

CS 331: Algorithms and Complexity (Spring 2024)

Unique numbers: 50930/50935/50940/50945

Discussion Section 7

Carpool Fairness.

Description: In this scenario, n people are sharing a carpool for m days. Each person may choose whether to participate in the carpool on each day.

Example. Figure 1 describes a carpool in which 4 people share a carpool 5 days. X 's indicate days when people participate in the carpool.

Our goal is to allocate the daily driving responsibilities 'fairly.' One possible approach is to split the responsibilities based on how many people use the car. So, on a day when k people use the carpool, each person incurs a responsibility of $\frac{1}{k}$. That is, for each person i , we calculate his or her driving obligation O_i as shown below. We can then require that person i drives no more than $\lceil O_i \rceil$ times every m days. Figure 2 shows the calculation of these O_i and their ceilings.

Person	Days:	1	2	3	4	5
1		X	X	X		
2		X		X		
3		X	X	X	X	X
4			X	X	X	X

Figure 1: Example of a carpool

Person	Days:	1	2	3	4	5	O_i	$\lceil O_i \rceil$
1		$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$			$\frac{1}{2}$	1
2		$\frac{1}{3}$		$\frac{1}{4}$			$\frac{1}{2}$	1
3		$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	2
4			$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	2
Σ		1	1	1	1	1	-	-

Figure 2: Driver Responsibilities.

Prove that there always exists a fair solution.