Endogenous Yield Curve and the Business Cycle

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English 316, Section 007
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Mrs. Pritchard,

I am submitting the attached technical report titled "The Endogenous Yield Curve and the Business Cycle". In this report I develop a theoretical model that endogenously prices risk free bonds and constructs a yield curve. I also interpret my results and provide some intuition regarding what they mean.

This report fulfills an assignment I have been given in English 316, Technical Writing. Furthermore, because it original, my research has the potential to be useful to many in the academic economics community. The main contributions this research makes to the field are the explaining, at least partially, the theoretical relationship between the yield curve and the business cycle and the ability to solve difficult non-linear systems with occasionally binding constraints.

In conclusion, the report will define a theoretical model and lay out a method for how the model can be solved. The findings presented in this model match what can be seen in real world data. This indicates that the methods presented are both accurate and meaningful.

Sincerely,

Spencer Lyon
Research Assistant, BYU Economics Department

Table of Contents

List of Figures and Tables....4
Abstract....5
Introduction....1
The Model....4
    The Households' Problem....4
    The Firm's Problem....7
    Market Clearing and Equilibrium....8
The Solution Method....9
    The GSSA Algorithm....10
        Setup....10
        Initialization....10
        Iteration....11
Pricing the Bonds....12
Results....13
    Policy Functions....13
    Risk Free Bonds and their Real World Counterpart....15
Conclusion....16
References....17
Appendices....18
    Euler Equation Derivation....18
    Policy Function Explanation....18
    Bond Pricing Theory....18

List of Figures

Illustration 1: The policy functions....14
Illustration 2: Correlation between yield curve and productivity....15
Illustration 3: Actual yield curve in the business cycle....15

List of Tables

Policy Functions....13

List of Equations

Budget Constraints....4
Objective Function....5
Euler Equations....6
Production Function....6
Firm's Objective....6
Firm's Solution....6
Market Clearing Conditions....7
Policy Function Notation....9
Policy Functions....9

Abstract

In this paper I develop a

Endogenous Yield Curve and the Business Cycle

Introduction

The overall movement of the economy affects everyone, whether they know it or not. This movement has been observed to have a fairly regular, repeating pattern. Much like a sinusoidal wave, the business cycle follows an up-and-down trend that repeats every few years. This pattern of fluctuations is what economists refer to as the business cycle. Despite being easily observed in historical data and having substantial implications for people, the business cycle is extremely hard to forecast.

One of the best approaches economists have found in their attempt to predict the business cycle is to find leading indicators of the economy. Leading indicators are economic variables that tend to move before the economy as a whole. In other words, if a leading indicator starts to decline, it is likely that the economy will soon decline also. There are many proposed leading indicators: interest rates, stock price indexes, and monetary aggregates (Estrella & Mishkin, 1996). There has been a general consensus in the literature, however, that the difference between returns on short-term and long-term United States treasury notes is the best leading indicator (Estrella & Trubin, 2006). These notes are backed by the US government and are very close to being risk free. The interest rate spread between the bonds (or notes), is known as the yield curve. Because of its predictive abilities, the yield curve has been the focus of extensive empirical analysis and study. Perhaps the most interesting fact regarding the study of the yield curve as a leading indicator is that there has yet to be a model that provides a strong theoretical foundation for why its relationship to the business cycle exists.

All of the models developed to date have various flaws. The first major flaw is that all of them are at least somewhat atheoretical (Gürkaynak & Wright, 2012). These atheoretical models have done a lot to show the statistical relationship between the yield curve and the business cycle, but they don't provide any motivating theory for the existence of the relationship. A second major flaw in prior models is that they are only accurate predictors of future GDP in times of economic stability. This means then whenever the economy is more turbulent, the models fail to accurately forecast what GDP will be the future. This is a big flaw because it is often in times of uncertainty that individuals and policy makers want to forecast the future path of the economy. A final drawback with the previous models is that the inclusion of theory has come in the form of imposing a relationship between the yield curve and the movement of the economy (Ang, Piazzesi, & Wei, 2006). This imposition eliminates the possibility for the model to uncover the theoretical relationship we are looking to find.

