

# Erasable Itemset Mining Using Multiple Maximum Thresholds Under Different Constraints

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113 / 04 / 10



# Outline

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Introduction



Related Work



Method



Experiment



Conclusion



# 0 1

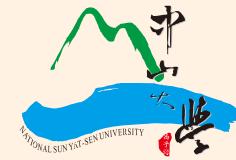
PART ONE

# Introduction

- ✓ Erasable itemset mining



# Introduction

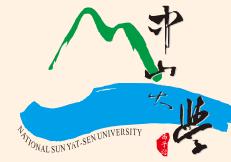


PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

- **Erasable itemset mining**
- **Product database**
- **Control loss**



# Trouble



Funding decrease



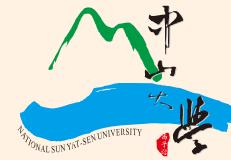
Insufficient storage space



Limited logistics



# Introduction



PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

- Erasable itemset
- Low loss



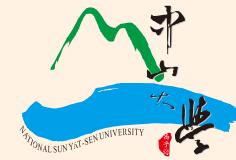
# 1 2

PART TWO

## Related work



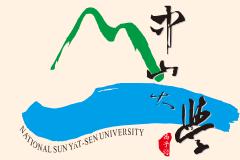
# Erasable itemset mining



PID	Items	Profit
P1	ABC	200
P2	D	200
P3	AD	100
P4	AB	400
P5	BCE	100

## META algorithm

- [Deng et al., 2009]
- Apriori-base method
- Downward closure



# Erasable itemset mining

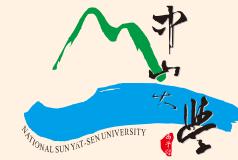
PID	Items	Profit
P1	ACD	200
P2	B	300
P3	AB	100
P4	AD	400
P5	CDE	400

## Gain

Losses when certain items cannot purchase

$$\rightarrow \text{Gain (A)} = 200 + 100 + 400 = 700$$

# Erasable itemset mining



PID	Items	Profit
P1	ABC	200
P2	D	200
P3	AD	100
P4	AB	400
P5	BCE	100

## MERA algorithm

Gain  $\leq$  Maximum gain threshold (MGT)

- Total profit =  $200 + 200 + 100 + 400 + 100 = 1000$
- Threshold = 0.4
- Maximum gain threshold =  $1000 * 0.4 = 400$



# Erasable itemset mining

## META algorithm

PID	Items	Profit
P1	ABC	200
P2	D	200
P3	AD	100
P4	AB	400
P5	BCE	100

MGT = 400



Candidate 1-itemsets	
A	700
B	700
C	300
D	300
E	100

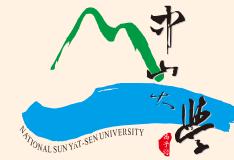


Erasable 1-itemsets	
C	300
D	300
E	100

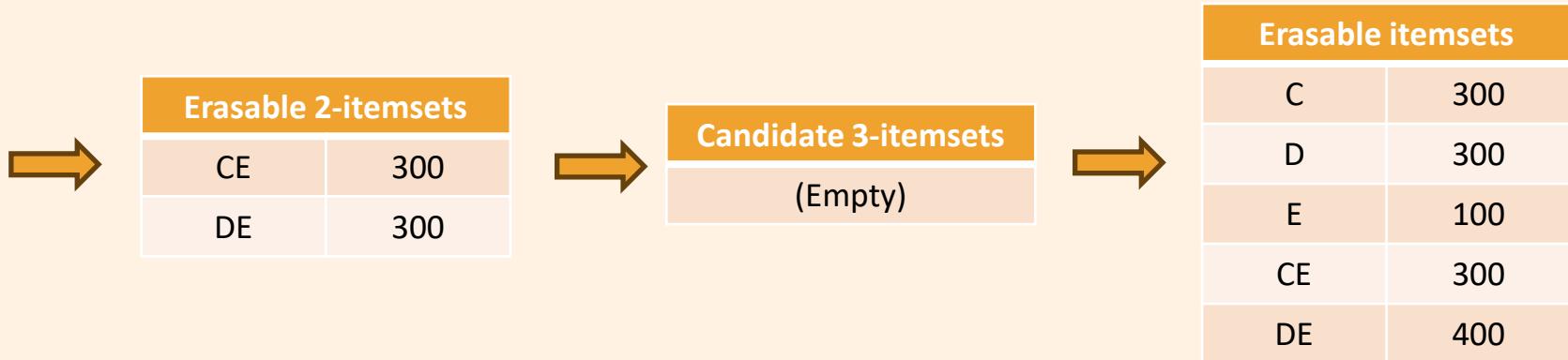
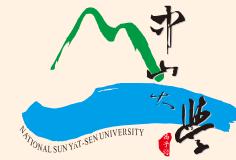


Candidate 2-itemsets	
CD	600
CE	300
DE	400

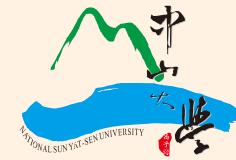
Downward  
closure



# Erasable itemset mining

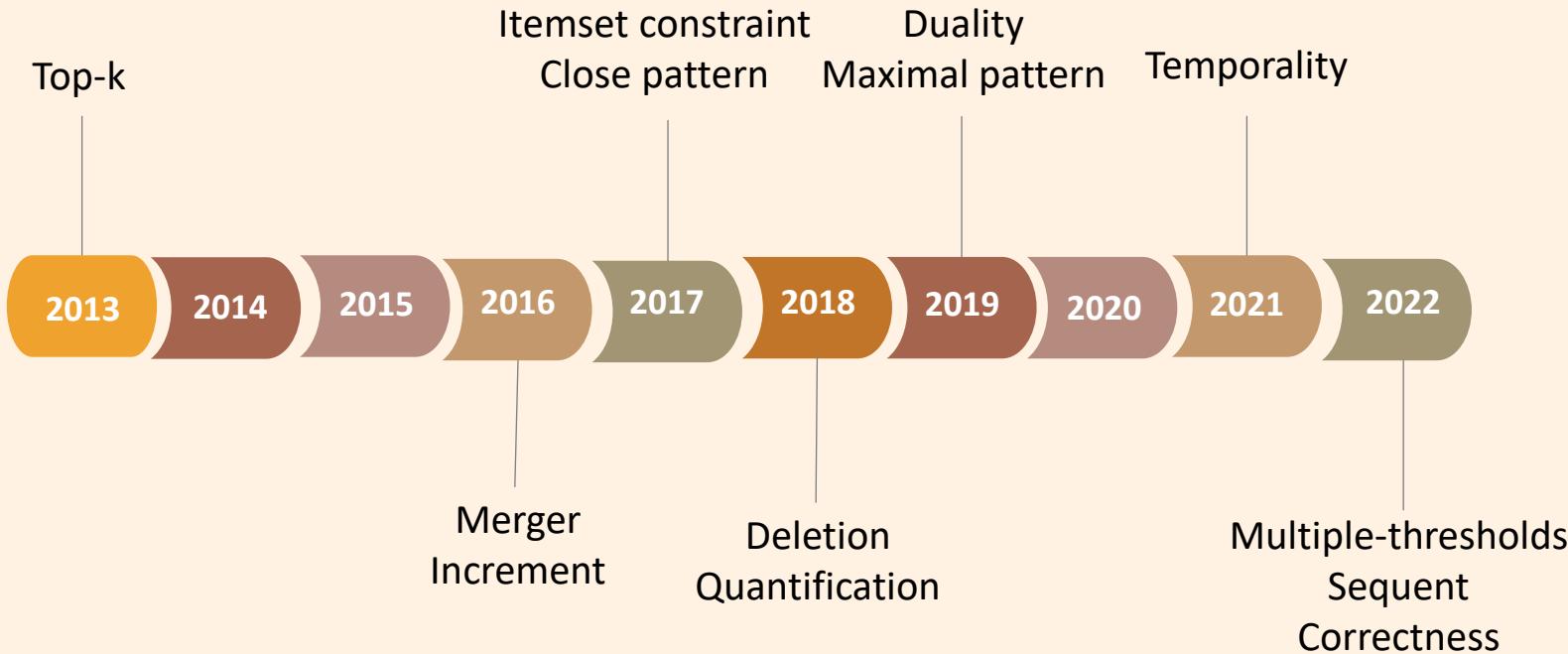
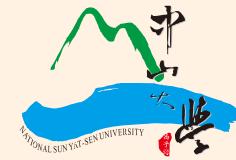


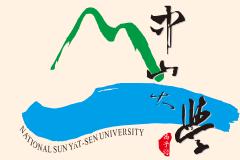
# Performance



Algorithm	Year	Author
VME	2010	Deng et al.
MERIT	2012	Deng et al.
MERIT+	2013	Le et al.
dMERIT+	2013	Le et al.
MEI	2014	Le and Bo
BERM	2021	Hong and Huang et al.

# Various themes



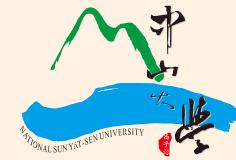


# Multiple-threshold itemset mining

- Single threshold → the same standard for each item
- Considering other attributes of the item:
  1. Volume and weight
  2. Quantity
  3. Storage method

Item	A	B
$\lambda$	0.4	0.2





# Problem of multiple-thresholds

Itemset

$$\lambda(A) = 0.4$$

One item



$$\lambda(B) = 0.2$$

More than one item



Item	A	B
$\lambda$	0.4	0.2



# Constraints of MGT

Item	A	B
$\lambda$	0.4	0.2



**Constraint 1: minimum constraint**

$$\lambda_{\min}(AB) = \min (0.4, 0.2) = 0.2$$

**Constraint 2: maximum constraint**

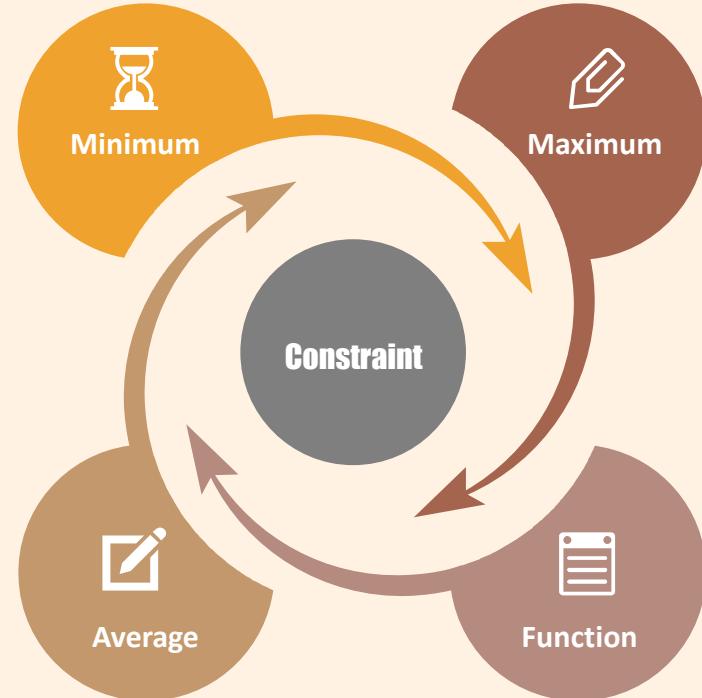
$$\lambda_{\max}(AB) = \max (0.4, 0.2) = 0.4$$

**Constraint 3: average constraint**

$$\lambda_{\text{avg}}(AB) = (0.4+0.2) / 2 = 0.3$$

**Constraint 4: function constraint**

$$\lambda_{\text{fun}}(AB) = \text{function} (0.4, 0.2)$$





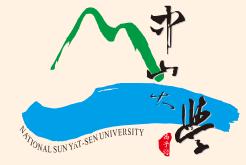
1 3

## PART THREE

# Method

- ✓ Minimum Constraint
- ✓ Maximum Constraint
- ✓ Average Constraint
- ✓ Function Constraint



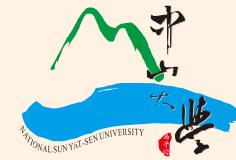


# 01

## Minimum Constraint



# Multiple-threshold erasable itemset mining



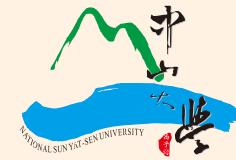
PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

- **Minimum constraint**
- **Downward closure**

Item	A	B	C	D	E	F	G
$\lambda$	0.6	0.1	0.7	0.9	0.5	0.8	0.4



# Step 1



PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

## Scan database

### (1) Total profit:

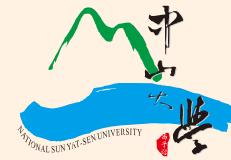
$$200+300+100+200+400+500+100+200 = 2000$$

### (2) Gain of each item:

Item	A	B	C	D	E	F	G
Gain	700	600	1100	800	1300	1000	1300

Item	A	B	C	D	E	F	G
$\lambda$	0.6	0.1	0.7	0.9	0.5	0.8	0.4

## Step 2



PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

Calculate MGT of each item

Total profit = 2000

$$\text{MGT (A)} = 2000 * 0.6 = 1200$$

Item	A	B	C	D	E	F	G
MGT	1200	200	1400	1800	1000	1600	800

Item	A	B	C	D	E	F	G
$\lambda$	0.6	0.1	0.7	0.9	0.5	0.8	0.4



# Step 3

PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

## Verify candidate 1-itemsets

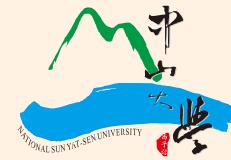
$\text{Gain} \leq \text{MGT}$

Item	A	B	C	D	E	F	G
MGT	1200	200	1400	1800	1000	1600	800
Gain	700	600	1100	800	1300	1000	1300

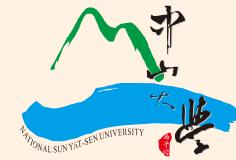


Erasable 1-itemsets			
A	C	D	F

Item	A	B	C	D	E	F	G
MGT	1200	200	1400	1800	1000	1600	800



# Step 4



PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

## Generate candidate 2-itemsets

**Erasable 1-itemsets**

A	C	D	F
---	---	---	---

↓

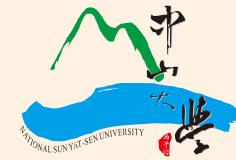
**Candidate 2-itemsets**

AC	AD	AF
CD	CF	DF

Item	A	B	C	D	E	F	G
MGT	1200	200	1400	1800	1000	1600	800



# Step 5



PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

Scan database to calculate gain

Item	AC	AD	AF	CD	CF	DF
Gain	700	600	1100	800	1300	1000

Calculate MGT<sub>min</sub>

$$MGT_{\min}(AC) = \min(MGT(A), MGT(C))$$

$$= \min(1200, 1400) = 1200$$

Item	AC	AD	AF	CD	CF	DF
MGT <sub>min</sub>	1200	1200	1200	1400	1400	1600

Item	A	B	C	D	E	F	G
MGT	1200	200	1400	1800	1000	1600	800



# Step 6



PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

## Verify candidate 2-itemsets

$\text{Gain} \leq \text{MGT}_{\min}$

Itemset	AC	AD	AF	CD	CF	DF
$\text{MGT}_{\min}$	1200	1200	1200	1400	1400	1600
Gain	1200	1100	1600	1200	1900	1000



### Erasable 2-itemsets

AC	AD	CD
----	----	----

Item	A	B	C	D	E	F	G
MGT	1200	200	1400	1800	1000	1600	800



# Step 7



PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

## Generate candidate 3-itemsets

Erasable 2-itemsets		
AC	AD	CD



Candidate 3-itemsets	
ACD	

Item	A	B	C	D	E	F	G
MGT	1200	200	1400	1800	1000	1600	800



# Step 8

PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

## Verify candidate 3-itemsets

$\text{Gain} \leq \text{MGT}_{\min}$

Itemset	ACD
$\text{MGT}_{\min}$	1200
Gain	1300

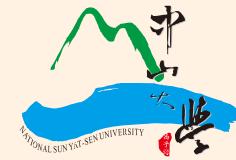


Erasable 3-itemsets
(Empty)

