

A New Approach to Analyzing Coevolving Longitudinal Networks in International Relations[☆]

Abstract

Previous models of international conflict have suffered two shortfalls. They tend not to embody dynamic changes, focusing rather on static slices of behavior over time across a single relational dimension. These models have also been empirically evaluated in ways that assumed the independence of each country, when in reality they are searching for the interdependence among all countries. A number of approaches are available now for analyzing relational data such as international conflict in a network context and a number of these can even handle longitudinal relational data, but none are developed to the point of exploring how networks can coevolve over time. We illustrate a solution to the limitations of existing approaches and apply this novel, dynamic, network based approach to study the dependencies among the ebb and flow of daily international interactions using a newly developed, and openly available, database of events among nations.

Keywords: Dynamic networks, time series, international crises, event data, tensor products

1. Motivation

What are the tools available to us for studying endogenous relational data structures in international relations? The canonical research design in the field of international relations for the analysis of relational data remains the directed dyadic approach. The units of analysis in this design are N countries that have been paired together to form a dataset of K directed dyads. This procedure dramatically increases the size of the dataset as every country is paired with another resulting in $N \times N - N$ dyads. The size of the dataset is then further increased by the incorporation of the dyadic frame into a panel context in which each of

the K dyads is observed over T years, resulting in hundreds of thousands or millions of supposedly independent observations. Though this design has remained standard for decades, this approach has been repeatedly argued to produce biased statistical results (Hoff, 2005; Snijders, 2011; Cranmer et al., 2012). The bias results from the fact that in the context of most relational data, the interactions between actors are interdependent (Hoff & Ward, 2004; Cranmer & Desmarais, 2011). More explicitly, the directed dyadic design assumes that given the other covariates in the model, the formation of a tie between any two countries is independent of the formation of ties among other countries in the network. However, if dependencies between pairs of actors exist, then the model is misspecified and biased parameter estimates result, as the key assumption underlying most common regression

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frameworks is observations are independent of one another. Additionally, not only are biased parameter estimates likely to result, but the non-independence of observations also leads to inflated standard errors and, consequently, a greater chance of making Type I errors (Erikson et al., 2014).

The statistical issues associated with dyadic analyses have not been completely lost on the literature. A handful of studies recognize the need to study the international system in a network context, for example, see work on trade flows (Ward & Hoff, 2007), alliances (Warren, 2010), and preferential trade agreements (Manger et al., 2012). Yet, even the approaches utilized by these papers cannot explain how multiple networks may coevolve in a longitudinal context. More problematic, however, is that since the 1980s the standard approach remains constructing directed dyadic datasets and simply ignoring the issue of interdependencies in relational datasets. Jung (2014), for instance, employs a directed dyadic dataset with almost a million of observations to argue that the presence of a rising power, or hegemon, plays a role in the relationship between domestic unrest and the initiation of interstate conflict. Mitchell & Trumbore (2014) also employ this research design to test whether rogue states, defined as states that regularly violate international human rights norms, are more likely to challenge the territorial claims of other countries. The same type of design is used in recent work by Fuhrmann & Sechser (2014) and Weisiger & Yarhi-Milo (2015). Examples of virtually the same design – using virtually the same data – include Maoz & Abdolali (1989), Bremer (1992), Maoz & Russett (1993), and Bremer & Cusack (1995), as well as many, many others.

The theoretical and empirical mistake that authors continue to make in employing this design is that it is preferable to treat, for exam-

ple, “a dispute initiated by Japan against North Korea ... as distinct from a dispute initiated by North Korea against Japan” (Fuhrmann & Sechser, 2014, p. 924). In many ways, this idea is emblematic of the problem that we are referring to, there is little to reason assume that an action from Japan to North Korea is independent of how North Korea is acting towards Japan. Further the extant approaches assume that dyadic relationships are unconnected across the various time points at which they are measured. Thus, the 1990 Japan→North Korea dyad is not linked in any way to the same dyad measured in 1991 or 2001. The dyads are also not connected to similar or even neighboring dyads. The 1990 dyad of South Korea→North Korea is independent of the 1990 dyad between the Japan→North Korea, despite sharing North Korea as the target of a particular action.

Succinctly, each dyad is assumed in empirical estimations to be independent of every other dyad. The formal representation of this data structure is given by a time series of vectors comprising the measured dyadic relations at each point in time; for each dyad $i \rightarrow j$ and time t , $y_{i,j,t}$ denote the relation of the dyad at that time. Such a structure doesn’t necessarily preserve, but rather generally destroys, the dependencies among the nodes and across time. Further the assumption that every dyad is independent of every other is often not tenable statistically, and, perhaps more importantly, simply assuming that all dyads are independent closes off alternative avenues of theoretical interest. Specifically, it inhibits our ability to explore the ways in which the actions of actors in the international system are strategically interdependent (Signorino, 1999). Ignoring the role of interdependence among states in the international system is a serious oversight that plagues our attempt to understand, let alone predict, the course of world politics.

Another factor that has little changed since the 1980s, is the fact that we continue to use the same database in most of these studies. While it has been continuously updated since its creation, virtually all quantitative research on international conflict and cooperation at the dyadic level has used the same, canonical database: the Militarized Interstate Dispute (MID) data that were initially created by the Correlates of War project in the 1980s (Palmer et al., 2015). Thus these data and this research design have been a constant of research over the past several decades. What is clear is that it is hard to underestimate the resilience of the standard regression framework in spite of demonstrations that it can lead to faulty conclusions in the presence of higher-order dependencies in dyadic data structures: Krackhardt (1988); Hoff et al. (2002).¹

In this paper, we outline a different approach to the regression framework that embraces the dependency across actors as well as over time, permitting a tensor regression based approach to estimating the parameters of longitudinal and endogenous networks of political actors. This approach builds and expands on earlier latent space network based approaches in a number of important ways. First, like many existing approaches, it provides a way to understand how and why networks change in a longitudinal context. Second, unlike extant approaches, it enables us to estimate how multiple networks may evolve endogenously. Third, also unlike previous work, it enables us to differentiate the network dynamics underlying each predictor variable entered into the regression framework. We apply this novel framework to a new dyadic database, which is completely independent of the COW MID data, on conflict and cooperative events among coun-

tries.

2. Modeling Approach

Let relational data that is longitudinal be represented as a series of matrices $\{\mathbf{Y}_t : t = 1, \dots, n\}$, with each \mathbf{Y}_t representing the actions or flows between the pairs of actors, in our case countries, at time t . That is, \mathbf{Y}_t is the $m \times m$ matrix whose $(i, j)^{th}$ entry is $y_{i,j,t}$, the measured interaction between actors i and j at time t . If these data are binary – such as onsets of war or militarized disputes – this is a different but equivalent representation of the canonical data in the quantitative study of war, as in the case of Jung (2014). But such binary data are also considered by a different research tradition to represent *networks*. One such matrix \mathbf{Y}_t is shown in Figure 1. Here at time t country i has initiated disputes with j and k , and k has initiated a dispute with i .

$$Y_t = \begin{matrix} & i & \dots & j & \dots & k \\ \begin{matrix} i \\ \vdots \\ j \\ \vdots \\ k \end{matrix} & \begin{bmatrix} 0 & \dots & 1 & \dots & 1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 1 & \dots & 0 & \dots & 0 \end{bmatrix} \end{matrix}$$

Figure 1. Adjacency matrix representation of a dyadic, relational measure for one time point.

However, for many dyadic, relational data in international relations we do not have just a snapshot of interactions in time, but an actual time series of observations. Studying dynamical networks such as this has become a topic of increasing interest in a variety of fields (Holme & Saramäki, 2012). The focus on examining longitudinal networks is of particular relevance to the study of international relations as much of what we are interested in involves how and why interactions between

¹See also Cranmer & Desmarais (2011) and Snijders et al. (2012).

states change over time. In this vein, a number of approaches have been introduced to deal with longitudinal network data.

