Question 7) Proving that the median minimizes the absolute loss function

$m = m_{t+H}$

$y = y_{t+H}$, $\hat{y} = \hat{y}_{t+H}$ in the following derivations

$$E(\hat{y}-y) = \int_{-\infty}^{\infty} |\hat{y}-y|\, f(x)\,dx$$

Can split the integral into two.

$$= \int_{-\infty}^{y_{t+H}} (y-\hat{y}) f(x)\,d(x) + \int_{y}^{+\infty} (\hat{y}-y) f(x)\,dx$$

If the median $> y$, further lets us split the second integral

$$= \int_{-\infty}^{y} (y-\hat{y}) f(x)\,dx + \int_{y}^{M} (\hat{y}-y) f(x)\,dx + \int_{M}^{\infty} (\hat{y}-y) f(x)\,dx$$

If median $< y$, lets us split first integral

$$= \int_{-\infty}^{M} (y-\hat{y}) f(x)\,dx - \int_{y}^{M} (y-\hat{y}) f(x)\,dx + \int_{y}^{M} (\hat{y}-y) f(x)\,dx + \int_{M}^{+\infty} (\hat{y}-y) f(x)\,dx$$

$$= \int_{-\infty}^{M} (y-\hat{y}) f(x)\,dx + 2\int_{y}^{M} (\hat{y}-y) f(x)\,dx + \int_{M}^{+\infty} (\hat{y}-y) f(x)\,dx$$

Need to ~~se~~ evaluate the term $E(|\hat{y}-M|)$

$$= \int_{-\infty}^{M} (y-M+M-\hat{y}) f(x)\,dx + 2\int_{y}^{M} (\hat{y}-y) f(x)\,dx + \int_{M}^{+\infty} (\hat{y}-M+M-y) f(x)\,dx$$

$$= \int_{-\infty}^{M} (y-M) f(x)\,dx + \int_{-\infty}^{M} (M-\hat{y}) f(x)\,dx + 2\int_{y}^{M} (\hat{y}-y) f(x)\,dx + \int_{M}^{+\infty} (\hat{y}-M) f(x)\,dx + \int_{M}^{+\infty} (M-y) f(x)\,dx$$

$$\int_{-\infty}^{M} (\hat{y}-M) f(x)\,dx + \int_{M}^{\infty} (M-\hat{y}) f(x)\,dx = E(|\hat{y}-M|)$$

$$= E(|\hat{y}-M|) + \int_{-\infty}^{M} (y-M) f(x)\,dx + 2\int_{y}^{M} (\hat{y}-y) f(x)\,dx + \int_{M}^{\infty} (M-y) f(x)\,dx$$

$$\int_{-\infty}^{M} f(x)\,dx = \int_{-\infty}^{M_{t+m}} f(y_{t+m} \mid I_T, \theta)\, dy_{T+H} = \tfrac{1}{2} \text{ from the question}$$

$$\therefore \int_{-\infty}^{M} (y-M) f(x)\,dx \Rightarrow (y-m)\tfrac{1}{2}$$

$$= E(|\hat{y}-M|) + 2\int_{y}^{M} (\hat{y}-y) f(x)\,dx + (y-m)\tfrac{1}{2} + (m-y)\tfrac{1}{2}$$

$$= \text{''} \quad \text{''} + \tfrac{1}{2}y - m\tfrac{1}{2} + \tfrac{1}{2} - y\tfrac{1}{2}$$

$$= 0$$

$$= E(|\hat{y}-M|) + 2\int_{y}^{M} (\hat{y}-y) f(x)\,dx$$

$\therefore$ The value which mimizes the above term is when the integral takes its smallest value.

$$\boxed{M=y}$$

Question 8) a) As it is vital ~~when~~ for each ~~a~~ attendee of the meeting to get a copy, and only a ~~fa~~ linear loss of ~~0.5~~ $0.05 x ~~fo~~ loss for each additional copy, the loss function is greater when less copies are brought and is therefore not ~~sy~~ symmetrical.