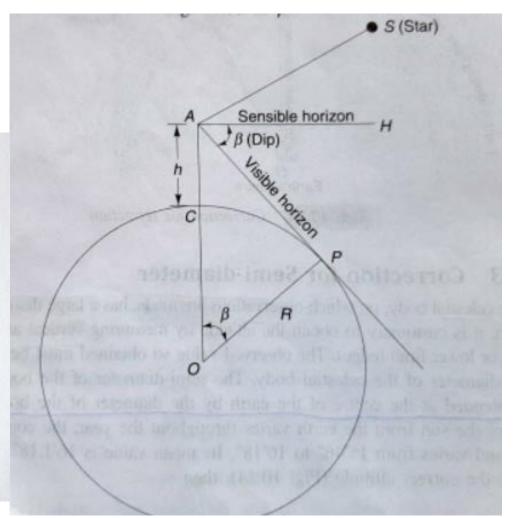


- Additive = if observation is
 - made for the lower limb of sun
- Subtractive = if observation made for the upper limb

Correction for Dip

- When the observation is made with sextant at the sea, the altitude is observed from the visible or sea horizon .
- The angle between sensible and visible horizon is known as Dip
- Angle of dip = $\langle HAP = \langle AOP = \beta \rangle$
- In triangle AOP





Question

• When observing a sun upper limb from the top of mount Everest, the observed altitude was 27 deg 54min 24sec. The semi-diameter of the sun at a time of observation was 16min 1.8sec. Find the true altitude of the sun?