Number of Equity Futures Contracts Needed for a Target Portfolio Beta

Basic Form for Synthetic Equity or Synthetic Cash

Number of Equity Futures Contracts Needed for Synthetic Cash Position

Number of Futures Contracts Needed to Achieve for a Target Portfolio Duration

Synthetic cash or synthetic equity is modeled using long or short offsetting positions in equity futures.

- A synthetic risk-free asset is formed with a long stock position and a short stock index futures position.
- A synthetic equity position is formed with a long risk-free asset and a long stock index futures position.

The formula for beta of a particular asset is a is

$$\beta_a = \frac{\text{cov}(a, M)}{\sigma_M^2}$$

where:

$$\begin{aligned} \text{cov}(a, M) &= \text{covariance of returns on asset } a \text{ with the market} \\ \sigma_M^2 &= \text{variance of the market returns} \end{aligned}$$

The number of contracts needed to achieve target portfolio beta, β_T, is

$$\text{number of contracts} = \left(\frac{\beta_T - \beta_P}{\beta_f}\right)\left(\frac{V_P}{P_f \times \text{multiplier}}\right)$$

where:

$$\begin{aligned} \beta_T &= \text{desired portfolio beta} \\ \beta_P &= \text{portfolio beta} \\ \beta_f &= \text{equity futures contract beta} \\ V_P &= \text{current value of the portfolio} \\ P_f &= \text{futures price} \end{aligned}$$

$$\text{number of contracts} = (\text{yield beta})\left(\frac{\text{MD}_T - \text{MD}_P}{\text{MD}_F}\right)\left(\frac{V_P}{P_f \times \text{multiplier}}\right)$$

where:

$$\begin{aligned} V_P &= \text{current value of the portfolio} \\ P_f &= \text{futures price} \\ \text{MD}_T &= \text{target modified duration} \\ \text{MD}_P &= \text{modified duration of the portfolio} \\ \text{MD}_F &= \text{modified duration of the futures contract} \end{aligned}$$

An equity position can be converted to a synthetic cash position for T years by using

$$\text{number of equity contracts} = -\frac{V_P(1 + R_F)^T}{P_f}$$

where:

$$\begin{aligned} V_P &= \text{value of the equity position} \\ P_f &= \text{total futures price (quoted price times multiplier)} \\ R_F &= \text{risk-free rate} \\ T &= \text{designated period of time} \end{aligned}$$

Steps to Synthetically Change Equity and Bond Allocations

Preinvesting

Three Types of Foreign Exchange Risk

Hedging Foreign Market and Foreign Currency Risks

Preinvesting is taking a long futures position to create an exposure converting a future cash inflow into a synthetic equity or bond position.

- To reallocate from equity to bonds
  1. Remove all systematic risk (target a beta of zero) by shorting equity futures.
  2. Add duration to the position (target a modified duration of more than zero) by going long bond futures.
- To reallocate from bonds to equity
  1. Remove all duration (target a modified duration of zero) by shorting bond futures.
  2. Add systematic risk to the position (target a beta of more than zero) by going long equity futures.

- Hedging foreign market risk
  - Can sell short (sell forward) the foreign market index.
  - The effectiveness depends on correlation of the portfolio and the market index. Perfect correlation will return the foreign risk-free rate.
  - Applying a currency hedge on top of this will return the domestic risk-free rate.
- Hedging foreign currency risk
  - Problem is uncertainty of future value. Strategies to deal with this are
    - Hedge a minimum future value.
    - Hedge the estimated future value.
    - Hedge the initial value of the portfolio.

1. **Transaction exposure** is present when a cash flow in a foreign currency occurs at a future date. Can be hedged by selling forward in the case of a receipt or buying forward in the case of a payment.
2. **Economic exposure** refers to situations when changes in currency value affect competitiveness. Some examples are companies that
   - Export and sell products in foreign markets. Domestic currency appreciation means less competitive products internationally.
   - Purchase and import from foreign markets. Domestic currency depreciation means costs in the domestic currency increase.
   - Operate domestically, but have competitors or suppliers who are affected by changes in currency value.

   This can be desirable to hedge, but difficult to quantify.
3. **Translation exposure** is the risk of converting foreign financial statements into domestic currency units. Generally not hedged as it's not seen as a real cash flow risk.

