NANYANG TECHNOLOGICAL UNIVERSITY

AI6104 – MATHEMATICS FOR ARTIFICIAL INTELLIGENCE

ASSIGNMENT 2

ACADEMIC HONESTY PLEDGE

I pledge that this is fully my work and I have not given nor received any unauthorised assistance to complete this assignment.

Name: ______________________________

Sign: ______________________________.

INSTRUCTIONS

1. This assignment comprises 4 pages.

2. You can use Internet and computer to finish this assignment, but you are not allowed to discuss with anyone, including your classmates.

3. Answer **ALL** questions.

4. Total is 100 marks.

5. If any notations or questions are not clear, write down your assumptions and continue.

6. You are not necessary to type your answers.

7. Please submit your answers in a pdf file.

8. You must submit your answers and this page, including the signed **ACADEMIC HONESTY PLEDGE** before 18 April 2020, 6:00pm through NTU Learn. If you have technical problems to submit the assignment through NTU Learn. You can also send your answers to adamskong@ieee.org with the email title, AI6104 assignment 2 submission.

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1. (a) Find a vector equation for the line L that passes through the point (5, 1, 3) and is parallel to the line passing through (0, 0, 0) and (-1, 1, 2). Then, determine the points on the line L passing through the surface of the sphere, $x^2 + y^2 + z^2 = 25$.

(5 marks)

(b) Find a plane containing the points, (5, 1, 3), (1, 1, 1), and (5, 5, 9) and find the point on the plane, which is closest to the point (2,1,0).

(5 marks)

(c) Evaluate the following limits:

(i) $\lim_{(x,y)\to(1,1)} \frac{y}{x^2+y^2}$

(ii) $\lim_{(x,\theta)\to(0,\infty)} xsin(\theta)$

(iii) $\lim_{(x,y)\to(0,0)} \frac{2x^2+y^2}{x^2-y^2}$

(6 marks)

(d) Compute the partial derivatives of the following functions:

(i) $f(x,y) = \left(\frac{x}{y}\right)^2 + xy$

(ii) $f(x_1, \cdots, x_n) = \sum_{i=1}^{n-1}(x_i x_{i+1} + x_i^2)$

(6 marks)

(e) Find the tangent plane to the surface z = $4x^2 + 2y^2 + xy + 4y + 1$ at the point (1, 1).

(3 marks)

2. (a) Use the Chain rule to compute the partial derivatives, $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, \frac{\partial z}{\partial w}$, at $u = 2$, $v = 1$, $w = 0$, where $z = x^2 + xy^2 + 1$, $x = uv^2 + w^2$, $y = u + ve^w$.

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(7 marks)

Note: Question No. 2 continues on Page 2

(b) Compute the rate of change of $f(x,y) = x^2 lny$ at $P = (3,1)$ in the direction of vector $\boldsymbol{u} = i + j$. (Hint: $\frac{d}{dx} lnx = \frac{1}{x}$).

(4 marks)

(c) Find the absolute minimum and maximum values of $f(x,y) = x^2 + y^2 - 2x$ on the closed triangular region with vertices (2, 0), (0,2) and (0, 0).

(7 marks)

(d) Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be $n \times 1$ vectors and $\mathbf{A}$ be an $n \times n$ matrix. Compute $\frac{\partial}{\partial x} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|$, where $\| \quad \|$ represents L2 norm.

(7 marks)

3. (a) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 8 & 3 \\ 2 & 7 \end{bmatrix}$.

(5 marks)

(b) Let $\boldsymbol{B}$ be an invertible matrix and λ and $\boldsymbol{x}$ be respectively an eigenvalue and the corresponding eigenvector of $\boldsymbol{A}$. Show that λ is an eigenvalue of $\boldsymbol{BAB^{-1}}$ and that its corresponding eigenvector is $\boldsymbol{Bx}$.

(5 marks)

(c) Suppose $\boldsymbol{A} = \boldsymbol{X\Lambda X^{-1}}$, where $\boldsymbol{X}$ is the eigenvector matrix of $\boldsymbol{A}$ and $\boldsymbol{\Lambda}$ is its eigenvalue matrix. Compute the eigenvalue matrix and eigenvector matrix for $\boldsymbol{A} + 5\boldsymbol{I}$.

(5 marks)

(d) Let $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ be $n \times n$ matrices. Given that $\boldsymbol{A} \sim \boldsymbol{B}$ and $\boldsymbol{B} \sim \boldsymbol{C}$, prove $\boldsymbol{A} \sim \boldsymbol{C}$.

(5 marks)

(e) Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be $n \times n$ matrices with eigenvalues λ and μ, respectively. Both λ and μ have the same eigenvector $\boldsymbol{x}$. Show that $\lambda + \mu$ and $\lambda \times \mu$ are the eigenvalues of $\boldsymbol{A} + \boldsymbol{B}$ and $\boldsymbol{AB}$, respectively.

(5 marks)

4. (a) Determine whether the set $\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}\right\}$ is linearly independent or linearly dependent.

(8 marks)

(b) Extend $\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}\right\}$ to a basis of $\mathfrak{R}^4$.

(10 marks)

(c) Compute the rank and nullity of $\begin{bmatrix} 1 & 3 & 1 \\ -1 & 3 & 1 \\ 0 & 6 & 2 \end{bmatrix}$.

(7 marks)

END OF PAPER