CS 331: Algorithms and Complexity (Spring 2024)
Unique Number: 50930, 50935 50940, 50945

Assignment 3 - Solution

Due on Thursday, 8 Febrauary, by 11.59pm

Problem 1: Short Answer Section

(10 pts) True or false. If true, briefly justify, otherwise, provide a counter example . When justifying, restrict answers to no more than a few sentences.

1. **(1 pt)** True, a greedy option picks the best option at each step, without considering the future.

2. **(2 pts)** True, you argue that the differences between the two algorithms are irrelevant to the quality of the solution.

3. **(2 pts)** True, since the shortest path is the path with the fewest edges.

4. **(2 pts)** False, Counterexample:

   ```
   (1, 2), (2, 3), (1, 3)
   Pick (1, 3) since it has the largest time interval
   Since it only conflicts with (1, 2) and (2, 3), we can pick it
   However, the optimal solution is to pick (1, 2) and (2, 3)
   ```

5. **(3 pts)** No, the shortest path is not necessarily the path with the fewest edges, Counterexample:

   ```
   (a, b) = 1; (b, c) = 1; (c, d) = 1; (a, d) = 4
   Shortest path: a -> b -> c -> d with weight 3
   Increment all edges by 1, we get: a -> b -> c -> d with weight 6
   However, the shortest path is a -> d with weight 5
   ```

Problem 2

(10 points) I will denote tasks as (p(i), t(i)) where p(i) is the value of the task and t(i) is the duration of the task.

1. (Smallest duration first) Pick task i that has the minimum duration $t(i)$, or

   Proof. This is not optimal.
   Counterexample:

   ```
   (1, 1), (10, 2)
   Pick (1, 1) first, then (10, 2)
   This yields (1 * 1) + (10 * 3) = 31
   However, the optimal solution is to pick (10, 2) first, then (1, 1)
   This yields (10 * 2) + (1 * 3) = 23
   ```

   □

2. (Most valuable first) Pick task i that has maximum $p(i)$, or

   Proof. This is not optimal.
   Counterexample:

   ```
   (1, 1), (2, 3)
   Pick (2, 3) first, then (1, 1)
   This yields (2 * 3) + (1 * 4) = 10
   However, the optimal solution is to pick (1, 1) first, then (2, 3)
   This yields (1 * 1) + (2 * 4) = 9
   ```

   □

3. (Maximum time-scaled value first) Pick task i that has maximum $p(i)/t(i)$.

   Proof. This is optimal. □

Problem 3

(10 points) Yes