Active Return and Tracking Risk

Advantages and Disadvantages of Bond Portfolio Management Strategies

Four Criteria for Selecting a Benchmark Bond Index

Stratified Sampling

Strategy	*Advantages*	*Disadvantages*
Pure Bond Indexing	• Zero tracking error • Same risk factors as index • Low fees	• Costly and difficult • Lower expected return than index
Enhanced indexing by matching primary risk factors	• Less costly • Higher expected return • Same risk factors as index	• Higher management fees • Higher tracking error • Lower expected return than index
Enhanced indexing by small risk factor mismatches	• Same duration as index • Higher expected return • Lower restrictions	• Higher risk • Higher tracking error • Higher management fees
Active management by larger risk factor mismatches	• Higher expected return • Lower restrictions • Manage duration	• Higher risk • Higher tracking error • Higher management fees
Full-blown active management	• Higher expected return • Few restrictions • No duration limits	• Higher risk • Higher tracking error • Higher management fees

Active return is the difference between portfolio and index returns.

Tracking risk is the standard deviation of active return, and measures the variability of portfolio excess return.

First, create a matrix of bonds according to risk factors such as sector, rating, duration, callability, etc.

Measure the total value of the bonds in each cell, and determine each cell's weight in the index.

Select a sample of bonds from each cell in an amount that produces the same weight in the portfolio as that cell's weight in the index.

1. *Market value risk* varies with duration. More risk aversion equates to lower market risk and shorter benchmark duration.
2. *Income risk* varies indirectly with maturity. Higher dependence on stable income equates to longer maturity of the benchmark. Often is opposite from market value risk.
3. *Credit risk* of the benchmark should closely match that of the portfolio.
4. *Liability framework risk* applies only to portfolios matching a liability structure and should be minimized.

Risk Factors Used in Bond Benchmark Multifactor Models

Summary of Factors Used for Bond Risk Exposure

Present Value Distribution of Cash Flows

Classical Immunization

	Primary Risk Factors					
Risk	*Interest Rate*	*Yield Curve*		*Spread*	*Credit*	*Optionality*
What is Measured	Exposure to yield curve shifts	Exposure to yield curve twists		Exposure to spread changes	Exposure to credit changes	Exposure to call or put
Measurement used	Duration	PVD	Key rate duration	Spread duration	Duration contribution by credit rating	Delta

- **Duration.** Estimates the change in value given small, parallel yield curve shifts. Convexity must also be considered.
- **Key rate duration.** Measures sensitivity to yield curve twists.
- **Present value distribution of cash flows.** Measures the proportion of the index's total duration attributable to cash flows falling within selected time periods. Matching the PVD will create the same sensitivities to yield curve twists and shifts.
- **Sector and quality percent.** Matches the weights of sectors and qualities in the index.
- **Sector duration contributions.** Matches the proportion of the index duration contributed by each sector.
- **Quality spread duration contribution.** Matches the proportion of the index duration contributed by rating categories.
- **Sector/coupon/maturity cell weights.** Matching sector, coupon, and maturity weights mimics callability of bonds in the index.
- **Issuer exposure.** Use enough securities such that risk attributable to any particular issuer is minimized.

The process of structuring a bond portfolio that balance any change in value with the return from reinvestment of coupon and principal received throughout the investment period. The goal of classical immunization is to form a portfolio such that:

- If interest rates increase, the gain in reinvestment income exceeds the loss in portfolio value.
- If interest rates decrease, the gain in portfolio value exceeds the loss in investment income.

Without rebalancing, classical immunization only works for a one-time instantaneous change in interest rates. Portfolios cease to be immunized when:

- Interest rates fluctuate more than once.
- Time passes and the durations differ.

PVD measures the proportion of an index's total duration attributable cash flows (both coupons and redemptions) falling within selected time periods.

Assuming the time periods are six month intervals, first find the PV of cash flows from the index that fall in every six-month period. Divide by total PV of the index cash flows to get the weights of each period.

Multiply the duration of each period by its weight to get duration contribution for each period. Divide by the index duration and repeat for all time periods. This is the present value distribution.

Matching the PVD of the index will match sensitivities to both shifts and twists in the yield curve.

Immunization of a Single Liability

Bond Characteristics to Consider When Immunizing

Immunization Risk

Effective Duration of a Portfolio

- *Credit rating.* Immunization implicitly assumes that none of the bonds will default.
- *Embedded options.* Make duration difficult to estimate because cash flows are difficult to forecast.
- *Liquidity.* Important because rebalancing requires selling bonds.

1. Select a bond or portfolio with an effective duration equal to the liability's duration. If the liability is payable on a single date, the duration is the time horizon until payment.
2. Set the PV of the bond equal to the PV of the liability.

The effective duration of a portfolio is the weighted average of the bonds in the portfolio.

$$D_p = \sum_{i=1}^{n} w_i D_i = w_1 D_1 + w_2 D_2 + w_3 D_3 + \cdots + w_n D_n$$

where:

D_p = the effective duration of the portfolio
w_i = the weight of bond i in the portfolio
D_i = the effective duration of bond i

A measure of how much the end value of an immunized portfolio falls short of its target value because of nonparallel changes in interest rates.

