MEEN 432 –Automotive Engineering

Fall 2026

Instructor: Dr. Arnold Muyshondt

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Lecture 8: Vehicle Longitudinal Dynamics

- Vehicle Dynamics
- Gears and Transmission

Vehicle Dynamics (revisit)

- $$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v_y\omega + (F_{x_{Wf}}(\lambda_f)\cos\delta - F_{y_{Wf}}(\alpha_f)\sin\delta + \ F_{x_{Wr}}(\lambda_r) - F_{drag} - F_{gravity}\ )/m \\ -v_x\omega + (F_{x_{Wf}}(\lambda)\sin\delta\ + F_{y_{Wf}}(\alpha)\cos\delta + F_{y_{Wr}}(\alpha)\ )/m \\ ((F_{x_{Wf}}(\lambda)\sin\delta\ + F_{y_{Wf}}(\alpha)\cos\delta)\, l_f \qquad - \ F_{y_{Wr}}(\alpha)\cos\delta\ \ l_r)/I \\ \omega \end{bmatrix}$$
- Assume:
  - no lateral velocity i.e., $v_y = 0; \dot{v}_y = 0;$
  - Assume we don’t care abut heading for now.
  - Assume no steer i.e., $\delta = 0$
  - Then the remaining dynamics is the “longitudinal” dynamics:
- $\dot{v}_x = (F_{x_{Wf}}(\lambda_f)\ +\ F_{x_{Wr}}(\lambda_r) - F_{drag} - F_{gravity}\ )/m$
- We talked about the tire force component in detail – namely $F_{x_{W_f}}$ and $F_{x_{W_r}}$
- Next we will talk about the drag: F_{drag}

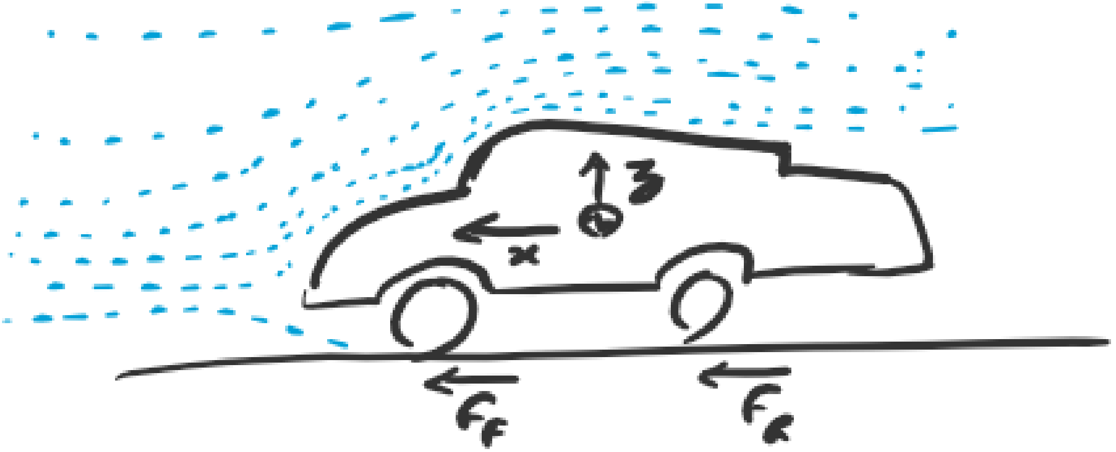
(Simple) Longitudinal Dynamics Model

- The Vehicle is treated as one single rigid body
  - Assume no pitching, heave independent of road
  - Assume no lateral movement
  - Assume single axle drive
- Vehicle Dynamics
  - $m\,a = m\,\dot{v} = F_x - F_{drag} - mg\sin\theta$
  - $F_{drag} = c_0 + c_1 v_x + c_2 v_x^2$
- We will assume that vehicle is in contact with the road, and thus grade is a function of position on road:
  - $\theta = f_{road}(X)$

