AI6104 - Mathematics for AI

Tutorial 4 - Optimization

Problem 1

Find the critical points of each of the following functions

(a) $f(x, y) = \sqrt{4y^2 - 9x^2 + 24y + 36x + 36}$

(b) $g(x, y) = x^2 + 2xy - 4y^2 + 4x - 6y + 4$

(c) $h(x, y) = x^3 + 2xy - 2x - 4y$

(d) $f(x, y, z) = x + \dfrac{y^2}{4x} + \dfrac{z^2}{y} + \dfrac{2}{z}$ where $x, y, z > 0$

Problem 2

Find and classify the critical points of the function

$$f(x, y) = 10x^2 y - 5x^2 - 4y^2 - x^4 - 2y^4$$

Problem 3

Find the absolute maximum and minimum values of the following functions in the specified domains

(a) $f(x, y) = x^2 - 2xy + 4y^2 - 4x - 2y + 24$ on the domain defined by $0 \le x \le 4$ and $0 \le y \le 2$

(b) $g(x, y) = x^2 + y^2 + 4x - 6y$ on the domain defined by $x^2 + y^2 \le 16$