Assignment # 2: Appendix

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1 Question 5, part (b)

1. $N_t \in \{1,2,3,4,5\} \to N_{t+1} \in \{1,2,3,4,5\} | x_t, d_{it} = 1$

   - **First Row**
   - $\mathbb{P}(1 \to 1 | x_t, d_{it} = 1) = 1 - \Phi(\overline{\gamma}(1, x_t); \theta_\gamma)$ # no entry
   - $\mathbb{P}(1 \to 2 | x_t, d_{it} = 1) = \Phi(\overline{\gamma}(1, x_t); \theta_\gamma)$ # one entry
   - $\mathbb{P}(1 \to 3 | x_t, d_{it} = 1) = \mathbb{P}(1 \to 4 | x_t, d_{it} = 1) = \mathbb{P}(1 \to 5 | x_t, d_{it} = 1) = 0$

   - **Second Row**
   - $\mathbb{P}(2 \to 1 | x_t, d_{it} = 1) = [1 - \Phi(\overline{\gamma}(2, x_t); \theta_\gamma)] \times [1 - \Phi(\overline{\mu}(2, x_t); \theta_\mu)]$ # no entry, one exit
   - $\mathbb{P}(2 \to 2 | x_t, d_{it} = 1) = [1 - \Phi(\overline{\gamma}(2, x_t); \theta_\gamma)] \times [\Phi(\overline{\mu}(2, x_t); \theta_\mu)]$ # no entry, no exit
     $+ [\Phi(\overline{\gamma}(2, x_t); \theta_\gamma)] \times [1 - \Phi(\overline{\mu}(2, x_t); \theta_\mu)]$ # one entry, one exit
   - $\mathbb{P}(2 \to 3 | x_t, d_{it} = 1) = [\Phi(\overline{\gamma}(2, x_t); \theta_\gamma)] \times [\Phi(\overline{\mu}(2, x_t); \theta_\mu)]$ # one entry, no exit
   - $\mathbb{P}(2 \to 4 | x_t, d_{it} = 1) = \mathbb{P}(2 \to 5 | x_t, d_{it} = 1) = 0$

   - **Third Row**
   - $\mathbb{P}(3 \to 1 | x_t, d_{it} = 1) = [1 - \Phi(\overline{\gamma}(3, x_t); \theta_\gamma)] \times [1 - \Phi(\overline{\mu}(3, x_t); \theta_\mu)]^2$ # no entry, two exits
   - $\mathbb{P}(3 \to 2 | x_t, d_{it} = 1) = [\Phi(\overline{\gamma}(3, x_t); \theta_\gamma)] \times [{}_2C_2 \cdot (1 - \Phi(\overline{\mu}(3, x_t); \theta_\mu))^2]$ # one entry, two exits
     $+ [1 - \Phi(\overline{\gamma}(3, x_t); \theta_\gamma)] \times [{}_2C_1 \cdot \Phi(\overline{\mu}(3, x_t); \theta_\mu) \cdot (1 - \Phi(\overline{\mu}(3, x_t); \theta_\mu))]$ # no entry, one exit
   - $\mathbb{P}(3 \to 3 | x_t, d_{it} = 1) = [1 - \Phi(\overline{\gamma}(3, x_t); \theta_\gamma)] \times [{}_2C_2 \cdot (\Phi(\overline{\mu}(3, x_t); \theta_\mu))^2]$ # no entry, no exits
     $+ [\Phi(\overline{\gamma}(3, x_t); \theta_\gamma)] \times [{}_2C_1 \cdot \Phi(\overline{\mu}(3, x_t); \theta_\mu) \cdot (1 - \Phi(\overline{\mu}(3, x_t); \theta_\mu))]$ # one entry, one exit
   - $\mathbb{P}(3 \to 4 | x_t, d_{it} = 1) = [\Phi(\overline{\gamma}(3, x_t); \theta_\gamma)] \times [\Phi(\overline{\mu}(2, x_t); \theta_\mu)]^2$ # one entry, no exit
   - $\mathbb{P}(3 \to 5 | x_t, d_{it} = 1) = 0$

