Your Trusted Financial Brain

The Black-Litterman Approach for Asset Allocation

Dr. Supakorn Soontornkit

With the Ministry of Finance as Shareholder

Agenda

Motivation & Intuition

Black-Litterman Model

B-L Model in Practice

Result & Conclusion

Agenda

MFC Asset Management plc.

Part 1

Motivation & Intuition

Motivation & Intuition

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- **Principles of Mean-Variance Optimization**
  - Diversification : The risk of a portfolio can be decreased by combining assets whose returns move in different directions under certain market condition
  - Markowitz discovered that an investor can reduce the volatility of a portfolio and increase its return at the same time by using Markowitz Mean-Variance Optimization

However, Markowitz optimization still has some problems !

Motivation & Intuition

- **The Problem of Markowitz Optimization**
  - Highly-concentrated portfolios
    - Extreme portfolio
  - Input-sensitivity
    - Unstable
    - Estimation error maximization
  - Unintuitive
    - No way to incorporate investor's view
    - No way to incorporate investor's confidence level
    - No intuitive starting point for expected return
    - Complete set of expected return is required

What is the solution ?

Agenda

Part 2

Black-Litterman Model

Black-Litterman model

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- **What is the B-L Model**
  - **The B-L model was created by Fisher Black and Robert Litterman of Goldman Sachs**
  - **The B-L model uses a Bayesian approach to combine the subjective view of an investor regarding the expected return of one or more assets with the market equilibrium vector of expected returns (the prior distribution) to form a new mixed estimate of expected return (the posterior distribution)**
  - **However, it's not a source of Alpha itself**

Black-Litterman model

MFC Asset Management plc.

- **How does B-L model work**

1. Start with the Market Returns

2. Apply your own unique views of how certain markets are going to behave

Market Return

View

Black Litterman Forecast Returns

3. The result is a set of expected returns of assets as well as the optimal portfolio

Black-Litterman model

- **How does B-L model work**
  - **Investors are unnecessary to have all assets view**
  - **If investors do not have view, they hold the market portfolio (the benchmark)**
  - **The additional view will tilt the expected return from the equilibrium market return and weights away from the market portfolio**

Black Litterman model

- **B-L model formula**

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

$E[R]$	= New Combined Return Vector (N x 1 column vector)
τ	= Scalar
Σ	= Covariance Matrix (N x N matrix)
P	= View Participation Matrix (K x N matrix that identifies the asset involved in the views)
Ω	= Diagonal covariance matrix of error terms from the expressed view representing the uncertainty in each view (K x K matrix)
Π	= Implied Excess returns over the risk free rate (N x 1 column vector)
Q	= View Vector (K x 1 column vector)

Black Litterman model

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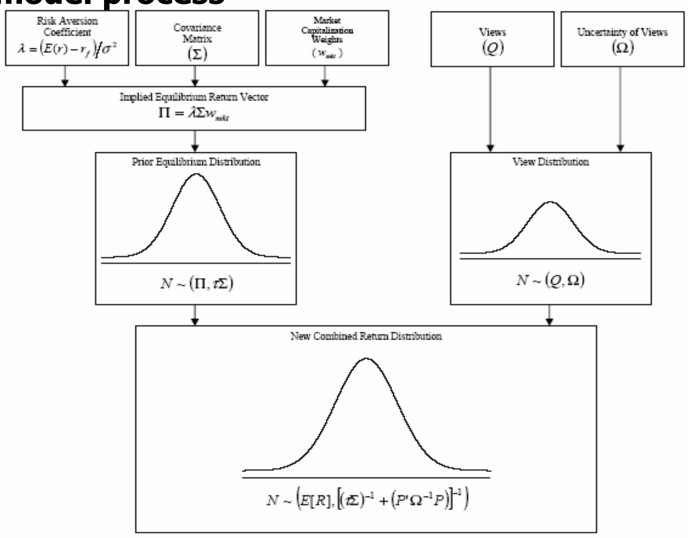
- **B-L model process**



Agenda

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Part 3

B-L Model in Practice

B-L Model in Practice

MFC Asset Management plc.

