MEEN 432 –Automotive Engineering

Fall 2026

Instructor: Dr. Arnold Muyshondt

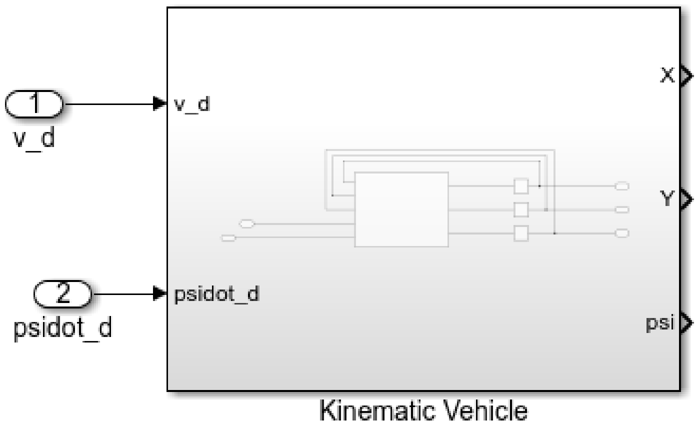
Acknowledgement: Most of the material for this class was developed by Dr. Swami Gopalswamy

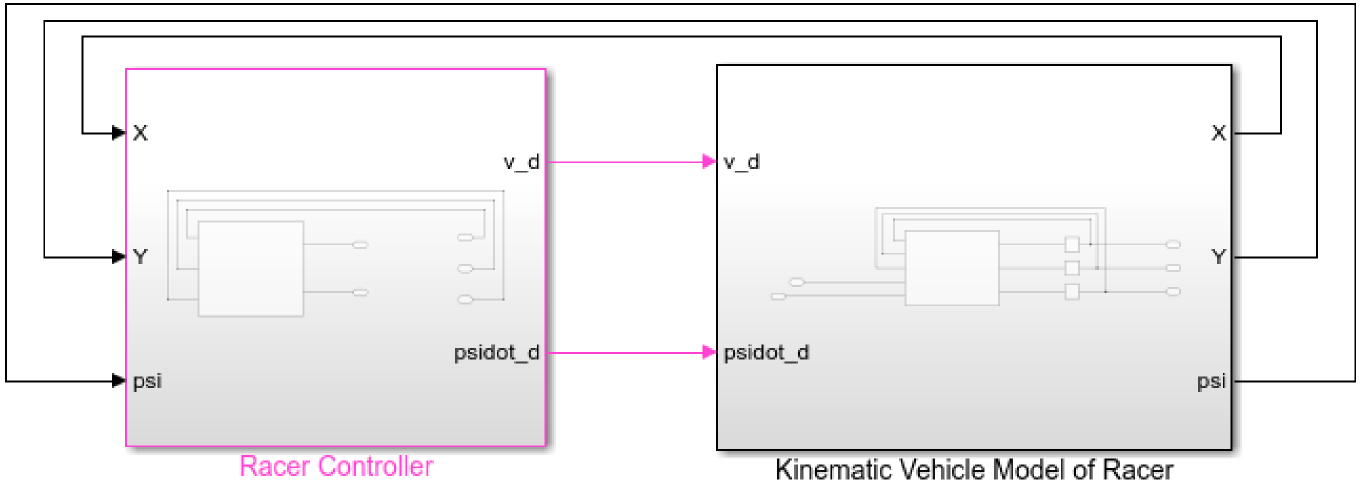
Lecture 6: Vehicle Lateral Dynamics

- Tire Forces

Kinematic Model and Control

- How do you control your race vehicle so that it follows the track?
- Set v_d based on desired velocity
- Set $\dot{\psi}_d$ based on track curvature as below:
  - $\dot{\psi}_d = v_d/R$!!
- Key is to detect what R is based on your location X, Y