   - **Fourth Row** (Keep in mind the binomial..)
   - $\mathbb{P}(4 \to 1 | x_t, d_{it} = 1) = \mathbb{P}(\text{no entry} \cap \text{three exit})$
   - $\mathbb{P}(4 \to 2 | x_t, d_{it} = 1) = \mathbb{P}(\text{no entry} \cap \text{two exit \& one remain}) + \mathbb{P}(\text{one entry} \cap \text{three exit})$
   - $\mathbb{P}(4 \to 3 | x_t, d_{it} = 1) = \mathbb{P}(\text{no entry} \cap \text{one exit \& two remain})$
     $+ \mathbb{P}(\text{one entry} \cap \text{two exit \& one remain})$

- $\mathbb{P}(4 \to 4|x_t, d_{it} = 1) = \mathbb{P}(\text{no entry} \cap \text{no exit}) + \mathbb{P}(\text{one entry} \cap \text{one exit \& three remain})$
- $\mathbb{P}(4 \to 5|x_t, d_{it} = 1) = \mathbb{P}(\text{one entry} \cap \text{no exit})$

- **Fifth Row** (Keep in mind the binomial and entry is not allowed..)
- $\mathbb{P}(5 \to 1|x_t, d_{it} = 1) = \mathbb{P}(\text{four exit})$
- $\mathbb{P}(5 \to 2|x_t, d_{it} = 1) = \mathbb{P}(\text{three exit \& one remain})$
- $\mathbb{P}(5 \to 3|x_t, d_{it} = 1) = \mathbb{P}(\text{two exit \& two remain})$
- $\mathbb{P}(5 \to 4|x_t, d_{it} = 1) = \mathbb{P}(\text{one exit \& three remain})$
- $\mathbb{P}(5 \to 5|x_t, d_{it} = 1) = \mathbb{P}(\text{no exit})$

2. $N_t \in \{0, 1, 2, 3, 4\} \to N_{t+1} \in \{1, 2, 3, 4, 5\}|x_t, e_{it} = 1$
   Calculating procedure is essentially same with the above, except for the fact that we only care about incumbent's decision in this part (since entrant's decision is already decided as $e_{it} = 1$). Still keep in mind the binomial coefficients.

   - **First Row**
   - $\mathbb{P}(0 \to 1|x_t, e_{it} = 1) = 1$
   - $\mathbb{P}(0 \to 2|x_t, e_{it} = 1) = \mathbb{P}(0 \to 3|x_t, e_{it} = 1) =$
     $\mathbb{P}(0 \to 4|x_t, e_{it} = 1) = \mathbb{P}(0 \to 1|x_t, e_{it} = 5) = 0$

   - **Second Row**
   - $\mathbb{P}(1 \to 1|x_t, e_{it} = 1) = \mathbb{P}(\text{the one and only incumbent exits})$
   - $\mathbb{P}(1 \to 2|x_t, e_{it} = 1) = \mathbb{P}(\text{the one and only incumbent does not exit})$
   - $\mathbb{P}(1 \to 3|x_t, e_{it} = 1) = \mathbb{P}(1 \to 4|x_t, e_{it} = 1) = \mathbb{P}(1 \to 5|x_t, e_{it} = 1) = 0$

   - **Third Row**
   - $\mathbb{P}(2 \to 1|x_t, e_{it} = 1) = \mathbb{P}(\text{all the two incumbents exit})$
   - $\mathbb{P}(2 \to 2|x_t, e_{it} = 1) = \mathbb{P}(\text{one of the two incumbents exits \& the other remains})$
   - $\mathbb{P}(2 \to 3|x_t, e_{it} = 1) = \mathbb{P}(\text{all the two incumbents remain})$
   - $\mathbb{P}(2 \to 4|x_t, e_{it} = 1) = \mathbb{P}(2 \to 5|x_t, e_{it} = 1) = 0$

   - **Fourth Row**
   - $\mathbb{P}(3 \to 1|x_t, e_{it} = 1) = \mathbb{P}(\text{all the three incumbents exit})$
   - $\mathbb{P}(3 \to 2|x_t, e_{it} = 1) = \mathbb{P}(\text{two of the three incumbents exit \& the other one remains})$
   - $\mathbb{P}(3 \to 3|x_t, e_{it} = 1) = \mathbb{P}(\text{one of the three incumbents exits \& the other two remain})$
   - $\mathbb{P}(3 \to 4|x_t, e_{it} = 1) = \mathbb{P}(\text{all the three incumbents remain})$
   - $\mathbb{P}(3 \to 5|x_t, e_{it} = 1) = 0$