- **Compositions of Black-Litterman model**

Market Return

- Π
- Σ

View

- τ
- Q
- P
- Ω

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

$E[R]$	= New Combined Return Vector (N x 1 column vector)
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Π	= Implied Excess returns over the risk free rate (N x 1 column vector)
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B-L Model in Practice

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- **Market Return**
  - In Black-Litterman model, we use the Implied Excess Equilibrium Returns which are derived from know information using "Reverse optimization"

$$\Pi = \lambda \sum w_{mkt}$$

Π	(Pi) is the excess market return over the risk free rate
λ	(Lambda) is the risk aversion coefficient
$\sum$	(Sigma) is the covariance matrix of excess returns
w_{mkt}	Is the market capitalization weight of the asset

B-L Model in Practice

- **Market Return**
  - Risk Aversion Coefficient λ
    - Is the rate at which more return is required for more risk (Risk-return trade-off)

$$\lambda = \frac{R_{mkt} - r_f}{\sigma^2_{Mkt}} = \frac{Risk\ \mathrm{Pr}\,emium}{Variance}$$

R_{Mkt} is return of the market or benchmark

r_f is Risk-free rate

σ^2_{Mkt} is Variance of the market or benchmark

B-L Model in Practice

- **Market Return**
  - Market Capitalization or Benchmark Weights w_{mkt}

Ex : Flexible Fund Benchmark

	Asset Class	Market Capitalization Estimate		Weight
1. Bond				**50%**
2. Stock				50%
	Agro & Food Industry	1,118,732	7%	**3.6%**
	Consumer Products	143,798	1%	**0.5%**
	Financials	3,136,108	20%	**10.2%**
	Industrials	1,727,804	11%	**5.6%**
	Property & Construction	2,096,000	14%	**6.8%**
	Resources	4,497,231	29%	**14.7%**
	Services	816,320	5%	**2.7%**
	Technology	1,808,058	12%	**5.9%**
Total				**100%**

B-L Model in Practice

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- **Market Return**
  - Risk Aversion Coefficient λ

$$\lambda = \frac{Risk\ Premium}{Variance}$$

$$\lambda = \frac{1.90\%}{1.52\%}$$

$$\lambda = 1.25$$

- Assume : Risk free rate = 3%
- The Historical average market (benchmark) return = 4.9%
- The Historical long-term variance = 1.52%

Note : use historical 72 months return from 2004-2009

B-L Model in Practice

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- **Market Return**
  - Covariance Matrix of excess returns Σ

	ZRR3Y	Agro & Food Industry	Consumer Products	Financials	Industrials	Property & Construction	Resources	Services	Technology
ZRR3Y	0.13%	-0.10%	-0.05%	-0.09%	-0.19%	-0.04%	-0.14%	-0.08%	-0.06%
Agro & Food Industry	-0.10%	3.91%	1.58%	3.98%	4.96%	4.62%	4.54%	3.70%	2.65%
Consumer Products	-0.05%	1.58%	1.18%	1.50%	2.03%	2.04%	1.91%	1.61%	1.11%
Financials	-0.09%	3.98%	1.50%	6.83%	6.96%	6.67%	6.23%	4.89%	4.03%
Industrials	-0.19%	4.96%	2.03%	6.96%	10.29%	8.09%	8.29%	6.29%	4.71%
Property & Construction	-0.04%	4.62%	2.04%	6.67%	8.09%	7.91%	6.99%	5.66%	4.53%
Resources	-0.14%	4.54%	1.91%	6.23%	8.29%	6.99%	9.43%	5.57%	4.81%
Services	-0.08%	3.70%	1.61%	4.89%	6.29%	5.66%	5.57%	5.00%	3.84%
Technology	-0.06%	2.65%	1.11%	4.03%	4.71%	4.53%	4.81%	3.84%	4.73%

