AI6104 - Mathematics for AI

Systems of Equations

Problem 1: Solve the following linear systems of equations

(i) $$\begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 1 \\ 4 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 4 \end{bmatrix}$$

(ii) $$\begin{bmatrix} 1 & -3 & -2 \\ -1 & 2 & 1 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(iii) $$\begin{bmatrix} 2 & 1 \\ 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

Problem 2: Determine if the vector **v** is a linear combination of the remaining vectors.

(i) $$v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, u_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

(ii) $$v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Problem 3:

(a) Suppose that vector **w** is a linear combination of vector $u_1, \cdots u_k$ and that each u_i is a linear combination of vectors $v_1, \cdots v_m$. Prove that **w** is a linear combination of $v_1, \cdots v_m$ and therefore $span(u_1, \cdots u_k) \subseteq span(v_1, \cdots v_m)$.

(b) In part (a), suppose in additional that each v_i is also linear combination of $u_1, \cdots u_k$. Prove that $span(u_1, \cdots u_k) = span(v_1, \cdots v_m)$.

(c) Use the results of part (b) to prove that

$$\Re^3 = span\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right)$$

Problem 4:

(a) If the columns of an $n \times n$ matrix **A** are linearly independent as vectors in $\Re^n$, what is the rank of **A**? Explain.

(b) If the rows of an $n \times n$ matrix **A** are linearly independent as vectors in $\Re^n$, what is the rank of **A**? Explain.

Problem 5: Show that $\Re^3 = span\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right)$