A 2nd Real Business Cycle Model

Major Features of the Model

Add a labor-leisure decision with continuous hours to model 1
One source of uncertainty: z
Stochastic technology growth about a deterministic trend

We first define the important terms that will be used throughout:
Endogenous variables that change over time:

z	productivity (temporary or permanent)
K	capital stock owned by household
H	labor supplied by household
C	consumption
w	wage rate
r	interest rate
Y	output of final goods

Parameters:

α	capital share in output from a Cobb-Douglas production function
δ	rate of depreciation
β	time discount factor; $\beta<1$
a	trend in z
γ	elasticity of substitution, $\gamma>0$
D	leisure weight in utility
ρ	autocorrelation parameter for z; $0<\rho<1$
σ	standard deviations of the shocks to z; $0<\sigma$

Nonstationary Model

Given information on prices and shocks, $\mathbf{\Omega}=\{w,r,z\}$, the household solves the following non-linear program when the factor markets clear.

$$V(K,\mathbf{\Omega})=\underset{K',H}{Max}\, u(C,1-H)+\beta E\{V(K',\mathbf{\Omega}')\}$$

where:

$$C=wH+(1-\delta+r)K-K' \tag{1.1}$$

The first-order conditions are:

$$u_c(C,1-H)(-1)+\beta E\{V_K^i(K',\Omega')\}=0$$

$$u_c(C,1-H)w-u_h(C,1-H)=0$$

The envelope condition from this problem is as follows.

$$V_K(K,\Omega)=u_c(C,1-H)(1-\delta+r)$$

The Euler equations are:

$$u_c(C,1-H)=\beta E\{u_c(C',1-H')(1-\delta+r')\} \tag{1.2}$$

$$u_c(C,1-H)w=u_h(C,1-H) \tag{1.3}$$

Picking functional form of $u(C,L)=\frac{1}{1-\gamma}(C^{1-\gamma}-1)+D\frac{1}{1-\gamma}[(e^{at}L)^{1-\gamma}-1]$

$u_c(C,L)=C^{-\gamma}$ & $u_h(C,L)=BL^{-\gamma}$

Rewriting (1.2) & (1.3)

$$1=\beta E\left\{\left(\frac{C}{C'}\right)^{\gamma}(1-\delta+r')\right\} \qquad (1.2')$$

$$C^{-\gamma}w=De^{at}(1-H)^{-\gamma} \qquad (1.3')$$

Additional Behavioral Equations

The law of motion for *z* is:

$z'=\rho z+\varepsilon'$; where ε' is distributed normal with a mean of 0 and a variance of σ^2 (1.4)

An assumption of competition in the goods market along with a Cobb-Douglas production function gives the following shares in output for labor & capital. Market clearing conditions have already been imposed.

$$Y=K^{\alpha}(e^{at+z}H)^{1-\alpha} \qquad (1.5)$$

$$wH=(1-\alpha)Y \qquad (1.6)$$

$$rK=\alpha Y \qquad (1.7)$$

Definitions for Later Use

$$I\equiv K'-(1-\delta)K \qquad (1.8)$$

$$A\equiv e^{at+z} \qquad (1.9)$$

Eqs (1.1)-(1.9) are the system.

Transformation & Simplifications

If z is stationary ($\rho < 1$):

Transform the problem by dividing all growing variables by $A \equiv e^{at}$, denoting with a carat.

$z' = \rho z + \varepsilon'$; where ε' is distributed normal with a mean of 0 and a variance of σ^2 (2.1)

$$\hat{C} = \hat{w}H + (1-\delta+r)\hat{K} - (1+a)\hat{K}' \tag{2.2}$$

$$1 = \beta E\left\{\left(\frac{\hat{C}}{(1+a)\hat{C}'}\right)^{\gamma}(1-\delta+r')\right\} \tag{2.3}$$

$$\hat{C}^{-\gamma}\hat{w} = D(1-H)^{-\gamma} \tag{2.4}$$

$$\hat{Y} = \hat{K}^{\alpha}(e^{z}H)^{1-\alpha} \tag{2.5}$$

$$\hat{w}H = (1-\alpha)\hat{Y} \tag{2.6}$$

$$r\hat{K} = \alpha\hat{Y} \tag{2.7}$$

Variables are

These are the equations we will use in Dynare.

The endogenous variables are $\hat{C}, \hat{K}, H, \hat{Y}, \hat{w}, r, \hat{I}, \hat{A}$ & z .

The exogenous variable is ε.

The parameters are $\alpha, \delta, \beta, a, \gamma, \rho, \sigma$ & D .