



# Introduction To Reinforcement Learning

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Centre for Advance Robotics Technology Innovation

April 2025

# Outline

- Recent progresses in RL.
- Overview of RL
- Basic methods to solve the RL problems
- Tutorials
  - Tic-tac-toe. Complete MDP with *Value Iteration* method.
  - Cartpole. Small scaled DRL problem → benchmarking and analysis

# Objectives

- Exposure to mathematical formulism of RL.
- Familiarize with basic concepts of Reinforcement Learning (RL).

In the context of RL...

- Agent, environment, observations, state, reward, action, value, return, discount ...
- Evaluation, Iteration, Improvement, Value Iteration ...
- Monte Carlo, Off-policy
- Temporal Difference, Q-learning, Sarsa
- Function Approximation
- Policy Gradient Methods

# Deep Reinforcement Learning Doesn't Work Yet

Feb 14, 2018

June 24, 2018 note: If you want to cite an example from the post, please cite the paper which that example came from. If you want to cite the post as a whole, you can use the following BibTeX:

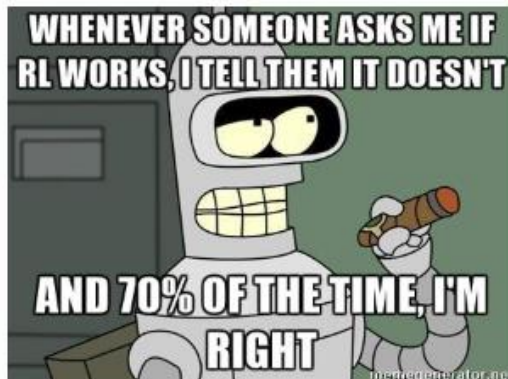
```
@misc{rlblogpost,  
  title={Deep Reinforcement Learning Doesn't Work Yet},  
  author={Irpan, Alex},  
  howpublished={\url{https://www.alexirpan.com/2018/02/14/rl-hard.html}},  
  year={2018}  
}
```

This mostly cites papers from Berkeley, Google Brain, DeepMind, and OpenAI from the past few years, because that work is most visible to me. I'm almost certainly missing stuff from older literature and other institutions, and for that I apologize - I'm just one guy, after all.

## Introduction

Once, on Facebook, I made the following claim.

Whenever someone asks me if reinforcement learning can solve their problem, I tell them it can't. I think this is right at least 70% of the time.



<https://www.alexirpan.com/2018/02/14/rl-hard.html>



**Alexander Irpan** ✓ · 3rd

Research Scientist at Google

Google · University of California, Berkeley

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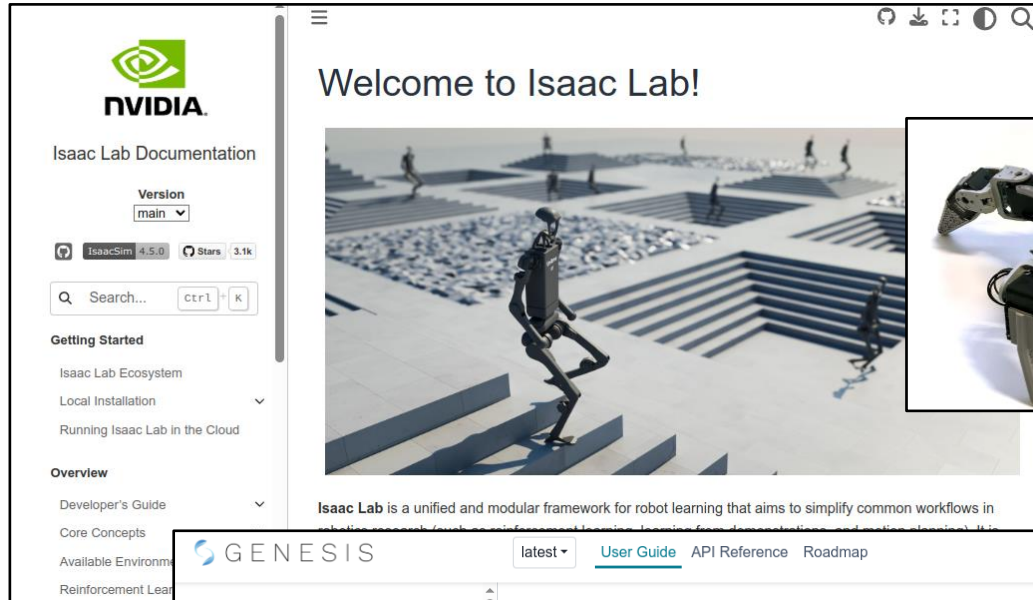
 Connect

# Why RL ain't work?

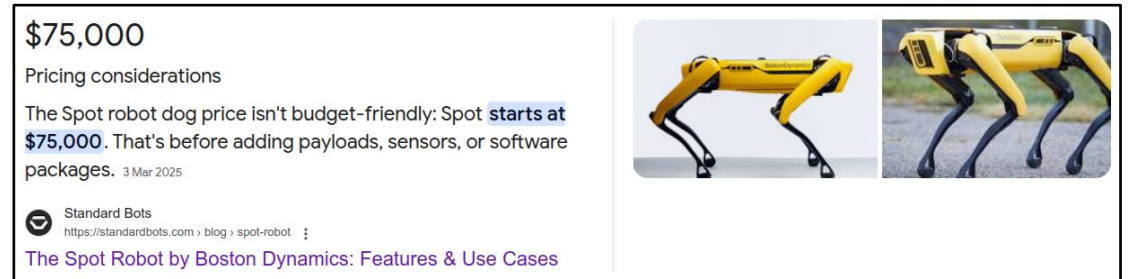
-  Sample Inefficient
-  Can be solved by other methods
-  Always requires a reward function
-  Reward function design is difficult
-  Local optima hard to escape
-  Overfitting
-  Unstable and hard to reproduce

# Why RL works now?

- Sample Inefficient → **Cost of experiment ↓**



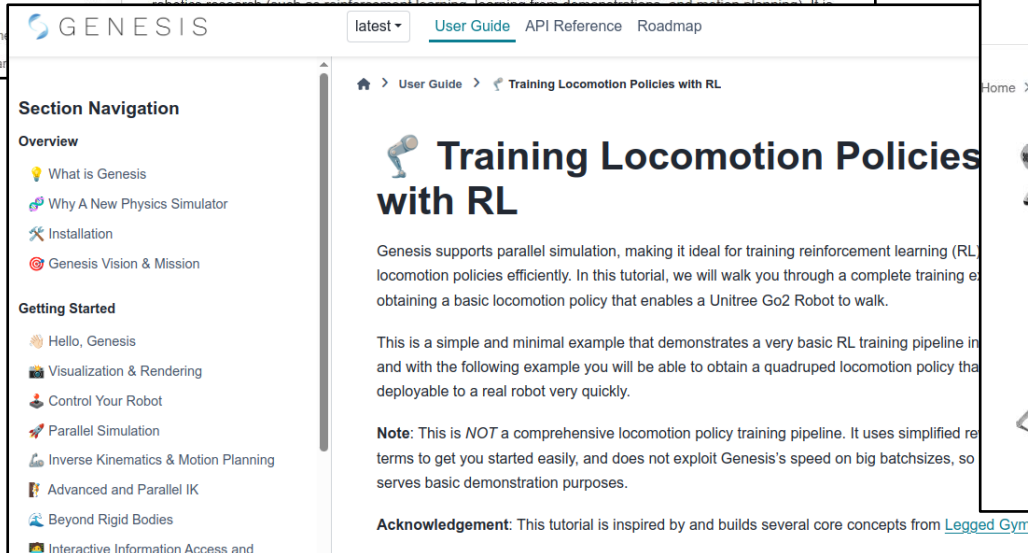
The screenshot shows the NVIDIA Isaac Lab documentation page. The header includes the NVIDIA logo and the text "Isaac Lab Documentation". Below this, there's a "Version" dropdown set to "main". A search bar is present with the text "Search..." and a "ctrl+K" shortcut. The left sidebar contains a "Getting Started" section with links to "Isaac Lab Ecosystem", "Local Installation", and "Running Isaac Lab in the Cloud". The main content area has a large image of a robot walking on a grid and the text "Welcome to Isaac Lab!".



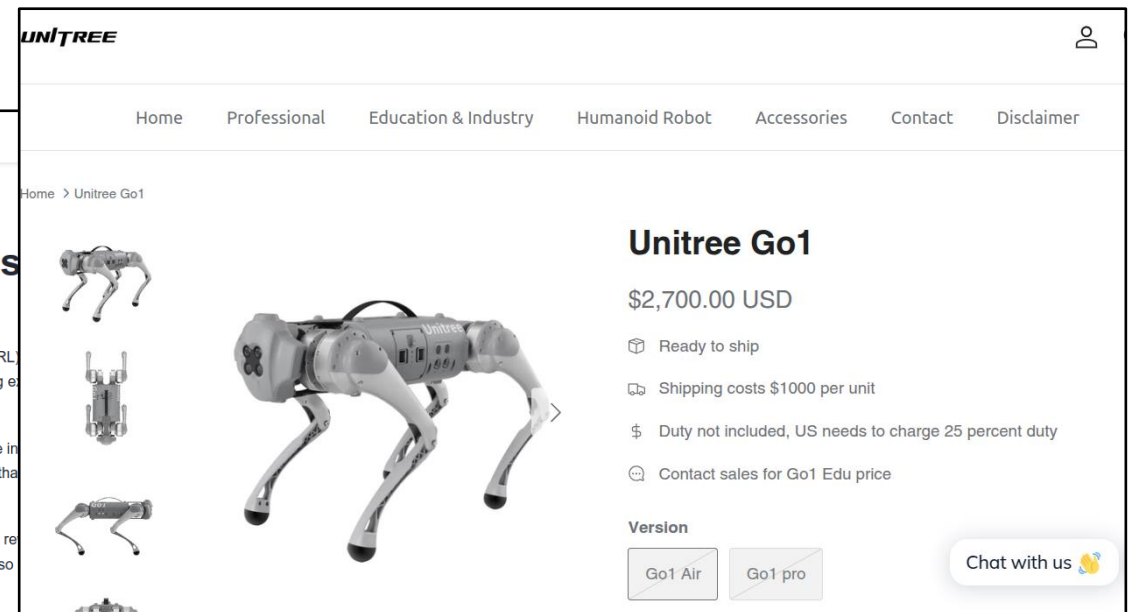
The screenshot shows a blog post titled "The Spot Robot by Boston Dynamics: Features & Use Cases". The price is listed as "\$75,000". The text states: "Pricing considerations The Spot robot dog price isn't budget-friendly: Spot starts at \$75,000. That's before adding payloads, sensors, or software packages." There are two images of the yellow Spot robot.



The ANYmal robot, a four-legged robot made by ANYbotics for industrial customers, costs around \$150,000 and includes the full autonomy platform with LIDAR and a docking station, but excludes payloads, self-charging docks, and autonomous capabilities.




The screenshot shows the Genesis documentation page. The header includes the Genesis logo and the text "Genesis". Below this, there's a "latest" dropdown and a "User Guide" link. The left sidebar contains a "Section Navigation" section with links to "What is Genesis", "Why A New Physics Simulator", "Installation", "Genesis Vision & Mission", "Hello, Genesis", "Visualization & Rendering", "Control Your Robot", "Parallel Simulation", "Inverse Kinematics & Motion Planning", "Advanced and Parallel IK", "Beyond Rigid Bodies", and "Interactive Information Access and". The main content area has a large image of a robot walking on a grid and the text "Training Locomotion Policies with RL".



The screenshot shows the Unitree Go1 product page. The header includes the Unitree logo and the text "Unitree". Below this, there's a navigation bar with links to "Home", "Professional", "Education & Industry", "Humanoid Robot", "Accessories", "Contact", and "Disclaimer". The main content area has a large image of the Unitree Go1 robot and the text "Unitree Go1 \$2,700.00 USD". There are also smaller images of the robot in different poses.

# Why RL *works* now?

- Sample Inefficient → **Cost of experiment** ↓
- Some problems can be solved by other methods  
→ **and many others can be solved by RL**




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Published on 14 Nov 2023

**NBS Knowledge Lab Interdisciplinary Distinguished Speaker Series webinar: Reinforcement Learning for Quantitative Trading**


## AlphaGo

AlphaGo mastered the ancient game of Go, defeated a Go world champion, and inspired a new era of AI systems.