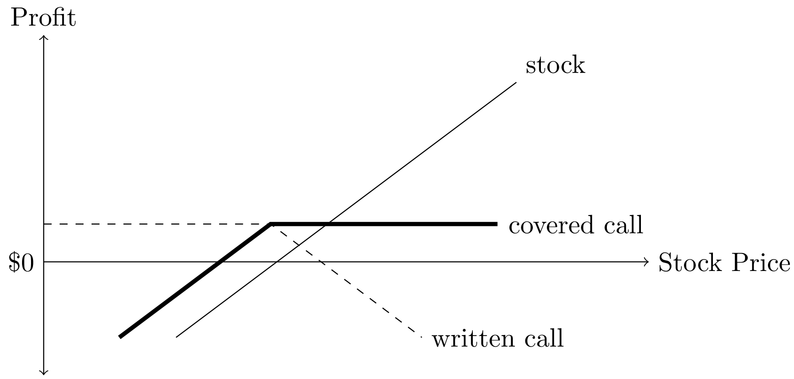
Hedging with Forwards Versus Futures

Covered Call

Protective Put

Bull Spread

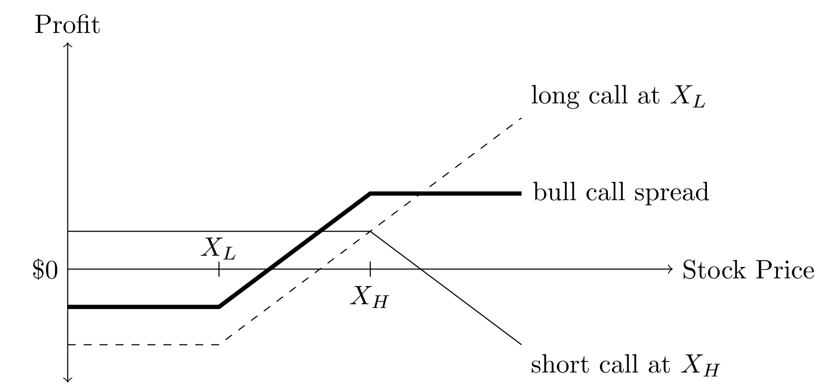
Buy the underlying and sell a call option. Used to generate income when the underlying price is expected to remain unchanged.



$$\text{profit} = -\max(0, S_T - X) + S_T - S_0 + C_0$$
$$\text{maximum profit} = X + C_0 - S_0$$
$$\text{maximum loss} = S_0 - C_0$$
$$\text{breakeven price} = S_0 - C_0$$

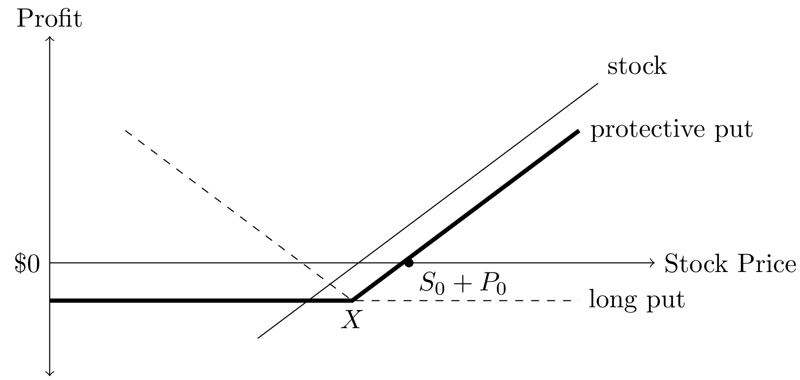
- Practical differences
  - Futures are standardized while forwards are customized.
  - Forwards have counterparty risk while futures use clearinghouses.
  - Futures are more regulated and transparent.
  - Futures require margin.
- Empirical differences
  - Most bond and equity hedging is done with futures, but this creates cross-hedge and basis risk.
  - Interest payment and currency hedges usually use forwards so exact amounts and dates can be hedged.
  - Eurodollar futures are a large market that is mostly used by dealers and market makers to hedge their own business needs.

Purchase a call option with a low exercise price, X_L and sell a call with a higher exercise price, X_H. At inception, $X_L < X_H$ and $C_{L,0} < C_{H,0}$. The investor expects the stock price to end up between X_L and X_H. This provides limited upside if the stock rises, with a limited downside.



$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L,0} + C_{H,0}$$
$$\text{maximum profit} = X_H - X_L - C_{L,0} + C_{H,0}$$
$$\text{maximum loss} = C_{L,0} - C_{H,0}$$
$$\text{breakeven price} = X_L + C_{L,0} - C_{H,0}$$

Buy the underlying and buy a put option. Limits downside risk at the cost of the put premium, P_0.



$$\text{profit} = \max(0, X - S_T) + S_T - S_0 - P_0$$
$$\text{maximum profit} = S_T - S_0 - P_0$$
$$\text{maximum loss} = S_0 - X + P_0$$
$$\text{breakeven price} = S_0 + P_0$$

