

Construction Economics & Finance

Module 2

Lecture-1

Comparison of alternatives:-

For most of the engineering projects, equipments etc., there are more than one feasible alternative. It is the duty of the project management team (comprising of engineers, designers, project managers etc.) of the client organization to select the best alternative that involves less cost and results more revenue. For this purpose, the economic comparison of the alternatives is made. The different cost elements and other parameters to be considered while making the economic comparison of the alternatives are initial cost, annual operating and maintenance cost, annual income or receipts, expected salvage value, income tax benefit and the useful life. When only one, among the feasible alternatives is selected, the alternatives are said to be mutually exclusive.

As already mentioned in module-1, the cost or expenses are generally known as cash outflows whereas revenue or incomes are generally considered as cash inflows. Thus in the economic comparison of alternatives, cost or expenses are considered as negative cash flows. On the other hand the income or revenues are considered as positive cash flows. From the view point of expenditure incurred and revenue generated, some projects involve initial capital investment i.e. cash outflow at the beginning and show increased income or revenue i.e. cash inflow in the subsequent years. The alternatives having this type of cash flow are known as investment alternatives. So while comparing the mutually exclusive investment alternatives, the alternative showing maximum positive cash flow is generally selected. In this case, the investment is made at the beginning to gain profit at the future period of time. Example for such type alternatives includes purchase of a dozer by a construction firm. The construction firm will have different feasible alternatives for the dozer with each alternative having its own initial investment, annual operating and maintenance cost, annual income depending upon the production capacity, useful life, salvage values etc. Thus the alternative which will yield more economic benefit will be

selected by the construction firm. There are some other projects which involve only costs or expenses throughout the useful life except the salvage value if any, at the end of the useful life. The alternatives having this type of cash flows are known as cost alternatives. Thus while comparing mutually exclusive cost alternatives, the alternative showing minimum negative cash flow is generally selected. Example for such type alternatives includes construction of a government funded national highway stretch between two regions. For this project there will be different feasible alternatives depending upon length of the stretch, type of pavement, related environmental, social and regulatory aspects etc. Each alternative will have its initial cost of construction, annual repair and maintenance cost and some major repair cost if any, at some future point of time. The alternative that will exhibit lowest cost will be selected for the construction of the highway stretch.

The differences in different parameters namely initial capital investment, annual operation cost, annually generated revenue, expected salvage value, useful life, magnitude of output and its quality, performance and operational characteristics etc. may exist among the mutually exclusive alternatives. Thus the economic analysis of the mutually exclusive alternatives is generally carried out on the similar or equivalent basis since each of the feasible alternatives will meet the desired requirements of the project, if selected.

The economic comparison of mutually exclusive alternatives can be carried out by different equivalent worth methods namely present worth method, future worth method and annual worth method. In these methods all the cash flows i.e. cash outflows and cash inflows are converted into equivalent present worth, future worth or annual worth considering the time value of money at a given interest rate per interest period.

Comparison of alternatives by present worth method:

In the present worth method for comparison of mutually exclusive alternatives, the future amounts i.e. expenditures and incomes occurring at future periods of time are converted into equivalent present worth values at a certain rate of interest per interest period and are added to present worth occurring at '0' time. The converted equivalent present worth values are always less than the respective future amounts since the rate of interest is normally greater than zero. The cash flow of the mutually exclusive alternatives may consist of future expenditures and incomes in different forms namely randomly placed single amounts, uniform amount series commencing from end of year 1, randomly placed uniform amount series i.e. commencing at time period other than end of year 1, positive and negative uniform gradient series starting either from end of year 1 or at different time periods and geometric gradient series etc. The different compound interest factors namely single payment present worth factor, uniform series present worth factor and present worth factors for arithmetic and geometric gradient series etc. will be used to convert the respective future amounts to the equivalent present worth values for different alternatives.

The methodology for the comparison of mutually exclusive alternatives by the present worth method depends upon the magnitude of useful lives of the alternatives. There are two cases; a) the useful lives of alternatives are equal and b) the useful lives of alternatives are not equal. The alternatives having equal useful lives are designated as equal life span alternatives whereas the alternatives having unequal life spans are referred as different life span alternatives.

a) Equal life span alternatives

The comparison of mutually exclusive alternatives having equal life spans by present worth method is comparatively simpler than those having different life spans. In case of equal life span mutually exclusive alternatives, the future amounts as already stated are converted into the equivalent present worth values and are added to the present worth occurring at time zero. Then the alternative that exhibits maximum positive equivalent present worth or minimum negative equivalent present worth is selected from the considered feasible alternatives.

a) Different life span alternatives

In case of mutually exclusive alternatives, those have different life spans, the comparison is generally made over the same number of years i.e. a common study period. This is because; the comparison of the mutually exclusive alternatives over same period of time is required for unbiased economic evaluation of the alternatives. If the comparison of the alternatives is not made over the same life span, then the cost alternative having shorter life span will result in lower equivalent present worth i.e. lower cost than the cost alternative having longer life span. Because in this case, the cost of the short span alternative is considered only for a shorter period of time, even though this alternative may not be economical. In case of mutually exclusive investment alternatives, the alternative with longer life span will result in higher equivalent present worth i.e. higher positive equivalent worth, as the costs, revenues, savings through reduced costs is considered over a longer period of time than the alternative with shorter life span. Thus in order to minimize the effect of such kind of discrepancy on the selection of best alternative from the considered feasible alternatives, the comparison is made over the same life span.

The two approaches used for economic comparison of different life span alternatives are as follows;

- i) Comparison of mutually exclusive alternatives over a time period that is equal to least common multiple (LCM) of the individual life spans
- ii) Comparison of mutually exclusive alternatives over a study period which is not necessarily equal to the life span of any of the alternatives.

In the first approach the comparison is made over a time period equal to the least common multiple of the life spans of mutually exclusive alternatives. The cash flow of the alternatives i.e. cash flow of the first cycle is repeated and the number of repetitions depends upon the value of least common multiple of life spans between the mutually exclusive alternatives. It may be noted here that the cash flow i.e. all the costs and revenues of the alternatives in the successive cycle will be exactly same as that in the first cycle. For example if there are two alternatives with useful lives of 4 years and 5 years.

Then the alternatives will be compared over a period of 20 years (least common multiple of life spans) at the given rate of interest per year. Thus the cash flow of the alternative having the life span of 4 years will be repeated 5 times including the first cycle whereas the cash flow of the alternative with life span of 5 years will be repeated 4 times including the first cycle. After that the most economical alternative will be selected. Taking another example, there are two alternatives with life spans of 5 years and 10 years. In this case the alternatives will be compared over a period of 10 years (LCM). Thus the alternative with life span of 5 years will be analyzed for 2 cycles whereas the alternative with 10 year life span will be analyzed for one cycle only at the given rate of interest per year.

In the second approach, a study period is selected over which the economic comparison of mutually exclusive alternatives is carried out. The length of the study period will depend on the overall benefit of the project i.e. it may be shorter or longer (as compared to useful lives of the individual alternatives) depending upon the short-term or long-term benefits as desired for the project. Thus the cash flows of the alternatives occurring during the study period are only considered for the economic comparison. However if any alternative possesses salvage value at the end of its useful life and that occurs after the study period, then its equivalent value must be included in the economic analysis. The values of equivalent present worth of the mutually exclusive alternatives are calculated over the selected study period and the alternative showing maximum positive equivalent present worth or minimum negative equivalent present worth is selected.

Lecture-2

Comparison by present worth method:-

Now some examples showing the use of present worth method for comparison of mutually exclusive alternatives are presented. First the comparison of equal life span mutually exclusive alternatives by present worth method will be illustrated followed by comparison of different life span alternatives. The following examples are formulated only to demonstrate the use of different methods for comparison of alternatives. The values of different cost and incomes mentioned in the examples are not the actual ones pertaining to a particular item. In addition it may also be noted here that the cash flow diagrams have been drawn not to the scale. These are merely graphical representations.

Example -1

There are two alternatives for purchasing a concrete mixer. Both the alternatives have same useful life. The cash flow details of alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.3,00,000, Annual operating and maintenance cost = Rs.20,000, Expected salvage value = Rs.1,25,000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.2,00,000, Annual operating and maintenance cost = Rs.35,000, Expected salvage value = Rs.70,000, Useful life = 5 years.

Using present worth method, find out which alternative should be selected, if the rate of interest is 10% per year.

Solution:

Since both alternatives have the same life span i.e. 5years, the present worth of the alternatives will be compared over a period of 5 years. The cash flow diagram of Alternative-1 is shown in Fig. 2.1.

As already mentioned Module-1, the cash outflows i.e. costs or expenditures are represented by vertically downward arrows whereas the cash inflows i.e. revenue or income are represented by vertically upward arrows. The same convention is adopted here.

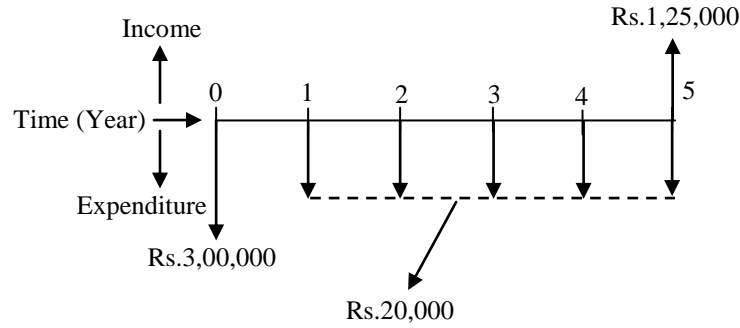


Fig. 2.1 Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 i.e. PW_I is calculated as follows;

The initial cost, $P = \text{Rs.}3,00,000$ (cash outflow),

Annual operating and maintenance cost, $A = \text{Rs.}20,000$ (cash outflow),

Salvage value, $F = \text{Rs.}1,25,000$ (cash inflow).

$$PW_I = -3,00,000 - 20,000(P/A, i, n) + 1,25,000(P/F, i, n)$$

$$PW_I = -3,00,000 - 20,000(P/A, 10\%, 5) + 1,25,000(P/F, 10\%, 5)$$

Now putting the mathematical expressions of different compound interest factors (as mentioned in Module-1) in the above expression for PW_I (in Rs.) results in the following;

$$PW_I = -3,00,000 - 20,000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 1,25,000 \times \frac{1}{(1+i)^n}$$

$$PW_I = -3,00,000 - 20,000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 1,25,000 \times \frac{1}{(1+0.1)^5}$$

$$PW_I = -3,00,000 - 20,000 \times 3.7908 + 1,25,000 \times 0.6209$$

$$PW_I = -3,00,000 - 75,816 + 77,613$$

$$PW_I = - \text{Rs.}2,98,203$$

The cash flow diagram of Alternative-2 is shown in Fig. 2.2.

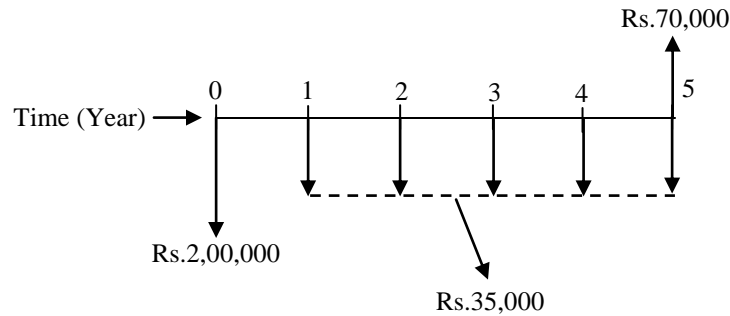


Fig. 2.2 Cash flow diagram of Alternative-2

Now the equivalent present worth of Alternative-2 i.e. PW_2 (in Rs.) is calculated as follows;

The initial cost, $P = \text{Rs.}2,00,000$ (cash outflow),

Annual operating and maintenance cost, $A = \text{Rs.}35,000$ (cash outflow),

Salvage value, $F = \text{Rs.}70,000$ (cash inflow).

$$PW_2 = -2,00,000 - 35,000(P/A, i, n) + 70,000(P/F, i, n)$$

$$PW_2 = -2,00,000 - 35,000(P/A, 10\%, 5) + 70,000(P/F, 10\%, 5)$$

$$PW_2 = -2,00,000 - 35,000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 70,000 \times \frac{1}{(1+i)^n}$$

$$PW_2 = -2,00,000 - 35,000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 70,000 \times \frac{1}{(1+0.1)^5}$$

$$PW_2 = -2,00,000 - 35,000 \times 3.7908 + 70,000 \times 0.6209$$

$$PW_2 = -2,00,000 - 1,32,678 + 43,463$$

$$PW_2 = - \text{Rs.}2,89,215$$

Comparing the equivalent present worth of both the alternatives, it is observed that Alternative-2 will be selected as it shows lower negative equivalent present worth compared to Alternative-1 at the interest rate of 10% per year.

The equivalent present worth of both the alternatives can also be calculated by using the values of compound interest factors from interest tables. The equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows;

$$PW_1 = -3,00,000 - 20,000(P/A, i, n) + 1,25,000(P/F, i, n)$$

$$PW_1 = - 3,00,000 - 20,000(P/A, 10\%, 5) + 1,25,000(P/F, 10\%, 5)$$

The values of compound interest factors i.e. $(P/A, i, n)$ and $(P/F, i, n)$ can be obtained from the interest tables (discrete compounding) available in texts cited in the list of references [1, 7, 14]. Now referring to the interest table for 10% interest rate, the values of compound interest factors i.e. $(P/A, 10\%, 5)$ and $(P/F, 10\%, 5)$ at interest rate (' i ') of 10% and for interest period (' n ') of 5 years are obtained at the intersection of these factors and interest period ' n ' equal to 5 i.e. the values are obtained from P/A column and P/F column at ' n ' equal to 5 from the interest table (discrete compounding) corresponding to 10% interest rate. The obtained values of $(P/A, 10\%, 5)$ and $(P/F, 10\%, 5)$ are 3.7908 and 0.6209 respectively (same as those obtained using mathematical expressions of these factors).

Now putting the values of compound interest factors in the above expression, the equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows;

$$PW_1 = -3,00,000 - 20,000 \times 3.7908 + 1,25,000 \times 0.6209$$

$$PW_1 = - 3,00,000 - 75,816 + 77,613$$

$$PW_1 = - \text{Rs.} 2,98,203$$

Now the calculation of equivalent present worth of Alternative-2 i.e. PW_2 (in Rs.) is presented below.

$$PW_2 = - 2,00,000 - 35,000(P/A, i, n) + 70,000(P/F, i, n)$$

$$PW_2 = - 2,00,000 - 35,000(P/A, 10\%, 5) + 70,000(P/F, 10\%, 5)$$

Now putting the values of compound interest factors in the above expression (same as above) the equivalent present worth of Alternative-2 i.e. PW_2 is calculated as follows;

$$PW_2 = -2,00,000 - 35,000 \times 3.7908 + 70,000 \times 0.6209$$

$$PW_2 = - 2,00,000 - 1,32,678 + 43,463$$

$$PW_2 = - \text{Rs.} 2,89,215$$

It may be noted that in the above example only cost components and the salvage value of the alternatives were considered for comparison. In the next example, same problem as mentioned in Example-1 will be discussed by taking into account the annual revenues of the alternatives.

Example -2

Alternative-1: Initial purchase cost = Rs.300000, Annual operating and maintenance cost = Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost = Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

The annual revenue to be generated from production of concrete (by concrete mixer) from Alternative-1 and Alternative-2 are Rs.50000 and Rs.45000 respectively. Compute the equivalent present worth of the alternatives at the same rate of interest as in Example-1 i.e. 10% per year and find out the economical alternative.

Solution:

The cash flow diagram of Alternative-1 is shown in Fig. 2.3.

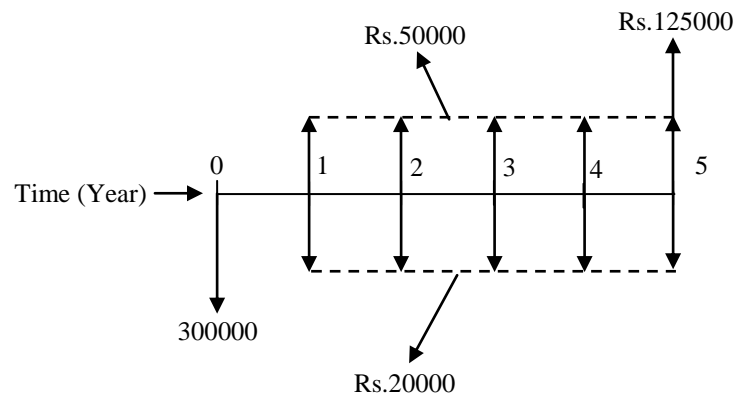


Fig. 2.3 Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 is calculated as follows;

$$PW_1 = -300000 - 20000(P/A, i, n) + 50000(P/A, i, n) + 125000(P/F, i, n)$$

$$PW_1 = -300000 - 20000(P/A, 10\%, 5) + 50000(P/A, 10\%, 5) + 125000(P/F, 10\%, 5)$$

$$PW_1 = -300000 + (50000 - 20000)(P/A, 10\%, 5) + 125000(P/F, 10\%, 5)$$

$$PW_1 = -300000 + 30000(P/A, 10\%, 5) + 125000(P/F, 10\%, 5)$$

$$PW_1 = -300000 + 30000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 125000 \times \frac{1}{(1+i)^n}$$

$$PW_1 = -300000 + 30000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 125000 \times \frac{1}{(1+0.1)^5}$$

$$PW_1 = -300000 + 30000 \times 3.7908 + 125000 \times 0.6209$$

$$PW_1 = -300000 + 113724 + 77613$$

$$PW_1 = - \text{Rs.}108663$$

The cash flow diagram of Alternative-2 is shown in Fig. 2.4.

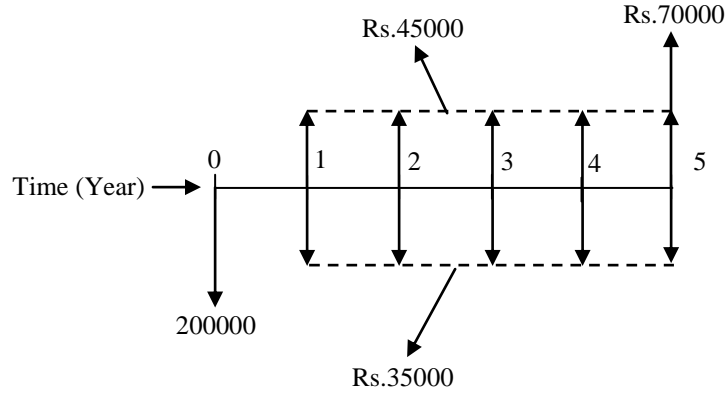


Fig. 2.4 Cash flow diagram of Alternative-2

Now the equivalent present worth of Alternative-2 i.e. PW_2 (in Rs.) is calculated as follows;

$$PW_2 = - 200000 - 35000(P/A, i, n) + 45000(P/A, i, n) + 70000(P/F, i, n)$$

$$PW_2 = - 200000 - 35000(P/A, 10\%, 5) + 45000(P/A, 10\%, 5) + 70000(P/F, 10\%, 5)$$

$$PW_2 = - 200000 + (45000 - 35000) (P/A, 10\%, 5) + 70000(P/F, 10\%, 5)$$

$$PW_2 = - 200000 + 10000(P/A, 10\%, 5) + 70000(P/F, 10\%, 5)$$

$$PW_2 = - 200000 + 10000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 70000 \times \frac{1}{(1+i)^n}$$

$$PW_2 = - 200000 + 10000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 70000 \times \frac{1}{(1+0.1)^5}$$

$$PW_2 = - 200000 + 10000 \times 3.7908 + 70000 \times 0.6209$$

$$PW_2 = - 200000 + 37908 + 43463$$

$$PW_2 = - \text{Rs.}118629$$

Comparing the equivalent present worth of the both the alternatives, it is observed that Alternative-1 will be selected as it shows lower cost compared to Alternative-2. The annual revenue to be generated by the alternatives made the difference as compared to the outcome obtained in Example-1.

When there are more than two alternatives for the selection of the best economical alternative by present worth method, the same procedure as mentioned earlier for the case of two alternatives is followed and illustrated in the next example.

Example -3

A construction contractor has three options to purchase a dump truck for transportation and dumping of soil at a construction site. All the alternatives have the same useful life. The cash flow details of all the alternatives are provided as follows;

Option-1: Initial purchase price = Rs.2500000, Annual operating cost Rs.45000 at the end of 1st year and increasing by Rs.3000 in the subsequent years till the end of useful life, Annual income = Rs.120000, Salvage value = Rs.550000, Useful life = 10 years.

Option-2: Initial purchase price = Rs.3000000, Annual operating cost = Rs.30000, Annual income Rs.150000 for first three years and increasing by Rs.5000 in the subsequent years till the end of useful life, Salvage value = Rs.800000, Useful life = 10 years.

Option-3: Initial purchase price = Rs.2700000, Annual operating cost Rs.35000 for first 5 years and increasing by Rs.2000 in the successive years till the end of useful life, Annual income = Rs.140000, Expected salvage value = Rs.650000, Useful life = 10 years.

Using present worth method, find out which alternative should be selected, if the rate of interest is 8% per year.

Solution:

The cash flow diagram of Option-1 is shown in Fig. 2.5.

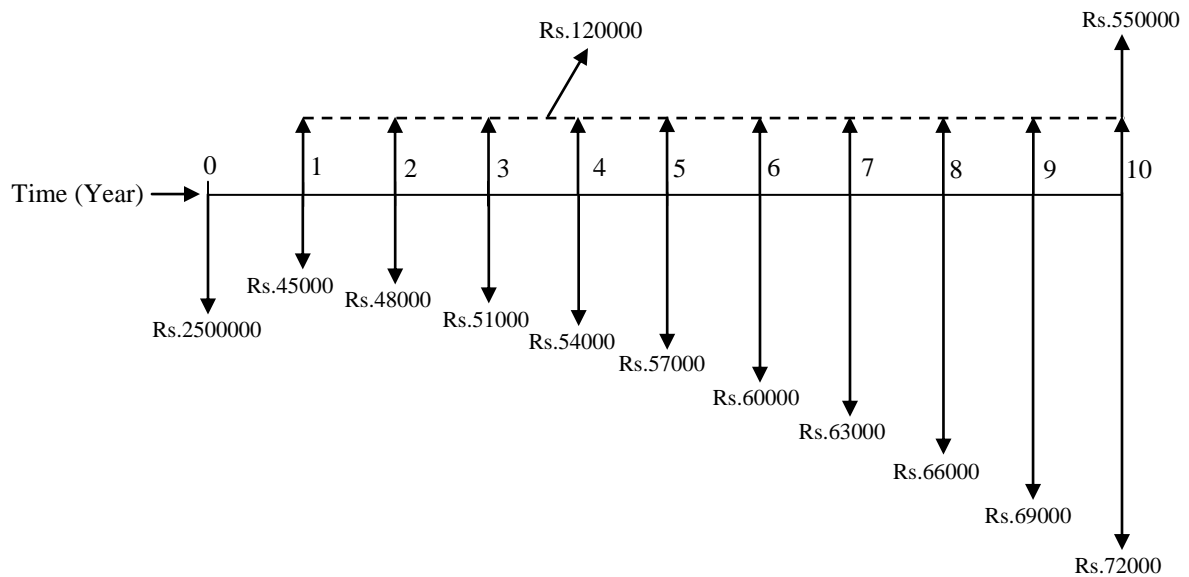


Fig. 2.5 Cash flow diagram of Option-1

For Option-1, the annual operating cost is in the form of a positive uniform gradient series with gradient starting from end of year '2'. The operating cost at the end of different years can be split into the uniform base amount of Rs.45000 and the gradient amount in multiples of Rs.3000 as shown in Fig. 2.6.

