

Names: Ian Chen

Group Average

Proximity between two clusters is the average pairwise distance between all pairs of points in the two clusters

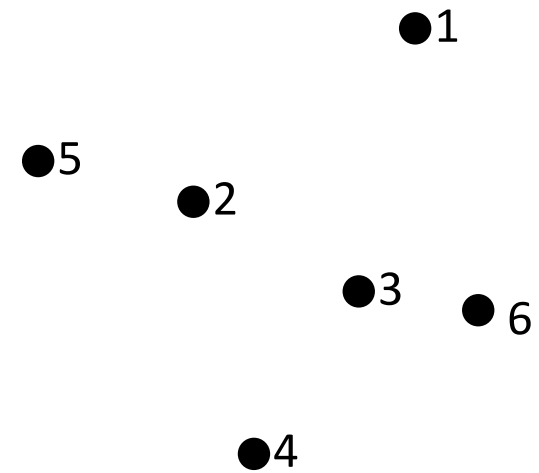
| Point | x Coordinate | y Coordinate |
|-------|----------------|----------------|
| p1 | 0.40 | 0.53 |
| p2 | 0.22 | 0.38 |
| p3 | 0.35 | 0.32 |
| p4 | 0.26 | 0.19 |
| p5 | 0.08 | 0.41 |
| p6 | 0.45 | 0.30 |

Table 8.3. xy coordinates of 6 points.

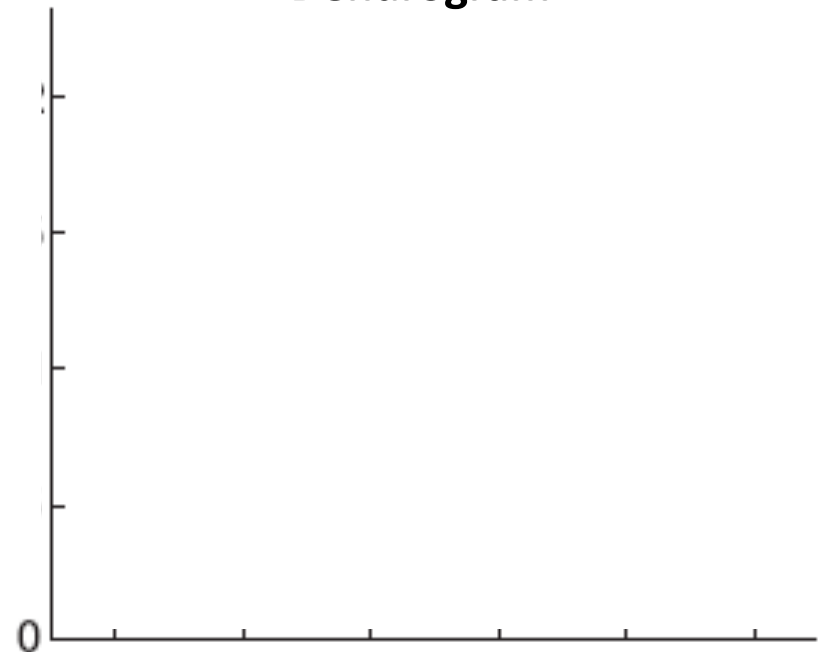
| | p1 | p2 | p3 | p4 | p5 | p6 |
|----|------|------|------|------|------|------|
| p1 | 0.00 | 0.24 | 0.22 | 0.37 | 0.34 | 0.23 |
| p2 | 0.24 | 0.00 | 0.15 | 0.20 | 0.14 | 0.25 |
| p3 | 0.22 | 0.15 | 0.00 | 0.15 | 0.28 | 0.11 |
| p4 | 0.37 | 0.20 | 0.15 | 0.00 | 0.29 | 0.22 |
| p5 | 0.34 | 0.14 | 0.28 | 0.29 | 0.00 | 0.39 |
| p6 | 0.23 | 0.25 | 0.11 | 0.22 | 0.39 | 0.00 |

Table 8.4. Euclidean distance matrix for 6 points.

