MEEN 432 –Automotive Engineering

Fall 2026

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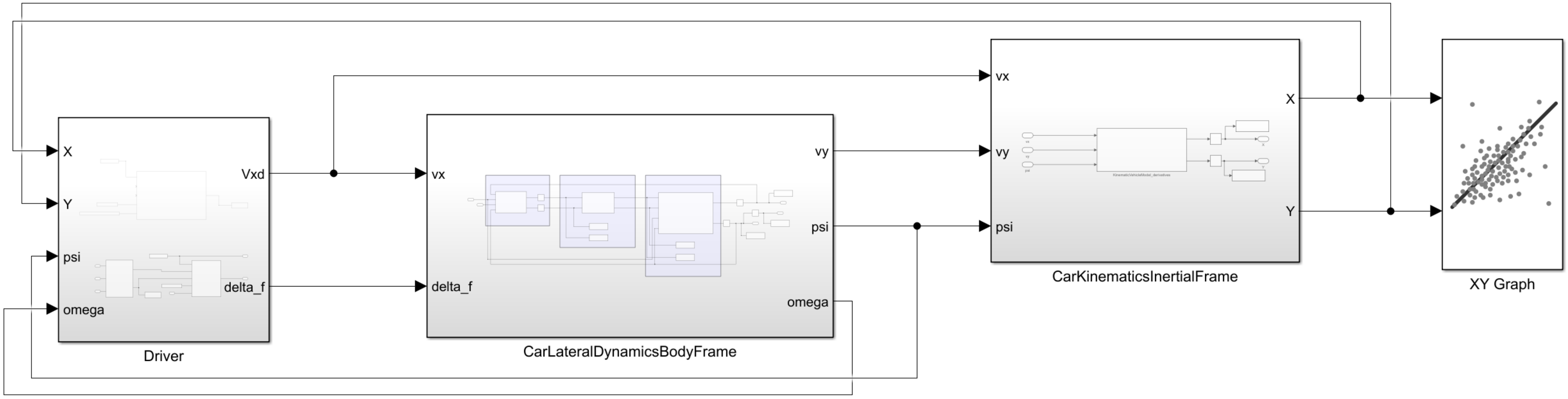
Acknowledgement: Most of the material for this class was developed by Dr. Swami Gopalswamy

Lecture 7: Vehicle Lateral Dynamics

- Advanced Driver Model – Pure Pursuit Control
- Ackerman steering

Demo

Simplest Driver Model (recall)



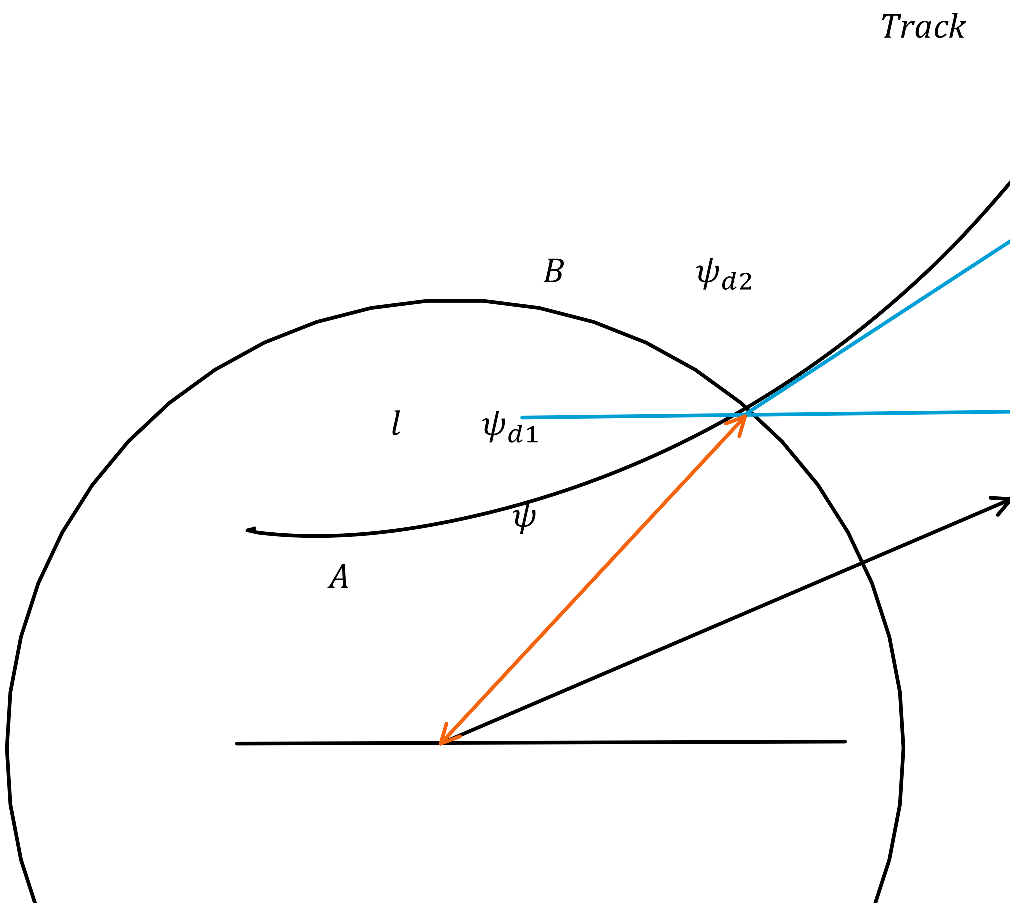
- Simplest Driver Model
  - Set commanded velocity to desired velocity
  - If on straight-away set steering to 0
  - If on curve, set steering to some $\delta_d > 0$