In this report I will present a theoretical model that doesn't suffer from any of the shortcomings mentioned in the previous paragraph. This model will be a dynamic stochastic general equilibrium (DSGE) model. It is dynamic in the sense that at each stage of the model there is interaction between the modeled parties and that interaction depends on past information as well as expectations regarding future events. It is stochastic because there are parts of the model that are random, or are dependent on random variables. Finally, it is a general equilibrium because in multiple markets there will be supply and demand coming together to produce a complete market where no goods are wasted.

The structure of the rest of this paper is as follows. First I will present the theoretical model I will be using to answer the question. This is divided into three sections: the households' problem, the firm's, and market clearing and equilibrium conditions. I will then explain the method by which the model will be solved. The next section will be dedicated to the theory behind the risk free bonds and how they fit into the model. I will then present the results of my calculations. Finally I will conclude with overall implications of this research and potential for future work to enhance what I have done here.

The Model

For my research I will be using the simplest three-period overlapping generations (OLG) model with aggregate productivity shocks. These OLG models are standard in modern economic modeling and have a few defining characteristics. First they allow for heterogeneity among the agents in the model. This is important because in the real world, not all people are alike so allowing for agents in my model to be different allows my to more accurately mimic reality. Second, the aggregate shocks provide a mechanism whereby the overall movement of the model can match the overall movement of the real economy. Specifically, the shocks in my model adjust the productivity of the workers and thereby affect total output in the economy. Third, my OLG model falls into a class of models known as general equilibrium (GE) models. GE models reach an equilibrium as consumers and producers coordinate to align wages and interest rates with the supply of labor and capital. As such, these models are generally introduced in three stages: the households' problem, the firm's problem, and equilibrium conditions. I will follow this convention in defining my OLG model.

The Households' Problem

As hinted at in the classification of the model, an OLG model features distinct agents of different ages or generations. In my model I assume that at all times there are three living agents: one young, one middle aged, and one old. This means that each agent lives for 3-time periods. The problem each agents faces is how to optimally allocate their income between spending and consumption to maximize the sum of their expected utility over the 3 periods of their life. Utility can be thought of as a mathematical tool that measures how happy someone is with particular decisions. In this model agents decide how much they will consume each period. I define a utility function that relates a particular level of consumption to a particular level of happiness.

In each period of life an agent must decide how much of their income to consume in that period and how much to save for a later date. We make two assumptions about this decision. First, we assume that the young agents start their life with no initial savings. In other words, agents do not inherit any wealth. Second, we assume that all agents die with zero savings, or that they consume the rest of their wealth when they are old. These assumptions allow us to define a distinct budget constraints, one for each aged individual.

$$\begin{aligned} c_{1,t} + k_{2,t+1} &= w_t l_1 \\ c_{2,t+1} + k_{3,t+1} &= w_{t+1} l_2 + (1 + r_{t+1} - \delta) k_{2,t+1} \\ c_{3,t+2} &= w_{t+2} l_3 + (1 + r_{t+2} - \delta) k_{3,t+2} \end{aligned} \tag{1}$$

In the set of equations (1) the variables are defined as follows.

- $c_{s,t}$ stands for consumption by an agent currently in period s of their life during period t
- $k_{s,t+1}$ represents the savings made by an individual currently of age s to be consumed tomorrow, or in period t+1
- w_t stands for the wage earned per unit of labor in period t
- l_s is how much labor is supplied by an agent currently in period s of their life
- r_t is the interest rate (returns) paid out in period t on capital that was saved in period t-1
- δ is a depreciation rate, or ~~. This is how~~ much less valuable capital is next period because it is one period older
- $k_{s,t}$ is the amount of capital savings an agent currently age s brings in to period t. This was saved in period t-1 to earn interest and be consumed in period t

In addition to these budget constraints, we need to define the objective function for the households. An objective function is simply a mathematical representation something an entity want to either minimize or maximize. The households in this model try to maximize their expected lifetime utility. We will use the notation $u(c_t)$ to represent the utility gained from a certain level of consumption in period t. We can now define the objective function:

$$\max_{\{c_{s,t+s-1}\}_{s=1}^{3},\{k_{s+1,t+s}\}_{s=1}^{2}} \quad u(c_{1,t}) + \beta E\Big[u(c_{2,t+1})\Big] + \beta^2 E\Big[u(c_{3,t+2})\Big] \tag{2}$$

In equation (2) β is a discount factor. A discount factor incorporates the idea that people are impatient. Generally, people care more about how much they can consume today, when compared with how much they will be able to consume at a later point in their life. Including β in our model allows us to capture this piece of human nature. The E in equation (2) is simply the expectation operator. This is necessary because at the start of someone's life, when they are in period 1, they do not know what the nature of the economy will be in the future. The expectation operator simply means that agents make consumption decisions using the best possible information and forecasting tools available to them. Finally, the max statement at the beginning of the equation simply means agent are maximizing by choosing how much to consume in periods 1, 2, and 3 of their life as well as deciding how much to save in periods 1 and 2.

