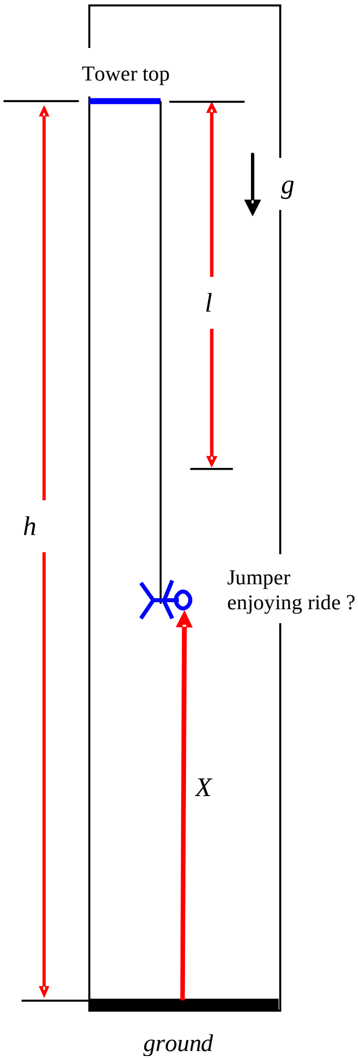
MEEN 363 – SPRING 25 Computational Assignment 1 due 2/19/2025

Modeling the Dynamics of Bungee Cord Jumping



Consider the motion of a bungee jumper as seen from ground (see Figure). The jumper falls from a structure at a height *(h)* above ground. A typical bungee cord undeflected length (*l*) is 18 m, and the average weight (W) and height of a sizable adult is 70 kg and 1.70 m, respectively. During the jumper free fall and subsequent "ride", assume that air drag can be modeled with a viscous damping coefficient (D_a) equal to 8 N.s/m.

Derive appropriate equations for the motion for the jumper.

Investigate the dynamics of the bungee cord jumper for cord stiffnesses (K) ranging from 40 N/m to 240 N/m. The rubber cord has structural type damping which is proportional to the cord stiffness. This damping is related to a material "loss" factor (η), ranging from 0.15 to 0.30 for most cords. The relationship between the cord "equivalent" viscous damping coefficient (D_c) and the cord stiffness (K) is given by,
$D_c = K\eta/\omega_n$, with $\omega_n=(K/M)^{1/2}$.

Note that when the bungee cord is stretched, and as the jumper falls having fun (hopefully), the total viscous damping coefficient (D) retarding the jumper motion equals (D_a+D_c).

Explain (in jargon-free English) how bungee jumpers can fall without sustaining serious injury from a high tower with an elastic cord connected to their feet or waist. Also indicate when (during the jump) the maximum acceleration of the body occurs, and when the body has its maximum speed. How is the cord stiffness (K) related to the jumper maximum acceleration and speed?

Your report must show predictions for the downward displacement traveled by the jumper, speed, and acceleration (+ and -), maximum and minimum values, and the times when these events occur versus the cord stiffness (K) and loss factor (η). Specify the minimum height of the high tower for each case.

Explain (in jargon-free English) what is the effect of cord stiffness and hysteretic damping on the peak force exerted on the jumper? How would you minimize this force while still ensuring a safe and pleasant ride, i.e., with many up/downs? Are the dynamics of the bungee cord jumper most important on the first ½ period of motion when the cord is being stretched? Explain why.

I recommend you to perform an analysis for soft and stiff cords and with small and high loss factors, i.e., encompassing the range of values given. Among other things you need to: select appropriately the coordinate system(s) describing the jumper motion; derive and solve equation for jumper during free fall and predict the velocity of the jumper just before the cord begins to stretch; derive and solve the equation of motion while the cord is being stretched, and predict maximum deflection of cord, maximum acceleration, final position of jumper, etc. **The maximum acceleration (upwards) is of utmost importance for the survival of the jumper.** You may consider performing some physical observations as well.

This is an **open end problem** that has many applications including packaging of electronic components, for example. Note any particular items of interest or your attempts to solve the "actual" problem. **Bonus points** given for including information on "hysteretic" type damping, give details on bungee cord manufacturing and mechanical properties, ride expectations, etc.