Improving driver models

- Model 1
  - "Anticipating" a curve, et steering angle to be the Ackerman angle
    - $\delta = \frac{L}{R}$
- Model 2
  - "Knowing" your vehicle, set steering to be velocity compensated Ackermann angle
    - $\delta = \frac{L}{R} + \frac{m}{LR}\left(\frac{l_r}{C_{\alpha_f}} - \frac{l_f}{C_{\alpha_r}}\right) v_x^2$
- A fundamental problem with all the above is that we assume we can observe the environment exactly and react to it instantaneously
  - In reality, there will be sensor noise and so measurements will not be perfect
  - The controller can respond only with a certain lag (arising from digital hardware, or physical hardware)
  - Numerical simulations also introduce errors because of quantization, integration approximations, etc.
- All these can lead to divergence of performance from desired
  - Especially as you push your system (for e.g. go faster!)

Feedback Controlling Drivers

- Vehicle at point A needs to follow the path given by $Track$
- Driver "looks ahead" a distance l
  - This intersects the path at point B
- Driver steers "towards" B
  - Current heading of vehicle ψ
  - Desired heading – without considering the future path: ψ_{d1}
  - Desired heading – if we consider the (tangent of the) path at point B is ψ_{d2}
  - Actual desired heading is somewhere in-between
    - Closer to A, $\psi_d \rightarrow \psi_{d1}$; closer to B, $\psi_d \rightarrow \psi_{d2}$
- How to steer?
  - Look at deviation $\psi_d - \psi$
  - Steering will be based on this deviation



Pure Pursuit Control

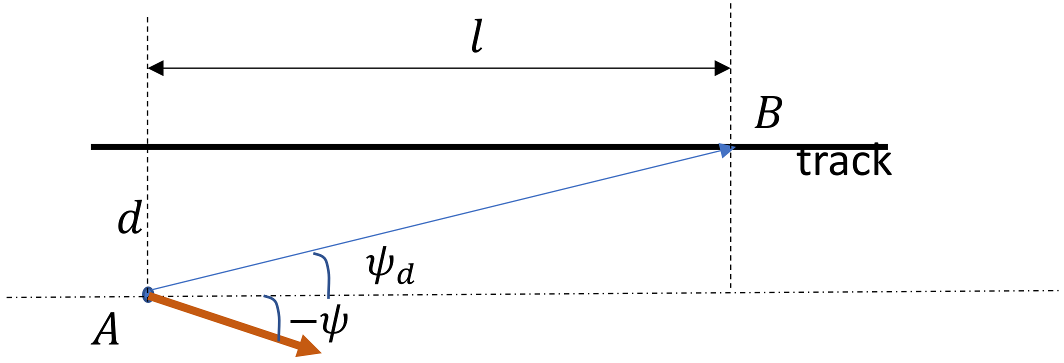
- The strategy to pursue this imaginary look-ahead point on the path is called "Pure Pursuit" control (amongst other names)
- Simplest Strategy:
  - We want $\psi \to \psi_d$. Therefore we try to make $\dot{\psi} = k_\psi(\psi_d - \psi)$ for some $k_\psi > 0$
  - Recall:
    - For constant radius circular motion, $\delta = \frac{L}{R} = \frac{\dot{\psi}L}{v}$
    - Or, loosely, steering would directly be proportional to yaw rate (instead of yaw)
  - Thus we synthesize the following strategy: $\delta = k\ (\psi_d - \psi)$ for some $k > 0$
    - k is often "calibrated" (or tuned) through experiments

