OIS Discounting and Dual-Curve Stripping Methodology at Bloomberg

Abstract

The recent Credit Crises led to profound changes in OTC derivatives market practice, as it is now common that transactions are collateralized in order to mitigate counterparty credit risk. These changes together with issues related to the LIBOR quality/credibility, led to market adoption of the overnight index swap (OIS) as the "new risk-free swap curve." This paper describes the methodology used at Bloomberg for the evaluation of interest rate swaps in the post credit crunch era. It is a compilation of several existing Bloomberg documents and covers topics such as the new swap math for OIS discounting and dual-curve (DC) stripping, impacts on forward projection, extension of OIS curves beyond the maturity of available market quotes, ramifications of OIS discounting on cross-currency swaps (CCS), among others.

Introduction

The credit crisis that began in 2007 led to various changes in the over-the-counter (OTC) interest rate (IR) derivatives market, which is the largest market of the world with $550 trillion outstanding notional and comprises 80 percent of all OTC derivatives [1]. As financial firms began racking up losses and write downs that totaled more than $1.7 trillion by late 2009, questions about the liquidity and creditworthiness of big banks affected how deals such as interest-rate swaps are entered into and valued. Deteriorating credit and liquidity issues in the LIBOR swap market were accompanied by a widening of various risk indicators such as the LIBOR/LIBOR and LIBOR/OIS basis spreads, reflecting both idiosyncratic and systematic risk through the banking system.

Market participants began to question whether the London interbank offered rate could maintain its role as the short-term interest rate benchmark for OTC derivatives. Dealers were under increasing pressure to mitigate counterparty risk associated with OTC derivative transactions, especially after the Lehman collapse and its default on OTC derivative counterparty obligations, and moved toward use of more stringent CSA agreements requiring daily margin/collateral calculations and maintenance. Consistent with these market trends, and consistent with subsequent regulation (such as Dodd Frank in the U.S.), from 2009 onwards, market participants increased their use of Central Counterparties (CCP’s) such as LCH Clearnet, which requires daily collateral maintenance.

Central banks, such as the U.S. Federal Reserve, continued to provide abundant liquidity via their bank lending windows using Fed Funds (“cash”) and short dated T-Bills/Govies. These short dated “risk free” assets became the acceptable deliverable assets for collateral maintenance. Additionally, the continued decline of unsecured short-term funding (in conjunction with regulatory investigations into the computation of LIBOR by large banks that are members of the BBA panels) led to distrust of LIBOR as an "accurate and fair" market rate fixing. The combination of these various market trends led toward market adoption of the overnight index swap (OIS) curve as the "new risk-free swap curve.”

New OIS/DC Swap Math

OIS discounting means discounting the expected cash flows of a derivative using a nearly risk free curve such as an overnight index swap (OIS) curve. With OIS-discounting, swap rates are calculated using a different formula than its single-curve counterpart, and the OIS-discounted IR curves are built using a dual-curve (DC) stripping technique. Presented below is a summary of the derivation of OIS/DC swap equation as described in [2] and [3].

Given $t \leq T_{i-1} < T_i$, we denote by $L(T_{i-1}, T_i)$ the LIBOR rate with tenor $T_i - T_{i-1}$ set at time T_{i-1}. The associated year fraction for the interval $(T_{i-1}, T_i]$ is denoted by t_i. Let us define the forward LIBOR $L_i(t)$ as the fixed rate to be exchanged at time T_i for the LIBOR rate $L(T_{i-1}, T_i)$ so that the swap has zero value at time t. It can be shown that $L_i(t)$ has the following properties:

1. It coincides with the classically-defined forward rate in the limit case of a single interested rate curve.
2. At the reset time T_{i-1}, $L_i(T_{i-1})$ coincides with the LIBOR rate $L(T_{i-1}, T_i)$.
3. Its time-0 value $L_i(0)$ can be stripped from market data.

Now let us consider an interest rate swap IRS where the floating leg pays in arrears (at time T_i) the LIBOR rate $L(T_{i-1}, T_i)$ set in advance (at time T_{i-1}), where $i = a+1, \cdots, b$, and the fixed leg pays the fixed rate K at times $T_{c+1}^S, \cdots, T_d^S$ with t_j^S denoting the year fraction of $(T_{j-1}^S, T_j^S]$. Then the time-t IRS value to the fixed-rate payer is given by

$$\mathrm{IRS}\left(t, K;\ T_a\ , \cdots, T_b\ , T_c^S, \cdots, T_d^S\right) = \sum_{i=a+1}^{b} L_i(t)\, t_i\, D(t, T_i) - K\ \sum_{j=c+1}^{d} t_j^S\, D\left(t, T_j^S\right)$$

where $D(t, T)$ denotes the OIS discount factor from $t\ to\ T$. Therefore the corresponding forward swap rate, which is the fixed rate *K* that make IRS value equal to zero at time *t*, is define by

$$S_{a,b,c,d}(t) = \frac{\sum_{i=a+1}^{b} L_i(t)\, t_i\, D(t, T_i)}{\sum_{j=c+1}^{d} t_j^S\, D(t, T_j)} \tag{1}$$