B-L Model in Practice

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- **Implied excess return over risk-free**

$$\Pi = \lambda \sum w_{mkt}$$

$$\begin{vmatrix} 0.02\% \\ 2.54\% \\ 1.05\% \\ 3.68\% \\ 4.51\% \\ 4.11\% \\ 4.39\% \\ 3.16\% \\ 2.69\% \end{vmatrix} = 1.25 \begin{vmatrix} 0.13\% & -0.10\% & -0.05\% & -0.09\% & -0.19\% & -0.04\% & -0.14\% & -0.08\% & -0.06\% \\ -0.10\% & 3.91\% & 1.58\% & 3.98\% & 4.96\% & 4.62\% & 4.54\% & 3.70\% & 2.65\% \\ -0.05\% & 1.58\% & 1.18\% & 1.50\% & 2.03\% & 2.04\% & 1.91\% & 1.61\% & 1.11\% \\ -0.09\% & 3.98\% & 1.50\% & 6.83\% & 6.96\% & 6.67\% & 6.23\% & 4.89\% & 4.03\% \\ -0.19\% & 4.96\% & 2.03\% & 6.96\% & 10.29\% & 8.09\% & 8.29\% & 6.29\% & 4.71\% \\ -0.04\% & 4.62\% & 2.04\% & 6.67\% & 8.09\% & 7.91\% & 6.99\% & 5.66\% & 4.53\% \\ -0.14\% & 4.54\% & 1.91\% & 6.23\% & 8.29\% & 6.99\% & 9.43\% & 5.57\% & 4.81\% \\ -0.08\% & 3.70\% & 1.61\% & 4.89\% & 6.29\% & 5.66\% & 5.57\% & 5.00\% & 3.84\% \\ -0.06\% & 2.65\% & 1.11\% & 4.03\% & 4.71\% & 4.53\% & 4.81\% & 3.84\% & 4.73\% \end{vmatrix} \begin{vmatrix} 50.00\% \\ 3.65\% \\ 0.47\% \\ 10.22\% \\ 5.63\% \\ 6.83\% \\ 14.65\% \\ 2.66\% \\ 5.89\% \end{vmatrix}$$

B-L Model in Practice

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- **Market Return**

Asset Class	Implied Excess Return	Risk-free Rate	Total Implied return
Bond	0.02%	3.00%	3.02%
Agro & Food Industry	2.54%	3.00%	5.54%
Consumer Products	1.05%	3.00%	4.05%
Financials	3.68%	3.00%	6.68%
Industrials	4.51%	3.00%	7.51%
Property & Construction	4.11%	3.00%	7.11%
Resources	4.39%	3.00%	7.39%
Services	3.16%	3.00%	6.16%
Technology	2.69%	3.00%	5.69%

B-L Model in Practice

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- **View**
  - **The scalar** τ
    - The scalar is more or less inversely proportional to the relative weight given to the Implied equilibrium return vector Π
    - It's depended on the confidence in the expressed view of the investor
    - It's an abstract parameter
    - Many practitioner say that τ = 0.3 is plausible

B-L Model in Practice

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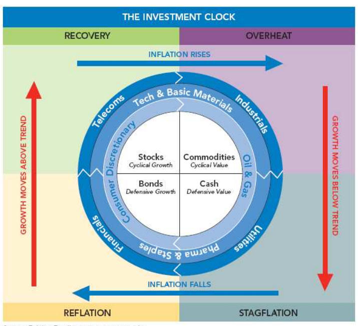
- **View**
  - **View Vector** Q **(K x 1) ; K is number of view**
    - Investors can incorporate their views regarding the expected return of some of the asset in a portfolio, which differ from the Implied equilibrium return
    - The views can be expressed in either absolute or relative terms.



Source: Fidelity. For illustration purposes only

<u>Example of view</u>

As market is starting to the recovery phase, investor manager has several views

View 1: Bond will have an absolute excess return of 0%

(less than 0.02% of equilibrium implied return)

View 2: Resources will outperform Consumer product by 2%

(less than 3.34% outperform in equilibrium implied return)

View 3: Technology and Industrial will outperform Financial and Resources by 0.1%

(Actually, Technology and Industrial are underperformed to Financial and Resources by 0.52% in equilibrium implied return)

