MEEN 432 –Automotive Engineering

Fall 2026

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Lecture 11: Transmission 1

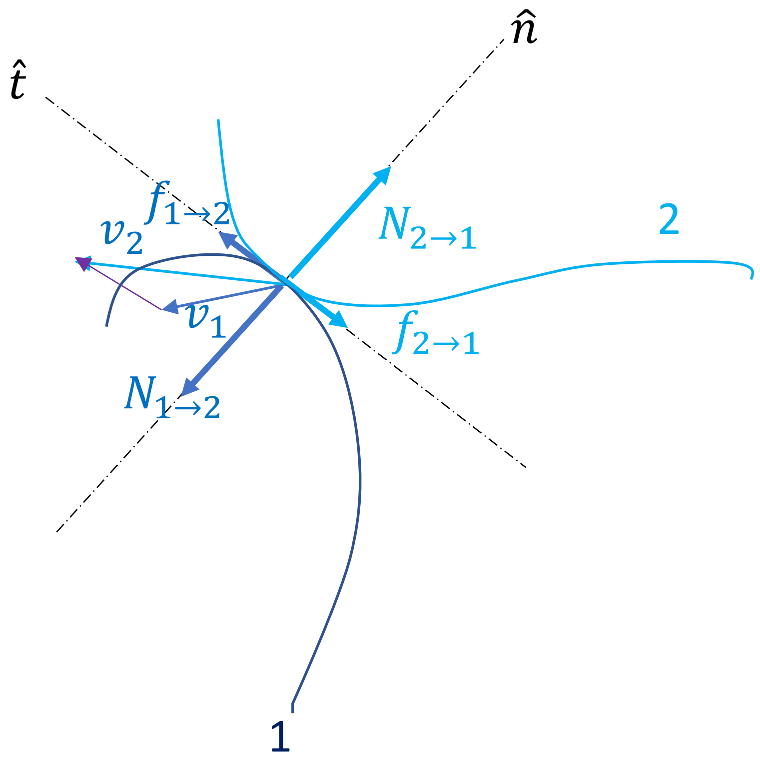
- Gears

Transmission

- The purpose of the transmission is to transmit power from one source of power to another
  - Delivered power is always mechanical in automotive systems, so there is a corresponding torque and angular speed (or force and linear speed):
    - $p_o = \tau_o \, \omega_o$
  - Input power could be either mechanical or electrical:
    - $p_i = \tau_i \, \omega_i$ or $p_e = i \; V$
- Mechanical Power Input to Mechanical Power Input is achieved through mechanical transmissions
  - A common mechanical transmission is composed of “gears”

