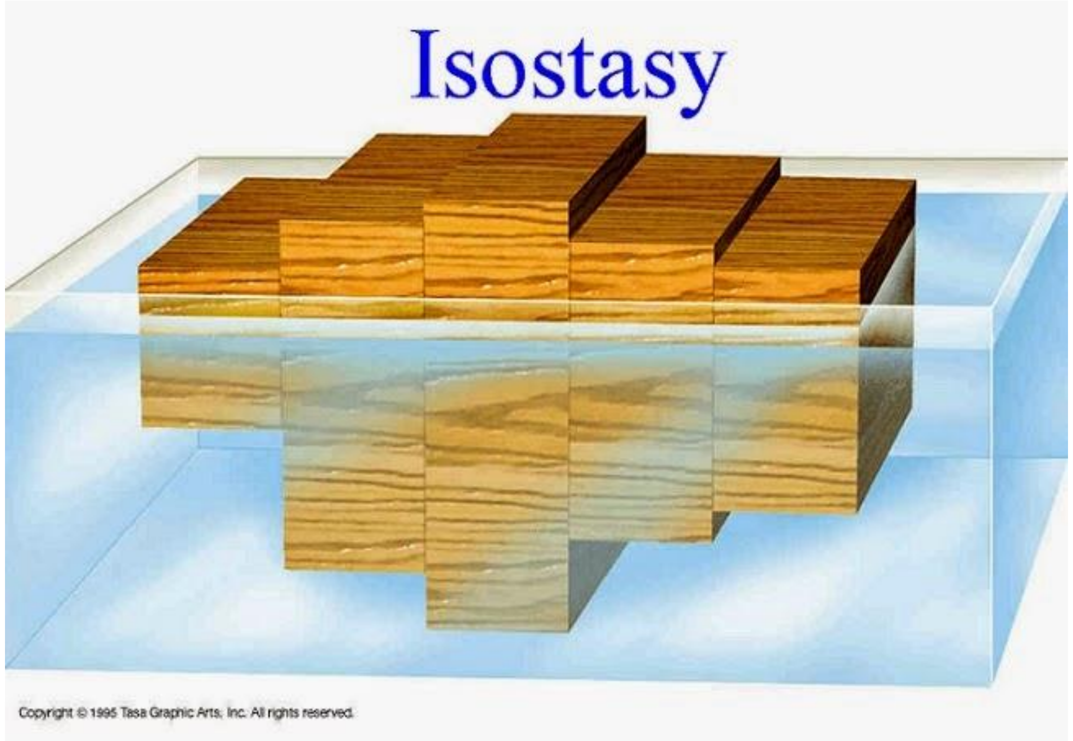
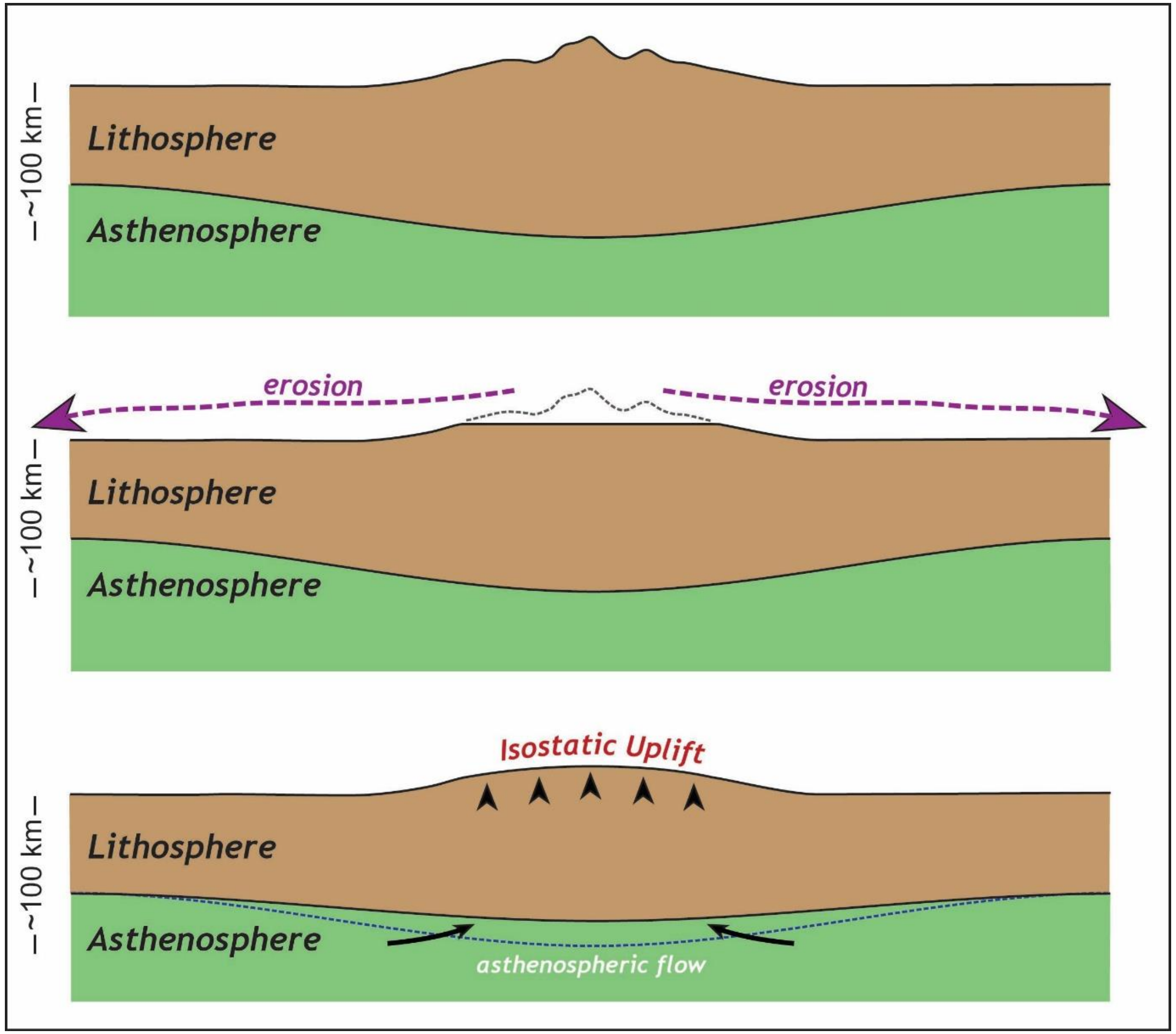
Gravity Reduction

Isostasy





Gravity Reduction

- Gravity measured on earth surface must be distinguished from normal gravity measured on ellipsoid.
- Also to refer ‘g’ to the sea level a reduction is necessary
- Reduction method depends on how to deal with the topographic masses that are above the sea level
- Mainly gravity reduction helps in

✓Determination of geoid

✓Interpolation and extrapolation of gravity

✓Investigation of earth’s crust

Types

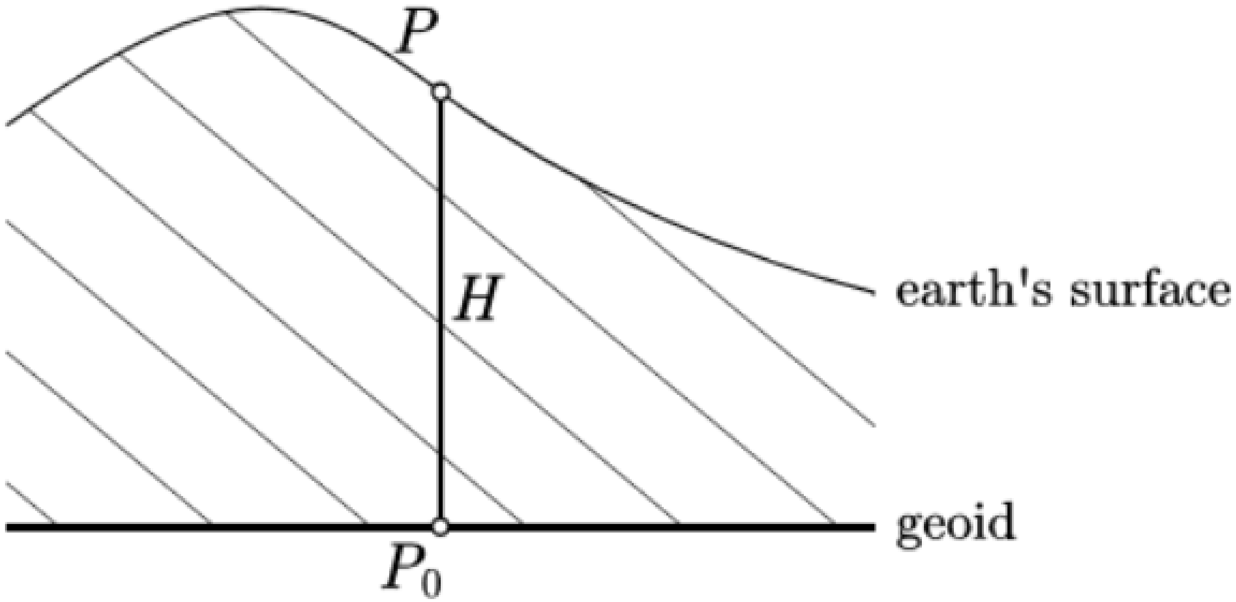
1. Non-isostatic reduction
   a. Free air reduction
   b. Bouguer reduction
   c. Terrain correction
   d. Poincare and Prey correction
2. Iso-static reduction
   a. Pratt-Hayford System
   b. Airy-Heiskanen System
   c. Vening-Meinesz regional system

Cont…

- Generally gravity reduction assume the following

i. Topo mass outside the geoid are completely removed or shifted below the sea level

ii. Then the gravity station is lowered from the earth's surface (point P) to the geoid (point P_0)

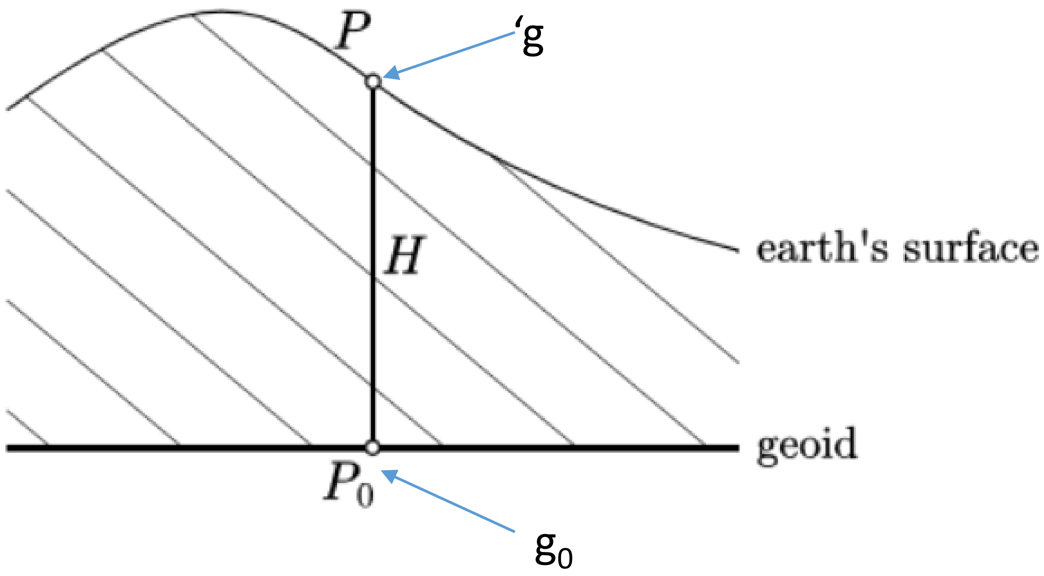


- How ever such type of computation requires knowledge of the topographic masses, which itself is very complex and problematic.

Free-air reduction

- In free-air reduction, to find the theoretically correct reduction of gravity to the geoid, we need $\partial g/\partial H$ (the vertical gradient of gravity)
- If 'g' is the observed value at the surface of the earth, then the value 'g_0' at the geoid may be obtained by Taylor's series expansion as::

$$g_0 = g - \frac{\partial g}{\partial H} H \ldots \ldots \ldots . \qquad (1)$$



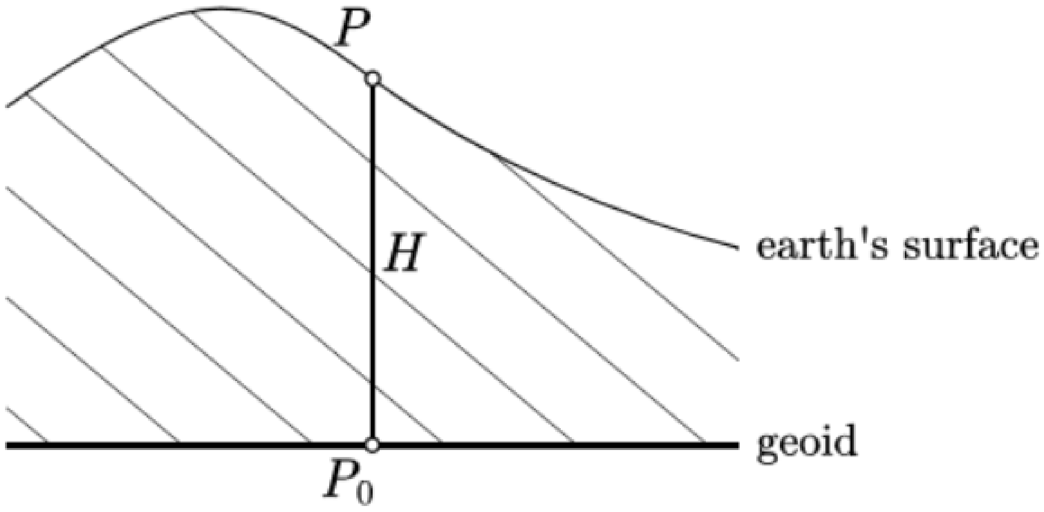
where,

H = height between P, the gravity station above geoid, and P_0, the correspoinding point on geoid.