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[DeepSeek-R1](#)



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**Discovering faster matrix multiplication algorithms with reinforcement learning**

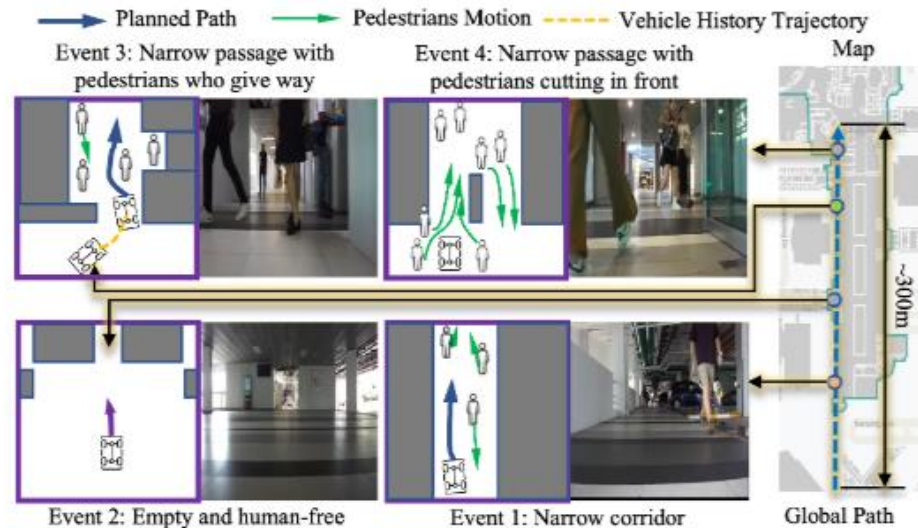


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## Learning Dynamic Weight Adjustment for Spatial-Temporal Trajectory Planning in Crowd Navigation

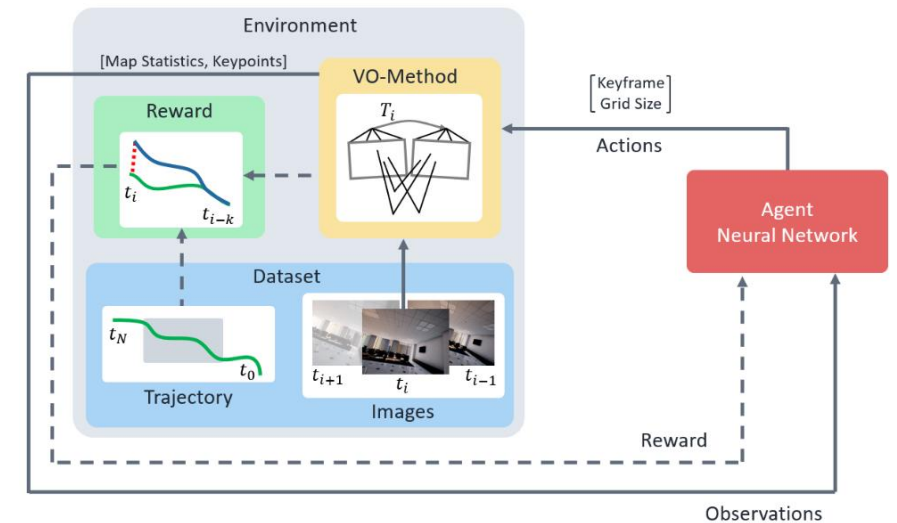
Muqing Cao\*, Xinhang Xu\*, Yizhuo Yang\*, Jianping Li, Tongxing Jin, Pengfei Wang, Tzu-Yi Hung, Guosheng Lin, and Lihua Xie<sup>1</sup> *Fellow, IEEE*



## Reinforcement Learning Meets Visual Odometry

Nico Messikommer\*, Giovanni Cioffi\*, Mathias Gehrig\*, and Davide Scaramuzza\*

Dept. of Informatics, University of Zurich  
 {nmessi,cioffi,mgehrig,sdaveide}@ifi.uzh.ch





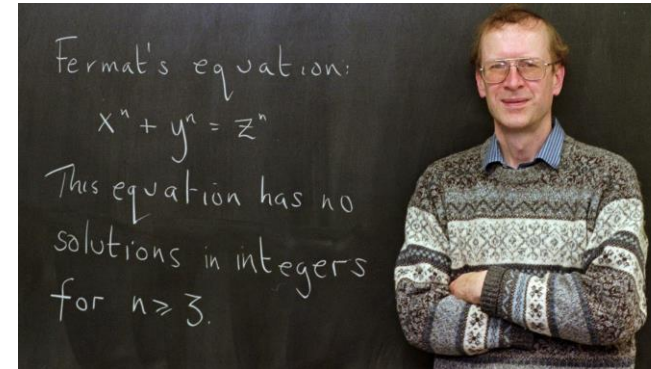
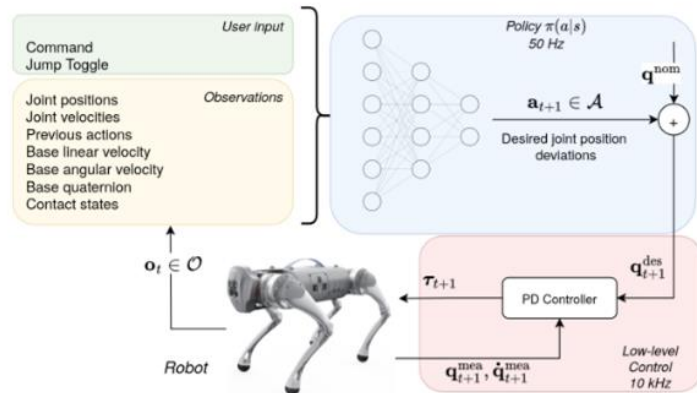
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- Unstable and hard to reproduce



## Curriculum-Based Reinforcement Learning for Quadrupedal Jumping: A Reference-free Design

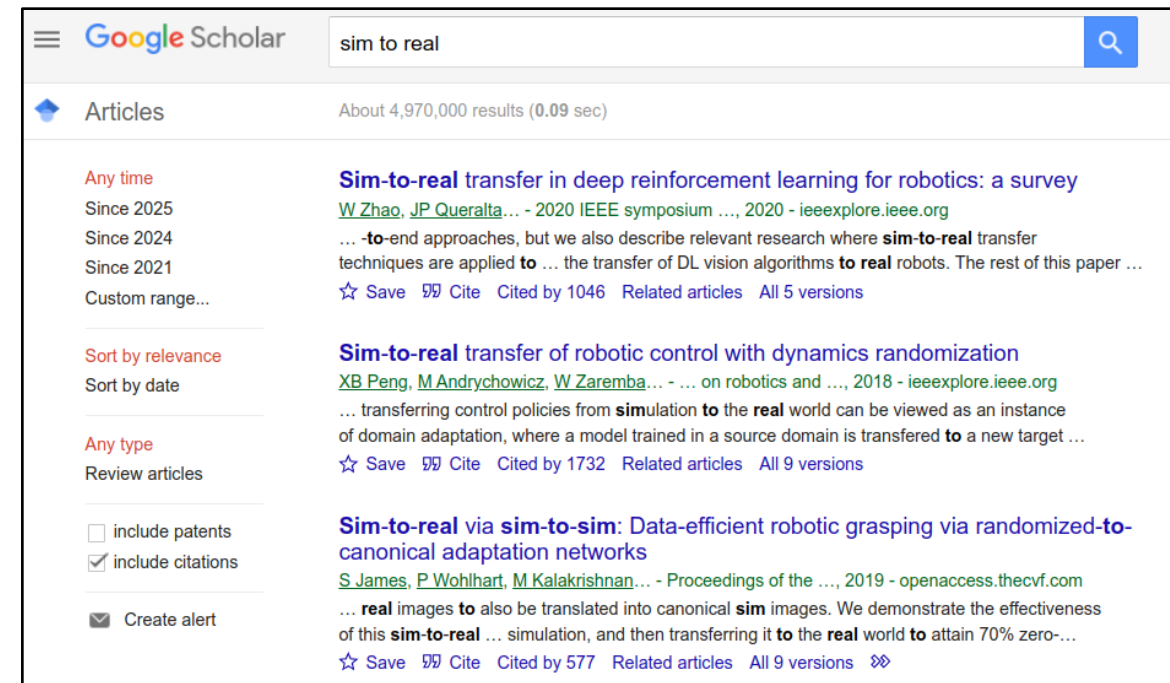
Vassil Atanasov\*, Jiatao Ding\*, Jens Kober, Ioannis Havoutis, Cosimo Della Santina

REWARDS DEFINITION. THE LIGHT ORANGE COLOUR INDICATES TASK-BASED REWARDS, WHILE THE LIGHT PURPLE SHADE DESCRIBES REGULARISATION REWARDS.  $w_x$  IS THE WEIGHT,  $\sigma_x$  IS A SCALING FACTOR FOR THE EXPONENTIAL KERNEL,  $e(\cdot)$  AND  $\log(\cdot)$  SEPARATELY DENOTE THE EXPONENT AND LOGARITHM OPERATION.

Name	Type	Stance	Flight	Landing
Landing position	Single	0	0	$w_p(e(-\sum   p_{land} - p_{des}  ^2)/\sigma_{p,land})$
Landing orientation	Single	0	0	$w_{ori}(e(-  \log(\bar{q}_{land}^{-1} * \bar{q}_{des})  ^2)/\sigma_{ori,land})$
Max height	Single	0	0	$w_h(e(  h_{max} - 0.9  ^2)/\sigma_{p_z,max})$
Jumping	Single	0	0	$w_{jump}$
Base Position	Continuous	$w_{p_z,st}(e(-  p_z - 0.20  ^2/\sigma_{p_z,st}))$	$w_{p_z,f}(e(-  p_z - 0.7  ^2/\sigma_{p_z,f}))$	$w_{p,l}(e(-\sum   p - p_{des}  ^2/\sigma_{p,l}))$
Orientation Tracking	Continuous	$w_{ori,st}(e(-  \log(\bar{q}_{base}^{-1} * \bar{q}_{des})  ^2/\sigma_{ori,st}))$	0	$w_{ori,l}(e(-  \log(\bar{q}_{base}^{-1} * \bar{q}_{des})  ^2/\sigma_{ori,l}))$
Base linear velocity	Continuous	0	$w_{v_x,y}(e(-\sum   v_{x,y} - v_{des}  ^2/\sigma_v))$	0
Base angular velocity	Continuous	0	$w_{\omega}(e(-\sum   \omega - \omega_{des}  ^2/\sigma_{\omega}))$	$0.1w_{\omega}(e(-\sum   \omega  ^2/\sigma_{\omega}))$
Feet clearance	Continuous	0	$w_{feet}( p_{feet} - p_{feet}^0 + [0.0, 0.0, -0.15] ^2)$	0
Symmetry	Continuous	$w_{sym}(\sum_{joint}   q_{left} - q_{right}  ^2)$		
Nominal pose	Continuous	$w_q(e(-\sum_{joint}   q_j - q_{j,nom}  ^2/\sigma_q))$	$0.1w_q(e(-\sum_{joint}   q_j - q_{j,nom}  ^2/\sigma_q))$	$w_q(e(-\sum_{joint}   q_j - q_{j,nom}  ^2/\sigma_q))$
Energy	Continuous	$w_{energy}(\tau^T \dot{q})$		
Base acceleration	Continuous	$w_{acc} \dot{v} ^2$		
Contact change	Continuous	$w_c \sum_{feet} (c_{foot}(t) - c_{foot}(t-1))$		
Maintain Contact	Continuous	$w_{contact} \sum_{feet} c_{foot}(t)$	0	0
Contact forces	Continuous	$w_{F_e} \sum_{i=0}^{n_f}  F_i - \bar{F} $		
Action rate	Continuous	$w_a \sum_{joint}  a(t) - a(t-1) ^2$		
Joint acceleration	Continuous	$w_{\ddot{q}} \sum_{joint}  \ddot{q}_j ^2$		
Joint limits	Continuous	$w_{qlim} \sum_{joint}  q_j - q_{j,lim} ^2$		

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- Sample Inefficient → **Cost of experiment** ↓
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  - Always requires a reward function
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  - Local optima hard to escape
  - Overfitting
  - Unstable and hard to reproduce
- **Active research areas**



Google Scholar search results for "sim to real". The search bar shows "sim to real" and the results are approximately 4,970,000 (0.09 sec).

**Articles**

**Any time**  
Since 2025  
Since 2024  
Since 2021  
Custom range...

**Sort by relevance**  
Sort by date

**Any type**  
Review articles

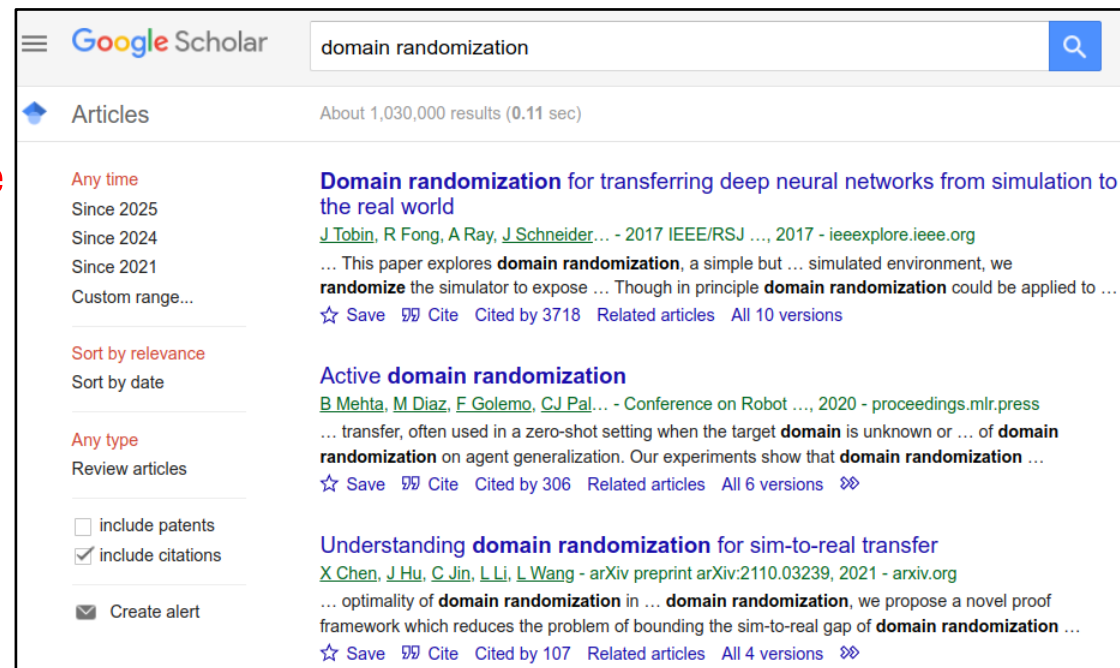
☐ include patents  
☒ include citations

☒ Create alert

**Sim-to-real transfer in deep reinforcement learning for robotics: a survey**  
[W Zhao, JP Queralta...](#) - 2020 IEEE symposium ..., 2020 - [ieeexplore.ieee.org](#)  
... -to-end approaches, but we also describe relevant research where **sim-to-real** transfer techniques are applied to ... the transfer of DL vision algorithms to **real** robots. The rest of this paper ...  
☆ Save Cite Cited by 1046 Related articles All 5 versions

**Sim-to-real transfer of robotic control with dynamics randomization**  
[XB Peng, M Andrychowicz, W Zaremba...](#) - ... on robotics and ..., 2018 - [ieeexplore.ieee.org](#)  
... transferring control policies from **simulation** to the **real** world can be viewed as an instance of domain adaptation, where a model trained in a source domain is transferred to a new target ...  
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**Sim-to-real via sim-to-sim: Data-efficient robotic grasping via randomized-to-canonical adaptation networks**  
[S James, P Wohlhart, M Kalakrishnan...](#) - Proceedings of the ..., 2019 - [openaccess.thecvf.com](#)  
... **real** images to also be translated into canonical **sim** images. We demonstrate the effectiveness of this **sim-to-real** ... simulation, and then transferring it to the **real** world to attain 70% zero-...  
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Google Scholar search results for "domain randomization". The search bar shows "domain randomization" and the results are approximately 1,030,000 (0.11 sec).

