Phys 512 Assignment 6

Ian Hendricksen

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My code loops over every template and its associated events for both the Hanford and Livingston detectors. It prints the answer to parts (a) $\rightarrow$ (f) for every event seen by both detectors; therefore, there are 8 answers to each question. I have included the file `A6 Print Output.txt` in my GitHub repo containing all the answers for each question (it should be a little more self-explanatory than I'm making it at the moment). I will not produce each answer here for each event for each detector, but I will write in this document relevant explanations for questions when necessary, as well as display some plots (for example, each matched filter).

(a) We use the Tukey Window I created at the top of my script (using my own function) so that we force the edges of our data to go to 0, but we have a "band" in the center of the data of some width (set by alpha) so that the majority of the data retains its original scale after applying the window. This way we don't lose important information in the middle where our signal probably is and important noise information, but the edges are forced to 0. An example of the Tukey window I use throughout the script is shown in Figure 1.

I smooth the power spectrum by convolving it with a small rectangular pulse. This ensures that we don't lose information about where certain peaks are in the power spectrum, but said peaks are made less noisy. Figure 2 compares an example of a portion of the smoothed to original power spectra. It's apparent that the original power spectrum is much noisier, and that the smoothed power spectrum still preserves the overall behavior of the original.

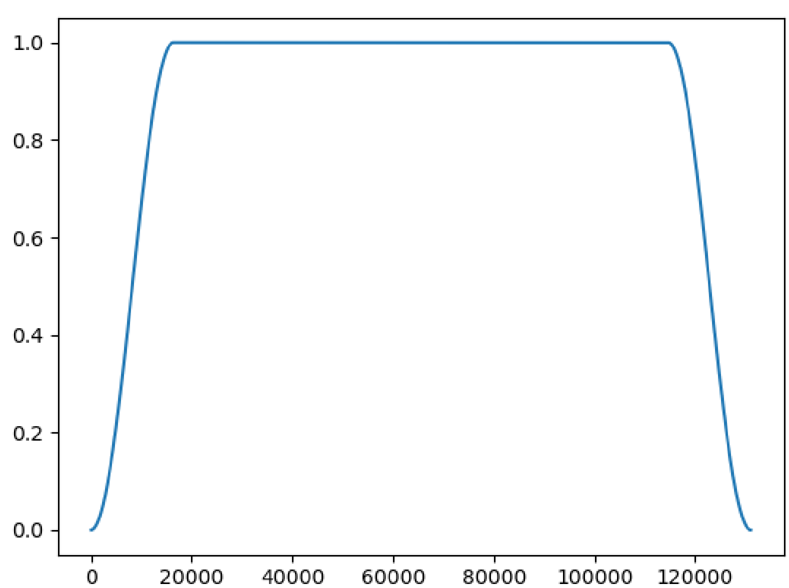


Figure 1: A Tukey window.

