Keen - jl

This code simulates the Keen model as specified in Grasselli and Costa Lima (2012)
Authors: B. Costa Lima and M. Grasselli

Links & source

Grasselli, M. R., and B. Costa Lima. 2012. "**An Analysis of the Keen Model for Credit Expansion, Asset Price Bubbles and Financial Fragility.**" Mathematics and Financial Economics 6 (3): 191–210. https://doi.org/10.1007/s11579-012-0071-8 (https://doi.org/10.1007/s11579-012-0071-8).

https://ms.mcmaster.ca/~grasselli/GrasselliCostaLima_MAFE_online.pdf (https://ms.mcmaster.ca/~grasselli/GrasselliCostaLima_MAFE_online.pdf)
https://www.oecd.org/naec/projects/naecinnovationlabevents/matheus_grasselli_OECD_masterclass
(https://www.oecd.org/naec/projects/naecinnovationlabevents/matheus_grasselli_OECD_masterclas
https://ms.mcmaster.ca/~grasselli/keen.html (https://ms.mcmaster.ca/~grasselli/keen.html)

https://diffeq.sciml.ai/stable/tutorials/ode_example/ (https://diffeq.sciml.ai/stable/tutorials/ode_example/)

- Grasselli, M. R., and B. Costa Lima. 2012. "An Analysis of the Keen Model for Credit Expansion, Asset Price Bubbles and Financial Fragility." Mathematics and Financial Economics 6 (3): 191–210. https://doi.org/10.1007/s11579-012-0071-8 (https://doi.org/10.1007/s11579-012-0071-8).
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- Keen, Steve. 1995. "Finance and Economic Breakdown: Modeling Minsky's 'Financial Instability Hypothesis.'" Journal of Post Keynesian Economics 17 (4): 607–35. http://econpapers.repec.org/article/mespostke/v_3a17_3ay_3a1995_3ai_3a4_3ap_3a607-635.htm (http://econpapers.repec.org/article/mespostke/v_3a17_3ay_3a1995_3ai_3a4_3ap_3a607-635.htm).
- ———. 2013. "A Monetary Minsky Model of the Great Moderation and the Great Recession." Journal of Economic Behavior & Organization 86 (February): 221–35. https://doi.org/10.1016/j.jebo.2011.01.010 (https://doi.org/10.1016/j.jebo.2011.01.010).

Preamble

"C:\Users\ibuckley\OneDrive - CSTO\Code\Grasselli\2012 Keen"

In [12]:
```julia
# using Pkg;
# Pkg.add("ParameterizedFunctions")
# Pkg.add("DifferentialEquations")
# Pkg.add("PlotlyBase")
```

In [111]:
```julia
using DifferentialEquations # for ODEProblem
using DataFrames
using Plots
plotly() # Choose Plotly as the backend to Plots
```

```
┌ Info: Precompiling DataFrames [a93c6f00-e57d-5684-b7b6-d8193f3e46c0]
└ @ Base loading.jl:1260
```

Out[111]:
```
Plots.PlotlyBackend()
```

In [ ]:
```julia
#using Plotly
#Plotly.signin("IanBuckley","Z4HJZpdeW4wss1tnizq8")
```

Keen model

Notation

- D the amount of debt in real terms
- r is a constant real interest rate and
- $d = D/Y$ is the debt ratio in the economy

Capital stock

- The key insight provided by Minsky is that current cash-flows validate past liabilities and form the basis for future ones.
- In other words, high net profits lead to more borrowing whereas low net profits (possibly negative) lead to a deleveraging of the economy.
- Keen formalizes this insight by taking the **change in capital stock** to be

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K$$

where the rate of new investment is a nonlinear increasing function κ of the net profit share $\pi = (1 - \omega - rd)$ and δ is a constant depreciation rate as before.

Total output

$$\frac{\dot{Y}}{Y} = \frac{\kappa(1-\omega-rd)}{v} - \delta := g(\omega, d)$$

Employment rate

$$\frac{\dot{\lambda}}{\lambda} = \frac{\kappa(1-\omega-rd)}{v} - \alpha - \beta - \delta$$

Debt

The new dynamic variable in this model is the amount of debt, which changes based on the difference between new investment and net profits.

