Number of Equity Futures Contracts Needed for a Target Portfolio Beta

Basic Form for Synthetic Equity or Synthetic Cash

Number of Equity Futures Contracts Needed for Synthetic Cash Position

Number of Futures Contracts Needed to Achieve for a Target Portfolio Duration

Synthetic cash or synthetic equity is modeled using long or short offsetting positions in equity futures.

- A synthetic risk-free asset is formed with a long stock position and a short stock index futures position.
- A synthetic equity position is formed with a long risk-free asset and a long stock index futures position.

The formula for beta of a particular asset is a is

$$\beta_a = \frac{\text{cov}(a, M)}{\sigma_M^2}$$

where:

$\text{cov}(a, M)$ = covariance of returns on asset a with the market

σ_M^2 = variance of the market returns

The number of contracts needed to achieve target portfolio beta, β_T, is

$$\text{number of contracts} = \left(\frac{\beta_T - \beta_P}{\beta_f}\right)\left(\frac{V_P}{P_f \times \text{multiplier}}\right)$$

where:

β_T = desired portfolio beta

β_P = portfolio beta

β_f = equity futures contract beta

V_P = current value of the portfolio

P_f = futures price

$$\text{number of contracts} = (\text{yield beta})\left(\frac{\text{MD}_T - \text{MD}_P}{\text{MD}_F}\right)\left(\frac{V_P}{P_f \times \text{multiplier}}\right)$$

where:

V_P = current value of the portfolio

P_f = futures price

MD_T = target modified duration

MD_P = modified duration of the portfolio

MD_F = modified duration of the futures contract

An equity position can be converted to a synthetic cash position for T years by using

$$\text{number of equity contracts} = -\frac{V_P(1 + R_F)^T}{P_f}$$

where:

V_P = value of the equity position

P_f = total futures price (quoted price times multiplier)

R_F = risk-free rate

T = designated period of time

Steps to Synthetically Change Equity and Bond Allocations

Preinvesting

Three Types of Foreign Exchange Risk

Hedging Foreign Market and Foreign Currency Risks

Preinvesting is taking a long futures position to create an exposure converting a future cash inflow into a synthetic equity or bond position.

- To reallocate from equity to bonds
  1. Remove all systematic risk (target a beta of zero) by shorting equity futures.
  2. Add duration to the position (target a modified duration of more than zero) by going long bond futures.
- To reallocate from bonds to equity
  1. Remove all duration (target a modified duration of zero) by shorting bond futures.
  2. Add systematic risk to the position (target a beta of more than zero) by going long equity futures.

- Hedging foreign market risk
  - Can sell short (sell forward) the foreign market index.
  - The effectiveness depends on correlation of the portfolio and the market index. Perfect correlation will return the foreign risk-free rate.
  - Applying a currency hedge on top of this will return the domestic risk-free rate.
- Hedging foreign currency risk
  - Problem is uncertainty of future value. Strategies to deal with this are
    - Hedge a minimum future value.
    - Hedge the estimated future value.
    - Hedge the initial value of the portfolio.

1. **Transaction exposure** is present when a cash flow in a foreign currency occurs at a future date. Can be hedged by selling forward in the case of a receipt or buying forward in the case of a payment.
2. **Economic exposure** refers to situations when changes in currency value affect competitiveness. Some examples are companies that
   - Export and sell products in foreign markets. Domestic currency appreciation means less competitive products internationally.
   - Purchase and import from foreign markets. Domestic currency depreciation means costs in the domestic currency increase.
   - Operate domestically, but have competitors or suppliers who are affected by changes in currency value.

   This can be desirable to hedge, but difficult to quantify.
3. **Translation exposure** is the risk of converting foreign financial statements into domestic currency units. Generally not hedged as it's not seen as a real cash flow risk.

