1 Introduction

Engineering economics is the application of economic techniques for the evaluation of design and engineering alternative. The role of engineering economic is to

- Access the appropriateness of a given project.
- Estimate its value
- Justify it from an engineering point of view

It involves the systematic and technical evaluation of analysis, with emphasis on the economic aspects and has the objective of assisting in decision making.

Helping

1.1 Origin of Engineering Economy

- The perspective that ultimate economy is a concern to the engineer and the availability of sound techniques to address this concern differentiate this aspect of modern engineering practice from that of the past.
- Pioneer: Arthur M. Wellington, civil engineer in latter part of nineteenth century. He addressed role of economic analysis in the engineering project. The main area of interest for him was railroad building, followed by other contributions which emphasized techniques depending on financial and actuarial mathematics.
- Later his work was followed by other Eugene Grant who published the first edition of his book which was the milestone in the development of engineering economy.
- In 1942 Woods and Degarmo published the first edition of the book, later entitled engineering economy.

1.2 Principles of Engineering Economics

Mainly there are seven principles related to engineering economics.

Principle1: Develop the Alternatives

There must be more than one alternatives, the choice is made among these after subsequent analysis.

Principle 2: Focus on the difference

Only the differences in expected outcomes among the alternatives are relevant to their comparison & should be considered in decision.

Principle 3: Use a consistent viewpoint

The likely outcomes of the various alternatives, initial cost, future saving etc. should be consistently developed from a defined view point.

Principle 4: Use a common unit of measure

Using a common unit of measurement to specify as many of the prospective outcomes as possible will make easier the analysis & comparison of the alternatives.

Principle 5: Consider all relevant criteria

Selection of best alternative required consideration of almost all relevant criteria. This includes both the outcomes specified in the monetary unit & those expressed in some other unit of measurement.

Principle 6: Make uncertainty explicit

Uncertainty is inbuilt in projecting the future outcomes of the alternatives and it should be honored in their analysis and comparison.

Principle 7: Revisit the decision

The initial projected outcomes of the selected alternative should be subsequently compared with the actual results achieved. Revisiting of decision ensure the good result of the final decision.

1.3 Role of Engineer's in Decision Making

Decision making is always challenging. Analysis of each alternatives by considering all relevant criteria, technically, economically (initial cost and future saving).is must necessary. The role of engineers in the decision making is that evaluate the condition, feasibility and appropriateness of an alternatives of a given project. Estimates its value and justify it from an engineering stand point and finally discover the best alternative for implementation.

Role of engineers in decision making (stepwise duty)

- Understand the problem and define the objectives
- Collect relevant information
- Define the feasible alternative solution and make realistic estimates. उचित
- Identify the criteria for decision making using one or more attribute.
- Evaluate each alternative, using sensitivity analysis to enhance the evaluation
- Select the best alternatives
- Implement the solution and monitor the result.

1.4 Cash flow

Cash flow is the statement which shows inflows and outflows of cash and cash equivalents during life of project. i.e. actual rupees coming into a firm or actual rupees going out from firm in different time periods.

It is the basis for the evaluation of different alternatives.

Cash flow diagram (CFD):

The graphical representation of different cash flow streams is known as cash flow diagram (CFD).

It should show three things:

i) A time interval divided into an appropriate number of equal periods.

ii) All cash out flows in each period.

iii) All cash inflows for each period.

[Note: Unless otherwise indicated, all such cash flows are considered to occurs at the end of their respective periods.]

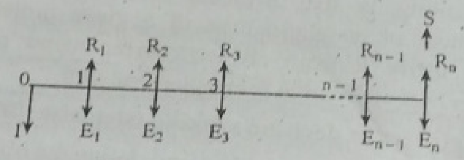

Fig. Cash flow diagram

Where, I = Initial investment

R = Revenue (Benefit)

E = Expenses (cost)

S = Salvage value

How to draw cash flow diagram?

Steps of drawing a cash flow diagram:

1. Horizontal line is time scale with progression of time moving from left to right. (Normally starts from 0 to year N) where end of previous year coincide with beginning of next year.
2. Inflows & outflows of money is expressed by upward and downward arrows respectively. Here inflows of cash represent annual benefit (revenue), salvage values etc and outflows represent initial capital investment, annual cost etc.
3. It is better to show two or more cash flows occurring in the same year individually to make clear connection in between problem statement and cash flows in the diagram.

[Note: Always kept in mind, arrow lengths are approximate proportional to the magnitude of their respective cash flows:]

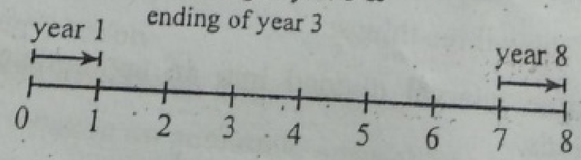

Fig. Typical CFD for 8 years

It shows cash flows begins from year 0 to year N = 8.

2016 S(L) + 2017 fall

Old Question Solution

1. **Define engineering economics and enlist the principles of engineering economics.** **[2069 Bhadra]**

Solution:

Engineering economics is the application techniques for the evaluation of design and engineering alternatives. The role of engineering economics is to

- Access the appropriateness of a given project.
- Estimate its value
- Justify it from an engineering point of view

2016 fall

Principles of Engineering: *Refer to [1.2]*

2. **Explain the roles of engineers in the decision making examples:** [2070 Magh]

Solution:

Refer to [1.3]

3. **Scarcity is the emerging issue in engineering field. How does the study of economics helps an engineer for decision making?** [2070 Bhadra]

Solution:

Scarcity is a naturally occurring limitation on the resources of that cannot be replenished. Scarcity may be classified as:

i) Financial scarcity

ii) Material scarcity

iii) Labour scarcity

iv) Technology scarcity

v) Machine scarcity

vi) Scarcity in time.

Scarcity is the emerging issue in the field of engineering. To implement any kind of project it demands a lot of financial resources, material, machine, long time various kinds of manpower. Proper arrangement, management and coordinations were all those factors is much necessary for any project to succeed. It is always desirable to complete the project within possible short period, economically without compromising on quality. For this wisely decision making plays vital role & engineers must be responsible for this.

For Role of engineers in Decision making: Refer to (1.3)

2015 H

4. **List out the principles of engineering economics. [2071 Magh]**

Solution:

Refer to [1.2] 2015 S

5. **Defined engineering economics. Write down the principles of engineering economic analysis. [2071 Bhadra]**

Solution: 2015(S)

Refer to [2069 Bhadra]

6. **Explain why the subject of engineering economics is important to civil engineer. [2072 Ashwin]**

Solution:

Fields of civil engineering involves construction of big projects with huge investment which directly related to the economy of the society. There occur a lot of alternatives; selection of best alternatives is always a very important task. So, decision making is always challenging and tough. Analysis of each alternatives by considering all relevant criteria, technically, economically (initial cost, future saving) is must necessary. Civil engineers can play vital role in decision making to evaluate the condition, feasibility an appropriateness of an alternatives of a given project, estimate its value and justify it from an engineering stand point & finally discover the best alternative for implementation. For this knowledge about both in the field of civil engineering and engineering economics is necessary which clearly shows the important of engineering economics to civil engineer.

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2 Interest & Time Value of Money

2.1 Introduction to Time Value of Money

The idea that a rupees today worth more than a rupees in the future because the rupees today can earn interest.

So, it is defined as "Time value of money is the time dependent value of money stemming both from changes in purchasing power of money (inflation and deflation) and from the real earning potential of alternative investment overtime." Since money has the ability to earn interest. It's value increase with time. Hence it is the relationship between interest and time.

The value of a currency expressed in terms of the amount of goods or services that one unit of money can buy is called purchasing power. As time lefts purchasing power of money is reduced.

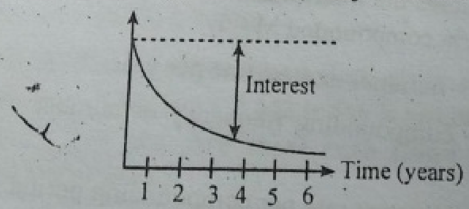


Fig.: Time vs value of money

2.2 Simple interest

Simple interest is interest earned on only the principal amount during each interest period. In other words, with simple interest, the interest earned during each interest period does not earn additional interest in the remaining periods, even though you don't withdraw it.

In general, for a deposit of P dollars at a simple interest rate of r for N periods, the total earned interest would be,

$$I = (r \times P)\,N$$

2.3 Compound Interest

When the interest earn in each period is calculated on the basis of the total amount at the end of the previous period & this total amount includes the original principal plus the accumulated interest that has been left in the account is called compound interest.

So, the total amount, at the end of N period when sum P is deposited at interest rate 'i' is $F = P\,(1 + i)^N$

2.3.1 Nominal Interest Rate

The nominal interest rate is periodic interest rate times the number of periods per year. For example, a nominal annual interest rate of 12% based on monthly compounding means 1% interest rate per month.

If a financial institution uses a unit of time other than a year. A month or quarter (eg: when calculating interest payments), the institution usually quotes the interest rate on annual basis. Commonly this rate is stated as r% compounded M – ly

Where, r = nominal interest rate per year

M = compounding frequency or number of interest periods per year

r/M = interest rate per compounding period

2.3.2 Effective Interest Rate

The actual rate of interest earned during one year is known effective rate and it is also expressed on the annual basis unless specifically stated otherwise; Effective interest rate is represented by 'i'

$$\text{Then } i = \left(1 + \frac{r}{m}\right)^m - 1$$

Where, m = compounding period per year

Example: From above formula the effective interest rate for 12% Compounded semi – annually.

$$i = \left(1 + \frac{0.12}{2}\right)^2 - 1 = 12.36\% \qquad [\because r = 12\% \ \& \ m = 2]$$

2.3.3 Continuous compounding

Continuous compounding can be thought of as making compounding period infinitesimally small. Here cash flows occurs at discrete interval but it is compounded continuously.

Therefore it can be achieved by taking limit of m (no of compounding periods in year) to infinity.

$$\text{So, } i = \lim_{m \to \infty} \left[\left(1 + \frac{r}{M}\right)^m - 1\right]$$

$$\therefore \boxed{i = e^r - 1} \ \ldots\ldots\ldots\ldots\ldots (a)$$

$$\text{or, } e^r = (1 + i)$$

We have,

$$\boxed{F = P(1 + i)^N}$$

$$\text{or, } F = P\, e^{rN}$$

$$F = P\,\{E/P, r\%, N\}$$

r % is used to denote the nominal rate and the use of continuous compounding.

2.4 Economic Equivalence

The process of comparing two different cash amounts at different points in time is called economic equivalence.

Different sums of money at different times are equal in economic value

Calculations for determining the economic effects of one or more cash flows are based on the concept of economic equivalence. Economic equivalence refers to the fact that a cash flow. Whether a single payment or a series of payments can be converted to an equivalent cash flow at any point in time.

For example, we could find the equivalent future value of F or a present amount P at interest rate i at period n; or we could determine the equivalent present value of P of n equal payment A.

Equivalence Calculations : General Principles

Prin 1: Equivalence calculation made to compare alternatives require a common time basis:

To establish an economic equivalence between two cash flow amounts, a common base period must be selected.

Prin 2: Equivalence depends on interest rate:

The equivalence between two cash flows is a function of the magnitude and timing of individual cash flows and the interest rate will destroy the equivalence between these two sums, as we will demonstrate in Example 4

Prin 3: Equivalence calculation may require the conversion of multiple payment cash flows to a single cash flow.

Prin 4: Equivalence is maintained regardless of point of view.

Solved Example

1. **Suppose you deposit \$1000 in a bank savings accounts that pays interest rate of 10% compounded annually. Assume that you don't withdraw the interest earned at the end of each period (one year), but let it accumulate. How much would you have at the end of year 3?**

Solution:

Given: P = \$1000, N = 3 years, and i = 10% per year

Find: F, $F = \$1000\,(1 + 0.10)^3 = \$1.33\%$ [∵ from $F = P(1 + i)^N$]

2. **In 1626, Peter of the Dutch west company paid \$24 to purchase Manhattan Island in New York from the Chinese. In retrospect, if peter had invested \$24 in a saving account**

that earned 8% interest rate, how much would it be worth in 2007?

Solution:

Given $P = \$24$, $i = 8\%$ per year & $N = 381$ yrs (2007 – 1626)

Find F, based on (a) 8% simple interest, $F = \$24\ [1 + 0.08 \times 381] = \$755.52$

(b) 8% compound interest, $F = \$24\ (1 + 0.08)^{381}$

$= \$130{,}215{,}319{,}909{,}015$

3. **Suppose you are offered the alternative of receiving either \$ 3,000 at the end of 5 yrs or P dollars today. There is no question that the \$ 3,000 will be paid in full (no risk). Because you have no current need for money, you deposit the p dollars in an account that pays 8% interest. What value of P would make you indifferent to choose between p dollars today and the promise of \$ 3,000 at the end of 5yrs?**

Solution:

Given: F = \$ 3,000 N = 5 yrs and i = 8% per year

Find: P $F = P(1 + i)^N$

$$P = \frac{F}{(1+i)^N} = \frac{3000}{(1+0.08)^5} = \$\ 2{,}042$$

So, it is clear that if P is anything less than \$ 2,042 you would prefer the promise of \$ 3,000 in five years to P dollars today. Otherwise prefer \$ 3,000

4. **In example 2.3, we determined that, given an interest rate of 8% per year, receiving \$ 2,042 today is equivalent to receiving \$ 3,000 in five yrs. Are these cash flows equivalent at an interest rate of 10%?**

Solution:

Given: P = \$ 2,042 i = 10% & N = 5 yrs

Find: F $F = P(1 + i)^N = 2042\ [1 + 0.10]^5 = \$\ 3{,}289$

Since the amount is greater than \$ 3,000, the change in interest rate destroys the equivalence between the two cash flows.

5. **Suppose you have invested Rs. 1000 at present. How long does it take for your investment to double if interest rate is 8% compounded annually?**

Solution:

Let investment F will become 2P after N yrs

$F = 2P$

From $F = P(1 + i)^N$

$2P = P(1 + i)^N$

or, $2 = (1 + 0.08)^N$

$1.08^N = 2$

$N \log 1.08 = \log 2$ [$\because$ taking log on both sides]

$\therefore$ N = 9 yrs.

6. **Find effective interest rate when nominal rate of interest is 18% per year & compounding is (i) Monthly (ii) Daily (iii) Hourly (iv) Continuously.**

Solution:

(i) Monthly, $i = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 19.561\%$

(ii) Daily, $i = \left(1 + \frac{0.18}{365}\right)^{365} - 1 = 19.716\%$

(iii) Hourly, $i = \left(1 + \frac{0.18}{365 \times 24}\right)^{365 \times 24} - 1 = 19.721\%$

(iv) Continuously, (from equation (a) from 2.33)

$i = e^r - 1 = e^{0.18} - 1 = 19.7217\%$

2.5 Development of Interest Formula

As we begin to compare series of cash flow instead of single payments the required analysis become more complicated. However, when patterns in cash flow transactions can be identified, we can take advantage of these patterns by developing concise expression for computing either present or future worth of the series. We will classify the five major categories of cash flow transactions, develop interest formulas for them.

1. **Single Cash flow**

It involves the equivalence of a single present amount and its future worth. So it deals with only a single present amount P and its future worth F and is given by

$$F = P(1 + i)^N$$

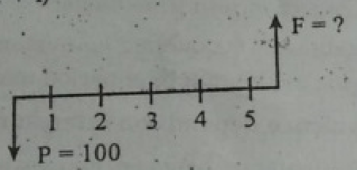


Fig.: Single Cash Flows

2. Equal payment series:

This describes the cash flow of the common installment loan contract, which arranges repayment of the loan in equal periodic installments.

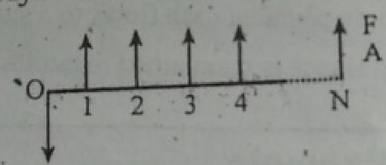


Fig: Uneven Series

$$F = A\frac{[(1 + i)^N - 1]}{i}$$

where, A = Annual payments or incomes

3. Linear gradient series:

- The amounts are not always uniform, when many transactions involves series of cash flows but they vary in some regular difference.
- One common pattern of variation occurs when each other cash flow in a series increases (or decreases) by a fixed amount.

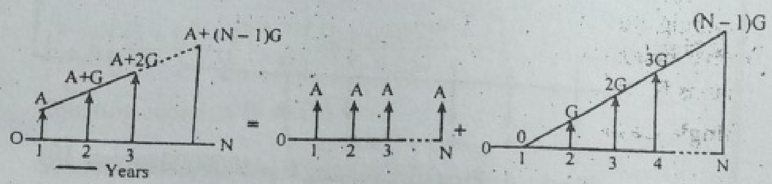


Fig: Linear Gradient Series

4. Geometric Gradient series:

When the series in cash flow is determined not by a fixed amount like above but by some fixed rate, then the series is called Geometric gradient series.

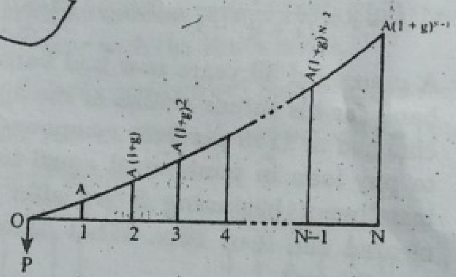


Fig. Geometric Gradient Series

Here, $F_1 = A(1 + i)^{N-1}$

$F_2 = A(1 + g)\ (1 + i)^{N-2}$

. . .

. . .

. . .

$F_3 = A(1 + g)^2\ (1 + i)^{N-3}$

$F_N = A(1 + g)^{N-1}$

So, $F = F_1 + F_2 + \ldots\ldots\ldots + F_N$

$= A(1 + i)^{N-1} + A(1 + g)\ (1 + i)^{N-2} + A(1 + g)^2\ (1 + i)^{N-3} + \ldots\ A(1 + g)^{N-1}$

(From sequence and series formula)

$$\text{Sum} = \frac{a(1-r^n)}{1-r} \qquad [\because r = \text{common ratio}]$$

$$F = A(1+i)^{N-1}\frac{\left[1-\left(\frac{1+g}{1+i}\right)^N\right]}{1-\left(\frac{1+g}{1+i}\right)} \quad \text{here } r = \left(\frac{1+g}{1+i}\right)$$

$$= A(1+i)^{N-1}\frac{[(1+i)^N-(1+g)^N]}{1+i-1-g} \times \frac{(1+i)}{(1+i)^N}$$

$$= A\left[\frac{(1+i)^N-(1+g)^N}{i-g}\right]$$

$$\therefore \boxed{F = A\left[\frac{(1+i)^N-(1+g)^N}{i-g}\right]}$$

5. **Unequal payment series/Irregular series:** A series of cash flow which are irregular and doesn't exhibit an overall regular pattern.

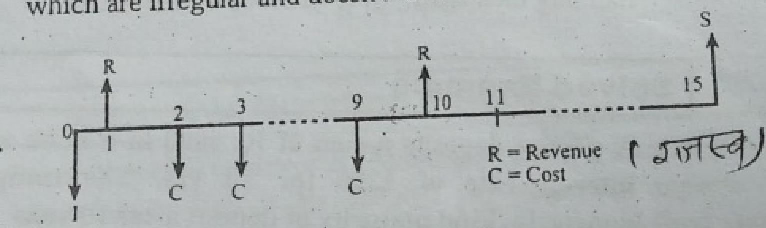


Fig. Uneven Series

<u>**Some formulae:**</u>

1. **Equal payment series:** $F = A\left[\frac{(1+i)^N-1}{i}\right]$

2. **Linear gradient series:** $F = \frac{G}{i}\left[\frac{(1+i)^N-1}{i}\right] - \frac{NG}{i}$

$$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N-1}\right]$$

& $$P = \frac{G}{i^2}\left[\frac{(1+i)^N - Ni - 1}{(1+i)^N}\right]$$

3. **Geometric gradient series:** $F = A\left[\frac{(1+i)^N-(1+g)^N}{i-g}\right]$

4. **For compounding:**

Rule 1: From higher compounding period to lower compounding period.

Eg: $i_{quarter} = \frac{i_{year}}{4}$ or $i_{quarter} = \frac{i_{semi}}{2}$

$i_{semi} = \frac{i_{year}}{2}$

Rule2: From lower compounding period to higher compounding period.

eg: $i_{semi} = (1 + i_{quarter})^2 - 1$

$i_{year} = (1 + i_{month})^{12} - 1$

For withdrawn: if i = interest rate per year and withdraw is monthly then $i_{month} = (1 + i_{year})^{1/12} - 1$

Solved Example

1. **A person deposits a sum of Rs 5000 in a bank at a nominal interest rate of 12% for 10 yrs. The compounding is quarterly. Find maturity of deposit after 10 yrs.**

Solution: Given, P = Rs 500, N = 10 yrs, r = 10% compounded quarterly

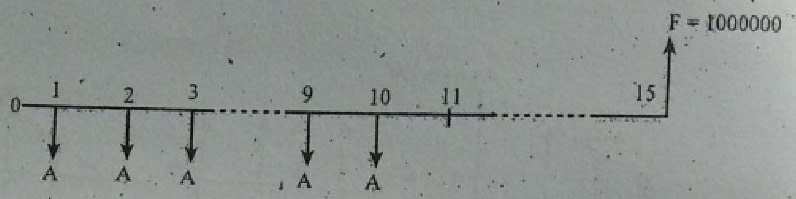
$$\text{So, } i_{effective} = \left(1 + \frac{r}{m}\right)^m = \left(1 + \frac{0.12}{4}\right)^4 = 12.5508\% \text{ per year.}$$

$$F = P(1 + i)^N = 5000\,(1 + 0.1255)^{10} = \text{Rs } 16310$$

2. **Mr. Jha wants to have \$ 1000000 for the studies of his daughter after period of 15 yrs. How much rupees does he has to deposit each year for 10 continuous yrs in a saving account that earns 8% interest annually.**

Solution:

Given: F = \$1000000, N = 15 yrs i = 8% per year



Find : A First discounting F = 1000000 to year 10

$F_{10} = F \times (1 + i)^{-N} = 1000000 \times (1 + 0.08)^{-5} = \$ \ 680,583.19$

Now, from equal payment series formula

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] \qquad (\because F \text{ is } F_{10})$$

$$\therefore A = F \times \frac{i}{(1+i)^N - 1} = 680583.19 \times \frac{0.08}{(1.08)^{10} - 1} = \$ \ 46,980.3$$

3. **A man aged 40 years now had borrowed Rs 500000 from a bank for his future studies at the age of 20 yrs. Interest was changed at 11% per year compounded quarterly. He wished to pay loan in semiannual equal installments with the first installment beginning 5 yrs after receiving loan. He just cleared the loan now. What amount did he pay in each installment?**

Solution:

Given, P = Rs. 500000 I = 11% per year & compounding quarterly.

N = 20 years A = ?

Quarterly interest rate,

$$i_q = \frac{i_{year}}{4} = \frac{11\%}{4} = 2.75\% \text{ [From formula No.4]}$$

Semi–annual interest rate

$$i_{semi} = (1 + i_q)^2 - 1$$

$$= (1 + 0.0275)^2 - 1 = 5.57\%$$

Using single payment compound amount factor

$$F = P(1 + i)^N = 500000(1 + 0.057)^{40} \ldots\ldots\ldots\ldots\ldots (1)$$

Using uniform series compound amount factor

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A\left[\frac{(1+0.057)^{30} - 1}{0.057}\right] \ldots\ldots\ldots\ldots\ldots(2)$$

Equating equation (i) & (ii)

$$500000\,(1.057)^{40} = A\left[\frac{(1.057)^{30} - 1}{0.057}\right]$$

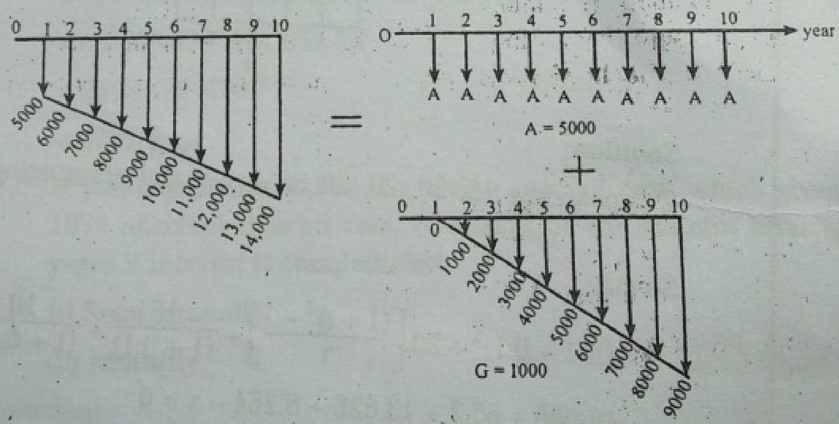
$\therefore$ A = Rs. 61217.32 is semi-annual payment.

4. **A person is planning for his retired life and has 10 more years of service. He would like to deposit 30% of his salary, which is Rs 5000 at the end of first year and thereafter he wishes to deposit the amount with an annual increase of Rs. 1000 for next 9 yrs with an interest rate of 15%. Find the total amount at end of 10^{th} year with the above series.**

Solution: Given: (It is the problem of Linear gradient series)

G = Rs. 1000, A = Rs. 5000, N = 10 yrs and i = 15% per year

Find: F = future worth



$F = F_A + F_G$

F_A = Future worth from Equal payment series

F_G = Future worth from Linear gradient series

$$\text{So, } F = A\left[\frac{(1+i)^N - 1}{i}\right] + \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

$$= 5000\left[\frac{(1+0.15)^{10}-1}{0.15}\right] + \frac{1000}{0.15}\left[\frac{(1+0.15)^{10}-1}{0.15}\right] - \frac{10 \times 1000}{0.15}$$

$= \text{Rs. } 170209.33$

5. An investment of Rs. 100,000 is made in a company. The first year of the investment produce net revenue of Rs. 20,000. Over a 40 year period, the net revenue received from the investment decreased by 10% each year. If the interest rate is 12%, what is the present worth for the investment?

Solution:

Given A = 20,000, I = 12%, g = −10%, N = 40 years.

$$Pw = -100000 + 20000\left[\frac{1-(1-g)^N (1+i)^{N}}{i-g}\right]$$

$$= -100000 + 20000\,\frac{(1-0.9^{40} \times 1.12^{-40})}{0.12-(-0.1)} = \text{Rs. } 13{,}618.3$$

Old Question Solution

1. Ramesh, a civil engineer is planning to plane save a total of 20% of his salary, which is Rs 250,000 per year. He expects 7% increase in salary for next 15 years if the mutual fund results in 10% annual return, what will be the amount at the end of 15 years. If salary increases by 25000/ year, what will be the amount? [2069 Bhadra]

Solution:

For the first case, the flow is geometric

A = 20% of 250,000 = 50000

g = 0.07

i = 0.10

N = 15

We have,

$$P = A \times \frac{1-(1+g)^N (1+i)^{-N}}{i-g}$$

$$= 50000 \times \frac{1-(1.07)^{15} \times 1.10^{-15}}{0.10-0.07}$$

$$= 565849.642$$

$$F = P(1+i)^N$$

$$= 2363694.381$$

b) If salary increases by Rs.25,000/year than

$$A = 50000$$

$$G = 5000/\text{year}$$

$$F = 50000\ (F/A,\ 10\%,\ 15) + \frac{5000}{0.10}\left(\frac{1.10^{15}-1}{0.10}\right)\frac{-15 \times 5000}{0.10}$$

$$= 50000 \times \frac{1.10^{15}-1}{0.10} + 50000\left(\frac{1.10^{15}-1}{0.10}\right)\frac{-15 \times 5000}{0.10}$$

$$F = \text{Rs. } 24,27,248.164$$

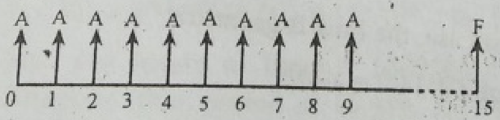
2. **Define nominal and effective interest rate.** **[2071 Bhadra]**

Solution:

See at 2.3.1 and 2.3.2

3. **Evaluate FW at the end of 15 yrs with 10% interest rate compounded monthly on a cash flow of 50,000 at the beginning of each first years.** **[2071 Bhadra]**

Solution:



Given: $A = 50000$, $i = 10\%$

At the end of 9^{th} year or begging of 10^{th} year

$$FW = A\left[\frac{(1+i)^N-1}{i}\right] = 50000\left[\frac{(1+0.1)^{10}-1}{0.1}\right]$$

$$= \text{Rs. } 796871.23$$

So, the value of FW at the end of 15^{th} year is

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$F = P(1 + i)^N = 79687.23\ (1 + 0.1)^6 = \text{Rs. } 141170.789$

4. **What will be the amount of money at the end, if you deposit Rs. 5000 per month for five years continuously. If nominal interest rate is 10% & compounded quarterly.** **[2071 Magh]**

Solution:

Given: A = Rs. 5000 per month for five yrs

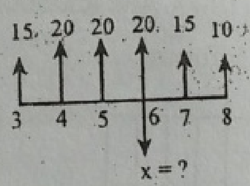
$i_{nominal} = 10\%$ & compounded quarterly

$$\text{so, } i_{quarter} = \frac{i_{year}}{4} = \frac{10}{4} = 2.50\%$$

$$i_{monthly} = (1 + i_{quarter})^{1/4} - 1 = 0.62\%$$

$$\text{So, FW} = A\left[\frac{(1+i)^N - 1}{i}\right] = 5000\left[\frac{(1 + 0.0062)^{60} - 1}{0.0062}\right] = 362070.93$$

5. **Calculate the value of x from the following figure. (i = 10%)** **[2071 Magh]**



Solution:

Compounding 15, 20, 20, 20 to year–6 and discounting 15 & 10 to year–6

So that,

$$15(1 + 0.1)^3 + 20\left[\frac{(1+i)^3 - 1}{i}\right] + \frac{15}{(1 + 0.1)} + \frac{10}{(1 + 0.1)^2} - x = 0$$

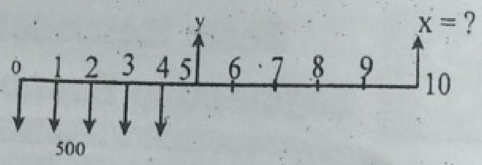
$$19.965 + 66.2 + 13.636 + 8.264 - x = 0$$

$$\therefore x = 108.065$$

6. **What is the effective and nominal interest rate. Evaluate FW at the end of 10 yrs with 8% interest rate compounded continuously with cash flow of Rs. 500 at the beginning of each year for first five years.** **[2070 Bhadra]**

Solution:

For theory part See at 2072 Ashwin



Transforming the amount to end of each year

$T = 500 \times e^{0.08}$

The worth of all at the end of 5th year is

$Y = 500 \times e^{0.08}$ (F/A 8% 5)

$= 500 \times e^{0.08} \times \dfrac{e^{0.08 \times 5} - 1}{e^{0.08} - 1}$

$= 3198.499$

The worth of it at the end of 10 years is

$x = 3198.439$ (F/e, 8%, 5)

$= 3198.439 \times e^{0.08 \times 5}$

$= 4771.6$

7. **If you deposit Rs 10,000 is a saving account now, which gives 10% nominal interest rate, what will be the amount after 5 years if interest is compounded**

   (i) Semi annually

   (ii) Monthly **[2070 Magh]**

Solution:

i) If interest is compounded semi–annually then m = 2 and the effective interest rate is

$i_{eff} = (1 + r/m)^m - 1$

$= \left(1 + \dfrac{0.10}{2}\right)^2 - 1$

$= 10.25\%$

$F = P$ (F/P, 10.25%, 5)

$$= 10000(1.1025)^2$$
$$= \text{Rs. } 16288.952$$

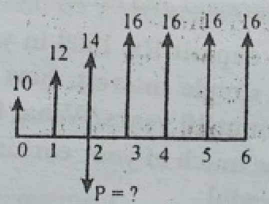
ii) For monthly compounding

$$i_{eff} = \left(1 + \frac{0.10}{12}\right)^{12} - 1 = 10.47\%$$

$$F = P(F/P, 10.47, 5)$$
$$= 10000 \times 1.1047^5$$
$$= \text{Rs. } 16452.116$$

9. Find the value of P if i = 10%, use gradient formula also.

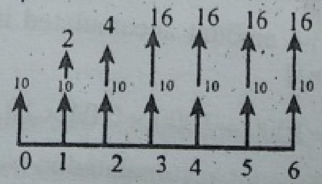
[2070 Magh]



Solution:

Discounting all cash flow to year 2,

$$P = 10(F/A, 10\%, 3) + [\frac{G}{i}(F/G, 10\%, 3) - \frac{NG}{i}] + 16(P/A, 10\%, 4)$$



$$P = 10 \times \frac{(1+i)^N - 1}{i} + \left[\frac{2}{0.10} \times \frac{(1+i)^N - 1}{i} - \frac{NG}{i}\right] + 16 \times \frac{(1+i)^N - 1}{i(1+i)^N}$$

$$= 10 \times \frac{1.10^3 - 1}{0.10} + \left(\frac{2}{0.10} \times \frac{1.10^3 - 1}{0.10} - \frac{3 \times 2}{0.10}\right) + 16 \times \frac{1.10^4 - 1}{0.10 \times 1.10^4}$$

$$= \text{Rs. } 90.004$$

10. What is difference between nominal and effective interest rate?

[2072 Ahswin]

Solution:

Nominal: Interest rates are the stated, advertised or quoted rates where no time period is started, than per year (also known as per annum) is assumed.

Effective: Interest rates are what borrows have to actually pay, and depend on how frequently the nominal rate is compounded. (i.e which means adding interest to the balance of the loan).

For formula and examples: Refer at 2.3.1 and 2.3.2

11. **You deposit Rs. 1000 in your bank account. If the bank pays 4% simple interest, how much will you accumulate in your account 10 years? What if the bank pays compound interest? How much of your earnings will be interest on interest? [2072 Ahswin]**

Solution:

Principle (P) = 1000

Simple interest rate (i) = 4%

Time (N) = 10 years

The amount accumulated in bank @ 4% Simple interest = P + PiN

$= 1000 + 1000 \times 0.04 \times 10 = \text{Rs. } 1400$

Amount accumulated in bank @ 4% compound interest;

$= P(1 + i)^N = 1000\ (1 + 0.04)^{10} = \text{Rs. } 1480.24$

Interest on interest = 1480.24 – 14000 = Rs. 80.24

3 Basic Methodologies of Engineering Economic Analysis

3.1 Determining minimum Attractive (Acceptable) Rate of Return (MARR)

MRRR is the interest rate at which firm can always earn or borrow money. MARR can also be regarded as the minimum return required to get investors to put up the money. MAAR can be developed from existing projects and may be different from time to time within the same firm.

Factors influence the determination of MARR

1. The amount of fund available for investment and its source.
2. The nature of investment alternatives.
3. The amount of risk perceived in the investment.
4. The type of organization involved. (government, public, private)

- MAAR is determined from the opportunity cost viewpoint.

Example: Consider the following schedule, which shows prospective annual rates of profit for a company's portfolio of capital investment projects.

