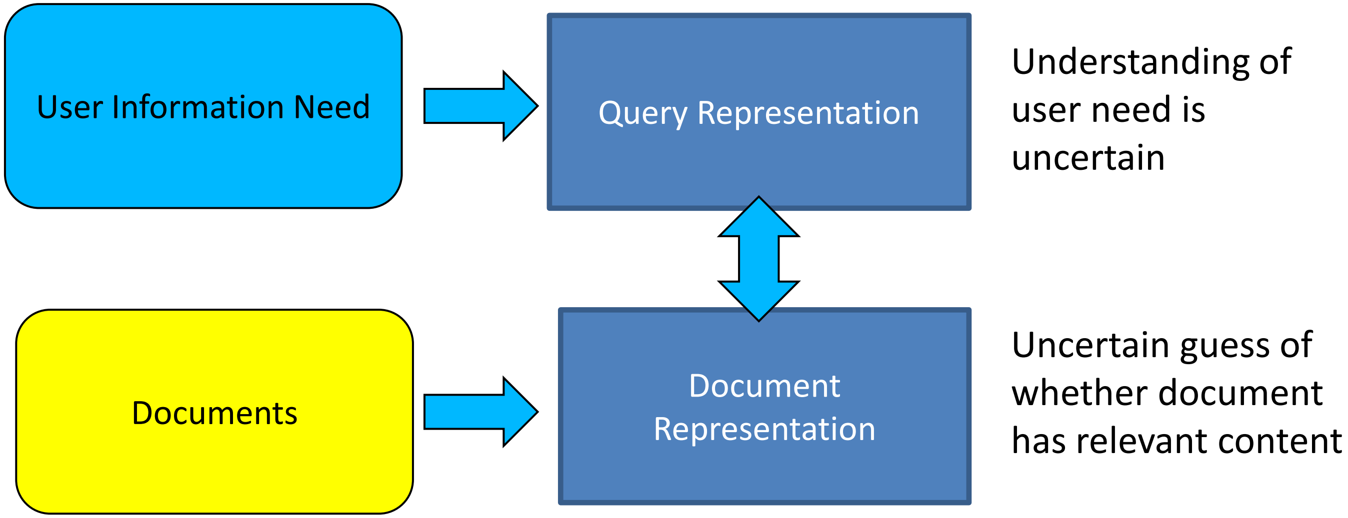
AI6122 Text Data Management & Analysis

Topic: Probabilistic Ranking

Why Probabilistic IR?

- In traditional IR systems, matching between each document and a query is attempted in a **semantically imprecise space** of index terms.
- Can we use probabilities to quantify the uncertainties?



Probabilistic IR topics

- Classical probabilistic retrieval model
- Language model approach to IR

Recall a few probability basics

- Bayes' Rule: for events a and b:

$$P(a,b) = P(a \cap b) = P(a|b)P(b) = P(b|a)P(a)$$

$$P(b) = P(b|a)P(a) + P(b|\bar{a})P(\bar{a})$$

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)} = \frac{P(b|a)P(a)}{\sum_{x=a,\bar{a}} P(b|x)P(x)}$$

- Odds:

$$O(a) = \frac{P(a)}{P(\bar{a})} = \frac{P(a)}{1 - P(a)}$$

Overview

- Probabilistic Approach to Retrieval
- Basic Probability Theory
- Probability Ranking Principle [**For your information only**]
- Language Models for Information Retrieval

The Document Ranking Problem

- Settings: We have a collection of documents
  - User issues a query
  - A list of documents need to be returned

- Ranking method is core of an IR system
  - In what order do we present documents to the user?
  - "Best" documents to be placed top

- Idea: rank by probability of relevance of the document w.r.t. information need
  - $P(Relevance|document, query)$

Probability ranking principle (PRP)

SKIP

- Let x be a document in the collection
- Let R represent **relevance** of a document w.r.t. given query and let NR represent **non-relevance**

- Need to find $P(R|x)$:
  – the probability that a document x is relevant to the query:

$$P(R|x) = \frac{P(x|R)P(R)}{P(x)} \qquad P(NR|x) = \frac{P(x|NR)P(NR)}{P(x)}$$

- $P(R|x) + P(NR|x) = 1$

Probability ranking principle (PRP)

SKIP

- Bayes' optimal decision rule: x is relevant iff $P(R|x) > P(NR|x)$
- PRP in action: Rank all documents by $P(R|x)$
- How do we compute all those probabilities?
  - Do not know exact probabilities, have to use **estimates**
  - Simplest model: Binary Independence Retrieval (BIR)
    - Binary means Boolean: document/query is represented by binary term vectors
- (Questionable) Assumptions
  - Relevance of each document is independent from relevance of other documents
    - how about duplicates?
  - Single step information need
    - how about query reformulation?