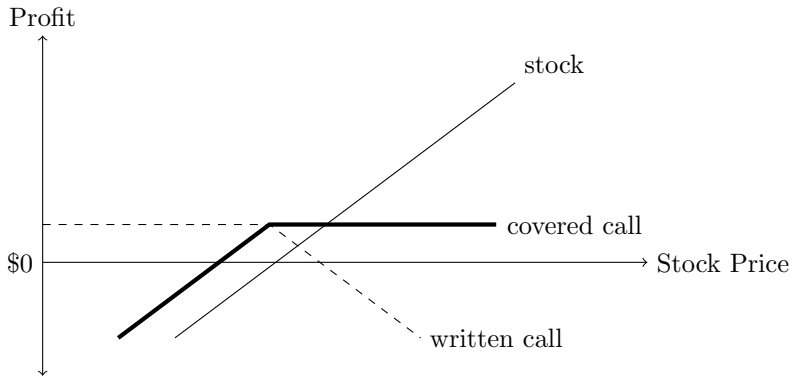
Hedging with Forwards Versus Futures

Covered Call

Protective Put

Bull Spread

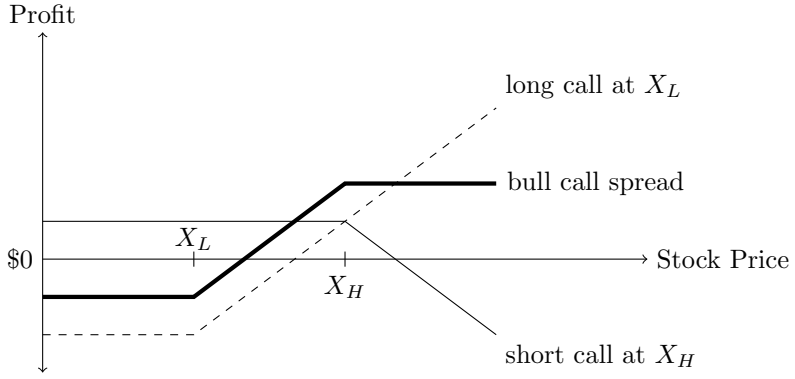
Buy the underlying and sell a call option. Used to generate income when the underlying price is expected to remain unchanged.



$$\begin{aligned}\text{profit} &= -\max(0, S_T - X) + S_T - S_0 + C_0 \\ \text{maximum profit} &= X + C_0 - S_0 \\ \text{maximum loss} &= S_0 - C_0 \\ \text{breakeven price} &= S_0 - C_0\end{aligned}$$

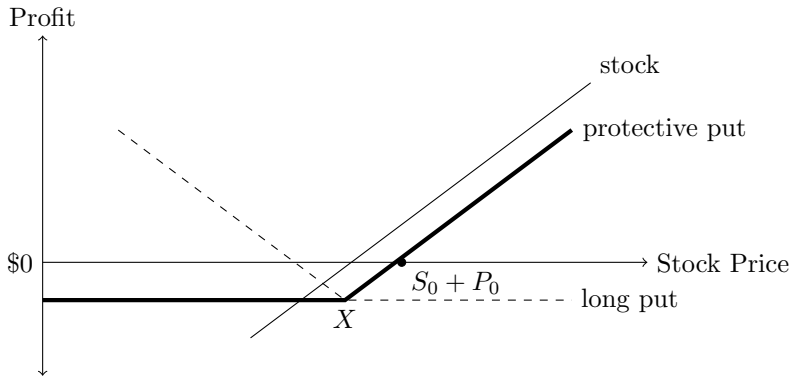
- Practical differences
  - Futures are standardized while forwards are customized.
  - Forwards have counterparty risk while futures use clearinghouses.
  - Futures are more regulated and transparent.
  - Futures require margin.
- Empirical differences
  - Most bond and equity hedging is done with futures, but this creates cross-hedge and basis risk.
  - Interest payment and currency hedges usually use forwards so exact amounts and dates can be hedged.
  - Eurodollar futures are a large market that is mostly used by dealers and market makers to hedge their own business needs.