Expected Annual Rate of Profit	Investment Requirements	Cumulative Investment
4% and over	Rs. 2,200	Rs. 2,200
30- 39.9%	3,400	5,600
20- 29.9%	6,800	12,400
10 – 19.9%	14,200	26,600
Below 10%	22,800	49,400

If the supply of capital obtained from internal and external sources has a cost of 15% per year for the first Rs. 5,000,000 invested and then increases 1% for every Rs. 5,000,000 thereafter, what is this company's MAAR when using an opportunity cost viewpoint?

Solution:

Cumulative capital demand versus supply can be plotted against prospective annual rate of profit, as shown in figure. The point of intersection is approximately 18% per year, which represents a realistic estimate of this company's MARR when using the opportunity cost view point.

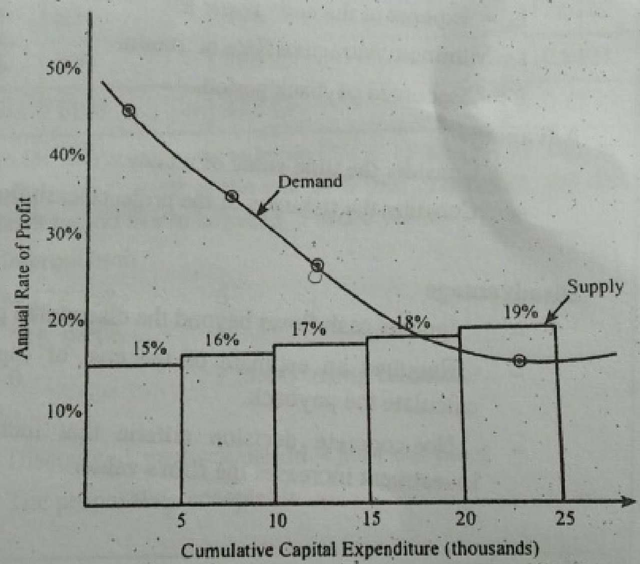


Fig: Cumulative capital expenditure

3.2 Payback Period method

Payback period is the length of time needed before an investment makes enough to recoup the initial investments. The payback method calculates the number of years required for cash inflows to just equal cash outflows.

Based on the way we compute the payback period, it can be classified in to two types.

1. Simple or conventional payback period.
2. Discounted payback period.

1. Simple payback period (SP)

Simple payback period doesn't consider the time value of money (i = 0) It measures a projects liquidity and is computed as follows.

$$SP = \frac{\text{Initial Investemt}}{\text{Expected savings or Reveneue Per year}}$$

In other way, SP period for a project having one time investment at time 0,

$$\sum_{k=1}^{\theta} (R_k - E_k) - I > o$$

Where, I = Capital investment.

R_k = Revenues at the end of year k.

E_k = Expenses at the end of year k.

θ = Simple payback period.

Advantages

- Simplicity
- It doesn't require any assumption.
- Reduces information search by focusing on that time at which the firm expects to recover the initial investment.

Disadvantages

- Fails to measure profitability
- It takes no account of the time value of money.
- It takes no account of the residual value in the capital asset.

Example: Calculate simple payback period for the given cash flow of the project.

End of Year (EOY)	Net cash flow (Rs)
0	- 25,000
1	8,000
2	8,000
3	8,000
4	8,000
5	13,000

Solution:

End of Year (E0)	Net cans flow (Rs)	Cumulative cash flow (Rs)
0	- 25,000	– 25, 000
1	8,000	– 17, 000
2	8,000	– 9, 000
3	8,000	– 1, ,000
4	8,000	+ 7, 000
5	13,000	+ 20, 000

Here the cumulative cash flow turns to positive in year 4. Therefore payback period lie between year 3 and 4. By interpolation,

Payback period = 3.125 years

[Note: *Interpolation Technique by fx – 991ES plus calculation*

Mode $\Rightarrow$ *3 (STAT)* $\Rightarrow$ *2 (A + Bx)* $\Rightarrow$

For example : x *y*

– 1, 000 *3*

+7, 000 *4*

$\Rightarrow$ *AC* $\Rightarrow$ *SHIFT* $\Rightarrow$ *1 (STAT)* $\Rightarrow$ *5 (Reg.)* $\Rightarrow$

5$(\hat{y})$ $\Rightarrow$ *(Interpolation value)* $\hat{y}$

0$\hat{y}$ = *3.125*]

2. **Discounted payback period**

It is defined as the number of years required to recover the investment from discounted cash flows. i.e. considering the time value of money.

Discounted payback period for a project having one time investment at time zero can be computed as

$$\sum_{k=1}^{\theta} (R_k - E_k)\ (P/F, i\%, k) - I > 0$$

Where, I = capital investment

R_k = Revenues at the end of year k.

E_k = expense at the end of year k.

i = Minimum Attractive Rate of return.

θ = Discounted payback period.

Advantage

- Consider the time value of money.
- Consider the riskiness of the project's cash flow.

Disadvantage

- Ignores cash flows beyond the discounted payback period.
- Requires an estimate of the cost of capital in order to calculate the payback.
- No concrete decision criteria that indicate whether to investment increases the firm's value.

Example

Evaluate the acceptability of the following proposal if maximum allowable discounted payback is 6 years. Assume MARR = 15%

EOY	0	1	2	3	4	5	6	7
Amount (in thousand)	-1500	200	400	450	450	600	900	1100

Solution:

EOY	Net cash how	PW at 15%	Cumulative PW
0	– 1500	–1500	–1500
1	200	$200/(1+0.15)^1 = 173.91$	–1326.09
2	400	$400/(1+0.15)^2 = 302.46$	–1023.63
3	450	$450/(1+0.15)^3 = 295.88$	–727.747
4	450	257.29	–470.458
5.	600	298.31	–172.152
6	900	389.09	216.9431
7	1100	413.53	630.47

From the above table, cumulative PW at 15% has – ve sign at the end of 5 year and + ve at the end of 6 year. Therefore, the discounted payback period lies in between 5 and 6 years.

By interpolation

– 172.152	5
216. 943	6
0	? (5.44) using calculator.

Decision:

Discounted payback period = 5.44 < 6 years

The proposal is acceptable.

3.3 Equivalent worth Methods.

3.3.1 Present worth method.

Present worth method discounts future amounts to the present by using the interest rate over the appropriate study period as

$$PW(i\%) = F_o (1 + i)^0 + F_1 (1 + i)^{-1} + F_2 (1 + i)^{-2} + \ldots\ldots$$
$$F_k (1 + i)^{-k} + \ldots\ldots\ldots + F_N (1 + i)^{-N}$$

$$= \sum_{k=0}^{N} F_N (1 + i)^{-k}$$

Where, i = effective interest rate or MAAR

k = index $(0 \le k \le N)$

F_k = future cash flow of at the end of period k

N = Number of compounding periods in study period.

- While evaluating a project of by PW method, the following rule appreciable

If PW (i%) > 0, accept the project.

If PW (i%) = 0 remain indifferent

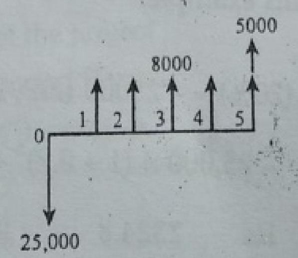
If PW (i%) < 0, reject the project.

Example:

A piece of new equipment has been proposed by engineers to increase the productivity of certain manual welding operation. The investment cash is Rs. 25,000 and the equipment will have a market value of 5,000 at the end of a study period of Five years. Increased productivity attributable to the equipment will amount to Rs.8, 000 per year after extra operating costs have been subtracted from the revenue generated by the additional production, is this proposal a sound one? MAAR = 20%, use PW method.

Solution:

Cash flow diagram



Net PW = PW of cash in inflows – PW of cash flows

= 8,000 (P/A, 20%, 5) + 5, 000 (P/F, 20% 5) – 2500

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$$= 8{,}000\frac{[(1+0.2)^5 - 1]}{0.2 \times (1+0.2)^5} + 5{,}000 \times \frac{1}{(1+0.2)^5} - 2500$$

$$= 23924.89 + 2009.38 - 25{,}000 = \text{Rs } 934.27$$

PW (20 %) > 0 this equipment is economically justified.

3.3.2 Future worth method

Future worth of a project is the equivalent worth of all cash flows, both inflows and outflows, at the end of the planning horizon of an interest rate that is generally the MAAR. The FW is calculated as,

$$FW\ (i\%) = F_0\ (1+i)^N + F_1\ (1+i)^{N-1} + \ldots\ldots + F_N\ (1+i)^0$$

$$= \sum_{k=0}^{N} F_k\ (1+i)^{N-k}$$

Where, I = effective interest rate or MAAR

k = index for each compounding period.

F_k = future cash flows at the end of period k.

N = Number of compounding periods in study period.

Decision rule:

If FW (i%) > 0, accept the project.

If FW (i%) = 0 remain indifferent.

If FW (i%) < 0, reject the project.

<u>Example :</u> Evaluate the FW of the potential improvement project describe in above example. Show relationship between FW and PW for this example.

Solution:

Net FW (20%) = – 25000 (F/P, 20%,5) + 8,000(F/A, 20%5) + 5,000

$$= -25{,}000 \times (1+0.2)^5 + 8{,}000\frac{[(1+0.2)^5 - 1]}{0.2} + 5000$$

$$= \text{Rs } 2324.8$$

As, NFW (20%) > 0, this equipment is economically justified

$$PW\ (20\%) = 2{,}324.8\ (P/F,\ 20\%,\ 5) = \frac{2324.8}{(1+0.2)^5} = 934.29$$

∴ Same result obtained.

3.3.3 Annual worth method

Annual worth is the equivalent worth of a lump-sum amount converted into a series of equal payments at the end of each period and is calculated as,

$$AW\ (i\%) = R - E - CR\ (i\%)$$

Where, R = equivalent revenues

E = equivalent expenses

$CR(i\%)$ = capital recovery amount

Decision rule:

If $AW\ (i\%) > 0$, accept the project

If $AW\ (i\%) = 0$, remain indifferent

If $AW\ (i\%) < 0$, reject the project

Applicable of AW

- Asset replacement
- Breakeven analysis
- Make or buy decision
- studies dealing with manufacturing costs
- economic value added analysis (EVA)

Capital Recovery amount

The annual equivalent of the capital cost covering the loss in the value of the asset and interest on invested capital is known as capital recovery amount. Capital recovery amount is calculated as,

$$CR(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N)$$

where, I = initial investment for the period

S = salvage value at the end of the study period

N = project study period

Example: **Solution the same above problem by AW method.**

Solution:

$$\text{Net } AW(20\%) = \underbrace{8000}_{R-E} - \underbrace{[25000\ (A/P, 20\%, 5) - 5000\ (A/F, 20\%, 5)]}_{CR}$$

$$= 8000 - 25000 \times \frac{0.2 \times (1 + 0.2)^5}{[(1 + 0.2)^5 - 1]} + 5000 \times \frac{0.2}{[(1 + 0.2)^5 - 1]}$$

$$= \text{Rs. } 312.4$$

NAW (20%) > 0, economically justified.

3.4 Rate of Return method

3.4.1 Internal Rate of return method

Internal rate of return of a project is the breakeven interest rate, i, at which equivalent worth of the project's cash flow is zero. This method solves for the interest rate that equates the equivalent worth of an alternative's cash flows (receipts or saving) to the equivalent worth of cash outflows (expenditures, including investment costs).

Based on PW formulation to find out this interest rate, the corresponding equation becomes,

$$PW(i\%) = \sum_{K=0}^{N} R_k\,(P/F, i\%, k) - \sum_{K=0}^{N} E_k\,(P/F, i\%, k) = 0$$

where,

R_k = revenues for the K^{th} year

E_k = expenditures (including investment) for the K^{th} year

N = Project life or study period

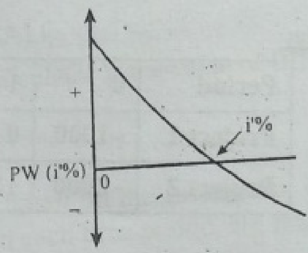
Similarly at the IRR,

FW (i%) = 0

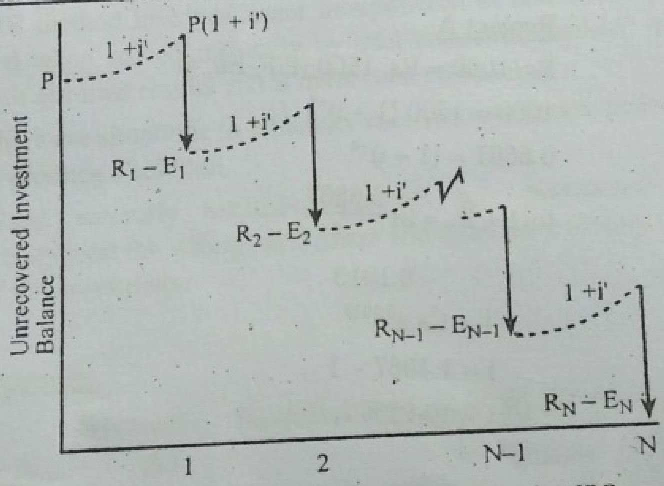
AW (i%) = 0

Though IRR can be computed by equating any equivalent worth to zero, it is advisable to use the PW formulation as it is found excessively used.

Graphically representation



Representation by investment balance diagram



Investment balance diagram showing IRR

Decision Rule

If IRR > MARR, Accept the project

If IRR = MARR, Remain indifferent

If IRR < MARR, Reject the project

Method of finding IRR

1) Direct solution method

A project with only a two flow transaction or service life of 2 years of return, we can apply direct mathematical solution for determining the rate of return.

Example:

Period	0	1	2	3	4
Project 1	–1000	0	0	0	1500
Project 2	–2000	1300	1500	–	–

Solution:

Project A

Rs. 1000 = Rs. 1500 (P/F, i%, 4)

$1000 = 1500\,(1 + i)^{-4}$

$0.6667 = (1 + i)^{-4}$

$\ln(1 + i) = \ln \frac{0.6667}{-4}$

$= 0.1013$

$1 + i = e^{0.1013}$

$i = 1.1067 - 1$

$i = 10.67\%$

Project B

$$PW\,(i) = -2000 + \frac{1.300}{(1 + i)} + \frac{1500}{(1 + i)^2} = 0$$

let $\frac{1}{1 + i} = x,$

$-2000 + 1300\,x + 1500\,x^2 = 0$

Solving, $x = 0.8$ or -1.667

$\therefore\ i = 25\%$ for $x = 0.8$

$i = -160\%$ for $x = -1.667,$

$\therefore$ the project's $i = 25\%$

2) Trial and error method

Example:

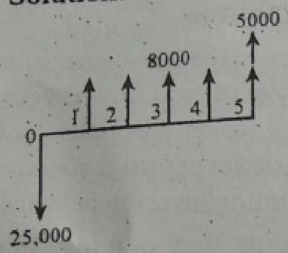
Initial investment = Rs. 25000

Salvage value = Rs. 5000 after 5 years

Net revenue = Rs. 8000 per year MARR = 20%

Is the investment good? Use IRR trial and error method

Solution:



PW(i%) = 0

5000 (P/F, i%, 5) + 8000 (P/A, i%, 5) − 25000 = 0

or, $\frac{5000}{(1+i)^5} + 8000 \times \frac{[(1+i)^5 - 1]}{[i \times (1+i)^5]} - 25000 = 0$ ....(1)

By trial and error,

At i = 20% PW = 934.3

At i = 25% PW = −1847.1

By interpolation, i = 21.67% at PW = 0

Putting i = 21.67% in equation (1), PW = −58.2

IRR lies between 20% and 21.67%

Again using linear interpolation

i = 21.8%

putting i = 21.58% in equation (1)

PW (21.58%) = −1.4

IRR lies between 20% and 21.58%

Again using linear interpolation i = 21.577%

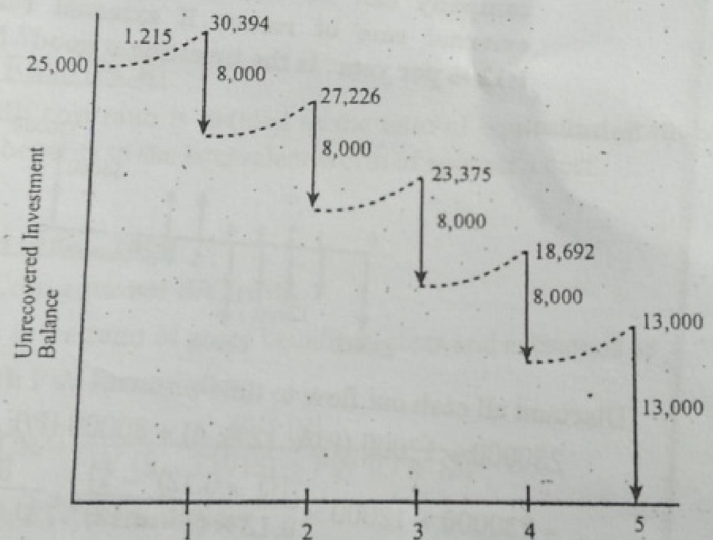
PW (21.577%) ≈ 0

Here, IRR > MARR (20%), Investment is acceptable

Unrecovered project balance calculation

Year	Unpaid balance at beginning year (Rs.)	Return of unpaid balance (Rs.)	Payment received	Unpaid balance at end of year (Rs.)
0	−25000	0	0	−25000
1	−25000	−25000 × 0.21577 = −5394.25	8000	(−25000 − 5394 + 8000) = −22394.25
2	−22394.25	−4832	8000	−19226.25
3	−19226.25	−4148.45	8000	−15374.7
4	−15374.7	−3317.4	8000	−10692
5	−10692	−2307	13000	0

Unrecovered Balance



Investment balance diagram showing IRR

3. **Computer solution method**

Consider the same example

PW (i%) = 0

8000 (P/A, i%, N) + 5000 (P/F, i%, N) −25000 = 0

$$8000 \times \frac{[(1+i)^5 - 1]}{[(1+i)^5 \times i]} + \frac{5000}{(1+i)^5} - 25000 = 0$$

Write the equation in calculator as,

$8((1 + i)^5 - 1) / (1 + i)^5 \times i)) + 5/(1 + i) - 25 = 0$

- Press Shift $\Rightarrow$ CALC

$\Rightarrow$The value in the calculator is the value of IRR

Drawbacks of IRR

1. IRR method is based on the assumption that recovered funds, if not consumed in each time period, are reinvested at rather than at MARR. This is not always practical.
2. IRR method involves linear interpolation of non–linear function and when solved manually by trial and error method, may not give accurate results and is more time consuming.
3. There are situations in which its iterative calculation process fails to produce a solution.
4. When naturally exclusive projects are considered it can recommend the wrong investment and does not consider the scale of the investment.

For Example:

EOY	Mutually Exclusive Project Cash flow A1	A2
0	-1000	-5000
1	2000	7000
IRR	100%	40%
PW(10%)	818	1364

Both projects are acceptable at MARR = 10%, but A2 with higher PW is worth more to the stockholders, whereas from IRR point of view, A1 seems better.

5. **When the algebraic sign of the cash flow changes in the middle of the series it is possible to obtain two right answers.**

EOY	Net cash flow
0	–1000
1	2300
2	–1320

We find IRR = 10% and 20% but both of them are in correct. So, use NPW criteria to make the decision.

3.4.2 External/modified rate of return method

IRR has a serious drawback of reinvestment assumption which is not always practical for cash borrowed or released from a project to be reinvested to yield a rate of return equal to that received from the project.

Steps of ERR calculation

1. Discount all expenses to the present at the external reinvestment rate, $\in\%$

$$\sum_{K=0}^{N} E_k\,(P/F, \in\%, k)$$

2. Project all income to the future at the external reinvestment rate $\in\%$

$$\sum_{K=0}^{N} R_K(F/P, \in\%, N-k)$$

3. ERR, i% is the interest rate that establishes equivalence between these terms

$$\sum_{K=0}^{N} E_k\,(P/F, \in\%, k)\,(E/P, i\%\ N) = \sum_{K=0}^{N} R_k\,(F/P, \in\%, N-k)$$

Where,

R_k = net revenues in period k

E_k = net expenses in period k

N = project's life

$\in\%$ = External reinvestment rate

Decision rule

If ERR > MARR, Accept the project

If ERR = MARR, remain indifferent

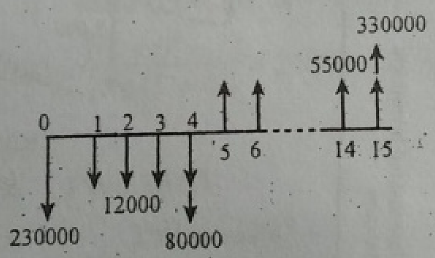
If ERR < MARR, reject the project

Advantages of ERR over IRR

1. It does not need trial and error process to solve for i%
2. There is no possibility of multiple rates of return.

Example: A consultancy opened now for Rs. 230000 will lose Rs.12000 each year the first four years. An additional Rs. 80000 invested in the consultancy during the fourth year will result in a profit of Rs. 55000 each year from the fifth year to the fifteenth year. At the end of 15 years the company can be sold for Rs. 330000. Calculate the external rate of return if external reinvestment rate is12% per year. Is the investment good? MARR = 15%

Solution:



Discount all cash out flow to time zero.

$$230000 + 12000\ (P/A,\ 12\%,\ 4) + 80000\ (P/F,\ 12\%,\ 4)$$

$$= 230000 + 12000 \times \frac{[(1+0.12)^4 - 1]}{0.12 \times (1+0.12)^4} + \frac{80000}{(1+0.12)^4}$$

$$= \text{Rs. } 317288$$

Compound all cash inflows to N i.e 15 years at 12%

$$55000\ (F/A,\ 12\%,\ 11) + 330000$$

$$= 55000 \times \frac{(1+0.12)^{11} - 1}{0.12} + 330000 = \text{Rs. } 1466003$$

Equivalence,

317288 (F/P, i%, 15) = 1466003

i% = 10.74 i.e. ERR = 10.74% < MARR

Therefore, investment is not good.

3.5 Public sector economic analysis (Benefit cost ratio method)

When evaluating public project, taxes and pay off periods are considered less significant and indirect effects, interest rate, over counting, tolls, fees, user charges etc are more significant. In the evaluation of such projects, we often use benefit cost analysis (BCA). BCA also provides information about the relative economic efficiency of alternatives.

B/C Ratio (BCR)

Benefit cost ratio is defined as the ratio of equivalent worth of user's benefits to the equivalent worth of sponsor's cost.

Types of B/C ratio :

1. **Conventional B/C ratio**

   It is the ratio of gross benefit to costs and expressed as

a) **With PW formulation**

$$BCR = \frac{PW(B)}{PW(I) - PW(S) + PW(O\ \&\ M)}$$

b) **With FW formulation**

$$BCR = \frac{FW(B)}{FW(I) - FW(S) + FW(O\ \&\ M)}$$

c) **With AW formulation**

$$BCR = \frac{AW(B)}{AW(I) - AW(S) + AW(O\ \&\ M)}$$

2. **Modified B/C ratio**

   It is the ratio of net benefits to the costs and expressed as,

a) With PW formulation

$$BCR = \frac{PW(B) - PW\ (O\ \&\ M)}{PW(I) - PW(S)}$$

b) With FW formulation

$$BCR = \frac{FW(B) - FW\ (O\ \&\ M)}{FW\ (I) - FW\ (S)}$$

c) With AW formulation,

$$BCR = \frac{AW\ (B) - AW\ (O\ \&\ M)}{AW\ (I) - AW\ (S)}$$

Where, B = benefit

I = investment

S = salvage

O & M = Operation and maintenance

Decision Rule

IF BCR > 1, Accept the project

If BCR = 1, remain indifferent

If BCR < 1, reject the project.

Project to be feasible if

Benefit (B) > cost (C)

OR, BCR > 1

Example

Find both types of B/C ratio using PW, FW and AW method.

Initial investment = Rs. 2000

Revenue/year = Rs. 1000

Expenses/year = Rs. 440

Salvage value = Rs. 400

Useful life = 5 years

MARR = 8%

Solution:

A) Using PW method

$$PW\ (B) = 1000\ (P/A, 8\%, 5) = 1000 \frac{(1.08^5 - 1)}{1.08^5 \times 0.08}$$

$$= Rs.\ 3992.7$$

$$PW\ (S) = 4000\ (P/F, 8\%, 5)$$

$$= \frac{400}{1.08^5} = Rs.\ 272.23$$

$$PW\ (O\ \&\ M) = 440\ (P/A, 8\%, 5)$$

$$= 440 \frac{(1.08^5 - 1)}{1.08^5 \times 0.08} = Rs.\ 1756.79$$

$$\text{Conventional B/C ratio} = \frac{PW(B)}{PW(I) - PW(S) + PW\ (O\ \&\ M)}$$

$$= \frac{3992.7}{2000 - 272.23 + 1756.79} = 1.146 > 1,\ Ok$$

$$\text{Modified B/C ratio} = \frac{PW(B) - PW\ (O\ \&\ M)}{PW\ (I) - PW\ (S)}$$

$$= \frac{3992.7 - 1756.79}{2000 - 272.23} = 1.294\ (Ok)$$

2. Using AW method

$$\text{Conventional B/C ratio} = \frac{AW\ (B)}{CR + AW\ (O\ \&\ M)}$$

$$CR = I - S$$

$$CR = 2000\ (A/P, 8\%, 5) - 400\ (A/F, 8\%, 5)$$

$$= 2000 \times \frac{1.08^5 \times 0.08}{(1.08^5 - 1)} - 400 \times \frac{0.08}{(1.08^5 - 1)}$$

$$= 432.7$$

$$\text{Conventional B/C ratio} = \frac{1000}{(432.7 + 440)} = 1.146 > 1\ \text{(ok)}$$

$$\text{Modified B/C ratio} = \frac{AW\ (B) - AW\ (O\ \&\ M)}{CR}$$

$$= \frac{1000 - 440}{432.7} = 1.294 > 1\ \text{(ok)}$$

- **Detailed design stage**

Include detailed plans for resources, capital, human, facilities, information system, marketing etc.

Operation phase

All activities are functioning product and services are available

- **Construction and implementation stage:**

Includes purchase, construction and implementation of system.

- **Usage stage:** Generate products or services
- **phase out and disposal stage:** removal/recycling of old system

3.7 Introduction to financial and economical analysis

Financial analysis

Capital requirement, source of fund, projected cash flows profitability and project's capacity to meet financial obligations are the area focused by financial analysis

Breakeven analysis and sensitivity analysis is done to test the effect of changes in variables such as cost, price, time etc.

Economical analysis

It analyze the economic viability of the project, for private sector– profitability is the major concern and satisfactory rate of return is needed and for public sector profitability is in terms of contribution to national economy is considered.

IRR > MARR, BCR > 1, NPV > 0 are the decision criteria.

Analysis
i. IEE (Initial Environmental Examination) → For Small Project
ii. EIA (Environmental Impact Assessment → For Large Project

Old Question Solution

"2069 Bhadra"

1. From the following cash flow

EOY	0	1	2	3	4	5
Cash flow	-3000	800	1000	1100	1210	1464

Calculate both type of payback period. MARR = 10%

Solution:

Year	Cash flow	Cumulative Flow (CF)	PW of cash flow	CF
0	-3000	-3000	-3000	-3000
1	800	-2200	$800/1.1 = 727.27$	-2272.73
2	1000	-1200	$1000/1.1^2 = 826.44$	-1446.29
3	1100	-100	$1100/1.1^3 = 826.44$	-619.85
4	1210	1110	$1210/1.1^4 = 826.44$	206.85
5	1464	2574	$1464/1.1^5 = 909.028$	1033.03

a) Simple payback period

Year	CF
3	-100
4	1110
? (3.083)	0

by interpolation

Hence simple payback period = 3.083 years

b) Discounted payback period

CF	Year
-619.85	3
206.85	4
0	? (3.75)

Hence discounted payback period = 3.75 years

2. Equipment costs 250000 and has salvage value of 50000 at the end of its expected life 5 years. Annual expenses will be 40000. It will produce revenue of 120000 per year.

   MARR = 20% = ∈

   i) Evaluate IRR using AW formulation

   ii) Evaluate both type of B/C ratio with FW formulation

   iii) Find ERR

Solution:

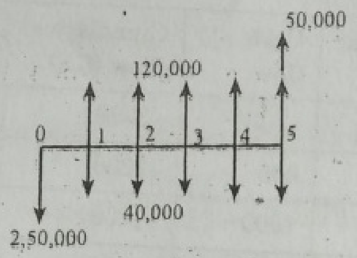


Fig. Cash Flow Diagram

a) For IRR

Let i% be the IRR, then

$AW(i\%) = 0$

$-250000\,(A/P, i\%, 5) + 120000 - 40000 + 50000\,(A/F, i\%, 5) = 0$

$$\text{or, } -250000 \times \frac{i \times (1+i)^5}{[(1+i)^5 - 1]} + 80000 + 50000 \times \frac{i}{[(1+i)^5 - 1]} = 0$$

on solving, we get

$i = 0.2157 = 21.57\% > \text{MARR } (20\%)$ (justified)

b) **BCR**

$FW(I) = 250000\,(F/P, 20\%, 5)$

$= 250000 \times (1 + 0.2)^5 = 622080$

$FW\,(O\,\&\,M) = 40000\,(F/A, 20\%, 5)$

$$= 40000 \times \frac{((1+0.2)^5 - 1)}{0.2} = 297{,}664$$

$FW\,(B) = 120000\,(F/A, 20\%, 5)$

$$= 120000 \times \frac{[(1+0.2)^5 - 1]}{0.2}$$

$$= 892992$$

Now, Convention B/C $= \frac{FW(B)}{FW(I) - FW(S) + FW(O \& M)}$

$$= \frac{892992}{622080 - 50000 + 297664} = 1.026 > 1 \text{ (justified)}$$

Modified B/C $= \frac{FW(B) - FW(O \& M)}{FW(I) - FW(S)}$

$$= \frac{892997 - 297664}{622080 - 50000} = 1.0406 > 1 \text{ (justified)}$$

C) ERR

a) Discounting all cash out flow at time zero

$= -250,000 - 40000\ (P/A, 20\%, 5)$

$$= -250000 - 40000 \times \frac{1.2^5 - 1}{0.2 \times 1.2^5} = -369624.48$$

b) Discounting all cash inflows to 5 years (N)

$= 120000 \times (F/A, 20\%, 5) + 50000$

$$= 120000 \times \frac{1.2^5 - 1}{0.2} + 50000$$

$= 942992.00$

Equating with ERR i%

$369624.48\ (1 + i)^5 = 942992.00$

on solving, we get

$i = ERR = 20.6\%$

"2070 Magh"

Define equivalent worth and rate of return method. How much rupees should you deposit now in a bank account that gives 8% interest per year if you wish to draw Rs. 10000 per month for 10 years?

Solution:

Equivalent worth:

The equivalent value of all the cash in and out flows of an organization at year 0 or annual basis or at year N is called the equivalent worth. Types of equivalent worth.

1. Present worth method
2. Future worth method
3. Annual worth method

Rate of return

Rate of return is the break–even interest rate, i which equates the present worth of a project's. Cash outflows to the present worth of its cash inflows.

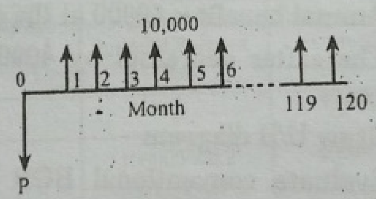
i_{eff} = 8% per year

If $i_{monthly}$ be the interest rate for month then

$i_{monthly} = (1 + i_{eff})^{1/12} - 1$

$= (1 + 0.08)^{1/12} - 1$

$= 0.643\%$



Using NPW (i_m%) = 0

$-P + 10000\ (P/A,\ 0.643\%,\ 120) = 0$

$$P = 10000 \times \frac{1.00643^{120} - 1}{0.00643 \times 1.00643^{120}}$$

= Rs. 8,34,500.6

∴ We have to deposit a sum of Rs. 834500.60 at the beginning to draw Rs. 10000 each for next 10 years.

2. **What is the different between financial and economic analysis? Determine both type of B/C ratio from the following cashflow**

Initial investment = 300000

Annual revenue = 85000

Annual cost = 15000

Salvage value = 20% of initial investment

Useful life = 6 years

MARR = 10%

Solution:

Economic Analysis	Financial Analysis
1. It is done by government project.	1. It is done by private project.
2. Evaluates the monitory and non–monitory benefits also convert social benefits.	2. Evaluates monitory benefits only.
3. Consumer oriented	3. Investor oriented.
4. Profit is never a goal.	4. Profit is most.
5. B/C ratio analysis is most used.	5. ROR and PW are most used.

Using PW approach,

PW (I) = 300000

(S) = $0.2 \times 300000 = 60000$

PW (S) = 60000 (P/F, 10%, 6)

$$= \frac{60000}{1.1^6} = 33868.44$$

PW (O & M) = 15000 (P/A, 10%, 6)

$$= 15000 \times \frac{[(1+0.1)^6 - 1]}{0.1 \times (1+0.1)^6}$$

$$= 65328.91$$

PW (B) = 85000 (P/A, 10%, 6)

$$= 85000 \times \frac{(1.1^6 - 1)}{0.1 \times 1.1^6} = 370192.16$$

Now, B/C conventional $= \dfrac{PW(B)}{PW(I) - PW(S) + PW(O \& M)}$

$= \dfrac{370197.16}{300000 - 33868.44 + 65328.391}$

$= 1.12 > 1$ (Justified)

Modified B/C $= \dfrac{PW(B) - PW(O \& M)}{PW(I) - PW(S)}$

$= \dfrac{370197.16 - 65328.91}{300000 - 33868.44} = 1.145 > 1$ (Justified)

3. **Compute IRR by using trial and error process of the following projects. Determine also investment decision.**

   Initial investment = 25000

   Annual income = 8000

   Salvage value = 5000

   Useful life = 5 year

   MARR = 20%

Solution:

Using PW method,

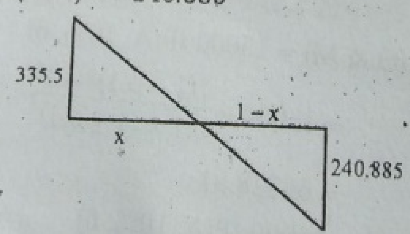
PW (i%) = 0

PW (i%) = −25000 + 8000 (P/A, i%, 5) + 5000 (P/F, 1%, 5000)

$= -25000 + 8000 \dfrac{(1+i)^5 - 1}{i(1+i)^5} + \dfrac{5000}{(1+i)^5}$ ....(1)

Put i = 21%, PW (21%) = 335.50

I = 22%, PW (22%) = −240.885



$\dfrac{1-x}{x} = \dfrac{240.885}{335.5}$

Solving x = 0.582

At i = 21 + 0.582 = 21.582, PW (21.582) = −2.59

∴ i lies between 21% and 21.582%

Interpolation

21%	335.5
21.582%	−2.59
(21.577)?	0

Now, i = 21.577 at equation (1)

PW (21.577) ≈ 0

∴ IRR = 21.577 > MARR (20%)

∴ Economically justified for investment

"2070 Bhadra"

1. Initial investment = Rs. 100000
   Salvage value = 0
   Annual O & M cost = Rs. 20000
   Useful life = 5 years
   Annual benefit = 60000 at the end of first year;
   Thereafter decreases by 4000 each year for the remaining years.

