MEEN 432 –Automotive Engineering

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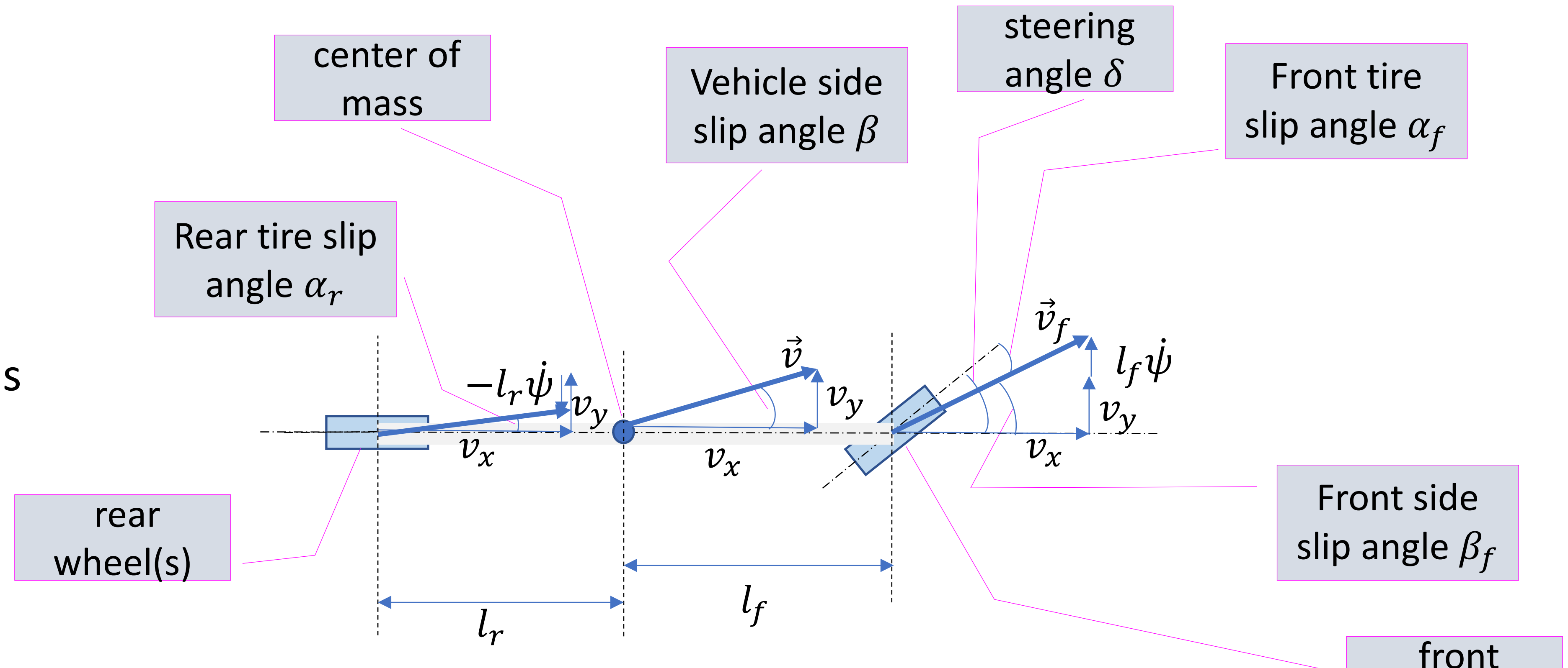
TEXAS A&M UNIVERSITY

Lecture 6: Vehicle Lateral Dynamics

- Race Car – simulation system architecture
- Tire Forces
- Vehicle Forces

Bicycle (!) Model of a Car

- Vehicle "collapsed" along longitudinal line of symmetry
  - Two wheels collapse into each other



- $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$ ($\hat{\imath}, \hat{\jmath}$ are BFF unit vectors) – velocity of vehicle
- $\vec{v}_f = v_x\,\hat{\imath} + \left(v_y + l_f\dot{\psi}\right)\hat{\jmath}$ - velocity of vehicle at front axle
- $\vec{v}_r = v_x\,\hat{\imath} + \left(v_y - l_r\dot{\psi}\right)\hat{\jmath}$ - velocity of vehicle at rear axle

