

CS 331: Algorithms and Complexity (Spring 2024)
Unique numbers: 50930/50935/50940/50945

Discussion Section 9

Problem 1

The Mute Prison problem

An international prison manager, trying to keep swearing off his institution, wants to place inmates into cells so that no two inmates speak the same language. Given a table T of n inmates and m languages where $T[i, j] = 1$ if inmate i speaks language j and 0 otherwise; the manager wants to know whether there exists a subset S of the inmates where $|S| \geq k$ and no two inmates in S speak the same language. Let's call this problem The Mute Prison problem. Show that the Independent Set problem can be reduced in polynomial time to The Mute Prison problem.

Problem 2

The Nonsense Prerequisites problem

University of Chaos computer system crashed. While recovering it, you noticed that the prerequisite data was corrupted. Some prerequisites were added and shouldn't have been. For instance, the original prerequisite path to get to CSC373 was to take $165 \rightarrow 236 \rightarrow 263 \rightarrow 373$. After the crash, the prerequisite $373 \rightarrow 236$ was added, thus creating a cycle from $236 \rightarrow 263 \rightarrow 373 \rightarrow 236$. At University of Chaos, we don't really care that much if prerequisites make sense, but rather just want to avoid having cycles. Your job is to break these cycles. In the example above, you could do so by removing one the following prerequisites: $236 \rightarrow 263$, $263 \rightarrow 373$ or $373 \rightarrow 236$.

More formally, we define The Nonsense Prerequisites problem as follows: Suppose you are given a list n courses, you can construct a directed graph $G(V, E)$ where every vertex is a course, and the edge (i, j) directed $i \rightarrow j$ means course i is a prerequisite for course j . You are also given a positive integer k , and you want to return 1 if and only if there exists a subset $E' \subseteq E$ of edges whose removal makes G acyclic and $|E'| \leq k$. Give a reduction from VERTEX COVER to this problem.