

Statistical solutions and astrophysical simulations

Ian Hawke

heavily based on work of U Fjordholm and S Mishra

@IanHawke

github.com/IanHawke

orcid.org/0000-0003-4805-0309

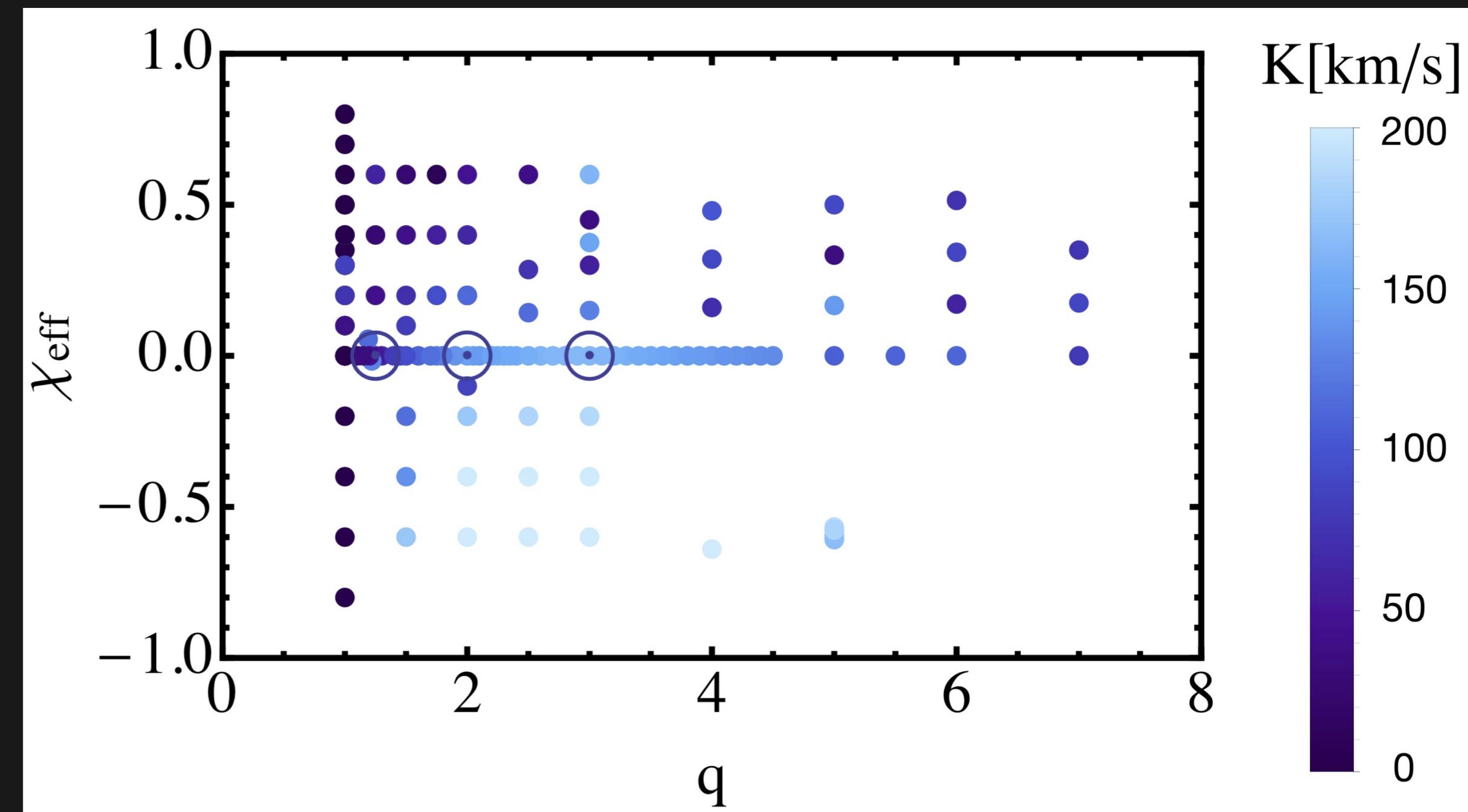
STAG, University of Southampton

Uncertainty quantification

To match observations from simulations:

1. set initial data;
2. simulate observable;
3. repeat across parameter space;
4. interpolate.