Purchase a call option with a low exercise price, X_L and sell a call with a higher exercise price, X_H. At inception, $X_L < X_H$ and $C_{L,0} < C_{H,0}$. The investor expects the stock price to end up between X_L and X_H. This provides limited upside if the stock rises, with a limited downside.



$$\begin{aligned}\text{profit} &= \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L,0} + C_{H,0} \\ \text{maximum profit} &= X_H - X_L - C_{L,0} + C_{H,0} \\ \text{maximum loss} &= C_{L,0} - C_{H,0} \\ \text{breakeven price} &= X_L + C_{L,0} - C_{H,0}\end{aligned}$$

Buy the underlying and buy a put option. Limits downside risk at the cost of the put premium, P_0.



$$\begin{aligned}\text{profit} &= \max(0, X - S_T) + S_T - S_0 - P_0 \\ \text{maximum profit} &= S_T - S_0 - P_0 \\ \text{maximum loss} &= S_0 - X + P_0 \\ \text{breakeven price} &= S_0 + P_0\end{aligned}$$

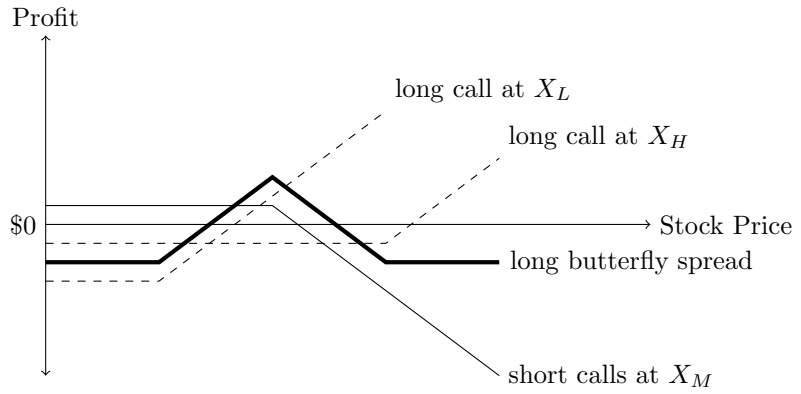
Bear Spread

Butterfly Spread with Calls

Straddle

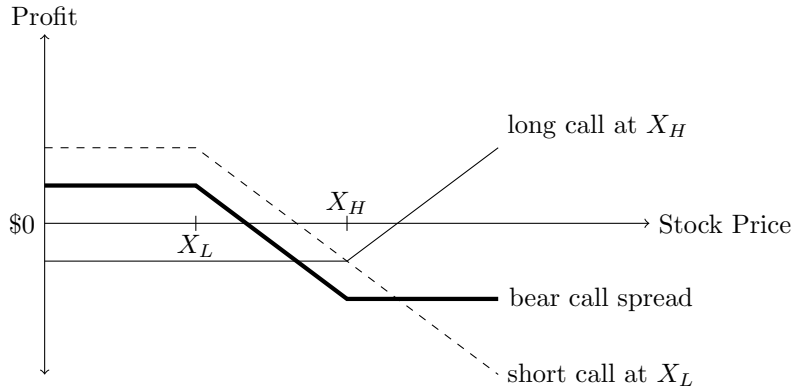
Collar

Buy two calls at strike prices X_L and X_H and write two calls at strike price X_M with $X_L < X_M < X_H$. The investor expects the stock price to stay near X_M, but the downside loss is limited by the purchased calls.



