AI6121 Computer Vision

Spatial Image Filtering

Contents and Learning Objectives

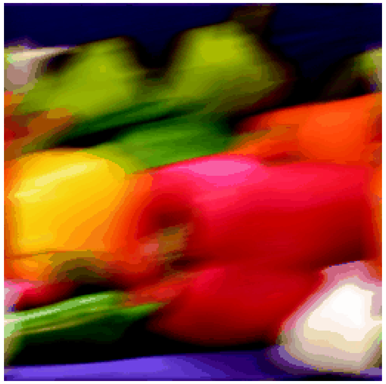
1. Spatial Filtering
2. Point Image Processing
3. Area Image Filtering
4. Applications

1. Why Image Enhancement

When you are not happy with the image

- Low contrast
- Noisy
- Blurred
- With missing parts
- …

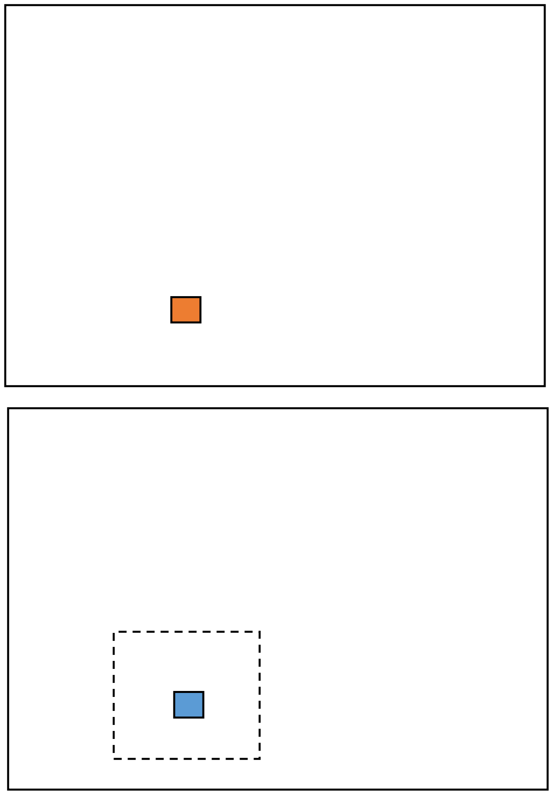


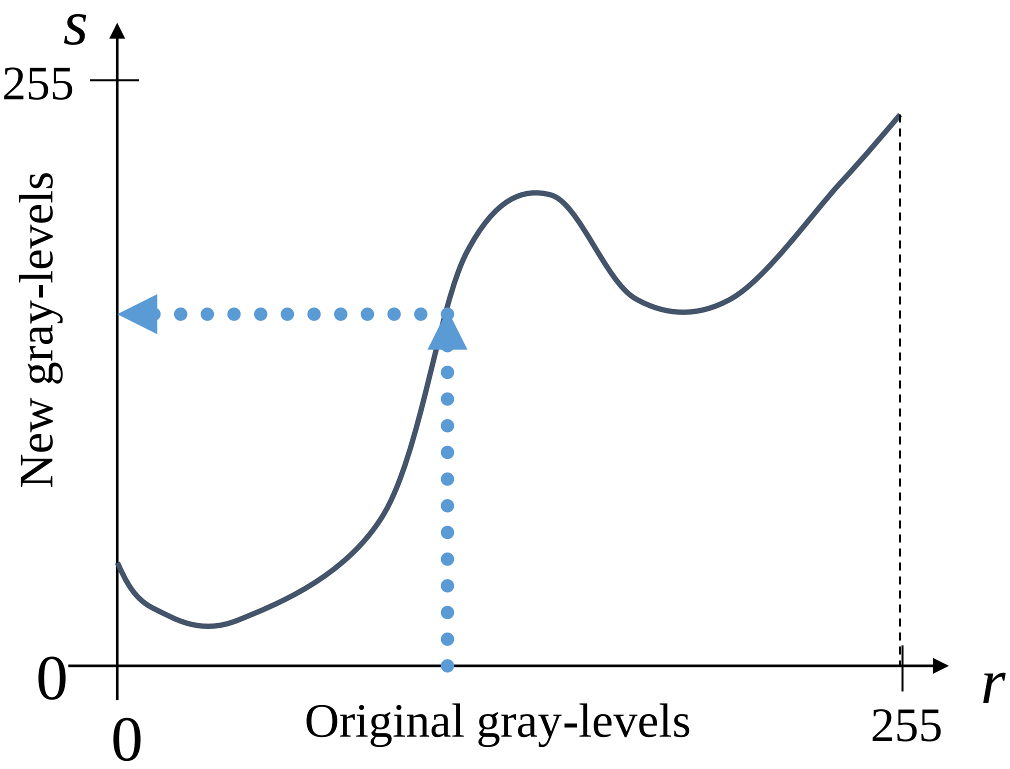



2. Point Image Processing (PIP)

- Each pixel's new value does not depend on other pixel's values.
- Pixel gray-level transformation is expressed as a function:

$$s = T(r)$$





2. PIP: Examples and Illustrations

original

$$x$$

darken

$$x - 128$$

lower contrast

$$\frac{x}{2}$$

non-linear low contrast

$$\left(\frac{x}{255}\right)^{1/3} \times 255$$

invert

$$255 - x$$

lighten

$$x + 128$$

raise contrast

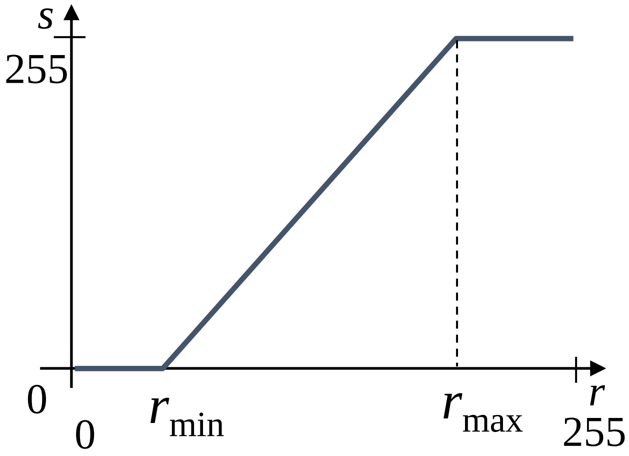
$$x \times 2$$

non-linear raise contrast

$$\left(\frac{x}{255}\right)^{2} \times 255$$

2. PIP: Linear Contrast Stretching

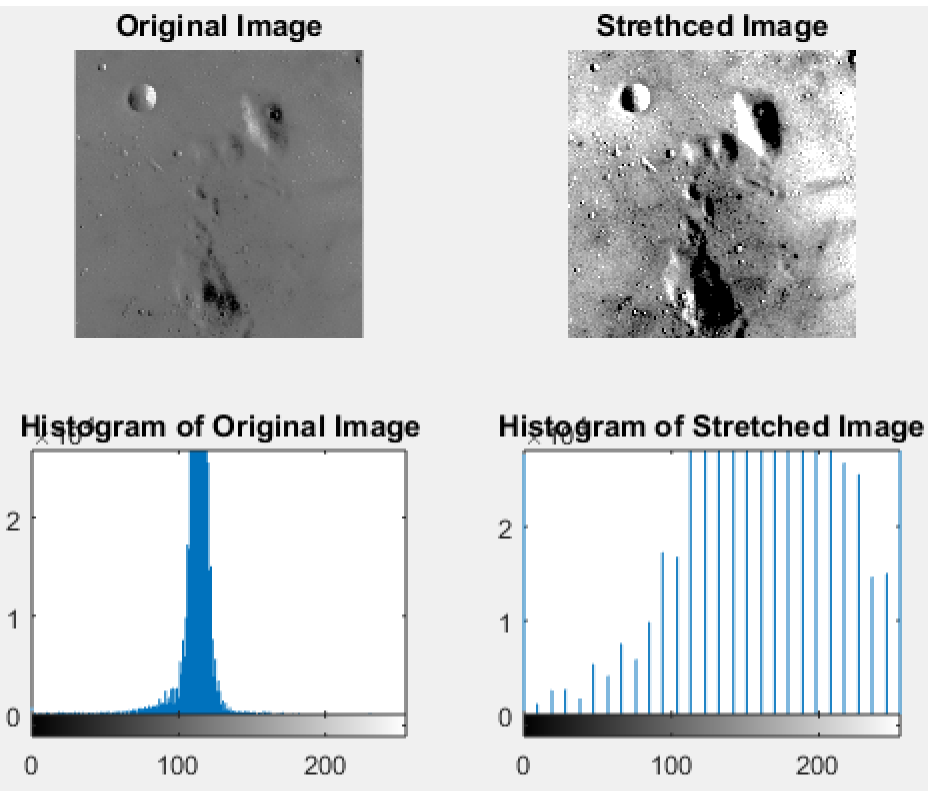
Increase contrast of images captured under poor illumination, by stretching [r_{min}, r_{max}] to [0,255]