Nested cluster diagram



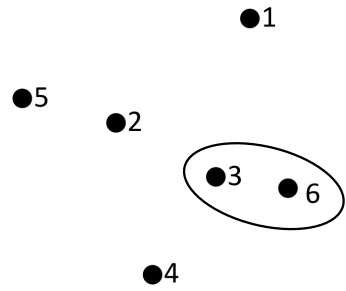
Dendrogram



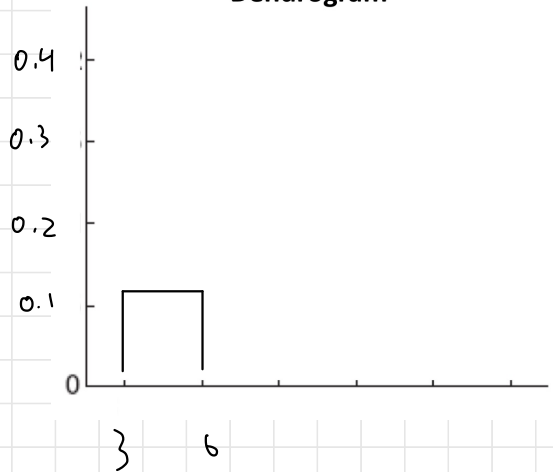
| | p1 | p2 | p3 | p4 | p5 | p6 |
|----|----|------|------|------|------|------|
| p1 | | 0.24 | 0.22 | 0.37 | 0.34 | 0.23 |
| p2 | | | 0.15 | 0.20 | 0.14 | 0.25 |
| p3 | | | | 0.15 | 0.28 | 0.11 |
| p4 | | | | | 0.29 | 0.22 |
| p5 | | | | | | 0.39 |
| p6 | | | | | | |

p3-p6 is the closest
with distance 0.11

Nested cluster diagram



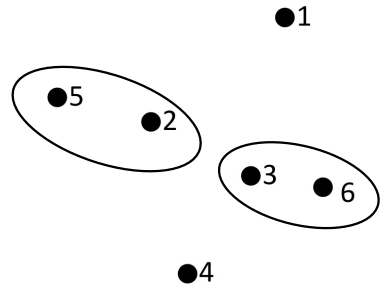
Dendrogram



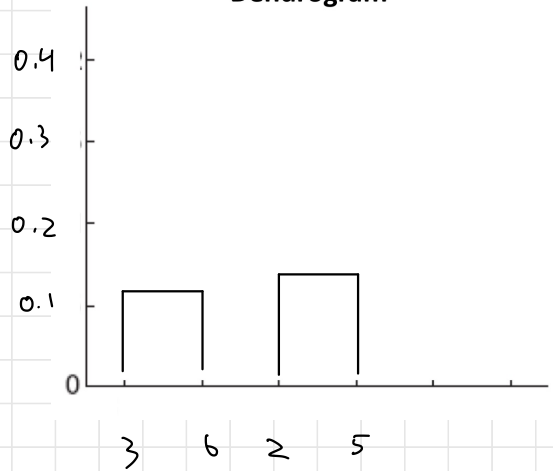
$\{p_3, p_6\}$

| | p_1 | p_2 | p_4 | p_5 |
|----------------|-------|-------|-------|-------|
| p_1 | ○ | 0.24 | 0.225 | 0.37 |
| p_2 | | ○ | 0.15 | 0.20 |
| $\{p_3, p_6\}$ | | | ○ | 0.185 |
| p_4 | | | | ○ |
| p_5 | | | | |

Nested cluster diagram



Dendrogram



$$\frac{(p_1 - p_3) + (p_1 - p_6)}{2} = \frac{(0.22 + 0.23)}{2} = 0.225$$

$$\frac{(p_2 - p_3) + (p_2 - p_6)}{2} = \frac{(0.15 + 0.25)}{2} = 0.15$$

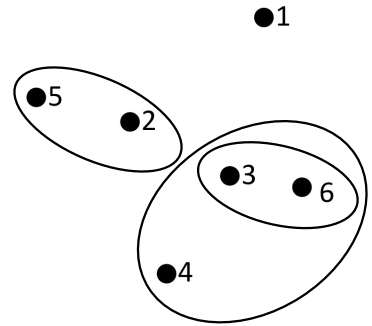
$$\frac{(p_4 - p_3) + (p_4 - p_6)}{2} = \frac{(0.15 + 0.22)}{2} = 0.185$$

