

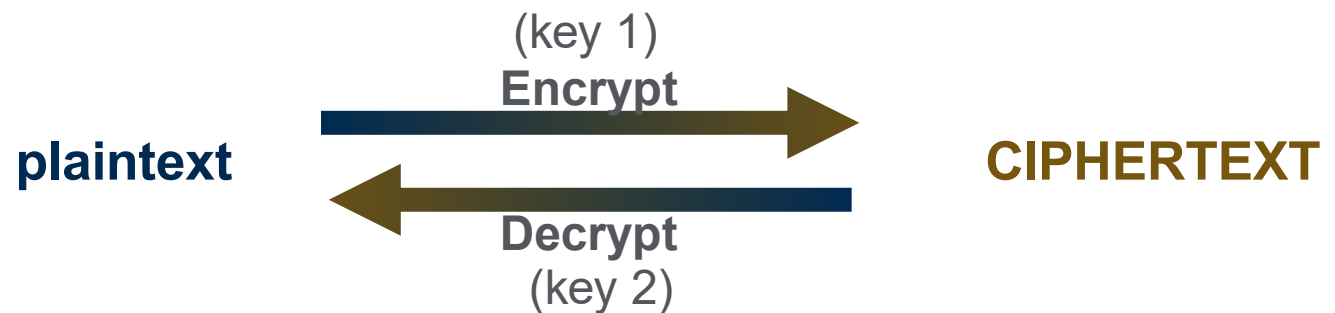
Lesson 2 – Stream Ciphers

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- Terminologies

- Cryptography: making “secret codes” (secret message)
- Cryptanalysis: breaking “secret codes”
- Cryptology: making and breaking “secret codes”
- Crypto: a synonym for any of the above and more!

- Cipher system (cryptosystem)



- Key used for encryption and decryption can be different
(Symmetric cipher vs. Asymmetric cipher)

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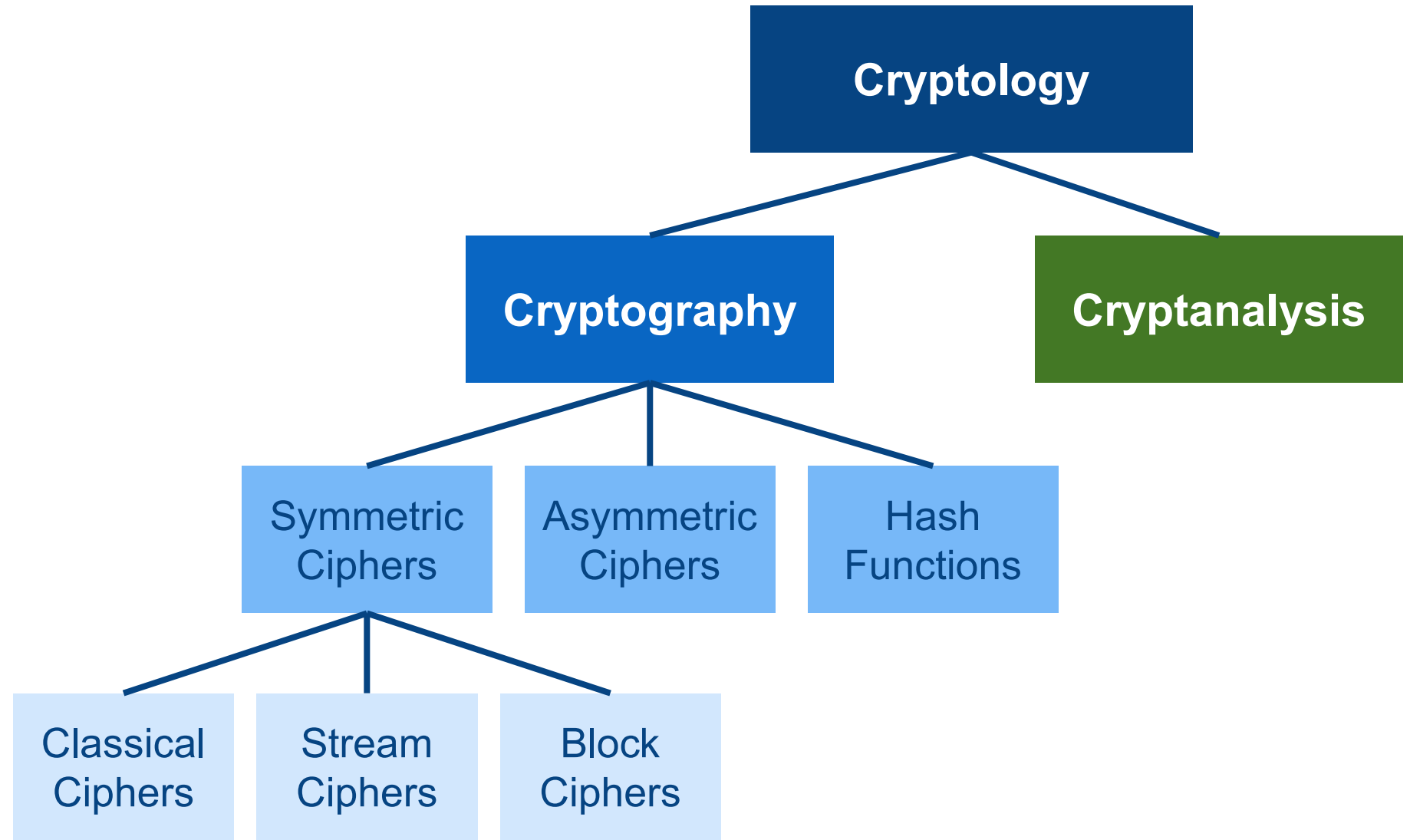
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- Kerckhoffs' principle: the strength of a cryptosystem depends ONLY on the key
 - Trudy only doesn't know the key (and of course, the plaintext)
- More terminologies...
 - Keyspace: the set of all possible values of the key
 - Exhaustive key search (brute-force attack): check the whole keyspace – always available to Trudy!
 - Definition for “secure”:
A cipher system is secure if best know attack is to try all keys
A cipher system is insecure if any shortcut attack is known



