AI6122 Text Data Management & Analysis

Topic: POS and HMM

Topics

- Word classes
- Part of speech tagging
- Use HMMs for POS tagging

Word Classes: Parts of Speech

- Parts of speech (POS)
  - Noun, verb, adjective, preposition, adverb, article, interjection, pronoun, conjunction, etc.
  - Also Called: parts-of-speech, lexical categories, word classes, morphological classes, lexical tags...

N	noun	chair, bandwidth, pacing
V	verb	study, debate, munch
ADJ	adjective	purple, tall, ridiculous
ADV	adverb	unfortunately, slowly
P	preposition	of, by, to
PRO	pronoun	I, me, mine
DET	determiner	the, a, that, those

POS Tagging

- The process of assigning a part-of-speech or lexical class marker to each word in a sequence.
  - First step of a vast number of practical tasks
- Information extraction
  - Finding names, e.g., people, organization -- N.
- Machine Translation
  - E.g. result/N, result/V -> kyol-kwa/N, kyol-kwa-lul-ne-da/V
- Parsing
  - Helpful to know parts of speech before you start parsing, e.g. subject-verb-object

WORD	tag
the	DET
koala	N
put	V
the	DET
keys	N
on	P
the	DET
table	N

POS Tagging: Choosing a Tagset

- To do POS tagging, we need to choose a standard **set of tags**
  – Could pick very coarse tagsets (e.g., N, V, Adj, Adv.)
  – More commonly used set is the finer grained, “Penn TreeBank tagset”, 45 tags

Tag	Description	Example	Tag	Description	Example
CC	coordin. conjunction	*and, but, or*	SYM	symbol	+,%, &
CD	cardinal number	*one, two, three*	TO	“to”	*to*
DT	determiner	*a, the*	UH	interjection	*ah, oops*
EX	existential ‘there’	*there*	VB	verb, base form	*eat*
FW	foreign word	*mea culpa*	VBD	verb, past tense	*ate*
IN	preposition/sub-conj	*of, in, by*	VBG	verb, gerund	*eating*
JJ	adjective	*yellow*	VBN	verb, past participle	*eaten*
JJR	adj., comparative	*bigger*	VBP	verb, non-3sg pres	*eat*
JJS	adj., superlative	*wildest*	VBZ	verb, 3sg pres	*eats*
LS	list item marker	*1, 2, One*	WDT	wh-determiner	*which, that*
MD	modal	*can, should*	WP	wh-pronoun	*what, who*
NN	noun, sing. or mass	*llama*	WP$	possessive wh-	*whose*
NNS	noun, plural	*llamas*	WRB	wh-adverb	*how, where*
NNP	proper noun, singular	*IBM*	$	dollar sign	$
NNPS	proper noun, plural	*Carolinas*	#	pound sign	#
PDT	predeterminer	*all, both*	“	left quote	‘ or “
POS	possessive ending	*’s*	”	right quote	’ or ”
PRP	personal pronoun	*I, you, he*	(	left parenthesis	[, (, {, <
PRP$	possessive pronoun	*your, one’s*	)	right parenthesis	],), }, >
RB	adverb	*quickly, never*	,	comma	,
RBR	adverb, comparative	*faster*	.	sentence-final punc	. ! ?
RBS	adverb, superlative	*fastest*	:	mid-sentence punc	: ; ... – -
RP	particle	*up, off*			

Using the Penn Tagset

- The/**DT** grand/**JJ** jury/**NN** commented/**VBD** on/**IN** a/**DT** number/**NN** of/**IN** other/**JJ** topics/**NNS** ./**.**
- Prepositions and subordinating conjunctions marked IN ("although/**IN** I/**PRP**..")
- Except the preposition "to" is just marked "**TO**".

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POS Tagging

- Words often have more than one POS: ***back***
  - The ***back*** door = JJ (adj)
  - On my ***back*** = NN
  - Win the voters ***back*** = RB (adv)
  - Promised to ***back*** the bill = VB (verb, base form)

- The POS tagging problem is to determine the POS tag for a particular **instance** of a word.

How Hard is POS Tagging? Measuring Ambiguity

- Number of word types with different levels of POS ambiguity from the Brown corpus

Many **ambiguous** words appear **frequently** in corpus

		87-tag Original Brown		45-tag Treebank Brown	
Unambiguous (1 tag)		**44,019**		**38,857**	
Ambiguous (2–7 tags)		**5,490**		**8844**	
Details:	2 tags	4,967		6,731	
	3 tags	411		1621	
	4 tags	91		357	
	5 tags	17		90	
	6 tags	2	(*well*, *beat*)	32	
	7 tags	2	(*still*, *down*)	6	(*well*, *set*, *round*, *open*, *fit*, *down*)
	8 tags			4	(*'s*, *half*, *back*, *a*)
	9 tags			3	(*that*, *more*, *in*)

POS Tagging as Sequence Classification

- **Input**: We are given a sentence (an "observation" or "sequence of observations")

- **Output**: What is the best sequence of tags that corresponds to this sequence of observations?

The/DT grand/JJ jury/NN commented/VBD on/IN a/DT number/NN of/IN other/JJ topics/NNS ./.

- Probabilistic view:
  - Consider all possible sequences of tags
  - Out of this universe of sequences, choose **the tag sequence** which is **most probable** given the observation sequence of n words $w_1 \ldots w_n$.

Road to HMMs

- Out of all sequences of n tags $t_1 \ldots t_n$ the single tag sequence such that $P(t_1 \ldots t_n | w_1 \ldots w_n)$ is highest.

$$\hat{t}_1^n = arg \max_{t_1^n} P(t_1^n | w_1^n)$$

  - Hat ^ means "our estimate of the best one"
  - $arg \max_x f(x)$ means "the x such that $f(x)$ is maximized"

- But how to compute this value?
  - Bayesian inference: Use Bayes rule to transform this equation into a set of other probabilities that are easier to compute

Using Bayes Rule

$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}}\, P(t_1^n|w_1^n)$$

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} \frac{P(w_1^n|t_1^n)P(t_1^n)}{P(w_1^n)}$$

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}}\, P(w_1^n|t_1^n)P(t_1^n)$$

Likelihood and Prior

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} \overbrace{P(w_1^n|t_1^n)}^{\text{likelihood}} \overbrace{P(t_1^n)}^{\text{prior}}$$

The/**DT** yellow/**JJ** hat/**NN**

$$P(w_1^n|t_1^n) \approx \prod_{i=1}^{n} P(w_i|t_i)$$

The probability of a word appearing depends only on **its own POS tag** P (the | DT)

$$P(t_1^n) \approx \prod_{i=1}^{n} P(t_i|t_{i-1})$$

The probability of a tag appearing depends only **on the previous tag** P(NN|JJ)

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} P(t_1^n|w_1^n) \approx \underset{t_1^n}{\operatorname{argmax}} \prod_{i=1}^{n} P(w_i|t_i) P(t_i|t_{i-1})$$

Two Kinds of Probabilities

- Tag transition probabilities $p(t_i|t_{i-1})$

$$P(t_i|t_{i-1}) = \frac{C(t_{i-1}, t_i)}{C(t_{i-1})}$$

  - Example: determiners likely to precede adjectives and nouns
    - That/DT flight/NN
    - The/DT yellow/JJ hat/NN
  - So we expect $P(NN|DT)$ and $P(JJ|DT)$ to be high, but not $P(DT|JJ)$ to be high
  - Compute $P(NN|DT)$ by counting in a labeled corpus:

$$P(NN|DT) = \frac{C(DT, NN)}{C(DT)} = \frac{56,509}{116,454} = .49$$

Two Kinds of Probabilities

- Word likelihood probabilities $p(w_i|t_i)$

$$P(w_i|t_i) = \frac{C(t_i, w_i)}{C(t_i)}$$

  - Example: VBZ (3rd person singular present verb) likely to be "is"
  - Compute $P(is|VBZ)$ by counting in a labeled corpus:

$$P(is|VBZ) = \frac{C(VBZ, is)}{C(VBZ)} = \frac{10,073}{21,627} = .47$$

Example: The Verb "race"

