Brigham Young University Department of Economics

Economics 581 – Advanced Macroeconomics

Homework #1
Due 9/5

Solow Growth Model

Simulate the stochastic version of the Solow growth model described in section 1.4 of the McCandless text. However, modify the law of motion for technology to the following:

$$A_t = \bar{A}e^{g^t + a_t};\;\; a_t = \rho a_{t-1} + \varepsilon_t,\;\; \varepsilon_t \sim N(0, \omega^2)$$

Also, impose labor-augmenting technology:

$$Y_t = K_t{}^{\alpha}(A_t H_t)^{1-\alpha}$$

Use the following parameters:

$\bar{A}$=1, g =.025, n=.01, δ=.1, θ=.33, σ=.1, ρ=.9 & ω^2=.0004.

Run a series of 1000 Monte Carlos with a sample size of 120 observations each. For each Monte Carlo:

1) generate a series of random normally distributed shocks, ε_t.
2) calculate an exact time series for k_t using the appropriate version of equation (1.2).
3) calculate an approximate time series for $\tilde{k}_t$, using the appropriate version equation (1.4).
4) using the exact series for k_t, calculate exact time series for $\{y_t, c_t, i_t, A_t\}$
5) using the approximate series for $\tilde{k}_t$, calculate an approximate series for k_t, and then use this to recalculate approximate series for $\{y_t, c_t, i_t\}$.
6) for both sets (exact & approximate) calculate the following sample moments for each time series, $\{y_t, c_t, i_t, k_t\}$, in each Monte Carlo: 1) mean, 2) standard deviation, 3) correlation with y_t, 4) correlation with A_t, 5) autocorrelation. Calculate the averages of each of these moments over the 1000 Monte Carlos.

Repeat the analysis above for the following sets of alternative parameters:

$\bar{A}$=1, g =.025, n=.01, δ=.1, θ=.33, σ=.1, ρ=0 & ω^2=.0004.
$\bar{A}$=1, g =.025, n=.01, δ=.1, θ=.33, σ=.2, ρ=.9 & ω^2=.0004.

Submit the following in hard copy for grading:

1) The MATLAB code you used to generate the moments.
2) Three tables reporting the 40 moments in question for each set of parameters. As follows:

variable	mean	standard deviation	correlation –w– y_t	correlation –w– A_t	autocorrelation
exact					
y_t					
c_t					
i_t					
k_t					
approximate					
y_t					
c_t					
i_t					
k_t					

Brigham Young University Department of Economics

Economics 581 – Advanced Macroeconomics

Homework #2
Due 9/12

Ramsey-Cass-Koopmans Model

Problem Set 1
Using the continuous version of the Ramsey-Cass-Koopmans model, consider the transition from one steady state to another, when a change is anticipated in advance. Use the phase diagram to explain your answer. In particular, suppose we start off in one steady state and then a parameter of the model is scheduled to change at some future point in time, S. Assume we have drop in g, so that technology grows more slowly. Use the phase diagram to explain the time paths of k & c from the old steady state to the new one. Pay particular attention to what happens immediately and what happens at date S.

Problem Set 2
Simulate the non-stochastic version of the discrete-time Ramsey-Cass-Koopmans model as explained in the handout.

Use the following parameters:
g =.025, n=.01, δ=.1, θ=.33, γ=1.0, ρ=.05

Solve for the steady state values of k, y, c, w & r.

Starting at an initial value of k that is 30% below the steady state value for k, use the iteration method discussed in the handout to simulate the transition of the economy from this initial state to the steady state. Assume that the steady state arrives in $T = 50$ periods. Plot the time paths for k, y, c, w & r.

Repeat the analysis above for a starting value of k that is 50% above the steady state.

Repeat both simulations for the following sets of alternative parameters:
g =.025, n=.01, δ=.1, θ=.33, $\boldsymbol{\gamma}$**=2.5**, ρ=.05
g =.025, n=.01, δ=.1, θ=.33, γ=1.0, $\boldsymbol{\rho}$**=.10**

Brigham Young University Department of Economics

Economics 581 – Advanced Macroeconomics

Homework #4
Due 9/24

Overlapping Generations Models

Problem Set 1

Simulate a stochastic version of the overlapping generations model described in section 2.3 of the McCandless text. Use the parameter values listed on page 31. Consider the following alterations to the model:

Let McCandless' productivity shock, λ_t, be defined as: $\lambda_t \equiv e^{z_t}; z_t = \gamma z_t + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2)$.

