Image Reconstruction from a Sinogram

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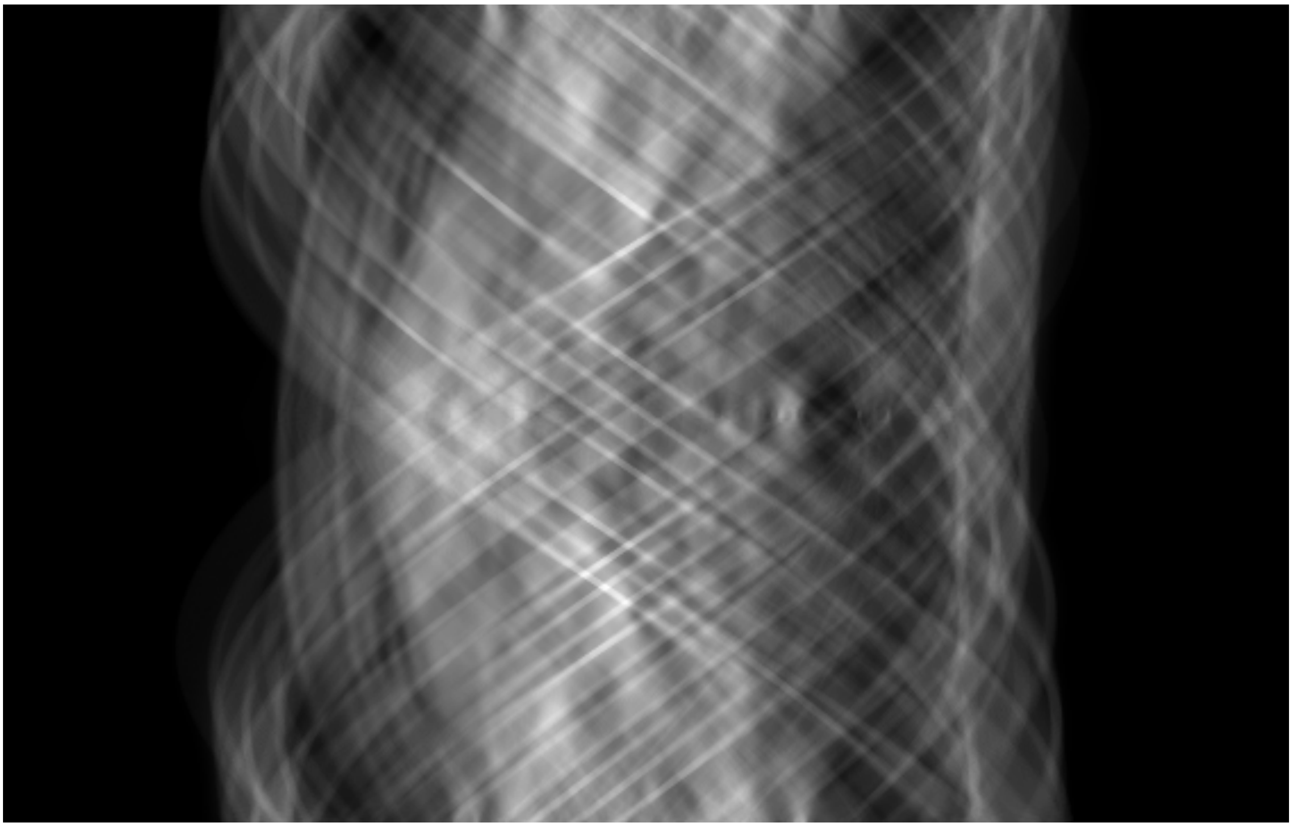
Introduction

A **sinogram** is a discrete representation of the Radon transform of an image. For this project, the objective was to reconstruct an image from a sinogram. The sinogram used for the image reconstruction is shown below:



Original Sinogram Image

In it’s current state, the sinogram doesn’t give us any meaningful information about the image it represents. A number of operations need to be performed so that we can examine the original image.