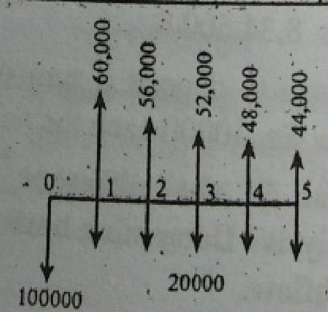
a) Draw U/B diagram

b) Evaluate conventional BCR using PW formulation. Take salvage value = 10000

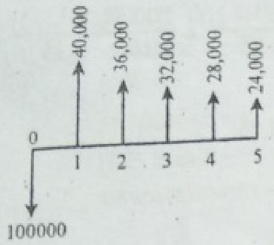
c) Evaluate discounted payback period. Take standard (cut off) payback period = 3 years.

Solution:

<u>UIB (Unrecovered Investment Balance) Diagram:</u>



This can also represented by



Let i% be the internal rate of interest then,

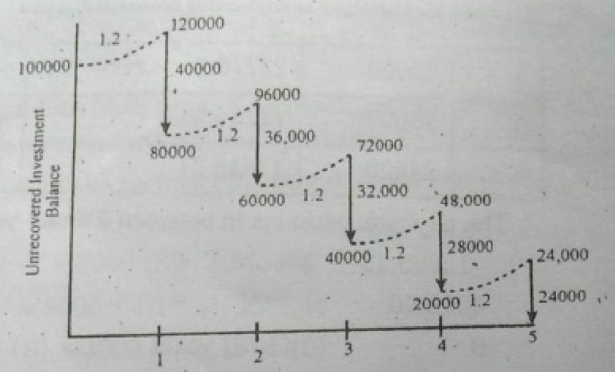
PW(i%) = 0

or, $-100000 + \frac{40000}{(1+i)} + \frac{36000}{(1+i)^2} + \frac{32000}{(1+i)^3} + \frac{28000}{(1+i)^4} + \frac{24000}{(1+i)^5}$

On solving,

i = 0.2 = 20% = MARR. Remain indifferent.

Year	Unpaid balance at beginning year (Rs.)	Return on unpaid Balance (Rs.)	Payment received	Unpaid balance at end of year (Rs.)
0	–100000	0	0	– 100000
1	–100000	100000 × 0.2 = –20000	40000	– 80000
2	–80000	–80000 × 0.2 = – 16000	36000	– 60000
3	–60000	–12000	32000	– 40000
4	–40000	–8000	28000	– 20000
5	–20000	–4000	24000	0



Investment balance diagram

b) B/C ratio , MARR = 12%

$$PW(B) = \frac{60000}{1.12} + \frac{56000}{1.12^2} + \frac{52000}{1.12^3} + \frac{48000}{1.12^4} + \frac{44000}{1.12^5}$$

$$= 190628.5$$

$$PW(S) = \frac{10000}{1.12^5} = 5674.26$$

$$PW(O \& M) = 20000 \times \frac{(1.15^6 - 1)}{0.12 \times 1.12^5} = 72095.52$$

$$\text{Conventional B/C ratio} = \frac{PW(B)}{PW(I) + PW(O \& M) - PW(S)}$$

$$= \frac{190698.5}{100000 + 72096.5 - 5674.26}$$

$$= 1.145 > 1$$

Economically justified

C) Discounted payback period

Year	Cash flow	PW of CF	Cumulative CF
0	-100000	-100000	-100000
1	40000	40000/1.12 = 35714	-64285.2
2	36000	$36000/1.12^2$ = 28698.9	-35586.2

3	32000	22776.9	-12809.25
4	28000	17794.5	4985.50
5	24000	13618.24	18603.44

∴ The payback period is in between 3 and 4 years.

-12809.25 3

4985.50 4

0 ? (3.72) years

∴ Discounted Payback Period = 3.72 years > 3 years

∴ Economically justified

"2071 Magh"

1. **Calculate both types of BCR using FW formulation**

   Initial investment is Rs. 50000

   Income is Rs. 10000 at the end of first year and increasing by 10% per year

   Annual expenditure is Rs. 2000 at the end of first year and increasing by Rs. 200 per year

   Useful life is 6 years

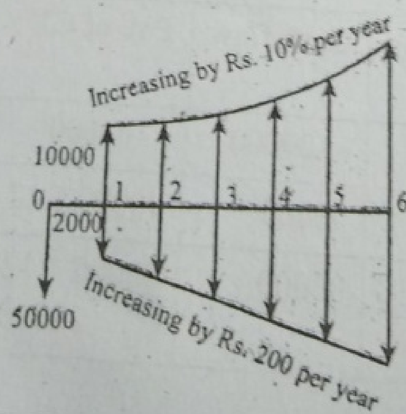
   MARR = 15%

Solution:

FW(I) = 50000 (F/P, 15%, 6)

$= 50000 \times 1.15^6$

= 115653.038



$$FW\ (O\ \&\ M) = 2000\ (F/A,\ 15\%\ 6) + \frac{G}{i}\ (F/G,\ 15\%,\ 6) - \frac{NG}{i}$$

$$= 2000 \times \frac{(1.15^6 - 1)}{0.15} + \frac{200}{0.15}\frac{(1.15^6 - 1)}{0.15} - \frac{6 \times 200}{0.15}$$

$$= 21179.128$$

$$i = 15\%,\ g = 10\%$$

$$FW\ (B) = A\frac{((1 + i)^N - (1 + g)^N)}{i - g}$$

$$= 10000 \times \frac{(1.15^6 - 1.1^6)}{0.15 - 0.1}$$

$$= 108299.95$$

∴ From conventional BCR method

$$FW\ (B) = \frac{FW(B)}{FW(I) - FW\ (S) + FW(O\ \&\ M)}$$

$$= \frac{108299.953}{115653 - 20000 + 21179.128}$$

$$= 0.926 < 1$$

Not justified.

$$\text{Modified BCR} = \frac{FW(B) - FW(O\ \&\ M)}{FW(I) - FW(S)}$$

$$= \frac{108299.953 - 21179.128}{115653 - 20000}$$

$$= 0.910 < 1$$

Not justified

"2071 Bhadra"

1. **Define IRR. Find d IRR and ERR of the following project. MARR = ∈ = 15%**

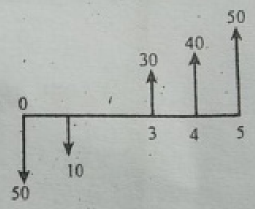
Year	0	1	2	3	4
Cash flow	-50	-10	30	40	50

Solution:

IRR is the breakeven interest rate 'i' which equates the present worth of a project's cash out flows to the present worth of its cash inflows.

In another way,

IRR is defined as the interest rate earned on the unpaid balance of an installment loan.



PW (i%) = 0

$$-50 - \frac{10}{(1+i)} + \frac{30}{(1+i)^3} + \frac{40}{(1+i)^4} + \frac{50}{(1+i)^5} = 0$$

Solving,

$i = 16.21\% > \text{MARR } (15\%)$

Discounting all the cash outflow at present with $\in = 15\%$

$$= -5 - \frac{100}{1.15} = -58.695$$

Discounting all the cash inflow at the end of 5th year

$$30 \times 1.15^2 + 40 \times 1.15 + 50 = 135.675$$

establishing equivalence

$58.695 \times (1+i)^5 = 135.68$, solving $i = 0.1824$

$i = 18.24\% > \text{MARR}$

$\therefore$ ERR = 18.24%

2.

	Machine A
Initial investment	**Rs. 6000**
Annual benefits	**Rs. 3000**
O & M cost	**Rs. 1000**
Salvage value	**Rs. 1500**
MARR	**10%**

a) Evaluate both type of BCR (FW formulation). Take useful life = 10 years.

b) Evaluate both type of payback period. If useful life = 5 years. (Take standard payback period = 3 years).

c) Explain the factors affecting determination of MARR

Solution:

$$FW(I) = 6000 (F/P, 10\%, 10)$$

$$= 6000 \times 1.1^{10} = 15562.45$$

$$FW(B) = 3000 (F/A, 10\%, 10)$$

$$= 3000 \times \frac{(1.1^{10} - 1)}{0.1} = 47812.27$$

$$FW(O \& M) = 1000 (F/A, 10\%, 10)$$

$$= 1000 \times \frac{1.1^{10} - 1}{0.1} = 15937.42$$

$$FW(S) = 1500$$

$$\text{Conventional B/C ratio} = \frac{FW(B)}{FW(I) - FW(S) + FW(O \& M)}$$

$$= \frac{47812.27}{15562.45 - 1500 + 15937.42}$$

$$= 1.59 > 1, \text{justified}$$

$$\text{Modified B/C ratio} = \frac{FW(B) - FW(O \& M)}{FW(I) - PW(S)}$$

$$= \frac{47812.27 - 15937.42}{15562.45 - 1500} = 2.26 > 1$$

b) Payback period

Year	CF	Cumulative CF	PW of CF	Cumulative CF (Discounted)
0	-6000	-6000	-6000	-6000
1	3000 - 1000 = 2000	-4000	1818.18	-4181.82
2	2000	-2000	1652.89	-2528.93
3	2000	0	1502.63	-1026.33

4	2000	2000	1366.026	339.726
5	2000 + 1500 = 3500	55000	2173.22	2512.946

i) Simple payback period = 3 years = standard payback period.

Decision → Remain indifferent

ii) Discounted payback period

−1026.33 3

339.726 4

0 ? (3.75)

Discounted payback period = 3.75 > 3 years

Economically not justified.

iii) Refer at 3.1

"2072 Ashwin"

1. Calculate IRR from the following cash flow and draw investment balance diagram.

Year	0	1	2	3	4	5
Cash flow	-800	250	300	400	-150	600

Solution:

PW(i%) = 0

$$-800 + \frac{250}{(1+i)} + \frac{300}{(1+i)^2} + \frac{400}{(1+i)^3} - \frac{150}{(1+i)^4} + \frac{600}{(1+i)^5} = 0$$

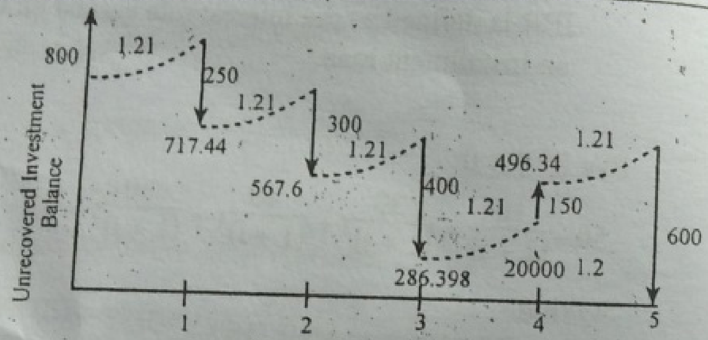
Solving, we get, i = 0.2093

i = 20.93%

Unrecovered project balance calculation

Year	Unpaid balanced at beginning year (Rs.)	Return on unpaid Balance (Rs.)	Payment received	Unpaid balance at end of year
0	0	0	-800	-800
1	-800	-167.44 (800 × 0.2093)	250	-717.44

2	-717.44	-150.16 (717.44 × 0.2093)	300	-567.6
3	-567.6	-118.79	400	-286.398
4	-286.398	-59.94	-150	-496.34
5	-496.34	-103.8	600	0.14 ≈ 0

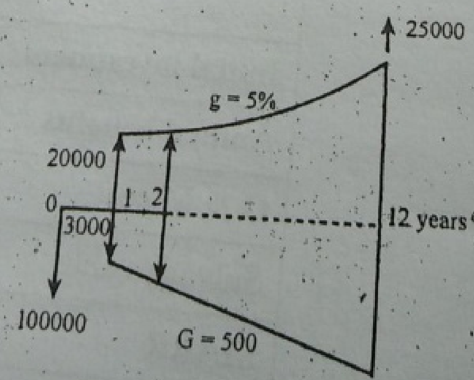


Investment balance diagram

2. Calculate both types of BCR of a project with following details: MARR = 12%

Initial investment	Annual income	Annual cost	Useful life	Salvage
Rs. 100000	Rs. 20000 at the end of first year and increased by 5% per year	Rs. 3000 at the end of first year and increased by Rs. 500 per year	12 years	25000

Solution:



Using a FW formulation:

$$\text{FW (I)} = 100000\ (F/A,\ 12\%,\ 12)$$

$$= 100000 \times 1.12^{12} = 389597.599$$

$$\text{FW (B)} = A\,\frac{(1+i)^N - (1+g)^N}{i - g}$$

$$= 20000\,\frac{[(1+0.12)^{12} - (1+0.05)^{12}]}{0.12 - 0.05}$$

$$= 600034.19$$

$$\text{FW (O \& M)} = 3000\ (F/A,\ 12\%,\ 12) + \frac{G}{i}\,(F/G,\ i\%,\ N) - \frac{NG}{i}$$

$$= 3000 \times \frac{(1.12^{12} - 1)}{0.12} + \frac{500}{0.12} \times \frac{1.12^{12} - 1}{0.12} - \frac{12 \times 500}{0.12}$$

$$= 122954.12$$

$$\text{FW(S)} = 25000$$

$$\text{Conventional B/C ratio} = \frac{\text{FW(B)}}{\text{FW(I)} - \text{FW(S)} + \text{FW(O \& M)}}$$

$$= \frac{600034.19}{389597.599 - 25000 + 122954.12}$$

$$= 1.23 > 0$$

Economically justified.

$$\text{Modified B/C ratio} = \frac{\text{FW(B)} - \text{FW(O \& M)}}{\text{FW(I)} - \text{FW(S)}}$$

$$= \frac{600034.19 - 122954.12}{389597.599 - 25000}$$

$$= 1.3 > 1$$

Justified

Extra Questions solutions

1. Evaluate machine XYZ on the basis of the PW and FW methods, when the MARR is 12% per year. Pertinent cost data are as shown follows:

Investment cost (Rs)	150000
Salvage value (Rs.)	35000
Useful life (yrs)	15
Annual expenses(Rs.)	12000
Overhead cost (Rs.) – end of 5^{th} year	24000
Overhead cost (Rs.) – end of 10^{th} year	60000

Solution:

PW(method)

PW(12%) = –150000 – 12000 (P/A, 12%,15) – 24000 (P/F, 12%, 5) – 60000 (P/F, 12%, 10) + 35000 (P/F, 12%, 15)

$$= -150000 - 12000 \times \frac{(1.12^{15}-1)}{1.12^{15} \times 0.12} - \frac{24000}{1.12^{5}} - \frac{60000}{1.12^{10}} + \frac{35000}{1.12^{15}}$$

$= -158272.64$

Since PW (12%) < 0 the project is not acceptable

FW(method)

FW(12%) = –150000 (F/P,12%,15) – 12000 (F/A,12%,15) – 24000 (F/P,12%,10) – 60000 (F/P,12% 5) + 35000

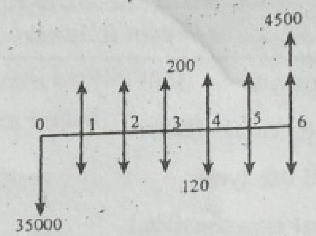
$= -866315.72$

Since FW (12%) < 0, the project is not acceptable

2. Suppose that you purchased a building five years ago for Rs. 350000. Its annual maintenance expenses has been Rs. 120000 per year. At the end of five years, you sell the building for Rs. 4500000. During the period of ownership you rented the building for Rs. 200000 per year at the beginning of each

year. Use the AW method to evaluate this investment when your MARR is 12% per year.

Solution:



Calculation in Thousands

AW (12%) = –3500 (A/P,12%,5) + (200 – 120) + 4500 (A/F,12%, 5)

= – Rs. 183

Since AW (12%) < 0, the project is not acceptable.

3. **If a machine will be operated according to varying hours. 1500 hrs in the first year, 2500 hrs in the second year, and 2000 hrs. in the third year. Compute the annual equivalent saving or cost per machine hour if the firm's MARR is 15% and AW(15%) = Rs. 6896**

Solution:

Let 'X' Rs/hr. be the equivalent annual saving per machine hour, which is to be determined. Equivalent annual saving in terms of X can be computed as,

AW(15%) = [15000x (P/F, 15%, 1) + 2500x (P/F, 15%, 2) + 2000x (P/F, 15%, 3)] (A/P, 15%, 3)

Calculating,

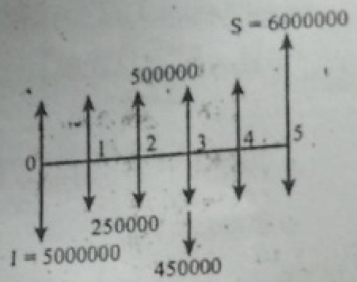
6896 = 1975.227 x

X = Rs. 3.49/hr

4. **You purchased a building 5 years ago for Rs. 50,00,000. Annual Maintenance cost is Rs. 250,000/year. At the end of 3 years Rs. 450,000 was spent on roof repairs. At the end of 5 years you sell a building for Rs. 6000000. During the period of ownership, you rented the building for Rs. 500000 per year**

paid at the beginning of each year. Use AW method. MARR = 12%

Solution:



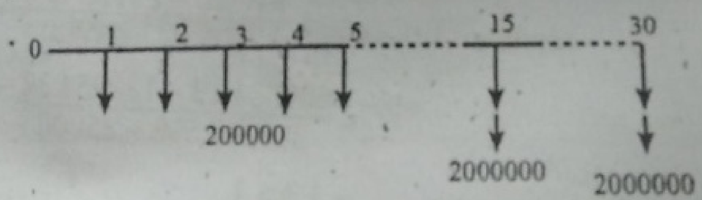
AW (12%) = −5000000 (A/P,12%, 5) − 450000 (F/P, 12% , 2) (A/F, 12%, 5) + 6000000 (A/F, 12 % 5) + 500000 − 250000

= −221520.9

AW is −ve, investment is not economically justified.

5. **Maintenance cost for a new communication tower with an expected 50 years life are estimated to be Rs. 200000 each years for the first 5 years, followed by a Rs. 2000000 expenditure in the 15th year and another Rs. 2000000 expenditure in the 30th year. If MARR = 10% what is the equivalent uniform annual cost over the entire so period?**

Solution:



Discounting all cash flows to the present

PW(10%) = 200000 (P/A, 10%, 5) + 2000000 (P/F, 10%, 15) + 2000000 (P/F, 10%, 30)

$= 200000 \times 3.7908 + 2000000 \times 0.2394 + 2000000 \times 0.0573$

= 1351560

AW(10%) = 1351560 (A/P, 10%,50) = Rs. 136318.34

6. A housing company is considering building apartments at Kalanki, Kathmandu. The company has reached to a following

No. of apartment	50
Occupancy capacity	80%
Land cost	Rs. 30000000
Construction cost	Rs. 95000000
Study period	20 years
Annual taxes & insurance	15% of total investment
Salvage values	1.2 times the land cost

Determine minimum monthly rate to be charged per unit apartment. MARR = 15%

Solution:

(Amounts are in 00000s)

AW of costs

Capital recovery costs = (300 + 950) (A/P, 15%, 20) + 300 × 1.2 × (A/F, 15%, 20)

= Rs. 196.222

Annual taxes = 15% of (300 + 350) = Rs. 187.5

Therefore, equivalent AW costs = Rs. 383.722 (196.222 + 187.5)

AW of revenues

Let x be the monthly rent to be charged

AW of revenues = x × 12 × 50 × 80% = 480 x

Equating,

480 x = 383.722

x = 0.799 ≈ 0.8

x = 80000 per month (Approximately)

***

4 Comparative Analysis of Alternatives

4.1 Comparing mutually exclusive alternatives having same useful life by:

4.1.1 Payback period method and equivalent worth method:

Payback period:

In payback period, the length of time required to recover the cost of an investment for two or more alternative projects are compared and one with least payback period is selected.

(Note: Types of payback period is already studied in chapter 3)

Example: Autonumeries company has two mutually exclusive project. Select the best project using payback period method. Study period = 5 yrs and NARR = 15%

Alternatives	A	B
Initial Investment	850000	400000
Net Annual Income	120000	110000

Take salvage value = 20% of initial investment

Solution:

For alternative A

Periods	Cash flows	PW of net cash flows (i = 15%)	Cumulative cash flows
0	– 850000	– 850000	– 850000
1	120000	104347.82	– 745652.17
2	120000	90737.24	– 654914.93
3	120000	78901.94	– 576012.98
4	120000	68610.38	– 507402.59
5	120000 + 170000	144181.25	– 36322134

[At year 5 salvage value = 170000]

Here, the cumulative Balance does changes into positive in 5 years therefore payback period is more than 5 yrs for alternative A.

For Alternative 'B'

No. of period	Cash flow	PW of net cash flow	Cumulative cash flow
0	– 400000	– 400000	– 400000
1	110000	95652.17	– 304347.82
2	110000	83175.8	– 220872.03
3	110000	72326.78	– 148545.25
4	110000	62892.85	– 85652.4
5	110000 + 80000	94463.58	8811.18

[At 5 year salvage value = 80000]

Here, cumulative balance turns positive in the year 5. So payback period lies between year 4 and year 5 by interpolation.

∴ Payback period = 4.90 yrs.

[Note: Interpolation can be done by using calculator as in chapter 3]

Equivalent worth method:

In equivalent worth method two or more alternatives are compared on the basis of (PW, AW or FW) i.e. equivalent worth. And the alternative with highest equivalent worth is selected as best alternatives.

Example: Three mutually exclusive investment alternatives for implementing an office automation plan in an engineering design form are being considered. Each alternative meets the same service requirement but capital investment & benefits are different for each alternative. The study period is 10 yrs. MAPR = 10% per year, which alternatives should be selected in view of the following investment.

Alternatives	A	B	C
Capital investment	– Rs 390,000	– Rs 920,000	– Rs 666,000
Annual cost saving	69,000	167,000	133,500

Solution:

By PW method:

$$PW(10\%)_A = -\text{Rs } 390{,}000 + \text{Rs } 69000\,(P/A,\ 10\%,\ 10)$$

$$= -390000 + 69000 \times \left[\frac{(1+0.1)^{10}-1}{0.1}\right] \times \frac{1}{1.1^{10}}$$

$$\left[\because F = A\left[\frac{(1+i)^n - 1}{i}\right] \text{ and } p = \frac{F}{(1+i)^n}\right]$$

$$= -390000 + 423975.13$$

$$= \text{Rs. } 33975.13$$

$PW(10\%)_B = -$ Rs. 920,000 + Rs. 167,000 (P/A, 10%, 10)

$$= -920{,}000 + 167{,}000 \times \left[\frac{(1+0.1)^{10}-1}{0.1}\right] \times \frac{1}{1.1^{10}}$$

= Rs. 106,148

PW (10%) C = – Rs. 660,000+Rs. 133,500 (P/A, 10%, 10) = Rs. 160,304

So, Based on PW method, Alternatives C would be selected because it has largest PW value (Rs. 160,304)

By AW method

$AW(10\%)_A$ = – Rs.390,000 (A/P, 10%, 10) + Rs.69,000

$$= -390{,}000 \times \left[\frac{(1+0.1)^{10} \times 0.1}{(1+0.1)^{10}-1}\right] + \text{Rs. } 69{,}000$$

$$\left[\because F = P(1+i)^n \;\&\; A = \frac{F \times i}{(1+i)^n - 1}\right]$$

= – 63470.7 + 69000

= Rs. 5529.30

$AW(10\%)_B$ = – 920,000 (A/P, 10%, 10) + 167,000 = Rs. 17,316

$AW(10\%)_C$ = – 660,000 (A/P, 10%, 10) + 133,500 = Rs. 26,118

Alternative C is choose because it has largest AW value (Rs. 26,118)

By FW method:

$FW(10\%)_A$ = – 390,000 (F/P, 10%, 10) + 69000 (F/A, 10%, 10)

$$= -390{,}000 \times (1.1)^{10} + 69000 \times \left[\frac{(1+0.1)^{10}-1}{0.1}\right]$$

= Rs. 88,138

$FW(10\%)_B$ = – 920,000 (F/P, 10%, 10) + 167,000 (F/A, 10%, 10)

= Rs. 275,342

$FW(10\%)_C$ = – 660,000 (F/P,10%,10) + 133,500 (F/A,10%,10)

= Rs. 415,801

So, based on the FW method, the choice is again alternative C, because it has the largest FW value (Rs. 415,801).

Example: A company is planning to install a new automated plastic molding press, four different presses are available. The initial capital investment, annual expenses & salvage values are given as & (MARR = 10%). Select best one.

Press	P1	P2	P3	P4
Capital investment	– Rs. 24000	– Rs. 30,400	– Rs. 49,600	– Rs. 52,000
Useful life	5 yrs	5 yrs	5 yrs	5 yrs
Annual expenses	– Rs. 31,200	– Rs. 29,128	– Rs. 25,192	– Rs. 22,880
Salvage	10,000	18000	20000	25000

Solution:

Here we solve (By the FW method):

$FW\ (10\%)_{P1} = -24000\ (F/P,\ 10\%,\ 5) - 31200\ (F/A,\ 10\%\ 5) + 10000$

$= -24000 \times (1 + 0.1)^5 - 31200 \left[\frac{(1+0.1)^5 - 1}{0.1}\right] + 10000$

$= -229,131 + 10000$

$= -219131$

Similarly,

$FW\ (10\%)_{P2} = -30,400\ (F/P,\ 10\%,\ 5) - 29.128\ (F/A,\ 10\%,\ 5) + 18000$

$= -226,788 + 18000$

$= -208788$

$FW\ (10\%)_{P3} = -49,600\ (F/P,\ 10\%,5) - 25,192 (F/A, 10\%,\ 5) + 20000$

$= -233,689 + 20000$

$= -213689$

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$FW\ (10\%)_{P4} = -52{,}000\ (F/P,\ 10\%,\ 5) - 22{,}880\ (F/A,\ 10\%,5) + 25000$

$= -223{,}431 + 25000$

$= -198431$

So, Based on FW analysis preference ranking is (P4 > P2 > P3 > P1) and the best option is press 4.

[Note: It can solve by using PW and FW method also]

4.1.2 Rate of Return method and Benefit cost ratio method:

The projects having highest value of IRR, ERR and BCR may not be the preferred alternative, they are just relative measure and cannot be in the way as (PW, AW and FW) method. For this purpose we adopt incremental analysis.

<u>Procedure for incremental analysis:</u>

1. Arrange the feasible alternative, based on increasing capital investment.
2. Establish a base alternative:
   a. Cost alternative: The first alternative (Least capital investment) ∴ is base.
   b. Investment alternatives: If the first alternative is acceptable (IRR > MARR), select it as the base. If the first alternative is not acceptable, choose the next alternative in order of increasing capital investment & check the profitability criteria (PW etc.) values. Continue until an acceptable alternative is obtained. If none is obtained, the do nothing alternative is selected.
3. Use iteration to evaluate differences between alternative until no more alternative exist.
   a) if the incremental cash flow between the next (higher capital investment) alternative and the current selected alternative is acceptable, choose the next alternative as the current best alternative. Otherwise, retain the last acceptable alternative as the current best.
   b) Repeat, and select the preferred alternative the last one for which the incremental cash flow was acceptable.

Example: The estimated capital investment and the annual expenses for four alternatives design of a diesel powered air compressor are shown. The study period is five years and the MARR is 20% per year. Based on this information determine the preferred design alternative using IRR on incremental investment.

Design alternatives	D1	D2	D3	D4
Capital investment	– Rs. 100000	– Rs. 140600	– Rs. 148200	– Rs. 122000
Annual expenses	– 29000	– 16900	–14800	– 22,100
Useful Life	5	5	5	5
Market value	10,000	14,000	25,600	14,000

[Note: Market value is salvage value]

Solution:

Arrange the four mutually exclusive cost alternatives based on their increasing capital investment costs.

So order of alternatives for incremental analysis is D1, D4, D2 and D3.

Take D1 as the base alternatives

Now, the first increment analysis is between D1 and D4 i.e. D4 – D2 and present in tabular form.

Increment covered	D4 – D1	D2 – D4	D3 – D4
Capital investment	– 22000	– 18.600	– 26.200
Annual expenses	6900	5200	7,300
Market value	4000	0	11,600

Calculate IRR of increment cash flow of D4 – D1

$$PW\ (i\%) = 0$$

$$-22000 + 6900\ (P/A,\ i\%,\ 5) + 4000\ (F/P,\ i\%,5) = 0$$

$$-22000 + 6900\left[\frac{(1+i)^5 - 1}{i}\right] \times \frac{1}{(1+i)^5} + 4000 \times \frac{1}{(1+i)^5} = 0$$

Solving by calculator,

$i = 20.5\% > MARR (20\%)$ So, eliminate D1

Again, incremental analysis is between D4 and D2.

Calculate IRR of incremental cash flow of D2 – D4.

FW (i%) = 0 [∵ for calculating IRR any of AW, PW & FW is made equal to zero]

$$-18,600\ (F/P,\ i\%,5) + 5200\ (F/A,i\%,5) + 0 = 0$$

$$-18,600 \times (1+i)^5 + 5200\left[\frac{(1+i)^5 - 1}{i}\right] = 0$$

Solving by calculator

$i = 12.3\% < MARR$ (20%0 So eliminate D2.

Again, make incremental analysis between D3 and D4.

Calculate IRR of incremental cash flow of D3 – D4.

FW (i%) = 0

$$-26,200\ (F/P,\ i\%,\ 5) + 7,300\ (F/A,\ i\%,\ 5) + 11,600 = 0$$

$$-26,200 \times (1+i)^5 + 7300\left[\frac{(1+i)^5 - 1}{i}\right] + 11600 = 0$$

Solving by calculator

$i = 20.4\% > MARR (20\%)$ So eliminate D4 and Select D3 as the best alternative.

Example: An engineering firm is considering following exclusive alternatives.

EOY	Alternatives		
	A1	A2	A3
0	– 4000	– 2000	– 6000
1	3000	1600	3000
2	2000	1000	4000
3	1600	1000	2000

Determine the best alternatives by using IRR incremental analysis.

Assuming MARR = 19%

Solution: This is the case of investment alternatives so,

First calculate IRR for all alternatives

For A1 PW (i%) = 0

$$-4000 + \frac{3000}{(1+i)} + \frac{2000}{(1+i)^2} + \frac{1600}{(1+i)^3} = 0$$

$\therefore$ I = 34.34% > MARR, (accepted)

Similarly for A2 PW (i%) = 0

$$-2000 + \frac{1600}{(1+i)} + \frac{1000}{(1+i)^2} + \frac{1000}{(1+i)^3} = 0$$

i = 40.76% > MARR, (accepted)

& for A3 PW (i%) = 0

$\therefore$ i = 24.81% > MARR (accepted)

Taking A2 as the base alternative & order for incremental analysis is A2, A1, A3.

Now, the first increment analysis is between A1 and A2 i.e. A1–A2

EOY	A1 – A2	A3 – A1
0	– 2000	– 2000
1	1400	0
2	1000	2000
3	600	400
IRR	27.61%	8.8%

Calculate IRR of incremental cash flow of (A1 – A2)

Determine the best alternatives by using IRR incremental analysis.

Assuming MARR = 19%

Solution: This is the case of investment alternatives so,

First calculate IRR for all alternatives

For A1 PW (i%) = 0

$$-4000 + \frac{3000}{(1+i)} + \frac{2000}{(1+i)^2} + \frac{1600}{(1+i)^3} = 0$$

$\therefore$ I = 34.34% > MARR, (accepted)

Similarly for A2 PW (i%) = 0

$$-2000 + \frac{1600}{(1+i)} + \frac{1000}{(1+i)^2} + \frac{1000}{(1+i)^3} = 0$$

i = 40.76% > MARR, (accepted)

& for A3 PW (i%) = 0

$\therefore$ i = 24.81% > MARR (accepted)

Taking A2 as the base alternative & order for incremental analysis is A2, A1, A3.

Now, the first increment analysis is between A1 and A2 i.e. A1–A2

EOY	A1 – A2	A3 – A1
0	– 2000	– 2000
1	1400	0
2	1000	2000
3	600	400
IRR	27.61%	8.8%

Calculate IRR of incremental cash flow of (A1 – A2)

PW (i%) = 0

$$-2000 + \frac{1400}{(1+i)} + \frac{1000}{(1+i)^2} + \frac{600}{(1+i)^3} = 0$$

Solving by calculator

i = 27.61% > MARR (15%); accept A1 & eliminate A2

Again, increment analysis between A1 & A3 i.e (A3 – A1)

PW(i%) = 0

$$-2000 + 0 + \frac{2000}{(1+i)^2} + \frac{400}{(1+i)^3} = 0$$

i = 8.8% < MARR (15%); accept A1, Eliminate A3.

So the best alternative is A1.

4.2 Comparing mutually exclusive alternatives having different useful lives by

When the useful lives of mutually exclusive alternatives are different, the repeatability assumption may be used in their comparison if the study period can be infinite in length or a common multiple of the useful lives.

4.2.1 Repeatability Assumption

Two or more alternatives having different useful life are changed into projects having same useful life by expanding their life upto atleast common year.

The economic consequences that are estimated to happen in an alternative's initial useful life span will also happen in all succeeding life spans replacements)

- Actual situations in engineering practice seldom meet both conditions.

Example: The following data have been estimated for two mutually exclusive investment alternatives A and B, associated with a small engineering project for which revenues as well as expenses are involved. They have useful lives of four and six years respectively. If the MARR = 10% per year, show which alternative is more desirable by using equivalent worth method. Use repeatability assumptions.

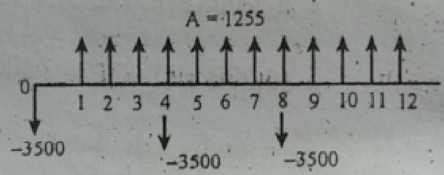
Alternative	A	B
Capital investment	– Rs. 3500	– Rs. 5000
Annual revenue	1900	2500
Annual expenses	– 645	– 1020
Useful life (year)	4	6
Market value at the end of useful life	0	0

Solution:

The LCM of useful lives of alternatives A and B is 12 years. using repeatability assumptions and a 12 years study period.

For Alternative A

Net revenue = Revenue – expenses = 1900 – 645 = Rs.1255



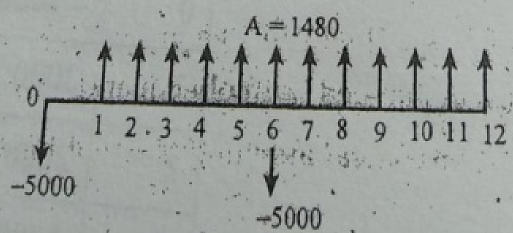
Now, PW(10%)A = – 3500 – 3500 [(P/F,10%,4) + (P/F,10%,8)] + 1255 (P/A,10%,12)

$$= -3500 - 3500\,[(1+0.1)^{-4} + (1+0.1)^{-8}] + 1255 \times \left[\frac{1.1^{12}-1}{0.1}\right] \times \frac{1}{1.1^{12}}$$

= Rs. 1028

For alternative B,

Net revenue = Revenue – expenses = 2500 – 1020 = Rs. 1480



Now, PW(10%)B = – 5000 – 5000 (P/F, 10%,6) + 1480 (P/A, 10%,12)

= Rs. 2,262

Based on PW method we would select alternative B because it has larger value (2,262)

[Note: Similarly we can apply AW & FW method also]

4.2.2 Co–terminated Assumption:

→ A finite and identical study period is used for all alternatives.

→ This planning horizon, combined with appropriate adjustments to the estimated cash flows, puts the alternatives on a common and comparable basis.

→ Used when repeatability assumption is not applicable.

