AI6104 - Mathematics for AI

Additional exercises

1. Compute the rank, nullity and determinant of the following matrixes.

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Ans: (2, 1, 0), (rank, nullity and determinant)

(b) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -1 \\ 2 & 3 & 1 \end{bmatrix}$

Ans: (3, 0, -8), (rank, nullity and determinant)

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

(d) Ans: (3, 0, 4), (rank, nullity and determinant)

2. Solve the following linear systems

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

Ans: x=-1.5, y=1 and 0.5

(b) $\begin{bmatrix} 2 & -1 & 0 \\ 3 & 0 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

Ans: x=-1/3, y=1/3 and z=-2/3

3) Compute the inverse of the following matrixes

(a) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 8 & 4 & 2 \end{bmatrix}$

Ans: $\begin{bmatrix} 1 & 4 & 0.5 \\ -3 & -11 & -1 \\ 2 & 6 & 0.5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 4 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

(c) Ans: $\begin{bmatrix} 0.2 & -2.2 & 0.4 & -0.8 \\ -0.4 & 0.4 & 0.2 & 1.6 \\ 0 & 1 & 0 & 0 \\ 0.4 & -0.4 & -0.2 & -0.6 \end{bmatrix}$

4) Compute the eigenvectors and eigenvalues of the following matrix

(a) $\begin{bmatrix} 0 & -2 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Ans: eigenvalues (1, 0, 0) and eigenvectors $\begin{bmatrix} 0 & 0.7071 & 0.7071 \\ 0 & 0 & 0 \\ 1 & -0.7071 & -0.7071 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & -2 \end{bmatrix}$

Ans: eigenvalues (-2, 3, 2) and eigenvectors $\begin{bmatrix} 0 & 0.9806 & 0 \\ 0 & 0 & 1 \\ 1 & 0.1961 & 0 \end{bmatrix}$

5. Compute

(a) $\begin{bmatrix} 0 & -2 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}^{10} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Ans: $\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{9} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Ans: $\begin{bmatrix} 512 \\ -1 \\ 1 \end{bmatrix}$

6. Determine whether the following matrixes diagonalizable.

(a) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

Ans: yes diagonalizable

(b) $\begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$

Ans: yes diagonalizable