B-L Model in Practice

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- **View**
  - **View Vector Q (K x 1) ; K is number of view**

$$Q = \begin{bmatrix} 0\% \\ 2\% \\ 0.1\% \end{bmatrix} \begin{matrix} \text{View 1} \\ \text{View 2} \\ \text{View 3} \end{matrix}$$

Example of view

View 1: Bond will have an absolute excess return of 0% (Confidence of View = 60%)

View 2: Resources will outperform Consumer product by 2% (Confidence of View = 30%)

View 3: Technology and Industrial will outperform Financial and (Confidence of View = 15%)

B-L Model in Practice

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- **View**
  - **The view participation matrix P (K x N) ; N is number of asset class**
    - Each row of the matrix represent the views expressed in the view vector Q
    - If an absolute view is presented, sum of that row will equal to 1
    - In the case of relative view, that row sums to 0

$$\begin{array}{c} \begin{matrix} \text{Bond} & \text{Agro \& Food Industry} & \text{Consumer Products} & \text{Financials} & \text{Industrials} & \text{Property \& Construction} & \text{Resources} & \text{Services} & \text{Technology} \end{matrix} \\ P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.41 & 0.49 & 0 & -0.59 & 0 & 0.51 \end{bmatrix} \begin{matrix} \text{View 1} \\ \text{View 2} \\ \text{View 3} \end{matrix} \end{array}$$

Example of view

View 1: Bond will have an absolute excess return of 0% (Confidence of View = 60%)

View 2: Resources will outperform Consumer product by 2% (Confidence of View = 30%)

View 3: Technology and Industrial will outperform Financial and (Confidence of View = 15%)

B-L Model in Practice

- **View**
  - **The view participation matrix P (K x N) ; N is number of asset class**
    - **View 1 is an example of an absolute view of Bond**
    - **View 2 and 3 represent relative view**
      - In view 2, the outperformed asset by the view is marked by 1 ,on the other hand, the underperformed asset is marked by -1
      - In view 3, we use weighted average among the set of outperformed asset for positive marks in the matrix and weighted average among the set of underperformed asset for negative marks in the matrix

$$
\begin{array}{c}
\begin{array}{ccccccccc} \text{Bond} & \text{Agro \& Food Industry} & \text{Consumer Products} & \text{Financials} & \text{Industrials} & \text{Property \& Construction} & \text{Resources} & \text{Services} & \text{Technology} \end{array} \\
P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -0.41 & 0.49 & 0 & -0.59 & 0 & 0.51 \end{bmatrix} \begin{array}{l} \text{View 1} \\ \text{View 2} \\ \text{View 3} \end{array}
\end{array}
$$

B-L Model in Practice

- **View**
  - **The Covariance matrix of error terms of the expressed view Ω (K x K) ; K is number of view**
    - Ω is assumed to be diagonal matrix because model assumes that the views are independent of one another

$$\Omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix}$$

    - The variance of an individual view portfolio ω_k is $p_k \Sigma p_k$

Variance of the view Portfolio

View	Formula	Variance
1	$p_1\Sigma p_1$	0.13%
2	$p_2\Sigma p_2$	6.79%
3	$p_3\Sigma p_3$	1.32%

B-L Model in Practice

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- Now investors can input all variables in Black-Litterman formula

$$E[R]=\left[(\tau\Sigma)^{-1}+P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi+P'\Omega^{-1}Q\right]$$

- The combined return vector $E[R]$ will be used to find the optimal weight of the portfolio with investors views

- Rearrange $\mu=\lambda\sum w_{mkt}$ formula

  The solution to the unconstrained maximization problem : $\max_{w} w'\mu-\lambda w'\Sigma w/2$