Bicycle (!) Model of a Car

- $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$
- $\vec{v}_f = v_x\,\hat{\imath} + \left(v_y + l_f\dot{\psi}\right)\hat{\jmath}$
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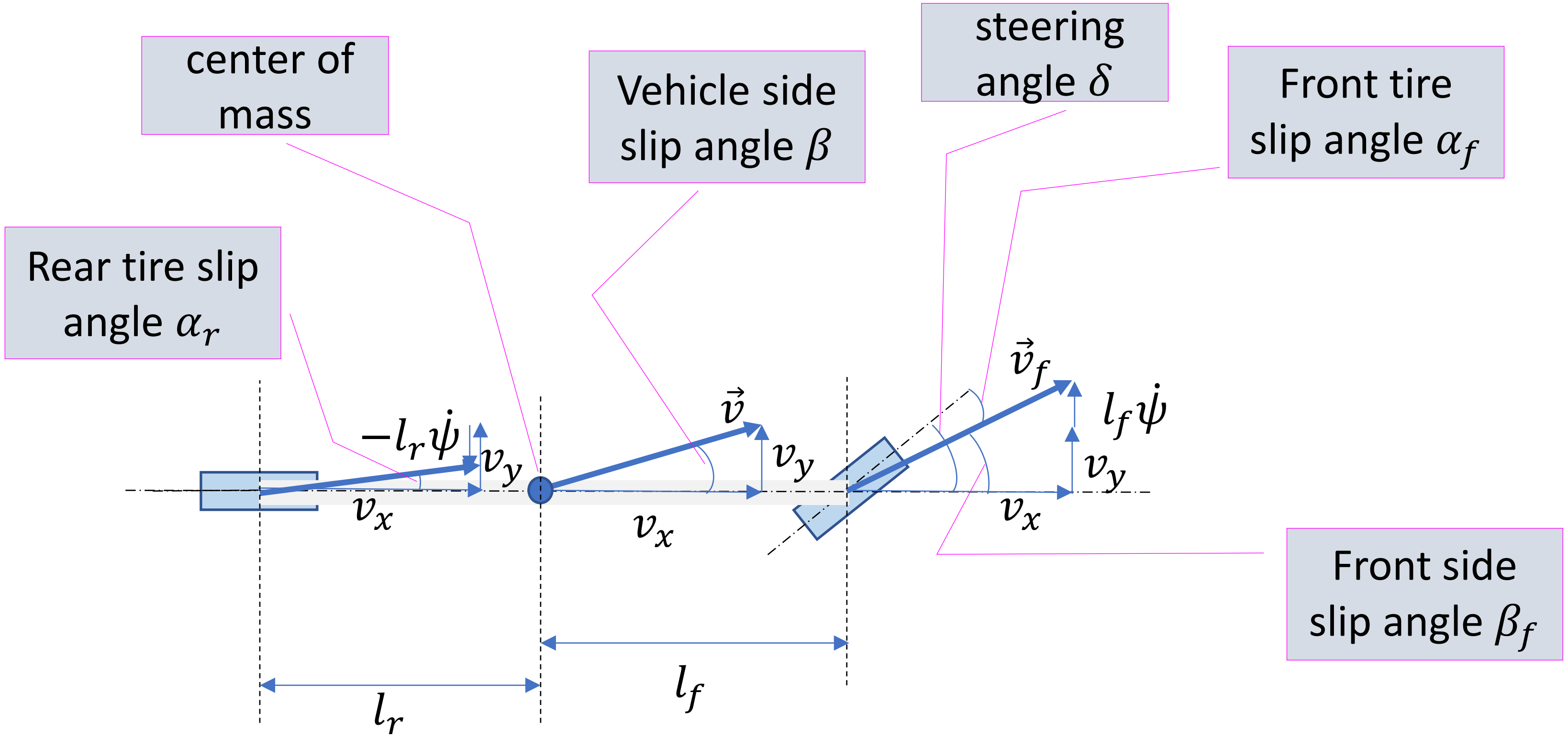

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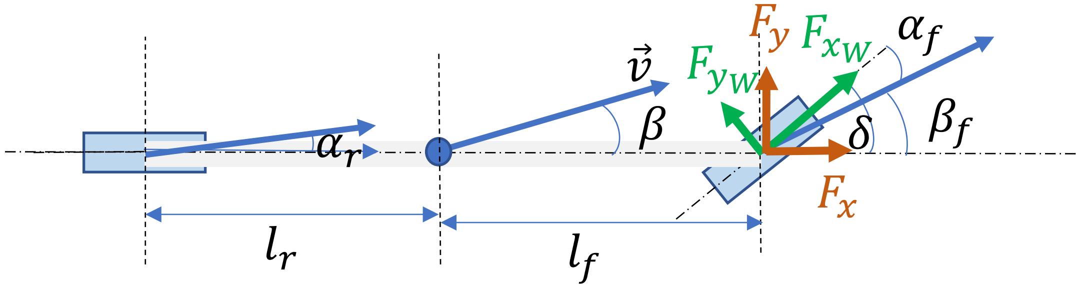
- $\beta = \tan^{-1}\frac{v_y}{v_x}$
- $\beta_f = \tan^{-1}\frac{v_y + l_f\dot{\psi}}{v_x}$
- $\beta_r = \tan^{-1}\frac{v_y - l_r\dot{\psi}}{v_x}$