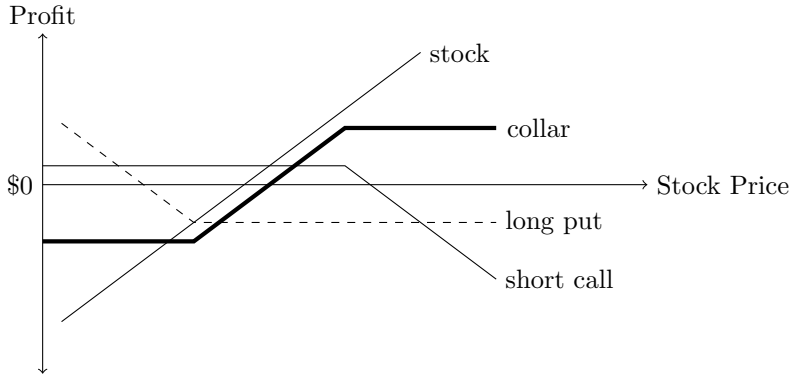
$$\begin{aligned}\text{profit} &= \max(0, S_T - X_L) - 2\max(0, S_T - X_M) \\ &\quad + \max(0, S_T - X_H) - C_{L,0} + 2C_{M,0} - C_{H,0} \\ \text{maximum profit} &= X_M - X_L - C_{L,0} + 2C_{M,0} - C_{H,0} \\ \text{maximum loss} &= C_{L,0} - 2C_{M,0} + C_{H,0} \\ \text{breakeven price} &= X_L + C_{L,0} - 2C_{M,0} + C_{H,0} \text{ and } 2X_M - X_L - C_{L,0} + 2C_{M,0} - C_{H,0}\end{aligned}$$

Sell a call with a low exercise price, X_L and buy a call with a higher exercise price, X_H. This provides limited upside if the stock falls, with a limited downside. As prices fall, the investor keeps the premium of the written call, net of the long call premium.



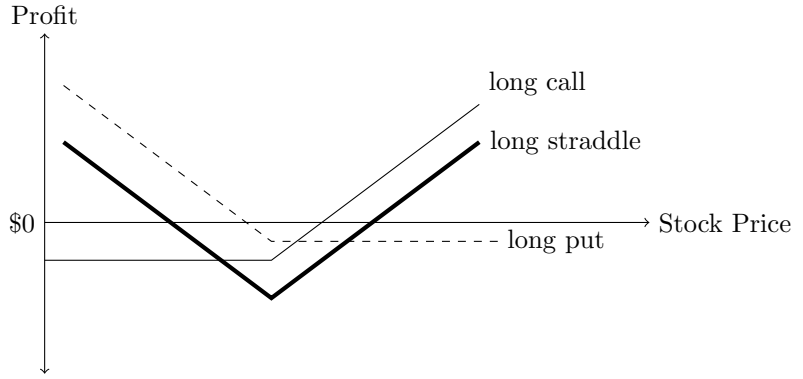
$$\begin{aligned}\text{profit} &= \max(0, S_T - X_H) - \max(0, S_T - X_L) + C_{L,0} - C_{H,0} \\ \text{maximum profit} &= X_H - X_L + C_{L,0} - C_{H,0} \\ \text{maximum loss} &= C_{L,0} - C_{H,0} \\ \text{breakeven price} &= X_L + C_{L,0} - C_{H,0}\end{aligned}$$

Combination of a protective put and a covered call. Can be zero-cost if call and put premia are equal. Usually put has lower strike X_L and call has higher strike X_H. Both the upside and downside are limited by the call and the put respectively.



$$\begin{aligned}\text{profit} &= \max(0, X_L - S_T) - \max(0, S_T - X_H) + S_T - S_0 \\ \text{maximum profit} &= X_H - S_0 \\ \text{maximum loss} &= S_0 - X_L \\ \text{breakeven price} &= S_0\end{aligned}$$

Buy both a put and a call with the same strike price and expiration on the same asset. The investor expects a large price move in some direction, and will incur a loss if the price remains static.



$$\begin{aligned}\text{profit} &= \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0 \\ \text{maximum profit} &= S_T - X - C_0 - P_0 \\ \text{maximum loss} &= C_0 + P_0 \\ \text{breakeven price} &= X - C_0 - P_0 \text{ and } X + C_0 + P_0\end{aligned}$$

Box Spread

Combination of a bull call spread and a bear put spread. That is, a long call and short put at X_L and a long put and a short call at X_H. The payoff is always the same, regardless of the underlying price, which means assuming the prices are correct, the payoff is the risk-free rate.