$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y$$

$$\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{Y}}{Y} = \frac{\kappa(1-\omega-rd) - (1-\omega-rd)}{d} - \frac{\kappa(1-\omega-rd)}{v} + \delta.$$

Parameters

State variables

- ω - wage share of the economy
- λ - employment rate
- d - debt (or reciprocal debt: $u = 1/d$)
- Y - output

Fundamental economic constants Equation (24)

- ν = 3 capital-to-output ratio
- α = 0.025 productivity growth rate
- β = 0.02 population growth rate
- δ = 0.01 depreciation rate

In [16]:
```
ν = 3              # capital-to-output ratio
α = 0.025          # productivity growth rate
β = 0.02           # population growth rate
δ = 0.01;          # depreciation rate
```

Phillips curve - Equation (25)

$$\Phi(\lambda) = \frac{\phi_1}{(1-\lambda)^2} - \phi_0$$

In [15]:
```
φ0 = 0.04/(1-0.04^2)         # Phillips curve parameter,  Phi(0.96)=0
φ1 = 0.04^3/(1-0.04^2);       # Phillips curve parameter, Phi(0)=-0.04
```

Net profit share π

$$\pi = (1 - \omega - rd)$$

Investment function - Equation (64)

$$\kappa(x) = \kappa_0 + \kappa_1 e^{\kappa_2 x}$$

$$\kappa_0 = -0.0065, \quad \kappa_1 = e^{-5}, \quad \kappa_2 = 20$$

In [20]:
```
r = 0.03                     # real interest rate  κ
κ0 = -0.0065                 # investment function parameter
κ1 = exp(-5)                 # investment function parameter
κ2 = 20;                     # investment function parameter
```

Define Phillips curve Φ, & investment function κ

Phillips curve - Equation (25)

In [17]:
```
Φ(λ) = φ1./(1-λ).^2- φ0                # Phillips curve, equation (25)
Φ′(λ) = 2*φ1./(1-λ).^3                 # derivative of Phillips curve
Φ_inv(λ) = 1 - (φ1/(λ+φ0)).^(1/2);      # inverse of Phillips curve
```

Investment function - Equation (64)

In [18]:
```
κ(x) = κ0+κ1.*exp(κ2.*x)               # Investment function, equation (64)
κ′(x) = κ1.*κ2.*exp(κ2.*x)             # Derivative of investment function
κ_inv(x) = log((x-κ0)./κ1)./κ2;        # Inverse of investment function
```

Numerical Values of Interest

- investment at interior equilibrium
- profit at interior equilibrium, equation (41)
- debt ratio at interior equilibirum, equation (42)
- wage share at interior equilibrium, equation (42)

- employment at interior equilibirum, equation (42)
- solution to equation (40) given in equation (66)
- eigenvalues of the Jacobian at the interior, first d0 & second d0 equilibrium

https://www.mathworks.com/help/symbolic/vpasolve.html
(https://www.mathworks.com/help/symbolic/vpasolve.html)
Symbolic Math Toolbox (https://www.mathworks.com/help/symbolic/index.html)

Equation (35)

$$\begin{aligned}
\dot{\omega} &= \omega[\Phi(\lambda) - \alpha] \\
\dot{\lambda} &= \lambda\left[\tfrac{\kappa(1-\omega-rd)}{\upsilon} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \tfrac{\kappa(1-\omega-rd)}{\upsilon} + \delta\right] + \kappa(1 - \omega - rd) - (1 - \omega)
\end{aligned}$$

Equation 40

$$d\left[r - \frac{\kappa(1 - rd)}{\upsilon} + \delta\right] + \kappa(1 - rd) - 1 = 0$$

Equation (42) - equilibrium point

$$\bar{\pi}_1 = \kappa^{-1}(\nu(\alpha + \beta + \delta))$$

$$\begin{aligned}
\bar{\omega}_1 &= 1 - \bar{\pi}_1 - r\tfrac{\upsilon(\alpha+\beta+\delta)-\bar{\pi}_1}{\alpha+\beta} \\
\bar{\lambda}_1 &= \Phi^{-1}(\alpha) \\
\bar{d}_1 &= \tfrac{\upsilon(\alpha+\beta+\delta)-\bar{\pi}_1}{\alpha+\beta}
\end{aligned}$$

Equation (47)

$$\begin{aligned}
\dot{\omega} &= \omega[\Phi(\lambda) - \alpha] \\
\dot{\lambda} &= \lambda\left[\tfrac{\kappa(1-\omega-r/u)}{\upsilon} - \alpha - \beta - \delta\right] \\
\dot{d} &= d\left[r - \tfrac{\kappa(1-\omega-r/u)}{\upsilon} + \delta\right] + \kappa(1 - \omega - r/u) - (1 - \omega)
\end{aligned}$$

