CS 331: Algorithms and Complexity (Spring 2024)

Unique numbers: 51310/51315

Discussion Section 2

Problem 1

A binary tree is a tree in which each node has at most two children. A full node in a binary tree is a node with two children. Prove by mathematical induction that the number of full nodes is one less than the number of leaves in any non-empty binary tree.

Problem 2

We have a connected graph $G = (V, E)$, and a specific vertex $u \in V$. Suppose we compute a depth-first search tree rooted at u, and obtain a tree T that includes all nodes of G. Suppose we then compute a breadth-first search tree rooted at u, and obtain the same tree T. Prove that $G = T$. (In other words, if T is both a depth-first search tree and a breadth-first search tree rooted at u, then G cannot contain any edges that do not belong to T.)

Problem 3

For an undirected and acyclic graph G, prove that the BFS-forest T produced by running BFS on G is identical to G.

Problem 4

Given a directed graph $G = (V, E)$ such that
$V = \{a, b, c, d, e, f\}$ and $E = \{(a, b), (a, d), (b, c), (d, e), (c, f), (e, f)\}$.
Is G a DAG? If Yes, write down all topological orderings for G.