Snijders (2001) developed an agent based approach, referred to as the stochastic actor-oriented model (SAOM), to modeling such binary networks that considers the linkages among the actors in terms of decisions, such that utility models could be implemented to generate the linkages. This strategy does enable the specification of a variety of network dynamics that may exist in the data, but the decision logic is homogeneous, with the same utility applying to each actor unless nodal covariates are specified. Current versions of this approach are perhaps better known as Jackson-Wolinsky networks (Jackson & Wolinsky, 1996), though the estimation of the parameters is typically given, rather than estimated. The SAOM technique has become a popular technique to modeling longitudinal networks in political science (e.g., see Manger et al., 2012; Kinne, 2013; Berardo, 2013).² The temporal exponential random graph model (TERGM) developed by Hanneke & Xing (2007) and introduced to political science by Cranmer & Desmarais (2011) is a similar approach used to model binary longitudinal relational data (e.g., see Cranmer et al., 2012; Corbetta, 2013). TERGM and SAOM share a similar mathematical core, the exponential random graph model (ERGM), but differ in their estimation approach and in how they deal with temporal dynamics (Leifeld & Cranmer, 2015).

An alternative to this ERGM based framework is the multiple regression quadratic assignment procedure (MRQAP) introduced by

Krackhardt (1988) and refined by Dekker et al. (2007). The goal of the MRQAP procedure is different from ERGM based methods in that it focuses on examining the relationship between dyadic variables, whereas ERGM helps to examine relationships between dyadic variables and also accounts for structural tendencies in the network. A benefit of the MRQAP approach, however, is that it enables the study of weighted networks, whereas most implementations of ERGM based approaches are only developed for binary networks.

Another popular approach to modeling longitudinal networks has been to specify dynamic latent variables, in which each \mathbf{Y}_t is modeled as a function of actor specific latent variables. The goal of this approach is to collapse information contained in the network about higher-order dependencies to a lower dimensional latent space. Embedding the network information onto this lower dimensional space has the added benefit of providing meaningful visualizations of how likely actors are to interact with each other due to network influences. This approach was first introduced in the political science literature by Ward & Hoff (2007) and expanded by Ward et al. (2012) to enable inference in a longitudinal context. Similar, recent approaches include Durante & Dunson (2014) and Sewell & Chen (2014). With the exception of MRQAP, common to each of these approaches is an effort to account for higher-order dependencies that can emerge in network data. Examples of these dependencies include concepts such as reciprocity and transitivity that are familiar to the international relations literature. Broadly, these dependencies can be placed along two dimensions: second and third-order dependencies.

Second-order dependencies refer to what is often described as reciprocity in the context of directed relationships. Reciprocity is a not a new concept to the field of international stud-

²Additionally, in the 2016 Journal of Peace Research special issue on network analysis three out of twelve articles employed the SAOM approach: Chyzyh, 2016; Warren, 2016; Kinne, 2016.

ies; it has its roots in previous theories of co-operation and the evolution of norms between states (Richardson, 1960; Choucri & North, 1972; Ward, 1981; Cusack & Ward, 1981; Ward, 1984; Goldstein, 1991). This concept has particular relevance in the conflict literature, as we would expect that if, for instance, Iran behaved aggressively towards Saudi Arabia that this would induce Saudi Arabia to behave aggressively in return. The prevalence of these types of potential interactions within directed dyadic data structures directly challenges the basic assumption of observational independence.

An example of a third-order dependency is transitivity, which follows the familiar logic of “a friend of a friend is a friend”. The importance of these types of relationship is being increasingly recognized in the literature as well (see, for example, Lai, 1995; Manger et al., 2012; Kinne, 2013). In binary data, transitivity describes the dependence among three actors i , j , k in which i and j are more likely to be linked if linkages already exist between $i - k$ and $j - k$. The principal idea behind these dependencies is that knowing something about the relationships between $i - k$ and $j - k$ may reveal information about the relationship between $i - j$, even if it is not directly observed. This type of network dynamic might be particularly important for understanding the development of cooperative relationships. The consequence of not incorporating this type of dependency into our modeling framework is the same as above, and, as a result our models, will suffer from misspecification bias and will be more likely to result in type 1 errors.

While both the ERGM based and latent space approaches effectively incorporate statistical interdependence among the actors, the ERGM based approaches assume that this effect is homogeneous among all dyads. The latent variable based approach allows hetero-

geneity between dyads. The downside, however, of earlier work using the latter approach is that explicit parameterizations of the effect of reciprocity and transitivity could not be calculated due to the simple structure of the latent variables. Instead all of the information contained within those higher order dependences was collapsed into a lower dimensional latent space. A more fundamental limitation of the extant approaches mentioned here is that none are suitable for explaining how a set of networks may evolve endogenously over time. Brandt & Sandler (2012) have already noted the need to account for endogenous and dynamic relationships when using count series from event data and the need for such an approach is no less absent in the study of longitudinal networks. The realization that actors are connected across a number of possible interdependent relational dimensions is not a novel argument in the study of networks (Wasserman & Faust, 1994), but it is one that increasingly occupies the attention of the field (De Domenico et al., 2013; Kivelä et al., 2014; Boccaletti et al., 2014; D’Agostino & Scala, 2014). The implication of this expanding field of research is that empirical research into networks should begin to acknowledge that the networks we are interested in studying are not independent, but rather may be a part of an interdependent set of networks, a “multiplex”, that need to be studied holistically.

Thus in this paper we present a novel approach developed by Hoff (2015) that encompasses the benefits of many of these earlier approaches and also begins to address some of the limitations of each. Namely, the method that we review here provides: a way to analyze longitudinal networks, provides an explicit representation of network dependencies, such as reciprocity and transitivity, between dyads while still allowing for heterogeneity between dyads, and enables us to model “mul-

tiplex” networks in a regression context.

3. Tensor Model

To begin our discussion on this approach we first introduce some notation, say that (\mathbf{y}_t) represents a time series of vectors representing the dyadic interactions for m countries through the period $t = 1, \dots, n$, so that $\mathbf{y}_t \in \mathbb{R}^{m^2}$ (this is just the matrix \mathbf{Y}_t rewritten as a vector). A simple way to model the evolution of this time series is as a function of past behavior. A modeling framework that lends itself to this type of set up is a vector autoregression (VAR) model. The VAR model has been employed in macroeconomic research ever since Sims (1980) introduced the approach, and it remains a useful tool for political scientists, as evidenced by the work of Brandt & Freeman (2006); Davies (2014); Tahk (2015). The VAR model is a generalization of the univariate autoregressive model that accounts for linear interdependencies among multiple time series. The benefit of this approach is that it models a set of time series as resulting from a dynamic and endogenous process, rather than imposing a priori assumptions of exogeneity between the various explanatory variables.

A first-order VAR model posits that:

$$\mathbf{y}_t = \Theta \mathbf{y}_{t-1} + \mathbf{e}_t \quad (1)$$

$$E[\mathbf{e}_t] = 0 \quad (2)$$

$$E[\mathbf{e}_t \mathbf{e}_s^T] = \begin{cases} \Sigma & \text{if } t = s, \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where \mathbf{y}_t has been demeaned to eliminate intercepts such that $\sum_t y_{i,j,t}/n \equiv 0$. This is easy to estimate, but has extreme data requirements since Θ has m^4 entries. One basic solution to this is to use a bilinear model such that the regression matrix is given by $\Theta = \mathbf{B} \otimes \mathbf{A}$, where \otimes is the Kronecker product. As a result of this

specification we obtain a basic specification that looks relatively familiar: $\mathbf{Y}_t = \mathbf{A} \mathbf{Y}_{t-1} \mathbf{B}^T$.