**Articles**

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Since 2021  
Custom range...

**Sort by relevance**  
Sort by date

**Any type**  
Review articles

☐ include patents  
☒ include citations

☒ Create alert

**Domain randomization for transferring deep neural networks from simulation to the real world**  
[J Tobin, R Fong, A Ray, J Schneider...](#) - 2017 IEEE/RSJ ..., 2017 - [ieeexplore.ieee.org](#)  
... This paper explores **domain randomization**, a simple but ... simulated environment, we **randomize** the simulator to expose ... Though in principle **domain randomization** could be applied to ...  
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**Active domain randomization**  
[B Mehta, M Diaz, F Golemo, C J Pal...](#) - Conference on Robot ..., 2020 - [proceedings.mlr.press](#)  
... transfer, often used in a zero-shot setting when the target **domain** is unknown or ... of **domain randomization** on agent generalization. Our experiments show that **domain randomization** ...  
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**Understanding domain randomization for sim-to-real transfer**  
[X Chen, J Hu, C Jin, L Li, L Wang](#) - arXiv preprint arXiv:2110.03239, 2021 - [arxiv.org](#)  
... optimality of **domain randomization** in ... **domain randomization**, we propose a novel proof framework which reduces the problem of bounding the sim-to-real gap of **domain randomization** ...  
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# Why RL *works* now?

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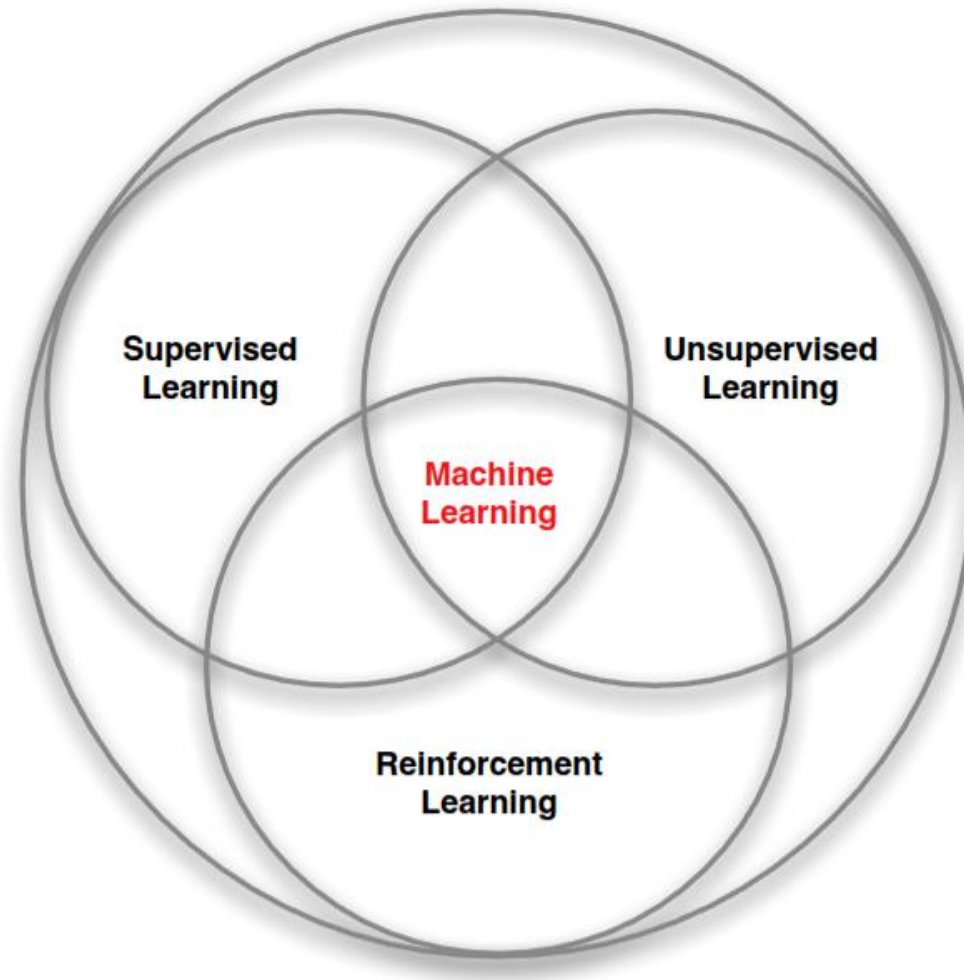


In the context of RL...

- Agent, environment, observations, state, reward, action, value, return, discount ...
- Evaluation, Iteration, Improvement, Value Iteration ...
- Monte Carlo, Off-policy
- Temporal Difference, Q-learning, Sarsa
- Function Approximation
- Policy Gradient Methods



Have labels of objects →  
make **learning model**  
that predicts the label of  
new objects

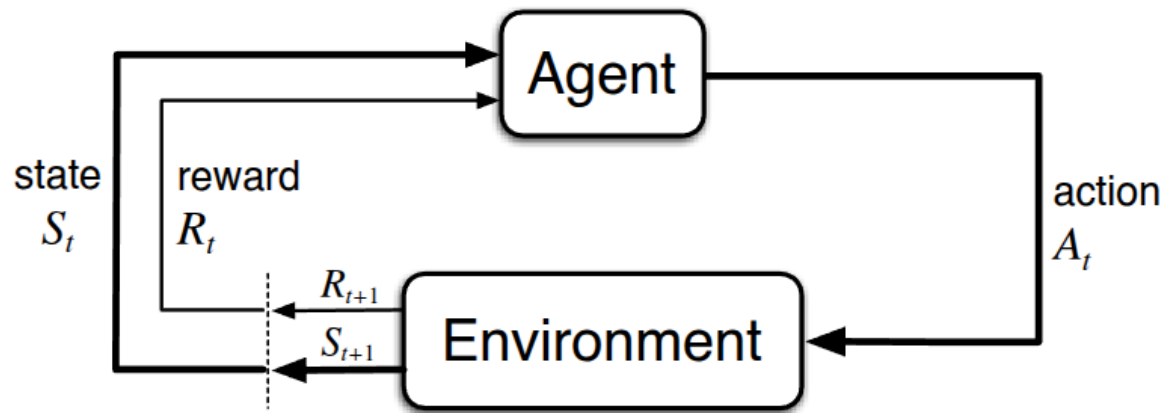


Have data with assumed  
**correlation model** →  
discovering the model

Have agent-environment **dynamics**  
→ find the policy that yields optimal  
*return*.

# Agent and Environment

- **Agent:** receives **observations** and **rewards**, generates **action**.
- **Environment:** receives **action**, produces **observation** and **reward**.



The robot belongs to which category?

*"All goals can be described by the maximization of expected cumulative reward"*

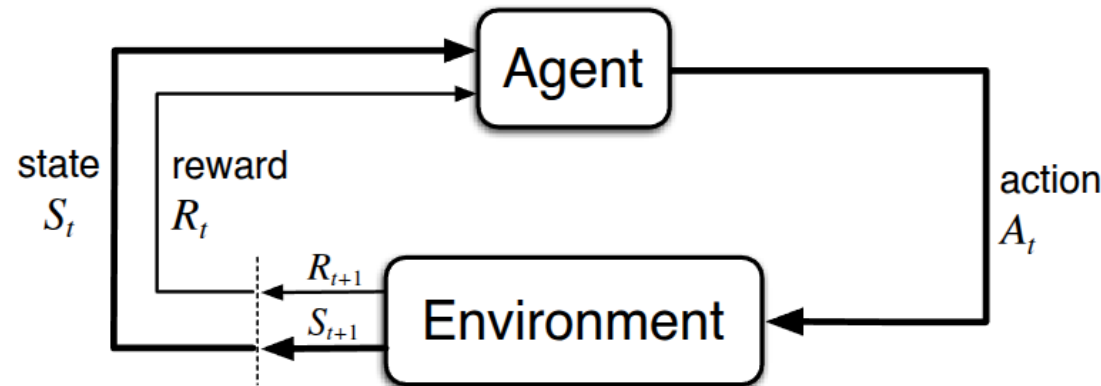


main IsaacLab / source / isaacsim\_tasks / isaacsim\_tasks / direct / anymal\_c / anymal\_c\_env\_cfg.py

Code Blame 148 lines (130 loc) · 4.48 KB · 1

```
53 class AnymalCFlatEnvCfg(DirectRLEnvCfg):
54     decimation = 4
55     action_scale = 0.5
56     action_space = 12
57     observation_space = 48
58     state_space = 0
59
60     # simulation
61
62     > sim: SimulationCfg = SimulationCfg( ...
63     )
64
65     > terrain = TerrainImporterCfg( ...
66     )
67
68     # scene
69     scene: InteractiveSceneCfg = InteractiveSceneCfg(num_envs=4096, env_spacing=4.0, replicate_physics=True)
70
71     # events
72     events: EventCfg = EventCfg()
73
74     # robot
75     robot: ArticulationCfg = ANYMAL_C_CFG.replace(prim_path="/World/envs/env_*/Robot")
76     contact_sensor: ContactSensorCfg = ContactSensorCfg(
77         prim_path="/World/envs/env_*/Robot/.*", history_length=3, update_period=0.005, track_air_time=True
78     )
79
80     # reward scales
81     lin_vel_reward_scale = 1.0
82     yaw_rate_reward_scale = 0.5
83     z_vel_reward_scale = -2.0
84     ang_vel_reward_scale = -0.05
85     joint_torque_reward_scale = -2.5e-5
86     joint_accel_reward_scale = -2.5e-7
87     action_rate_reward_scale = -0.01
88     feet_air_time_reward_scale = 0.5
89     undesired_contact_reward_scale = -1.0
90     flat_orientation_reward_scale = -5.0
```

# Concepts (1)



## Definition

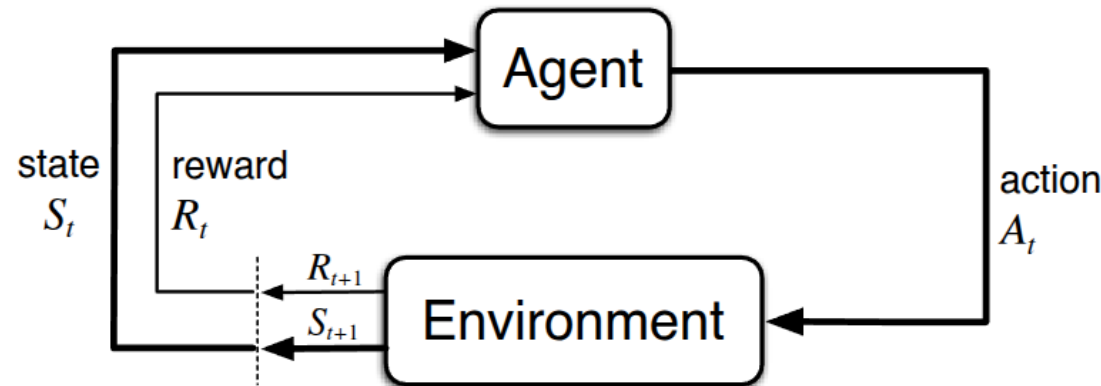
- A finite Markov Decision Process:
  - $R_t$  is the **reward**, a **scalar signal**
  - $A_t$  is the **action**, e.g., torque command, velocity command, chess moves ...
  - $S_t$  is the **state**. Some states are called *terminal states*.
  - $t \in \{0, 1, 2 \dots\}$ ,  $s \in \mathcal{S}$ ,  $a \in \mathcal{A}(s)$ ,  $r \in \mathcal{R} \subset \mathbb{R}$

- The dynamics between agent and environment is summarized in:

$$\mathcal{P} \triangleq p(s', r | s, a) = \text{Prob}(S_{t+1} = s', R_{t+1} = r | S_t = s, A_t = a)$$

- $H_t \triangleq (S_0, R_0, A_0 \dots S_t, R_t, A_t)$ , the **trajectory**.
- $O_t = h(H_t)$  is the **observation**, e.g., image, can be IMU reading, lidar scan ...
- Often, we need to estimate the state from the observation  $S_t = f(O_t)$

# Concepts (1)



## Definition

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# Concepts (2) <sup>[1]</sup>

## Definition

- Policy function  $\pi(\cdot)$ :

- Deterministic policy:

$$a = \pi(s)$$

- Stochastic policy:

$$\pi(a|s) = \text{Prob}(A_t = a|S_t = s)$$

- The return  $G_t$ :

$$G_t \triangleq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots$$

- The discount factor:

$$\gamma \in [0, 1)$$

## Definition

- (State-)value function under policy  $\pi$ ,  $v_\pi(s)$ :

$$v_\pi(s) \triangleq E_\pi(G_t|S_t = s)$$

- The action-value function:

$$q_\pi(s, a) \triangleq E_\pi(G_t|S_t = s, A_t = a)$$

## Note

$S_t, A_t, R_t, G_t$  are all random variables that can take value  $s, a, r, g$  in their respective domain

# Somewhere in the multiverse...

- Jiedi Wan is an “RL researcher” at SpadeX, he runs an experiment and sends this data to his boss Yilong Ma:

$$o_0, a_0, r_0, o_1, a_1, r_1, o_2, a_2, r_2, o_3 \dots, a_{t_{terminal}}$$

- Yilong fires Jiedi Wan...