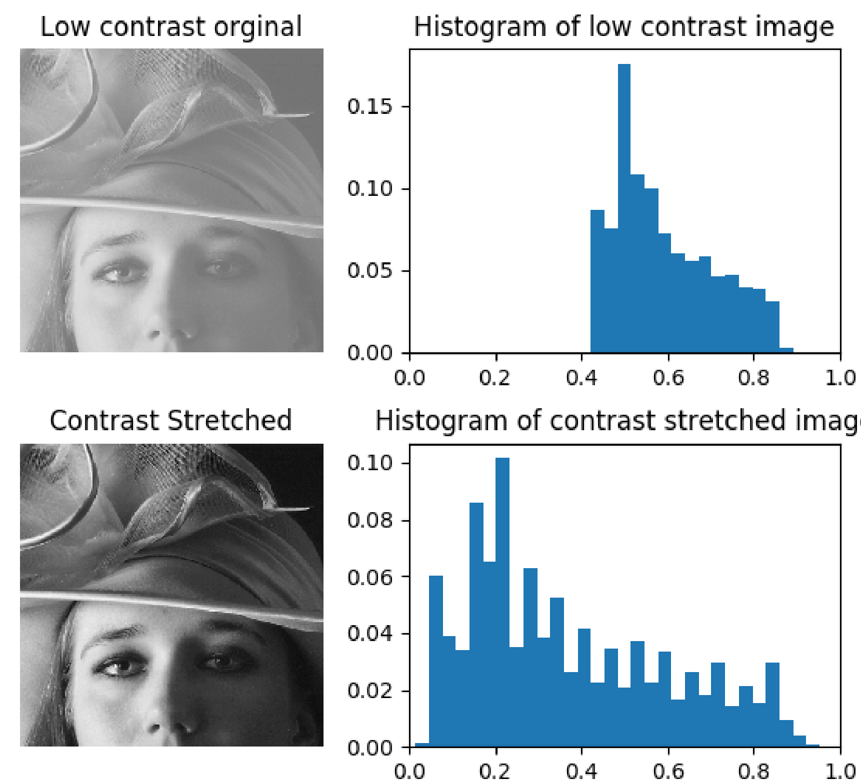


$$s = \frac{255(r - r_{min})}{r_{max} - r_{min}}$$

2. PIP: Linear Contrast Stretching

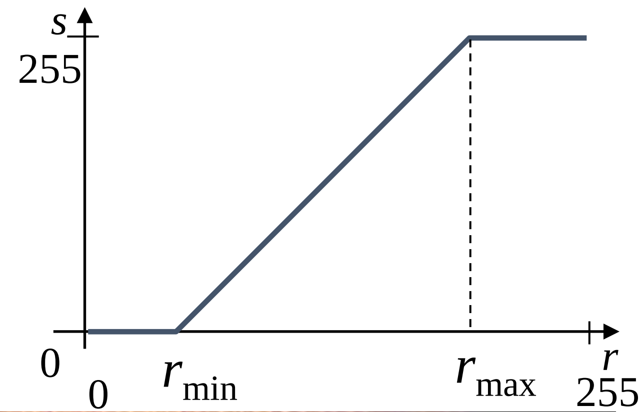
The contrast enhancement of images can be observed from the corresponding image histograms.





2. PIP: Linear Contrast Stretching

Although the technique is quite useful and simple to implement, it is vulnerable to outlier pixel values. Suppose the image has all the pixels in the range [200 - 255] except one which is completely black i.e has a value of 0, then the contrast stretching won't work at all.



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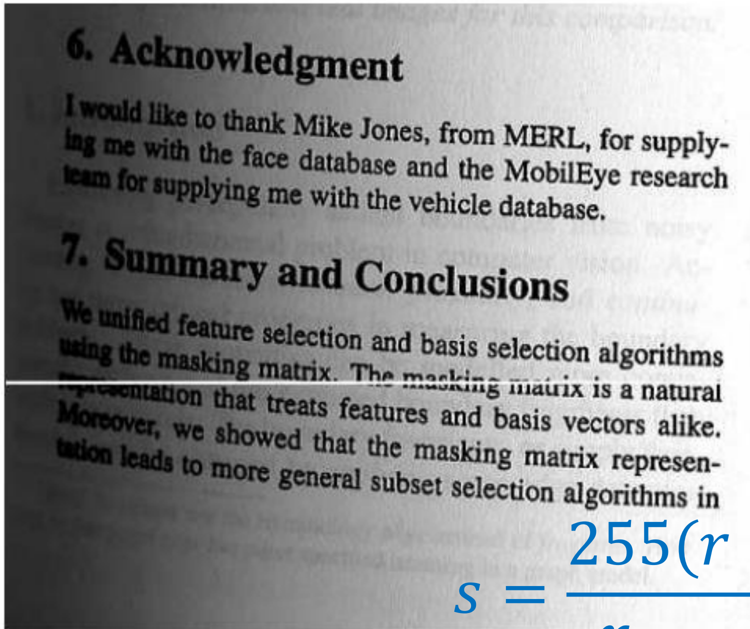
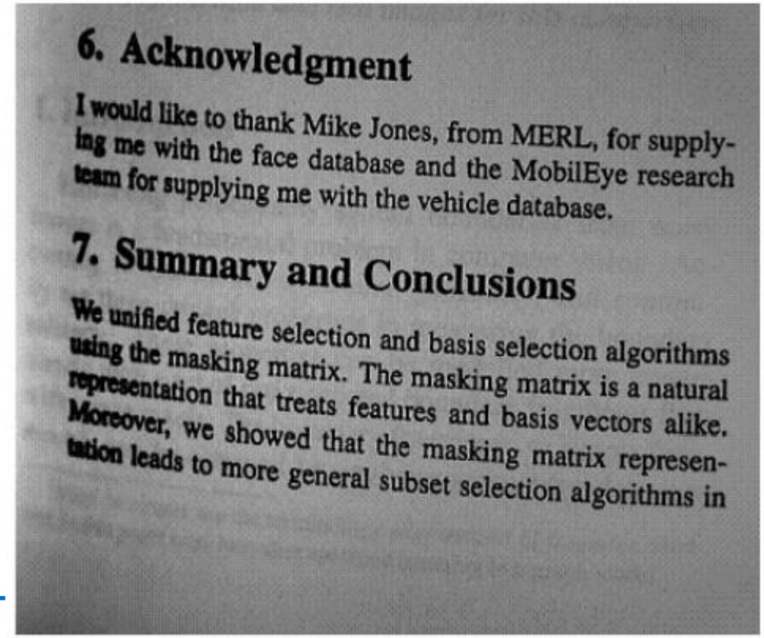


6. Acknowledgment

I would like to thank Mike Jones, from MERL, for supplying me with the face database and the MobilEye research team for supplying me with the vehicle database.

7. Summary and Conclusions

We unified feature selection and basis selection algorithms using the masking matrix. The masking matrix is a natural representation that treats features and basis vectors alike. Moreover, we showed that the masking matrix representation leads to more general subset selection algorithms in



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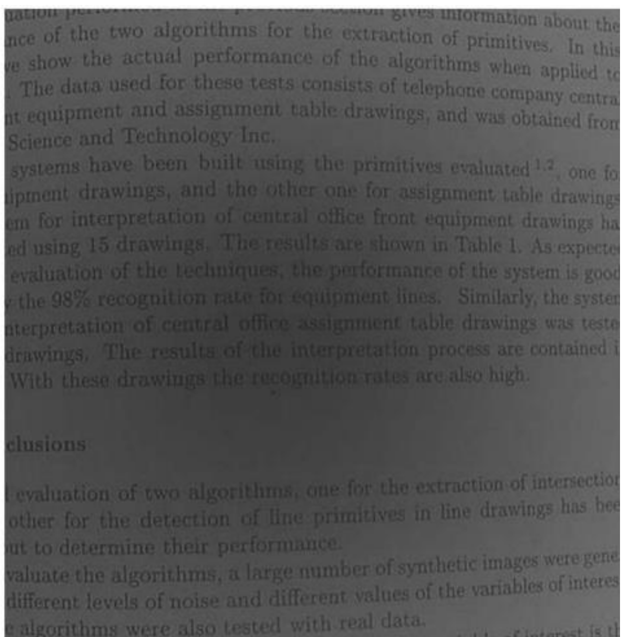

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ve show the actual performance of the algorithms when applied to
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nt equipment and assignment table drawings, and was obtained from
Science and Technology Inc.
systems have been built using the primitives evaluated [1,2], one for
ipment drawings, and the other one for assignment table drawings
em for interpretation of central office front equipment drawings has
ed using 15 drawings. The results are shown in Table 1. As expected
evaluation of the techniques, the performance of the system is good
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With these drawings the recognition rates are also high.

