MEEN 432 –Automotive Engineering

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L14: Transmission (Multi speed)

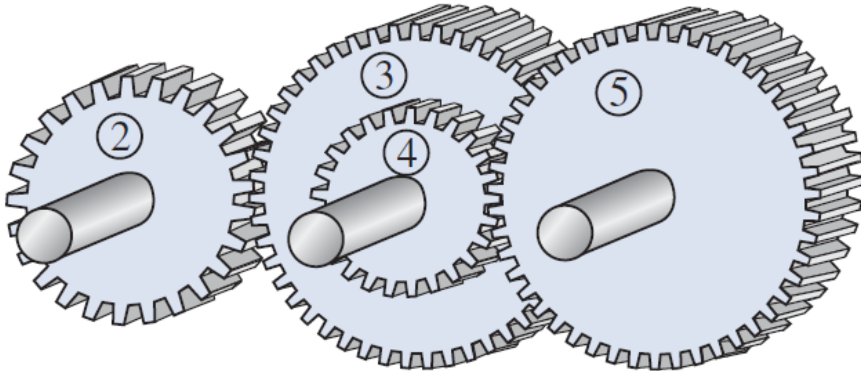
- Planetary Gears
- Motor/Engine Matching

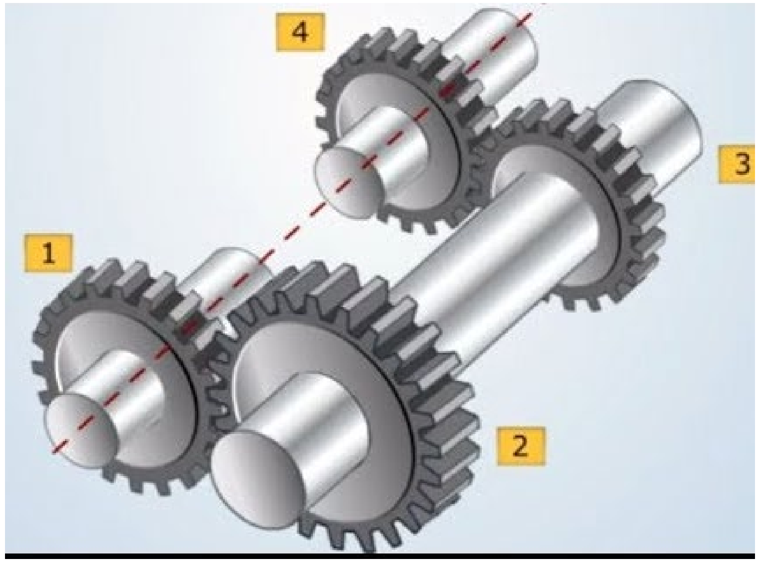
Transmission

- The purpose of the transmission is to transmit power from one source of power to another
  - Delivered power is always mechanical in automotive systems, so there is a corresponding torque and angular speed (or force and linear speed):
    - $p_o = \tau_o \, \omega_o$
  - Input power could be either mechanical or electrical:
    - $p_i = \tau_i \, \omega_i$ or $p_e = i \; V$
- Mechanical Power Input to Mechanical Power Input is achieved through mechanical transmissions
  - A common mechanical transmission is composed of “gears”

Combinations of gears – a gear train

- A simple gear is approximately characterized by one gear mesh
- Complex gears can be decomposed into a series of simple gears that are "kinematically attached" so the attachment shaft speeds are the same
- For example:
  - Top gear train: $\omega_3 = \omega_4$
  - Bottom gear train: $\omega_2 = \omega_3$
- Each simple gear decomposition follows its own speed relations
- Together the entire "gear train" can be considered as having one "effective" gear ratio
- Example:
  - Top gear train: $G \coloneqq \frac{\omega_2}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_3}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_4}{\omega_5} = G_{23}G_{45}$
  - Bottom gear train: $G \coloneqq \frac{\omega_1}{\omega_4} = \frac{\omega_1}{\omega_2}\frac{\omega_2}{\omega_4} = \frac{\omega_1}{\omega_2}\frac{\omega_3}{\omega_4} = G_{12}G_{34}$