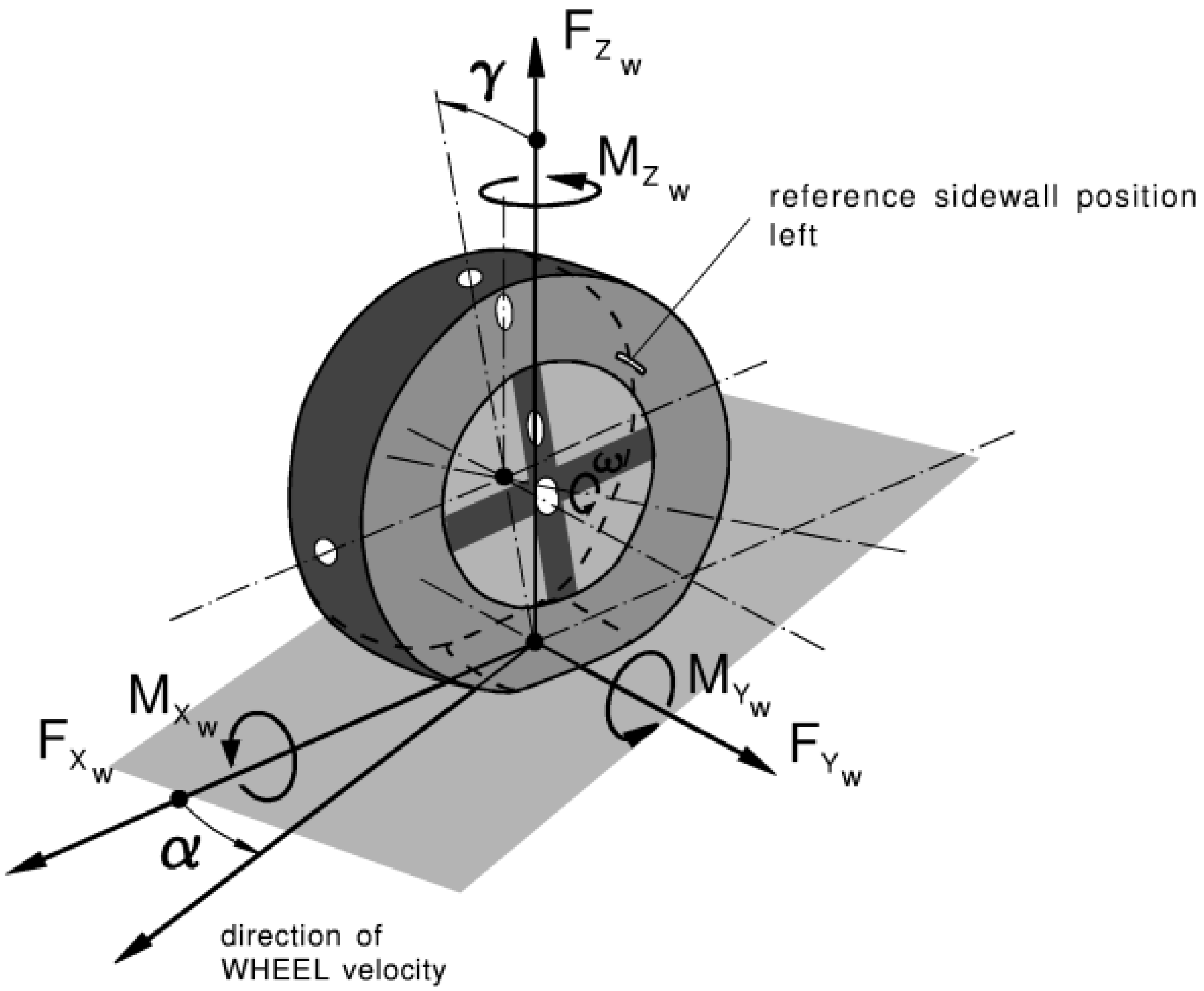
Vehicle Dynamics (Recall)