P.S. Material searched online must be acknowledged appropriately.

SOME predictions for you to compare against

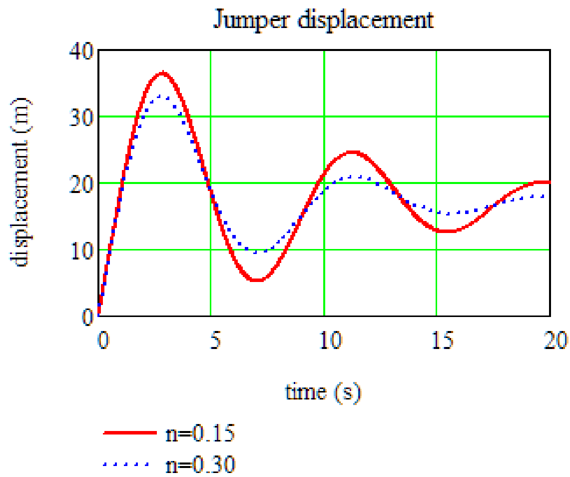
Below origin of the coordinate system (z) is at the free length of the bungee cord; i.e., position just before being stretched

PREDICTIONS:

VARY LOSS FACTOR - KEEP CORD STIFFNESS

$$K := 40 \cdot \frac{N}{m}$$

$$\eta_1 := 0.15 \qquad \eta_2 := 0.3$$



$$D_{crit} := 2 \cdot (K \cdot M)^{.5}$$

end displacement:

$$z_{ss} := \frac{W}{K} \qquad z_{ss} = 17.162\,m$$

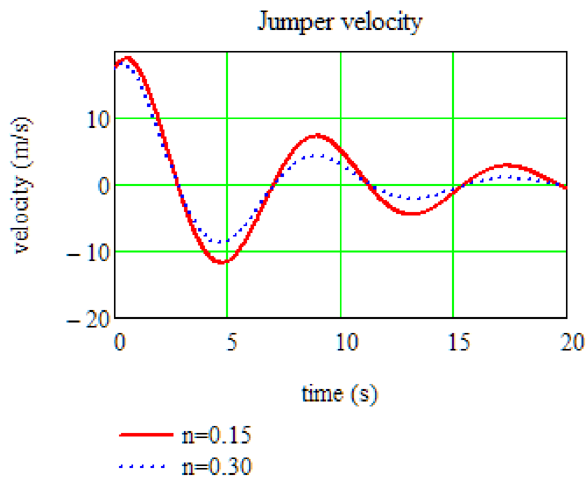
z>0 always; z<0 means cord is compressed (clearly not possible with a bungee cord)

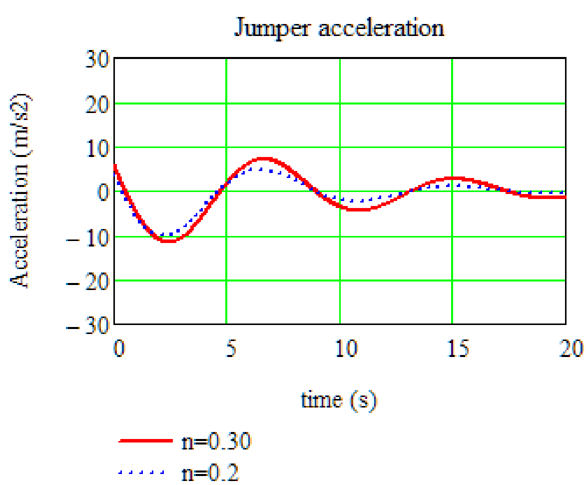
Maximum displacement & time it occurs

[m] [s]

$$ZZ_{max}(K, \eta_1) = (36.449 \quad 2.825)$$

$$ZZ_{max}(K, \eta_2) = (32.966 \quad 2.797)$$





Maximum (pull up acceleration)

[m/s2] [s]

$$AZ_{max}(K, \eta_1) = (-11.541 \quad 2.42)$$

$$AZ_{max}(K, \eta_2) = (-10.035 \quad 2.179)$$