Item	A	B	C	D	E	F	G
MGT	1200	200	1400	1800	1000	1600	800



# Step 9

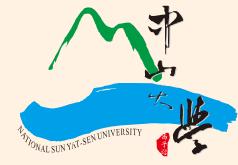


## Output all erasable itemsets

PID	Items	Profit
P1	ABC	200
P2	CDE	300
P3	AF	100
P4	BFG	200
P5	ACDEG	400
P6	EFG	500
P7	DE	100
P8	BCFG	200

Erasable itemsets
A
C
D
F
AC
AD
CD

Item	A	B	C	D	E	F	G
MGT	1200	200	1400	1800	1000	1600	800



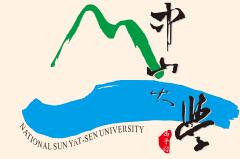
01

Minimum  
Constraint

02

Maximum  
Constraint

# Multiple-threshold erasable itemset mining



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

- Maximum constraint
- No downward closure property
- Sorted closure property

Item	A	B	C	D	E	F	G
$\lambda$	0.4	0.1	0.5	0.6	0.3	0.6	0.7

# Step 1

PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Sort the thresholds in descending order

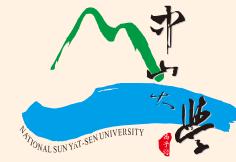
Item	A	B	C	D	E	F	G
$\lambda$	0.4	0.1	0.5	0.6	0.3	0.6	0.7



Item	G	D	F	C	A	E	B
$\lambda$	0.7	0.6	0.6	0.5	0.4	0.3	0.1



# Step 2



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Scan database

### (1) Total profit:

$$200+300+100+200+400+500+100+200 = 2000$$

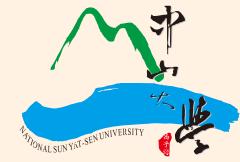
### (2) Gain of each item:

Item	G	D	F	C	A	E	B
Gain	1500	600	400	500	500	900	1300

Item	G	D	F	C	A	E	B
$\lambda$	0.7	0.6	0.6	0.5	0.4	0.3	0.1



# Step 3



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Calculate MGT of each item

Total profit = 2000

Item	G	D	F	C	A	E	B
$\lambda$	0.7	0.6	0.6	0.5	0.4	0.3	0.1



Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 4

PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Find the first erasable itemset

$\text{Gain} \leq \text{MGT}$



Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200
Gain	1500	600	400	500	500	900	1300

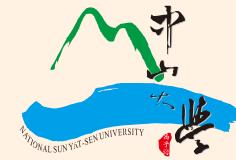


Candidate 1-itemsets

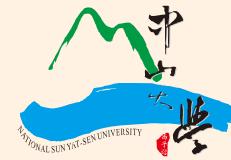
D

$$\text{MGT}(D) = 1200$$

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 5



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Verify itemset sorted after {D}

$\text{Gain} \leq \text{MGT}(D)$

Item	D	F	C	A	E	B
MGT(D)	1200	1200	1200	1200	1200	1200
Gain	600	400	500	500	900	1300



Candidate 1-itemsets

D	F	C	A	E
---	---	---	---	---

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 6



## Verify candidate 1-itemsets

PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Item	D	F	C	A	E
MGT	1200	1200	1000	800	600
Gain	600	400	500	500	900

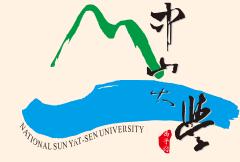


Erasable 1-itemsets				
D	F	C	A	

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 7



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 2-itemset

Candidate 1-itemsets				
D	F	C	A	E

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 7



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200

Generate candidate 2-itemset



Item	D	F	C	A	E
MGT	1200	1200	1000	800	600
Gain	600	400	500	500	900

$\text{Gain}(F) \leq \text{MGT}(D)$

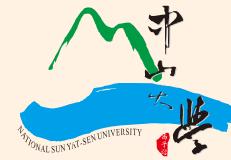


Candidate 2-itemsets

DF



# Step 7

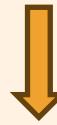


PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Generate candidate 2-itemset



Item	D	F	C	A	E
MGT	1200	1200	1000	800	600
Gain	600	400	500	500	900



Candidate 2-itemsets			
DF	DC	DA	DE

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 7



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200

## Generate candidate 2-itemset

Item	D	F	C	A	E
MGT	1200	1200	1000	800	600
Gain	600	400	500	500	900



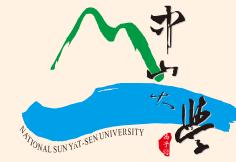
$\text{Gain}(E) > \text{MGT}(A)$

### Candidate 2-itemsets

AE



# Step 7



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 2-itemsets

Candidate 1-itemset

D	F	C	A	E
---	---	---	---	---

↓

Candidate 2-itemsets

DF	DC	DA	DE	FC
FA	FE	CA	CE	

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 8

## Verify candidate 2-itemsets



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>max</sub>	Gain
DF	1200	900
DC	1200	800
DA	1200	1100
DE	1200	1100
FC	1200	900
FA	1200	600
FE	1200	1200
CA	1000	800
CE	1000	1100

Gain  $\leq$  MGT<sub>max</sub>

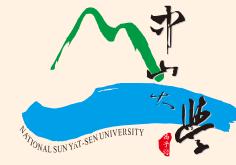


Erasable 2-itemsets
DF
DC
DA
DE
FC
FA
FE
CA

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 9



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 3-itemset

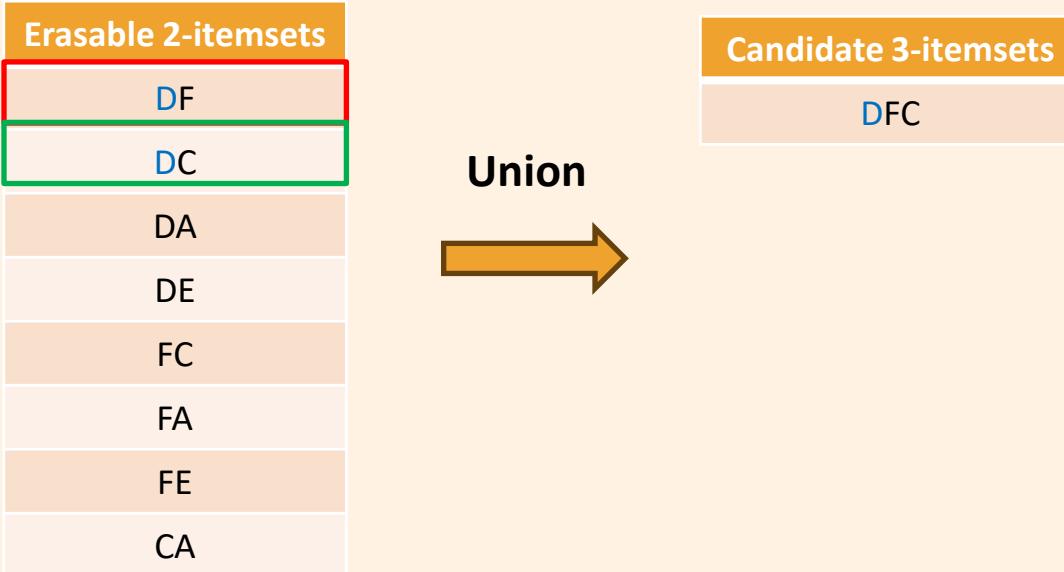
Erasable 2-itemsets			
DF	DC	DA	DE
FC	FA	FE	CA

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



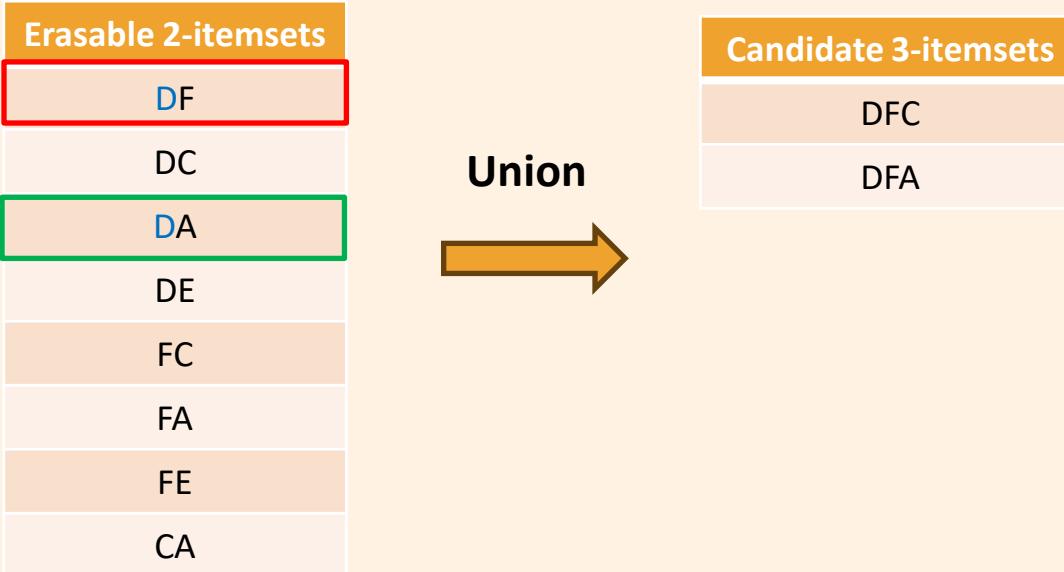


## Step 9-1 Union of any two itemsets with the same front (k-1) items.





## Step 9-1 Union of any two itemsets with the same front (k-1) items.



## Step 9-1

Union of any two itemsets with the same front ( $k-1$ ) items

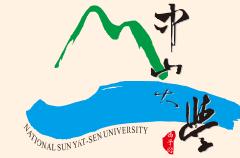


Erasable 2-itemsets
DF
DC
DA
DE
FC
FA
FE
CA



Candidate 3-itemsets
DFC
DFA
DFE
DCA
DCE
DAE
FCA
FCE





## Step 9-2 Prune candidate itemsets

Case 1:  $MGT(item_1) = MGT(item_2)$

Candidate 3-itemsets
DFC
DFA
DFE
DCA
DCE
DAE
FCA
FCE

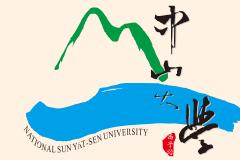
All (k-1)-subsets  
MGT(D)=MGT(F)



2-subset
DF
DC
FC

Erasable 2-itemsets
DF
DC
DA
DE
FC
FA
FE
CA

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



## Step 9-2 Prune itemsets

Case 2:  $MGT(item_1) \neq MGT(item_2)$

Candidate 3-itemsets
DFC
DFA
DFE
DCA
DCE
DAE
FCA
<b>FCE</b>

(k-1)-subsets contain item<sub>1</sub>

$MGT(F) \neq MGT(C)$

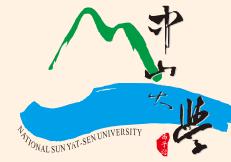


2-subset
FC
FE
CE <span style="color:red;">X</span>

Erasable 2-itemsets
DF
DC
DA
DE
FC
FA
FE
CA

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200

# Step 9



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 3-itemset

Erasable 2-itemsets			
DF	DC	DA	DE
FC	FA	FE	CA



Candidate 3-itemsets				
DFC	DFA	DFE	DCA	DCE
DAE	FCA	FCE	FAE	

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 10

## Verify candidate 3-itemsets



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>max</sub>	Gain
DFC	1200	1100
DFA	1200	1100
DFE	1200	1400
DCA	1200	1100
DCE	1200	1300
DAE	1200	1600
FCA	1200	900
FCE	1200	1400
FAE	1200	1400

Gain  $\leq$  MGT<sub>max</sub>

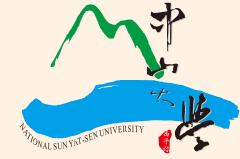


Erasable 3-itemsets
DFC
DFA
DCA
FCA

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 11



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 4-itemsets

Erasable 3-itemsets			
DFC	DFA	DCA	FCA



Candidate 4-itemsets	
DFCA	

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 12

## Verify candidate 4-itemset



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>max</sub>	Gain
DFCA	1200	1100

$\text{Gain} \leq \text{MGT}_{\text{max}}$

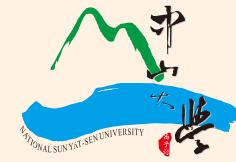


Erasable 4-itemset
DFCA

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 13



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 5-itemsets

Erasable 4-itemsets
DFCA



Candidate 5-itemsets
(Empty)

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 14



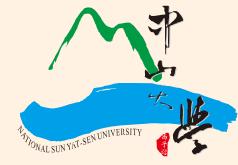
## Output all erasable itemsets

PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200

Erasable itemsets	
D	FA
F	FE
C	CA
A	DFC
DF	DFA
DC	DCA
DA	FCA
DE	DFCA
FC	





01

Minimum  
Constraint

02

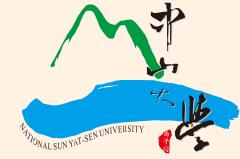
Maximum  
Constraint

03

Average  
Constraint



# Multiple-threshold erasable itemset mining

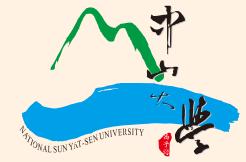


PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

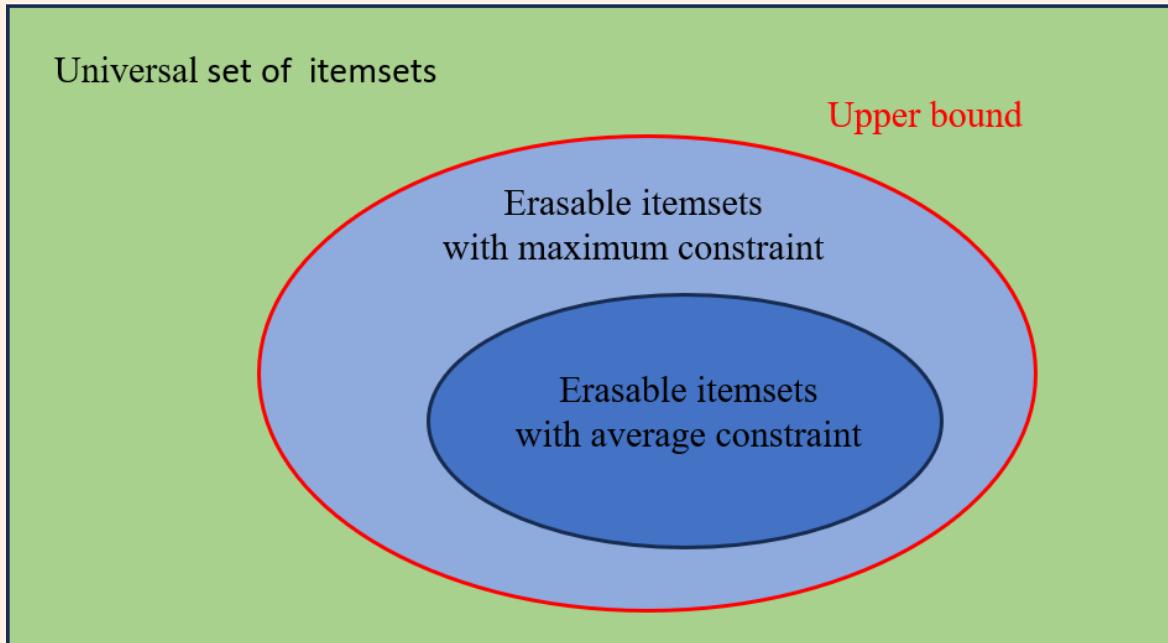
- Average constraint
- No downward closure property
- Upper bound strategy

Item	A	B	C	D	E	F	G
$\lambda$	0.4	0.1	0.5	0.6	0.3	0.6	0.7





# Upper bound strategy



# Step 1

PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Sort the thresholds in descending order

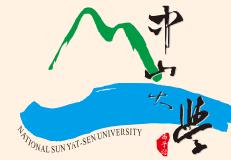
Item	A	B	C	D	E	F	G
$\lambda$	0.4	0.1	0.5	0.6	0.3	0.6	0.7



Item	G	D	F	C	A	E	B
$\lambda$	0.7	0.6	0.6	0.5	0.4	0.3	0.1



# Step 2



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Scan database

### (1) Total profit:

$$200+300+100+200+400+500+100+200 = 2000$$

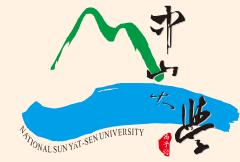
### (2) Gain of each item:

Item	G	D	F	C	A	E	B
Gain	1500	600	400	500	500	900	1300

Item	G	D	F	C	A	E	B
$\lambda$	0.7	0.6	0.6	0.5	0.4	0.3	0.1



# Step 3



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Calculate MGT of each item

Total profit = 2000

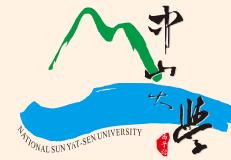
Item	G	D	F	C	A	E	B
$\lambda$	0.7	0.6	0.6	0.5	0.4	0.3	0.1



Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 4



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Find the first erasable itemset

→

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200
Gain	1500	600	400	500	500	900	1300



Candidate 1-itemsets

D

$$MGT(D) = 1200$$

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 5



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Verify itemset sorted after {D}

Item	D	F	C	A	E	B
MGT(D)	1200	1200	1200	1200	1200	1200
Gain	600	400	500	500	900	1300



Candidate 1-itemsets				
D	F	C	A	E

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 6



## Verify candidate 1-itemsets

PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Item	D	F	C	A	E
MGT	1200	1200	1000	800	600
Gain	600	400	500	500	900

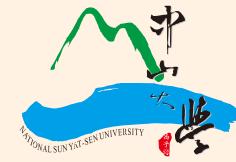


Erasable 1-itemsets				
D	F	C	A	

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200

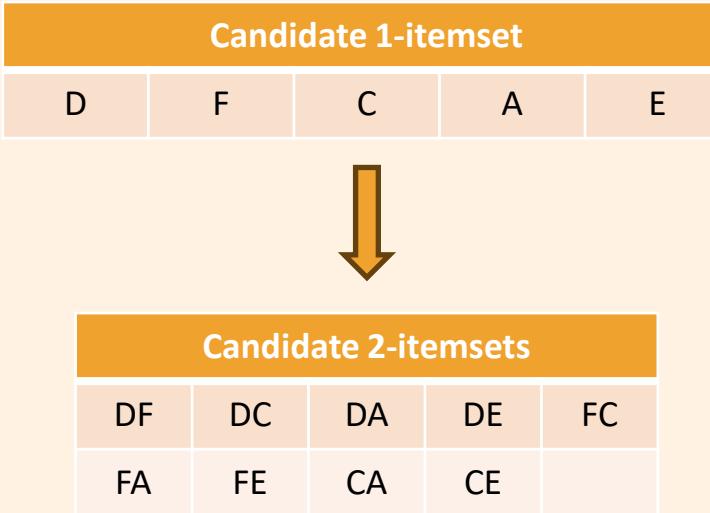


# Step 7



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 2-itemsets

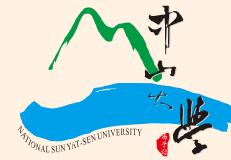


Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 8

## Verify candidate 2-itemsets



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>max</sub>	MGT <sub>avg</sub>	Gain
DF	1200	1200	900
DC	1200	1100	800
DA	1200	1000	1100
DE	1200	900	1100
FC	1200	1100	900
FA	1200	1000	600
FE	1200	900	1200
CA	1000	900	800
CE	1000	800	1100

Case 1:

$\text{Gain} \leq \text{MGT}_{\text{max}}$  &  $\text{Gain} \leq \text{MGT}_{\text{avg}}$

Add to the erasable itemsets



Erasable 2-itemsets
DF
DC
FC
FA
CA

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 8

## Verify candidate 2-itemsets



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>max</sub>	MGT <sub>avg</sub>	Gain
DF	1200	1200	900
DC	1200	1100	800
DA	1200	1000	1100
DE	1200	900	1100
FC	1200	1100	900
FA	1200	1000	600
FE	1200	900	1200
CA	1000	900	800
CE	1000	800	1100

Case 2:

$\text{Gain} \leq \text{MGT}_{\text{max}}$  &  $\text{Gain} > \text{MGT}_{\text{avg}}$

Do nothing

Ex: {D, A}, {D, E}, {F, E}

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 8

## Verify candidate 2-itemsets



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>max</sub>	MGT <sub>avg</sub>	Gain
DF	1200	1200	900
DC	1200	1100	800
DA	1200	1000	1100
DE	1200	900	1100
FC	1200	1100	900
FA	1200	1000	600
FE	1200	900	1200
CA	1000	900	800
CE	1000	800	1100

Case 3:

Gain > MGT<sub>max</sub>

Prune the candidate itemset

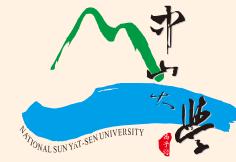
Ex: {C, E}

Candidate 2-itemsets
DF
DC
DA
DE
FC
FA
FE
CA

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 9



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 3-itemset

Candidate 2-itemsets			
DF	DC	DA	DE
FC	FA	FE	CA



Candidate 3-itemsets				
DFC	DFA	DFE	DCA	DCE
DAE	FCA	FCE	FAE	

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 10

## Verify candidate 3-itemsets



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>max</sub>	MGT <sub>avg</sub>	Gain
DFC	1200	1133.3	1100
DFA	1200	1066.6	1100
DFE	1200	1000	1400
DCA	1200	1000	1100
DCE	1200	933.3	1300
DAE	1200	866.6	1600
FCA	1200	1000	900
FCE	1200	933.3	1400
FAE	1200	866.6	1400

Candidate 3-itemsets
DFC
DFA
DCA
FCA



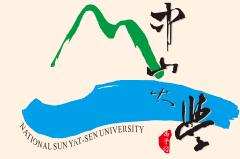
Erasable 3-itemsets
DFC
FCA



Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 11



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 4-itemsets

Candidate 3-itemsets			
DFC	DFA	DCA	FCA



Candidate 4-itemsets	
DFCA	

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 12

## Verify candidate 4-itemset



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>max</sub>	Gain
DFCA	1200	1100

Candidate 4-itemset

DFCA



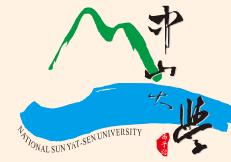
Erasable 4-itemset

(Empty)

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 13



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 5-itemsets

Candidate 4-itemset
DFCA

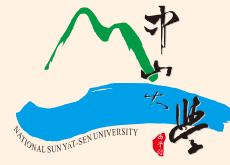


Candidate 5-itemset
(Empty)

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200



# Step 14



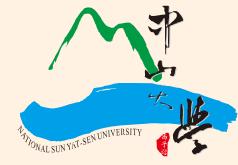
## Output all erasable itemsets

PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Erasable itemsets	
D	FC
F	FA
C	CA
A	DFC
DF	FCA
DC	

Item	G	D	F	C	A	E	B
MGT	1400	1200	1200	1000	800	600	200





**01**

Minimum  
Constraint

**02**

Maximum  
Constraint

**03**

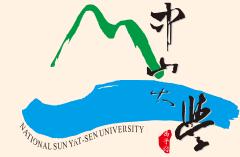
Average  
Constraint

**04**

Function  
Constraint



# Multiple-threshold erasable itemset mining



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

- Function constraint
- No downward closure property
- Upper bound strategy
- Extend from average constraint

Item	A	B	C	D	E	F	G
$\lambda$	0.4	0.1	0.5	0.6	0.3	0.6	0.7

# Idea

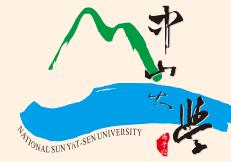


- $0 \leq \lambda \leq 1$
- Function : sum of  $\lambda$
- $\lambda_{\text{sum}}(\text{ABD}) = 0.4 + 0.1 + 0.6 = 1.1 > 1$

Item	A	B	C	D	E	F	G
$\lambda$	0.4	0.1	0.5	0.6	0.3	0.6	0.7



# Idea



- Highest threshold  $\lambda_h = 0.7$
- $\lambda_{\text{sum}}(\text{ABD}) = 1.1 > \lambda_h \rightarrow 0.7$
- $\lambda_{\text{sum}}(\text{AB}) = 0.5 < \lambda_h \rightarrow 0.5$

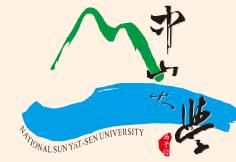
Downward  
closure

Item	A	B	C	D	E	F	G
$\lambda$	0.4	0.1	0.5	0.6	0.3	0.6	0.7

$$f_{\text{fun}}(A) = \begin{cases} \lambda_h & , f(A) > \lambda_h \\ f(A) & , 0 \leq f(A) \leq \lambda_h \\ 0 & , f(A) < 0 \end{cases}$$



# Step 1



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Scan database

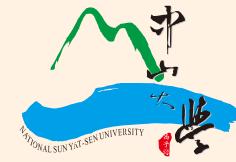
### (1) Total profit:

$$200+300+100+200+400+500+100+200 = 2000$$

### (2) Gain of each item:

Item	A	B	C	D	E	F	G
Gain	500	1300	500	600	900	400	1500

# Step 2



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Calculate MGT of each item

Total profit = 2000,  $\lambda_h = 0.6$

Item	A	B	C	D	E	F	G
$\lambda$	0.4	0.1	0.5	0.6	0.3	0.6	0.7



Item	A	B	C	D	E	F	G
MGT	800	200	1000	1200	600	1200	1400

$$MGT_h = 2000 * 0.6 = 1200$$



# Step 3



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Verify itemset with $MGT_h$

$Gain \leq MGT_h$

Item	A	B	C	D	E	F	G
$MGT_h$	1200	1200	1200	1200	1200	1200	1200
Gain	500	1300	500	600	900	400	1500



### Candidate 1-itemsets

A	C	D	E	F
---	---	---	---	---

Item	A	B	C	D	E	F	G
MGT	800	200	1000	1200	600	1200	1400



# Step 4



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Verify candidate 1-itemsets

$\text{Gain} \leq \text{MGT}$

Item	A	C	D	E	F
MGT	800	1000	1200	600	1200
Gain	500	500	600	900	400

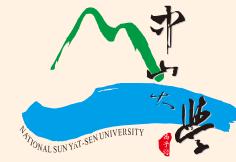


Erasable 1-itemsets			
A	C	D	F

Item	A	B	C	D	E	F	G
MGT	800	200	1000	1200	600	1200	1400

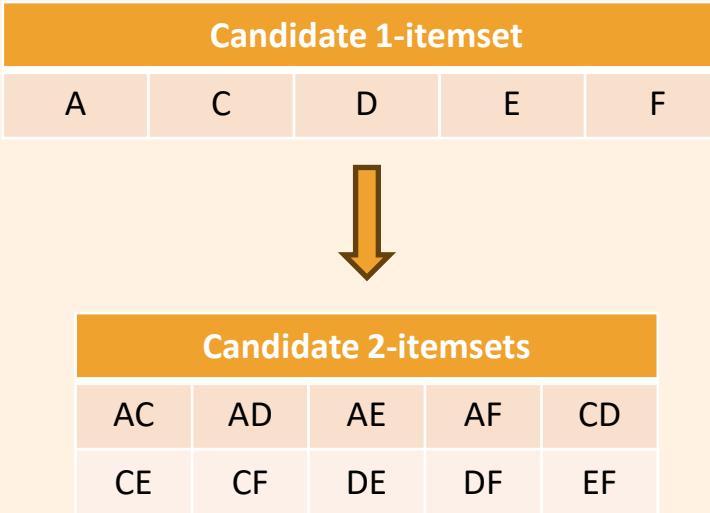


# Step 5



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 2-itemsets



Item	A	B	C	D	E	F	G
MGT	800	200	1000	1200	600	1200	1400



# Step 6

## Verify candidate 2-itemsets

PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>h</sub>	MGT <sub>avg</sub>	Gain
AC	1200	900	800
AD	1200	1000	1100
AE	1200	700	1400
AF	1200	1000	600
CD	1200	1100	800
CE	1200	800	1100
CF	1200	1100	900
DE	1200	900	1100
DF	1200	1200	900
EF	1200	900	1200

### Candidate 2-itemsets

AC

AD

AF

CD

CE

CF

DE

DF

EF

### Erasable 2-itemsets

AC

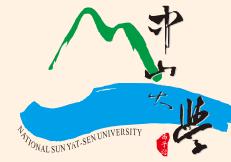
AF

CD

CF

DF

# Step 7



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Generate candidate 3-itemset

Candidate 2-itemsets				
AC	AD	AF	CD	CE
CF	DE	DF	EF	



Candidate 3-itemsets				
ACD	ACF	ADF	CDE	CDF
CEF	DEF			

Item	A	B	C	D	E	F	G
MGT	800	200	1000	1200	600	1200	1400



# Step 8

## Verify candidate 3-itemsets



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Itemset	MGT <sub>h</sub>	MGT <sub>avg</sub>	Gain
ACD	1200	1000	1100
ACF	1200	1000	900
ADF	1200	1066.6	1100
CDE	1200	933.3	1300
CDF	1200	1133.3	1100
CEF	1200	933.3	1300
DFG	1200	1000	1400

Candidate 3-itemsets
ACD
ACF
ADF
CDF



Erasable 3-itemsets
ACF
CDF

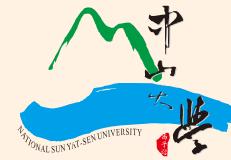


Item	A	B	C	D	E	F	G
MGT	800	200	1000	1200	600	1200	1400



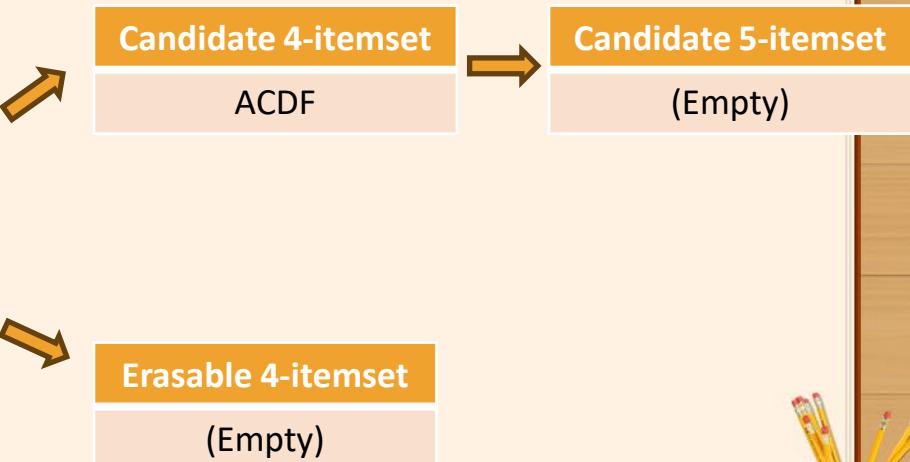
# Step 9

## Verify candidate 3-itemsets



PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

Candidate 3-itemsets
ACD
ACF
ADF
CDF



Item	A	B	C	D	E	F	G
MGT	800	200	1000	1200	600	1200	1400

# Step 10



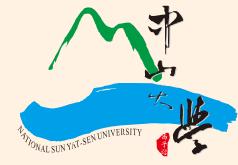
PID	Items	Profit
P1	ABCG	200
P2	CDE	300
P3	AF	100
P4	AFG	200
P5	BG	400
P6	BEG	500
P7	DEF	100
P8	BDG	200

## Output all erasable itemsets

Erasable itemsets	
A	CD
C	CF
D	DF
F	ACF
AC	CDF
AF	

Item	A	B	C	D	E	F	G
MGT	800	200	1000	1200	600	1200	1400





01

Minimum  
Constraint

02

Maximum  
Constraint

03

Average  
Constraint

04

Function  
constraint

05

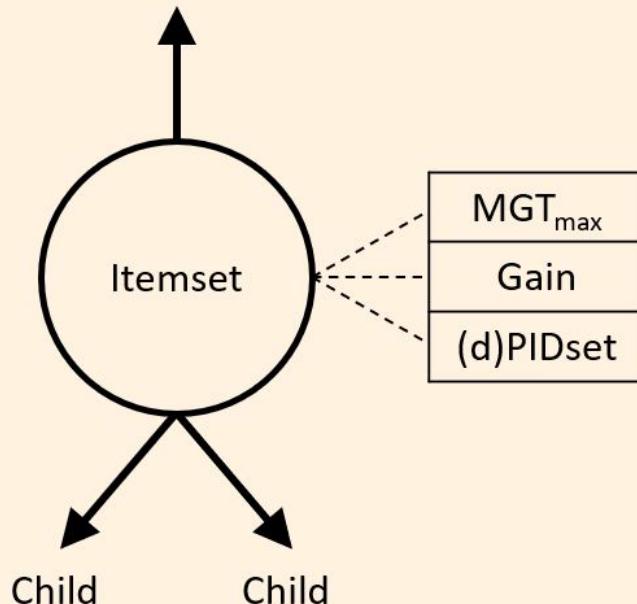
Efficient  
maximum  
constraint



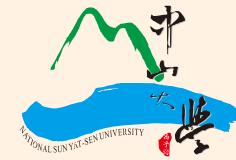
# Node structure



Additional information :