Uncertainty from simulation and parameter space sampling.



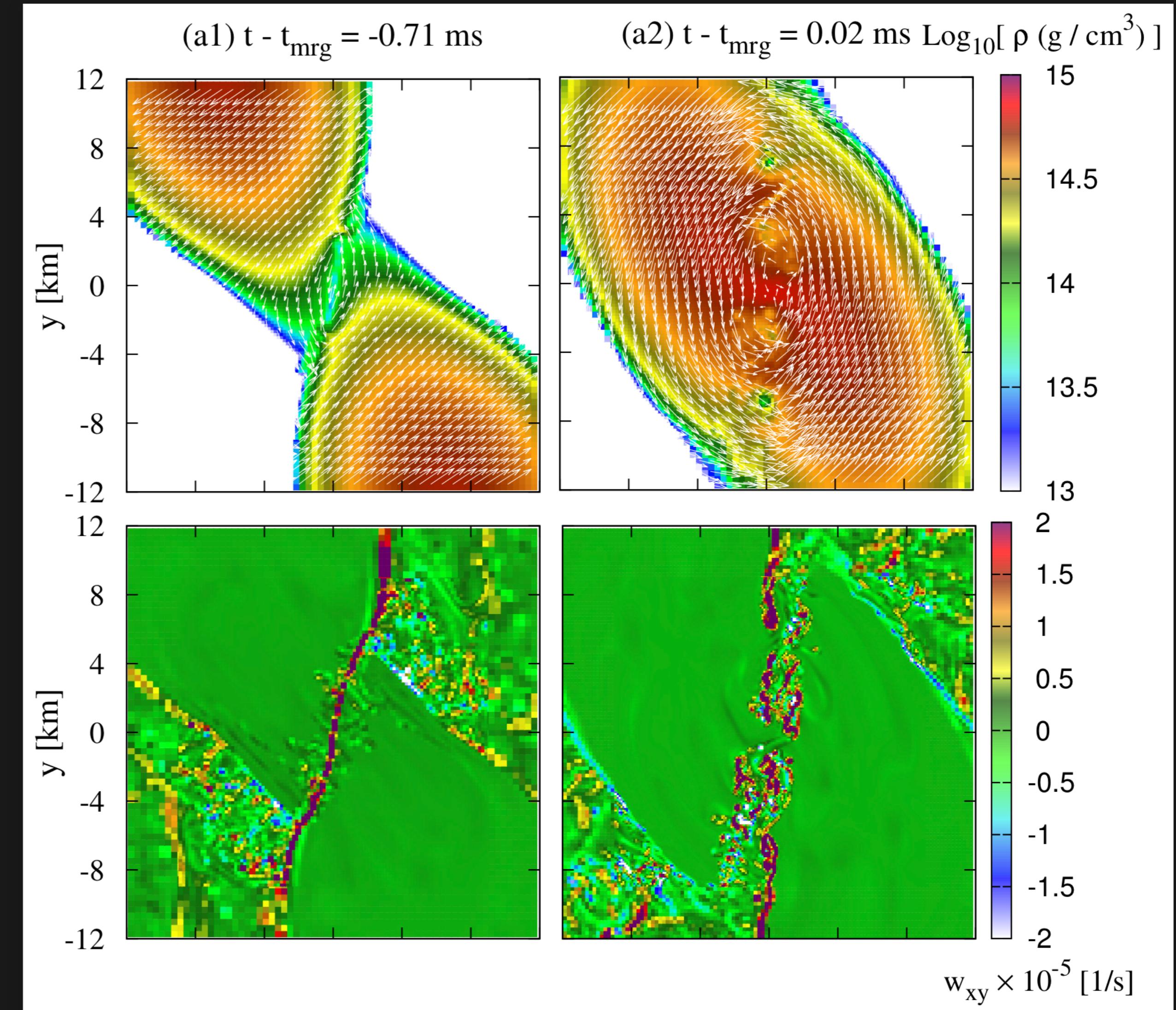
Bustillo et al, 1806.11160

Matter

Matter simulations:

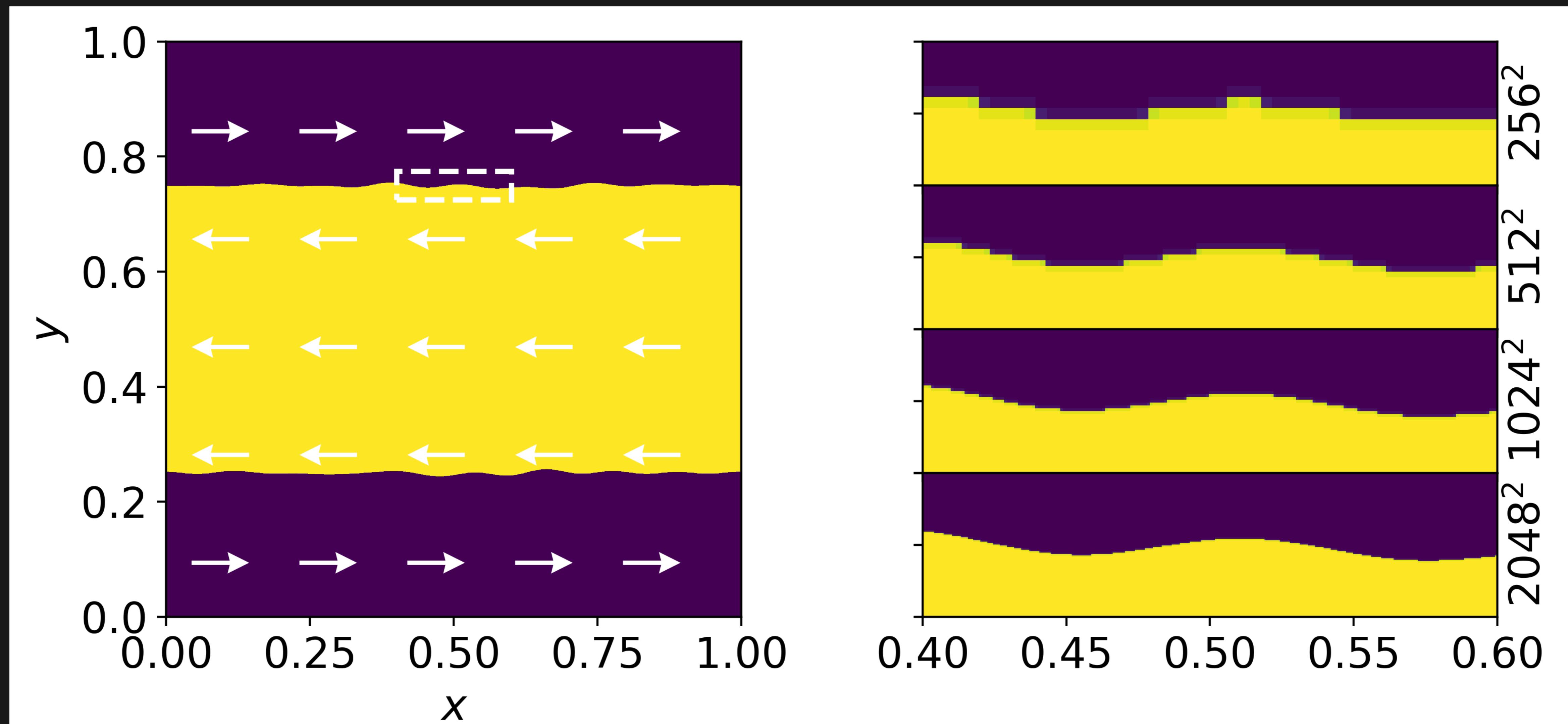
1. have shocks;
2. "turbulence";
3. more observables;
4. little on well-posedness.

Can we get reliable predictions?