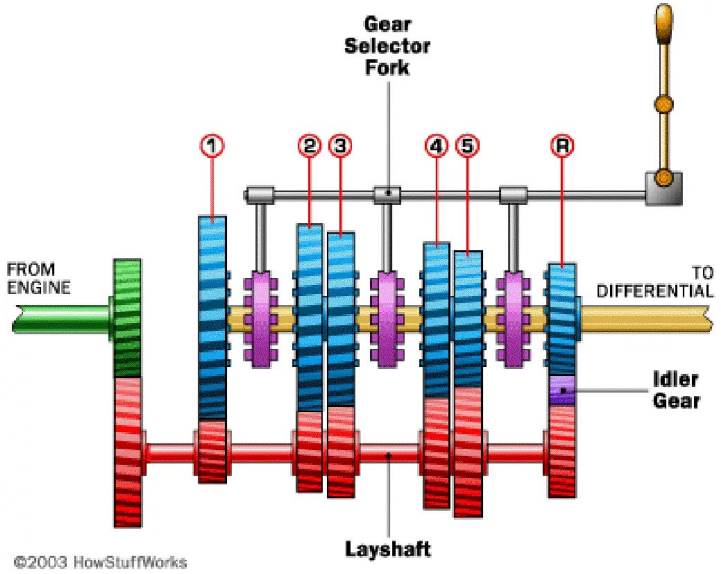




Gear Trains – 8

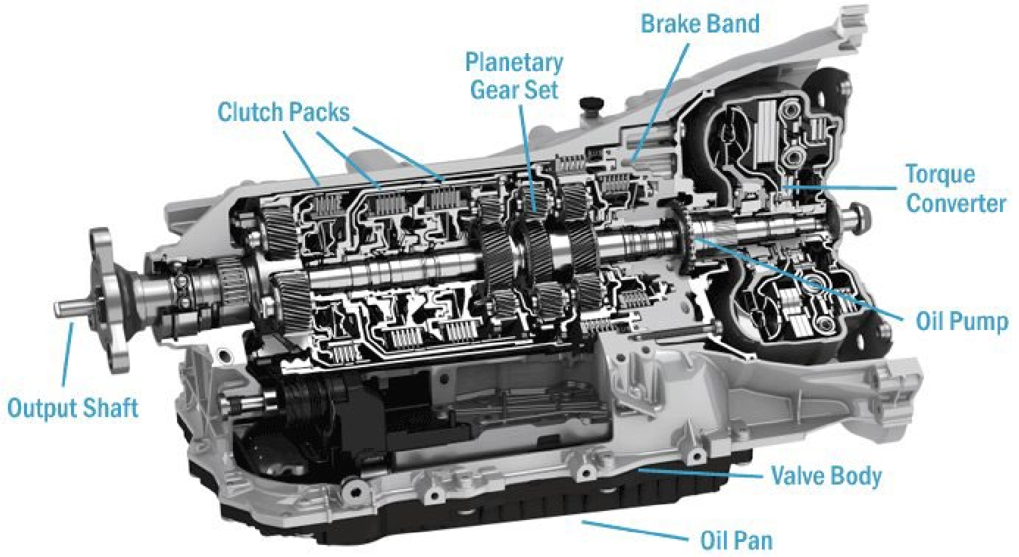
- Often a Transmission consists of several gear pathways that can be "chosen" on demand
  - Manual Transmission – Human Driver chooses gear by manually shifting
  - Automatic Transmission – shifting achieved through electro-hydraulic actuators
- We will assume transmission shifting is instantaneous for this class scope
  - In reality, this is a complex process that can take anywhere from a 200ms to 1.5s, with varying impacts on shift quality and durability
- For the project we will assume that a controller can simply "specify" the Gear, and we can achieve the desired gear
  - That is why we keep the gear as an "input" in the model
- Often there is confusion between "Gear (number)" and "Gear Ratio" – watch out
  - For example, Gear 1 may have gear ratio 3, Gear 2 : 1.7, Gear 3 : 1.0, Gear 4 : 0.6, …

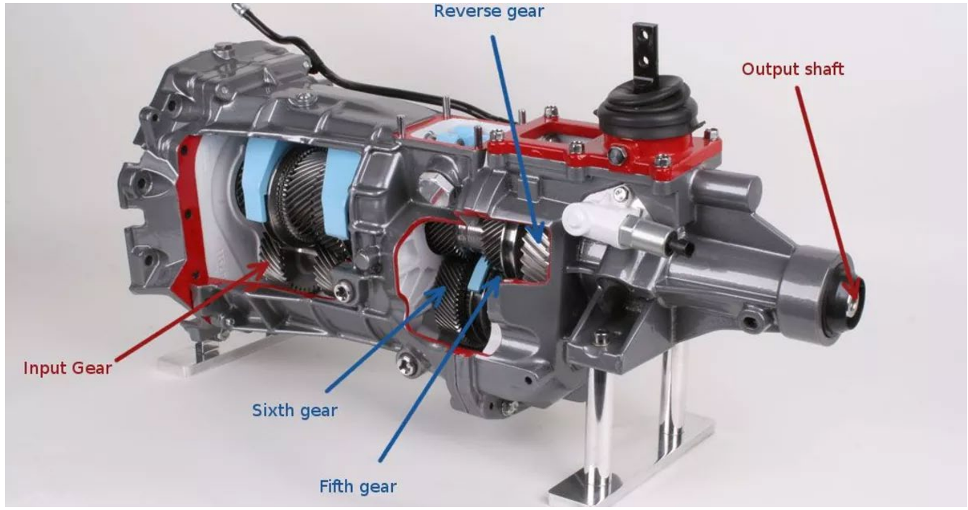
https://www.dnatube.com/video/7881/An-Animation-of-Manual-Transmission



Automatic Transmission

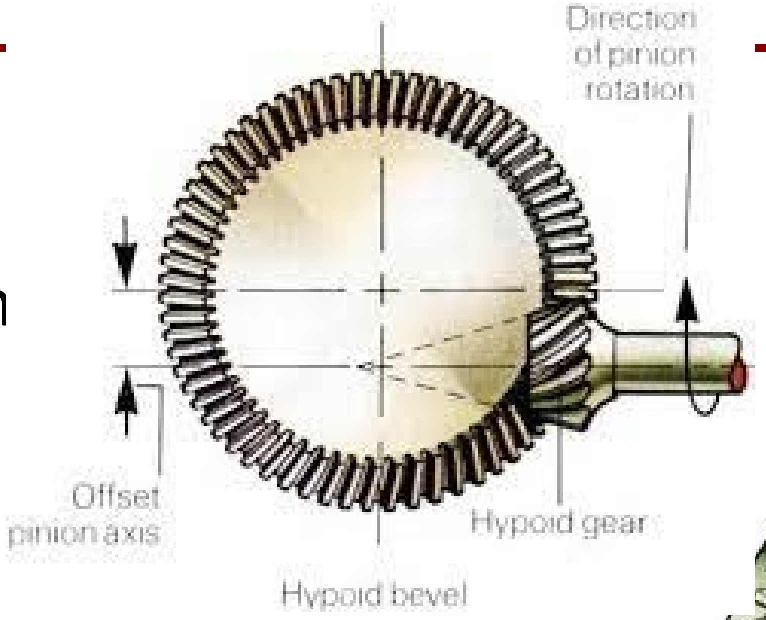
- A typical vehicle transmission consists of multiple gear pairs that could be "selected" in an algorithmic way
  - "Clutches" used to selectively connect/disconnect shafts, and correspondingly which gear train is active
  - A "torque converter" or a "starting clutch" couples the engine with the gear train part of the transmission
- We will not discuss the design of a Transmission for this class scope
  - Instead a transmission will be characterized by its gear ratios (and corresponding input and output inertias, and efficiencies)
  - And we will assume that we can magically control the gear ratio by simply issuing a command
    - In reality, such command will lead a to coordinated actuation of multiple solenoids, which will in turn engage different clutches, to realize the gear change!