- $\alpha_f = \delta - \beta_f = \delta - \tan^{-1}\frac{v_y + l_f\dot{\psi}}{v_x}$
- $\alpha_r = -0 - \beta_r = -\tan^{-1}\frac{v_y - l_r\dot{\psi}}{v_x}$

Tire Forces in BFF

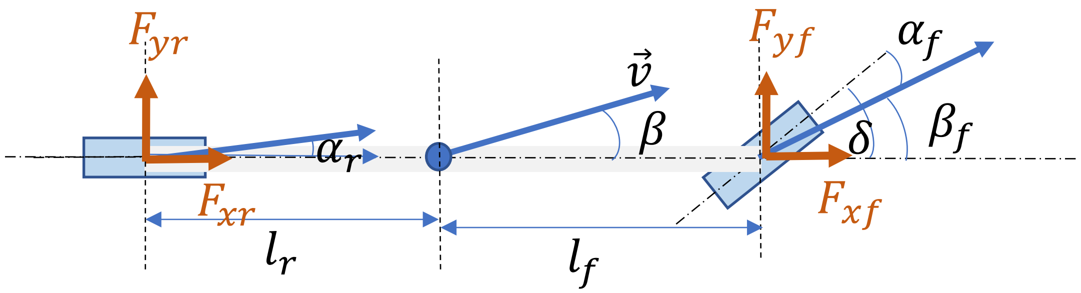
- We previously described the tire forces in the TFF – now we transform it to the BFF:
  - Tire forces $F_{xw}(\lambda)$ and $F_{yw}(\alpha)$ are in TFF
  - Their impact on the vehicle F_x (the longitudinal traction force) and F_y (the lateral force) are in BFF
  - $F_x = F_{xw}(\lambda)\cos\delta - F_{yw}(\alpha)\sin\delta$
  - $F_y = F_{xw}(\lambda)\sin\delta + F_{yw}(\alpha)\cos\delta$



- Often a "small angle approximation" is employed: δ and α are small
  - $\cos\delta \approx 1, \cos\alpha \approx 1$
  - $\sin\delta \approx \delta, \sin\alpha \approx \alpha$
- Then:
  - $F_x = F_{xw} - F_{yw}\delta$
  - $F_y = F_{xw}\delta + F_{yw}$

Vehicle Forces

- We consider the tire forces at both the front and rear, in BFF
- NOTE: We need to consider both the right and left wheels together, i.e. sum the forces at each axle
- Net Forces:
  - $\Sigma F_x = F_{xf} + F_{xr} - F_{drag} - F_{gravity}$
  - $\Sigma F_y = F_{yf} + F_{yr}$
- Net Moment:
  - $\Sigma M = F_{yf} l_f - F_{yr} l_r$



- Expanding (assuming Front Wheel Steer):
  - $\Sigma F_x = F_{x_{Wf}}(\lambda_f)\cos\delta - F_{y_{Wf}}(\alpha_f)\sin\delta + F_{x_{Wr}}(\lambda_r) - F_{drag} - F_{gravity}$
  - $\Sigma F_y = F_{x_{Wf}}(\lambda)\sin\delta + F_{y_{Wf}}(\alpha)\cos\delta + F_{y_{Wr}}(\alpha)$
  - $\Sigma M = (F_{x_{Wf}}(\lambda)\sin\delta + F_{y_{Wf}}(\alpha)\cos\delta)\, l_f - F_{y_{Wr}}(\alpha)\cos\delta\ l_r$

Vehicle Dynamics

- $\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \\ \dot{\psi} \end{bmatrix} =$

$$\begin{bmatrix} v_y\omega + (F_{x_{Wf}}(\lambda_f)\cos\delta - F_{y_{Wf}}(\alpha_f)\sin\delta + F_{x_{Wr}}(\lambda_r) - F_{drag} - F_{gravity})/m \\ -v_x\omega + (F_{x_{Wf}}(\lambda)\sin\delta + F_{y_{Wf}}(\alpha)\cos\delta + F_{y_{Wr}}(\alpha))/m \\ ((F_{x_{Wf}}(\lambda)\sin\delta + F_{y_{Wf}}(\alpha)\cos\delta)\, l_f \qquad - F_{y_{Wr}}(\alpha)\cos\delta\; l_r)/I \\ \omega \end{bmatrix}$$