Binary Independence Model (BIM)

- Traditionally used in conjunction with the PRP
- ‘Binary’ (=Boolean)
  - Documents and queries represented as binary term incidence vectors
  - Document d represented by vector $\vec{x} = (x_1, \ldots, x_M)$, where $x_t = 1$ if term t occurs in d and $x_t = 0$ otherwise

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Binary Independence Model (BIM)

SKIP

- 'Independence': no association between terms
  - not true, but practically works
  - 'naive' assumption of Naive Bayes models

- Similar to Bernoulli Naïve Bayes model
  - One feature X_w for each word in dictionary
  - X_w=True in document d if w appears in d
  - Naïve Bayes assumption
    - Given the document's topic, appearance of one word in the document tells us nothing about the chances that another word appears.

Binary Independence Model (BIM)

SKIP

- Queries: binary term incidence vectors

- Given query $\vec{\boldsymbol{q}}$,
  - for each document $\boldsymbol{d}$ (represented as $\vec{\boldsymbol{x}}$), compute $\boldsymbol{P}(\boldsymbol{R}|\vec{\boldsymbol{q}},\vec{\boldsymbol{x}})$
  - Interested only in ranking

- Will use odds and Bayes's Rule:

$$O(R|\vec{q},\vec{x}) = \frac{P(R|\vec{q},\vec{x})}{P(NR|\vec{q},\vec{x})} = \frac{\frac{P(R|\vec{q})P(\vec{x}|R,\vec{q})}{P(\vec{x}|\vec{q})}}{\frac{P(NR|\vec{q})P(\vec{x}|NR,\vec{q})}{P(\vec{x}|\vec{q})}} = \frac{P(R|\vec{q})}{P(NR|\vec{q})} \times \frac{P(\vec{x}|R,\vec{q})}{P(\vec{x}|NR,\vec{q})}$$

Deriving a Ranking Function for Query Terms

SKIP

- Using Independence Assumption:

$$\frac{P(\vec{x}|R,\vec{q})}{P(\vec{x}|NR,\vec{q})} = \prod_{i}^{n} \frac{P(x_i|R,\vec{q})}{P(x_i|NR,\vec{q})}$$

- Then we have:

$$O(R|\vec{q},\vec{x}) = \frac{P(R|\vec{q})}{P(NR|\vec{q})} \times \prod_{i}^{n} \frac{P(x_i|R,\vec{q})}{P(x_i|NR,\vec{q})}$$

Document independent

Deriving a Ranking Function for Query Terms

SKIP

- Since x_i is either 0 or 1: $$O(R|\vec{q},\vec{x}) = O(R|\vec{q}) \cdot \prod_{i=1}^{n} \frac{P(x_i|R,\vec{q})}{P(x_i|NR,\vec{q})}$$

$$O(R|\vec{q},\vec{x}) = O(R|\vec{q}) \cdot \prod_{x_i=1} \frac{P(x_i=1|R,\vec{q})}{P(x_i=1|NR,\vec{q})} \cdot \prod_{x_i=0} \frac{P(x_i=0|R,\vec{q})}{P(x_i=0|NR,\vec{q})}$$

- Assume: Ignore all terms not occurring in the query (q_i=*0*)

$$O(R|\vec{q},\vec{x}) = O(R|\vec{q}) \cdot \prod_{x_i=1,q_i=1} \frac{P(x_i=1|R,\vec{q})}{P(x_i=1|NR,\vec{q})} \cdot \prod_{x_i=0,q_i=1} \frac{P(x_i=0|R,\vec{q})}{P(x_i=0|NR,\vec{q})}$$