Kiuchi et al, 1509.09205

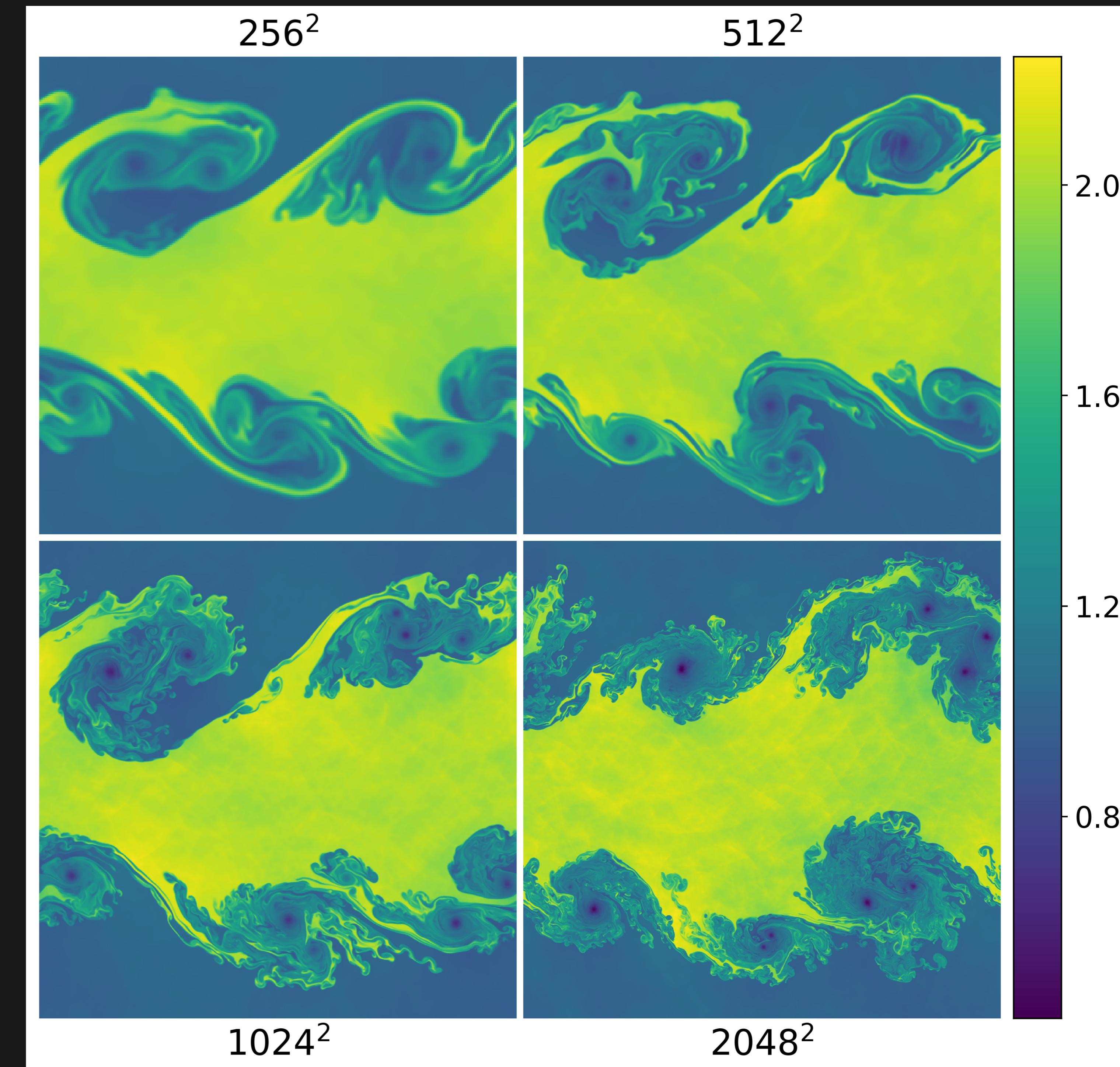
Kelvin-Helmholtz instability



Lax

- Code consistent;
- Numerics stable;
- No convergence.

Lax's theorem means the system
is *not* well-posed.