Cont…

- Suppose there are no masses above the geoid and neglecting all terms but the linear one, we have

$$g_0 = g - \frac{\partial g}{\partial H} H \longrightarrow g_0 = g + F \qquad (2)$$



where,

$$F = -\frac{\partial g}{\partial H} H \longrightarrow (3)$$

- This $F = -\frac{\partial g}{\partial H} H$ is the free-air reduction to the geoid

$$g0 = g - \frac{\partial g}{\partial H} H \ldots\ldots\ldots.$$

Remark :: The assumption of no mass above geoid in a sense

that the masses have been mathematically removed,

so that this reduction is indeed carried out "in free-air"

Cont.....

- For many practical purpose geodesist also use normal gradient of gravity (i.e associated with the ellipsoidal height h) $\partial\gamma/\partial h$ instead of $\partial g/\partial H$ as ::

$$F = -\frac{\partial\gamma}{\partial h} h = +0.3086h \text{ mgal} \longrightarrow (4)$$

We have $g = \frac{GM}{R^2}$

$$\frac{dg}{dR} = -\frac{2GM}{R^3} = -\frac{2g}{R}$$

$$F = -\frac{\partial g}{\partial H} H = +0.3086\,H$$

At 45 degree latitude, $\frac{2g}{R}$ = 0.3086 mgal

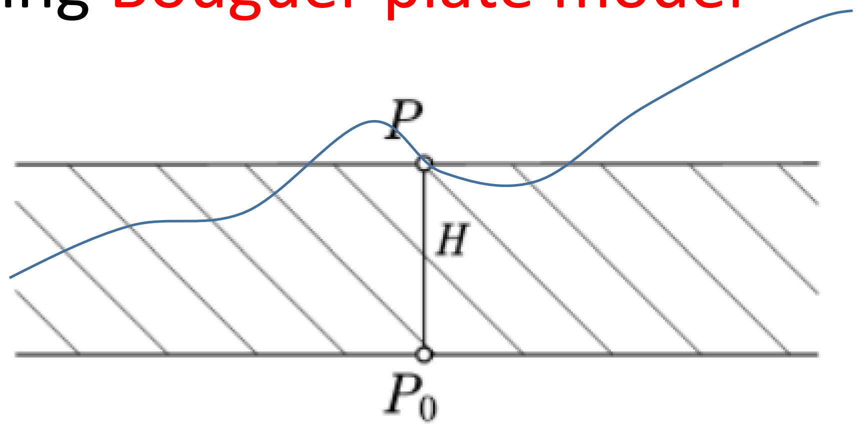
Finally, the free air correction is the amount that must be added to the measurement at height h to correct it a reference level

$$::: F = \frac{2g}{R} * h = 0.3086 * h \; Mgal$$

Bouguer reduction

- In this reduction, it assume the complete removal of the topographic masses, that is the masse outside the geoid using Bouguer plate model

❖ Bouger plate model

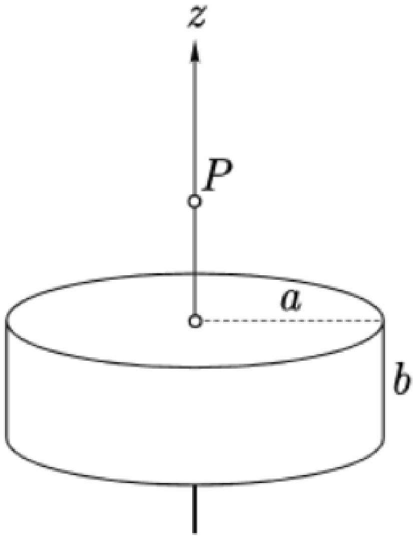


- Assume the area around the gravity station 'P' to be completely flat and horizontal, having the masses between geoid and earth's surface having constant density ' ρ'

Cont….

- We have, for the case of a cylinder for a point on the cylinder the potential is given as::

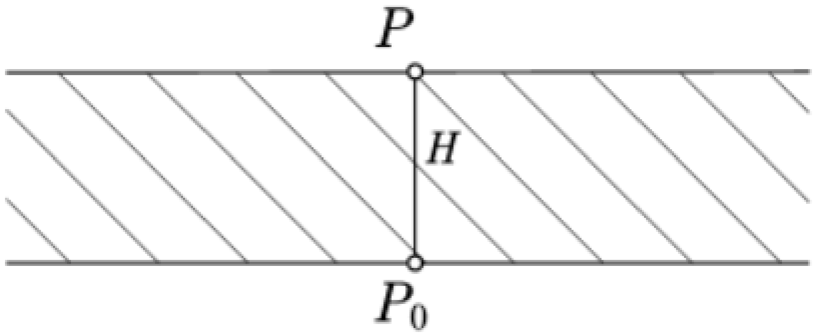
$$A_0 = 2\pi G \varrho \left[a + b - \sqrt{a^2 + b^2}\right]$$



- In case of Bouguer plate let say,

a $\longrightarrow$ ∞

b $\longrightarrow$ H

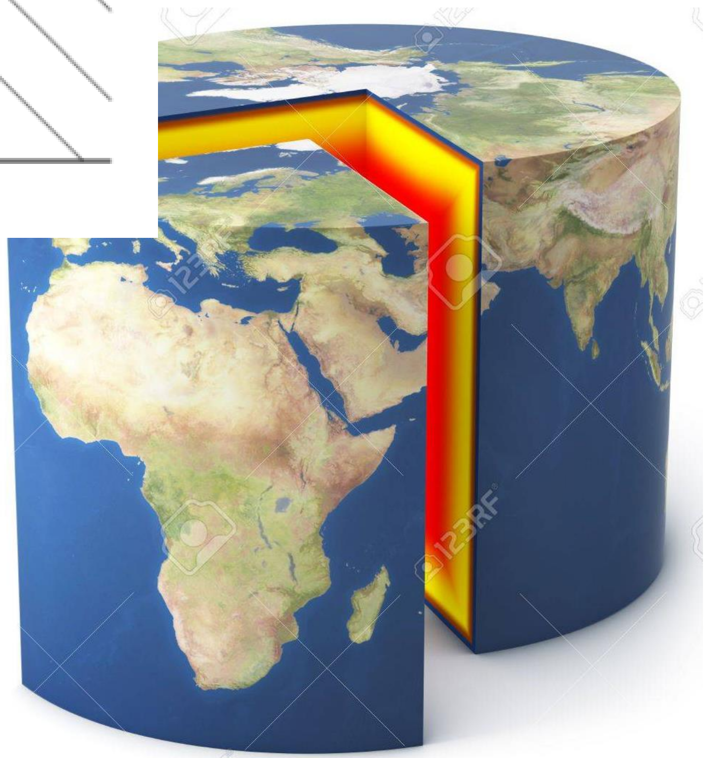


Than the total potential will be::

$$A_B = 2\pi G \varrho H$$

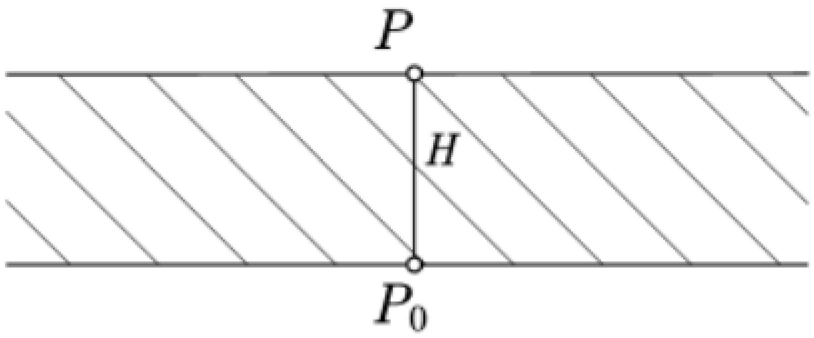
where,

ρ=avg. crustal density = 2.67gcm^{-3}



Cont…

- So, the attraction of the infinite Bouguer plate with 2.67gcm^{-3} become:::



$$A_B = 2\pi G \varrho H$$

or, $A_B = 0.1119\, H$ [mgal]

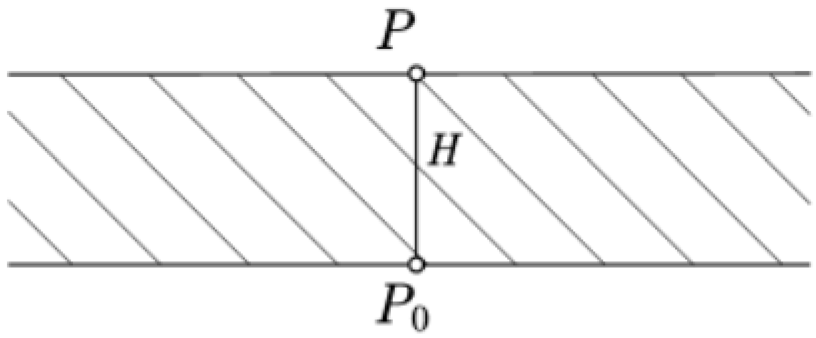
- Subtracting the Bouguer attraction from the observed gravity = incomplete gravity reduction
- To obtained the complete Bouguer reduction we have to apply the Free-air reduction F.
- This combined process of removing topographic masses and applying free-air reduction is called complete Bouguer reduction

i.e $g_0 = g - A_B + F \longrightarrow$ (5)

Cont…

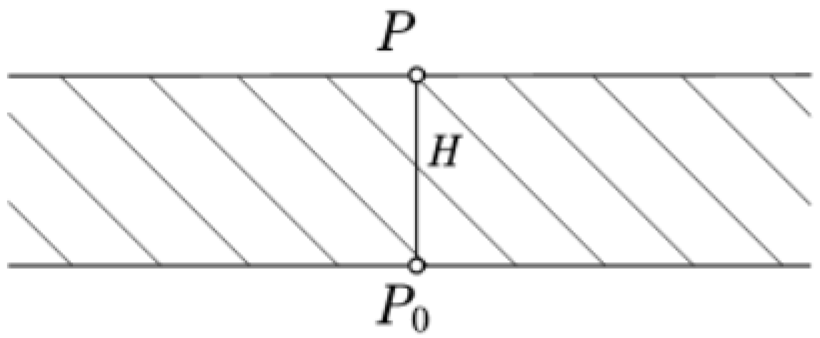
- Thus

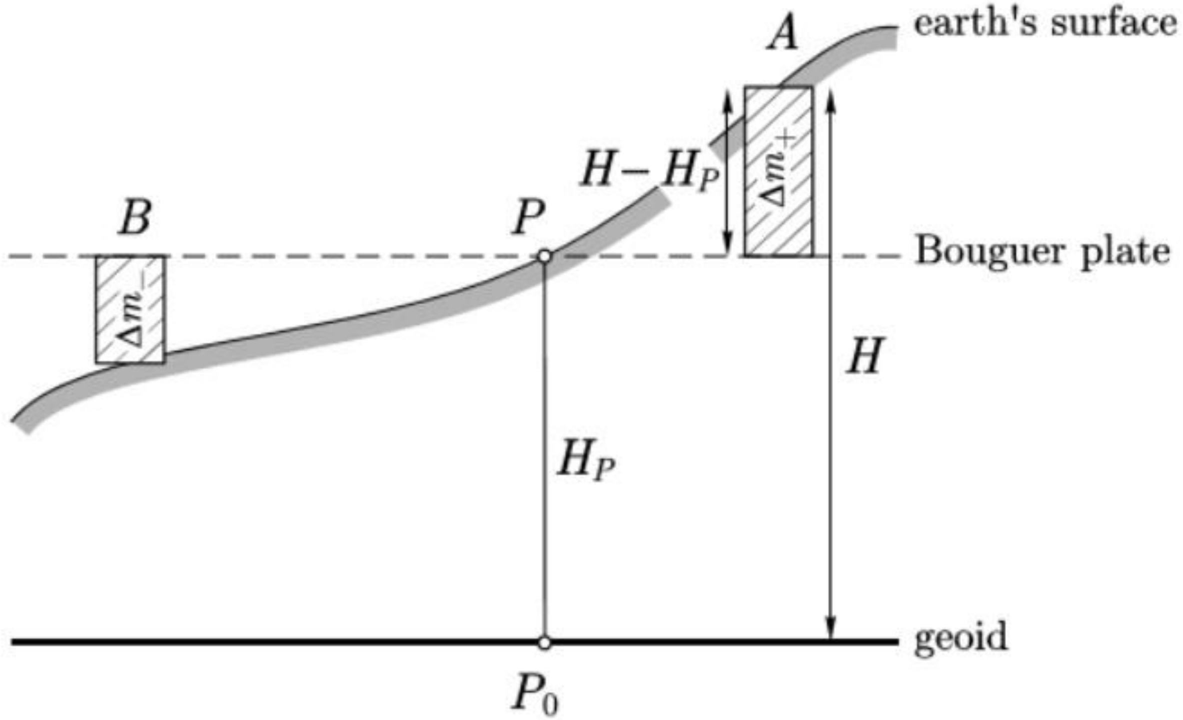
gravity measured at P	g
minus Bouguer plate	$-0.1119\ H$
plus free-air reduction	$+0.3086\ H$
Bouguer gravitya t P_0	$g_B = g + 0.1967\ H.$



Terrain correction

- The deviation of actual topography from Bouguer plate P is called terrain correction or Topographic correction.