Interior equilibrium point expressions

See Equation (42)

```
In [21]: κ_eq = v*(α+β+δ)                  # investment at interior equilibrium
         π_eq = κ_inv(v*(α+β+δ))             # profit at interior equilibrium, equation (41
         d_eq = (κ(π_eq)-π_eq)/(α+β)       # debt ratio at interior equilibirum, equation (
         ω_eq = 1-π_eq-r*d_eq              # wage share at interior equilibrium, equation (
         λ_eq = Φ_inv(α);                     # employment at interior equilibirum, equatio
```

```
In [ ]: # TO DO!
        #d_sols(1) = vpasolve(d*(r+delta-fun_kappa(1-r*d)/nu)+fun_kappa(1-r*d)-1,d) ;
        #d_sols(2) = vpasolve(d*(r+delta-fun_kappa(1-r*d)/nu)+fun_kappa(1-r*d)-1,[20 40])
        #d_sols
```

Equation (59)

$$
r\left[\frac{\kappa'(\bar{\pi}_1)}{v}(\bar{\pi}_1-\kappa(\bar{\pi}_1)+v(\alpha+\beta))-(\alpha+\beta)\right]>0
$$

Jacobian ~ equation (59)

$$
J(\omega,\lambda,u)=\begin{bmatrix} \Phi(\lambda)-\alpha & \omega\Phi'(\lambda) & 0 \\ -\frac{\lambda\kappa'(\pi)}{v} & \frac{\kappa(\pi)-v(\alpha+\beta+\delta)}{v} & -\frac{r\lambda\kappa'(\pi)}{u^2 v} \\ \frac{(vu^2-u)\kappa'(\pi)-vu^2}{v} & 0 & \frac{\kappa(\pi)(1-2u)+r\kappa'(\pi)(1/u-1)+2uv(1-\omega)-v(r+\delta)}{v} \end{bmatrix}
$$

```
In [8]: J(ω,λ,u)= [Φ(λ)-α ω.*Φ'(λ) 0;
            -λ.*κ'(1-ω-r*z)/ν (κ(1-ω-r*z)-ν*(α+β+δ))/ν -r*λ.*κ'(1-ω-r*z)/ν;
            ((z-ν).*κ'(1-ω-r*z)+ν)/ν 0 (ν*(r+δ)-κ(1-ω-r*z)+r*(z-ν).*κ'(1-ω-r*z))/ν];
```

```
In [ ]: # TO DO!!!
        #E1=eig(J(omega_eq,lambda_eq,d_eq)) # eigenvalues of the Jacobian at the interior
        #E2=eig(J(0,0,d_sols(1)))          # eigenvalues of the Jacobian at the first de
        #E3=eig(J(0,0,d_sols(2)))          # eigenvalues of the Jacobian at the second d
```

```
In [22]: cond59= κ'(π_eq)/v*(π_eq-v*δ)-(α+β) # expression in brackets in condition (59)
```

```
Out[22]: 0.10573866662583055
```

Sample paths of the Keen model using `DifferentialEquations`

[https://diffeq.sciml.ai/stable/tutorials/ode_example/ (https://diffeq.sciml.ai/stable/tutorials/ode_example/)](https://diffeq.sciml.ai/stable/tutorials/ode_example/)

In [52]:
```
function keen!(dz,z,p,t)
    log_ω, tan_λ, π_n = z                                    # state variables
    α, β, δ, v, r, (φ0,φ1), (κ0,κ1,κ2) = p                   # parameters & functions κ, Φ
                                                             # Recover λ & ω
    λ = atan(tan_λ)/π + 0.5
    ω = exp(log_ω)

    # Functions
    Φ(λ)  = φ1./(1-λ).^2 - φ0                        # Phillips curve, equation (25)
    κ(x)  = κ0 + κ1.*exp(κ2.*x)                       # Investment function, equation (64)

    g_Y = κ(π_n)/v - δ              # Output function (31)

    # Equation (47)
    dz[1] = dlog_ω = Φ(λ) - α                                            #d(log_ω)/d
    dz[2] = dtan_λ = (1 + tan_λ^2)*π*λ*(g_Y - α - β)                    #d(tan_λ)/d
    dz[3] = dπ_n = -ω*dlog_ω - r*(κ(π_n) - π_n) + (1 - ω - π_n)*g_Y   #d(π_n)/dt
end
```

Out[52]: keen! (generic function with 1 method)

convert [...]

restore [...]

