CS 331: Algorithms and Complexity (Spring 2024)
Unique Number: 50930, 50935 50940, 50945

Assignment 1

Due on Thursday, 25 January, by 11.59pm

Problem 1

(a) (3 points) Arrange the following list of functions in increasing order of growth rate. That is, if function $g(n)$ follows function $f(n)$, then it should be the case that $f(n)$ is $O(g(n))$ (the base of logarithms is 2). Justify in simple words. (You do not have to show detailed calculations.)

$$f_1(n) = 17n + (\frac{2}{n^2})$$
$$f_2(n) = 7n \log \log n$$
$$f_3(n) = 20n \log^3 n + 5n^2$$
$$f_4(n) = 4\sqrt{n} + 3\log(n^2)$$
$$f_5(n) = 2^{\sqrt{n}}$$
$$f_6(n) = 2^{(3 \log n)}$$

(b) (3 points) Let $a_1 = 1, a_2 = 8$ and $a_n = a_{n-1} + 2a_{n-2}$, where $n \geq 3$. Use induction to prove that $a_n = 3 \cdot 2^{n-1} + 2(-1)^n$ for all $n \in \mathbb{N}$. *Note: you will need to use strong induction (sometimes known as complete induction) to solve this problem.*

(c) (4 points)
Alice and Bob, two students of CS 331, were asked to provide algorithms for a simple program. The program is supposed to receive a stream of integers as input. After each input integer, the program must report the average of the integers it has seen so far. Alice and Bob came up with the following algorithms to do this simple task, as in Algorithm 1 and 2, respectively.

State whether Alice and Bob have provided correct algorithms, ignoring truncation due to integer division. Also, either prove the algorithm to be right, or show how it is wrong. If an algorithm is correct then prove the correctness of the algorithm using the concept of loop invariant. For the incorrect algorithm it is enough to specify how to fix the algorithm and no need to prove the correctness of your suggested fix.

Algorithm 1: Alice's Algorithm

```
Initially sum is 0 and count of integers read is 0 ;
while the input stream is not empty do
    Read the integer i ;
    Add i to the sum ;
    Report average as sum / count ;
    Increase count by 1;
```

Algorithm 2: Bob's Algorithm

```
Initially average is 0 and count of integers read is 0 ;
while the input stream is not empty do
    Read the integer i ;
    Modify average as (average * count + i) / (count + 1) ;
    Increase count by 1 ;
    Report average;
```

Problem 2

(6 points) Assume you are given an array-like data structure, E, such that E is arbitrarily large and there is no way to know the size of it. The array E is populated with integer values, from the beginning up to the nth cell, sorted in increasing order. The rest of the cells after the nth contain a very large integer call it max and you don't know n. Your goal is to describe an algorithm that receives an integer value x ($x < max$) and returns the position of that integer in E in $O(\log n)$. You can assume that x exists in E.

Problem 3

(14 points) The CS department has received several applications for teaching assistant positions from its students. The applicants are provided with the list of available courses and asked to submit a list of their preferences. The TA committee at UTCS prepares a sorted list of the applicants (according to the applicants' background and experience) for each course and the total number of TAs needed for the course.

Note that there are more students than available TA spots. There can also be multiple TAs per course, though a single TA can only be assigned to 1 course.

Making matter more complicated, certain applicant-course are *disallowed* since an applicant does not have enough background in a specific course.

We will say that A (an applicant) prefers C (a course) to C' iff C is ranked higher than C' on his/her preference list. The committee wants to apply an algorithm to produce stable

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assignments such that each student is assigned to at most one course. The assignment of the TAs to courses is stable if none of the following situations arises.

1. If an applicant A and a course C are allowed and not matched, but A prefers C more than his/her assigned course, and C prefers A more than, at least, one of the applicants assigned to it.

2. If an applicant, A, and a course, C, are allowed such that A is unmatched, and C has an empty spot or an applicant A' assigned to it such that C prefers A to A'.

Your task is to do the following:

i **(7 points)** Modify the Gale-Shapley algorithm to find a stable assignment for the TAs-Courses matching problem. Write pseudo code of modified algorithm.

ii **(7 points)** Prove that the modified algorithm produces stable TAs-Courses assignments.