$$\frac{(p_5 - p_3) + (p_5 - p_6)}{2} = \frac{(0.28 + 0.39)}{2} = 0.335$$

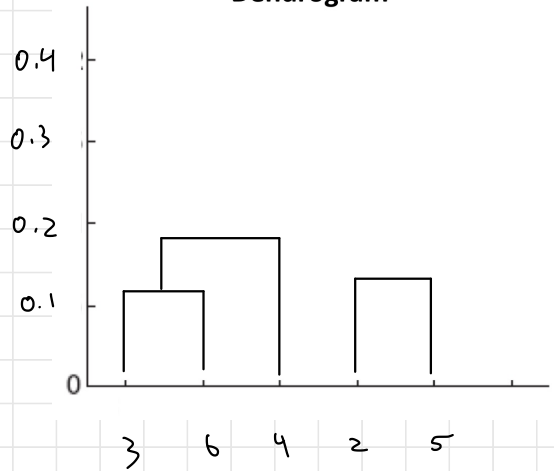
Closest is $p_2 - p_5$ with distance 0.14

| | | |
|----------------|----------------|-----------------|
| | $\{p_2, p_5\}$ | $\{p_3, p_6\}$ |
| p_1 | \downarrow | \downarrow |
| p_1 | 0 | 0.29 0.225 0.37 |
| $\{p_2, p_5\}$ | 0 | 0.2675 0.295 |
| $\{p_3, p_6\}$ | 0 | 0.185 |
| p_4 | | 0 |

Nested cluster diagram



Dendrogram



$$\frac{(p_1 - p_2) + (p_1 - p_5)}{2} = \frac{(0.24 + 0.34)}{2} = 0.29$$

$$\frac{((p_2 - p_3) + (p_2 - p_6) + (p_5 - p_3) + (p_5 - p_6))}{4} = \frac{(0.15 + 0.25 + 0.28 + 0.39)}{4} = 0.2675$$

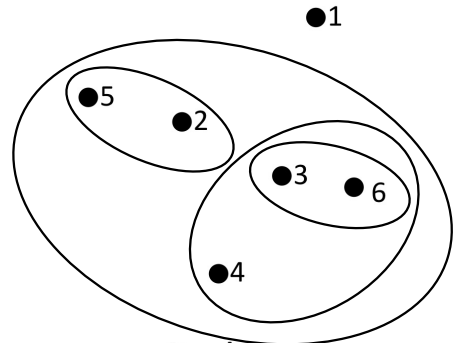
$$\frac{((p_4 - p_2) + (p_4 - p_5))}{2} = \frac{(0.20 + 0.29)}{2} = 0.295$$

Closest is $\{p_3, p_6\} - p_4$
with distance 0.185

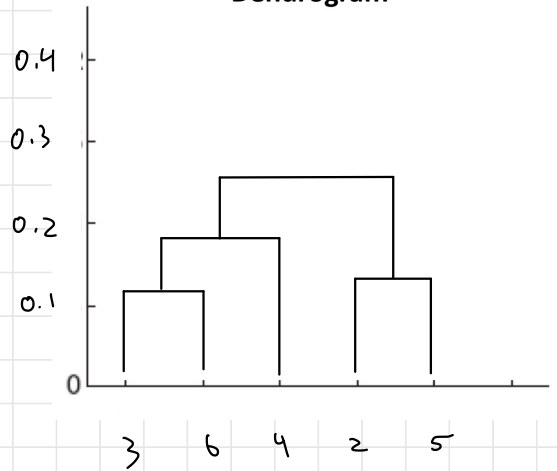
$$\begin{array}{c} \{p_2, p_5\} \quad \{p_3, p_4, p_6\} \\ \begin{array}{|c|} \hline p_1 \\ \hline \end{array} \end{array}$$

| | | | |
|---------------------|---------|--------|--------|
| p_1 | \circ | 0.29 | 0.37 |
| $\{p_2, p_5\}$ | \circ | 0.26 | |
| $\{p_3, p_4, p_6\}$ | \circ | | 0 |