- We can summarize the previous discussions as below:
- $$\begin{bmatrix} a_x \\ a_y \\ \alpha \end{bmatrix} = \begin{bmatrix} \dot{v}_x - v_y\omega \\ \dot{v}_y + v_x\omega \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\Sigma F_x}{m} \\ \frac{\Sigma F_y}{m} \\ \frac{\Sigma M}{I} \end{bmatrix}$$
  - where we are yet to define the forces and moments, except that they be in BFF
- In order to simulate the system we want to write it in the normal ODE form:
  - $$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v_y\omega + \Sigma F_x/m \\ -v_x\omega + \Sigma F_y/m \\ \frac{\Sigma M}{I} \\ \omega \end{bmatrix}$$

- Often we also need to keep track of the position of the vehicle – in the global frame!
- We use the standard coordinate transformations:
  - $$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \mathcal{R}(-\psi) \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$
- With this, we have all the information needed to simulate a vehicle
  - IF we know the forces and moments on the vehicle

Tire Coordinate Frame

- Tire forces are specified in a coordinate frame attached to the Tire!
  - Tire/Wheel Fixed Frame
  - The origin is at the center of the wheel (when undeformed and vertical)
  - The x_W axis is along the radius in the horizontal forward direction
  - The z_W axis is along the radius in the vertical up direction
  - The y_W axis is along the axis in the horizontal direction
- The angle between x_W and the direction of wheel travel is the “tire side slip” angle α
- The angle between inertial vertical and the z_W axis is the “camber” angle γ
  - For this course, we will ignore camber effects, and set $\gamma \rightarrow 0$

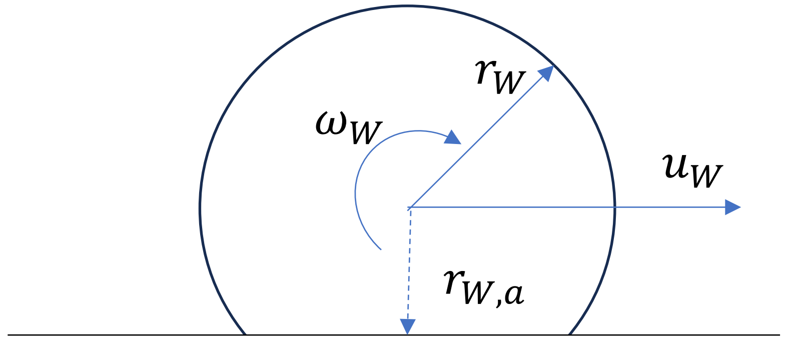


Tire Forces

- Tire forces are generated by shear stresses at the contact patch
- Longitudinal Force F_{x_W}
  - This is the force along the wheel x_W axis, acting at the contact surface
  - This is the primary propulsive force on the vehicle
- Lateral Force F_{y_W}
  - This is the force along the wheel y_W axis, acting at the contact surface
  - This is the primary steering force on the vehicle
- Normal Force F_{z_W}
  - This is the force along the wheel z_W axis, acting at the contact surface
  - This is usually a dynamic force that depends on the vehicle vertical acceleration as well as the suspension characteristics
  - For this class, we will assume this is simply the statically distributed weight of the vehicle
- Wheel axis Moment M_{y_W}
  - This is the torque on the wheel, based on the force gradients along the contact surface
  - For our class, we will neglect this: $M_{y_W} \to 0$