- **Fifth Row**
- $\mathbb{P}(4 \to 1|x_t, e_{it} = 1) = \mathbb{P}(\text{all the four incumbents exit})$
- $\mathbb{P}(4 \to 2|x_t, e_{it} = 1) = \mathbb{P}(\text{three of the four incumbents exit \& the other one remains})$
- $\mathbb{P}(4 \to 3|x_t, e_{it} = 1) = \mathbb{P}(\text{two of the four incumbents exit \& the other two remain})$
- $\mathbb{P}(4 \to 4|x_t, e_{it} = 1) = \mathbb{P}(\text{one of the four incumbents exits \& the other three remain})$
- $\mathbb{P}(4 \to 5|x_t, e_{it} = 1) = \mathbb{P}(\text{none of the four incumbents exits \& all the four remain})$

2 Question 11, part (a) & (b)

From now on assume that a set of parameters $\theta := (\gamma, \sigma_\gamma^2, \mu, \sigma_\mu^2)$ is given from the outer loop.

- **Initial Step: Calculate entry and exit threshold**
- **Step (a): Calculate the remaining decision's PDV under N and x**
  1. Assume an initial state (N_0, x_0) where $N_0 \in \{1, 2, 3, 4, 5\}$
  2. At $t = 0$, i's exit decision is fixed as 0 (Keep in mind that we are calculating the PDV of remaining in the market).
     (a) Decide $(N_0 - 1)$ many incumbents' exit decisions.
     (b) Decide the potential entrant's entry decision.
     (c) update $N_1 = N_0 - (\#\text{ of exit decisions among } (N_0 - 1) \text{ incumbents}) + \mathbb{1}(\gamma_{e0} \leq \overline{\gamma}(N_0, x_0; \theta))$
     (d) update x_1 using the markov transition matrix.
  3. From $t = 1$, i's exit decision is also endogenized.
     (a) Decide i's exit decision.
        - if $d_{it} = 0$(i.e, i exits), break the procedure and store the scrap value.
        - else, move on to the next step.
     (b) Conditional on i's remaining decision, decide $(N_t - 1)$ many incumbents' exit decisions.
     (c) Decide the potential entrant's entry decision.
     (d) update $N_{t+1} = N_t - (\#\text{ of exit decisions among } (N_t - 1) \text{ incumbents}) + \mathbb{1}(\gamma_{et} \leq \overline{\gamma}(N_t, x_t; \theta))$
     (e) update x_{t+1} using the markov transition matrix.
  4. If 3 doesn't break, do the step 3 until t hits 1,000.
  5. Do 1-4 for many times and get the average of the PDV outcomes.
- **Step (b): Calculate the entry decision's PDV under N and x**
  1. Assume an initial state (N_0, x_0) where $N_0 \in \{0, 1, 2, 3, 4\}$

2. At $t = 0$, e's entry decision is fixed as 1 (Keep in mind that we are calculating the PDV of entering the market).
   (a) Decide N_0 many incumbents' exit decisions.
   (b) update $N_1 = N_0 - (\#\text{ of exit decisions among } N_0 \text{ incumbents}) + 1$
   (c) update x_1 using the markov transition matrix.
3. From $t = 1$, e's exit decision is also endogenized.
   (a) Decide e's exit decision.
      – if $d_{it} = 0$(i.e, e exits), break the procedure and store the scrap value.
      – else, move on to the next step.
   (b) Conditional on e's remaining decision, decide $(N_t - 1)$ many incumbents' exit decisions.
   (c) Decide the potential entrant's entry decision.
   (d) update $N_{t+1} = N_t - (\#\text{ of exit decisions among } (N_t - 1) \text{ incumbents}) + \mathbb{1}(\gamma_{et} \leq \overline{\gamma}(N_t, x_t))$
   (e) update x_{t+1} using the markov transition matrix.
4. If 3 doesn't break, do the step 3 until t hits 1,000.
5. Do 1-4 for many times and get the average of the PDV outcomes.