Equations (1) and (2) fully characterize the households' problem. We can make one simplification to this problem before proceeding to define its solution. Although agents maximize with respect to c and k each period, there are really only two decisions to be made during their life. Taking a closer look at the budget constraints in equation (1) we see that each period the agents simply need to decide how to allocate their total income (the right hand side of the equation) between spending and saving (the left hand side). If we look at the problem in this way we see that if the agents can decide how much of their income to save in the first and second periods of their life, the choice of how much to consume is already made. They will simply consume the rest of their income in each period. We have now decomposed this problem to two choice variables. To find a solution to the problem we need two equations that determine appropriate values of those choice variables.

An appeal to calculus will give us the 2 equations we need in order to solve this problem. First, we insert the budget constraints (1) into the objective function (2). We then take partial derivatives with respect to our choice variables ($k_{2,t+1}$, $k_{3,t+1}$) and set the resultant expressions equal to zero. After some tedious algebra (see appendix A) we arrive at two Euler equations.

$$\begin{aligned} u'(c_{1,t}) &= \beta E\left[(1+r_{t+1}-\delta)u'(c_{2,t+1})\right] \\ u'(c_{2,t}) &= \beta E\left[(1+r_{t+1}-\delta)u'(c_{3,t+1})\right] \end{aligned} \quad , \tag{3}$$

where u' stands for the partial derivative of the utility function with respect to consumption.

The Firm's Problem

I now turn to the other half of the economy, the firm. This firm takes the capital saved by households as well as the labor they supply each period to produce the consumption good. These firms borrow the capital at a rate r_t and pay a wage of w_t. The inputs of capital (K) and labor (L) are combined produce goods (Y) according to a Cobb-Douglass production function with an aggregate productivity shock each period. This equation is below.

$$Y_T = e^{z_t} K_t^{\alpha} L_t^{1-\alpha} \quad , \tag{4}$$

where z_t is a shock to the workers productivity and follows the auto-regressive process $z_t = \rho z_{t-1} + (1-\rho)\mu + \epsilon_t$ with $\epsilon \sim N(0, \sigma^2)$, $\rho \in [0,1)$, and μ and σ^2 are the mean and variance of z, respectively.

The problem the firm faces is to maximize their profits by choosing the correct amounts of capital and labor to hire. Their objective function takes the following form

$$\max_{K_t, L_t} \; e^{z_t} K_t^{\alpha} L_t^{1-\alpha} - r_t K_t - w_t L_t \tag{5}$$

Taking partial derivatives with respect to K and L, we can solve for the interest rate and wage implied by firms choosing the optimal values of K and L:

$$\begin{aligned} w_t &= (1-\alpha)e^{z_t}\left(\frac{K_t}{L_t}\right)^{\alpha} \\ r_t &= \alpha e^{z_t}\left(\frac{L_t}{K_t}\right)^{1-\alpha} \end{aligned} \tag{6}$$

Market Clearing and Equilibrium

In order for economic activity to occur, both parties must feel they are better off as a result of entering into a transaction than they would be if they remained independent. What this boils down to is a set of market clearing and equilibrium conditions that, when met, ensure all parties optimize their objective functions subject to their various constraints. The market clearing conditions can be summarized as follows:

$$\begin{aligned} L_t &= l_1 + l_2 + l_3 = 1 \\ K_t &= k_{2,t} + k_{3,t} \\ Y_t &= C_t + K_{t+1} - (1-\delta)K_t \end{aligned} \tag{7}$$

The first condition simply states that the total labor used by the firm in the production process is the sum of the labor contribution from each agent. The second condition follows the same logic, but for the capital used in production. The third condition implies that all the goods that are produced must be consumed or saved by agents each period.