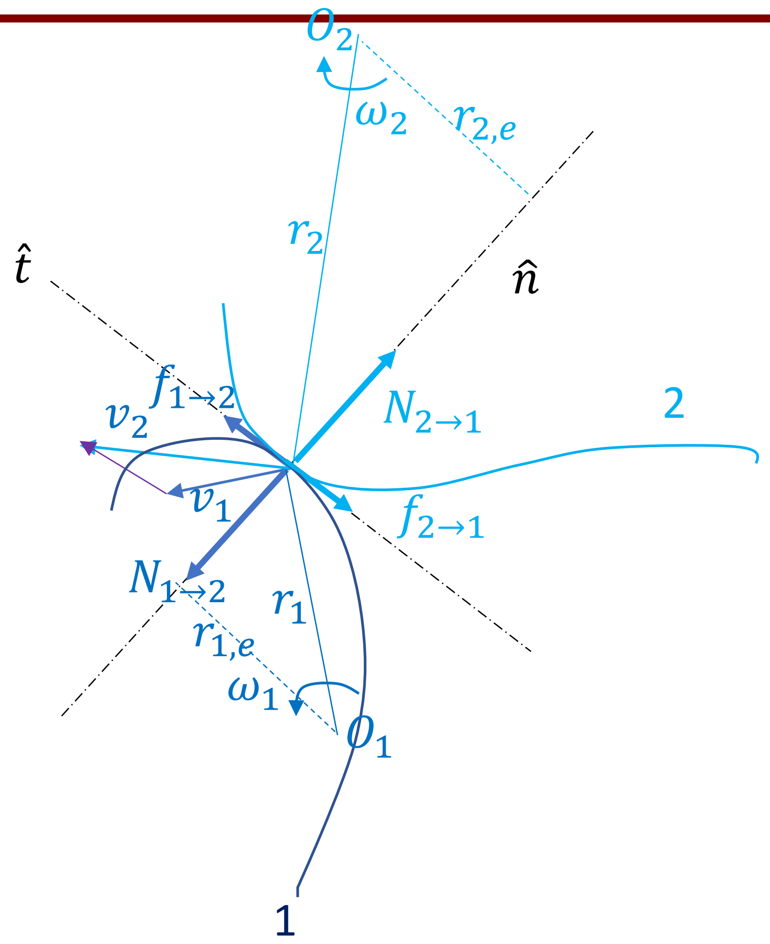
Contact force transmission

- When two surfaces contact each other to transmit force, we consider this a positive engagement
- The interaction force can be considered along two dimensions:
  - Normal to the surface
  - Tangential to the surface
- The tangential component of the force arises because of (and only if there is) friction at the surface
  - Relative motion at point of contact is possible
- The normal component of the force arises because the materials are sufficiently rigid and solid that both materials cannot exist at the same place at the same time!
  - Relative motion at point of contact is not possible without breaking contact
  - Action and Reaction: $N_{1\to 2} = N_{2\to 1}$ and $f_{1\to 2} = f_{2\to 1}$
- In general, we would like to design the contact profile so that useful power is transmitted along the normal, and sliding force is minimized by good surface finishes



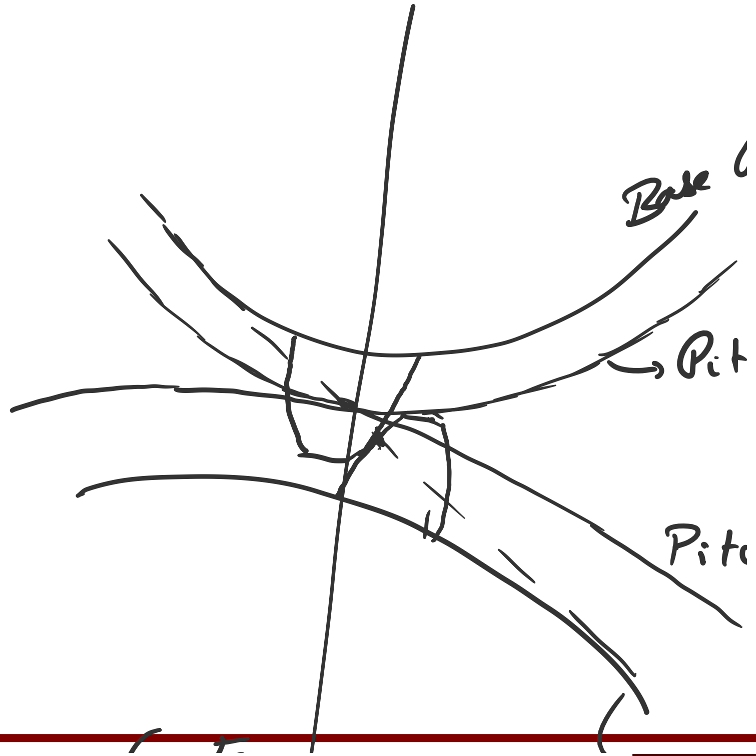
Contact force transmission

- Now consider that the two bodies 1 and 2 are rotating about centers of rotation at O_1 and O_2 , with angular velocities ω_1 and ω_2 respectively as shown
- Additionally let the distance from the center of rotation to the point of contact be r_1 and r_2 respectively
- Consider the case when there is no friction, i.e. $f_{1\to 2} = f_{2\to 1} = 0$
- Torque on each of the body is then:
  - $\tau_2 = \vec{r}_2 \times N_{2\to 1}$ and $\tau_1 = \vec{r}_1 \times N_{1\to 2}$
- Alternatively, we can consider the normal force acting tangential to two circles of effective radii $r_{1,e}$, $r_{2,e}$:
  - $\tau_2 = r_{2,e} N$ and $\tau_1 = r_{1,e} N$ with $N = |N_{2\to 1}| = |N_{1\to 2}|$
- Since relative velocity along normal is zero, we have:
  - $\omega_2 r_{2,e} = \omega_1 r_{1,e}$