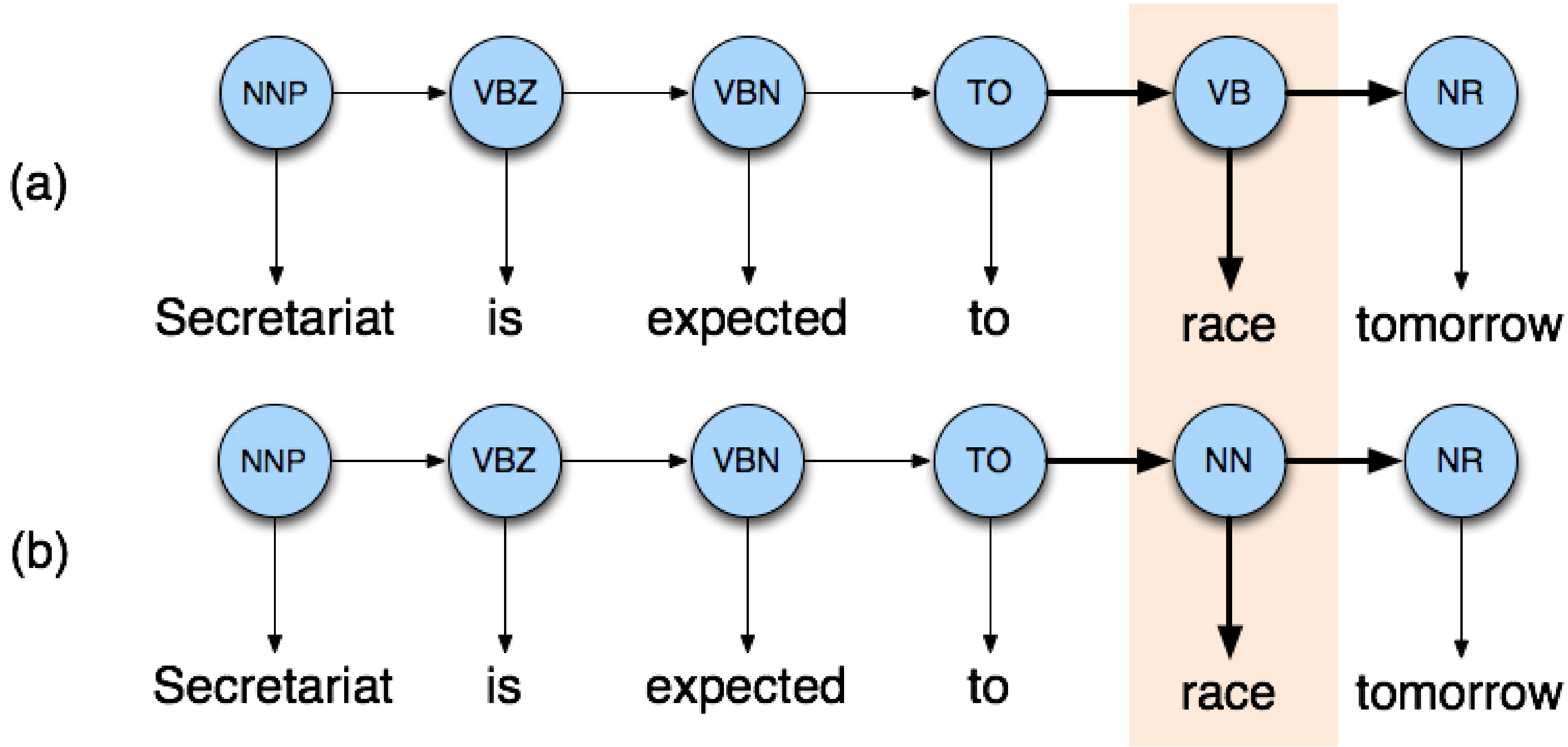
- Secretariat/**NNP** is/**VBZ** expected/**VBN** to/**TO** **race**/**VB** tomorrow/**NR**

- People/**NNS** continue/**VB** to/**TO** inquire/**VB** the/**DT** reason/**NN** for/**IN** the/**DT** **race**/**NN** for/**IN** outer/**JJ** space/NN

- How do we pick the right tag for word "race" in each sentence?

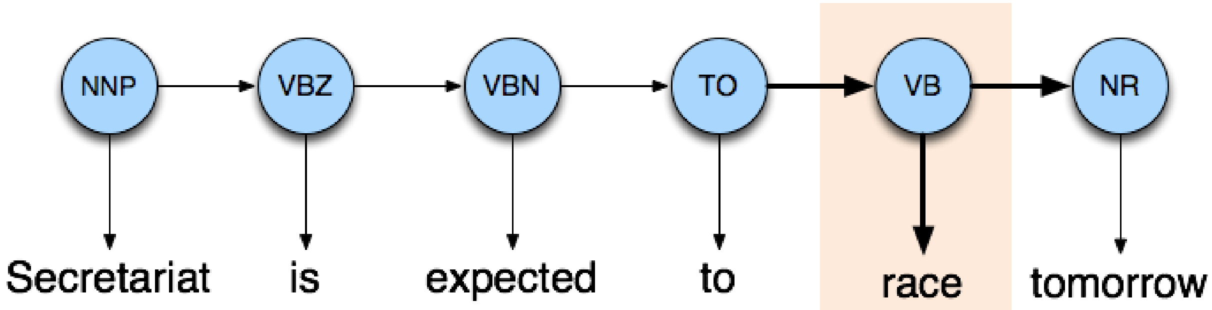
Disambiguating “race”

- Assuming tags of other words are known (for this example)



Calculating estimated probability

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} P(t_1^n|w_1^n) \approx \underset{t_1^n}{\operatorname{argmax}} \prod_{i=1}^{n} P(w_i|t_i)P(t_i|t_{i-1})$$



$$p(Secretariat|NNP) * p(NNP|Start)$$
$$* p(is|VBZ) * p(VBZ|NNP)$$
$$* p(expected|VBN) * p(VBN|VBZ)$$
$$* p(to|TO) * p(TO|VBN)$$
$$* p(race|VB) * p(VB|TO)$$
$$* p(tomorrow|NR) * p(NR|VB)$$

Example

- From a training corpus, we know
  - $P(NN|TO)$ = .00047 $P(VB|TO)$ = .83 $P(race|NN)$ = .00057
  - $P(race|VB)$ = .00012 $P(NR|VB)$ = .0027 $P(NR|NN)$ = .0012

- How to use the above information to do POS tagging?
  - $P(VB|TO)P(NR|VB)P(race|VB)$ = .00000027
  - $P(NN|TO)P(NR|NN)P(race|NN)$ = .00000000032

- So we (correctly) choose verb for race in this sentence

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} P(t_1^n|w_1^n) \approx \underset{t_1^n}{\operatorname{argmax}} \prod_{i=1}^{n} P(w_i|t_i)P(t_i|t_{i-1})$$

Summary

- Parts of speech, Tagsets, and tagging

- Next: HMM Tagging
  - Hidden Markov Models
  - Viterbi decoding

- The next two slides are about linguistics and are for your references

Open and Closed Classes

Skip

- Closed class: a small(ish) fixed membership
  - Usually function words (short common words which play a role in grammar)

  - prepositions: *on, under, over, …*
  - particles: *up, down, on, off, …*
  - determiners: *a, an, the, …*
  - pronouns: *she, who, I, ..*
  - conjunctions: *and, but, or, …*
  - auxiliary verbs: *can, may should, …*
  - numerals: *one, two, three, third, …*

- Open class: new ones can be created all the time
  - English has 4: Nouns, Verbs, Adjectives, Adverbs
    - Many languages have these 4, but not all!
  - Nouns are typically where the bulk of the action is with respect to new items

