Risk Analysis using ARIMA and (G)ARCH Models

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April 9, 2013

Risk Aversion Example

- Everyone gets to "invest" in one of two assets:
  1. A guaranteed $51,209
  2. A 50/50 chance of getting either $50,000 or $100,000.
- Raise hands and vote

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EPP History

- Seminal 1985 paper titled *The Equity Premium: A Puzzle* by Mehra and Prescott
- Discovered odd relationship between demand for risky and risk free assets
- Can be summarized with 1 of 2 questions:
  1. Why is demand for risk free government bonds so high when returns on those assets are so low?
  2. Why is demand for stocks, and hence stock prices, so low when returns are relatively high expected returns?

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Possible Explanations

- Some say it doesn't exist. The data says it does!
- Risk Aversion:
  - Stocks are risky assets so people are less willing to invest in them.
  - Very Incomplete → In 1991, Mankiw showed that investors would be indifferent between our assets from the beginning using standard risk aversion coefficients.
- Market Failures: things like transaction costs or liquidity constraints
- Volatility arguments: Risky assets are much more volatile - people don't like that.
- Tax policy

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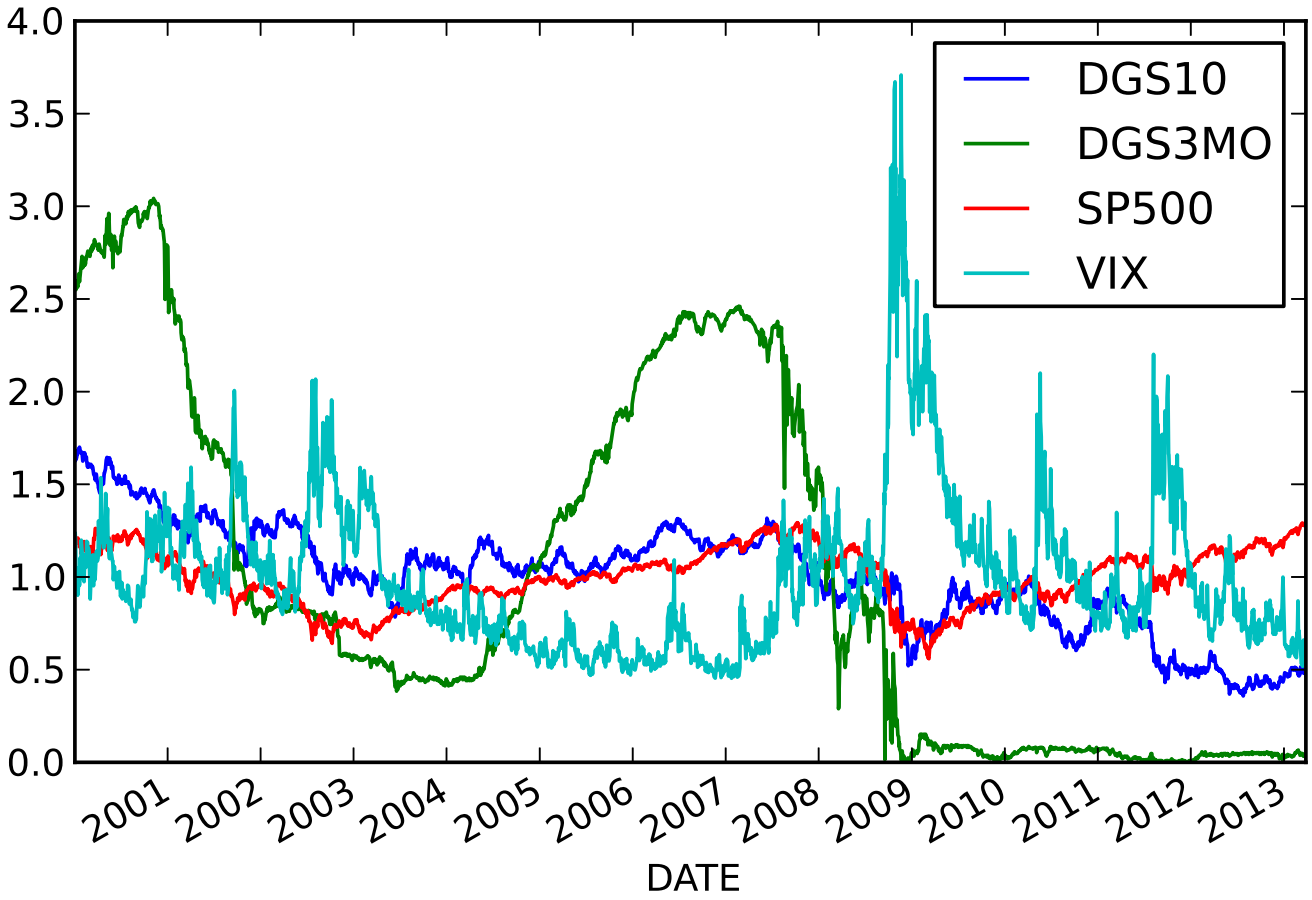
My Data

- Risky Assets:
  - Prices for SP500 (^GSPC)
  - Value of VIX (^VIX)
- Risk Free Assets:
  - 3-Month US T-bill (DGS3MO)
  - 10-Year US T-bill (DGS10)
- Frequency for all data is Business Day
- Collected from 1-1-1990 to 4-8-2013