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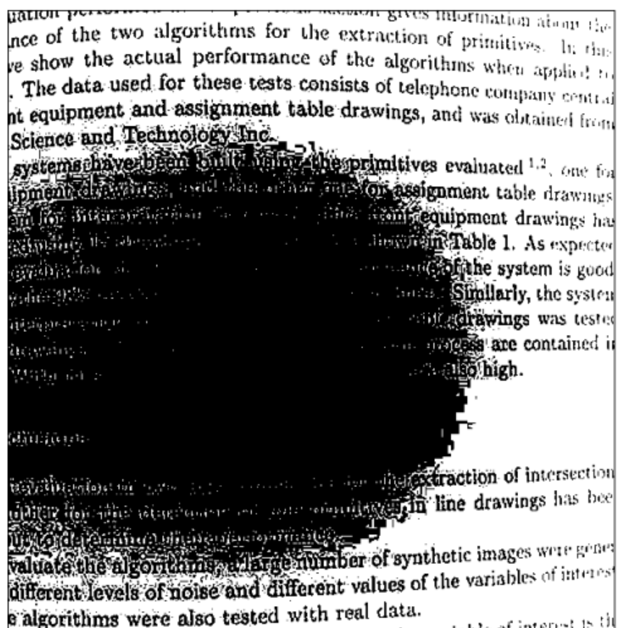

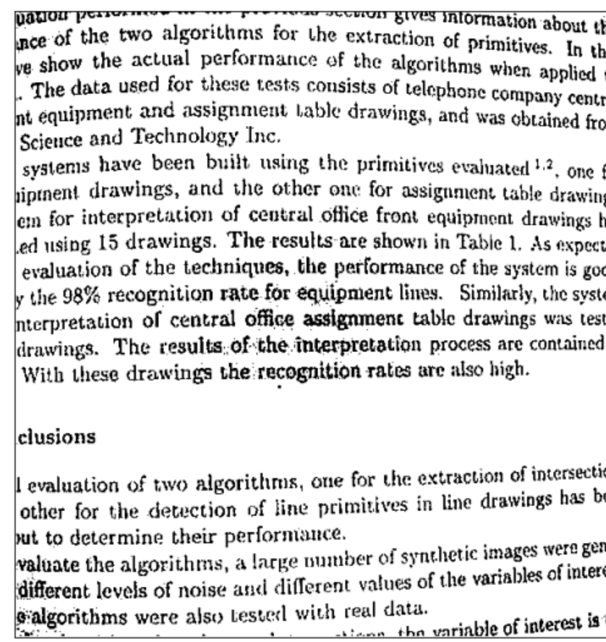

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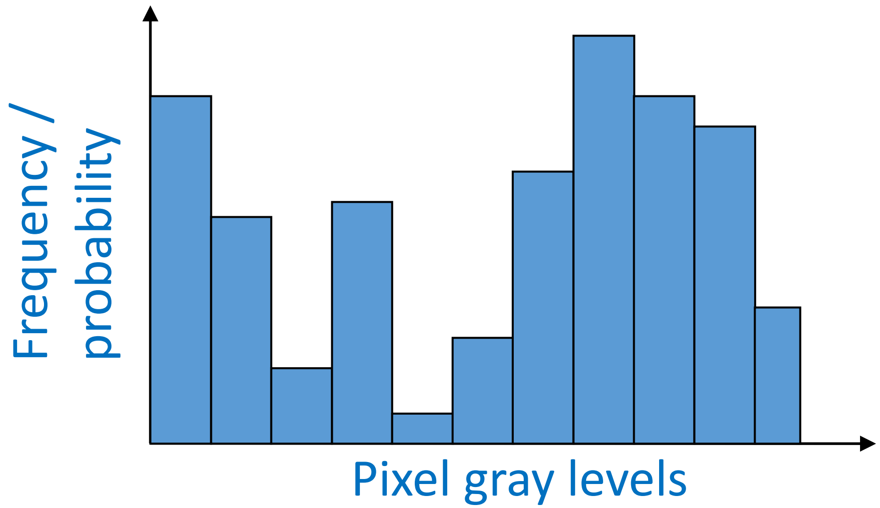
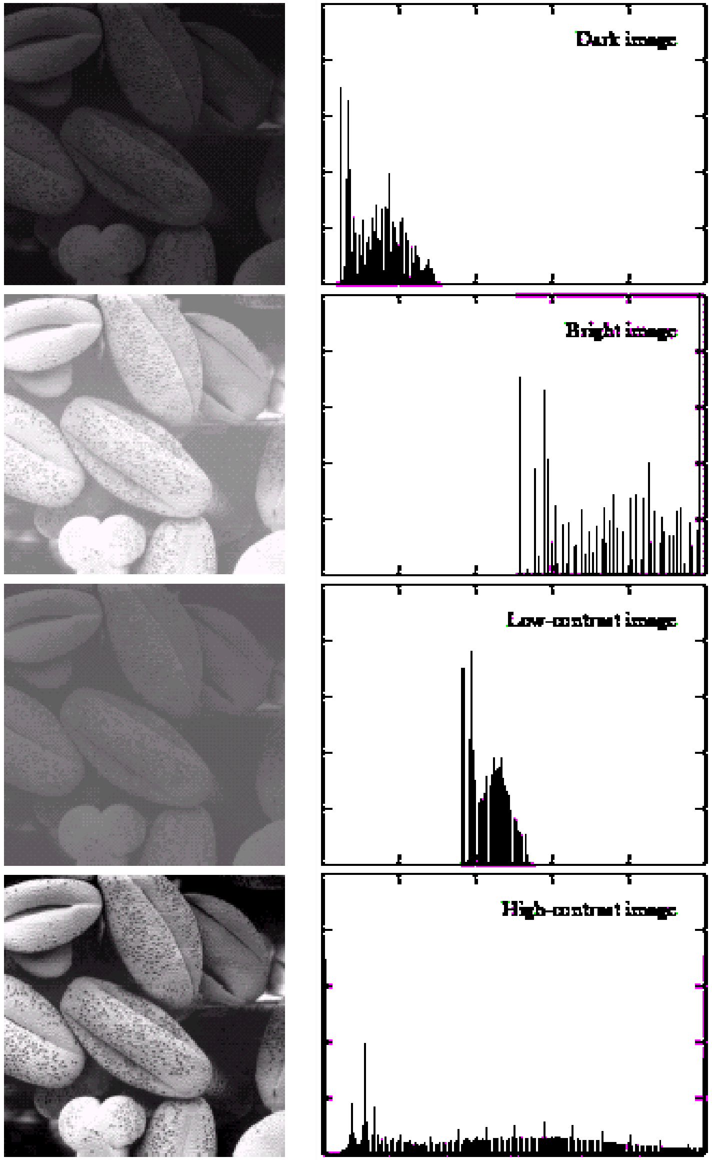
2. PIP: Histogram Equalization

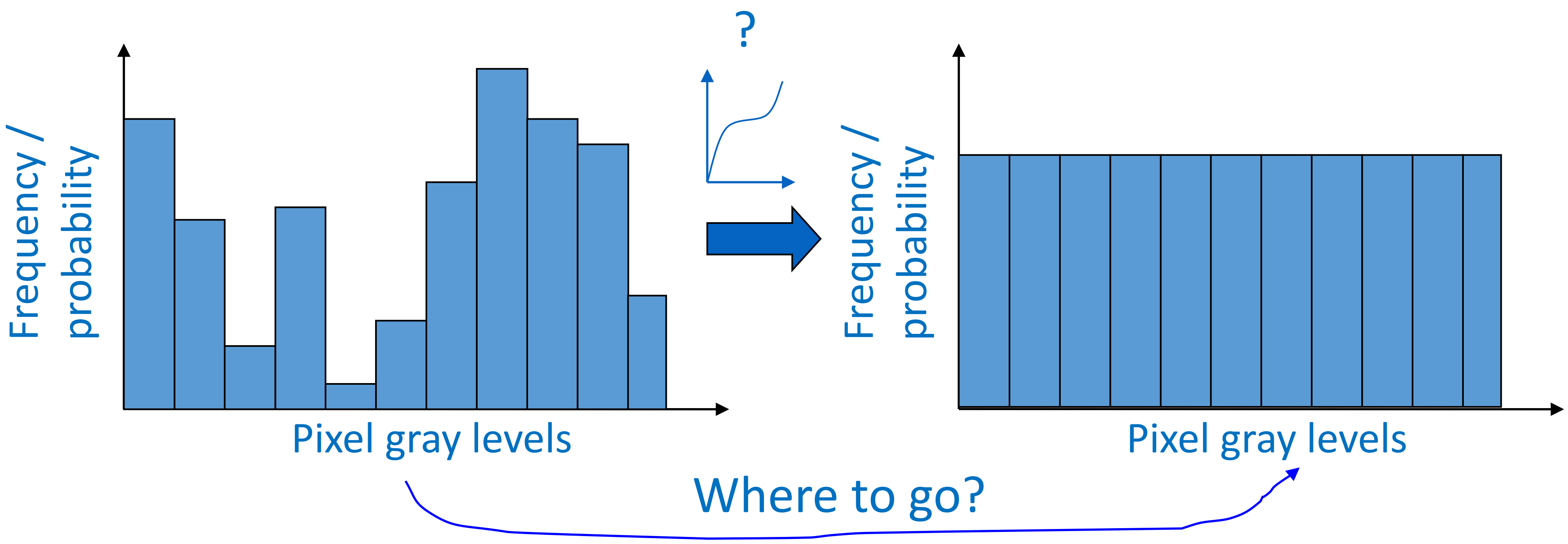
- A histogram shows how frequently different gray-levels appear in an image
- Can be represented as bin counts, or probability distributions if divided by total number of pixels in the image
- Example:





2. PIP: Histogram Equalization

Histogram equalization attempts to flatten the gray-level histogram through a gray-level transformation



2. PIP: Histogram Equalization

- After equalization, how many pixels are in each bin? $$\frac{MN}{L-1}$$

- How many pixels' values are small than or equal to *k*? $$\sum_{j=0}^{k} N_j$$

- How many bins can be filled? $$\sum_{j=0}^{k} N_j \div \frac{MN}{L-1} = \frac{L-1}{MN} \sum_{j=0}^{k} N_j$$