Open Class Words

Skip

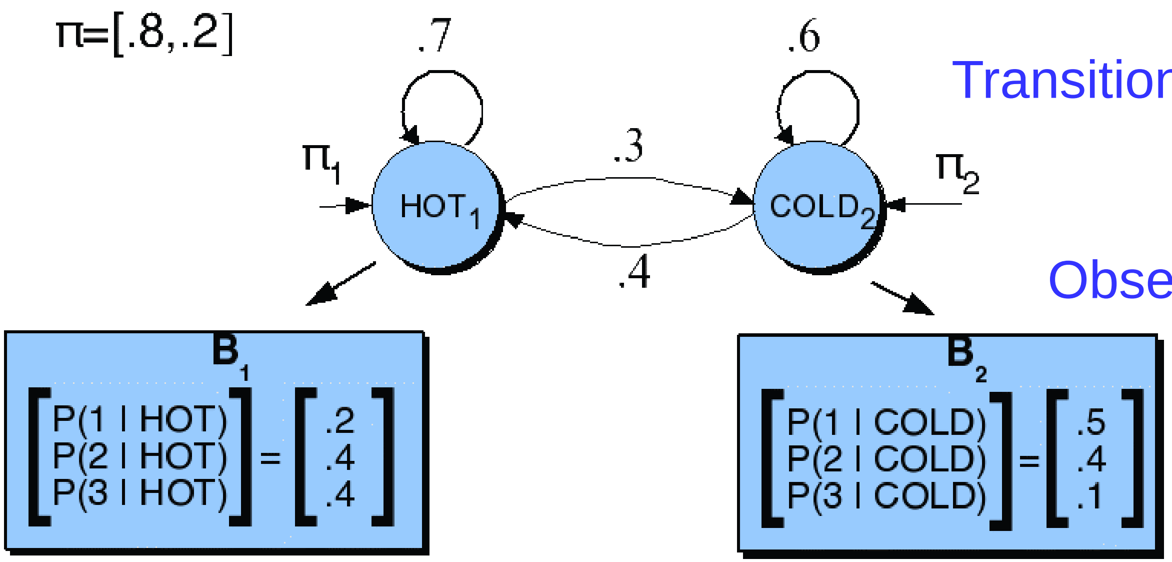
- Nouns
  - Proper nouns (Boulder, Granby, Beyoncé) -- English capitalizes these.
  - Common nouns (the rest)
  - Count nouns and mass nouns
    - Count: have plurals, get counted: goat/goats, one goat, two goats
    - Mass: don't get counted (snow, salt, communism) (*two snows)
- Adverbs: tend to modify things
  - Unfortunately, John walked home extremely slowly yesterday
  - Directional/locative adverbs (here,home, downhill)
  - Degree adverbs (extremely, very, somewhat)
  - Manner adverbs (slowly, slinkily, delicately)
- Verbs: In English, have morphological affixes (eat/eats/eaten)
  - With differing patterns of regularity

HMM for Ice Cream

- You are a climatologist in the year 2799 studying global warming
- You can’t find any records of the weather in Singapore for summer of 2018
- But you find your grandma’s diary which lists how many ice-creams she ate every date that summer
- Your job: figure out whether each day was cold/hot

Example of sequence prediction

- Can the number of ice cream eaten be used to **predict** the weather?
  - Ice cream observation sequence: 2,1,3,2,2…
  - Weather Sequence: H,C,H,H,C…



Transition Probabilities

Observation Likelihoods

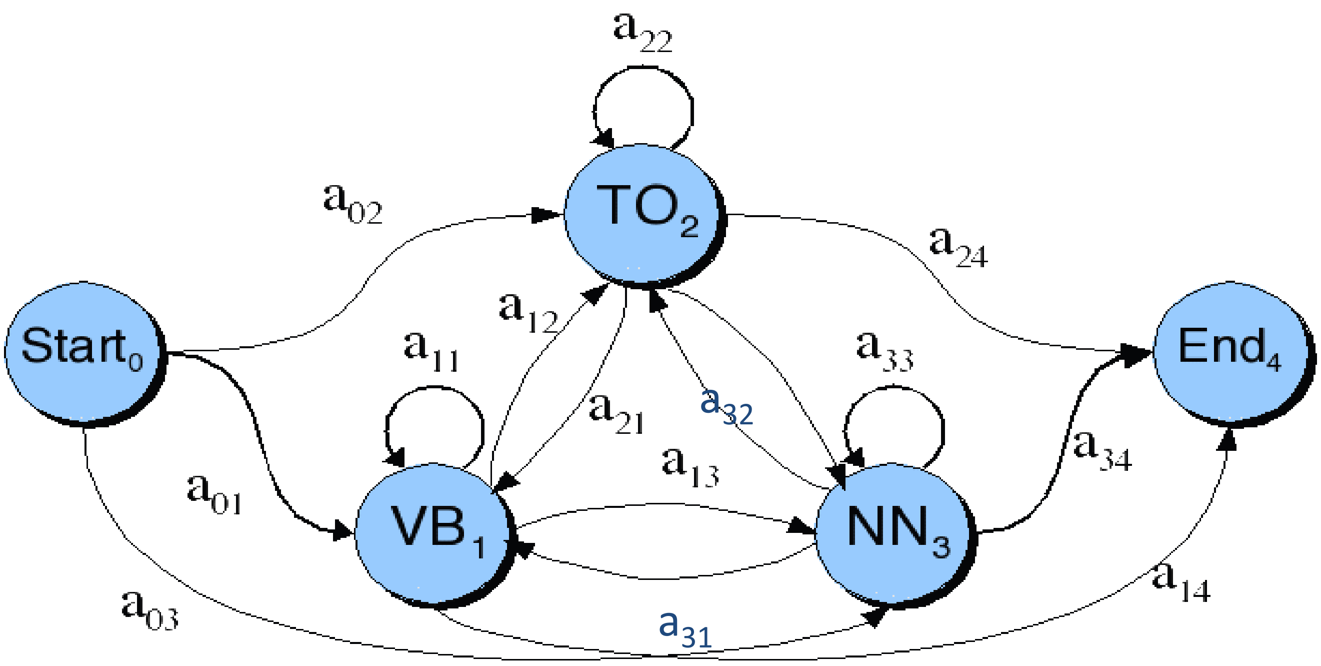
Hidden Markov Models

- What we've described with these two kinds of probabilities is a Hidden Markov Model (HMM)
  - Transition Probabilities
  - Observation Likelihoods

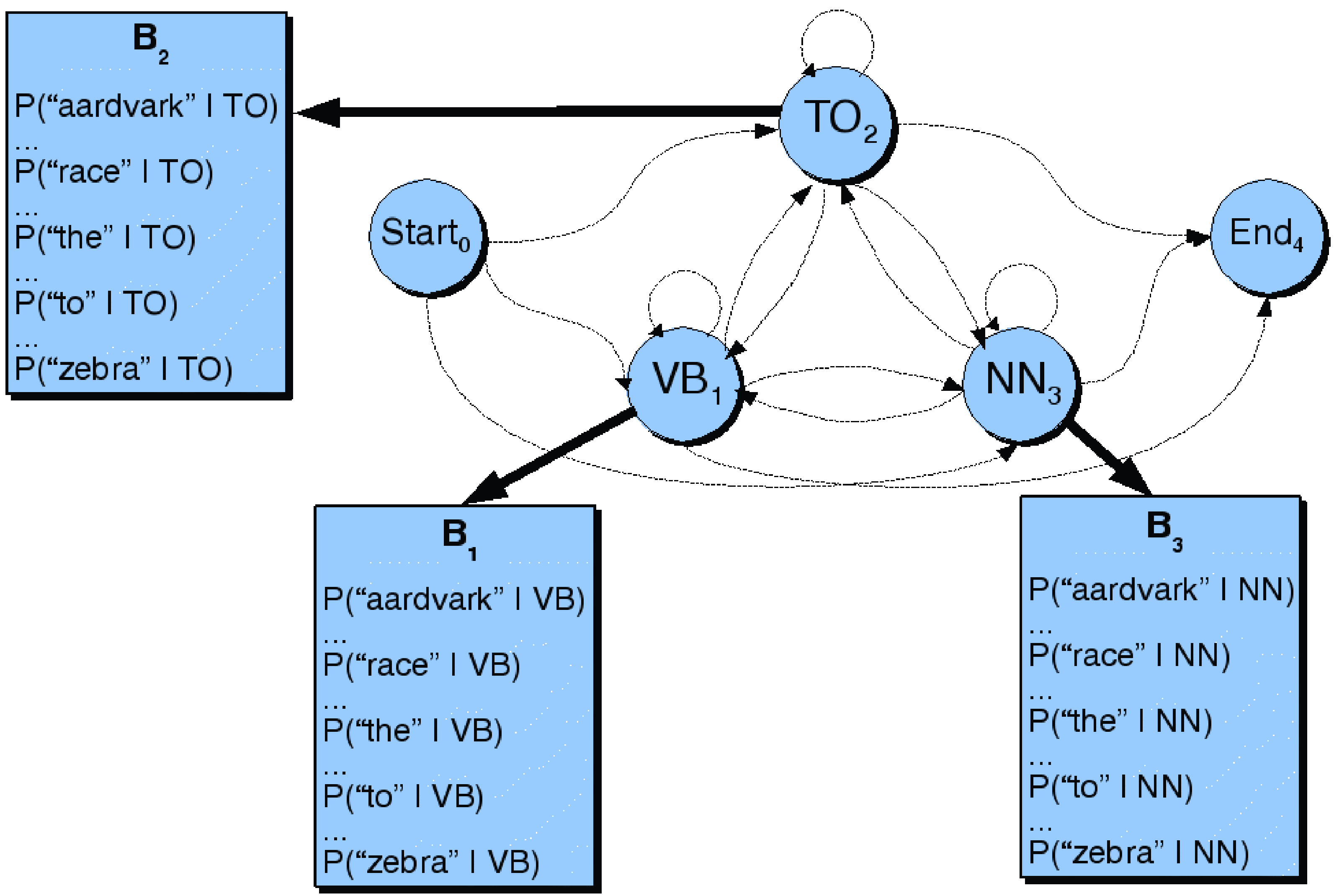
- Formalizing HMM:
  - A **weighted finite-state automaton** where each arc is associated with a probability
  - The probability indicates how likely a path is to be taken