Gears

- Gears transmit power through toothed wheels
  - Ideally at a constant ratio of speeds
- How to design a tooth profile such that a constant speed ratio be achieved?
  - This will be achieved if the Normal remains tangential to circles of radii of the desired ratio
  - An “involute” profile is an answer
    - The locus of a thread unwinding from a spooled roll!
- The Gears then behave like they are two “circles rolling on top of each other”!
  - These “effective” circles are called the “pitch circles”
- Dropping the “effective” subscripts we have:
  - $\omega_1 r_1 = \omega_2 r_2$
- Sometimes, pitch diameters are used:
  - $\omega_1 d_1 = \omega_2 d_2$



Simple Gear

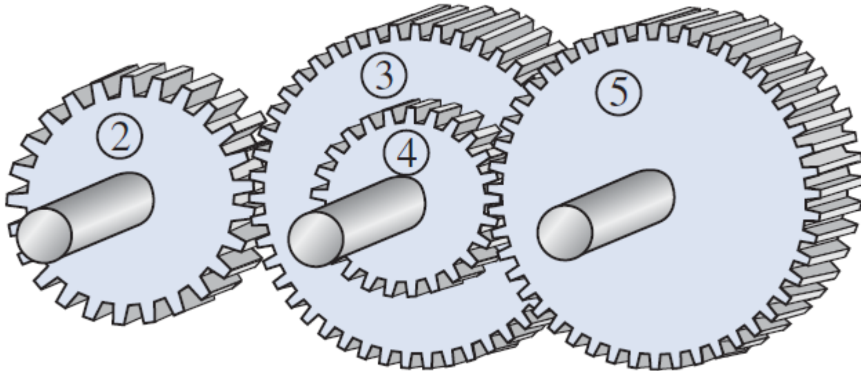
- Ratio of pitch circle diameters
- = Ratio of base circle diameters
- = ratio of number of teeth (assuming they all are equally sized)
- $\frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{N_2}{N_1} := G$
- G is used to define a "gear ratio"
  - The ratio of the speed of the input (upstream) shaft to the output (downstream) shaft
- What we have discussed is referred to as a "simple gear"

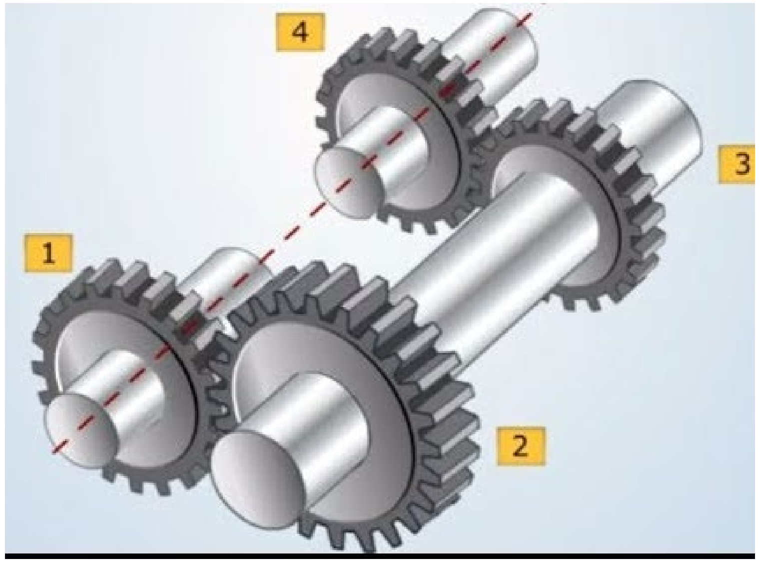
Power Flow through an "Ideal" Simple Gear

- An "ideal gear" is defined by the following assumptions:
  - Gear teeth surface have no friction
  - Gears have no inertia
- Then no energy can stored or lost in a gear – so:
  - $p_1 = \tau_1\omega_1 = p_2 = \tau_2\omega_2$
  - Using the definition of gear ratio G:
    - $\tau_2 = \tau_1 \frac{\omega_1}{\omega_2} = \tau_1 \, G$
- The assumption of having no inertias also leads to the ideal gear being called the "kinematic gear"