# PIDset

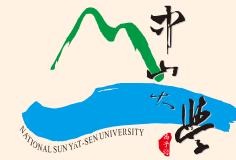


PID	Items	Profit
P1	ABC	200
P2	BE	200
P3	CD	100
P4	BEF	300
P5	ABE	100
P6	ACE	100

PIDset (A)= {1, 5, 6}

PIDset (B)= {1, 2, 4, 5}

# New method for calculating gain



PID	Items	Profit
P1	ABC	200
P2	BE	200
P3	CD	100
P4	BEF	300
P5	ABE	100
P6	ACE	100

## Itemset {A, B}

$$\begin{aligned} dPIDset(AB) &= PIDset(B) - PIDset(A) \\ &= \{1, 2, 4, 5\} - \{1, 5, 6\} \\ &= \{2, 4\} \end{aligned}$$

$$\begin{aligned} Gain(AB) &= Gain(A) + (P2.profit + P4.profit) \\ &= 400 + (200 + 300) \\ &= 900 \end{aligned}$$

Item	A	B
Gain	400	800
PIDset	1, 5, 6	1, 2, 4, 5



# Step 1

PID	Items	Profit
P1	ABC	200
P2	BE	200
P3	CD	100
P4	BEF	300
P5	ABE	100
P6	ACE	100

Sort the thresholds in descending order

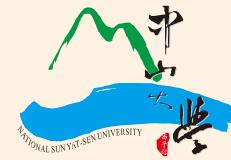
Item	A	B	C	D	E	F
$\lambda$	0.5	0.3	0.4	0.4	0.6	0.2



Item	E	A	C	D	B	F
$\lambda$	0.6	0.5	0.4	0.4	0.3	0.3



# Step 2



PID	Items	Profit
P1	ABC	200
P2	BE	200
P3	CD	100
P4	BEF	300
P5	ABE	100
P6	ACE	100

## Scan database

**(1) Total profit:**

$$200+200+100+300+100+100 = 1000$$

**(2) Gain and PIDset of each item:**

Item	E	A	C	D	B	F
Gain	700	400	400	100	600	300
PIDset	2, 5, 6	1, 5, 6	1, 3, 6	3	1, 2, 4, 5	4

Item	E	A	C	D	B	F
$\lambda$	0.7	0.6	0.6	0.5	0.4	0.3



# Step 2

PID	Items	Profit
P1	ABC	200
P2	BE	200
P3	CD	100
P4	BEF	300
P5	ABE	100
P6	ACE	100

Item	E	A	C	D	B	F
$\lambda$	0.7	0.6	0.6	0.5	0.4	0.3

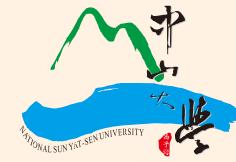
## Scan database

(3) The profit hash table:

Key	Value
P1	200
P2	200
P3	100
P4	300
P5	100
P6	100



# Step 3



PID	Items	Profit
P1	ABC	200
P2	BE	200
P3	CD	100
P4	BEF	300
P5	ABE	100
P6	ACE	100

## Calculate MGT of each item

Total profit = 1000

Item	E	A	C	D	B	F
$\lambda$	0.6	0.5	0.4	0.4	0.3	0.3



Item	E	A	C	D	B	F
MGT	600	500	400	400	300	300



# Step 4

PID	Items	Profit
P1	ABC	200
P2	BE	200
P3	CD	100
P4	BEF	300
P5	ABE	100
P6	ACE	100

## Find the first erasable itemset

Gain  $\leq$  MGT



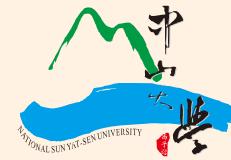
Item	E	A	C	D	B	F
MGT	600	500	400	400	300	300
Gain	700	400	400	100	600	300



Candidate 1-itemsets

A

$$MGT(A) = 500$$



# Step 5

PID	Items	Profit
P1	ABC	200
P2	BE	200
P3	CD	100
P4	BEF	300
P5	ABE	100
P6	ACE	100

## Verify itemset sorted after {A}

$\text{Gain} \leq \text{MGT}(A)$  

Item	A	C	D	B	F
MGT(A)	500	500	500	500	500
Gain	400	400	100	600	300



### Candidate 1-itemsets

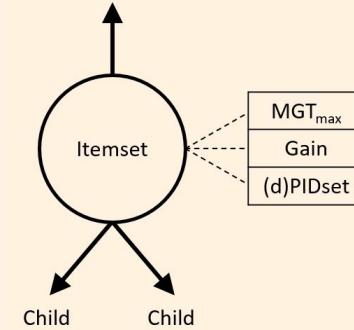
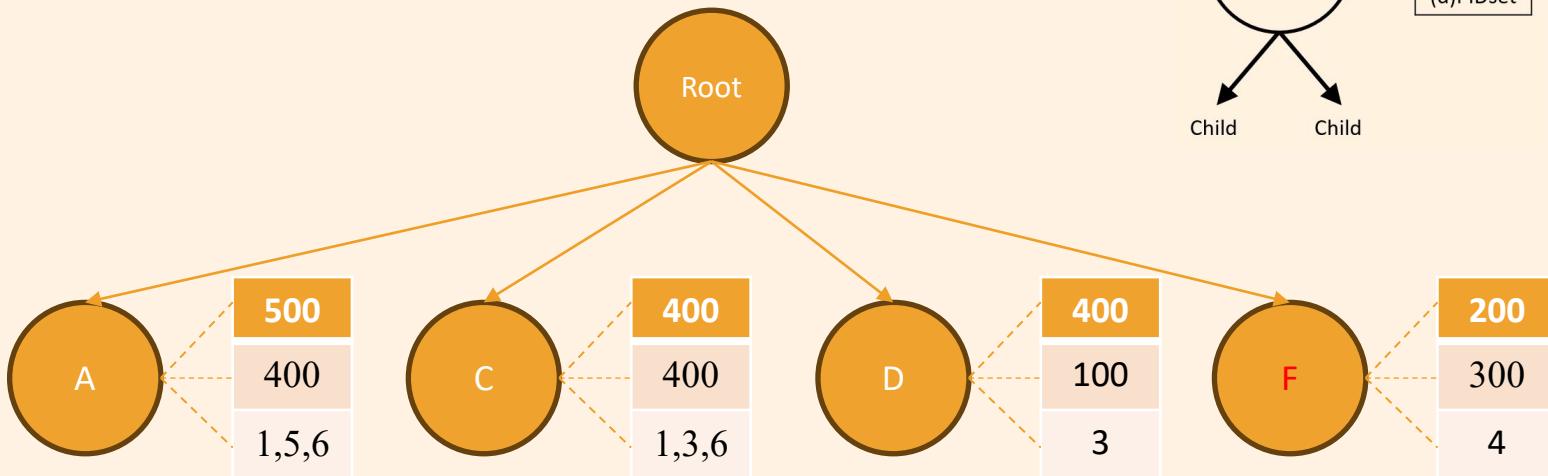
A	C	D	F
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# Step 6 Insert candidate 1-itemsets

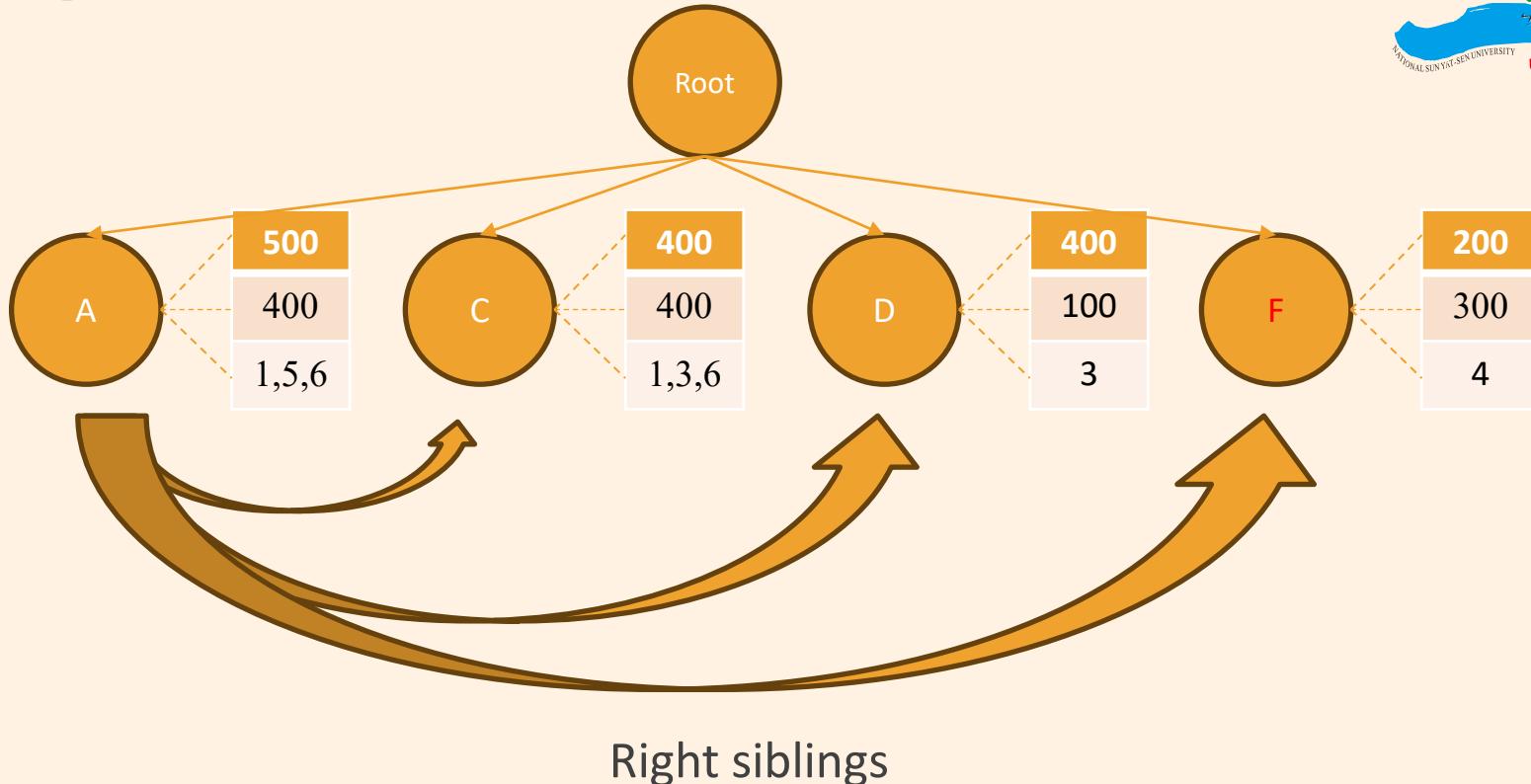
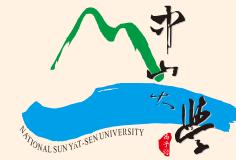


Item	A	C	D	F
MGT	500	400	400	200
<i>Gain</i>	400	400	100	300
PIDset	1, 5, 6	1, 3, 6	3	4

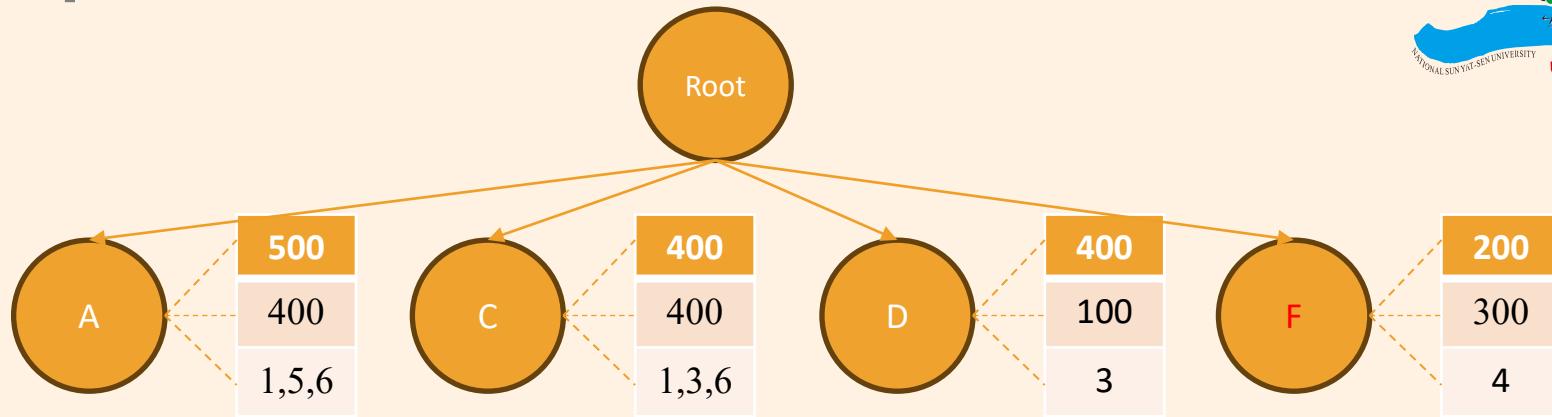
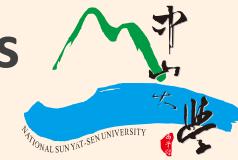


# Step 7

## Generate candidate itemsets for {A}



# Step 8 Calculate information of candidate itemsets

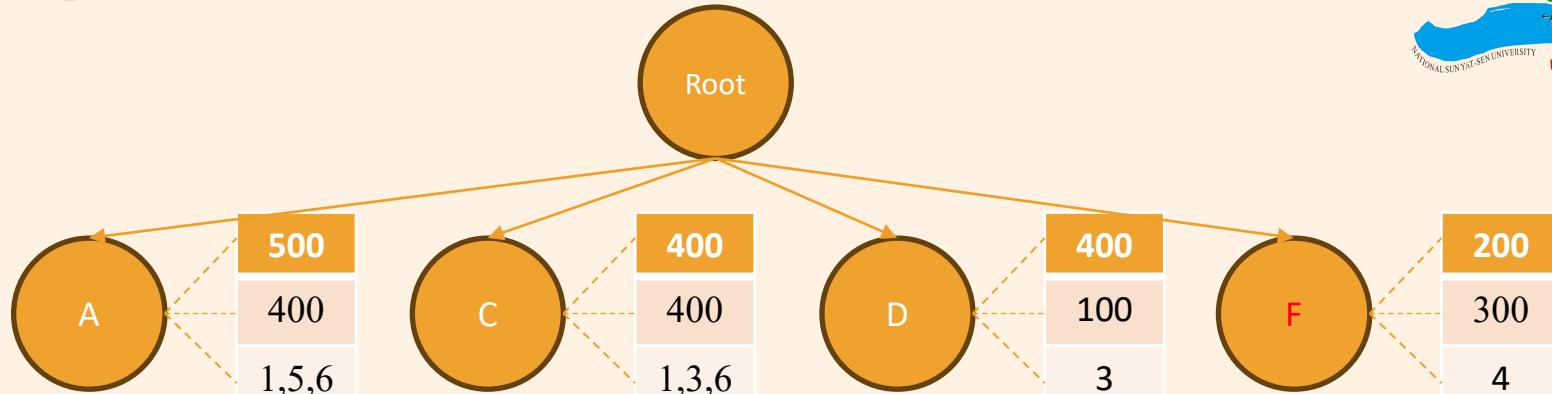


$$\begin{aligned} \text{MGT}_{\max}(\text{AC}) &= \text{MGT}_{\max}(\text{AC.parent}) \\ &= \text{MGT}_{\max}(\text{A}) \\ &= 500 \end{aligned}$$

