

**Problem 1**

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^2}{x^2+y^2}$

*Solution:*

Consider two lines where  $y = 0$  and  $y = x$ . When  $y = 0$ , the above limit becomes

$$\lim_{x \rightarrow 0} \frac{x^2}{x^2} = 1$$

and when  $y = x$ , the limits becomes

$$\lim_{x \rightarrow 0} \frac{4x^2}{2x^2} = 2$$

Since  $1 \neq 2$ , the limit does not exist.

(b)  $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy - xz + yz}{x^2 + y^2 + z^2}$

*Solution:*

Consider  $y = 0, z = 0$  and  $y = x, z = 0$ . When  $y = 0, z = x$ , we have

$$\lim_{x \rightarrow 0} \frac{-x^2}{2x^2} = -\frac{1}{2}$$

while when  $y = x, z = 0$ , we have

$$\lim_{x \rightarrow 0} \frac{x^2}{2x^2} = \frac{1}{2}$$

Since  $\frac{1}{2} \neq -\frac{1}{2}$ , the above limit does not exist.