

# Handout 1

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January 18, 2024

## 1 Problem 1

**Theorem 1.** *Prove that for any tree, the number of edges is one less than the number of nodes, i.e.,  $n - m = 1$ .*

*Proof.* Default tree: 1 node, 0 edges. Adding a node requires adding an edge, so the number of edges always equals the number of nodes minus 1.  $\square$

## 2 Problem 2

**Theorem 2.** *Prove that such an algorithm cannot possibly exist.*

*Proof.* Assume there is, then we can recursively apply it and get smaller files.  $\square$

## 3 Problem 3

**Theorem 3.** *Prove that a bishop placed on that square can go to any black colored square on the chessboard.*

*Proof.* Bishop can move diagonally, so it can move to any black square, which are placed in diagonal patterns.  $\square$

## 4 Problem 4

**Theorem 4.** *Prove that this new board cannot be tiled with dominoes|that is, any attempt to cover the chessboard with dominoes must always have either an uncovered square or a domino hanging off the edge.*

*Proof.* Removed 2 black squares, and since dominoes cover 1 black and 1 white square, it is impossible.  $\square$