Plot Data


4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
DATE
DGS10
DGS3MO
SP500
VIX

Plans

- My Question: Will standard, modern time series techniques forecast data in line with the EPP or against it?
- I will use ARIMA, GARCH, and ARCH models to generate forecasts for the data
- I will then see what the implied equity premium is on my forecasted data

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Setting up ARIMA

- Need to identify p, q, d for arima.
- For finding p and q see Figure on next slide (p=1, q=0 for all)
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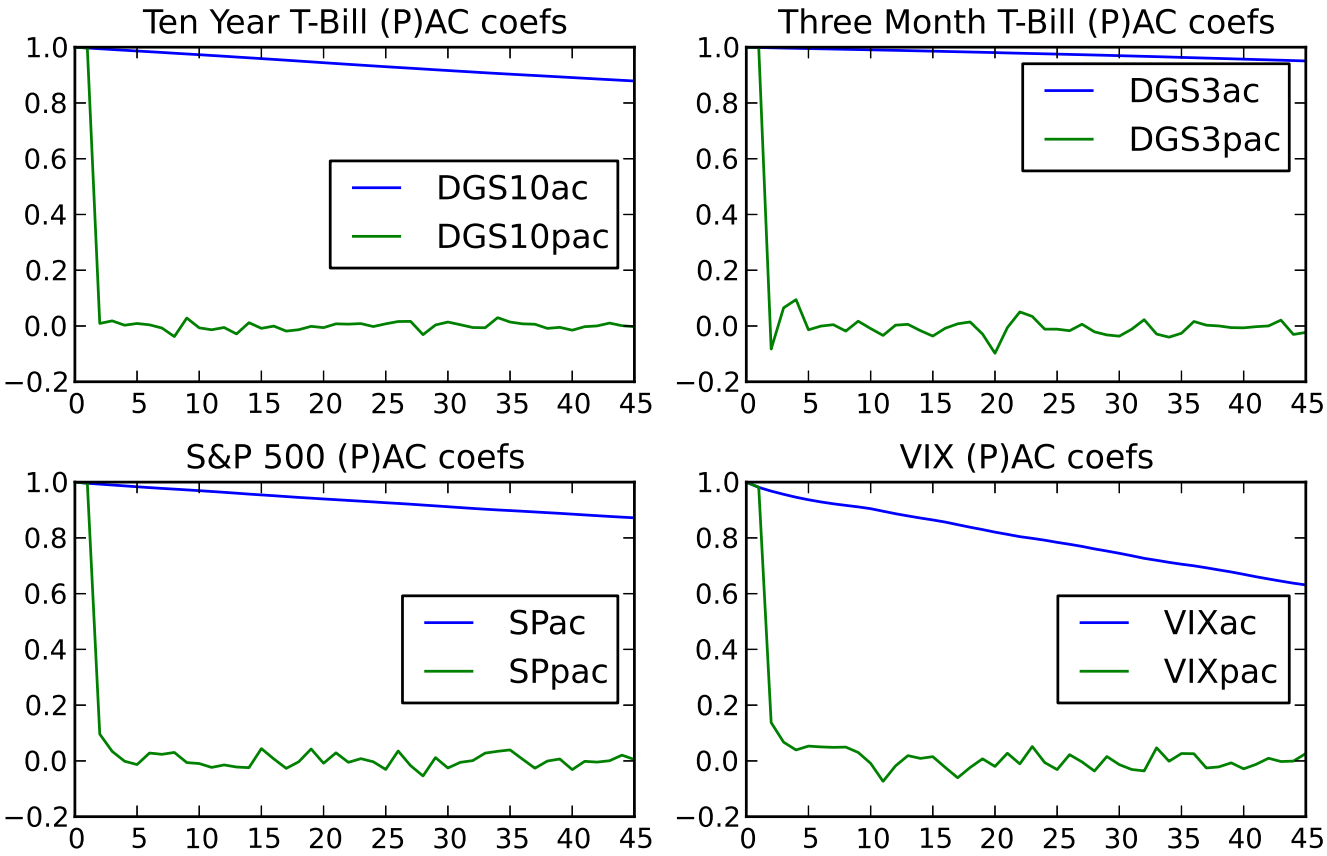
PAC and AC


Ten Year T-Bill (P)AC coefs
DGS10ac
DGS10pac
Three Month T-Bill (P)AC coefs
DGS3ac
DGS3pac
S&P 500 (P)AC coefs
SPac
SPpac
VIX (P)AC coefs
VIXac
VIXpac

Moments for Differences

DataSet	Lags	Mean	Variance	DataSet	Lags	Mean	Variance
DGS10	1	-0.001425545	0.003990852	SP500	1	0.03449455	224.2908
	2	-0.002824705	0.007977331		2	0.08398426	406.3557
	3	-0.00424894	0.01156277		3	0.1329679	574.528
	4	-0.005655862	0.01518609		4	0.1779158	748.1064
DGS3MO	1	-0.001637409	0.003366346	VIX	1	-0.003483656	3.13206
	2	-0.003254617	0.007774232		2	-0.007680896	5.355797
	3	-0.00488189	0.01147704		3	-0.01181405	7.261939
	4	-0.006498031	0.01407682		4	-0.01544986	9.018458

GARCH

I still need to do this