Rigidity at the Point of No Return: Completing the Finite Reduction of the Collatz Conjecture

Ian Adams
GPT-5.2

Abstract

We compile below a consolidated "final track" writeup for the corridor-entry → corridor-entry rigidity approach, together with a gate-recurrence sublemma and a finite-certificate style closing theorem, following the notation and narrative used in the supplied notes.

1 Corridor-entry → corridor-entry: final rigidity track

Assume for contradiction that the avoidance sets never vanish:

$$B_m \neq \quad \forall m \geq 16.$$

By the compactness lemma, this produces a genuine 2-adic obstruction. By the corridor machinery, this forces an infinite minimal-exit deep-reentry tower.

We now analyze that tower once, and isolate the single rigidity inequality that would finish the argument.

1.1 Canonical tower variables (exactly the setup)

At corridor entries:
$$x_k = 2^{T_k} q_k - 1, \qquad T_k \geq 2, \quad q_k \text{ odd}.$$

Define the corridor exit quantity:
$$u_k := 3^{T_k} q_k.$$

The deep-reentry condition is:
$$u_k + 1 = 2^{T_{k+1}+1} q_{k+1}.$$

Let
$$n_k := v_2(u_k + 1) = T_{k+1} + 1 \to \infty.$$

So
$$u_k = -1 + 2^{n_k} a_k, \qquad a_k := q_{k+1} \text{ odd}.$$

1.2 Key normal form (hinge equation)

The defining equation becomes
$$3^{T_k} a_{k-1} + 1 = 2^{n_k} a_k.$$

⋆ (1)

This is the only equation that matters.

1.3 Resonance cylinder decomposition (arithmetic form)

Fix k. The deep condition means

$$3^{T_k} a_{k-1} \equiv -1 \pmod{2^{n_k}}.$$

This has a unique solution modulo 2^{n_k}:

$$a_{k-1} \equiv -3^{-T_k} \pmod{2^{n_k}}.$$

Write a 2-adic expansion

$$a_{k-1} = a_0 + 2^{n_k} t_k.$$

Substitute into $(\star)$:

$$3^{T_k}(a_0 + 2^{n_k} t_k) + 1 = (3^{T_k} a_0 + 1) + 2^{n_k} 3^{T_k} t_k.$$

Divide by 2^{n_k} to obtain the exact affine recursion

$$a_k = s_k + 3^{T_k} t_k, \qquad s_k := \frac{3^{T_k} a_0 + 1}{2^{n_k}}. AFF \tag{2}$$

1.4 Rigidity heuristic as a checkable condition

To get the next deep re-entry one requires

$$3^{T_{k+1}} a_k \equiv -1 \pmod{2^{n_{k+1}}}.$$

Substituting (AFF) gives

$$3^{T_{k+1}}(s_k + 3^{T_k} t_k) \equiv -1 \pmod{2^{n_{k+1}}}.$$

Since coefficients are units modulo powers of 2, this forces a congruence of the form

$$t_k \equiv \tau_k \pmod{2^{n_{k+1}}},$$

so each deeper corridor consumes n_{k+1} fresh binary digits of t_k.

2 Gate recurrence sublemma (the τ-countdown law)

2.1 Statement

Fix m_0. Let the gate set at level m_0 be

$$I_{m_0} := \{(2^{m_0} - 1, \ell) \in V_{m_0} : \ \ell \ odd\},$$

corresponding to the congruence $x \equiv -1 \pmod{2^{m_0+1}}$, i.e.

$$\tau(x) = v_2(x+1) \geq m_0 + 1.$$

Assume there is an infinite deep-reentry tower so that along the associated re-entry states u_k we have

$$n_k := \tau(u_k) = v_2(u_k + 1) \to \infty.$$

Then along the corresponding true odd orbit (iterating U), the level-m_0 lift-state visits the gate I_{m_0} infinitely often, and these visits occur during steps where $b(x) = v_2(3x + 1) = 1$.

2.2 Proof (explicit countdown)

Write an odd integer x as

$$x = -1 + 2^m y, \qquad y \ odd, \quad m = \tau(x) = v_2(x+1).$$

If $m \geq 1$, then

$$3x + 1 = 3(-1 + 2^m y) + 1 = -2 + 3 \cdot 2^m y = 2(-1 + 3 \cdot 2^{m-1} y),$$

so $b(x) = v_2(3x+1) = 1$ and

$$U(x) = \frac{3x+1}{2} = -1 + 3 \cdot 2^{m-1} y, \qquad \tau(U(x)) = m - 1.$$

Iterating yields the corridor formula

$$\tau(U^j(x)) = m - j \quad for j = 0, 1, \ldots, m - 1.$$

Now apply this with $x = u_k$ and $m = n_k$. If $n_k \geq m_0 + 1$, then for each $j = 0, 1, \ldots, n_k - (m_0 + 1)$ one has

$$\tau(U^j(u_k)) = n_k - j \geq m_0 + 1,$$

equivalently $U^j(u_k) \equiv -1 \pmod{2^{m_0+1}}$. Along this corridor segment, $b(U^j(u_k)) = 1$.

3 Finite-state bridge and certificate closure (template)

3.1 Finite determinism bridge

Let $V_{m_0} = (Z/2^{m_0}Z)^\times \times \{0, \ldots, 15\}$ be the lift-state space and let F_{m_0} denote the deterministic update induced by U. If a single induced orbit in V_{m_0} visits I_{m_0} infinitely often, then by finiteness and determinism its eventual directed cycle must intersect I_{m_0}.

3.2 Certificate criterion

A finite certificate excluding all towers at level m_0 is:

$$\text{``}Nodirectedcyclein(V_{m_0}, F_{m_0})intersectsI_{m_0}.\text{''}$$

Combined with the gate-recurrence sublemma, this yields:

$$(towerexists) \implies (cycleintersectingI_{m_0}exists),$$

so the certificate implies that no tower exists.

4 Notes on scope

The text above is a compiled narrative of the supplied notes. Any global conclusion (e.g. a proof of the Collatz conjecture) depends on the correctness and completeness of the upstream reduction and on providing a checkable finite certificate in the exact formal model used in that reduction.