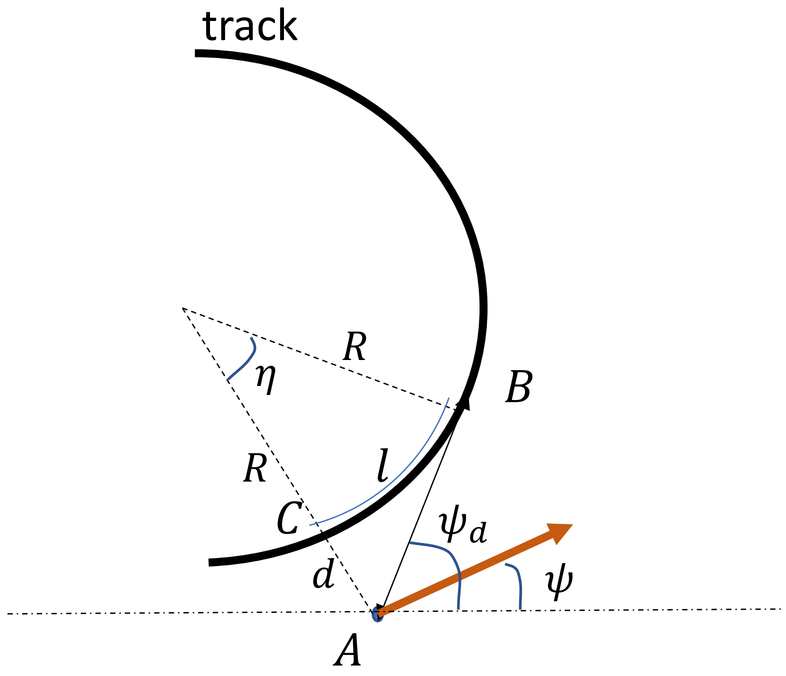
Pure Pursuit

- A more general approach would be to extend the P control to PID:
  - $e \coloneqq \psi_d - \psi$
  - $\delta = PID(e) = k_p e + k_d \dot{e} + k_i \int e$
- Often the approach above is not anticipating the curve well enough
  - An alternate is to add "feed forward" terms in the control
  - $\delta = PID(e) + L/R$
    - If you are picky, you might want to consider dependence on car design:
      - $\delta = PID(e) + \frac{L}{R} + \frac{m}{LR}\left(\frac{l_r}{C_{\alpha_f}} - \frac{l_f}{C_{\alpha_r}}\right) v_x^2$

Special Cases for Pure Pursuit

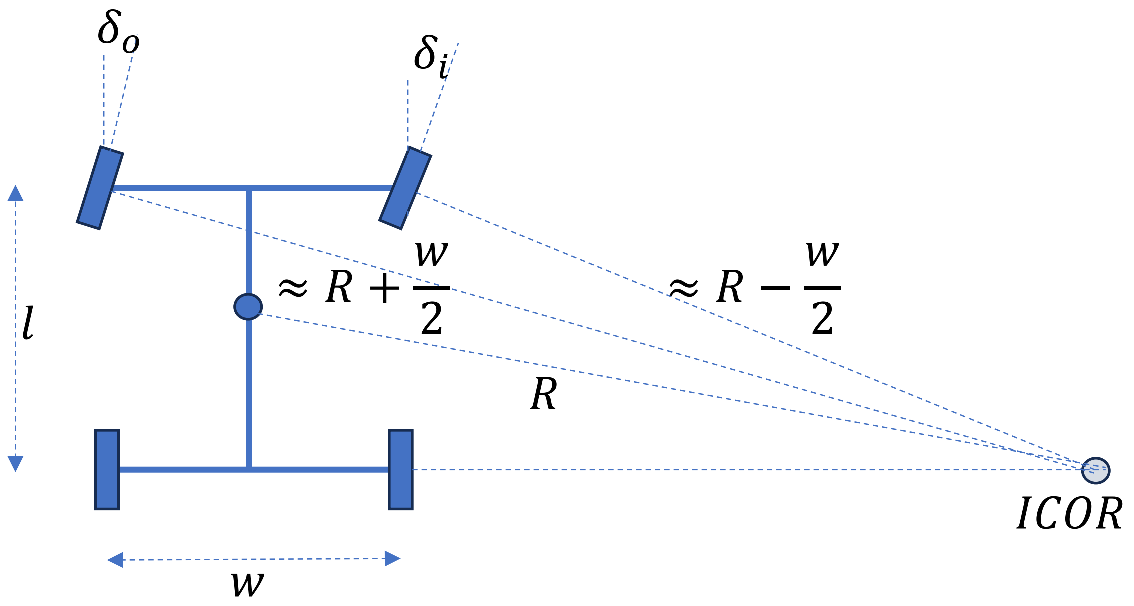
- Choose look ahead along path
- Straight Line
  - $\psi_d = \tan^{-1} d/l$
- Circular Path
  - $\eta = l/R$
  - Draw a line from A (the vehicle's current location) to the center of the curve, to intersect the circle at C.
  - Rotate by angle η to find B at distance l along path
  - ψ_d is the angle that AB makes with the horizontal