Transition Probabilities

- The sum of the probabilities leaving any arc must sum to one
  - For example, $a_{01} + a_{02} + a_{03} \ = 1$

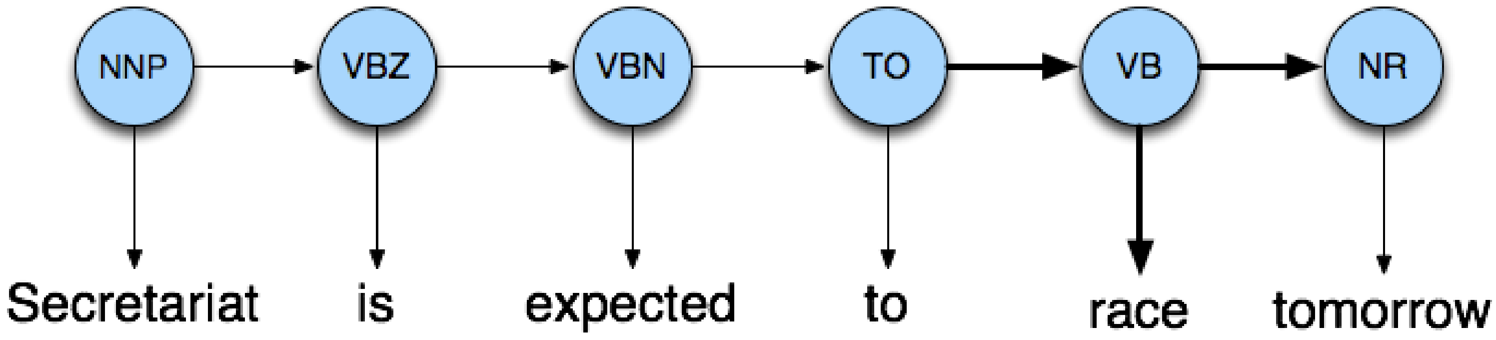


Observation Likelihoods



Hidden Markov Model

- In part-of-speech tagging
  - The input symbols are **words**
  - But the hidden states are **part-of-speech tags**



- It has many other applications
  - Named entity recognition, gene prediction, etc

Hidden Markov Models

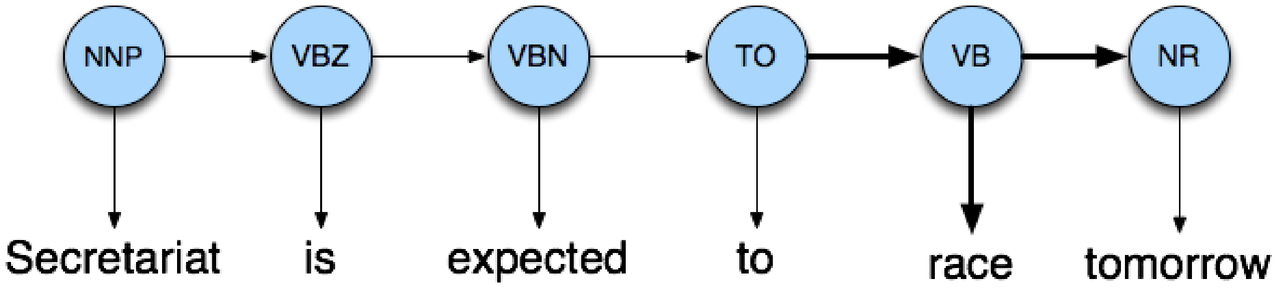
- States $Q = q_1, q_2 \dots q_N$; and the start and end states $\boldsymbol{q_0}, \boldsymbol{q_F}$
- Observations $O = o_1, o_2 \dots o_T$;
  - Each observation is a symbol from a vocabulary $V = \{v_1, v_2, \dots v_V\}$
  - s_i: the state of the i-th observation;
  - $q_0,\ q_F$ are not associated with observations

- Transition probabilities: Transition probability matrix $A = \{a_{ij}\}$;
  - $a_{ij} = \mathrm{P}(\mathrm{s_t} = \mathrm{j} | \mathrm{s_{t-1}} = \mathrm{i})\ \ 1 \le i, j \le N$
- Observation likelihoods: Output probability matrix $B = \{b_i(k)\}$;
  - $b_i(k) = P(X_t = o_k | s_t = i)$
- Special initial probability vector π;
  - $\pi_i = P(s_1 = i)\ 1 \le i \le N$

Hidden Markov Model

Recall

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} P(t_1^n | w_1^n) \approx \underset{t_1^n}{\operatorname{argmax}} \prod_{i=1}^{n} P(w_i|t_i) P(t_i|t_{i-1})$$



$$\begin{aligned}
&p(Secretariat|NNP) * p(NNP|Start)\\
&* p(is|VBZ) * p(VBZ|NNP)\\
&* p(expected|VBN) * p(VBN|VBZ)\\
&* p(to|TO) * p(TO|VBN)\\
&* p(race|VB) * p(VB|TO)\\
&* p(tomorrow|NR) * p(NR|VB)
\end{aligned}$$