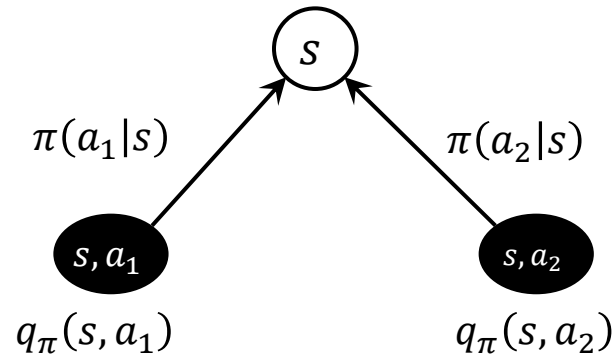


<https://tinyurl.com/tmnRL2025>

# Bellman Equation

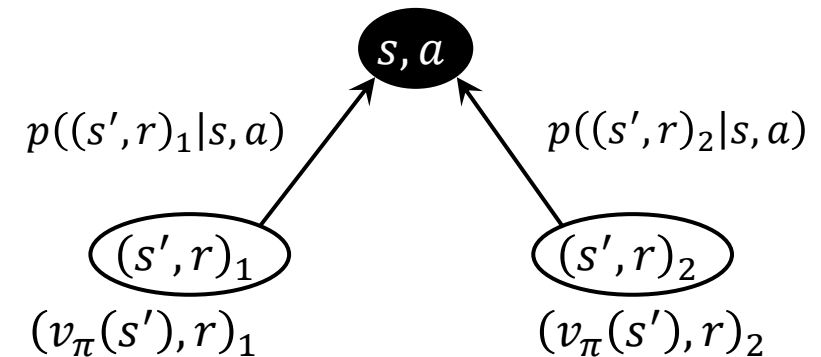
- For state-value function:

$$\begin{aligned} v_{\pi}(s) &= E_{\pi}(G_t | S_t = s) \\ &= \sum_{a \in \mathcal{A}} \pi(a|s) q_{\pi}(s, a) \end{aligned}$$



- For action-value function:

$$\begin{aligned} q_{\pi}(s, a) &= E_{\pi}(G_t | S_t = s, A_t = a) = E_{\pi}[R_{t+1} + \gamma v_{\pi}(S_{t+1}) | s, a] \\ &= \sum_{(s', r) \in \mathcal{S} \times \mathcal{R}} p(s', r | s, a) [r + \gamma v_{\pi}(s')] \end{aligned}$$



# Bellman Equation

- For state-value function:

$$\begin{aligned}
 v_{\pi}(s) &= E_{\pi}(G_t | S_t = s) \\
 &= \sum_{a \in \mathcal{A}} \pi(a|s) q_{\pi}(s, a) \\
 &= \sum_{a \in \mathcal{A}} \pi(a|s) \left[ \sum_{(s', r) \in \mathcal{S} \times \mathcal{R}} p(s', r | s, a) [r + \gamma v_{\pi}(s')] \right]
 \end{aligned}$$

- For action-value function:

$$\begin{aligned}
 q_{\pi}(s, a) &= E_{\pi}(G_t | S_t = s, A_t = a) = E_{\pi}[R_{t+1} + \gamma v_{\pi}(S_{t+1}) | s, a] \\
 &= \sum_{(s', r) \in \mathcal{S} \times \mathcal{R}} p(s', r | s, a) [r + \gamma v_{\pi}(s')] \\
 &= \sum_{(s', r) \in \mathcal{S} \times \mathcal{R}} p(s', r | s, a) \left[ r + \gamma \left[ \sum_{a' \in \mathcal{A}(s')} \pi(a'|s') q_{\pi}(s', a') \right] \right]
 \end{aligned}$$



# Optimal Value Functions & BOE

## Definition

- $\pi > \pi' \Rightarrow v_\pi(s) > v_{\pi'}(s), \forall s$
- The optimal state-value function  $v_*(s)$ :
- The optimal action-value function  $q_*(s, a)$ :
- For any optimal  $\pi_*$ , all  $s \in \mathcal{S}$ , all  $a \in \mathcal{A}(s)$ :

$$v_*(s) = \max_{\pi} v_\pi(s)$$

$$q_*(s, a) = \max_{\pi} q_\pi(s, a)$$

$$v_*(s) = \max_{a \in \mathcal{A}(s)} q_*(s, a)$$

$$q_*(s, a) = \sum_{(s', r) \in \mathcal{S} \times \mathcal{R}} p(s', r | s, a) [r + \gamma v_*(s')]$$

## Theorem

For any MDP:

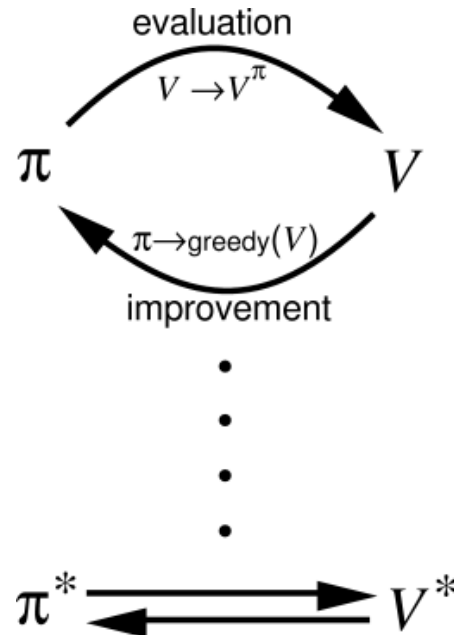
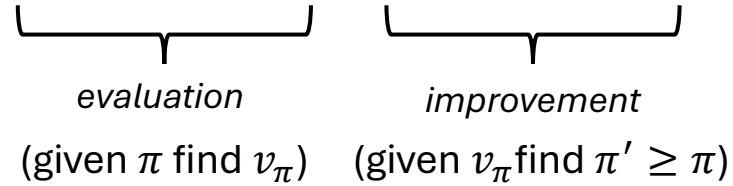
- $\exists \pi_*, \pi_* \geq \pi, \forall \pi$
- $v_{\pi_*}(s) = v_*(s), \forall \pi_*$
- $q_{\pi_*}(s, a) = q_\pi(s, a), \forall \pi_*$

In the context of RL...

- Agent, environment, observations, state, reward, action, value, return, discount ...
- Evaluation, Iteration, Improvement, Value Iteration ...
- Monte Carlo, Off-policy
- Temporal Difference, Q-learning, Sarsa
- Function Approximation
- Policy Gradient Methods

# Solving the MDP

- Policy iteration: from some  $\pi \rightarrow$  evaluate  $\pi \rightarrow$  improve  $\pi$ , repeat until  $\pi \approx \pi^*$



- Value iteration: a direct approach that achieves faster convergence.

# Solving the MDP

## Policy *Evaluation*:

Given a policy  $\pi(a|s)$

- For  $k = 0 \dots K - 1$ :

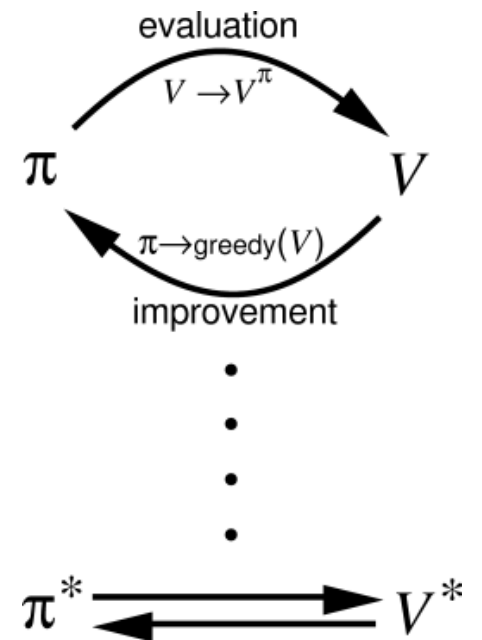
$$\forall s \in \mathcal{S}: V_{k+1}(s) \leftarrow \sum_{a \in \mathcal{A}} \pi(a|s) \sum_{(s', r) \in \mathcal{S} \times \mathcal{R}} p(s', r|s, a) [r + \gamma V_k(s')],$$

- $V_k(s) \xrightarrow{K \rightarrow \infty} v_\pi(s)$

## Policy *Improvement*:

Given a value function  $v_\pi(s)$ :

- $\pi_* = \text{greedy}(v_\pi(s))$



**Policy Iteration**

# Solving the MDP

## Policy *Evaluation*:

Given a policy  $\pi(a|s)$

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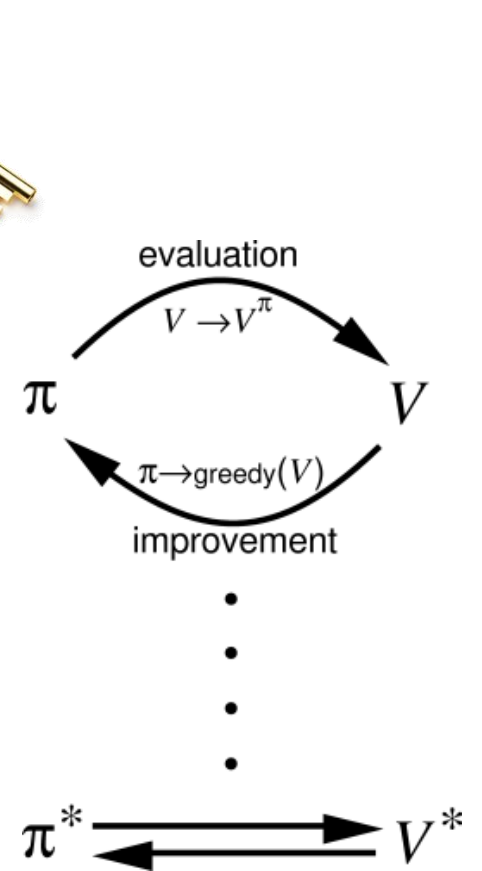
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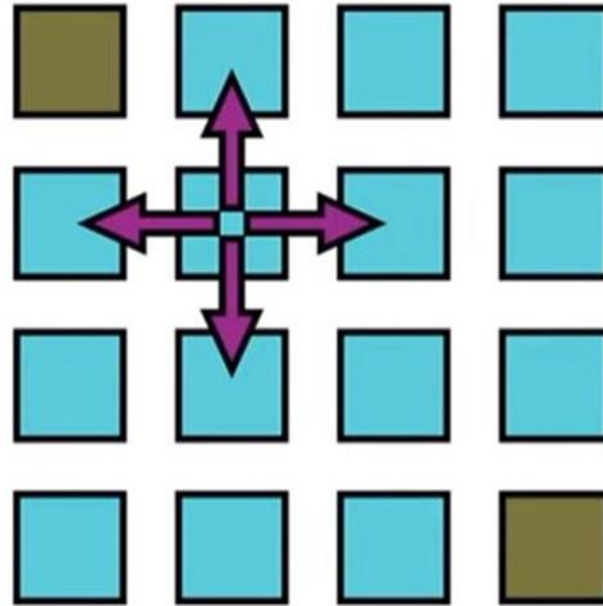
- $\pi_* = \text{greedy}(v_\pi(s))$



**Policy Iteration**

# Policy Evaluation – Grid World Example

Compute  $v_\pi(s)$  or  $q_\pi(s, a)$  for a given  $\pi$ .



$$R_t = -1$$

$$\pi(a|s) = 0.25$$

# Policy Evaluation – Grid World Example

Compute  $v_\pi(s)$  or  $q_\pi(s, a)$  for a given  $\pi$ .

0.0	0.4	0.2	0.1
-0.2	0.3	-0.1	0.8
0.9	-0.2	0.6	0.1
0.1	0.9	-0.9	0.0

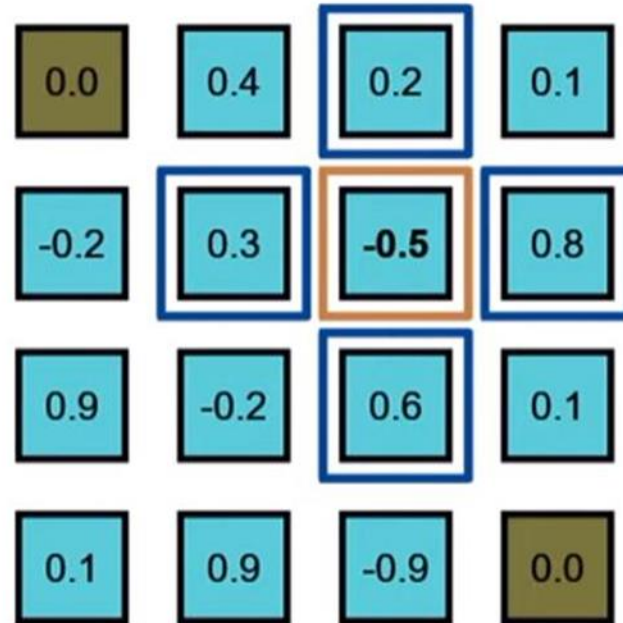
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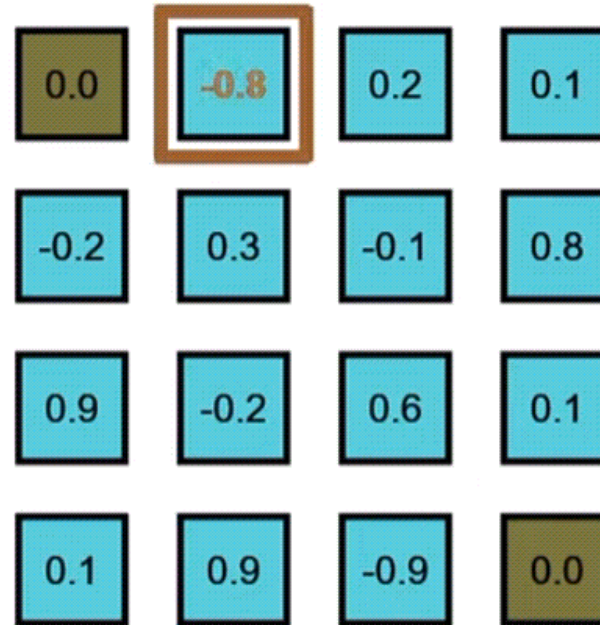
$$R_t = -1$$

$$\pi(a|s) = 0.25$$

$$V(s) \leftarrow \sum_{a \in \mathcal{A}(s)} \pi(a|s) \sum_{\substack{s' \in \mathcal{S} \\ r \in \mathcal{R}}} p(s', r|s, a) [r + \gamma V(s')]$$

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Compute  $v_\pi(s)$  or  $q_\pi(s, a)$  for a given  $\pi$ .

