

CS 331: Algorithms and Complexity (Spring 2024)
Unique Number: 50930, 50935 50940, 50945

Assignment 6 - Solution

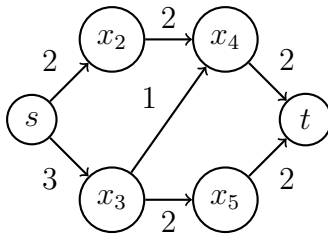
Due on Tuesday, 19 March, by 11.59pm

Problem 1

(10 points) State whether the following are true or false. Justify if true. If false, give a counter-example.

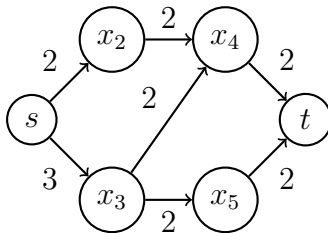
a) (1 point)

False, it can stay the same
Consider the following graph:



Max flow: 4

Add 1 to edge with minimum capacity



However, the max flow remains the same since $x_4 \rightarrow t$ and $s \rightarrow x_3$ can't pass more flow

b) (2 point)

True, we can describe the max flow as the min-cut where $s \in A$ and $t \in B$

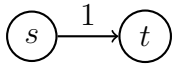
Since the sum of integers divisible by 3 is divisible by 3, we can say that every cut is divisible by 3, and therefore the min-cut is also divisible by 3.

Therefore, the max flow is divisible by 3.

c) **(1 point)**

False, it can still be unique

Consider this graph:



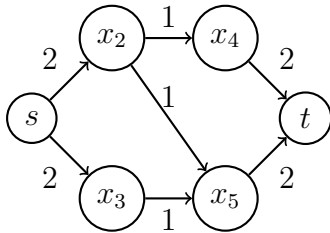
There is one edge, so all edges have the same capacity

There is only one possible min-cut, so it's unique

d) **(2 points)**

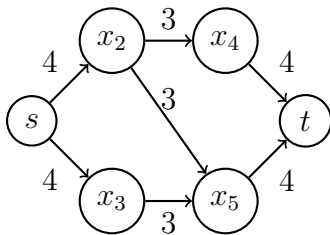
False, the min-cuts may change

Consider the following graph:



The min-cuts are $\{s, x_3\}$ and $\{x_2, x_4, x_5, t\}$ with a capacity of 3 and $\{s, x_2, x_3\}$ and $\{x_4, x_5, t\}$ with a capacity of 3 and $\{s, x_2, x_3, x_5\}$ and $\{x_4, t\}$ with a capacity of 3

Let's add 2 to all edges



However, the min-cuts are now $\{s, x_3\}$ and $\{x_2, x_4, x_5, t\}$ with a capacity of 7 and $\{s, x_2, x_3, x_5\}$ and $\{x_4, t\}$ with a capacity of 7

e) **(2 points)**

True, the min-cuts stay the same

Assume this is false

Now we have 2 cases: A min-cut previously is no longer a min-cut, or a new min-cut is added to the sets

Assume a previous min-cut is no longer a min-cut, let's call it α

Then, some other cut, let's call it β has a smaller capacity than α

$\therefore, \text{cap}(\beta) < \text{cap}(\alpha)$, and since every edge was multiplied by a constant factor c , then this implies $\frac{\text{cap}(\beta)}{c} < \frac{\text{cap}(\alpha)}{c}$

However, α was a min-cut before, so $\frac{\text{cap}(\beta)}{c} \geq \frac{\text{cap}(\alpha)}{c}$

This is a contradiction, so a previous min-cut can't be no longer a min-cut

Now for the second case, assume a new min-cut is added to the sets, lets call it γ

Therefore, for every cut ξ in the transformed graph, $cap(\gamma) \leq cap(\xi)$

Divide both sides by c , which yields $\frac{cap(\gamma)}{c} \leq \frac{cap(\xi)}{c}$

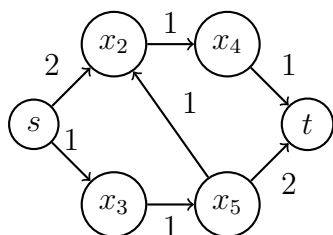
This means that γ is a min-cut in the original graph, which is a contradiction

Therefore, the min-cuts stay the same

f) **(2 points)**

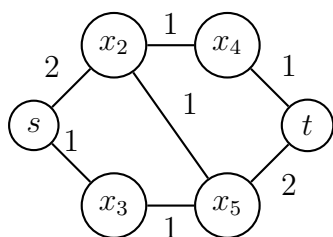
False, the max flow may change

Consider the following graph:



The max-flow is 2

Now we transform this directed graph to an undirected graph



The max flow is now 3

Problem 2

(10 points)

Problem 3

(10 points)