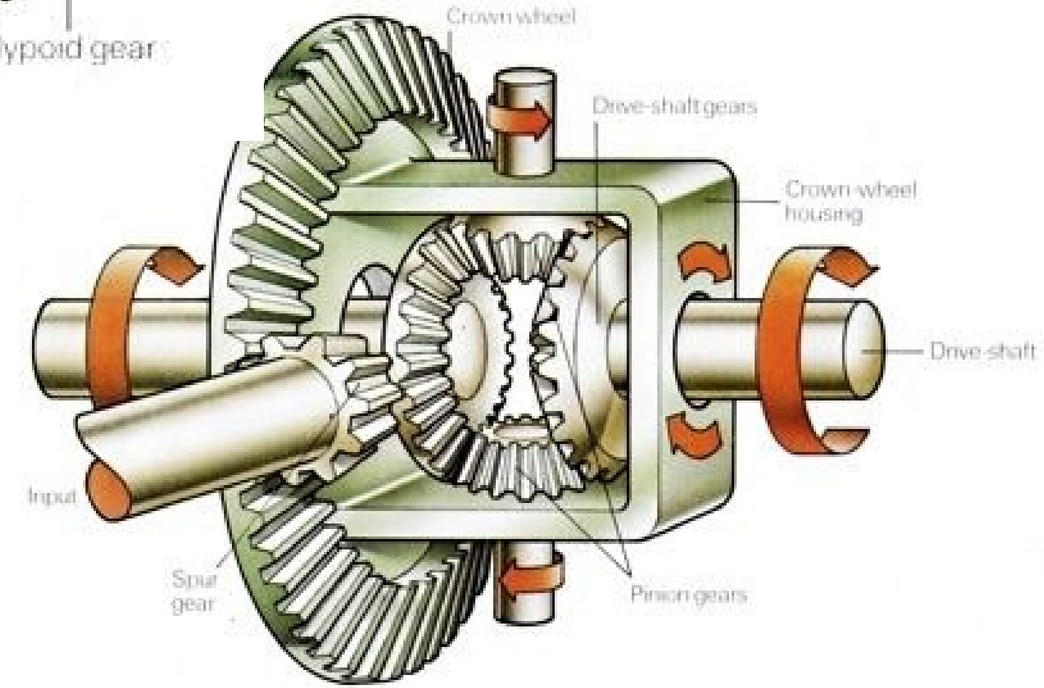




Final Drive

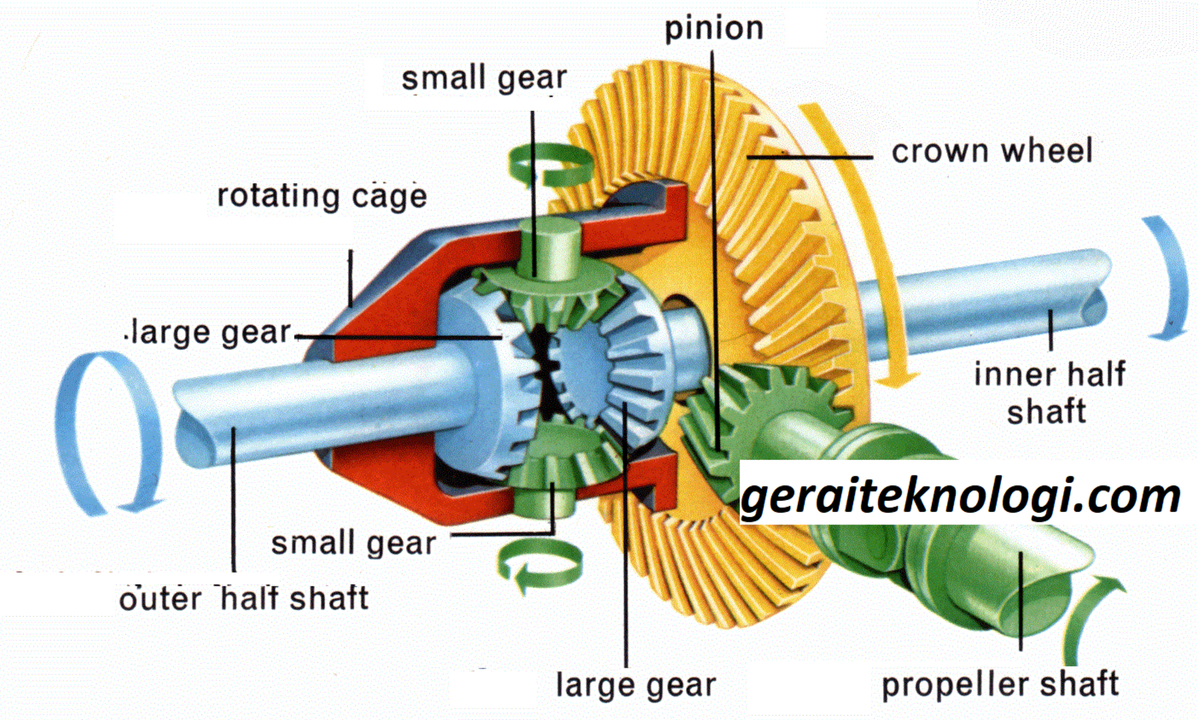
- Provides torque multiplication
- Changes direction of rotation (from longitudinal axis of car to along wheel shaft axes)
- Allows for differential rotation of wheels
  - Two-degree of freedom gears (planetary gears)
- For the class scope – just another gear train (with fixed gear ratio!)