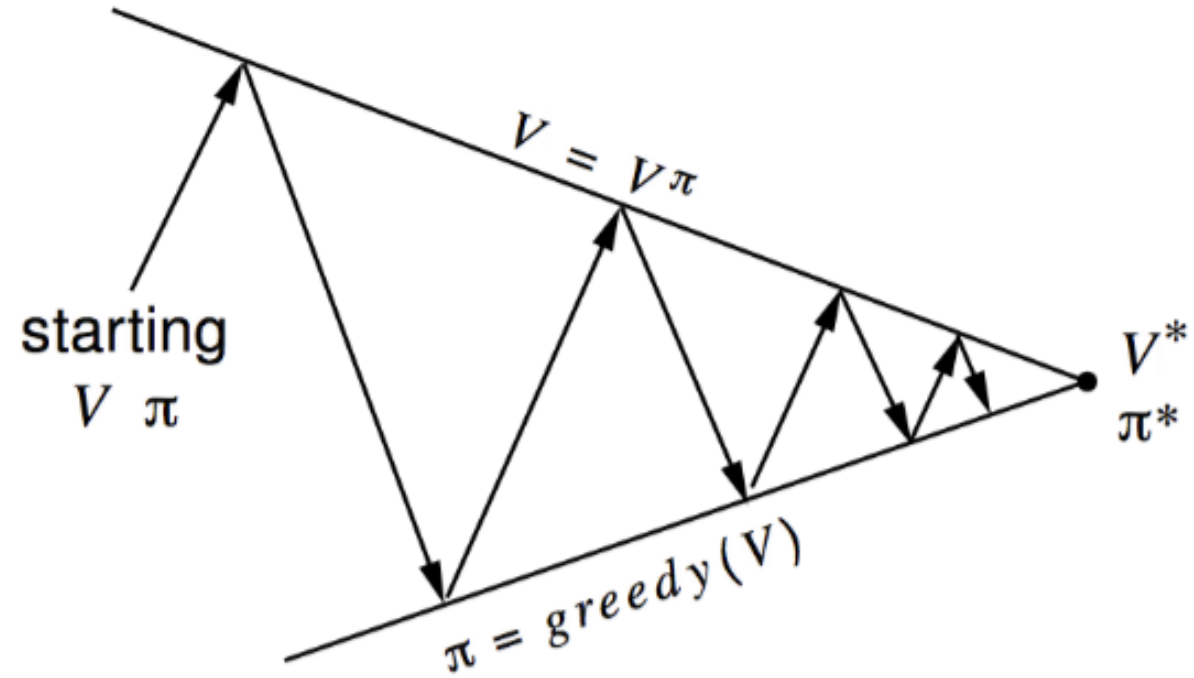


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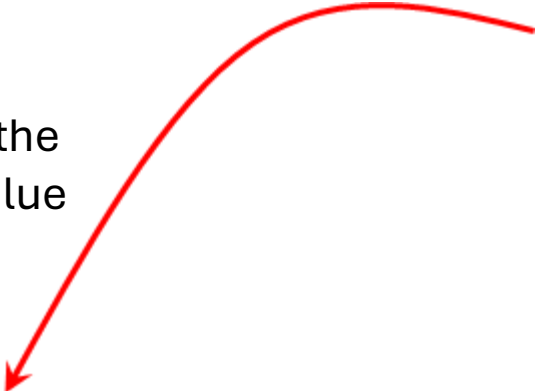
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# Policy Iteration



# Value Iteration

Or by invoking the  
state-action value



## Value Iteration:

Find the optimal policy  $\pi_*$ :

- Given  $V_0(s)$ :
- Repeat:
  - For each  $s \in \mathcal{S}$
  - $V_{k+1}(s) \leftarrow \max_{a \in \mathcal{A}(s)} \sum_{(s',r) \in \mathcal{S} \times \mathcal{R}} p(s',r|s,a)[r + \gamma V_k(s')]$

## Value Iteration:

Find the optimal policy  $\pi_*$ :

- Given  $V_0(s)$ :
- Repeat:
  - For each  $s \in \mathcal{S}$ :
  - For each  $a \in \mathcal{A}(s)$ :
  - $Q(s,a) \leftarrow \sum_{(s',r) \in \mathcal{S} \times \mathcal{R}} p(s',r|s,a)[r + \gamma V_k(s')]$
  - $V_{k+1}(s) \leftarrow \max_{a \in \mathcal{A}(s)} Q(s,a)$

# Tutorial: Tic-Tac-Toe by Value Iteration

## Notes:

- 'x' goes first w.l.o.g.
- For 3x3 game, neither player can lose if they play optimally:
  - Do not train the AI, play dumb and see that it takes dumb move.
  - Do train the AI, play dumb, and lose to it.
  - Do Train the AI, play smart, and never win over it.

## Notes:

- $\mathcal{S} = \{1, -1, 0\}^9$
- $R_t = \begin{cases} 1, & \text{if } s_t \text{ win} \\ -1, & \text{if } s_t \text{ loses} \\ 0, & \text{otherwise} \end{cases}$
- $p(s', r | s, a) = \begin{cases} \frac{1}{|\text{legal}(s')|}, & \text{if } (s', r) \text{ is possible} \\ 0, & \text{otherwise} \end{cases}$
- Transition: ...

O	O	
	X	X
	O	

$s$

O	O	X
	X	X
	O	

$s, a$

O	O	X
X	X	X
	O	

O	O	X
	X	X
X	O	

O	O	X
	X	X
	O	X

$\{s'\}$

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# Monte Carlo Methods

- In real world, most of the time we have imperfect knowledge → estimate.
- Monte Carlo methods are *model-free*



# Monte Carlo Evaluation

- Goal: Given the *data acquired under  $\pi$* , estimate  $q_\pi$ .
- Approach: Express  $q_\pi$ -estimation problem as  $v_\pi$ -estimation problem,
  - Define a new problem where:

$$\bar{S}_t = (S_t, A_t)$$

→ Estimating  $v(\bar{s})$  is equivalent to estimating  $q_\pi(s, a)$ .

- Data =  $\{H_m = (\bar{s}_0, \bar{s}_1, \dots, \bar{s}_{T_m}), m = 1 \dots M\}$ .  
→ *Markov Reward Process*.
-

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→ *Markov Reward Process*.

- 
- Idea: Use averages to approximate  $v_\pi(s) \approx V(s)$ :
    - Batch update:

$$v_\pi(s) = E_\pi(G_t | S_t = s) \approx \frac{1}{C(s)} \sum_{m=1}^M \sum_{\tau=0}^{T_m-1} \mathbb{I}[s_\tau^m = s] g_\tau^m \triangleq V(s)$$

# Monte Carlo Evaluation

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- Iterative update after the  $m$ -th sample:

$$V(s_t^m) \leftarrow V(s_t^m) + \frac{1}{C(s_t^m)} (g_t^m - V(s_t^m))$$

- Or simply use a constant step size:

$$V(s_t^m) \leftarrow V(s_t^m) + \alpha (g_t^m - V(s_t^m))$$

$$\begin{aligned} \mu_k &= \frac{1}{k} \sum_{j=1}^k x_j \\ &= \frac{1}{k} \left( x_k + \sum_{j=1}^{k-1} x_j \right) \\ &= \frac{1}{k} (x_k + (k-1)\mu_{k-1}) \\ &= \mu_{k-1} + \frac{1}{k} (x_k - \mu_{k-1}) \end{aligned}$$

# MC Control

## Constant- $\alpha$ MC for estimating $\pi \approx \pi^*$

Algorithm inputs:

$\epsilon$                        $\alpha$                        $M$

Initialize arbitrarily:

$\pi \leftarrow$  some  $\epsilon$ -soft policy

$Q(s, a) \leftarrow$  some value for  $s \in \mathcal{S}, a \in \mathcal{A}(s)$

For  $m = 1, \dots, M$ :

Under  $\pi$  sample:  $s_0^m, a_0^m, r_1^m \dots a_{T_m-1}^m, r_{T_m}^m$

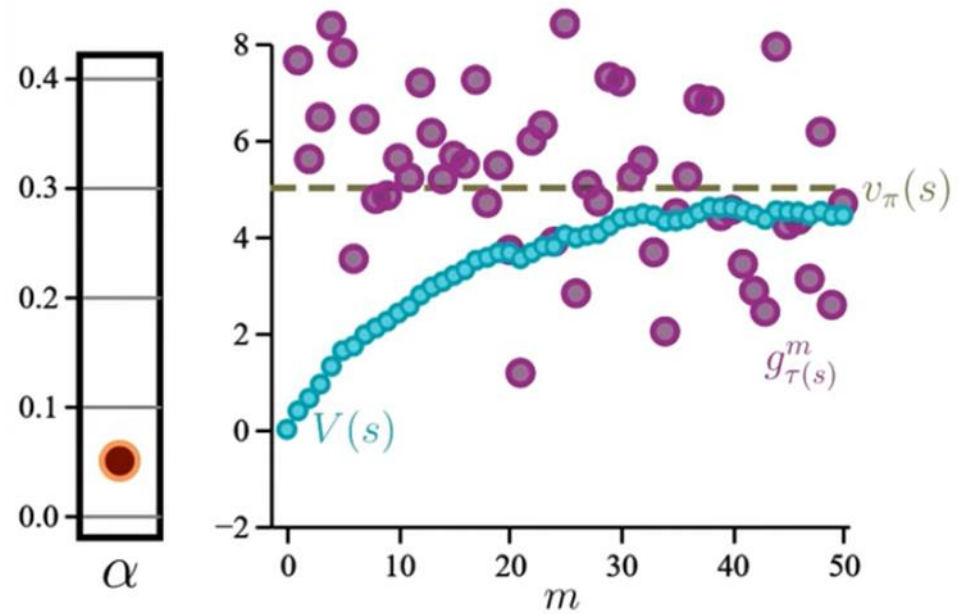
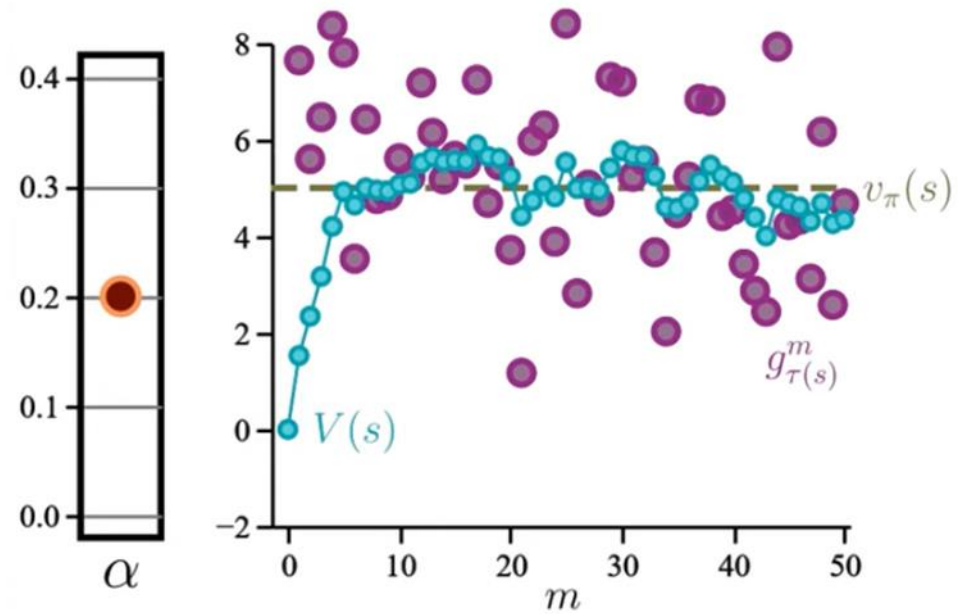
For  $t = 0, \dots, T_m - 1$ :

$g_t^m \leftarrow r_{t+1}^m + \gamma r_{t+2}^m + \dots$

$Q(s_t^m, a_t^m) \leftarrow Q(s_t^m, a_t^m) + \alpha(g_t^m - Q(s_t^m, a_t^m))$

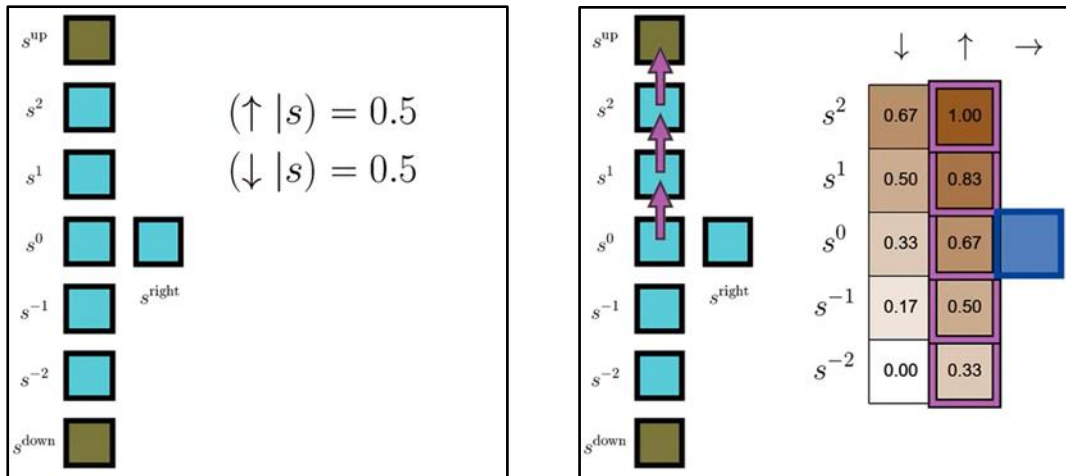
$\pi \leftarrow \epsilon$ -greedy( $Q$ )

# Monte Carlo Evaluation



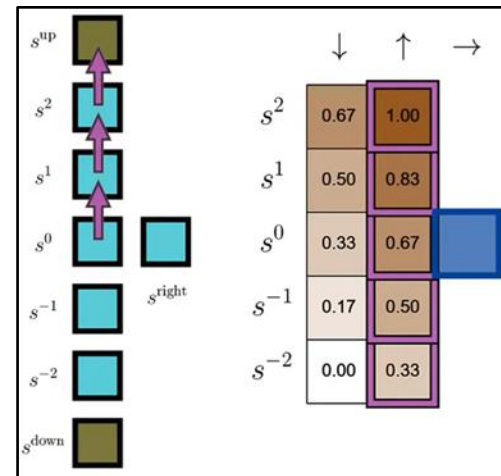
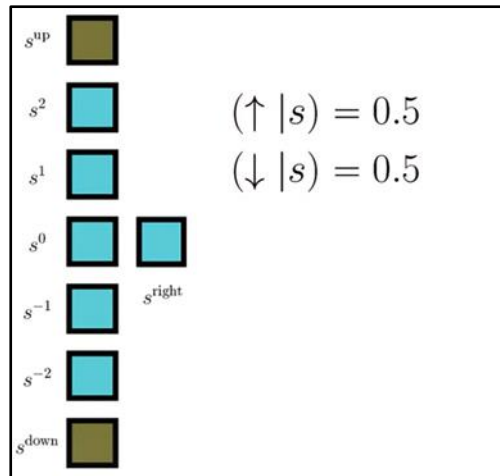
# Caveats of MC

- Trajectories have to terminate
- Exploration-Exploitation *dichotomy*:
  - To discover optimal policies, we must **explore** all state-action pairs.
  - To get high returns we must **exploit** known high state-action pairs.