Nested cluster diagram



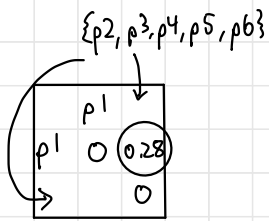
Dendrogram



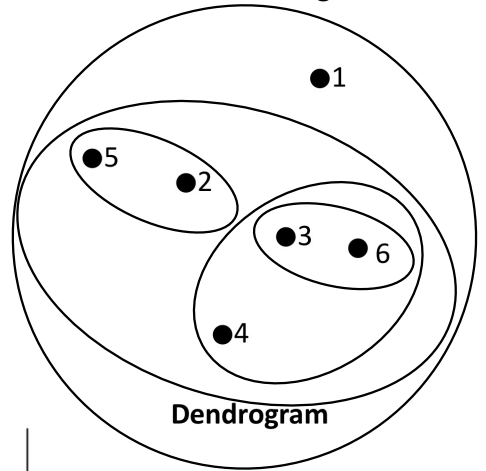
$$\begin{aligned} & ((p_1 - p_3) + (p_1 - p_4) + (p_1 - p_6)) / 3 \\ &= (0.22 + 0.37 + 0.23) / 3 = 0.27\bar{3} \end{aligned}$$

$$\begin{aligned} & ((p_2 - p_3) + (p_2 - p_4) + (p_2 - p_6) + (p_5 - p_3) + \\ & \quad (p_5 - p_4) + (p_5 - p_6)) / 6 \\ &= (0.15 + 0.20 + 0.25 + 0.28 + 0.29 + 0.39) / 6 = 0.26 \end{aligned}$$

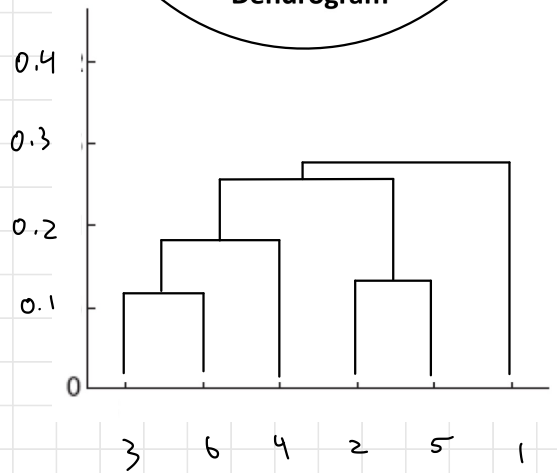
Closest is $\{p_2, p_5\} - \{p_3, p_4, p_6\}$
with distance 0.26



Nested cluster diagram



Dendrogram



$$\frac{(p_1 - p_2) + (p_1 - p_3) + (p_1 - p_4) + (p_1 - p_5) + (p_1 - p_6)}{5} \\ = \frac{(0.24 + 0.22 + 0.37 + 0.34 + 0.23)}{5} = 0.28$$

Closest is $p_1 - \{p_2, p_3, p_4, p_5, p_6\}$
with distance 0.28

Names: Ian Chen

Centroid Method

Proximity between two clusters is the distance between their centroids

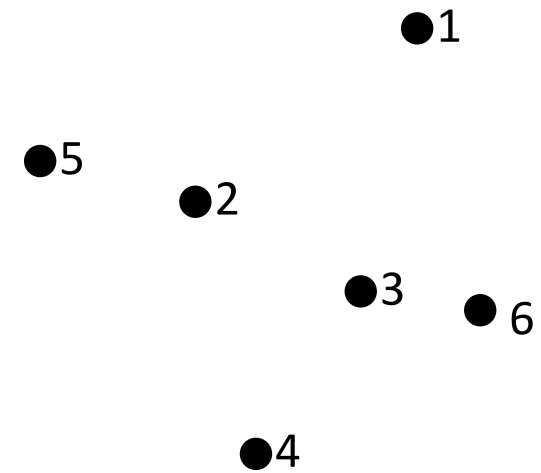
| Point | <i>x</i> Coordinate | <i>y</i> Coordinate |
|-------|---------------------|---------------------|
| p1 | 0.40 | 0.53 |
| p2 | 0.22 | 0.38 |
| p3 | 0.35 | 0.32 |
| p4 | 0.26 | 0.19 |
| p5 | 0.08 | 0.41 |
| p6 | 0.45 | 0.30 |

Table 8.3. *xy* coordinates of 6 points.