In terms of this formulation we have, for each i and j , $E[y_{i,j,t} | \mathbf{Y}_{t-1}] = \sum_{i'} \sum_{j'} a_{i,i'} b_{j,j'} y_{i',j',t-1}$, which makes clear that the *regression parameters are multiplicative functions* of the basic bilinear parameters. The $a_{i,i'}$ capture how previous actions of i' affect i and $b_{j,j'}$ shows how actions that target j are influenced by prior actions toward j' .

Given that we are interested in estimating the endogenous relationships between how a set of network variables about the interactions of states evolve over time, it will be useful to quickly go over how the data we are modeling is structured. Recall that in Figure 1, the matrix contains a snapshot of interactions on a particular dimension occurring between a set of m actors at a given point in time. Now we are moving to a setting through which we can estimate how a network evolves over time, thus we have a time series of matrices $\{\mathbf{Y}_t : t = 1, \dots, n\}$. Additionally, we want to estimate a model in which we are not required to make a priori assumptions about exogeneity among our networks of interest, so we employ a modeling framework that allows us to model the endogenous effects of a set of v relational parameters over time.

Thus the data that we are modeling no longer takes the shape of a simple series of two dimensional matrices, but instead should be thought of as a tensor, or multiway array, \mathbf{Y} , of dimension $m \times m \times v \times n$, where m corresponds to the number of countries, v to the number of variables (i.e., relational measures), and n to the number of time periods. The use of tensors in this context has a long history and is increasingly used for modeling “multiplex” or longitudinal networks (e.g., see Mucha et al., 2010; Dunlavy et al., 2011). Figure 2 provides an example of this for the simple case of four countries, three time points, and two relational

measures.

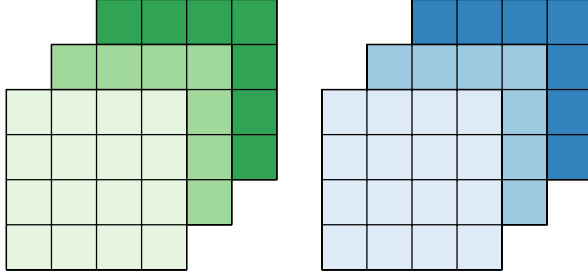


Figure 2. Tensor representation of longitudinal dyadic, representational measures. The green and blue colors represent different relational measures and darker shading indicates later time periods. Specifically, we show a tensor with dimensions of $4 \times 4 \times 2 \times 3$, where 4 represents the number of actors, 2 the number of relational measures, and 3 the number of time points.

Now take \mathbf{Y}_t to be the time t component of \mathbf{Y} (so now, instead of an $m \times m$ matrix, a tensor incorporating each of the v variables). The regression problem then becomes one of regressing the relational tensor \mathbf{Y}_t from time t on the tensor $\mathbf{X}_t = \mathbf{Y}_{t-1}$ from time $t - 1$ in a parsimonious way. Hoff (2015) proposes and develops the following multilinear generalization of the bilinear regression model:

$$\mathbf{Y} = \mathbf{X} \times \{\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3\} + \mathbf{E}, \quad (4)$$

where “ \times ” is a multilinear operator known as the “Tucker product”. The Tucker product is a multilinear operator that is used for higher-order singular value decomposition (SVD), the same way that matrix multiplication is used for matrix SVD (De Lathauwer et al., 2000; Kolda & Bader, 2009).³ Here \mathbf{Y} and \mathbf{X} are

³To illustrate how the Tucker product is calculated, say that we want to get the following expression: $\mathbf{Y} = \mathbf{X} \times \{\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3\}$, where \mathbf{Y} has dimensions $a_1 \times a_2 \times a_3$ and \mathbf{X} has dimensions $b_1 \times b_2 \times b_3$. The first step involves reshaping \mathbf{X} so that it is a matrix with dimensions $b_1 \times (b_2 \times b_3)$, then we multiply on the left by \mathbf{B}_1 , next we reshape the result to an $a_1 \times b_2 \times b_3$ matrix.

$m \times m \times v \times n$ tensors and each entry for \mathbf{X} is lagged by one period. The model for a single country-country-variable observation is shown in equation 5, where i represents a sender country, j a target, k a particular relational variable, and t a time point.

$$y_{ijvt} = \sum_i \sum_j \sum_k b_{1ii'} b_{2jj'} b_{3kk'} x_{i'j'k't} \quad (5)$$

In this model \mathbf{B}_1 and \mathbf{B}_2 are $m \times m$ matrices of regressions coefficients, and are similar to the parameters \mathbf{A} and \mathbf{B} , respectively, discussed above. The key difference, however, is that we are no longer running separate models for every v ; rather we are leveraging each of the v dyadic, relational variables to help predict the others. Thus \mathbf{B}_1 and \mathbf{B}_2 represent how previous actions along any of the v parameters affects future interactions. For example, the ij coefficient in \mathbf{B}_1 measures the effect of an interaction from actor j to actor k along any of the v parameters on the likelihood of an interaction from i to k , in any of the parameters, in the next time period.⁴

\mathbf{B}_3 is a v by v matrix of coefficients representing the direct effects of the lagged dyadic, relational parameters on one another. In other words, the uw coefficient of \mathbf{B}_3 provides a measure of the effect of the value of parameter w in a dyad on the value of parameter u in that dyad during the next time period. In order to incorporate explicit representations of higher-

This procedure would be applied iteratively among the remaining dimensions. Further details on this operator can be found in: Kolda (2006); Hoff (2011)

⁴For example, for each county i , the coefficient b_{1iUSA} describes how predictive the actions of the USA are of the future actions of country i . Heterogeneity of these coefficients across countries i describes heterogeneity of the nodes in terms of their dependence on the actions of USA.

order dependencies we extend the \mathbf{X} tensor along its third dimension.

First, we add in reciprocity, which captures the tendency for actions to be reciprocated from one node, i , to another, i' for a variable w . More explicitly, if $y_{i,i',w,t}$ (that is, the i to i' relationship in variable w at time t) is large, and reciprocal relationships exist, we would expect $y_{i',i,w,t+1}$ to be large as well. In the formulation of \mathbf{X} we have presented so far, its third dimension has length equal to v ; we add v additional slices to the tensor along its third mode to incorporate reciprocity, so that it now has dimension $m \times m \times (2 * v) \times n$. The first v slices across this third mode are the same as before, $y_{i,i',w,t-1}$, but the last v slices are defined using the reciprocal lagged elements, $y_{i',i,w,t-1}$. The general form of our tensor regression remains the same as what was shown in Equation 4, but now \mathbf{B}_3 is a v by $(2 * v)$ matrix, in which the last v columns capture the tendencies of actions to be reciprocated by other actions at the next time point.

Next, we add in a measure of transitivity to account for the possibility of third-order dependencies. An implication of transitivity is that existing linkages $y_{i,i'',w,t}$ and $y_{i',i'',w,t}$, imply that $y_{i,i',w,t+1}$ may form a linkage as well. To incorporate this additional term, we again add v slices to the third mode of our \mathbf{X} tensor. The elements of these last v slices are $\sum_{i''} (y_{i,i'',w,t} + y_{i'',i,w,t})(y_{i',i'',w,t} + y_{i'',i',w,t})$. Similar to when reciprocity was incorporated, \mathbf{B}_3 now expands to a v by $(3 * v)$ matrix, in which the last v columns capture how relations that i and i' share with common targets leads to future interactions between i and i'' .