Reconstruction with <u>no</u> Ramp Filtering

If the image is back-projected directly from the sinogram, the result shown to the right is obtained. This back-projection gives us an idea of what the original image looks like, but there is a severe issue with this reconstruction - a large amount of blurring has been added. This is caused by the large amount of back-projections which are required for the reconstruction. As these back-projections are summed and normalised, this characteristic blurring appears in the image. Obviously, this reconstruction is not clear enough to be an acceptable representation of the original image.

Back Projection with Ramp Filtering

To suppress the blurring, a number of steps need to be taken. Using the Fourier Slice theorem and a ramp filter, we should be able to get back a much clearer representation of the original image using the following steps:

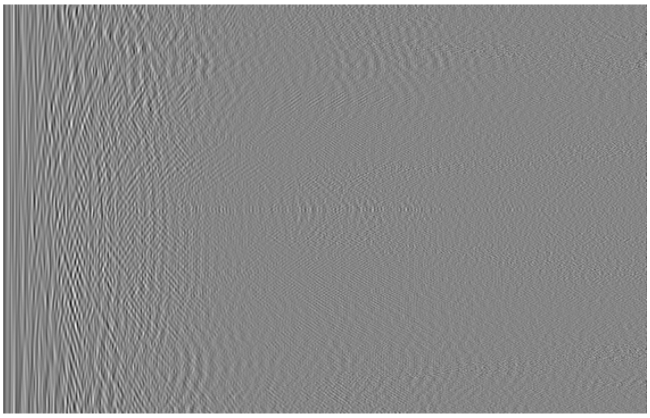
1. Translate the image projections (sinogram) into the frequency domain using the Fast Fourier Transform (FFT)
2. Filter the Frequency domain projections by multiplying each one by the frequency domain ramp filter
3. Use the Inverse FFT to convert back to the spatial domain
4. Construct the 2D image by back projecting the filtered projections

Note that we need to translate the sinogram from the spatial domain to the frequency domain - this is because the ramp filter is not band-limited and has no meaningful spatial domain representation. Also note that this method will add a certain amount of noise to the reconstruction, as well as some level shifting.

As we move through the steps numbered above, the following outputs were returned:

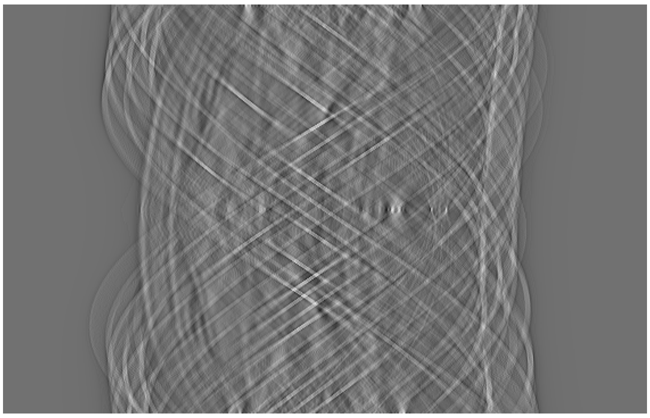
Step 1 - Translation to Frequency Domain

It's difficult to see in this image (full resolution images are in the 'images' folder), but a number of frequency components can be made out on the left hand side of this image. While this may not look like much to the human eye, this is still a good representation of the sinogram. To get this output, all we had to do is pass the sinogram (which is a numpy array) to the fft.rfft method available in the scipy.fft routine - this computes the one-dimensional, n-point discrete Fourier transform of an array. The output of this function is an array of 1D FFTs of the projections, where each projection is one row of the array.



Step 2 - Filtration of Frequency Domain Projections with Ramp Filter

In this step, each of the frequency domain projections is filtered using a ramp function. This is accomplished by multiplying each 1D projection (row) by the ramp function. Also note that the ramp filter has to account for mixed real and imaginary elements in the FFTs of the projections. This gives us a filtered frequency domain representation of the image. Again, this representation doesn't look like anything significant to the human eye but the following steps will begin to transform this representation into something meaningful.



Step 3 - Translation back to Spatial Domain

This step uses an inverse Fourier Transform to return the frequency domain representation to the spatial domain. Note the similarity of this image to the original sinogram - the shapes and general outline of the sinogram have been preserved but the colours have been washed out and the lines have become sharper. This representation of the sinogram will give us a much clearer version of the original image.

