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A Brief Description of a Simplified BL Surplus Optimizer

– Theory, Data and Step-by-Step Calculations for the Demo Version –

Toshiki Yotsuzuka

1. Background

Individual investors have vastly different investment objectives and constraints depending on their expectations of future incomes, retirement pension benefits and basic consumption needs, none of which appears in the standard single-period MVO framework. Although such issues can in principle be dealt with in a generalized intertemporal optimization framework, formulating the problem and obtaining numerical solutions tends to be a complex, computationally intensive process. A much simpler approximate solution, however, can be obtained by extending the Surplus Framework, originally proposed by Leibowitz and Henriksson and explored by Sharpe and Tint (1990) in the context of pension fund asset-liability management.

The basic insight is to apply the mean-variance approach to the **"surplus"** (as defined below) of the investor. In pension fund ALM, "surplus" means "financial assets minus the PV of pension liabilities", but our definition is more general. We recognize that the typical investor has substantial "basic consumption needs", but also holds human capital (PV of future incomes) and can expect to receive retirement pension benefits. Thus, the natural definition of his/her ***net liability*** would be the PV of basic consumption needs minus a conservative estimate of human capital, with the PV of pension benefits to be further subtracted. If net liability is negative, then we can ignore it and revert to the standard mean-variance approach.

In practice, we would need to estimate the values and risk exposures of such liabilities based on the investor's responses to a few simple questions. Although estimation errors will inevitably be significant, we believe it should still be significantly better to use rough estimates than to ignore all such information. We also plan to incorporate housing assets and mortgages, but this extension will be implemented at a later stage.

Denote the return on financial assets by $\boldsymbol{r_A}$ and the return on net liability (% changes in the PV of net liability) by $\boldsymbol{r_L}$. Then, our objective function is

$$\boldsymbol{Max}\left[\boldsymbol{E}\left(\boldsymbol{r_S}\right)-\frac{\gamma}{2}\boldsymbol{Var}\left(\boldsymbol{r_S}\right)\right] \qquad \textbf{where } \boldsymbol{r_S}=\boldsymbol{r_A}-\boldsymbol{q}\boldsymbol{r_L} \qquad \textbf{(1)}$$

which is equivalent to

$$\boldsymbol{Max}\left[\boldsymbol{E}\left(\boldsymbol{r_A}\right)-\frac{\boldsymbol{\gamma}}{\mathbf{2}}\mathbf{var}\left(\boldsymbol{r_A}\right)+\boldsymbol{\gamma q}\,\mathbf{cov}\left(\boldsymbol{r_A},\boldsymbol{r_L}\right)\right] \qquad \mathbf{(2)}$$

where γ is the Investor's risk aversion coefficient (not necessarily equal to the Global Investor's risk aversion coefficient δ) and $\boldsymbol{q}$ is a non-negative constant that reflects the ***importance of net liability***.

In the context of pension fund ALM, the Sharpe-Tint formulation is

$$\boldsymbol{q}=\boldsymbol{k}\frac{\boldsymbol{L}}{\boldsymbol{A}} \qquad \mathbf{(3)}$$

which means

$$\boldsymbol{r_S}=\boldsymbol{r_A}-\boldsymbol{k}\frac{\boldsymbol{L}}{\boldsymbol{A}}\boldsymbol{r_L} \qquad \mathbf{(4)}$$

where $\boldsymbol{A}$ is the amount of financial assets, $\boldsymbol{L}$ is the PV of net liabilities, and $\boldsymbol{k}$ is the weight given to liability considerations. In a full pension fund ALM framework, $\boldsymbol{k}$ would be set to 1 and $\boldsymbol{q}$ would therefore be equal to the inverse of the funding ratio. In the current context, we could calculate $\boldsymbol{q}$ if we had sufficient information on $\boldsymbol{k}$, $\boldsymbol{A}$ and $\boldsymbol{L}$; otherwise we could simply assign values to $\boldsymbol{q}$ based on questionnaire responses.