# Caveats of MC

- Trajectories have to terminate
- Exploration-Exploitation dichotomy:
  - To discover optimal policies, we must **explore** all state-action pairs.
  - To get high returns we must **exploit** known high state-action pairs.



With infinite data,  $\pi_*$  is always discoverable if the policy is **SOFT**:

$$\pi(a|s) > 0 \quad \forall s \in \mathcal{S} \quad \forall a \in \mathcal{A}(s)$$

**$\epsilon$ -GREEDY POLICY OF  $Q$ :** With probability  $\epsilon$ , take an action selected uniformly from  $\mathcal{A}(s)$ , otherwise take  $\text{argmax}_a Q(s, a)$ .

# Off-policy method

- Goal:

Estimate  $q_{\pi}(s, a) = E_{\pi}[G_t | S_t = s, A_t = a]$

- Issue:

Data exists but collected under  $b$

- Remedy:

$$q_{\pi}(s, a) = E_b \left[ \frac{p_{\pi}(G_t)}{p_b(G_t)} G_t | S_t = s, A_t = a \right]$$

$$\rho = \prod_{\tau=t+1}^{T-1} \frac{\pi(A_{\tau} | S_{\tau})}{b(A_{\tau} | S_{\tau})}$$

$$\pi(a, s) > 0 \Rightarrow b(a, s) > 0$$

**BEHAVIOR POLICY:** Generates the data:

$$b(a|s)$$

**TARGET POLICY:** To be improved/evaluated:

$$\pi(a|s)$$

**ON-POLICY  
METHODS**

$$b = \pi$$

**OFF-POLICY  
METHODS**

$$b \neq \pi$$



# Off-policy MC

## Constant- $\alpha$ MC for estimating $\pi \approx \pi^*$

Algorithm inputs:

$\epsilon$                        $\alpha$                        $M$

Initialize arbitrarily:

$\pi \leftarrow$  some  $\epsilon$ -soft policy

$Q(s, a) \leftarrow$  some value for  $s \in \mathcal{S}, a \in \mathcal{A}(s)$

For  $m = 1, \dots, M$ :

Under  $\pi$  sample:  $s_0^m, a_0^m, r_1^m \dots a_{T_m-1}^m, r_{T_m}^m$

For  $t = 0, \dots, T_m - 1$ :

$g_t^m \leftarrow r_{t+1}^m + \gamma r_{t+2}^m + \dots$

$Q(s_t^m, a_t^m) \leftarrow Q(s_t^m, a_t^m) + \alpha(g_t^m - Q(s_t^m, a_t^m))$

$\pi \leftarrow \epsilon$ -greedy( $Q$ )

## Off-Policy Constant- $\alpha$ MC for $\pi \approx \pi^*$

Algorithm inputs:

$b$                        $\alpha \in (0, 1]$                        $M \in \mathbb{N}$

Initialize arbitrarily:

$\pi \leftarrow$  some policy

$Q(s, a) \leftarrow$  some value for  $s \in \mathcal{S}, a \in \mathcal{A}(s)$

For  $m = 1, \dots, M$ :

Under  $b$  sample:  $s_0^m, a_0^m, r_1^m \dots a_{T_m-1}^m, r_{T_m}^m$

For  $t = 0, \dots, T_m - 1$ :

$\rho_t^m \leftarrow \prod_{\tau=t+1}^{T_m-1} \frac{\pi(a_\tau^m | s_\tau^m)}{b(a_\tau^m | s_\tau^m)}$  (or 1 if  $t+1 > T_m - 1$ )

$g_t^m \leftarrow \rho_t^m (r_{t+1}^m + \gamma r_{t+2}^m + \dots)$

$Q(s_t^m, a_t^m) \leftarrow Q(s_t^m, a_t^m) + \alpha(g_t^m - Q(s_t^m, a_t^m))$

$\pi(s_t^m) \leftarrow \operatorname{argmax}_a Q(s_t^m, a)$  (ties broken arbitrarily)

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- Policy Gradient Methods

# Temporal Difference Learning

- *a priori:*


$$q_{\pi}(s, a) = E_{\pi}[\textcolor{red}{G}_t | (S_t, A_t) = (s, a)] = E_{\pi}[\textcolor{red}{R}_{t+1} + \gamma q_{\pi}(\textcolor{red}{S}_{t+1}, \textcolor{red}{A}_{t+1}) | (S_t, A_t) = (s, a)]$$

- *Just read through the maths:*  $Q(s_t, a_t) \approx g_t \approx r_t + \gamma Q(s_{t+1}, a_{t+1})$

- MC approach:

$$g_t^m = r_{t+1}^m + \gamma r_{t+2}^m \dots \gamma^{T_m-1} r_{T_m}^m$$

target


$$Q(s_t^m, a_t^m) \leftarrow Q(s_t^m, a_t^m) + \alpha(g_t^m - Q(s_t^m, a_t^m))$$

# Temporal Difference Learning

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- *Just read through the maths:*  $Q(s_t, a_t) \approx g_t \approx r_t + \gamma Q(s_{t+1}, a_{t+1})$

- MC approach:

$$g_t^m = r_{t+1}^m + \gamma r_{t+2}^m \dots \gamma^{T_m-1} r_{T_m}^m$$

target

$$Q(s_t^m, a_t^m) \leftarrow Q(s_t^m, a_t^m) + \alpha(g_t^m - Q(s_t^m, a_t^m))$$

- 1-step TD approach:

$$\hat{g}_t^m = r_{t+1}^m + \gamma \boxed{Q(s_{t+1}, a_{t+1})}$$

bootstrap

$$Q(s_t^m, a_t^m) \leftarrow Q(s_t^m, a_t^m) + \alpha(\hat{g}_t^m - Q(s_t^m, a_t^m))$$

target

# SARSA

## Constant- $\alpha$ MC for estimating $\pi \approx \pi^*$

Algorithm inputs:

$\epsilon$                        $\alpha$                        $M$

Initialize arbitrarily:

$\pi \leftarrow$  some  $\epsilon$ -soft policy

$Q(s, a) \leftarrow$  some value for  $s \in \mathcal{S}, a \in \mathcal{A}(s)$

For  $m = 1, \dots, M$ :

Under  $\pi$  sample:  $s_0^m, a_0^m, r_1^m \dots a_{T_m-1}^m, r_{T_m}^m$

For  $t = 0, \dots, T_m - 1$ :

$g_t^m \leftarrow r_{t+1}^m + \gamma r_{t+2}^m + \dots$

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$\pi \leftarrow \epsilon$ -greedy( $Q$ )

## On Policy TD Control: n-step SARSA

Changes:

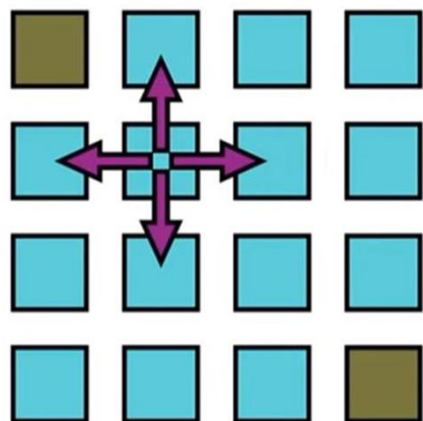
- Approximating the rewards beyond the  $n$ -th step with the current value of  $Q(s, a)$  (bootstrapping):

$$g_{t:t+n}^m = r_{t+1}^m + \dots + \gamma^{n-1} r_{t+n}^m + \gamma^n Q(s_{t+n}^m, a_{t+n}^m)$$

$$Q(s_t^m, a_t^m) \leftarrow Q(s_t^m, a_t^m) + \alpha(g_{t:t+n}^m - Q(s_t^m, a_t^m))$$

- Updates happen *during* the episode, Interweaving between  $(S, A, R)$  tuples, with an  $n$  step delay.
- The policy is updated in similar manner with MC