Saving convergence

We don't see convergence to *weak solutions* of

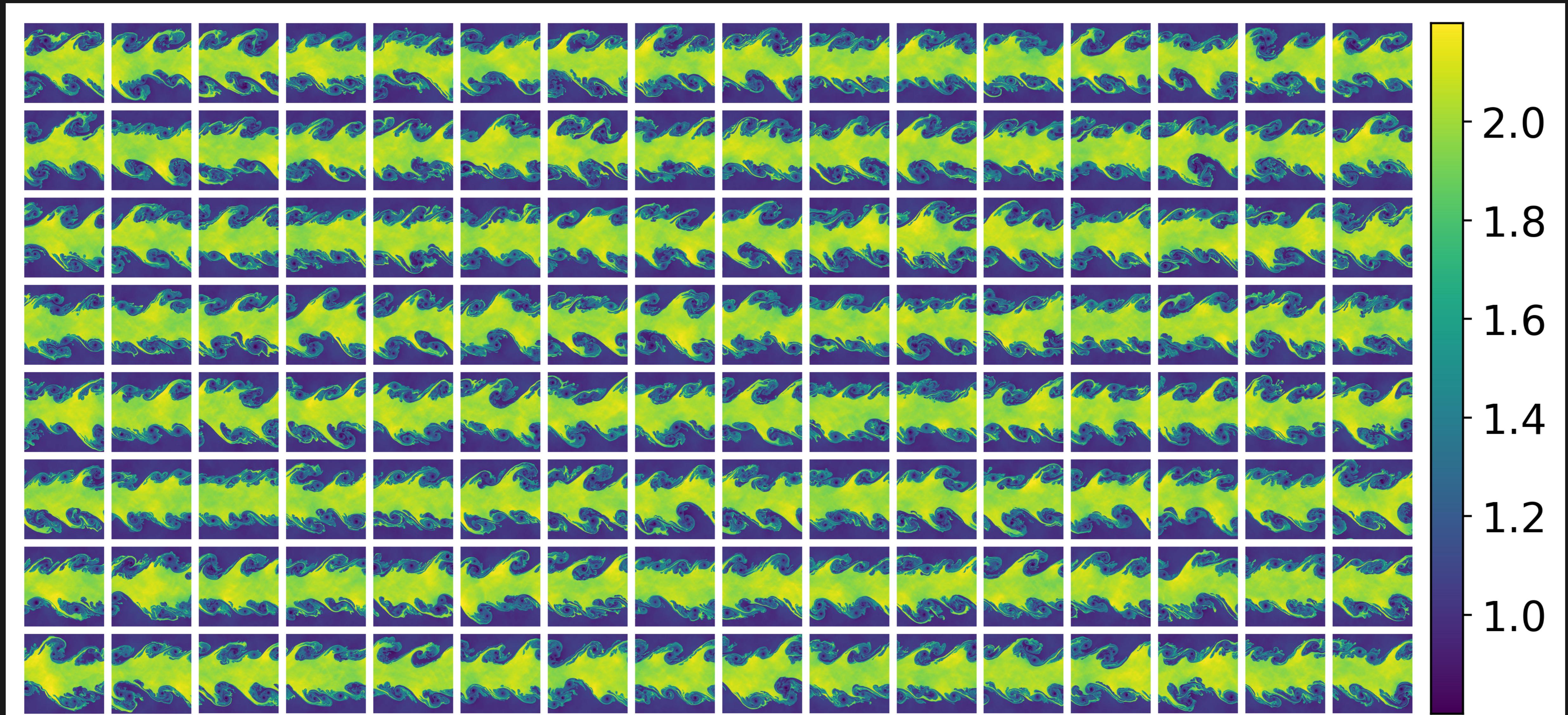
$$\partial_t q + \partial_i f^{(i)}(q) = s(q).$$

Instead use a spacetime-dependent probability measure ν solving

$$\partial_t \langle \nu, \text{id} \rangle + \partial_i \langle \nu, f^{(i)} \rangle = \langle \nu, s \rangle.$$

Need to ensure appropriate entropy solution and small scale features.