As with homework #1, run a series of 1000 Monte Carlos with a sample size of 120 observations each.
For each Monte Carlo:

1) generate a series of randomly distributed shocks, ε_t, that are bounded between zero and some upper bound. Then construct a series of technology shocks, λ_t, using the equation above.
2) calculate a time series for K_t using the transition equation near the bottom of page 30. Use a value of $\sigma^2 = .0004$ for the z process.
3) calculate time series for $\{Y_t, C_t, I_t, w_t, r_t\}$.
5) calculate the following sample moments for each time series, $\{Y_t, C_t, I_t, w_t, r_t, K_t, \lambda_t\}$ in each Monte Carlo: 1) mean, 2) standard deviation, 3) correlation with Y_t, 4) correlation with λ_t, 5) autocorrelation. Calculate the averages of each of these moments over the 1000 Monte Carlos.

Submit a table reporting the 35 moments in question for each set of parameters. As follows:

variable	mean	standard deviation	correlation –w– Y_t	correlation –w– λ_t	autocorrelation
Y_t					
I_t					
C_t					
w_t					
r_t					
K_t					
λ_t					

Brigham Young University Department of Economics
Economics 581 – Advanced Macroeconomics

Homework #5
due 10/1

Filtering Data

1. Go to the following sources:
Bureau of Economics Analysis (BEA) website: http://www.bea.gov/interactive.htm
Bureau of Labor Statistics (BLS) website: http://bls.gov/data/
Federal Reserve Economic Data (FRED) website: http://research.stlouisfed.org/fred2/

Look through the list of available tables. Choose time-series data from these sources that best match the following concepts from the model: capital (k), labor (h), output (y), investment (i), consumption (c), interest rates (r) and wages (w). All series should be at quarterly frequency and inflation adjusted as necessary. Discuss any adjustments (interpolations, extrapolations, combining of series, etc.) that you made to the data.

Download this data for as long a time period as you can. Report exactly which series you used for each concept.

Using this data, construct a series of Solow residuals that correspond to the level of technology. A production function of $y_t = {k_t}^{\theta}(\lambda_t \ell_t)^{1-\theta}$, gives this formula for the Solow residual, $\lambda_t = {y_t}^{\frac{1}{1-\theta}} {k_t}^{\frac{-\theta}{1-\theta}} {\ell_t}^{-1}$.

For each of the series in your data set do the following:

1) Take the natural log of the time-series.
2) Filter this transformed data with each of the following filters:
   a) A linear OLS trend.
   b) The Hodrick-Prescott filter.
   c) A band pass filter.
   d) First-differencing.
3) Calculate and report the following moments for each time-series and filter:
   a) Standard deviation of the series.
   b) Standard deviation relative to y.
   c) Correlation of the series with y.
   d) Correlation of the series with λ.
   e) Autocorrelation of the series.

Brigham Young University Department of Economics

Economics 581 – Advanced Macroeconomics

Homework #6
due 10/8

Working with Recursive Models and Markov Processes

1. Consider an infinitely-lived Robinson Crusoe economy with the following functional form assumptions.

$Y_t = AK_t{}^{\theta}\ ;\ K_{t+1} = I_t;\ C_t = Y_t - I_t;\ u(C) = \ln C$

Write down the Bellman equation treating K as the state variable.
Find the first-order condition for the maximization problem.
Find the envelope condition for the Bellman equation.
Combine these two conditions to obtain an Euler equation.
Verify that the transition function in this case is $K_{t+1} = \beta\theta AK_t{}^{\theta}$.
Find the functional form of the value-function, $V\{K_t\}$.

2. Using the same data from your homework #5, perform the following adjustments on the raw data.

i) Find the average growth rate over the sample size and report it.
ii) Remove this trend from the data (i.e. filter the data).
iii) For the filtered data, calculate and report the standard deviation and the autocorrelation.

Now, consider a statistical model of the log of GDP per capita that includes a Markov process.

$y_t = g(s_{t-1}) + y_{t-1} + \varepsilon_t$
Where s_{t-1} is the Markov state, and $\varepsilon_t \sim N(0, \sigma^2)$

Suppose the Markov process has two states, with $g(1) = .01$ and $g(2) = -.03$.
Assume also that the probability matrix is $\begin{bmatrix} .9 & .1 \\ .5 & .5 \end{bmatrix}$. Let $\sigma^2 = .0004$.

Simulate this process for 254 periods starting in state 1 with a value of $y_1 = 0$. Filter this data and calculate the same moments that you did for the US data. Run 10,000 Monte Carlos and report the average moments as in previous homework.