- Where do pixels with intensity *k* go? $$s_k = (L-1) \sum_{j=0}^{k} p_j$$

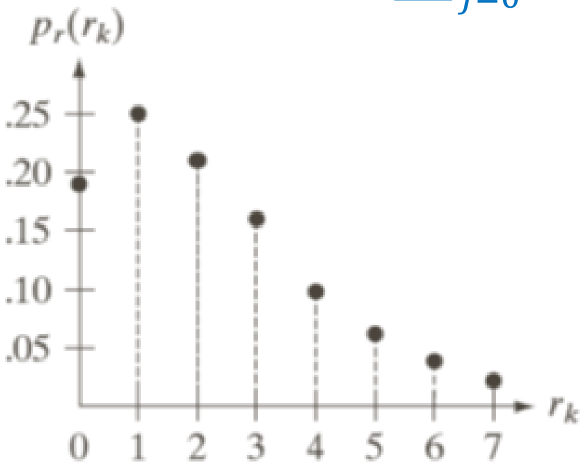
Notes: 1) $M*N$ is the image size; 2) round s_k to an integer; 3) do it for k=0,1,…,L-1

2. PIP: Histogram Equalization

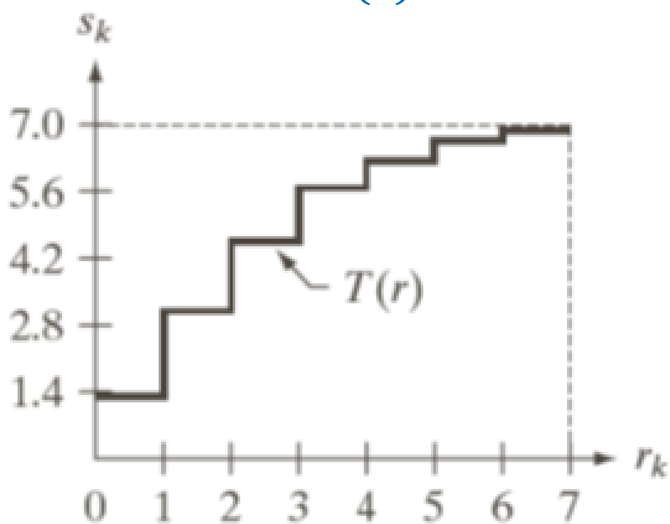
r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_k = (L-1)\sum_{j=0}^{k} p_j$$

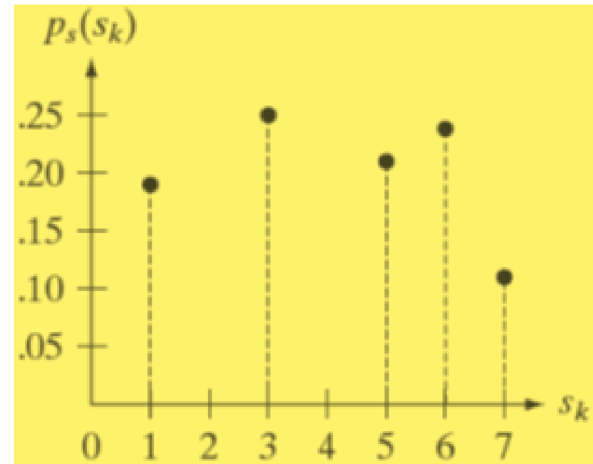
$$s = T(r)$$



Original histogram



Transformation function s_k

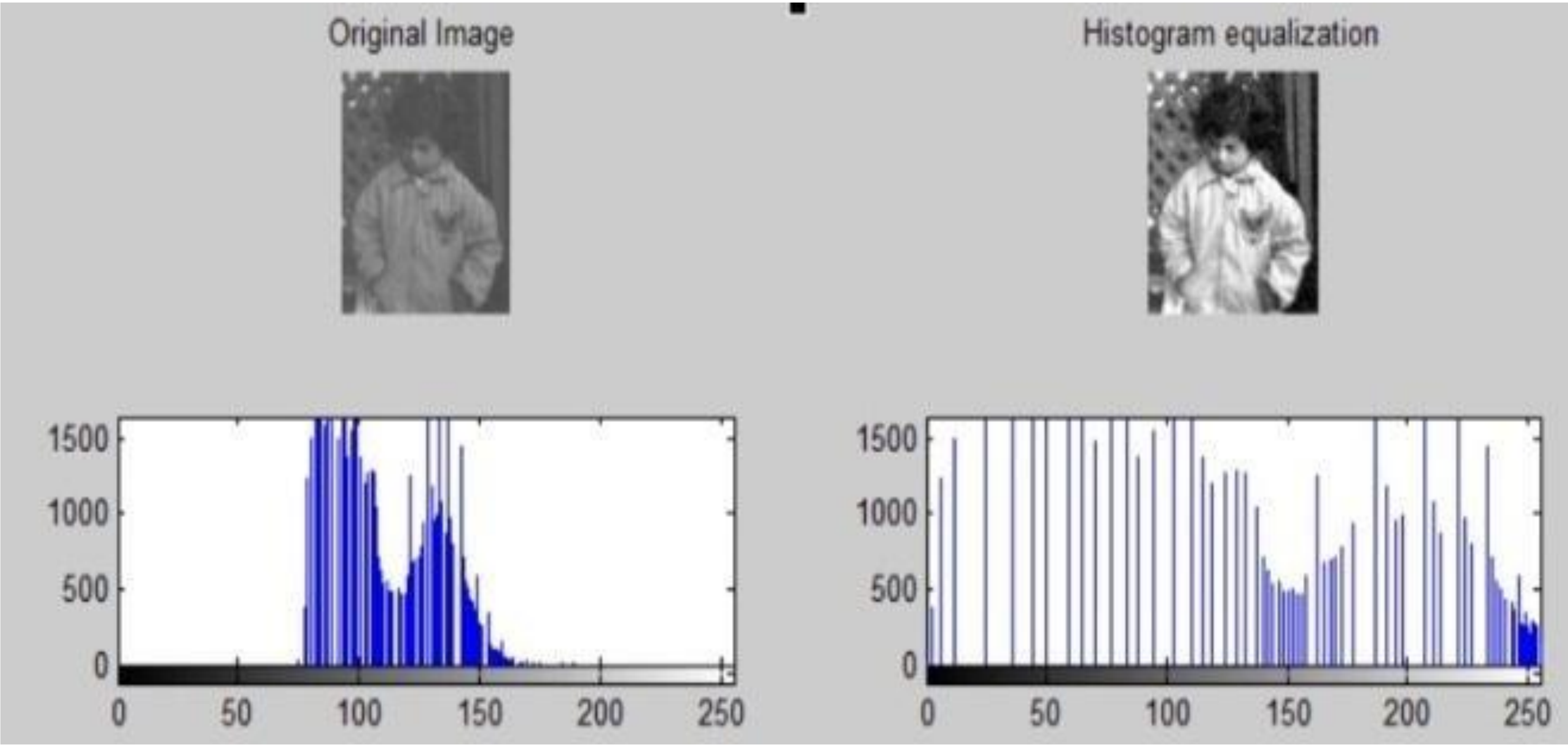


Equalized histogram

r	0	1	2	3	4	5	6	7
s	1.33	3.08	4.55	5.67	6.23	6.65	6.86	7.00
Rounded s	1	3	5	6	6	7	7	7

2. PIP: Histogram Equalization

An example is shown below. HE works like to transport pixels in the middle to the left and right ends.



2. PIP: Optimal Transport (optional)

1. I How to move dirt from one place to another while minimizing the effort?

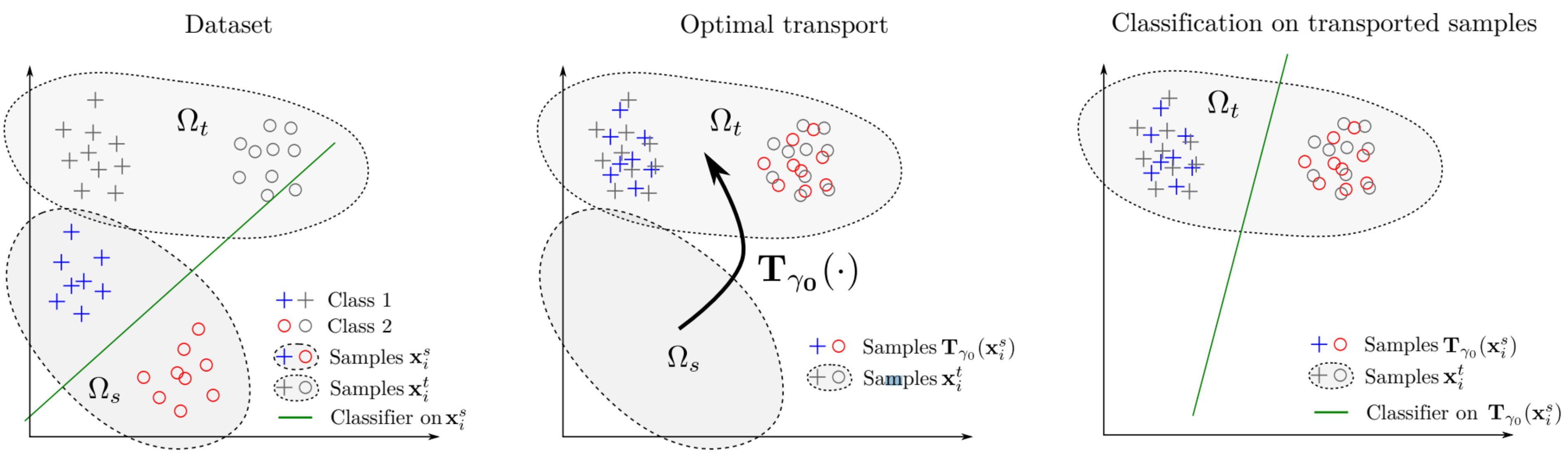
2. Find a mapping *T* between the two distributions of mass (transport).