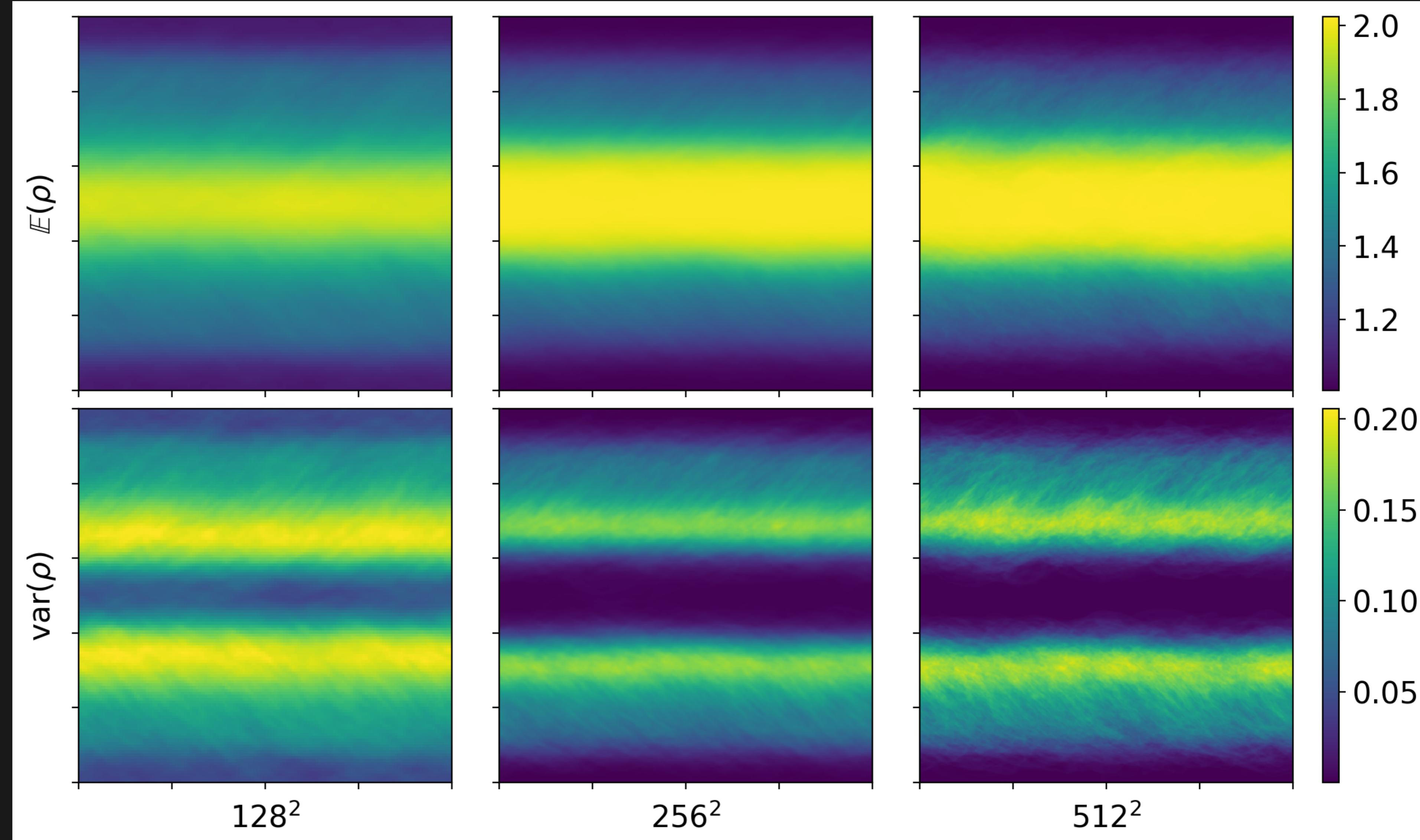
Constructing the measure



\mathbb{E} & var

Both mean and variance clearly converge.