→ This is the approach most frequently used in engineering practice.

a) Useful life shorter than study period (useful life < study period)

i) <u>Cost alternatives:</u> Each cost alternative must provide same level of service as study period:

1) Control for service or least equipment for remaining time.
2) Repeat part of useful life of original alternative until study period ends.

ii) <u>Investment alternatives:</u> Assume all cash flows reinvested in other opportunities of MARR to end of study period.

This is mostly applicable in our syllabus numerical problems.

<u>Some Definition:</u>

1. Imported market value Technique: Used for useful life > study period

When current market place data is unavailable for an asset, it is sometimes necessary to estimate the market value of an asset.

Referred to as an imputed or implied market value.

Estimating is based on logical assumption about the remaining life for the asset.

IMV_T = [EW at the end of year T of remaining capital recovery amounts]

[EW at the end of year T of original market value at the end of useful life]

T < useful life

Example: A large corporation is considering the funding of following mutually exclusive water supply projects. Based on given information determine which alternative is more desirable to implement by using co–terminated assumption.

Alternative	A	B
Investment	70000	100000
Annual Revenue	38000	50000
Annual cost	12900	27660
Useful life	4 yrs	8 yrs
Salvage value	0	0

[Take MARR = 10%]

Solution:

Taking study period as 8 years

(Note: always take study period equal or greater than useful lives of all alternatives, if not given in the question]

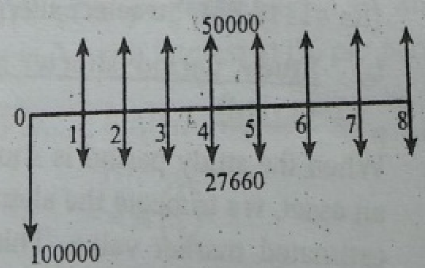


Fig.: Cash Flow Diagram of Project B

No adjustment is required for alternative B because study period is equal to useful life.

Now,

$$FW(10\%)B = -10000[F/P, 10\%, 8] + (50000 - 27660)[F/A, 10\%, 8]$$

$$= -10000(1+0.1)^8 + 22340\left[\frac{(1+0.1)^8 - 1}{0.1}\right]$$

$$= \text{Rs. } 41118.85$$

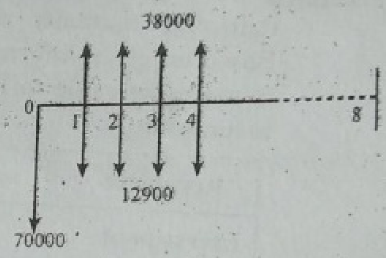


Fig: Cash Flow Diagram of Project A

For alternative A, the cash flow accumulated at end of useful life is reinvested for extended periods.

$$FW(10\%) = [-70000(F/P, 10\%, 4) + (38000 - 12900)(F/A, 10\% 4)] \times [F/p, 10\% 4]$$

$$= [-70000(1+0.1)^4 + 25100 \times \left(\frac{1.1^4 - 1}{0.1}\right)] \times (1+0.1)^4$$

$$= \text{Rs. } 20500.47$$

Here based on co-terminated assumption FW for project B is higher (Rs 41118.85). So select alternative 'B' for implementation.

b) <u>**Study period shorter than useful life: (Useful life > study period)**</u>

When the study period is shorter than the useful life of any project or an asset, we truncate the alternative at the end of study period using an estimated market value. This assumes that disposable assets will be sold at the end of study period at that value.

For this purpose we adopt imputed market value technique, which is sometimes called the implied market value & estimated by imputed market value (IMV_T) = [PW at the end of year T of remaining capital recover amounts] + [PW at the end of T of original market value at the end of useful life]

Example: Use co-terminate (i.e. the imputed market value technique) method to develop an estimated market value and determine the best alternative project by taking study period of 5 yrs for given information.

Alternative	X	Y
Capital investment	– 35000	– 50000
Annual expenses	1500	2500
Annual revenue	13000	17500
Salvage Value	3500	5000
Useful life	5 years	8 years

Use MARR = 10% per year

Solution:

For alternative X:

Net annual revenue = Revenue – expenses

$= 13000 - 1500 = 11500$

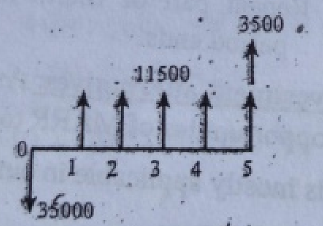


Fig. Cash Flow Diagram of X

$$FW(10\%)x = -35000\ (F/P,\ 10\%,\ 5) + 11500\ (F/A,\ 10\%,\ 5) + 3500$$

$$= -35000\ (1 + 0.1)^5 + 11500 \left[\frac{1.1^5 - 1}{0.1}\right] + 3500$$

$$= \text{Rs. } 17340.8$$

For alternative Y:

Net annual revenue = 17500 – 2500 = 15000

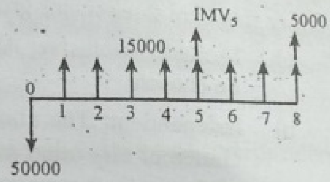


Fig: Cash Flow Diagram of Y

By using imputed market value (IMV) technique

capital recovery, CR(10%) = 50000 (A/P, 10%, 8) – 5000 (A/F, 10%, 8)

$$= 50000\ (1.1)^8 \times \frac{0.1}{1.1^8 - 1} - 5000 \times \frac{0.1}{1.1^8 - 1}$$

$$= 9372.2 - 437.2$$

$$= \text{Rs. } 8935 \qquad \left[\because A = \frac{F \times i}{(1+i)^n - 1}\right]$$

Now,

PW at the end of 5th year for remaining 3 years,

$$PW\ (10\%)_{CR} = 8935\ (P/A,\ 10\%,\ 3)$$

$$= 8935 \times \left[\frac{1.1^3 - 1}{0.1}\right] \times \frac{1}{1.1^3}$$

$$= \text{Rs. } 22220$$

Again,

PW at the end of 5th year of original market value at the end of useful life (8 years),

i.e. at the end of 5th year of salvage value (Rs. 5000)

So, PW (10%) MV = 5000 (P/F, 10%, 3)

$$= 5000 \times (1 + 0.1)^{-3}$$

$$= \text{Rs. } 3756.50$$

Now, imputed Market value (IMV) = 22220 + 3756.5

= Rs. 25976.5

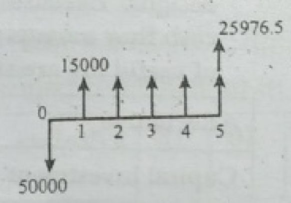


Fig: Revised Cash Flow Diagram of Y

$$FW\ (10\%)y = -50000\ (F/P,\ 10\%,\ 5) + 15000\ (F/A, 10\%\ 5) + 25976.5$$

$$= -50000 \times (1 + 0.1)^5 + 15000\left[\frac{(1+0.1)^5 - 1}{0.1}\right] + 25976.5$$

$$= \text{Rs. } 37027.5$$

Based on co–terminated assumption FW for alternative Y is higher (Rs. 37027.5)

So, alternative Y is best selection.

4.2.3 Capitalized worth method:

→ The process in which present worth of all revenues and for expenses over an infinite length of time is calculated is called capitalized worth (CW) method.

→ This is a convenient basis for comparing mutually exclusive alternatives when the period of needed service is indefinitely long end repeatability assumption is applicable.

→ The CW of a perpetual series of end of period uniform payments A, with interest at i% per period is A (P/A, i%, ∞)

$$\text{So, } CW(i\%) = PW_{N\to\infty} = A(P/A,\ i\%,\ \infty) = A\left[\lim_{N\to\infty} \frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$$= A\left(\frac{1}{i}\right)$$

$$\therefore \boxed{CW\ (i\%) = \frac{A}{i}}$$

Hence the CW of a project with interest i% per year is the annual equivalent of the project over its useful life divided by i.

Example: A selection is to be made between two structured designs. Because revenues do not exist, only negative cash flow amounts (cost) and the market value at the end of useful life are situated as follows:

Alternative	Structure M	Structure N
Capital investment	– 12,000	40000
Market value	0	10000
Annual expenses	– 22000	– 1000
Useful life	10 yrs	25 yrs

By using the repeatability assumption & the CW method of analysis, determine which structure is better if the MARR is 15% per year.

Solution:

The annual worth (AW) over the useful life of each alternative structure at MARR = 15%, is calculated as,

$$AW(15\%)\ M = -12000\ (A/P,\ 15\%, 10) - 2200$$

$$= -12000 \times (1.15)^{10} \times \frac{0.15}{1.15^{10} - 1} - 2200 = -4592$$

$$AW(15\%)\ N = -40000(A/P,\ 15\%, 25) + 10000\ (A/F, 15\%, 25) - 1000$$

$$= -40000 \times (1.15)^{25} \times \frac{0.15}{1.15^{25} - 1} + 10000 \times \frac{0.15}{1.15^{25} - 1} - 1000$$

$$= -7,141$$

Then the CWs of structure M & N are as follows:

$$CW(15\%)M = \frac{AW_M}{i} = \frac{-4592}{0.15} = -30,613$$

$$CW(15\%)N = \frac{AW_M}{i} = \frac{-7141}{0.15} = -47,607$$

So, based on the CW of each structural design, alternative M should be selected because it has the lesser negative value of CW (–30,613)

4.3 Comparing mutually exclusive, contingent and independent projects in combination

Mainly there are three major groups of investment opportunities

a) Mutually exclusive: At most one project out of the group can be chosen.

b) Independent: The choice of a project is independent of the choice of any other project in the group, so that all or none of the projects may be selected or some number in between.

c) Contingent: The choice of a project is conditional on the choice of one or more other projects.

Formation of mutually exclusive alternatives:

1) If A, B are two independent projects then the mutually exclusive combination is:

Mutually exclusive combination	A	B	Remarks/Decision
1	0	0	Do nothing
2	1	0	Accept A
3	0	1	Accept B
4	1	1	Accept Both A & B

2) If A, B, C is three projects where C is contingent on acceptance of B and acceptance of B is contingent of acceptance of A, we can make following combination.

Mutually exclusive combination	A	B	C	Decision
1	0	0	0	Do nothing
2	1	0	0	Accept A
3	1	1	0	Accept A & B
4	1	1	1	Accept A, B & C

Example: The following are five proposed projects being considered by an engineer in an integrated transportation company for upgrading an intermodal shipment transfer facility for less than truckload lots of consumer goods. The inter relationships among the projects are:

Project B1
Project B2 } mutually exclusive and independent of C set

Project C1
Project C2 } mutually exclusive and dependent (contingent) on the acceptance of B2.

Project D is contingent on the acceptance of C1.

The respective cash flow is shown on table. Using PW method & MARR = 10% per year, determine what combination of projects is best if the capital to be invested is (a) unlimited and (b) limited to 48000.

Cash flows for end of year (Rs.)

Projects/EOY	0	1	2	3	4
B1	– 50000	20000	20000	20000	20000
B2	– 30000	12000	12000	12000	12000
C1	– 14000	4000	4000	4000	4000
C2	– 15000	5000	5000	5000	5000
D	– 10000	6000	6000	6000	6000

Solution:

The PW for each project is calculated as

PW (10%) B1 = – 50,000 + 20000 (P/A, 10%, 4)

$$= -50000 + 20000 \times \left[\frac{1.1^4 - 1}{0.1}\right] \times \frac{1}{1.1^4} = 13,400$$

Similarly,

PW (10%) B2 = –30000 + 12000 (P/A, 10%, 4) = 8000

PW (10%) C1 = −14000 + 4000 (P/A, 10%, 4) = −1300

PW (10%) C2 = −15000 + 5000 (P/A, 10%, 4) = 800

PW (10%) D = −10000 + 6000 (P/A, 10%, 4) = 9000

Mutually exclusive project combinations is:

Mutually exclusive combination	B1	B2	C1	C2	D
1	0	0	0	0	0
2	1	0	0	0	0
3	0	1	0	0	0
4	0	1	1	0	0
5	0	1	0	1	0
6	0	1	1	0	1

Combined project cash flows and PW is:

Mutually exclusive combination	Cash flow for EOY (× 1000)					Invested capital (× 1000)	Pw (× 1000)
	0	1	2	3	4		
1	0	0	0	0	0	0	0
2 (B1)	− 50	20	20	20	20	50	13.4
3 (B2)	− 30	12	12	12	12	30	8.0
4 (B2, C1)	− 44	16	16	16	16	44	6.7
5 (B2, C2)	− 45	17	17	17	17	45	8.9
6 (B2, C1, D)	− 54	22	22	22	22	54	15.7

Note: Here, for combination 1 no project is selected.

For combination 2, project B1 is selected so,, combined cash flows is same as of B1.

For combination 3, project B2 is selected so, combined cash flows is same as of B2

For combination 4, project B2 & C1 is selected so, combined cash flow is (cash flow of B2 + cash flow of C1)

similarly for combination 5 and 6

PW calculation for combined project is:

$PW(10\%)1 = 0$

$PW\ (10\%)2 = -50 + 20\ (P/A, 10\%, 4) = 13.4$

$PW\ (10\%)3 = -30 + 12\ (P/A, 10\%, 4) = 8.0$

$PW\ (10\%)4 = -44 + 16\ (P/A, 10\%, 4) = 6.7$

Similarly,

$PW\ (10\%)5 = 8.9$

$PW\ (10\%)6 = 15.7$

(all these values shown in above table)

So,

a) Based on mutually exclusive combination the best combination is 6(B2, C1, D) which has the highest PW if capital available is unlimited.

b) If, capital available is limited to 48000 mutually exclusive combination 2 & 6 are not feasible. Of the remaining mutually exclusive combinations, 5 is best, means combination of B2 and C2 is selected with a PW = $8.9 \times 1000 = 8900$

Old Question Solution

[2069 Bhadra]

QN.1 From the following information select the best project

	Project A	Project B
Initial investment	35000	50000
Annual revenue	16450	25000
Annual cost	3000	13,830
Useful life	4 yrs	8 yrs
Salvage value at the end of useful life	0	0

MARR = 10%

When service period required is

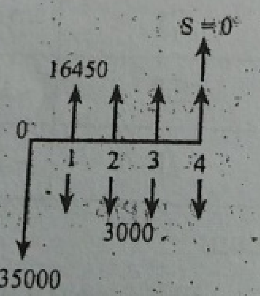
(i) 4 years by FW method

(ii) 8 years by IRR method with PW formulation

Solution:

i) 4 years by FW method

For Project A



$$PW(10\%) = -35000 + (16450 - 3000)(P/A, 10\%, 4)$$

$$= -35000 + 13450 \times \frac{(1.1^4 - 1)}{0.1} \times \frac{1}{1.1^4}$$

$$= -35000 + 13450 \times 3.169$$

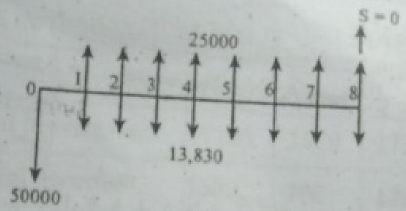
$$= \text{Rs. } 7634.69$$

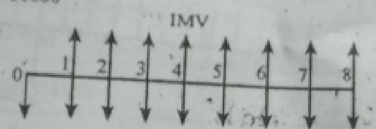
$\therefore$ FW $= 7634.69 \times 1.1^4$

$= \text{Rs. } 11177.95$

(or we can direct calculate FW instead of PW)

For Project B: Study period (4 yrs) is less than useful life so, use co-terminated assumption. (Calculating imputed market value)





IMV = (PW at end of year T of remaining capital recovery amount) + (PW at end of year T of original market value at the end of useful life]

for savage value = 0

Capital recovery (CR) = 50000 (A/P, 10%, 8)

$$= 5000 \times 1.1^8 \times \frac{0.1}{1.1^8 - 1}$$

$= \text{Rs. } 9372.2$

PW at the end of 4th year for remaining 4 years

PW $(10\%)_{CR}$ = Rs. 9372.2 (P/A, 10% 4)

$$= 9372.2 \times \frac{1.1^4 - 1}{0.1} \times \frac{1}{1.1^4}$$

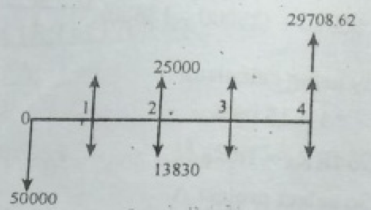
$= \text{Rs. } 29708.62$

Salvage value is zero so,

PW at end of 4th year of original market value = 0

So, IMV $= 9732.2 + 0 = \text{Rs. } 9732.2$

Now,



$\therefore$ PW(10%)4 = − 50000+(25000−13830)(P/A,10%,4)+ 29708.62 (P/F,10%4)

$= -50000 + 11170 \times 3.169 + 29708.62 \times 0.6383$

= Rs. 5688.29

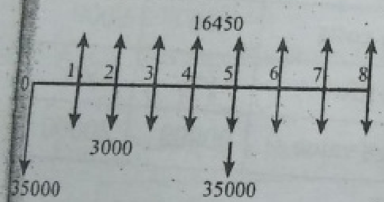
$\therefore$ FW $= 5688.29 \times 1.1^4$ = Rs. 8328.22

Here FW $(10\%)_A >$ FW $(10\%)_B$

So, select project A having higher FW.

(ii) 8 year by IRR with PW formulation

For Project A:



Considering IRR = i%

Then PW $(i\%)_8 = 0$

$$35000 - \frac{35000}{(1+i)^4} + (16450 - 3000)\,\frac{(1+i)^8 - 1}{i} \times \frac{1}{(1+i)^8} = 0$$

By solving calculator,

i = 19.73%

For Project B:

PW $(i\%)_8 = 0$

$$-50000 + (25000 - 13830)\frac{(1+i)^8 - 1}{i} \times \frac{1}{(1+i)^8} = 0$$

By using calculator

$i = 15.08\%$

So $IRR_A > IRR_B$

So select project A.

[2070 Magh]

QN.1 **State and explain about the cases of mutually exclusive contingent and independent projects with example. Compare the following projects by using repeatability assumption when MARR = 12%**

Project	A	B
Initial Investment	200000	300000
Annual revenue	25000	30000
Annual costs	7000	9000
Useful life	6 yrs	8 yrs
Salvage value	90000	20000

Solution:

Mutually exclusive project:

At most one project, out of no. of alternatives to achieve desired go can be chosen. This kind of opportunity is called mutually exclusive.

Contingent:

Among the available alternative, the choice of one project conditional on the choice of one or more other projects is call contingent.

Eg: Purchasing of printer is dependent on purchasing computer.

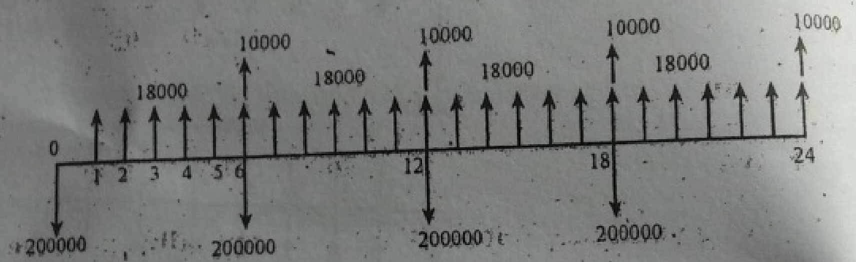
Independent:
The choice of a project is independent of the choice of any other project in the group. i.e. Project can be accepted or rejected without influencing the acceptance or rejection of other.

Eg: Buying machinery, furniture & truck are 3 independent projects.

Here, common study period for project A & B is = LCM of 6 & 8
= 24 years.

Using repeatability assumption:

For project A: Net Revenue = Annual Revenue − Annual costs
= Rs. 18000

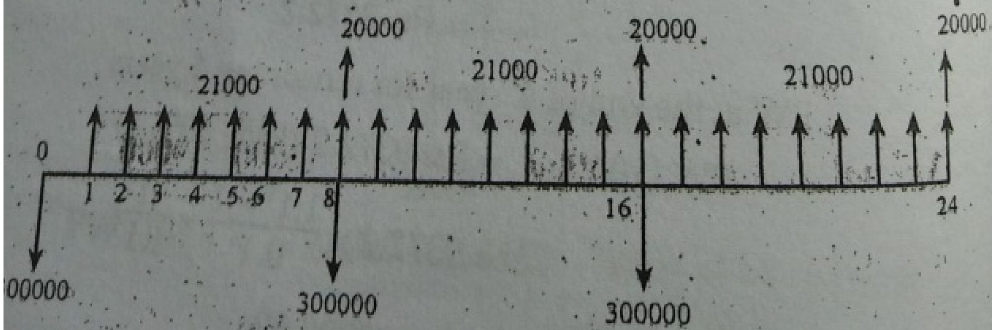


Future worth, $FW(12\%)_{24} = -20000\ [(F/P,12\%,24) + (F/P,12\%,18) + (F/P,12\%,12) + (F/P,12\%,6)] + 10000\ [(F/P,12\%,18) + (F/P,12\%,12) + (F/P,12\%,6) + 1] + 18000\ (F/A,12\%,24)$

$= -200000\ [1.12^{24} + 1.12^{18} + 1.12^{12} + 1.12^{6}] + 10000\ [1.12^{18} + 1.12^{12} + 1.12^{6} + 1] + 18000 \times \dfrac{1.12^{24} - 1}{0.12}$

= Rs. 3476118.53

For Project B: Net Revenue = 30000 − 9000 = 21000



$PW(12\%)24 = -300000\ [(F/P,12\%,24) + (F/P,12\%,16) + (F/P,12\%,8)]$

$= -300000\,[15.17 + 6.13 + 2.476] + 20000\,[6.13 + 2.476 + 1]$
$+ 21000 \times 118.15$

$= -$ Rs. 4459530

Here FW of A > FW of B

So Project A would be best selection

[2070 Bhadra]

QN) Use IRR method to select best project MARR = 12%

	A	B	C	D
Initial investment	1100	1500	2750	2000
Annual income	500	700	1200	950
Useful life	4	4	4	4
Salvage value	250	500	800	1000

Select the best combination if A, B & C are mutually exclusive

Solution:

Solution is similar to the [2071 Magh]

Refer to next question (below):

[2071 Magh]

QN. Use IRR method to select the best project. Use MARR = 12%. Select best combination if A, B & C are mutually exclusive.

	A	B	C	D
Initial investment	10000	15000	27000	20000
Annual income	5000	7000	12000	9000
Useful life	4	4	4	4
Salvage value	2500	5000	8000	10000
MARR	15	15	15	15

Solution:

Calculating IRR of Projects

For Project A:

FW (i%) = 0

$$-10000 \times (1+i)^4 + 5000 \times \frac{(1+i)^4 - 1}{i} + 2500 = 0$$

$\therefore i = 39.34\% >$ MARR, accepted

For Project B:

FW (i%) = 0

$$-15000 \times (1+i)^4 + 12000 \times \frac{(1+i)^4 - 1}{i} + 8000 = 0$$

By solving calculator,

$\therefore i = 36.87\% >$ MARR, accepted

Similarly, (with the help of calculator)

For project C: $i = 33.63\% >$ MARR, accepted.

For project D: $i = 37.74\% >$ MARR, accepted.

Again, Arrange the projects according to their capital investment

A, B, D and C

And Select A as a base project then

Increment	B – A	D – B	C – D
Initial investment	5000	5000	7000
Annual income	2000	2000	3000
Salvage	2500	5000	– 2000

For B – A

FW (i%) = 0

$$-5000 \times (1+i)^4 + 2000 \times \frac{(1+i)^4 - 1}{i} + 2500 = 0$$

$\therefore i = 32.16\% >$ MARR So accept B and eliminate A.

Again, Incremental analysis between B and D. i.e. (D – B)

Calculating IRR for B – A

FW (i%) = 0

$$-5000\,(1+i)^4 + 2000\,\frac{(1+i)^4-1}{i} + 5000 = 0$$

$\therefore i = 40.00\% > \text{MARR}$ So, select D and eliminate B.

Again, incremental analysis between C & D i.e. (C – D)

IRR for C – D

FW (i%) = 0

$$-7000\,(1+i)^4 + 3000\,\frac{(1+i)^4-1}{i} + (-2000) = 0$$

$\therefore i = 18.32\% > \text{MARR}$, So, select C & eliminate D.

Based upon IRR method analysis the best project would be project C.

Again, To select best combination:

Here, Project A, B, C & D are mutually exclusive projects.

So,

Combination of Project A and Project D will be the best combination because these two have highest value of IRR as calculated earlier.

[2071 Bhadra]

QN. Use repeatability assumption to select the best project from the following three projects.

Project	A	B	C
Initial investment	100000	200000	250000
Annual expenditure	25000	20000	15000
Useful life	3	5	7
Salvage value	40000	50000	60000
MARR	14%	14%	14%

Solution:

Here common year for A, B & C is = LCM of 3, 5 & 7

= 105 years

Study period is very large, here project A, B & C has to be repeated 35 times, 21 times and 15 times respectively.

For this case we have to use summation approach.

For Project A:

$$PW(14\%)_A = \sum_{i=0}^{34} -\frac{100000}{(1+i)^{3i}} + (-25000) \times \frac{(1+i)^{105}-1}{i} \times \frac{1}{(1+i)^{105}} + \sum_{i=1}^{35} \frac{40000}{(1+i)^{3i}}$$

To convert all the capital investment to year '0'	To convert all annual cost to year '0'	To convert all average value to year '0'

Now, solve by using calculator.

$$PW_A = \sum_{i=0}^{34} -\frac{100000}{(1+0.14)^{3i}} + (-25000) \times \left[\frac{(1+0.14)^{105}-1}{0.14}\right] \times \frac{1}{(1+0.14)^{105}} + \sum_{i=1}^{35} \frac{40000}{(1+0.14)^{3i}}$$

$= -307665.02 - 178571.24 + 83066.05$

$= -$ Rs. 403170.21

Similarly,

For Project B:

$$PW(14\%)B = \sum_{i=0}^{20} -\frac{200000}{1.14^{5i}} + (-20000) \times \left[\frac{1.14^{105}-1}{0.14}\right] \times \frac{1}{1.14^{105}} + \sum_{i=1}^{21} \frac{50000}{1.14^{5i}}$$

$= -416118.91 - 142857.0 + 54029.78$

$= -$ Rs. 504946.13

For Project C:

$$PW(14\%)C = \sum_{i=0}^{14} -\frac{250000}{1.14^{7i}} + (-15000) \times \left[\frac{1.14^{105}-1}{0.14}\right] \times \frac{1}{1.14^{105}} + \sum_{i=1}^{15} \frac{60000}{1.14^{7i}}$$

$= -416414.52 - 107142.0 + 39939.5 = -$ Rs. 483617.02

So Based on PW, the project A would be best selection. It has lowest value of present worth (PW) = – Rs. 403170.21

[2072 Ashwin]

QN.1 a) Select the best project by ERR method. Take MARR = 10% & E = 20%.

EOY	0	1	2	3	4	5	6
Project A	– 64,000	26200	29000	30,200	31,000	31,000	26,000
Project B	– 68,000	– 4,000	39,200	38,000	38,000	38,000	38,000
Project C	– 75,500	20,500	40,600	40,000	39,000	39,000	32,400

Solution:

For Project A:

PW of all cost = – Rs. 64000

FW of all benefit $= 26200 \times 1.2^5 + 29000 \times 1.2^4 + 30200 \times 1.2^3 + 31000 \times 1.2^2 + 31000 \times 1.2^1 + 26000$

$= \text{Rs. } 285353.984$

Now, $PW \times (1 + i)^6 = FW$

$64000 \times (1 + i)^6 = 285353.984$

Solving,

$i = 28.29\% >$ MARR, accepted

For Project B:

PW of all cost $= 68000 + 4000 \times 1.2^{-1} = \text{Rs. } 71333.33$

FW of all benefit $= 39200 \times 1.2^4 + 38000 \times (1.2^3 + 1.2^2 + 1.2 + 1)$

$= \text{Rs. } 285269.12$

Now, $PW (1 + i)^6 = FW$

$71333.33 (1 + i)^6 = 285269.12$

$\therefore I = 25.99\% >$ MARR, accepted.

For Project C:

PW of all cost = Rs. 75,500

FW of all benefit = $20500 \times 1.2^5 + 40600 \times 1.2^4 + 40000 \times 1.2^3 + 39000 \times (1.2^2 + 1.2) + 32400$

= Rs. 339678.72

Now, PW $(1 + i)^6$ = FW

$75500 \times (1 + i)^6 = 339678.72$

$\therefore$ i = 28.48% > MARR, accepted

Again, select project A as the base project and arrangement of project according to initial investment is A, B & C.

Incremental analysis between A & B i.e. (B – A)

Increment	0	1	2	3	4	5	6
B – A	– 4000	– 30200	10200	7800	7000	7000	12000
C – B	– 7500	24500	1400	2000	1000	1000	– 5600

For (B – A)

$$\text{PW of all cost} = -4000 - \frac{30200}{1.2} = \text{Rs. } -29166.67$$

FW of all benefit = $10200 \times 1.2^4 + 7800 \times 1.2^3 + 7000\ (1.2^2 + 1.2) + 12000$ = Rs. 65109.12

Now, PW $(1 + i)^6$ = FW

$29166.67\ (1 + i)^6 = 65109.12$

$\therefore$ i = 14.32 > MARR so accept B, eliminate A

Again, Incremental analysis between B and C i.e. (C – B)

For C – B: PW of all cost = $-7500 - \dfrac{5600}{1.26}$

= Rs. – 9375.43

FW of all benefit = $24500 \times 1.2^5 + 1400 \times 1.2^4 + 2000 \times 1.2^3 + 1000 \times (1.2^2 + 1.2)$

= Rs. 83767.106

Now, PW $(1 + i)^6$ = FW

$9375.43\,(1+i)^6 = 83767.106$

$\therefore I = 44\% > \text{MARR}$ Select C & Eliminate B

So, Based on ERR method, the project C would be best selection.

b) Co-terminating both project at 5 years and select the best project by modified BCR (using AW formulation). Take salvage value of each project = 10% of first cost. MARR = 15%

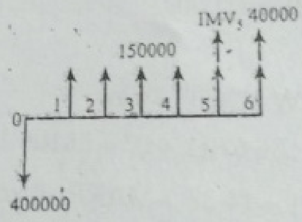
Project	First cost	Annual Benefits	Annual O & M costs	Useful life
A	Rs. 400000	Rs. 175,000	Rs. 25000	6 yrs
B	Rs. 700000	Rs. 250,000	Rs. 35000	8 yrs

Solution:

Here, for both project study period is lesser than the useful life. So, we have to use imputed market value (IMV) technique.

For project A: Salvage = 10% of 4000000 = 40000 &

Net benefit = 175000 − 25000 = 150000



Capital recovery CR (10%) = 400000 (A/P, 10%,6) − 40000 (A/F, 10%,6)

$$= 400000 \times 1.1^6 \times \frac{0.1}{1.1^6 - 1} - 40000 \times \frac{0.1}{1.1^6 - 1}$$

= Rs. 86658.65

PW at the end of 5th year for remaining years

$PW(10\%)_{CR} = 86658.65\ (P/A, 10\%, 1)$

$$= 86658.65 \times \frac{1}{1.1^1}$$

= Rs. 78780.60

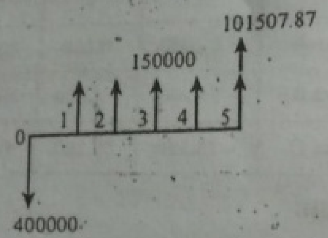
\PW at the end of 5^{th} year of original market value at the end of life (6 yrs)

$$PW\ (10\%)MV = 25000\ (P/F\ 10\%,1)$$

$$= 25000 \times \frac{1}{1.1} = Rs.\ 22727.27$$

Now, Imputed market value (IMV) = 78780.60 + 22727.27

= Rs. 101507.87



$$\text{We have, modified BCR} = \frac{AW(B) - AW\ (O\ \&\ M)}{AW\ (I) - AW\ (S)}$$

$$AW\ (I) - AW\ (S) = (400000 \times 1.1^5 - 101507.87) \times \frac{0.1}{1.1^5 - 1}$$

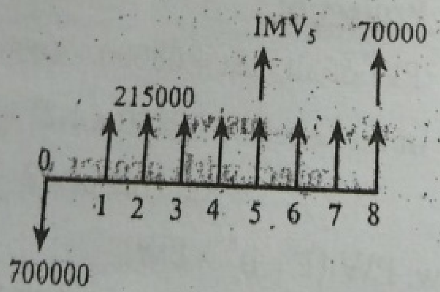
$$= Rs.\ 88892.258$$

$$\text{So, modified BCR} = \frac{150000}{88892.258} = 1.687 > 1,\ \text{ok}$$

For project B:

Salvage = 10% of 700000 = Rs. 70000 Net benefit

= 250000 – 35000 = 215000



$$(10\%) = 700000\ (A/P,10\%,8) - 70000(A/F,10\%,8)$$

$$= 700000 \times 0.187 - 70000 \times 0.087$$

$$= Rs.\ 124810$$

PW $(10\%)_{CR}$ for remaining yrs = 124810 (P/A 10%,3)

= 124810 × 2.486

= Rs. 310384

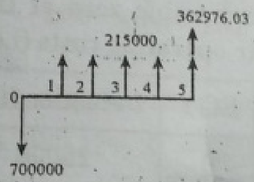
Also, PW$(10\%)_{MV}$ = 70000 (P/F,10%,3)

= 70000 × 0.7513

= Rs. 52592.03

Now, IMV = 310384 + 52592.03 = Rs. 362976.03

So,



$$AW(I) - AW(S) = (700000 \times 1.1^5 - 362976.03) \times \frac{0.1}{1.1^5 - 1}$$

= Rs. 125203.677

AW(B) – AW(O & M) = 215000

So, modified BCR = $\frac{215000}{125203.677}$ = 1.72 > 1 So, ok

Now, based on modified BCR Method

BCR of B > BCR of A

So, project B would be best selection s

[Note: If more than two alternative project were given then you have to proceed similarly as for IRR & ERR) in the case of BCR also]

QN.2 Define mutually exclusive project, independent project and contingent project with proper combination.

Solution:

Refer [2070 Magh]

***

5 Replacement Analysis

5.1 Fundamentals of Replacement Analysis

5.1.1 Basic concepts and Terminology

Replacement analysis is a choice between the present asset (called defender) and new alternatives (called challenger).

Concepts of Replacement analysis refers to selection of similar but new assets to replace the existing assets to meet current and future requirement more economically.

Reasons for Replacement:

1. Physical impairment (Deterioration): These are changes that occur in the physical condition of the asset. It includes wear and tear of equipment. Normally aging result, in the operation of an asset becoming less efficient.
2. Inadequacy: Capital assets are used to produce goods and services that satisfy human wants. When the demand for a service is increases then existing asset does not have sufficient capacity to fill that.
3. Obsolescence: Due to impact of changes in technology result in more frequent replacement of existing assets with new and better challengers.
4. Financing: Financial factors involve economic opportunity changes external to the physical operation. For example, the rental (lease) of assets may become more attractive than ownership.

Some definition:

- **Economic service life:** Economic service life of assets is defined as the period of time that results in the minimum equivalent uniform annual cost of owning and operating an asset.

- **Ownership life:** It is the period between the date of acquisition and the date of disposal by a specific owner.
- **Physical Life:** It is the period between original acquisition and final disposal of an asset over its succession of owners.
- **Useful life:** It is the time period, that an asset is kept in productive service. It is the estimate of how long an asset is expected to be used in business to produce income.
- **Sink cost:** A sink cost is money that has already been spent and cannot be recovered. Sink costs are also called retrospective costs.