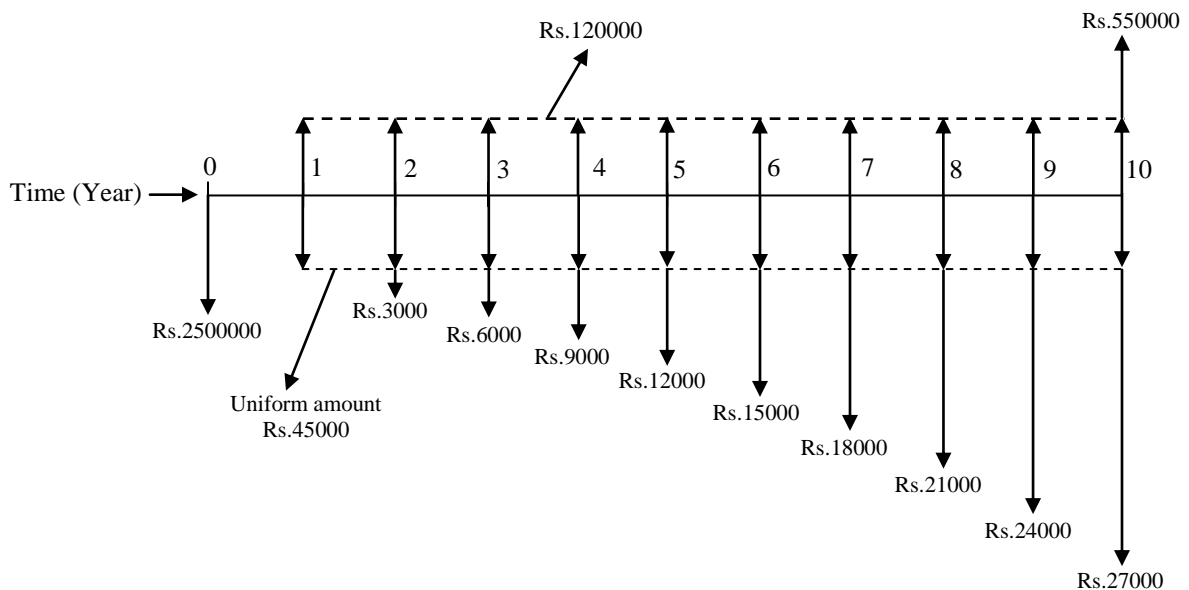


Fig. 2.6 Cash flow diagram of Option-1

with annual operating cost split into uniform base amount and gradient amount

The present worth of the uniform gradient series will be located at the beginning i.e. in year '0' i.e. 2 years before the commencement of the uniform gradient.

Now the equivalent present worth (in Rs.) of Option-1 is calculated as follows;

$$PW_I = - 2500000 - 45000(P/A, i, n) - 3000(P/G, i, n) + 120000(P/A, i, n) + 550000(P/F, i, n)$$

$$PW_I = - 2500000 - 45000(P/A, 8\%, 10) - 3000(P/G, 8\%, 10) + 120000(P/A, 8\%, 10) + 550000(P/F, 8\%, 10)$$

$$PW_I = - 2500000 + (120000 - 45000) (P/A, 8\%, 10) - 3000(P/G, 8\%, 10) + 550000(P/F, 8\%, 10)$$

Now putting the values of different compound interest factors (the expressions in terms of 'i' and 'n' already stated in Module-1) in the above expression for PW_I results in the following;

$$PW_I = - 2500000 + 75000 \times 6.7101 - 3000 \times 25.9768 + 550000 \times 0.4632$$

$$PW_I = - 2500000 + 503258 - 77930 + 254760$$

$$PW_I = - \text{Rs.1819912}$$

The cash flow diagram of Option-2 is shown in Fig. 2.7.

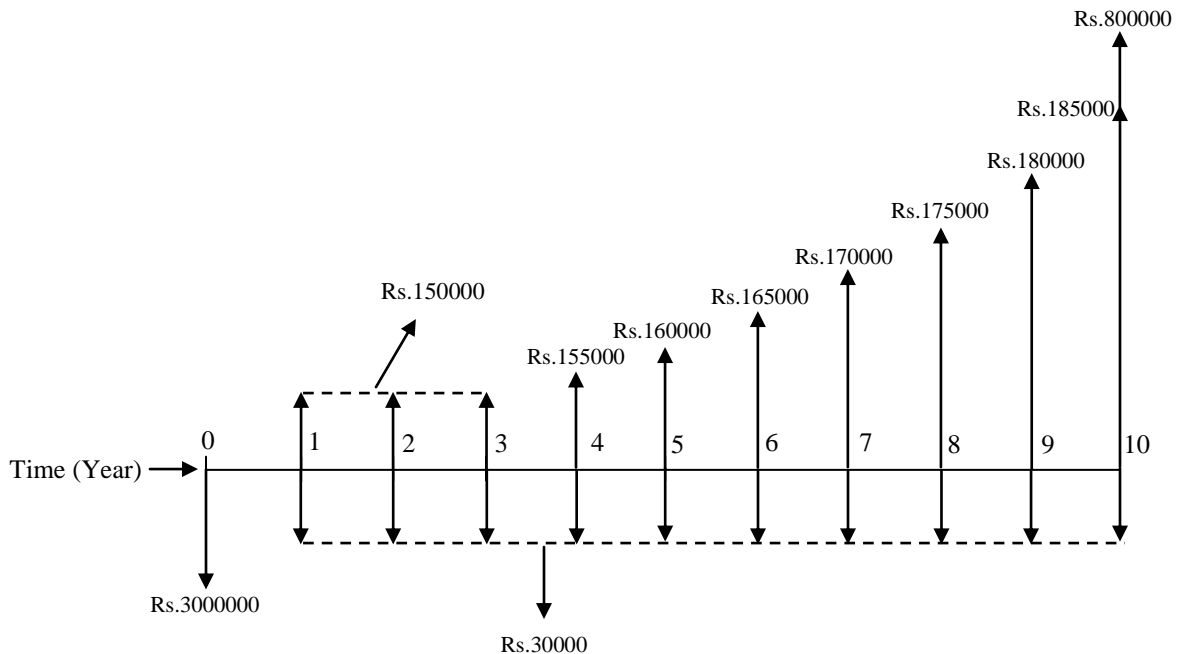


Fig. 2.7 Cash flow diagram of Option-2

For Option-2, the annual income is in the form of a positive uniform gradient series with gradient starting from end of year '4'. The annual income can be split into the uniform base amount of Rs.150000 and the gradient amount in multiples of Rs.5000 starting from end of year '4' and is shown in Fig. 2.8.

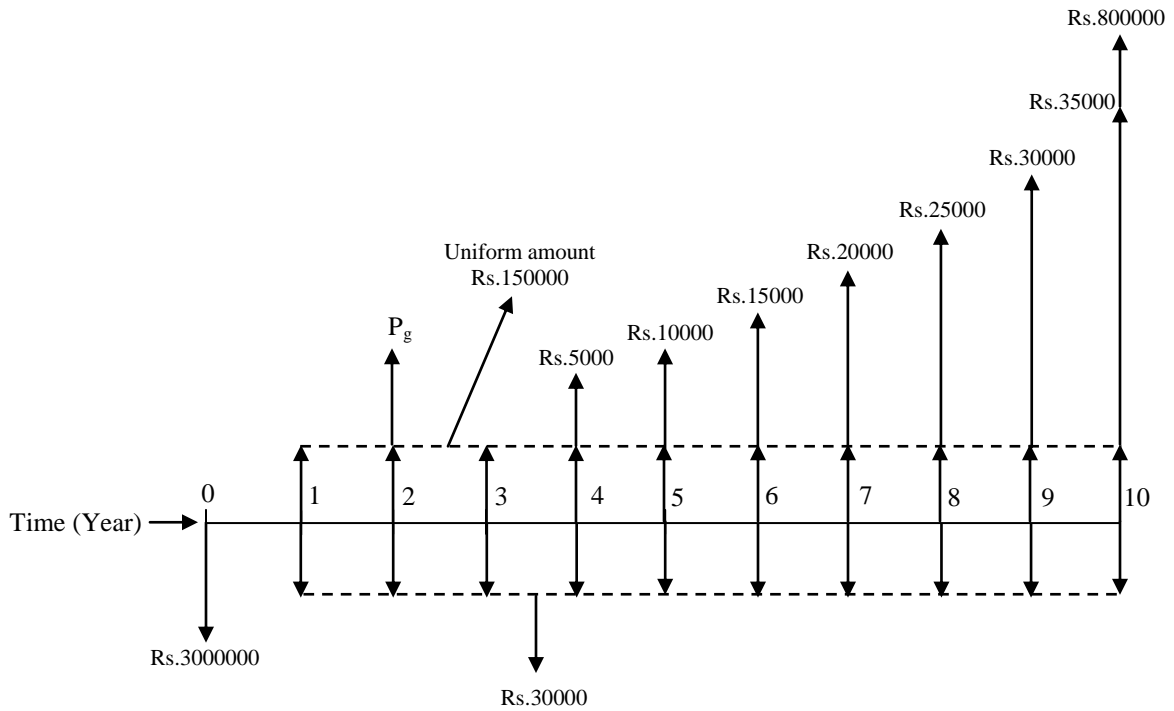


Fig. 2.8 Cash flow diagram of Option-2 with annual income split into uniform base amount and gradient amount

The equivalent present worth of the gradient series (of the annual income) starting from end of year '4' will be located at the end of year '2' i.e. 2 years before the start of the gradient. Further the present worth of this amount at beginning i.e. at time '0' will be obtained by multiplying the equivalent present worth ' P_g ' (shown in Fig. 2.8) at the end of year '2' (which is a future amount) with the single payment present worth factor (P/F , i , n).

Now the equivalent present worth (in Rs.) of Option-2 is determined as follows;

$$PW_2 = - 3000000 - 30000(P/A, 8\%, 10) + 150000(P/A, 8\%, 10) + P_g (P/F, 8\%, 2) + 800000(P/F, 8\%, 10)$$

Now in the above expression, P_g will be replaced by $G (P/G, i, n)$ i.e. $5000(P/G, 8\%, 8)$.

$$PW_2 = - 3000000 - 30000(P/A, 8\%, 10) + 150000(P/A, 8\%, 10) + 5000(P/G, 8\%, 8) (P/F, 8\%, 2) + 800000(P/F, 8\%, 10)$$

$$PW_2 = - 3000000 + (150000 - 30000) (P/A, 8\%, 10) + 5000(P/G, 8\%, 8) (P/F, 8\%, 2) + 800000(P/F, 8\%, 10)$$

Now putting the values of different compound interest factors in the above expression for PW_2 results in the following;

$$PW_2 = - 3000000 + 120000 \times 6.7101 + 5000 \times 17.8061 \times 0.8573 + 800000 \times 0.4632$$

$$PW_2 = - 3000000 + 805212 + 76326 + 370560$$

$$PW_2 = - \text{Rs.1747902}$$

The cash flow diagram of Option-3 is shown in Fig. 2.9.

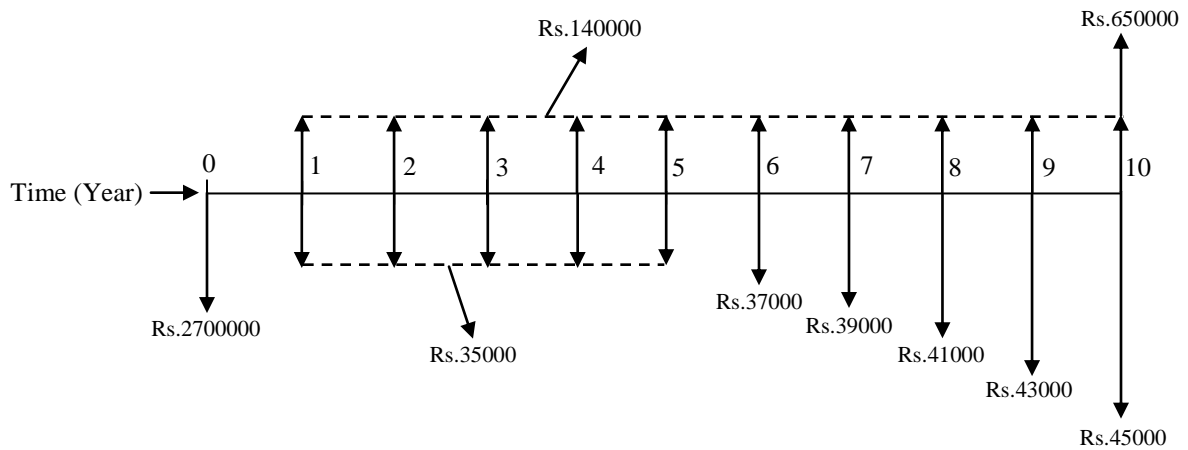
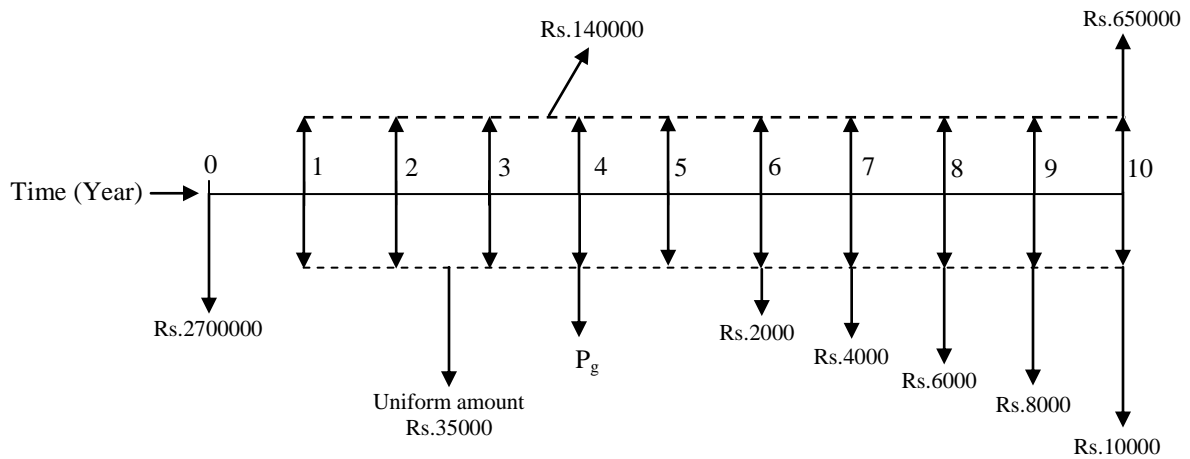


Fig. 2.9 Cash flow diagram of Option-3

For Option-3, the annual operating cost is in the form of a positive uniform gradient series with gradient starting from end of year '6'. The annual operating cost can thus be split into the uniform base amount of Rs.35000 and the gradient amount in multiples of Rs.2000 starting from end of year '6' (shown in Fig. 2.10).

The equivalent present worth of the gradient series for the annual operating cost starting from end of year '6' will be located at the end of year '4'. Further the present worth of this amount at time '0' will be determined by multiplying the equivalent present worth ' P_g ' (shown in Fig. 2.10) at the end of year '4' with the single payment present worth factor $(P/F, i, n)$.



**Fig. 2.10 Cash flow diagram of Option-3 with
annual operating cost split into uniform base amount and gradient amount**

The equivalent present worth (in Rs.) of Option-3 is obtained as follows;

$$PW_3 = - 2700000 - 35000(P/A, 8\%, 10) - P_g (P/F, 8\%, 4) + 140000(P/A, 8\%, 10) + 650000(P/F, 8\%, 10)$$

Now in the above expression, P_g will be replaced by $G (P/G, i, n)$ i.e. $2000(P/G, 8\%, 6)$.

$$PW_3 = - 2700000 - 35000(P/A, 8\%, 10) - 2000(P/G, 8\%, 6) (P/F, 8\%, 4) + 140000(P/A, 8\%, 10) + 650000(P/F, 8\%, 10)$$

$$PW_3 = - 2700000 + (140000 - 35000) (P/A, 8\%, 10) - 2000(P/G, 8\%, 6) (P/F, 8\%, 4) + 650000(P/F, 8\%, 10)$$

Now putting the values of different compound interest factors in the above expression, the value of PW_3 is given by;

$$PW_3 = - 2700000 + 105000 \times 6.7101 - 2000 \times 10.5233 \times 0.7350 + 650000 \times 0.4632$$

$$PW_3 = - 2700000 + 704561 - 15469 + 301080$$

$$PW_3 = - \text{Rs.1709828}$$

From the comparison of equivalent present worth of all the three mutually exclusive alternatives, it is observed that Option-3 shows lowest negative equivalent present worth as compared to other options. Thus Option-3 will be selected for the purchase of the dump truck.

Lecture-3

Comparison by present worth method:-

After the illustration of comparison of equal life span mutually exclusive alternatives, now some examples illustrating the use of present worth method for comparison of different life span mutually exclusive alternatives are presented.

Example -4

A material testing laboratory has two alternatives for purchasing a compression testing machine which will be used for determining the compressive strength of different construction materials. The alternatives are from two different manufacturing companies. The cash flow details of the alternatives are as follows;

Alternative-1: Initial purchase price = Rs.1000000, Annual operating cost = Rs.10000, Expected annual income to be generated from testing of different construction materials = Rs.175000, Expected salvage value = Rs.200000, Useful life = 10 years.

Alternative-2: Initial purchase price = Rs.700000, Annual operating cost = Rs.15000, Expected annual income to be generated from testing of different construction materials = Rs.165000, Expected salvage value = Rs.250000, Useful life = 5 years.

Using present worth method, find out the most economical alternative at the interest rate of 10% per year.

Solution:

The alternatives have different life spans i.e. 10 years and 5 years. Thus the comparison will be made over a time period equal to the least common multiple of the life spans of the alternatives. In this case the least common multiple of the life spans is 10 years. Thus the cash flow of Alternative-1 will be analyzed for one cycle (duration of 10 years) whereas the cash flow of Alternative-2 will be analyzed for two cycles (duration of 5 years for each cycle). The cash flow of the Alternative-2 for the second cycle will be exactly same as that in the first cycle.

The cash flow diagram of Alternative-1 is shown in Fig. 2.11.

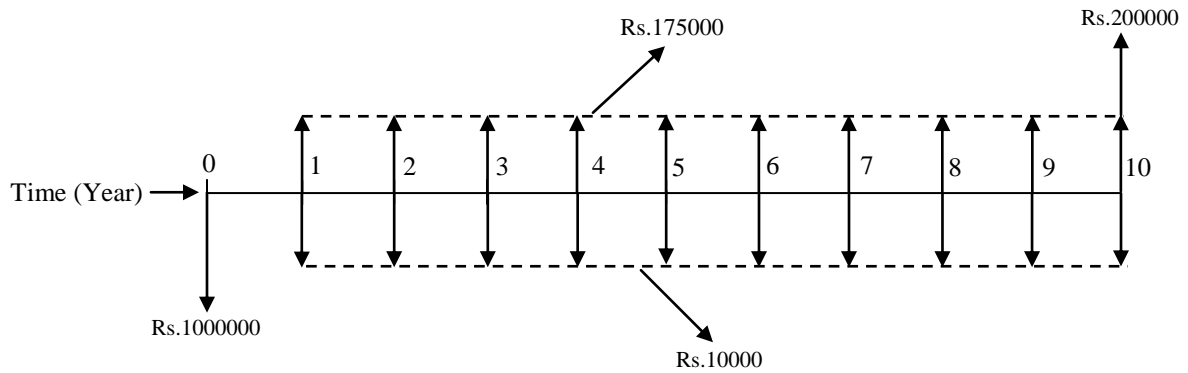


Fig. 2.11 Cash flow diagram of Alternative-1

The equivalent present worth PW_I (in Rs.) of Alternative-1 is calculated as follows;

$$PW_I = -1000000 - 10000(P/A, i, n) + 175000(P/A, i, n) + 200000(P/F, i, n)$$

$$PW_I = -1000000 - 10000(P/A, 10\%, 10) + 175000(P/A, 10\%, 10) + 200000(P/F, 10\%, 10)$$

$$PW_I = -1000000 + (175000 - 10000) (P/A, 10\%, 10) + 200000(P/F, 10\%, 10)$$

Putting the values of different compound interest factors in the above expression for PW_I ;

$$PW_I = -1000000 + 165000 \times 6.1446 + 200000 \times 0.3855$$

$$PW_I = -1000000 + 1013859 + 77100$$

$$PW_I = \text{Rs.90959}$$

The cash flow diagram of Alternative-2 is shown in Fig. 2.12. As the least common multiple of the life spans of the alternatives is 10 years, the cash flow of Alternative-2 is shown for two cycles with each cycle of duration 5 years.

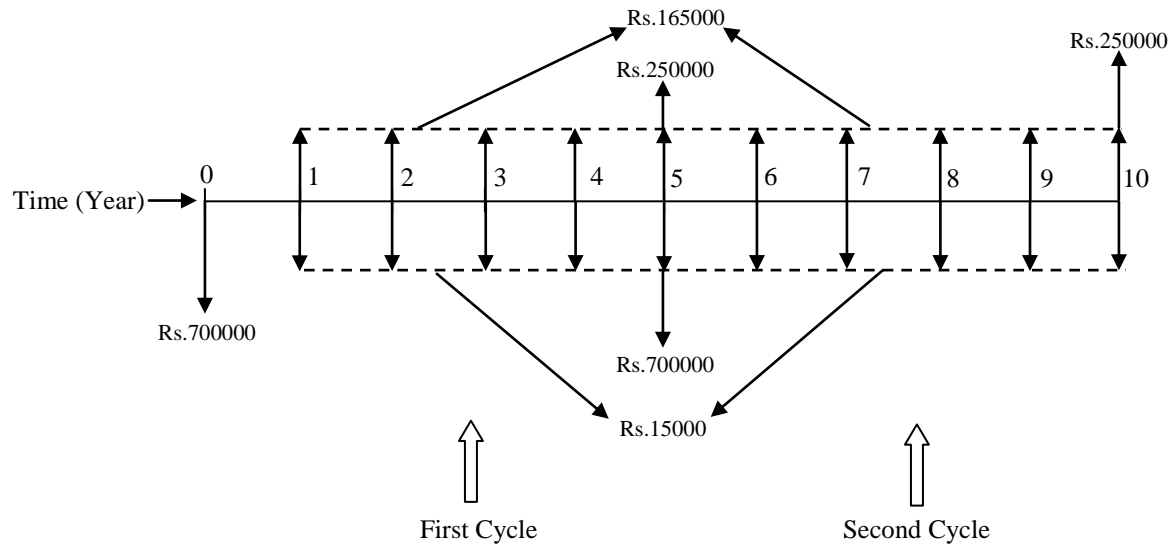


Fig. 2.12 Cash flow diagram of Alternative-2 for two cycles

In the cash flow diagram of Alternative-2, the initial purchase price of Rs.700000 is again located at the end of year '5' i.e. at the end of first cycle or the beginning of the second cycle. In addition the annual operating cost and the annual income are also repeated in the second cycle from end of year '6' till end of year '10'. Further the salvage value of Rs.250000 is also located at end of year '10' i.e. at the end of second cycle.

The equivalent present worth PW_2 (in Rs.) of Alternative-2 is determined as follows;

$$PW_2 = -700000 - 15000(P/A, 10\%, 10) + 165000(P/A, 10\%, 10) + 250000(P/F, 10\%, 5) - 700000(P/F, 10\%, 5) + 250000(P/F, 10\%, 10)$$

$$PW_2 = -700000 + (165000 - 15000)(P/A, 10\%, 10) - (700000 - 250000)(P/F, 10\%, 5) + 250000(P/F, 10\%, 10)$$

Putting the values of different compound interest factors in the above expression for PW_2 results in the following;

$$PW_2 = -700000 + 150000 \times 6.1446 - 450000 \times 0.6209 + 250000 \times 0.3855$$

$$PW_2 = -700000 + 921690 - 279405 + 96375$$

$$PW_2 = \text{Rs.}38660$$

Thus from the comparison of equivalent present worth of the alternatives, it is evident that Alternative-1 will be selected for purchase of the compression testing machine as it shows the higher positive equivalent present worth.