- m - mass of the vehicle
- a - longitudinal acceleration
- F_x - axle tractive force
- F_{drag} - drag force on vehicle (includes aero drag and rolling resistance)
  - c_0, c_1, c_2 are coefficients
- τ, τ_b drive torque and brake torque
- r effective tire radius
- $\dot{\omega}$ is the angular acceleration at the front or the rear wheel
- I_{PT}, I_w Inertia of the upstream powertrain and the wheels
- θ grade

(Simple) Longitudinal Dynamics Model

- Aerodynamic Drag
  - $c_2 = \frac{1}{2}\rho A C_d$
    - ρ is the density of air (where you drive)
    - A is the "projected" frontal area of the vehicle (style, shape)
    - C_d is the drag coefficient (style, shape, roughness of surface, ..
- Also, it should be noted that the drag is based on "relative velocity"
  - If there is a head wind velocity v_w then drag force is:
  - $\frac{1}{2}\rho A\, C_d(v_x + v_w)^2$



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(Simple) Longitudinal Dynamics Model

- Rolling Resistance
  - c_1
    - Its nature is slightly different from viscous damping
    - It is because of the energy loss due to constant compression and release of the tires as they roll
  - c_0
    - This refers to the coulombic friction throughout the powertrain reflected at the vehicle. Note that this force is always opposite the motion direction.
    - When at rest, there is no "net" force component from the c_0 term

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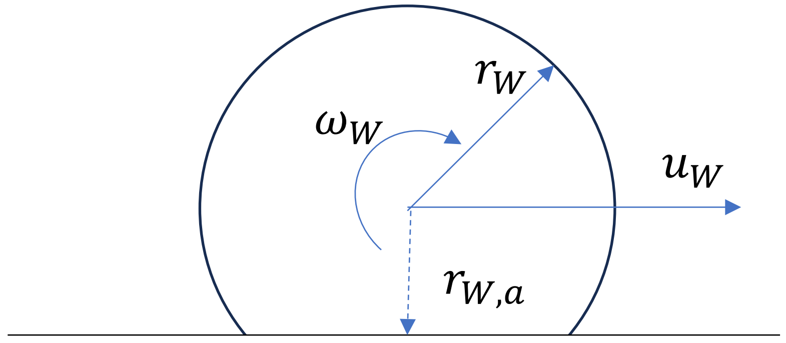
Longitudinal Force (Recall)

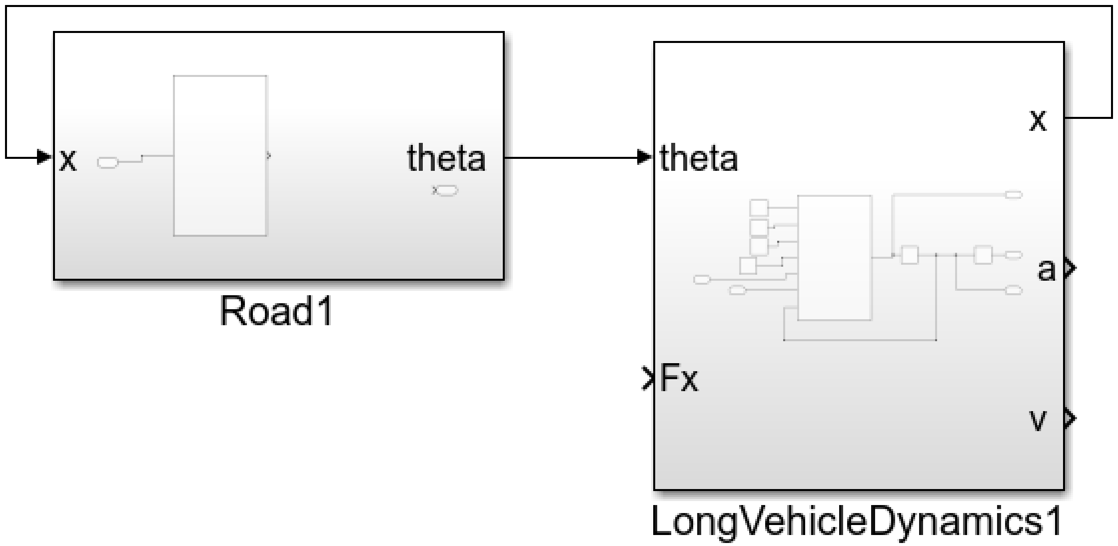
- Consider a tire of radius r_W rotating about its axis at angular speed ω_W with linear speed of u_W at its axis
- If the tire was rolling without any deflections, the velocity at the axle "would" have been $r_W \omega_W$
- However, because the tire deforms, there is an "apparent" radius $r_{W,a}$ less than r_W
- This in turn creates an "apparent slip" given by $r_W \omega_W - u_W$
    - When accelerating $r_W \omega_W - u_W \geq 0$
    - When decelerating $r_W \omega_W - u_W < 0$



