

# AI6104 - MATHEMATICS FOR AI

## EXTRA PRACTICE QUESTIONS - LIMITS & DERIVATIVES

### Problem 1

Evaluate the following limits

(a)  $\lim_{(x,y) \rightarrow (0,0)} x^4 \sin \left( \frac{1}{x^2 + |y|} \right)$

(b)  $\lim_{(x,y) \rightarrow (0,0)} e^{x+y^2}$

(c)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$

(d)  $\lim_{(x,y) \rightarrow (-1,0)} \frac{x^2 + xy + 3}{x^2y - 5xy + y^2 + 1}$

(e)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^4}{x^2 + y^4}$

(f)  $\lim_{(x,y) \rightarrow (0,0)} \frac{e^x e^y}{x + y + 2}$

(g)  $\lim_{(x,y) \rightarrow (2,0)} \frac{x^2 - y^2 - 4x + 4}{x^2 + y^2 - 4x + 4}$

(h)  $\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^2}{x^2 + y^2}$

(i)  $\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2 + y^2}{x^2 + y^2}$

(j)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 2xy + y^2}{x + y}$

(k)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^4}{x^2 + y^2}$

(l)  $\lim_{(x,y) \rightarrow (0,0), x \neq y} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

(m)  $\lim_{(x,y,z) \rightarrow (0, \sqrt{\pi}, 1)} e^{xz} \cos y^2 - x$

(n)  $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy - xz + yz}{x^2 + y^2 + z^2}$

(o)  $\lim_{(x,y,z) \rightarrow (0,0,0)} x^2 + 2xy + yz + z^3 + 2$

### Problem 2

Calculate the partial derivatives with respect to all variables in following functions

(a)  $f(x, y) = xy^2 + x^2y$

(b)  $f(x, y) = \sin xy + \cos xy$

- (c)  $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$
- (d)  $f(x, y) = \cos x^3 y$
- (e)  $f(x, y) = x e^y + y \sin(x^2 + y)$
- (f)  $f(x, y) = e^{x^2 + y^2}$
- (g)  $f(x, y) = \frac{x^3 - y^2}{1 + x^2 + 3y^4}$
- (h)  $f(x, y) = \ln(x^2 + y^2)$
- (i)  $f(x, y) = \ln\left(\frac{x}{y}\right)$
- (j)  $f(x, y, z) = \frac{x - y}{y + z}$
- (k)  $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$
- (l)  $f(x, y, z) = \frac{x + y + z}{(1 + x^2 + y^2 + z^2)^{3/2}}$
- (m)  $f(x, y, z) = \frac{x^3 + yz}{x^2 + z^2 + 1}$

### Problem 3

Find the second-order partial derivatives for the following functions

- (a)  $f(x, y) = x^3 y^7 + 3xy^2 - 7xy$
- (b)  $f(x, y) = e^{y/x} - ye^{-x}$
- (c)  $f(x, y) = 1/(\sin^2 x + 2e^y)$
- (d)  $f(x, y) = y \sin x - x \cos y$
- (e)  $f(x, y) = x^2 e^y + e^{2z}$
- (f)  $f(x, y, z) = x^2 yz + xy^2 z + xyz^2$
- (g)  $f(x, y, z) = e^{ax} \sin y + e^{bx} \cos z$

### Problem 4

Find the gradient  $\nabla f$  of the following functions

- (a)  $f(x, y) = x^2 y + e^{y/x}$
- (b)  $f(x, y) = \frac{x-y}{x^2+y^2+1}$
- (c)  $f(x, y) = e^{xy} + \ln(x - y)$
- (d)  $f(x, y, z) = \sin xyz$
- (e)  $f(x, y, z) = xy + y \cos z - x \sin yz$
- (f)  $f(x, y, z) = \frac{x+y}{e^z}$
- (g)  $f(x, y, z) = \cos z \ln(x + y^2)$
- (h)  $f(x, y, z) = \frac{xy^2 - x^2 z}{y^2 + z^2 + 1}$

### Problem 5

Find the matrix  $Df$  of partial derivatives for the following functions

- (a)  $f(x, y) = \frac{x}{y}$
- (b)  $f(x, y, z) = x^2 + x \ln(yz)$

- (c)  $f(x, y, z) = (2x - 3y + 5z, x^2 + y, \ln(yz))$
- (d)  $f(x, y, z) = \left(xyz, \sqrt{x^2 + y^2 + z^2}\right)$
- (e)  $f(t) = (t, \cos 2t, \sin 5t)$
- (f)  $f(x, y, z, w) = (3x - 7y + z, 5x + 2z - 8w, y - 17z + 3w)$
- (g)  $f(x, y) = (x^2y, x + y^2, \cos \pi xy)$
- (h)  $f(s, t) = (s^2, st, t^2)$

**Problem 6**

- (a) Let  $f(x, y) = ye^{3x}$ . Give general formulas for  $\partial^n f / \partial x^n$  and  $\partial^n f / \partial y^n$ , where  $n \geq 2$ .
- (b) Let  $f(x, y, z) = \ln\left(\frac{xy}{z}\right)$ . Give general formulas for  $\partial^n f / \partial x^n$ ,  $\partial^n f / \partial y^n$  and  $\partial^n f / \partial z^n$ , where  $n \geq 1$ .