Suppose the Markov process has three states, with $g(1) = .02$, $g(2) = .01$ and $g(3) = -.03$
Assume also that the probability matrix is $\begin{bmatrix} .5 & .45 & .05 \\ .05 & .85 & .10 \\ .25 & .25 & .50 \end{bmatrix}$. Let $\sigma^2 = .0004$.

Repeat the Monte Carlo experiment for this case.

3. Using the two-state Markov process from above try to calibrate the model to fit the US data.

The process has 5 parameters: $p_{11},\ p_{22},\ g_1,\ g_2,\ \sigma$. Choose these to match the three moments (average growth, standard deviation, & autocorrelation) from the Monte Carlo experiment to the three moments from the US data as closely as possible. Plot one of your Monte Carlo time series and compare it to the US data.

Brigham Young University Department of Economics

Economics 581 – Advanced Macroeconomics

Homework #7
due 10/22

Solving and Simulating Hansen's Model

1. Consider a version of Hansen's basic model with no utility from leisure and with the following functional forms.

$u(c_t) = \frac{1}{1-\gamma}({c_t}^{1-\gamma} - 1)$; $f(k_t, z_t) = A{k_t}^{\theta}(e^{z_t}\bar{h})^{1-\theta}$
$z_t = \rho z_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma^2)$

Write down a Bellman equation for this model's households.
Write down the first-order and envelope conditions.
Write down the Euler equation.

What are the exogenous state variables?
What are the endogenous state variables?
List your endogenous non-state variables.

Write down the set of behavioral equations that define this model.

Set the parameters of the model to the following values:
$\gamma = 2.0, \beta = 0.995, \delta = 0.025, \theta = 0.33, \bar{h} = 0.3, A = 1, \rho = 0.9, \sigma = 0.02$

Use the behavioral equations to find the steady state values of:
capital (*k*), output (*y*), investment (*i*), consumption (*c*), the interest rate (*r*), the wage rate (*w*).
Did you find the solution analytically or numerically?

Log-linearize the behavioral equations analytically and write each next to the equation from which it was derived.
Given the parameter values above, write down the numerical values of the coefficient matrices **A** through **N**, which come from the log-linearized equations. Did you find the values analytically or numerically?

Write down values of the policy coefficient matrices **P** through **S**.

Simulate this model with 1000 Monte Carlos of 250 observations each.
For *k*, *y*, *i*, *c*, *r*, *w*, λ (the level of technology, Ae^z) & *u* (within period utility) report the following moments: mean, standard deviation, coefficient of variation, correlation with *y*, correlation with λ, and autocorrelation.

2. Repeat the analysis above using the basic Hansen model with utility from leisure. Use the following functional forms.

$u(c_t) = \frac{1}{1-\gamma}({c_t}^{1-\gamma} - 1) + B\frac{1}{1-\mu}[(1-h_t)^{1-\mu} - 1]$; $f(z_t, k_t, h_t) = A{k_t}^{\theta}(e^{z_t}h_t)^{1-\theta}$
Let $B = 2.5$ & $\mu = 1$ in your simulation. Use the same values as in part 1 for the other parameters.
Include hours worked in the list of variables for which you find steady state values and moments.

Brigham Young University Department of Economics

Economics 581 – Advanced Macroeconomics

Homework #8
due 11/1

Solving and Simulating Versions of Hansen's Model using Dynare

1. Consider a version of Hansen's basic model with utility from leisure and divisible labor. Use the following functional forms.

$u(c_t) = \frac{1}{1-\gamma}(c_t{}^{1-\gamma} - 1) + B\frac{1}{1-\mu}[(1-h_t)^{1-\mu} - 1]$; $f(z_t, k_t, h_t) = Ak_t{}^{\theta}(e^{z_t}h_t)^{1-\theta}$

Set the parameters of the model to the following values:
$\gamma = 2.0, \beta = 0.995, \delta = 0.025, \theta = 0.33, \bar{h} = 0.3, A = 1, \rho = 0.9, \sigma = 0.02, B = 2.5$ & $\mu = 1$

Use Dynare to find the steady state values of: capital (*k*), output (*y*), investment (*i*), consumption (*c*), the interest rate (*r*), the wage rate (*w*).
For *k*, *y*, *i*, *c*, *r*, *w*, λ (the level of technology, Ae^z) & *u* (within period utility) report the following moments: mean, standard deviation, coefficient of variation, correlation with *y*, correlation with λ, and autocorrelation.
Show the responses of each of these variables to an innovation in the *z* process.
How do your moments compare with the answers from homework #7 question 2.