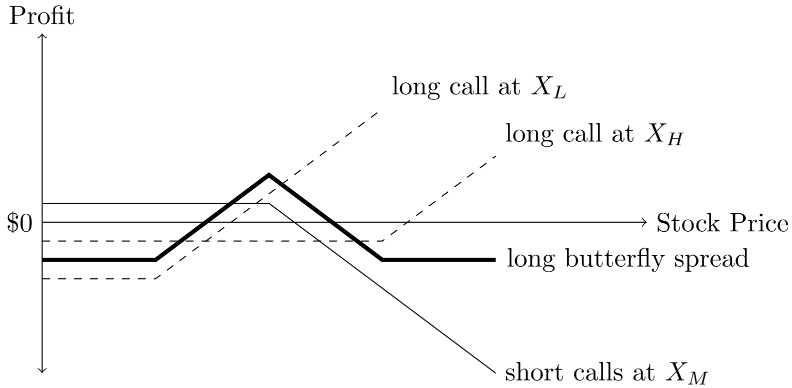
Bear Spread

Butterfly Spread with Calls

Straddle

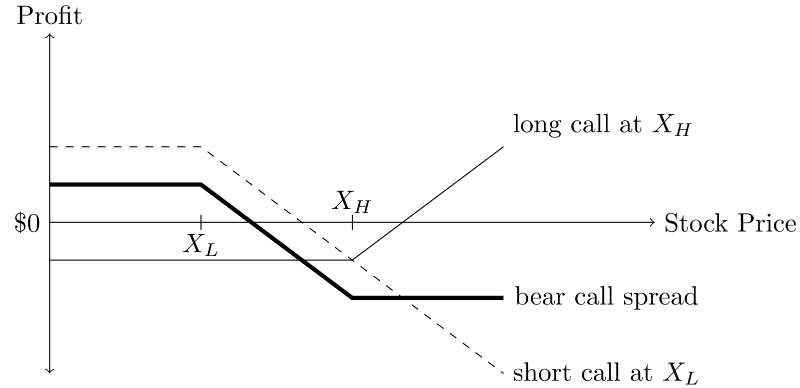
Collar

Buy two calls at strike prices X_L and X_H and write two calls at strike price X_M with $X_L < X_M < X_H$. The investor expects the stock price to stay near X_M, but the downside loss is limited by the purchased calls.



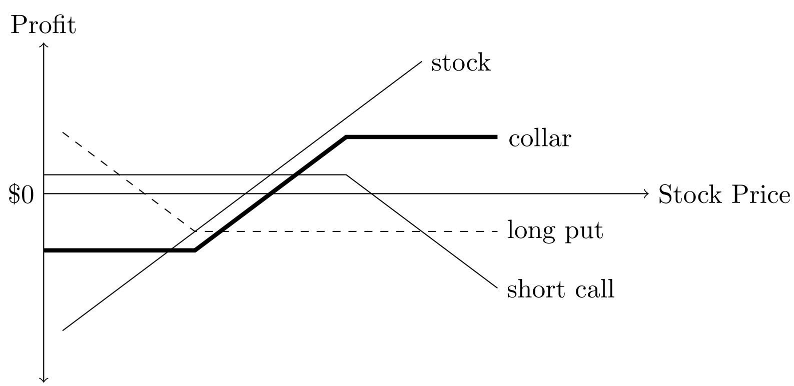
$$\begin{aligned}
\text{profit} &= \max(0, S_T - X_L) - 2\max(0, S_T - X_M) \\
&\quad + \max(0, S_T - X_H) - C_{L,0} + 2C_{M,0} - C_{H,0} \\
\text{maximum profit} &= X_M - X_L - C_{L,0} + 2C_{M,0} - C_{H,0} \\
\text{maximum loss} &= C_{L,0} - 2C_{M,0} + C_{H,0} \\
\text{breakeven price} &= X_L + C_{L,0} - 2C_{M,0} + C_{H,0} \text{ and } 2X_M - X_L - C_{L,0} + 2C_{M,0} - C_{H,0}
\end{aligned}$$

Sell a call with a low exercise price, X_L and buy a call with a higher exercise price, X_H. This provides limited upside if the stock falls, with a limited downside. As prices fall, the investor keeps the premium of the written call, net of the long call premium.