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- Simple substitution: substitute another character for each character in plaintext
 - (Simplest) Caesar cipher: shift left by alphabet by 3
 - Parameterize the key: $\text{key} \in \{1, 2, \dots, 25\}$
 - General: any permutation of letters
- Cryptanalysis
 - Caesar: once algorithm known, key is known (too weak!)
 - Parameterized: 25 attempts (worst); 13 attempts (average)
 - General: $26! \approx 2^{88}$ attempts (worst); 2^{87} attempts (average)

Or statistical attack: English letter frequency!

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- (Simplified) Double transposition cipher
 - Put plaintext into a matrix and permute the rows & columns
 - Key is the matrix size and permutations
 - Pros: hide the statistic (“diffusion”)Several letters is substituted for the same letter
 - Cons: only shuffle the order, NOT disguise the letters
- One-time pad: key only used once, and use XOR
 - Preparation: encode each letter to a binary string
 - Key generation: random string of bits with the same size of the encoded plaintext

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- One-time pad (continued)
 - Encryption: $\text{CIPHER} = \text{plain} \oplus \text{key}$, then decode to text
 - Decryption: $\text{plain} = \text{CIPHER} \oplus \text{key}$, then decode to text
 - Pros: provably secure (if random & one-time)
 - Cons: not practical (key too large)
- Codebook: dictionary-like book filled with “codewords”
 - Words (plaintext) and corresponding codewords (ciphertext)
 - The code book itself is the key
 - Security depends on the physical security of the codebook
 - Usually use with “additive”: $\text{new cipher} = \text{old cipher} + \text{MI}$

- Recall: Stream ciphers uses the idea of “one-time pad”
 - Keep the pros: “random” & one-time key
 - Solve the cons: use a small key instead of large one...
And “stretch” the small key to the size needed!
 - Keystream K: the “stretched” key
 - Used just like a one-time pad ($C = p \oplus K$, $p = C \oplus K$)
 - Algorithm to generate keystream from the key is the “heart” of each stream cipher
 - More random, more like one-time pad, the better!
- 💡 Provably secure? Or when it will be insecure?