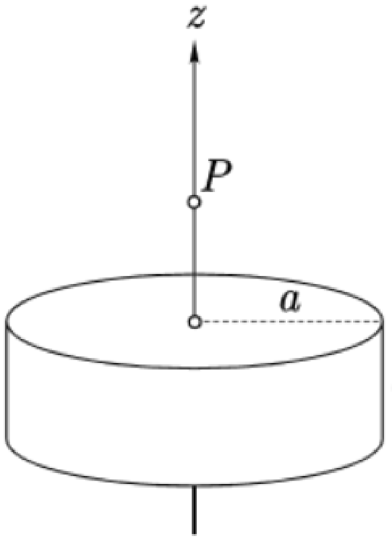


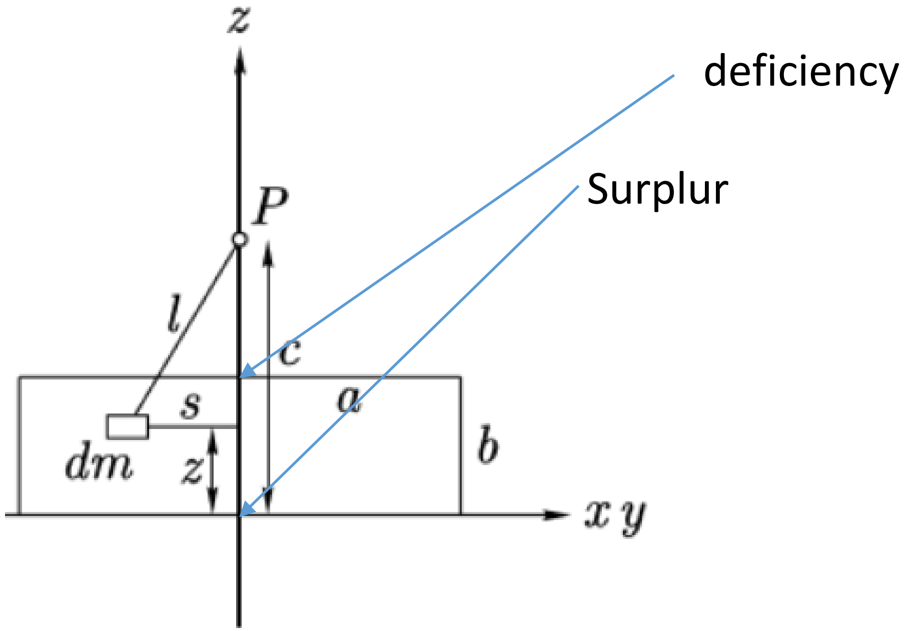
- At ‘A’ the mass surplus Δm+ which attracts upward is removed, causing ‘g’ at ‘P’ to increase.
- At ‘B’ the mass deficiency Δm- is made up, causing ‘g’ at ’P’ to increase
- The terrain correction is always +ve

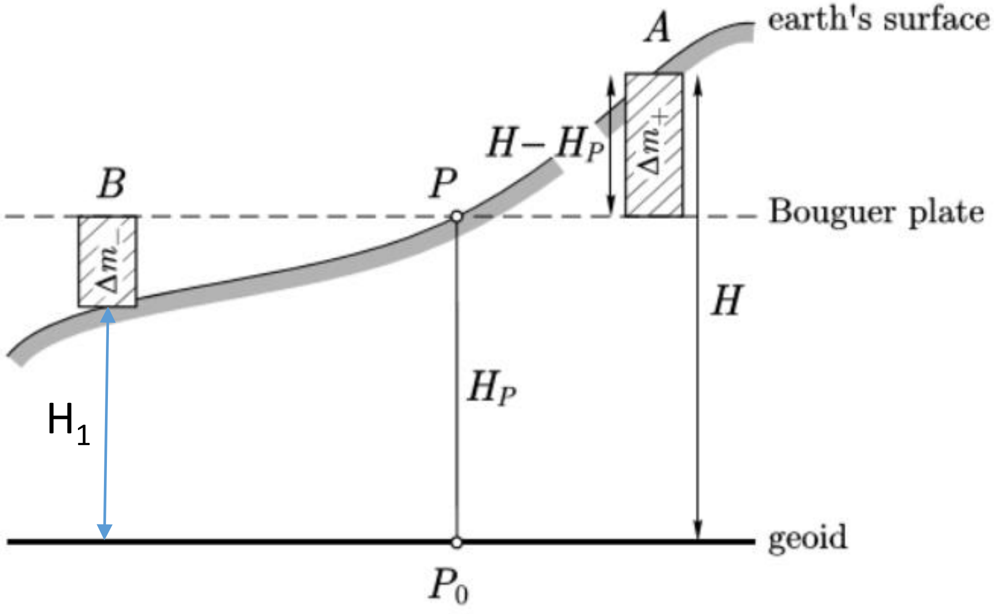
Cont…

- The terrain correction (A_t) is done by adding the effect of the individual compartment

i.e $A_t = \sum \Delta A.$







For surplus mass Δm+ $H>H_P$ $b= H-H_P$ $c=0$	For mass deficiency Δm- $H_1<H_P$ $b = Hp-H_1 = c$

Cont….

- Using the above mentioned height for the computation of potential of total compartment

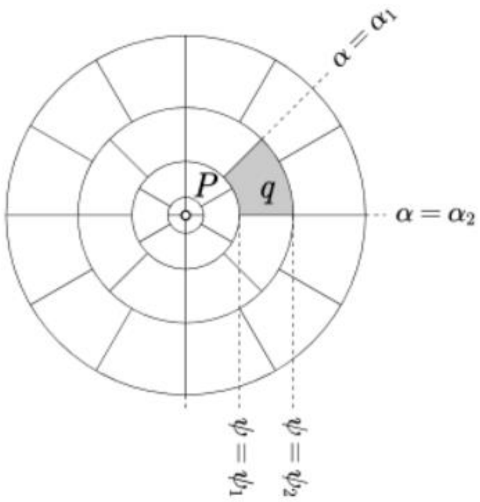
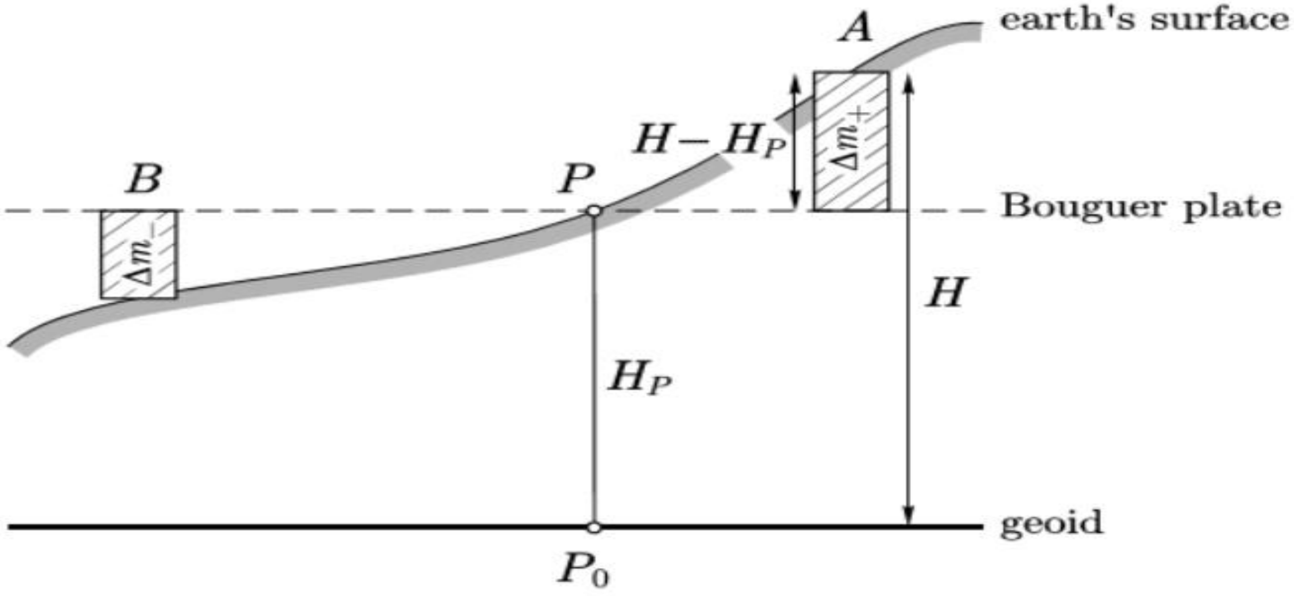
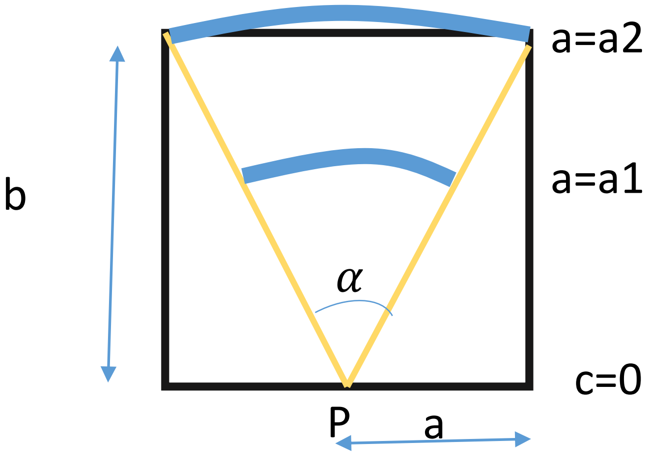
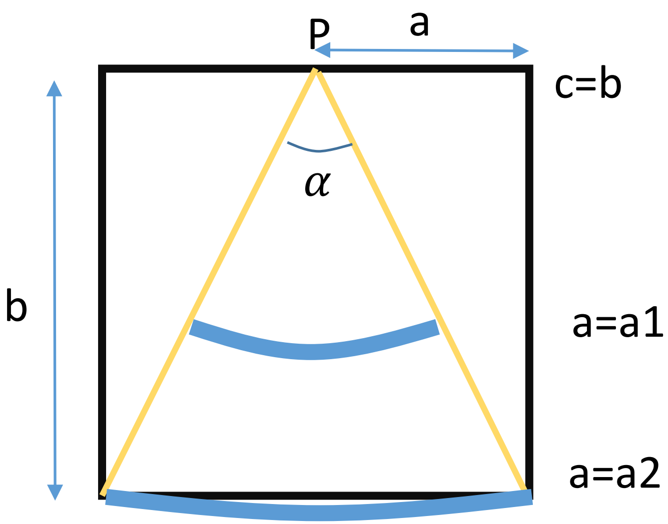


Fig. 2.20. A template







$$\Delta A = \frac{2\pi}{n} G\varrho \left[\sqrt{a_2^2 + (c-b)^2} - \sqrt{a_1^2 + (c-b)^2} - \sqrt{a_2^2 + c^2} + \sqrt{a_1^2 + c^2}\right].$$

Cont….