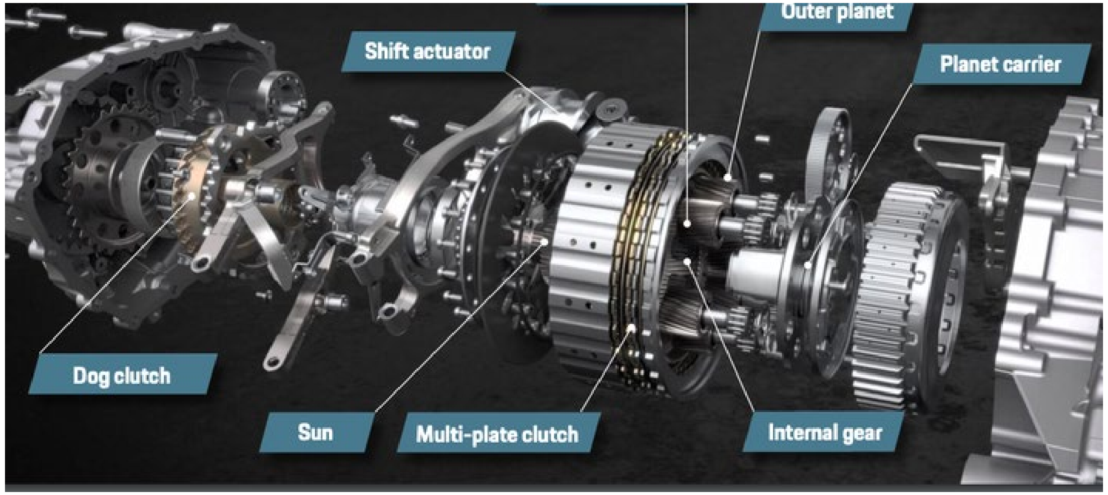
Gear Trains - 9

- A Final Drive is a fixed gear ratio + a “differential” that splits the axle torque between the two wheels
  - A 2 d.o.f. mechanism, that allows for two different wheel speeds (for a given axle speed)
- For our class we model it as a fixed gear, since we collapse the left and right wheels together for modeling purposes

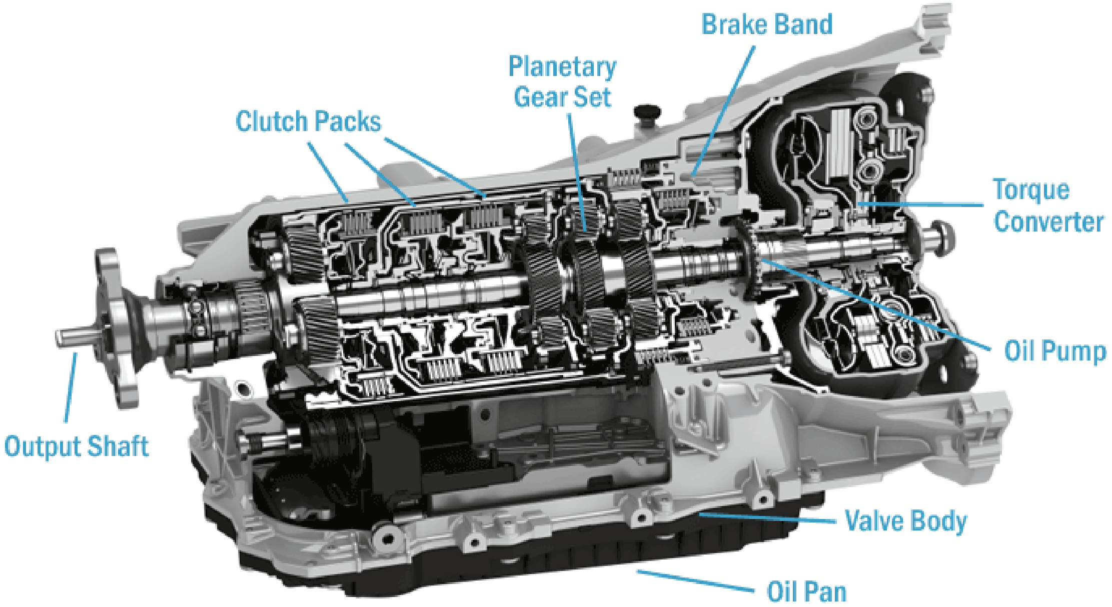


Multi-Speed Transmissions in EVs

- As OEMs try to eke out higher efficiencies, multi-speed transmissions are becoming more common even in Electri Vehicle Drive trains
  - Example: Porsche Taycan

