Masakazu H.W #4

Ian Jaeyeon Kim

1 DGP

1. Randomly draw: x, z_1, z_2, ξ, e
2. Calculate optimal price of each firm:

$$
\begin{aligned}
\frac{\partial \pi_j}{\partial p_j} &= s_j + \frac{\partial s_j}{\partial p_j}(p_j - mc_j) \overset{!}{=} 0 \\
&\Rightarrow p_j = mc_j - (\frac{\partial s_j}{\partial p_j})^{-1} \cdot s_j \quad \cdots (*) \\
\text{where } s_j &\approx \frac{1}{1,000} \sum_{i=1}^{1,000} \frac{exp(\alpha_j + \gamma x_j + \overline{\beta} p_j + \xi_j + \sigma_\beta \nu_i p_j)}{1 + \sum_{k=1}^{5} exp(\alpha_k + \gamma x_k + \overline{\beta} p_k + \xi_k + \sigma_\beta \nu_i p_k)} \\
\frac{\partial s_j}{\partial p_j} &\approx \frac{1}{1,000} \sum_{i=1}^{1,000} (\overline{\beta} + \sigma_\beta \nu_i)[\frac{exp(\sim)}{1 + \sum_{k=1}^{5} exp(\sim)}][1 - \frac{exp(\sim)}{1 + \sum_{k=1}^{5} exp(\sim)}] \\
mc_j &= \omega_0 + \omega_1 x_j + \omega_2 z_{1j} + \omega_3 z_{2j} + e_j
\end{aligned}
$$

   apply fixed point algorithm to $(*)$ to search for the optimal price vector.

3. Calculate true share using the derived optimal prices.

2 Demand Side Estimation

1. Outer loop searches for σ_β with GMM estimation.
2. Inner loop searches for
   - Fixed point δ_j^* associated with σ_β thrown from the outer loop.
   - $(\{\alpha_j\}_j, \gamma, \overline{\beta})$ through 2SLS.
   - Estimate α_1 using constant, and α_{-1} using dummies.

3 Supply Side Estimation

1. Assume the equilibrium concept: Bertrand-Nash.
   - i.e, Assume that a firm's pricing decision comes from $(*)$.
2. Under the assumption, compute mc_j using the formula $(*)$.

- We observe p_j and s_j^o from the data.
- We can calculate $\dfrac{\partial s_j}{\partial p_j} = -\hat{\bar{\beta}} \cdot s_j^o(1-s_j^o)$, where $\hat{\bar{\beta}}$ comes from the demand estimation.
- Hence, $\hat{mc}_j = p_j + \dfrac{s_j^o}{\hat{\bar{\beta}} \cdot s_j^o(1-s_j^o)} = p_j + \dfrac{1}{\hat{\bar{\beta}} \cdot (1-s_j^o)}$

3. Do the linear regression to recover cost side parameters.
   - Recall the specification for marginal cost: $mc_j = \omega_0 + \omega_1 x_j + \omega_2 z_{1j} + \omega_3 z_{2j} + e_j$
   - Replace mc_j with $\hat{mc}_j$ and do the linear regression using observed data on x, z_1, z_2.

