Chapter 1

Introduction

1.1 Optimisation

Optimisation is the mathematical procedure for finding the best choice of a set of variables subject to a range of constraints. It is a huge subject, and we will focus on two specific model problems as motivation.

The first is the *diet problem*. We have a population that we want to keep healthy. We assume they are isolated, or resource limited in some way: think of astronauts, or explorers at sea, or a population at war. That means we want to provide food that meets their essential nutritional needs (enough energy, protein, and so on) whilst minimising its cost (which might be in terms of money, or weight, or volume, or something else). The best choice has the smallest cost. The constraints we need to satisfy are the essential nutritional needs.

The second is the *shortest path problem*. We want to travel between two points with the minimum cost. We are thinking of a road or other transport network as the underlying example here. The best choice again has the smallest cost, which again might be in terms of money, or time, or distance. The constraints are the costs needed to go between any two underlying points, or along one single path, on the network.

1.1.1 Purpose of these notes

These notes will give the minimum expected material for this course. They are mainly here to develop the theory and the analysis. For (much) more detail see the books

- $1. \ \ Operations \ Research, \ Winston, \ QA264.2 \ WIN.$
- 2. Linear Programming, Chvátal, QA265 CHV.

Neither are needed, but those (and other similar books in the library and elsewhere) will give many additional ideas, examples, and exercises.

1.1.2 Links to other modules

Whilst it may seem that a modules whose purpose is finding extreme values would naturally link with calculus, this is in fact only a small link. The main

connection will be to Linear Algebra, particularly the use of matrices and vectors to formulate the problem and for key steps in the arguments. You will need to be comfortable with matrices and matrix operations, linear independence, and basis vectors. If in doubt refer to your Linear Algebra notes.