In the following example, the comparison of different life span mutually exclusive alternatives having expenditure or income in the form of gradient series by present worth method is illustrated.

Example -5

A construction firm has decided to purchase a dozer to be employed at a construction site. Two different companies manufacture the dozer that will fulfill the functional requirement of the construction firm. The construction firm will purchase the most economical one from one of these companies. The alternatives have different useful lives. The cash flow details of both alternatives are presented as follows;

Company-A Dozer: Initial purchase cost = Rs.3050000, Annual operating cost Rs.40000 at end of 1st year and increasing by Rs.2000 in the subsequent years till the end of useful life, Annual income = Rs.560000, Expected salvage value = Rs.1050000, Useful life = 6 years.

Company-B Dozer: Initial purchase cost = Rs.4000000, Annual operating cost = Rs.55000, Annual revenue to be generated Rs.600000 at the end of 1st year and increasing by Rs.5000 in the subsequent years till the end of useful life, Expected salvage value = Rs.1000000, Useful life = 12 years.

Using present worth method, find out the most economical alternative at the interest rate of 7% per year.

Solution:

Since the alternatives have different life spans i.e. 6 and 12 years, the comparison will be made over a time period equal to the least common multiple of the life spans of the alternatives i.e. 12 years. The cash flow of Company-A Dozer will be analyzed for two cycles i.e. duration of 6 years for each cycle. The cash flow of Company-B Dozer will be analyzed for one cycle i.e. duration of 12 years.

The cash flow diagram of Company-A Dozer is shown in Fig. 2.13. Since the least common multiple of the life spans of the alternatives is 12 years, the cash flow is shown for two cycles.

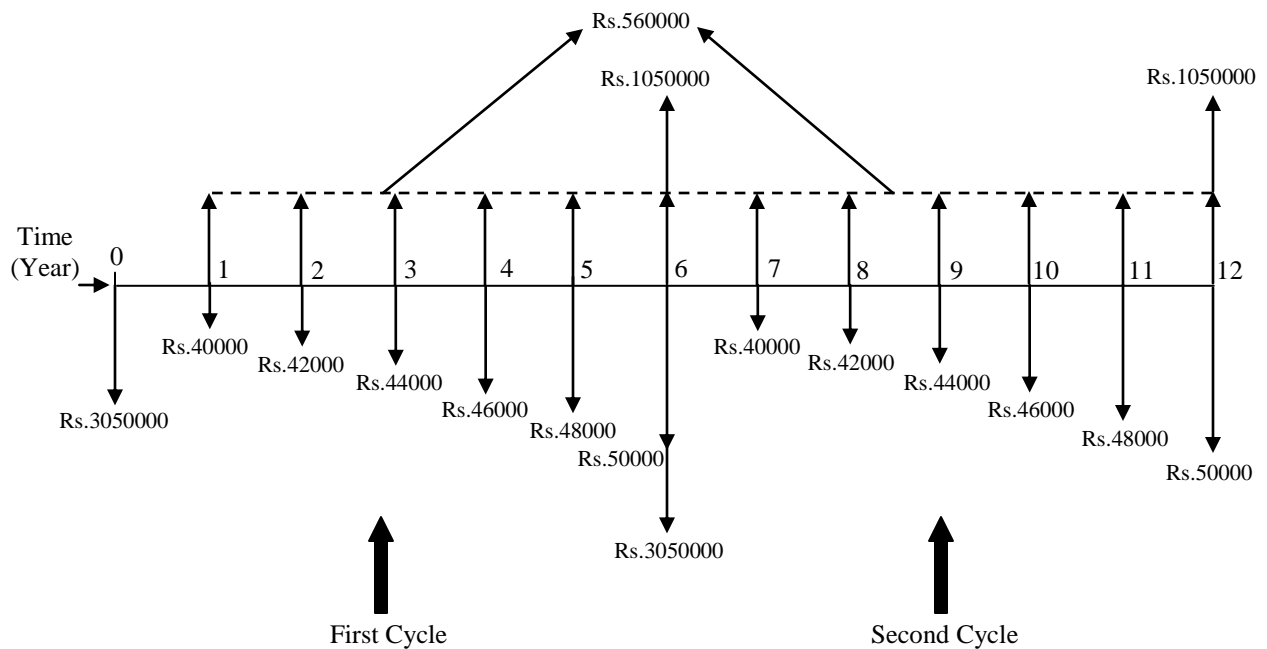
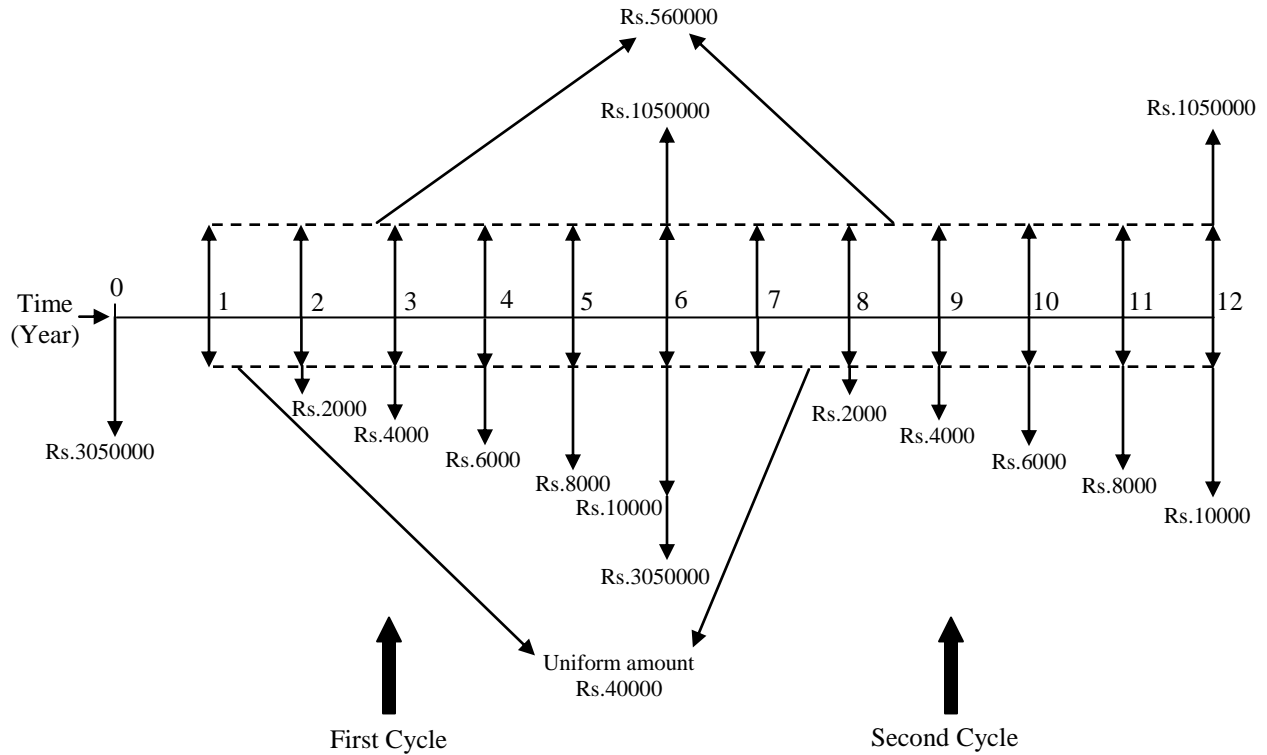


Fig. 2.13 Cash flow diagram of Company-A Dozer for two cycles

For Company-A Dozer, the annual operating cost is in the form of a positive uniform gradient series which can be split into the uniform base amount of Rs.40000 and the gradient amount in multiples of Rs.2000 starting from end of year '2' for first cycle as shown in Fig. 2.14. The equivalent present worth of this gradient for cycle one will be located at the beginning i.e. in year '0'. However for second cycle, the equivalent present worth of the gradient for the annual operating cost starting from end of year '8' (shown in Fig. 2.14) will be located at the end of year '6'. Further the present worth of this amount at time '0' will be determined by multiplying the equivalent present worth of the gradient at the end of year '6' with the single payment present worth factor ($P/F, i, n$).



**Fig. 2.14 Cash flow diagram of Company-A Dozer for two cycles
with annual operating cost split into uniform base amount and gradient amount**

The equivalent present worth PW_A (in Rs.) of Company-A Dozer is calculated as follows;

$$PW_A = -3050000 - 40000(P/A, 7\%, 12) - 2000(P/G, 7\%, 6) + 560000(P/A, 7\%, 12) + 1050000(P/F, 7\%, 6) - 3050000(P/F, 7\%, 6) - 2000(P/G, 7\%, 6)(P/F, 7\%, 6) + 1050000(P/F, 7\%, 12)$$

$$PW_A = -3050000 + (560000 - 40000)(P/A, 7\%, 12) - 2000(P/G, 7\%, 6) - (3050000 - 1050000)(P/F, 7\%, 6) - 2000(P/G, 7\%, 6)(P/F, 7\%, 6) + 1050000(P/F, 7\%, 12)$$

Putting the values of different compound interest factors in the above expression;

$$PW_A = -3050000 + 520000 \times 7.9427 - 2000 \times 10.9784 - 2000000 \times 0.6663 - 2000 \times 10.9784 \times 0.6663 + 1050000 \times 0.4440$$

$$PW_A = -3050000 + 4130204 - 21957 - 1332600 - 14630 + 466200$$

$$PW_A = \text{Rs.177217}$$

The cash flow diagram of Company-B Dozer is shown in Fig. 2.15.

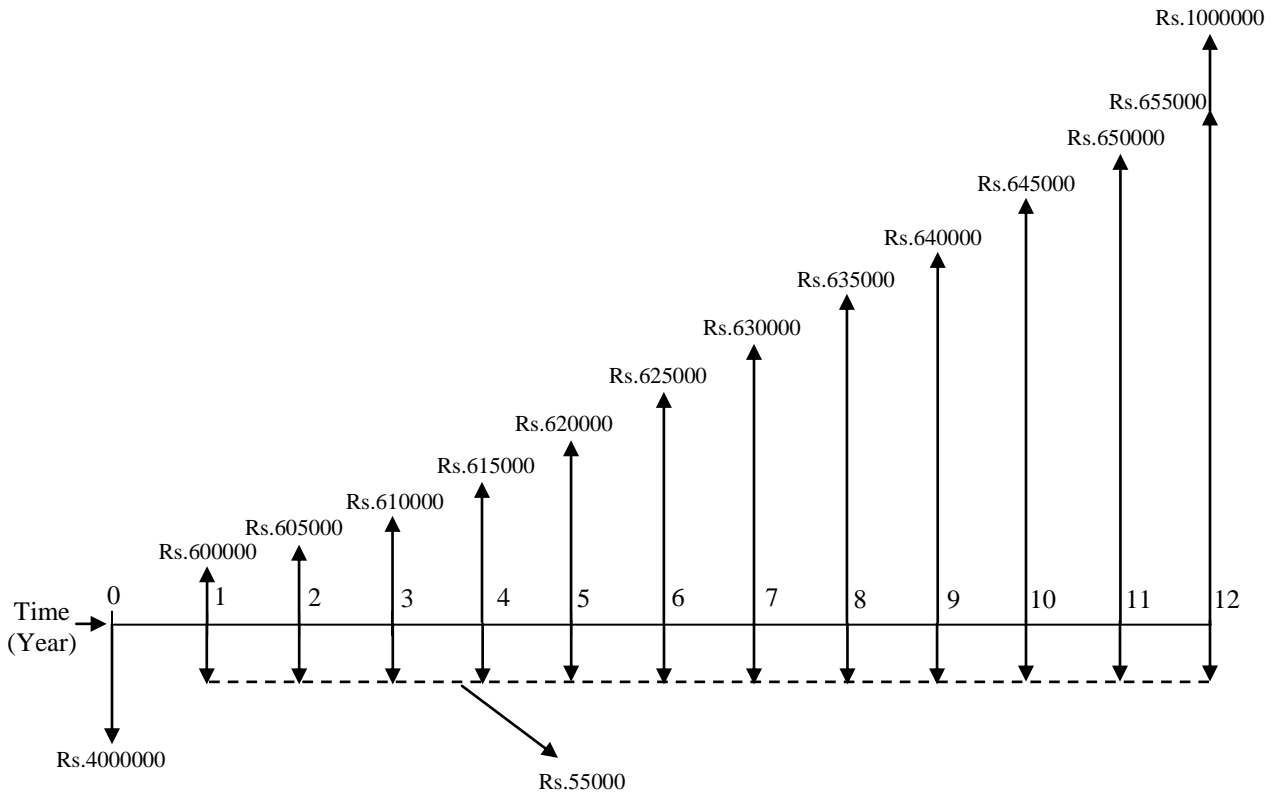
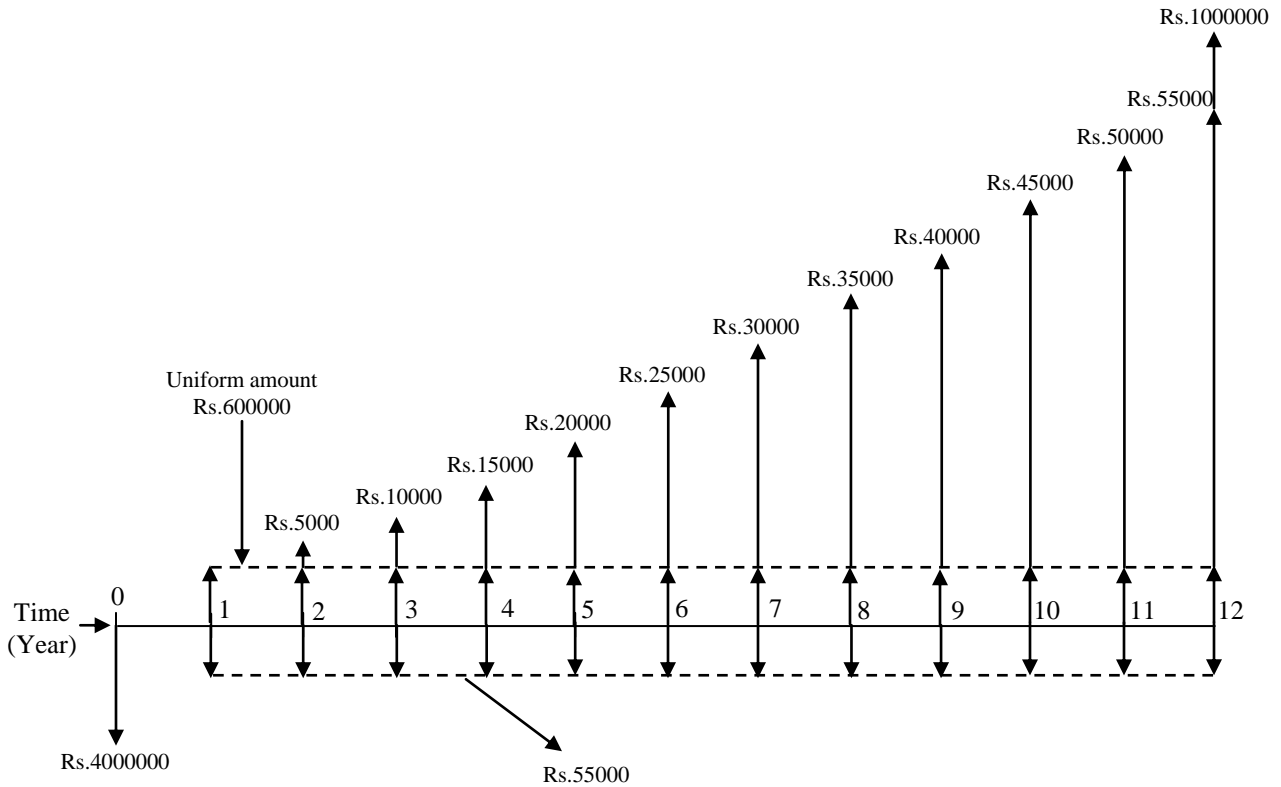


Fig. 2.15 Cash flow diagram of Company-B Dozer

For Company-B Dozer, the annual revenue is in the form of a positive uniform gradient series that can be split into the uniform base amount of Rs. 600,000 and gradient amount in multiples of Rs. 5,000 as shown in Fig. 2.16. The equivalent present worth of this gradient amount will be located at the beginning i.e. in year '0'.



**Fig. 2.16 Cash flow diagram of Company-B Dozer
with annual revenue split into uniform base amount and gradient amount**

The equivalent present worth PW_B (in Rs.) of Company-B Dozer is determined as follows;

$$PW_B = -4000000 - 55000(P/A, 7\%, 12) + 600000(P/A, 7\%, 12) + 5000(P/G, 7\%, 12) + 1000000(P/F, 7\%, 12)$$

$$PW_B = -4000000 + (600000 - 55000)(P/A, 7\%, 12) + 5000(P/G, 7\%, 12) + 1000000(P/F, 7\%, 12)$$

Now putting the values of different compound interest factors in the above expression for PW_B results in the following;

$$PW_B = -4000000 + 545000 \times 7.9427 + 5000 \times 37.3506 + 1000000 \times 0.4440$$

$$PW_B = -4000000 + 4328772 + 186753 + 444000$$

$$PW_B = \text{Rs. } 959525$$

Thus from the comparison of equivalent present worth of the alternatives, it is evident that the construction firm should select Company-B Dozer over Company-A Dozer, as it shows higher positive equivalent present worth i.e. $PW_B > PW_A$.

Lecture-4

Comparison of alternatives by future worth method:

In the future worth method for comparison of mutually exclusive alternatives, the equivalent future worth (i.e. value at the end of the useful lives of alternatives) of all the expenditures and incomes occurring at different periods of time are determined at the given interest rate per interest period. As already mentioned, the cash flow of the mutually exclusive alternatives may consist of expenditures and incomes in different forms. Therefore the equivalent future worth of these expenditures and incomes will be determined using different compound interest factors namely single payment compound amount factor, uniform series compound amount factor and future worth factors for arithmetic and geometric gradient series etc.

The use of future worth method for comparison of mutually exclusive alternatives will be illustrated in the following examples. Similar to present worth method, first the comparison of equal life span alternatives by future worth method will be illustrated followed by comparison of different life span alternatives. Some of the examples already worked out by the present worth method will be illustrated using the future worth method in addition to some other examples.

Example -6 (Using data of Example-1)

There are two alternatives for purchasing a concrete mixer. Both the alternatives have same useful life. The cash flow details of alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.300000, Annual operating and maintenance cost = Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost = Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

Using future worth method, find out which alternative should be selected, if the rate of interest is 10% per year.

Solution:

The future worth of the mutually exclusive alternatives will be compared over a period of 5 years. The equivalent future worth of the alternatives can be obtained either by

multiplying the equivalent present worth of each alternative already obtained by present worth method with the single payment compound amount factor or determining the future worth of expenditures and incomes individually and adding them to get the equivalent future worth of each alternative.

The equivalent future worth of Alternative-1 is obtained as follows;

$$FW_1 = PW_1(F/P, i, n)$$

PW_1 is the equivalent present worth of Alternative-1 which is equal to - Rs.298203 (referring to Example-1). $(F/P, i, n)$ is the single payment compound amount factor.

$$FW_1 = -298203(F/P, 10\%, 5)$$

Now putting the value of single payment compound amount factor in the above expression;

$$FW_1 = -298203 \times 1.6105$$

$$FW_1 = \text{-Rs.480256}$$

The equivalent future worth of Alternative-1 can also be determined in the following manner (Referring to cash flow diagram of Alternative-1, Fig. 2.1);

$$FW_1 = -300000(F/P, 10\%, 5) - 20000(F/A, 10\%, 5) + 125000$$

Now putting the values of different compound interest factors in the above expression;

$$FW_1 = -300000 \times 1.6105 - 20000 \times 6.1051 + 125000$$

$$FW_1 = -483150 - 122102 + 125000$$

$$FW_1 = \text{-Rs.480252}$$

Now it can be seen that the calculated future worth of Alternative-1 by both ways is same. The minor difference between the values is due to the effect of decimal points in the calculations.

The equivalent future worth of Alternative-2 is calculated as follows;

$$FW_2 = PW_2(F/P, i, n)$$

PW_2 is the equivalent present worth of Alternative-2 which is equal to - Rs.289215 (referring to Example-1).

$$FW_2 = -289215(F/P, 10\%, 5)$$

Now putting the value of single payment compound amount factor in the above expression;

$$FW_2 = -289215 \times 1.6105$$

$$FW_2 = \text{-Rs.465781}$$

The equivalent future worth of Alternative-2 can also be determined in the same manner as in case of Alternative-1 and is presented as follows (Referring to cash flow diagram of Alternative-2, Fig. 2.2);

$$FW_2 = -200000(F/P, 10\%, 5) - 35000(F/A, 10\%, 5) + 70000$$

Now putting the values of different compound interest factors in the above expression;

$$FW_2 = -200000 \times 1.6105 - 35000 \times 6.1051 + 70000$$

$$FW_2 = -322100 - 213679 + 70000$$

$$FW_2 = \text{-Rs.465779}$$

Thus the future worth of Alternative-2 obtained by both methods is same. In this case also the minor difference between the values is due to the effect of the decimal points in the calculations.

Comparing the equivalent future worth of the both the alternatives, it is observed that Alternative-2 will be selected as it shows lower negative equivalent future worth as compared to Alternative-1. This outcome of the comparison of the alternatives by future worth method is same as that obtained from the present worth method (Example-1). This is due to the equivalency relationship between present worth and future worth through compound interest factors at the given rate of interest per interest period.

Example -7

There are two alternatives for a construction firm to purchase a road roller which will be used for the construction of a highway section. The cash flow details of the alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.1500000, Annual operating cost = Rs.35000 starting from the end of year '2' (negligible in the first year) till the end of useful life, Annual revenue to be generated = Rs.340000 for first 4 years and then Rs.320000

afterwards till the end of useful life, Expected salvage value = Rs.430000, Useful life = 8 years.

Alternative-2: Initial purchase cost = Rs.1800000, Annual operating cost = Rs.25000, Annual revenue to be generated = Rs.365000, Expected salvage value = Rs.550000, Useful life = 8 years.

Find out the most economical alternative on the basis of equivalent future worth at the interest rate of 9.5% per year.

Solution:

The cash flow diagram of Alternative-1 is shown in Fig. 2.17.

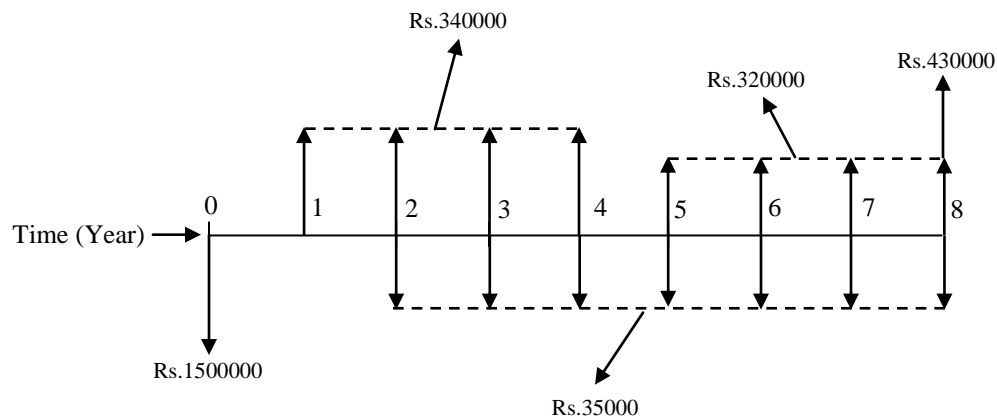


Fig. 2.17 Cash flow diagram of Alternative-1

From Fig. 2.17, it is observed that there are two uniform amount series for the annual income i.e. first series with Rs.340000 from end of year '1' till end of year '4' and second one with Rs.320000 from end of year '5' till end of year '8'. For the first series, the equivalent present worth at time '0' will be calculated first and then it will be multiplied with single payment compound amount factor i.e. $(F/P, i, n)$ to calculate its equivalent future worth. For the second uniform series with Rs.320000, the future worth will be calculated by multiplying the uniform amount i.e. Rs.320000 with uniform series compound amount factor by taking the appropriate 'n' i.e. number of years.

