

## Practice Problem 2

Ian Chen

February 6, 2024

**Problem 1** — Find the equation for the least-squares regression line for this dataset by hand and show your work.

**Answer**

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\hat{\beta}_1 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$$

$X$	$Y$
2	50
3	71
4	87
5	95

$$\bar{X} = \frac{2+3+4+5}{4} = 3.5$$

$$\bar{Y} = \frac{50+71+87+95}{4} = 75.75$$

$$\sum (X_i - \bar{X})^2 = (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 = (-1.5)^2 + (-0.5)^2 + (0.5)^2 + (1.5)^2 = 4.78125$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = (2-3.5)(50-75.75) + (3-3.5)(71-75.75) + (4-3.5)(87-75.75) + (5-3.5)(95-75.75) = (-1.5)(-25.75) + (-0.5)(-4.75) + (0.5)(11.25) + (1.5)(19.25) = 75.5$$

$$\hat{\beta}_1 = \frac{75.5}{4.78125} = 15.79084967$$

$$\hat{\beta}_0 = 75.75 - 15.79084967(3.5) = 20.48202614$$

**Problem 2** — Using your regression model, predict the exam grade for someone who studied for 30 minutes.

**Answer**

$$Y = \hat{\beta}_1 X + \hat{\beta}_0$$

$$30 \text{ minutes} = 0.5 \text{ hours}$$

$$Y = 15.79084967(0.5) + 20.48202614 = 28.37745098$$

**Problem 3** — Calculate the  $R^2$  of your model by hand and show your work.

**Answer**

$$\text{mean} = \bar{y} = \frac{50+71+87+95}{4} = 75.75$$

$$\text{Var}(\text{mean}) = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n}$$

$$\text{Var}(\text{mean}) = \frac{(50-75.75)^2 + (71-75.75)^2 + (87-75.75)^2 + (95-75.75)^2}{4} = \frac{1182.75}{4} = 295.6875$$

$$\text{Var}(\text{fit}) = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}$$

$$\text{Best fit line: } f(x) = 15.79084967x + 20.48202614$$

$$\text{Var}(\text{fit}) = \frac{(50-f(2))^2 + (71-f(3))^2 + (87-f(4))^2 + (95-f(5))^2}{4}$$

$$\begin{aligned}
&= \frac{(50-52.06372549)^2+(71-67.85457516)^2+(87-83.64542484)^2+(95-99.43627451)^2}{4} \\
&= \frac{(-2.06372549)^2+(3.14542484)^2+(3.35457516)^2+(-4.43627451)^2}{4} \\
&= \frac{4.258962898+9.893697424+11.2531745+19.68053153}{4} \\
&= \frac{45.08636635}{4} = 11.27159159 \\
R^2 &= \frac{Var(mean)-Var(fit)}{Var(mean)} = \frac{295.6875-11.27159159}{295.6875} = 0.9618800538
\end{aligned}$$