Candidate itemsets		
AC	AD	AF



# Step 8 Calculate information of candidate itemsets



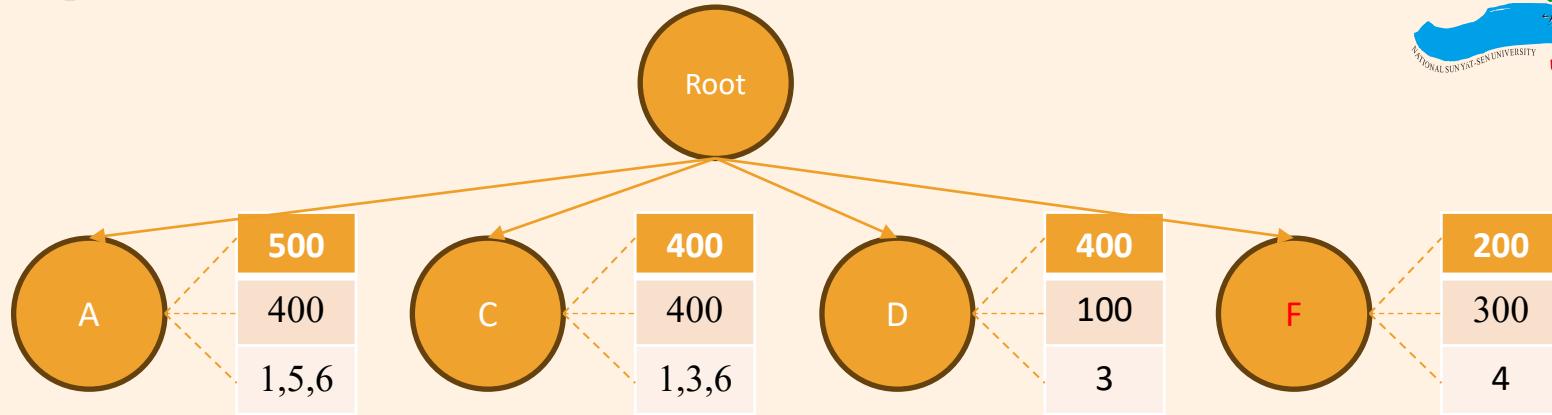
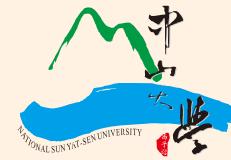
Candidate itemsets		
AC	AD	AF



Item	AC	AD	AF
MGT <sub>max</sub>	500	500	500
Gain	500	500	700
dPIDset	3	3	4



# Step 9 Verify candidate itemsets for {A}



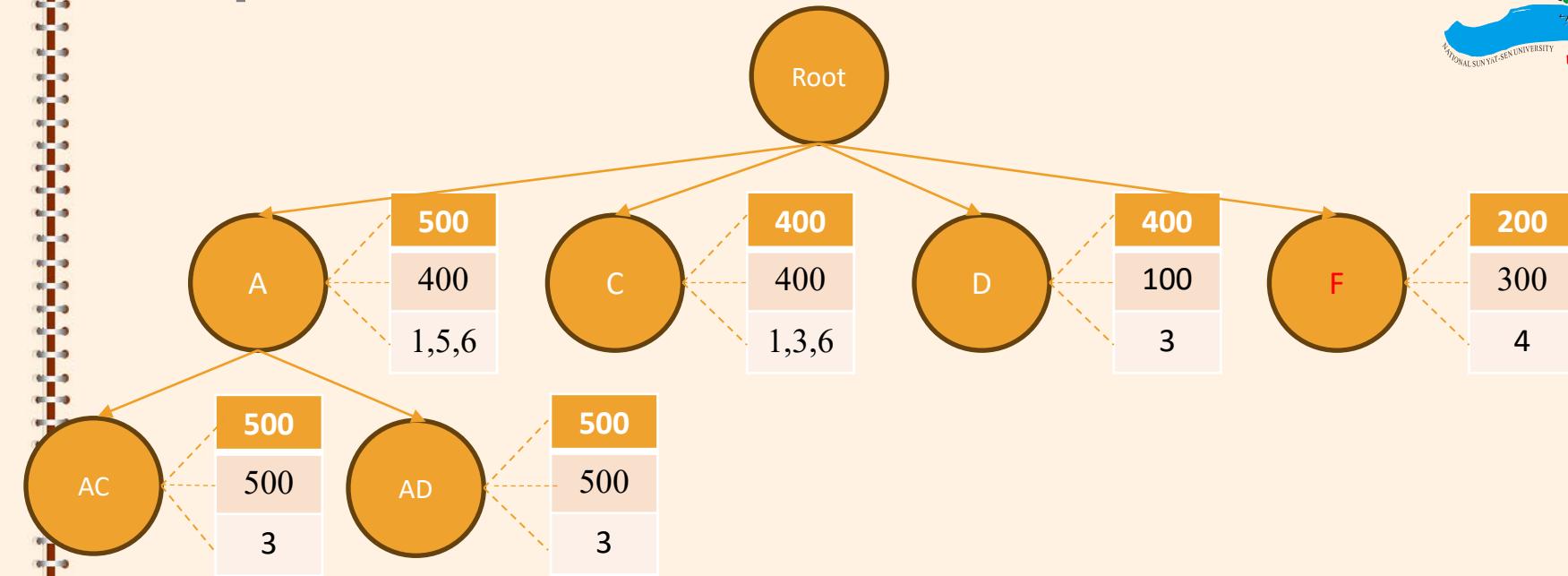
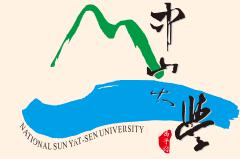
Item	AC	AD	AF
MGT <sub>max</sub>	500	500	500
Gain	500	500	700
dPIDset	3	3	4



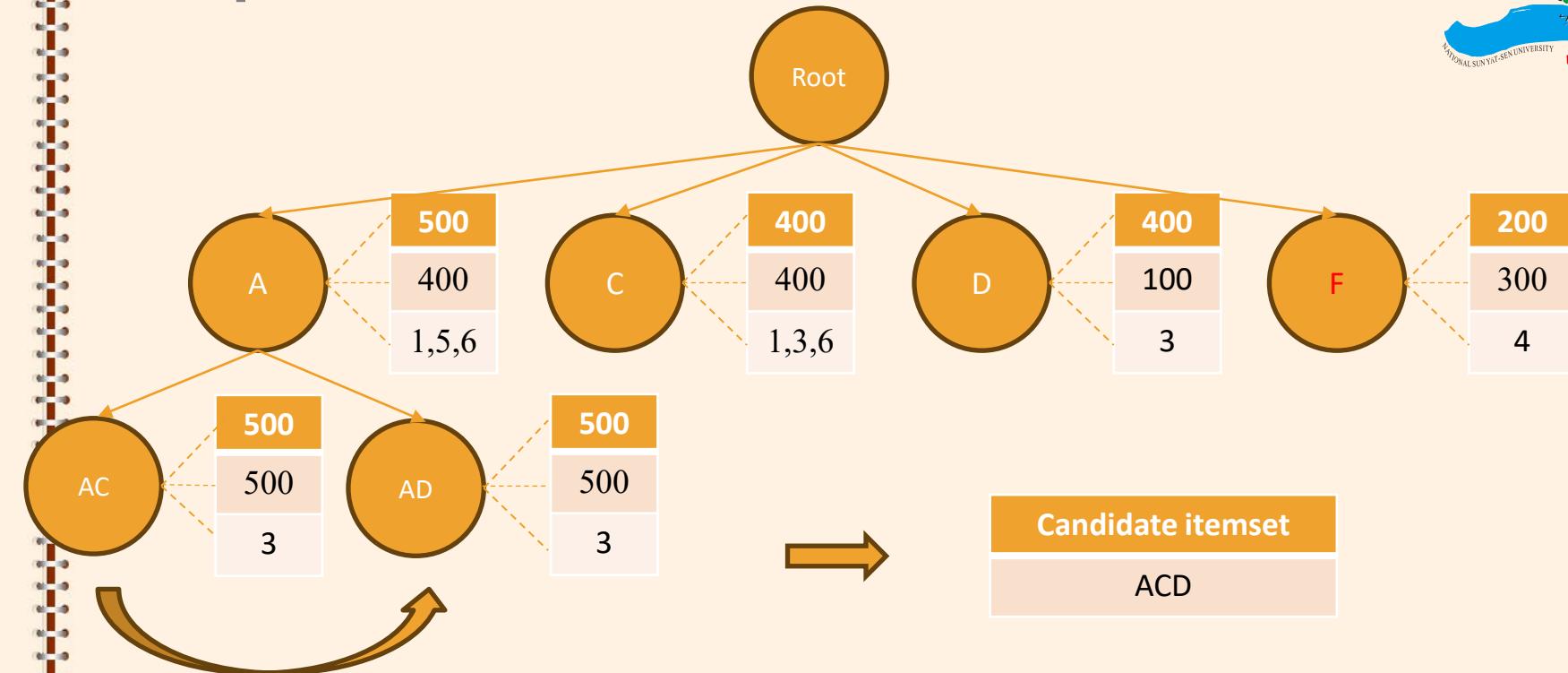
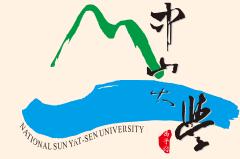
Erasable itemsets	
AC	AD



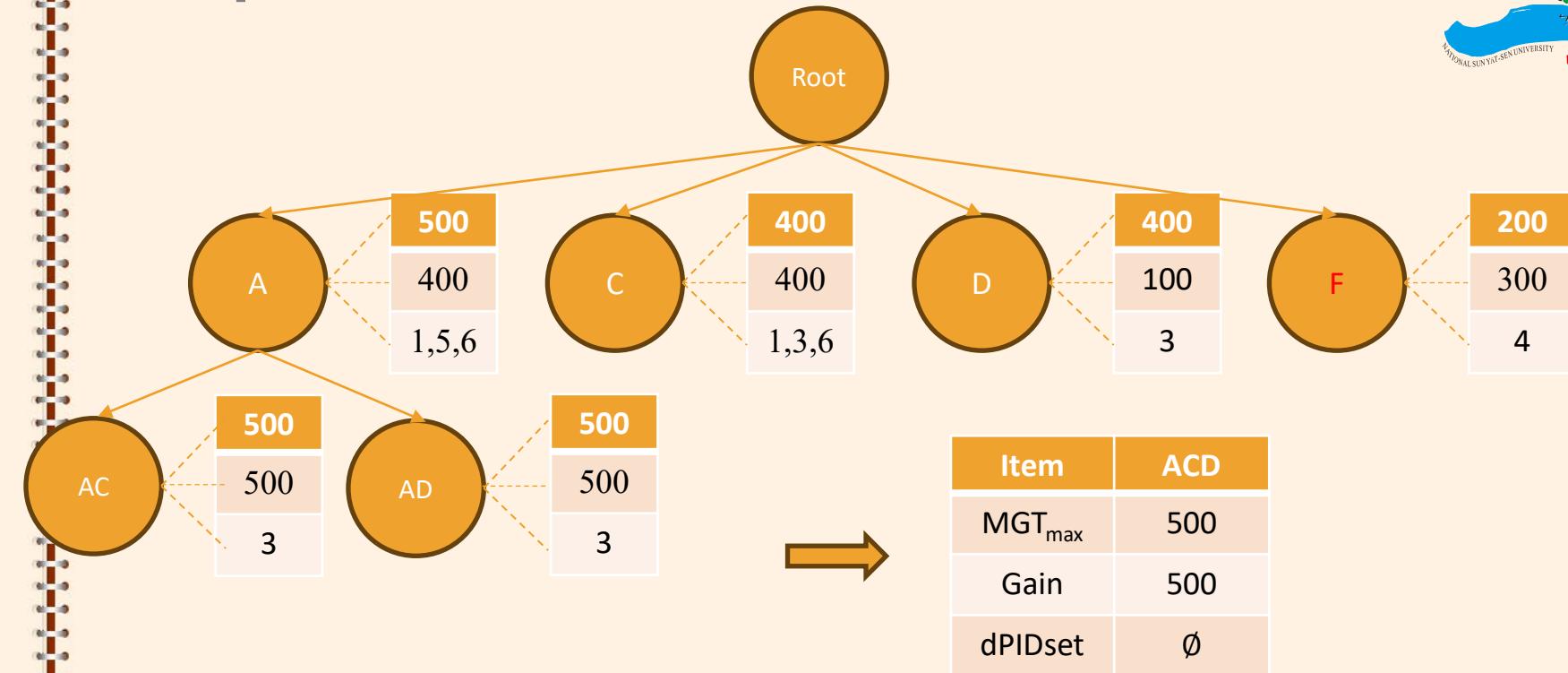
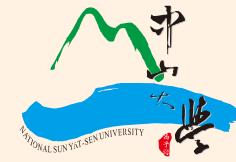
# Step 10 Insert erasable itemsets for {A}



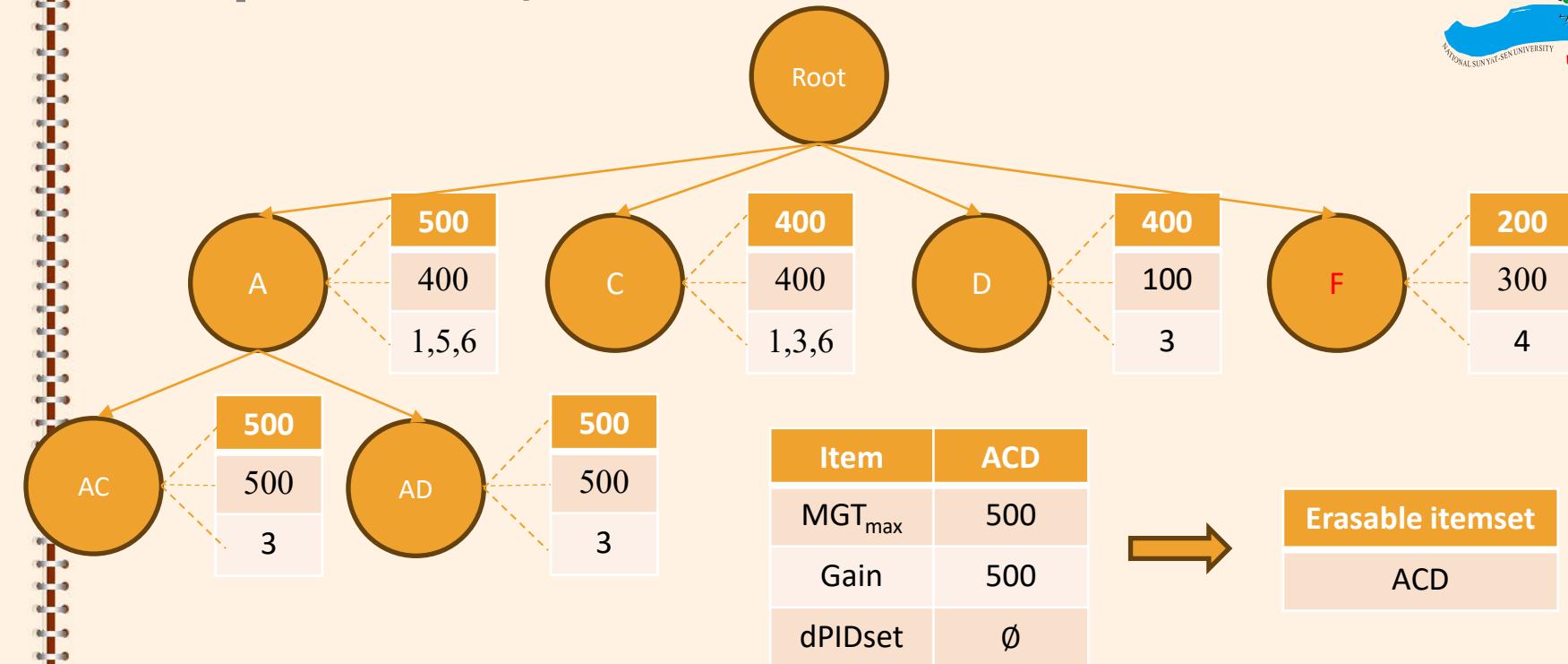
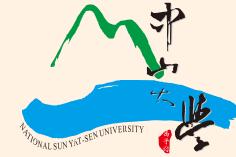
# Step 11 Generate candidate itemsets for {A, C}