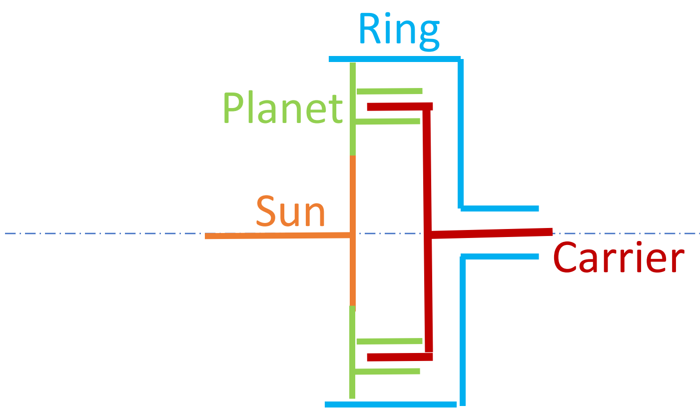


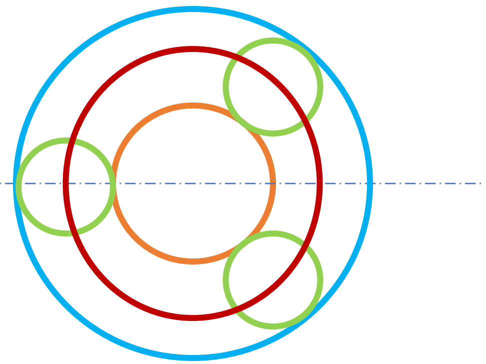
- Planetary Gears are a common theme in multi-speed transmissions



Planetary Gears – 1

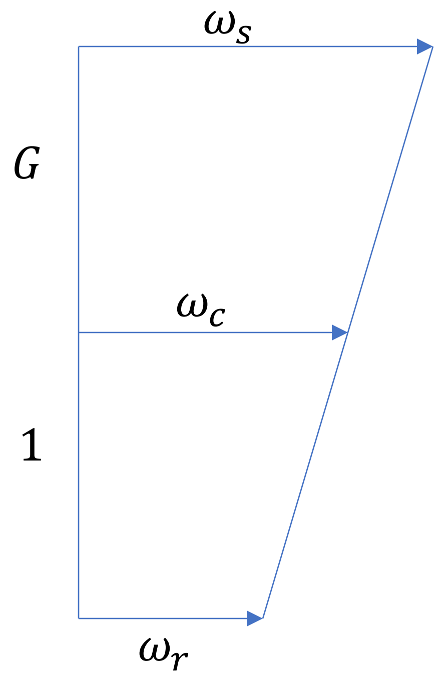
- https://www.youtube.com/watch?v=ARd-Om2VyiE
- Stick and Lever Diagram for analysis of gear train speeds
  - *Challenge question: how do you relate the ratio of speeds to the number of teeth on the gears?*
- Radius of the different gears: Ring: r_r, Sun: r_s, Planet: r_p, Carrier r_c:
  - Note: $r_p = \frac{(r_r - r_s)}{2}$ and $r_c = r_s + r_p$ ➔ only the ring and sun radii can be specified independently





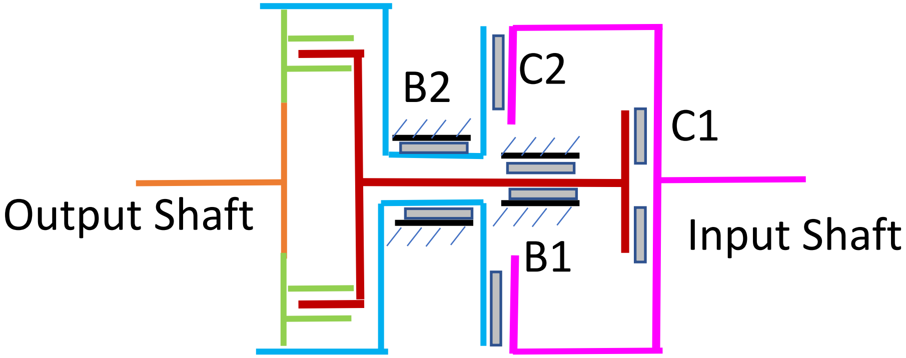
Planetary Gears – 2

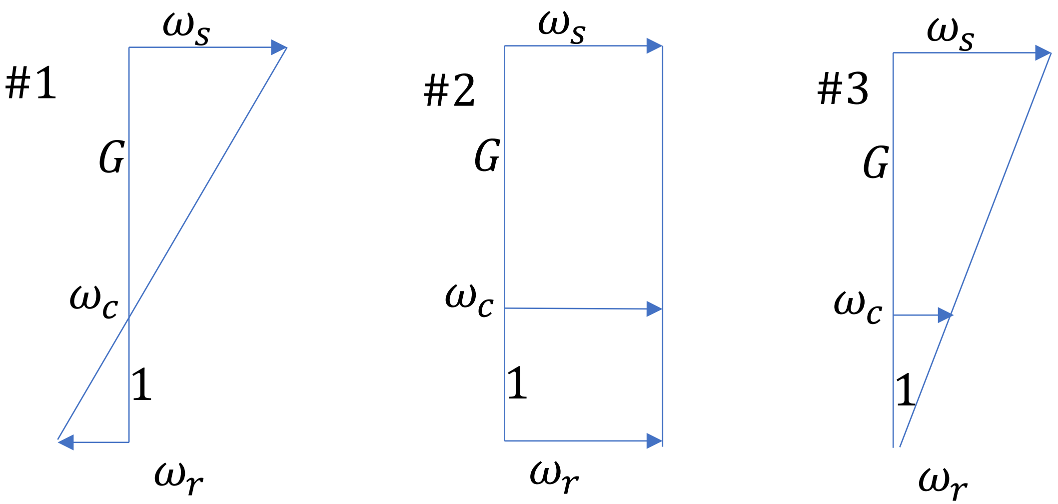
- $\omega_s r_s = \omega_c(r_s + r_p) - \omega_p r_p$
- $\omega_r r_r = \omega_c(r_s + r_p) + \omega_p r_p$
- *Together:* $\omega_s r_s + \omega_r r_r = \omega_c(r_s + r_r)$
- *Or,* $\omega_s = -\omega_r \frac{r_r}{r_s} + \omega_c(1 + \frac{r_r}{r_s})$
- *Defining* $G = \frac{r_r}{r_s}$ *we have:*
- $\omega_s = -G\,\omega_r + \omega_c(1 + G)$
  - *Note that* G *does not depend on* r_p
- *Rewriting,* $(\omega_s - \omega_r) = (1 + G)(\omega_c - \omega_r)$