The equilibrium conditions are very similar. In order for an economic equilibrium to occur all agents and the firm must maximize their respective objective functions subject to their corresponding constraints. For the agents this means the young and middle aged people choose consumption and savings based on the Euler equations (3). For the firm, this means they effectively set the wage and interest rate according to their first order conditions using equations (6). These equilibrium conditions and the market clearing conditions in (7) fully characterize the solution to this model.

The Solution Method

Although the conditions governing the solution to the model are well-defined algebraically, it is extremely difficult to compute the solution. All mathematical systems or equations can be categorized as being either linear or non-linear. Mathematicians have, for quite some time, had success in defining and computing solutions to linear systems. Non-linear systems, however, often do not have representations that can be written down as a set of equations. In these situations, solutions to non-linear systems can only be approximated numerically. Most of these approximation methods are fairly restrictive in that they either impose constraints on the types of equations you can model, or the type of data that can be used to estimate the functions. I will be using a solution technique presented in 2011 that provides the flexibility I need to solve my non-linear system (Judd, Maliar, & Maliar, 2011).

There are a few technical matters I need to present before explaining the solution method. The first is the idea of a state vector. In an economic model, certain variables are fundamental to characterizing the model in any given time period. Other variables or parameters can be derived if the values of the fundamental variables are known. These fundamental variables are called state variables and putting all of them into one object results in the state vector. The term state vector is appropriate because the state vector contains all the necessary information required to completely characterize the state of the economy. It turns out that for my model the state vector has three elements in it: the savings made by the young each period ($k_{2,t}$), the savings made by the middle aged each period ($k_{3,t}$), and the productivity shock in the production function each period (z_t).

The second technical definition I need to provide before moving into the solution algorithm is the idea of a policy function. Taking a closer look at the state vector we see that of the 3 elements in the vector, only 2 of them are decided by agents in the model: $k_{2,t}$ and $k_{3,t}$. z_t is a random variable and is not chosen by households or the firm. That being said, if I can define how much young and middle aged agents want to save for tomorrow as a function of the state vector today, I will have a solution to the model. Specifically I am searching for policy functions, or functions that define the optimal savings policy for the choice variables ($k_{2,t+1}$ and $k_{3,t+1}$) in terms of the current state vector. Going forward I will use the following notation when talking about policy functions.

$$\begin{aligned} k_{2,t+1} &= \psi_2(k_{2,t}, k_{3,t}, z_t) \\ k_{3,t+1} &= \psi_3(k_{2,t}, k_{3,t}, z_t) \end{aligned} \tag{8}$$

THE GSSA ALGORITHM

The process by which I will find the policy functions in (8) is called the generalized stochastic simulation algorithm (GSSA). This algorithm has three main stages: (1) setup, (2) initialization, (3) iteration.

Setup

The setup part of GSSA really consists of a single step: choosing a functional form for the policy functions. While apparently simple, this step is actually quite difficult and extremely important. After a lot of trial and error I decided on the following functional form for the policy function.

$$\begin{aligned} \psi_2(k_{2,t}, k_{3,t}, z_t) &= (\frac{u_{2,t} - l_{2,t}}{\pi}) \left[\arctan(P_{2,n}(k_{2,t}, k_{3,t}, z_t)\Gamma_{2,t}) + \frac{\pi}{2}\right] + l_{2,t} \\ \psi_3(k_{2,t}, k_{3,t}, z_t) &= (\frac{u_{3,t} - l_{3,t}}{\pi}) \left[\arctan(P_{3,n}(k_{2,t}, k_{3,t}, z_t)\Gamma_{3,t}) + \frac{\pi}{2}\right] + l_{3,t} \end{aligned}, \tag{9}$$

where $P_{s,n}(k_{2,t}, k_{3,t}, z_t)$ is a standard *n*th polynomial expansion about the state for the agent currently in period *s* of his life. For example if $n = 1$, then the form of P would be $P_{s,1} = a_1 + a_2 k_{2,t} + a_3 k3, t + a_4 z_t$, where a_1, a_2, a_3, and a_4 are constants to be estimated. For a more detailed description of each term in (9) and why it was included in the policy function see appendix B.

Initialization

The initialization stage is quite simple and consists of 4 self-explanatory steps. This stage, as well as the setup is performed only once within the GSSA algorithm.