The annual operating cost is in the form of a uniform amount series, which starts from end of year '2' till the end of useful life i.e. the uniform amount series is shifted by one year.

The equivalent future worth of the Alternative-1 i.e. FW_1 is computed as follows;

$$FW_1 = -1500000(F/P, 9.5\%, 8) - 35000(F/A, 9.5\%, 7) + 340000(P/A, 9.5\%, 4)(F/P, 9.5\%, 8) + 320000(F/A, 9.5\%, 4) + 430000$$

Putting the values of different compound interest factors in the above expression results in the following;

$$FW_1 = -1500000 \times 2.0669 - 35000 \times 9.3426 + 340000 \times 3.2045 \times 2.0669 + 320000 \times 4.6070 + 430000$$

$$FW_1 = -3100350 - 326991 + 2251950 + 1474240 + 430000$$

FW₁ = Rs.728849

The cash flow diagram of Alternative-2 is shown in Fig. 2.18.

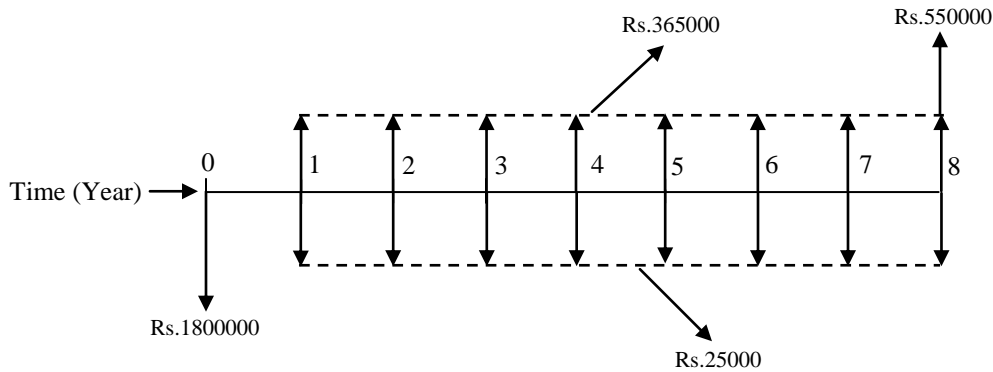


Fig. 2.18 Cash flow diagram of Alternative-2

The equivalent future worth of the Alternative-2 i.e. FW_2 is calculated as follows;

$$FW_2 = -1800000(F/P, 9.5\%, 8) - 25000(F/A, 9.5\%, 8) + 365000(F/A, 9.5\%, 8) + 550000$$

Now putting the values of different compound interest factors in the above expression results in the following;

$$FW_2 = -1800000 \times 2.0669 + (365000 - 25000) \times 11.2302 + 550000$$

$$FW_2 = -3720420 + 3818268 + 550000$$

FW₂ = Rs.647848

Comparing the equivalent future worth of the alternatives, it is observed that Alternative-1 shows higher positive equivalent future worth as compared to Alternative-2. Thus Alternative-1 will be selected for purchase of the road roller.

Lecture-5

Comparison by future worth method:-

In the following example, the comparison of three mutually exclusive alternatives by future worth method will be illustrated. The data presented in Example-3 will be used for comparison of the alternatives by the future worth method.

Example -8 (Using data of Example-3)

A construction contractor has three options to purchase a dump truck for transportation and dumping of earth at a construction site. All the alternatives have the same useful life. The cash flow details of all the alternatives are presented as follows;

Option-1: Initial purchase price = Rs.2500000, Annual operating cost Rs.45000 at the end of 1st year and increasing by Rs.3000 in the subsequent years till the end of useful life, Annual income = Rs.120000, Salvage value = Rs.550000, Useful life = 10 years.

Option-2: Initial purchase price = Rs.3000000, Annual operating cost = Rs.30000, Annual income Rs.150000 for first three years and increasing by Rs.5000 in the subsequent years till the end of useful life, Salvage value = Rs.800000, Useful life = 10 years.

Option-3: Initial purchase price = Rs.2700000, Annual operating cost Rs.35000 for first 5 years and increasing by Rs.2000 in the successive years till the end of useful life, Annual income = Rs.140000, Expected salvage value = Rs.650000, Useful life = 10 years.

Using future worth method, find out which alternative should be selected, if the rate of interest is 8% per year.

Solution:

The cash flow diagram of Option-1 is shown here again for ready reference.

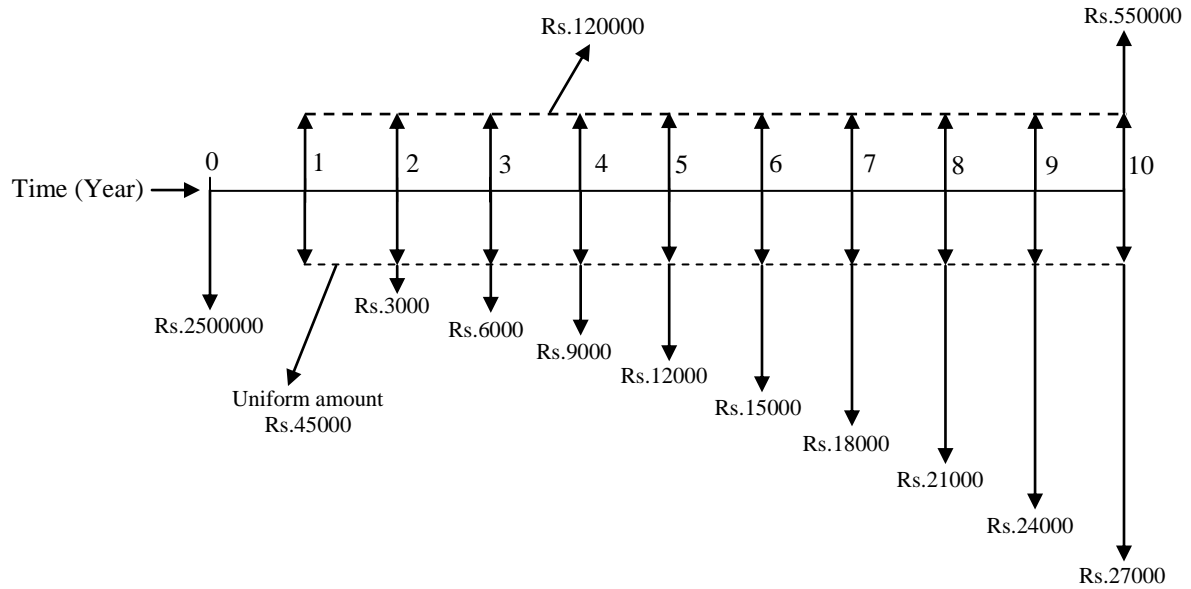


Fig. 2.6 Cash flow diagram of Option-1 with annual operating cost split into uniform base amount and gradient amount (shown for ready reference)

The equivalent future worth (in Rs.) of Option-1 is determined as follows;

$$FW_1 = -2500000(F/P, 8\%, 10) - 45000(F/A, 8\%, 10) - 3000(F/G, 8\%, 10) + 120000(F/A, 8\%, 10) + 550000$$

$$FW_1 = -2500000(F/P, 8\%, 10) + (120000 - 45000)(F/A, 8\%, 10) - 3000(F/G, 8\%, 10) + 550000$$

Now putting the values of different compound interest factors in the above expression for

FW_1 results in the following;

$$FW_1 = -2500000 \times 2.1589 + 75000 \times 14.4866 - 3000 \times 56.0820 + 550000$$

$$FW_1 = -5397250 + 1086495 - 168246 + 550000$$

$$FW_1 = -\text{Rs.}3929001$$

The cash flow diagram of Option-2 is shown again for ready reference.

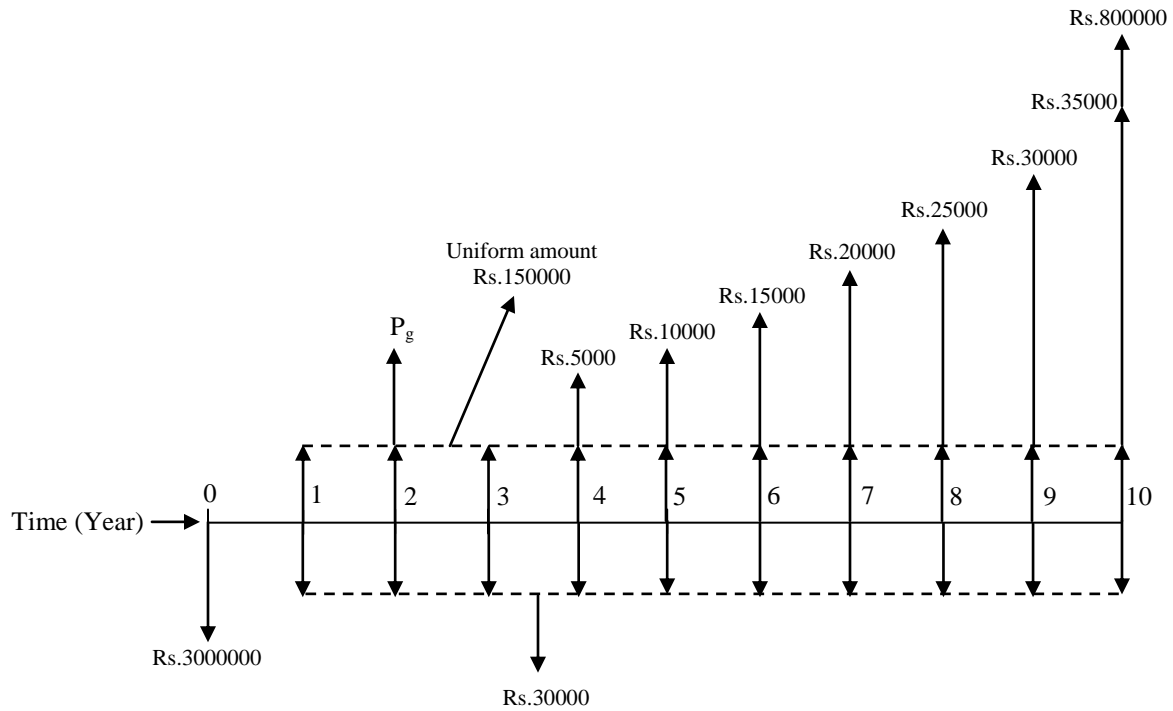


Fig. 2.8 Cash flow diagram of Option-2 with annual income split into uniform base amount and gradient amount (shown for ready reference)

The equivalent present worth of the gradient series (of the annual income) starting from end of year '4' will be located at the end of year '2'. The future worth of this amount at end of year '10' will be obtained by multiplying the equivalent present worth ' P_g ' (shown in Fig. 2.8) at the end of year '2' with the single payment compound amount factor (F/P , i , n).

The equivalent future worth (in Rs.) of Option-2 is determined as follows;

$$FW_2 = -3000000(F/P, 8\%, 10) - 30000(F/A, 8\%, 10) + 150000(F/A, 8\%, 10) + P_g(F/P, 8\%, 8) + 800000$$

Now replacing P_g with $G(P/G, i, n)$ i.e. $5000(P/G, 8\%, 8)$ in the above expression;

$$FW_2 = -3000000(F/P, 8\%, 10) + (150000 - 30000)(F/A, 8\%, 10) + 5000(P/G, 8\%, 8)(F/P, 8\%, 8) + 800000$$

It may be noted here that, in the above expression, $5000(P/G, 8\%, 8)(F/P, 8\%, 8)$ can be replaced by $5000(F/G, 8\%, 8)$ and will result in the same value.

Now putting the values of different compound interest factors in the above expression;

$$FW_2 = -3000000 \times 2.1589 + 120000 \times 14.4866 + 5000 \times 17.8061 \times 1.8509 + 800000$$

$$FW_2 = -6476700 + 1738392 + 164787 + 800000$$

$$FW_2 = -\text{Rs.}3773521$$

The cash flow diagram of Option-3 is shown here again for ready reference.

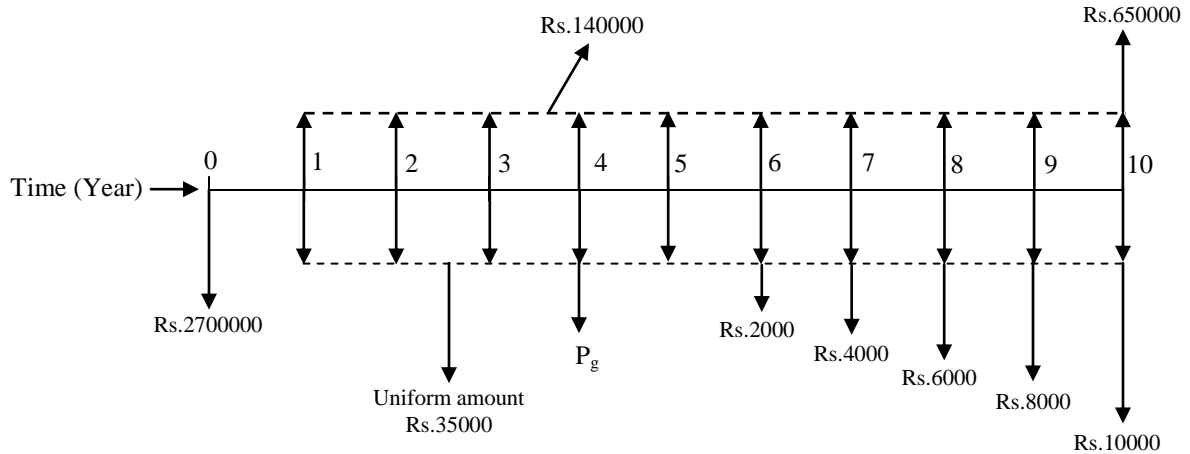


Fig. 2.10 Cash flow diagram of Option-3 with annual operating cost split into uniform base amount and gradient amount (shown for ready reference)

For the annual operating cost, the equivalent present worth of the gradient series starting from end of year '6' will be located at the end of year '4'. The future worth of this amount at end of year '10' will be determined by multiplying the equivalent present worth ' P_g ' (shown in Fig. 2.10) at the end of year '4' with the single payment compound amount factor ($F/P, i, n$).

The equivalent future worth (in Rs.) of Option-3 is determined as follows;

$$FW_3 = -2700000(F/P, 8\%, 10) - 35000(F/A, 8\%, 10) - P_g(F/P, 8\%, 6) + 140000(F/A, 8\%, 10) + 650000$$

Now replacing P_g with $G(P/G, i, n)$ i.e. $2000(P/G, 8\%, 6)$ in the above expression;

$$FW_3 = -2700000(F/P, 8\%, 10) + (140000 - 35000)(F/A, 8\%, 10) - 2000(P/G, 8\%, 6)(F/P, 8\%, 6) + 650000$$

In the above expression, $2000(P/G, 8\%, 6)(F/P, 8\%, 6)$ can also be replaced by $2000(F/G, 8\%, 6)$.

Now putting the values of different compound interest factors in the above expression;

$$FW_3 = -2700000 \times 2.1589 + 105000 \times 14.4866 - 2000 \times 10.5233 \times 1.5869 + 650000$$

$$FW_3 = -5829030 + 1521093 - 33399 + 650000$$

$$FW_3 = - \text{Rs.}3691336$$

Comparing the equivalent future worth of all the three alternatives, it is evident that Option-3 shows lowest negative equivalent future worth as compared to other options. Thus Option-3 will be selected for the purchase of the dump truck. This outcome obtained by future worth method is same as that obtained from the present worth method (Example-3) i.e. Option-3 is the most economical alternative.

After carrying out the comparison of equal life span mutually exclusive alternatives, now the illustration of future worth method for comparison of different life span mutually exclusive alternatives is presented.

Example -9 (Using data of Example-4)

A material testing laboratory has two alternatives for purchasing a compression testing machine which will be used for determining the compressive strength of different construction materials. The alternatives are from two different manufacturing companies. The cash flow details of the alternatives are as follows;

Alternative-1: Initial purchase price = Rs.1000000, Annual operating cost = Rs.10000, Expected annual income to be generated from testing of different construction materials = Rs.175000, Expected salvage value = Rs.200000, Useful life = 10 years.

Alternative-2: Initial purchase price = Rs.700000, Annual operating cost = Rs.15000, Expected annual income to be generated from testing of different construction materials = Rs.165000, Expected salvage value = Rs.250000, Useful life = 5 years.

Find out the most economical alternative at interest rate of 10% per year by future worth method.

Solution:

As the alternatives have different life spans i.e. 10 years and 5 years, the comparison will be made over a time period equal to the least common multiple of the life spans of the alternatives i.e. 10 years. Thus the cash flow of Alternative-1 is analyzed for one cycle

(duration of 10 years) whereas that of cash flow of Alternative-2 is analyzed for two cycles of duration 5 years each (already mentioned in Example-4).

The cash flow diagram of Alternative-1 is shown here again for ready reference.

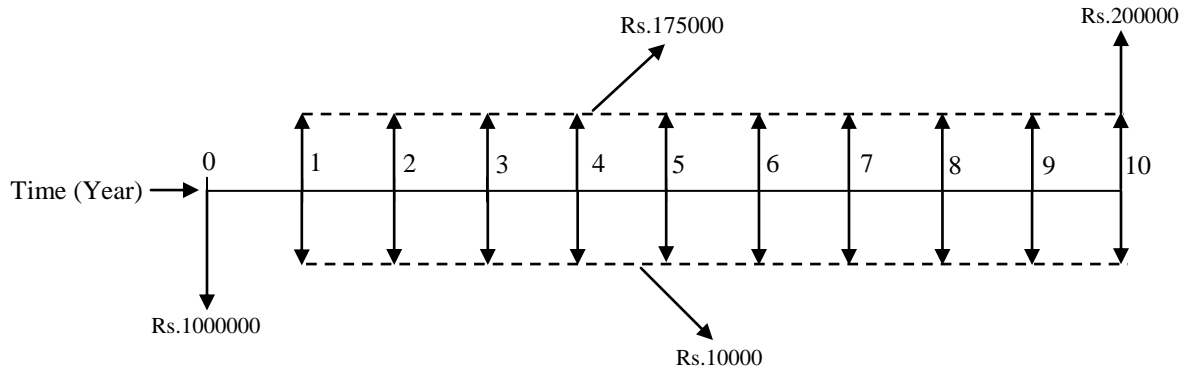


Fig. 2.11 Cash flow diagram of Alternative-1 (shown for ready reference)

The equivalent future worth FW_1 (in Rs.) of Alternative-1 is determined as follows;

$$FW_1 = -1000000(F/P, 10\%, 10) - 10000(F/A, 10\%, 10) + 175000(F/A, 10\%, 10) + 200000$$

$$FW_1 = -1000000(F/P, 10\%, 10) + (175000 - 10000)(F/A, 10\%, 10) + 200000$$

Putting the values of different compound interest factors in the above expression;

$$FW_1 = -1000000 \times 2.5937 + 165000 \times 15.9374 + 200000$$

$$FW_1 = -2593700 + 2629671 + 200000$$

$$FW_1 = \text{Rs.235971}$$

The cash flow diagram of Alternative-2 is shown here again for ready reference.

The equivalent future worth FW_2 (in Rs.) of Alternative-2 is determined as follows;

$$FW_2 = -700000(F/P, 10\%, 10) - 15000(F/A, 10\%, 10) + 165000(F/A, 10\%, 10) + 250000(F/P, 10\%, 5) - 700000(F/P, 10\%, 5) + 250000$$

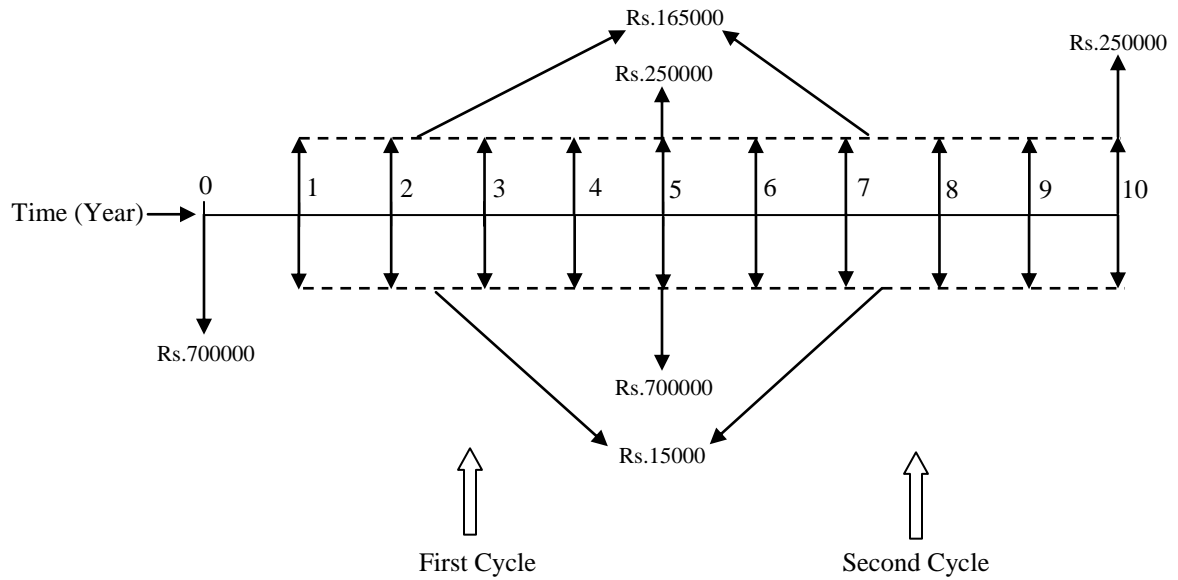


Fig. 2.12 Cash flow diagram of Alternative-2 for two cycles

$$FW_2 = -700000(F/P, 10\%, 10) + (165000 - 15000)(F/A, 10\%, 10) - (700000 - 250000)(F/P, 10\%, 5) + 250000$$

Putting the values of different compound interest factors in the above expression for FW_2 results in the following;

$$FW_2 = -700000 \times 2.5937 + 150000 \times 15.9374 - 450000 \times 1.6105 + 250000$$

$$FW_2 = -1815590 + 2390610 - 724725 + 250000$$

$$FW_2 = \text{Rs. } 100295$$

Thus Alternative-1 will be selected for purchase of the compression testing machine, as it shows the higher positive equivalent future worth as compared to Alternative-2.