Planetary Gears – 3

- We can get different speed ratios depending on which clutch /brake is applied
  - Assume G = 2

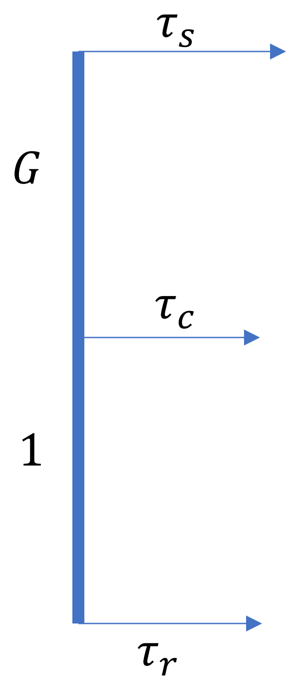




#	C1	C2	B1	B2	Speed Ratio
1	off	On	on	off	-0.5
2	on	on	off	off	1
3	on	off	off	on	0.3
4	off	off	off	off	0 (neutral)

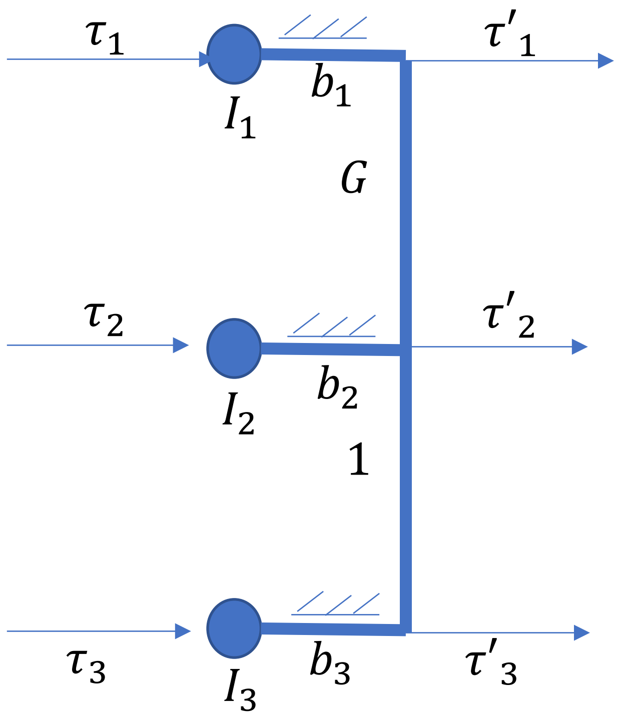
Planetary Gears – 4

- Assume an "ideal" gear – i.e. no losses
- Power Balance: $\tau_s\omega_s + \tau_c\omega_c + \tau_r\omega_r = 0$
  - Net power "into" the gear should be equal to 0
- Torque Balance: $\tau_s + \tau_c + \tau_r = 0$
- Or, $\tau_s(\omega_s - \omega_r) + \tau_c(\omega_c - \omega_r) = 0$
- $(1 + G)\tau_s + \tau_c = 0$ or $\tau_c = -(1 + G)\tau_s$
- And, $G\tau_s = \tau_r$
- These are the same as a static balance around the "lever" shown right, with torques standing in for the forces on the lever
  - This is called the "lever analogy"