Decoding

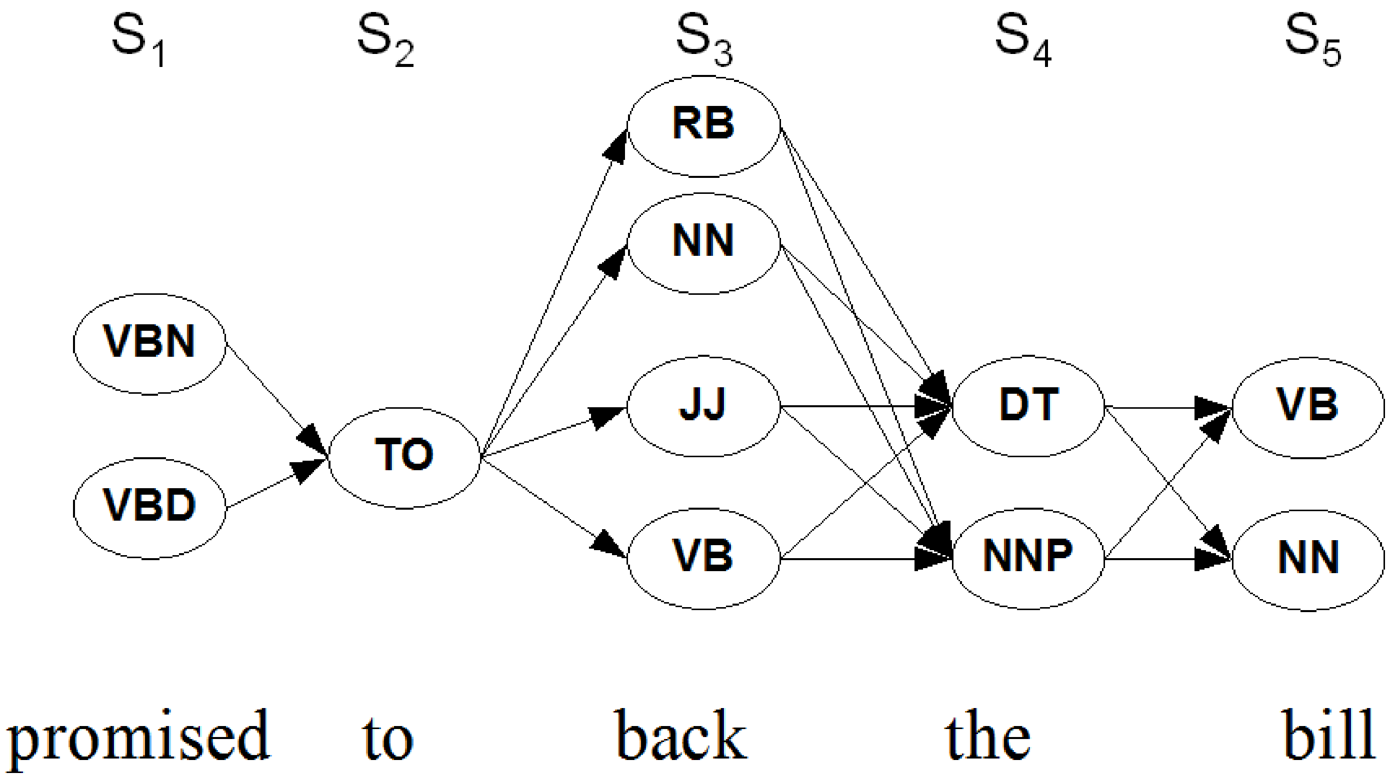
- Now we have a complete model and we need to get

$$\hat{t}_1^n = arg \max_{t_1^n} P(t_1^n | w_1^n)$$

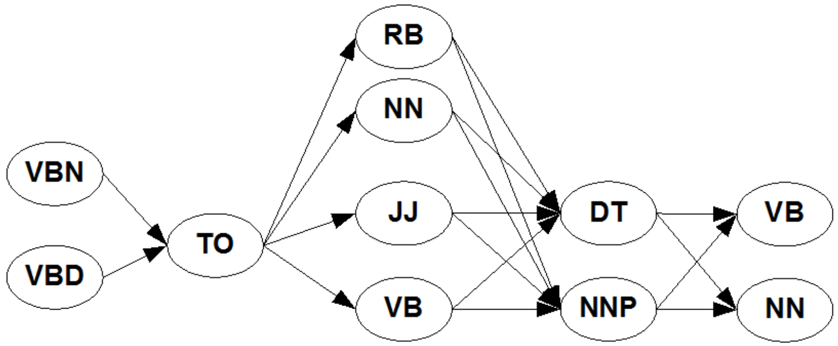
- **Determine** sequences of variables, **given** sequence of observations

- We could just enumerate all paths given the input and use the model to assign probabilities to each.
  - Not a good idea. 1 2 -- HH, HC,CC,CH
  - N^T : N (number of states) T (size of sequence)
  - Dynamic programming helps us here

Example sentence



Enumerate all paths



- VBN TO RB DT VB
- VBN TO RB DT NN
- VBN TO RB NNP VB
- VBN TO RB NNP NN
- VBN TO NN DT VB
- VBN TO NN DT NN
- VBN TO NN NNP VB
- VBN TO NN NNP NN
- VBN TO JJ DT VB
- VBN TO JJ DT NN
- VBN TO JJ NNP VB
- VBN TO JJ NNP NN
- VBN TO VB DT VB
- VBN TO VB DT NN
- VBN TO VB NNP VB
- VBN TO VB NNP NN
- VBD TO RB DT VB
- VBD TO RB DT NN
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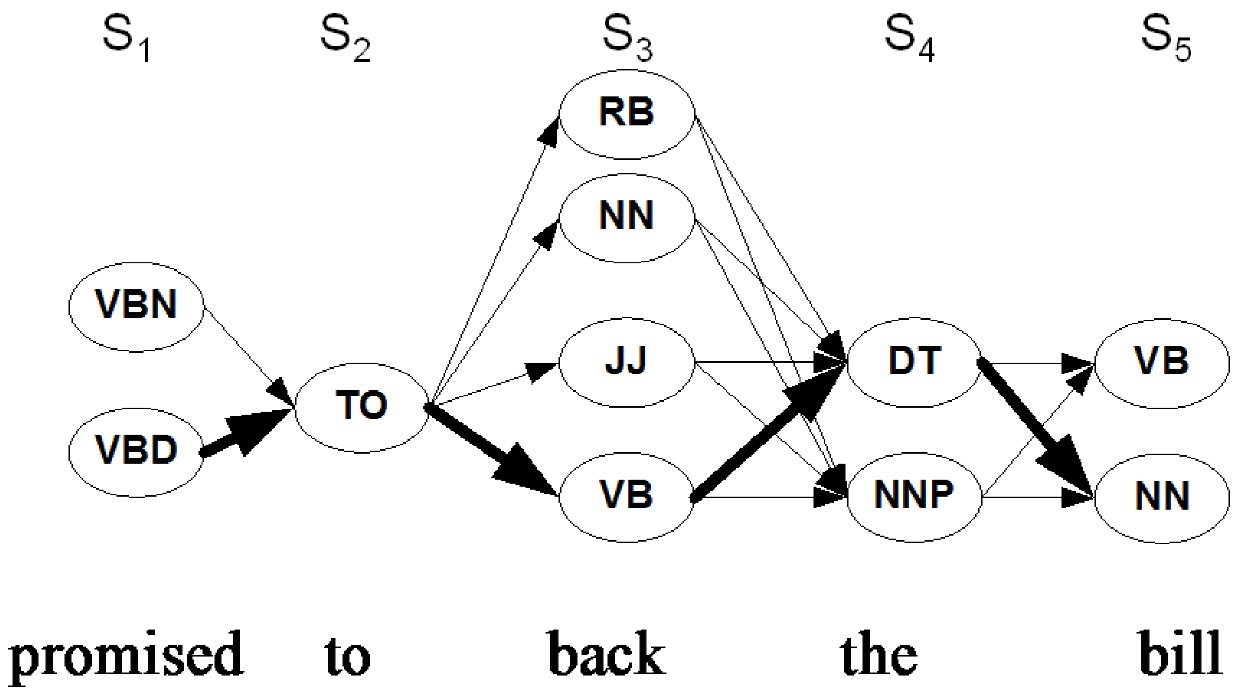
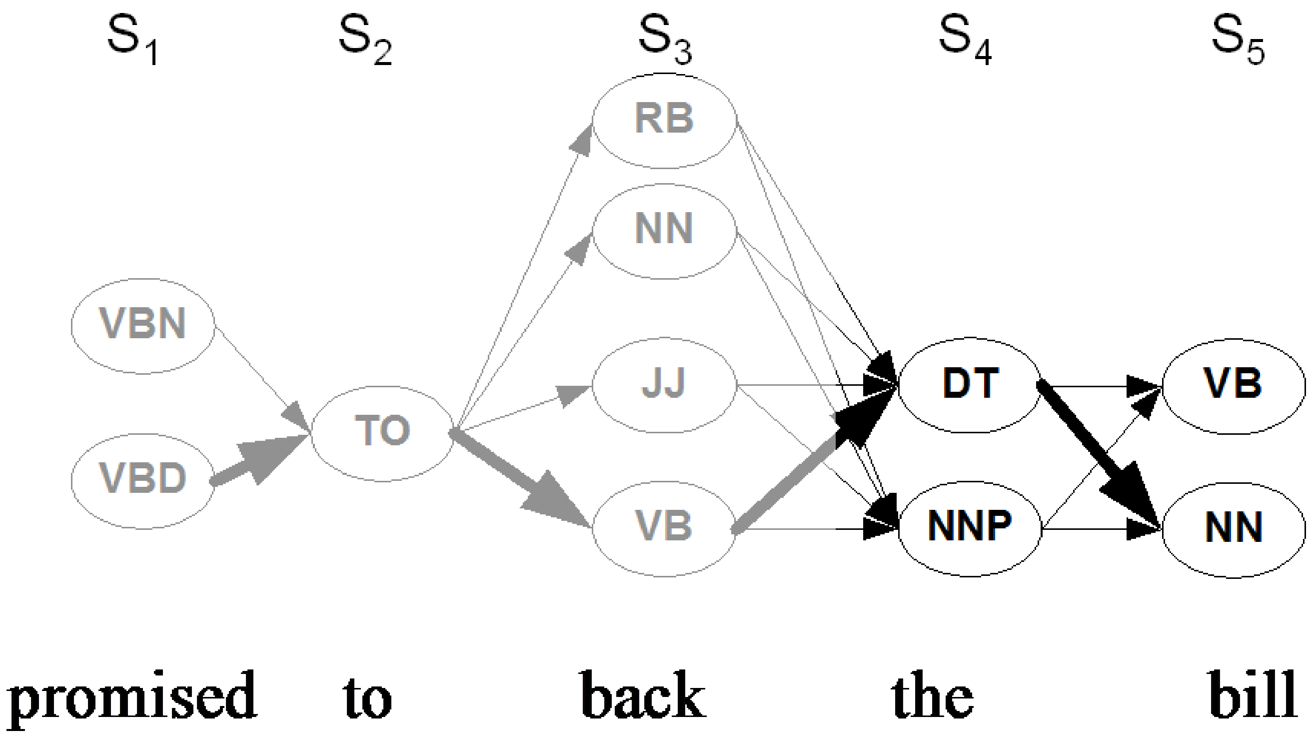
- VBD TO NN NNP NN
- VBD TO JJ DT VB
- VBD TO JJ DT NN
- VBD TO JJ NNP VB
- VBD TO JJ NNP NN
- VBD TO VB DT VB
- VBD TO VB DT NN
- VBD TO VB NNP VB
- VBD TO VB NNP NN

