MEEN 432 –Automotive Engineering

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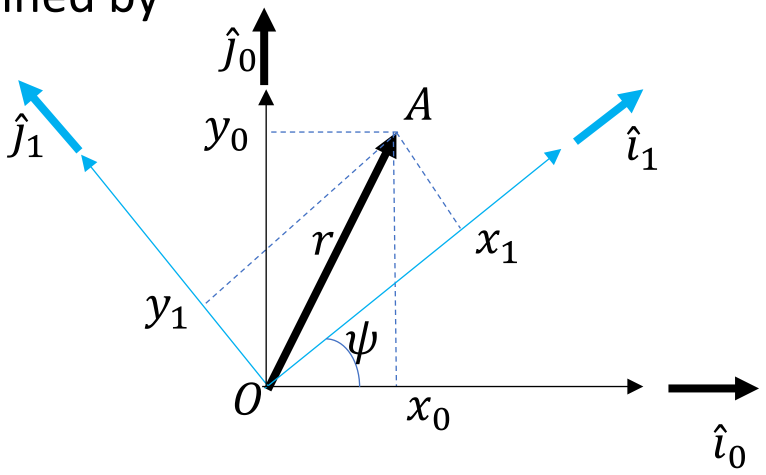
Lecture 4: Vehicle Kinematic Model

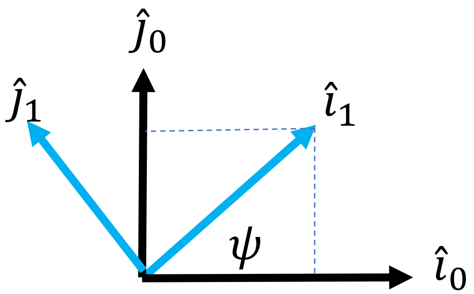
- Kinematics

Vector Transformation under Frame Rotation

- Consider frames $\mathrm{F}_0 = (O, (\hat{\imath}_0, \hat{\jmath}_0))$ and $\mathrm{F}_1 = (O, (\hat{\imath}_1, \hat{\jmath}_1))$, with F_1 obtained by rotating F_0 by ψ about the z axis



- Consider a vector $\overrightarrow{OA} = x_0\hat{\imath}_0 + y_0\hat{\jmath}_0 = x_1\hat{\imath}_1 + y_1\hat{\jmath}_1$
- Placing the unit vectors at the origin:
  - $\hat{\imath}_1 = \cos\psi\ \hat{\imath}_0 + \sin\psi\ \hat{\jmath}_0$
  - $\hat{\jmath}_1 = -\sin\psi\ \ \hat{\imath}_0 + \cos\psi\ \hat{\jmath}_0$



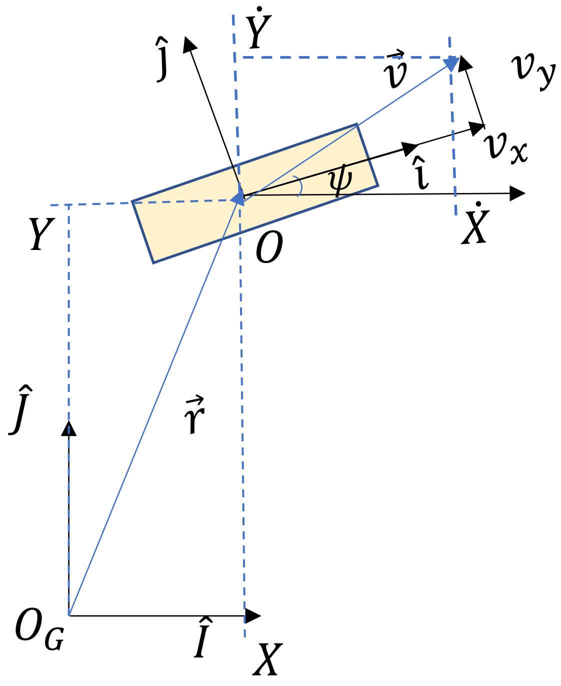
- This yields:
  - $x_0\hat{\imath}_0 + y_0\hat{\jmath}_0 = x_1(\cos\psi\ \hat{\imath}_0 + \sin\psi\ \hat{\jmath}_0) + y_1(-\sin\psi\ \ \hat{\imath}_0 + \cos\psi\ \hat{\jmath}_0)$
- Separating the individual dimensions, we get:
  - $x_1\cos\psi\ - y_1\sin\psi = x_0;\ x_1\sin\psi + y_1\cos\psi = y_0$
- With some algebra and trigonometry,
  - $x_1 = x_0\cos\psi + y_0\ \sin\psi;\ y_1 = -x_0\sin\psi + y_0\cos\psi$
  - Writing in vector form:
    - $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \coloneqq \mathcal{R}(\psi) \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$; and $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \coloneqq \mathcal{R}(-\psi) \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

Rotation Matrix

Body Fixed Frame

- Assume 2D
- Consider:
  - A global (inertial) frame $F_G \coloneqq (O_G, \hat{I}, \hat{J})$
  - A vehicle defined as a rectangle (top view)
    - With a "Body Fixed Frame" $F_B \coloneqq (O, \hat{\imath}, \hat{\jmath})$.
    - Vehicle Heading ψ
- Location of vehicle is $\vec{r}$ and is often represented in Global Frame:
  - $\vec{r} = X\,\hat{I} + Y\,\hat{J}$
- Velocity of vehicle (always w.r.t. F_G) $\vec{v}$ is the derivative of vehicle position relative to inertial frame:
  - $\vec{v} = \frac{d}{dt}\vec{r} = \dot{X}\,\hat{I} + \dot{Y}\,\hat{J}$
- However, it is usually represented in Body Fixed Frame:
  - $\vec{v} = v_x\,\hat{\imath} + v_y\hat{\jmath}$

- In vector notation
  - $\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \mathcal{R}(-\psi) \begin{bmatrix} v_x \\ v_y \end{bmatrix}$



The “Kinematic” Vehicle Model

- Assume that:
  - You could control the vehicle forward (Longitudinal) velocity, i.e. v_x to some desired velocity v_d
  - There is no side-slip, the vehicle moves only in the direction it is pointing, i.e. $v_y = 0$
    - This also implies that the overall vehicle velocity $\vec{v} \coloneqq v_x\,\hat{\imath} + v_y\,\hat{\jmath} = v_x\,\hat{\imath}$
  - You have the ability to the “steer” the vehicle, such that you can control the “yaw rate” $\dot{\psi}$ to some desired value $\dot{\psi}_d$
- Equations of motion for the vehicle:
  - $\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \mathcal{R}(-\psi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_d \\ 0 \\ \dot{\psi}_d \end{bmatrix}$