- Finally from computation of A_t, and applying this correction to

$$g_B=g-A_B+F$$

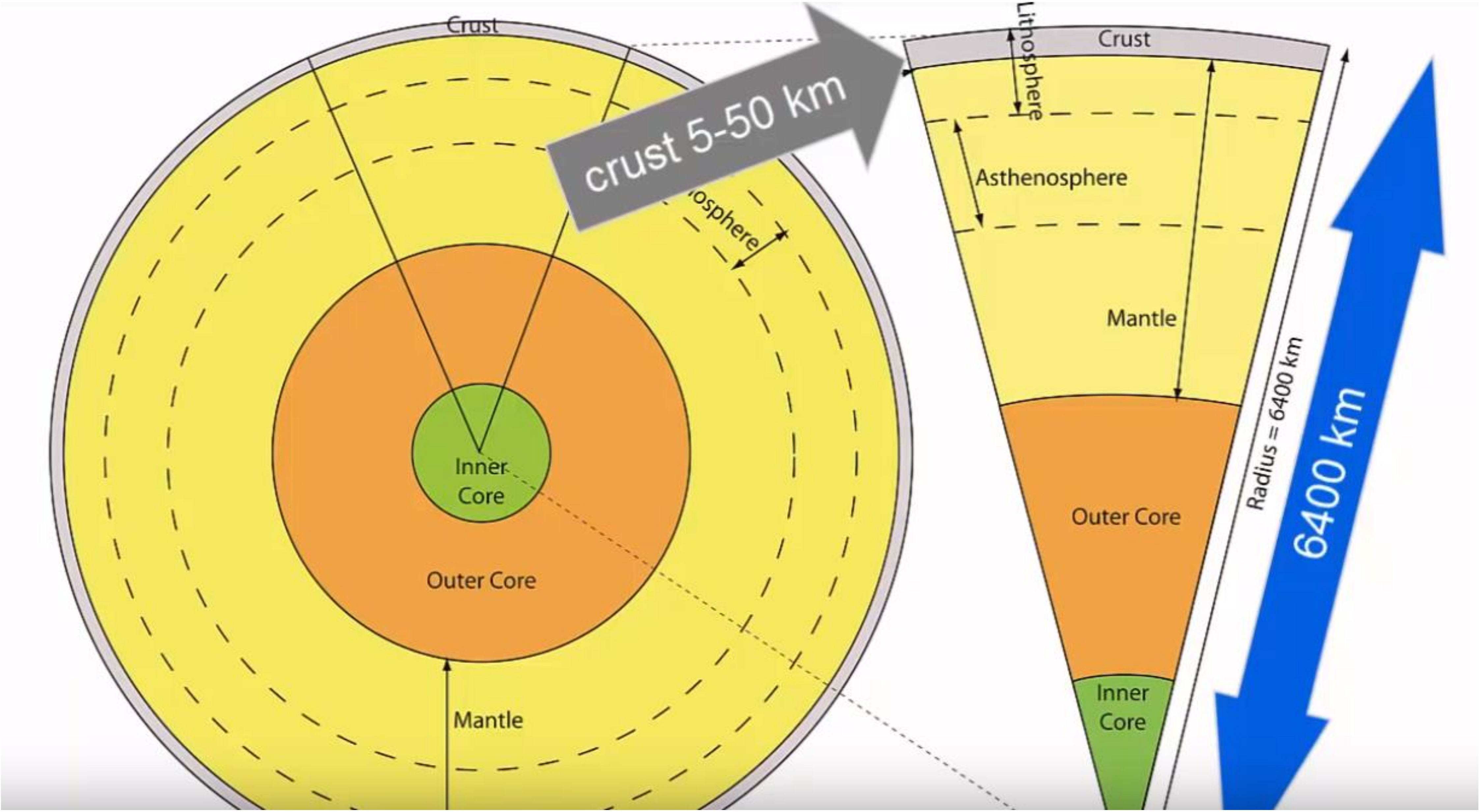
and and as it is +ve

$$g_B=g-A_B+F+A_t$$

- This is known as refined or simple Bouguer gravity reduction

Poincare and Prey reduction= Assignment

Isostasy



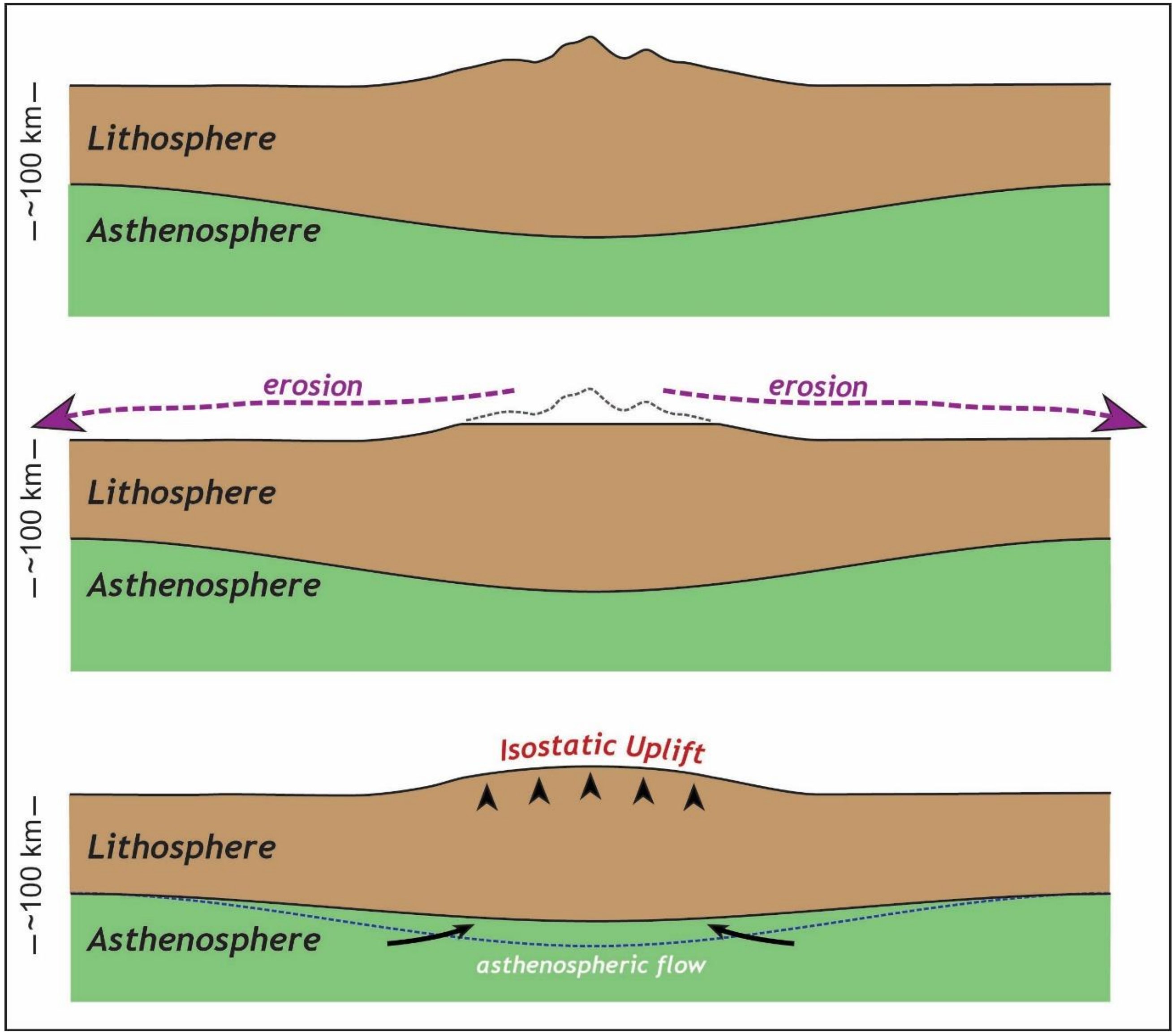
Crust
crust 5-50 km
osphere
Inner
Core
Outer Core
Mantle
Lithosphere
Crust
Asthenosphere
Mantle
Outer Core
Inner
Core
Radius = 6400 km
6400 km

Isostasy

- Term first coined by American geologist Dutton in 1859 then discovered by Bouguer and later confirmed by Pratt
- It is simply a state of gravitational equilibrium between earth crust and mantle such that crust floats at an elevation that depends on its thickness and density
- It is based on the principle of upthrust where an object immersed in a fluid floats with a force equal to the weight of the displaced fluid

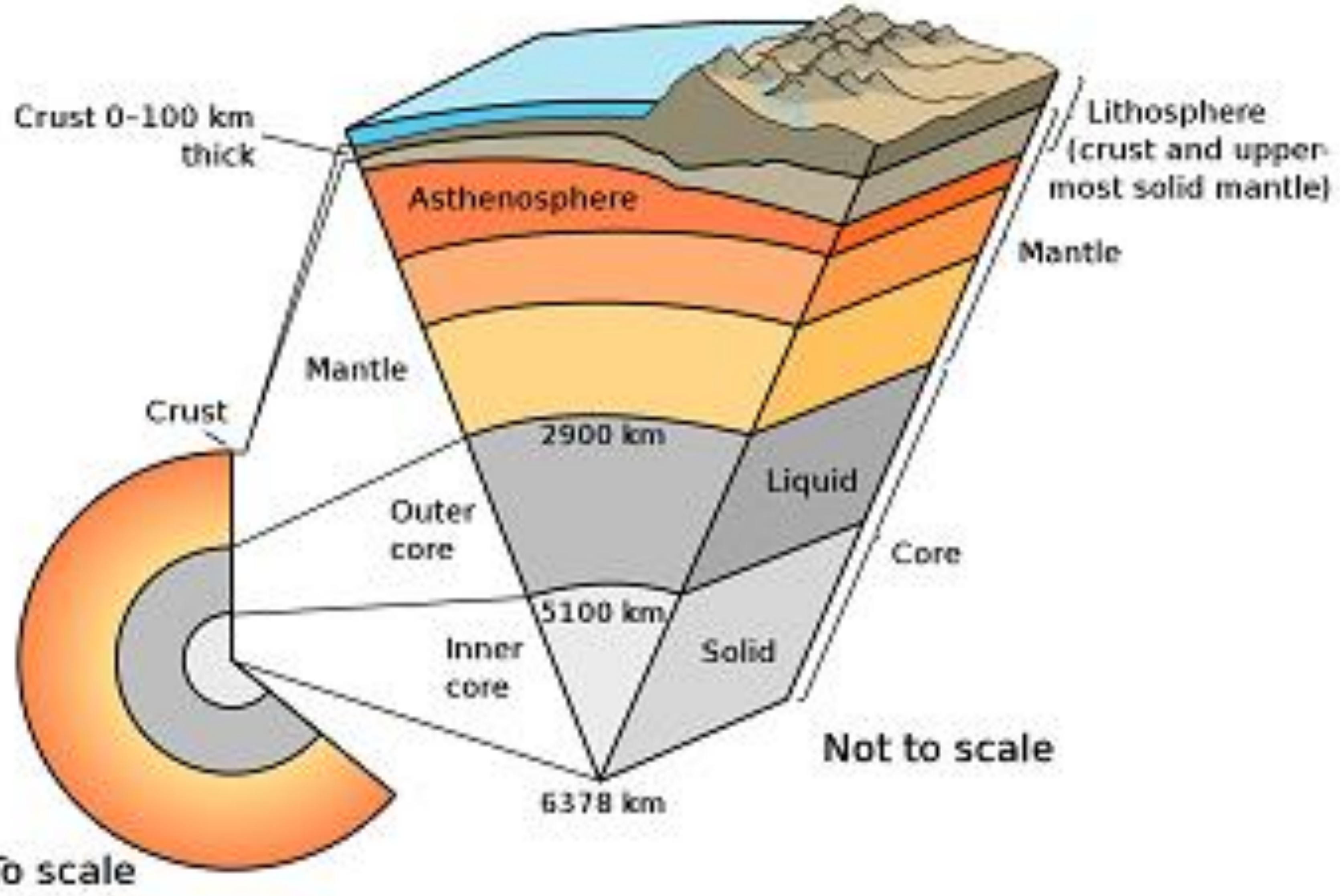
Isostasy

Cont….