2. Redo your work above using an indivisible labor version of Hansen's model. Calibrate as in McCandless when setting the fixed work hours, h_0.

3. Redo you work from question 1 using the following production function:
$f(z_t, k_t, h_t) = A[\theta k_t{}^{\eta} + (1-\theta)(e^{z_t}h_t)^{\eta}]^{1/\eta}; \eta = 2.0$

4. Redo you work from question 2 using the production function from question 3.

Brigham Young University Department of Economics
Economics 581 – Advanced Macroeconomics

Homework #9
due 11/7

Solving and Simulating Monetary Models

1. Consider a model with a cash-in-advance constraint like that presented in class, with the following utility function. $u(c_t) = \frac{1}{1-\gamma}\left({c_t}^{1-\gamma} - 1\right) \times \frac{h}{h_0}\left\{\frac{1}{1-\gamma}\left[(1-h_0)^{1-\gamma} - 1\right]\right\}^B$. Assume the number of workers in each household is growing each period by rate *n*, and technology is growing at rate a.

Write down a Bellman equation for this model's households.
Write down the first-order and envelope conditions.
Write down the Euler equations.

Write down the set of behavioral equations that define this model.
Include the following equations for the money process.
$g_{t+1} = (1-\psi)\bar{g} + \psi g_t + \varepsilon_{gt}$
$M_{t+1} = g_{t+1} M_t$

Transform these variables appropriately so that the model has a steady state, and write each transformed equation next to the original behavioral equation from which it is derived. Be careful when stationarizing prices.

Set the parameters of the model to the following values:
$\gamma = 2.0, \beta = 0.995, \delta = 0.025, \theta = 0.33, \bar{h} = 0.3, A = 1, B = 2.5, h_0 = 0.583, n = .00125, a = .00375, \bar{g} = .01,$
$\rho = 0.995, \sigma_z = 0.02, \psi = 0.9, \sigma_g = 0.01$.
Use Dynare and for *K*, *M*, *H*, *Y*, *I*, *C*, *r*, *w*, λ (the level of technology, e^z) & u (within period utility) report the following moments: mean, standard deviation, correlation with *y*, correlation with λ, and autocorrelation.

2. Consider a model with money in the utility function like that presented in class, with the following utility function. $u(c_t) = \frac{1}{1-\gamma}\left({c_t}^{1-\gamma} - 1\right) \times \frac{h}{h_0}\left\{\frac{1}{1-\gamma}\left[(1-h_0)^{1-\gamma} - 1\right]\right\}^B \times \left\{\frac{1}{1-\gamma}\left[\left(\frac{m_t}{N_t P_t}\right)^{1-\gamma} - 1\right)\right]\right\}^D$
Let all other aspects of the model be the same as above. Repeat the analysis from section 1 for this model. Use $D = .01$

Brigham Young University Department of Economics
Economics 581 – Advanced Macroeconomics

Homework #10
due 11/14

Solving and Simulating Sticky Price Models

1. Consider a model with a cash-in-advance constraint like that in homework #9. Use the handout on sticky prices to establish the notation and functional form for monopoly production of intermediate goods. Assume, as in the handout, that each intermediate producer faces a Calvo-style price-setting arrangement.

List the sets of exogenous state variables, endogenous state variables, jump variables, definitions, and stochastic shocks.

Write down the set of stationarized behavioral equations that define this model.

Use Dynare, MATLAB or Python to solve and simulate the model. For K, M, h, Y, I, C, r, w, e^z & u report the following moments: mean, standard deviation, correlation with y, correlation with e^z, and autocorrelation.

For simulation, set the parameters of the model to the following values:

a	quarterly growth rate of technology	.00375
n	quarterly growth rate of the labor force	.00125
D	utility weight on leisure	2.5
h_0	fixed hours worked per worker	0.583
M_0	initial money supply	1.0
α	capital share in GDP	0.33
β	quarterly discount factor	0.995
γ	CES utility parameter	2.0
δ	quarterly rate of depreciation	0.025
λ	Calvo probability	0.75
μ	quarterly average growth rate of money	0.01
ρ_z	autocorrelation of technology	0.95
ψ	CES production aggregator parameter	11.0
σ_z	standard deviation of technology shocks	0.02
ρ_g	autocorrelation of money growth	0.90
σ_g	standard deviation of money shocks	0.01