5.1.2 Approaches for comparing defender and challenger

→ Basic opportunity cost approach is considered for comparing defender and challenger. Replacement analysis is carried out comparing annual equivalent cost (AEC) of defender and challenger.

→ After comparison, the option having lower value of annual equivalent cost is accepted. Where AEC is calculated, by considering salvage value of an asset (if any) with the help of annual worth (AW) of all cost throughout it's life.

→ The total annual equivalent cost (AEC) of owning & operating an Asset (AEC) (i%) is the summation of the capital recovery cost and annual equivalent of operating cost of an assets.

i.e. AEC (i%) = CR (i%) + OC (i%)

5.2 Economic service life of challenger and defender:

<u>Economic service life of challenger:</u> The economic life of an assets minimizes the equivalent uniform annual cost of owning and operating an asset, and is often shorter than the useful or physical life. It is essential to know a challenger's economic life in view of the principle

that new and existing assets should be compared over their economic lives.

Economic service life of the defender:

The economic life of the defender is often one year. Generally defender involves a lower initial cost than purchasing challenger but it requires more annual operating (repair & maintenance also) cost due to old technology and continuing use (aging).

Example: Determine the choice between a defender that has a current market value of Rs. 25000 and challenger that can be purchased for 37500. Both have a same life of 3 years with salvage value zero at the end of life. Take MARR = 12%. Their operating costs (i.e. Cash flow) values are gives as:

EOY	Defender (D)	Challenger (C)
0	25000	37500
1	8500	2500
2	10000	5500
3	12500	6500

Solution:

Determine annual equivalent cost for both defender and challenger

For Defender (D)

The market value of 25000 is considered as capital investment.

Then,

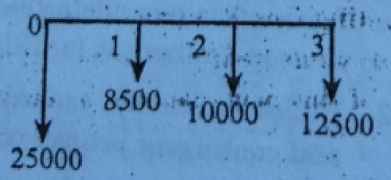


Fig.: CFD of defender (D)

$AEC\ (12\%)_D = [-25000\ (F/P,12\%,3) - 8500\ (F/P,\ 12\%2) - 10000\ (F/P,12\%,1) - 12500] \times (A/F,\ 12\%,\ 3)$

$= [-25000\ (1 + 0.12)^3 - 8500\ (1 + 0.12)^2 - 10000\ (1 + 0.12) - 12500]\ (A/F, 12\%, 3)$

$= -49460\ (A/F, 12\%, 3)$

$$= -69485.60 \times \frac{0.12}{1.12^3 - 1}$$

$= \text{Rs.} -20591.98$

For challenger (C)

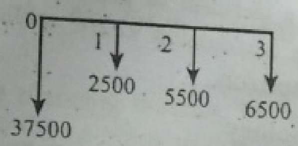


Fig.: CFD of challanger (C)

$AEC\ (12\%)_C = [-37500\ (F/P, 12\%, 3) - 2500\ (F/P, 12\%, 2) - 5500\ (F/P, 12\%, 1) - 6500] \times (A/F, 12\%, 3)$

$$= -68564.19 \times \frac{0.12}{1.12^3 - 1}$$

$= \text{Rs.} -20295$

Hence, $AEC\ (12\%)_D > AEC\ (12\%)_C$

So replacement should be made.

Example

One electric company has decided to replace existing generator with a new one having initial cost of 54000 with useful life of 8 years and having operating cost of Rs. 9000 in the first year. For the remaining year operating cost increases by 15% over the previous year's operating cost & salvage value declines each year by 20% from the previous year's salvage value. Determine the economic service life of this new machine by considering MARR = 12% per year.

Solution:

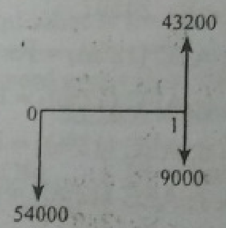
Here to determine economic service life with MARR = $12\%_8$ for given cash flow information, we can use the following equation of AEC

Annual equivalent cost AEC(i%) = CR (i%) + OC (i%)

Now,

N	Operation cost	Salvage cost
0	-	54000
1	9000	43200
2	10350	34560
3	11902.50	27648
4	13687.87	22118.4
5	15741.05	17694.72
6	18102.21	14155.77
7	20817.5	11324.62
8	23940.8	9059.70

For year N = 1;



$$AEC\ (12\%)_1 = CR\ (12\%) + OC\ (12\%)$$

$$= [54000\ (F/P,12\%,1) - 43200 + 9000] \times (A/F,\ 12\%,\ 1)$$

$$= (54000 \times 1.12 - 43200 + 9000) \times \frac{0.12}{1.12^1 - 1}$$

$$= \text{Rs. } 26280$$

Note: We can also calculate CR & OC separately as,

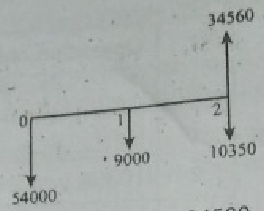
$$CR(12\%) = 54000\ (A/P,12\%,1) - 43200\ (A/F,12\%,1)$$

$$= \text{Rs. } 17280$$

$$OC(12\%) = 9000$$

Then $AEC\ (12\%) = 17280 + 9000 = \text{Rs. } 26280$

For year N = 2



$AEC\ (12\%)_2 = [54000\ (F/P,12\%,2) - 34560 + 9000\ (F/P,12\%,1) + 10350] \times (A/F,12\%,2)$

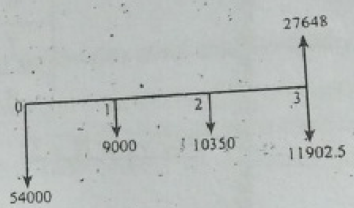
$= (54000\ (1.12)2 - 34560 + 9000\ (1.12) + 10350) \times (A/F,\ 12\%,2)$

$= 53607.6 \times \dfrac{0.12}{1.12^2 - 1}$

= Rs. 25287

Similarly,

For N = 3



$AEC\ (12\%)_3$ = Rs. 24597

& For N = 4; $AEC\ (12\%)_4$ = Rs. 24165

For N = 5; $AEC\ (12\%)_5$ = Rs. 23955

For N = 6; $AEC\ (12\%)_6$ = Rs. 23931

For N = 7; $AEC\ (12\%)_7$ = Rs. 24072

For N = 8; $AEC\ (12\%)_8$ = Rs. 24354

So, From above calculation AEC is decreases from N = 1 to 6, then after then it start to increase.

So, from analysis, economic service life (N) = 6 years even though, it has useful life = 8 yrs.

5.3 Replacement Analysis when required service life is long:

5.3.1 Requirement Assumption & decision Framework:

There are three assumption to determine the optimal time to replace the defender. They are

a) Planning horizon (study period)
b) Technology
c) Relevant cash flow information

(a) Planning horizon:

By the planning horizon, we simply mean the service period required by the defender and a sequence of future challengers. When we are simply unable to predict when the activity under consideration will be terminated, infinite planning horizon is used. But when the project will have a definite & predictable duration, the replacement policy should be formulated more realistically on the basis of a finite planning horizon.

(b) Technology:

Predictions of technological patterns over the planning horizon refer to the development of types of challengers that may replace those under study. A number of possibilities exist in predicting purchase cost, salvage value and operating cost that are dictated by the efficiency of new machine over the life of an existing asset.

(c) Revenue and cost patterns over the life of an Asset:

Many varieties of predictions can be used to estimate the patterns of revenue, cost of salvage value over the life of an asset. Sometimes revenue is constant, but assets increase, with salvage value decreases over the life of a machine. In other situation, a decline in revenue over the life of piece of equipment can be expected. The specific situation will determine whether replacement analysis is directed towards cost minimization or profit maximization. We formulate a replacement policy for an asset whose salvage value does not increase with age.

Decision framework:

Decision framework is developed for indicating a replacement sequence of an assets with notation (J_0,n_0) (J_1,n_1) (J_2,n_2).........(J_k,n_k) where such pair (J,n) indicates a types of asset and life time over which that asset will be retained.

Normally in the case of defender and challenger

Where, $J_0 \rightarrow$ defender

$J \rightarrow$ Challenger

If defender asset (J_0) replaced now then $n_0 = 0$ & it can be written as (J_0,0)

The sequence (J_0,2), (J_1,5) (J_2,3) indicates retaining the defender for two years and then replacing defender with an asset type J_1 (Challenger – I) & it is used for next 5 years & then again replacing J_1 with an asset type J_2 (Challenger– II) & using it for three years. Hence total planning horizon covers 2 + 5 + 3 = 10 yrs.

5.3.2 Replacement Analysis under infinite planning horizon:

Based on the infinite planning horizon, the service is required for a very long time. Either we continue to use the defender to provide the service, or we replace the defender with best available challenger for the same service requirement.

Procedure:

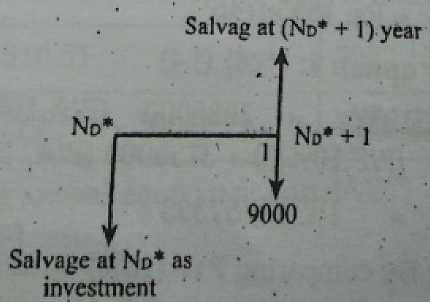
Step 1: Compute economic life for defender say N_D* and also compute economic life for challenger say N_C*. The corresponding annual equivalent cost is AEC_D* and AEC_C*:

Step 2: If $AEC_D^* > AEC_C^*$, Accept challenger

If AECD* < AECC*, if cost less to keep defender than replace it by challenger .

i.e. No replace on the year of (N_D*)

Step 3: Then compute the AEC for defender after N_D* year i.e. for (N_D* + 1) year



[∵ OC = Operational Cost]

Then compute AEC (i%) & if AEC (i%) > AEC_C* then replaced.

otherwise check for next (N_D* + 2) year.

5.3.3 Replacement Strategies under the Finite Planning Horizon

If the planning period is finite (for example, eight years), a comparison based on the AE method over a defender's economic service life does not generally apply. The procedure for solving such a problem with a finite planning horizon is to establish all "reasonable" replacement patterns and then use the PW values for the planning period to select the most economical pattern.

Example: A hydropower company has a contract to provide electricity to government of Nepal for the next 8 years. It can produce hydroelectricity using old turbine or the newly bought turbine. Consider old turbine as defender & new turbine as challenger and by using annual equivalent costs technique, compute their economic service life? And what is the best replacement strategy?

Number of years (N)	Annual equivalent cost at MARR = 10%	
	Old turbine	New turbine
1	2575000	3750000
2	2550000	3075000
3	2750000	2930000
4	2980000	2915000
5	3220000	2950000

Solution:

From inspection

For old turbine (defender):

Economic service life = 2nd year (year at minimum value of AEC)

For New turbine (challenger):

Economic service life = 4th year (year at minimum value of AEC)

Now,

Likely replacement patterns under finite planning horizon of 8 years be

Option 1: $(J_0,0)\ (J,4)(J,4)$

Option 2: $(J_0,1)\ (J,4)(J,3)$

Option 3: $(J_0,2)\ (J,4)(J,2)$

Option 4: $(J_0,3)\ (J,4)(J,1)$

Option 5: $(J_0,4)\ (J,4)$

Option 1: $(J_0,4)\ (J,4)$

Here, $J_0 \rightarrow$ Defender

$J \rightarrow$ Challenger

Note:

Here we prepared six options but result of best strategies always lies on where for both J_0 & J are within the limit of their economic life

i.e. most likely strategies are

option 1: $(J_0,0)\ (J,4)(J,4)$, option 2: $(J_0,1)\ (J,4)(J,3)$ & option 3: $(J_0,2)\ (J,4)(J,3)$

For option 1: $(J_0,0)\ (J,4)(J,4)$

$$PW(10\%)_1 = 2915000\ (P/A, 10\%, 8)$$

$$= 2915000\left(\frac{1.1^8 - 1}{0.1}\right) \times \frac{1}{1.1^8}$$

$$= 2915000 \times 5.33$$

$$= \text{Rs. } 15551309.87$$

For option 2: $(J_0,1)\ (J,4)\ (J,3)$

$PW(10\%)_2$ = 257000 (P/A,10%,1) + [2915000 (P/A,10%4)] (P/F,10%,1) + [2930000 (P/A, 10%,3)] (P/F,10%,5).

$$= 2575000\left[\frac{1.1^1-1}{0.1}\right]\times\frac{1}{1.1}+2915000\left[\frac{1.1^4-1}{0.1}\right]\times\frac{1}{1.1^4}\times\frac{1}{1.1}+2930000\times\frac{1}{1.1^3}\times\frac{1}{1.1^5}$$

= 2340932.5 + 8400320 + 4524260

= Rs. 15265512.50

For option 3: $(J_0,2)\ (J,4)\ (J,2)$

$PW(10\%)_3$ = 2550000 (P/A,10%,2) + [2915000 (P/A,10% 4)] (P/F,10%,2) + [3075000 (P/A, 10%,2)] (P/F,10%,6)

= 4425525 + 7636150 + 3012545 = Rs. 15074220

For option 4: $(J_0,3)\ (J,5)$

$PW(10\%)_4$ = 2750000 (P/A,10%,3) + 2950000 (P/A,10% 5) (P/F,10%,3)

= 6838976 + 8401680

= Rs. 15240655

Similarly,

For option 5: $(J_0,3)\ (J,4)\ (J,1)$

$PW(10\%)_5$ = 2750000 (P/A,10%,3) + 2915000 (P/A,10% 4) (P/F,10%,3) + 3750000 (P/A, 10%,1) (P/F,10%,7)

= Rs. 15530745

For option 6: $(J_0,4)\ (J,4)$

$PW(10\%)_6$ = 2980000 (P/A,10%,3) + 2915000 (P/A,10%4) (P/F,10%,3) + 3750000 (P/A, 10%,1) (P/F,10%,7)

= Rs. 15757395

So, By comparing PW (10%) of different strategies, the minimum PW is derived by option 3. Hence best replacement strategy would be $(J_0,2)\ (J,4)\ (J,2)$ having PW (10%) of AEC = Rs. 15074220

Old question solution

[2069 Bhadra]

1. **What is the economic service life of an asset? Find the economics service life of a new electric lift truck which costs $ 20000 have a operating cost of $1000 in the first year and salvage value of $12000 at the end of the first year. For the remaining years, operating costs increases each year by 10% over the previous years operating costs. Similarly the salvage value declines each year by 20% from the previous years salvage value. The lift truck has a maximum life of 7 years. An over out costing of $3000 and $5000 will be required during the fifth and seventh year of service respectively. The firm's required rate of return is 15% per year.**

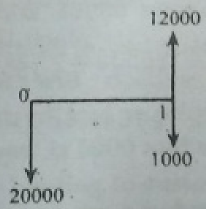
Solution: For theory part see at [2071 Bhadra]

Now,

Year	Annual operating cost	Salvage
1	$ 1000	$ 12000
2	$ 1000 × (1.1) = 1100	12000 × 0.8 = 9600
3	$ 1100 × (1.1) = 1210	9600 × 0.8 = 7680
4	1331	6144
5	1664.10 (+ 3000)	4915.20
6	1610.51	3932.16
7	1771.56 (+ 3000)	3145.728

Now, Calculating annual equivalent cost (AEC) = AEC (15%) = CR (15%) + OC (15%)

For year N = 1



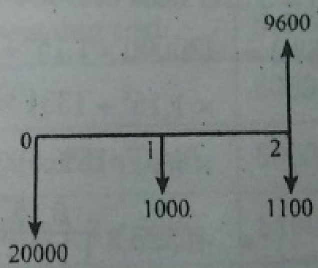
AEC $(15\%)_1$ = 20000 (A/P,15%,1) – 12000 (A/F,15%,1) + 1000 (A/F,15%,1)

$$= 20000 \times (1.15) \times \frac{0.15}{1.15^1 - 1} - 12000 \times \frac{0.15}{1.15^1 - 1} + 1000 \times \frac{0.15}{1.15^1 - 1}$$

$$= 23000 - 12000 + 1000$$

$$= \$\ 12000$$

For year N = 2



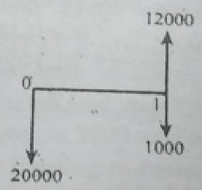
AEC $(15\%)_2$ = $(20000 \times 1.15^2 + 1000 \times 1.15 + 1100 - 9600)$ (A/F, 15%,2)

(Note: Here first convert all cash flow to future value & convert FW to AEC)

= 19100 (A/F,15%,2)

$$= 19100 \times \frac{0.15}{1.15^2 - 1}$$

For year N = 1



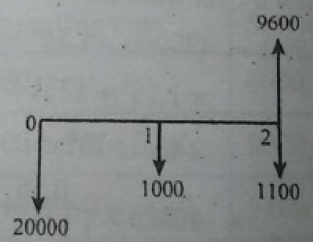
$$AEC(15\%)_1 = 20000\ (A/P,15\%,1) - 12000\ (A/F,15\%,1) + 1000\ (A/F,15\%,1)$$

$$= 20000 \times (1.15) \times \frac{0.15}{1.15^1 - 1} - 12000 \times \frac{0.15}{1.15^1 - 1} + 1000 \times \frac{0.15}{1.15^1 - 1}$$

$$= 23000 - 12000 + 1000$$

$$= \$\ 12000$$

For year N = 2



$$AEC(15\%)_2 = (20000 \times 1.15^2 + 1000 \times 1.15 + 1100 - 9600)\ (A/F,\ 15\%,2)$$

(Note: Here first convert all cash flow to future value & convert FW to AEC)

$$= 19100\ (A/F,15\%,2)$$

$$= 19100 \times \frac{0.15}{1.15^2 - 1}$$

$$= \$\ 8883.72$$

Similarly,

$$AEC(15\%)_3 = (20000 \times 1.15^3 + 1000 \times 1.15^2 + 1100 \times 1.15 + 1210$$

$- 7680)$ (A/F, 15%,3)

$= 26535 \times \frac{0.15}{1.15^3 - 1}$

$= \$ 7641.47$

<u>For year N = 4</u>

AEC $(15\%)_3 = (20000 \times 1.15^4 + 1000 \times 1.15^3 + 1100 \times 1.15^2 + 1210 \times 1.15 + 1331 - 6144)$ (A/F, 15%,4)

$= 34061.85 \times \frac{0.15}{1.15^4 - 1}$

$= \$ 6821.42$

<u>For year N = 5</u>

[You can direct write AEC (15%) by solving on calculator similarly as above, No need to write full expression]

At yearn = 5 over head cost of $ 3000 is required so,

AEC $(15\%) = (20000 \times 1.15^5 + 1000 \times 1.15^4 + 1100 \times 1.15^3 + 1210 \times 1.15^2 + 1331 \times 1.15 + 1664.10 + 3000 - 4915.20)$ $\times$ (A/F, 15%,5)

$= 46328 \times \frac{0.15}{1.15^5 - 1}$

$= \$ 6871.30$

For year N = 6, AEC $(15\%)_6 = \$ 6465.939$

For year N = 7, AEC $(15\%)_7 = \$ 5808.31$

So,

From above calculation AEC is decreases from year N = 1 to N = 7

So, economic service life = 7 years.

OR

A firm has a contract to provide printing service to IOE for next 8 years. It can provide the service using its old printing machine (the current defender) or the newly bought machine (the

challenger). After the contract work neither the old machine nor the new machine will be retained. Considering the annual equivalent costs of the old machine and new machine as follows, what are their economic service life? And what is the best replacement strategy?

Number of years (N)	Annual equivalent cost (Rs) at MARR = 10%	
	Old machine	New machine
1	5,15,000	7,50,000
2	5,10,000	6,15,000
3	5,50,000	5,86,000
4	5,96,000	5,83,000
5	6,44,000	5,90,000

Solution:

From the analysis of given data of AEC for old machine & new machine the value of AEC decline from N = 1 to N = 2 & then increasing for old machine, so economic service life = 2 years

And AEC decline from N = 1 to N = 4 from new machine, so economic service life = 4 years.

Replacement strategy: ($J_0 \rightarrow$ old machine, $J \rightarrow$ new machine)

Option 1: (J_0,0) (J,4) (J,4)

$$PW(10\%)_1 = 583000\ (P/A, 10\%, 8)$$

$$= 583000 \times \left(\frac{1.1^8 - 1}{0.1}\right) \times \frac{1}{1.1^8}$$

[First change A to F & then to P]

$$= \text{Rs. } 3110262$$

Option 2: (J_0,1) (J,4) (J,3)

$$PW(10\%)_2 = 515000\ (P/A, 10\%, 1) + 583000\ (P/A, 10\%, 4)\ (P/F, 10\%, 1) + 586000\ (P/A, 10\%, 3)\ (P/F, 10\%, 5)$$

$= 515000 \times 0.91 + 583000 \times 3.17 \times 0.91 + 586000 \times 2.49 \times 0.621 = \text{Rs. } 3053102.50$

Option 3: $(J_0,2)$ $(J,4)$ $(J,2)$

$PW\ (10\%)_2 = 510000\ (P/A,10\%,2) + 583000\ (P/A,10\%,4)\ (P/F,10\%,2) + 615000\ (P/A,10\%,2)\ (P/F,10\%,6)$

$= 510000 \times 1.735 + 583000 \times 3.17 \times 0.826 + 615000 \times 1.73 \times 0.56$

$= \text{Rs. } 3014844$

Option 4: $(J_0,3)$ $(J,5)$

$PW\ (10\%)_4 = 550000\ (P/A,10\%,3) + 590000\ (P/A,10\%,5)\ (P/F,10\%,3)$

$= 550000 \times 2.48 + 590000 \times 3.79 \times 0.75$

$= \text{Rs. } 3048131$

Option 5: $(J_0,3)$ $(J,4$ $(J,1)$

$PW\ (10\%)_5 = 55000\ (P/A,10\%,3) + 583000\ (P/A,10\%,4)\ (P/F,10\%,3) + 750000\ (P/A,10\%,1)\ (P/F,10\%,7)$

$= 550000 \times 2.48 + 583000 \times 3.169 \times 0.68$

$= \text{Rs. } 3106149$

Option 6: $(J_0,4)$ $(J,4)$

$PW\ (10\%)_6 = 596000\ (P/A,10\%,4) + 583000\ (P/A,10\%,4)\ (P/F,10\%,4)$

$= 596000 \times 3.169 + 583000 \times 3.169 \times 0.68$

$= \text{Rs. } 3151479$

When we compare PW (10%) of different strategies option, the minimum PW(10%) of Annual cost is obtained by option–3

So the best replacement strategy would be $(J_0,2)$ $(J,4)$ $(J,2)$ with PW(10%) = Rs. 3014844

[2070 Magh]

1. The new machine costs 10000 operating cost 2200 in first year, then increases by 20% per year. Market value is 6000 after one year and will decline by 115% each year N = 5 years. If required, old machine can work another 3 years. Market value now is 5000 and will decline by 25% each year. Immediate over hauling to restore to operable condition costs 1200. operating costs 2000 in the first years & increases by 1500 per year thereafter. MARR = 15%.

   i) Find the economic service life of this machine (new)

   ii) AEC of defender is as following:

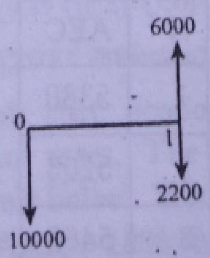
N	1	2	3	4
AEC	5380	5203	5468	5845

when should the old machine be replaced with the new machine.

Solution: (i) for a new machine

No. of year	Operating cost	Salvage
1	2200	6000
2	$2200 \times 1.2 = 2640$	$6000 \times 0.85 = 5100$
3	$2640 \times 1.2 = 3080$	$5100 \times 0.85 = 4335$
4	3520	3684.75
5	3960	3132

For year N = 1, AEC (15%) = CR (15%) + OC (15%)

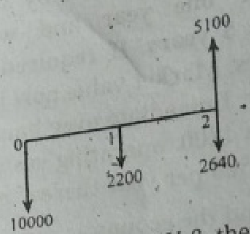


$AEC\ (15\%)_1 = [10000 \times 1.15 - 6000 + 2200]\ (A/F,\ 15\%,\ 1)$

$= 7700 \times 1$

$= 7700$

For year N = 2



[Note: First convert all cash flow to FW & then into AW for AEC]

$AEC(15\%)_2 = [10000 \times 1.15^2 + 2200 \times 1.15 + 2640 - 5100]$ (A/F, 15%,2)

$= 13295 \times \frac{0.15}{1.15^2 - 1}$

$=$ Rs. 6183.72

Similarly for,

year N = 3, $AEC(15\%)_3 = (10000 \times 1.15^3 + 2200 \times 1.15^2 + 2640 \times 1.15 + 3080 - 4335)$ (A/F, 15%,3)

also, (Just by using calculator)

For year N = 4, $AEC(15\%)_4$ = Rs. 5548.30

For year N = 5, $AEC(15\%)_5$ = Rs. 4889.34

So, by analyzing above calculated AEC (15%), it's value decreases from year N = 1 to N = 5, so, economic service life = 5 years.

(ii) From given value of AEC for defender.

N	AEC
1	5380
2	5203
3	5468
4	5845

Here, AEC decreases from year N = 1 to N = 2 & then it increases.

So, economic service life = 2 years

Here, AEC of defender for economic service life (2 years) is greater than the AEC of challenges for economic service life of (5 years) so, there is no need to use defender. It can be replaced at first year.

[2070 Bhadra]

QN.1 Explain about Reasons for replacement of asset. The annual equivalent of the defender and challenger are given in the table below. What is the best replacement strategy? Use MARR = 12%. The planning horizon of the project is 8 years.

EOY (n)	1	2	3	4	5	6
$(AEC)_D$	5300	5250	5400	5750	6200	6550
$(AEC)_C$	7700	6150	5700	5600	5675	5800

Solution:

For theory part See at (5.1.1)

As per above [2069 Bhadra] we can make 6 numbers of strategy but the economic strategy lies within the economic service life of defender and challenger we it has 3 strategy as.

$J_0 \rightarrow$ Defender

$J \rightarrow$ Challenger

Option 1: $(J_0, 2)$ $(J,6)$

Option 2: $(J,3)$ $(J,5)$

Option 3: $(J,4)$ $(J,4)$

Option 1: $(J_0, 2)$ $(J,6)$

$$PW(12\%)_1 = 5250\,(P/A,12\%,2) + 5800\,(P/A,12\%,6)\,(P/F,12\%,2)$$

$$= 5250 \times 1.69 + 5800 \times 4.11 \times 0.79$$

$$= \text{Rs. } 27704.52$$

Option 2: $(J_0, 3)$ $(J,5)$

$PW(12\%)_2 = 5400\ (P/A,12\%,3) + 5675\ (P/A,12\%,5)\ (P/F,\ 12\%,3)$

$= 5400 \times 2.40 + 5675 \times 3.6 \times 0.712$

$=$ Rs. 27506.16

Option 3: $(J_0, 4)\ (J,4)$

$PW(12\%)_3 = 5750\ (P/A,12\%,4) + 5600\ (P/A,12\%,4)\ (P/F,\ 12\%,4)$

$= 5750 \times 3.037 + 5600 \times 3.037 \times 0.63$

$=$ Rs. 28177.286

So, By comparison of PW(12%) of annual cost for above strategy, option 3 has minimum value of PW = Rs. 27506.16

Therefore the best replacement strategy is $(J_0,3)\ (J,5)$

[2071 Magh]

QN.1 **Annual equivalent cost of defender and challenger are given below.**

n	Defender	Challenger
1	5380	7700
2	5203	6184
3	5469	5756
4	5844	5625
5	6258	5631
6	6682	5721

Either the defender or challenger is required for next 8 years.

After the work, neither the defender nor the challenger will be retained. What is the best replacement strategy?

Solution:

$J_0 \rightarrow$ Defender

$J \rightarrow$ Challenger

Strategy are:

Economic service life for Defender → 2 yrs

Economic service life for challenger → 4 yrs

Option 1: $(J_0,2)$ $(J,6)$

Option 2: $(J_0,3)$ $(J,5)$

Option 3: $(J_0,4)$ $(J,4)$

[No need to check for other options such as $(J_0,0)$ $(J,4)$ $(J,4)$. $(J_0,3)$ $(J,4)$ $(J,1)$ etc.]

Calculating PW (i%) of Annual cost for combination as;

Option 1: $(J_0,2)$ $(J,6)$

Consider i = 12%

$$
\begin{aligned}
PW(12\%)_1 &= 5203\ (P/A,12\%,2) + 5721\ (P/A,12\%,6)\ (P/F,12\%,2) \\
&= 5203 \times 1.69 + 5721 \times 3.277 \\
&= \text{Rs. } 27540.78
\end{aligned}
$$

Option 2: $(J_0,3)$ $(J,5)$

$$
\begin{aligned}
PW(12\%)_2 &= 5469\ (P/A,12\%,3) + 5631\ (P/A,12\%,5)\ (P/F,12\%,3) \\
&= 5469 \times 2.4 + 5631 \times 3.6 \times 0.712 \\
&= \text{Rs. } 27558.98
\end{aligned}
$$

Option 3: $(J_0,4)$ $(J,4)$

$$
\begin{aligned}
PW(12\%)_3 &= 5844\ (P/A,12\%,4) + 5625\ (P/A,12\%,4)\ (P/F,12\%,4) \\
&= 5844 \times 3.037 + 5625 \times 3.037 \times 0.636 \\
&= \text{Rs. } 28613.09
\end{aligned}
$$

So, By comparison of PW(12%), the best strategy is option 1 having least PW = Rs. 27540.78

And the replacement strategy is $(J_0,2)$ $(J,6)$

[2071 Bhadra]

QN.1 Explain about sink cost, economic life and reasons for replacement of an asset. The AEC for defender and challenger are given below. What is the best replacement strategy? Take MARR = 10% & the planning horizon of 8 years.

Solution:

Sink cost: A sink cost is money that has already been spent and cannot be recovered. Sink costs are also called retrospective costs.

Companies in every industry have to spend money to make money. A company budget may allow for investing money in employee salaries, inventory, office space or any other cost of doing business. Once the company's money is spent, that money is considered a sink cost. Regardless of what money is spent on, sink costs are already spent & permanently lost. It cannot be refunded. For example, once rent is paid, that money is no longer recoverable which is sink.

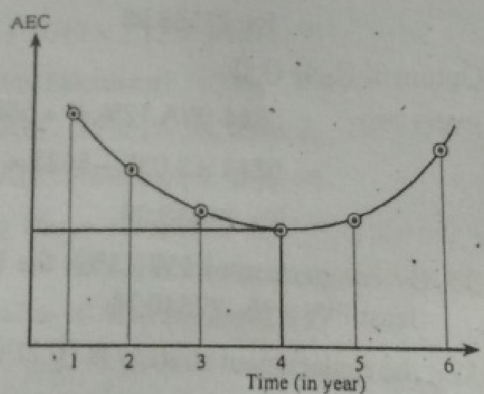
Economic service life:

The economic service life of an asset is defined as the period of useful life that minimize the annual equivalent cost (AEC), owning and operating the asset. To get exact economic service life, we need to find the value of N (years) that minimizes AEC as expressed in

AEC (i%) = CR(i%) + OC(i%)

where CR = Capital recovery cost, OC = Operating cost

Example:



Here life period = 6 yrs

But economic service life = 4 yrs (upto which AEC is on declining & after which AEC is increasing)

Reasons for replacement: See at [5.1.1]

For Numerical part: Refer to [2070 Bhadra]

[2072 Ashwin]

QN.1 An existing machine has market value of Rs. 10000 and decreases by Rs. 2000 per year. It's operating cost is Rs. 2500 in year 1 and increases by 20% each year for 4 years. New machine costs Rs. 20000 now and its market value will decrease by Rs. 20% per year from 4 years. Operating cost is Rs. 1500 in first year and increases by 30% each year. Calculate equivalent uniform annual cost of both existing and New machines. MAPR = 15%. Formulate the best replacement strategy if we need the machine for four years only.

Solution:

	For Defender		For challenger	
Year	Operation cost	Salvage	Operation cost	Salvage
0	–	10000	–	20000
1	2500	8000	1500	$20000 \times 0.8 = 160$[illegible]
2	3000	6000	1950	12800
3	3600	4000	2535	10240
4	4320	2000	3295.5	8192

For Defender:

For $N = 1$ $AEC(15\%)_1 = CR(15\%) + OC(15\%)$

$$= [10000\,(F/P, 15\%, 1) - 8000 + 2500] \times (A/F, 15\%, 1)$$

$$= (10000 \times 1.15 - 8000 + 2500) \times \frac{0.15}{1.15^1 - 1}$$

$= \text{Rs. } 6000$

For $N = 2$ $AEC(15\%)_2 = (10000 \times 1.15^2 - 6000 + 2500 \times 1.15 + 3000)\,(A/F, 15\%, 2)$

$= 13100 \times 0.465$

$= \text{Rs. } 6093.02$

For N = 3 AEC $(15\%)_3 = (10000 \times 1.15^3 - 4000 + 2500 \times 1.15^2 + 3000 \times 1.15 + 3600)$ (A/F, 15%,3)

$= 21565 \times 0.2879$

$= \text{Rs. } 6210.22$

For N = 4 AEC $(15\%)_3$ = (Directly from calculator) = Rs. 6352.36

For challenger:

For N = 1, $AEC(15\%)_1 = (20000 \times 1.15 - 16000 + 1500)$ (A/F,15%,1)

$= \text{Rs. } 8500$

For N = 2, $AEC(15\%)_2 = (20000 \times 1.15^2 - 12800 + 1500 \times 1.15 + 1950) \times$ (A/F,15%,2)

$= \text{Rs. } 8058.14$

For N = 3, $AEC(15\%)_3 = (20000 \times 1.15^3 - 10240 + 1500 \times 1.15^2 + 1950 \times 1.15 + 2535) \times$ (A/F,15%,3)

$= \text{Rs. } 7757.74$

For N = 4, $AEC(15\%)_4 = (20000 \times 1.15^4 - 8192 + 1500 \times 1.15^3 + 1950 \times 1.15^2 + 2535 \times 1.15 + 3295.5) \times$ (A/F,15%,4)

$= \text{Rs. } 7581.858$

So from above calculation,

Economic service life of defender is 1 year

Economic service life of challenge is 4 years

But AEC of challenger > AEC of defender for Economic service life.

Also, maximum AEC of defender < AEC of challenges for economic service life.

Therefore No need to replace the existing machine.

6 Risk Analysis

6.1 Origin / Sources of Project Risks

Risk: The chance that on investment's actual return will be different than expected.

Origin / Sources of Project Risks

1) **Cash Flow Estimate**

- inaccuracy of the cash flow estimates and measurements error

2) **Nature of Business**

- All business are not same nature because some types of business operation are less stable then others.

3) **Rate of Interest**

It depends on current health of economy and future expectation of economic condition.

4) **Study Period**

Long study period generally increases the uncertainty of a capital investment and economic return.

5) **Unclear specification**
6) **Volatile and unpredictable future**
7) **Social risks**
8) **Variability in output**

6.2 Methods of Describing Project risk

6.2.1 Sensitivity Analysis

Sensitivity analysis is a general non probabilistic methodology readily available, to provide information about potential impact of uncertainty in selected factor estimates. Its routine use is fundamental to developing economic information useful in the decision process. In general sensitivity means the relative magnitude of change in the measure of merit (PW or IRR) caused by one or more changes in estimated study factor values.