The parameters in our completed model formulation have the following dimensions:

- \mathbf{Y} : $m \times m \times v \times n$,
- \mathbf{X} : $m \times m \times (3 * v) \times n$,

- \mathbf{B}_1 and \mathbf{B}_2 : $m \times m$,

- \mathbf{B}_3 : $v \times (3 * v)$,

where m refers to the number of countries, v the number of relational parameters, and n the number of time periods. This framework can be referred to as a relational multilinear regression. Like previous work using latent spaces to incorporate network characteristics, we are able to model actor heterogeneity in the network, however, this framework now also enables the explicit measurement of higher-order dependencies. A full derivation of the model is beyond the scope of this paper, however, Hoff (2015) details a Bayesian implementation of this model via Gibbs sampling using semi-conjugate prior distributions to simulate parameter values from the posterior.⁵

4. ICEWS Dyadic Event Database

We use this modeling framework to estimate models for verbal and material cooperation and conflict between states. Information on these dimensions is collected from the Integrated Crisis Early Warning System (ICEWS) event data project. The ICEWS data along with extensive documentation are available on dataverse.org (Lautenschlager et al., 2015; Boschee et al., 2015). The ICEWS event data are constructed by applying natural language processing and graph theory techniques (Boschee et al., 2013) to a corpus of about 30 million media reports from about 275 local and global news sources in or translated to English. Each media report is coded in accordance with an ontology of events that is derived from the Conflict and Median Event Observation (CAMEO) scheme (Schrodt et al., 2009).

⁵Code for this Gibbs sampler can be found here: http://www.stat.washington.edu/people/pdhoff/Code/hoff_2014/.

This results in relevant reports being tagged as involving a set of actors and by the interactions that took place between them. We choose to focus on interactions among the top fifty countries by GDP during time period from the beginning of 2001 to the end of 2014. And, like all good social scientists, we use a 2×2 table to classify; herein, across the distinctions between actions and words on the one hand and conflict and cooperation on the other. Thus, we aggregate events for this set of countries and temporal span into the so-called “quad variables”, originally developed by Duval & Thompson (1980). Yonamine (2011) provides a definition for each:

- *Verbal Cooperation*: The occurrence of dialogue-based meetings (e.g. negotiations, peace talks), statements that express a desire to cooperate or appeal for assistance (other than material aid) from other actors.
- *Material Cooperation*: Physical acts of collaboration or assistance, including receiving or sending aid, reducing bans and sentencing, etc.
- *Verbal Conflict*: A spoken criticism, threat, or accusation, often related to past or future potential acts of material conflict.
- *Material Conflict*: Physical acts of a conflictual nature, including armed attacks, destruction of property, assassination, etc.

Given that the quad variables are generated using an event data approach, we have a very low level of temporal aggregation available to us through which to measure directed interactions between countries. In this analysis, we focus on the monthly level of aggregation, thus leaving us with a large time series of directed,

dyadic, relational observations for each quad variable.

In Figure 3, we show the total number of event counts falling under each of these variables at the monthly level. Looking across the panels we can see that verbal cooperation events between countries is much more frequent than the other relational variables we use here. Further, the verbal event variables, in general, are more frequently occurring than the material events. This itself may simply reflect the fact that verbal actions are less costly to engage in than material ones, thus they occur more frequently. Despite this the number of material conflictual events, that is, events involving a country engaging in activities such as imposing a blockade, are not rare. For many months during the time period shown, the number of material conflictual events involving the 50 countries in our analysis exceeded a 1,000. Material cooperation, on the other hand, is a much less frequent event within the context of our sample. This is likely due to the fact that by design we chose the fifty countries with the highest average GDPs during the period of 2001 to 2014, and these countries are less likely to provide material aid to each other than they would be to poorer countries.

An important benefit to using these relational measures is that they display interesting variation over time. Most measures used in the extant literature to estimate the dyadic relations between states exhibit relatively little temporal variation (e.g., MIDs), and, if they do, they are often not available at low levels of temporal aggregation without substantial loss of country coverage. The quad variables shown here suffer from neither of those drawbacks. Thus the addition of these variables to the set of databases used in political science research may enable the field to develop a richer understanding of how directed relationships between states evolve.

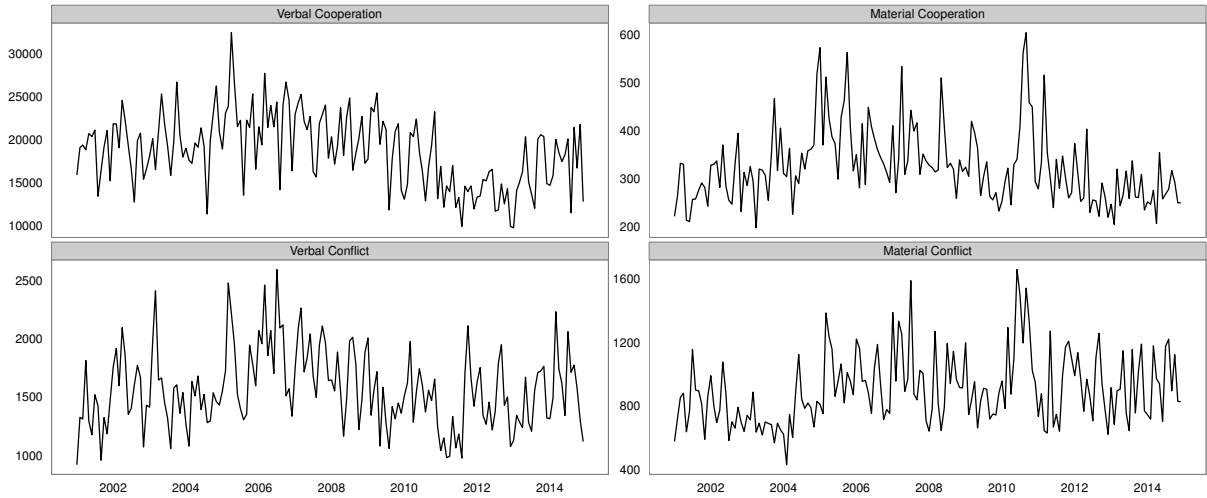


Figure 3. Quad variable counts over time.

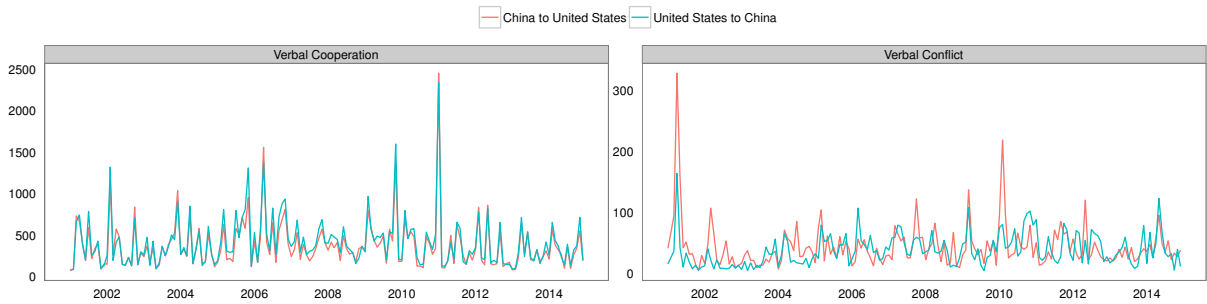


Figure 4. Quad variable counts over time for interactions between the United States and China.

An example of how we can trace interesting features of state interaction over time using these variables is shown in Figure 4. Here we visualize the directed relations between the United States and China for the two verbal quad variables at the monthly level across our period of analysis. The blue line in the left panel illustrates verbal cooperative actions that the United States sent to China, while the red line designates the same type of actions that China sent to the United States.

In the case of verbal cooperation, we can see that the two lines almost completely overlap with one another, indicating that many of

these events either occur in tandem or are reciprocated almost immediately. The panel on verbal conflict shows similar patterns of reciprocity as verbal cooperation, but unlike cooperative interactions between the United States and China verbal conflictual events are usually driven to a greater degree by one party or another. For example, in February of 2010, we can see a peak in the number of verbal conflictual events sent by China to the United States. This likely was related to the announcement of an arms sale package to Taiwan by the Obama administration.