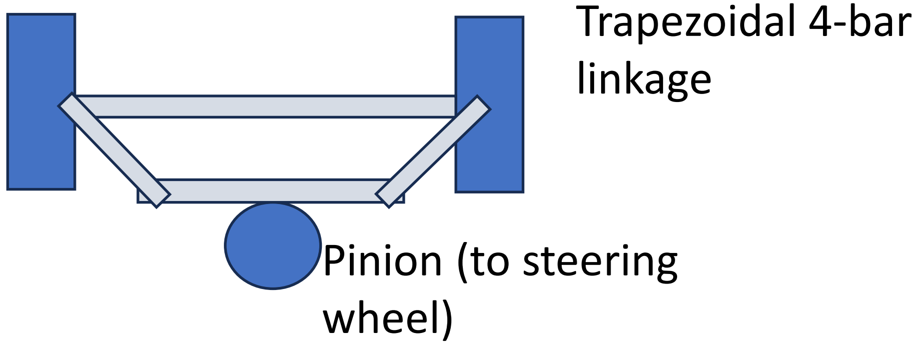




Ackerman Steering

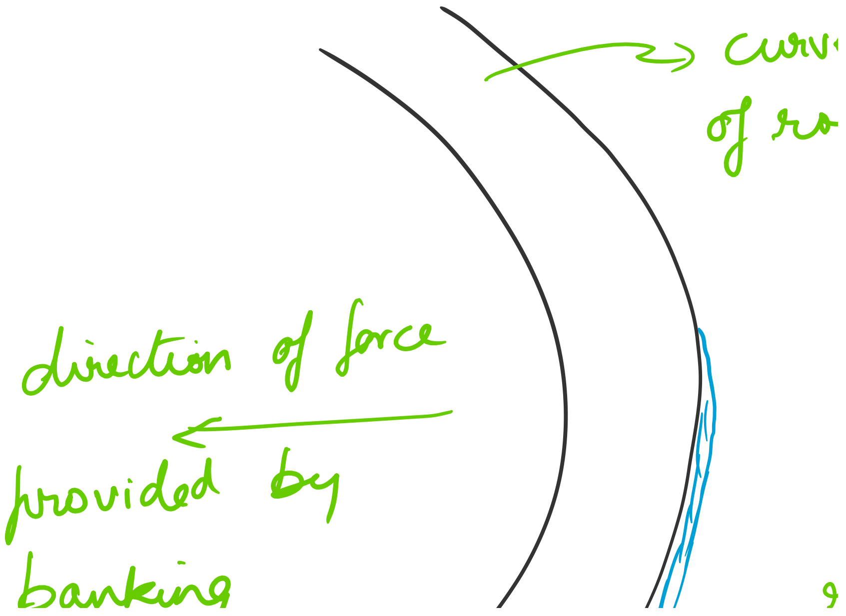
- Consider the two steering wheels separately (i.e. generalize the bicycle model)
- Assume steady cornering, large R:
  - No side slip at rear wheel (i.e. ICOR along rear axle)
  - For no side slip at front wheels, steering angle should make wheels perpendicular to radius
- With small angle approximations:
  - $\delta_i = \frac{l}{R-w/2}$ and $\delta_o = \frac{l}{R+w/2}$
  - $\Delta\delta \coloneqq \delta_i - \delta_o = \frac{lw}{R^2 - \frac{w^2}{4}} \approx \frac{lw}{R^2} = \delta^2 \frac{w}{l}$
    - Where δ is the Ackerman Angle
- If you have independent steering capabilities (wheel steering) you could either "mimic" the Ackerman steering, or develop your own strategy