The best choice?



From DT to NN?



S_1
S_2
S_3
S_4
S_5
RB
NN
VBN
JJ
DT
VB
TO
VBD
VB
NNP
NN
promised
to
back
the
bill

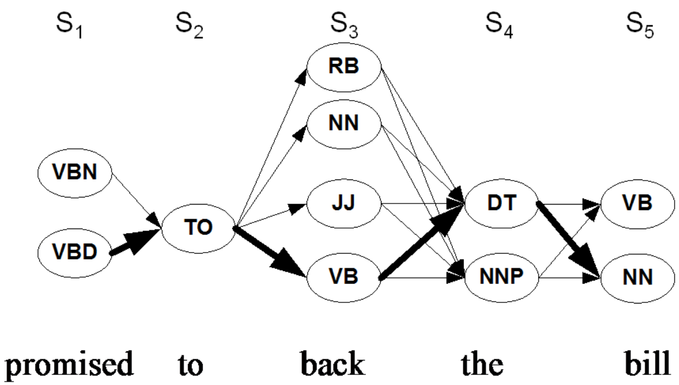
Intuition

- Consider a state sequence (tag sequence) that ends at time t with a particular tag i.
- The probability of that tag sequence can be broken into two parts
  - The probability of the **BEST** tag sequence up through $t-1$
  - Multiplied **by the transition probability** from the tag at the end of the $t-1$ sequence to i, and the observation probability of the word given tag i.
- Let j be the tag at the end of the $t-1$ sequence, and W be the word at time t

$$\underbrace{Viterbi[i,t]}_{v_t[i]} = Viterbi[j,t-1] \times \underbrace{p(i|j)}_{a_{j,i}} \times \underbrace{p(W|i)}_{b_i(W)}$$

Consider paths ending with bill:NN

From S4, we have two paths P1, P2 to reach NN



- $p_1 = v_4[DT] * p(NN|DT) * p(bill|NN)$
- $p_2 = v_4[NNP] * p(NN|NNP) * p(bill|NN)$

- $v_5[NN] = \max(p_1, p_2)$

- $$\begin{aligned} v_4[DT] = \max(&v_3[RB] * p(DT|RB) * p(the|DT), \\ &v_3[NN] * p(DT|NN) * p(the|DT), \\ &v_3[JJ] * p(DT|JJ) * p(the|DT), \\ &v_3[VB] * p(DT|VB) * p(the|DT)) \end{aligned}$$
  $$= \max(v_3[RB] * p(DT|RB), v_3[NN] * p(DT|NN), v_3[JJ] * p(DT|JJ), v_3[VB] * p(DT|VB)) * p(the|DT)$$

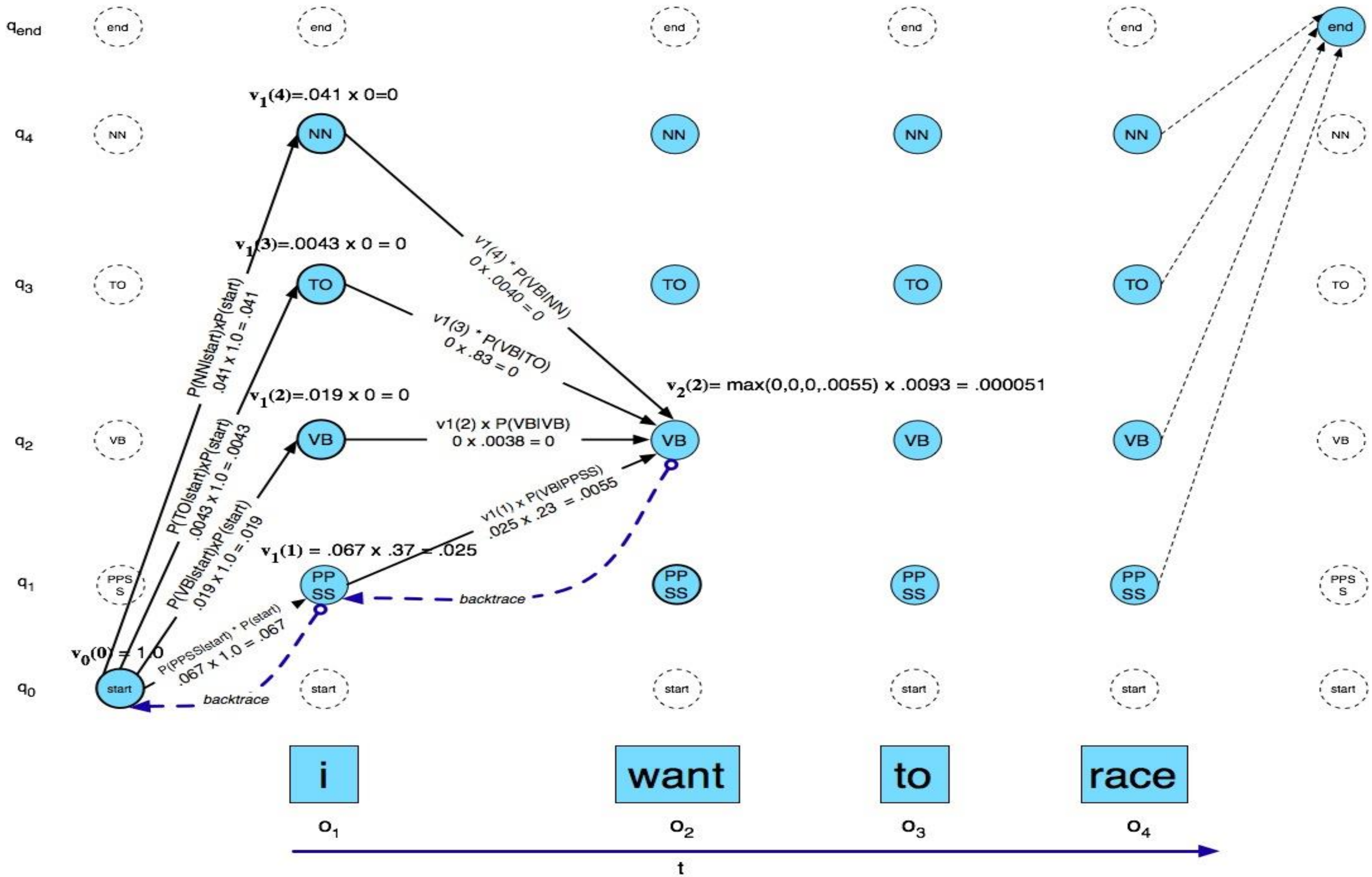
Main Idea

- We also have a matrix.
  - Each column– a time 't' (observation)
  - Each row – a state 'i'
  - For each cell $v_t[i]$, we compute the probability of the **best** path to the cell