To begin to model the processes driving the

quad variables we turn to the the relational multilinear regression framework introduced earlier. We run two separate models. In the first, we group together the verbal and material conflict variables, and, in the second, the verbal and material cooperation variables. We model the processes of cooperation and conflict separately in this iteration of the project to ease the interpretability of the parameter estimates. Additionally, if we run a tensor model using all four of the relational variables, as opposed to running two separate tensor models as discussed above, the model performance noticeably declines (higher root-mean-squared error (RMSE) and lower R^2).

For both models then, this leaves us with a \mathbf{Y} tensor that has dimensions $50 \times 50 \times 2 \times 167$, where 50 corresponds to the number of countries, 2 to the number of relational variables, and 167 to the number of months included in our analysis. Given that we are including explicit representations of higher-order dependencies, our \mathbf{X} tensor has dimensions $50 \times 50 \times 6 \times 167$. We also apply a normal quantile-quantile transformation to the time-series for every sender-target-variable type triplet and further centered and scaled each, so that their empirical distributions are approximately standard normal.

5. Results

Figure 5 shows the trace plots after running 8,000 iterations of the Gibbs sampler for each of the six \mathbf{B}_3 parameter estimates from the cooperation model.⁶ Trace plots for the conflict model produce results that are similar, in terms

⁶The time requirements to run the current iteration of the model are substantial. Running 8,000 iterations on a m4.large Amazon EC2 instance took approximately 28 hours. We are currently working on a C++ implementation of the model using the Rcpp framework.

of indicating that the iterative estimation algorithm quickly converged. The starting value for these Gibbs samplers were the least square estimates and we can see that each parameter converges to its stationary distribution almost immediately. We chose to use the first 1,000 iterations as burn-in, leaving us with 7,000 iterations for every parameter through which to conduct posterior analysis.

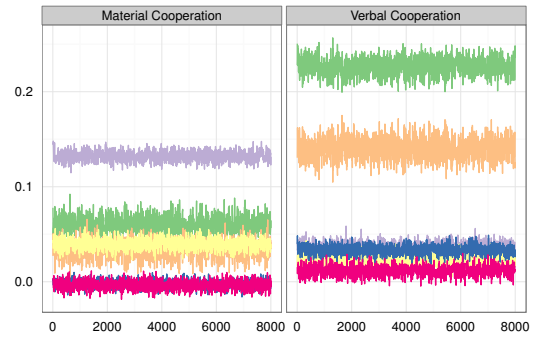


Figure 5. Trace plots for each of the six \mathbf{B}_3 parameter estimates from the cooperation model. Different colors correspond to different parameter estimates.

5.1. Direct, Reciprocal, & Transitive Effects

The posterior distributions of the \mathbf{B}_3 coefficients are summarized in Figure 6. The panel on the left corresponds to the tensor model on cooperation and the right to conflict. Also note that though we split up the verbal and material cooperation dependent variables in Figure 6a, they are estimated simultaneously using the modeling framework discussed earlier – the same holds for the conflict model. Additionally, given the transformations we made to the time series for each of our parameters, the relative importance of our parameters can be assessed by comparing their coefficient estimates.

Along the x-axis of this figure, we list the direct (ij), reciprocal (ji), and transitive lagged

estimates (ijk) for each variable. The direct estimate represents the effect of a particular variable from i to j on the future value of that variable from i to j . The reciprocal estimate designates the effect of actions from j to i on future actions from i to j . Last, the transitive estimate assesses the effect on action between i and k in the future given that i and j and j and k are already interacting with one another.

For each of the dyadic, relational variables examined in Figure 6, the largest estimated coefficient comes from the direct, lagged parameter, indicating that the strongest predictor of relations at a given point a time are what they were in the period before. For instance, the regression estimates for the Material Conflict variable, shown in the bottom half of Figure 6b, the parameter estimate for Material Conflict $_{ij,t-1}$ is twice as large as that of Verbal Conflict $_{ij,t-1}$. This general trend persists for the predictors of Verbal Conflict and for the results of the cooperation model.

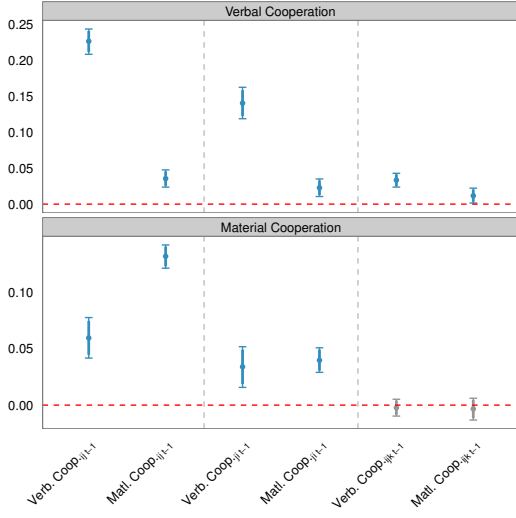
The lagged effect, however, is not the only powerful and precise predictor. For both the cooperation and conflict models, the effect of both of the direct, lagged variables is precisely estimated and positive. Thus, for example, in the case of our model on cooperation, we find that higher levels of verbal cooperation between a pair of countries is also likely to lead to higher levels of material cooperation and vice versa. The same finding holds for the parameters measuring the direct effects in the conflict model.

The results discussed so far correspond to the type of parameters that existing latent space approaches can already estimate. But the novel component of this new framework comes with the ability to estimate higher-order effects for each of these parameters. Thus we turn to the set of ji parameter estimates in the panels shown in Figure 6. In every case, we find that the reciprocal effects are precisely es-

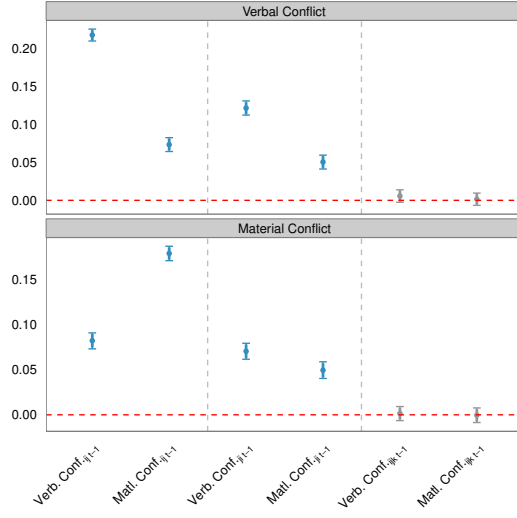
timated for both lagged parameters, and that there are a number of instances in which the reciprocal effects are of similar magnitude to the direct effects. The implication of this, though potentially not surprising, is that countries tend to reciprocate behavior. For instance, in the case of our cooperation model, we find that material cooperation in a ji pair is likely to lead to reciprocal material cooperation in the ij pair in the next period. More interestingly, we also find evidence in both our models that verbal actions are reciprocated by material ones and vice versa.

The key implication of these findings for the reciprocal parameters in our models is that second-order dependencies have a significant role to play in our understanding of interactions between states. As noted earlier, ignoring these types of dependencies not only closes off interesting theoretical avenues of analysis, but also can lead to inferential mistakes. The last set of coefficient estimates contained in \mathbf{B}_3 are the transitive effects of each of the lagged parameters. We find less support for this particular type of higher-order dependency in our analysis. In the conflict models, none of the parameters used to estimate transitive effects have a meaningful role in explaining verbal or material conflict. In the case of our cooperation model, we do find some evidence that transitivity comes into play in explaining verbal cooperation. Though the effect is small, the implication is that countries are likely to engage in verbal cooperation with countries to whom they they share indirect verbal or material ties already. The same effect, however, does not hold for material cooperation, likely because providing material support is more costly.

