MATH1058: Problem Sheet 6

Problem 1 (Production planning (subject to uncertainty)). *Consider a production planning problem for a single good over a time period of τ time steps. An intensive marketing campaign has revealed that, at each time period $t = 1, \ldots, \tau$, no more than d_t units can be sold. The production of a unit requires assembling components of n types, with a_i representing the number of components of type $i \in 1, \ldots, n$ that are needed to produce one unit of the single good and q_i representing their availability during the whole time period.*

We are asked to determine a production plan which maximizes the revenue of the company over the time period, subject to availability and demand constraints.

1. *Propose a linear programming model for the problem where c_t, for all $t = 1, \ldots, \tau$, denotes the price at which a unit of the single good is sold.*

2. *Consider now the case where, rather than knowing c with absolute precision, we have a set of κ estimates (*scenarios*) of it, where each estimate is a vector $c^k \in \mathbb{R}^\tau$, with $k = 1, \ldots, \kappa$, corresponding to the time series of unit prices that was forecast for the whole time period. Propose a linear programming formulation to find a production plan by which the profit is maximized* in the worst case, *i.e., a production plan $x \in \mathbb{R}^\tau$ by which, rather than cx, the following function is maximized:*

$$\phi(x) := \min_{k \in \{1, \ldots, \kappa\}} c^k x.$$

Note that, for any $x \in \mathbb{R}^\tau$, $\phi(x)$ corresponds to the smallest profit achieved with the production plan corresponding to x over all scenarios $k = 1, \ldots, \kappa$.

Problem 2 (Simplex method with Bland's rule I). *Solve the following linear program with the two-phase simplex method, applying Bland's rule:*

$$\begin{array}{llccccccc}
\min & z = & x_1 & - & 2x_2 & & & & \\
& & 2x_1 & & & + & 3x_3 & = 1 \\
& & 3x_1 & + & 2x_2 & - & x_3 & = 5 \\
& & x_1 & , x_2 & , x_3 & \geq 0. & & &
\end{array}$$

Problem 3 (Simplex method with Bland's rule II). *Solve the following LP with the two-phase simplex method, adopting Bland's rule:*

$$\begin{array}{lllll}
\min & x_1 & +x_2 & +x_3 & \\
s.t. & x_1 & +2x_2 & +3x_3 & = 3 \\
& -x_1 & +2x_2 & +6x_3 & = 2 \\
& & -3x_2 & -9x_3 & = -5 \\
& & x_1, x_2, x_3 \geq 0 & &
\end{array}$$