3. Optimize with respect to a displacement cost *c(x, y)* (optimal).

2. PIP: Optimal Transport (optional)

Optimal transport is a more general technique for transforming data from one distribution to another distribution of a different domain.

It has been successfully used in unsupervised domain adaptation (UDA) in deep learning, where the source and the target domains often have clear gaps and discrepancy, and optimal transport helps to align the distributions of the two domains for optimal classification.



Optimal Transport for Domain Adaptation, TPAMI, vol. 9, no. 39, 2017.

Assignment I

Histogram is a graphical representation of the intensity distribution of an image. It captures the occurrence frequency of pixel intensity values with which multiple image statistics can be calculated. Many image processing methods make use of image histograms.

Histogram Equalization (HE) is an image processing technique that has been widely adopted to improve the contrast in images. It accomplishes this by spreading out the most frequent intensity values, i.e. stretching out the intensity range of the image. It usually increases the global contrast of images when the image pixels fall within a narrow range of intensity values. This allows for areas of lower local contrast to gain a higher contrast.

Assignment I

This assignment consists of the following tasks:

1. Implement the HE algorithm in Matlab or Python or other computer programming languages, and apply your implemented HE algorithm to the 8 sample images. The submission of your solution should include your source-code algorithm implementation as well as the enhanced sample images by your implemented algorithm.
2. Discuss the pro and con of histogram equalization algorithm according to the enhanced sample images by your implemented HE algorithm. Discuss possible causes of some unsatisfactory contrast enhancement.
3. Discuss possible improvements of the basic HE algorithm. Implement and verify your ideas over the provided test images. This subtask is optional, and there will be bonus marks for good addressing of this subtask.

Assignment I

1. You need to submit your solution report in PDF format, and there are no standard templates for your report. Ensure you include your name and matriculation number clearly.
2. Similar to the direct reading, I will evaluate your report according to both contents and presentation.
3. You need to submit your assignment report through NTULearn before the deadline on Sept 15th 2021. There will be penalty for late submissions.

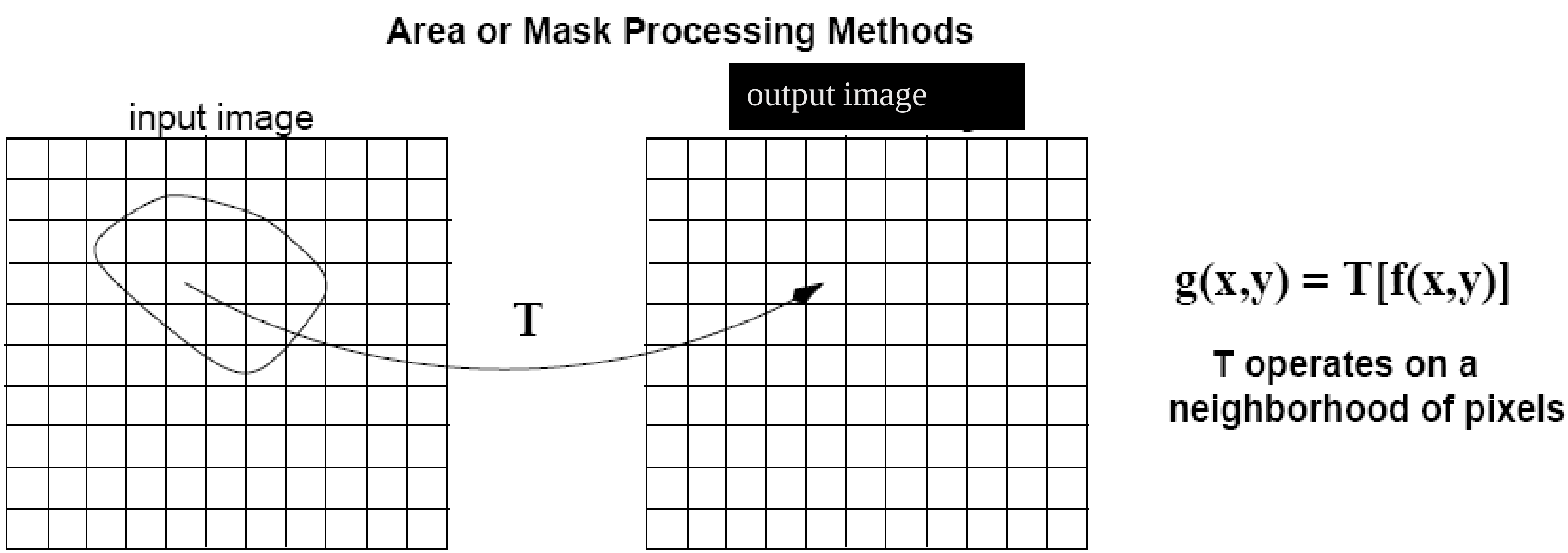
Summary

1. What does PIP means and what are typical point image processing operators?
2. How does contrast stretching work, and what is its limitation?
3. How does histogram equalization work?

3. Area Image Processing (AIP)

Area image processing (AIP) filters image pixels by considering their neighboring pixels.

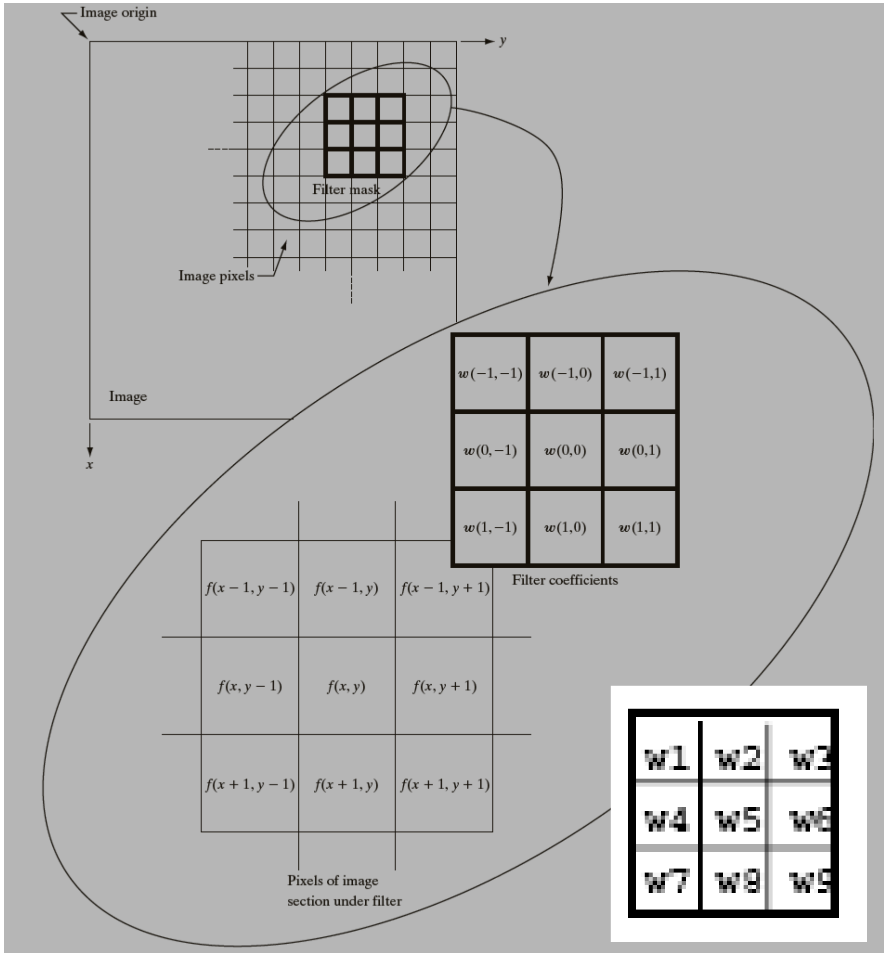
It need to define an area shape and area size of a local neighborhood and the operations that are to be applied to the image pixels within the defined neighborhood.



3. AIP: Area Shape, Area Size and Operations

Area shape is typically defined using a rectangular mask, and the area size is determined by the mask size, e.g. 3x3 or 5x5.

The operation is typically a linear combination (weighted sum) of pixel values within the mask. Different weights (or mask coefficient) give different operations, e.g. smoothing, sharpening, edge detection, etc.