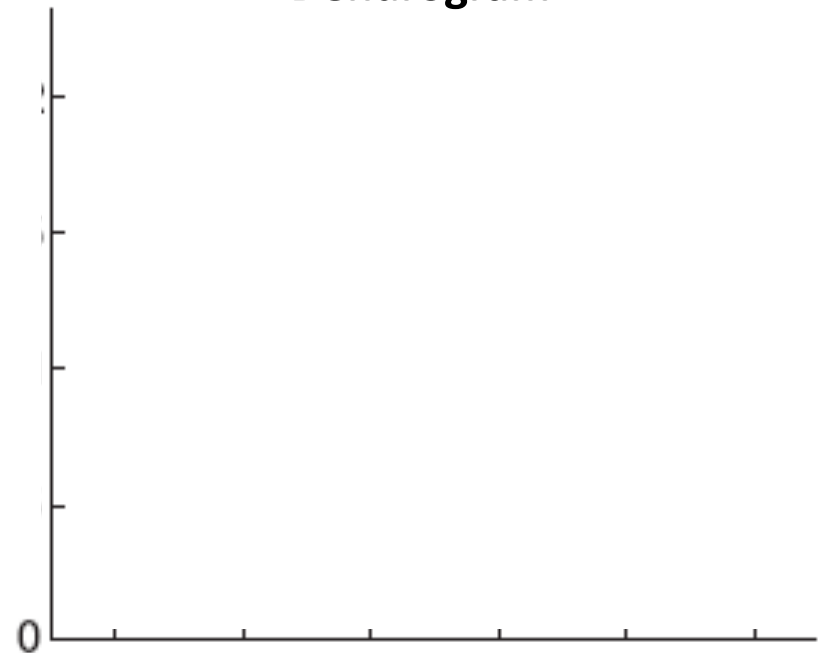
| | p1 | p2 | p3 | p4 | p5 | p6 |
|----|------|------|------|------|------|------|
| p1 | 0.00 | 0.24 | 0.22 | 0.37 | 0.34 | 0.23 |
| p2 | 0.24 | 0.00 | 0.15 | 0.20 | 0.14 | 0.25 |
| p3 | 0.22 | 0.15 | 0.00 | 0.15 | 0.28 | 0.11 |
| p4 | 0.37 | 0.20 | 0.15 | 0.00 | 0.29 | 0.22 |
| p5 | 0.34 | 0.14 | 0.28 | 0.29 | 0.00 | 0.39 |
| p6 | 0.23 | 0.25 | 0.11 | 0.22 | 0.39 | 0.00 |

Table 8.4. Euclidean distance matrix for 6 points.

Nested cluster diagram



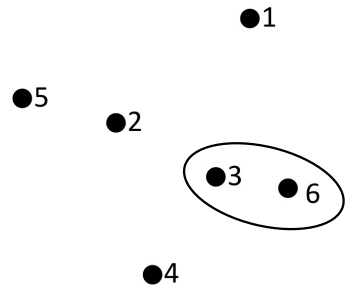
Dendrogram



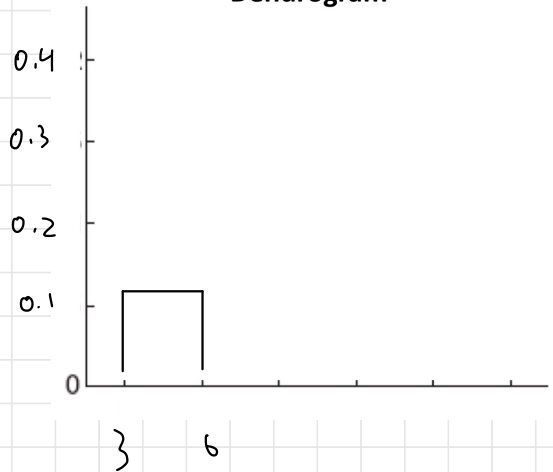
| | p1 | p2 | p3 | p4 | p5 | p6 |
|----|----|------|------|------|------|------|
| p1 | | 0.24 | 0.22 | 0.37 | 0.34 | 0.23 |
| p2 | | | 0.15 | 0.20 | 0.14 | 0.25 |
| p3 | | | | 0.15 | 0.28 | 0.11 |
| p4 | | | | | 0.29 | 0.22 |
| p5 | | | | | | 0.39 |
| p6 | | | | | | |

p3-p6 is the closest
with distance 0.11

Nested cluster diagram



Dendrogram



$$\{p_3, p_6\}: \frac{p_3 + p_6}{2} = \frac{(0.35 + 0.45, 0.32 + 0.30)}{2}$$

$$= (0.40, 0.31)$$

| | p_1 | p_2 | p_4 | p_5 |
|----------------|-------|-------|-------|-------|
| p_1 | ○ | 0.24 | 0.22 | 0.37 |
| p_2 | | ○ | 0.19 | 0.20 |
| $\{p_3, p_6\}$ | | | ○ | 0.18 |
| p_4 | | | | ○ |
| p_5 | | | | |