Lecture-6

Comparison of alternatives by annual worth method:

In this method, the mutually exclusive alternatives are compared on the basis of equivalent uniform annual worth. The equivalent uniform annual worth represents the annual equivalent value of all the cash inflows and cash outflows of the alternatives at the given rate of interest per interest period. In this method of comparison, the equivalent uniform annual worth of all expenditures and incomes of the alternatives are determined using different compound interest factors namely capital recovery factor, sinking fund factor and annual worth factors for arithmetic and geometric gradient series etc. Since equivalent uniform annual worth of the alternatives over the useful life are determined, same procedure is followed irrespective of the life spans of the alternatives i.e. whether it is the comparison of equal life span alternatives or that of different life span alternatives. In other words, in case of comparison of different life span alternatives by annual worth method, the comparison is not made over the least common multiple of the life spans as is done in case of present worth and future worth method. The reason is that even if the comparison is made over the least common multiple of years, the equivalent uniform annual worth of the alternative for more than one cycle of cash flow will be exactly same as that of the first cycle provided the cash flow i.e. the costs and incomes of the alternative in the successive cycles is exactly same as that in the first cycle. Thus the comparison is made only for one cycle of cash flow of the alternatives. This serves as one of greater advantages of using this method over other methods of comparison of alternatives. However if the cash flows of the alternatives in the successive cycles are not the same as that in the first cycle, then a study period is selected and then the equivalent uniform annual worth of the cash flows of the alternatives are computed over the study period.

Now the comparison of mutually exclusive alternatives by annual worth method will be illustrated in the following examples. First the data presented in Example-2 will be used for comparison of the alternatives by the annual worth method.

Example -10 (Using data of Example-2)

There are two alternatives for purchasing a concrete mixer and following are the cash flow details;

Alternative-1: Initial purchase cost = Rs.300000, Annual operating and maintenance cost = Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost = Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

The annual revenue to be generated from production of concrete (by concrete mixer) from Alternative-1 and Alternative-2 are Rs.50000 and Rs.45000 respectively. Compute the equivalent uniform annual worth of the alternatives at the interest rate of 10% per year and find out the economical alternative.

Solution:

The cash flow diagram of Alternative-1 i.e. Fig. 2.3 is shown here again for ready reference.

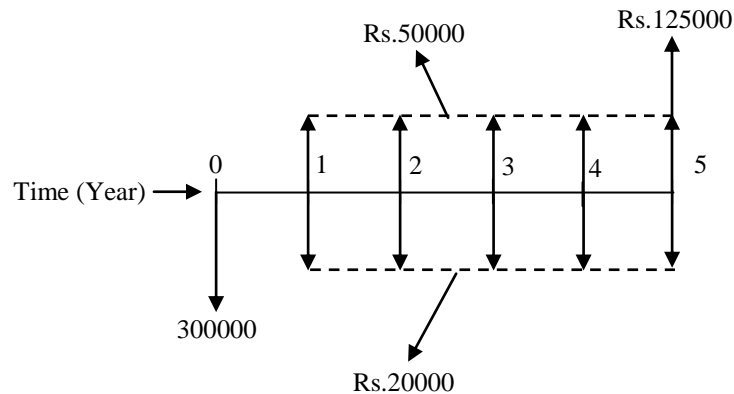


Fig. 2.3 Cash flow diagram of Alternative -1

The equivalent uniform annual worth of Alternative-1 i.e. AW_1 is computed as follows;

$$AW_1 = -300000(A/P, i, n) - 20000 + 50000 + 125000(A/F, i, n)$$

$$AW_1 = -300000(A/P, 10\%, 5) - 20000 + 50000 + 125000(A/F, 10\%, 5)$$

Here Rs.20000 and Rs.50000 are annual amounts.

Now putting the values of different compound interest factors;

$$AW_1 = -300000 \times 0.2638 + (50000 - 20000) + 125000 \times 0.1638$$

$$AW_1 = -79140 + 30000 + 20475$$

$$AW_1 = -\text{Rs.28665}$$

The cash flow diagram of Alternative-2 is shown here again for ready reference.

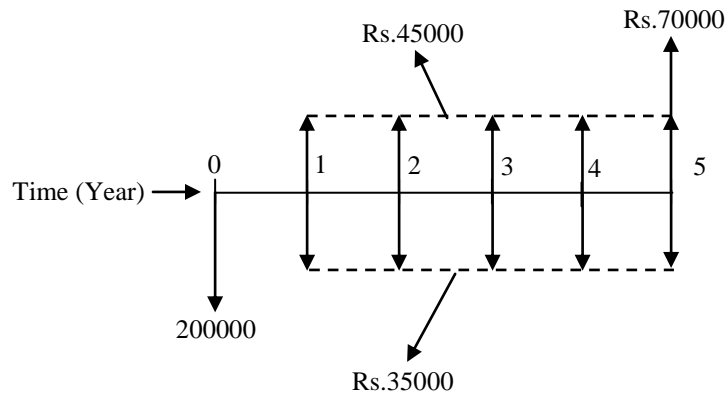


Fig. 2.4 Cash flow diagram of Alternative -2

Now the equivalent uniform annual worth of Alternative-2 i.e. AW_2 is calculated as follows;

$$AW_2 = -200000(A/P, i, n) - 35000 + 45000 + 70000(A/F, i, n)$$

$$AW_2 = -200000(A/P, 10\%, 5) - 35000 + 45000 + 70000(A/F, 10\%, 5)$$

For alternative-2, Rs.35000 and Rs.45000 are annual amounts.

Now putting the values of different compound interest factors in the above expression;

$$AW_2 = -200000 \times 0.2638 + (45000 - 35000) + 70000 \times 0.1638$$

$$AW_2 = -52760 + 10000 + 11466$$

$$AW_2 = - \text{Rs.}31294$$

From this comparison, it is observed that Alternative-1 will be selected as it shows lower negative equivalent uniform annual worth compared to Alternative-2. This outcome is in consistent with the outcome obtained by present worth method in Example-2.

Example -11

A material supply contractor has two options (i.e. from two different manufacturing companies, Company-1 and Company-2) to purchase a tractor for supply of construction materials. The details of cash flow of the two options are given below;

Company-1 Tractor: Initial purchase cost = Rs.2000000, Annual operating cost including labor and maintenance = Rs.50000, Cost of new set of tires to be replaced at the

end of year '3', year '6' and year '9' = Rs.110000 each, Expected salvage value = Rs.520000, Useful life = 10 years.

Company-2 Tractor: Initial purchase cost = Rs.2200000, Annual operating cost including labor and maintenance = Rs.27000, Cost of new set of tires to be replaced at the end of year '4' and year '8' = Rs.120000 each, Expected salvage value = Rs.700000, Useful life = 10 years.

Determine which company tractor should be selected on the basis of equivalent uniform annual worth at the interest rate of 12% per year.

Solution:

The cash flow diagram of Company-1 tractor is shown in Fig. 2.19.

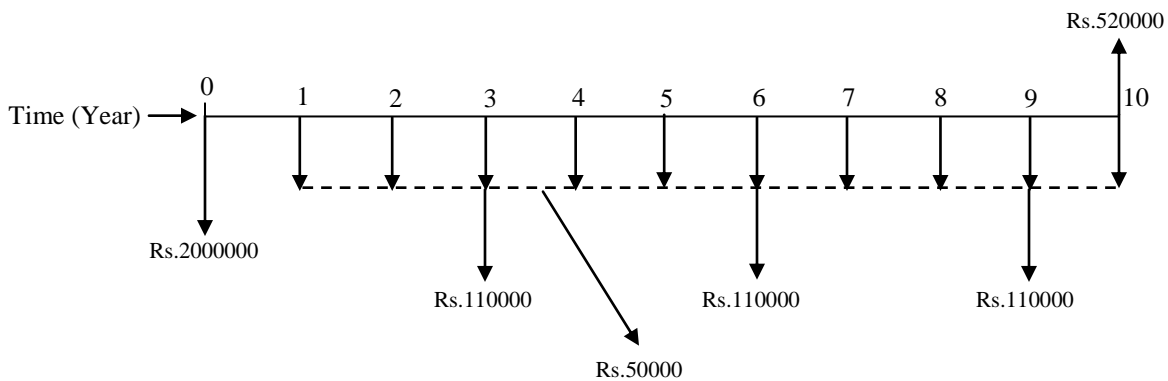


Fig. 2.19 Cash flow diagram of Company-1 Tractor

From the cash flow diagram it is noted that three single amounts i.e. Rs.110000 each are located at the end of year '3', year '6' and year '9'. For these amounts, first the equivalent present worth at time '0' is determined and then equivalent annual worth of this present worth is computed using the appropriate compound interest factor.

The equivalent uniform annual worth of Company-1 tractor is determined as follows;

$$AW_1 = -2000000(A/P, 12\%, 10) - 50000 - 110000(P/F, 12\%, 3)(A/P, 12\%, 10) - 110000(P/F, 12\%, 6)(A/P, 12\%, 10) - 110000(P/F, 12\%, 9)(A/P, 12\%, 10) + 520000(A/F, 12\%, 10)$$

Now putting the values of different compound interest factors in the above expression;

$$AW_1 = -2000000 \times 0.1770 - 50000 - 110000 \times 0.7118 \times 0.1770 - 110000 \times 0.5066 \times 0.1770 - 110000 \times 0.3606 \times 0.1770 + 520000 \times 0.0570$$

$$AW_1 = -354000 - 50000 - 13859 - 9864 - 7021 + 29640$$

$$AW_1 = -405104$$

The cash flow diagram of Company-2 tractor is shown in Fig. 2.20.

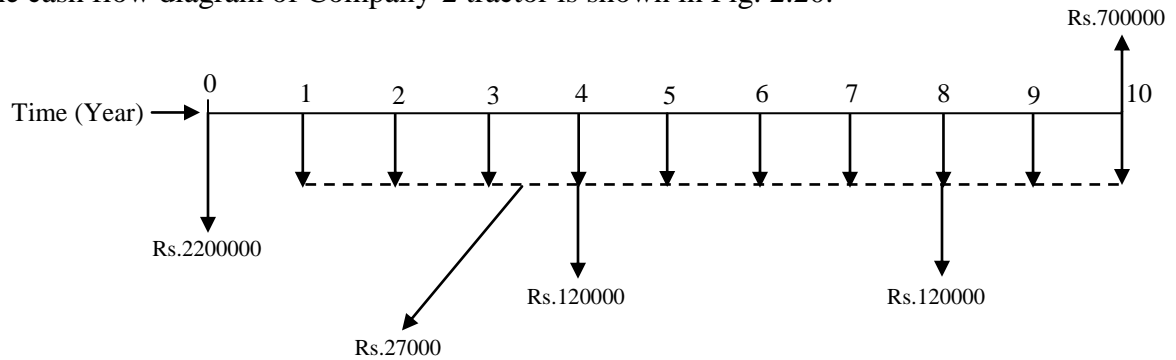


Fig. 2.20 Cash flow diagram of Company-2 Tractor

From Company-2 tractor, two single amounts i.e. Rs.120000 each are located at the end of year '4', and year '8'. Similar to first alternative, first the equivalent present worth at time '0' of these amounts is determined and then equivalent annual worth is computed.

The equivalent uniform annual worth of Company-2 tractor is computed as follows;

$$AW_2 = -2200000(A/P, 12\%, 10) - 27000 - 120000(P/F, 12\%, 4)(A/P, 12\%, 10) - 120000(P/F, 12\%, 8)(A/P, 12\%, 10) + 700000(A/F, 12\%, 10)$$

Now putting the values of different compound interest factors in the above expression;

$$AW_2 = -2200000 \times 0.1770 - 27000 - 120000 \times 0.6355 \times 0.1770 - 120000 \times 0.4039 \times 0.1770 + 700000 \times 0.0570$$

$$AW_2 = -389400 - 27000 - 13498 - 8579 + 39900$$

$$AW_2 = -398577$$

From the above comparison, it is observed that Company-2 Tractor shows lower negative equivalent uniform annual worth as compared to Company-1 tractor. Thus the contractor should select Company-2 Tractor for purchase.

Lecture-7

Comparison by annual worth method:

Now the comparison of alternatives with cash flows involving gradient series and randomly placed single amount by annual worth method will be illustrated followed by the comparison of different life span alternatives.

Example -12

Compare the following equipment on the basis of the equivalent uniform annual worth and find out the most economical one at the interest rate of 9.5% per year.

Equipment-A

Cash flow details:

Initial purchase cost = Rs.5000000

Annual operating cost = Rs.60000 at the end of year '1' and increasing by Rs.3000 in the subsequent years till the end of useful life.

Annual income = Rs.770000

Cost of one time major repair = Rs.200000 at the end of year '8'

Expected salvage value = Rs.1400000

Useful life = 12 years

Equipment-B

Cash flow details:

Initial purchase cost = Rs.4600000

Annual operating cost = Rs.75000

Annual income = Rs.710000 for the first 5 years and increasing by Rs.5000 in the subsequent years till the end of useful life.

Cost of one time major repair = Rs.230000 at the end of year '6'

Expected salvage value = Rs.1200000

Useful life = 12 years

Solution:

The cash flow diagram of Equipment-A is shown in Fig. 2.21.

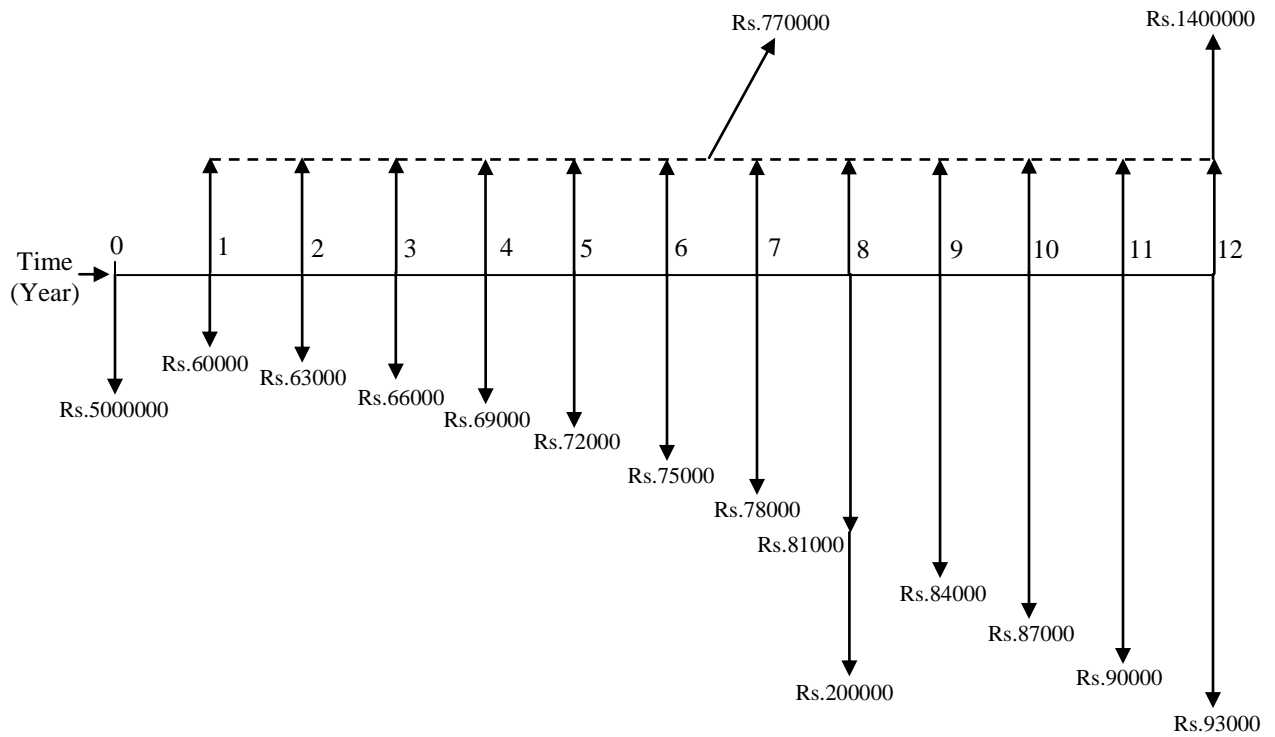


Fig. 2.21 Cash flow diagram of Equipment-A

The annual operating cost is in the form of a positive uniform gradient series. This can be split into the uniform base amount of Rs. 60,000 and gradient amount in multiples of Rs. 3,000 starting from end of year '2' till the end of useful life as shown in Fig. 2.22.

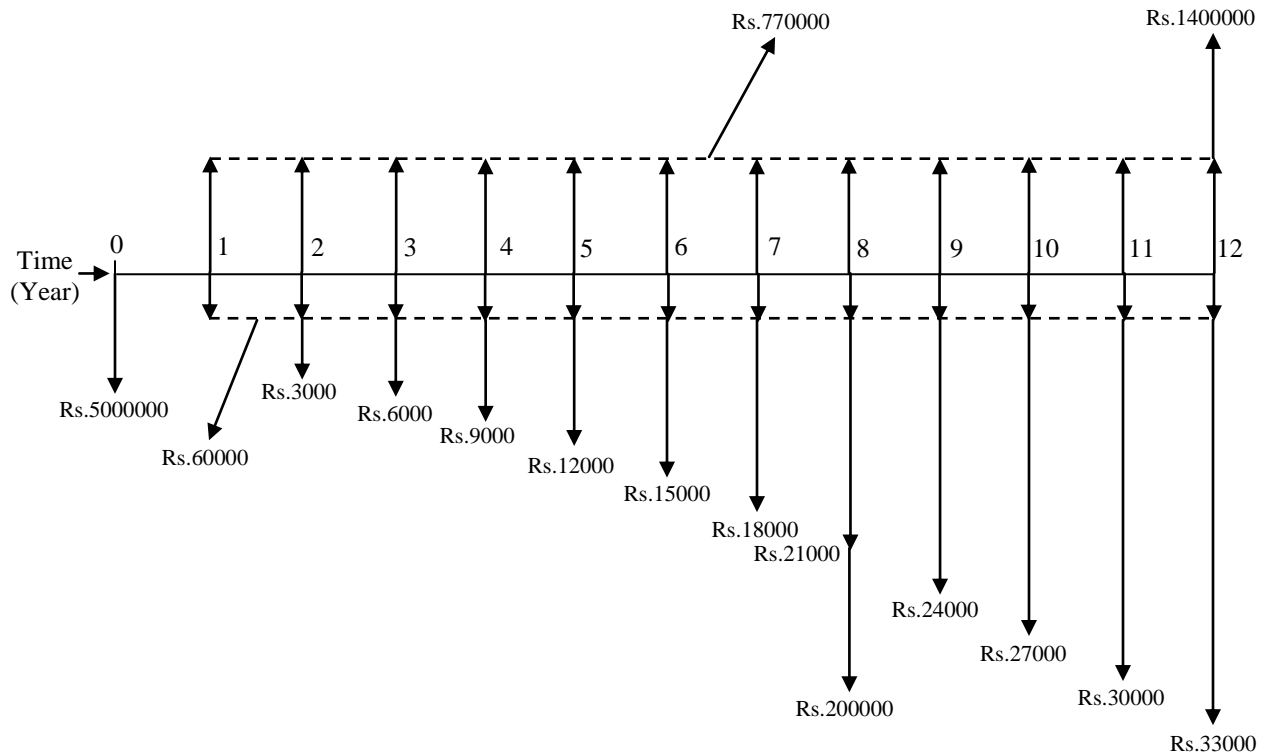


Fig. 2.22 Cash flow diagram of Equipment-A with annual operating cost split into uniform base amount and gradient amount

The equivalent uniform annual worth of Equipment-A is determined as follows;

$$AW_A = -5000000(A/P, 9.5\%, 12) - 60000 - 3000(A/G, 9.5\%, 12) - 200000(P/F, 9.5\%, 8)(A/P, 9.5\%, 12) + 770000 + 1400000(A/F, 9.5\%, 12)$$

Now putting the values of different compound interest factors in the above expression results in the following;

$$AW_A = -5000000 \times 0.1432 - 60000 - 3000 \times 4.4394 - 200000 \times 0.4838 \times 0.1432 + 770000 + 1400000 \times 0.0482$$

$$AW_A = -716000 - 60000 - 13318 - 13856 + 770000 + 67480$$

$$AW_A = \text{Rs.34306}$$

The cash flow diagram of Equipment-B is shown in Fig. 2.23.

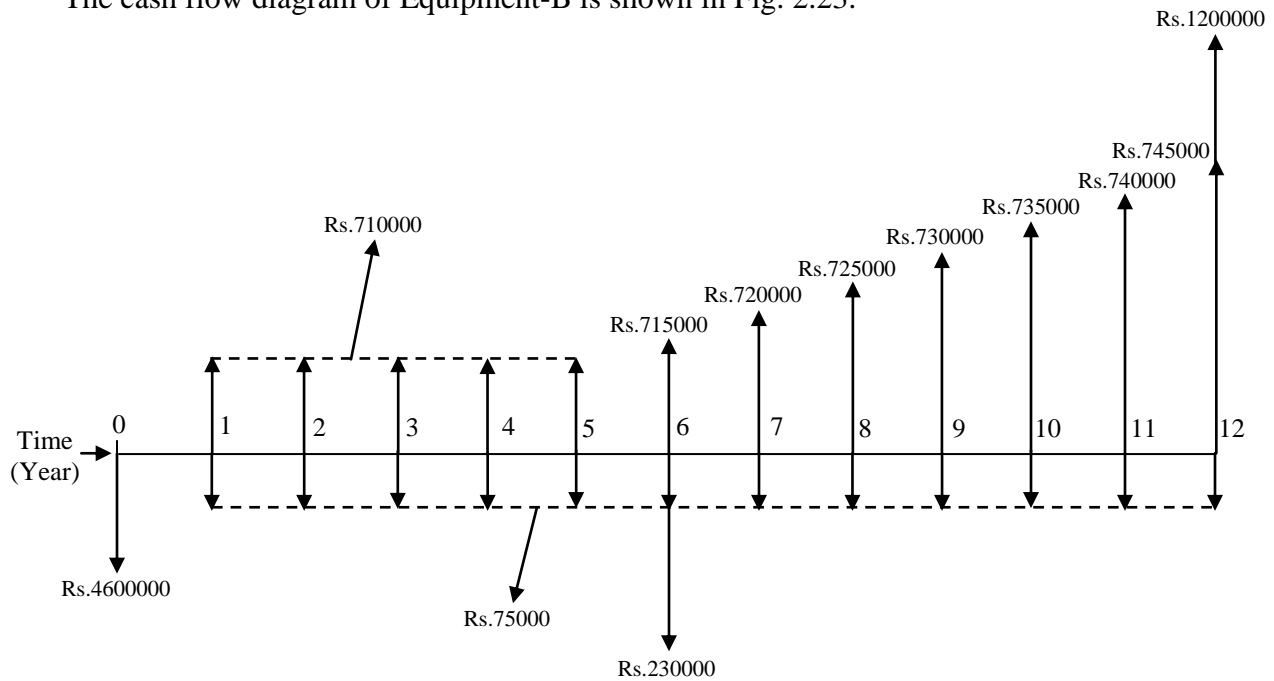


Fig. 2.23 Cash flow diagram of Equipment-B

For Equipment-B, the annual income is in the form of a positive uniform gradient series with gradient starting from end of year '6'. The annual income is split into the uniform base amount of Rs. 710,000 and the gradient amount in multiples of Rs. 5,000 starting from end of year '6' till the end of useful life as shown in Fig. 2.24.