- The longitudinal force F_{x_w} is created by the shear stress due to the tire deformations.
- This is a complex relation, that depends on the material properties and design of the tire, as well as environmental conditions
- In practice, F_{x_w} is "characterized" as a function of the apparent slip

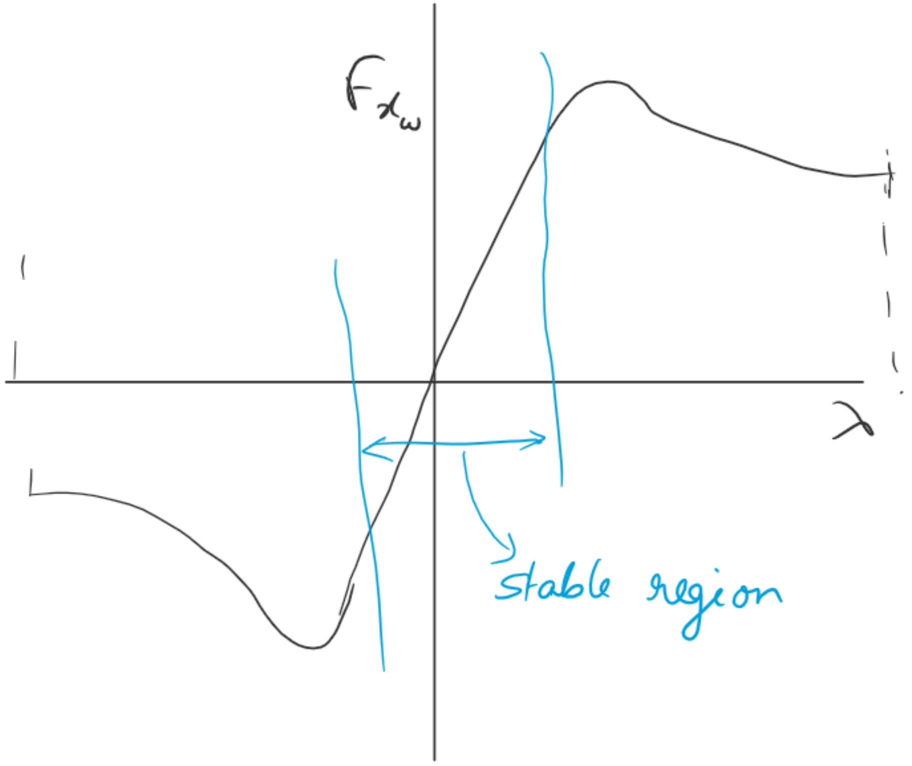
Longitudinal Dynamics Model

- Vehicle longitudinal dynamics is a straightforward implementation of the $\dot{v}$ equation
- Road grade can be implemented as a function of the position
  - Often table lookups are used
  - For Project 3, you will assume no road grade!