- Viterbi path probability at time t for state i
  - there are $|Q|$ number of paths from $t-1$ to $v_t[i]$
  - if we know the best path to each cell in $t-1$ (v_{t-1} $[j]$)

$$\arg\max_j v_{t-1}[j] \times P(i|j) \times P(s_t|i)$$

Viterbi Example: Variable $v_t[i]$ the Viterbi path probability at time t for state i



Example continue from previous slide

- $v_2[NN] = \max(v_1[NN] * p(NN|NN), v_1[TO] * p(NN|TO), v_1[VB] * p(NN|VB), v_1[PPSS] * p(NN|PPSS)) * p(want|NN)$
- $= \max(0 * 0.087, 0 * 0.00047, 0 * 0.047, 0.025 * 0.0012) * 0.000054$

	VB	TO	NN	PPSS
<s>	.019	.0043	.041	.067
VB	.0038	.035	.047	.0070
TO	.83	0	.00047	0
NN	.0040	.016	.087	.0045
PPSS	.23	.00079	.0012	.00014

	I	want	to	race
VB	0	.0093	0	.00012
TO	0	0	.99	0
NN	0	.000054	0	.00057
PPSS	.37	0	0	0

The Viterbi Algorithm

T: word N: tag

function VITERBI(*observations* of len T,*state-graph* of len N) **returns** *best-path*

create a path probability matrix *viterbi[N+2,T]*

for each state s **from** 1 **to** N **do** ; initialization step

$viterbi[s,1] \leftarrow a_{0,s} * b_s(o_1)$

$backpointer[s,1] \leftarrow 0$

for each time step t **from** 2 **to** T **do** ; recursion step

for each state s **from** 1 **to** N **do**

$$viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$$

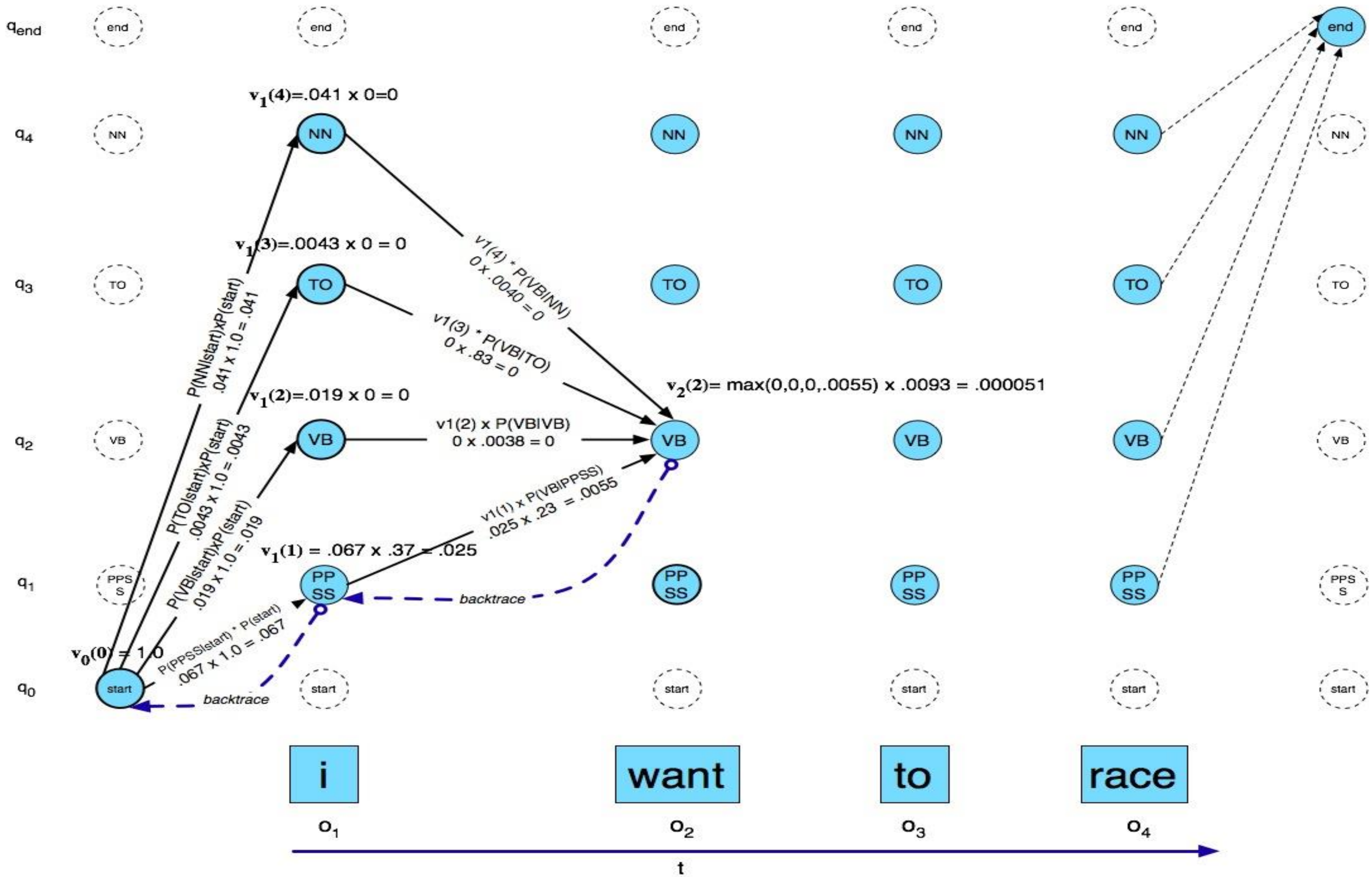
$$backpointer[s,t] \leftarrow \operatorname{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s}$$

$$viterbi[q_F,T] \leftarrow \max_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step

$$backpointer[q_F,T] \leftarrow \operatorname{argmax}_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step

return the backtrace path by following backpointers to states back in time from $backpointer[q_F,T]$

Viterbi Example: Variable $v_t[i]$ the Viterbi path probability at time t for state i



Viterbi Summary

- Create a matrix (two-dimensional array)
  - With columns corresponding to inputs
  - Rows corresponding to possible states

- Sweep through the array in one pass filling the columns **left to right** using our transition probs and observations probs

- Dynamic programming key is that we need only store the **MAX** prob path to each cell, (not all paths).

Summary

- HMM
  - Transition Probabilities
  - Observation Likelihoods
- Decoding
  - Viterbi

- Next
  - Evaluation
  - Assigning probabilities to inputs
    - Forward
  - Finding optimal parameters for a model

Evaluation

- The result is compared with a manually coded “Gold Standard”
  - Typically accuracy reaches 96-97%
  - This may be compared with result for a baseline tagger (one that uses no context).

- Important: 100% is impossible even for human annotators.

HMM

- Given this framework there are 3 problems that we can pose to an HMM
  - Given an observation sequence and a model, what is the most likely state sequence?
  - Given an observation sequence, what is the probability of that sequence given a model?
  - Given an observation sequence, infer the best parameters for model

Problem

- Most probable state sequence given a model and an observation sequence

Decoding: Given as input an HMM $\lambda = (A, B)$ and a sequence of observations $O = o_1, o_2, ..., o_T$, find the most probable sequence of states $Q = q_1 q_2 q_3 \ldots q_T$.