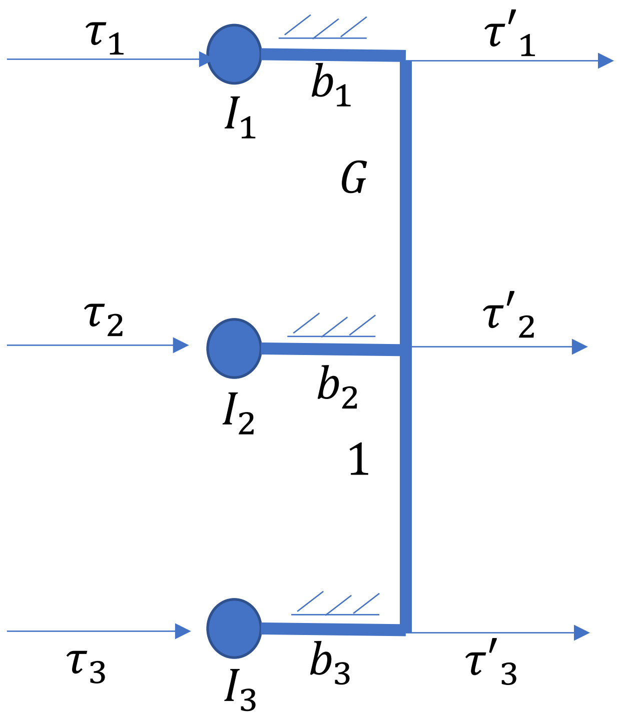
Planetary Gears - 5

- So far we have assumed that the planetary gears are ideal – i.e. no mass, and no losses.
  - To recognize that the planetary gear can be more general by connecting to additional gear trains, and to recognize the choice of sun, carrier or ring can be easily changed by appropriate definition of the “characteristic gear ratio”, we notate the three external shafts by 1, 2, and 3
- Now we relax those assumptions:
  - The effective inertias of the rotating components are allocated to the three shafts of the device: I_1, I_2, and I_3
  - Planetary losses are more complex and depend on the relative speeds between the shafts.
    - A first approximation is to consider the losses as a function of the shaft speeds, and allocate them to the three shafts also, as a viscous damping: $b_1\omega_1$, $b_2\omega_2$, and $b_3\omega_3$
- In the figure ‘ is used to denote the torque at the idealized gear train



Planetary Gears - 6

- The new torque equations:
  - $\tau_i - I_i\dot{\omega}_i - b_i\omega_i = \tau_i'$ with $i \in \{1, 2, 3\}$
  - $\tau_2' = -(1+G)\tau_1'$; and $G\tau_1' = \tau_3'$
- Or,
  - $\tau_2 - I_2\dot{\omega}_2 - b_2\omega_2 = -(1+G)(\tau_1 - I_1\dot{\omega}_1 - b_1\omega_1)$
  - $G(\tau_1 - I_1\dot{\omega}_1 - b_1\omega_1) = \tau_3 - I_3\dot{\omega}_3 - b_3\omega_3$
- We also know, from kinematics:
  - $\dot{\omega}_1 = -G\,\dot{\omega}_3 + \dot{\omega}_2(1+G)$
- Thus we can obtain two independent ODEs from the above that constitute the dynamics of the planetary
  - For example: (choosing to eliminate $\dot{\omega}_3$ from the torque equations)
    - Dynamic Equations:
      - $\dot{\omega}_1 = f_1(\tau_1, \tau_2, \tau_3, \omega_1, \omega_2, \omega_3)$, $\dot{\omega}_2 = f_2(\tau_1, \tau_2, \tau_3, \omega_1, \omega_2, \omega_3)$
    - Kinematic Equation: $\omega_3 = -\frac{\omega_1}{G} - \frac{\omega_2}{G}(1+G)$