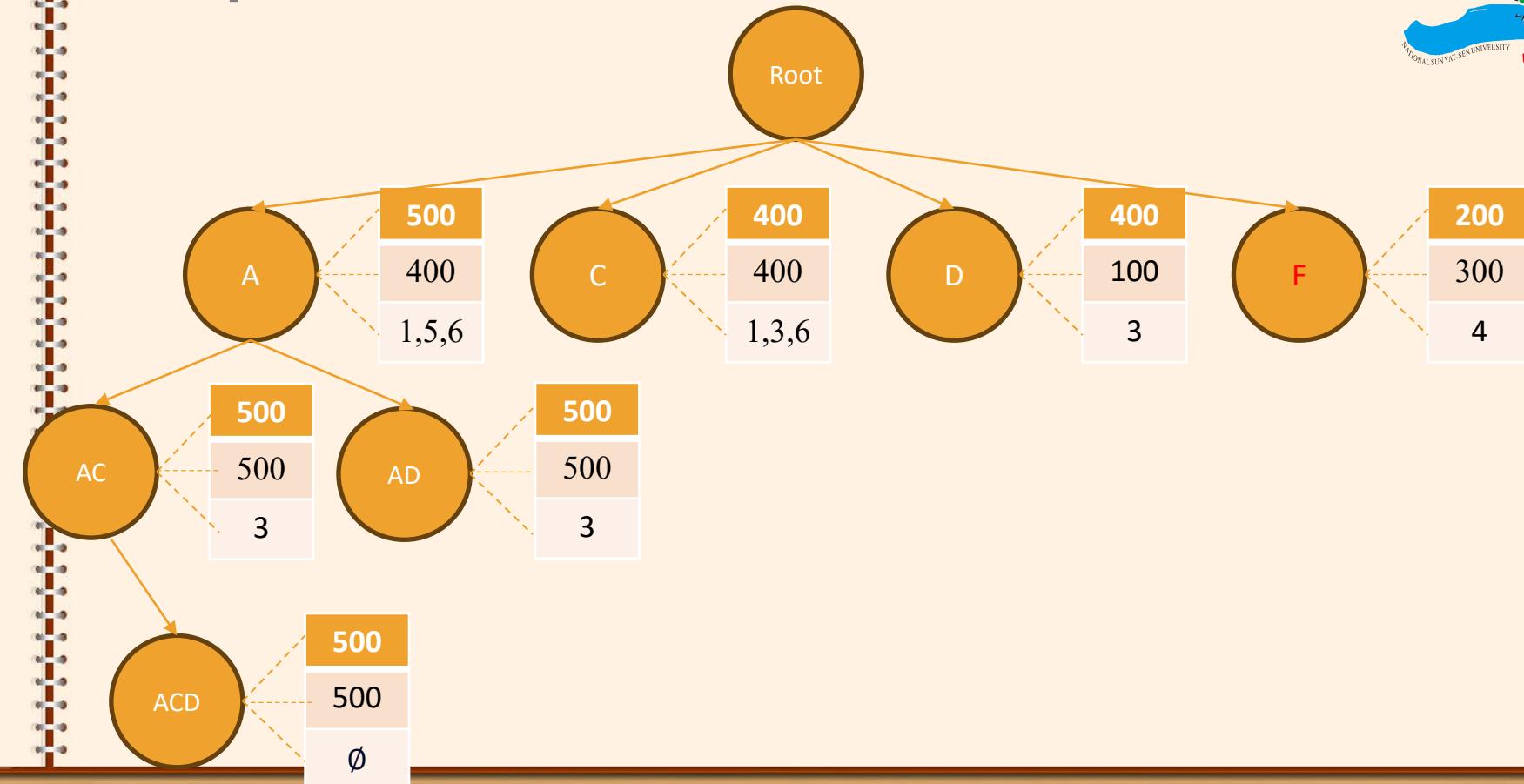
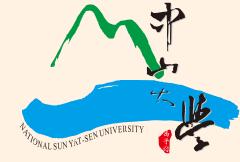
# Step 12 Calculate candidate itemsets information



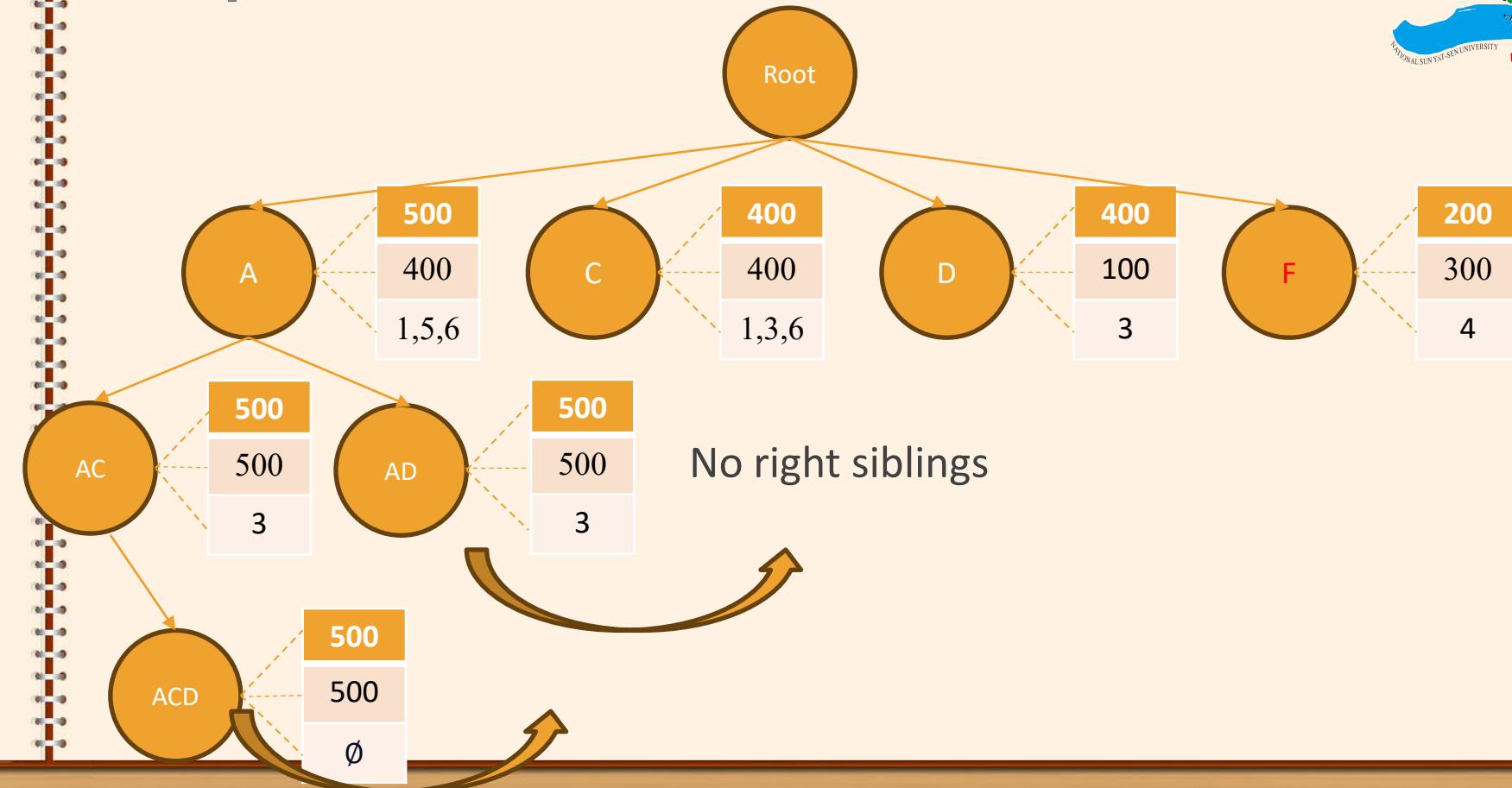
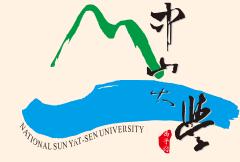
# Step 13 Verify candidate itemsets for {A, C}



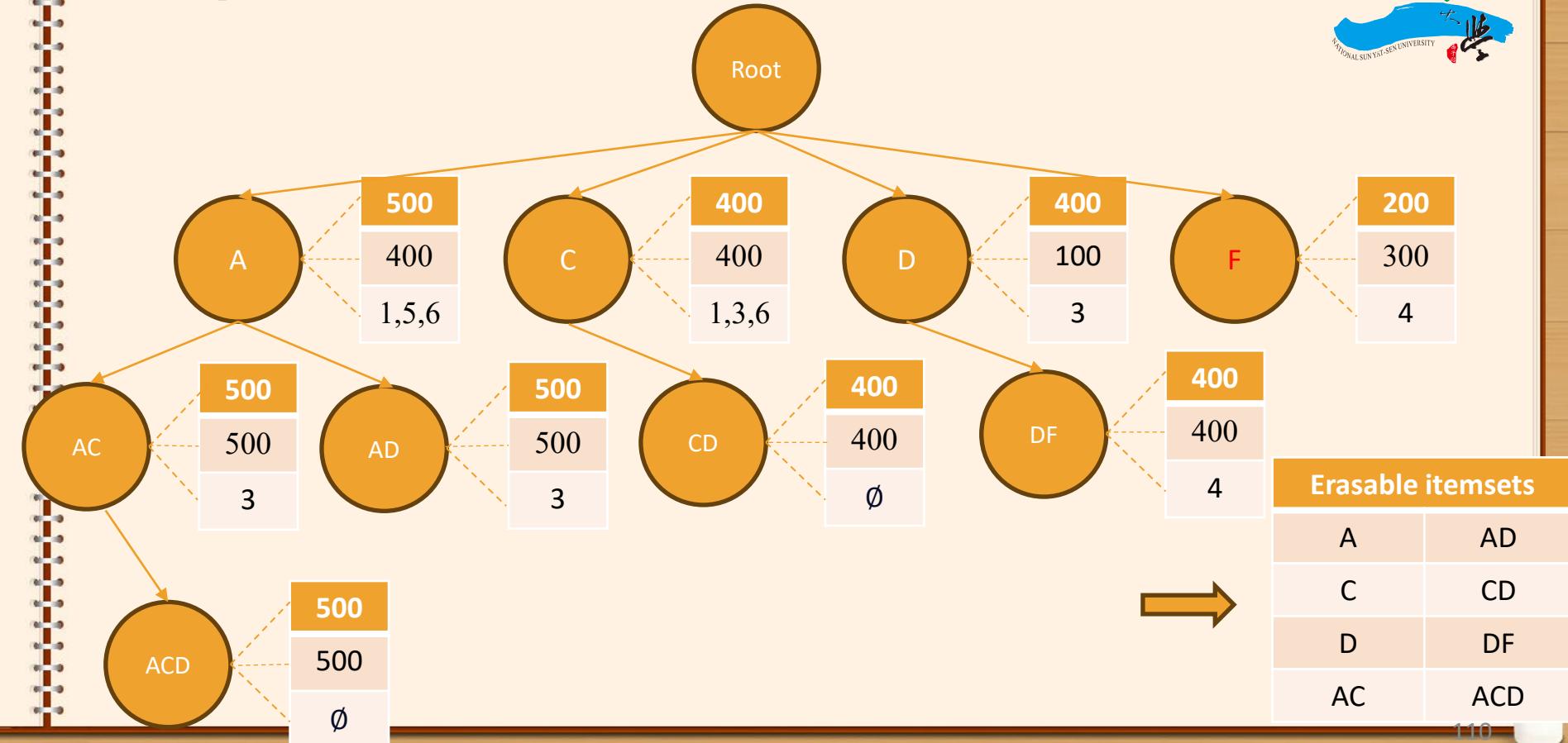
# Step 14 Insert erasable itemsets for {A, C}

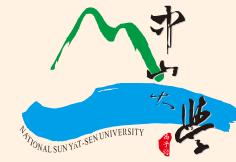


# Step 15 Generate candidate itemsets for {A, D}

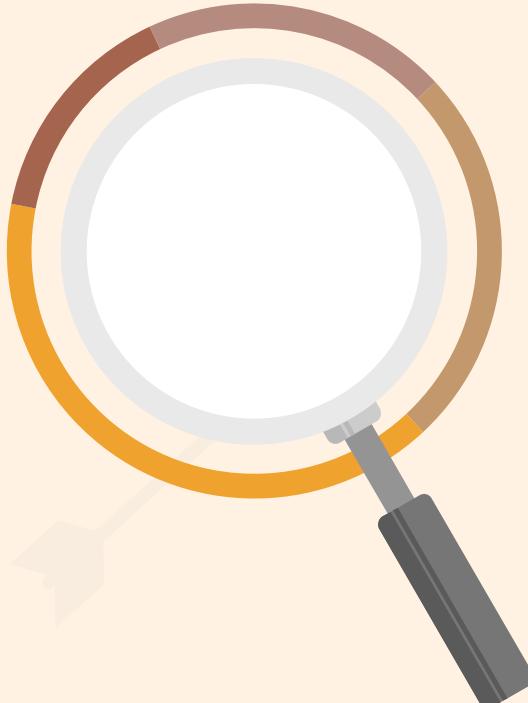


# Step 16 Final result





# The key for improving speed



- 01 Single database scan
- 02 Hash table for profit access
- 03 Calculate gain with dPIDset
- 04 Getting  $MGT_{max}$  from parent node



# 4

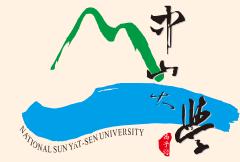
## PART FOUR

# Experiment

- ✓ Multiple threshold vs. Single threshold
- ✓ Apriori-base vs. MEI-base



# Environment



- Language: JDK 19.0.1
- CPU: Intel Core i7-9750H@2.60GHz
- Memory: 8G

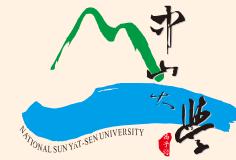


# Multiple-Thresholds versus Single-threshold

---



# Synthetic datasets



## IBM IBM data generator

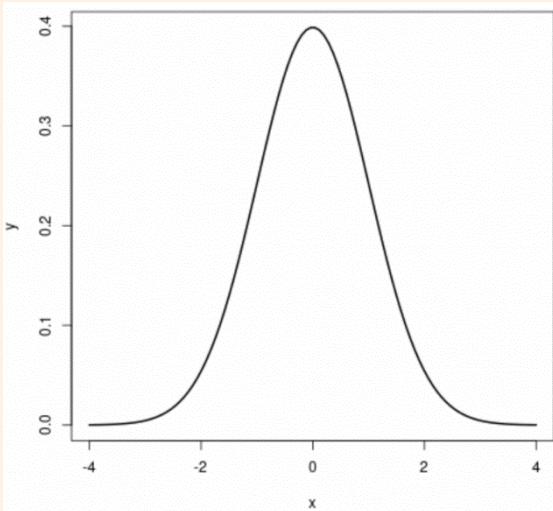
Parameter	Description
P	Number of products
I	Number of items
A	Average items per product