Tire “slip ratio” (Recall)

- To characterize the “apparent” slip, we define a “tire slip ratio” λ
  - $\lambda := \begin{cases} \dfrac{(r_W\omega_W - u_W)}{r_W\omega_W} & \text{when accelerating} \\ \dfrac{(r_W\omega_W - u_W)}{u_W} & \text{when braking} \end{cases}$
- Then the longitudinal force F_{x_W} is characterized as a function of λ
  - This function is determined experimentally, and many functions are used to “fit” the data
    - E.g. “Pacheka’s Tire model” is a popular curve fit
  - In general, the shape of the curve is as shown:

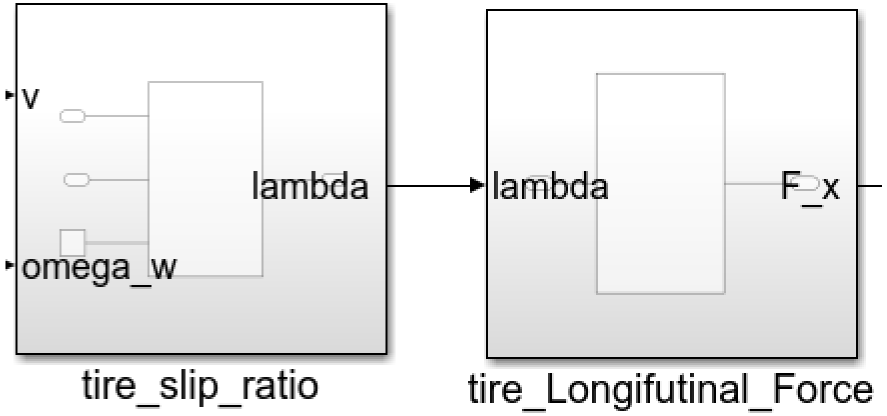


Small slip-ratio approximation (Recall)

- When tire slip ratio is small, we can approximate the curve by a straight line:
  - $F_{xW} = C_\lambda \lambda$
  - C_λ is sometimes called the longitudinal tire stiffness
- The value of C_λ is a function of
  - the tire material,
  - tire design,
  - the surface on which the tire is traveling
  - the normal force on tires.
- For our purposes, it is convenient to write the stiffness as
  - $C_\lambda = \bar{C}_\lambda \mu N$ where
    - $\bar{C}_\lambda$ is the normalized tire stiffness,
    - μ is a friction factor ($\mu = 1$ for high friction, $\mu \ll 1$ for low friction),
    - N is the normal load on the tire
- It is more realistic to saturate the achievable traction force by a maximum:
  - $\lambda = \lambda_{orig}$ if $|\lambda_{orig}| < \lambda_{max}$
  - $\lambda = \lambda_{max} \operatorname{sgn} \lambda_{orig}$