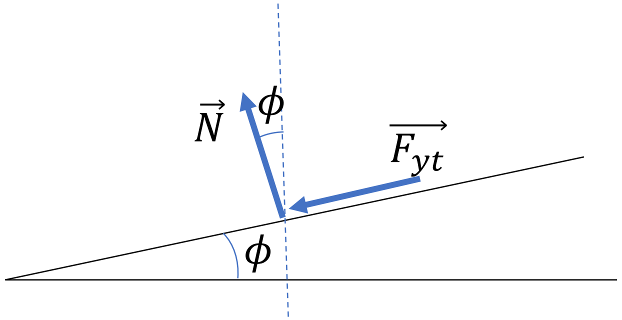




Banking

- Sometimes we want to reduce the responsibility of the tires to produce the lateral force required to steer the vehicle (e.g. on icy roads)
  - In those situations, we leverage “gravity” through “banking”
  - The road surface is (slightly) inclined towards the turn – as shown
- Net lateral force with banking:
  - $\Sigma F_y = F_{bank} = N \sin\phi + F_{yt} \cos\phi$
- Force Balance:
  - $N \cos\phi = mg + F_{yt} \sin\phi$
  - Or, $N = (mg + F_{yt} \sin\phi)\dfrac{1}{\cos\phi}$





Banking

- Thus we have
  - $\Sigma F_y = F_{bank} = N\sin\phi + F_{yt}\cos\phi = mg\tan\phi + F_{yt}(\frac{\sin^2\phi}{\cos\phi} + \cos\phi)$
  - Or, $\Sigma F_y = m\,g\tan\phi + \frac{F_{yt}}{\cos\phi}$
    - (Clearly this equation is not well defined when banking is at a right angle!)
  - Alternatively:
    - $F_{yt} = \Sigma F_y\cos\phi - mg\sin\phi$ i.e. tire force (required) is reduced by gravitational force!
- Plugging back into system dynamics:
- $$\begin{bmatrix} \dot{v}_y \\ \dot{\omega} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -v_x\omega + \frac{F_{y_{Wf}}(\alpha)+F_{y_{Wr}}(\alpha)}{m} \\ \frac{F_{y_{Wf}}(\alpha)l_f - F_{y_{Wr}}(\alpha)\,l_r}{I} \\ \omega \end{bmatrix} \rightarrow \begin{bmatrix} -v_x\omega + \frac{mg\,sin\phi + C_{\alpha_f}\alpha_f + C_{\alpha_r}\alpha_r}{m\cos\phi} \\ \frac{C_{\alpha_f}\alpha_f l_f - C_{\alpha_r}\alpha_r l_r}{I} \\ \omega \end{bmatrix}$$

Banking

- How should banking be designed for a given curve of a road?
  - Consider a maximum safe speed v_{xd} that you are designing for
  - What should the banking be so that gravity provides "all" the turning force, allowing the vehicle to negotiate that curve even on icy roads?
- We want: $F_{yt} = 0 \Rightarrow \quad \Sigma_{Fy} = F_{bank} = mg \tan \phi$
- But, from constant speed cornering we studied earlier: $\Sigma F_y = \frac{m}{R} v_{xd}^2$
- Thus:
  - $\phi = \tan^{-1} \frac{1}{gR} v_{xd}^2$
  - Example: For a 65 mph road turning on a 200 m radius (your race track) ,we need a banking of 23 degrees to achieve "no-hands-on-steering-wheel" turn!

Banking and Project

- For your project 2, you do not *need* banking. However you are allowed up to 10 degrees banking if you desire to bring up your max negotiation speed.
- For project 4, you will need banking.
- Note: Introduce banking gradually when a curve starts – so there are no discontinuities in the road surface

Project 3 (teaser)

- Build an electric vehicle that follows the federal test driving cycles, and determine its energy efficiency by calculating the energy consumed!
  - Longitudinal vehicle dynamics
  - Battery
  - Electric Motor
  - Transmission
  - Driveline