Step 4 - Reconstruction of Original Image

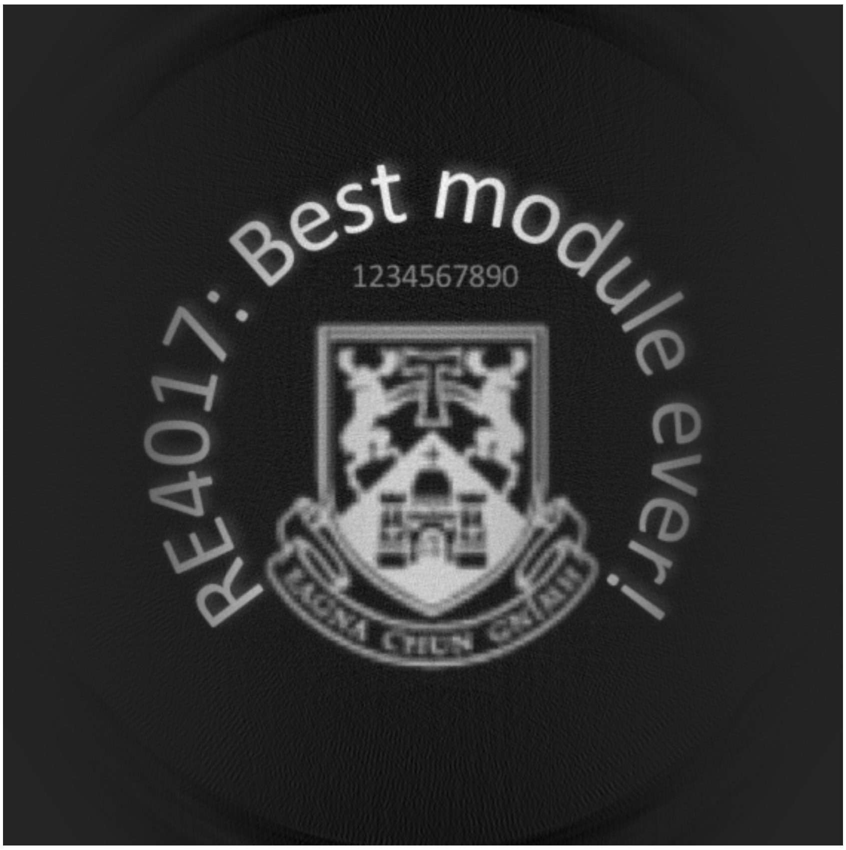
This image is a back-projection of filtered sinogram shown in step 4. Comparing this to the unfiltered back-projection, it’s clear that performing the filtration steps will give us back a much clearer representation of the original image.

However, note that this reconstruction is not perfect - the image shown to the right is a close up view of a section of the reconstruction. There is still a degree of blurring around the white letters in the image (although this reconstruction shows that the blurring is much less severe than in the original), an interference pattern is showing in the black background of the image, and characteristic ‘ringing’ artefacts appear around the edges of the image. This ringing occurs because the ramp filter emphasises high-frequency components in the output image, which leads to these artefacts.

So while the filtration steps can definitely increase the overall quality of the image, there are still a number of issues which need to be addressed.

Reconstruction Using a Hamming-Windowed Ramp Filter

Artefacts in the reconstruction are caused by the FFT filtering process. We can try and reduce these issues by windowing the FFT - in this case, we will be using a **Hamming Window** on the ramp function to try and reduce the severity of the artefacting.



Hamming-Windowed, Ramp-Filtered Reconstruction

This window has had mixed success - the ‘ringing’ artefacts have been drastically reduced (but not eliminated) and the interference pattern in the background is also much less severe. However, the blurring remains and the window appears to be emphasising the text in the centre of the image while darkening the text on the left and right.

Conclusion

It’s clear that filtering the sinogram is essential in reconstructing an image - these steps give a much clearer representation of the original image. We can also use standard window functions to improve the quality of the reconstruction even further (with some drawbacks). The Python code used to get these images has been included with this report, along with a .zip file containing full resolution versions of the images used in this project.