

THE UNIVERSITY OF HONG KONG
DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE

STAT6011/7611/8305 COMPUTATIONAL STATISTICS
(2021 Fall)

Assignment 1, due on October 5

All numerical computation **MUST** be conducted in Python, and attach
the Python code.

1. Question 2 (Metropolis-Hasting algorithm)

- (a) Assume $\mathbf{X} = (X_1, \dots, X_{10})$ follows a multivariate standard normal distribution $\pi(\mathbf{x})$. Build a Random Walk algorithm with the Gaussian kernel to estimate $E[D]$, where $D = f(\mathbf{X}) = \sqrt{\sum_{j=1}^{10} X_j^2}$. Firstly, tune the step size such that you have roughly 23.4% acceptance rate. What step size do you choose? Then, draw 10000 samples to do the estimation, set the initial point $\mathbf{x}_0 = \mathbf{0}$, set the burn-in parameter to be 1000 and set the thinning parameter to be 5.
- (b) A simple technique called batching can help us to build a confidence interval with MCMC samples. We divide the above obtained 10000 samples $\{d_1, \dots, d_{10000}\}$ into 20 batches, where $d_i = f(\mathbf{x}_i)$, and $\mathbf{x}_i = (x_{i1}, \dots, x_{i10})$ is a sample in the MCMC sequence. Do the estimation in each batch:

$$\bar{d}_b = \frac{1}{500} \sum_{i=500(b-1)+1}^{500b} d_i,$$

for $b = 1, \dots, 20$. Estimate that

$$s^2 = \frac{1}{20(20-1)} \sum_{b=1}^{20} (\bar{d}_b - \bar{d})^2.$$

So, the 95% confidence interval would be

$$\bar{d} \pm t_{(19)}^{0.975} s.$$

Please give this interval.

- (c) In the self-normalized IS, the optimal proposal is proportional to $\pi(\mathbf{x})|f(\mathbf{x}) - E[D]|$. Replace $E[D]$ by the estimate $\hat{E}[D]$ obtained in (a), draw samples from the optimal proposal based on the Random Walk, and weight each obtained sample by the weighting function

$$w(\mathbf{x}) = \frac{1}{|f(\mathbf{x}) - \hat{E}[D]|}.$$

Follow a similar procedure of (a) to calculate the estimate of $E[D]$ by these weighted samples. Do 100 repetitions to estimate the variance of these two estimation methods but only have 1000 samples each time. Do the optimal proposal make things better or worse? Explain why by comparing the two strategies' first 500 samples' trajectory.