This all makes a good deal of intuitive sense: if i and j are at a high level of conflict on issue A, and i and k are at war on issue B, why would we expect j and k to enter a con-



(a) Cooperation Model



(b) Conflict Model

Figure 6. Relational multilinear regression results for \mathbf{B}_3 . Darker colors indicate that the coefficient estimate is significantly different from zero at a 95% credible interval, while lighter the same for a 90%. Grey indicates that the estimate is not significantly different from zero at either of those intervals.

flict with one another? On the cooperative side of things, it seems more plausible. And this is exactly what the empirical estimates suggest.

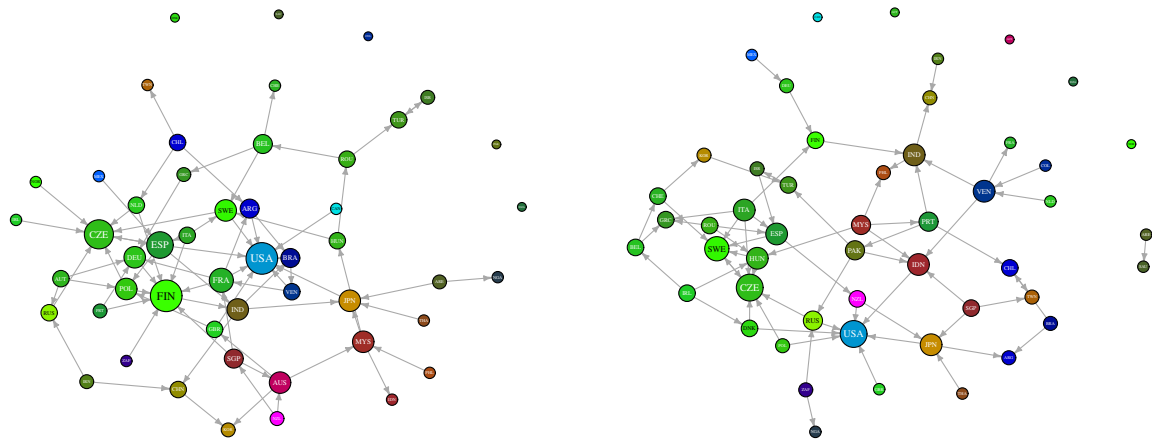
5.2. \mathbf{B}_1 & \mathbf{B}_2

The posterior distributions of \mathbf{B}_1 and \mathbf{B}_2 are summarized in Figures 7a for cooperation and 7b for the conflict models. Both these parameters contain $m \times m$ regression coefficients. The ij coefficient in \mathbf{B}_1 provides a measure of how likely i is to send, for example, a verbal or material cooperative event to a third party, k , given that j has already sent a cooperative event to k . Similarly, the ij coefficient in \mathbf{B}_2 provides a measure of how likely i is to receive a cooperative event from a third party, k , given that j has already received a cooperative event from k .

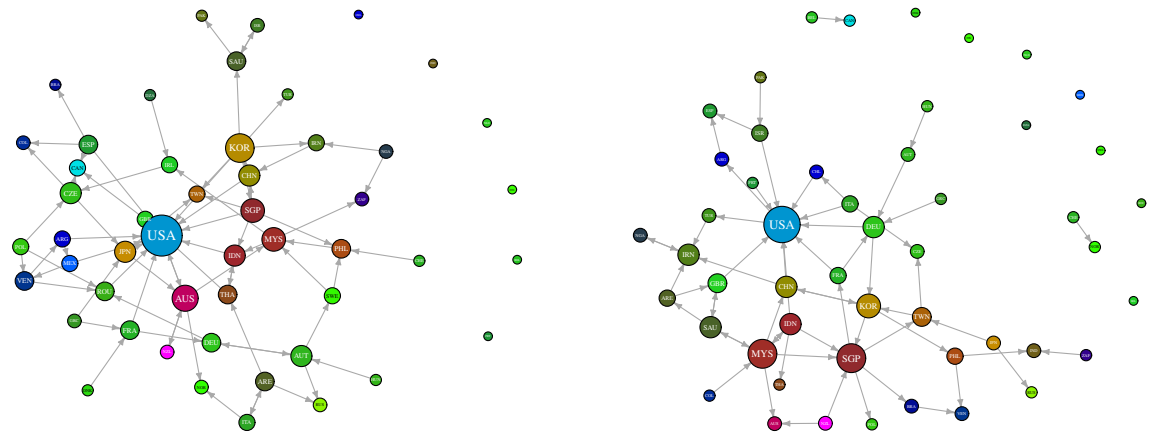
To visualize the posterior distributions for these parameters, we create a set of network plots in which linkages designate significant positive effects using a 99% credible interval.

Nodes are replaced with three letter country abbreviations and these pieces of text are colored by the geographic location of countries shown in Figure 7c. Results using a lower threshold for significance lead to a large number of linkages being drawn and the plots becoming illegible. Additionally, very few, less than 3%, of the ij estimates were significantly negative, so we exclude visualizing those here as well to improve readability.

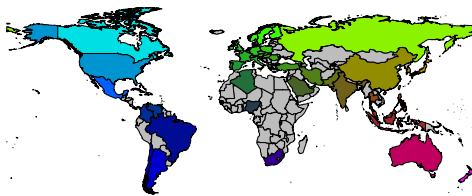
Another important feature missing from our visualizations of \mathbf{B}_1 and \mathbf{B}_2 in Figures 7a and 7b is the diagonal elements. The diagonal elements, in the case of \mathbf{B}_1 , represent the tendency of i to send a cooperative event to a third party, given that i has done so in the past.



(a) Cooperation Model



(b) Conflict Model



(c) Legend

Figure 7. \mathbf{B}_1 , on left, and \mathbf{B}_2 , on right. Colors are assigned to nodes by the geographic position of countries (see Figure 7c) and links are drawn between nodes when the corresponding parameter estimate for that ij pair is significant at a 99% credible interval. Arrows designate the direction of the relationship. The size of nodes correspond to the number of linkages sent and received.

Even though we do not visualize those relationships here, their effects are notable; specifically, we find that the posterior means of the diagonal elements of both \mathbf{B}_1 and \mathbf{B}_2 are approximately 10 times as large as the off-diagonal elements for the cooperation model. For the conflict model, the diagonal elements are about 12 times as large. This result points to a certain stickiness in how states relate to others in our sample.

Nonetheless, there are additional findings to be gleaned from the network visualizations we present in Figures 7a and 7b. First, it is clear that in each of these panels we see a noticeable amount of geographic clustering. For example, in the \mathbf{B}_1 posterior visualization from the cooperative model, the Philippines (PHL) are likely to send cooperative events to countries that have already received cooperative verbal or material support from Malaysia (MYS). Malaysia is more likely to do the same for countries that have received cooperative interactions from Indonesia (IDN). Israel (ISR) and Turkey (TUR) also both share this type of relationship, in that a third party that has received a cooperative event from one has a higher of probability receiving that type of event from the other at a future time point as well.

This type of geographic clustering is still apparent as we move to examining \mathbf{B}_2 from the conflict model, shown in the right panel of Figure 7b. Here we can see that Hungary (HUN) is more likely to receive conflictual events from countries that have sent those type of events to Austria (AUT). However, there are a number of relationships here that go beyond simple geographic clustering. For example, South Africa (ZAF) is significantly more likely to receive conflictual events from countries that have threatened India (IND); and Argentina is more likely to receive conflictual events from countries that have previously threatened Spain.

5.3. Performance analysis

Last, we undertake an assessment of the model's performance. To do this, we calculate the RMSE for the time series of every country-pair by variable. This generates an $(m \times m) - m$ number of RMSE values; we visualize the results of this analysis using the surface plot shown in Figure 8. Here we only show results for the conflict model – the performance of the cooperation model is similar.