Define ODEProblem and solve

In [33]:
```
# Sample paths of the Keen model
π_bar_n_K = κ_inv(v * (α + β + δ));
d_bar_K = (κ(π_bar_n_K) - π_bar_n_K)/(α + β);
ω_bar_K = 1 - π_bar_n_K - r * d_bar_K;
λ_bar_K = Φ_inv(α);
```

Initial values

In [36]:
```
ω0 = 0.75
λ0 = 0.75
d0 = 0.1
Y0 = 100
T = 300;
```

ODEProblem

In [104]:
```julia
u0 = [-0.288, 1.0, 0.247]
tspan = (0.0,300.0)
p = [α, β, δ, ν, r, (φ0,φ1), (κ0,κ1,κ2)]
prob = ODEProblem(keen!,u0,tspan,p)
```

Out[104]:
```
ODEProblem with uType Array{Float64,1} and tType Float64. In-place: true
timespan: (0.0, 300.0)
u0: [-0.288, 1.0, 0.247]
```

solve

In [119]:
```julia
sol = solve(prob,reltol=1e-6);
```

Convert to DataFrame

[...]

State variables

- ω - wage share of the economy
- λ - employment rate
- d - debt (or reciprocal debt: $u = 1/d$)
- Y - total output

In [122]:
```julia
#state_variables = map(u -> restore(u,r),sol.u);
```

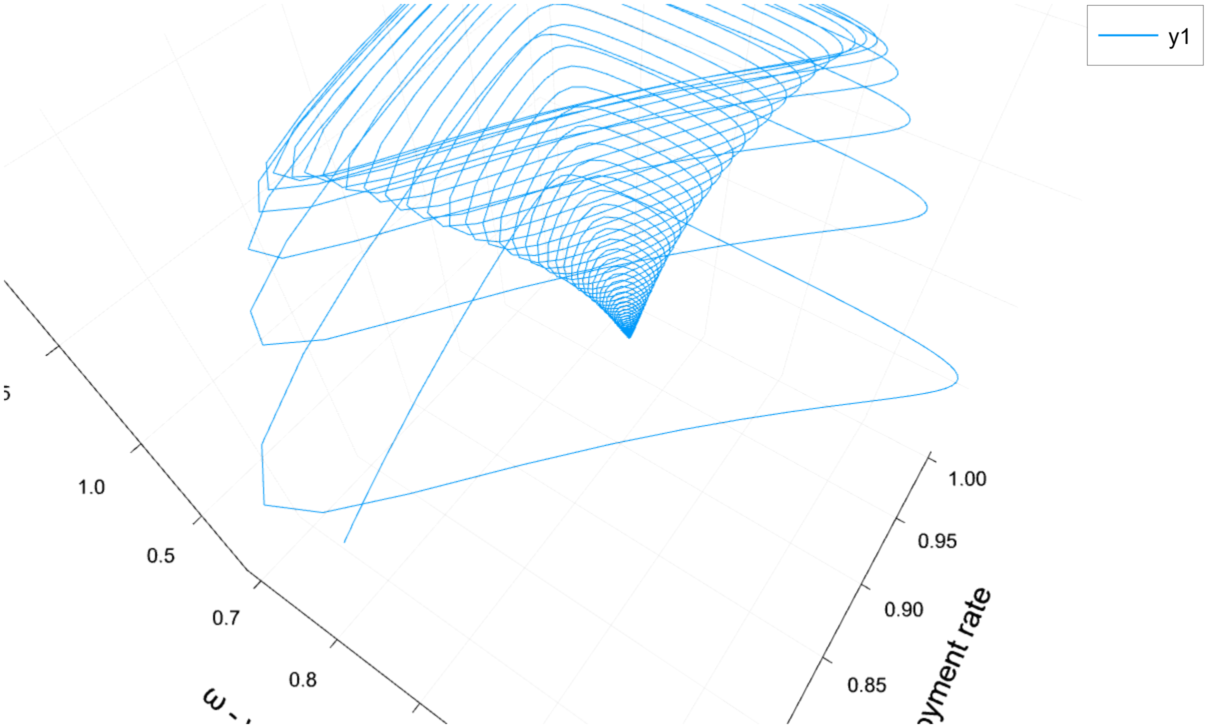
In [134]:
```julia
state_variables = [(restore(u,r),t) for (u,t) in zip(sol.u,sol.t)];
```

In [135]:
```julia
state_variables[2]
```

Out[135]:
```
((0.7461811191854018, 0.7649302031586827, 0.15779923733286), 0.074825915350504
6)
```

In [137]:
```
plot([u for (u,t) in state_variables])
xaxis!("ω - wage share of the economy")
yaxis!("λ - employment rate")
#zaxis!("d - debt")
```

Out[137]:



Experiments munging the result [...]

Calculate Y

In [7]:
```
Y_output = Y0*yK(:,2)/λ0.*exp((α+β)*tK);
```

```
Error using eval
Undefined function 'convert' for input arguments of type 'double'.
```

Figure 3 [...]

Figure 4 [...]

Figure 5 [...]

Auxiliary functions [...]

Test plotting

[…]