Using the Black-Litterman risk premia and the Black-Litterman covariance matrix as inputs, and utilizing Lakshmi's notations, the first-order condition for the optimum is written as:

$$\hat{\Pi}-\gamma\Psi_p\hat{w}+\gamma q\theta=\mathbf{0} \qquad \mathbf{(5)}$$

$$\textbf{where } \theta=\begin{bmatrix} Cov\left(r_1,r_L\right) \\ \vdots \\ Cov\left(r_n,r_L\right) \end{bmatrix} \textbf{ and } r_i = \textbf{ returns on asset class } i$$

The optimal portfolio weights are now given by:

$$\begin{aligned}\hat{w}&=\frac{\mathbf{1}}{\gamma}\Psi_p{}^{-1}\hat{\Pi}+q\Psi_p{}^{-1}\theta\\ &=\frac{\mathbf{1}}{\gamma}\times\textbf{Myopic MV Portfolio}+q\times\textbf{Liability Hedging Portfolio} \qquad \mathbf{(6)}\end{aligned}$$

Thus, the optimal asset allocation would be a linear combination of the myopic mean-variance portfolio (MVP) and the liability hedging portfolio (LHP). For a retiree with modest financial assets, q would be relatively large, and therefore the LHP would be an important component of his portfolio. On the other hand, for a relatively wealthy person, q would be relatively small, and therefore the optimal portfolio would be close to the standard (myopic) MVP.

In order to estimate $Cov(r_i, r_L)$, we can simulate the time series of r_L given the pattern of the Investor's projected net cash outflows (basic consumption expenditures minus conservative estimates of wage & pension incomes) evaluated with historical interest rate data, and taking its historical covariance with asset class i.

2. Data (Details to be supplied later)

The following data will be used as inputs to our simplified Black-Litterman Surplus Optimizer.

(A) Market Data

- Historical time series of ***unhedged*** JPY rates of return on n asset classes, based on the Topix, Nomura, MSCI, Citi and/or Barclays Capital indices. "Unhedged JPY" means that the returns are not currency-hedged, and are calculated from the perspective of a JPY-based investor using historical USD-JPY spot exchange rates.
- Historical time series of ***hedged*** JPY rates of return on m ($< n$) foreign asset classes, based on the MSCI, Citi and/or Barclays Capital indices. "Hedged JPY" means that the returns are currency-hedged with (short-term) FX forward contracts.
- Current market capitalization values for the n asset classes, based on the Topix, Nomura, MSCI, Citi and/or Barclays Capital indices.
- Historical time series of JPY LIBOR rates and JPY swap yields.
- Historical time series of JGB benchmark yields published by the Japanese Ministry of Finance.

(B) Investor Inputs

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Investor responses to the Questionnaire have to be translated into actual values for the following parameters. Those parameters, however, can be treated as exogenous when coding the Optimizer.

- Risk Aversion Coefficient $\boldsymbol{\gamma}$
- Importance of Net Liability $\boldsymbol{q}$

(C) Manager-Specified Parameters

- "Tau" (measure of uncertainty of the equilibrium variance) $\boldsymbol{\tau}$

3. Calculations (Details to be supplied later)

- We need to combine market data (2A) and user inputs (2B) to generate the returns $\boldsymbol{r_L}$ on net liability.
- This can be done by (1) calculating the time series of discount factors based on historical yield curve data, (2) evaluating the projected net cash outflows of the Investor using the discount factors, and then (3) generating the time series of $\boldsymbol{r_L}$. Since we might encounter the problem of singularity (division by zero) when net liability is sufficiently close to zero, we need to have some procedures for dealing with such cases.
- Assuming that historical covariance matrix Ψ has already been calculated, we can proceed as in Lakshmi's "Implementation Steps" memo once we have an estimate for $\boldsymbol{Cov(r_i, r_L)}$.

References

Sharpe, W. F., and L. G. Tint, "Liability --- A New Approach," *Journal of Portfolio Management*, Vol. 16, No. 2, 1990.