# Additional data



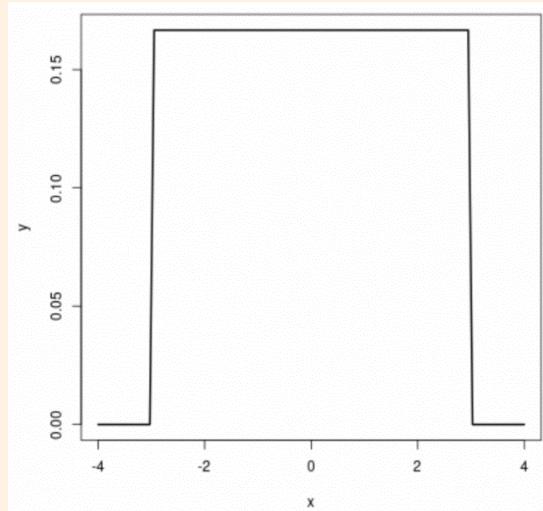
## Profit



Normal distribution

$$N(100, 20)$$

## Thresholds



Uniform distribution

$$U(L, H)$$



# Compare method

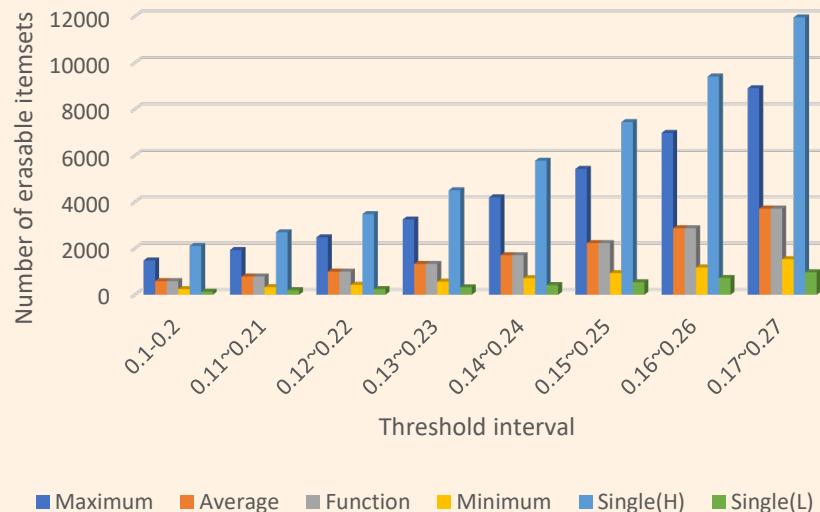
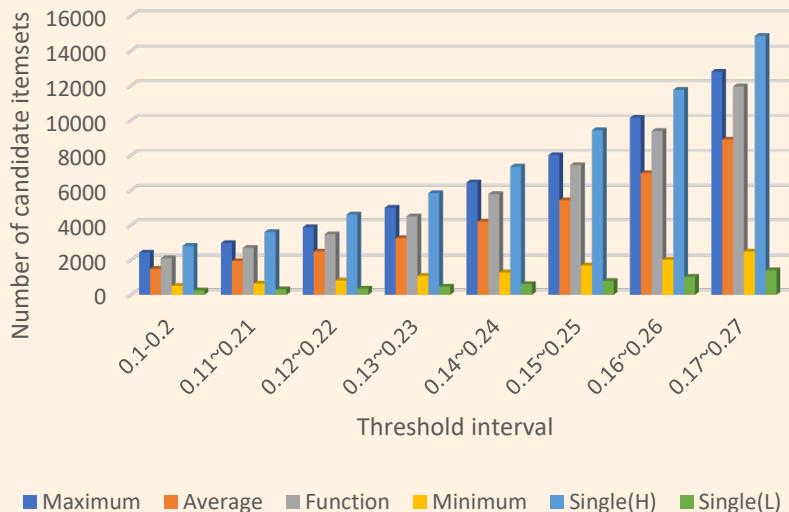
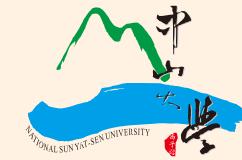


$U(L, H)$

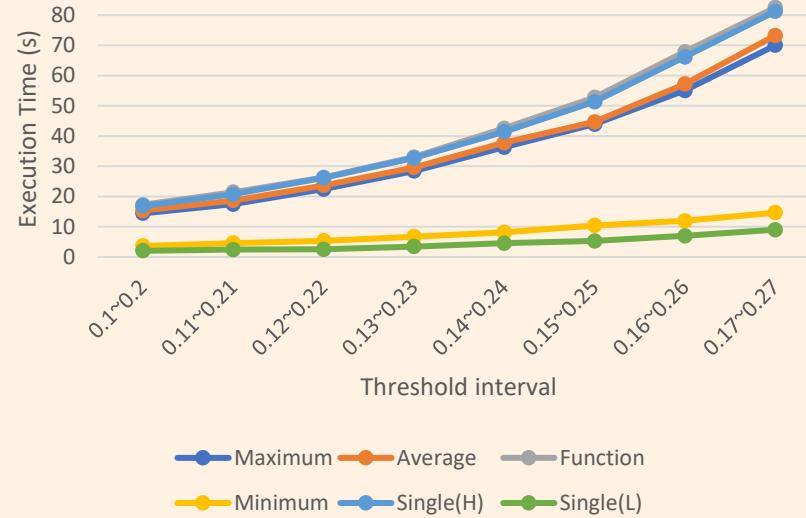
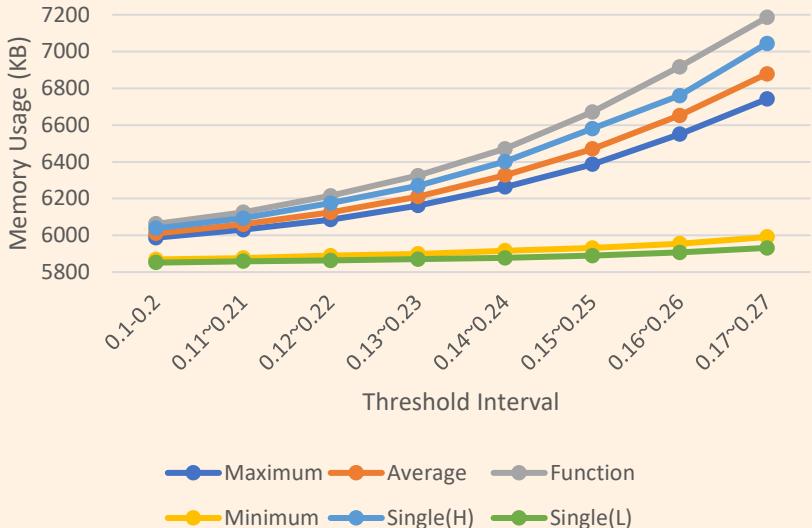
- 01 Minimum constraint
- 02 Maximum constraint
- 03 Average constraint
- 04 Function constraint (Average,  $\lambda_h = H$ )
- 05 Single( $L$ ) and Single( $H$ )



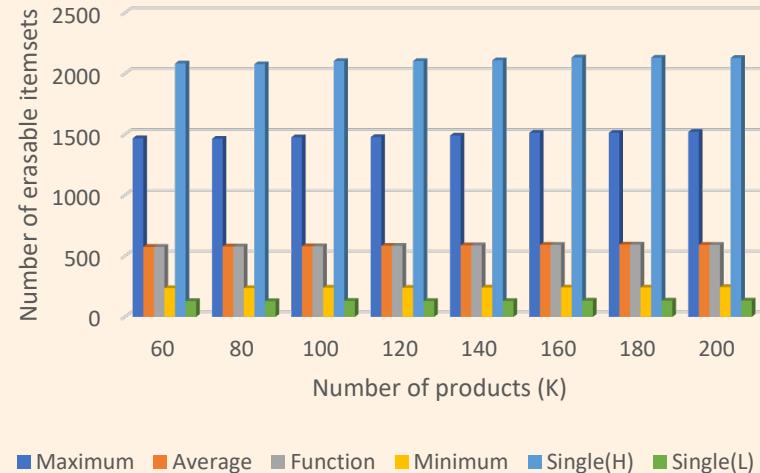
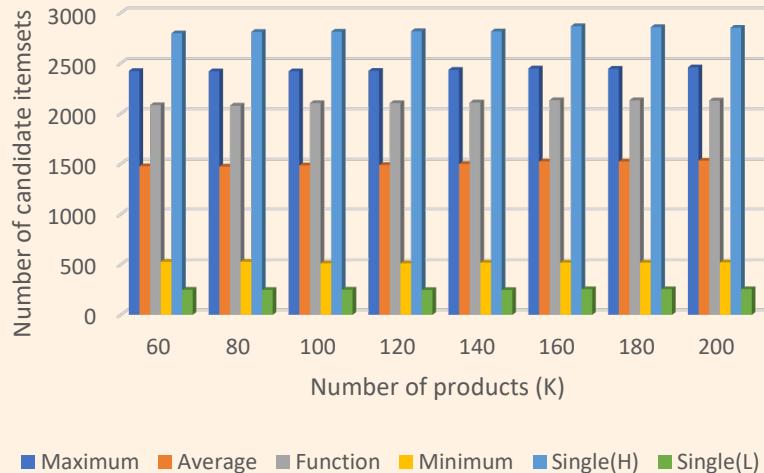
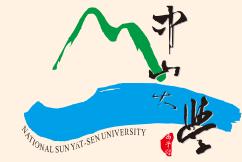
# Different threshold intervals



# Different threshold intervals



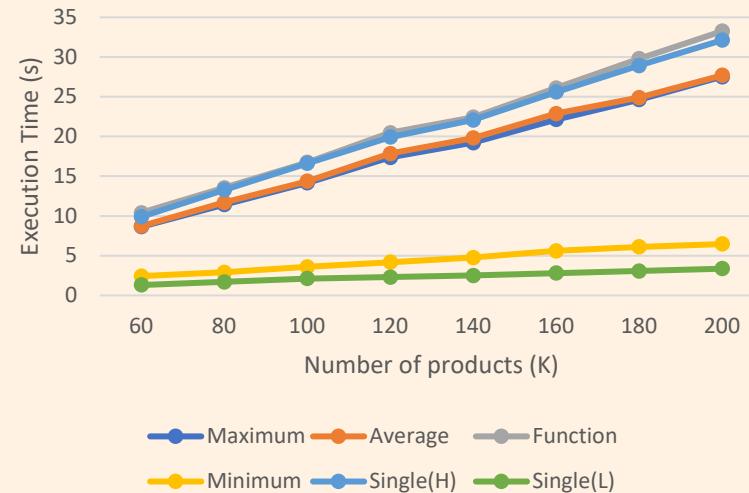
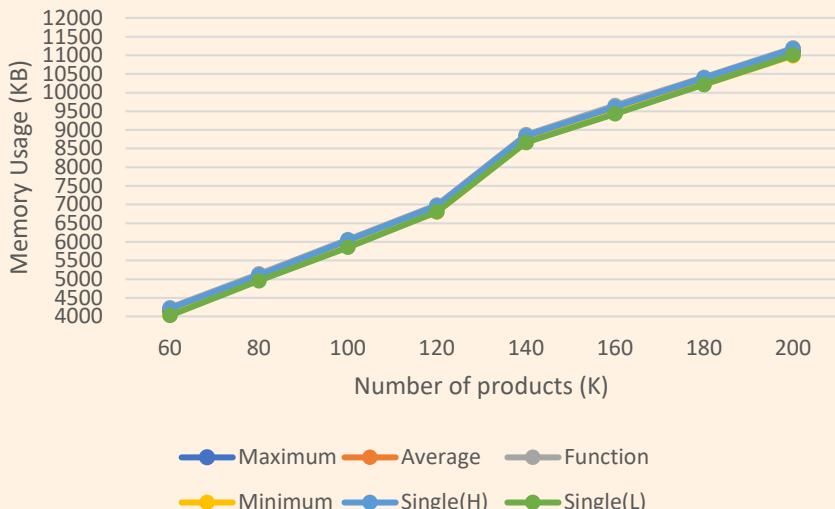
# Different data sizes



Threshold interval:  $U(0.1, 0.2)$



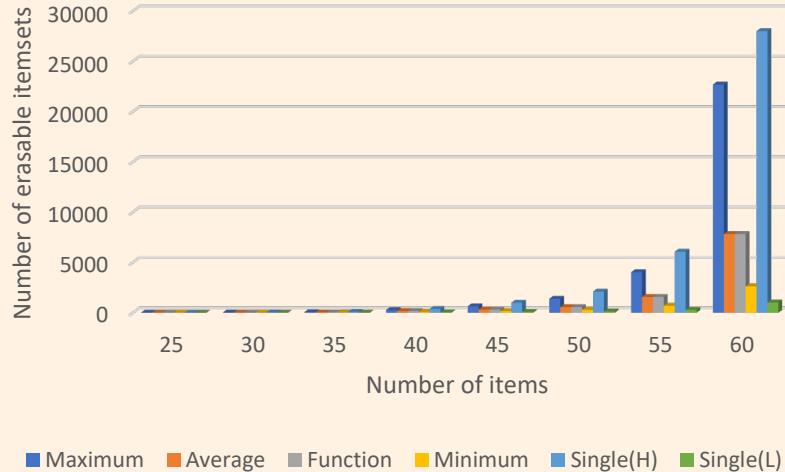
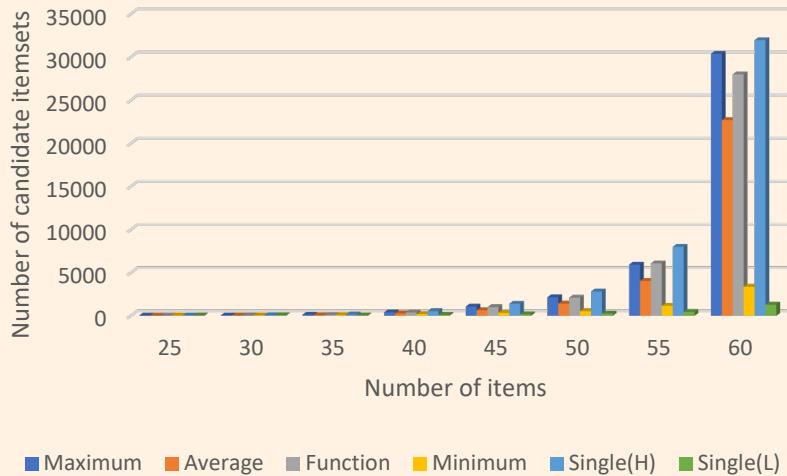
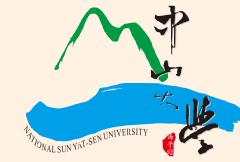
# Different data sizes



Threshold interval: U(0.1, 0.2)



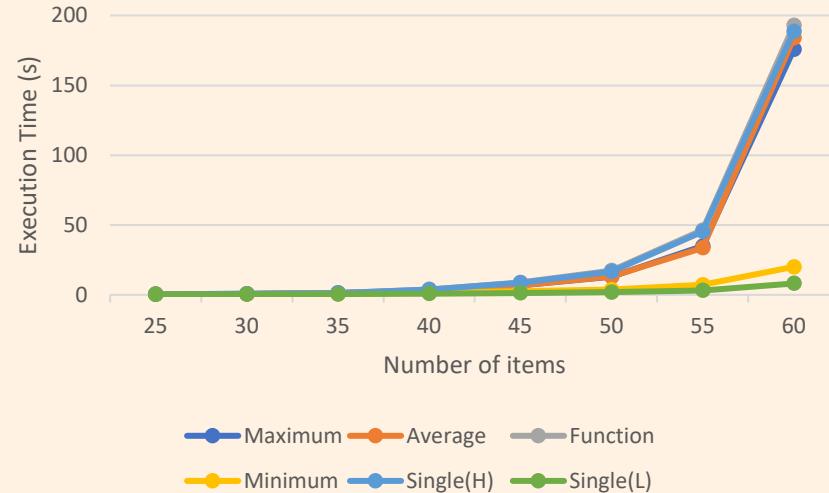
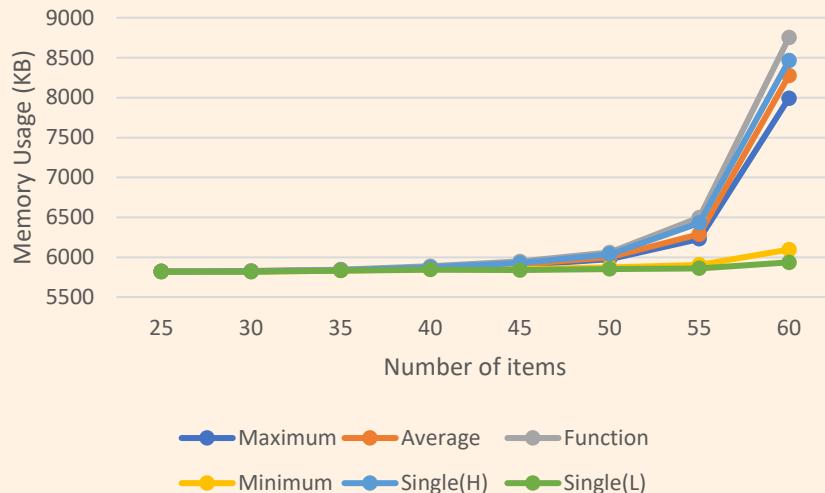
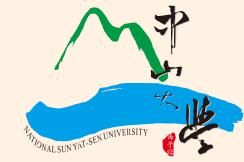
# Different numbers of items



Threshold interval: U(0.1, 0.2)