3. AIP: Cross-Correlation

For an average image smoothing example, assume the averaging window (2k+1)x(2k+1):

$$G[i,j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]$$

We can generalize this idea by allowing different weights for different neighboring pixels:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

This is called a cross-correlation operation and written:

$$G = H \otimes F$$

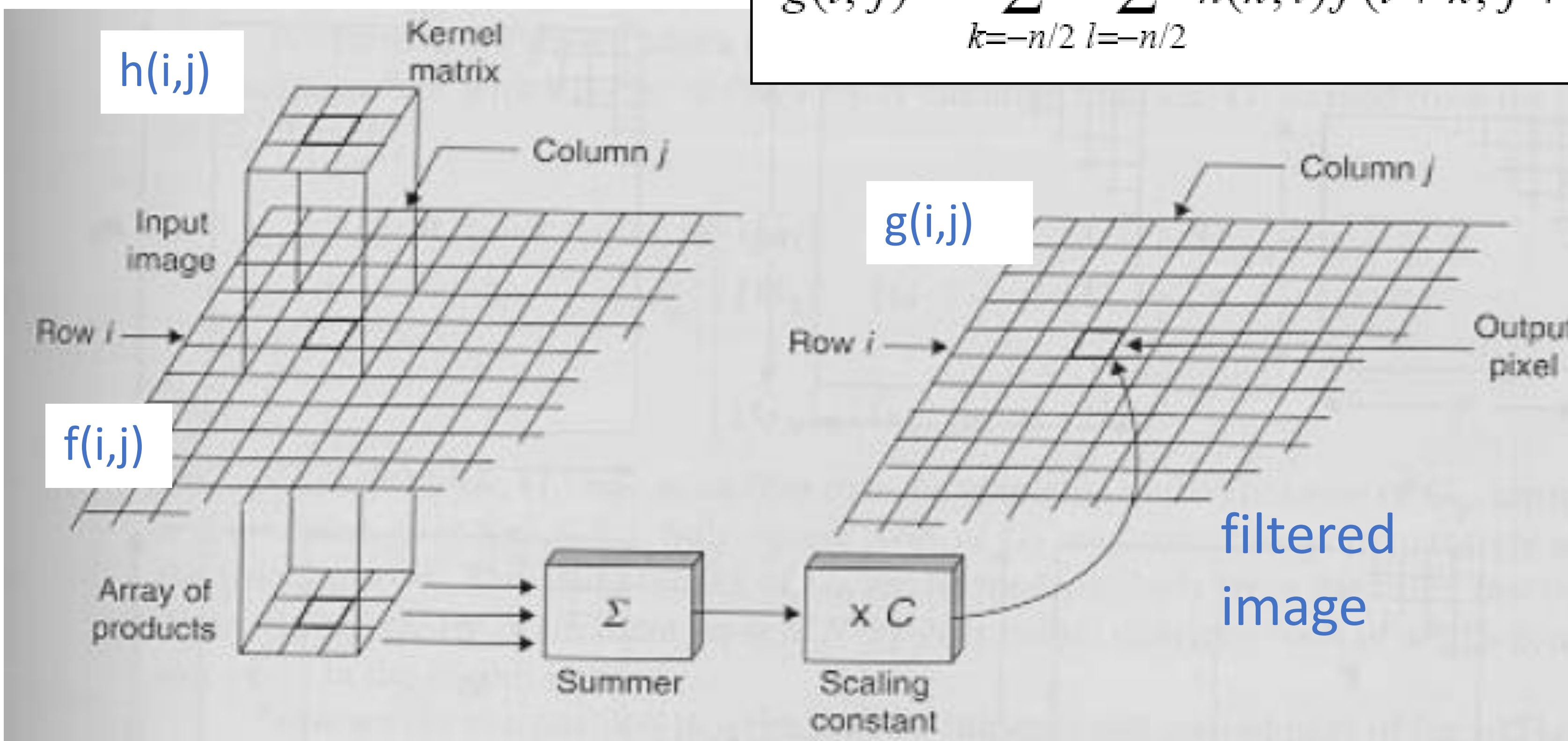
H is called the "filter", "kernel," or "mask."

3. AIP: Cross-Correlation

A filtered image is generated as the center of the mask visits every pixel in the input image.

$$g(i,j) = \sum_{k=-n/2}^{n/2} \sum_{l=-n/2}^{n/2} h(k,l) f(i+k, j+l)$$

n x n mask



3. AIP: Cross-Correlation

Cross-correlation has geometric interpretation. Suppose *x* and *y* are two n-dimensional vectors:

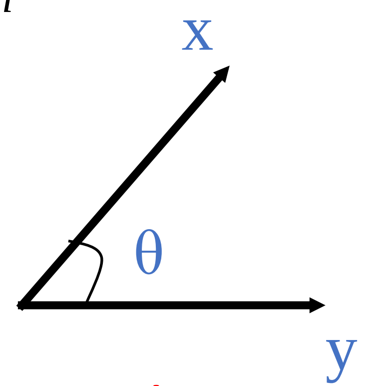
$$x = (x_1, x_2, \ldots, x_n) \qquad y = (y_1, y_2, \ldots, y_n)$$

The dot product of *x* with *y* is defined as:

$$x.y = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n$$

Using vector notation:

$$x\,.\,y = |\,x\,||\,y\,|\cos(\theta)$$



Correlation generalizes the notion of dot product

3. AIP: Convolution

Convolution is the same as correlation except that the mask is flipped, both horizontally and vertically.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow{H} \begin{bmatrix} 7 & 8 & 9 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} \xrightarrow{V} \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

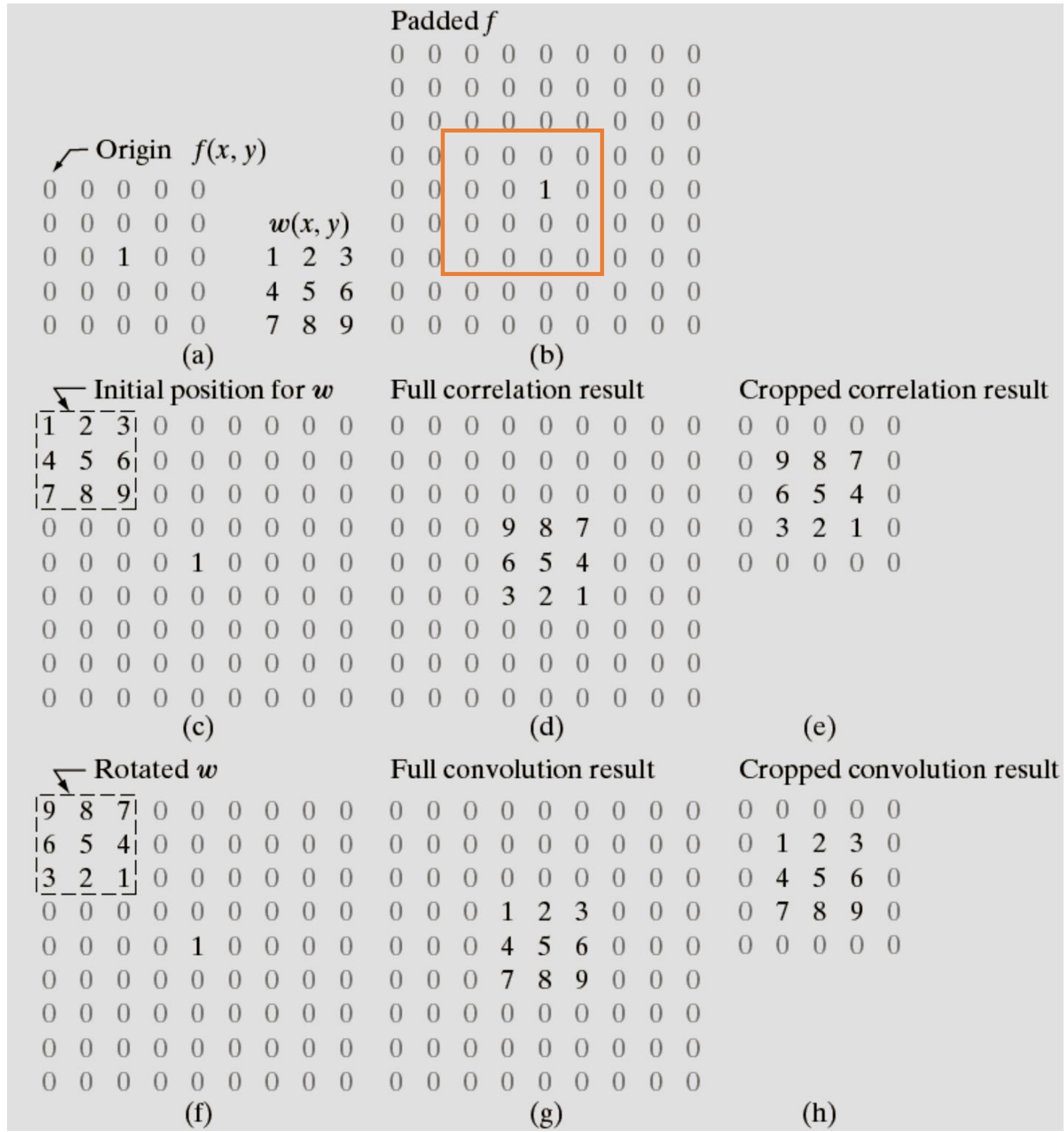
$$g(i,j) = \sum_{k=-\frac{n}{2}}^{\frac{n}{2}} \sum_{l=-\frac{n}{2}}^{\frac{n}{2}} h(k,l)\, f(i-k, j-l) = \sum_{k=-\frac{n}{2}}^{\frac{n}{2}} \sum_{l=-\frac{n}{2}}^{\frac{n}{2}} h(i-k, j-l)\, f(k,l)$$

For symmetric masks (i.e., $h(i,j) = h(-i,-j)$), convolution is equivalent to correlation!

3. AIP: Comparison of Correlation and Convolution

Cross-Correlation:

Convolution:



3. AIP: Typical Linear Filters

- Different filtering effects can be achieved by using different filters with different filter parameters.
- The filter parameters are often normalized by summing up to one (often for certain smoothing) or zero (often for enhancing contrast or detecting image changes).