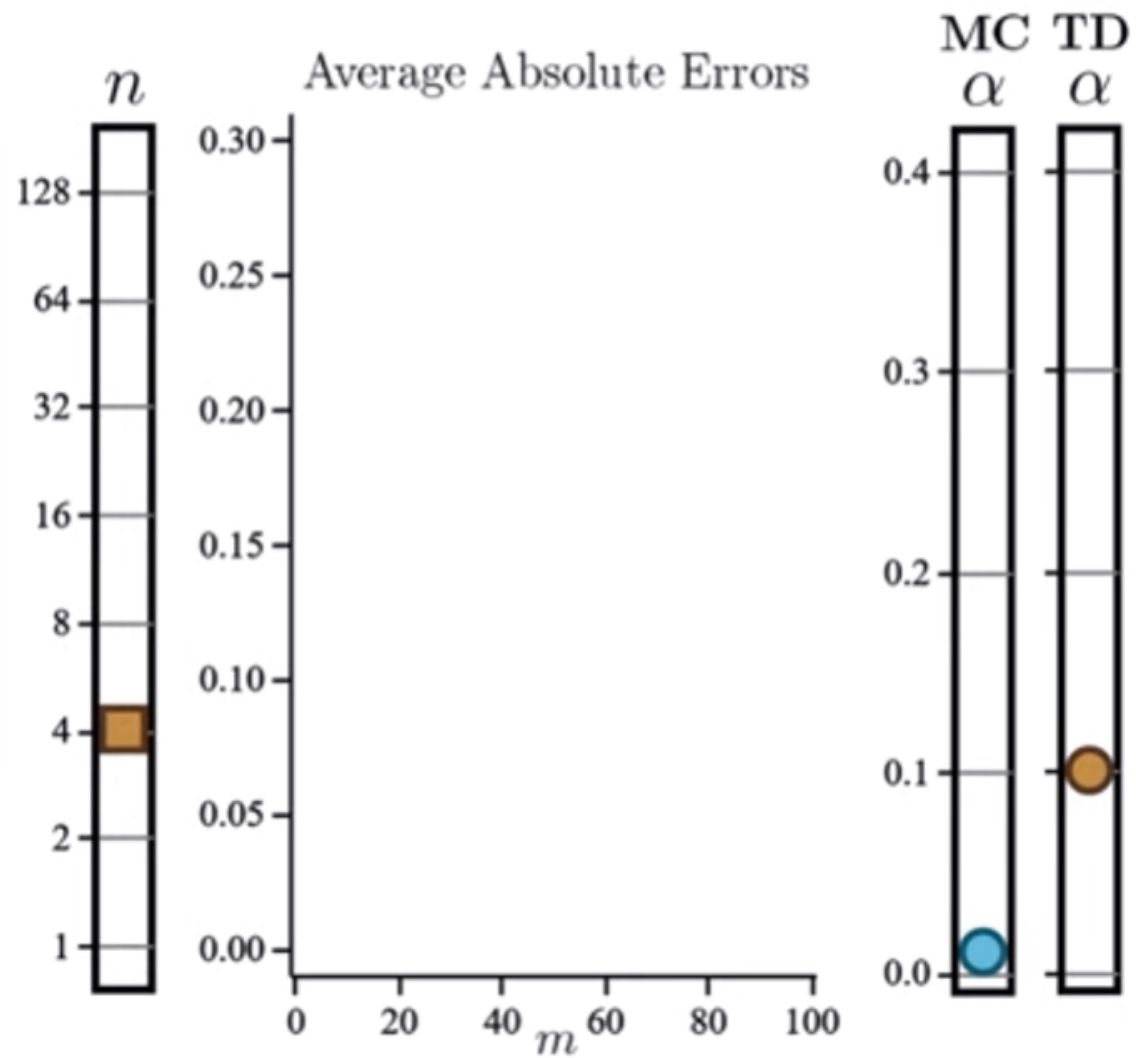
# TD $\ni$ MC



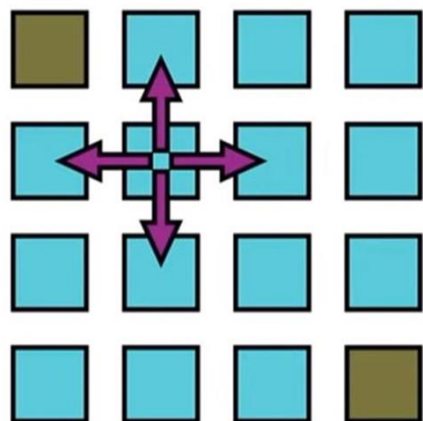
## Evaluation Example: MC vs TD

# States = 11

# Algo Runs = 200



# TD $\ni$ MC



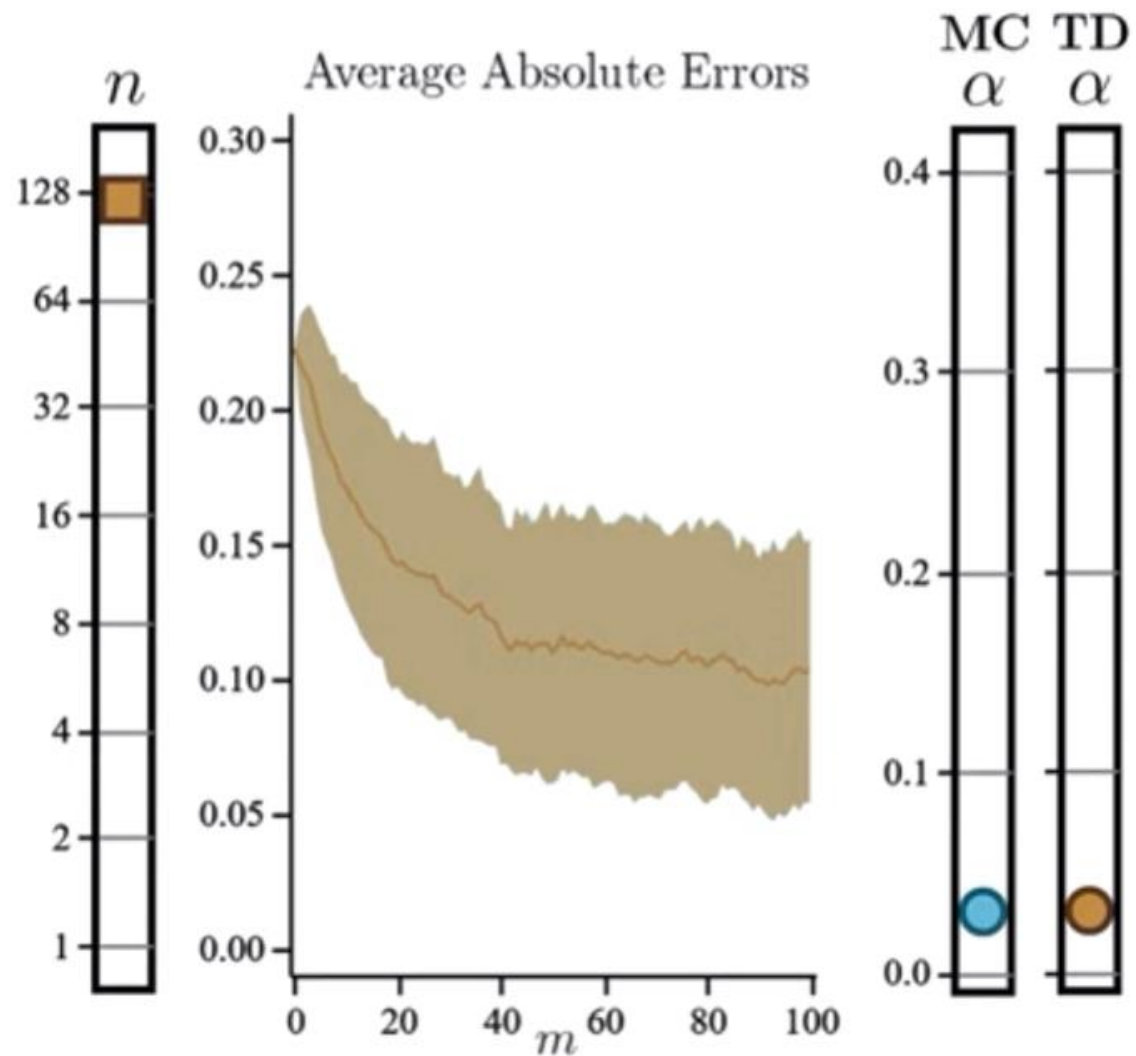
$$R_t = -1$$

$$\pi(a|s) = 0.25$$

## Evaluation Example: MC vs TD

# States = 11

# Algo Runs = 200



# Q-learning

### Constant- $\alpha$ MC for estimating $\pi \approx \pi^*$

Algorithm inputs:

M

Initialize arbitrarily:

$$\pi \leftarrow \text{some } \epsilon\text{-soft policy}$$
$$Q(s, a) \leftarrow \text{some value for } s \in \mathcal{S}, a \in \mathcal{A}(s)$$

For  $m = 1, \dots, M$ :

Under  $\pi$  sample:  $s_0^m, a_0^m, r_1^m \cdots a_{T_m-1}^m, r_{T_m}^m$

For  $t = 0, \dots, T_m - 1$ :

$$g_t^m \leftarrow r_{t+1}^m + \gamma r_{t+2}^m + \dots$$
$$Q(s_t^m, a_t^m) \leftarrow Q(s_t^m, a_t^m) + \alpha(g_t^m - Q(s_t^m, a_t^m))$$
$$\pi \leftarrow \epsilon\text{-greedy}(Q)$$

## Q-Learning

From 1-step **TD** Control, the primary adjustment is to the target:

$$\begin{array}{c} r_{t+1}^m + \gamma Q(s_{t+1}^m, a_{t+1}^m) \\ \downarrow \\ r_{t+1}^m + \gamma \max_a Q(s_{t+1}^m, a) \end{array}$$

The max operator means this is **off-policy**.

Under the behavior policy, we are targeting  $q_*$ .

There's also a change to the update's timing:

1-step **TD**:      update  $Q$     update  $Q$     update  $Q$   
 $s_0^m, a_0^m, r_1^m, s_1^m, a_1^m, r_2^m, s_2^m, a_2^m, r_3^m, s_3^m, a_3^m, r_4^m \dots$   
1-step **Q**:      update  $Q$     update  $Q$     update  $Q$



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- **Function Approximation**
- Policy Gradient Methods

# Function Approximation

- When  $\mathcal{S}$  is continuous  $\rightarrow$  never enough data.

- Example:

- Assume  $v_\pi(s) = \hat{v}(s, w)$ ,  $\hat{v}(s + \delta, w) = \hat{v}(s, w) + \left(\frac{\partial \hat{v}}{\partial s}\right)^\top \delta$ .

- Goal:

$$\min_w \sum_{s \in \{s_i\}} \|v_\pi(s_i) - \hat{v}(s_i, w)\|^2$$

- Update rule:

$$w \leftarrow w + \alpha [g_i - \hat{v}(s_i, w)] \nabla_w \hat{v}(s_i, w)$$

$$\nabla_w \hat{v}(s_i, w) = \frac{\partial \hat{v}}{\partial w}$$

- DRL  $\rightarrow w$  is param of DNN.

# Function Approximation

- Example:
  - Assume  $q_\pi(s) = \hat{q}(s, a, w)$
  - Goal:

$$\min_w \sum_{s \in \{s_i\}} \|q_\pi(s_i, a_i) - \hat{q}(s_i, a_i, w)\|^2$$

- Update rule:

$$w \leftarrow w + \alpha [G_i - \hat{q}(s_i, a_i, w)] \nabla_w \hat{q}(s_i, a_i, w)$$
$$\nabla_w \hat{q}(s_i, a_i, w) = \frac{\partial \hat{q}}{\partial w}$$

# Function Approximation

- What about the policy function?

In the context of RL...

- Agent, environment, observations, state, reward, action, value, return, discount ...
- Evaluation, Iteration, Improvement, Value Iteration ...
- Monte Carlo, Off-policy
- Temporal Difference, Q-learning, Sarsa
- Function Approximation
- Policy Gradient Methods

# Policy Gradient Methods

## REINFORCE

To specify upfront:

- Functional form:  $\pi(a|s, \boldsymbol{\theta})$
- Initial  $\boldsymbol{\theta}$
- Step size  $\alpha$

For  $m = 1, \dots, M$ :

Sample:  $s_0^m, a_0^m, r_1^m \dots a_{T_m-1}^m, r_{T_m}^m$

For  $t = 0, \dots, T_m - 1$ :

$$g_t^m \leftarrow r_{t+1}^m + \gamma r_{t+2}^m + \dots$$

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \gamma^t g_t^m \nabla \ln \pi(a_t^m | s_t^m, \boldsymbol{\theta})$$

- $\nabla_{\boldsymbol{\theta}} \ln \pi(a_t | s_t, \boldsymbol{\theta})$  gives the “direction” that increasing  $\boldsymbol{\theta}$  will increase  $\pi(a_t | s_t, \boldsymbol{\theta})$ .

$$\nabla \ln \pi(a_t^m | s_t^m, \boldsymbol{\theta}) = \frac{\nabla \pi(a_t^m | s_t^m, \boldsymbol{\theta})}{\pi(a_t^m | s_t^m, \boldsymbol{\theta})}$$

# Policy Gradient Methods

## REINFORCE

To specify upfront:

- Functional form:  $\pi(a|s, \theta)$
- Initial  $\theta$
- Step size  $\alpha$

For  $m = 1, \dots, M$ :

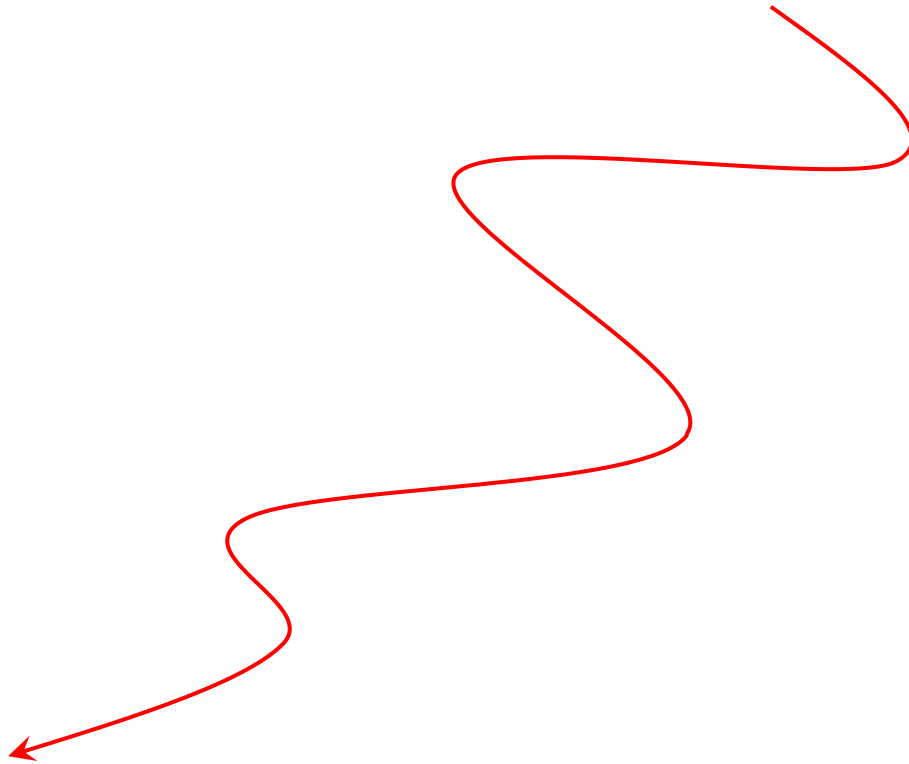
Sample:  $s_0^m, a_0^m, r_1^m \dots a_{T_m-1}^m, r_{T_m}^m$

For  $t = 0, \dots, T_m - 1$ :

$$g_t^m \leftarrow r_{t+1}^m + \gamma r_{t+2}^m + \dots$$

$$\theta \leftarrow \theta + \alpha \gamma^t g_t^m \nabla \ln \pi(a_t^m | s_t^m, \theta)$$

- $\nabla_{\theta} \ln \pi(a_t | s_t, \theta)$  gives the “direction” that increasing  $\theta$  will increase  $\pi(a_t | s_t, \theta)$ .
- The increase of  $\theta$  is  $\sim g_t \nabla_{\theta} \ln \pi(a_t | s_t, \theta)$



# Policy Gradient Methods

## REINFORCE

To specify upfront:

- Functional form:  $\pi(a|s, \theta)$
- Initial  $\theta$
- Step size  $\alpha$

For  $m = 1, \dots, M$ :

Sample:  $s_0^m, a_0^m, r_1^m \dots a_{T_m-1}^m, r_{T_m}^m$

For  $t = 0, \dots, T_m - 1$ :

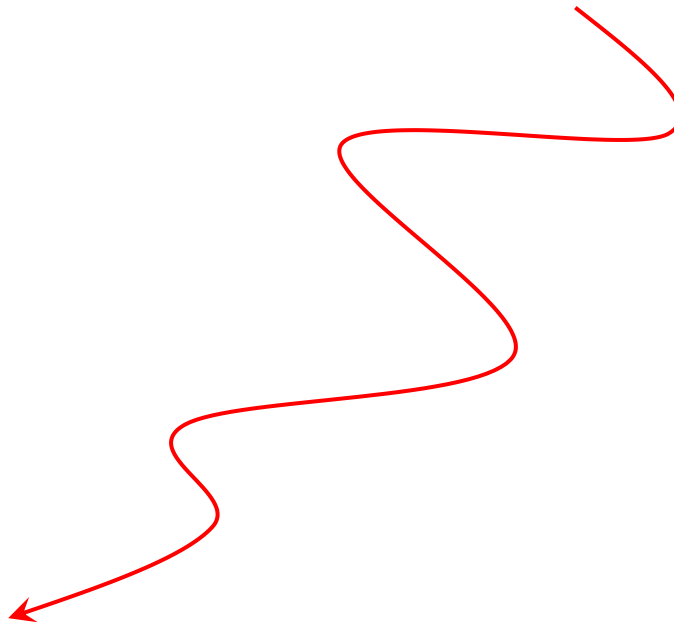
$$g_t^m \leftarrow r_{t+1}^m + \gamma r_{t+2}^m + \dots$$

$$\theta \leftarrow \theta + \alpha \gamma^t g_t^m \nabla \ln \pi(a_t^m | s_t^m, \theta)$$

- $\nabla_{\theta} \ln \pi(a_t | s_t, \theta)$  gives the “direction” that increasing  $\theta$  will increase  $\pi(a_t | s_t, \theta)$ .

- The increase of  $\theta$  is  $\sim g_t \nabla_{\theta} \ln \pi(a_t | s_t, \theta)$

→ the higher the return  $g_t$  an action  $a_t$  yields, the higher the probability of an action is *increased*.





# Policy Gradient Methods

- **Actor-Critic Methods** combine elements of policy-based methods and value-based methods.

