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Physics 321 HW 4 (Due 9-10-12)

Problems 4.3, 4.9, 4.14

4.3

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In[7]:= m1 = m2 = 5000; (* kg *)  
G = 6.67*^-11; (*N m^2kg^-2 *)
```

We know that $F = ma \rightarrow \frac{-m_1 m_2 G}{r^2} = m_1 a \rightarrow a = \frac{m G}{R^2}$.

The acceleration will be non-constant because it is dependent on the (changing) distance between the 2 spheres. The slowest acceleration occurs when the distance is a maximum, or it is equal to one. If we solve for that and then assume it is the constant velocity we will get a maximum on the time required for the spheres to come in contact.

```
In[13]:= a = (m1 * G) / 1^2 // N; Print["a = ", a, " m/s^2"]  
  
a = 3.335*10^-7 m/s^2
```

The constant acceleration equation for motion is $x = \frac{1}{2} a t^2 \rightarrow t = \sqrt{\frac{2x}{a}}$.

We will use this equation and note that the total distance the spheres need to move is $1 - 2(0.47) = 0.06$ m. This means that each sphere will cover 1/2 this distance or 0.03 m.

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In[16]:= t = Sqrt[(2 * 0.03) / a]; Print["Final time is that t = ", t, " seconds"]
```

Final time is that t = 424.158 seconds

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In[18]:= Print["If t < 425 true will be printed here: ", t < 425]
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If t < 425 true will be printed here: True

4.9

Equation (4.2) says that $F(z) = m \frac{dv}{dt} \rightarrow \frac{dv}{dt} = g$

Integrating this equation we get that $v = gt + C$. We know that the object was dropped from rest so at $t=0$ the velocity was also equal to 0. $v(0) = 0 = g(0) + C \rightarrow C = 0 \rightarrow v = gt$.

If we integrate this again, we see that $x = \frac{1}{2} g t^2 + C$. We now say that the ball's initial position at time $t=0$ before the ball was dropped was $x=0$. Using this we can solve for C: $x(0) = 0 = \frac{1}{2} g(0)^2 + C = 0 \rightarrow C = 0 \rightarrow x = \frac{1}{2} g t^2$.

Now we let h_1 be equal to the height from the top of the building to the top of the window. t_1 is the time required to travel this distance. We will use the equation we just derived to write expressions for the position and time of the ball when it reaches the top and bottom of the window.

$$h_1 = \frac{1}{2} g(t_1)^2 \quad [1]$$

$$h_1 + h = \frac{1}{2} g(t_1 + \tau)^2 = \frac{1}{2} g(t_1^2 + 2t_1\tau + \tau^2) \quad [2]$$

Subtracting equation [1] from equation [2] gives us: $h = \frac{1}{2} g(2t_1\tau + \tau^2)$.

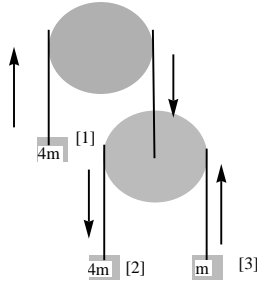
We will now solve this for t_1 and get this expression: $t_1 = \frac{h}{g\tau} - \frac{1}{2}\tau$.

Plugging this back into equation [1] we get: $h_1 = \frac{1}{2} g \left(\frac{h}{g\tau} - \frac{1}{2} \tau \right)^2 = \frac{1}{2} g \left(\frac{h^2}{g^2 \tau^2} - \frac{h}{g} + \frac{\tau^2}{4} \right)$

Simplifying we get: $h_1 = \frac{1}{2} g \left(4 \frac{h^2}{4 g^2 \tau^2} - \frac{4 h g \tau^2}{4 g^2 \tau^2} + \frac{g^2 \tau^4}{4 g^2 \tau^2} \right) = \frac{1}{2} g \left(\frac{4 h^2 - 4 h g \tau^2 + h^2 \tau^4}{4 g^2 \tau^2} \right) = \frac{1}{8 g \tau^2} (4 h^2 - 4 h g \tau^2 + g^2 \tau^4)$

Finally we get that $h_1 = \frac{1}{8 g \tau^2} (2 h - g \tau^2)^2$

4.14



The system is set up as in the figure above. The arrows indicate which direction the masses/2nd pulley system are moving.

We will define the first mass (weighing 4m moves upward) at velocity v_1 . It must be true that the second pulley moves down at this same velocity. We will then say that the second system of masses moves with a velocity, relative to the 2nd pulley, of v_2 . This makes mass [2] move at a velocity $v_1 + v_2$ relative to the first pulley and the mass [4] moves at a relative velocity of $v_1 - v_2$. We will also define tensions in the two strings to be T_1 for the upper pulley system and T_2 for the lower pulley system.

Using this info we can define three equations that describe the motion of the 3 blocks (all based on equation that $F = ma$):

$$4m \frac{dv_1}{dt} = T_1 - 4mg \quad (0.1)$$

$$4m \frac{d}{dt} (v_2 + v_1) = 4mg - T_2 \quad (0.2)$$

$$m \frac{d}{dt} (v_2 - v_1) = T_2 - mg \quad (0.3)$$

We will also need to define $a_1 = \frac{dv_1}{dt}$ and $a_2 = \frac{dv_2}{dt}$. We are still missing 1 equation because we have 4 unknowns, 2 accelerations and 2 tensions, but only three equations (the ones numbered above).

We do still have one more equation we can find, and that is the one defining the motion of the 2nd pulley. The only forces acting on are the two tensions. We know that the second tension acts twice as much as the first one and that because there is no mass, there is no net force, and therefore the two tensions must be equal to one another. This gives us our 4th equation :

$$T_1 = 2T_2 \quad (0.4)$$

We now make a lot of substitutions and see the following:

$$4m a_1 - T_1 = -4mg$$

$$4m(a_2 + a_1) + T_2 = 4mg$$

$$m(a_2 - a_1) - T_2 = -mg$$

$$T_1 - 2T_2 = 0$$

Transforming these 4 equations into a matrix we get:

$$\begin{pmatrix} 4m & 0 & -1 & 0 \\ 4m & 4m & 0 & 1 \\ -m & m & 0 & -1 \\ 0 & 0 & 1 & -2 \end{pmatrix} * \begin{pmatrix} a_1 \\ a_2 \\ T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} -4mg \\ 4mg \\ -mg \\ 0 \end{pmatrix}$$

Allowing mathematica to solve this system we get.

$$\begin{aligned} \text{In[34]:= } \begin{pmatrix} \mathbf{a1} \\ \mathbf{a2} \\ \mathbf{T1} \\ \mathbf{T2} \end{pmatrix} &= \text{Inverse} \left[\begin{pmatrix} 4m & 0 & -1 & 0 \\ 4m & 4m & 0 & 1 \\ -m & m & 0 & -1 \\ 0 & 0 & 1 & -2 \end{pmatrix} \right] \cdot \begin{pmatrix} -4mg \\ 4mg \\ -mg \\ 0 \end{pmatrix} \\ \text{Out[34]= } &\begin{pmatrix} -\frac{g}{9} \\ \frac{2g}{3} \\ \frac{32gm}{9} \\ \frac{16gm}{9} \end{pmatrix} \end{aligned}$$

Therefore we get that $a_1 = -\frac{1}{9}g$ and $a_2 = \frac{2}{3}g$.