$$\sqrt{(p_1 - \{p_3, p_6\})^2}$$

$$\Rightarrow \sqrt{(0.40 - 0.40)^2 + (0.53 - 0.31)^2} = 0.22$$

$$\sqrt{(p_2 - \{p_3, p_6\})^2}$$

$$\Rightarrow \sqrt{(0.22 - 0.40)^2 + (0.38 - 0.31)^2} = 0.19$$

$$\sqrt{(p_4 - \{p_3, p_6\})^2}$$

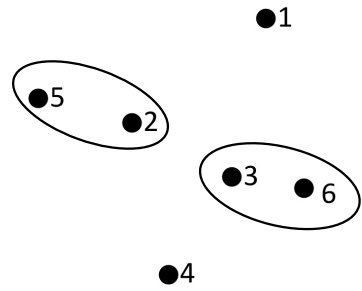
$$\Rightarrow \sqrt{(0.26 - 0.40)^2 + (0.19 - 0.31)^2} = 0.18$$

$$\sqrt{(p_5 - \{p_3, p_6\})^2}$$

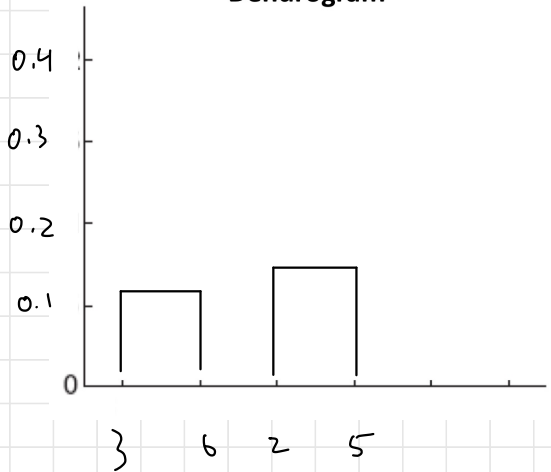
$$\Rightarrow \sqrt{(0.08 - 0.40)^2 + (0.41 - 0.31)^2} = 0.34$$

$p_2 - p_5$ is the closest
with distance 0.14

Nested cluster diagram



Dendrogram



$$\{p_2, p_5\} \quad \{p_3, p_6\}: (0.40, 0.31)$$

| | p_1 | p_4 |
|----------------|-------|-------|
| p_1 | 0 | 0.28 |
| $\{p_2, p_5\}$ | 0.22 | 0.37 |
| $\{p_3, p_6\}$ | 0.26 | 0.23 |
| p_4 | 0.18 | 0 |

$$\{p_2, p_5\}: \frac{p_2 + p_5}{2}$$

$$\frac{(0.22 + 0.08, 0.38 + 0.41)}{2}$$

$$= (0.15, 0.395)$$

$$\sqrt{(p_1 - \{p_2, p_5\})^2}$$

$$\Rightarrow \sqrt{(0.40 - 0.15)^2 + (0.53 - 0.395)^2} = 0.28$$

$$\sqrt{(\{p_3, p_6\} - \{p_2, p_5\})^2}$$

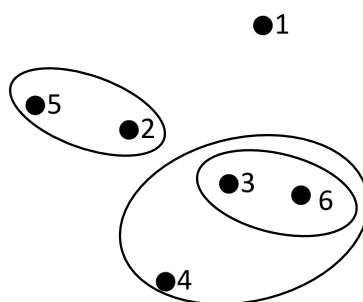
$$\Rightarrow \sqrt{(0.40 - 0.15)^2 + (0.31 - 0.395)^2} = 0.26$$

$$\sqrt{(p_4 - \{p_3, p_6\})^2}$$

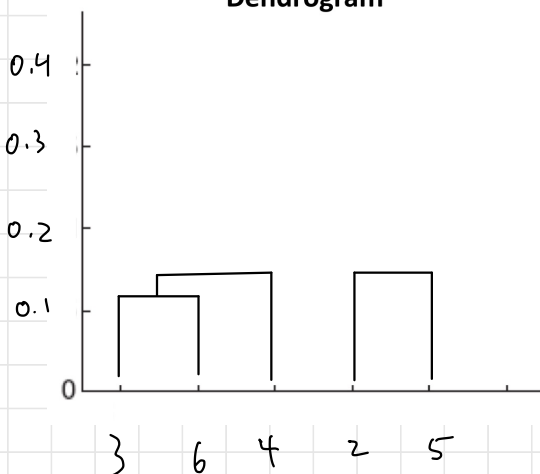
$$\Rightarrow \sqrt{(0.26 - 0.15)^2 + (0.19 - 0.395)^2} = 0.23$$