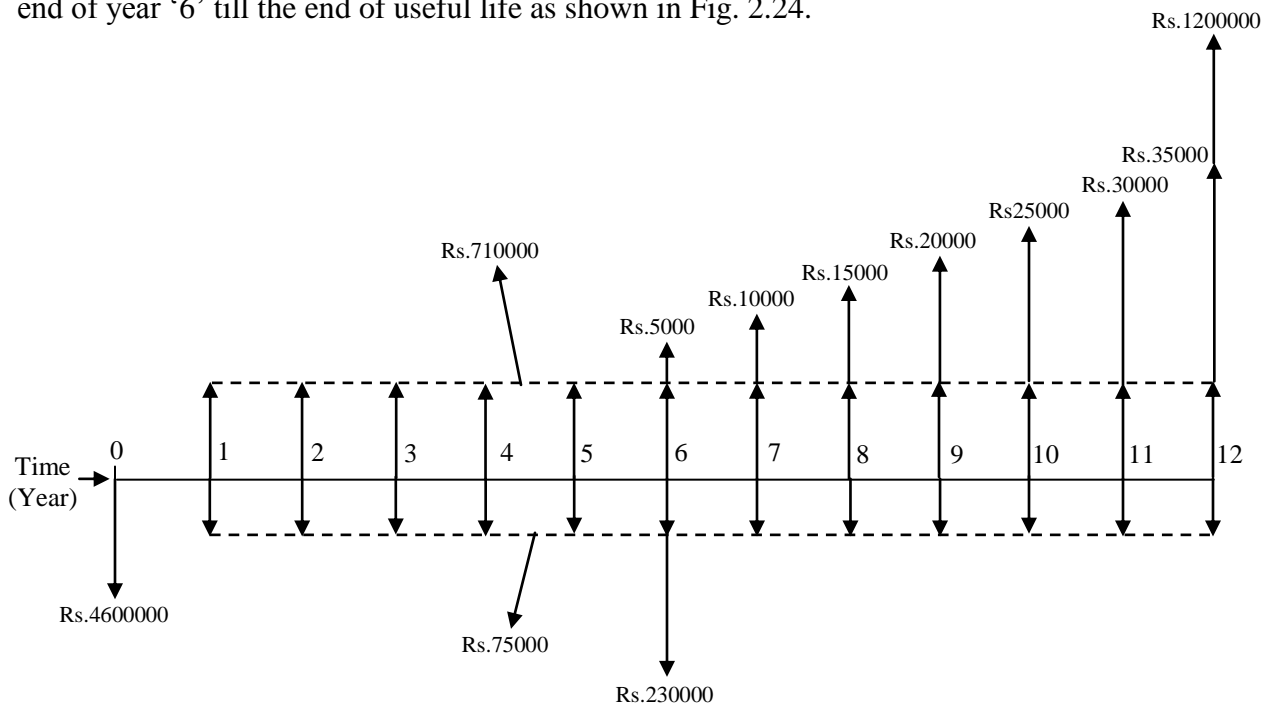


Fig. 2.24 Cash flow diagram of Equipment-B with annual income split into uniform base amount and gradient amount

For the gradient series (of annual income), the equivalent present worth will be located at the end of year '4' i.e. 2 years before the start of the gradient. Then the present worth of this amount at time '0' will be calculated and after that the equivalent annual worth of this amount will be determined.

The equivalent uniform annual worth of Equipment-B is determined as follows;

$$AW_B = -4600000(A/P, 9.5\%, 12) - 75000 - 230000(P/F, 9.5\%, 6)(A/P, 9.5\%, 12) + 710000 + 5000(P/G, 9.5\%, 8)(P/F, 9.5\%, 4)(A/P, 9.5\%, 12) + 1200000(A/F, 9.5\%, 12)$$

Now putting the values of different compound interest factors in the above expression;

$$AW_B = -4600000 \times 0.1432 - 75000 - 230000 \times 0.5801 \times 0.1432 + 710000 + 5000 \times 16.4510 \times 0.6956 \times 0.1432 + 1200000 \times 0.0482$$

$$AW_B = -658720 - 75000 - 19106 + 710000 + 8193 + 57840$$

$$AW_B = \text{Rs.23207}$$

From the comparison by annual worth method, it is observed that Equipment-A exhibits higher positive equivalent uniform annual worth as compared to Equipment-B. Thus the Equipment-A is the most economical alternative.

Example -13

Using the annual worth method, compare the following machines having different life spans at an interest rate of 11.5% per year.

Machine-1

Cash flow details:

Initial purchase price = Rs.1200000

Annual operating cost = Rs.38000

Annual revenue = Rs.210000 for first 6 years and then Rs.225000 afterwards till the end of useful life.

Expected salvage value = Rs.320000

Useful life = 14 years

Machine-2

Cash flow details:

Initial purchase price = Rs.1400000

Annual operating cost = Rs.26000

Annual revenue = Rs.290000

Expected salvage value = Rs.450000

Useful life = 7 years

Solution:

The cash flow diagram of Machine-1 is shown in Fig. 2.25.

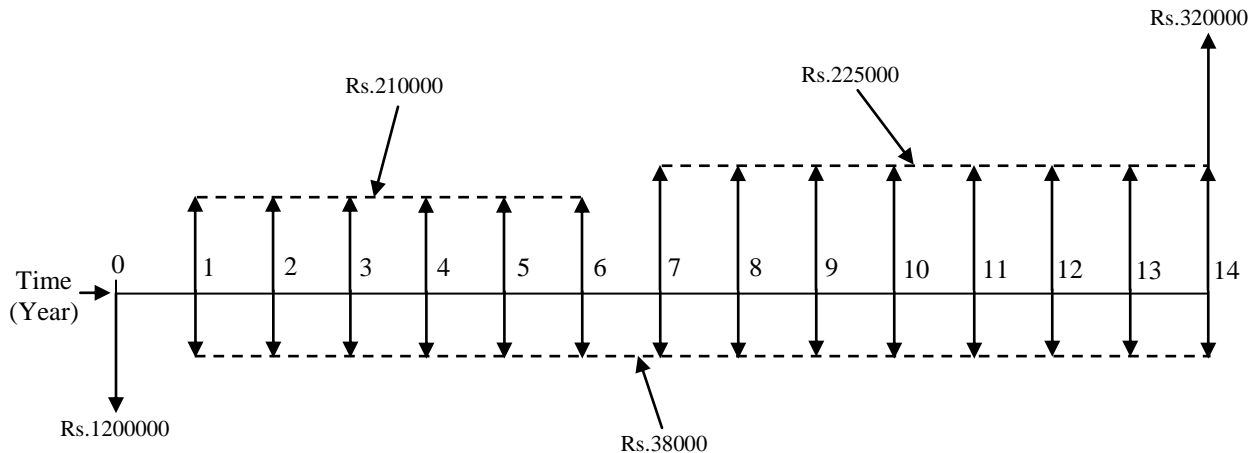


Fig. 2.25 Cash flow diagram of Machine-1

It may be noted here that the annual revenue is in the form of two uniform annual amount series i.e. one with annual amount of Rs.210000 from beginning till end of year '6' and the second one with annual amount of Rs.225000 from end of year '7' till the end of useful life (as shown in Fig. 2.25). The annual revenue in the cash flow diagram can also be represented as annual amount of Rs.210000 from beginning till the end of useful life and the annual amount of Rs.15000 from end of year '7' till the end of useful life as shown in Fig. 2.26.

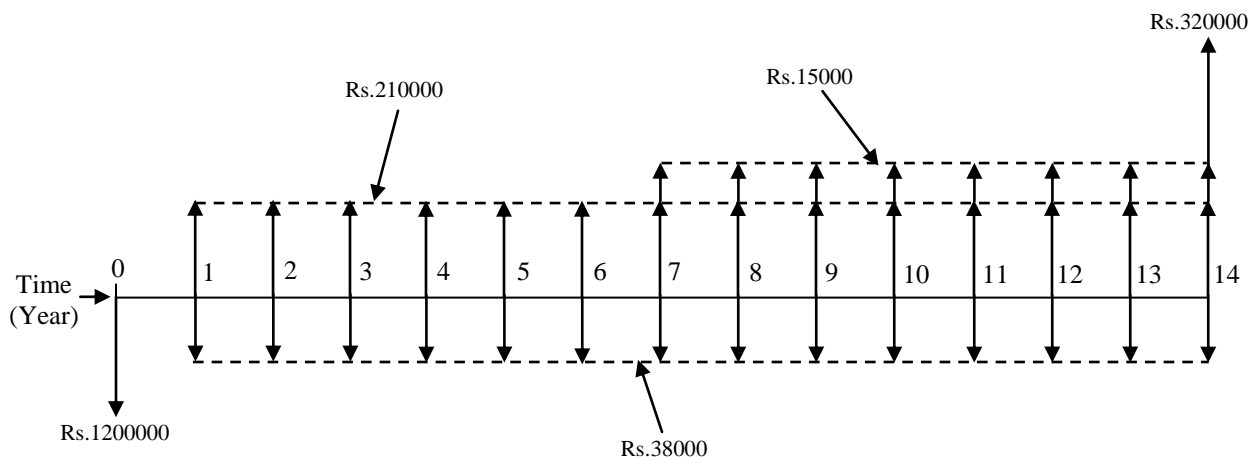


Fig. 2.26 Cash flow diagram of Machine-1 with simplified representation of annual revenue

For the annual revenue of Rs.15000 from end of year '7' till the end of useful life, first the equivalent future worth is calculated followed by the calculation of the equivalent annual worth.

The equivalent uniform annual worth of Machine-1 is computed as follows;

$$AW_1 = -1200000(A/P, 11.5\%, 14) - 38000 + 210000 + 15000(F/A, 11.5\%, 8)(A/F, 11.5\%, 14) + 320000(A/F, 11.5\%, 14)$$

Now putting the values of different compound interest factors;

$$AW_1 = -1200000 \times 0.14703 - 38000 + 210000 + 15000 \times 12.07744 \times 0.03203 + 320000 \times 0.03203$$

$$AW_1 = -176436 - 38000 + 210000 + 5803 + 10250$$

$$AW_1 = \text{Rs.11617}$$

The cash flow diagram of Machine-2 is shown in Fig. 2.27.

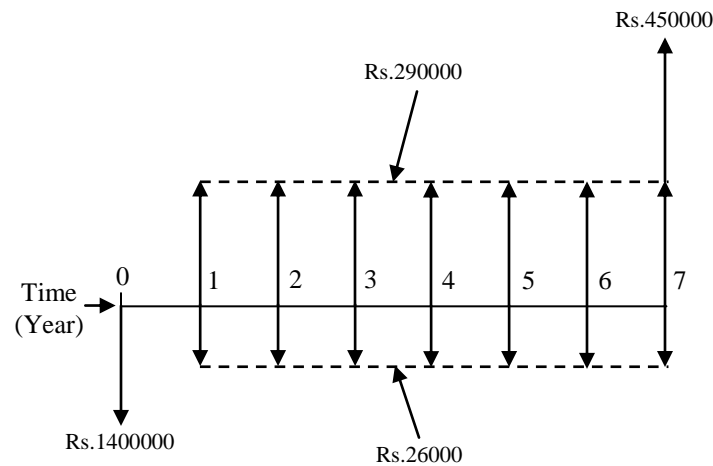


Fig. 2.27 Cash flow diagram of Machine-2

The equivalent uniform annual worth of Machine-2 is calculated as follows;

$$AW_2 = -1400000(A/P, 11.5\%, 7) - 26000 + 290000 + 450000(A/F, 11.5\%, 7)$$

Now putting the values of different compound interest factors in the above expression;

$$AW_2 = -1400000 \times 0.21566 - 26000 + 290000 + 450000 \times 0.10066$$

$$AW_2 = -301924 - 26000 + 290000 + 45297$$

$$AW_2 = \text{Rs.7373}$$

As already stated, the equivalent uniform annual worth for more than one cycle of cash flow will be same as that for the first cycle provided the cash flow in the successive

cycles is exactly same as that in the first cycle. For illustration of this note, the cash flow of Machine-2 is repeated for two cycles with life span of 7 years each (least common multiple of life spans of the alternatives is 14 years).

The cash flow diagram of Machine-2 for two cycles is shown in Fig. 2.28

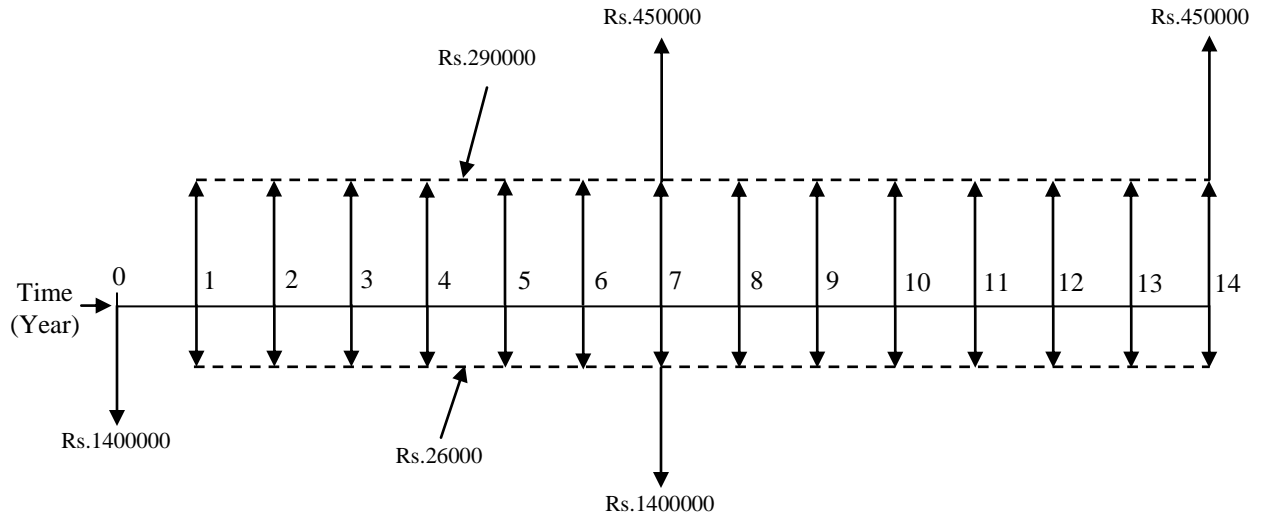


Fig. 2.28 Cash flow diagram of Machine-2 repeated for two cycles

The equivalent uniform annual worth of Machine-2 is determined as follows;

$$AW_2 = -1400000(A/P, 11.5\%, 14) - 26000 + 290000 + 450000(P/F, 11.5\%, 7)(A/P, 11.5\%, 14) \\ - 1400000(P/F, 11.5\%, 7)(A/P, 11.5\%, 14) + 450000(A/F, 11.5\%, 14)$$

Now putting the values of different compound interest factors in the above expression results in the following;

$$AW_2 = -1400000 \times 0.14703 - 26000 + 290000 + 450000 \times 0.46674 \times 0.14703 - \\ 1400000 \times 0.46674 \times 0.14703 + 450000 \times 0.03203 \\ AW_2 = -1400000 \times 0.14703 - 26000 + 290000 - (1400000 - 450000) \times 0.46674 \times 0.14703 + \\ 450000 \times 0.03203 \\ AW_2 = -205842 - 26000 + 290000 - 65194 + 14414$$

$$AW_2 = \text{Rs.7378}$$

Now it can be seen that, the calculated equivalent uniform annual worth of Machine-2 for two cycles of cash flow is same as that with only one cycle i.e. first cycle. The minor difference among the values is due to the effect of decimal points in the above calculations.

Lecture-8

Rate of return:-

The rate of return technique is one of the methods used in selecting an alternative for a project. In this method, the interest rate per interest period is determined, which equates the equivalent worth (either present worth, future worth or annual worth) of cash outflows (i.e. costs or expenditures) to that of cash inflows (i.e. incomes or revenues) of an alternative. The rate of return is also known by other names namely internal rate of return (IRR), profitability index etc. It is basically the interest rate on the unrecovered balance of an investment which becomes zero at the end of the useful life or the study period. In the following lectures, the rate of return is denoted by “ i_r ”.

Using present worth, the equation for rate of return can be written as follows;

$$PW_C = PW_I \dots\dots\dots (2.1)$$

PW_C = Present worth of cash outflows (cost or expenditure)

PW_I = Present worth of cash inflows (income or revenue)

As already stated in earlier lectures, cost or expenditures are considered as negative cash flows whereas income or revenues are considered as positive cash flows.

Equation (2.1) can be rewritten as;

$$0 = -PW_C + PW_I \dots\dots\dots (2.2)$$

In the above equation the net present worth is zero.

Now putting the expressions for present worth of cash outflows and that of cash inflows in equation (2.1) results in the following expression;

$$P_o + F_C(P/F, i_r, n) + A_C(P/A, i_r, n) = F_I(P/F, i_r, n) + A_I(P/A, i_r, n) \dots\dots\dots (2.3)$$

On left hand side of the above equation, P_o is the initial cost at time zero and F_C (single amount) and A_C (uniform amount series) are the expenditures occurring at future period of time. Similarly on the right hand side of the equation, F_I (single amount) and A_I (uniform amount series) are the incomes or revenues occurring at future period of time. The value of interest period ‘n’ will vary depending upon the occurrence of the future amounts (either expenditure or income).

Equation (2.3) can be rewritten as follows;

$$0 = -[P_o + F_C(P/F, i_r, n) + A_C(P/A, i_r, n)] + [F_I(P/F, i_r, n) + A_I(P/A, i_r, n)] \dots \dots \dots (2.4)$$

The value of rate of return ' i_r ' can be calculated by solving the above equation. The equation (2.4) can be solved either manually through trial and error process or using Microsoft Excel spreadsheet. The first method i.e. trial and error process for determination of the rate of return consumes more time whereas the second method is faster. However the trial and error method gives a clear understanding of the analysis of calculation for the rate of return. Similar to equivalent present worth, the rate of return can also be determined by finding out the interest rate at which the net future worth or net annual worth is zero.

After determination of the rate of return for a given alternative, it is compared with minimum attractive rate of return (MARR) to find out the acceptability of this alternative for the project. If the rate of return i.e. i_r is greater than or equal to MARR, then the alternative will be selected or else it will not be selected. The MARR is the minimum rate of return from the investment, which is acceptable. In other words it is the minimum rate of return below which the investment alternatives are economically not acceptable. The minimum attractive rate of return (MARR) serves as an important criteria while selecting a single alternative or comparing mutually exclusive alternatives whenever the investments are made. For an organization, it is governed by various parameters namely availability of financially viable projects, amount of fund available for investment along with the associated risk, and type of organization (i.e. government, public sector, private sector etc.).

The difference between equivalent worth methods (present worth method/future worth method/annual worth method) and rate of return method is that; in case of former, the equivalent worth of the cash inflows and cash outflows are determined at MARR whereas in case of latter, a rate is determined which equates the equivalent worth of cash inflows to that of the cash outflows and the resulting rate is compared against MARR. The rate of return and MARR are expressed in terms of percentage per period i.e. mostly percentage per year.

In the following example, the illustration of the procedure for determination of rate of return for an alternative is presented.

Example -14

A construction firm is planning to invest Rs.800000 for the purchase of a construction equipment which will generate a net profit of Rs.140000 per year after deducting the annual operating and maintenance cost. The useful life of the equipment is 10 years and the expected salvage value of the equipment at the end of 10 years is Rs.200000. Compute the rate of return using trial and error method based on present worth, if the construction firm's minimum attractive rate of return (MARR) is 10% per year.

Solution:

The cash flow diagram of the construction equipment is shown in Fig. 2.29.

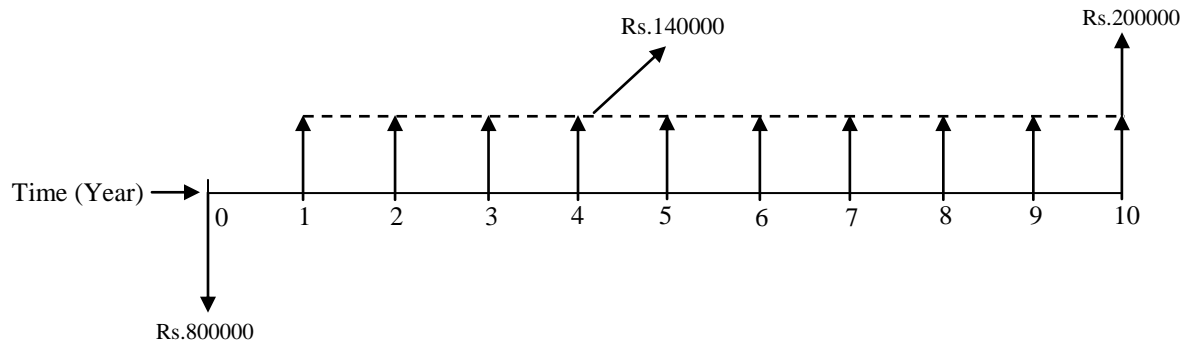


Fig. 2.29 Cash flow diagram of the construction equipment

For determination of rate of return ' i_r ' of the construction equipment, first the equation for net present worth of cash inflows and cash outflows is equated to zero. Then using the trial and error method the value of ' i_r ' is determined. The net present worth of cash inflows and cash outflows of the construction equipment is given by the following expression. $PW = -800000 + 140000(P/A, i_r, 10) + 200000(P/F, i_r, 10)$

For determining the value of ' i_r ' the net present worth is equated to zero.

$$0 = -800000 + 140000(P/A, i_r, 10) + 200000(P/F, i_r, 10)$$

Now the above equation will be solved through trial and error process to find out the value of i_r . Basically a positive value and a negative value of the net present worth will be

determined at rate of return values close to the actual one and then by linear interpolation between these two values, the actual rate of return will be calculated. For finding out the rate of return values (close to the actual one), those will give a positive value and a negative value of net present worth, one has to carry out a number of trial calculations at various values of i_r .

Since MARR is 10%, first assume a value of i_r equal to 8% and compute the net present worth. Now putting the values of different compound interest factors in the expression for net present worth at i_r equal to 8% results in the following;

$$PW = -800000 + 140000(P/A, 8\%, 10) + 200000(P/F, 8\%, 10)$$

$$PW = -800000 + 140000 \times 6.7101 + 200000 \times 0.4632$$

$$PW = \text{Rs.}232054$$

The above calculated net present worth at i_r equal to 8% is greater than zero, now assume a higher value of i_r i.e. 12% for the next trial and compute the net present worth.

$$PW = -800000 + 140000(P/A, 12\%, 10) + 200000(P/F, 12\%, 10)$$

$$PW = -800000 + 140000 \times 5.6502 + 200000 \times 0.3220$$

$$PW = \text{Rs.}55428$$

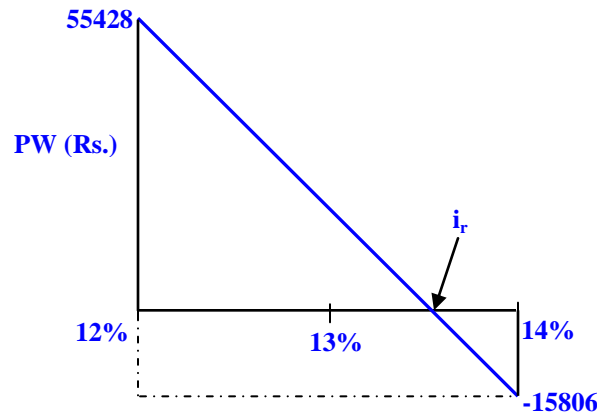
As observed from this calculation, the net present worth is decreased at higher value of i_r . Thus for getting a negative value of net present worth, assume further higher value of i_r than the previous trial and take 14% for the next trial and determine the net present worth.

$$PW = -800000 + 140000(P/A, 14\%, 10) + 200000(P/F, 14\%, 10)$$

$$PW = -800000 + 140000 \times 5.2161 + 200000 \times 0.2697$$

$$PW = \text{-Rs.}15806$$

Since a negative value of net present worth at i_r equal to 14% is obtained (as above), the actual value of rate of return is less than 14%. The actual rate of return is now obtained by doing linear interpolation either between 8% and 14% or between 12% and 14%. However for obtaining a more accurate value of rate of return, the linear interpolation is carried out between 12% and 14% and is given as follows;



PW = Rs.55428 at $i_r = 12\%$

PW = - Rs.15806 at $i_r = 14\%$

$$\frac{55428 - (-15806)}{14\% - 12\%} = \frac{55428 - 0}{i_r - 12\%}$$

On solving the above expression, the value of i_r is found to be 13.55% per year which is greater than MARR (10%). Now using the using Microsoft Excel spreadsheet and entering year-wise cash inflows and cash out flows, the value of rate of return is found to be 13.53% (using the function 'IRR'). However this minor difference in the value of i_r obtained from both the methods can be minimized by finding out the net present worth at narrow range of interest rate values and carrying out linear interpolation between these values (trial and error method) to find out the more precise value close to the actual rate of return.