For all matching query terms

For all non-matching query terms

Deriving a Ranking Function for Query Terms

SKIP

$$O(R|\vec{q},\vec{x}) = O(R|\vec{q}) \cdot \prod_{x_i=1,q_i=1} \frac{P(x_i=1|R,\vec{q})}{P(x_i=1|NR,\vec{q})} \cdot \prod_{x_i=0,q_i=1} \frac{P(x_i=0|R,\vec{q})}{P(x_i=0|NR,\vec{q})}$$

- Let

$$p_i = P(x_i = 1|R,\vec{q})$$
$$r_i = P(x_i = 1|NR,\vec{q})$$

	document	relevant (R)	nonrelevant (NR)
Term present	$x_i = 1$	p_i	r_i
Term absent	$x_i = 0$	$1 - p_i$	$1 - r_i$

P_i is the probability of term x_i appearing in a relevant document

$$O(R|\vec{q},\vec{x}) = O(R|\vec{q}) \cdot \prod_{x_i=q_i=1} \frac{p_i}{r_i} \cdot \prod_{x_i=0,q_i=1} \frac{1-p_i}{1-r_i}$$

$$= O(R|\vec{q}) \cdot \prod_{x_i=q_i=1} \frac{p_i(1-r_i)}{r_i(1-p_i)} \cdot \prod_{q_i=1} \frac{1-p_i}{1-r_i}$$

Including query terms found in documents into the right product and simultaneously dividing through by them in the left product

Document independent

Binary Independence Model (BIM)

SKIP

$$O(R|\vec{q},\vec{x}) = O(R|\vec{q}) \cdot \prod_{x_i=q_i=1} \frac{p_i(1-r_i)}{r_i(1-p_i)} \cdot \prod_{q_i=1} \frac{1-p_i}{1-r_i}$$

Constant for each query

- Retrieval Status Value (RSV)

$$RSV = \log \prod_{x_i=q_i=1} \frac{p_i(1-r_i)}{r_i(1-p_i)} = \sum_{x_i=q_i=1} \log \frac{p_i(1-r_i)}{r_i(1-p_i)} = \sum_{x_i=q_i=1} c_i$$

$$\text{where } c_i = \log \frac{p_i(1-r_i)}{r_i(1-p_i)}$$

Binary Independence Model (BIM)

SKIP

- For each term t, look at the table of document counts:

Documents	Relevant	Non-relevant	Total
Term present ($x = 1$)	s	$df_t - s$	df_t
Term absent ($x = 0$)	$S - s$	$(N - df_t) - (S - s)$	$N - df_t$
Total	S	$N - S$	N

$$RSV = \sum_{x_i = q_i = 1} c_i \quad \text{where } c_i = \log \frac{p_i(1 - r_i)}{r_i(1 - p_i)}$$

$$\text{Estimates: } p_i \approx \frac{s}{S}, \qquad r_i \approx \frac{df_t - s}{N - S}$$

$$c_i \approx \log \frac{\frac{s}{S} \times \left(1 - \frac{df_t - s}{N - S}\right)}{\frac{df_t - s}{N - S} \times \left(1 - \frac{s}{S}\right)} = \log \frac{{}^{s}/_{S - s}}{{}^{df_t - s}/_{N - S - df_t + s}}$$

Estimation – key challenge

SKIP

- r_i (prob. of term occurrence in non-relevant documents for query)
  - If non-relevant documents are approximated by the whole collection, then r_i is $\frac{df_i}{N}$ and
  - $\log\frac{(1-r_i)}{r_i} \approx \log\frac{(N-df_i)}{df_i} \approx \log\frac{N}{df_i} = IDF!$

- p_i (probability of term occurrence in relevant documents) can be estimated in various ways:
  - from relevant documents from user (relevance feedback)
  - constant (e.g. 0.5)
  - proportional to probability of occurrence in collection

$$c_i = \log\frac{p_i(1-r_i)}{r_i(1-p_i)}$$

PRP and BIR

- Among the oldest formal models in IR
  - Since an IR system cannot predict with certainty which document is relevant, we should deal with probabilities

- Requires restrictive assumptions
  - Boolean representation of documents/queries/relevance
  - Term independence
  - Out-of-query terms do not affect retrieval
  - Document relevance values are independent

- Some of the assumptions can be removed
  - A successful method: Okapi BM25 (later)

An Appraisal of Probabilistic Models

- The difference between ‘vector space’ and ‘probabilistic’ IR **is not that great**:
  - In either case you build an information retrieval scheme in the exact same way.