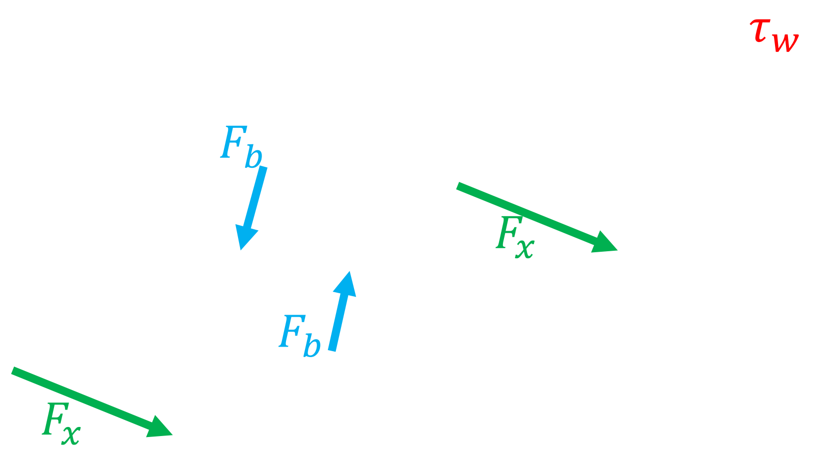
Tire Model

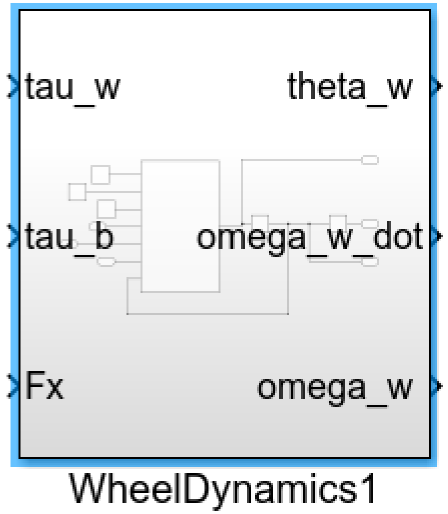
- Tire slip ratio and the longitudinal tire force are calculated directly from the equations
- Tire slip ratio calculation is numerically sensitive when speeds are low, so you use different approximation techniques.
  - Example:
    - $\frac{(r_W \omega_W - u_W)}{r_W \omega_W + \epsilon_1 \, \epsilon^{-|\omega_W|}}$ for small $\epsilon_1, \epsilon > 0$
- Tire longitudinal force calculation is approximated by two straight lines (similar to lateral force)
  - Initial linear increase with λ, then a constant maximum



Axle, Brakes and Wheel

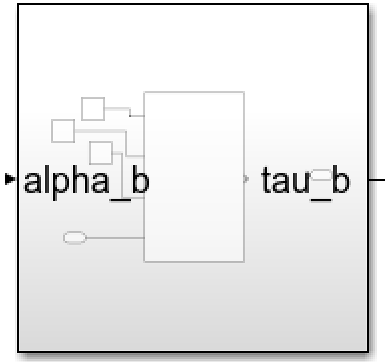
- The tire force depends on λ, which in turn depends on the wheel angular velocity ω_W and the linear velocity of its center v_x
- The angular velocity of the wheel is governed by the total torque on the wheel:
  - $I_W \dot{\omega}_W = \Sigma M_w = \tau_w - \ F_x r_W - F_b r_b = \tau_w - \tau_b - F_x r_W$
- The total torque on the wheel has the following components:
  - τ_w is the axle torque coming from the powertrain
  - F_x is the tractive force from the tire
  - r_W is the wheel radius
  - F_b is the effective tangential braking force (for disk brakes)
  - r_W is the effective disk brake radius
  - τ_b is the more general brake torque





Brake Torque

- Many braking mechanisms possible, but we will consider a disk braking system that just applies a normal force between a pad and the disk on the wheel
  - Force is primarily Couloumbic
- Brake can be in two states: “Locked (L)” or “Unlocked (U)”
- When Locked:
  - $\tau_b = \tau\_a$ Braking torque is equal to applied torque
- When Unlocked:
  - $\tau_b = \mu_b N_b g_b \operatorname{sgn} \omega_w$ with $N_b \coloneqq \alpha_b \, N_{b,max}$
  - where
    - $N_{b,max}$ is the maximum brake normal force achievable
    - g_b is a geometric design factor (that depends on the size and location of the pads relative to the disk)
    - μ_b is the friction on the pad (function of material, design, and usage)
    - α_b is a percent command of the maximum available brake – this is directly related to the brake pedal push
      - this is one of the control inputs for longitudinal vehicle dynamics



