MEEN 432 –Automotive Engineering

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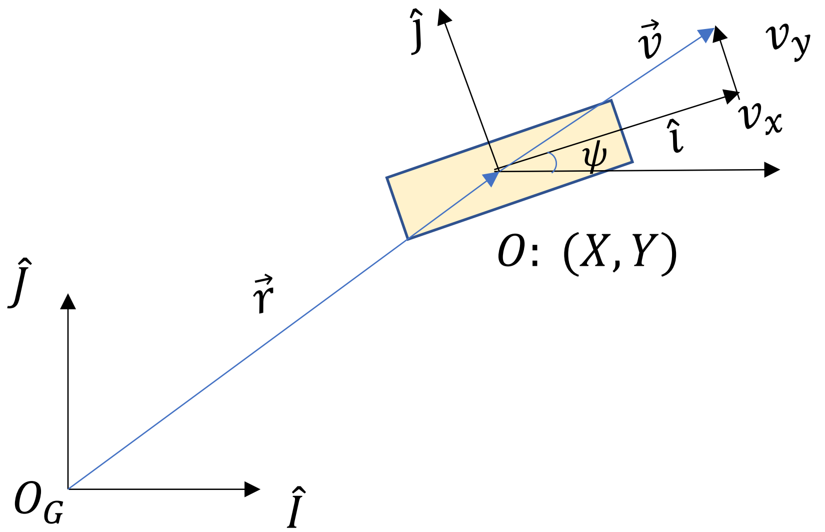
Lecture 4: Vehicle Lateral Dynamics

- Dynamics Equations

Vehicle Acceleration in Body Fixed Frame

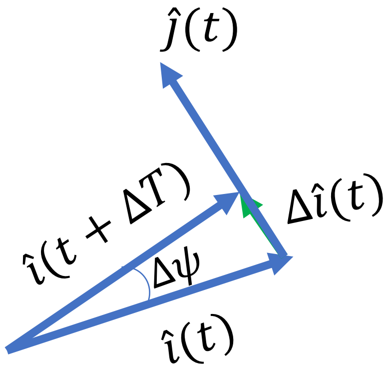
- We want to understand how the vehicle moves when subjected to different forces
  - Powertrain, road, atmosphere
- A first requirement is to define the "acceleration" of the vehicle
- Acceleration is the rate of change of velocity w.r.t. the inertial frame, i.e.
  - $\vec{a} := \frac{d}{dt}\vec{v}$
- Similar to the velocity, we would like to represent it in the Body Frame
  - $\vec{a} = a_x\,\hat{\imath} + a_y\,\hat{\jmath}$

- Why in Body Frame?
  - Because it is easy to describe forces and moments on the vehicle in the Body Frame
  - Example: Powertrain Traction Force, Brake Force, Aerodynamic Drag …



Derivative of a rotating vector

- Assume that a frame F is rotating at some angular velocity $\dot{\psi}$ about its z axis (i.e. angular velocity = $\dot{\psi}\hat{k}$)
- Then each of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$ is rotating at the same angular velocity.
- In a small time period $[\mathrm{t}, \mathrm{t} + \Delta t)$, $\hat{\imath}$ would have rotated by an angle $\Delta\psi \approx \dot{\psi}\Delta\mathrm{t}$
- Change of $\hat{\imath}$ is : $\Delta\hat{\imath}(t) \coloneqq \hat{\imath}(t + \Delta t) - \hat{\imath}(t) = \Delta\psi\ \hat{\jmath}$
- Then, $\dot{\hat{\imath}}(t) = \lim_{\Delta T \to 0} \frac{\Delta\hat{\imath}(t)}{\Delta T} = \dot{\psi}\hat{\jmath}$
- Similarly: $\dot{\hat{\jmath}}(t) = -\dot{\psi}\hat{\imath}$



Vehicle Acceleration

- Acceleration:
- $\vec{a} \coloneqq \frac{d}{dt}\vec{v} = \frac{d}{dt}\left(v_x\,\hat{\imath} + v_y\hat{\jmath}\right)$
- $= \dot{v}_x\,\hat{\imath} + v_x\,\dot{\hat{\imath}} + \dot{v}_y\hat{\jmath} + v_y\,\dot{\hat{\jmath}}$
- Utilizing the derivation we just did,
- $\vec{a} = \dot{v}_x\;\;\hat{\imath} + v_x\,(\dot{\psi}\hat{\jmath}) + \dot{v}_y\,\hat{\jmath} + v_y(-\dot{\psi}\hat{\imath})$ Or
- $\vec{a} = (\dot{v}_x - \dot{\psi}v_y)\,\hat{\imath} + (\dot{v}_y + \dot{\psi}v_x)\,\hat{\jmath}$

- Thus we can write:
- $\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} \dot{v}_x - v_y\dot{\psi} \\ \dot{v}_y + v_x\dot{\psi} \end{bmatrix}$
- Vehicle angular velocity and acceleration are:
  - $\omega \coloneqq \dot{\psi}$
  - $\alpha \coloneqq \ddot{\psi}$