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- Taxonomy: Stream ciphers are symmetric-key ciphers
- Stream ciphers were the king of crypto
 - Since it's efficient in hardware
 - Today, hardware gets larger and cheaper...more applications can be handled in software since processors are fast
 - So not as popular as block ciphers now
- Stream ciphers to be covered
 - A5/1: based on shift registers, used in GSM mobile phone
 - RC4: Based on a changing lookup table, used many places
 - Others (reading assignment 1): ORYX & PKZIP

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- A5/1: a stream cipher using shift register
 - Used in for GSM (Global System for Mobile Communication), a standard to describe the protocols for 2G cellular networks
 - Efficient in hardware, slow in software (was popular)
- At the beginning, Kerckhoffs' principle was violated...
 - They tried to keep this algorithm secret...
 - But as usual, the secret was reverse-engineered and leaked!
- Uses 3 linear feedback shift registers (LFSR)
 - Key is the initial fill of the registers
 - Keystream generation based on XOR and majority function

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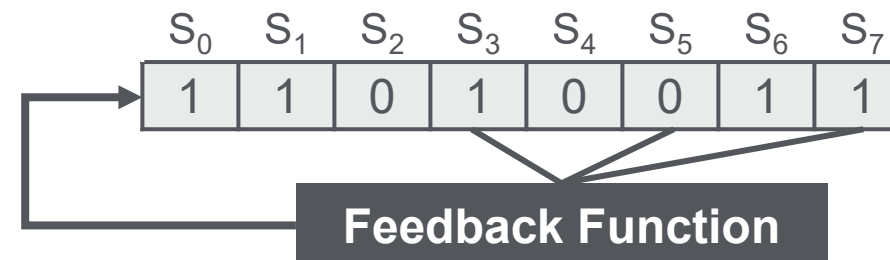
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- Shift register: a sequence of n bits
 - A temporary memory initialized with an n -bit binary number
 - Can shift left or right
 - Index starts from 0 (can kind of view it as an array)
- Feedback shift register: feedback based on the contents of the shift register and input it into the shift register
 - Leftmost bit calculated by a “feedback function” that takes in bits of the shift register (“taps”) and returns a new bit



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- Linear feedback shift register (LFSR): the feedback function is linear (only XOR operation is used)
- Step: an operation in feedback shift registers
 - Feedback function f produces a single bit P
 - P becomes the leftmost bit, and all contents shift 1 bit to right
 - Rightmost bit is shifted off (sometimes is taken as output)
 - Example: suppose taps are S_2 & S_5 , and $f = S_2 \oplus S_5$



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- Application of LFSR: pseudo-random binary generator
 - Pseudo-random: seems random but NOT truly random
- The pattern of the numbers generated are deterministic
 - If initialize the register with the same binary number (“seed”), the same sequence of numbers will be produced
- An n -bits register can have at most 2^n different binaries
 - If at any point, the content is all zero, then ...
the register will be all zero at every subsequent step
 - Therefore, the upper-bound of the period length of LFSRs is $2^n - 1$, where n is the number of bits

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- A5/1 uses LFSR to generate keystream
 - X: 19 bits, feedback function $F_X = x_{13} \oplus x_{16} \oplus x_{17} \oplus x_{18}$
 - Y: 22 bits, feedback function $F_Y = y_{20} \oplus y_{21}$
 - Z: 23 bits feedback function $F_Z = z_7 \oplus z_{20} \oplus z_{21} \oplus z_{22}$
 - 💡 What is the size of the key?
- Whether the register steps or not, based on the output of the majority function $m = \text{maj}(x, y, z)$
 - If more 1's than 0's (2/3 are 1), returns 1
 - If more 0's than 1's (2/3 are 0), returns 0
 - Examples: $\text{maj}(1, 1, 0) = 1$; $\text{maj}(0, 1, 0) = 0$

- To generate one bit in the keystream...
 - Calculate $m = \text{maj}(x_8, y_{10}, z_{10})$
 - X steps only if $x_8 = m$
 - Y steps only if $y_{10} = m$
 - Z steps only if $z_{10} = m$
 - Keystream bit = $x_{18} \oplus y_{21} \oplus z_{22}$ (XOR rightmost bit after step)
- Example:
 - Bold red: used for **maj**; bold and underline: **taps**

