AI6104 - Mathematics for AI

Tutorial 5 - Lagrange Multipliers, Jacobians

Problem 1

Use the method of Lagrange multipliers to find the minimum value of $f(x, y) = x^2 + 4y^2 - 2x + 8y$, subject to the constraint $x + 2y = 7$.

Problem 2

Find the shortest distance from the origin to the curve $x^2y = 16$.

Problem 3

Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.

Problem 4

Find the maximum and minimum values of $f(x, y, z) = xy + 2z$ on the intersection of the plane $x + y + z = 0$ and the sphere $x^2 + y^2 + z^2 = 24$.

Problem 5

Calculate the Jacobian Matrix of the following functions

(a) $f(x, y) = (x \sin(xy), \arctan(x + y))$

(b) $g(x, y) = \left(xy^2, \sin(x^2 - y), e^{x-2y}\right)$