- Difference:
  - For probabilistic IR, at the end, you score queries not by cosine similarity and *tf-idf* in a vector space, but by a slightly different formula motivated by probability theory

Okapi BM25: A Nonbinary Model

- The BIM was originally designed for short catalog records of fairly consistent length, and it works reasonably in these contexts

- For modern full-text search collections, a model should pay attention to term frequency and document length

- BestMatch25 (a.k.a BM25 or Okapi) is sensitive to these quantities

- From 1994 until today, BM25 is one of the most widely used and robust retrieval models

Okapi BM25: A Nonbinary Model

- The simplest score for document d is just idf weighting of the query terms present in the document:
  - $RSV_d = \sum_{t \in q} \log \frac{N}{df_t}$
- Improve this formula by factoring in the term frequency and document length:

$$RSV_d = \sum_{t \in q} \log \left[\frac{N}{df_t} \right] \times \frac{(k_1 + 1) \times tf_{td}}{k_1 \times \left((1 - b) + b \times \left(\frac{L_d}{L_{ave}} \right) \right) + tf_{td}}$$

  - tf_{td} : term frequency in document d
  - L_d (L_{ave}): length of document d (average document length in the whole collection)
  - k_1: tuning parameter controlling the document term frequency scaling
  - b: tuning parameter controlling the scaling by document length

Okapi BM25: A Nonbinary Model

- If the query is long, we might also use similar weighting for query term

$$RSV_d = \sum_{t \in q} \log\left[\frac{N}{df_t}\right] \times \frac{(k_1 + 1) \times tf_{td}}{k_1 \times \left((1 - b) + b \times \left(\frac{L_d}{L_{ave}}\right)\right) + tf_{td}} \times \frac{(k_3 + 1) \times tf_{tq}}{k_3 + tf_{tq}}$$

  - tf_{tq}: term frequency in the query q
  - k_3: tuning parameter controlling term frequency scaling of the query

- The above tuning parameters should ideally be set to optimize performance on a development test collection.
  - In the absence of such optimisation, experiments have shown reasonable values are to set k_1 and k_3 to a value between 1.2 and 2 and $b = 0.75$

Stochastic Language Models

- Model probability of generating strings (each word in turn) in a language
  - commonly all strings over alphabet ∑
  - Example: a unigram model

Model M	
0.20	the
0.10	a
0.01	man
0.01	woman
0.03	said
0.02	likes
…	…

the	**man**	**likes**	**the**	**woman**
0.2	0.01	0.02	0.2	0.01

$$P(s \mid M) = 0.00000008$$

Stochastic Language Models

- Model probability of generating any string

Model M_1	
0.20	the
0.01	class
0.0001	sayst
0.0001	pleaseth
0.0001	yon
0.0005	maiden
0.01	woman

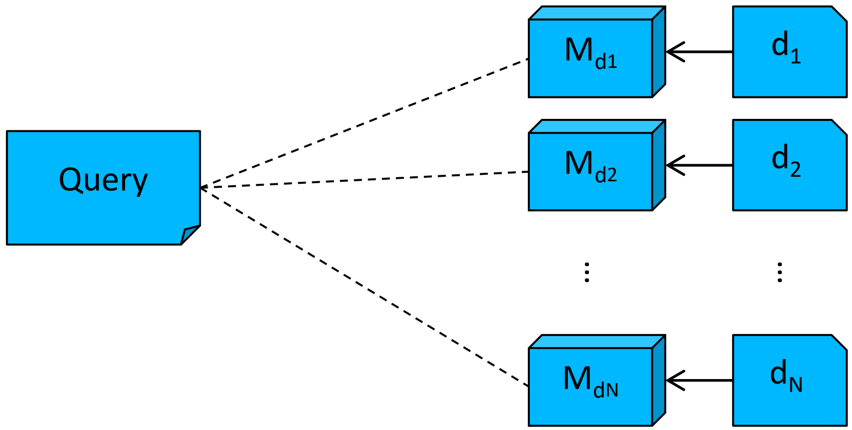
Model M_2	
0.20	the
0.0001	class
0.03	sayst
0.02	pleaseth
0.1	yon
0.01	maiden
0.0001	woman

the	class	pleaseth	yon	maiden
0.2	0.01	0.0001	0.0001	0.0005
0.2	0.0001	0.02	0.1	0.01

$$P(s|M_2) > P(s|M_1)$$

Language models (LMs) for IR

- We view a document as a generative model that generates the query
  - $p(Query|M_d)$

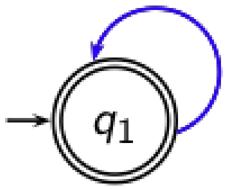


- How do we create such a model for a document?
- How do we generate a query from language model?