1. Choose initial policy function coefficients $a_1, a_2, a_3, a_4, \ldots$ for both policy functions.

2. Choose initial values for the state $k_{2,0}, k_{3,0}, z_0$

3. Choose a simulation length T. This is how many periods I will simulate the model each iteration.

4. Draw productivity shocks ϵ_t for all T periods and use them to compute z_t for $t = 1 \rightarrow T$.

Iteration

The iteration stage is the heart of the GSSA algorithm and is by far the most complicated part. For a deeper explanation of this stage of the algorithm I point the interested reader to the original paper by Judd, Maliar, & Maliar (Judd, et. al, 2011). Unlike the setup and initialization phases, the steps in the iteration stage are repeated multiple times. Each iteration consists of 7 steps.

1. Use the policy functions and coefficients from the previous iteration to generate $k_{2,t}$ and $k_{3,t}$ for all T periods.

2. Use the state data to compute data for all other variables in the model ($c_{1,t}, c_{2,t}, c_{3,t}, w_t,$ ect.).

3. Use the fixed point representation[1] of the Euler equations (3) to generate a second set of data for $k_{2,t}, k_{3,t}$. Call this $k'_{2,t}, k'_{3,t}$.

4. Find coefficients $\boldsymbol{a'} = \{a_1, a_2, a_3, a_4\}, \ldots$ that minimize ν in the equations

   $k'_{2,t+1} = \psi_2(k_{2,t}, k_{3,t}, z_t | \boldsymbol{a'}_2) + \nu_2$ and $k'_{3,t+1} = \psi_3(k_{2,t}, k_{3,t}, z_t | \boldsymbol{a'}_3) + \nu_3$.

5. Use a convex combination to update the coefficients for the next iteration:

1 The fixed point representation is not too difficult. By way of example we will use the first Euler equation $u'(c_{1,t}) = \beta E[u'(c_{2,t+1})(1 + r_{t+1} - \delta)]$ We will first divide both sides by $u'(c_{1,t})$ to get $1 = \frac{\beta E[u'(c_{2,t+1})(1+r_{t+1}-\delta)]}{u'(c_{1,t})}$. We will then multiply both sides by $k_{2,t+1}$ and get $k'_{2,t+1} = \frac{\beta E[u'(c_{2,t+1})(1+r_{t+1}-\delta)k_{2,t+1}]}{u'(c_{1,t})}$. On the right hand side we have an expectation operator so we will have to take into account all possible values of each of the variables inside the E and the probabilities associated with those values. Doing these operations will yield a $k'_{2,t+1}$ that is slightly different than $k_{2,t+1}$, but closer to where the Euler equations say savings should be.

$\boldsymbol{a}^{(i+1)} = \xi \boldsymbol{a'}^{i} + (1-\xi)\boldsymbol{a}^{i}$. In this equation i stands for iteration i and $i+1$ stands for the next iteration. This step is important because it allows the coefficients to slowly converge to their true value. Changing coefficients too quickly could result in instability.

6. Compute the average percentage change between the data generated in step 1 on this iteration and the data from the previous iteration.

7. If the average percentage change is larger than some pre-specified convergence criteria, continue. If it is smaller you can stop iterations because you have converged on the true solution.

Pricing the Bonds

After applying the GSSA algorithm to my model I had policy functions that give savings decisions for any possible state of the economy and was ready to price the riskless bonds. These bonds are supplied by large entity external to my model (i.e. the government) that cannot default on payments and supplies whatever quantity of bonds is demanded by the households. Because the supplying entity cannot default, the bonds are risk-free. Economic theory suggests that the net supply of any risk free asset must be non-negative. I also make the assumption that the bonds are held in zero-net supply. This assumption simply means that the sum of all bond holdings must be zero. This assumption with the non-negativity constraint, imply that the total holdings of all bonds are exactly zero. This greatly simplifies the computation because zero-net supply assets do not enter into the equilibrium and therefore can be priced after a solution to the model is found.

The actual theory behind how the bonds are priced is somewhat complicated and is left for the interested reader to pursue in appendix C.

Results

The end product of this theory and computation is a set of two tools that can be used to analyze the relationship between the yield curve and the business cycle. The first of these tools is a set of two policy functions that can tell what households will save given any state of the economy. These policy functions can be used to simulate the economy and produce data for all variables defined in the model. The second tool is a series of asset pricing functions that can return the price for bonds of different maturities given any state of the economy. In this section, I will examine what these policy functions look like, what prices are predicted for the risk free bonds, and how the predictions of my model match real world data.