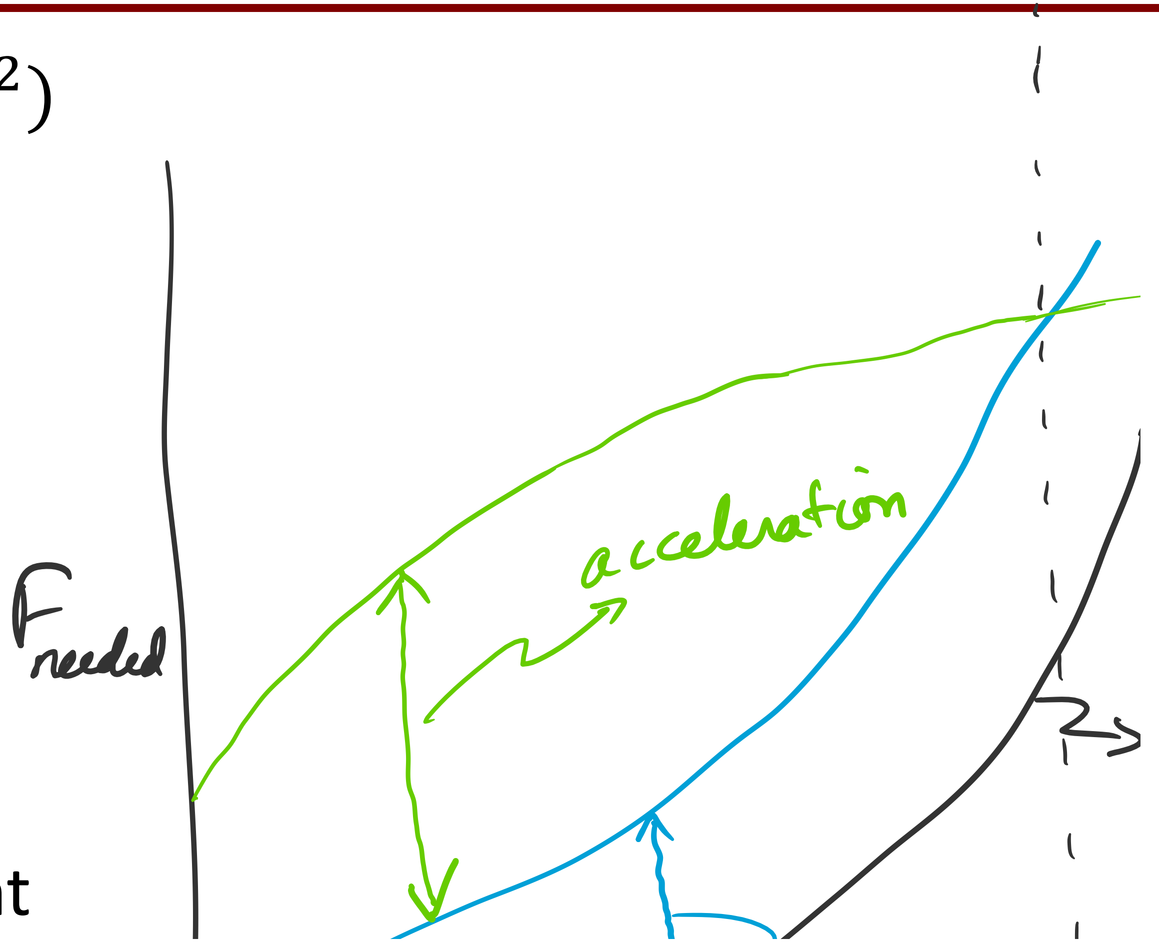


Planetary Gears - 7

- When a clutch (or a band) is applied (i.e it looks two shafts together), we decrease the degrees of freedom back to 1
- This is what happens when a transmission is “in gear”
- During shifting the dynamics are governed by independent ODEs
  - The way clutch torque is applied plays a big role in the dynamics “during a shift”
  - Good “Shift control” of the actuators (motor torques in EVs, electro-hydraulic solenoids in conventional Automatic Transmissions) is critical to achieve good “shift quality”

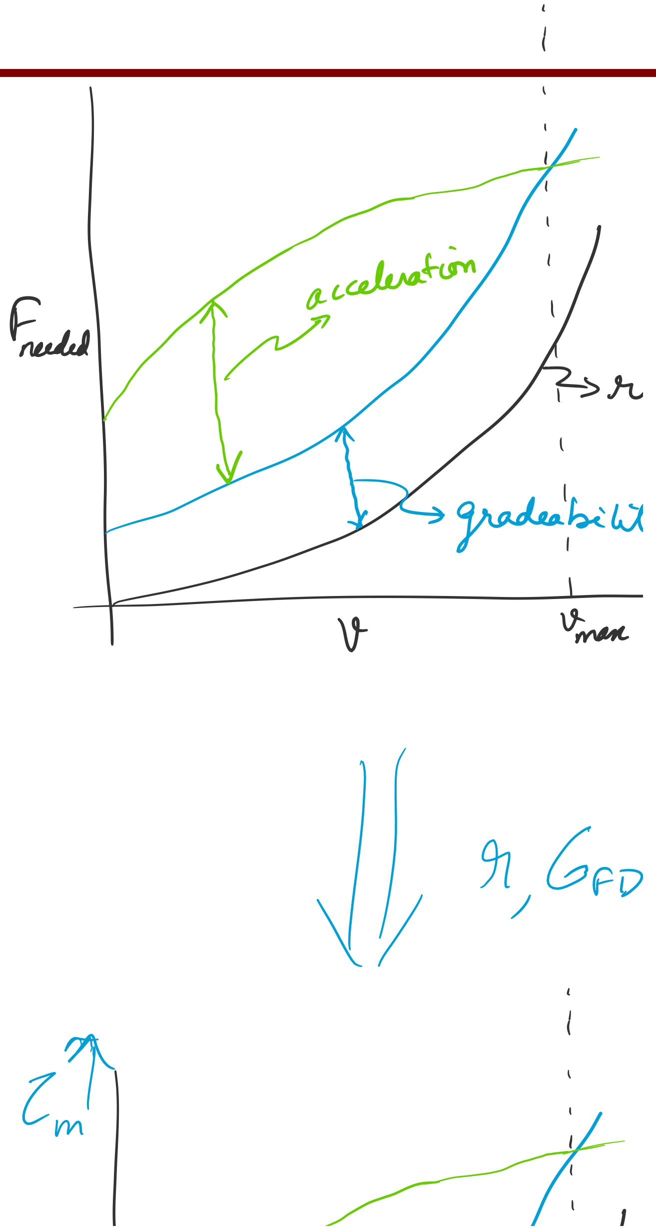
Vehicle Specifications – revisited - 1

- $F_x = M\,\dot{v} + Mg\sin(\theta) + (c_0 + c_1 v + c_2 v^2)$
  - θ the grade of road
- Vehicle Specifications:
  - Max speed v_{max}
  - Gradeability $gr\% = \tan(\theta)$
  - Max acceleration at different speeds $a(v)$
- Powertrain Requirements specified as a graph of tractive force required for different velocities



Vehicle Specifications – revisited - 2

- Neglecting inertia effects ,we can write:
  - $v = r\,\omega_w = \frac{r}{G_{FD}}\omega_t = \frac{r}{G_{FD}G}\omega_m$
  - $F = \frac{1}{r}\tau_w = \frac{G_{FD}}{r}\tau_t = \frac{G_{FD}G}{r}\tau_m$
- Then we can redraw the vehicle requirements (tractive force vs speed) in terms of motor requirements (tau vs omega)



Vehicle Specifications – revisited - 3

- We can overlay the motor requirements (based on the vehicle level specifications) on the motor capabilities
- If you have multiple gears, you can “choose” the gear to meet the vehicle specifications at optimal efficiency!

3

1

Powertrain Design Considerations

- Given a vehicle – identify power and torque needs from the specifications
- Given a set of motor choices, choose the one(s) that satisfy the max power need first
- Given the max speed and max torque curve of the motor:
  - Choose a gear ratio from available gear designs:
    - Confirm if the torque speed of requirement is satisfiable
      - Else iterate
    - If no single transmission can satisfy the requirement consider multiple gear ratios
      - You need to have a mechanism to "switch" gears
- Operational: (if you have multiple gears)
  - Choose operational gear to maximize some objective (e.g. maximize energy efficiency)