Extra Practice Questions – Answer

1(a)

$$\lim_{(x,y)\to(0,0)} x^4\sin(\frac{1}{x^2+y})$$

Since -1 $\leq \sin\theta \leq 1$, -1 $\leq \sin(\frac{1}{x^2+y}) \leq 1$. Thus,

$$-x^4 \leq x^4\sin(\frac{1}{x^2+y}) \leq x^4$$

$$\lim_{(x,y)\to(0,0)} -x^4 = \lim_{(x,y)\to(0,0)} x^4 = 0$$

$0 = \lim_{(x,y)\to(0,0)} -x^4 \leq \lim_{(x,y)\to(0,0)} x^4\sin(\frac{1}{x^2+y}) \leq \lim_{(x,y)\to(0,0)} x^4$=0

Thus, by sandwich theorem, $\lim_{(x,y)\to(0,0)} x^4\sin(\frac{1}{x^2+y}) = 0$

1b)

1(m), exp, cos and x are continuous function, thus,

$$\lim_{(x,y,x)\to(0,\sqrt{\pi},1)} e^{xz}\cos y^2 - x = e^0\cos\pi - 0 = -1$$

1(L) $\lim_{(x,y)\to(0,0),x\neq y} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}} = \lim_{(x,y)\to(0,0),x\neq y} \frac{x(x-y)}{\sqrt{x}-\sqrt{y}} = \lim_{(x,y)\to(0,0),x\neq y} \frac{x(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}{\sqrt{x}-\sqrt{y}}$

= $\lim_{(x,y)\to(0,0),x\neq y} x(\sqrt{x}+\sqrt{y}) = 0$

2(L) $f(x,y,z) = \left(\frac{x+y+z}{(1+x^2+y^2+y^2)^{3/2}}\right)$

$\frac{\partial f}{\partial x} = \frac{-2x^2-3xy-3xz+y^2+z^2+1}{(1+x^2+y^2+y^2)^{5/2}}$

$\frac{\partial f}{\partial y} = \frac{-2y^2-3xy-3xz+x^2+z^2+1}{(1+x^2+y^2+y^2)^{5/2}}$

$\frac{\partial f}{\partial z} = \frac{-2z^2-3xz-3yz+x^2+y^2+1}{(1+x^2+y^2+y^2)^{5/2}}$

https://www.symbolab.com/solver/partial-derivative-calculator/%5Cfrac%7B%5Cpartial%7D%7B%5Cpartial%20x%7D%5Cleft(%5Cfrac%7Bx%2By%2Bz%7D%7B%5Cleft(1%2Bx%5E%7B2%7D%2By%5E%7B2%7D%2Bz%5E%7B2%7D%5Cright)%5E%7B%5Cfrac%7B3%7D%7B2%7D%7D%7D%5Cright)?or=sug

2(m) $f(\mathrm{x,y,z}) = \left(\frac{\mathrm{x}^3+\mathrm{yz}}{\mathrm{x}^2+\mathrm{z}^2+1}\right)$

$$\frac{\partial f}{\partial x} = \frac{\mathrm{x}^4 + 3\mathrm{x}^2\mathrm{z}^2 + 3\mathrm{x}^2 - 2\mathrm{xyz}}{(\mathrm{x}^2 + \mathrm{z}^2 + 1)^2}$$

$$\frac{\partial f}{\partial \mathrm{y}} = \frac{\mathrm{z}}{\mathrm{x}^2 + \mathrm{z}^2 + 1}$$

$$\frac{\partial f}{\partial \mathrm{z}} = \frac{\mathrm{x}^2\mathrm{y} - \mathrm{yz}^2 + \mathrm{y} - 2\mathrm{x}^3\mathrm{z}}{(\mathrm{x}^2 + \mathrm{z}^2 + 1)^2}$$

https://www.symbolab.com/solver/partial-derivative-calculator/%5Cfrac%7B%5Cpartial%7D%7B%5Cpartial%20z%7D%5Cleft(%5Cfrac%7Bx%5E%7B3%7D%2Byz%7D%7Bx%5E%7B2%7D%2Bz%5E%7B2%7D%2B1%7D%5Cright)

3c) $f(\mathrm{x,y}) = \frac{1}{\sin^2\mathrm{x}+2\mathrm{e}^\mathrm{y}}$=$(\sin^2\mathrm{x} + 2\mathrm{e}^\mathrm{y})^{-1}$

$$f_\mathrm{x} = -(\sin^2 x + 2\mathrm{e}^\mathrm{y})^{-2} \times 2sin(x) \times cos(x)$$

$$f_\mathrm{x} = -(\sin^2 x + 2\mathrm{e}^\mathrm{y})^{-2} \times \sin(2\mathrm{x})$$

$$f_\mathrm{xx} = 2(\sin^2 x + 2\mathrm{e}^\mathrm{y})^{-3} \times \sin(2\mathrm{x}) \times 2sin(x) \times cos(x) - (\sin^2 x + 2\mathrm{e}^\mathrm{y})^{-2} \times 2\cos(2\mathrm{x})$$

$$f_\mathrm{xx} = 2(\sin^2 x + 2\mathrm{e}^\mathrm{y})^{-3} \times \sin^2(2\mathrm{x}) - (\sin^2 x + 2\mathrm{e}^\mathrm{y})^{-2} \times 2\cos(2\mathrm{x})$$

$$f_\mathrm{xx} = \frac{2(\sin^2(2\mathrm{x}) - \cos(2\mathrm{x}) \times (\sin^2 x + 2\mathrm{e}^\mathrm{y}))}{(\sin^2 x + 2\mathrm{e}^\mathrm{y})^3}$$

$$f_\mathrm{xy} = 2(\sin^2 x + 2\mathrm{e}^\mathrm{y})^{-3} \times \sin(2\mathrm{x}) \times 2\mathrm{e}^\mathrm{y}$$

$$f_\mathrm{xy} = f_\mathrm{yx} = \frac{4 \times \sin(2\mathrm{x}) \times \mathrm{e}^\mathrm{y}}{(\sin^2 x + 2\mathrm{e}^\mathrm{y})^3}$$

$$f_\mathrm{y} = -(\sin^2\mathrm{x} + 2\mathrm{e}^\mathrm{y})^{-2} \times 2\mathrm{e}^\mathrm{y}$$

$$f_\mathrm{yy} = 2(\sin^2\mathrm{x} + 2\mathrm{e}^\mathrm{y})^{-3} \times 2\mathrm{e}^\mathrm{y} \times 2\mathrm{e}^\mathrm{y} - (\sin^2\mathrm{x} + 2\mathrm{e}^\mathrm{y})^{-2} \times 2\mathrm{e}^\mathrm{y}$$

$$f_\mathrm{yy} = \frac{8\mathrm{e}^{2\mathrm{y}} - 2(\sin^2\mathrm{x} + 2\mathrm{e}^\mathrm{y})\mathrm{e}^\mathrm{y}}{(\sin^2\mathrm{x} + 2\mathrm{e}^\mathrm{y})^3}$$

$$f_\mathrm{yy} = \frac{4\mathrm{e}^{2\mathrm{y}} - 2\mathrm{e}^\mathrm{y}\sin^2\mathrm{x}}{(\sin^2\mathrm{x} + 2\mathrm{e}^\mathrm{y})^3}$$