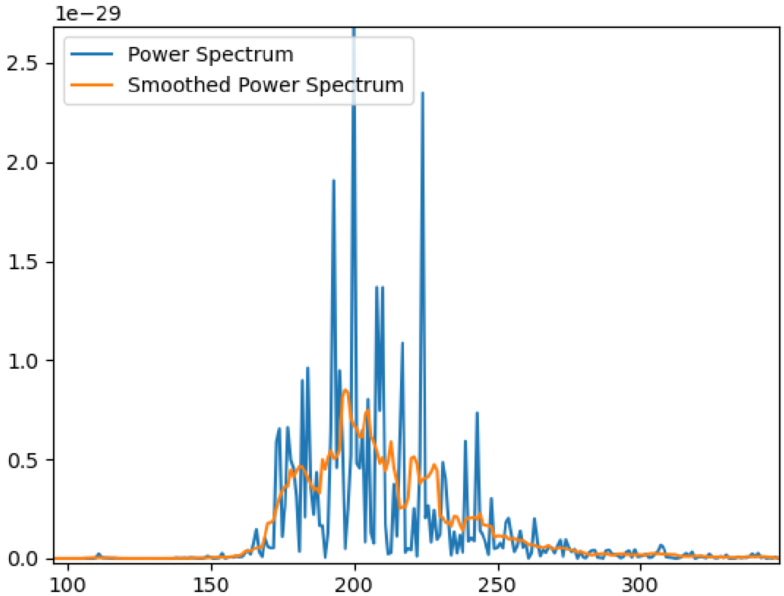


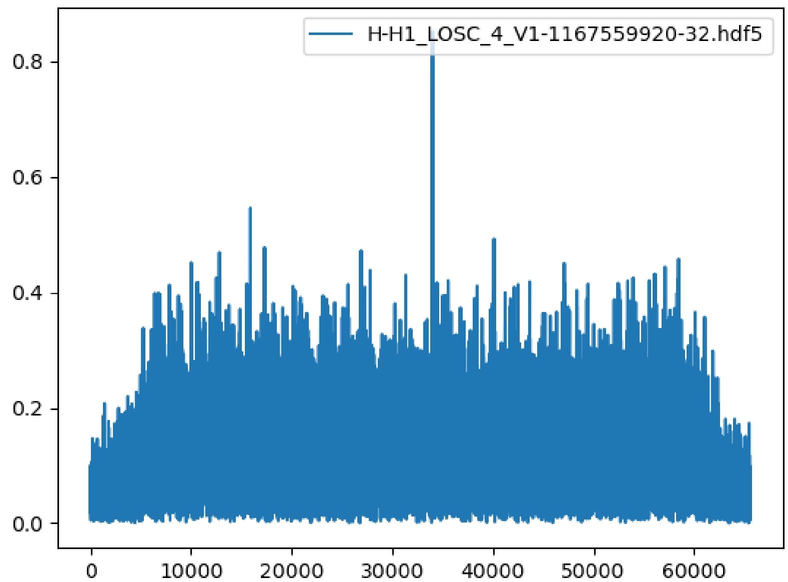
Figure 2: Power spectrum compared with its smoothed version.

I use the smoothed power spectrum to whiten the data and the template, by dividing each by the square root of the smoothed power spectrum.

(b) The following figures display my matched filters for each event, as observed by both the Hanford and Livingston detectors (same event figures are on the same page, and are labeled by their legends). The matched filter is calculated as

$$mf = \mathcal{F}^{-1}\left(S_f \cdot \mathcal{F}(T)^*\right), \tag{1}$$

where S_f is the windowed, smoothed, and whitened Fourier transform of the strain, and $\mathcal{F}(T)^*$ is the complex conjugate of the Fourier transform of the whitened template. Each matched filter shows a large peak that would never occur due to statistics alone, so we have found a gravitational wave event for each! Both detectors observe such a peak for the same event, supporting this claim.