Following technique are usually included in a discussion of sensitivity analysis.

i) Breakeven analysis

ii) Sensitivity graph

iii) Combination of factors

Steps for sensitivity graph analysis

a) Plot PW, FW, AW, IRR or BCR on y - axis

b) % error in estimate of parameter value is plotted on x - axis

c) The slope of the line shows level of sensitivity.

d) The steeper the slope, more sensitive.

Example:

A Project cost s Rs. 500000 and will have annual saving of Rs. 92500 per year for 10 years, salvage value is 10% of initial cost. Perform sensitivity analysis over a period of ± 40% in

i) Initial investment

ii) Annual Saving

iii) Project Life

Assume MARR = 10% using AW formulation

Solution:

Salvage = 10% of I = 50,000

AW = – 500000 (A/P,10 %, 10) + 92500 + 50000 (A/F, 10% 10)

$$= -500000 \times \frac{0.1 \times 1.1^{10}}{1.1^{10} - 1} + 92500 + 50,000 \times \frac{0.1}{1.1^{10} - 1}$$

= 14264.57

Writing in equation format

AW = – I (A/P, 10%, N) + B + 50,000 (A/F, 10%, N)

$$= -I * \frac{0.1 \times 1.1^N}{1.1^N - 1} + B + 50{,}000 \times \frac{0.1}{1.1^N - 1} \quad \text{..........(i)}$$

Where I = initial investment, B = annual saving, N = project life

Variation	10%	20%	30%	40%	-10%	-20%	-30%	-40%	Remarks
AW (I varies)	6127.3	-2009	-10147.2	-18284	22401.8	30539.1	38676.3	46813.7	B & N Constant
AW (B varies)	23514.57	32764.57	4204.57	51264.57	5014.57	-4235.47	-13485.42	-22735.4	I and N constant
AW (N varies)	18216.58	21456.5	2449.6	26434.2	9361.75	3150.19	-4932.47	-15823.23	B & I constant

Calculator Trick

For I varies

Enter equation (i) in calculator

i.e. $-x * \frac{0.1 \times 1.1^{10}}{(1.1^{10} - 1)} + 92500 + 50000 \times 0.1 / (1.1^{10} - 1)$

Press CALC, X = ?

Enter 500000 * 1.1 $\Rightarrow$ 6127.3 (For 10%)

Press CALC, X = ?

Enter 500000 * 1.2 $\Rightarrow$ - 2009.96 (For 20%)

And For negative sign

Enter 500000 * 0.9 $\Rightarrow$ 22401.84 (For - 10%)

Enter 500000 $\times$ 0.8 $\Rightarrow$ 30539.1 (For - 20%)

Similarly, For N

Enter equation (1) in this way

$$-500000 \times 0.1 \times \frac{1.1^x}{(1.1^x - 1)} + 92500 + 50000 \times \frac{0.1}{(1.1^x - 1)}$$

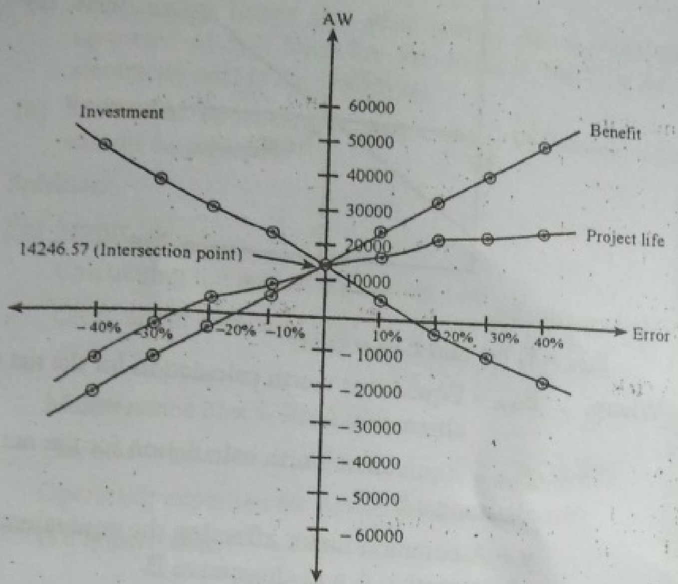
Enter CALC X ?

Enter 10* 1.1 $\Rightarrow$ 18216.58 (For 10%) and Similar for other

Enter 10 $\times$ 0.9 $\Rightarrow$ 9361.15 (For - 10%)

And Similar for B

And fill in the table and plot the graph



From graph benefit has a higher slope therefore, the most sensitive is benefit.

6.2.2 Breakeven Analysis

The usual breakeven problem involving two alternatives can be most easily approached mathematically by equating an equivalent worth of the two alternatives expressed as a function of the factor of interest. In short, the break - even point is the point at which revenue is exactly equal to costs.

Fixed cost

Remain constant over how many times you sell, and how many goods are produced. Such as land, rent, insurance etc.

Variable cost

Variable cost vary with number of items produce, production level, workforce size and other parameters. Such as labour, raw materials, fuel, advertise etc.

Total cost : Sum of fixed cost and variable cost

Let S be the selling price per unit

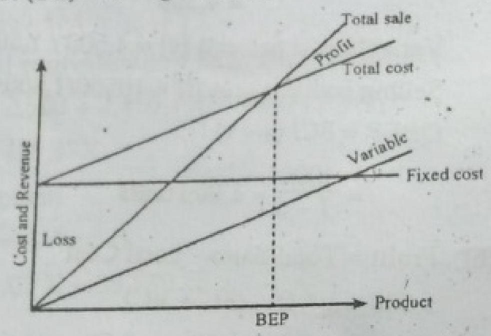
V be the variable cost per unit

FC is the fixed cost per period

Q is the quantity of production

Total sales (S) = s * Q .... (i)

Total cost (TC) = v * Q + FC (ii)



At intersection point

Total cost = total sales

$v * Q + FC = s * Q$

or, $FC = (s - v) Q$

or $Q = \frac{FC}{s - v}$

Example

From given data, determine

(a) BEP in terms of number of unit

(b) What should be the output if the profit desired is Rs. 5,000?

(c) What is the different between revenue and variable cost per unit at breakeven?

Fixed Cost (FC) = Rs. 4,000

Total Cost (TC) = Rs. 8,500

Total Sales (S) = Rs. 10,500

Sales Volume = 1500 units

Solution:

We have,

(a) Total Cost (TC) = Fixed Cost (FC) + Variable Cost (VC)

VC = TC – FC = 8,500 – 4,000

= 4,500

Variable cost per unit (v) = 4,500 / 1,500 = Rs. 3 per unit

Selling cost per unit (s) = 10500/1,500 = Rs. 7 pre unit

QBEP = FC/ (s – v)

$$= \frac{4000}{7-3} = 1{,}000 \text{ units}$$

(b) Profit = Total Sales - Total Cost

5,000 = s * Q – (FC + VC)

= s * Q – (400 + v * Q)

5000 + 4000 = (s – v) Q

$$Q = \frac{9000}{7-3} = 2{,}250$$

Therefore break even unit for the profit to be Rs. 5,000 is 2,250

(c) At break even profit = 0, Assume FC remains constant

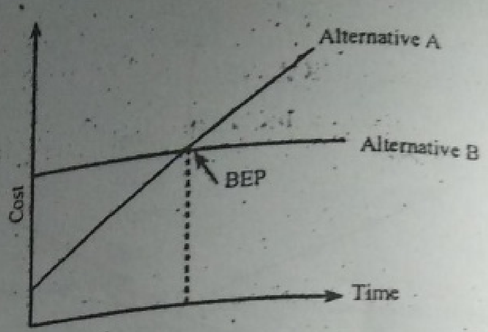
0 = s * Q – (4000 + v * Q)

4000 = (s – v) * 2000

s – v = Rs. 2 per unit

Break - Even Analysis for comparing two alternatives

When there are two investment opportunity under consideration and heavily dependent on a single and common factor obtained. This value by equating equivalent worth. That value is known as the break - even point.



Mathematically,

$E_{WA} = f_1\ (y)$ and $E_{WB} = f_2\ (y)$

Where, E_{WA} = Equivalent worth calculation for the net cash flow of alternative A.

F_{WB} = Equivalent worth calculation for the net cash flow of alternative B

y = A common factor affecting the equivalent worth values of alternative A and alternative B

At break evan point,

$E_{WA} = E_{WB}$

$\Rightarrow f_1\ (y) = f_2\ (y)$

Example

Consider the following two motors each of 100 hp output capacity item.

Item	Motor A	Motor B
Purchase Cost (Rs)	1,25,000	1,60,000
Efficiency (η)	74%	92%
Life (Years)	10	10
Maintenance cost (Rs)/Year	5,000	2,500
Annual tax and insurance	1.5% of investment	1.5 of investment
MARR	15%	15%

(a) How many hours per year would the motors have to be operated at full load for the annual costs to be equal? If electricity cost is Rs. 5/KW/hr.

(b) If annual operation is more than 55 hours, which motor should be selected?

Solution:

For Motor 'A'

Calculating the annual equivalent cost

CR (Capital recovery) cost = 1,25,00 (A 1P, 15%, 10)

= Rs. 24,906.5

Maintenance cost = Rs. 5,000

Tax and insurance = 1.5% of 1,25,000 = Rs. 1,875

Operating expenses for power (electricity cost),

We known that,

$$\text{Efficiency } (\nu) = \frac{\text{Output}}{\text{Input}}$$

Let 'X' be the number of hours of operation per year

Operating expenses = Input × rate * Hours

$$= \frac{\text{Output}}{\eta} \times \text{Rate} \times \text{Hours}$$

$$= \frac{100 \times 0.746}{0.74} \times 5 \times X \ [1 \text{ hp} = 0.746 \text{ kw}]$$

$$= 504.5\ X$$

∴ Total amount equivalent cost of motor A

= 24,906.5 + 5,000 + 1,875 + 504.05X

= 31,781.5 + 504.05X

For Motor B

Capital Recover (CR) = 1,60,000 (A/P, 15%, 10)

= Rs. 31, 888.32

Maintenance cost = Rs. 2,500

Tax and insurance = 1.5% of 1,60,000 = Rs. 2,400

Operating expenses for power,

Let 'X' be the number of hours of operation per year

$$\text{Operating expenses} = \frac{100 \times 0.746}{0.92} * 5 * X$$

$$= 405.43x$$

$\therefore$ Total annual equivalent cost of motor B (AW of B)

= 31,880.32 + 2,500 + 2,400 + 405.45X

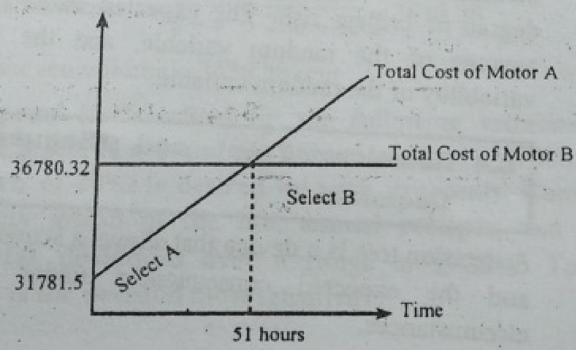
= 36,780.32 + 405.43X

At Breakeven point,

AW of A = AW of B

31,781.5 + 504.05 X = 36.780.32 + 405.43X

X = 51 hours



Figure

$\therefore$ If annual operation is more than 55 hours, motor B is selected.

6.2.3 Senario Analysis

A means of comparing a "base case" or expected project measurement (such as NPW) with one or more additional scenarios, such as the best case and the worst case, to identify the extreme and most likely project outcomes.

Both sensitivity and break-even analyses are useful, they have limitations. Often, it is difficult to specify precisely relationship between a particular variable and the NPW. The relationship is further complicated by interdependencies among the variables. Holding operating costs constant while varying unit sales may case the analysis, but in reality, operating costs do not behave in this manner. Yet, it may complicate the analysis too much to permit movement in more than one variable at a time.

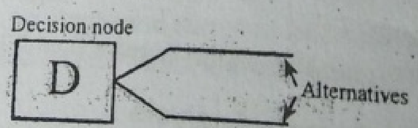
6.3 Probability Concept of Economic Analysis

The probability distribution of a random variable allows us to make a specific probability statement, a single value that may characterize the random variable and its probability distribution is often desirable. Such a quantity is the expected value of a random variable. We also want to know something about how the values of the random variable are dispersed about the expected value. (i.e. the variance). In investment analysis this dispersion information is interpreted as the degree of project risk. The expected value indicates the weighted average of the random variable, and the variance captures the variability of the random variable.

6.4 Decision Tree and Sequential Investment Decision

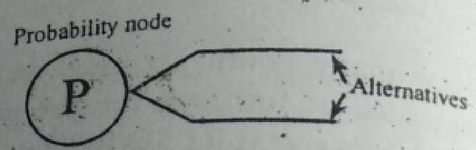
A decision tree is a device that shows a sequence of strategic decision and the expected consequences under each possible set of circumstances.

Components of Decision Tree



Probability node

A circle represents probability node with the possible outcomes and estimated probabilities on the branches.



Branch

- Line connecting nodes from the left to the right of the diagram.

Procedure for solving decision tree using PW analysis

- Start at the top right of the tree.
- Determine PW for each outcome branch.
- Calculate expected value for each decision alternative.

  $E(\text{decision}) = \Sigma\,(\text{outcome estimate}) \times P(\text{outcome})$
- Select best decision
- Continue towards root decision in order to select best alternative.

Example:

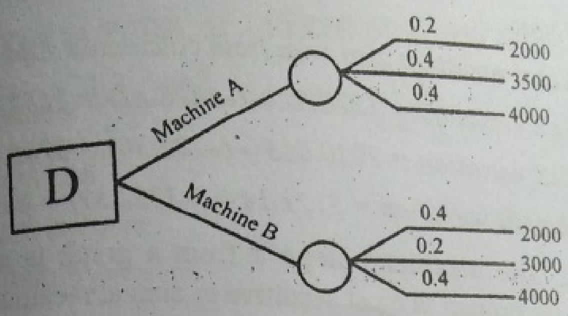
The income and corresponding probability of a machine A and machine B with Grade A, Grade B and Grade C, are provided as

	Machine A		Machine B	
	Probability	Income	Probability	Income
Grade C	0.2	2000	0.4	2000
Grade B	0.4	3500	0.2	3000
Grade A	0.4	4000	0.4	4000

The cost of machine A is 1050 and machine B is 1000. Find which machine should prefer?

Solution:

Decision tree



From the decision tree diagram,

Expected return Income from machine A

$= 0.2 \times 2000 + 0.4 \times 3500 + 0.4 \times 4000$

$= 3400$

Expected net profit from machine A = 3400 – 1050

= Rs. 2350

Expected return/income from machine B

$= 0.4 \times 2000 + 0.2 \times 3000 + 0.4 \times 4000$

$= 3000$

Expected net profit from machine B = 3000 – 1000 = Rs. 2000

Since, expected net profit from machine A is greater than that of machine B, the machine A should be prefer.

Old question solution

"2069 Bhadra"

1. **Calculate breakeven volume of a cable manufacturing company from the following data. Total cost = Rs. 1200000 Variable cost = Rs. 400000. Income from sales = Rs. 1500000 at production of 5000 unit.**

Solution:

Total cost (TC) = 1200000

Variable cost (VC) = 400000

Total cost = Fixed cost + variable cost

Fixed cost = 1200000 – 400000 = 800000

$$\text{Variable cost per unit (v)} = \frac{400000}{5000} = 80$$

$$\text{Sales per unit (s)} = \frac{1500000}{5000} = 300$$

$$\text{Now, Breakeven} = \frac{FC}{s-v} = \frac{800000}{300-80} = 3636.36$$

∴ Breakeven volume = 3636.36 unit

2. **A proposal is described by the following estimates: P = \$20000, S = 0, N = 5 and net annual receipts = \$7000. A rate of return of 20% is desired on such proposals. Construct a sensitivity graph of the life, annual receipts, and rate of return for deviations over a range of ± 20%. To which element is the decision most sensitive?**

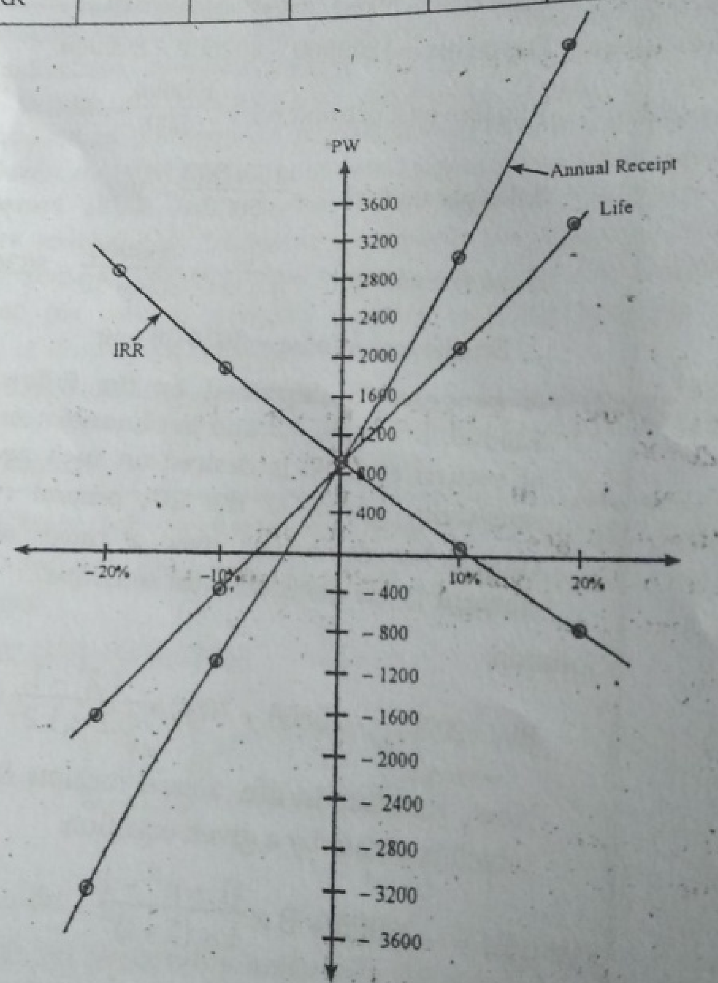
Solution:

$$PW\,(20\%) = -20000 + 7000 \times \frac{1.2^5 - 1}{0.2 \times 1.2^5} = 934.264$$

Now, variation in life, annual receipts & IRR are given in table which is obtain by a given equation.

$$PW\,(i\%) = -20000 + B \times \frac{(1+i)^N - 1}{i \times (1+i)^N}$$

Variation	10%	20%	-10%	-20%	Remarks
PW(N) Variation of life	2159.82	3278.57	-408.216	-1673.85	Annual receipts and IRR remains constant
PW(B) Variation of receipts	3027.713	5121.14	-1159.14	-3252.57	Life and IRR remains constant
PW (IRR) Variation of IRR	45.47	-782.3	1890.197	2920.055	Annual receipts & life remains constant



[Note:
Total variation also seen from calculated data
Life variation = 3278.57 – (–1878.81) = 5157.38
IRR variation = 2920.055 – (–782.30) = 3702.355
Benefit variation = 5121.14 – (–3252.57) = 8373.7 (more variable)]

∴ The maximum slope a from a graph is annual receipts. So, the project is most sensitive to annual receipts.

"2070 Magh"

1. **Explain decision tree analysis**

   Refer at 6.4

2. **Calculate breakeven hours of operation per year to become cost equal and recommended economic pump if it is to be operated 5 hours daily at full load.**

	KHASA Pump	SARVO Pump
Capacity	100 hp	100 hp
Purchase cost (Rs.)	500000	1000000
Tax per year (Rs.)	10000	15000
Maintenance cost per year (Rs.)	36500	29200
Efficiency	80%	90%
Life year	5	5
Salvage value	20% of purchase cost for both	
MARR	20% per year	20% per year
Electricity cost	Rs. 10/kwhr	Rs. 10/kwhr

Solution:

For KHASA Pump, salvage value = 0.2 × 500000 = 100000

$$CR = 500000\,(A/P,20\%,5) - 100000\,(A/F,20\%,5)$$

$$= 500000 \times \frac{1.2^5 \times 0.2}{1.2^5 - 1} - 100000 \times \frac{0.2}{1.2^5 - 1}$$

$$= 153751.88$$

Tax = 10000 per year

O & M = 36500 per year

$$\text{Operating expense of X hours} = \frac{(10 \times 100 \times 0.746 \times x)}{0.8} = 932.5\,x$$

$$\text{Total cost (TC)} = 932.5\,x + 10000 + 36500 + 153751.88$$

$$= 180826.22 + 932.5\,x \ldots\ldots\text{(i)}$$

For SERVO Pump,

$$\text{Salvage value} = 0.2 \times 1000000$$

$$= 200000$$

$$CR = 1000000\,(A/P,20\%,5) - 200000(A/F,20\%,5)$$

$$= 1000000 \times \frac{1.2^5 \times 0.2}{1.2^5 - 1} - 200000 \times \frac{0.2}{1.2^5 - 1}$$

$$= 307503.76$$

Tax = 15000 per year

O & M = 29200 per year

$$\text{Operating expenses for X hours} = \frac{(10 \times 100 \times 0.746 \times x)}{0.9}$$

$$= 828.88\,x$$

$$\text{Total cost} = 307503.76 + 15000 + 29200 + 828.88\,x$$

$$= 351703.76 + 828.88\,x \ldots\ldots\ldots\text{(II)}$$

For breakeven point, equate the total cost of two pump,

$$180826.22 + 932.5\,x = 351703.76 + 828.88\,x$$

$$103.62\,x = 170877.54$$

$$x = 1649.078 \text{ hrs/year}$$

$\therefore$ Break even hours of operation per year = 1649.078

"2070 Bhadra"

1. What are the sources of risk in engineering projects in Nepal? A real estate developer seeks to determine the most economical height for a new office building which will be sold after five years. The relevant net annual revenues and net resale values are as given below.

	Height	
	4 floors	5 floors
First cost	125000000	200000000
Annual revenues	19910000	37815000
Net Resale value	200000000	300000000

The developer is uncertain about the interest rate i to use, but is certain that it is in the range of 5 to 30%. For each building height, find the range of values of i for which that building height is the most economical. Draw sensitivity diagram to support your answer.

Solution:

First part, Refer at 6.1

Second part

For 4 floors building

PW = −125000000 + 19910000 (P/A, i%, 5) + 200000000 (P/F, i%, 5)

$$= -125000000 + 19910000 \times \frac{(1+i)^5 - 1}{i(1+i)^5} + \frac{200000000}{(1+i)^5}$$

Now varying i from 5% to 30%

i%	5	10	15	20	25	30
PW(i%)	117905113.8	74658829.19	41176755.06	14198602.11	-5920435.2	-22641991.37

From table,

PW	i%
14198602.11	20
–5920435.20	25
0	? (23.46) From interpolation

∴ The 4 floors building is economical for a ranges of 5% to 23.46% where PW is positive.

For 5 floors building

$$PW = -200000000 + 37815000 \times \frac{(1+i)^5 - 1}{i \times (1+i)^5} + \frac{300000000}{(1+i)^5}$$

Now varying i from 5 to 30%

i%	5	10	15	20	25	30
PW(i%)	198777010.4	129624998.6	75914765.67	33653269.68	–876.80	–27100207.52

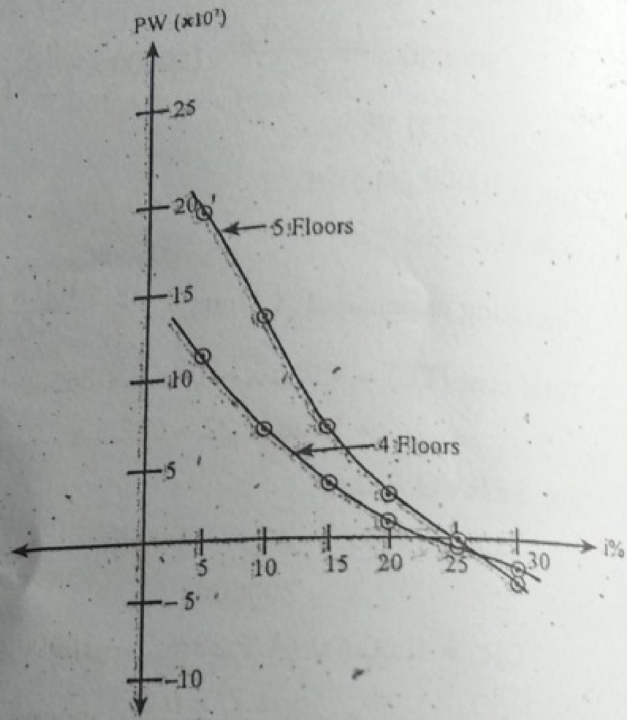
From table

PW	i%
33653269.68	20
–876.80	25
0	? (24.98) (By interpolation)

∴ 5 floors building is economical for a ranges of 5% to 24.98% where PW is positive.

Same result can be seen from sensitivity graph, economic interest rate is that where PW is positive.

1.



"2071 Magh"

1. What are the sources of risk in engineering project in Nepal? Perform sensitivity analysis of the following project over a range of ± 30% in a) initial investment b) Net annual revenue and c) useful life. Draw also sensitivity diagram. Use PW formulation.

Initial Investment (Rs.)	550000
Net Annual Revenue (Rs.)	150000
Salvage Value (Rs.)	80000
Useful life (years)	6
MARR	10%

Solution:

First part refer at 6.1

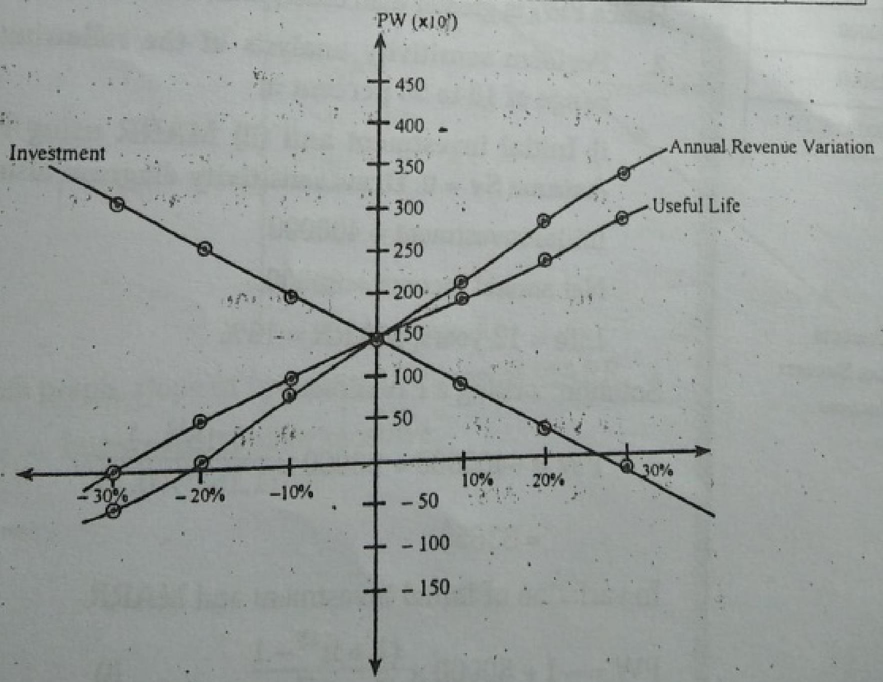
$$PW = -550000 + 150000\ (P/A,10\%,6) + 80000\ (P/F,10\%,6)$$

$$= -550000 + 150000 \times \frac{1.1^6 - 1}{1.1^6 \times 0.1} + \frac{80000}{1.1^6} = 148447.02$$

Now at variable I, B and N

$$PW = -I + B\frac{1.1^N - 1}{1.1^N \times 0.1} + \frac{80}{1.1^N} \ldots\ldots\ldots (i)$$

Variation	10%	20%	30%	-10%	-20%	-30%	Remarks
PW(I), investment variation	93477.02	38447.02	-16552.98	203447.02	258447.02	313447.02	B & N remain constant
PW(B), Annual revenue variation	213775.93	279104.84	344433.75	83118.10	17789.19	-47539.71	I & N remain constant
PW(N), useful life variation	192998.7	235074.13	274810.93	101273.34	51323.35	-1566.33	I & B remain constant



The steeper slope is obtained for annual revenue.

∴ The most sensitive is the variation in Net annual revenue.

2. **A machine has a fixed cost of Rs. 4000000. It has variable cost Rs. 45000 per unit. Find BEP both in volume and the value if selling price per unit is Rs. 60000. What would be the effect on profit/loss when fixed cost increases by 10% and selling price decreases by 5%.**

Solution:

Fixed cost (FC) = 400000

Variable cost per unit (v) = 45000

Selling price per unit (s) = 60000

$$Q_{BEP} = \frac{FC}{s - v} = \frac{4000000}{60000 - 45000} = 266.67 \text{ unit}$$

In terms of volume i.e. sales

$$Q_{BEP} = 266.67 \times 60000$$
$$= 16000000$$

Also, FC increases by 10%,

$$FC = 1.1 \times 4000000 = 4400000$$

Selling price decrease by 5%

$$SP(s) = 0.95 \times 60000 = 57000 \text{ per unit}$$

Total cost = FC + variable cost

$$= 4400000 + 266.67 \times 45000$$
$$= 16400000$$

Sales = 266.67 × 57000 = 15200000

Cost > Sales

$$\therefore \text{Loss } \% = \frac{16400000 - 15200000}{16400000} = 7.4\% \text{ of total cost}$$

3. **Explain the decision tree analysis**

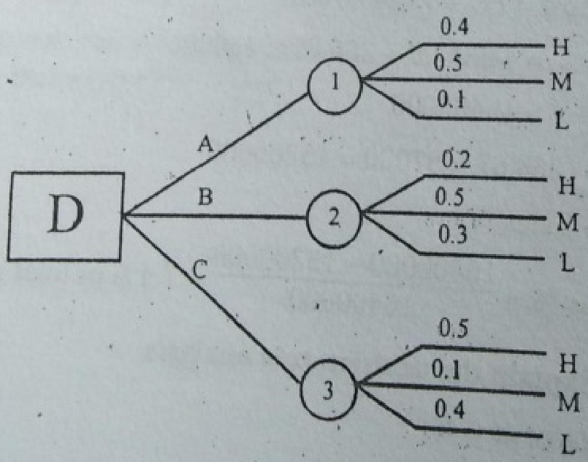
Refer at 6.4

"2070 Bhadra"

1. For the improvement of a manufacturing plant, following three alternatives are being considered. The estimated investments and the corresponding increment in income are also given as below. Draw decision tree diagram of the situation and decide on the best alternative using FW formulation. MARR = 15%, life of the project is 6 years.

Alternatives	Investment Cost	Sales	Probability	Annual income
A	1000000	High success	0.4	500000
		Medium success	0.5	300000
		Low success	0.1	125500
B	600000	High success	0.2	400000
		Medium success	0.5	250000
		Low success	0.3	100000
C	400000	High success	0.5	200000
		Medium success	0.1	125000
		Low success	0.4	50000

Solution:



H = High Success
M = Medium Success
L = Low Success

Annual incomes are obtained by multiplying incomes of different success with their probability.

For alternative A = $500000 \times 0.4 + 300000 \times 0.5 + 125500 \times 0.1 = 362500$

For alternative B = $400000 \times 0.2 + 250000 \times 0.5 + 100000 \times 0.3 = 235000$

For alternative C = $200000 \times 0.5 + 125000 \times 0.1 + 50000 \times 0.4 = 132500$

Now, PW for Alternative A = $-1000000 + 362500$ (P/A, 15%, 6)

$$PW_A = -1000000 + 362500 \times \frac{1.15^6 - 1}{1.15^6 \times 0.15}$$

$$= 371874.97$$

PW for Alternative B,

$$PW_B = -600000 + 235000 \times \frac{1.15^6 - 1}{1.15^6 \times 0.15} = 289340$$

PW for Alternative C,

$$PW_C = -400000 + 132500 \times \frac{1.15^6 - 1}{1.15^6 \times 0.15} = 101388$$

Hence PW_A is greater than other, thus, alternative A is most feasible.

2. **Perform sensitivity analysis of the following project over a range of 10 to 50 percent in**

   i) Initial investment and (ii) MARR using PW formulation. Assume Sv = 0. Draw sensitivity diagram also.

   Initial investment = 400000

   Net annual income = 80000

   Life = 12 years, MARR = 15%

Solution: writing a PW directly,

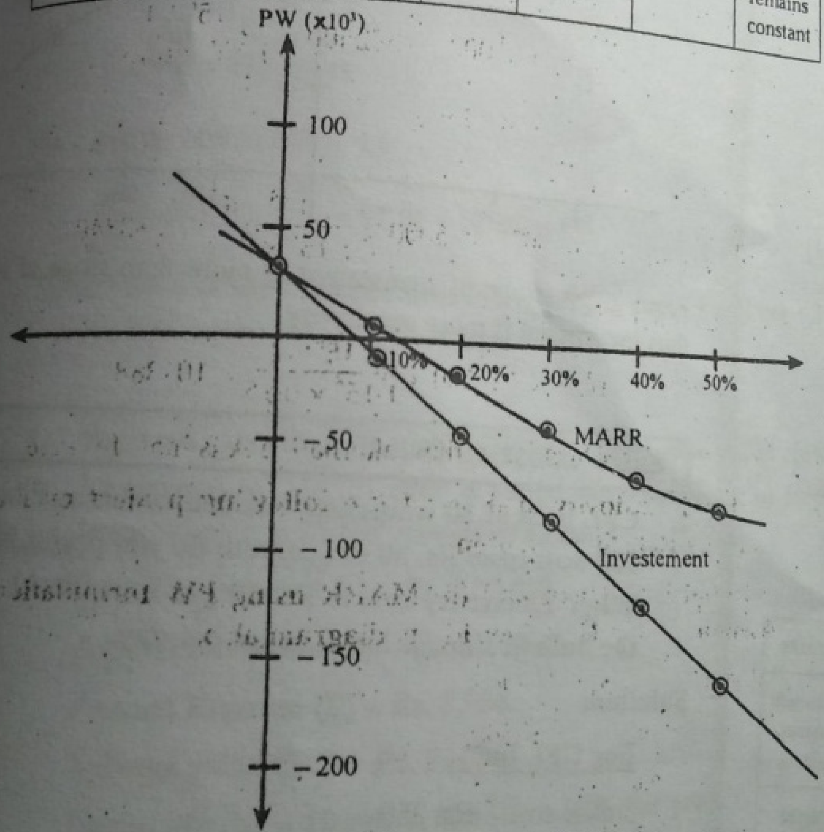
$$PW = -400000 + 80000 \times \frac{1.15^{12} - 1}{1.15^{12} \times 0.15}$$

$$= 33650$$

In variation of initial investment and MARR

$$PW = -I + 80000 \times \frac{(1+i)^{12} - 1}{(1+i)^{12} \times i} \quad \text{..........(i)}$$

Variation	10%	20%	30%	40%	50%	Remarks
PW (I): Investment in variation	-6350.48	-46350.48	-86350.48	-126350.48	-166350.48	MARR remains constant
PW (i%) MARR variation	7278.27	-16542	-38120.7	-57724.0	-75581.3	Investment remains constant



From graph, slope of investment > Slope of MARR

∴ Investment is more sensitive.