The net present worth of the construction equipment at MARR i.e. 10% is given by;

$$PW = -800000 + 140000(P/A, 10\%, 10) + 200000(P/F, 10\%, 10)$$

$$PW = -800000 + 140000 \times 6.1446 + 200000 \times 0.3855$$

PW = Rs.137344 at MARR (10%)

The net present worth of the construction equipment at MARR is greater than zero and the rate of return is greater than MARR. Thus the purchase of the construction equipment is economically justified. It may be noted here that when the equivalent worth of an investment is greater than zero at interest rate equal to MARR, then the rate of return of the investment is greater than MARR.

The rate of return ' i_r ' can also be determined by equating the net annual worth to zero. For the above construction equipment, the net equivalent annual worth at different values of i_r are calculated as follows;

At $i_r = 12\%$

$$AW = -800000(A/P, 12\%, 10) + 140000 + 200000(A/F, 12\%, 10)$$

$$AW = -800000 \times 0.1770 + 140000 + 200000 \times 0.0570$$

AW = Rs.9800

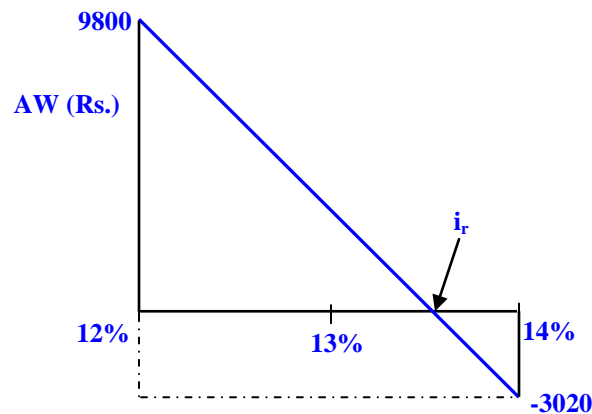
At $i_r = 14\%$

$$AW = -800000(A/P, 14\%, 10) + 140000 + 200000(A/F, 14\%, 10)$$

$$AW = -800000 \times 0.1917 + 140000 + 200000 \times 0.0517$$

AW = -Rs.3020

Now carrying out the linear interpolation between 12% and 14%;



AW = Rs.9800 at $i_r = 12\%$

AW = - Rs.3020 at $i_r = 14\%$

$$\frac{9800 - (-3020)}{14\% - 12\%} = \frac{9800 - 0}{i_r - 12\%}$$

On solving the above expression, the value of i_r is found to be 13.52% per year. The minor difference in the values of i_r from present worth and annual worth methods is due to the effect of decimal points in the calculations. Similar to present worth and annual worth methods, the rate of return ' i_r ' can also be determined by equating the net future worth to zero.

From the above example, a unique value of rate of return was obtained for the construction equipment (on the basis of its cash inflow and cash outflow). This is due to the fact that, there was only one sign change in the cash flows i.e. minus sign at time zero for the cash outflow followed by plus sign for cash inflows during the remaining periods. However in some cases, depending upon the cash flow it is possible to get multiple values of rate of return, those satisfy the rate of return equation of the equivalent worth of cash inflows and cash out flows. This may happen due to more than one sign change in the cash flows e.g. cash outflow (negative) at beginning (time zero) followed cash inflows (positive) at end of year 1 and 2 and then cash outflow (negative) at end of year 3 etc. Thus while selecting an alternative that has multiple values of rate of return (depending on the cash flow), other method of economic evaluation may be adopted to find out the economical suitability of the alternative.

Lecture-9

Incremental Rate of return:-

When the best alternative (economically suitable) is to be selected from two or more mutually exclusive alternatives on the basis of rate of return analysis, the incremental investment analysis is used. In incremental rate of return method, the alternative with larger investment is selected, provided the incremental (extra) investment over the lower investment alternative produces a rate of return that is greater than or equal to MARR. In other words if the additional benefits i.e. increased productivity, increased income, reduced operating expenditure etc. achieved at the expense of extra investment (associated with larger investment alternative) are more than that could have been obtained from the investment of same amount at MARR elsewhere by the organization, then this additional capital should be invested.

In incremental rate of return method, the economically acceptable lower investment alternative is considered as the base alternative against which the higher investment alternative is compared. The cash flow of higher investment alternative is considered equal to the cash flow of lower investment alternative plus the incremental cash flow i.e. difference in cash flow between the higher investment and lower investment alternatives.

When using rate of return method for comparing two or more mutually exclusive alternatives, the analysis must be done correctly, otherwise it may lead to incorrect ranking of the alternatives. However this problem is avoided in incremental investment rate of return analysis. In this technique, the individual rate of return values on total cash flow of the mutually exclusive alternatives are not compared against each other rather the rate of return (or IRR) of the mutually exclusive alternatives or the rate of return of the incremental investment is compared against MARR.

The procedures for comparison of mutually exclusive cost alternatives and that of mutually exclusive investment alternatives using incremental investment rate of return analysis are mentioned below. The details about cost and investment alternatives are already stated in Lecture-1 of Module 2.

Steps for comparison of cost alternatives:

- i)** First arrange the mutually exclusive cost alternatives on the basis of increasing initial capital investment. The lowest capital investment alternative is considered as the base alternative (B).
- ii)** The incremental cash flow is calculated between the base alternative (B) and the next higher capital investment alternative (H) over the useful life.
- iii)** Then the rate of return ' $i_{r(H-B)}$ ' of this incremental investment is calculated (procedure as stated earlier) by equating the net equivalent worth (present worth or annual worth or future worth) to zero.
- iv)** If the calculated ' $i_{r(H-B)}$ ' is greater than or equal to MARR, then alternative 'B' is removed from further analysis. Alternative 'H' now becomes the new base alternative and is compared against the next higher capital investment alternative. If ' $i_{r(H-B)}$ ' is less than MARR, then alternative 'H' is removed from further analysis and alternative 'B' remains as the base alternative and is compared against the next higher investment alternative (alternative with investment higher than 'H').
- v)** Steps ii) to iv) are repeated till only one alternative is left i.e. the best alternative which justifies the incremental investment associated with.

Steps for comparison of investment alternatives:

- i)** Arrange the mutually exclusive investment alternatives on the basis of increasing initial capital investment.
- ii)** Then the rate of return (IRR) on total cash flow of the lowest investment alternative is determined (procedure already stated earlier) to find out its acceptability as the base alternative. If the calculated rate of return is greater than or equal to MARR, this is selected as the base alternative. If the calculated rate of return is less than MARR, then this alternative is not considered for further analysis and the acceptability of the next higher investment alternative as base alternative is found out by calculating the rate of return on its total cash flow and comparing against MARR. This process is continued till the base alternative 'B' (acceptable alternative for which rate of return greater than or equal to MARR) is obtained. If no alternative is obtained in this manner i.e. rate of return less than MARR, then do-nothing alternative is selected. The do-nothing alternative

indicates that all the investment alternatives are rejected. Similar to the comparison of cost alternatives, the incremental cash flow is now calculated between the base alternative (B) and the next higher investment alternative (H) over the useful life.

Steps iii) to v) as mentioned above for the comparison of cost alternatives are then followed to select the best alternative.

The comparison of cost alternatives is illustrated in the following example.

Example -15

The development authority of a city has to select a pumping unit from four feasible mutually exclusive alternatives for supply of water to a particular location of the city. The details of cash flow and the useful life of all the alternatives are presented in the following table. The minimum attractive rate of return (MARR) is 20% per year. Select the best alternative using the incremental investment rate of return analysis.

Solution:

The cash flow and useful life of all the alternatives are presented in Table 2.1.

Table 2.1 Cash flow of alternatives

Alternative Cash flow	Alternative-1 A1	Alternative-2 A2	Alternative-3 A3	Alternative-4 A4
Initial capital investment (Rs.)	7800000	6600000	8100000	7400000
Annual operating and maintenance cost (Rs.)	850000	1185000	800000	970000
Salvage value (Rs.)	2050000	1780000	2200000	1865000
Useful life (Years)	10	10	10	10

As seen from the above table, these are cost alternatives involving all cash outflows (negative cash flows) except for the salvage value (positive cash flow) at the end of useful life. The alternatives are not in the increasing order of capital investment as

observed from Table 2.1. The alternatives are now arranged in the increasing order of capital investment as shown in Table 2.2 and cash outflows and cash inflows are shown with negative and positive signs respectively.

Table 2.2 Cash flow of alternatives in increasing order of initial capital investment

Alternative	Alternative-2	Alternative-4	Alternative-1	Alternative-3
Cash flow	A2	A4	A1	A3
Initial capital investment (Rs.)	-6600000	-7400000	-7800000	-8100000
Annual operating and maintenance cost (Rs.)	-1185000	-970000	-850000	-800000
Salvage value (Rs.)	+1780000	+1865000	+2050000	+2200000
Useful life (Years)	10	10	10	10

After arranging the alternatives in increasing order of capital investment, alternative-2 (A2) now becomes the base alternative (lowest capital investment with Rs.6600000) and it is compared with the next higher investment alternative i.e. alternative-4 (A4) with capital investment of Rs.7400000. The incremental cash flow between the two alternatives A2 and A4

is given as follows;

Incremental capital investment = -Rs.7400000 – (-Rs.6600000) = -Rs.800000 at beginning i.e. at time zero.

Incremental annual operating and maintenance cost from end of year 1 till end of year 10
= -Rs.970000 – (-Rs.1185000) = Rs.215000

Incremental salvage value = Rs.1865000 – Rs.1780000 = Rs.85000 at end of year 10

In order to find out the rate of return (IRR) of this incremental cash flow, the net present worth is equated to zero.

$$PW = 0 = -800000 + 215000(P/A, i_r, 10) + 85000(P/F, i_r, 10)$$

The value of rate of return ' i_r ' is now calculated by solving the above equation either manually through trial and error process with linear interpolation or using Microsoft Excel spreadsheet (already mentioned earlier). For faster calculation, the rate of return is calculated using Microsoft Excel spreadsheet after entering year-wise cash inflows and cash out flows. The value of rate of return is found to be 24.06% (using the function 'IRR' in Excel spreadsheet). As rate of return of the incremental cash flow is greater than MARR (20%), the incremental investment associated with alternative-4 (A4) is justified and alternative-2 (A2) is now removed from further analysis. Alternative-4 now becomes the new base alternative and is compared with next higher capital investment alternative i.e. alternative-1 (A1) with investment of Rs.7800000. The rate return of this incremental investment is calculated in same manner as above. The entire calculation is now presented in the Table 2.3.

Table 2.3 Comparison of cost alternatives using incremental rate of return analysis

Comparison	Between A4 and A2	Between A1 and A4	Between A3 and A1
Incremental cash flow			
Incremental capital investment (Rs.)	-800000	-400000	-300000
Incremental annual operating and maintenance cost i.e. savings (Rs.)	215000	120000	50000
Incremental salvage value (Rs.)	85000	185000	150000
Useful life (Years)	10	10	10
Rate of return on incremental investment	24.06% (> MARR)	28.65% (> MARR)	14.09% (< MARR)
Incremental investment justified	Yes	Yes	No
Present worth of incremental cash flow at MARR, 20% (Rs.)	115115 (> 0)	132978 (> 0)	-66150 (< 0)

In Table 2.3 the incremental values between the alternatives indicate the difference in cash flows between them.

The outcomes of the incremental investment analysis for the comparison of cost alternatives as presented in Table 2.3 are briefly described below.

- Comparison between alternatives **A2** (base alternative) and **A4** (next higher capital investment alternative). The obtained rate of return from the incremental investment analysis is 24.06% which is greater than MARR (20%). Alternative-2 (**A2**) is eliminated from further analysis and alternative-4 (**A4**) is the new base alternative.
- Now comparison between alternatives **A4** and **A1** (next higher capital investment alternative). The obtained rate of return from the incremental investment is 28.65% and is greater than MARR. Thus alternative-4 (**A4**) is eliminated from further analysis and alternative-1 (**A1**) is the new base alternative.
- Finally comparison between alternatives **A1** and **A3** (next higher capital investment alternative). The rate of return obtained from the incremental investment analysis is 14.09% which is less than MARR (20%). Thus the incremental investment associated with alternative-3 (**A3**) i.e. largest capital investment alternative is not justified and **hence alternative-1 (A1) is selected as the best alternative**, as no other alternative is left for comparison. In addition, the present worth of the incremental investment associated with alternative-1 (**A1**) over alternative-4 (**A4**) at MARR i.e. 20% is greater than zero i.e. $\text{Rs.}132978 > 0$.
- It can be seen here that, the largest capital investment alternative (**A3**) is not selected because the incremental investment associated with it results in a rate of return which is less than MARR. In addition the present worth of the incremental investment associated with alternative-3 (**A3**) over alternative-1 (**A1**) at MARR i.e. 20% is less than zero i.e. $-\text{Rs.}66150 < 0$.

Now the values of equivalent present worth of the total cash flow of the cost alternatives at MARR (20%) are found to be -Rs.11280643, -Rs.11165528, -Rs.11098700 and -Rs.11032550 for alternatives A2, A4, A3 and A1 respectively. Thus alternative A1 (the best alternative) exhibits lowest negative equivalent present worth as compared to other cost alternatives.

Lecture-10

Incremental Rate of return:-

The comparison of investment alternatives using incremental investment rate of return analysis is illustrated in the following example.

Example -16

A construction company is planning to invest for the purchase of a heavy construction equipment which will be used at a construction site. There are four feasible alternatives and the detailed cash flow of all the alternatives are presented in Table 2.4. Each alternative has the useful life of 8 years. If the company's MARR is 12% per year, select the best alternative using the incremental investment rate of return analysis.

Solution:

The cash flow and useful life of all the alternatives are presented in Table 2.4.

Table 2.4 Cash flow of alternatives

Alternative	Alternative-1	Alternative-2	Alternative-3	Alternative-4
Cash flow	B1	B2	B3	B4
Initial investment (Rs.)	2400000	3400000	2700000	3200000
Annual profit (after deducting expenditures) (Rs.)	415000	680000	525000	640000
Salvage value (Rs.)	590000	990000	710000	860000
Useful life (Years)	8	8	8	8

As observed from Table 2.4, these are investment alternatives having positive cash flows (cash inflows). As the alternatives are not in the increasing order of investment, they are now arranged in the increasing order of investment as shown in Table 2.5.

Table 2.5 Cash flow of alternatives in increasing order of investment

Alternative	Alternative-1	Alternative-3	Alternative-4	Alternative-2
Cash flow	B1	B3	B4	B2
Initial investment (Rs.)	-2400000	-2700000	-3200000	-3400000
Annual profit (after deducting expenditures) (Rs.)	415000	525000	640000	680000
Salvage value (Rs.)	590000	710000	860000	990000
Useful life (Years)	8	8	8	8

After arranging the alternatives in increasing order of investment, the acceptability of the lowest investment alternative i.e. alternative B1 as base alternative is checked by finding out the rate of return on its total cash flow. The rate of return is found out by equating net present worth of alternative B1 to zero.

$$PW = 0 = -2400000 + 415000(P/A, i_r, 8) + 590000(P/F, i_r, 8)$$

As already stated, the value of rate of return ' i_r ' is calculated by solving the above equation either manually through trial and error process with linear interpolation or using Microsoft Excel spreadsheet. The value of rate of return is found to be 10.91% (using the function 'IRR' in Excel spreadsheet), which is less than company's MARR i.e. 12%. Thus alternative B1 is eliminated from further calculation and the acceptability of next higher investment alternative i.e. B3 is checked in the same manner as above. The rate of return on total cash flow of alternative B3 is determined by equating the net present worth to zero.

$$PW = 0 = -2700000 + 525000(P/A, i_r, 8) + 710000(P/F, i_r, 8)$$

From this equation the value of rate of return is found to be 13.84% (using the function 'IRR' in Excel spreadsheet), which is greater than company's MARR i.e. 12%. Thus Alternative B3 becomes the base alternative and it is compared with the next higher investment alternative B4. Now the rate of return of the incremental cash flow between

the two alternatives B3 and B4 is calculated and compared with MARR to check suitability of the incremental investment associated with alternative B4. The entire calculation for selecting the best alternative is now presented in Table 2.6.

Table 2.6 Comparison of investment alternatives using incremental rate of return analysis

Alternatives	B1	B3	B4 – B3	B2 – B4
Cash flow				
Initial investment / Incremental value (Rs.)	-2400000	-2700000	-500000	-200000
Annual profit (after deducting expenditures)/ Incremental value (Rs.)	415000	525000	115000	40000
Salvage / Incremental value (Rs.)	590000	710000	150000	130000
Useful life (Years)	8	8	8	8
Rate of return on total cash flow/ Rate of return on incremental investment	10.91% (< MARR)	13.84% (> MARR)	18.52% (> MARR)	17.69% (> MARR)
Acceptable as base alternative / Incremental investment justified	No	Yes	Yes	Yes
Present worth of incremental cash flow at MARR 12% (Rs.)	-	-	131859 (> 0)	51211 (> 0)

In the above table the incremental values between the alternatives indicate the difference in their cash flows.

In Table 2.6, the total cash flows of the individual alternatives and incremental cash flows (comparison between two alternatives) are written in different colour fonts for ease of understanding. For alternative B1 and B3, total cash flows and for comparison between B4 – B3 and B2 – B4 the incremental cash flows are provided in the above table.

The outcomes of the incremental investment analysis for comparison of investment alternatives (presented in Table 2.6) are briefly described below.

- Checking the acceptance of lowest investment alternative **B1** as base alternative by calculating the rate of return on its total cash flow. The obtained rate of return is 10.91% which is less than MARR (12%). Alternative **B1** is now eliminated from further analysis.
- Now checking the acceptance of next higher investment alternative **B3** as the base alternative by finding out the rate of return on its total cash flow. The obtained rate of return is 13.84% which is greater than MARR (12%). Thus alternative **B3** now becomes the base alternative.
- Now comparison between alternatives **B3** and next higher investment alternative **B4**. The rate of return from the incremental investment analysis is found to be 18.52% which is greater than MARR (12%). Thus alternative **B3** is eliminated from further analysis and alternative **B4** is the new base alternative as the incremental investment is justified.
- Finally comparison between alternatives **B4** and **B2** (next higher investment alternative). The obtained rate of return from the incremental investment is 17.69% which is greater than MARR (12%). Thus the incremental investment associated with alternative **B2** is justified and **alternative B2 is selected as the best alternative**, as there is no other alternative remaining for comparison. Further the present worth of the incremental investment associated with alternative **B2** over alternative **B4** at MARR (12%) is greater than zero as observed from Table 2.6.

Now the values of equivalent present worth of the total cash flow of individual alternatives at MARR (12%) are found to be -Rs.100145, Rs.194759, Rs.326618 and Rs.377829 for alternatives B1, B3, B4 and B2 respectively. Thus alternative B2 exhibits highest positive equivalent present worth as compared to other alternatives which is in agreement with outcome obtained from the incremental investment analysis.

For comparison of mutually exclusive alternatives, those have different life spans, the comparison using incremental rate of return analysis must be made over the same number

of years i.e. least common multiple of the individual life spans as in case of comparison using present worth method in earlier lectures.

The important points to be noted for comparison of mutually exclusives alternatives using incremental investment rate of return analysis are as follows.

- The alternative with larger investment should be selected if the incremental (extra) investment associated with it over the lower investment alternative produces a rate of return that is greater than or equal to MARR.
- While comparing the alternatives, at first instance the alternative with highest rate of return on its total cash flow should not be selected as the best alternative. After carrying out the incremental investment rate of return analysis, the selection of the best alternative may match (depending on the cash flow) with the alternative with highest rate of return on its total cash flow. The rate of return of the alternatives B1, B2, B3 and B4 (Example 16) on their total cash flows are found to be 10.91%, 14.80%, 13.84% and 14.59% respectively (calculations not shown in the example). Thus Alternative B2 having highest rate of return (also greater than MARR i.e. $14.80\% > 12\%$) was not selected as the best alternative at first instance. However from outcome of incremental investment rate of return analysis, alternative B2 was selected as the best alternative that matched with the alternative having highest rate of return.
- Similarly the alternative with highest capital investment and has rate of return (on its total cash flow) greater than or equal to MARR, should not be selected as the best alternative at first instance. However the outcome of the incremental investment rate of return analysis for the best alternative may coincide with the highest investment alternative (with rate of return on its total cash flow greater than or equal to MARR). From incremental investment rate of return analysis, alternative B2 (Example 16) was selected as the best alternative, which happened to be the highest investment alternative with rate of return on its total cash flow greater than MARR.

- The higher capital investment alternative associated with the incremental capital investment that exhibits highest rate of return should not be selected as the best alternative. As observed from Example 16, incremental investment having highest rate of return (18.52%) was between alternatives B4 and B3 and the higher investment alternative associated with this incremental investment was B4. On the other hand, the rate of return on the incremental capital investment between B2 and B4 was 17.69% and the higher investment alternative associated with this incremental investment was B2. From the incremental rate of return analysis, alternative B2 was selected as the best alternative in place of B4, even though the rate of return on incremental investment associated with B4 (comparison between B4 and B3) was greater than the rate of return on the incremental investment associated with B2 (comparison between B2 and B4) i.e. $18.52\% > 17.69\%$.

Lecture-11

Capitalized cost analysis:-

Capitalized cost represents the present worth of an alternative for a project that is going to serve for a longer period of time i.e. for an infinite period of time. As the name indicates, it refers to the present worth of mainly cost or expenditures (cash outflows) of the alternative over infinite period of time. Capitalized worth refers to present worth of expenditures and revenues of an alternative over infinite period of time.

The capitalized cost method is used for comparison of mutually exclusive alternatives which have perpetual service life (assumed to serve forever). The examples of this kind of projects are bridges, dams, irrigation projects, water supply systems for cities, pipeline projects etc. This method can also be used for finding out the capitalized cost of permanent fellowship/scholarship endowment in educational institutes and other organizations.

As already stated, capitalized cost/worth is the present worth of an alternative that has a perpetual or permanent cash flow series. The capitalized cost/worth of a perpetual cash flow having uniform amount series with end of year payments 'A' is obtained as follows.

The present worth of a uniform amount series is given by;

$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

P = present worth, A = end of year payment of uniform amount series, i = interest rate per year and n = number of interest periods (already stated in Module 1)

The above equation can be rewritten as;

$$P = \frac{A}{i} \left[\frac{(1+i)^n}{(1+i)^n} - \frac{1}{(1+i)^n} \right]$$

$$P = \frac{A}{i} \left[1 - \frac{1}{(1+i)^n} \right]$$

In the above expression, when 'n' approaches infinity (i.e. for perpetual cash flow series), the term " $1/(1+i)^n$ " gets neglected and present worth 'P' becomes capitalized cost/worth, the expression of which is given by;

$$P = \frac{A}{i}$$

$$A = P \times i$$

Thus the above expression can be explained by a simple example.

If a person deposits an amount of Rs.100000 in a bank at an interest rate of 10% per year, then how much money the person can withdraw from the bank every year without reducing the original deposit.

- The amount of interest accumulated (i.e. amount of money the person can withdraw without reducing the deposit) at the end of 1st year is Rs.10000 (100000 x 0.10).
- After withdrawal of Rs.10000 at the end of 1st year, the amount that will remain in his bank account is Rs.100000, which will again earn an interest amount of Rs.10000 at the end of 2nd year. After withdrawal of Rs.10000 at the end of 2nd year, the person will have again the same amount of Rs.100000 in his bank account that will further earn the same interest amount at the end of 3rd year. This process can be continued perpetually generating an interest amount of Rs.10000 ($A = P \times i$) without changing the original deposit of Rs.100000. This example is similar to creating a scholarship endowment in a university that will give a scholarship of Rs.10000 every year indefinitely, by depositing Rs.100000 in a saving bank account at interest rate of 10% per year compounded annually.