H-H1_LOSC_4_V1-1167559920-32.hdf5
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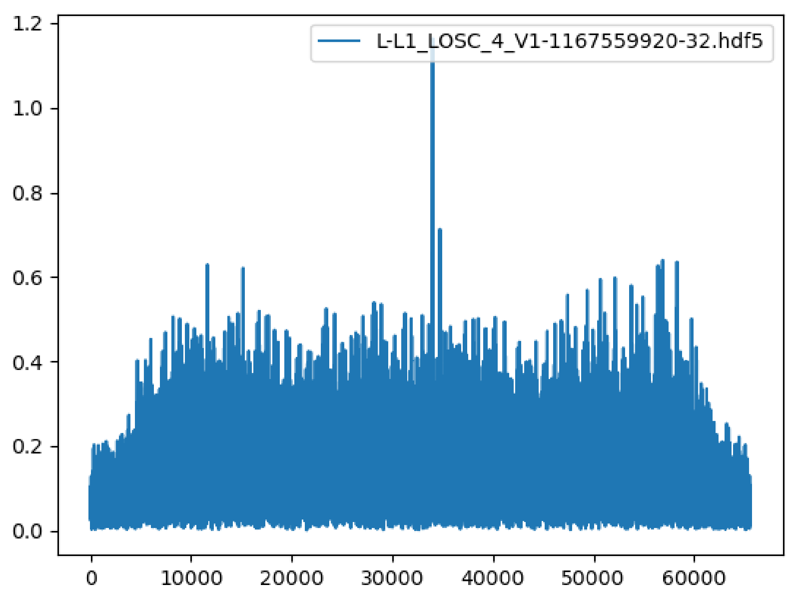
0.4
0.6
0.8
0
10000
20000
30000
40000
50000
60000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0.4
0.6
0.8
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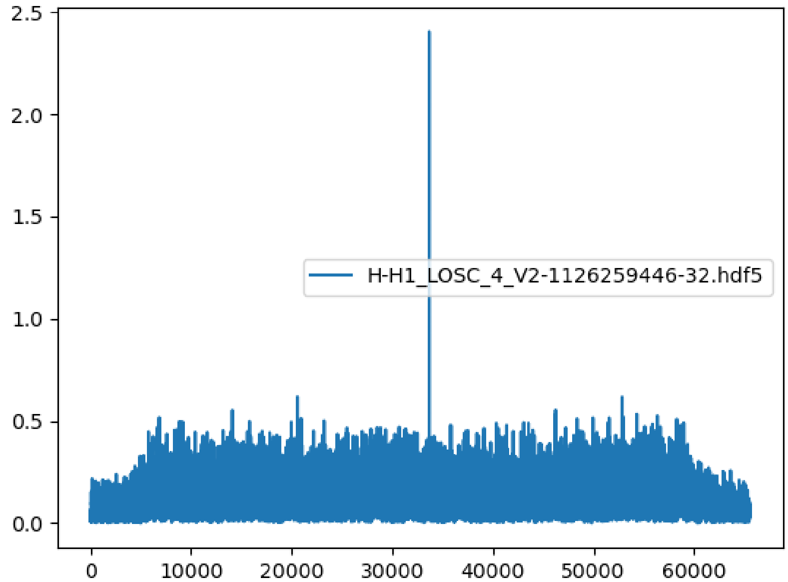
1.2
0
10000
20000
30000
40000
50000
60000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1.0
0.5
0.0
0
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20000
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40000
50000
60000
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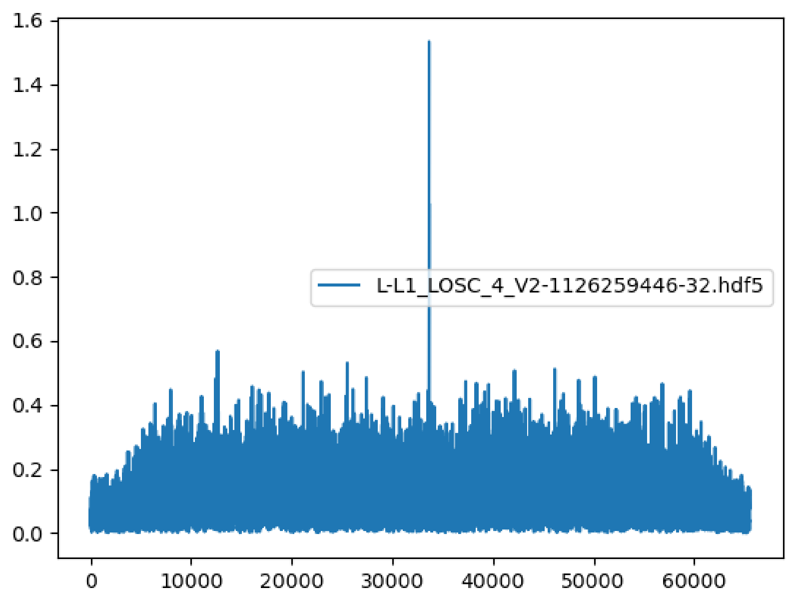
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1.4
1.2
1.0
0.8
0.6
0.4
0.2