Combinations of gears – a gear train

- A simple gear is approximately characterized by one gear mesh
- Complex gears can be decomposed into a series of simple gears that are "kinematically attached" so the attachment shaft speeds are the same
- For example:
  - Top gear train: $\omega_3 = \omega_4$
  - Bottom gear train: $\omega_2 = \omega_3$
- Each simple gear decomposition follows its own speed relations
- Together the entire "gear train" can be considered as having one "effective" gear ratio
- Example:
  - Top gear train: $G := \frac{\omega_2}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_3}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_4}{\omega_5} = G_{23}G_{45}$
  - Bottom gear train: $G := \frac{\omega_1}{\omega_4} = \frac{\omega_1}{\omega_2}\frac{\omega_2}{\omega_4} = \frac{\omega_1}{\omega_2}\frac{\omega_3}{\omega_4} = G_{12}G_{34}$

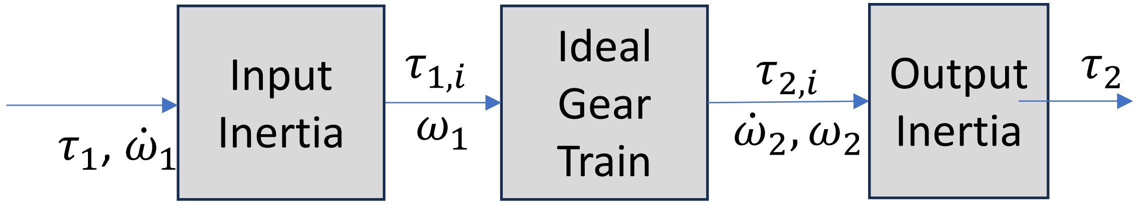




Gear Train Modeling

- An ideal gear train is modeled as a pure torque and speed ratio device, with two shafts 1 & 2
  - $\tau_{2,i} = G\,\tau_{1,i};\;\; \omega_1 = G\,\omega_2$
  - Here the subscript "i" denotes an ideal gear train

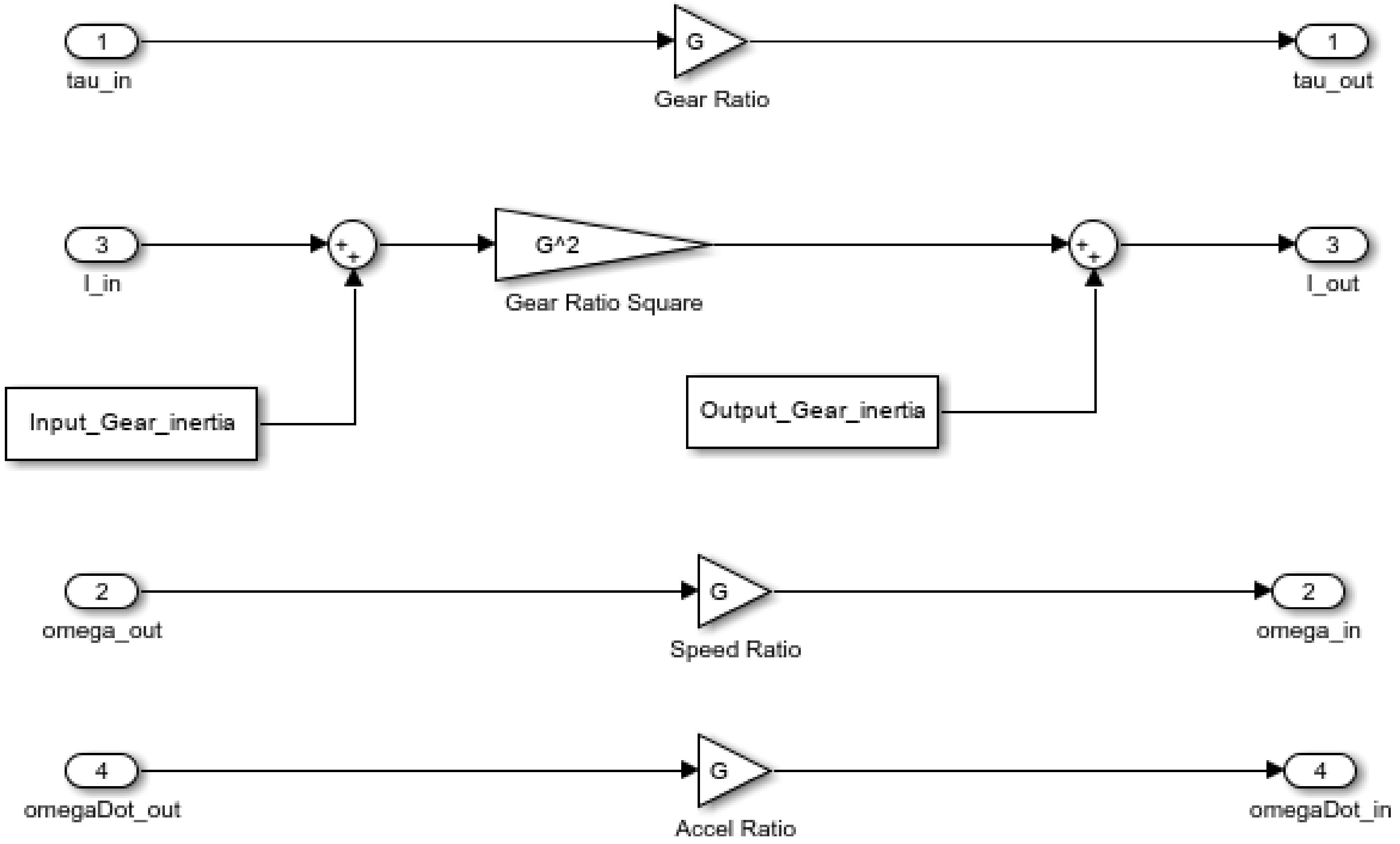
- A (non-ideal) real gear train is modeled by partitioning the inertias of all rotating components and assigning them to the two shafts
  - The inertias satisfy the standard equations of motion:
    - $\tau_1 - \tau_{1,i} = I_1\dot{\omega}_1$ and $\tau_{2,i} - \tau_2 = I_2\dot{\omega}_2$