In the particular case of spot-starting swap, with payment times for the floating and fixed legs given by $T_1\ , \cdots, T_b$ and $T_1^S, \cdots, T_d^S$, respectively, with $T_b\ = T_d^S$, the swap rate becomes

$$S_{0,b,0,d}(t) = \frac{\sum_{i=1}^{b} L_i(0)\, t_i\, D(0, T_i)}{\sum_{j=1}^{d} t_j^S\, D(0, T_j)} \tag{2}$$

where $L_1(0)$ is the constant first reset rate known at time 0. As traditionally done in any bootstrapping algorithm, equation (2) can be used to infer the expected (risk-free) rates L_i implied by the market quotes of spot-starting swaps, which by definition have zero value, given an already stripped OIS discount curve.

Impacts of OIS/DC [4]

Bloomberg has recently implemented OIS/DC stripping within its swaps/derivatives platform consistent with the above new swap math. When calibrated to the same set of swap rates, OIS/DC stripped swap curve will produce slightly different forward rates, and consequently forward swap rates, compared to those from the traditional single-curve approach. For example, 6M forwards for GBP, as shown in Figure 1, are adjusted down by up to 3 bps using OIS/DC. Hedging swap and fixed income portfolios has also become difficult by these valuation uncertainties, as OIS discounting of cash flows result in "longer durations" when compared to LIBOR swap curves. For example, for par spot-start euro 30-year swaps, the durations and DV01's etc, can be as much as five percent "longer" compared to those metrics obtained assuming traditional Libor swap discounting.

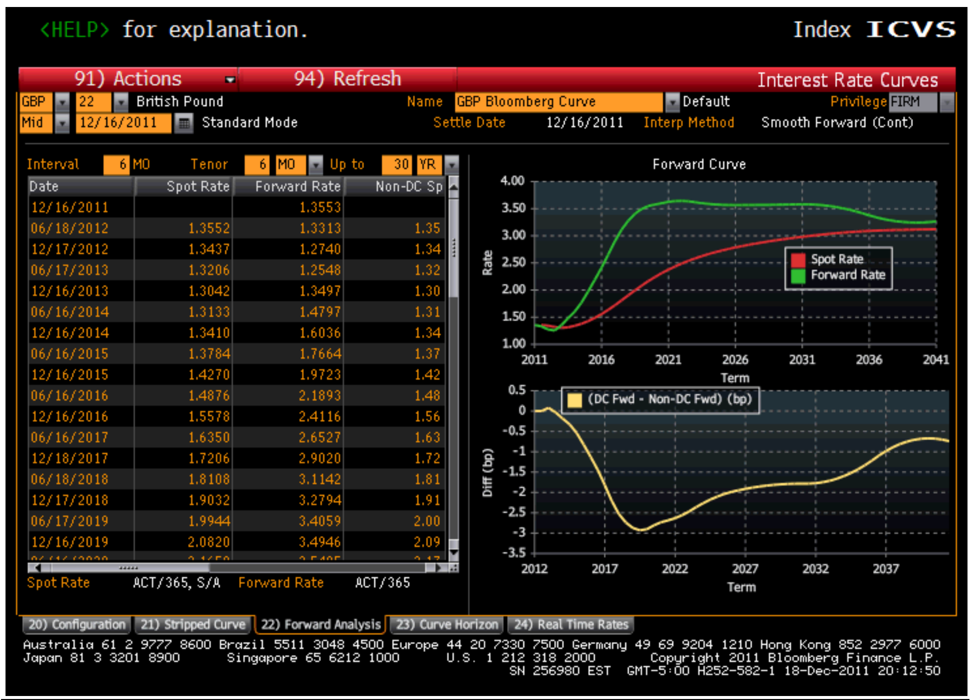


Figure 1 Comparing effect of OIS/DC and "Traditional" LIBOR (non-OIS/DC) stripping on 6MO GBP LIBOR forwards

As the buy side community has lagged behind the dealers in its adoption of the OIS/DC valuation, this has produced difficulty for the buy side to validate dealer quotes for various products that are significantly affected by the OIS/DC effect, such as for forward start swaps. For example, consider a corporate treasurer hedging their forward liabilities by receiving fixed coupons on 100 million-pound notional five-year swap forward-starting seven years from now. OIS/DC valuation implies a par coupon of 3.62 percent, (compared to a non-OIS/DC par coupon of 3.65 percent). This results in a difference of 120,000 pounds in swap market value that more closely matches the "street" quotes.