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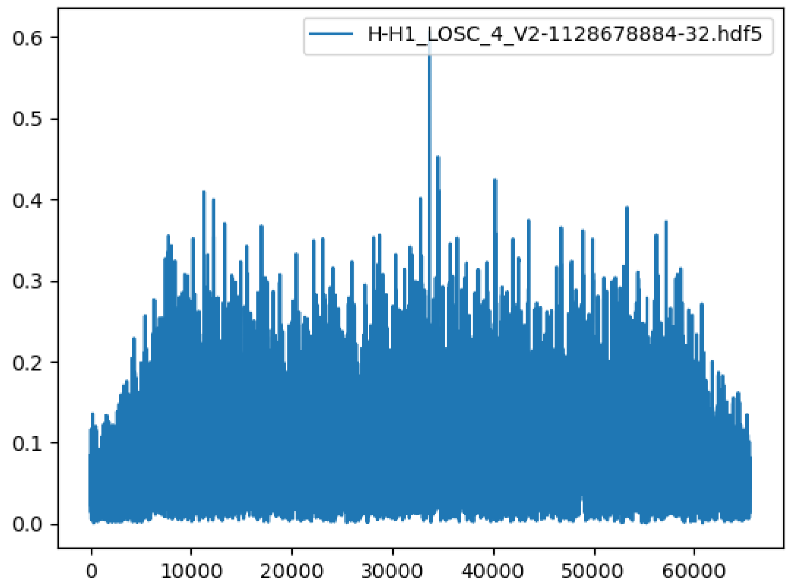
20000
30000
40000
50000
60000
L-L1_LOSC_4_V2-1126259446-32.hdf5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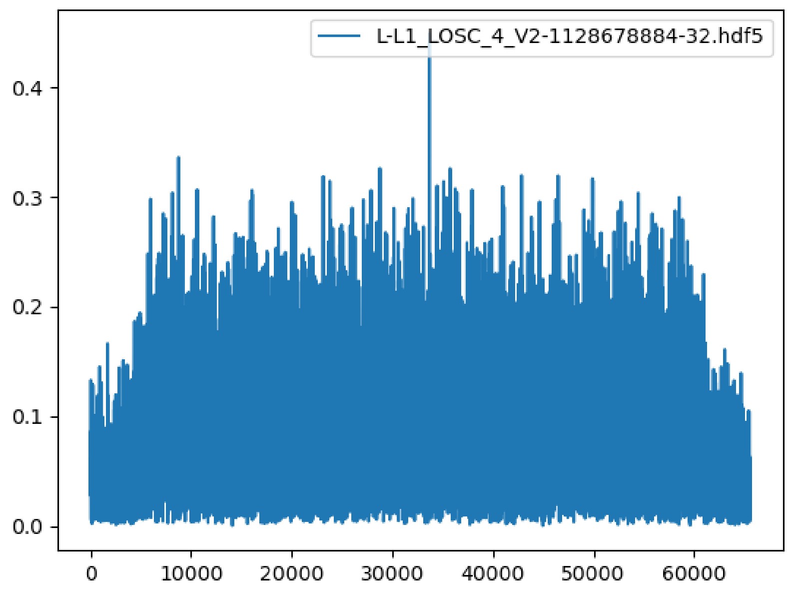
0
10000
20000
30000
40000
50000
60000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0
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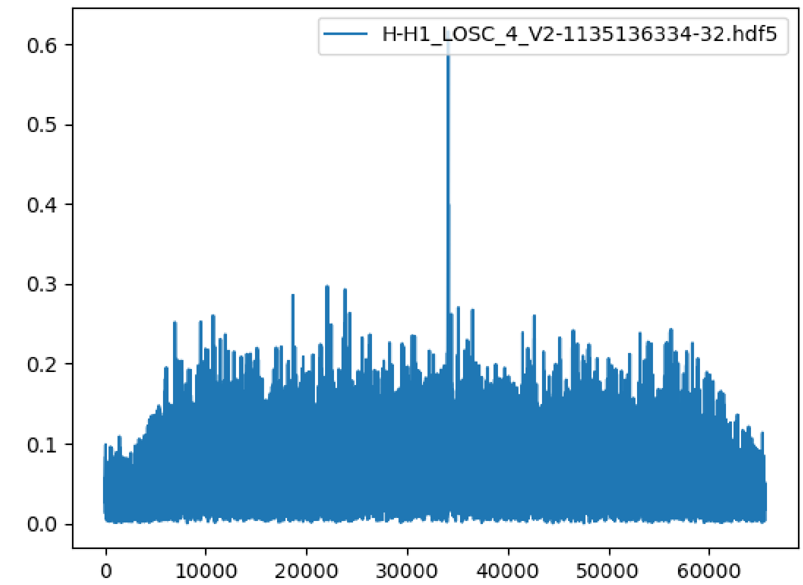
30000
40000
50000
60000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0.3
0.2
0.1
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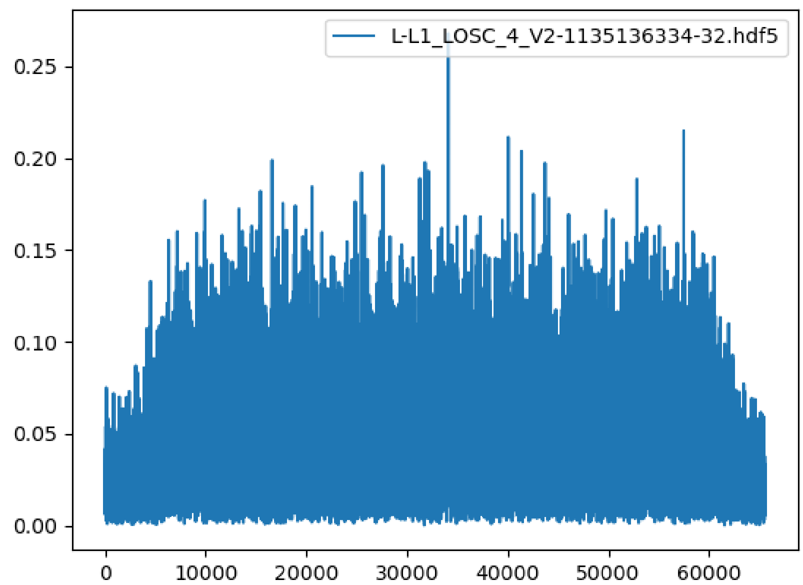
0
10000
20000
30000
40000
50000
60000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0.20
0.15
0.10
0.05
0.00
0
10000
20000
30000
40000
50000
60000