  is $\boxed{w=(\lambda\Sigma)^{-1}\mu}$

  if μ does not equal Π , w will not equal to w_{Mkt}

B-L Model in Practice

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- **The Optimal portfolio**

$$w=(\lambda\Sigma)^{-1}\mu$$

$$\begin{bmatrix} 42.00\% \\ 3.65\% \\ 6.79\% \\ 3.98\% \\ 13.05\% \\ 6.83\% \\ 1.00\% \\ 2.66\% \\ 13.15\% \end{bmatrix} = 1.25 \begin{bmatrix} 0.13\% & -0.10\% & -0.05\% & -0.09\% & -0.19\% & -0.04\% & -0.14\% & -0.08\% & -0.06\% \\ -0.10\% & 3.91\% & 1.58\% & 3.98\% & 4.96\% & 4.62\% & 4.54\% & 3.70\% & 2.65\% \\ -0.05\% & 1.58\% & 1.18\% & 1.50\% & 2.03\% & 2.04\% & 1.91\% & 1.61\% & 1.11\% \\ -0.09\% & 3.98\% & 1.50\% & 6.83\% & 6.96\% & 6.67\% & 6.23\% & 4.89\% & 4.03\% \\ -0.19\% & 4.96\% & 2.03\% & 6.96\% & 10.29\% & 8.09\% & 8.29\% & 6.29\% & 4.71\% \\ -0.04\% & 4.62\% & 2.04\% & 6.67\% & 8.09\% & 7.91\% & 6.99\% & 5.66\% & 4.53\% \\ -0.14\% & 4.54\% & 1.91\% & 6.23\% & 8.29\% & 6.99\% & 9.43\% & 5.57\% & 4.81\% \\ -0.08\% & 3.70\% & 1.61\% & 4.89\% & 6.29\% & 5.66\% & 5.57\% & 5.00\% & 3.84\% \\ -0.06\% & 2.65\% & 1.11\% & 4.03\% & 4.71\% & 4.53\% & 4.81\% & 3.84\% & 4.73\% \end{bmatrix}^{-1} \begin{bmatrix} 0.01\% \\ 2.21\% \\ 0.96\% \\ 3.12\% \\ 3.97\% \\ 3.61\% \\ 3.50\% \\ 2.80\% \\ 2.44\% \end{bmatrix}$$

Bond	Agro & Food Industry	Consumer Products	Financials	Industrials	Property & Construction	Resources	Services	Technology
42.00%	3.65%	6.79%	3.98%	13.05%	6.83%	1.00%	2.66%	13.15%

So now we have that optimal portfolio weight

Agenda

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Part4

Result & Conclusion

Result and Conclusion

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- **Returns Comparison**

Asset Class	Historical	Implied Equilibrium excess return Vactor	B-L Combined excess return vector
Bond	3.89%	0.02%	0.01%
Agro & Food Industry	16.39%	2.54%	2.21%
Consumer Products	4.04%	1.05%	0.96%
Financials	4.24%	3.68%	3.12%
Industrials	-0.86%	4.51%	3.97%
Property & Construction	-1.40%	4.11%	3.61%
Resources	12.25%	4.39%	3.50%
Services	5.25%	3.16%	2.80%
Technology	1.94%	2.69%	2.44%

- Assume : Risk free rate = 3%
- use historical 72 months return from 2004-2009

Result and Conclusion

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Portfolio Weight

Asset Class	Weight Based on Historical	Weight Based on Historical with constrain	Weight Based on Implied Equilibrium excess return Vactor (Mkt weight)	Weight based on B-L Combined excess return vector	Dif (B-L - Mkt Weight)
Bond	3442.96%	81.88%	50.00%	42.00%	-8.00%
Agro & Food Industry	1202.27%	18.12%	3.65%	3.65%	0.00%
Consumer Products	104.76%	0.00%	0.47%	6.79%	6.32%
Financials	378.49%	0.00%	10.22%	3.98%	-6.23%
Industrials	1.22%	0.00%	5.63%	13.05%	7.42%
Property & Construction	-1243.74%	0.00%	6.83%	6.83%	0.00%
Resources	222.38%	0.00%	14.65%	1.00%	-13.65%
Services	-32.28%	0.00%	2.66%	2.66%	0.00%
Technology	42.53%	0.00%	5.89%	13.15%	7.26%
High	3442.96%	81.88%	50.00%	42.00%	7.42%
Low	-1243.74%	0.00%	0.47%	1.00%	-13.65%