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|-----------------|----------|---|----------|----|----|-----------------|----|----|-----------------|-----------------|-----------------|----|-----------------|-----------------|-----------------|
| X | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | <u>0</u> | 1 | 0 | <u>1</u> | <u>0</u> | <u>1</u> | | | | |
| Y | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | <u>0</u> | <u>1</u> | |
| Z | 1 | 1 | 1 | 0 | 0 | 0 | 0 | <u>1</u> | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | <u>0</u> | <u>0</u> | <u>1</u> |

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• Example (continued)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|----------|---|---|----------|----|----|----------|----|----|----------|----------|----------|----|----------|----------|----------|
| X | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | <u>0</u> | 1 | 0 | <u>1</u> | <u>0</u> | <u>1</u> | | | | |
| Y | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | <u>0</u> | <u>1</u> | |
| Z | 1 | 1 | 1 | 0 | 0 | 0 | 0 | <u>1</u> | 1 | 1 | <u>1</u> | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | <u>0</u> | <u>0</u> | <u>1</u> |

- $m = \text{maj}(x_8, y_{10}, z_{10}) = \text{maj}(1, 0, 1) = 1$, so only X and Z step
- After X & Z step...

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|----------|---|---|----|----|----|----------|----|----|----------|----------|----------|----|----------|----------|----------|
| X | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | <u>1</u> | 0 | 1 | <u>0</u> | <u>1</u> | <u>0</u> | | | | |
| Y | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | <u>0</u> | <u>1</u> | |
| Z | 0 | 1 | 1 | 1 | 0 | 0 | 0 | <u>0</u> | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | <u>0</u> | <u>0</u> | <u>0</u> |

- keystream bit generated = $x_{18} \oplus y_{21} \oplus z_{22} = 0 \oplus 1 \oplus 0 = 1$

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- RC4: a stream cipher using lookup table
 - Invented by American cryptographer, Ronald Rivest in 1978
 - Also called “Ron’s Cipher” or “Ron’s Code”
 - Is remarkably simple, lived longer than A5/1
 - Was used in SSL, WEP, and WPA
 - Was broken but may still in use for non-security critical apps
- Each step of RC4 produces a byte (= 8 bits)
 - Efficient in software
 - vs. A5/1 –each step produces a bit: efficient in hardware
- Key size various: from 1 to 255 bytes

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- RC4 uses the 2 arrays of unsigned 8-bit numbers
 - `key[N]`: stores the N bytes key (N from 0 to 255)
 - `S[256]`: stores the permutations of numbers from 0 to 255
 - It's a self-modifying lookup table: change after each step

- Initialization (suppose key has N bytes)

- Initialize arrays:

```
for i = 0 to 255
    S[i] = i
    K[i] = key[i mod N]
next i
```

- Initial Permutation of S:

```
j = 0
for i = 0 to 255
    j = (j + S[i] + K[i]) mod 256
    swap(S[i], S[j])
next i
```

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- Then, to generate a byte...
 - Swaps elements in current lookup table ($i = 0, j = 0$ initially)

```
i = (i + 1) mod 256  
j = (j + S[i]) mod 256  
swap(S[i], S[j])
```

- Selects a keystream byte from table

```
t = (S[i] + S[j]) mod 256  
keystreamByte = S[t]
```

- Block ciphers
 - Feistel Cipher
 - DES
 - AES
 - TEA

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- Keystream
- A5/1
 - Shift register, feedback shift register, step, linear feedback shift register
 - Majority function
- RC4

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- For majority function $\text{maj}(x, y, z)$, ...
 - Write the truth table for this function
 - Derive the boolean function that is equivalent
- Most of the stream ciphers are efficient in hardware only. Why RC4 is also efficient in software?
- Find an upper bound for the number of different states that are possible for...
 - A5/1 cipher (Hint: how many possible states for each LFSR?)
 - RC4 cipher (Hint: it consists of a lookup table S , and two indices i and j . Count the number of possible distinct tables S and the number of distinct indices i and j , then compute the product of these numbers)
 - Why is the size of the state space important to know?

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