Gear Train Modeling

- Rewriting, we have:
  - $\tau_{1.i} = \tau_{2,i}/G$; $\tau_1 - \tau_{1,i} = I_1\dot{\omega}_1$; and $\tau_{2,i} = \tau_2 + I_2\dot{\omega}_2$
- Observing that $\dot{\omega}_1 = G\,\dot{\omega}_2$ we have:
  - $I_1\dot{\omega}_1 = \tau_1 - \tau_{1,ideal} = \tau_1 - \frac{\tau_{2,ideal}}{G} = \tau_1 - \frac{1}{G}(I_2\dot{\omega}_2 + \tau_2) = \tau_1 - \frac{1}{G}(I_2\frac{\dot{\omega}_1}{G} + \tau_2)$
  - Or $\left(I_1 + \frac{I_2}{G^2}\right)\dot{\omega}_1 = \tau_1 - \frac{1}{G}\tau_2$
- Alternatively, we can also derive:
  - $\left(I_2 + I_1G^2\right)\dot{\omega}_2 = G\tau_1 - \tau_2$
- Note the patterns that you have seen before:
  - The input shaft inertia appears squared by gear ratio when reflected on the output shaft

Gear Train Modeling – Implementation Example



Gear Losses

- In reality, there is friction (at the meshes)
  - The loss is characterized in many ways – example, a function of shaft speeds and transmitted torque
  - We will use a simple “Efficiency” η
- We will model gear losses external to the ratio transmission dynamics, and can be either done at the output or the input
  - $p_{loss} = |p_j|\,(1 \ -\eta)$ where $|p_1| > |p_2| \Rightarrow j = 1, else\ 2$
  - Then we can write $p_2 = p_1 \eta^{\operatorname{sgn} p_1}$
    - the signum function for negative power flow through the gear train.
  - Gear losses do not impact the kinematics, so we still have $\omega_1 = G\,\omega_2$
  - Thus: $\tau_2\omega_2 = \tau_2 \frac{\omega_1}{G} = \tau_1\,\omega_1 \eta^{\operatorname{sgn} \tau_1\omega_1}$
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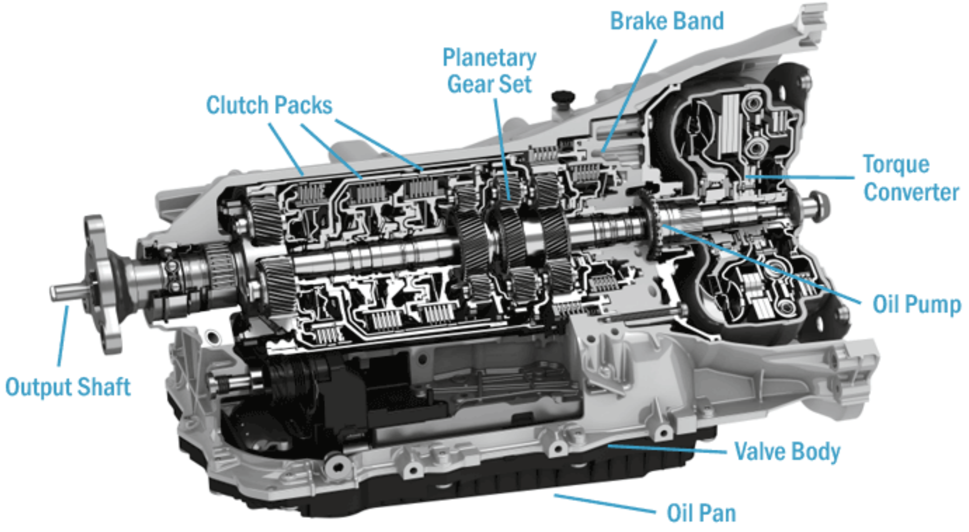
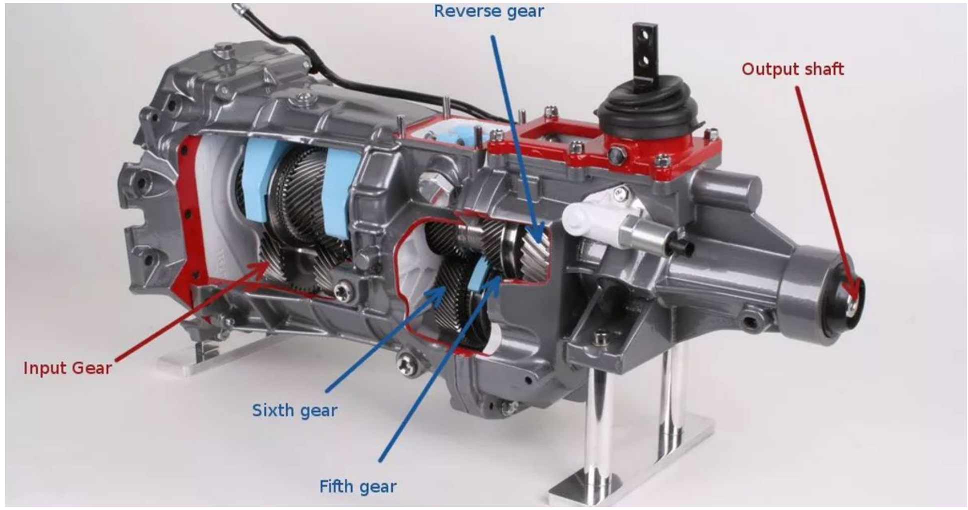
Automotive Transmission