Result and Conclusion

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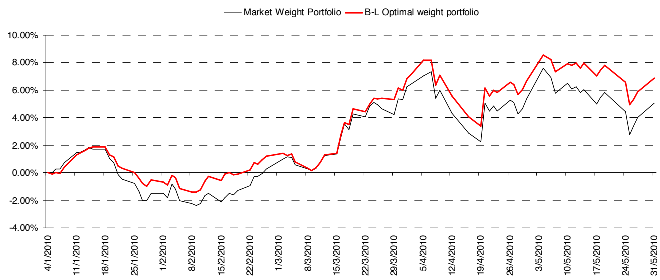
Performance

Chart : 2010 year to date performance (As of 31 may 2010)



	Market Weight Portfolio	B-L Optimal Weight Portfolio
Year to Date return	5.05%	6.90%
Average return (p.a.)	13.74%	18.58%
Standard deviation (p.a.)	11.45%	10.42%
Sharpe ratio	1.20	1.78

Final words ☺

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End

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Back up

Equilibrium Returns (1)

- Equilibrium Return
- =current Market collective forecasts of next period returns; i.e., the market's collective view on future returns
- =reverse optimized returns
- this Market View is to be combined with Our View; and the combination (using GLS) will take the estimation error of either views into consideration.

Equilibrium Returns (2)

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- **Assume Market has the following attributes**
  - **N assets**
  - **Expected Return vector μ[Nx1]**
  - **Expected covariance Matrix ∑[NxN]**

Equilibrium Returns (3)

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- **Today when the trades took place, market collectively reached the equilibrium (supply = demand).**
- **To do this it had ran the Markowitz mean-variance optimization and reached the optimized weights w[Nx1] – which are the current market capitalization weights**

Equilibrium Returns (4)

- Max $[w'\mu - (\lambda/2)w'\Sigma w]$
  - Note: This is derived from the utility theory and multivariate normal distribution – Financial Economics 101

  λ = risk aversion coefficient (E(M) –rf)/σ(mkt)^2)

  E(M) = Expected market or benchmark total return

  λ is found from historical data (approx = 3.07)
  - Solve $\delta w'\mu / \delta w - \delta((\lambda/2)w'\Sigma w)/ \delta w = 0$
  - They got $\mu = \lambda \Sigma w$
    - Note: two most important matrix derivation formula $\delta w'\mu / \delta w = \mu$ and $\delta(w'\Sigma w)/ \delta w = 2\Sigma w$

Equilibrium Returns using Implied Beta

- Equilibrium Returns can be calculated by using the "implied Beta" of assets.
  - μ = β(implied)*(risk premium of market)
  - Implied β = ∑*w(mkt)/(w(mkt)T*∑w(mkt))

    The denominator is basically the variance of market portfolio. The numerator is the covariance of the assets in the market portfolio. Asset weights are the equilibrium weights. Covariance matrix ∑ is historical covariance.

What is the estimation error of the Equilibrium Returns?

- A controversial issue in BL model.
- Since the equilibrium returns are not actually estimated, the estimation error cannot be directly derived.
- But we do know that the estimation error of the means of returns $\sigma_{E[r(i,t+1)]}$ should be less than the covariance of the returns.
- A scalar τ less than 1 is used to scale down the covariance matrix (Σ) of the returns.
- Some say that "τ =0.3 is plausible".

Forming Our View (1)

- **Our view is:**
- $Q=Pu+\eta,\ \mu\sim\Phi(0,\Omega)$
- Note: same as $Pu=Q+\eta$, because $\eta\sim\Phi(0,\Omega) \Leftrightarrow -\eta\sim\Phi(0,\Omega)$
- u is the expected future returns (a NX1 vector of random variables).
- Ω is assumed to be diagonal *(but is it necessary?)*

Forming Our View (2)

- **What does this $Q=P*u+\eta$, Or equivalently $P*u=Q+\eta$ mean?**
- **Look at P*u:**

  each row of P represents a set of weights on the N assets, in other words, each row is a portfolio of the N assets. (aka "view portfolio")

  u is the expected return vector of the N assets
- **P*u means we are expressing our views through k view portfolios.**

Forming Our View (3)