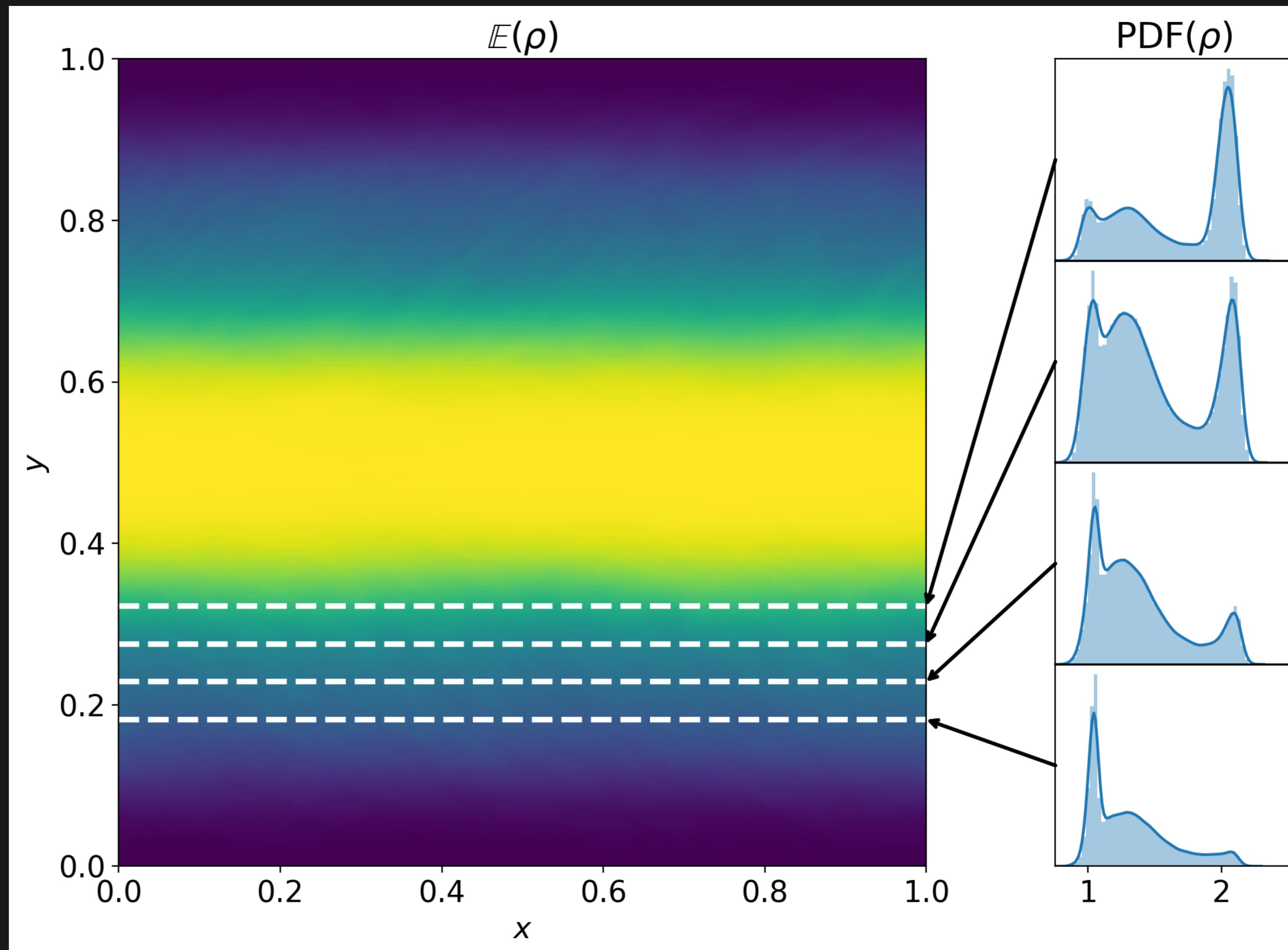
Characterize the uncertainty in the observable.



