Physics 441

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Contents

1 Course Info

The slides are available online, but they are password protected. The password is m@xwell.

TA Help sessions will be Thursday at Noon in N337

The Class website has good info.

The homework schedule is here.

2 Unit 1

2.1 Introduction

Dr. Berrando likes to use Clifford Algebras to solve these problems. Our book doesn't so in order for us to use them we need to be in class. He thinks they make this class easier, but did say that people either hate them or love them.

We will be studying Electricity and Magnetism as a single field. Maxwell has us think about vector fields and sources. His equations all take the form $\nabla \cdots = \ldots$, where the dots on the rhs stands for $\cdot$ or $\times$ some field. The dots on the right stand for a source. In this class they will all be static (time independent).

As an example of these principles and what things look like in a Clifford Algebra we would write:

$$\nabla \mathcal{F} = \tilde{J}$$

Clifford Algebras make solving this for $\mathcal{F}$ very easy:

$$\mathcal{F} = \nabla^{-1} J$$

2.2 Tools

Table 1: The rows of this table don't align. A table was just a compact way to show the data

Math	Physics
trigonometry	Trajectories r(t)
vectors: dot, cross, Clifford	Fields (scalar-vector, static-dynamic)
vector derivative operators (∇)	Sources (charge, current)
Dirac Delta function	Superposition of soureces
Discrete to continuum	Superposition of fields
integral theorems (stokes, gauss – inside cover)	unit point sources
cylindrical and spherical coords	maxwell's Equaitons
linearity	field lines
	charge conversation
	potentials

2.2.1 Math Review

- Sum of vectors
- dilation (multiplication by scalar)
- Linear combinations (put previous two points together)
- Scalar (dot) Product: $\boldsymbol{A} \cdot \boldsymbol{B} = ABcos(\theta)$, where $A = \sqrt{\boldsymbol{A} \cdot \boldsymbol{A}}$
- Cross product: $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{n}AB|sin(\theta)|$
- Orthonormalbasis: $\{e_1, e_2, e_3\} = \{i, j, k\}$
- Triple dot (scalar) product: One cross and a dot. It is cyclically constant. i.e. $A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B)$. It gives you the volume of the parallelipiped defined by the three vectors.
- The triple vector product has two crosses. It is non-associative. Rule: $A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$
- Rotation of a vector in 3-d. A unit vector (n) defines the rotation axis and ϕ defines the rotation angle. We can express this as \$r' $=$ e$^{\phi n \times}$ r $=$ e$^{\phi n \times}$ (r$_{\|}$ $+$ r$_{\perp}$) $=$ r$_{\|}$ $+$ e$^{\phi n \times}$ r$_{\perp}$ $=$ r$_{\|}$ $+$ cos(ϕ) r$_{\perp}$ $+$ sin(ϕ) n $\times$ r\$
  - Example: Rotate vector $e_1 + e_2$ by 45 degrees. Here $\phi = 45$, $r = (e_1 + e_2)$, $n = e_3$. Plugging it in we get $r' = (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} e_3 \times)(e_1 + e_2)$
  - Algorithm for finding rotation
    1. Make sure $\hat{n}$ is a unit vector
    2. Find $r_{\|} = (\hat{n} \cdot r)\hat{n}$
    3. Find $r_{\perp} = r - r_{\|}$
    4. Put it together: $r' = r_{\|} + (cos\phi)r_{\perp} + (sin\phi)\hat{n} \times r$

Example of something he spent a long time on (note we do a TS expansion of exponential):

$$e^{\alpha \frac{\partial}{\partial x}} f(x) = (1 + \alpha \frac{\partial}{\partial x} + \frac{\alpha^2}{2!} \frac{\partial^2}{\partial x^2} + \dots) f(x) = f(x) \alpha f'(x) + \frac{\alpha^2}{2!} f''(x) = f(x + \alpha)$$

2.2.2 Clifford Algebra Cl_3

We will define multiplication in this space as

$$AB = A \cdot B + iA \times B$$

, with $i = e_1e_2e_3$

There are 8 basis elements in a Clifford Space:

- In $\mathbb{R} : 1$
- In $i\mathbb{R} : i$
- In $\mathbb{R}^3 : e_1, e_2, e_3$
- In $i\mathbb{R}^3 : ie_1, ie_2, ie_3$

These relationships can be summarized in the following table:

grade	domain	basis elements	vector types (geometry)	who
0	$\mathbb{R}$	1	scalars	German
1	$\mathbb{R}^2$	$\boldsymbol{\hat{e_1}}$, $\boldsymbol{\hat{e_2}}$, $\boldsymbol{\hat{e_3}}$	vectors	French
2	$i\mathbb{R}^3$	$\boldsymbol{i\hat{e_1} = \hat{e_2}\hat{e_3}}$, $\boldsymbol{i\hat{e_2} = \hat{e_3}\hat{e_1}}$, $\boldsymbol{i\hat{e_3} = \hat{e_1}\hat{e_2}}$	bivectors	British
3	$i\mathbb{R}$	$i = \boldsymbol{\hat{e_1}\hat{e_2}\hat{e_3}}$	trivector (pseudo scalar)	Japanese

We can decompose the matrix product of a clifford algebra into a symmetric part and an anti-symmetric part. In other words

$$AB = (AB)_{sym} + (AB)_{non-sym} = A \cdot B + A \wedge B$$

- Sub-algebras
  Sub-algebras are simply subsets of an algebra where all objects are closed under multiplication and addition. For the clifford algebra, taking scalars and bivectors we end up with the even sub algebra (made up of grades 0 and 2).

- More facts
  The bivectors define oriented surfaces (oriented because $e_1e_2 = -(e_2e_1)$ – direction matters).

  The tri-vectors define an oriented volume.

2.2.3 Trig

We will need to know certain trig identities. Among them are the following (Note that bold letters are vectors, lower case letters are magnitude of vectors and capital letters are angles pointing to legs):

- Law of cosines: $\boldsymbol{c} = \boldsymbol{a} - \boldsymbol{b}$ and $c^2 = \mathrm{a}^2 + \mathrm{b}^2$ - 2 $\boldsymbol{a} \cdot \boldsymbol{b} Law of sines : \boldsymbol{c} + \boldsymbol{a} + \boldsymbol{b} = 0 \rightarrow |\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a} \times \boldsymbol{c}|$. We also know that $absin(C) = acsin(B) \rightarrow \frac{sin(B)}{b} = \frac{sin(C)}{c}$

2.2.4 Differential Calculus

I know all this stuff

2.3 Basic E&M

2.3.1 Summary

$$\nabla^{-}1 = \frac{\nabla}{\nabla^2}$$

$$\frac{1}{\nabla} = g*$$

$$g = \frac{1}{4\pi r} \text{ Solution to } \nabla^2 g = \delta(r)$$

$$\int_V \nabla \cdot \boldsymbol{E} d\tau = \oint_{dV} \boldsymbol{E} \cdot d\boldsymbol{a}$$

$$\int_S (\nabla \cdot \boldsymbol{B}) d\boldsymbol{a} = \oint_S \boldsymbol{B} \cdot d\boldsymbol{l}$$

$$\nabla \boldsymbol{E} = \frac{1}{\varepsilon_0}\rho \quad \rho \text{ is charge density}$$

2.4 Midterm

- Need to take it on Tuesday the 28th.
- There will be 4 problems like homework difficulty
- It is open book and open notes, open old homeworks
  - We will not be allowed to have internet or solutions manual
- We will go over it in class after it is graded
- Chapters 1-2 and part of 3.