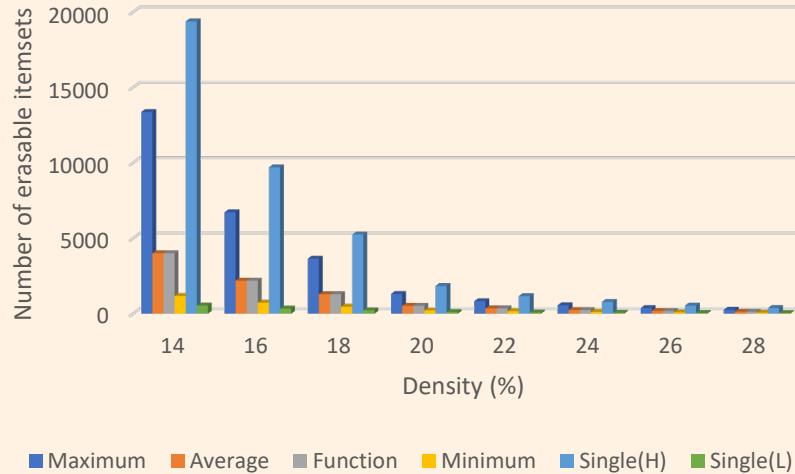
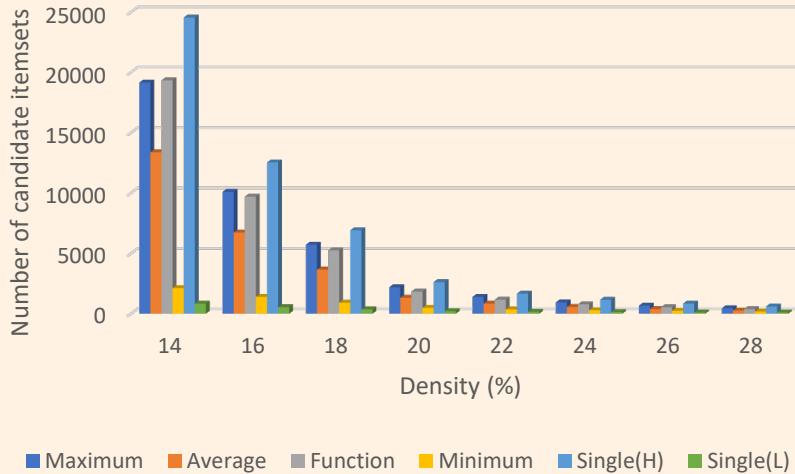
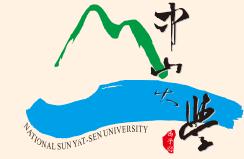
# Different numbers of items



Threshold interval: U(0.1, 0.2)



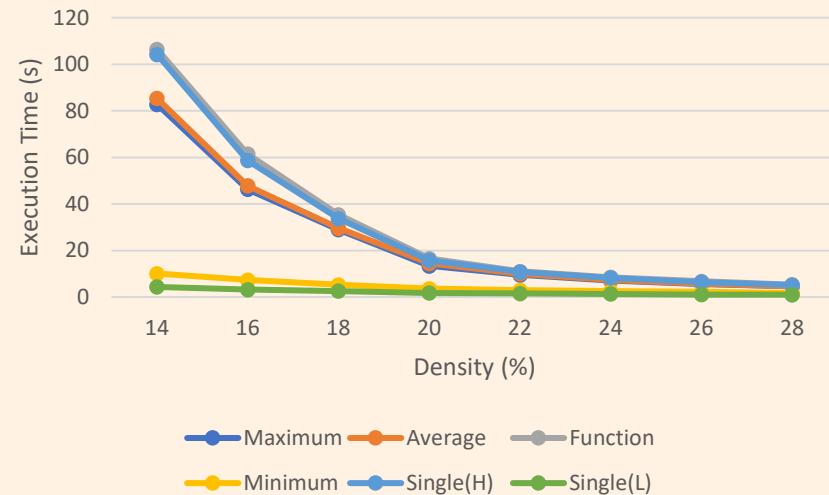
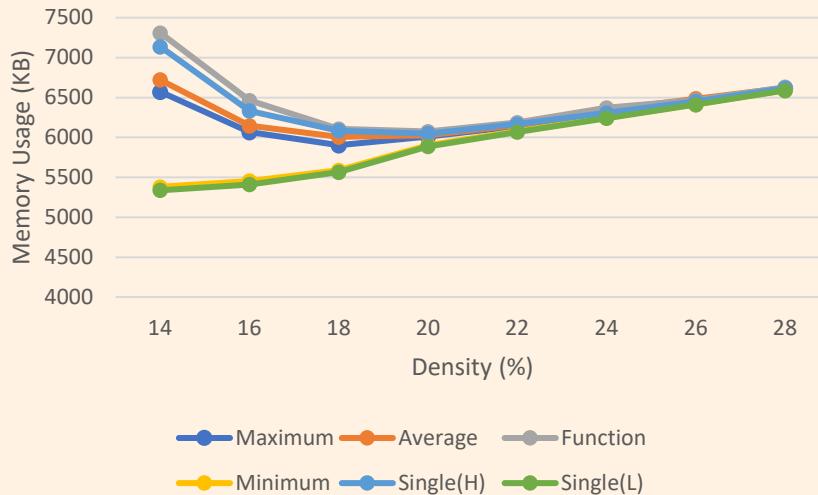
# Different densities



Threshold interval:  $U(0.1, 0.2)$



# Different densities



Threshold interval: U(0.1, 0.2)

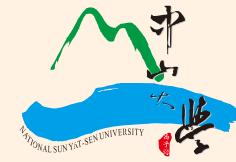


# Apriori-base versus MEI-base

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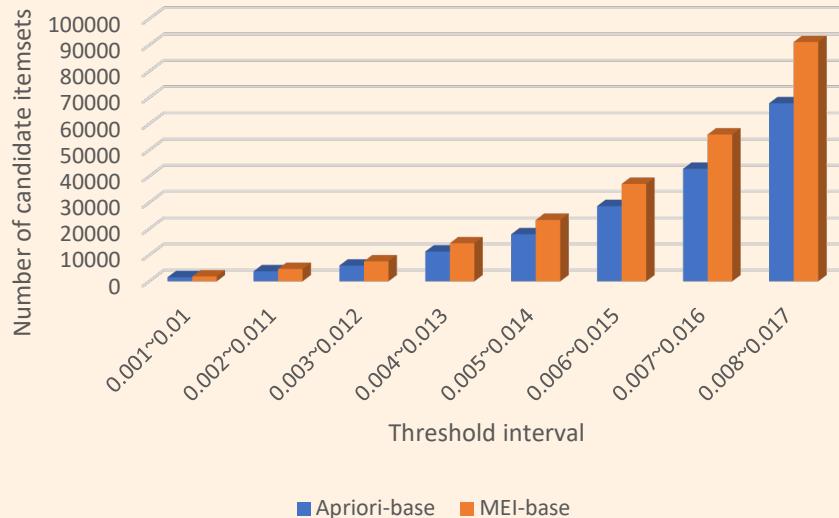
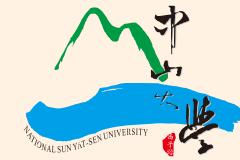
# Real-life datasets



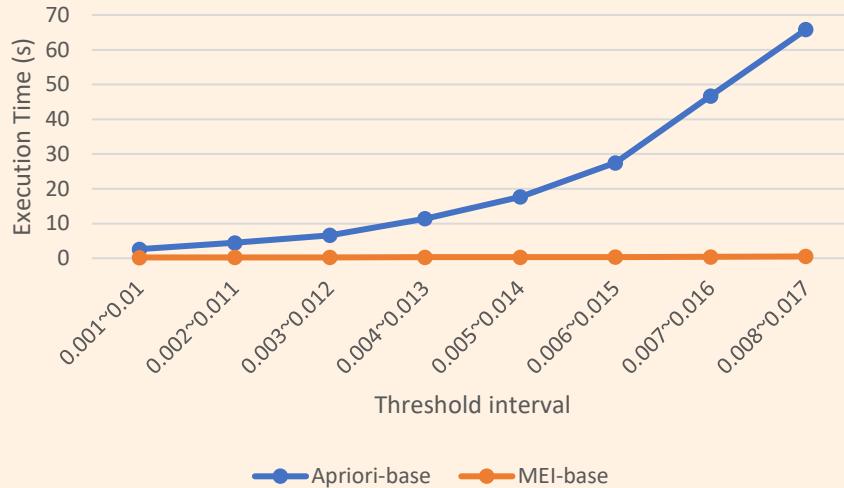
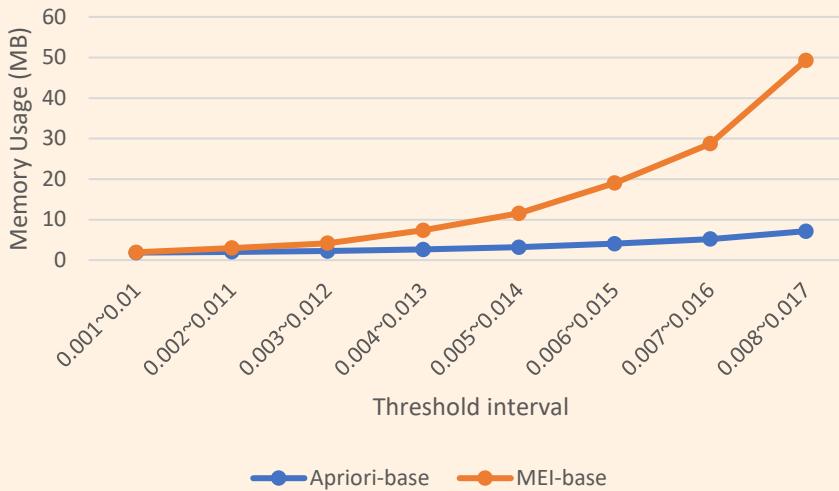
	Product	item	Average	Density
Mushrooms	8416	119	119	19.33%
Chess	3196	75	75	49.33%
Connect	67557	129	129	33.33%



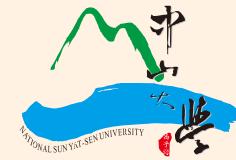
# Mushrooms



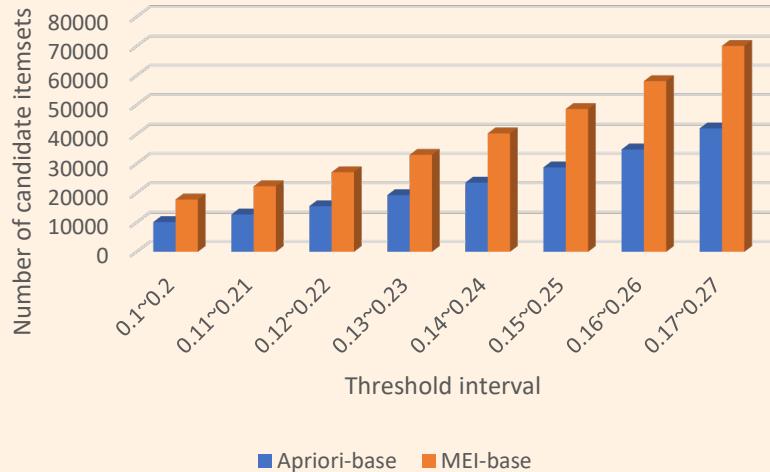
# Mushrooms



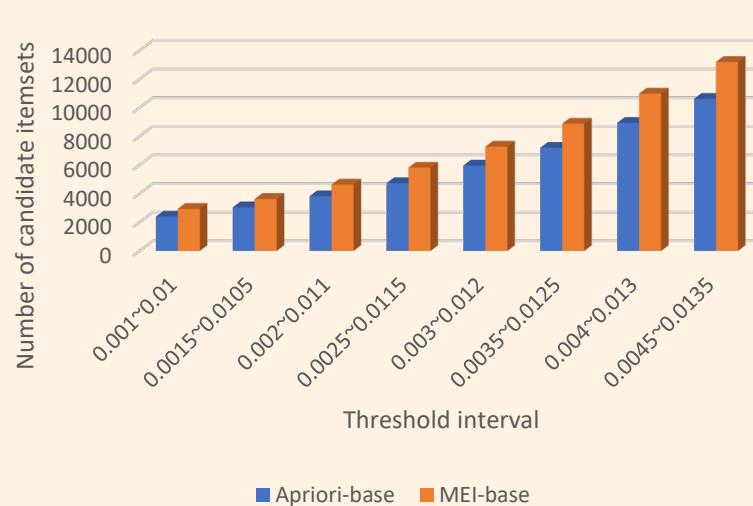
# Strategy comparison



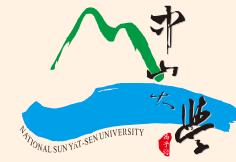
## Chess



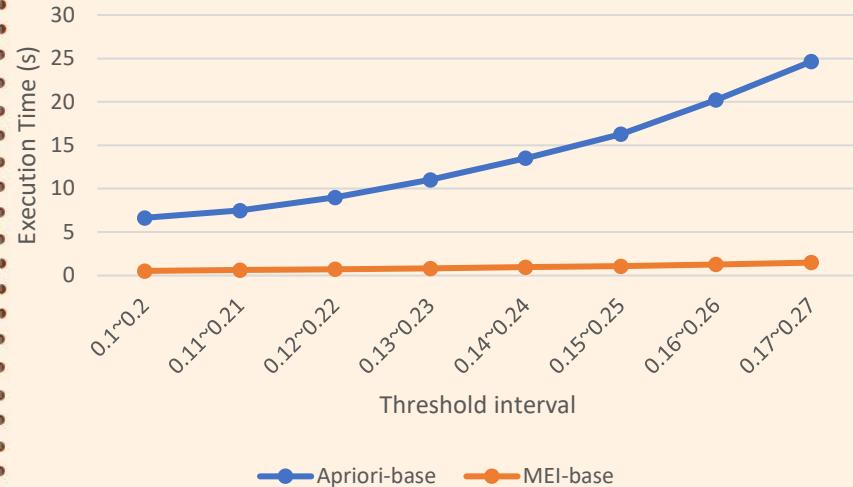
## Connect



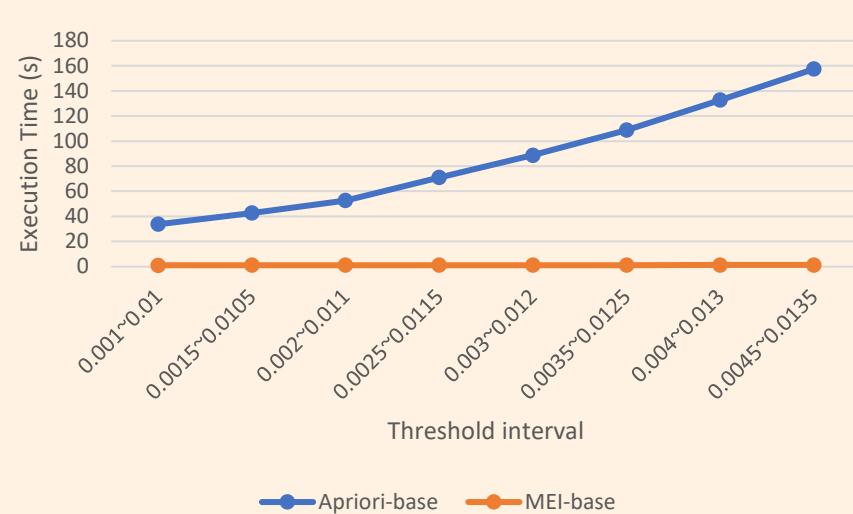
# Strategy comparison



## Chess



## Connect



	Product	item	Average	Density
Mushrooms	8416	119	119	19.33%
Chess	3196	75	75	49.33%
Connect	67557	129	129	33.33%



1 5

PART FIVE

# Conclusion

✓ Future work

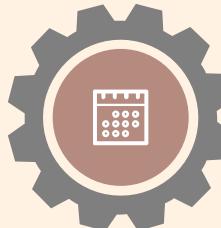


# Conclusion



## Maximum constraint

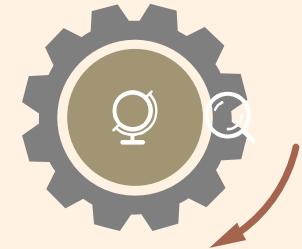
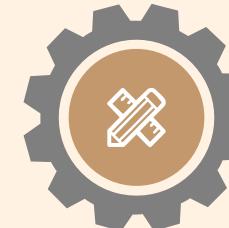
Sorted closure



## Function constraint

Upper-bound

Downward closure



## Minimum constraint

Downward closure

## Average constraint

Upper-bound

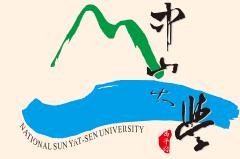
## Efficient maximum constraint

Tree structure

Hash table



# Future work



Better pruning skill



Reduce memory usage



Smaller upper bound



Appropriate threshold





# Thank you

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## Q & A