A probabilistic language model

- A language model is a function that puts a probability measure over strings drawn from some vocabulary
  - One-state <u>probabilistic</u> finite-state automaton
  - A unigram language model – and the state emission distribution for its one state q_1

- STOP is not a word, but a special symbol indicating that the automaton stops
  - e.g. P("frog said that toad likes frog STOP")
    = 0.01 · 0.03 · 0.04 · 0.01 · 0.02 · 0.01 · 0.2
    = 0.0000000000048



w	$P(w\|q_1)$	w	$P(w\|q_1)$
STOP	0.2	toad	0.01
the	0.2	said	0.03
a	0.1	likes	0.02
frog	0.01	that	0.04
		...	...

Using language models in IR

- Each document is treated as (the basis for) a language model.
- Given a query q, rank documents based on $P(d|q) = \frac{P(q|d)P(d)}{P(q)}$
  - $P(q)$ is the same for all documents, so ignore
  - $P(d)$ is the prior – often treated as the same for all d (how about PageRank?)
  - $P(q|d)$ is the probability of q given d.
    - the probability that a query would be observed as a random sample from the respective document model.
  - So to rank documents according to relevance to q, ranking according to $P(q|d)$ and $P(d|q)$ is equivalent.

How to compute $P(q|d)$: query likelihood model

- We will make the same conditional independence assumption as for Naive Bayes.

- $P(q|M_d) = P\left(< t_1, \dots, t_{|q|} > \middle| M_d\right) = \prod_{1 \leq k \leq |q|} P(t_k|M_d)$
  - $|q|$: length of q;
  - t_k : the token occurring at position k in q
  - This is equivalent to the following, where $tf_{t,\mathrm{q}}$: term frequency (# occurrences) of t in q
  - $P(q|M_d) = \prod_{distinct\ term\ t \in q} P(t_k|M_d)^{tf_{t,q}}$

Query generation probability

$$p(Q \mid M_d) = \prod_{t \in Q} p_{mle}(t \mid M_d) = \prod_{t \in Q} \frac{tf_{t,d}}{dl_d}$$

- Unigram assumption: Words appear independently in document
  - mle: Maximum likelihood estimation
  - $tf_{t,\mathrm{d}}$: term frequency of term t in document d
  - dl_d: the total number of tokens in document d
  - If a query term does not appear in a document?
  - Need smoothing

Smoothing

- Key intuition: a non-occurring term is possible
  - but no more likely than would be expected by chance in the collection.
- Notation:

$$p_{mle}(t \mid M_C) = \frac{cf_t}{T}$$

  - M_c: the collection model;
  - cf_t: the number of occurrences of t in the collection (or collection frequency)
  - $T = \sum_t cf_t$: the total number of tokens in the collection.
- We will use $\hat{P}(t|M_c)$ to "smooth" $P(t|d)$ away from zero.

Mixture model

$$p(Q \mid d) \propto \prod_{t \in Q} \left(\lambda \frac{tf_{t,d}}{dl_d} + (1-\lambda) \frac{cf_t}{T} \right)$$

- Model: $P(t|d) = \lambda P_{mle}(t|M_d) + (1 - \lambda) P_{mle}(t|M_c)$
- Mixes the probability from the document with the general collection frequency of the word.
  - High value of λ: “conjunctive-like” search – tends to retrieve documents containing all query words.
  - Low value of λ: more disjunctive, suitable for long queries
  - Correctly setting λ is very important for good performance.

LMs vs. Vector Space Model

- The two have common things
  - Both use term frequencies
  - Probabilities are inherently “length-normalized” [0,1]
  - Mixing term frequencies in document and collection has an effect similar to idf
    - Terms that are rare in the collection, but frequent in some documents have great influence on the ranking

- Differences
  - LMs: based on probabilistic theory, VSM: linear algebra notion
  - Collection frequency vs. document frequency
  - Other details: term frequency, length normalization