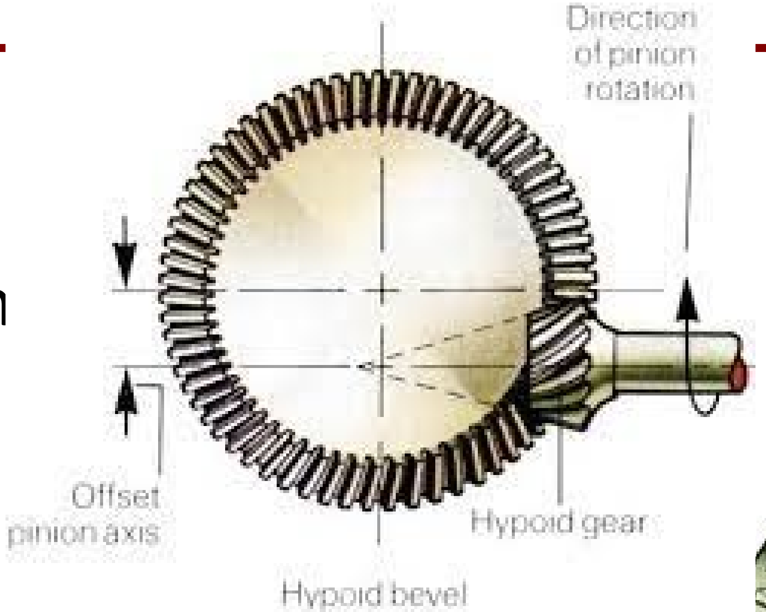
- A typical vehicle transmission consists of multiple gear pairs that could be “selected” in an algorithmic way
  - “Clutches” used to selectively connect/disconnect shafts, and correspondingly which gear train is active
  - A “torque converter” or a “starting clutch” couples the engine with the gear train part of the transmission
- We will not discuss the design of a Transmission for this class scope
  - Instead a transmission will be characterized by its gear ratios (and corresponding input and output inertias, and efficiencies)
  - And we will assume that we can magically control the gear ratio by simply issuing a command
    - In reality, such command will lead a to coordinated actuation of multiple solenoids, which will in turn engage different clutches, to realize the gear change!

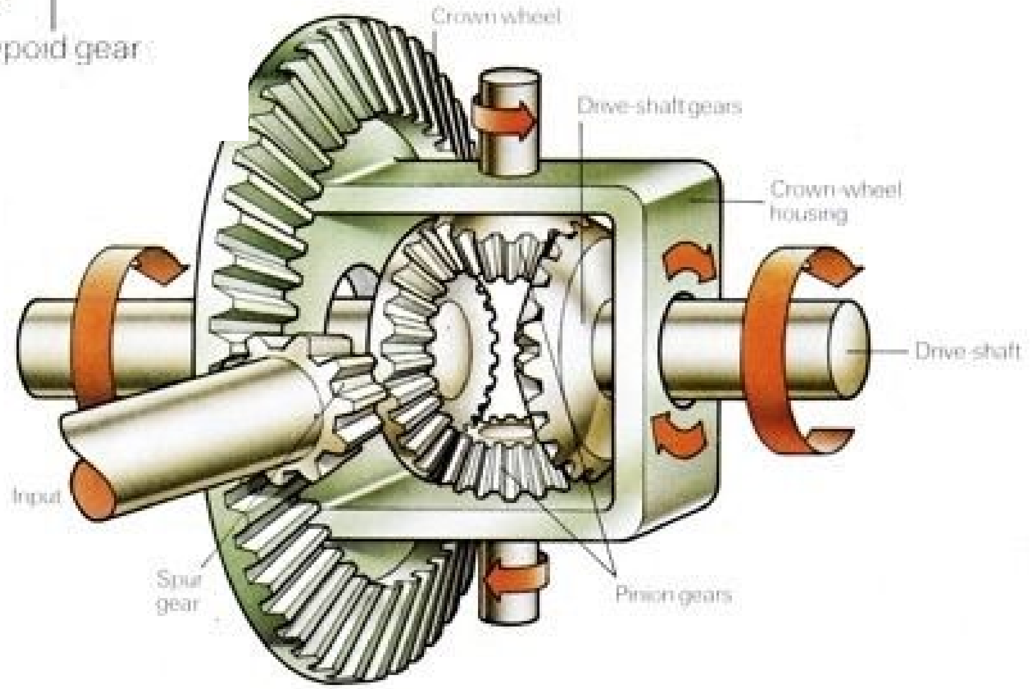




Final Drive

- Provides torque multiplication
- Changes direction of rotation (from longitudinal axis of car to along wheel shaft axes)
- Allows for differential rotation of wheels
  - Two-degree of freedom gears (planetary gears)
- For the class scope – just another gear train (with fixed gear ratio!)





Powertrain

- Now you know how to model everything about the longitudinal dynamics
  - Except the power plant – we will consider electric motors
- Components of the powertrain
  - Engine / Electric Motor
  - Transmission
  - Final Drive
  - Wheels
  - Brakes
  - Body
- Control capability (what can the driver (model) influence?)
  - Torque from Engine
  - Torque from Brake
  - Gear Cmd