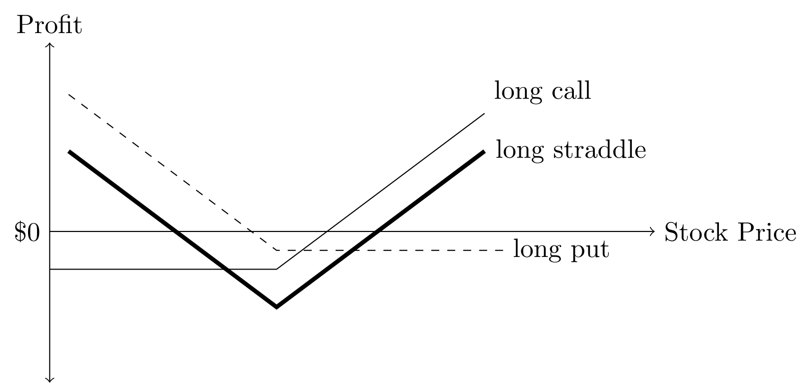
$$\begin{aligned}
\text{profit} &= \max(0, S_T - X_H) - \max(0, S_T - X_L) + C_{L,0} - C_{H,0} \\
\text{maximum profit} &= X_H - X_L + C_{L,0} - C_{H,0} \\
\text{maximum loss} &= C_{L,0} - C_{H,0} \\
\text{breakeven price} &= X_L + C_{L,0} - C_{H,0}
\end{aligned}$$

Combination of a protective put and a covered call. Can be zero-cost if call and put premia are equal. Usually put has lower strike X_L and call has higher strike X_H. Both the upside and downside are limited by the call and the put respectively.



$$\begin{aligned}
\text{profit} &= \max(0, X_L - S_T) - \max(0, S_T - X_H) + S_T - S_0 \\
\text{maximum profit} &= X_H - S_0 \\
\text{maximum loss} &= S_0 - X_L \\
\text{breakeven price} &= S_0
\end{aligned}$$

Buy both a put and a call with the same strike price and expiration on the same asset. The investor expects a large price move in some direction, and will incur a loss if the price remains static.



$$\begin{aligned}
\text{profit} &= \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0 \\
\text{maximum profit} &= S_T - X - C_0 - P_0 \\
\text{maximum loss} &= C_0 + P_0 \\
\text{breakeven price} &= X - C_0 - P_0 \text{ and } X + C_0 + P_0
\end{aligned}$$

Box Spread

Payoff of Interest Rate Calls and Puts

Interest Rate Caps and Floors

Considerations for Delta Hedging

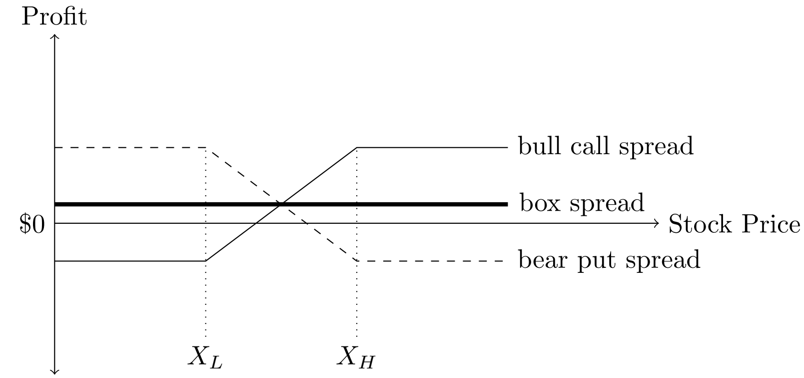
The payoff for an interest rate call is

$$\text{payoff} = \text{NP} \times \max(0, \text{LIBOR} - \text{strike rate}) \times (D/360)$$

where NP is the notional principal and D is the days in the theoretical underlying. The payoff for an interest rate put is

$$\text{payoff} = \text{NP} \times \max(0, \text{strike rate} - \text{LIBOR}) \times (D/360)$$

Combination of a bull call spread and a bear put spread. That is, a long call and short put at X_L and a long put and a short call at X_H. The payoff is always the same, regardless of the underlying price, which means assuming the prices are correct, the payoff is the risk-free rate.



- Delta is only an approximation and is less accurate for larger changes in the stock price.
- Delta changes as market conditions change, including changes in ΔS.
- Delta changes over time independent of other changes.
- In the Black-Scholes equation, $N(d_1)$ approximates delta.

Interest rate caps and floors are series of interest rate call and put options. Each cap and floor is called a caplet and floorlet.

Caps and floors are OTC contracts so they're tailored. The terms generally specify

- Reference rate—typically LIBOR.
- Cap or floor strike rate.
- Length of the agreement.
- Length of the agreement (D).
- Notional principal (NP).

Effects of Gamma on Delta Hedging

The greater the value of gamma the more risk in the position (i.e., the more variability in the value of the option.)

The gamma of an at-the-money option is greatest near the expiration. When gamma is large, option values are subject to large changes the position faces the most risk and the investor is most likely to use a two-option hedge. In this situation, two options are used to force both delta and gamma to zero.