$p_4 - \{p_3, p_6\}$ is the closest
with distance 0.18

Nested cluster diagram



Dendrogram



$\{p_2, p_5\}: (0.15, 0.395)$

$\{p_3, p_4, p_6\}$:

| | | | |
|---------------------|---|------|------|
| p_1 | 0 | 0.28 | 0.26 |
| $\{p_2, p_5\}$ | 0 | 0.24 | |
| $\{p_3, p_4, p_6\}$ | | | 0 |

$$\frac{(0.35 + 0.26 + 0.45, 0.32 + 0.19 + 0.30)}{3} = (0.35\bar{3}, 0.27)$$

$$\sqrt{(p_1 - \{p_3, p_4, p_6\})^2}$$

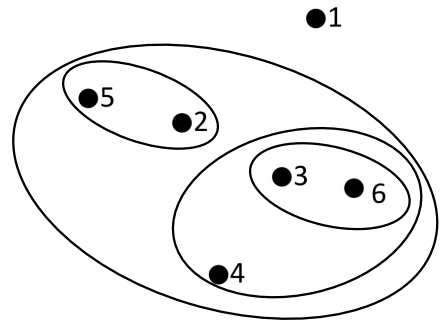
$$\Rightarrow \sqrt{(0.40 - 0.35\bar{3})^2 + (0.53 - 0.27)^2} = 0.26$$

$$\sqrt{(\{p_2, p_5\} - \{p_3, p_4, p_6\})^2}$$

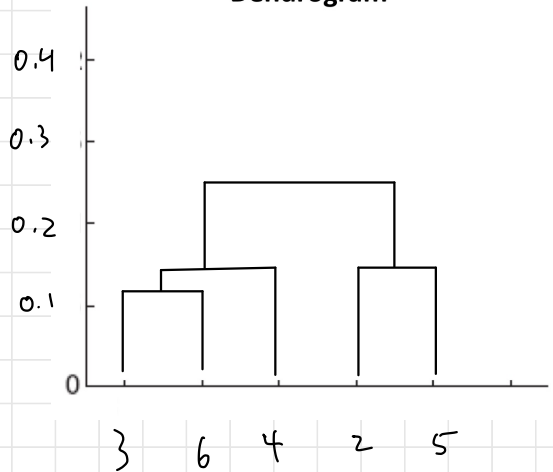
$$\Rightarrow \sqrt{(0.15 - 0.35\bar{3})^2 + (0.395 - 0.27)^2} = 0.24$$

$\{p_2, p_5\} - \{p_3, p_4, p_6\}$ is the closest with distance 0.24

Nested cluster diagram



Dendrogram



$\{p^2, p^3, p^4, p^5, p^6\}$:

| | | |
|-------|-------|--------|
| p^1 | p^1 | 0.25 |
| p^1 | 0 | 0 |

$$\frac{(0.22+0.35+0.26+0.08+0.45, 0.38+0.32+0.14+0.41+0.30)}{5}$$
$$=(0.272, 0.32)$$

$$\begin{array}{r} (0,22+0,35+0,26+0,08+0,45, \\ 0,38+0,32+0,19+0,41+0,30) \\ \hline 5 \\ = (0,272, 0,32) \end{array}$$

$$\sqrt{(p_1 - \{p_2, p_3, p_4, p_5, p_6\})^2}$$

$$\Rightarrow \sqrt{(0.40-0.27)^2 + (0.53-0.32)^2} = 0.25$$

$p_1 - \{p_2, p_3, p_4, p_5, p_6\}$ is the closest
with distance 0.25

| Number of incorrect answers | Relative frequency |
|-----------------------------|--------------------|
| 3 | 0.11 |
| 6 | 0.14 |
| 4 | 0.25 |
| 2 | 0.26 |
| 5 | 0.15 |
| 1 | 0.26 |