Appendix A

PROBABILITY DISTRIBUTIONS

This appendix gives the form of the pmf/pdf and summary statistics for common distributions, which are frequently used within statistical problems.

A.1 Discrete distributions

Distribution	Parameters	Mass function	Mean and variance
Binomial $\theta \sim Bin(n,p)$	sample size $n \in \mathbb{N}$ $p \in [0,1]$	$f(\theta) = \begin{pmatrix} n \\ \theta \end{pmatrix} p^\theta (1-p)^{n-\theta}$ $\theta = 0, 1, \ldots, n$	$\mathbb{E}(\theta) = np$ $Var(\theta) = np(1-p)$
Poisson $\theta \sim Poisson(\lambda)$	rate $\lambda > 0$	$f(\theta) = \lambda^\theta \exp(-\lambda)(\theta!)^{-1}$ $\theta = 0, 1, 2, \ldots$	$\mathbb{E}(\theta) = \lambda$ $Var(\theta) = \lambda$
Geometric $\theta \sim Geom(p)$	$p \in [0,1]$	$f(\theta) = p(1-p)^{\theta-1}$ $\theta = 0, 1, \ldots$	$\mathbb{E}(\theta) = 1/p$ $Var(\theta) = (1-p)/p^2$
Negative Binomial $\theta \sim Neg\text{-}Bin(\alpha,\beta)$	shape $\alpha > 0$ inverse scale $\beta > 0$	$f(\theta) = \begin{pmatrix} \theta+\alpha-1 \\ \alpha-1 \end{pmatrix} \left(\frac{\beta}{\beta+1}\right)^\alpha \left(\frac{1}{\beta+1}\right)^\theta$ $\theta = 0, 1, 2, \ldots$	$\mathbb{E}(\theta) = \alpha/\beta$ $Var(\theta) = \frac{\alpha}{\beta^2}(\beta+1)$
Multinomial $\boldsymbol{\theta} \sim MN(n, \boldsymbol{p})$	sample size $n \in \mathbb{N}$ $p_i \in [0,1]$; $\sum_{i=1}^k p_i = 1$	$f(\boldsymbol{\theta}) = \frac{n!}{\prod_{i=1}^k \theta_i!} \prod_{i=1}^k p_i^{\theta_i}$ $\theta_i = 0, 1, \ldots, n$; $\sum_{i=1}^k \theta_i = n$	$\mathbb{E}(\theta_j) = np_j$ $Var(\theta_i) = np_i(1-p_i)$

A.2 Continuous distributions

Distribution	Parameters	Density function	Mean and variance
Uniform $\theta \sim U[a,b]$	$b > a$	$f(\theta) = 1/(b-a)$ $\theta \in [a,b]$	$\mathbb{E}(\theta) = (a+b)/2$ $Var(\theta) = (b-a)^2/12$
Normal $\theta \sim N(\mu, \sigma^2)$	location μ scale $\sigma > 0$	$f(\theta) = \frac{\exp\left(-(\theta-\mu)^2/(2\sigma^2)\right)}{\sqrt{2\pi\sigma^2}}$ $\infty < \theta < \infty$	$\mathbb{E}(\theta) = \mu$ $Var(\theta) = \sigma^2$
log Normal $\theta \sim \log N(\mu, \sigma^2)$	μ $\sigma > 0$	$f(\theta) = \frac{\exp\left(-(\log\theta-\mu)^2/(2\sigma^2)\right)}{\sqrt{2\pi\sigma^2}\theta}$ $0 \leq \theta < \infty$	$\mathbb{E}(\theta) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$ $Var(\theta) = \exp(2\mu+\sigma^2)(\exp(\sigma^2)-1)$
Beta $\theta \sim Beta(\alpha, \beta)$	$\alpha > 0$ $\beta > 0$	$f(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}$ $\theta \in [0,1]$	$\mathbb{E}(\theta) = \frac{\alpha}{\alpha+\beta}$ $Var(\theta) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Exponential $\theta \sim Exp(\lambda)$	$\lambda > 0$	$f(\theta) = \lambda\exp(-\lambda\theta)$ $\theta > 0$	$\mathbb{E}(\theta) = 1/\lambda$ $Var(\theta) = 1/\lambda^2$
Gamma $\theta \sim \Gamma(\alpha, \beta)$	shape $\alpha > 0$ scale $\beta > 0$	$f(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}\exp(-\beta\theta)$ $\theta > 0$	$\mathbb{E}(\theta) = \alpha/\beta$ $Var(\theta) = \alpha/\beta^2$
Inverse Gamma $\theta \sim \Gamma^{-1}(\alpha, \beta)$	shape $\alpha > 0$ scale $\beta > 0$	$f(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{-(\alpha+1)}\exp(-\beta/\theta)$ $\theta > 0$	$\mathbb{E}(\theta) = \beta/(\alpha-1)$, for $\alpha > 1$ $Var(\theta) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$, $\alpha > 2$
Chi-squared $\theta \sim \chi^2_\nu$	df $\nu > 0$ (deg. of freedom)	$f(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)}\theta^{\frac{\nu}{2}-1}\exp(-\theta/2)$ $\theta > 0$ (same as $\Gamma\left(\alpha = \frac{\nu}{2}, \beta = \frac{1}{2}\right)$)	$\mathbb{E}(\theta) = \nu$ $Var(\theta) = 2\nu$
Inverse Chi-squared $\theta \sim \chi^{-2}_\nu$	df $\nu > 0$ (deg. of freedom)	$f(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)}\theta^{-\left(\frac{\nu}{2}+1\right)}\exp(-1/2\theta)$ $\theta > 0$ (same as $\Gamma^{-1}\left(\alpha = \frac{\nu}{2}, \beta = \frac{1}{2}\right)$)	$\mathbb{E}(\theta) = \frac{1}{\nu-2}$ $Var(\theta) = \frac{2}{(\nu-2)^2(\nu-4)}$
Dirichlet $\theta \sim Dir(\alpha_1, \ldots, \alpha_k)$	$\alpha_i > 0$; $\alpha_0 \equiv \sum_{i=1}^k \alpha_i$	$f(\theta) = \frac{\Gamma\left(\sum_{i=1}^k \alpha_i\right)}{\prod_{i=1}^k \Gamma(\alpha_i)}\prod_{i=1}^k \theta_i^{\alpha_i-1}$ $\theta_i > 0$; $\sum_{i=1}^k \theta_i = 1$	$\mathbb{E}(\theta_i) = \frac{\alpha_i}{\alpha_0}$ $Var(\theta_i) = \frac{\alpha_i(\alpha_0-\alpha_i)}{\alpha_0^2(\alpha_0+1)}$