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

Mean smoothing

1/16	1/8	1/16
1/8	1/4	1/8
1/16	1/8	1/16

Gaussian smoothing

0	1	0
1	-4	1
0	1	0

Laplacian sharpening

-1	0	+1
-2	0	+2
-1	0	+1

Sobel vertical edge

+1	+2	+1
0	0	0
-1	-2	-1

Sobel horizontal edge

3. AIP: Typical Linear Filters

Mean smoothing

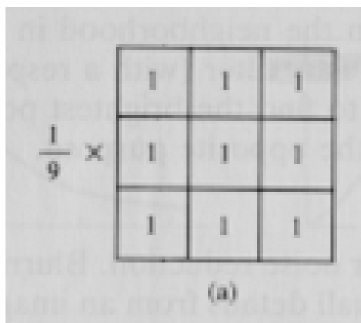
Median smoothing

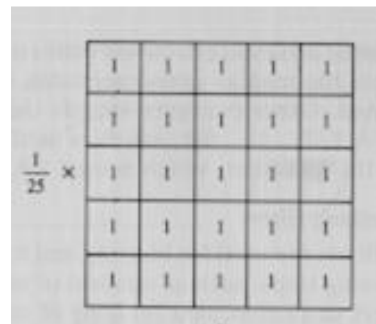
Gaussian smoothing

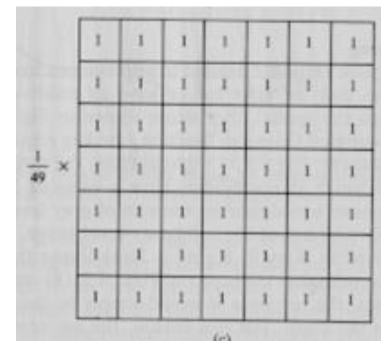
Sharpening smoothing

3. AIP: Averaging Smoothing

Idea: replace each pixel by the average of its neighbors. It's useful for reducing noise and unimportant details, and the size of the mask controls the amount of smoothing.







original

3x3

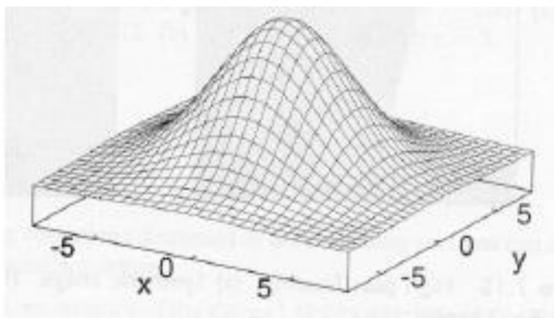
5x5

7x7

3. AIP: Gaussian Smoothing

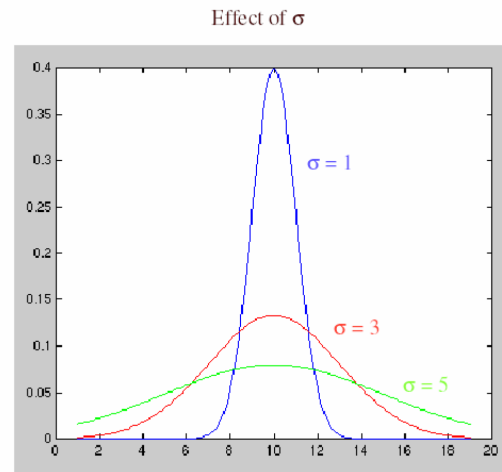
Idea: replace each pixel by a weighted average of its neighbors, where the mask weights are computed by sampling a Gaussian function. The mask size depend on δ which control the degree of smoothing.

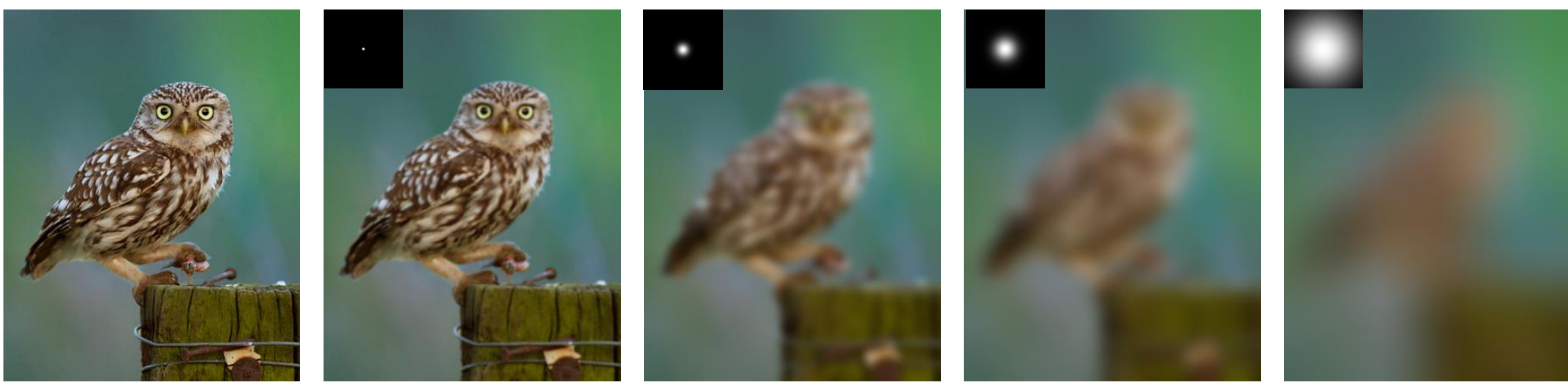
$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}}$$



7×7 Gaussian mask

1	1	2	2	2	1	1
1	2	2	4	2	2	1
2	2	4	8	4	2	2
2	4	8	16	8	4	2
2	2	4	8	4	2	2
1	2	2	4	2	2	1
1	1	2	2	2	1	1





σ = 1 pixel σ = 5 pixel σ = 15 pixel σ = 30 pixel

3. AIP: Sobel Gradient Filtering

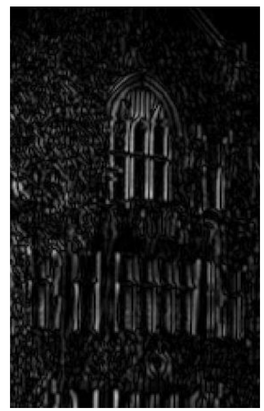
Sobel differential operators attempt to approximate the gradient at a pixel via Sobel masks. Thresholding the gradient prouces edge pixels as to be discussed in the next lecture.

INPUT IMAGE

Horizontal

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

G_x



G_y

Vertical

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}^T$$

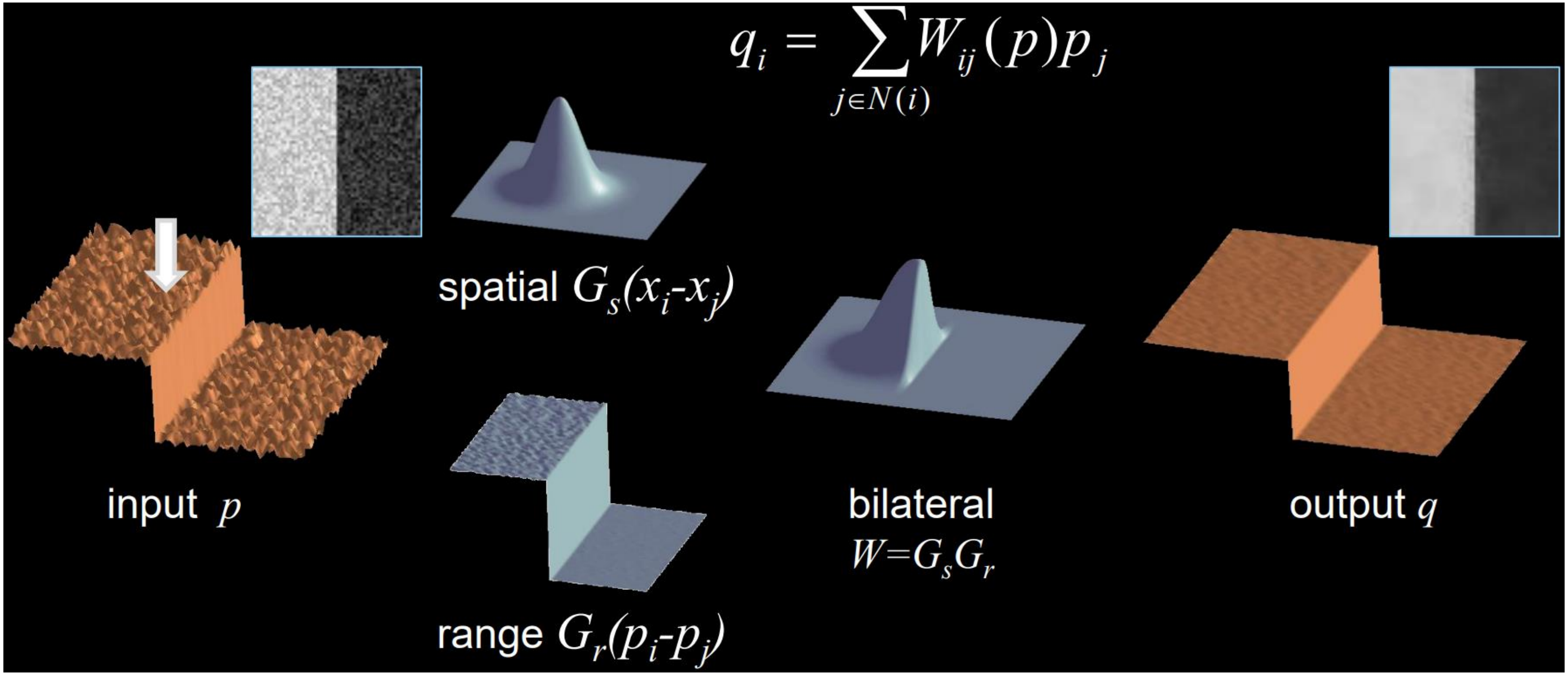
$$\sqrt{G_x^2 + G_y^2}$$

“GRADIENT” IMAGE

3. AIP: Edge-Preserving Filtering

Bilateral filtering is a nonlinear filtering technique that is capable of smoothing images but keep the main edge structures less affected.