- **Our Part 3 Numerical Example will show some examples of the process of expressing views.**
- **The Goldman Sachs Enigma is how they express views quantitatively.**

Forming **Our View** (4)

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- **Why is expressing views so important?**

- **Because the practical value of BL model lies in the View Expressing Scheme; the model itself is just a publicly available view combining engine.**
  - **Our view is the source of alpha.**
  - **Expressing views quantitatively means efficiently and effectively translate fundamental analyses into Views**

Combining Views (1)

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$$\begin{cases} \hat{\mu} = \mu + \varepsilon & \varepsilon \sim \Phi(0, \tau\hat{\Sigma}) \\ Q = P\mu + \eta & \eta \sim \Phi(0, \hat{\Omega}) \end{cases}$$

If we let $Y = \begin{pmatrix} \hat{\mu} \\ Q \end{pmatrix}$, $X = \begin{pmatrix} I \\ P \end{pmatrix}$, and $u = \begin{pmatrix} \varepsilon \\ \eta \end{pmatrix}$. Then firstly $u \sim \Phi(0, \Psi)$, where $\Psi = \begin{pmatrix} \tau\hat{\Sigma} & 0 \\ 0 & \hat{\Omega} \end{pmatrix}$. This results in the regression equation $Y = X\mu + u$, which leads to the following predictive mean for expected returns:

Generalized Least Square Estimator of μ $\longrightarrow$ $\mu^{Comb} = (X'\Psi^{-1}X)^{-1}X'\Psi^{-1}Y$

$$\mu^{Comb} = [\begin{pmatrix} I & P' \end{pmatrix} \begin{pmatrix} \tau\hat{\Sigma} & 0 \\ 0 & \hat{\Omega} \end{pmatrix}^{-1} \begin{pmatrix} I \\ P \end{pmatrix}]^{-1} \begin{pmatrix} I & P' \end{pmatrix} \begin{pmatrix} \tau\hat{\Sigma} & 0 \\ 0 & \hat{\Omega} \end{pmatrix}^{-1} \begin{pmatrix} \hat{\mu} \\ Q \end{pmatrix}$$

$$= [\begin{pmatrix} (\tau\hat{\Sigma})^{-1} & P'\hat{\Omega}^{-1} \end{pmatrix} \begin{pmatrix} I \\ P \end{pmatrix}]^{-1} \begin{pmatrix} (\tau\hat{\Sigma})^{-1} & P'\hat{\Omega}^{-1} \end{pmatrix} \begin{pmatrix} \hat{\mu} \\ Q \end{pmatrix}$$

$$= [(\tau\hat{\Sigma})^{-1} + P'\hat{\Omega}^{-1}P]^{-1}[(\tau\hat{\Sigma})^{-1}\hat{\mu} + P'\hat{\Omega}^{-1}Q]$$

Combining Views (1)

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$$\begin{cases}\hat{\mu} = \mu + \varepsilon & \varepsilon \sim \Phi(0, \tau\hat{\Sigma}) \\ Q = P\mu + \eta & \eta \sim \Phi(0, \hat{\Omega})\end{cases}$$

If we let $Y = \begin{pmatrix}\hat{\mu} \\ Q\end{pmatrix}$, $X = \begin{pmatrix}I \\ P\end{pmatrix}$, and $u = \begin{pmatrix}\varepsilon \\ \eta\end{pmatrix}$. Then firstly $u \sim \Phi(0, \Psi)$, where $\Psi = \begin{pmatrix}\tau\hat{\Sigma} & 0 \\ 0 & \hat{\Omega}\end{pmatrix}$. This results in the regression equation $Y = X\mu + u$, which leads to the following predictive mean for expected returns:

Generalized Least Square Estimator of μ

$$\mu^{Comb} = (X'\Psi^{-1}X)^{-1}X'\Psi^{-1}Y$$

$$\begin{aligned}\mu^{Comb} &= [\begin{pmatrix}I & P'\end{pmatrix}\begin{pmatrix}\tau\hat{\Sigma} & 0 \\ 0 & \hat{\Omega}\end{pmatrix}^{-1}\begin{pmatrix}I \\ P\end{pmatrix}]^{-1}\begin{pmatrix}I & P'\end{pmatrix}\begin{pmatrix}\tau\hat{\Sigma} & 0 \\ 0 & \hat{\Omega}\end{pmatrix}^{-1}\begin{pmatrix}\hat{\mu} \\ Q\end{pmatrix} \\ &= [\begin{pmatrix}(\tau\hat{\Sigma})^{-1} & P'\hat{\Omega}^{-1}\end{pmatrix}\begin{pmatrix}I \\ P\end{pmatrix}]^{-1}\begin{pmatrix}(\tau\hat{\Sigma})^{-1} & P'\hat{\Omega}^{-1}\end{pmatrix}\begin{pmatrix}\hat{\mu} \\ Q\end{pmatrix} \\ &= [(\tau\hat{\Sigma})^{-1} + P'\hat{\Omega}^{-1}P]^{-1}[(\tau\hat{\Sigma})^{-1}\hat{\mu} + P'\hat{\Omega}^{-1}Q]\end{aligned}$$

Combining Views (2)

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$$\hat{\mu} = \mu + \varepsilon \quad \varepsilon \sim \Phi(0, \tau\hat{\Sigma})$$
$$Q = P\mu + \eta \quad \eta \sim \Phi(0, \hat{\Omega})$$

If we let $Y = \begin{pmatrix}\hat{\mu} \\ Q\end{pmatrix}$, $X = \begin{pmatrix}I \\ P\end{pmatrix}$, and $u = \begin{pmatrix}\varepsilon \\ \eta\end{pmatrix}$. Then firstly $u \sim \Phi(0, \Psi)$, where $\Psi = \begin{pmatrix}\tau\hat{\Sigma} & 0 \\ 0 & \hat{\Omega}\end{pmatrix}$. This results in the regression equation $Y = X\mu + u$, which leads to the following predictive mean for expected returns:

$$\mathrm{Var}(\mu^{Comb}) = (X'\Psi^{-1}X)^{-1}$$

$$\begin{aligned}&= [\begin{pmatrix}I & P'\end{pmatrix}\begin{pmatrix}\tau\hat{\Sigma} & 0 \\ 0 & \hat{\Omega}\end{pmatrix}^{-1}\begin{pmatrix}I \\ P\end{pmatrix}]^{-1} \\ &= [\begin{pmatrix}(\tau\hat{\Sigma})^{-1} & P'\hat{\Omega}^{-1}\end{pmatrix}\begin{pmatrix}I \\ P\end{pmatrix}]^{-1} \\ &= [(\tau\hat{\Sigma})^{-1} + P'\hat{\Omega}^{-1}P]^{-1}\end{aligned}$$

Now we have a combined forecast of the expected returns.

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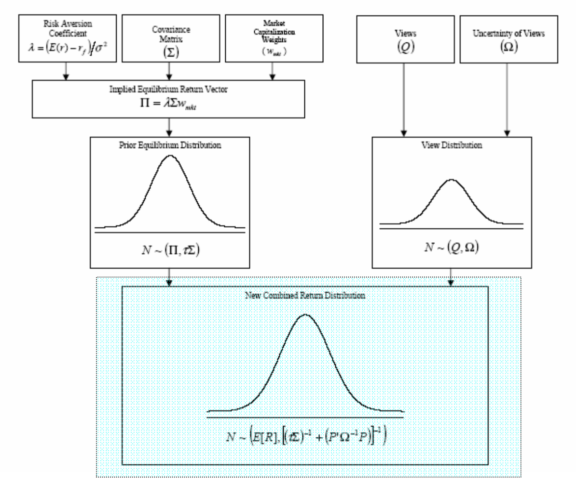

The next step is to do Markowitz Mean-Variance Optimization.

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- **By using the combined forecasted means**
- **and the foreca:** $\left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$
- **So we start with Markowitz (reverse optimization) and CAPM (implied beta).**
- **Go though Black-Litterman View Combining engine.**
- **And end up with Markowitz again with predictive means, (and forward looking return covariance matrix.)**

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