4 Counterfactual: Brand Value

1. Calculate $\hat{\xi}, \hat{e}$ from which we will draw each firm's demand and cost shocks to integrate them out.
   - Although it is not necessary in this practice (since iid across t is assumed), construct $\hat{\xi}_j$ and $\hat{e}_j$ separately across j to consider firm specific shock.
2. Fix t, and within the t randomly draw ξ^r and e^r for $r = 1, \cdots, R$.
   - Note that each ξ^r and e^r are 5×1 vectors.
3. Calculate the expected profit change with MC integration.
   - $\mathbb{E}_{\xi,e}[\Delta \pi_{jt}] \approx \frac{1}{R}\sum_{r=1}^{R} \pi_{jt}[p_t^{**}|\alpha_j = 0, \xi^r, e^r] - \pi_{jt}[p_t^*|\alpha_j = \hat{\alpha}_j, \xi^r, e^r]$
   - Note that p_t^{**} and p_t^* should be calculated for each pair of ξ^r, e^r.
   - In the procedure, store all the values of p_t^{**} and p_t^* for each pair of ξ^r, e^r to report price change.
   - I reported $\frac{1}{20}\sum_{t=1}^{20} \frac{1}{R}\sum_{r=1}^{R} \pi_{jt}[p_t^{**}|\alpha_j = 0, \xi^r, e^r] - \pi_{jt}[p_t^*|\alpha_j = \hat{\alpha}_j, \xi^r, e^r]$ for each j.
     - for the sake of time, I only referred first 20 markets.
4. Calculate the expected consumer welfare change.
   - $\mathbb{E}_{\xi,e}[\Delta CW_t] \approx \frac{1}{R}\sum_{r=1}^{R} CW_t^{\text{after}}(\xi^r, e^r) - CW_t^{\text{before}}(\xi^r, e^r)$
   - $CW_t^{\text{after}}(\xi^r, e^r) \approx \frac{1}{H}\sum_{h=1}^{H} \frac{1}{\hat{\bar{\beta}} + \hat{\sigma_\beta}\nu_i^h} \ln(1 + \sum_{k=1}^{5} exp(\tilde{\alpha}_k + \hat{\gamma} x_{kt} + (\hat{\bar{\beta}} + \hat{\sigma_\beta}\nu_i^h) p_{kt}(\xi^r, e^r) + \xi_{kt}^r))$
   - $CW_t^{\text{before}}(\xi^r, e^r) \approx \frac{1}{H}\sum_{h=1}^{H} \frac{1}{\hat{\bar{\beta}} + \hat{\sigma_\beta}\nu_i^h} \ln(1 + \sum_{k=1}^{5} exp(\hat{\alpha_k} + \hat{\gamma} x_{kt} + (\hat{\bar{\beta}} + \hat{\sigma_\beta}\nu_i^h) p_{kt}(\xi^r, e^r) + \xi_{kt}^r))$
   - Note that $\tilde{\alpha}_j = 0$ and $\tilde{\alpha}_{-j} = \hat{\alpha}_{-j}$ for counterfactual product j

5 Counterfactual: Merger

Assume that firm 1 and firm 2 merged.

1. For firm $j = 3, 4, 5$, we can still apply $(*)$ to calculate equilibrium price.

2. For the merged firm $j = 1, 2$, we should come up with a modified version of FOC:

$$\begin{aligned}
&\pi_1(p) = (p_1 - mc_1) \cdot s_1 + (p_2 - mc_2) \cdot s_2 \\
&\rightarrow \frac{\partial \pi_1}{\partial p_1} = s_1 + (p_1 - mc_1) \cdot \frac{\partial s_1}{\partial p_1} + (p_2 - mc_2) \cdot \frac{\partial s_2}{\partial p_1} \stackrel{!}{=} 0 \\
&\rightarrow p_1 = mc_1 - (\frac{\partial s_1}{\partial p_1})^{-1} \cdot s_1 - (p_2 - mc_2)(\frac{\partial s_1}{\partial p_1})^{-1} \cdot \frac{\partial s_2}{\partial p_1} \\
\text{likewise} &\rightarrow p_2 = mc_2 - (\frac{\partial s_2}{\partial p_2})^{-1} \cdot s_2 - (p_1 - mc_1)(\frac{\partial s_2}{\partial p_2})^{-1} \cdot \frac{\partial s_1}{\partial p_2} \\
\text{where } s_j &\approx \frac{1}{1,000} \sum_{i=1}^{1,000} \frac{exp(\alpha_j + \gamma x_j + \overline{\beta} p_j + \xi_j + \sigma_\beta \nu_i p_j)}{1 + \sum_{k=1}^{5} exp(\alpha_k + \gamma x_k + \overline{\beta} p_k + \xi_k + \sigma_\beta \nu_i p_k)} \\
\frac{\partial s_j}{\partial p_j} &\approx \frac{1}{1,000} \sum_{i=1}^{1,000} (\overline{\beta} + \sigma_\beta \nu_i)[\frac{exp(\sim)}{1 + \sum_{k=1}^{5} exp(\sim)}][1 - \frac{exp(\sim)}{1 + \sum_{k=1}^{5} exp(\sim)}] \\
\frac{\partial s_q}{\partial p_j} &\approx -\frac{1}{1,000} \sum_{i=1}^{1,000} (\overline{\beta} + \sigma_\beta \nu_i)[\frac{exp(q \text{ term})}{1 + \sum_{k=1}^{5} exp(\sim)}][\frac{exp(j \text{ term})}{1 + \sum_{k=1}^{5} exp(\sim)}] \\
mc_j &= \omega_0 + \omega_1 x_j + \omega_2 z_{1j} + \omega_3 z_{2j} + e_j
\end{aligned}$$