The results here highlight that the model's performance – particularly, for pairings involving countries like France (FRA), Germany (DEU), United Kingdom (GBR), and the United States (USA) – has meaningful room for improvement. However, the average error of the model is not large, though it does vary substantially across dyads. For example, the fit of the model with regards to the largest countries is likely a result of the fact that these countries are obviously quite different from others in the international system. Thus, even with a model as detailed as that presented, there is still work to be done in contextualizing the interaction patterns. In future work we hope to address issues involving the model's performance, by allowing it to apply even more differentially to different types of nation-pairs, in a kind of bloc partitioning.

Compared to the standard approaches, however, this novel approach generates a great deal of information about dyadic interactions that was simply lost before. Having that information also allows us to have a better way to get at first-order, as well as the second and third order interactions.

6. Lessons Learned

The field of International Relations has always had big data, data that are characterized as time-series cross-sections of relational data, often dyads. These data structures defeat most

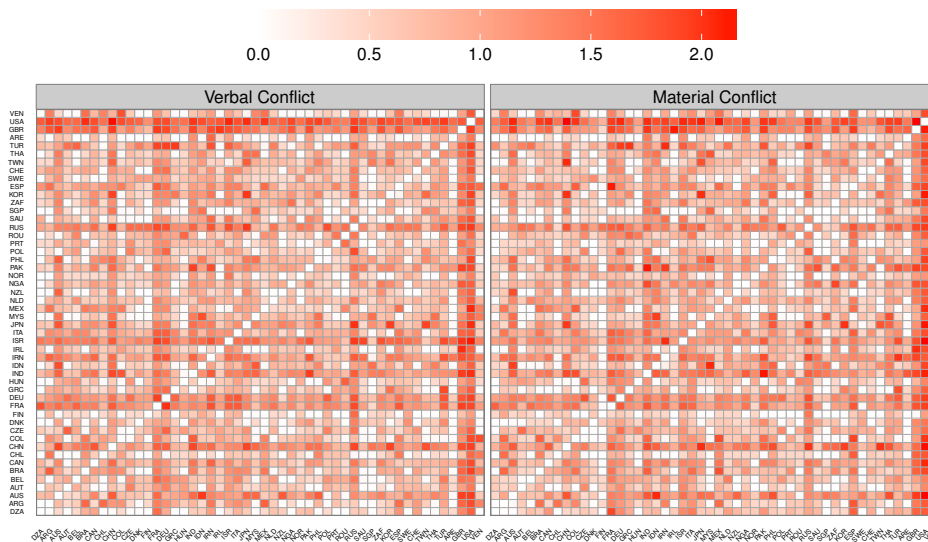


Figure 8. Root-mean-squared error values for every directed country-pair time series.

standard regression-based attempts to analyze their structure. In particular, some approaches such as latent variable models have been developed to handle the cross-sectional dependence and these have been applied in some of the literature, but ignored in much extant work. And some approaches have focused on the underlying structure of the time domain, but these too have been widely ignored.

At a substantive level, we know that the world around us is highly organized in hierarchical ways, and that some nodes are more powerful in their interactions than others (House & Ward, 1988a,b), but that figuring this out computationally or statistically was challenging exactly because of the dependencies. At the same time, finding a way to embed the longitudinal and endogenous aspects of world politics in these frameworks was often relegated to a simple inclusion of time counters into regression models or just ignored. But the framework herein allows this to be accomplished in a more straightforward approach that is quite familiar since it is es-

entially a type of regression, except with coevolving network matrices stacked over time.

The substantive findings that emerge from this suggest something both about the time dependencies and the network effects that permeate the ebb and flow of international politics.

References

- Berardo, Ramiro (2013) The coevolution of perceptions of procedural fairness and link formation in self-organizing policy networks. *The Journal of Politics* 75(3): 686–700.
- Boccaletti, Stefano; G Bianconi; R Criado; Charo I D Genio; J Gómez-Gardeñes; M Romance; I Sendina-Nadal; Z Wang & M Zanin (2014) The structure and dynamics of multilayer networks. *Physics Reports* 544(1): 1–122.
- Boschee, Elizabeth; Jennifer Lautenschlager; Sean O'Brien; Steve Shellman; James Starz & Michael D Ward (2015) ICEWS coded event data. <http://dx.doi.org/10.7910/DVN/28075> Harvard Dataverse Network [Distributor] V1 [Version].
- Boschee, Elizabeth; Premkumar Natarjan & Ralph Weischedel (2013) Automatic extraction of events from open source text for predictive forecasting. In: Venkatramana S. Subrahmanian (ed.) *Handbook*

- of *Computational Approaches to Counterterrorism*. New York: Springer.
- Brandt, Patrick T & John R Freeman (2006) Advances in Bayesian Time Series Modeling and the Study of Politics: Theory Testing, Forecasting, and Policy Analysis. *Political Analysis* 14(1): 1–36.
- Brandt, Patrick T & Todd Sandler (2012) A bayesian poisson vector autoregression model. *Political Analysis* 20(3): 292–315.
- Bremer, Stuart A (1992) Dangerous Dyads: Conditions Affecting the Likelihood of Interstate War, 1816–1965. *Journal of Conflict Resolution* 36(2): 309–341.
- Bremer, Stuart A & Thomas R Cusack (1995) *The Process of War: Advancing the Scientific Study of War*. Luxembourg and Philadelphia, PA: Gordon and Breach.
- Choucri, Nazli & Robert C North (1972) Dynamics of international conflict. *World Politics* 24(2): 80–122.
- Chyzh, Olga (2016) Dangerous liaisons: An endogenous model of international trade and human rights. *Journal of Peace Research* 52(2).
- Corbetta, Renato (2013) Cooperative and antagonistic networks: Multidimensional affinity and intervention in ongoing conflicts, 1946–2011. *International Studies Quarterly* 57(2): 370–384.
- Cranmer, Skyler & Bruce A Desmarais (2011) Inferential Network Analysis with Exponential Random Graph Models. *Political Analysis* 19(1): 66–86.
- Cranmer, Skyler J; Bruce A Desmarais & Elizabeth J Menninga (2012) Complex Dependencies in the Alliance Network. *Conflict Management and Peace Science* 29(3): 279–313.
- Cusack, Thomas R & Michael D Ward (1981) Military Spending in the United States, Soviet Union, and the People’s Republic of China. *Journal of Conflict Resolution* 25: 429–469.
- D’Agostino, Gregorio & Antonio Scala (2014) *Networks of networks: the last frontier of complexity* volume 340. Berlin: Springer.
- Davies, Graeme A (2014) Policy selection in the face of political instability do states divert, repress, or make concessions? *Journal of Conflict Resolution Online-First*: 1–25.
- De Domenico, Manlio; Albert Solé-Ribalta; Emanuele Cozzo; Mikko Kivelä; Yamir Moreno; Mason A Porter; Sergio Gómez & Alex Arenas (2013) Mathematical formulation of multilayer networks. *Physical Review X* 3(4).
- De Lathauwer, Lieven; Bart D Moor & Joos Vandewalle (2000) A multilinear singular value decomposition. *SIAM journal on Matrix Analysis and Applications* 21(4): 1253–1278.
- Dekker, David; David Krackhardt & Tom A Snijders (2007) Sensitivity of mrqap tests to collinearity and autocorrelation conditions. *Psychometrika* 72(4): 563–581.
- Dunlavy, Daniel M; Tamara G Kolda & W P Kegelmeyer (2011) Multilinear algebra for analyzing data with multiple linkages. *Graph Algorithms in the Language of Linear Algebra*, J. Kepner and J. Gilbert, eds., *Fundamentals of Algorithms*, SIAM, Philadelphia: 85–114.
- Durante, Daniele & David B Dunson (2014) Nonparametric bayes dynamic modelling of relational data. *Biometrika* 101(4): 883–898.
- Duval, Robert D & William R Thompson (1980) Reconsidering the aggregate relationship between size, economic development, and some types of foreign policy behavior. *American Journal of Political Science* 24(3): pp. 511–525.
- Erikson, Robert S; Pablo M Pinto & Kelly T Rader (2014) Dyadic analysis in international relations: A cautionary tale. *Political Analysis* 22(4): 457–463.
- Fuhrmann, Matthew & Todd S Sechser (2014) Signaling alliance commitments: Hand-tying and sunk costs in extended nuclear deterrence. *American Journal of Political Science* 58(4): 919–935.
- Goldstein, Joshua S (1991) Reciprocity in superpower relations: An empirical analysis. *International Studies Quarterly*: 195–209.
- Hanneke, Steve & Eric P Xing (2007) Discrete temporal models of social networks. In: *Statistical network analysis: Models, issues, and new directions*. Springer, 115–125.
- Hoff, Peter (2015) Multilinear tensor regression for longitudinal relational data. *The Annals of Applied Statistics Forthcoming*.
- Hoff, Peter D (2005) Bilinear mixed effects models for dyadic data. *Journal of the American Statistical Association* 100(469): 286–295.
- Hoff, Peter D (2011) Separable covariance arrays via the tucker product, with applications to multivariate relational data. *Bayesian Analysis* 6(2): 179–196.
- Hoff, Peter D; Adrian E Raftery & Mark S Handcock (2002) Latent space approaches to social network analysis. *Journal of the American Statistical Association* 97(460): 1090–1098.
- Hoff, Peter D & Michael D Ward (2004) Modeling dependencies in international relations networks. *Political Analysis* 12(2): 160–175.
- Holme, Petter & Jari Saramäki (2012) Temporal net-