"2072 Ashwin"

1. A company products an electronics timing switch that is used in consumer and commercial products made by several other manufacturing firms. The fixed cost and total cost are Rs. 40000 and Rs. 85000 respectively. The total sales are Rs. 105000 and sales volume is 15000 for this situation.

i) Find the breakeven point in terms of number of unit.

ii) What should be the output if the profit desired is Rs.50000?

Solution:

i) Total cost (TC) = Fixed cost (FC) + variable cost (VC)

$$VC = TC - FC = 85000 - 40000 = 45000$$

Variable cost per unit (v) $= \frac{45000}{15000} =$ Rs. 3 per unit

Selling cost per unit (s) $= \frac{105000}{15000} =$ Rs. 7 per unit.

$$Q_{BEP} = \frac{FC}{s - v}$$

$$= \frac{40000}{7 - 3} = 10000 \text{ units.}$$

ii) If the profit desired is Rs. 50000

Profit = Total sales – Total cost

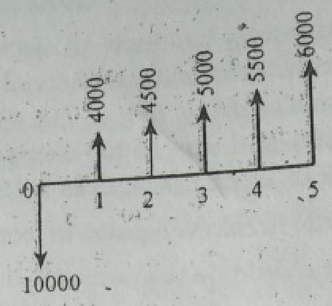
$$50000 = s \times Q - (40000 + v \times Q)$$

or, $90000 = (7 - 3) \times Q$

or, $Q = 22500$

Break even units for the profit to be Rs. 50000 is 22500

2. **Draw sensitivity chart using PW formulation of the following cash information's. It is desired to evaluate the sensitivity of PW to ± 30% change on: i) interest ii) investment**



Solution:

Assume MARR = 15%

$$PW = -10000 + 4000\ (P/A, 15\%\ 5) + G\ (P/G, 15\%, 5)$$

$$= -10000 + 4000 \times \frac{1.15^5 - 1}{1.15^5 \times 0.15} + \frac{500}{0.15^2} \times \frac{(1.15^5 - 5 \times 0.15 - 1)}{1.15^5}$$

$$= 6292.19$$

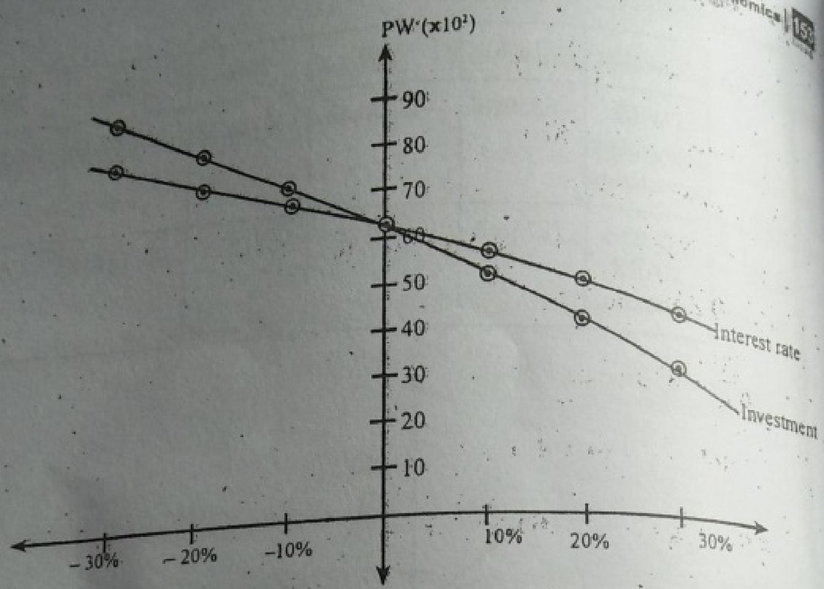
PW in variation of I & i

$$PW = -I + 4000 \times \frac{(1+i)^5 - 1}{(1+i)^5 \times i} + \frac{500}{i^2} \times \left[\frac{(1+i)^5 - 5\,i - 1}{(1+i)^5}\right]$$

or, this can be written as

$$PW = -I + \frac{4000}{(1+i)} + \frac{4500}{(1+i)^2} + \frac{5000}{(1+i)^3} + \frac{5500}{(1+i)^4} + \frac{6000}{(1+i)^5}$$

Variation	10%	20%	30%	–10%	–20%	–30%	Remarks
PW (i) Interest variation	5692.97	5124.3	4587.67	6936.72	7617.6	8342.18	Investment remains constant
PW (I) Investment variation	5296.19	4296.19	3296.19	7296.19	8296.19	9296.19	Interest remain constant



From graph, slope of investment is more than slope of interest rate, thus investment is more sensitivity.

Extra Question Solution

1. **CFL bulb of 20 watt costs Rs. 250 where as Filament bulb of 100 watt costs Rs. 30 which bulb do you prefer in your house? Why? Electricity cost is Rs. 10 per unit (Kwh). Assume light the bulb in a house is 4.5 hours per day.**

Solution:

For CFL bulb:

Initial cost = Rs. 250

Cost of use $= \left(\frac{20}{100}\right) \times x \times 10 = 0.2x$

Where x is the number of hours lighting the bulb

Total cost (x) = 250 + 0.2 x

For Filament bulb

Initial cost = 30

Cost of use = $\left(\frac{100}{1000}\right) \times x \times 10 = x$

Total cost (TC) = 30 + x

At breakeven point,

TC of CFC bulb = TC of Filament bulb

$250 + 0.2x = 30 + x$

$x = \frac{220}{0.8} = 275$ hours

In a day, light the bulb = 4.5

Therefore $x = \frac{275}{4.5} = 61.11 \approx 62$ days

Since I have to use for more than 275 hours (62 days), I will use CFL bulb in my house.

2. **Perform sensitivity analysis using IRR and BCR (with increment of 10%) over a range of ± 20 % in a) initial investment and b) Net annual revenue.**

Initial investment (I) = Rs. 2,00,000

Annual Revenues (R) = Rs. 50,000

Annual Expense (E) = Rs. 5,000

Salvage value (SV) = Rs. 25,000

Useful life (N) = 10 years, MARR = 12% per year

Solution:

Criteria of merit: IRR

Using AW formulation,

– 2,00,000 (A/P,i%,10) + (50,000 – 5000) + 25,000 (A/F,i%,10) = 0

$$\text{or, } -200000\frac{(1+i)^{10}\times i}{(1+i)^{10}-1} + 45{,}000 + 25{,}000 \times \frac{i}{(1+i)^{10}-1} = 0$$

IRR = 18.95%

When the capital investment varies by ± 20%

At 10%,

$$AW = -2{,}00{,}000 \times 1.1 \times \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45{,}000 + 25{,}000 \times \frac{i}{(1+i)^{10}-1} = 0$$

IRR = 16.37 %

At 20%,

$$AW = -2{,}00{,}000 \times 1.2 \times \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45{,}000 + 25{,}000 \times \frac{i}{(1+i)^{10}-1} = 0$$

IRR = 14.15 %

At – 10%,

$$AW = -2{,}00{,}000 \times 0.9 \times \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45{,}000 + 25{,}000 \times \frac{i}{(1+i)^{10}-1}$$

IRR = 21.99 %

Similarly, At – 20 %, IRR = 25.67 %

When net annual revenue (R – E) varies ± 20 %

R – E = 50000 – 5000 = 45000

At 10 %, $AW = -2{,}00{,}000 \times \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45{,}000 \times 1.1 + \frac{25000 \times i}{(1+i)^{10}-1} = 0$

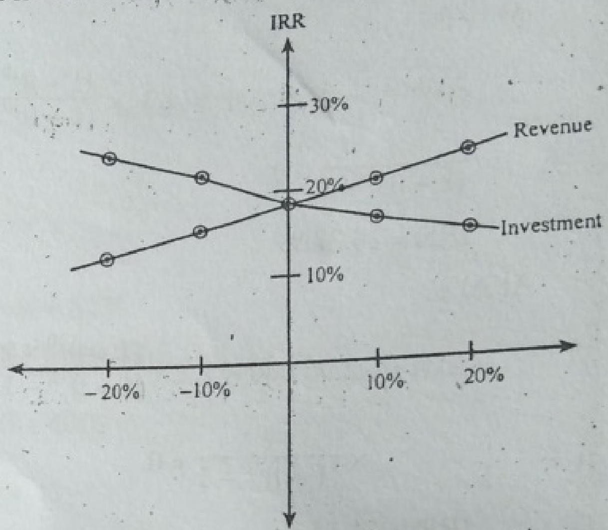
IRR = 20.66 %

Similarly, At 20 %, IRR = 23.36 %

At – 10%, $AW = -2{,}00{,}000 \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45000 \times 0.9 + \frac{25000 \times i}{(1+i)^{10}-1} = 0$

IRR = 16.18 %

Similarly, At – 20 %, IRR = 13.32 %



Different in investment = 25.67 – 14.15 = 11.52 %

Different in revenue = 23.36 – 13.32 = 10.64 %

∴ Investment is more sensitivity.

Criteria of merit: BCR

Using FW Formulation,

$$BCR_{modified} = \frac{(50000 - 5000) \times (F/A, 12\%, 10)}{200000 (F/P, 12\%, 10) - 25000}$$

$$= \frac{45000 \times (1.12^{10} - 1)/0.12}{200000 \times 1.12^{10} - 25000} = 1.32$$

When the investment (I) varies by ± 20%

$$\text{At 10 \%, BCR} = \frac{45000 \times (F/A, 12\%, 10)}{200000 \times 1.1 (F/P, 12\%, 10) - 25000} = 1.19$$

Similarly,

At 20%, BCR = 1.09

$$\text{At} - 10\%,\ BCR = \frac{45000 \times (F/A,\ 12\%,\ 10)}{200000 \times 0.9\ (F/P,\ 12\%,\ 10) - 25000} = 1.47$$

Similarly,

At – 20%, BCR = 1.67

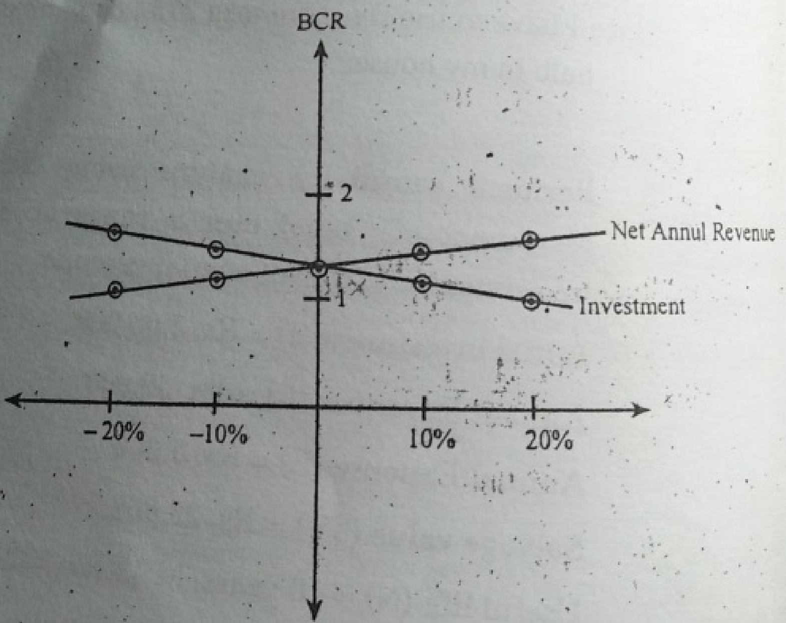
Where net annual revenue varies by ± 20%

$$\text{At } 10\%,\ BCR = \frac{4500 \times 1.1\ (F/A, 12\%, 10)}{200000\ (F/P, 12\%, 10) - 25000} = 1.45$$

$$\text{At } 20\%,\ BCR = \frac{4500 \times 1.2\ (F/A, 12\%,\ 10)}{200000\ (F/P,\ 12\%,\ 10) - 25000} = 1.58$$

$$\text{At} -10\%,\ BCR = \frac{4500 \times 0.9\ (F/A,\ 12\%,\ 10)}{200000\ (F/P, 12\%, 10) - 25000} = 1.19$$

Similarly, At – 20%, BCR = 1.05



Different in investment = 1.67 – 1.09 = 0.58

Different in revenue = 1.58 – 1.05 = 0.53

More slope in investment. Hence investment is more sensitivity.

3. Perform a sensitivity analysis by breakeven analysis under the following data.

Variable cost = Rs. 66,000

Output per unit = Rs. 3,000

Sales revenue per unit = Rs. 1,50,000

a) Selling price comes down to Rs. 40

b) Fixed cost increase to Rs. 40,000

c) Variable cost increase by 10%

Solution:

$$\text{Break even point (BEP)} = \frac{\text{Fixed cost}}{(\text{Selling price per unit} - \text{variable cost per unit})}$$

Variable cost per unit = 66,000/3,000 = Rs. 22

Selling cost per unit = 1,50,000/3,000 = Rs. 50

$$\therefore \text{BEP} = \frac{35000}{50-22} = 1{,}250 \text{ units}$$

a) Selling price down to Rs. 40

$$\text{BEP} = \frac{35000}{40-22} = 1{,}944.44 \text{ units}$$

$$\text{Reduction in selling price} = \frac{50-40}{50} \times 100 = 20\%$$

$$\text{Increase in BEP} = \frac{(1944.44-1250)}{1250} \times 100 = 56\%$$

b) Cost increase to Rs. 40,000

$$\text{BEP} = \frac{40000}{50-22} = 1{,}428.57$$

$$\text{Increase in FC} = \frac{40000-35000}{35000} \times 100 = 14.21\%$$

$$\text{Increase in BEP} = \frac{1428.57-1250}{1250} \times 100 = 14.28\%$$

c) Variable cost increase by 10%

i.e. variable cost = $1.1 \times 22 = 24.2$

$$BEP = \frac{35000}{50 - 24.2} = 1356.58 \text{ units}$$

$$\text{Increase in variable cost} = \frac{(24.2 - 22)}{22} \times 100$$

$$= 10\%$$

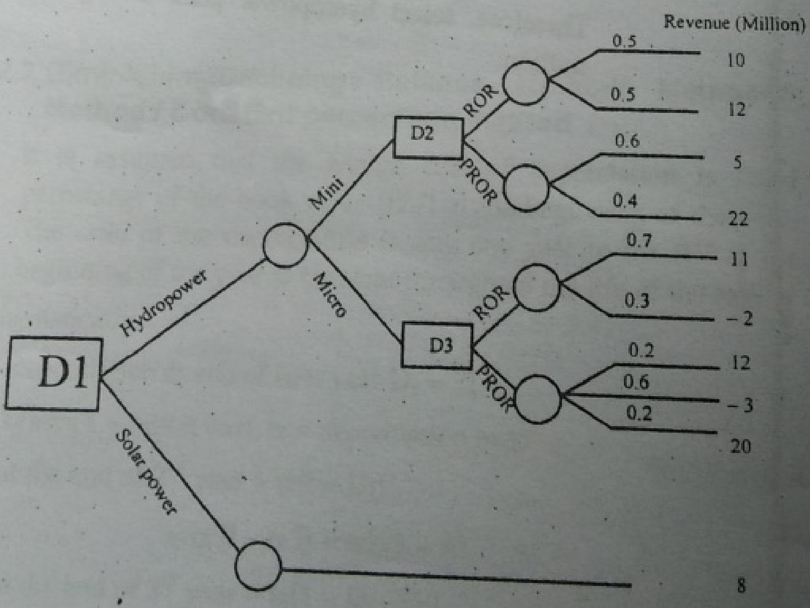
$$\text{Increase in BEP} = \frac{(1356.58 - 1250)}{1250} \times 100 = 8.5\%$$

Results;

- 20%, increase in selling price results in 56% increase of BEP
- 14.21% increase in FC results in 14.28% increase in BEP
- 10% increase in variable cost results in 8.5% increase of BEP

Out of three factors (Selling price, fixed cost, variable cost) BEP is more sensitive in selling price.

4. **Determine the best decision at the decision node D1.**



Solution:

Selection step by step from right to left.

Decision at node D_2

$ROR = 0.5 \times 10 + 0.5 \times 12 = 11$

$PROR = 0.6 \times 5 + 0.4 \times 22 = 11.8$

Select PROR as 11.8

Decision at node D_2

$ROR = 0.7 \times 11 + 0.3 \times (-2) = 7.1$

$PROR = 0.2 \times 12 + 0.6 \times (-3) + 0.2 \times 20 = 4.6$

Select ROR as 7.1

Selection at D_1.

From hydropower = $11.8 \times 0.4 + 7.1 \times 0.6 = 8.98$

From solar power = 8

Therefore, select hydropower plant as higher revenue of 8.98 million.

* * *

1. **Book value**

   Represent the remaining, un–depreciated capital investment on the books determined at the end of each year.

2. **Unadjusted cost**

   Initial cost of the assets including purchase price, delivery, installation fees etc.

3. **Recovery period**

   Depreciable life 'n' of the assets in years.

4. **Salvage value (SV)**

   Estimate value of the property at the end of its useful life.

5. **Market value**

   The amount that will be paid by the willing buyer to willing seller for a property where each has equal advantage and no compulsion to buy or sell.

7.2 Basic methods of depreciation

7.2.1 Straight line method

It assumes that a constant amount is depreciated each year over the depreciable life of the asset.

Mathematically,

$$D_n = \frac{(I - S)}{N}$$

Where, D_n = Annual depreciation deduction in the year N.

I = Initial cost

S = Salvage value

N = Life of the asset

Example: . **A machine costing of Rs. 4,000 is estimated to have life 10 years? and salvage value is 1,000. Find**

i) Annual depreciation

ii) Rate of depreciation

iii) **Depreciation amount for the 6^{th} year**

iv) **Accumulated depreciation and book value at end of 6^{th} year**

Solution:

Annual depreciation $= \dfrac{4000 - 1000}{10} =$ Rs. 300 per year

Rate of depreciation $= \dfrac{300}{3000} \times 100 = 10\%$ per year

OR,

Rate of depreciation $= \dfrac{1}{N} \times 100 = \dfrac{1}{10} \times 100 = 10\%$ year

iii) Depreciation for 6^{th} year = Annual depreciation = Rs. 300

iv) Accumulated depreciation amount for the 6^{th} year $= \dfrac{(4000 - 1000)}{10} \times 6$

$= 3000 \times 6 =$ Rs. 1800

Book value at end of 6^{th} year $= 4000 - 1800 =$ Rs. 2200

7.2.2 Diminishing/Declining Balance Method/ Matheson Method / Constant percentage method

It is assumed that the annual cost of depreciation is fixed percentage of the book value (BV) at the beginning of the year. The ratio of the depreciation in any one year to the BV at the beginning of the year is constant throughout the life of the asset.

Mathematically,

Depreciation at end of first year $D_1 = 2I$

Where I = initial cost, α = depreciation ratio

at the end of 2^{nd} year $= \alpha(I - D_1)$

$= \alpha\ (I - \alpha\ I) = \alpha I\ (1 - \alpha)$

at the end of 3^{rd} year $= \alpha(1 - D_1 - D_2)$

$$= \alpha I (1-\alpha)^2$$

For any year n

$D_n = \alpha I (1-\alpha)^{n-1}$

Book value $= I(1-\alpha)^n$

α = Declining depreciation rate = 2 × straight line depreciation rate.

Issues Regarding salvage value

- Salvage value must be estimated at the beginning of depreciation analysis.
- If final book value (B_n) doesn't equal to estimated salvage value (S), we have to make the adjustment in our depreciation analysis method.

Case I $B_N > S$

To reduce Final book value (B_N) to salvage value by switching from declining balance to straight line.

Switching rule

If depreciation by declining balance in any year is less than or equal to the depreciation by straight line, then we would switch to and remain with straight line for the duration of the project's depreciable life.

Straight line depreciation in any year n (D_n)

$$D_n = \frac{\text{book value at the beginning of year n} - \text{salvage value}}{\text{Remaining useful life at beginning of year n}}$$

Case II $B_N < S$

If the book value is lower than S, at any period, then depreciation amount are adjusted as $B_N = S$.

Example:

Consider the following accounting information for a computer system.

Cost basis of the asset, I = Rs. 20,000

Useful life (N) = 5 years

i) Salvage value (S) = Rs. 500

ii) Salvage value = Rs. 2000

Solution:

Declining balance rate (α) = $\frac{1}{5} \times 2 = 40\%$

Calculating the depreciation by Double Decline Balance (DDB) for each year

N	B_{N-1}	$D_N = \alpha \times B_{N-1} = 0.4 \times B_{N-1}$	$B_N = B_{N-1} - D_N$
1	20000	8000	12000
2	12000	4800	7200
3	7200	2880	4320
4	4320	1728	2592
5	2592	1036.8	1555.2

i) Here book value is Rs. 1552.2 at the end of 5 which is greater than Rs. 500. Therefore we use the switching.

So, compute the SL depreciation for each year and compare with DDB and use the decision rule.

N	SL Depreciation	DB depreciation	Decision
1	(20000 – 500)/5 = 3900	< 8000	Do not switch
2	(12000 – 500)/4 = 2875	< 4800	Do not switch
3	(7200 – 500)/3 = 2233.33	< 2880	Do not switch
4	(4320 – 500)/2 = 1910	> 1728	Switch to SL

The depreciation schedule is

N	DDB with switching to SL	End of year Book value (Rs.)
1	8000	12000
2	4800	7200
3	2880	4320
4	1910	2410
5	1910	500

ii) Book value < Salvage value (2000)

depreciation amount are adjusted as $B_N = S$

N	D_N	B_N (at end of year)
1	8000	12000 > 2000
2	4800	7200 > 2000
3	2880	4320 > 2000
4	1728	2592 > 2000
5	1036.8 > 592	2000 (2592 – 592)

7.2.3 Sinking Fund method

The fixed sum depreciated at the end of every time period earns an interest rate of i% compounded annually.

Let,

I = initial cost, S = salvage value, N = Life of the asset

i = Rate of return compounded annually

A = The annual equivalent amount of depreciation charge

B_N = Book value of the asset at the end of period n

D_n = Depreciation charge at the end of the period n

To find the annual equivalent amount (A) = (I – S) × (A/F, i%, N)

To Find the depreciation charge (D_n) = (I – S) × (A/F, i%, N)×(F/p, i%, n – 1)

To find the book value at the end of period t = I – (I – s) × (A/F, i%, N) × (F/P,i%, n)

Example: **Compute the depreciation charge and book value in each year by using sinking fund method with following information.**

Initial cost = Rs. 2,00,000

Salvage value = Rs. 20,000

Life of the asset = 6 years

i = 12%

Solution:

Fixed annual depreciation (A) = (I – S) (A/F,12%,6)

$$= (200000 - 20000) \times \frac{0.12}{1.12^6 - 1}$$

$$= 22180.63$$

Net depreciation for the year 1

d_1 = (I – S) (A/F, 12%,6) × (F/P,12%, n –1)

= 22180.63 × 1 (n = 1)

= 22180.63

Net depreciation for the year 2

d_2 = (I – S) (A/F, 12%,6) × (F/P, 12%, 2 – 1)

$= 22180.63 \times (1 + 0.12)^1$

= 24842.3

On continuing, we obtain the depreciation value & book value in following table

Year	Fixed depreciation (A)	Net depreciation (D_n)	Book value (B_n)
1	22180.63	22180.63	$200000 - 22180.63 = 177819.37$
2	22180.63	$22180.63 \times 1.2 = 24842.3$	$177819.37 - 24842.3 = 152977$
3	22180.63	$22180.63 \times 1.2^2 = 27823.38$	$152977 - 27823.38 = 125153.69$
4	22180.63	$22180.63 \times 1.2^3 = 31162.18$	93991.51
5	22180.63	34901.65	59089.86
6	22180.63	39089.84	$20000.02 \approx (20000)$ (ok)

7.2.4 Sum of the year Digit method (SOYD)

- Larger depreciation charges during the beginning years of asset and smaller depreciation charges as asset getting old.
- Depreciation charge is calculated from the ratio of the sum of the years digit for the total useful life and remaining useful life at the beginning of particular year.

$$\text{SOYD deprecation} = \frac{\text{Remaining useful life at the beginning of particular year}}{\text{SOYD for the total useful life}} \times (I - S)$$

$$D_n = \frac{(N - n + 1)\ (I - S)}{SOYD}$$

$$SOYD = 1 + 2 + 3 + 4 + \ldots\ldots\ldots N = \frac{N(N+1)}{2}$$

Example: Compute the SOYD depreciation schedule for the following:

Cost basis of the asset, I = Rs. 20,000

Useful life N = 5 years

Salvage value, SV = Rs, 2000

Solution: $SOYD = \frac{N(N+1)}{2} = \frac{5\ (5+1)}{2} = 15$

Year	Fixed depreciation (A)	Net depreciation (D_n)	Book value (B_n)
1	22180.63	22180.63	200000 – 22180.63 = 177819.37
2	22180.63	$22180.63 \times 1.2 = 24842.3$	177819.37 – 24842.3 = 152977
3	22180.63	$22180.63 \times 1.2^2 = 27823.38$	152977 – 27823.38 = 125153.69
4	22180.63	$22180.63 \times 1.2^3 = 31162.18$	93991.51
5	22180.63	34901.65	59089.86
6	22180.63	39089.84	20000.02 = (20000) (ok)

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$$\text{SOYD deprecation} = \frac{\text{Remaining useful life at the beginning of particular year}}{\text{SOYD for the total useful life}} \times (I - S)$$

$$D_n = \frac{(N - n + 1)\ (I - S)}{SOYD}$$

$$SOYD = 1 + 2 + 3 + 4 + \ldots\ldots\ldots N = \frac{N(N+1)}{2}$$

Example: Compute the SOYD depreciation schedule for the following:

Cost basis of the asset, I = Rs. 20,000

Useful life N = 5 years

Salvage value, SV = Rs, 2000

Solution: $SOYD = \frac{N(N+1)}{2} = \frac{5\ (5+1)}{2} = 15$

Year	SOYD depression D_n	Book value B_N
1	$\frac{5}{15}(20000-2000) = 6000$	20000 – 6000 = 14000
2	$\frac{4}{15}(20000-2000) = 4800$	14000 – 4800 = 9200
3	$\frac{3}{15}(20000-2000) = 3600$	9200 – 3600 = 5600
4	$\frac{2}{15}(20000-2000) = 2400$	5600 – 2400 = 3200
5	$\frac{1}{15}(20000-2000) = 1200$	3200 – 1200 = 2000

7.2.5 Modified Accelerated Cost Recovery System (MACRS)

- Historically, for tax purpose and accounting an asset's depreciable life was determined by its estimated useful.
- The MACRS scheme, totally abandon this practice and simpler guidelines were set which created several classes of assets each with more or less arbitrary life called recovery period.
- The salvage value of property is always zero.
- MARCS scheme includes 8 categories of assets: 3 years, 5 year, 7 year, 10 year, 15 year, 20 year, 27.5 year and 39 year

Half year convention

- It is assumed that all assets are placed at mid–year and they have zero salvage value.
- Half year depreciation is allowed for the first year and full year's depreciation is allowed in each of the remaining years of the assets recovery period and finally remaining half year depreciation in the end year of recovery period.
- The MACRS asset is depreciated initially by the declining balance and then to straight line method.

MACRS Depreciation Rates

Year	3 year	5 year	7 year	10 year	15 year	20 year
1	33.33	20.00	14.29	10.00	5.00	3.75
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81	19.20	17.49	14.40	8.55	6.667
4	7.41	11.52	12.49	11.52	7.70	6.177
5		11.52	8.93	9.22	6.93	5.75
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55	5.90	4.888
8			4.46	6.56	5.90	4.522
9				6.55	5.91	4.462
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

Example:

A tax payer wants to place in service a Rs. 20,000 asset that is assigned to the 5 year class. Compute the MACRS percentage depreciation amounts and book value for the asset.

Solution:

[Note: MACRs is obtain from given table but in exam table may not be given. Thus calculation is required which is shown below.]

MACRs deduction percentage, beginning with the first taxable year and ending with the 6th year.

Straight line rate = 1/5 = 0.2

Double declining balance rate (α) = $\frac{1}{5} \times 200\% = 40\%$

Salvage value in MACRS = 0

Year	Calculation (%)	MACRs	Decision
1	$\frac{1}{2}$ year DDB dep = $0.5 \times 0.4 \times 100$	20%	
2	DDB dep = $0.4 \times (100 - 20)$	32%	Do not switch
	SL dep. = $\frac{1}{4.5} \times (100 - 20)$	17.78%	
3	DDB dep = $0.4 \times (100 - 20 - 32)$	19.2%	Do not switch
	SL dep. = $\frac{1}{3.5} \times (100 - 20 - 32)$	13.71%	
4	DDB dep = $0.4 \times (100 - 71.2)$	11.52%	Switch to SL
	SL dep. = $\frac{1}{2.5} \times (100 - 71.2)$	11.52%	
5	SL dep = $\frac{1}{1.5}(100 - 82.72)$	11.52%	
6	$\frac{1}{2}$ year SL dep = 0.5×11.52	5.76%	

In the year 4, SL depreciation is ≥ DDB depreciation and we switch to SL

Calculation the depreciation amounts from the percentages

Year	MARCS (%)	Depreciation (D_N)	Depreciation Amount (B_N)
1	20	$0.2 \times 20000 = 40000$	$20000 - 4000 = 16000$
2	32	$0.32 \times 20000 = 6400$	$16000 - 6400 = 9600$
3	19.2	$0.192 \times 20000 = 3840$	$9600 - 3840 = 5760$
4	11.52	$0.1152 \times 20000 = 2304$	$5760 - 2304 = 3456$
5	11.52	$0.1152 \times 20000 = 2304$	$3456 - 2304 = 1152$
6	5.76	$0.0576 \times 20000 = 1152$	$1152 - 1152 = 0$

7.3 Introduction to corporate income tax

Individuals as well as corporations are subjected to income tax. Income tax on corporation is known as the corporate income tax or corporate tax. Corporation is a word used to cover a variety of business enterprises, all of which are entities having a legal personality and distinct from their owners. There are various types of business enterprises but corporations and companies have only a separate legal personality. It is only a way of including all corporate source of income in the personal income tax base.

7.4 After tax cash flow estimate

It is measure of a company's ability to generate positive cash flow after deducting taxes.

The general formula for After tax cash flow is

ATCF = Net income + Depreciation + Amortization

Sometimes analysis also add back other non-cash items and proceeds from debt or equity issuance

The amount remaining after expenses, mortgage payments, and income taxes have been deducted from the gross income of an investment property is the after tax cash flow.

7.5 General procedure for making after tax economic analysis

Let R_k = revenues from the project during period k

E_k = Expenses during year k

d_k = depreciation and depletion during year k

t = Effective income tax rate

T_k = income tax consequences during year k

$ATCF_k$ (After tax cash flow) = ATCF from the project during year k

Because the NIBT (i.e. taxable income) is $(R_k - E_k - d_k)$, the ordinary income tax consequences during year k are computed with

$$T_k = -t\,(R_k - E_k - d_k)$$

Therefore, when $R_k > (E_k + d_k)$, a tax liability occurs when $R_k < (E_k + d_k)$, a decrease in the tax amount occurs.

$$NIAT_k = \underbrace{(R_k - E_k - d_k)}_{\text{Taxable income}} - \underbrace{t\,(R_k - E_k - d_k)}_{\text{Income taxes}}$$

$$NIAT_k = (R_k - E_k - d_k)\,(1 - t)$$

The ATCF associated with a project equals the NIAT plus non- cash items such as depreciation.

$$ATCF_k = NIAT_k + d_k$$

$$= (R_k - E_k - d_k)\,(1 - t) + d_k$$

$$ATCF_k = (1 - t)\,(R_k - E_k) + td_k$$

Example: If the revenue from a project is \$10,000 during a tax year, out–of–pocket expenses are \$4000 and depreciation deductions for income tax purposes are \$ 2,000 what is the ATCF when t = 0.4? What is the NIAT?

Solution:

$$ATCF = (1 - t)\ (R_k - E_k) + td_k$$

$$= (1 - 0.4)\ (10000 - 4000) + 0.4 \times 2000 = \$\ 4{,}400$$

Also, $ATCF_k = NIAT_k + d_k$

$$NIAT = ATCF - d_k = 4400 - 2000 = \$\ 2400$$

Old Question Solutions

'2069 Bhadra'

1. **Define depreciation and list out important methods of calculating depreciation deductions.**

Solution: Definition of depreciation : Refer at 7.1

Important methods of calculating depreciation

1. Straight line method
2. Declining balance method
3. Sinking fund method
4. SOYD method
5. Modified accelerate cost recovery system (MACRs)
6. Service output method

2. **A machine costs Rs. 15000. Its useful life is 5 years and salvage value is Rs. 900. Compute the annual depreciation allowance and resulting book values using double declining balance depreciation methods.**

Solution: Rate of depreciation $= \frac{1}{N} \times 2 = 0.4$

$$= 40\%$$

End of year	Depreciation (DB)	SL Dep.	Decision	Book value
1	$0.4 \times 15000 = 6000$	$\frac{(15000-900)}{5} = 2820$	Not switch	$15000 - 6000 = 9000$
2	$0.4 \times 9000 = 3600$	$\frac{(9000-900)}{4} = 2025$	Not switch	$9000 - 3600 = 5400$
3	$0.4 \times 5400 = 2160$	$\frac{(5400-900)}{3} = 1500$	Not switch	$5400 - 2160 = 3240$
4	$0.4 \times 3240 = 1296$	$\frac{(3240-900)}{2} = 1170$	Not switch	$3240 - 1296 = 1944$
5	$0.4 \times 1944 = 777.6$	$\frac{(1944-900)}{1} = 1044$	Switch	$1944 - 1044 = 900$

"2070 Magh"

1. **Define depreciation. What are the causes for it? It a machine costing of Rs. 1,50,000 is purchased by expecting salvage value Rs. 40,000 at the end of 6th year. Calculate the depreciation amount for each year by,**

   i) SOYD

   ii) Declining balance

Solution: First part Refer at 6.1

$$SOYD = \frac{N(N+1)}{2} = \frac{6 \times 7}{2} = 21$$

Year	Depreciation (D_N)	Book value (B_N)
1	$\frac{6}{21}(1{,}50{,}000 - 40{,}000) = 31428.57$	$150000 - 31428.57 = 118571.43$
2	$\frac{5}{21}(1{,}50{,}000 - 40{,}000) = 26190.47$	$118571.43 - 26190.47 = 92380.96$

3	$\frac{4}{21}$ (1,50,000 – 40,000) = 20952.38	92380.96 – 20952.38 = 71428.58
4	$\frac{3}{21}$ (1,50,000 – 40,000) = 15714.28	71428.58 – 15714.28 = 55714.3
5	$\frac{2}{21}$ (1,50,000 – 40,000) = 10476.19	55714.3 – 10476.19 = 45238.09
6	$\frac{1}{21}$ (1,50,000 – 40,000) = 5238.095	45238.09 – 5238.09 = 40000

ii) Declining balance $\alpha = \frac{1}{N} \times 2 = \frac{1}{6} \times 2 = 0.33$

FOY	Decline dep. (Dn)	SL dep	Decision	Book value
1	0.33 × 150000 = 50000	$\frac{(150000-40000)}{6}$ = 18333.33	not switch (DB)	150000 – 50000 = 100000
2	0.33 × 100000 = 33333.33	$\frac{(100000-40000)}{5}$ = 12000	not switch (DB)	100000 – 33333.33 = 66666.67
3	66666.66 × 0.33 = 22222.22	$\frac{(66666.67-40000)}{4}$ = 6666.67	not switch (DB)	66666.67 – 22222.22 = 44444.45
4	44444.45 × 0.33 = 14666.67	$\frac{(44444.45-40000)}{3}$ = 1481.48	SV = 40000 44444.45 – 40000 = 4444.45 < DB, Switch to SL	44444.45 – 4444.45 = 40000
5	0	–	–	40000
6	0	–	–	40000

2. Suppose an equipment purchased for Rs. 10,00,000. It is expected to generate income of Rs. 3,50,000 per year during 5 years and corporate income tax rate is 25% per year. Under recovery periods depreciation are as follows.