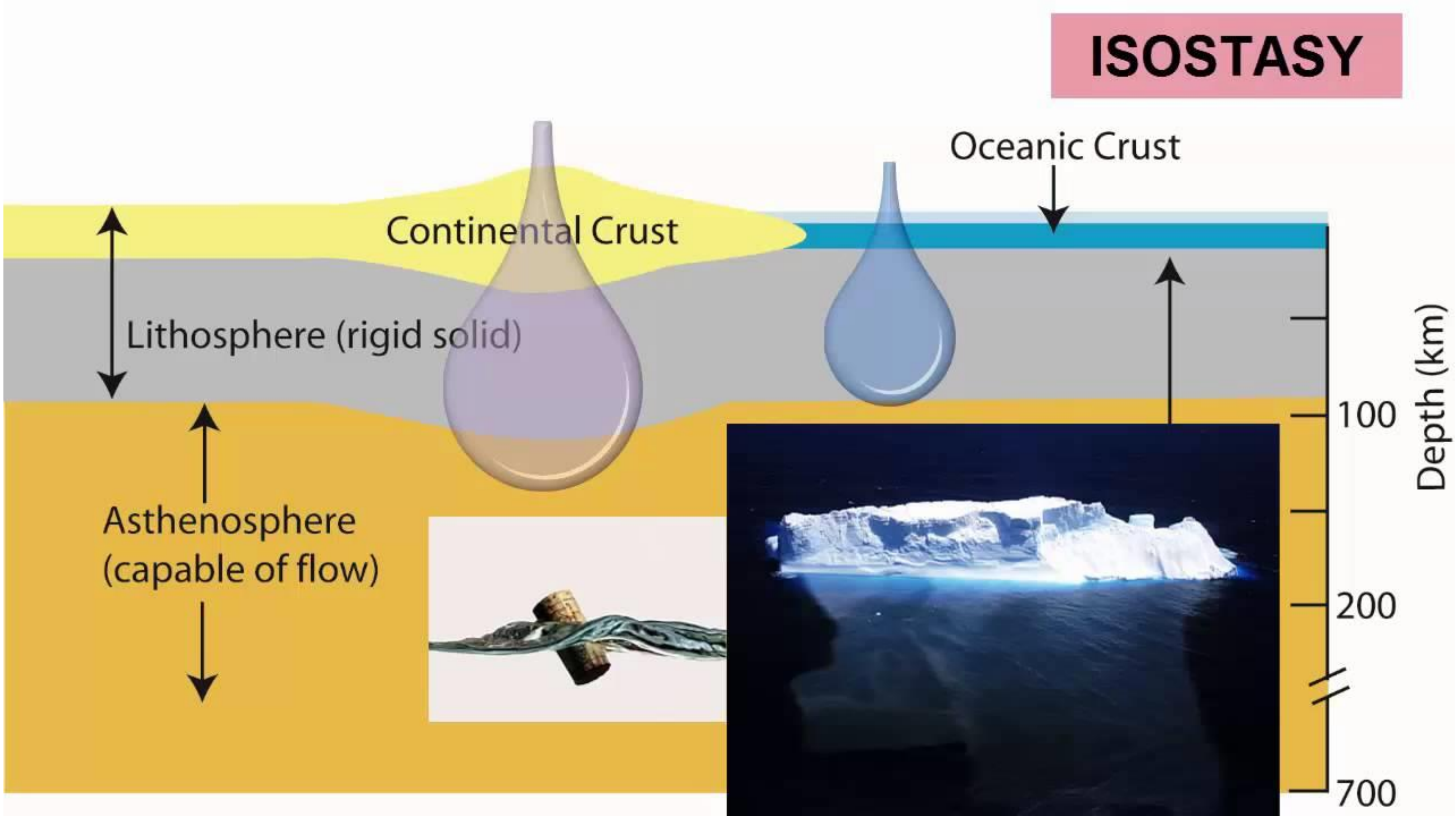
~100 km
Lithosphere
Asthenosphere
erosion
erosion
~100 km
Lithosphere
Asthenosphere
Isostatic Uplift
~100 km
Lithosphere
Asthenosphere
asthenospheric flow

Isostasy



Crust 0-100 km thick
Asthenosphere
Lithosphere (crust and upper most solid mantle)
Mantle
Mantle
Crust
2900 km
Liquid
Outer core
Core
5100 km
Inner core
Solid
Not to scale
6378 km
To scale

Isostasy


ISOSTASY
Oceanic Crust
Continental Crust
Lithosphere (rigid solid)
Asthenosphere
(capable of flow)
100
200
700
Depth (km)

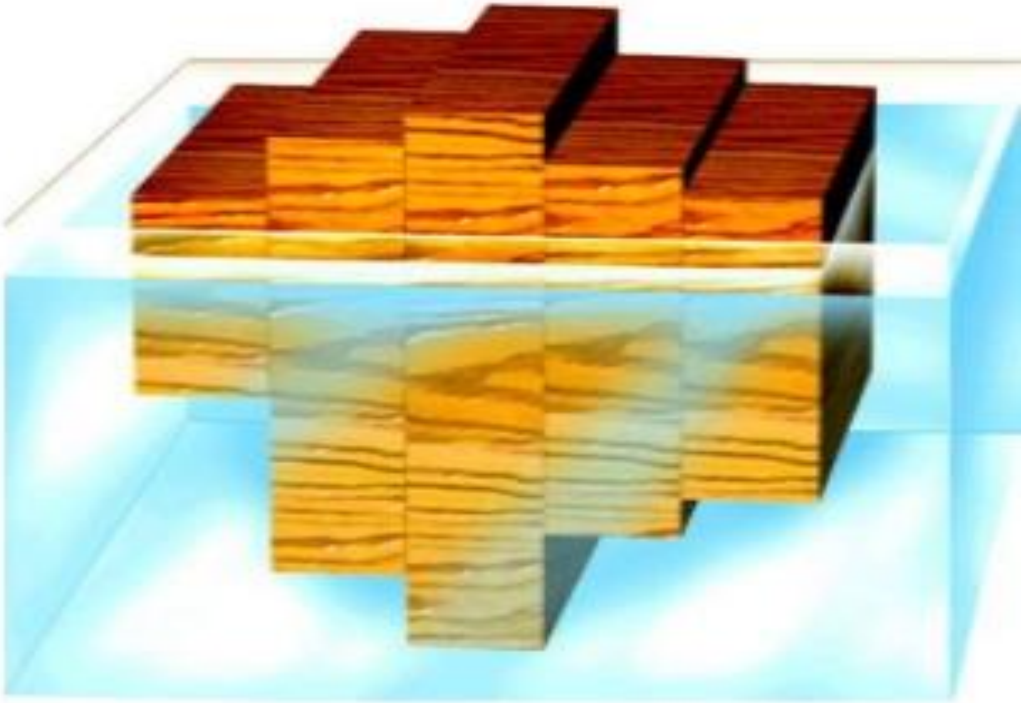
Contd

Isostasy

- **Isostasy** (Greek *ísos* "equal", *stasis* "standstill")
- Term used in geology to refer to the state of gravitational equilibrium between the earth's lithosphere and asthenosphere.

➢Isostasy: a state of gravitational equilibrium in which an area of crust "floats" in a balanced way on the denser rock of the mantle below.

➢The elevation of any part of the Earth's crust is a function of the THICKNESS and DENSITY of the crust.

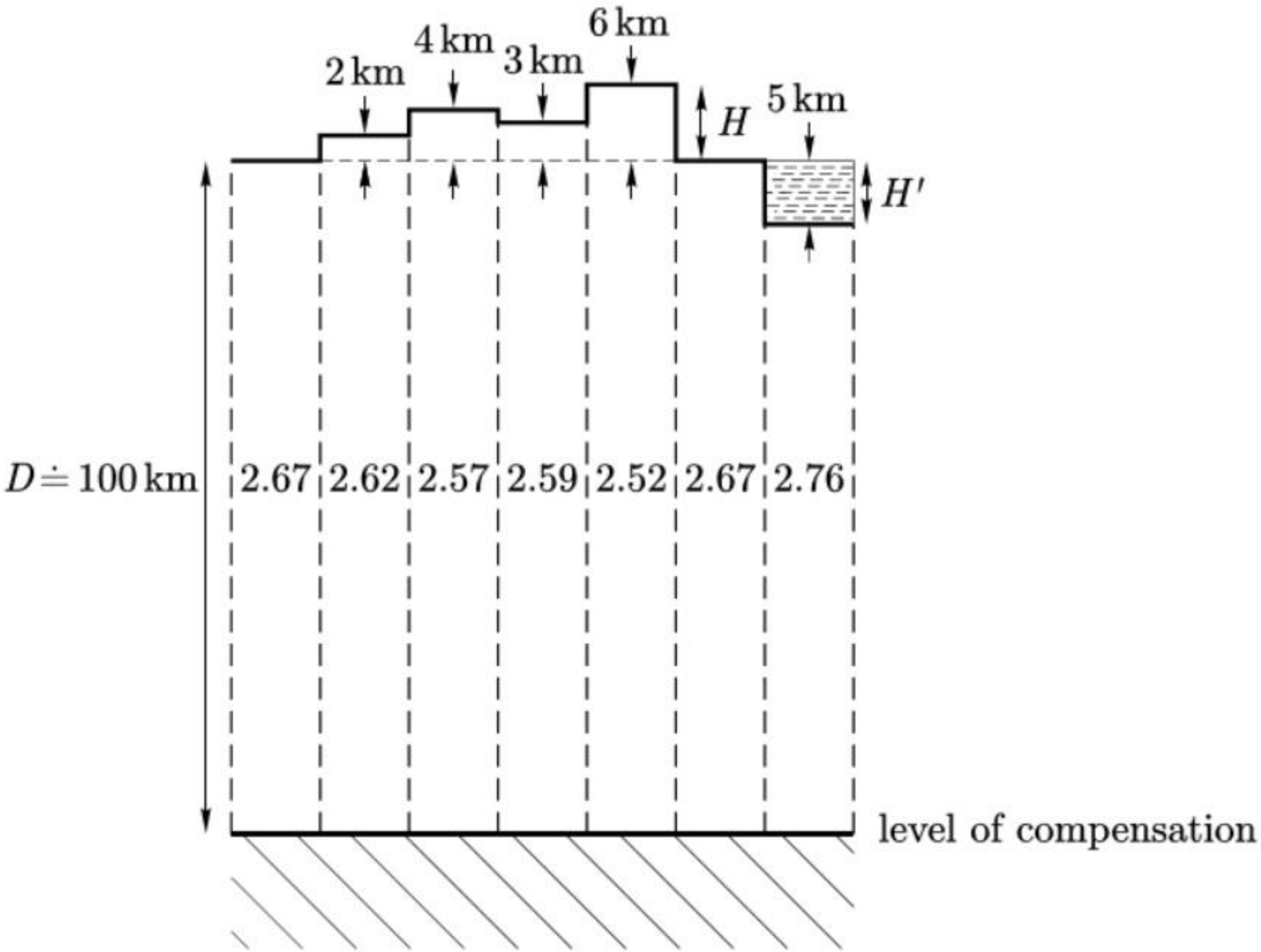


Contd…

- The earth surface is continuously rises and sink isostatically as it is removed or added.
- Mainly 3 basic model for isostasy
  1. Pratt-Hayford System
  2. Airy-Heiskanen System
  3. Vening meinesz Regional System

Pratt-Hayford System

- Fig



Cont…

- This model is outlined by Pratt and put into a mathematical form by Hayford
- Underneath the level of compensation there is uniform density.
- Above the mass of each column of the same cross section is equal
- Let D = depth of level of compensation, reckoned from sea level

  ρ_0 = density of column of height D

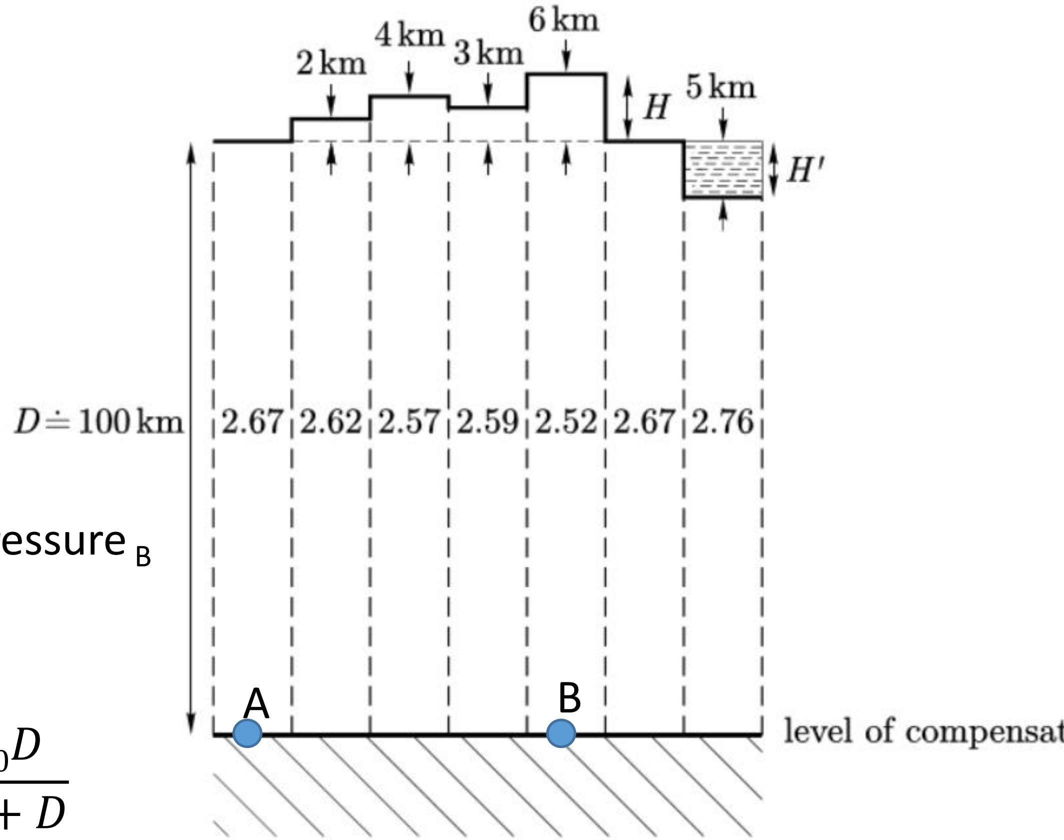
  Than the density ' ρ' of column of height

  (D+H) satisfy the following equation;

$$(D+H)\,\varrho = D\varrho_0$$

This is the condition for equilibrium

- Using ρ0 = 2.67gcm-3
- ::::Thus $\rho = 2.67\dfrac{D}{D+H}$



Pressuer $_A$ = Pressure $_B$

ρgh

ρ_0D=(H+D) ρ

$$\rho = \frac{\rho_0 D}{H+D}$$

Cont….