  - Typically used in tagging problems, where the tags correspond to hidden states
  - Viterbi solves problem

Problem

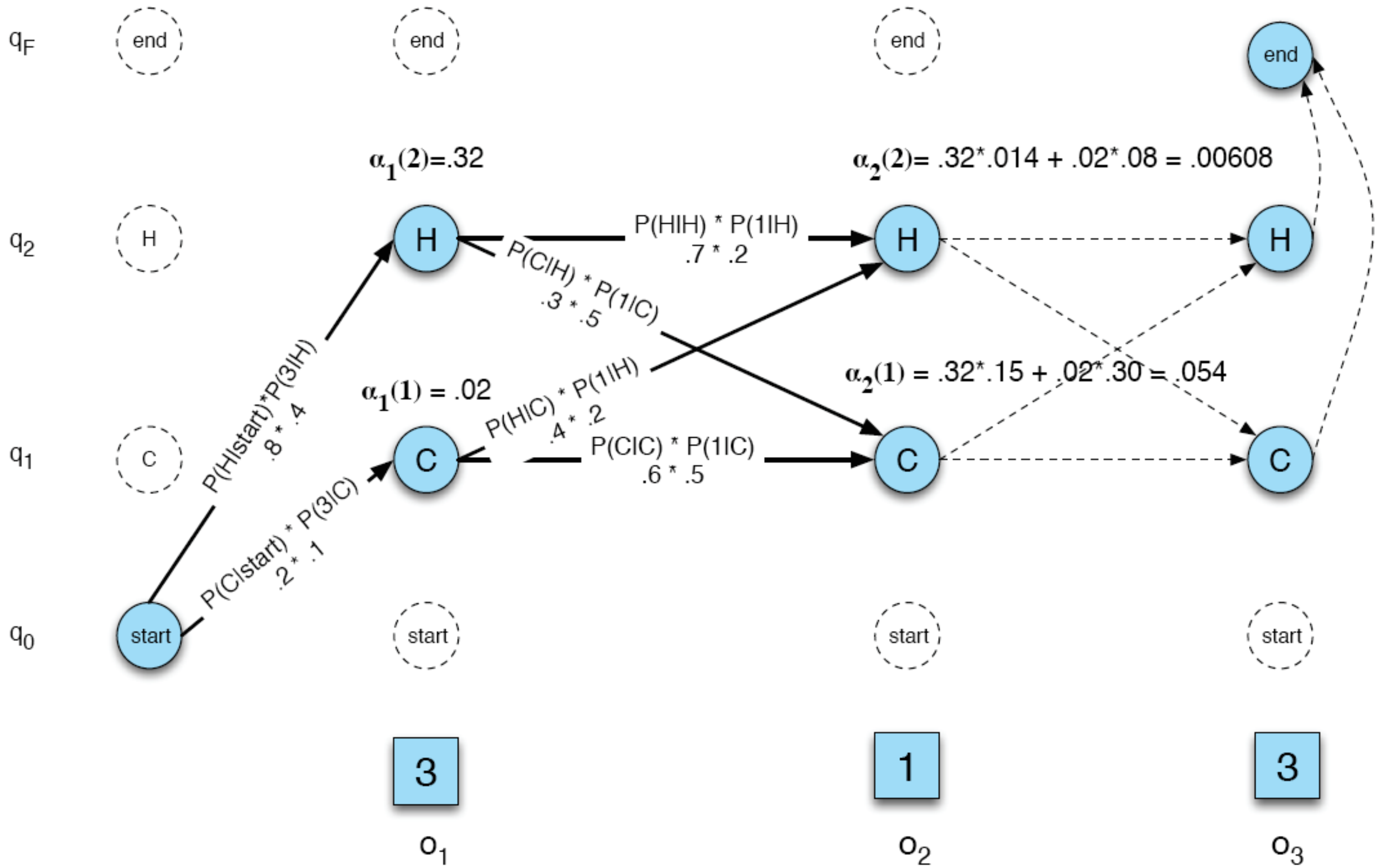
- The probability of a sequence given a model.. $P(seq|model)$.

Computing Likelihood: Given an HMM $\lambda = (A, B)$ and an observation sequence O, determine the likelihood $P(O|\lambda)$.

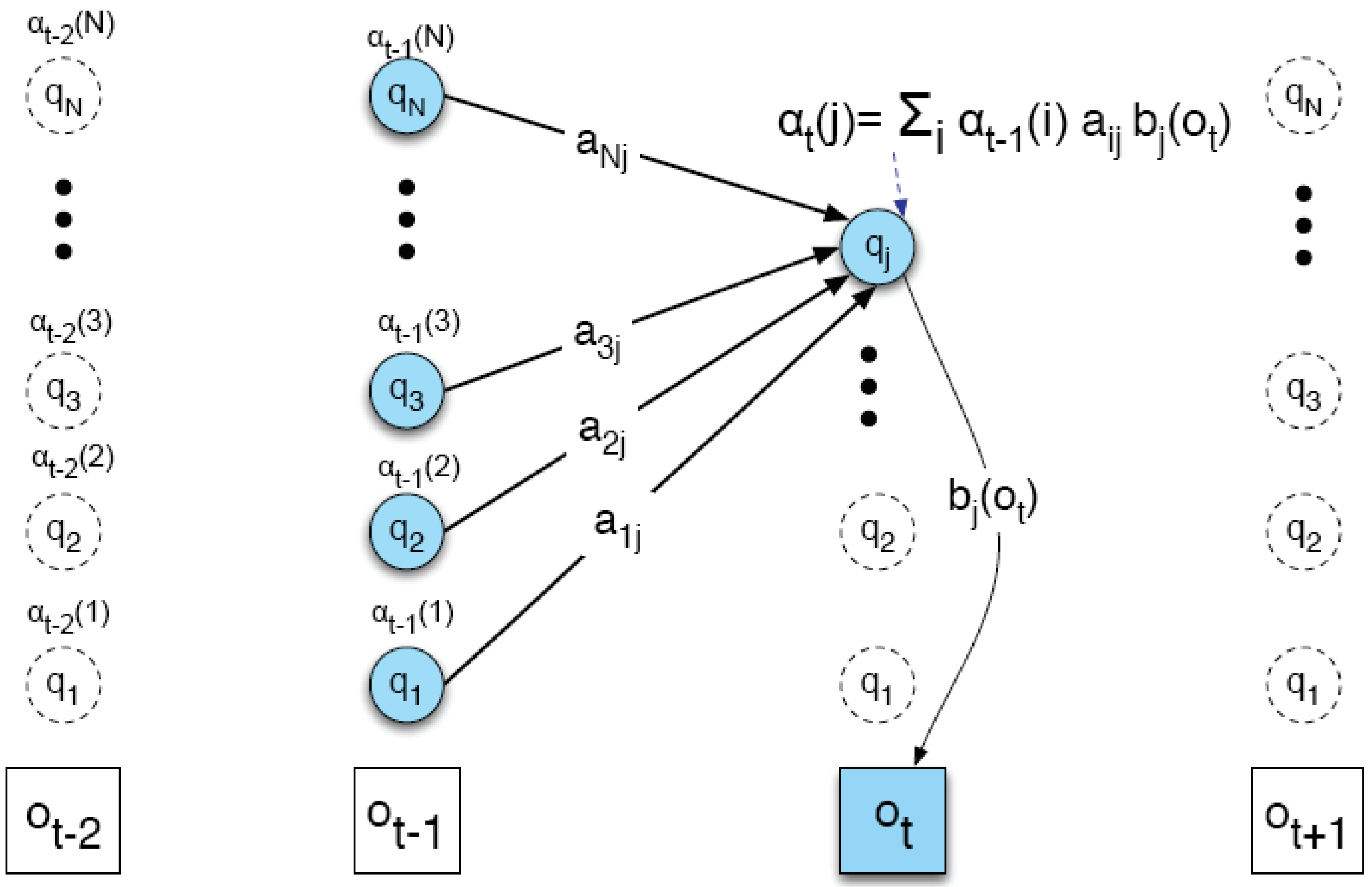
- Forward algorithm
  - Efficiently computes the probability of an observed sequence given a model
  - $P(sequence|model)$
  - Nearly identical to Viterbi: replace the MAX with a SUM

Ice Cream Example

Variable $a_t[i]$ the forward path probability at time t for state i



Forward algorithm: SUM



Forward

function FORWARD(*observations* of len T, *state-graph* of len N) **returns** *forward-prob*

create a probability matrix *forward[N+2,T]*

for each state s **from** 1 **to** N **do** ; initialization step

$$forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$$

for each time step t **from** 2 **to** T **do** ; recursion step

for each state s **from** 1 **to** N **do**

$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,T] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F}$$; termination step

return $forward[q_F,T]$

Summary

- HMM model- two probabilities
- Viterbi algorithm
- Evaluation
- Three problems in HMM model