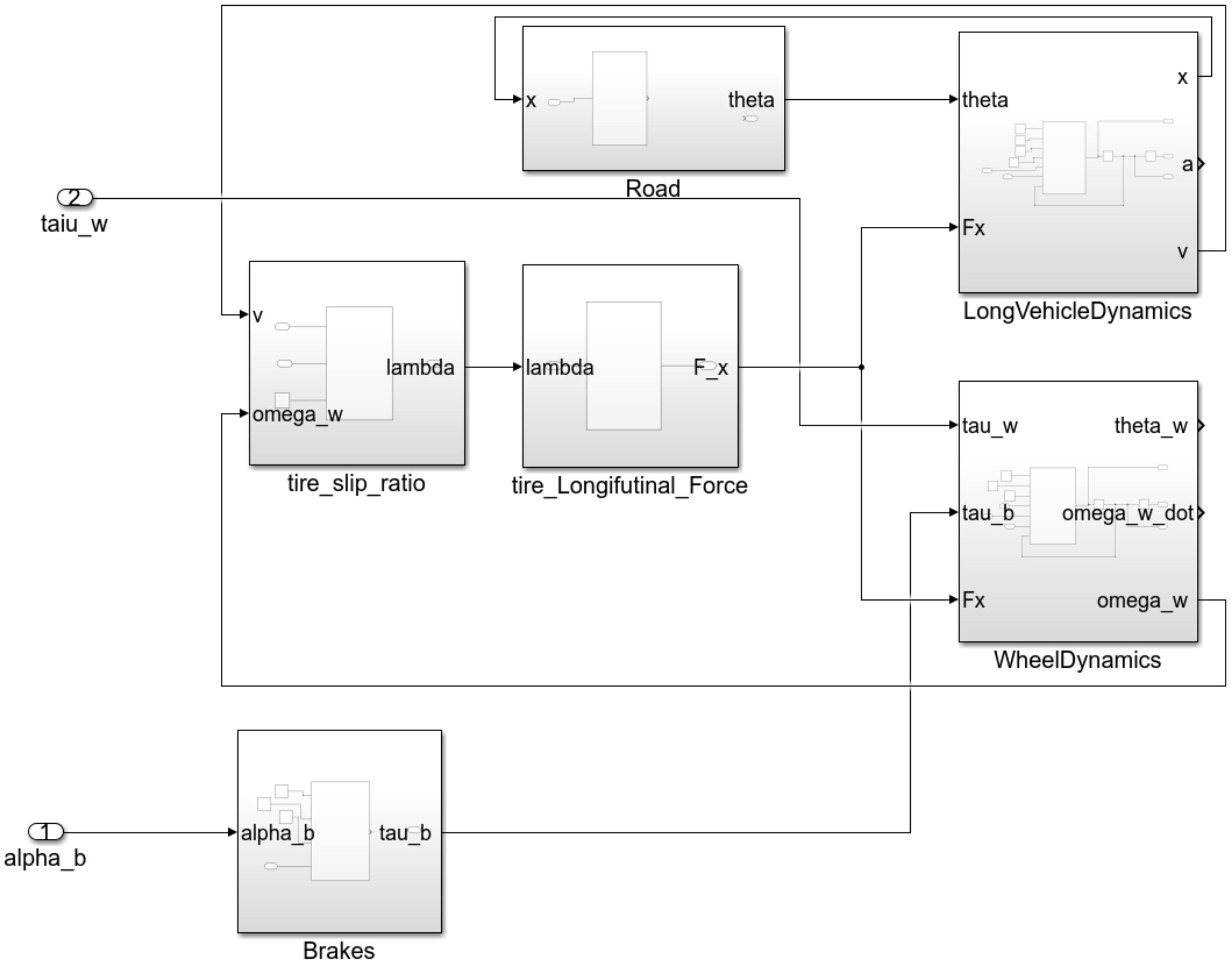
Brakes

Approximations

- It is numerically challenging to implement the coulombic friction model
- Often we use a simple approximation – that is fairly accurate when the vehicle is moving, but not so at rest:
- $\tau_b = \mu_b N_b g_b \operatorname{sgn} \omega_w \approx \mu_b N_b g_b \, \tilde{s}(\omega_w)$
- Where the function $\tilde{s}(.)$ approximates the $\operatorname{sgn}()$ function:
  - $\tilde{s}(x) = \frac{x}{|x|+\epsilon}$ for some small $\epsilon > 0$
    - The larger the value of ϵ is, the larger the approximation
    - The smaller the value of ϵ, the more challenging is numerical simulation
- For your simulations you will accept that the vehicle never "really" stops
  - More sophisticated implementations are required if you want higher accuracy in your simulation results