Longitudinal Force

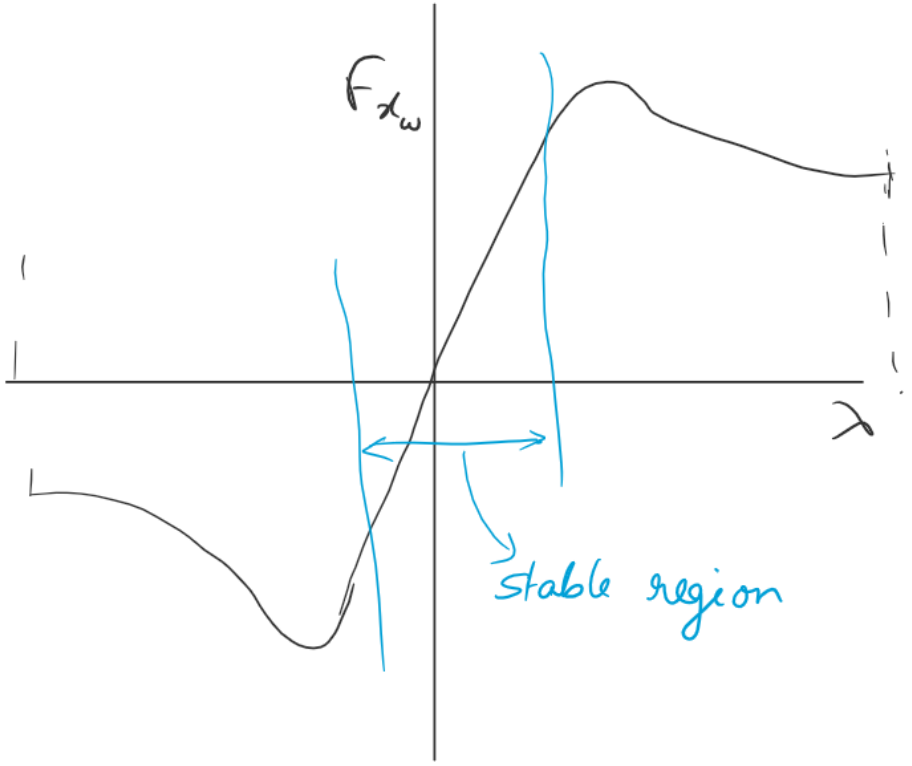
- Consider a tire of radius r_W rotating about its axis at angular speed ω_W with linear speed of u_W at its axis
- If the tire was rolling without any deflections, the velocity at the axle "would" have been $r_W\omega_W$
- However, because the tire deforms, there is an "apparent" radius $r_{W,a}$ less than r_W
- This in turn creates an "apparent slip" given by $r_W\omega_W - u_W$
    - When accelerating $r_W\omega_W - u_W \geq 0$
    - When decelerating $r_W\omega_W - u_W < 0$



- The longitudinal force F_{x_w} is created by the shear stress due to the tire deformations.
- This is a complex relation, that depends on the material properties and design of the tire, as well as environmental conditions
- In practice, F_{x_w} is "characterized" as a function of the apparent slip

Tire “slip ratio”

- To characterize the “apparent” slip, we define a “tire slip ratio” λ
  - $\lambda := \begin{cases} \dfrac{(r_W \omega_W - u_W)}{r_W \omega_W} & \text{when accelerating} \\ \dfrac{(r_W \omega_W - u_W)}{u_W} & \text{when braking} \end{cases}$
- Then the longitudinal force F_{x_W} is characterized as a function of λ
  - This function is determined experimentally, and many functions are used to “fit” the data
    - E.g. “Pacheka’s Tire model” is a popular curve fit
  - In general, the shape of the curve is as shown:

