Exercise session 0

Topics: matrices and vectors manipulation, plots of functions and Matlab programming language

1. Define the vector `x=[1:-0.1:0]`. Type the following `MATLAB` instructions, understanding their meaning:

   a) `x([1 4 3]);`

   b) `x([1:2:7 10])=zeros(1,5);`

   c) `x([1 2 5])=[0.5*ones(1,2) -0.3];`

   d) `y=x(end:-1:1).`

2. Define the matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}$$

   and type the following `MATLAB` instructions, understanding their meaning:

   a) `size(A);`

   b) `A(1:2,4), A(:,3), A(1:2,:), A(:,[2,4]), A([2 3 3],:);`

   c) `A(3,2)=A(1,1);`

   d) `A(1:2,4)=zeros(2,1);`

   e) `A(2,:)=A(2,:)-A(2,1)/A(1,1)*A(1,:).;`

3. Define the matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 7 & 8 & 9 & 10 \\ 9 & 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 & 20 \end{pmatrix}$$

   After that:

   a) assemble the matrix **B** made by the columns of **A** in reverse order (that is to say the first column of **B** is the 6^{th} of **A**, the 2^{nd} of **B** is the 5^{th} of **A**, ...).

   b) assemble the matrix made by the even columns of **A**;

   c) assemble the matrix made by the odd rows of **A**;

   d) assemble the matrix made by rows 1,4,3 and columns 5,2 of **A**;

   e) assemble the vector made by the diagonal elements $a_{kk},\ k = 1, ..., 4$ of **A**;

4. Use the `MATLAB` command `diag` to define a 10×10-tridiagonal matrix **B** in which the main diagonal elements are all equal to 5 while the sub-diagonal elements are equal to -1 and the super-diagonal elements are equal to 3. Set the elements of the intersection between column 6 and 9 and rows 5 and 8 equal to 2.

5. Use the Matlab command `plot` to graphically represent the following mathematical functions:

   $f(x) = \sin(x) \qquad x \in [-\pi, \pi]\,;$

   $f(x) = e^{x} \qquad x \in [-1, 1]\,;$

   $f(x) = e^{-x^2} \qquad x \in [-\pi, \pi]\,;$

- $f(x) = \frac{\sin(x)}{x} \qquad x \in (0, 4\pi]\,;$
- $f(x) = x\sin(\frac{1}{x}) \qquad x \in (0, 2]\,;$

6. Graphically represent the mathematical function:

$$f(x) = \sqrt{\frac{100(1-0.01x^2)^2 + 0.02x^2}{(1-x^2)^2 + 0.1x^2}} \qquad x \in [0.1,\ 100]\,,$$

   using the `MATLAB` commands `plot` and `loglog`. Evaluate the function at sufficiently large number of points in the range of interest. Comment the results.

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7. Write a `MATLAB` *function* which evaluates the mathematical function:

$$f(x) = \begin{cases} -2x, & x < 0, \\ 0, & x = 0, \\ 2x, & x > 0, \end{cases}$$

   both at a generic point x and at each point of a vector. After that, graphically represent the function f in the interval [-1,1].

8. Write a `MATLAB` *function* in order to approximate the value of the function $f(x) = e^x$ in a neighborhood of $x = 0$ by means of Taylor polynomial

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

   of degree n, 0-centered. Stop the summation when the term $\frac{x^i}{i!}$ is less than a fixed tolerance *tol*. Run the function with $x = 1$, $tol = 1.0e - 10$ and compute the relative error linked to the value of the polynomial at x, using as exact value the one given by the Matlab function `exp(x)`.