(c) The SNR is taken to be the matched filter divided by the estimate of the noise of the matched filter, which is taken to be simply the standard deviation of the matched filter. The maximum SNR (i.e. that of the peak) is reported in the printed output file. The combined SNR for both detectors is taken to be the sum of the individual SNR's, and is also reported at the end of the printed text for a given template.

(d) The difference between my estimated SNR and the analytical SNR is enormous. I suspect that there is something incorrect in the way I am computing the analytical SNR (this is absolutely the source since the estimated SNR is reasonable), although I can't seem to find where the bug is. I am computing the analytical SNR using a similar method to the used in the example code provided by the LIGO folks. Namely, I am computing the noise σ of the matched filter by taking the sum of the whitened template multiplied by its conjugate divided by the smoothed power spectrum, and taking the absolute value of this multiplied by the frequency step of our data (and of course taking the square root of the whole thing since what I just described is σ^2). I then divide the matched filter by the smoothed power spectrum and σ. This results in an enormous number, entirely incorrect. I believe I may have missed some large factor that would bring this larger number down somewhere, but I hunted for quite some time and couldn't find the issue.

(e) My code may provide a bit of a better explanation for how I calculate the "half-weight" frequency (as I refer to it there), but it essentially is computed by first assuming that the weight is the square root of the smoothed power spectrum, since we use this to whiten the data and the template. I then generate two new arrays, one which contains the sum of the weights (square root of the power spectrum) below a given index and one above that index. These arrays are both divided by the total sum of the weights to normalize them. I then search for the index at which these arrays are closest to 0.5 (not equal, since we won't have exactly 0.5), which should be some index i for the "below" array and $i+1$ for the "above" array given the way I wrote it. We then print the frequency of the power spectrum at the i^{th} position, and take this to be the half-weight frequency. These values are in the printed output file.

(f) If we look closely at the matched filters, we find that there isn't actually a sharply defined peak for each gravitational wave event, but rather some rounded peak of several values. In order to find the "peak time" at which the gravitational wave event occurred, I take the SNR of points that have an SNR > SNR_{max} - 1, i.e. those SNR's that are within a tolerance of 1 less than the maximum SNR, since these should be the SNR's of our rounded peak. I then multiply dt by the *indices* of these values and then average them, where dt is the time step of our data. This gives a rough estimate of our event detection time for each detector.

Given that the detectors are some thousands of kilometers apart, the typical positional uncertainty should be something on the order of arcmin or arcsec, since we are observing very long-wavelength signals, but with a pseudo radio "interferometer" that has a baseline of several thousand kilometers.