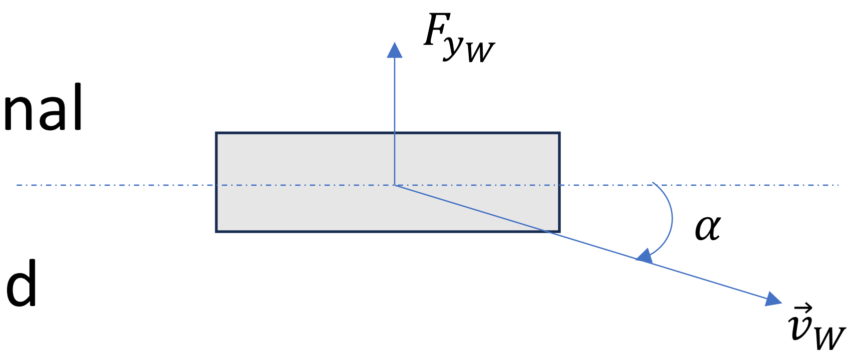


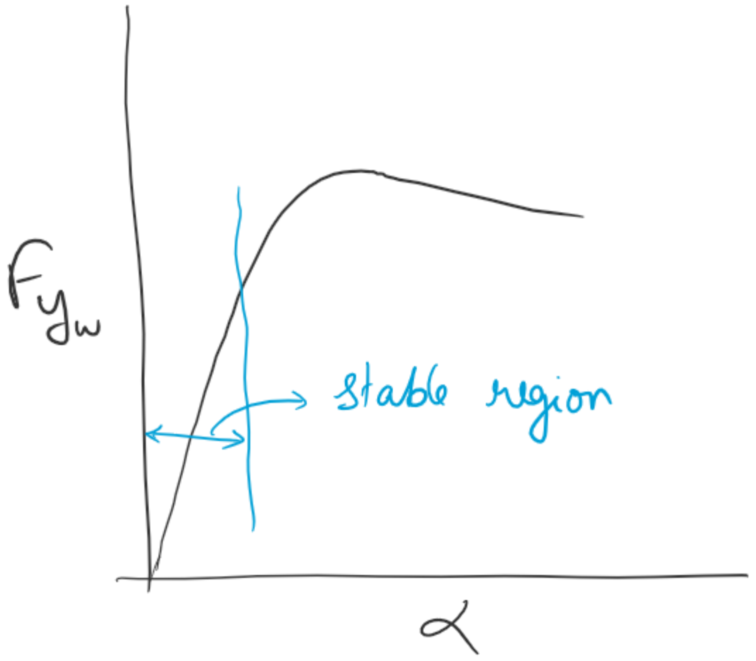
Small slip-ratio approximation

- When tire slip ratio is small, we can approximate the curve by a straight line:
  - $F_{x_W} = C_\lambda \lambda$
  - C_λ is sometimes called the longitudinal tire stiffness
- The value of C_λ is a function of
  - the tire material,
  - tire design,
  - the surface on which the tire is traveling
  - the normal force on tires.
- For our purposes, it is convenient to write the stiffness as
  - $C_\lambda = \bar{C}_\lambda \mu N$ where
    - $\bar{C}_\lambda$ is the normalized tire stiffness,
    - μ is a friction factor ($\mu = 1$ for high friction, $\mu \ll 1$ for low friction),
    - N is the normal load on the tire
- It is more realistic to saturate the achievable traction force by a maximum:
  - $\lambda = \begin{cases} \lambda_{orig} & \text{if } |\lambda_{orig}| < \lambda_{max} \\ \lambda_{max} \operatorname{sgn} \lambda_{orig} & else \end{cases}$

Lateral Tire Force

- Consider the top-view of the tire in motion
- The side slip angle α is the angle the tire longitudinal axis makes with the wheel velocity vector $\vec{v}_W$
- The mis-alignment between the wheel velocity and the tire longitudinal axis causes shear of the tire contact patch in the lateral direction
  - This yields the lateral force F_{yW} as shown
  - This force is characterized as a function of α
- The characterization is done empirically through tests and curve fits of data
  - A popular curve fit is "Pacheka's magic tire formula"!
  - The general shape looks as shown:





Small slip-angle approximation

- Similar to longitudinal dynamics, we can approximate the lateral force by a linear relation for small side slip angle:
  - $F_{y_W} = C_\alpha \alpha$
  - C_α is often referred to as the "Cornering Stiffness"
- As before we would limit side slip angle as below:
  - $\alpha = \begin{cases} \alpha_{orig} & \text{if } |\alpha_{orig}| < \alpha_{max} \\ \alpha_{max} \operatorname{sgn} \alpha_{orig} & else \end{cases}$
- Cornering Stiffness is also a function of the tire material, design, and driving surface. So we usually write:
  - $C_\alpha = \bar{C}_\alpha \mu N$ where $\bar{C}_\alpha$ is the normalized tire stiffness

Tire Force Control

- What can be actively controlled to change the tire force?
  - The angular velocity of the wheels
    - This is controlled by the vehicle powertrain and brakes
  - The velocity of the tire
    - This is dictated by the velocity of the (axle) vehicle.
      - In turn, this requires control of vehicle tire forces – and so not a useful control factor
  - The slip angle of the tire
    - This can be controlled (mostly) independent of the vehicle motion using “Steering”
      - i.e. the wheel can be rotated about the vertical axis using a separate actuator