Answer 6.1

From lecture note, if $C = A*B$, $c_{ij} = \sum_{k=1}^{n} a_{ik} * b_{kj}$.

Now, we consider $A^{-1}*A = I$. Thus, we know, $\sum_{k=1}^{n} (a^{-1})_{ik} * a_{kj} = 0$ if $i \neq j$ and $\sum_{k=1}^{n} (a^{-1})_{ik} * a_{kj} = 1$ if $i = j$, where $(a^{-1})_{ik}$ is the (i,k) element in A^{-1}, not $1/(a_{ik})$

Let $(a^{-1})_{i\cdot}$ be the i-th row vector of A^{-1} and $a_{\cdot j}$ be the j-the column vector of A.

Thus, $\sum_{k=1}^{n} (a^{-1})_{ik} * a_{kj} = (a^{-1})_{i\cdot} \circ a_{\cdot j}$, where ° represents dot product.

$$\frac{\partial (a^{-1})_{i\cdot} \circ a_{\cdot j}}{\partial \alpha} = (a^{-1})_{i\cdot} \circ \frac{\partial a_{\cdot j}}{\partial \alpha} + \frac{\partial (a^{-1})_{i\cdot}}{\partial \alpha} \circ a_{\cdot j}$$

(page 48 in the note)

Put them in matrix form, we have

$$\frac{\partial A^{-1} A}{\partial \alpha} = A^{-1} \frac{\partial A}{\partial \alpha} + \frac{\partial A^{-1}}{\partial \alpha} A = \frac{\partial I}{\partial \alpha} = 0$$

Simplifying it, we have

$$\frac{\partial A^{-1}}{\partial \alpha} = -A^{-1} \frac{\partial A}{\partial \alpha} A^{-1}$$