- It introduces an advantage function

$$A(s_i, a_i) = Q(s, a) - V(s)$$

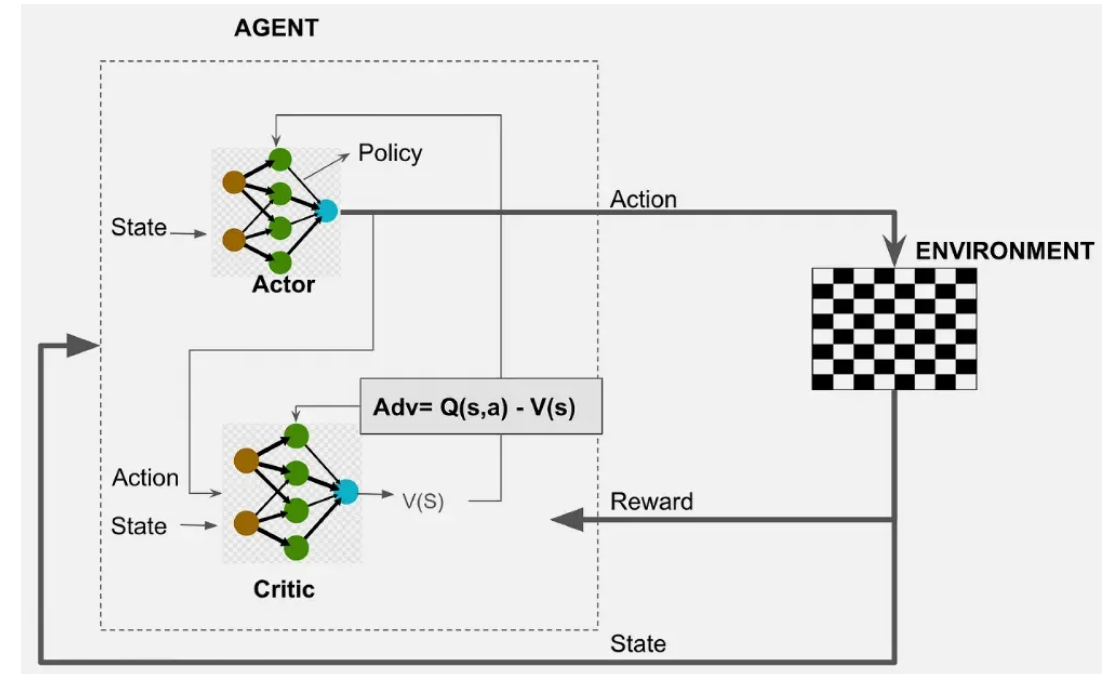
→ provides a measure of how “good” and action is compared with the average action.

- “Actor” gradient:

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_i \nabla_{\theta} \ln(\pi(a_i | s_i, \theta)) A(s_i, a_i)$$

- “Critic” gradient:

$$\nabla_w J(w) \approx \frac{1}{N} \sum_i \nabla_w A(s_i, a_i)^2$$

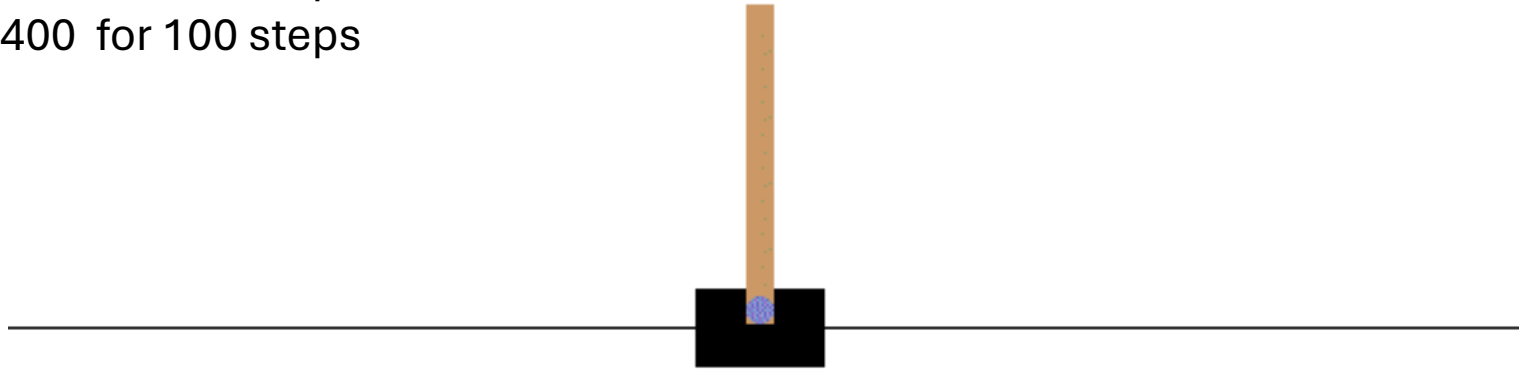


# Tutorial: CartPole by REINFORCE method

[https://www.gymnasium.dev/environments/classic\\_control/cart\\_pole/](https://www.gymnasium.dev/environments/classic_control/cart_pole/)

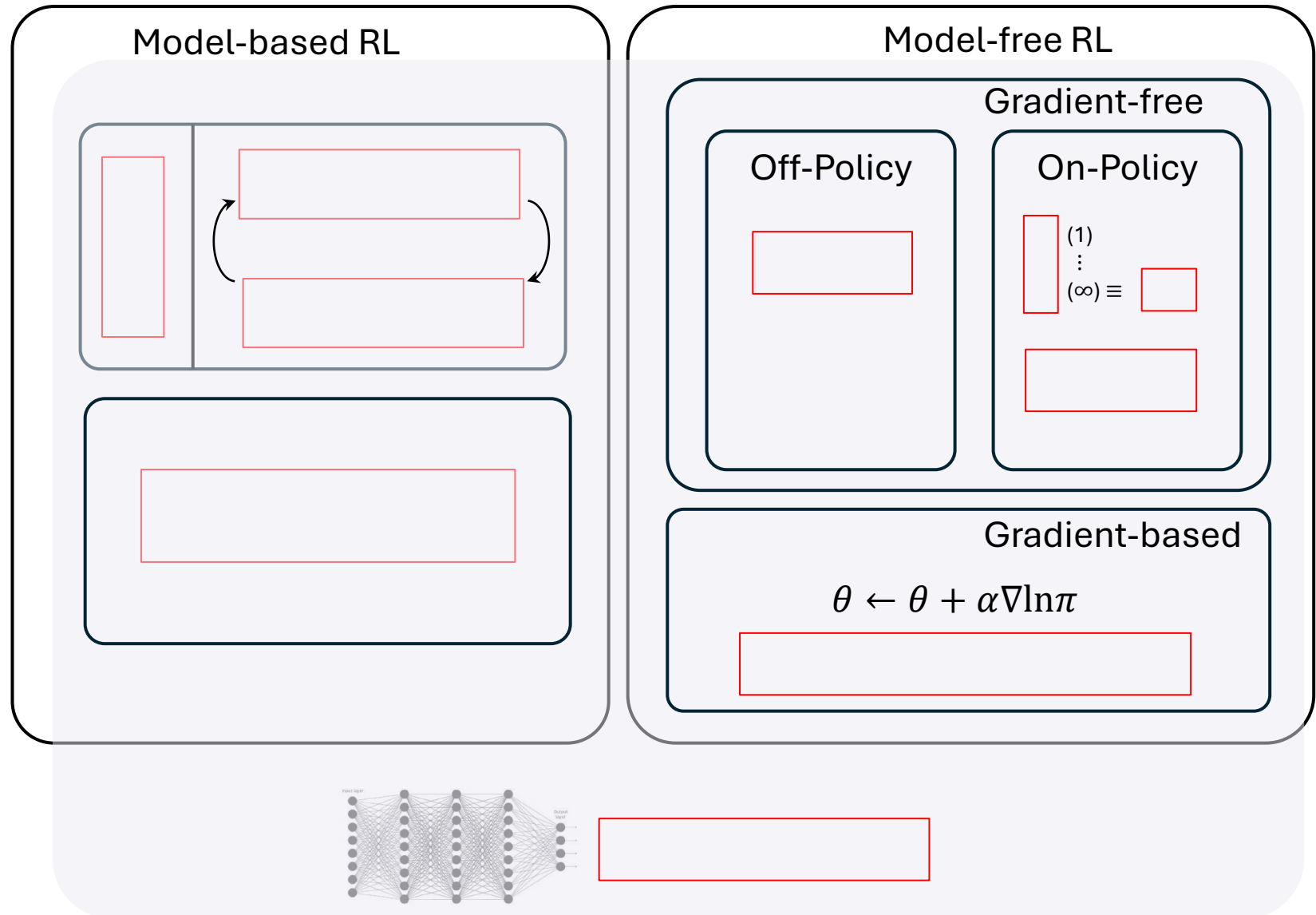
<https://tinyurl.com/tmn2025DRLCartpole>

- End when score = 050 for 100 steps
- End when score = 100 for 100 steps
- End when score = 200 for 100 steps
- End when score = 400 for 100 steps



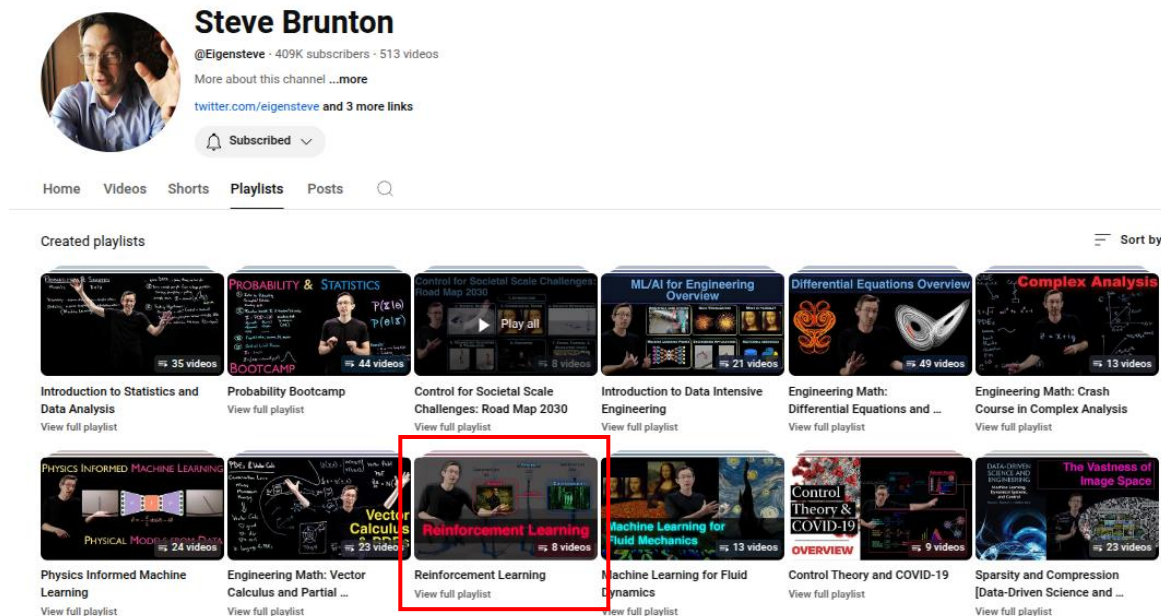
# Summary

- Value Evaluation, Policy Iteration, Policy Improvement, Value Iteration...
- MC
- TD, Q-learning, Sarsa
- Function Approximation (Deep RL)
- Policy Gradient Methods



# References

- [Reinforcement Learning: An Introduction, Sutton](#)
- [David Silver RL Lectures](#)
- [Zhao Shiyu RL Lectures](#)
- [OpenAI Introduction to RL](#)



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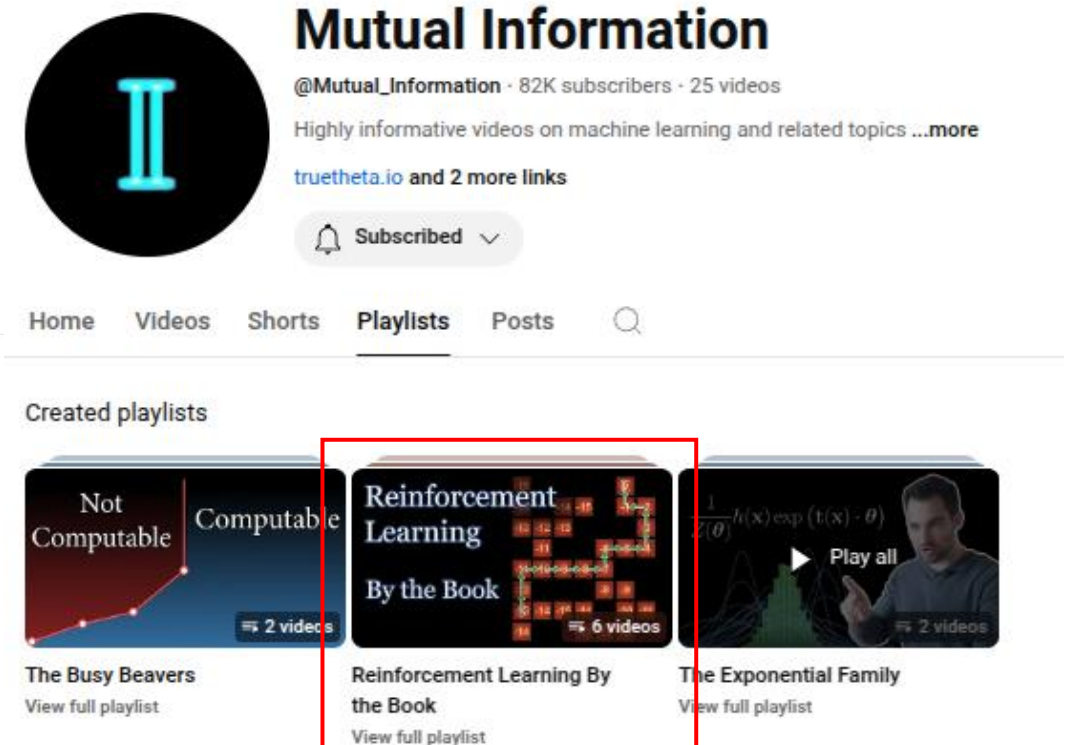
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