Econ 581 Final

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December 8, 2012

1 The Households' Problem

1.1 Population Growth

Let $\lambda_{s,t}$ represent the size of the generation of workers of age s in period t. Without loss of generality I make the following normalization:

$$\lambda_{S,0} = 1$$

Looking at all other generations in period zero I get the following:

$$\begin{aligned} \lambda_{S-1,0} &= (1+n)\lambda_{S,0} = (1+n) \\ \lambda_{S-2,0} &= (1+n)\lambda_{S-1,0} = (1+n)^2 \\ &\dots \\ \lambda_{S-s,0} &= (1+n)^s \end{aligned} \tag{1}$$

A similar analysis holds for the S aged generation in different periods t:

$$\begin{aligned} \lambda_{S,1} &= \lambda_{S-1,0} = (1+n) \\ \lambda_{S,2} &= \lambda_{S-2,0} = (1+n)^2 \\ &\dots \\ \lambda_{S,t} &= (1+n)^t \end{aligned} \tag{2}$$

Combining these two results I can an expression for the size of any generation in any time period

$$\lambda_{S-s,t} = (1+n)^{s+t}$$

1.2 Objective Function

Each period, households make two decisions: (1) how much of their income to consume in the current period, (2) how much labor to supply. Consumption must be strictly positive and labor is non-negative. If an agent

decides to supply 0 labor in a period, they have effectively made the decision to retire and their labor in each subsequent period will also be 0.
The budget constraint a household of age s in period t faces has the form

$$c_{s,t} + k_{s+1,t+1} = w_t f_s l_{t,s}(1-\tau) + (1 + r_t - \delta) k_{s,t} \tag{3}$$

Where

- $c_{s,t}$ is the consumption of a household of age s in period t
- $k_{s+1,t+1}$ is the savings by an agent currently of age s for the next period $(t+1)$
- w_t is the wage in period t
- f_s is the productivity of an agent in the sth period of his life
- l_{wst} is the labor supplied by the agent
- τ is the tax rate on labor income
- r_t is the interest rate of capital
- $k_{s,t}$ is what the agent currently of age s saved last period $(t-1)$ to consume in period t

Because agents have productivity according to their age only, I will, without loss of generality, represent each generation with a single representative agent. This simplification allows me to conclude that in all periods t there are exactly S households involved in the economy. I make two more assumptions about households and their budget constraints: first that agents begin their lives with no capital $(k_{1,t} = 0 \forall t)$ and second that agents consume all their income in their last period of life $(k_{S,t} = 0 \forall t)$.
Agents have a utility function defined in terms of consumption and leisure.

$$u(c_{s,t}, 1 - l_{s,t}) = \frac{1}{1-\gamma}(c^{1-\gamma} - 1) + Bln(1 - l_{s,t}) \tag{4}$$

2 The Firm's Problem

Note about firms.
I believe that I need to include population growth in the production function. The way to do it is to say that $L_t = \sum_{s=1}^{S} \lambda_{s,t} \times l_{s,t}$

3 The Governments Behavior

The behavior of the government is just a simple identity. Each period they will collect $T = \sum_{s=1}^{S}$

4 Market Clearing Conditions

There are three market clearing conditions that must be met in equilibrium

- Capital market $K_t = \sum_{s=2}^{S} k_{s,t}$
- Labor market: $L_t = \sum_{s=1}^{S} l_{s,t}$

- Goods market: $Y_t = C_t + K_{t+1} - (1-\delta)K_t$

One of these conditions is redundant in equilibrium by Walras' law. Generally, the goods market conditions isn't directly used in solving the model.

5 Stationarized Equations

6 Dynare Equations

7 Variables and Parameterization

List of variables

- Endogenous state
  - $k_{s+1,t+1}\ \forall s, t$
  - $l_{s,t}\ \forall s, t$
- Exogenous State
  - z_t

Below is a table of parameters and potential values for them

Item		
Parameter	Description	Value
α	Capital share of income	0.35
δ	depreciation rate	$1-(1-0.05)^{60/S}$
β	discount factor	$0.96^{60/S}$
σ	standard deviation of shock process	$(\sum_{i=1}^{60/S} \rho^{2(\frac{60}{S}-i)}) - \sigma_{ann}$
μ	mean of shock process	0
ρ	persistence of shock process	$0.95^{4\frac{60}{S}}$
γ	coefficient of relative risk aversion	2
B	LOOK THIS UP	???
b	slope term in f_s	no standard value
c	intercept term in f_s	no standard value
n	growth rate of generations	0.01 (1%)

Table 7.1: Parameters in the model and potential values

8 Steady State