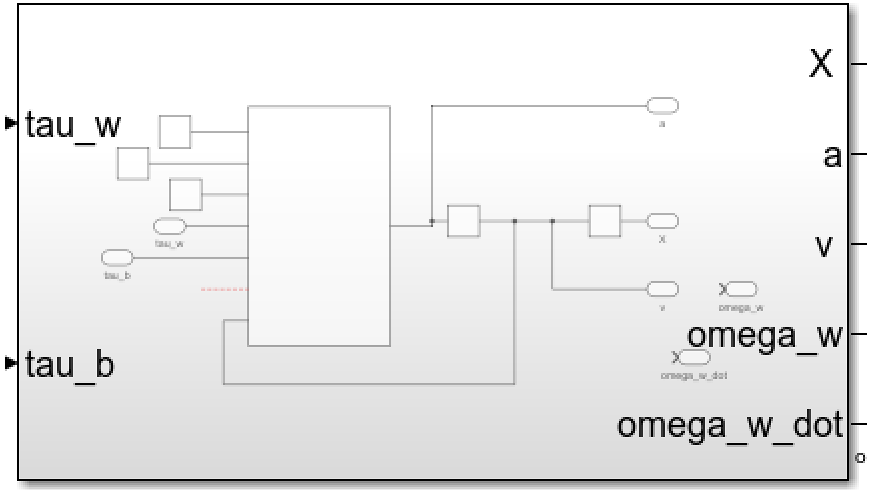
Modeling the Vehicle, Wheels and Brakes

- α_b is the input from the driver model, and corresponds to the brake pedal inputs, or desired brake torque percentage
- τ_w is the wheel torque coming from the powertrain
  - For week 1 you can assume that you can directly control this torque, and so you can assume it comes from the driver model!



Simplification with no-tire-slip assumption

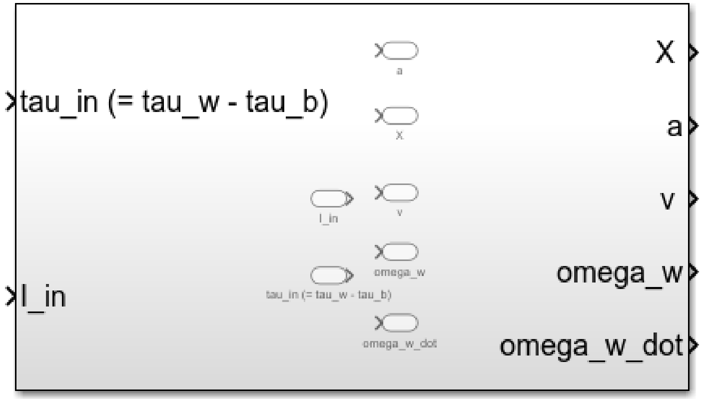
- If we assume no tire slip, then:
  - $v = r\,\omega$; Importantly, this implies: $\dot{v} = r\,\dot{\omega}$
- We lose a degree of freedom!
  - Thus we can combine the two dynamic equations using the above:
  - $m\,\dot{v} = F_x - F_{drag} - mg\sin\theta$ and $I_w\dot{\omega} = \tau_w - \tau_b - F_x r$ yield:
  - $mr\,\dot{\omega} = F_x - F_{drag} - mg\sin\theta$ or
  - $mr^2\,\dot{\omega} = F_x r - F_{drag} r - mgr\sin\theta$
  - $(I_w + mr^2)\dot{\omega} = \tau_w - \tau_b - rF_{drag} - r\,mg\sin\theta$, or
  - $\frac{(I_w+mr^2)\dot{v}}{r} = \tau_w - \tau_b - F_{drag} - mg\sin\theta$, or
  - $\left(\frac{I_w}{r^2} + m\right)\dot{v} = (\tau_w - \tau_b)/r - F_{drag} - mg\sin\theta$