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)\, I_{\mathbf{q}},$$



3. AIP: Edge-Preserving Filtering

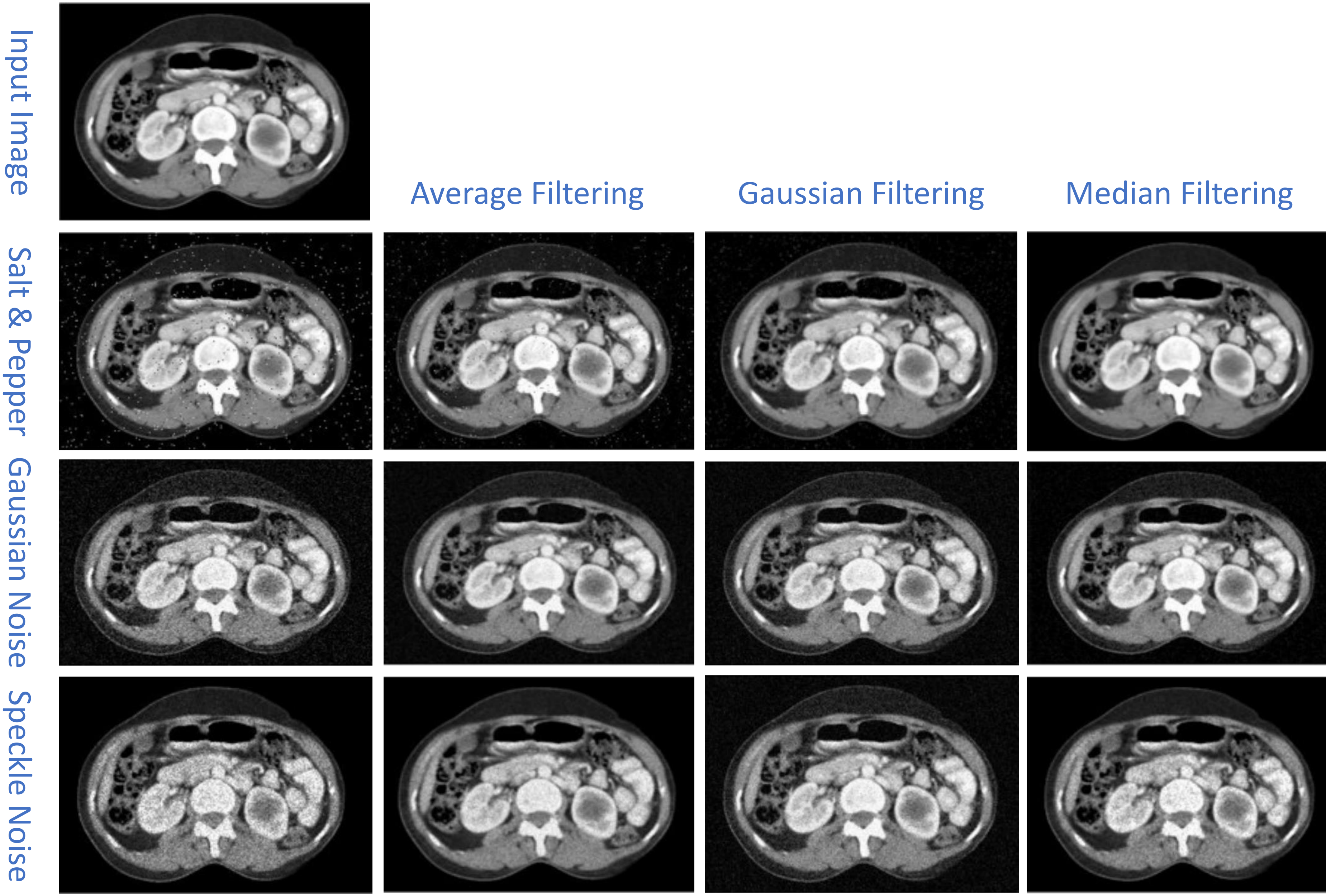
Below are two examples that show how bilateral filter filters images.

Input image

Filtered image

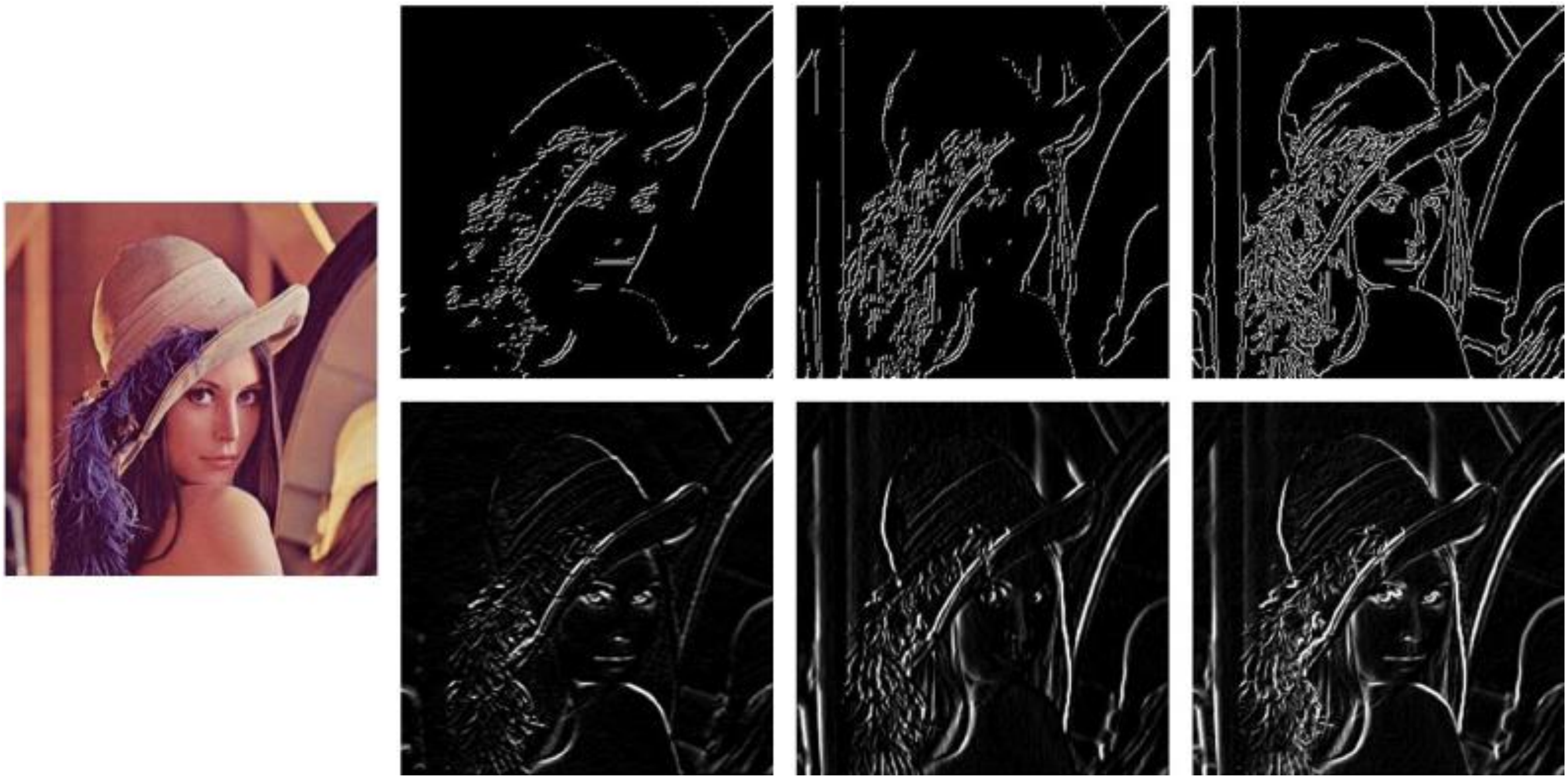
3. AIP: Edge-Preserving Filtering

4. Application: Noise Removal



4. Applications: Edge Detection

Image filtering can be used to detect edges. With the filtered image gradient, edge pixels can be detected by different detection methods, such as Sobel detector and Canny detector as shown below.

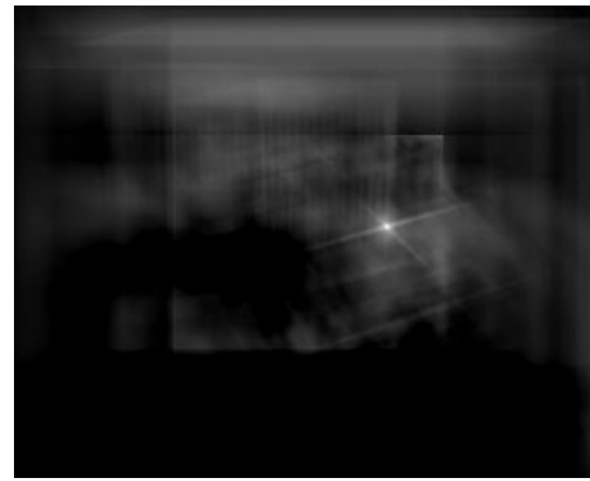


4. Application: Object Detection

Measure the similarity between images or parts of images.

mask

=



Traditional correlation cannot handle changes due to size, orientation, shape (e.g., deformable objects), etc.

?

Summary

1. Why Image Filtering and Enhancement
2. Point Image Processing
3. Area Image Filtering
4. Applications