Vehicle Dynamic Equations

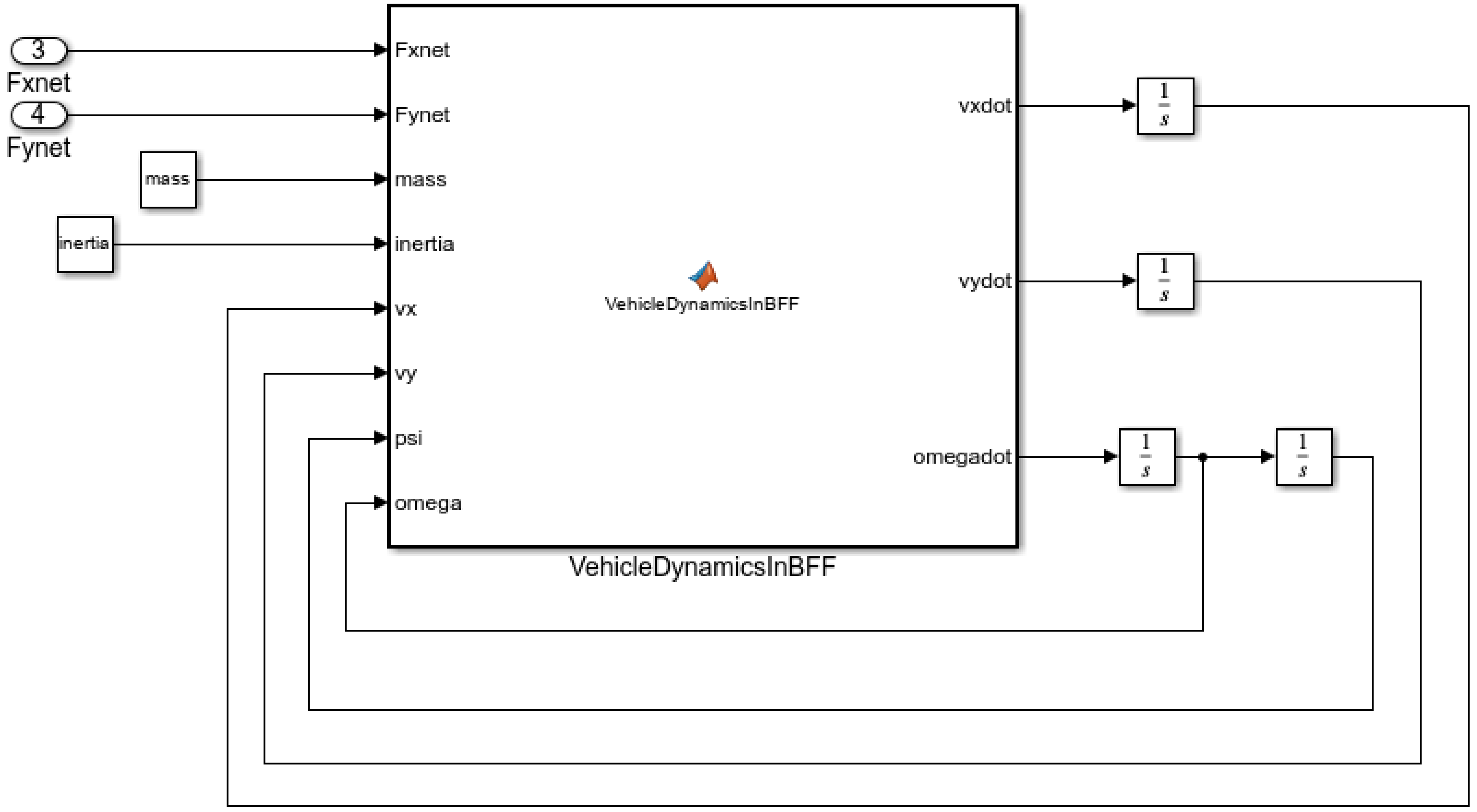
- We model the vehicle as a rigid body of mass m and inertia I !!
- The equations of motion fall out of Newton:
  - $m\ a_x = \Sigma F_x$
    - linear acceleration in body x direction is based on total forces along x
  - $m\ a_y = \Sigma F_y$
    - linear acceleration in body y direction is based on total forces along y
  - $I\ \alpha = \Sigma\ M$
    - angular acceleration around body z axis is based on total moment about z
- Body x axis is often referred to as the "Longitudinal Axis"
  - Motion and dynamics constrained to x axis referred to as "Longitudinal Dynamics"
- Motion and dynamics focused on y axis referred to as "Lateral Dynamics"
- Motion about z axis referred to as "Yaw" (Yaw angle, Yaw rate, Yaw acceleration)

Vehicle Dynamics

- We can summarize the previous discussions as below:
- $$\begin{bmatrix} a_x \\ a_y \\ \alpha \end{bmatrix} = \begin{bmatrix} \dot{v}_x - v_y\omega \\ \dot{v}_y + v_x\omega \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\Sigma F_x}{m} \\ \frac{\Sigma F_y}{m} \\ \frac{\Sigma M}{I} \end{bmatrix}$$
  - where we are yet to define the forces and moments, except that they be in BFF
- In order to simulate the system we want to write it in the normal ODE form:
  - $$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v_y\omega + \Sigma F_x/m \\ -v_x\omega + \Sigma F_y/m \\ \frac{\Sigma M}{I} \\ \omega \end{bmatrix}$$

- Often we also need to keep track of the position of the vehicle – in the global frame!
- We use the standard coordinate transformations:
  - $$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \mathcal{R}(-\psi) \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$
- With this, we have all the information needed to simulate a vehicle
  - IF we know the forces and moments on the vehicle

A Matlab Implementation Shell



Vehicle Dynamics

- What is left to analyze (simulate and predict) the motion of the vehicle is the understanding of the forces and moments:
  - F_x, F_y and M
- These are the forces that act on the vehicle – from the external world
  - Tire Forces
  - Aero Dynamic (Drag) Forces
  - (These don't directly include any powertrain forces … why?)
- Those forces are usually specified in the BFF
- Hence the derivation of the dynamics to be in the BFF