SimpleLongVehicleWithNoTireSlipAssumption

Effective Inertia (or Mass)

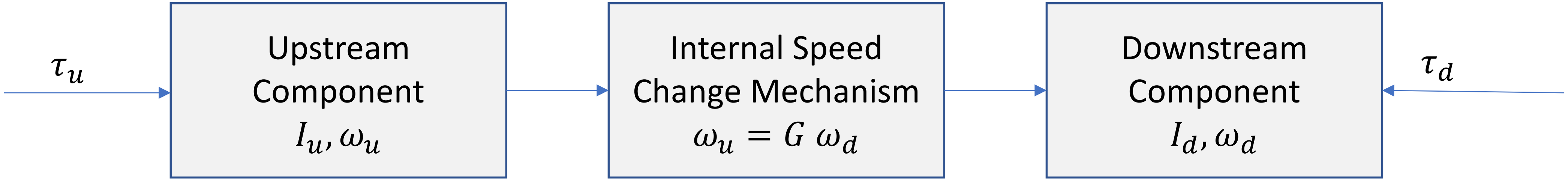
- Vehicle Dynamics looking from the vehicle point of view:
  - $\left(\frac{I_w}{r^2} + m\right)\dot{v} = (\tau_w - \tau_b)/r - F_{drag} \; - mg\sin\theta$
  - The “effective mass” of the vehicle is higher, and includes the inertia of the (upstream) wheel ***multiplied*** by the square of the speed ratio ($\frac{\omega}{v}$) i.e.
    - $m_{eq} = m + I_w \left(\frac{\omega}{v}\right)^2$
  - The “effective upstream force” is the upstream torque multiplied by the speed ratio:
    - $F_{eq} = (\tau_w - \tau_b)\frac{\omega}{v}$
- Vehicle Dynamics looking from the rotating axle point of view:
  - $(I_w + mr^2)\dot{\omega} = \tau_w - \tau_b - rF_{drag} \; - r\, mg\sin\theta$
  - The “effective inertia” of the wheel is higher, and includes the inertia of the (downstream) vehicle ***divided*** by the square of the speed ratio, i.e.
    - $I_{eq} = I_w + \frac{m}{\left(\frac{\omega}{v}\right)^2}$
  - “Effective downstream torque” is the downstream force divided by the speed ratio:
    - $\tau_{eq} = \tau_w - \tau_b - (F_{drag} + mg\sin\theta)\,\frac{1}{(\frac{\omega}{v})}$



SimpleLongVehicleWithNoTireSlipAssumption1

Generalization of connected components

- If two components are kinematically coupled so that there is a loss in the degrees of freedom of motion, we can couple the systems to evaluate the combined dynamics as below:
  - $\left(I_u G^2 + I_d\right)\dot{\omega}_d = \tau_u G \; - \tau_d$ (downstream perspective)
  - $\left(I_u + I_d / G^2\right)\dot{\omega}_u = \tau_u \; - \tau_d / G$ (upstream perspective)



(Simple) Longitudinal Dynamics Model

- Extending the ideas to include a powertrain inertia upstream yields:
  - $\dot{v} = \frac{1}{M_{eq}}((\tau_a - \tau_b)/r - F_{drag} - m\, g \sin\theta)$ with $M_{eq} := \frac{I_{PT} + I_w}{r^2} + m$
- We will next see how τ_a is obtained from the engine and what I_{PT} contains

Powertrain

- The powertrain will usually include the components that generate and transmit power to the wheels, and provide the wheel or axle torque τ_w
  - Powerplant:
    - Engine (Internal Combustion Engine, ICE)
    - Electric Motor
    - Hydraulic Motor
    - Human Legs
  - Transmission:
    - Gears (Gear Trains)
    - Chains
    - Differentials
  - Energy Storage
    - Batteries (electric)
    - Fuel Tank
    - Accumulators (hydraulic)
    - … the Sun?
- We will study a subset of the above:
  - Electric Motors, Gears, Batteries