CS 331: Algorithms and Complexity (Spring 2024)
Unique Number: 50930, 50935 50940, 50945

Assignment 6 - Solution

Due on Tuesday, 19 March, by 11.59pm

Problem 1

(10 points) State whether the following are true or false. Justify if true. If false, give a counter-example.

a) **(1 point)** If value of the edge with minimum capacity is increased by 1, then the max-flow must be increased by 1.

b) **(2 point)** If all capacities are an integer multiple of 3, then the max-flow is also a multiple of 3.

c) **(1 point)** If all capacities are the same, then the min-cut is not unique.

d) **(2 points)** If a constant $c \in \mathbb{R}_{>0}$ is added to each capacity, then the min-cuts are the same as the original graph.

e) **(2 points)** If a constant $c \in \mathbb{R}_{>0}$ is multiplied to each capacity, then the min-cuts are the same as the original graph.

f) **(2 points)** Let $G = (V, E)$ be a directed graph where $s, t \in V$ and $(u, v) \in E \implies (v, u) \notin E$ (i.e. s and t are vertices in G and if there's an edge from u to v then there's no edge from v to u). Each edge has some capacity. True or false, if G is transformed to be an undirected graph keeping the same vertices and edges the max s-t flow is the same.

Problem 2

(10 points) The organizers of a year-end award show are interested in maximizing the profit from the show. They can invite any subset of a set of n artists $\mathcal{A} = \{A_1, \ldots, A_n\}$, where each artist A_i costs c_i dollars to invite. Furthermore, the organizers have lined up a set of m potential stages $\mathcal{S} = \{S_1, \ldots, S_m\}$, where each stage S_i requires a subset $P_i \subseteq \mathcal{A}$ of artists to be invited and generates a revenue of r_i dollars if performed.

Using a suitable reduction to network flow, determine which stages should be performed and which artists should be invited in order to maximize the profit, i.e. the total revenue of the stages minus the total cost of inviting the artists.

Problem 3

(10 points) A farm sells m different kinds of produce. Let $P = p_1, \ldots, p_m$ denote how much of each produce the farm can supply in a particular week.

The farm supplies to n different stores. Every week, each store i sends a list $B_i = b_i^1, \ldots, b_i^m$ indicating how much of each produce that store is willing to buy from each of the m different types of produce. Additionally, each store i specifies a total budget T_i that denotes how much produce in total can fit in that store.

Now the farm need to figure out an efficient way of allocating produce to each store, so that the total amount of produce sold is maximized. Note that the assignment must remain within the total budgets of the stores.

For example, Let $P = \{10, 4, 6\}$, $B_1 = \{4, 5, 3\}$, $B_2 = \{4, 6, 4\}$, $T_1 = 10$ and $T_2 = 10$. Then the farm will be able to sell produce of total value 18 (some of the first item will remain unsold).

Describe an algorithm which takes $P, B_1, \ldots B_n$ and budgets $T_1, \ldots T_n$, and produces an assignment of items to stores such that the total amount sold is maximized. You must represent the problem as a maximum-flow problem and then explain why/how it is a maximum flow problem. Show the time complexity.