Policy Functions

I chose to use a second order polynomial expansion inside each of the two policy functions. This means I needed a total of ten coefficients, one for each of the following: an intercept term, $k_2, k_3, z, k_2^2, k_3^2, z^2, k_2k_3, k_2z$, and k_3z. The coefficients for both of the policy functions are presented in **Table 1** below.

	Intercept	k_2	k_3	z	k_2^2	k_3^2	z^2	k_2k_3	k_2z	k_3z
ψ_2	-4.79	29.21	10.18	0.99	-75.10	-13.44	-0.12	-71.93	-0.07	-1.57
ψ_3	-1.26	14.65	3.72	0.02	-48.42	088	-0.19	-26.28	-1.37	3.57

Table 1: Policy function coefficients

I used these coefficient values to sample what the policy functions looked like for different state vectors. Because I have 3 state variables, I can't show a plot that has all 3 state variables, plus the output for the policy functions. For this reason I decided to set z equal to its average value (0) and plot the policy functions against the other two state variables. These plots appear in **Illustration 1** below.

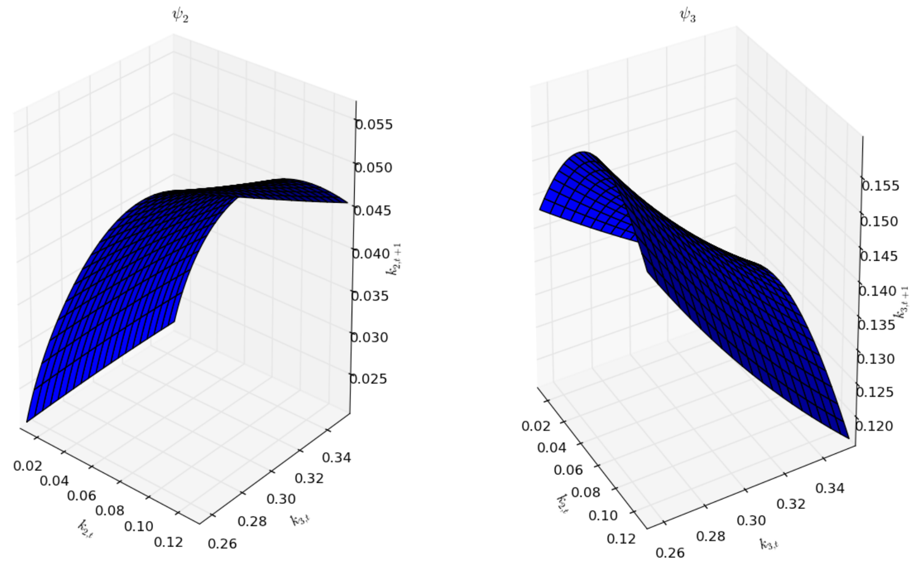

Illustration 1: The policy functions

These figures have a very intuitive interpretation. You can see that for ψ_2, the function is increasing in $k_{2,t}$ and decreasing in $k_{3,t}$. That this function is increasing in $k_{2,t}$ makes sense because if the young people last period had high savings, you would expect the young people today to have high savings. This is true because incentives from one period to the next remain fairly constant. The fact that the function is decreasing in $k_{3,t}$ also makes sense. Firms will demand a certain amount of total capital. If the middle aged people are saving a lot in one period, the young people have an incentive to save less. The same analysis is applicable to the ψ_3 policy function.

Risk Free Bonds and their Real World Counterpart

The results from the bond pricing were also encouraging. Remember that the spread between long term and short term bonds, the yield curve, should decrease just before a recession and start to increase through the middle and end of the recession. This means that the correlation between the yield curve and the productivity of the economy should be negative. For the data produced with my model the correlation coefficient between these two variables was -0.953. This relationship can be seen very clearly in **Illustration 2**.