The capitalized cost of a single amount occurring at regular intervals in future period of time is calculated by first finding out the equivalent uniform annual worth 'A' of the uniform series and then dividing by the interest rate 'i'.

The calculation of capitalized cost of an alternative is illustrated in the following example.

Example -17

A public project has an initial cost of Rs.11000000 and annual operating and maintenance cost of Rs.700000. Further the project will have one time major repair work of Rs.2000000 at the end of 15 year. Find out the capitalized cost of the alternative if interest rate is 12% per year.

Solution:

The capitalized cost of the alternative is equal to sum of the initial cost, present worth of one time major repair cost and capitalized cost of the annual operating and maintenance cost.

The total capitalized cost of the alternative is given by;

$$\text{Capitalized Cost} = -11000000 - 2000000(P/F, i, n) - 700000/i$$

$$\text{Capitalized Cost} = -11000000 - 2000000(P/F, 12\%, 15) - 700000/0.12$$

$$\text{Capitalized Cost} = -11000000 - 2000000 \times 0.1827 - 5833333$$

Capitalized cost = -Rs.17198733

Therefore the capitalized cost of the alternative for the public project is found to be **Rs.17198733.**

Lecture-12

Capitalized cost analysis:-

The capitalized cost can also be used for comparison of two or more mutually exclusive alternatives which are assumed to serve perpetually. In this case the comparison of the alternatives is made over same time period i.e. infinite period of time. The alternative that shows lowest capitalized cost is selected as the best alternative.

In the following examples, the calculation of capitalized cost of an alternative and the comparison of mutually exclusive alternatives on the basis of capitalized cost are illustrated.

Example -18

The initial cost of an infrastructure development project which is expected to serve residents of a city perpetually is Rs.15000000. The annual operating cost is Rs.800000 for first 10 years and Rs.900000 in the subsequent years (i.e. from year 11 onwards). The expected cost of renovation at the end of every 15 years is Rs.1800000. Find out the capitalized cost of the project at interest rate of 8% per year.

Solution:

The cash flow diagram of the project for a part of service life is shown in Fig. 2.30

The total capitalized cost of the project is equal to sum of the initial cost and capitalized cost of annual operating cost and renovation cost.

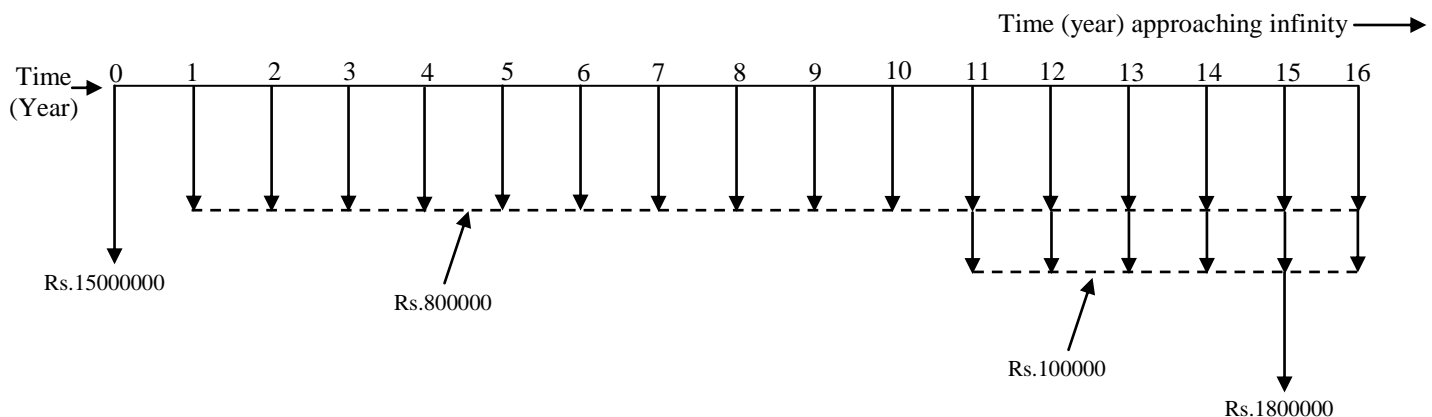


Fig. 2.30 Cash flow diagram of the project

The annual operating cost is Rs.800000 for first 10 years followed by Rs.900000 thereafter. This can be represented as Rs.800000 from end of year 1 to infinite period of

time and Rs.100000 from end of year 11 to infinity as shown in Fig. 2.30. Thus the capitalized cost of the annual operating cost is equal to the sum of capitalized cost of these two components.

Capitalized cost of the annual operating cost:

$$\text{Capitalized Cost} = -\frac{800000}{i} - \frac{100000}{i}(P/F, i, 10)$$

In the above expression, the capitalized cost of Rs.100000 from end of year 11 till infinity is located at the end of year 10. Now the present worth (i.e. amount at time zero) of this amount is calculated by multiplying it with single payment present worth factor.

$$\text{Capitalized Cost} = -\frac{800000}{0.08} - \frac{100000}{0.08}(P/F, 8\%, 10)$$

$$\text{Capitalized Cost} = -10000000 - \frac{100000 \times 0.4632}{0.08}$$

Capitalized cost = -Rs.10579000

The capitalized cost of the annual operating cost can also be calculated by considering Rs.800000 from end of year 1 till end of year 10 and Rs.900000 from end of year 11 till infinity. The calculation is shown below.

$$\text{Capitalized Cost} = -800000(P/A, i, 10) - \frac{900000}{i}(P/F, i, 10)$$

In this expression, first the present worth of uniform series with annual amount of Rs.800000 for first 10 years is calculated. Then the capitalized cost of Rs.900000 from end of year 11 till infinity is calculated in the same manner as for Rs.100000 in the first approach.

$$\text{Capitalized Cost} = -800000(P/A, 8\%, 10) - \frac{900000}{0.08}(P/F, 8\%, 10)$$

$$\text{Capitalized Cost} = -800000 \times 6.7101 - \frac{900000 \times 0.4632}{0.08}$$

Capitalized cost = -Rs.10579080

Thus it can be seen that the capitalized cost of annual operating cost by both ways is same. The minor difference between the values is due to the effect of decimal points in the calculations.

Capitalized cost of renovation component:

The renovation will take place at the end of every 15 years i.e. at the end of 15th year, 30th year, 45th year etc. In order to calculate the capitalized cost of renovation component (Rs.1800000) at the end of every 15 years, first the uniform annual amount 'A' of the equivalent uniform series for the first 15 years is calculated. This amount 'A' will be same for the subsequent intervals i.e. from end of year 16 till end of year 30, from end of year 31 till end of year 45 and so on (shown in Fig. 2.31). Then the uniform amount 'A' is divided by the interest rate.

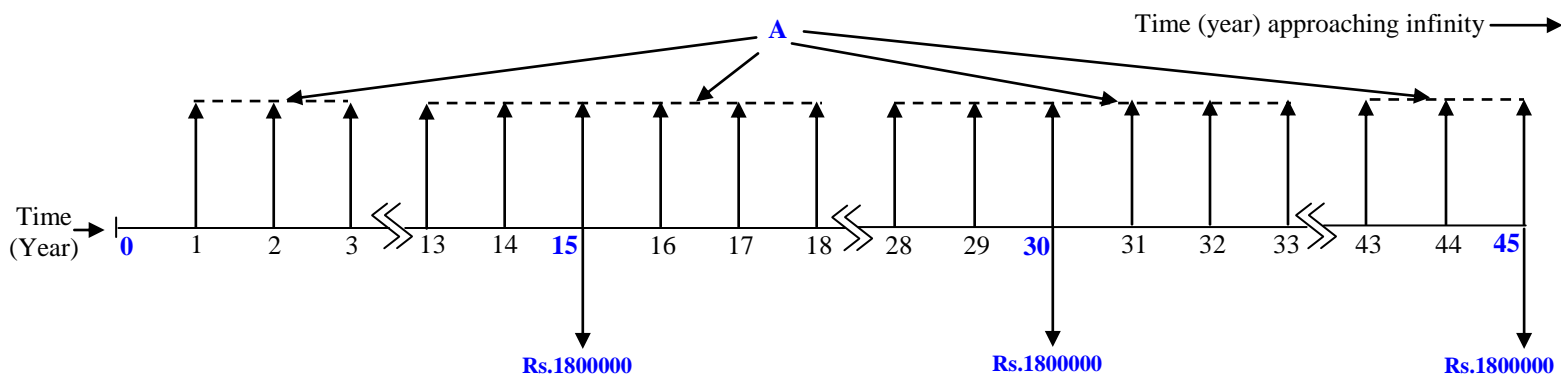


Fig. 2.31 Cash flow diagram for periodic renovation cost of the project

$$\text{Capitalized Cost} = \frac{-1800000(A/F, 8\%, 15)}{0.08}$$

$$\text{Capitalized Cost} = \frac{-1800000 \times 0.0368}{0.08}$$

Capitalized cost = -Rs.828000

Now the total capitalized cost of the project is equal to initial cost plus capitalized cost of annual operating cost and renovation cost.

Capitalized cost = -Rs.15000000 -Rs.10579000 -Rs.828000

Capitalized cost = -Rs.26407000

Therefore the capitalized cost of the infrastructure development project is **Rs.26407000.**

Example -19

There are two alternatives for a water supply project in a city. The details of cash flow of the alternatives are shown below.

Alternative-1

Initial cost = Rs.20000000

Annual operating cost = Rs.1600000

Cost of renovation = Rs.2500000 at the end of every 17 years

One time upgrading cost = Rs.3200000 at the end of 22 year

Alternative-2

Initial cost = Rs.26000000

Annual operating cost = Rs.1200000

Cost of renovation = Rs.3500000 at the end of every 20 years

Compare the alternatives on the basis of capitalized cost and find out the economical alternative if the rate of interest is 9% per year

Solution:

The capitalized cost of Alternative-1 will be equal to initial cost plus the capitalized cost of annual operating cost, periodic renovation cost and one time upgrading cost.

The capitalized cost of Alternative-1 is calculated as follows;

$$\text{Capitalized Cost} = -20000000 - \frac{1600000}{i} - \frac{2500000(A/F, i, 17)}{i} - 3200000(P/F, i, 22)$$

$$\text{Capitalized Cost} = -20000000 - \frac{1600000}{0.09} - \frac{2500000(A/F, 9\%, 17)}{0.09} - 3200000(P/F, 9\%, 22)$$

$$\text{Capitalized Cost} = -20000000 - 17777778 - \frac{2500000 \times 0.0270}{0.09} - 3200000 \times 0.1502$$

$$\text{Capitalized Cost} = -20000000 - 17777778 - 750000 - 480640$$

Capitalized cost = -Rs.39008418 (Alternative-1)

The capitalized cost of Alternative-2 will be equal to initial cost plus the capitalized cost of annual operating cost and periodic renovation cost.

The capitalized cost of Alternative-2 is obtained as follows;

$$\text{Capitalized Cost} = -26000000 - \frac{1200000}{i} - \frac{3500000(A/F, i, 20)}{i}$$

$$\text{Capitalized Cost} = -26000000 - \frac{1200000}{0.09} - \frac{3500000(A/F, 9\%, 20)}{0.09}$$

$$\text{Capitalized Cost} = -26000000 - 13333333 - \frac{3500000 \times 0.0195}{0.09}$$

$$\text{Capitalized Cost} = -26000000 - 13333333 - 758333$$

Capitalized cost = -Rs.40091666 (Alternative-2)

The capitalized cost of alternative-1 and Alternative-2 are found to be **Rs.39008418** and **Rs.40091666** respectively. Thus Alternative-1 is the economical as it shows lower capitalized cost (lower negative value) as compared to Alternative-2.

Lecture-13

Benefit-cost analysis:-

The benefit-cost analysis method is mainly used for economic evaluation of public projects which are mostly funded by government organizations. In addition this method can also be used for economic evaluation of alternatives for private projects. The main objective of this method is used to find out desirability of public projects as far as the expected benefits on the capital investment are concerned. As the name indicates, this method involves the calculation of ratio of benefits to the costs involved in a project.

In benefit-cost analysis method, a project is considered to be desirable, when the net benefit (total benefit less disbenefits) associated with it exceeds its cost. Thus it becomes imperative to list out separately the costs, benefits and disbenefits associated with a public project. Costs are the expenditures namely initial capital investment, annual operating cost, annual maintenance cost etc. to be incurred by the owner of the project and salvage value if any is subtracted from the costs. Benefits are the gains or advantages whereas disbenefits are the losses, both of which are experienced by the owner in the project. In case of public projects which are funded by the government organizations, owner is the government. However this fund is generally taxpayers' money i.e. tax collected by government from general public, thereby the actual owners of public projects are the general public. Thus in case of public projects, the cost is incurred by the government whereas the benefits and disbenefits are mostly experienced by the general public.

In order to know the costs, benefits and disbenefits associated with a public project, consider that a public sector organization is planning to set up a thermal power plant at a particular location. The costs to be incurred by the public sector organization are cost of purchasing the land required for the thermal power plant, cost of construction of various facilities, cost of purchase and installation of various equipments, annual operating and maintenance cost, and other recurring costs etc. The benefits associated with the project are generation of electric power that will cater to the need of the public, generation of revenue by supplying the electricity to the customers, job opportunity for local residents, development other infrastructure in the nearby areas etc. The disbenefits associated with project are loss of land of the local residents on which the thermal power plant will come

up. If it is agricultural land, then the framers will lose their valuable land along with the annual revenue generated from farming, even though they get money for their land from the public sector organization at the beginning. The other disbenefits to the local residents are greater likelihood of air pollution in the region because of the thermal power plant, chances of contamination of water in the nearby water-bodies etc.

In benefit-cost analysis method, the time value of money is taken in to account for calculating the equivalent worth of the costs and benefits associated with a project. The benefit-cost ratio of a project is calculated by taking the ratio of the equivalent worth of benefits to that of the costs associated with that project. Either of present worth, annual worth or future worth methods can be used to find out the equivalent worth of costs and benefits associated with the project.

The benefit-cost ratio of projects is determined in different forms namely conventional benefit-cost ratio and modified benefit-cost ratio. The benefit-cost ratio is generally designated as **B/C ratio**.

Conventional B/C ratio

The conventional benefit-cost ratio of a project is mentioned as follows;

$$\text{Conventional B/C ratio} = \frac{\text{Equivalent worth of Benefits} - \text{Equivalent worth of Disbenefits}}{\text{Equivalent worth of total cost} - \text{Equivalent worth of salvage value}}$$

The disbenefits associated with the project are subtracted from the benefits in the numerator of the ratio to obtain the net benefit associated with the project. Similarly the equivalent worth of salvage value of the initial investment is subtracted from equivalent worth of cost in the denominator of the ratio. The total cost mainly consists of initial cost (initial capital investment) plus the operating and maintenance cost.

As already stated the equivalent worth may be calculated either by present worth method, annual worth method or future worth method. Thus the expression for conventional benefit-cost ratio (B/C ratio) is mentioned as follows;

$$\text{Conventional B/C ratio} = \frac{\text{PW of Benefits} - \text{PW of Disbenefits}}{\text{Initial cost} + \text{PW of operating and maintenance cost} - \text{PW of salvage value}}$$

Or

$$\text{Conventional B/C ratio} = \frac{\text{AW of Benefits} - \text{AW of Disbenefits}}{\text{AW of initial cost} + \text{AW of operating and maintenance cost} - \text{AW of salvage value}}$$

Or

$$\text{Conventional B/C ratio} = \frac{\text{FW of Benefits} - \text{FW of Disbenefits}}{\text{FW of initial cost} + \text{FW of operating and maintenance cost} - \text{Salvage value}}$$

In the above expressions, PW, AW, and FW refer to equivalent present worth, annual worth and future worth respectively.

Modified B/C ratio

In the modified benefit-cost ratio method, the operating and maintenance cost is subtracted from the benefits in the numerator of the ratio. In other words, operating and maintenance cost is considered similar to the disbenefits. The expression for modified benefit-cost ratio using PW, AW or FW is given as follows;

$$\text{Modified B/C ratio} = \frac{\text{PW of Benefits} - \text{PW of Disbenefits} - \text{PW of operating and maintenance cost}}{\text{Initial cost} - \text{PW of salvage value}}$$

Or

$$\text{Modified B/C ratio} = \frac{\text{AW of Benefits} - \text{AW of Disbenefits} - \text{AW of operating and maintenance cost}}{\text{AW of initial cost} - \text{AW of salvage value}}$$

Or

$$\text{Modified B/C ratio} = \frac{\text{FW of Benefits} - \text{FW of Disbenefits} - \text{FW of operating and maintenance cost}}{\text{FW of initial cost} - \text{Salvage value}}$$

A project is considered to be acceptable when the conventional or modified B/C ratio is greater than or equal to 1.0. The illustration of conventional and modified B/C ratio methods is described in the following example.

Example -20

The cash flow details of a public project is as follows

Initial cost = Rs.21000000

Annual operating cost = Rs.1600000

Worth of annual benefits = Rs.5000000

Worth of annual disbenefits = Rs.1100000

Salvage value = Rs.4000000

Interest rate per year = 8% and useful life = 30 Years

Using benefit-cost ratio method (both conventional and modified), find out the economical acceptability of the public project. Use PW, AW and FW methods to find out the equivalent worth of costs, benefits and disbenefits.

Solution:

First the conventional benefit-cost ratio (B/C ratio) of the project is computed.

Conventional B/C ratio using Present worth:

The conventional benefit-cost ratio of the public project is calculated as follows;

$$\text{Conventional B/C ratio} = \frac{\text{PW of benefits} - \text{PW of disbenefits}}{\text{Initial cost} + \text{PW of operating cost} - \text{PW of salvage value}}$$

$$\text{Conventional B/C ratio} = \frac{5000000(P/A, i, n) - 1100000(P/A, i, n)}{21000000 + 1600000(P/A, i, n) - 4000000(P/F, i, n)}$$

$$\text{Conventional B/C ratio} = \frac{5000000(P/A, 8\%, 30) - 1100000(P/A, 8\%, 30)}{21000000 + 1600000(P/A, 8\%, 30) - 4000000(P/F, 8\%, 30)}$$

$$\text{Conventional B/C ratio} = \frac{5000000 \times 11.2578 - 1100000 \times 11.2578}{21000000 + 1600000 \times 11.2578 - 4000000 \times 0.0994}$$

$$\text{Conventional B/C ratio} = 1.137$$

Conventional B/C ratio using Annual worth:

$$\text{Conventional B/C ratio} = \frac{\text{AW of benefits} - \text{AW of disbenefits}}{\text{AW of initial cost} + \text{AW of operating cost} - \text{AW of salvage value}}$$

$$\text{Conventional B/C ratio} = \frac{5000000 - 1100000}{21000000(A/P, i, n) + 1600000 - 4000000(A/F, i, n)}$$

$$\text{Conventional B/C ratio} = \frac{5000000 - 1100000}{21000000(A/P, 8\%, 30) + 1600000 - 4000000(A/F, 8\%, 30)}$$

$$\text{Conventional B/C ratio} = \frac{5000000 - 1100000}{21000000 \times 0.0888 + 1600000 - 4000000 \times 0.0088}$$

$$\text{Conventional B/C ratio} = 1.137$$

Conventional B/C ratio using Future worth:

$$\text{Conventional B/C ratio} = \frac{\text{FW of benefits} - \text{FW of disbenefits}}{\text{FW of initial cost} + \text{FW of operating cost} - \text{Salvage value}}$$

$$\text{Conventional B/C ratio} = \frac{5000000(F/A, i, n) - 1100000(F/A, i, n)}{21000000(F/P, i, n) + 1600000(F/A, i, n) - 4000000}$$

$$\text{Conventional B/C ratio} = \frac{5000000(F/A, 8\%, 30) - 1100000(F/A, 8\%, 30)}{21000000(F/P, 8\%, 30) + 1600000(F/A, 8\%, 30) - 4000000}$$

$$\text{Conventional B/C ratio} = \frac{5000000 \times 113.2832 - 1100000 \times 113.2832}{21000000 \times 10.0627 + 1600000 \times 113.2832 - 4000000}$$

$$\text{Conventional B/C ratio} = 1.137$$

As calculated above, the conventional benefit-cost ratio is found to be same by using any of the equivalent worth methods i.e. PW method, AW method or FW method. As the benefit-cost ratio of the public project is 1.137 (i.e. greater than 1.0), the project is acceptable.

Now the modified benefit-cost ratio (B/C ratio) of the project is calculated.

Modified B/C ratio using Present worth:

The modified benefit-cost ratio of the public project is calculated as follows;

$$\text{Modified B/C ratio} = \frac{\text{PW of benefits} - \text{PW of disbenefits} - \text{PW of operating cost}}{\text{Initial cost} - \text{PW of salvage value}}$$

$$\text{Modified B/C ratio} = \frac{5000000(P/A, i, n) - 1100000(P/A, i, n) - 1600000(P/A, i, n)}{21000000 - 4000000(P/F, i, n)}$$

$$\text{Modified B/C ratio} = \frac{5000000(P/A, 8\%, 30) - 1100000(P/A, 8\%, 30) - 1600000(P/A, 8\%, 30)}{21000000 - 4000000(P/F, 8\%, 30)}$$

$$\text{Modified B/C ratio} = \frac{5000000 \times 11.2578 - 1100000 \times 11.2578 - 1600000 \times 11.2578}{21000000 - 4000000 \times 0.0994}$$

$$\text{Modified B/C ratio} = 1.257$$

Modified B/C ratio using Annual worth:

$$\text{Modified B/C ratio} = \frac{\text{AW of benefits} - \text{AW of disbenefits} - \text{AW of operating cost}}{\text{AW of initial cost} - \text{AW of salvage value}}$$

$$\text{Modified B/C ratio} = \frac{5000000 - 1100000 - 1600000}{21000000(A/P, i, n) - 4000000(A/F, i, n)}$$

$$\text{Modified B/C ratio} = \frac{5000000 - 1100000 - 1600000}{21000000(A/P, 8\%, 30) - 4000000(A/F, 8\%, 30)}$$

$$\text{Modified B/C ratio} = \frac{5000000 - 1100000 - 1600000}{21000000 \times 0.0888 - 4000000 \times 0.0088}$$

$$\text{Conventional B/C ratio} = 1.257$$

Modified B/C ratio using Future worth:

$$\text{Modified B/C ratio} = \frac{\text{FW of benefits} - \text{FW of disbenefits} - \text{FW of operating cost}}{\text{FW of initial cost} - \text{Salvage value}}$$

$$\text{Modified } B/C \text{ ratio} = \frac{5000000(F/A, i, n) - 1100000(F/A, i, n) - 1600000(F/A, i, n)}{21000000(F/P, i, n) - 4000000}$$

$$\text{Modified } B/C \text{ ratio} = \frac{5000000(F/A, 8\%, 30) - 1100000(F/A, 8\%, 30) - 1600000(F/A, 8\%, 30)}{21000000(F/P, 8\%, 30) - 4000000}$$

$$\text{Modified } B/C \text{ ratio} = \frac{5000000 \times 113.2832 - 1100000 \times 113.2832 - 1600000 \times 113.2832}{21000000 \times 10.0627 - 4000000}$$

Modified B/C ratio = 1.257

The modified benefit-cost ratio of the public project is found to be 1.257.

As observed from above calculations, the B/C ratio of the project from both methods (conventional and modified) is greater than 1.0, although the value is different. It may be noted here that, although the magnitude of benefit-cost ratio differs between two methods i.e. conventional B/C ratio and modified B/C ratio, but the decision to select or not a project is not changed by use of any of the two methods.