- For oceanic case, the condition for equilibrium is

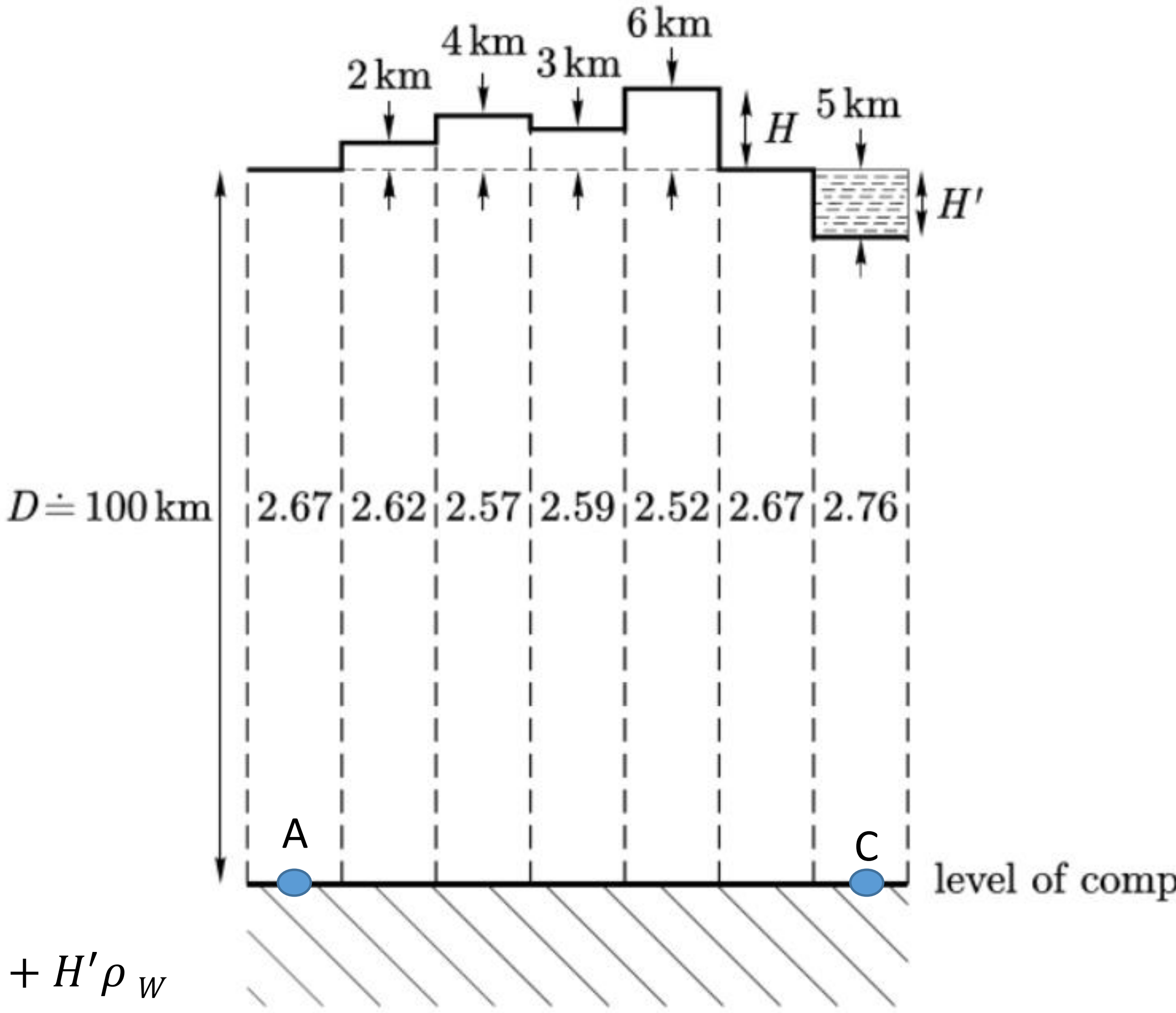
$$(D - H')\,\varrho + H'\,\varrho_w = D\varrho_0$$

where,

ρw= 1.027gcm-3

H'=depth of ocean

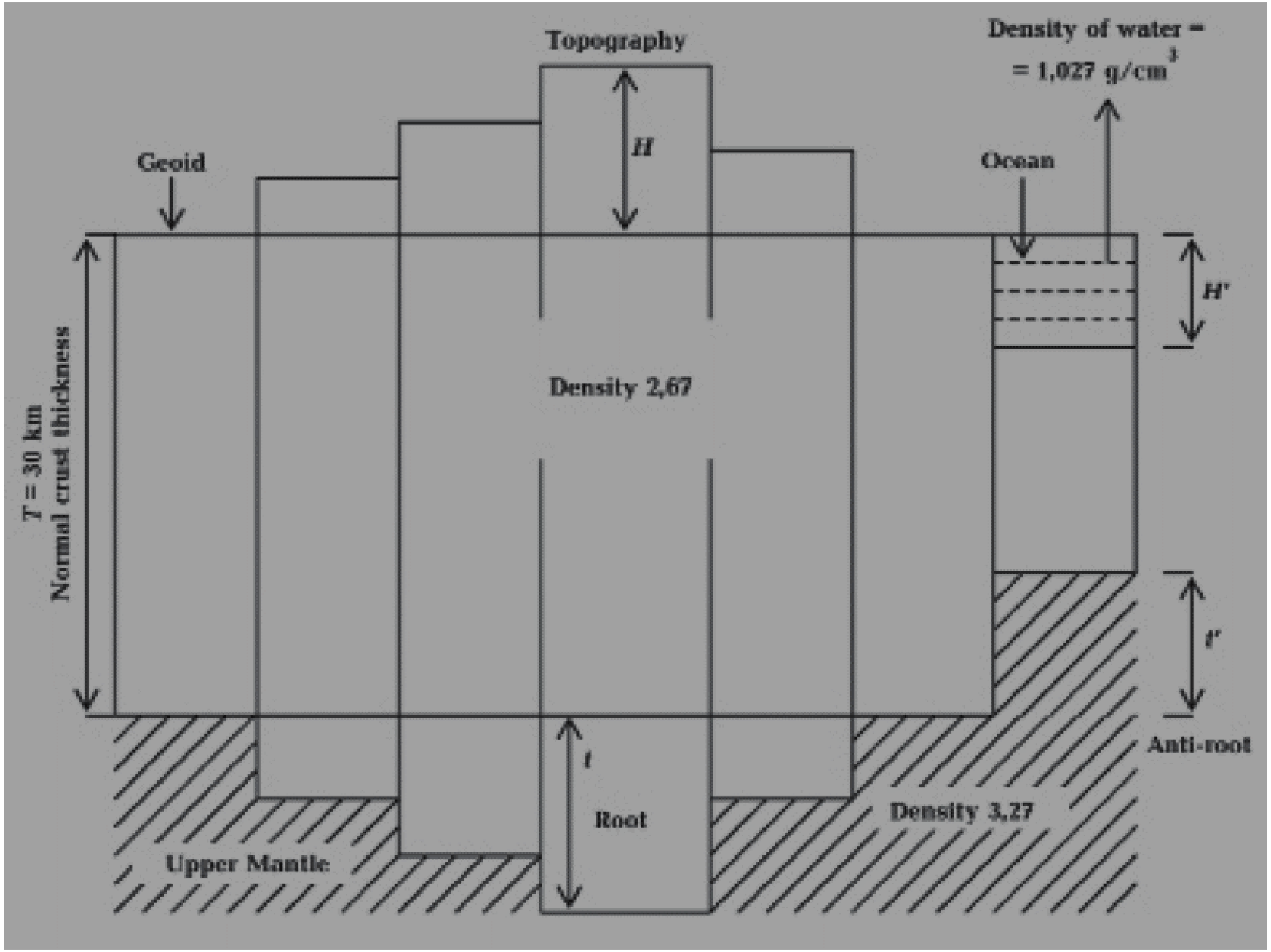
- Thus $\rho = \frac{1}{D-H'}(D\,\rho_0 - H'\,\rho_w)$



Press $_A$ = Press $_C$

$$\rho_0 D = (D - H')\rho + H'\rho_W$$

Airy-Heiskanen



Airy-Heiskanen system

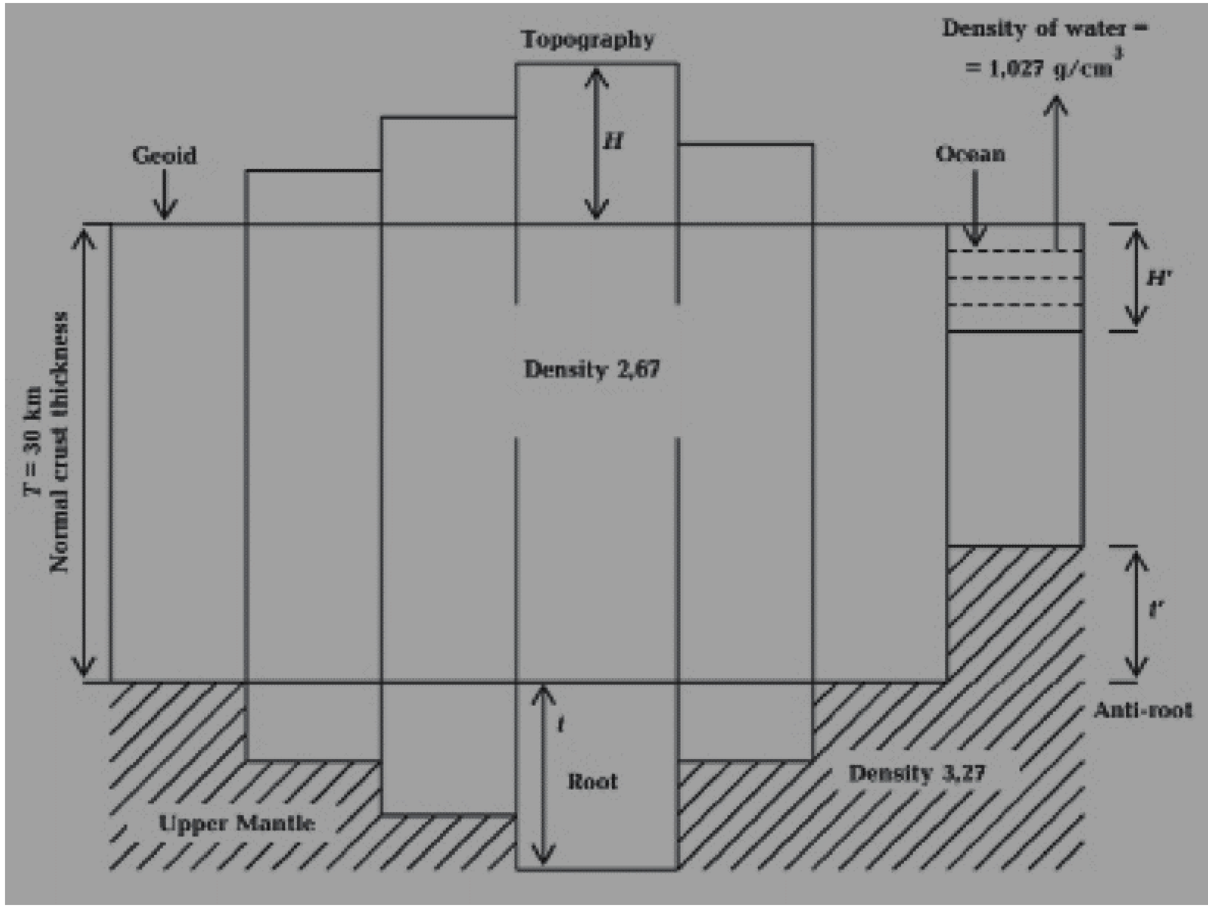
- Airy proposed this model and Heiskanen gave its precise formulation
- The mountain of constant density $\rho 0=2.67gcm^{-3}$ floats on a denser underlying of constant density $\rho 1=3.27gcm^{-3}$
- The higher they are, the deeper they sink
- The density difference

$$\Delta\rho = \rho 1 - \rho 0 = 0.6gcm^{-3}$$

- Denoting root by 't' than the condition of floating equilibrium is

$$t\Delta\varrho = H\varrho_0$$

or, $$t = \frac{\varrho_0}{\Delta\varrho} H = 4.45\ H$$



Contd….