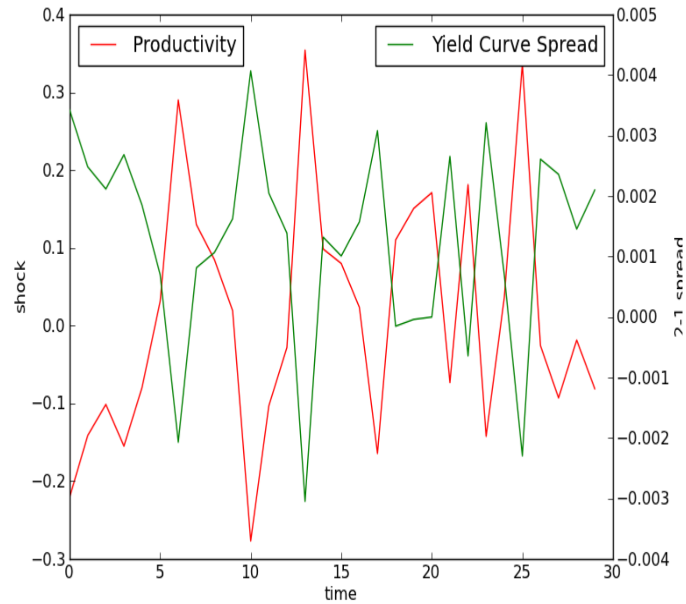

Illustration 2: Correlation between yield curve and productivity.

The veracity of the results mentioned above can also be seen by looking at a real data. **Illustration 3.** In this figure the actual yield curve is plotted from 1967 to 2012. To help the reader see where the economy starts to decline, all time periods officially recognized as economic recessions are shaded in grey. Key features of this plot are that in the years preceding each recession, the yield curve declines, and during recessions it starts to climb again. This is reminiscent of **Illustration 2**.

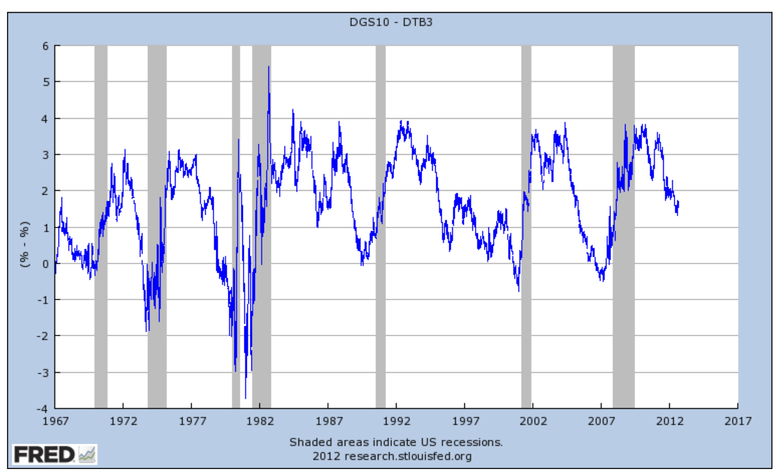

Illustration 3: Actual yield curve in the business cycle.

Conclusion

In summary, in this report I have presented a model, its solution, and an analysis of the results that fills a void in the yield curve literature. The model is able to endogenously price assets and generate a yield curve with characteristics similar to the real yield curve. As has been manifest in the federal reserves policy response to the most recent recession, interest rates play a powerful and active role in monetary policy. My model could be used directly by policy makers to test potential policies and see what effects they might have on the overall economy. Persons in position of power could take that analysis further and add their own enhancement to this model and use the solution method that was developed by Judd, Maliar, and Maliar and was applied in this paper.

Although this research has allowed us made great strides towards understanding the theoretical foundation for the correlation between the yield curve and the business cycle, there is still more that can be done. One major improvement I am currently working on is to extend this model so that households live for more than just 3 periods. This model tries to capture the behavior of adults that actively participate in labor and consumption markets. It is reasonable to suggest that the age horizon we are looking at is between 20 and 80 years old. The current model allows agents to live for 3 periods, which would make each period 20 years long. The bond pricing algorithm is limited by how many years per period the model captures. Looking at the spread between 1 period and 2 period bonds is a lot like looking at the spread between a 20-year and a 40-year bond. This will not give a very accurate approximation of the yield curve, which is usually defined as the spread between a 3-month bond and a 10-year bond. The current focus of my work is to solve a 60 period OLG model. This will allow me to look at bonds that are separated in maturity by only 1-year and approximate the actual yield curve with more accuracy.

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Appendices

Euler Equation Derivation

Derivation of the Euler equations.

Policy Function Explanation

Explanation of policy function terms.

Bond Pricing Theory

Bond pricing theory