Practice Problem 4

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Problem 1 — Do I believe that a patient with the following symptoms has the flu?
$\{Y_c, N_r, Mild_h, Y_f, ?_{flu}\}$

Answer
$P(C \mid X) = \frac{P(X|C)P(C)}{P(X)}$
We don't calculate $P(X)$ *since it is the same for all cases.*
$P(Y_{flu} \mid \{Y_c, N_r, Mild_h, Y_f\}) \propto P(Y_c \mid Y_{flu}) \cdot P(N_r \mid Y_{flu}) \cdot P(Mild_h \mid Y_{flu}) \cdot P(Y_f \mid Y_{flu}) \cdot P(Y_{flu})$
$P(Y_c \mid Y_{flu}) = \frac{3}{5}$
$P(N_r \mid Y_{flu}) = \frac{0}{5}$
$P(Mild_h \mid Y_{flu}) = \frac{2}{5}$
$P(Y_f \mid Y_{flu}) = \frac{4}{5}$
$P(Y_{flu}) = \frac{5}{8}$
This yields the expression $\frac{3}{5} \cdot \frac{0}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} \cdot \frac{5}{8}$
After smoothing, this is $\frac{5}{8} \cdot \frac{4}{7} \cdot \frac{1}{7} \cdot \frac{3}{8} \cdot \frac{5}{7} = \frac{75}{5488} = 0.01366618076$
$P(N_{flu} \mid \{Y_c, N_r, Mild_h, Y_f\}) \propto P(Y_c \mid N_{flu}) \cdot P(N_r \mid N_{flu}) \cdot P(Mild_h \mid N_{flu}) \cdot P(Y_f \mid N_{flu})$
$P(Y_c \mid N_{flu}) = \frac{1}{3}$
$P(N_r \mid N_{flu}) = \frac{2}{3}$
$P(Mild_h \mid N_{flu}) = \frac{1}{3}$
$P(Y_f \mid N_{flu}) = \frac{1}{3}$
Multiplying these together gives us $\frac{3}{8} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$
After smoothing, this is $\frac{3}{8} \cdot \frac{2}{5} \cdot \frac{3}{5} \cdot \frac{2}{6} \cdot \frac{2}{5} = \frac{3}{250} = 0.012$
$P(Y_{flu} \mid \{Y_c, N_r, Mild_h, Y_f\}) > P(N_{flu} \mid \{Y_c, N_r, Mild_h, Y_f\})$
$\therefore$*, the patient likely has the flu.*