Year	1	2	3	4	5
Depreciation amount	1,00,000	2,00,000	2,00,000	2,00,000	1,00,000

Calculate ATCFs and determine profitability (IRR) when MARR is 15% by using PW method

Solution:

Year	Cash flow	Depreciation	Taxable income (Cash flow Depreciation)	Tax = 0.25 × Taxable income	ATCF = Cash flow tax
0	– 10,00,000	–	–	–	– 10,00,000
1	3,50,000	1,00,000	2,50,000	62500	287500
2	3,50,000	2,00,000	1,50,000	37500	312500
3	3,50,000	2,00,000	1,50,000	37500	312500
4	3,50,000	2,00,000	1,50,000	37500	312500
5	3,50,000	1,00,000	2,50,000	62500	287500

<u>Calculation of profitability (IRR) ATCF</u>

$$PW(i\%) = -10{,}00{,}000 + \frac{287500}{(1+i)} + \frac{31250}{(1+i)^2} + \frac{312500}{(1+i)^3} + \frac{312500}{(1+i)^4} + \frac{287500}{(1+i)^5} = 0$$

On solving, we get

$i = 15.56\% > \text{MARR } (15\%)$,

Economically justified.

"2070 Bhadra"

An asset has installed value of 45,000. $S_s = 0$. It is classed as a 5 year property. Determine approximate MACRS depreciation schedule. Over 6 years it is estimated to generate revenue of Rs. 23,000 per year with annual operating cost 7300. Required rate of return = 15% after tax. Tax rate = 40%. Evaluate after tax IRR with annual worth method.

Solution:

Installed Value = 45,000 Revenue = 23,000/year. O & M = 7300/year

Year	MACRs % calculation	MACRs	Depreciation amount	BTCF	Taxable income	Tax $0.4 \times TI$	ATCF (BTCF − tax)
0.	–	Switching	$45000 \times$ MACRs	− 45000	–	–	− 45000
1. (half year)	$DB = \frac{1}{2} \times 0.2 \times 100 = 20\%$ $SL = \frac{1}{2} \times \frac{1}{5} \times 100 = 10\%$	20% (Do not switch)	$0.2 \times 45000 = 9000$	$\frac{1}{2} \times (23000 - 7300) = 7850$	$7850 - 9000 = -1150$	–	7850
2.	$DB = 0.4 \times (100 - 80) = 32\%$ $SL = \frac{1}{4.5} \times 80 = 17.78\%$	32% (Do not switch)	$0.32 \times 45000 = 14400$	$(23000 - 7300) = 15700$	$15700 - 14400 = 1300$	$0.4 \times 1300 = 520$	$15700 - 520 = 15180$
3.	$DB = 0.4 \times 48 = 19.2$ $SL = \frac{1}{3.5} \times 48 = 13.71$	19.2% (Do not switch)	$0.192 \times 45000 = 8640$	$(23000 - 7300) = 15700$	$15700 - 8640 = 7060$	$0.4 \times 7060 = 2824$	$15700 - 2824 = 12876$
4.	$DB = 0.4 \times 28.8 = 11.52$ $SI = \frac{28.8}{2.5} = 11.52$	11.52 Switch	$0.1152 \times 45000 = 5184$	$(23000 - 7300) = 15700$	$15700 - 5184 = 10516$	$0.4 \times 10516 = 4206.4$	$15700 - 4206.4 = 11493.6$
5.	$SL = \frac{17.28}{1.5} = 11.52$	11.52	$0.1152 \times 45000 = 5184$	$(23000 - 7300) = 15700$	$15700 - 5184 = 10516$	$0.4 \times 10516 = 4206.4$	$15700 - 4206.4 = 11493.6$
6	Half year dep $= \frac{1}{2} \times 11.25 = 5.76$	5.76	$0.0576 \times 45000 = 2592$	$\frac{1}{2} \times (23000 - 7300) = 7850$	$7850 - 2592 = 5258$	$0.4 \times 5258 = 2103.2$	$7850 - 2103.2 = 5746.8$

Where BTCF = Before Tax cash flow

ATCF = After Tax cash flow

[Note: If taxable income is negative, tax can be neglected]

Also, PW of ATCF (15%) $= -45000 + \frac{7850}{1.15} + \frac{15180}{1.15^2} + \frac{12876}{1.15^3} + \frac{11493.6}{1.15^4} + \frac{11493.6}{1.15^5} + \frac{5746.8}{1.15^6} = -3459.22\ (< 0)$, Economically not justified.

"2071 Magh"

1. **Compute the book value at the end of 3 years (BV3) by all the methods of depreciation except MACRS method. Cost basis of a machine is Rs. 10,000. SV = 0. Useful life = 5 years. MARR = 10%**

Solution:

I) Straight line method

$$\text{Depreciation} = \frac{10000 - 0}{5} = 2000 \text{ per year}$$

$$\text{Book value at the end of 3 year} = 10000 - 3 \times 2000 = 4000$$

II) Declining balance method

$$\text{rate of depreciation } (\alpha) = \frac{1}{5} \times 2 \times 100 = 40\%$$

$$\text{Book value at the end of 3 year} = 1000\,(1 - \alpha)^3 = 10000\,(1 - 0.4)^3 = 2160$$

III) Sinking fund method

Book value at the end of 3 year

$$I - (I - S) \times (A/F, i\%, N) \times (F/A, i\%, t)$$

$$= 10000 - (10000 - 0)\, \frac{0.1}{1.1^5 - 1} \times \frac{1.1^3 - 1}{0.1} \quad \begin{bmatrix} t = 3 \\ N = 5,\ i = 10 \end{bmatrix}$$

$$\fallingdotseq 4578.3$$

IV) SOYD

$$SOYD = \frac{5(5+1)}{2} = 15 \text{ (For N = 3)}$$

Sum of year = 1 + 2 + 3 = 6 (For n = 3)

$$\text{Book value at the end of 3 year} = 10000 - \frac{6}{15} \times (10000 - 0)$$

$$= 6000$$

2. **Evaluate after tax PW. The cost basis for a machine is Rs. 10,000 the machine is 5 years MACRs property over 6 years, it is estimated to save Rs. 4500 per year in maintenance costs with annual operating cost being Rs. 1000. It will be depreciated by MACRS method. SV = 0 Tax rate = 30%,** MARR = 15%.

Solution:

Total cost (I) = 10000

Revenue = 4500, O & M = 1000

Year	MACRs (Same as 2070 Bhadra)	Depreciation (Dn) amount 10000 × MACRS	BTCF	Taxable income BTCF − D_n	Tax Taxable income × 0.3	ATCF (BTCF − Tax)
0	-	-	− 10000	-	-	− 10000
1	20%	0.2 × 10000 = 2000	$\frac{1}{2}$ × (4500 − 1000) = 1750	(1750 − 2000) = − 250	-	1750 − 0 = 1750
2	32%	0.32 × 10000 = 3200	4500 − 1000 = 3500	3500 − 3200 = 300	0.3 × 300 = 90	3500 − 90 = 3410
3	19.2%	1920	45000 − 1000 = 3500	3500 − 1920 = 1580	0.3 × 1580 = 474	3500 − 474 = 3026
4	11.52%	1152	3500	3500 − 1152 = 2348	0.3 × 2348 = 704.4	3500 − 704.4 = 2795.6
5	11.52%	1152	3500	3500 − 1152 = 2348	0.3 × 2348 = 704.4	3500 − 704.4 = 2795.6
6	5.676%	576	$\frac{1}{2}$ (4500 − 1000) = 1750	1750 − 576 = 1174	0.3 × 1174 = 352.2	1750 − 352.2 = 1397.8

$$PW = -1000 + \frac{1750}{1.15} + \frac{3410}{1.15^2} + \frac{3026}{1.15^3} + \frac{2795.6}{1.15^4} + \frac{2795.6}{1.15^5} + \frac{1397.8}{1.15^6}$$

$$= -317.55 < 0$$

Economically not justified.

"2071 Bhadra"

1. **What do you mean by depreciation? Explain about the causes of it. Explain about any three methods of deprecation calculation that are used commonly. A machine purchased for Rs. 60000 by expecting useful life of 10 years. Calculate the depreciation amount for each year by using deciding balance method when rate of depreciation is 20% per year.**

Solution: Theory part, Refer at 7.1 and 7.2

Investment = 60000, life = 10

depreciation rate = 20% per year

Salvage value is not given, so assume book value at the end of 10 year is exactly equal to the salvage value, (but its value is not less than 2000)

$\alpha = \frac{1}{N} \times 2 = 20\%$ per year = Double deciding balance rate

Year	B_{N-1}	Depreciation D_n	Book value B_N
1	60000	$0.2 \times 60000 = 12000$	48000 (60000 – 12000)
2	48000	$0.2 \times 48000 = 9600$	38400
3	38400	7680	30720
4	30720	6144	24576
5	24576	4915.3	19660.8
6	19660.8	3932.1	15728.64

7	15728.64	3145.8	12582.9
8	1258.29	2516.5	10066.3
9	10066.3	2013.2	8053.1
10	8053.1	1610.62	6442.48 (>0) ok

Salvage value at the end of 10 year = 6442.48

"2072 Ashwin"

1. **Explain the general procedure for after tax economic analysis with suitable example.**

   Refer at 7.5

2. **Considering the following information, compute the annual depreciation and book value of each year by. (i) SL method (ii) DB method (iii) SOYD method and (iv) sinking fund method**

Cost basis	Salvage value	Useful life	MARR
$ 7000	$ 2000	5 years	10%

Solution:

i) SL method

$$Dn = \frac{(7000 - 2000)}{5} = 1000$$

N	B_{N-1}	D_n	B_N
1	7000	1000	6000
2	6000	1000	5000
3	5000	1000	4000
4	4000	1000	3000
5	3000	1000	2000

ii) DB method

$\alpha = \frac{1}{5} \times 2 = 0.4 = 40\%$

N	B_{N-1}	D_n	Decision	B_N
1	7000	$0.4 \times 7000 = 2800$		4200
2	4200	$0.4 \times 4200 = 1680$		2520
3	2520	1008	Dn > (2520 – 2000 = 520)	2000
4	2000	0		2000
5	2000	0		2000

iii) SOYD method

$\text{SOYD} = \frac{5(5+1)}{2} = 15$

N	B_{N-1}	D_n	B_N
1	7000	$\frac{5}{15}(7000 - 2000) = 1666.67$	5333.33
2	5333.33	$\frac{4}{15}(7000 - 2000) = 1333.33$	4000
3	4000	$\frac{3}{15}(7000 - 2000) = 1000$	3000
4	3000	$\frac{2}{15}(7000 - 2000) = 666.67$	2333.33
5	2333.33	$\frac{1}{15}(7000 - 2000) = 333.33$	2000 (OK)

iv) **Sinking Fund method**

Fixed annual depreciation (A) = (I – S) (A/F,10%, 5)

$$= (7000 - 2000) \times \frac{0.1}{1.1^5 - 1}$$

$$= 818.98$$

Net depreciation for the year 1

= (I – S) (A/F, 10%,5) × (F/P,10%,1 –1)

= 818.98 × 1 = 818.98

Depreciation for the year 2

= 818.98 × 1.1 = 900.88

Similar for others, in table

N	B_{N-1}	D_n	B_N
1	7000	$818.98 \times 1 = 818.98$	6181.02
2	6181.02	$818.98 \times 1.1 = 900.88$	5280.14
3	5280.14	$818.98 \times 1.1^2 = 990.96$	4289.18
4	4289.18	$818.98 \times 1.1^3 = 1090.06$	3199.12
5	3199.12	$818.98 \times 1.1^4 = 1199.06$	2000.06 ≈ 2000

Extra Question Solutions

1. **A machine costing of Rs. 4000 is estimated to have life of 10 years. Find**

   i) Depreciation amount for the 6^{th} year

   ii) Accumulated depreciation throughout 6^{th} year

   iii) Book value at the end of 6^{th} year

If Rate of depreciation = 20% and there is no salvage value by using Matheson method.

Solution:

i) depreciation amount for the 6^{th} year =

$$d_6 = \alpha I (1 - \alpha)^{n-1}$$

$$= 0.2 \times 4000 (1 - 0.2)^{6-1} = \text{Rs. } 262.144$$

ii) Accumulated depreciation throughout 6^{th} year =

$$D_6 = I [1 - (1 - \alpha)^n]$$

$$= 4000 [1 - (1 - 0.26)^6]$$

$$= \text{Rs. } 2951.42$$

iii) Book value at the end of 6^{th} year,

$$Bv_6 = I (1 - \alpha)^n$$

$$= 4000 (1 - 0.2)^6 = \text{Rs. } 1048.576$$

2. **Say you have a property with ten tenants each paying Rs. 10000 per month. You estimate a vacancy loss of 7%. The property has operating expenses of Rs. 45000 per year and a first mortgage payment of Rs. 36,326 per year. In month six, you add a new roof at the cost of Rs. 20000 and take out Rs. 20000 second mortgage to cover the cost of that construction. Your payment on this loan totals Rs. 881 for the remaining six months. What is your property's cash flow before tax (CFBT)?**

   If your tax liability in year one is 5000, then what is your property's cash flow after taxes (CFAT)?

Solution:

Gross Schedule income = 1000 per month

= 120000 per month

Vacancy loss = $0.07 \times 12000 = 8400$

Gross operating income = 120000 – 8400 = 111600

operating expenses = 45000

Gross operating income – operating expenses = 111600 – 45000 = 66600

$\therefore$ Net operating income = Rs. 66600

Cash flow before tax (CFBT) = Net operating income – debt service – capital addition + loan proceed =

= 66600 – 36326 – 20000 + 20000 = Rs. 30274

Cash flow after tax (CFAT) = CFBT – tax liability

= 30274 – 5000

= Rs. 25274

***

8 Inflation & Its Impact on Project Cash Flows

8.1 Concept of inflation

Inflation is the rate at which the general level of prices, good and services is rising, and subsequently purchasing power is falling.

Historically, the general economy has usually fluctuated in such a way as to exhibit inflation, a loss in the purchasing power of money over time. Inflation means that the cost of an item tends to increase over time or dollar amount buys less of an item over time.

Deflation:

Deflation is the opposite of inflation, is that prices usually decreases over time, hence a specific dollar amount gains in purchasing power.

8.2 Measuring inflation:

Generally, inflation is measured in percentage term which is simply obtained by calculating the percentage change in current price index over the previous year. Price indices are developed on the basis of market prices of various goods and services under the study and it is used for measuring inflation.

Average inflation

To account for the effect of varying yearly inflation rates over a period of several years, we can compute a single rate that represents an average inflation rate.

The most commonly used indices are:

1. **Consumer price index (CPI)**

CPI is an inflationary indicator that measures the change in the cost of a fixed basket of products and services.

2. **Wholesale price index (WPI)**
3. **GDP deflator**

Example: Calculate the average inflation rate for a 2-years period. The first year's inflation rate is 5% and the second year's rate is 10% using a base price of Rs. 100.

Solution:

Step 1: To find the price at the end of the second year we use the process of compounding

At the end of first year = 100 (1 + 0.05)

At the end of second year = 100 (1 + 0.05) (1 + 0.1) = 115.5

Step 2: To find the average inflation rate (f), we establish the equivalence relation.

At the end of second year,

Rs. $100 (1 + f)^2 = 115.5$

Solving for f,

$f = 7.47\%$

Money in one period of time t, can be brought to the same value as money in another period of time t_2, by using the relation.

$$\underbrace{\text{Dollars in period } t_2}_{\text{Future dollars}} = (\text{Inflation rate between } t_1 \text{ and } t_2) \times \underbrace{\text{Dollars in period } t_2}_{\text{Today dollars}}$$

8.3 Equivalence calculation under inflation.

To introduce the factor in changes in purchasing power i.e. inflation, we use either

i) Constant dollar analysis

ii) Actual dollar analysis

i) **Actual dollar analysis (An)**

Out of pocket dollars paid at the time of purchasing goods and service.

ii) **Constant dollar analysis (A_n')**

Dollars as if in some base year, used to adjust for the effects of inflation.

- **Conversion from constant to actual cash flow**

  $A_n = A_n' (1 + f)^n$

- **Conversion from actual to constant cash flow**

  $$An' = \frac{An}{(1+f)^n}$$

Different interest rate

i) **Market interest rate (i)/inflation adjusted MARR**

The interest rate quoted by financial institutions that accounts for both earning purchasing power. This is the interest rate that has been adjusted to take inflation into account.

ii) **Inflation-free interest (i')**

This rate is an estimated of the true earning power of the money when the inflation effect has been removed. This rate is commonly known as real interest rate. In the absence of inflation, the market interest rate is the same as the inflation-free rate.

iii) **Inflation rate (f):**

This is measure of the rate of change in the value of money.

Example:

a) **Convert the project's cash flows into the equivalent actual dollars. Inflation rate = 5%**

Period	0	1	2	3	4
Net cash flow in constant dollar	– 450000	200000	220000	220000	230000

Solution:

Period	Constant dollar	Actual dollar
0	– 450000	$-450000\ (1+0.05)^0 = -450000$
1	200000	$200000 \times 1.05^1 = 210000$
2	220000	$220000 \times 1.05^2 = 242550$
3	220000	$220000 \times 1.05^3 = 254677.5$
4	230000	$230000 \times 1.05^4 = 279566.43$

b) The project is expected to generate the following cash flow in actual dollars.

Year	0	1	2	3	4
Actual dollar	– 750000	320000	357000	328000	290000

i) What are the equivalent constant dollars? f = 5%

ii) Compute the present worth of these cash flows in constant dollars at i = 10%.

Solution:

Year	Actual dollar	Cash flow in constant dollar, f = 5%	Cash flow in constant dollar f = 10%
0	– 750000	$-750000 \times 1.05^{-0} = -750000$	$-750000 \times 1.1^{-0} = -750000$
1	320000	$320000 \times 1.05^{-1} = 304761.9$	$304761.9 \times 1.1^{-1} = 277056.27$
2	357000	$357000 \times 1.05^{-2} = 323809.5$	$323809.5 \times 1.1^{-2} = 267611.17$
3	328000	$328000 \times 1.05^{-3} = 283338.73$	$283338.73 \times 1.1^{-3} = 212876.58$
4	290000	$290000 \times 1.05^{-4} = 238583.72$	$238583.72 \times 1.1^{-4} = 162955.88$
		Total PW =	170499.9

Adjusted–discount method

The two step process shown in previous example can be solve in one step by adjusted discount method.

Mathematically,

$$A_n' = \frac{\frac{An}{(1+f)^n}}{(1+i')^n} = \frac{An}{(1+f)^n(1+i')^n}$$

$$A_n' = \frac{An}{(1+f)^n(1+i')^n} \ldots\ldots(1)$$

Also, from market interest rate

$$A_n' = \frac{An}{(1+i)^n} \ldots\ldots\ldots\ldots(2)$$

From (1) & (2)

$$\frac{An}{\{(1+f)\,(1+i')\}^n} = \frac{An}{(1+i)^n}$$

This leads to the following relationship among f, i, I'

$$(1+i) = (1+f)\,(1+i')$$

or, $1 + i = 1 + i' + f \times i' + f$,

$\therefore\ i = i' + i' \times f + f$

Example Compute the equivalent PW of a previous example by adjusted discount method.

Solution: Given, $i' = 0.1$, $f = 0.05$

where f = inflation rate, i' = inflation – free interest

Market interest rate $(i) = i' + i' \times f + f$

$= 0.1 + 0.1 \times 0.05 + 0.05$

$= 0.155$

$= 15.5\%$

Year	Actual dollar	Constant dollar, i = 15.5%
0	–750000	$-750000 \times 1.155^{-0} = -750000$
1	320000	$320000 \times 1.155^{-1} = 277056.27$
2	357000	$357000 \times 1.155^{-2} = 267611.17$

3	328000	$328000 \times 1.155^{-3} = 212876.58$
4	290000	$290000 \times 1.155^{-4} = 162955.88$
	Total PW =	170499.9 (Same result)

8.4 Impact of inflation on economic evaluation

Impact of inflation on economic evaluation are listed below

1. Fixed income groups are hit hard while flexible income groups like businessman, speculators are benefited.
2. Resources are diverted from productive to unproductive sectors during production.
3. During inflation social evils like corruption, gambling, black marketing etc. flourish.
4. It creates uncertainties in the economy.
5. It raises the cost of holding money.
6. It creates socio–political unrests and people do not trust the government.
7. Inflation redistributes d income and wealth in the economy because rich becomes richer and poor becomes poorer.

There are two methods of evaluation

1. Estimate inflation effect by converting all cash flows to money units that have constant purchasing power i.e. constant (real) dollars
2. Estimate cash flows in the amount of money units actually exchanged at the time of each transaction. i.e. actual (future) dollars.

Old Question Solutions

"2069 Bhadra"

1. **Define inflation. List out its effect. If the inflation rate is 5% per year and the market interest rate is 13% per year. What is the implied interest (inflation free) rate in inflationary economy?**

Solution:

Inflation is the rate at which the general level of prices, goods and services is rising and subsequently purchasing power is falling

Effects of inflation: Refer at 8.4

Given,

Market rate (i) = 13%

Inflation rate (f) = 5%

Inflation free rate (i) = ?

We have,

$$i = i' \times f \times i' + f$$

$$0.13 = i' + 0.05i' + 0.05$$

Solving $i' = 0.0761 \approx 7.61\%$

2. **A series of five constant dollar (or real dollar) income (beginning with \$5000 at the end of the first year) are increasing at the rate of 7% per year for five years. Inflation free interest rate is 5% and inflation is 8%. Is it feasible investment if investment cost is \$20000?**

Solution:

Inflation free interest rate (i') = 5%

Inflation (f) = 8%

$$\text{Market rate } (i) = f + f \times i' + i'$$

$$= 0.08 + 0.08 \times 0.05 + 0.05$$

$$= 0.134$$

= 13.4%

Investment = $20000

Income = $ 5000 at the end of 1^{st} year and increasing 7% per year.

Year	Cash flow (constant dollar)
0	−20000
1	5000
2	5000 (1 + 0.07) = 5350
3	$5000\ (1 + 0.07)^2 = 5724.5$
4	$5000\ (1 + 0.07)^3 = 6125.22$
5	$5000\ (1 + 0.07)^4 = 6553.98$

$$PW = -20000 + \frac{5000}{1.134} + \frac{5350}{1.134^2} + \frac{5724.5}{1.134^3} + \frac{6125.22}{1.134^3} + \frac{6553.98}{1.134^5}$$

$$= -321.41$$

Since, NPW < 0, Investment is not economically feasible.

"2070 Magh"

1. **Evaluate the PW of the following project:**

   Initial investment = Rs. 1,00,000

   Annual sales income = Rs. 40,000

   Annual labour cost = Rs. 3,000

   Annual material X = Rs. 2,000

   Annual material Y = Rs. 1,000

   Salvage value = 20% of initial investment

   All are in the constant dollars.

   Inflation rate for sales income, labour cost, material X, material Y and salvage value are 5%, 8%, 0%, 6% and 3% respectively for the project period. Take market interest rate = 20% project life is 4 years.

Solution:

Market interest rate (i) = 20%

Note: when cash flow are in constant amount we should use inflation free rate (i') of interest.

Project life = 4

$i = i' + i' \times f + f$

$i'(1+f) = i - f$

$i' = \frac{i-f}{1+f} = \frac{0.2-f}{1+f}$

S.N.	Cash flow item	i'	Cash flow
1	Investment	–	–100000
2	Sales	(0.2 – 0.05) /(1 + 0.05) = 0.143	40000 (annually)
3	Labour	(0.2 – 0.08) /(1 + 0.08) = 0.11	3000 (annually)
4	Material X	(0.2 – 0) /(1 + 0) = 0.2	2000 (annually)
5	Material Y	(0.2 – 0.06) /(1 + 0.06) = 0.132	1000 (annually)
6	Salvage value	(0.2 – 0.03) /(1 + 0.03) = 0.165	0.2 × 100000 = 20000

$$PW = -1{,}00{,}000 + 40000 \times \frac{1.143^4 - 1}{0.143 \times 1.143^4} - 3{,}000 \times \frac{1.11^4 - 1}{0.11 \times 1.11^4} - 2000 \times \frac{1.2^4 - 1}{0.2 \times 1.2^4} - 1000 \times \frac{1.132^4 - 1}{0.132 \times 1.132^4} + \frac{20000}{1.165^4}$$

$$= 9245.79 > 0$$

"2070 Bhadra"

The annual fuel cost required to operate a small solid waste treatment plant are projected to be Rs. 200000 without considering any future inflation. The best estimate indicates that the annual inflation free interest rate i' will be 6% and the general inflation rate, f, will be 5%, if the plant has the remaining useful life of four years. What is the present equivalent of its fuel costs? Use actual dollar analysis.

Solution:

$i' = 6\%$, $f = 5\%$

$i = i' + i' \times f + f = 0.06 + 0.06 \times 0.05 + 0.05 = 0.113 = 11.3\%$

Using actual dollars analysis.

EOY	Cash flow	Cash flow in actual dollar
1	200000	$200000\ (1 + 0.113)^1 = 222600$
2	200000	$200000\ (1 + 0.113)^2 = 247753.8$
3	200000	$200000\ (1 + 0.113)^3 = 275749.98$
4	200000	$200000\ (1 + 0.113)^4 = 306909.75$
	Total PW =	1053013.5

∴ Total present equivalent of fuel cost = Rs. 1053013.5

"2071 Magh"

1) **First cost = \$ 80000, Sv = 10% of first cost. The general inflation rate = 5%**

EOY	**1**	**2**	**3**	**4**	**5**
Net cash flow in actual dollars	**32000**	**35000**	**33000**	**29000**	**50000**

Evaluate the PW by deflection method, if inflation free interest rate = 10%

Solution:

Salvage value at the end of 5th year = 0.1×80000

$= 8000$

Total cash flow at the end of 5th year = $50000 + 8000$

$= 58000$

Using deflation method,

EOY	Net cash flow in actual dollars	Constant dollars at 5% inflation rate	Constant dollar at 10% inflation free rate
0	–80000	$-80000\ (1+0.05)^{-0} = -80000$	$-80000\ (1+0.1)^{-0} = -80000$
1	32000	$32000\ (1+0.05)^{-1} = 30476.2$	$30476.2\ (1+0.1)^{-1} = 27705.6$
2	35000	$35000\ (1+0.05)^{-2} = 31746$	$31746 \times 1.1^{-2} = 26236.38$
3	33000	$33000\ (1+0.05)^{-3} = 28506.64$	$28506.64 \times 1.1^{-3} = 21417.46$
4	29000	$29000\ (1+0.05)^{-4} = 23858.37$	$23858.37 \times 1.1^{-4} = 16295.58$
5	58000	$58000\ (1+0.05)^{-5} = 45444.52$	$45444.52 \times 1.1^{-5} = 28217.5$
		Total PW	39872.52

$\therefore$ PW = \$ 39872.52

"2071 Bhadra"

Define constant dollar amount and actual dollar amount. Suppose you borrowed Rs. 100000 from a bank to buy a bike and you have promised to pay Rs. 5500 per month for two years. What is the inflation free interest rate you are supposed to pay if average inflation rate is 0.75% per month?

Solution: Theory part see at 8.3

Let i be the market interest rate per month.

Useful life = 2 years = 24 months.

Fig:

Now, PW(i%) = 0

$$-100000 + 5500 \times \frac{[(1+i)^{24} - 1]}{i(1+i)^{24}} = 0$$

$$i = 0.0235 = 2.35\%$$

Average inflation rate (f) = 0.75%

inflation free rate (i') = ?

$$i = i' + f \times i' + f$$

$$i' = \frac{0.0235 - 0.0075}{1 + 0.0075} = 0.0158$$

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$\therefore$ Inflation free interest rate = 0.0158 = 1.58%

"2072 Ashwin"

1. Choose the best project from the following alternatives

Project	Material X	Material Y
First cost	1500000	2000000
Life	7 years	7 years
Salvage value	200000	300000
Annual operation and maintenance cost	300000	250000

Assume an average inflation of 5% for the next five years and interest rate is 15%/years.

Solution:

Inflation rate (r) = 5%

Market interest rate (i) = 15%

Inflation free interest (i') = ?

$$i' = \frac{0.15 - 0.05}{1 + 0.05} = 0.095 = 9.5\% \text{ for next five years and}$$

$$i' = \frac{0.15 - 0}{1 + 0} = 15\% \text{ for 2 years}$$

Assume constant dollar analysis

Note: Inflation is only for 5 years.

PW of X Machine = –15,00000+300000 (P/A, 9.5%,5)+ 300000 (P/F, 5%, 6)

+ 300000 (P/F, 15%, 7) + 200000 (P/F, 15%, 7)

$$= -15{,}00000 + 300000 * \frac{1.095^5 - 1}{1.095^5 * 0.095} + \frac{300000}{1.15^6}$$

$$+\frac{300000}{1.15^7}+\frac{200000}{1.15^7}$$

$$= -30420.56$$

Also,

PW of Y machine

$$= -20,00000 + 250000 * \frac{1.095^6 - 1}{1.095^5 \times 0.095} + \frac{250000}{1.15^6} + \frac{25000}{1.15^7} + \frac{300000}{1.15^7}$$

$$= -725225.53$$

Economically both projects aren't justified

but NPW of X machine > NPW of Y machine

So, choose a machine X.

Extra Solution:

1) **Calculate rate of inflation for each year and average inflation over the years from the following information.**

Year	Costs
0	504
1	538.4
2	577
3	629.5

Solution:

Inflation rate during year 1

$$f_1 = \frac{538.4 - 504}{504} = 6.83\%$$

Similarly, $f_2 = \dfrac{577 - 538.4}{538.4} = 7.71\%$

$$f_3 = \frac{629.5 - 577}{577} = 9.1\%$$

Average inflation rate (f) over the years

$$F = p(1+f)^N$$

$$629.5 = 504(1+f)^3$$

$$1+f = 1.0769$$

$$f = 7.69\%$$

$\therefore$ Average inflation rate (f) = 7.69%

2. **Consider a loan that can be arranged at a nominal interest rate of 10% compounded monthly. If the inflation rate is expected to be 5%. What will be the market interest rate?**

Solution:

If nominal of interest (i_m) and compounding period (m) is given

$$i_{iff} = \left(1+\frac{i_m}{m}\right)^{eff} - 1$$

$$= \left(1+\frac{0.1}{12}\right)^{12} - 1 = 0.1047 = 10.47\%$$

Inflation rate (f) = 5%

Now market rate (i) $= i_{eff} + f \times i_{eff} + f$

$$= 0.1047 + 0.05 \times 0.1047 + 0.05$$

$$= 0.1599 = 15.99\%$$

3. **The projected sales and net cash flows in constant dollar are as follows:**

Period	Unit Sales	Net cash flow in constant $
0	-	- 2,50,000
1	1,000	1,00,000
2	1,100	1,10,000
3	1,200	1,20,000
4	1,300	1,30,000
5	1,200	1,20,000

Calculate PW if inflation free rate = 12%

Solution:

When cash flows are in constant amount we should use inflation free rate of interest

PW (12%) = – 2,50,000 + 1,00,000 (P/A, 12%, 5)
+ 10,000 (P/G,12%, 4) + 20,000 (P/F, 12%, 5)
= $ 1,63,099

4. **Determine the best alternative from following condition by using constant and actual dollars analysis;**

Alternative 1: Development cost will be Rs. 300,000 the first year and will increase at a rate of 5% over the 5 - year period

Alternative 1: Development costs will be a constant Rs. 300,000 per year in terms of today's dollar over the 5 year period.

Assume inflation rate = 3.5% and MARR = 25%

Solution:

The costs for each of the two alternative

Year	Future Rs. started by A	Constant Rs. started by B
1	$300,000\ (1 + 0.05)^0 = 300,0000$	300,000
2	$300,000\ (1 + 0.05)^1 = 315000$	300,000
3	$300,000\ (1 + 0.05)^2 = 330750$	300,000
4	$300,000\ (1 + 0.05)^3 = 347287.5$	300,000
5	$300,000\ (1 + 0.05)^4 = 364651.87$	300,000

Using Constant Dollar Analysis

Converting the actual dollar of alternative 1 to the constant dollar by using deflation factor.

Year	Future Rs. started by A	Constant Rs. started by B
1	$300{,}000\,(1+0.035)^{-1} = 289855$	300,000
2	$315000\,(1+0.035)^{-2} = 294055.87$	300,000
3	$330750\,(1+0.035)^{-3} = 298317.55$	300,000
4	$347287.5\,(1+0.035)^{-4} = 302640.99$	300,000
5	$364651.87\,(1+0.035)^{-5} = 307027.08$	300,000

For constant dollar, calculate real interest rate i.e. inflation free interest rate (i')

$$i = i' + i' \times f + f$$

$$i' = \frac{(i-f)}{(i+f)},\ i = 25\% \text{ and } f = 3.5\%$$

$$\therefore\ i' = \frac{(0.25 - 0.035)}{(1+0.035)}$$

$$= 0.208$$

$$= 20.8\%$$

$$\text{PW of alternative 1} = \frac{289855}{(1+0.208)^1} + \frac{294055.87}{1.208^2} + \frac{298317.55}{1.208^3} + \frac{302640.99}{1.208^4} + \frac{307027.08}{1.208^5} = \text{Rs. } 872163$$

$$\text{PW of alternative 2} = 300{,}000\ (P/A,\ 20.8\%,\ 5)$$

$$= 300{,}000 \times \frac{1.208^5 - 1}{1.208^5 \times 0.248}$$

$$= \text{Rs. } 881617.12$$

Using Actual Dollar Analysis

Year	Future Rs. started by A	Constant Rs. started by B
1	300,000	$300000\ (1+0.035)^1 + = 310500$
2	315,000	$300000\ (1.035)^2 + = 321367.5$
3	330750	$300000°\ (1.035)^3 = 332615.36$
4	347287.5	$300000(1.035)^4 = 344256.9$
5	364651.87	$300000\ (1.035)^5 = 356305.89$

$$\text{PW of alt. 1} = \frac{300{,}000}{(1+0.25)^1} + \frac{315{,}000}{1.25^2} + \frac{330750}{1.25^3} + \frac{347287.5}{1.25^4} + \frac{364651.87}{1.25^5} = \text{Rs. } 872682$$

$$\text{PW of alt. 2} = \frac{310500}{1.25} + \frac{321367.5}{1.25^2} + \frac{332615.36}{1.25^3} + \frac{344256.9}{1.25^4} + \frac{356305.89}{1.25^5} = \text{Rs. } 882136.2$$

Using either Constant dollar or actual dollar, alternative 1 (minimum cost) should be choose.

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