- So if we consider a Line of compensation passing through the end portion of roots then,

pressure at A = Pressure at B

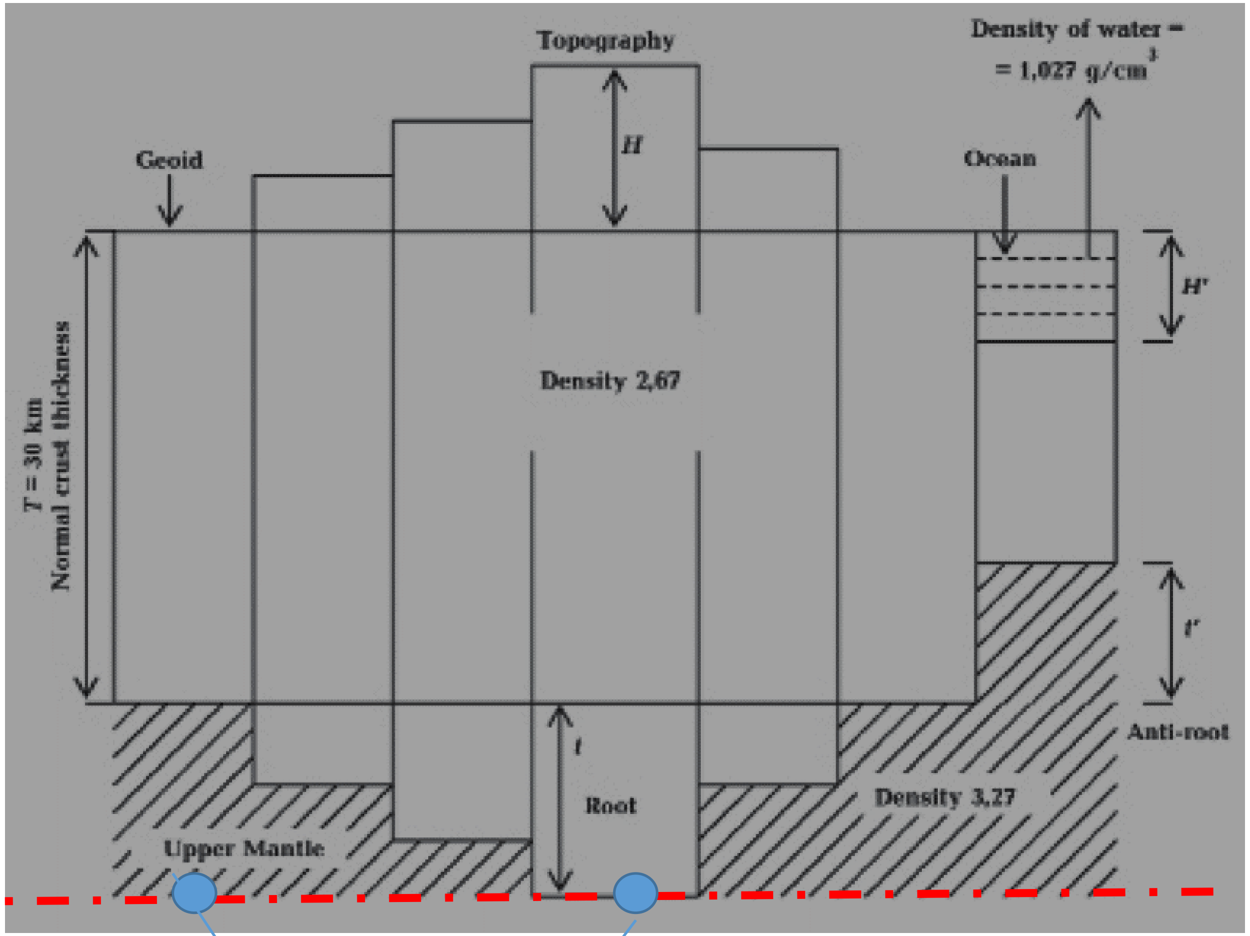
$$\rho_0 * T + \rho_1 * t = (H+T+t)\,\rho_0$$

$$\rho_0 T + \rho_1 t = H\rho_0 + T\rho_0 + t\rho_0$$

$$t(\rho_1 - \rho_0) = H\rho_0$$

$$t = \frac{H\rho_0}{\rho_1 - \rho_0}$$

$$t = \frac{\varrho_0}{\Delta\varrho} H = 4.45\ H$$



Press A = Press B (ρgh)

$\rho 0 * T + \rho 1 * t = (H+T+t)\ \rho 0$

Cont….

- For oceanic areas, the equilibrium condition is given as

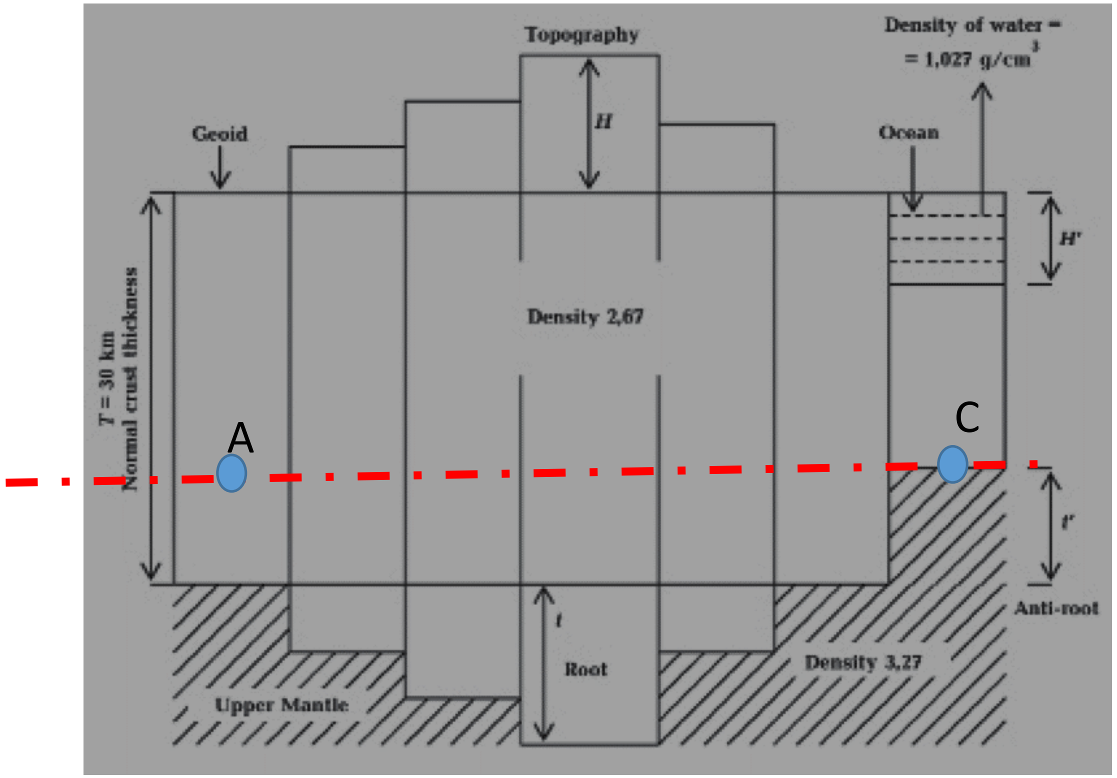
$$t' \Delta\varrho = H' (\varrho_0 - \varrho_w)$$

where,

H’ = depth of ocean

$\rho w = density\ of\ water$

t’ = anti-root (under ocean)



- Thus

$$t' = \frac{\varrho_0 - \varrho_w}{\varrho_1 - \varrho_0} H' = 2.73\ H'$$

Press $_A$ = Press $_C$

$$\rho_0 T - t'\rho_1 = (T - H' - t')\rho_0 + H'\rho_W$$

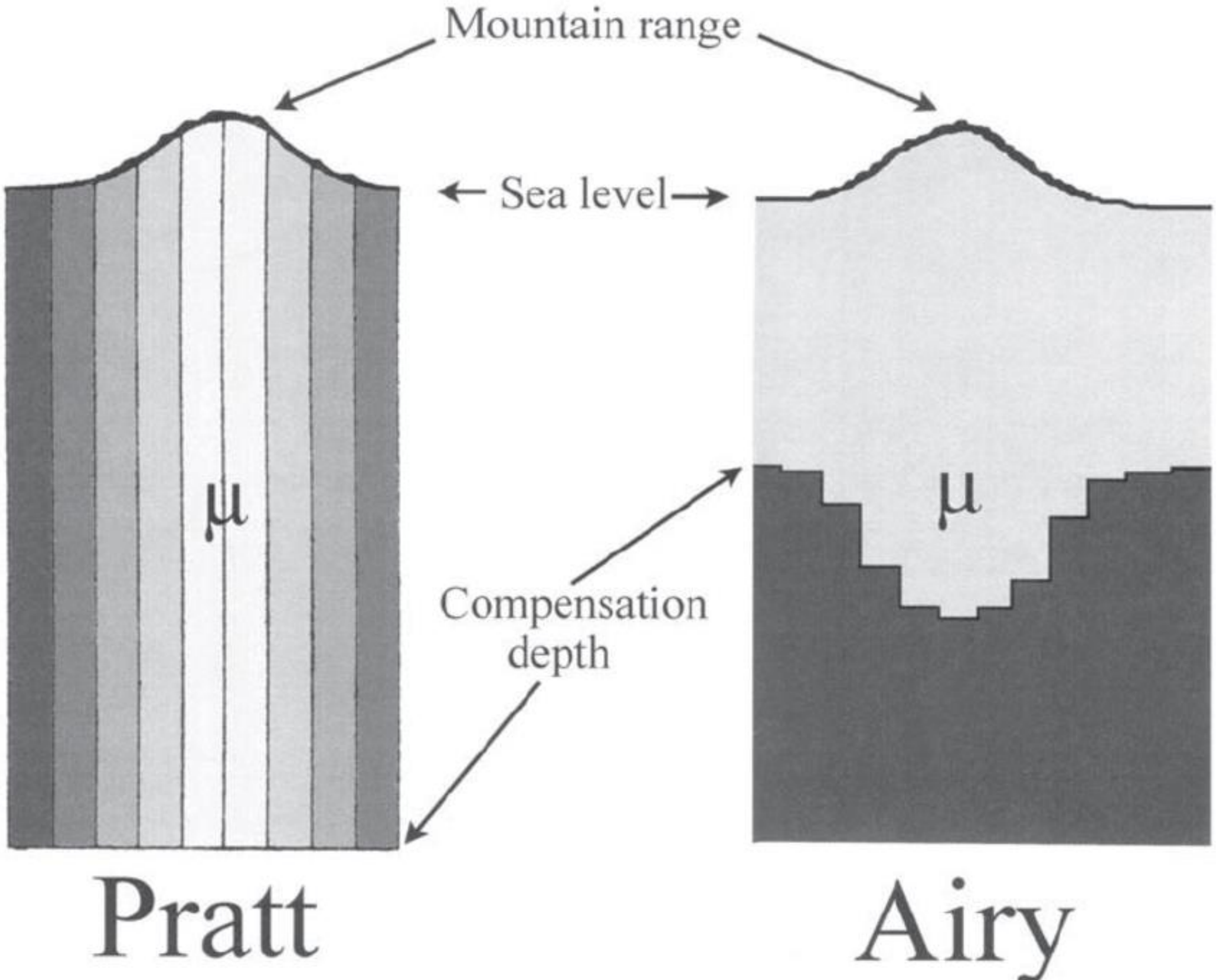
$$t' = \frac{\rho_0 - \rho_w}{\rho_1 - \rho_0} \mathrm{H}'$$

Therefore,

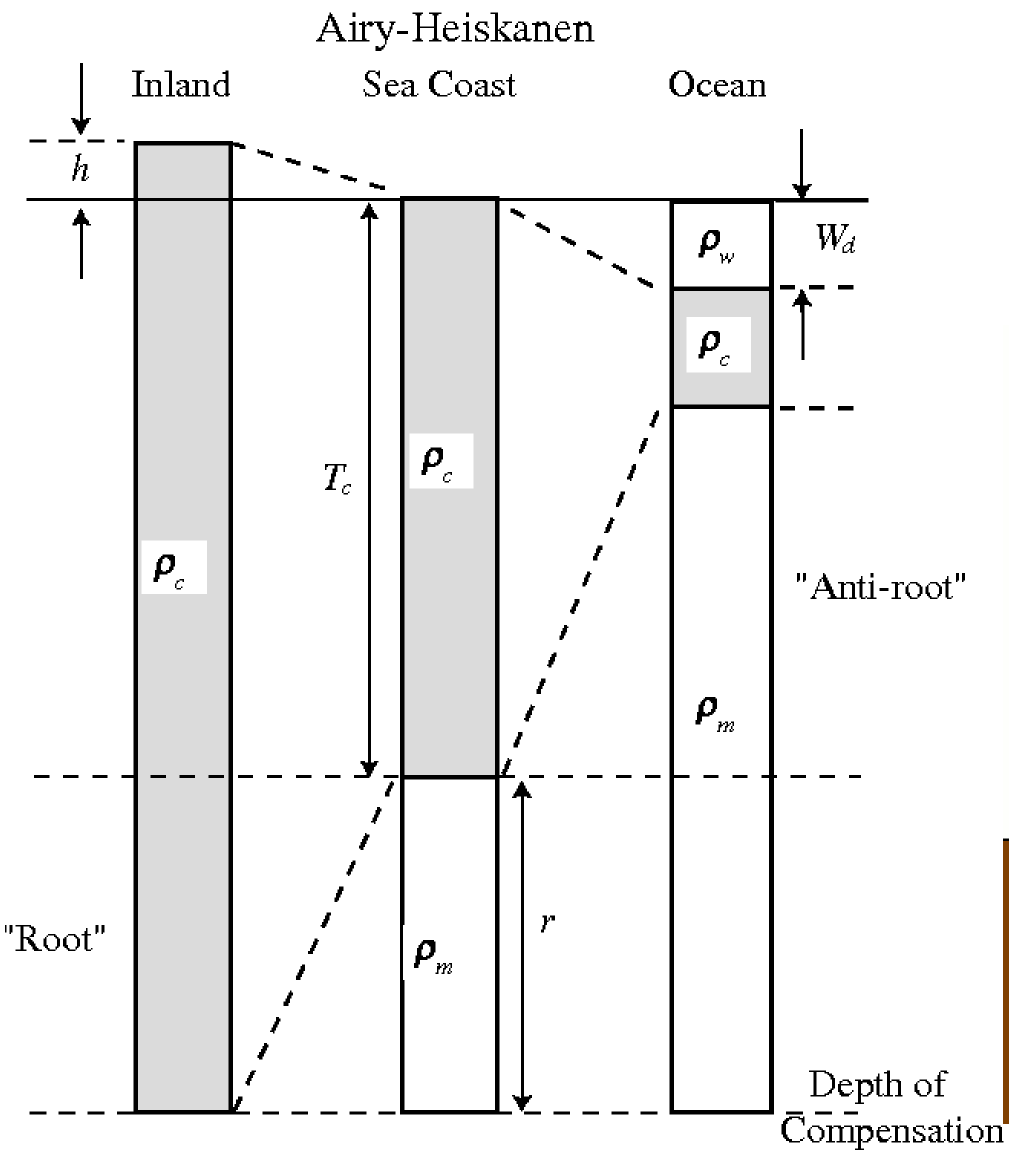
t = 4.45H

t’ = 2.73H’