- works. *Physics reports* 519(3): 97–125.
- House, Lewis L & Michael D Ward (1988a) The behavioral power of nations. *Quality and Quantity* 22: 167–188.
- House, Lewis L & Michael D Ward (1988b) A theory of the behavioral power of nations. *Journal of Conflict Resolution* 32(1): 3–36.
- Jackson, Matthew O & Asher Wolinsky (1996) A strategic model of social and economic networks. *Journal of economic theory* 71(1): 44–74.
- Jung, Sung C (2014) Foreign targets and diversionary conflict. *International Studies Quarterly* 58(3): 566–578.
- Kinne, Brandon (2013) Network dynamics and the evolution of international cooperation. *American Political Science Review* 107(4): 766–785.
- Kinne, Brandon (2016) Agreeing to arm: Bilateral weapons agreements and the global arms trade. *Journal of Peace Research* 52(2).
- Kivelä, Mikko; Alex Arenas; Marc Barthelemy; James P Gleeson; Yamir Moreno & Mason A Porter (2014) Multilayer networks. *Journal of Complex Networks* 2(3): 203–271.
- Kolda, Tamara G (2006). Multilinear operators for higher-order decompositions. Technical Report SAND2006-2081 Sandia National Laboratories.
- Kolda, Tamara G & Brett W Bader (2009) Tensor decompositions and applications. *SIAM review* 51(3): 455–500.
- Krackhardt, David (1988) Predicting with networks: Nonparametric multiple regression analysis of dyadic data. *Social networks* 10(4): 359–381.
- Lai, David (1995) A structural approach to alignment: A case study of the China-Soviet-U.S. strategic triangle, 1971–1988. *International Interactions* 20(4): 349–374.
- Lautenschlager, Jennifer; Steve Shellman & Michael D Ward (2015) ICEWS coded event aggregations. <http://dx.doi.org/10.7910/DVN/28117> Harvard Dataverse Network [Distributor] V1 [Version].
- Leifeld, Philip & Skyler J Cranmer (2015) A theoretical and empirical comparison of the temporal exponential random graph model and the stochastic actor-oriented model. *arXiv preprint arXiv:1506.06696*.
- Manger, Mark; Mark Pickup & Tom Snijders (2012) A hierarchy of preferences a longitudinal network analysis approach to PTA formation. *Journal of Conflict Resolution* 56(5): 853–878.
- Maoz, Zeev & Nasrin Abdolali (1989) Regime types and international conflict, 1816–1976. *Journal of Conflict Resolution* 33(1): 3–35.
- Maoz, Zeev & Bruce M Russett (1993) Normative and structural causes of democratic peace, 1946–1986. *American Political Science Review* 87(3): 624–38.
- Mitchell, Sara M & Peter F Trumbore (2014) Rogue states and territorial disputes. *Conflict Management and Peace Science* 31(3): 323–339.
- Mucha, Peter J; Thomas Richardson; Kevin Macon; Mason A Porter & J-P Onnela (2010) Community structure in time-dependent, multiscale, and multiplex networks. *science* 328(5980): 876–878.
- Palmer, Glenn; Vito d’Orazio; Michael Kenwick & Matthew Lane (2015) The mid4 data set, 2002–2010: Procedures, coding rules, and description. *Conflict Management and Peace Science* 32(2): 222–242.
- Richardson, Lewis F (1960) *Arms and Insecurity*. Chicago and Pittsburgh, PA: Quadrangle/Boxwood.
- Schrodt, Philip A; Debora J Gerner & Omur Yilmaz (2009) Conflict and mediation event observations (CAMEO): An event data framework for a post cold war world. In: Jacob Bercovitch & Scott Sigmund Gartner (eds.) *International Conflict Mediation: New Approaches and Findings*. New York: Routledge.
- Sewell, Daniel K & Yuguo Chen (2014) Latent space models for dynamic networks. *Journal of the American Statistical Association*: Forthcoming.
- Signorino, Curtis S (1999) Strategic interaction and the statistical analysis of international conflict. *American Political Science Review* 93(2): 279–297.
- Sims, Christopher A (1980) Macroeconomics and reality. *Econometrica*: 1–48.
- Snijders, Tom A (2011) Statistical models for social networks. *Annual Review of Sociology* 37: 131–153.
- Snijders, Tom A. B (2001) The statistical evaluation of social network dynamics. *Sociological Methodology* 31: 361–395.
- Snijders, Tom A. B; Johan Koskinen & Michael Schweinberger (2012) Maximum likelihood estimation for social network dynamics. *Annals of Applied Statistics* tbd(tbd): tbd.
- Tahk, Alexander M (2015) A continuous-time, latent-variable model of time series data. *Political Analysis* 23(2): 278–298.
- Ward, Michael D (1981) Seasonality, reaction, expectation, adaption, and memory in cooperative and conflictual foreign policy. *International Interactions* 8: 229–246.
- Ward, Michael D (1984) Differential paths to parity: A study of the contemporary arms race. *American Political Science Review* 78: 297–317.
- Ward, Michael D; John S Ahlquist & Arturas Rozenas (2012) Gravity’s rainbow: A dynamic latent space

- model for the world trade network. *Network Science* 1(1): 95–118.
- Ward, Michael D & Peter D Hoff (2007) Persistent patterns of international commerce. *Journal of Peace Research* 44(2): pp. 157–175.
- Warren, Camber (2010) The geometry of security: Modeling interstate alliances as evolving networks. *Journal of Peace Research* 47(6): 697–709.
- Warren, Camber (2016) Modeling the co-evolution of international and domestic institutions: Alliances, democracy, and the complex path to peace. *Journal of Peace Research* 52(2).
- Wasserman, Stanley & Katherine Faust (1994) *Social network analysis: Methods and applications* volume 8. Cambridge university press.
- Weisiger, Alex & Keren Yarhi-Milo (2015) Revisiting reputation: How past actions matter in international politics. *International Organization* 69: 473–495.
- Yonamine, Jay (2011). Working with event data: A guide to aggregation choices. Penn State University: Working Paper.