In general the portfolio that has the lowest reinvestment risk will be the best for immunization.

- An immunized portfolio of zero-coupon bonds that mature at the investment horizon has zero immunization risk because there is zero reinvestment risk.
- If cash flows are concentrated around the horizon(e.g., bullet), reinvestment and immunization risk will be low.
- If cash flows are highly dispersed around the horizon (e.g., barbell), reinvestment and immunization risk will be high.

Dollar Duration

Steps to Adjust a Portfolio's Dollar Duration

Three Spread Duration Measures for Fixed-rate Bonds

Four Extensions to Classical Immunization

1. Calculate the new dollar duration of the portfolio.
2. Calculate the rebalancing ratio and use it to calculate the required percentage change in the value of the portfolio.

$$\text{rebalancing ratio} = \frac{\text{old } DD}{\text{new } DD}$$

3. The required percentage change in the value of the portfolio is $1 -$ rebalancing ratio.

A measure of exposure in dollars.

$$DD = -(\text{modified or effective duration})(0.01)(\text{price})$$

$$DD_p = \sum_{i=1}^{n} DD_i = DD_1 + DD_2 + DD_3 + \cdots + DD_n$$

where:

$DD_p =$ the dollar duration of the portfolio

$DD_i =$ the dollar duration of bond or sector i

1. **Multifunctional duration** or key rate duration by focusing on certain key interest rate maturities.
2. **Multiple-liability immunization** aims to immunize several liabilities by monitoring portfolio value and duration for multiple investment horizons.
3. **Increased risk** by lowering the minimum risk requirement of classical immunization allows for excess portfolio value.
4. **Contingent immunization** mixes active and passive immunization strategies.

1. **Nominal spread** is teh spread between the nominal yield on a non-Treasury bond and a Treasury of the same maturity.
2. **Zero-volatility spread** or static spread is the spread that must be added to the Treasury spot rate curve to make the PV of the bond's cash flows equal to its price.
3. **Option-adjusted spread** is determined using a binomial interest rate tree. It measures the spread with the option removed.

Contingent Immunization

Three Risks Associated with Managing Against a Liability Structure

Maturity Variance

Three Conditions for Multiple Liability Immunization

1. **Interest rate risk** is the primary concern when managing a fixed-income portfolio. To avoid match duration and convexity of the liability. Convexity can be difficult to measure, particularly for bonds with negative convexity such as MBS and callable corporates.
2. **Contingent claim risk** (a.k.a. call risk or prepayment risk) is highest when interest rates have fallen. This causes a loss in steady coupon payments as well as the necessity to reinvest at lower rates. To avoid, the convexity of the bonds must be taken into account.
3. **Cap risk** is present if bonds in the portfolio have floating rates. If the coupon doesn't fully adjust upward for rising interest rates, the market value of the bond adjusts downward. The assets and liabilities will not adjust in sync and the surplus will deteriorate.

- Cusion spread is the difference in the immunization rate and the minimum required return.
- Determine the PV of liability and compare this to the PV of the assets to determine the surplus value.
- Continue to compute the value of the surplus as time passes. As long as it's positive, the portfolio can be actively managed.

1. Assets and liabilities have the same present values.
2. Assets and liabilities have the same aggregate durations.
3. The range of the distribution of durations of individual assets exceeds the distribution of liabilities. This is necessary to use the cash flows generated from our assets to meet our cash outflow needs.

These conditions will assure immunization only against parallel rate shifts. For nonparallel shifts, linear programming models can be used to create minimum-risk immunized portfolios.

The variance of the differences in the maturities of the bonds used in the immunization strategy and the maturity date of the liability. Written as M-Square or M^2.

Cash Flow Matching

Differences Between Cash Flow Matching and Multiple-liability Immunization

Combination Matching

- Cash flow matching depends on the cash flows of the portfolio, so expectations of reinvestment rates and borrowing rates are important. Deviations from a perfect match should be small, which can increase the cost. Immunization by matching duration may cost less.
- In a cash-flow-matched portfolio, only cash flows occurring before a liability may be used to meet that obligation. In a duration-matched portfolio, the portfolio only needs to have sufficient value on each liability payment.

Cash flow matching is used to construct a portfolio that will a fund a stream of liabilities with coupons and maturity values. It will cause the durations to be matched, but it is more stringent than immunization by matching duration alone because the timing and amount of cash flows must match the liabilities. To construct the portfolio:

- Select a bond with maturity equal to the last liability payment date.
- Buy enough so that the final payments fully fund that liability.
- Using a recursive procedure, select another bond based on maturity value and coupon so that its cash flows plus the coupon of the previous bond fund the latest unfunded liability.
- Continue until all liabilities have been funded.

Also known as horizon matching. Is a combination of multiple-liability immunization and cash flow matching. A portfolio is duration matched, and in the early years is also cash flow matched in order to make sure assets are properly dispersed to meet near-term obligations.

Combination matching has the following advantages over multiple-liability immunization.

- Provides liquidity in the initial period.
- Reduced the risk from nonparallel shifts in the yield curve. The initial cash needs are met with cash flows, so there is no rebalancing needed to meet the initial cash requirements.

The primary disadvantage of combination matching is that it tends to be more expensive than multiple-liability matching.