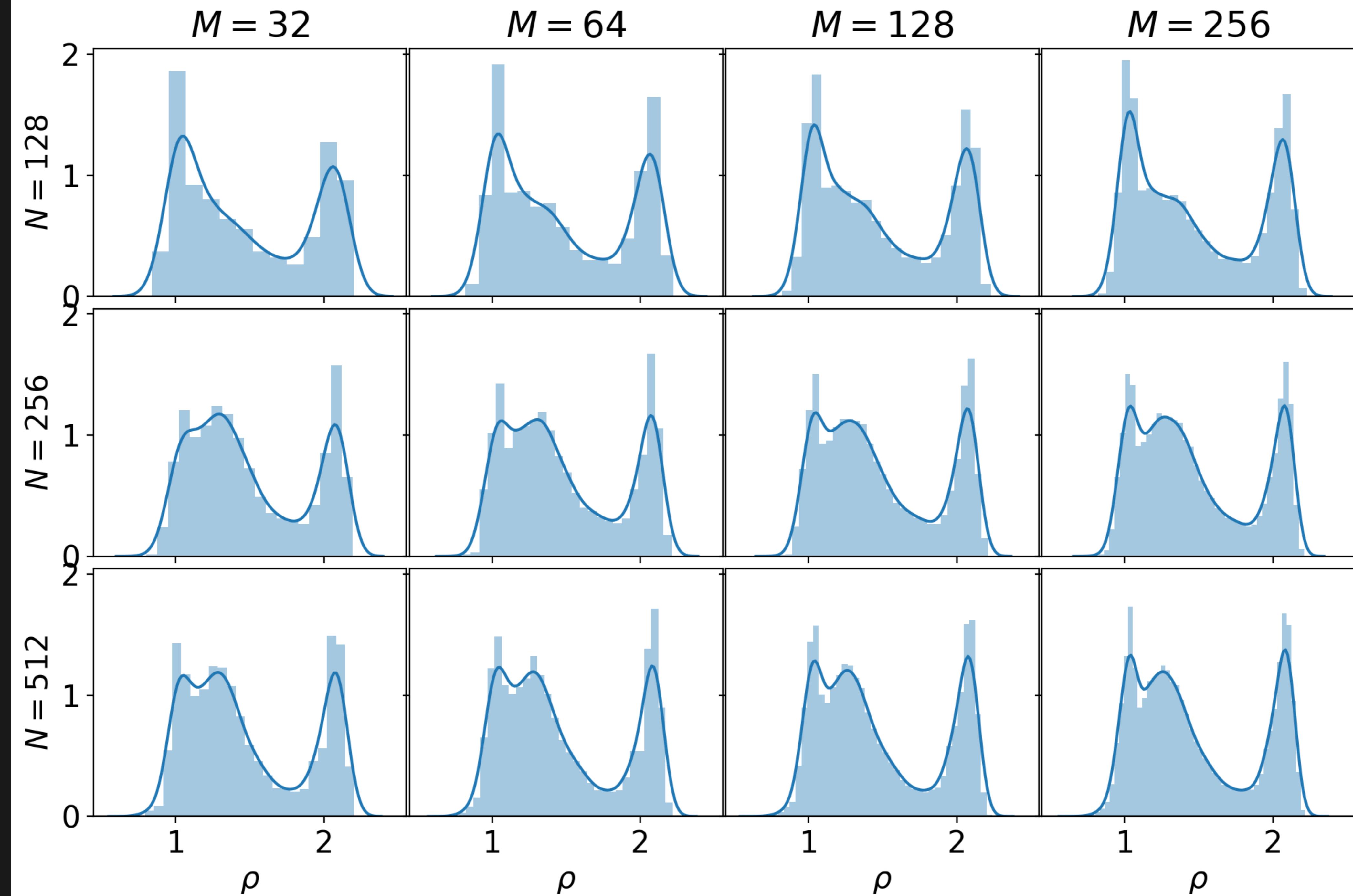
PDF

Can directly estimate the PDF,
eg along a line. These also
converge rapidly.

Some features not obvious
from \mathbb{E} and variance.



$PDF(\rho)$ at $y = 0.277$



Quantifying the problem

For "standard" case, measure numerical error as

$$\mathcal{E} \simeq C_1(\Delta x)^s.$$

For statistical solutions, simple Monte-Carlo, expect

$$\mathcal{E} \simeq C_1(\Delta x)^s + C_2 M^{-1/2}.$$

Measure C_i 's, see what error dominates. If variance large, prediction hard!

Summary

- Euler equations not globally well-posed;
- Statistical solutions mean simulations can still make predictions;
- Quantifying uncertainty will be essential when shocks interact with "turbulence", in eg binary neutron star mergers;
- May be that large variance means numerics can't make robust predictions.