3. Remaining part is essentially same with the part 4.

6 Results

6.1 Demand side estimation

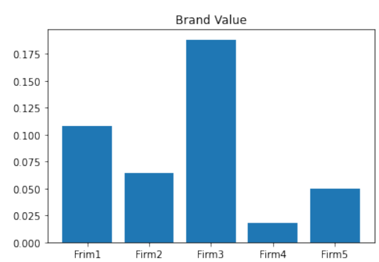
- $\hat{\alpha}_1 = 4.427, \hat{\alpha}_2 = 3.633, \hat{\alpha}_3 = 5.037, \hat{\alpha}_4 = 2.40, \hat{\alpha}_5 = 3.57$
- $\hat{\gamma} = 1.814, \hat{\overline{\beta}} = -1.115, \hat{\sigma_\beta} = 0.229$
- From the GMM estimation, I recovered all the demand side parameters. Level of firm specific intercepts deviate a bit from the true value, but the relative difference is preserved.

6.2 Supply side estimation

- $\hat{\omega}_0 = 4.443, \hat{\omega}_1 = 1.057, \hat{\omega}_2 = 1.027, \hat{\omega}_3 = 0.350$

6.3 Counterfactual: Brand Value

- (1) Brand value of each firm: Randomness of x and z are averaged out across t. So the brand value is highly correlated with the firm specific intercepts α_j



- (2) Price change in percentage (in the order of j = 1,2,3,4,5)
  - from killing product 1's brand value: -1.87 0.71 1.33 0.45 0.64
  - from killing product 2's brand value: 0.53 -1.12 0.78 0.26 0.36
  - from killing product 3's brand value: 1.68 1.31 -3.18 0.79 1.08
  - from killing product 4's brand value: 0.15 0.12 0.22 -0.33 0.10
  - from killing product 5's brand value: 0.44 0.31 0.59 0.18 -0.91

  As observed, price decreases by far as brand value increases.

- (3) Calculated welfare change:
  - from killing product 1's BV: -0.13
  - from killing product 2's BV: -0.08
  - from killing product 3's BV: -0.22
  - from killing product 4's BV: -0.02
  - from killing product 5's BV: -0.06

  Note that α_j directly goes into the consumer's utility function. And the loss from killing the brand value turns out to be bigger than the decrease in price.

6.4 Counterfactual: Merge

- (1) Calculated profit change (in the order of j = 1,2,3,4,5)
  - from merge between firm 1 and 2: 0.001, 0.000, 0.002, 0.000, 0.000
  - from merge between firm 1 and 3: 0.002, 0.003, 0.005, 0.001, 0.002
- (2) Price change in percentage (in the order of j = 1,2,3,4,5)
  - from merge between firm 1 and 2: 1.58 2.52 0.09 0.03 0.04
  - from merge between firm 1 and 3: 4.35 0.14 2.86 0.08 0.11

  As observed, when two big firms merge (1 and 3), they have more room to increase their markup because they can internalize the loss from increasing one product's price with high diversion ratio. From there, they can exploit more profit compared to previous status. One notable thing is the other firms' behavior. When big firms merge, smaller firm also increase their price. And when big merged firm acts its markup, smaller firms could earn additional profit from the diverted share.

- (3) Calculated welfare change
  - from merge between firm 1 and 2: -0.0409
  - from merge between firm 1 and 3: -0.0498

  As expected, there occur consumer welfare loss in both case. When large two firms merge, the magnitude of welfare loss was bigger.