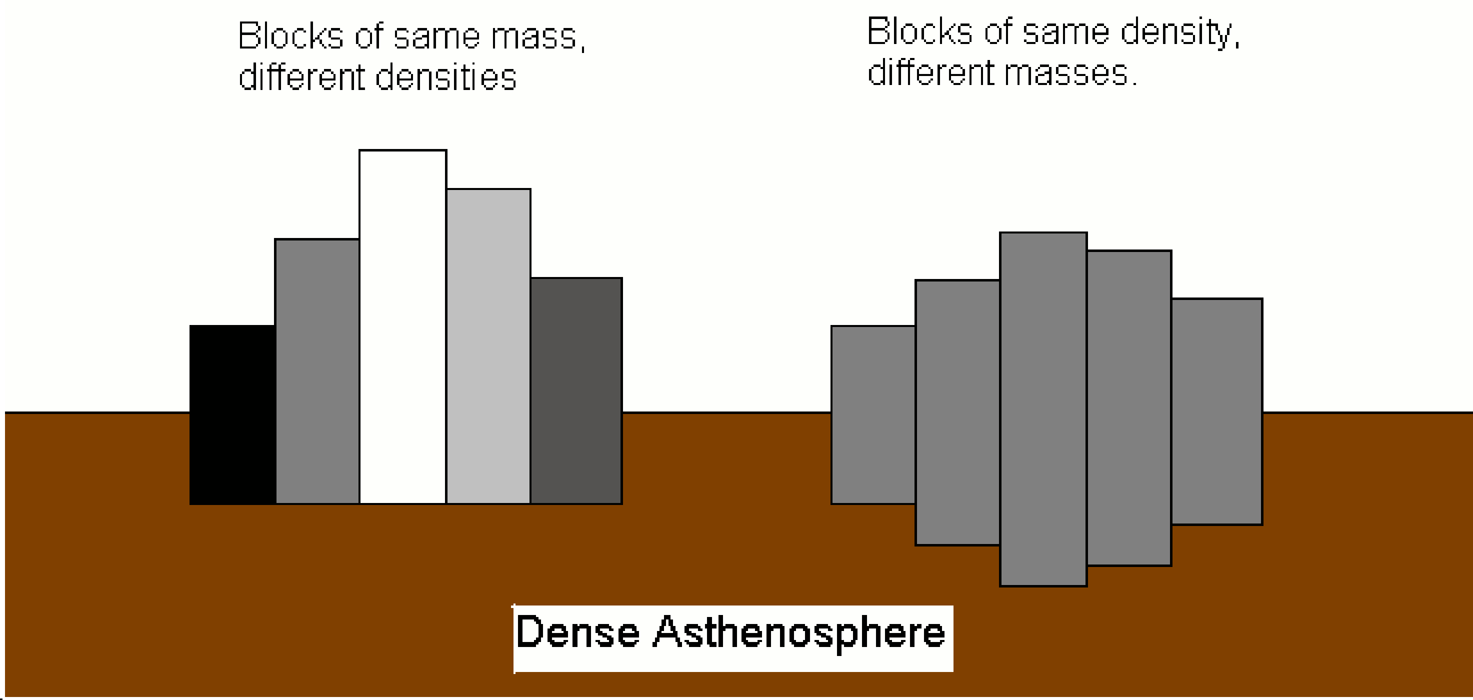
Contd…



Contd...

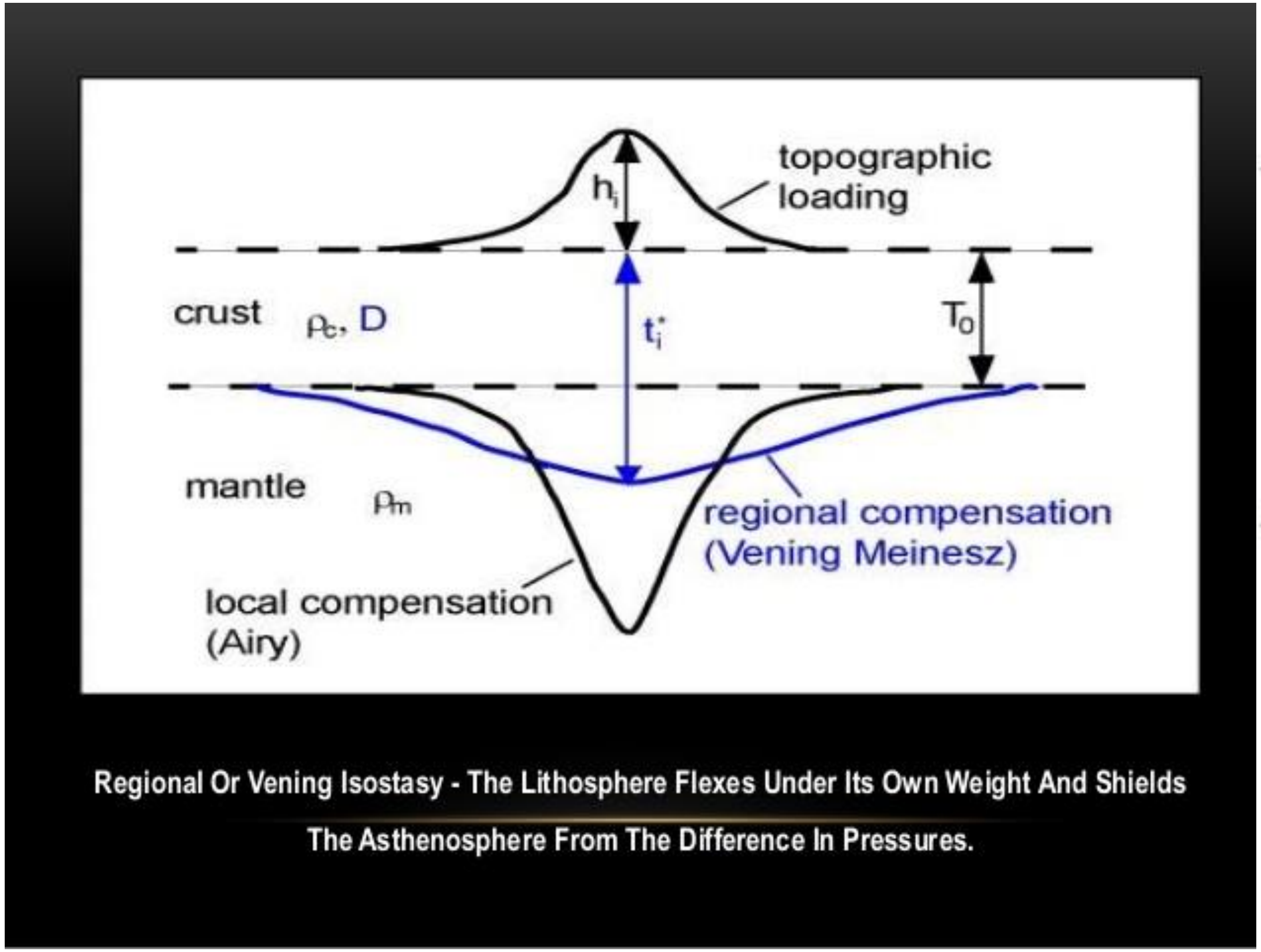


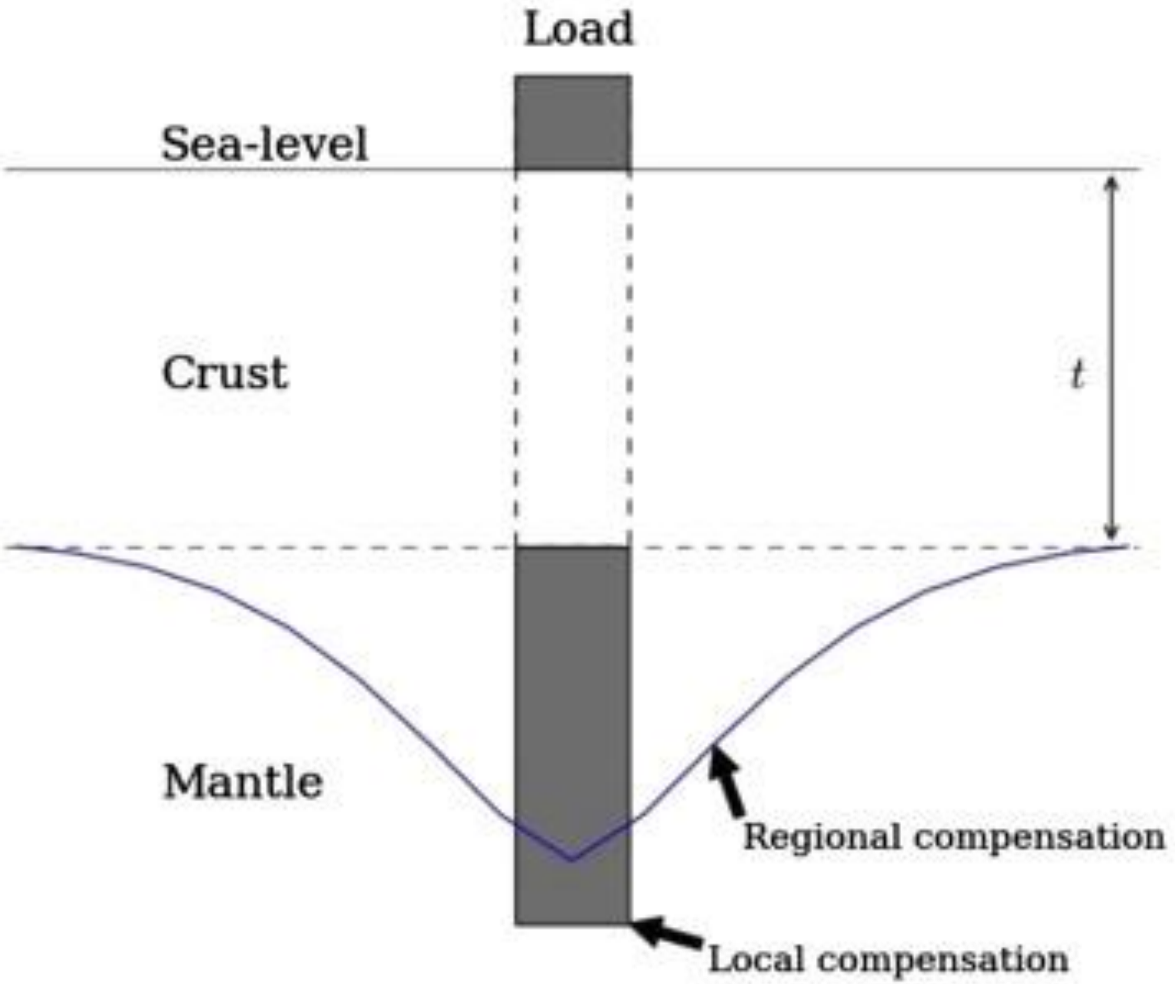
Airy-Heiskanen
Inland
Sea Coast
Ocean
h
T_c
ρ_c
ρ_c
ρ_w
W_d
ρ_c
"Anti-root"
ρ_m
"Root"
ρ_m
r
Depth of Compensation



Blocks of same mass, different densities
Blocks of same density, different masses.
Dense Asthenosphere

Vening - Meinesz regional system





Cont....

- Both Pratt-Hayford and Airy-Heiskanen system assume the compensation takes place along vertical column i.e local compensation
- But this method of reduction pre-suppose free mobility of masses introducing the regional compensation instead of local compensation