Additionally, spot swap unwind values (particularly for swaps significantly away from "par") are significantly affected by the discount curve choice. Consider an insurance company that hedged its bond assets (possibly via an asset swap or some other structured hedge) six months ago, by paying fixed 3.31percent, for 100 million Euros for 10 years. With 9 1/2 years of swap maturity remaining, and as Euro market rates have dropped around 90 basis points, the market value loss incurred on this swap can vary from 9.15 million Euros via LIBOR (EURIBOR) swap discounting to as much as 9.35 million Euros via OIS/DC and 9.55 million Euros via USD/EUR implied cross currency funding.

Extension of USD OIS Curves

Currently for USD, OIS rates are not widely available in the marketplace beyond the 10-year maturity. In order to support OIS/DC stripping, the OIS curve is extended beyond the 10-year maturity by harnessing USD Fed Funds (FF) basis swap quotes, that are available to 30-year maturity, since both OIS and FF basis are stripped to project forwards of the FF Effective Rate, FEDL01, and sensible inferences can be made from each other in an arbitrage-free environment Two methods have been developed to imply OIS rates from FF basis swaps [5].

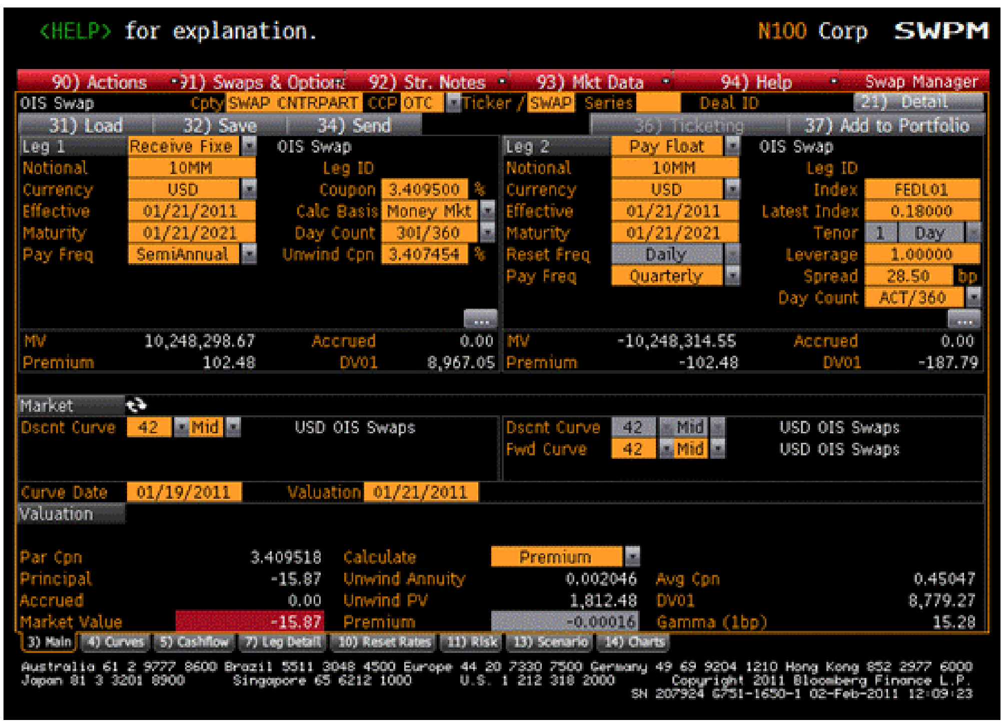


Figure 2 Illustration of method 1 that bootstraps OIS rates to match FF basis quotes

Method 1 bootstraps the OIS curve by evaluating FF basis swaps with FEDL01 projected from the OIS curve. Figure 2 illustrates this bootstrapping process using a custom {SWPM –FF <go>} screen, with its implementation details found in Appendix A. Suppose hypothetically that OIS swaps are quoted only to the 5Y maturity and we would like to calculate the 10Y OIS rate from the available market quote of 10Y FF basis spread (in conjunction with the 10Y LIBOR swap rate). A 10Y fixed-float FF basis swap can be set up as follows: a fixed leg receives 10Y Libor swap (USSWAP10), and a floating leg pays daily averaged FF rate plus basis spread (USBG10). The PVs of both fixed and floating legs are dependent on OIS discount curve S42 in which all swap rates are known except the 10Y maturity. Given an interpolation method, the 10Y OIS rate can therefore be solved by equating the PVs of both legs. In other words, the discount factor at 10Y is calibrated so that the resulting FF basis spread matches the market quote.

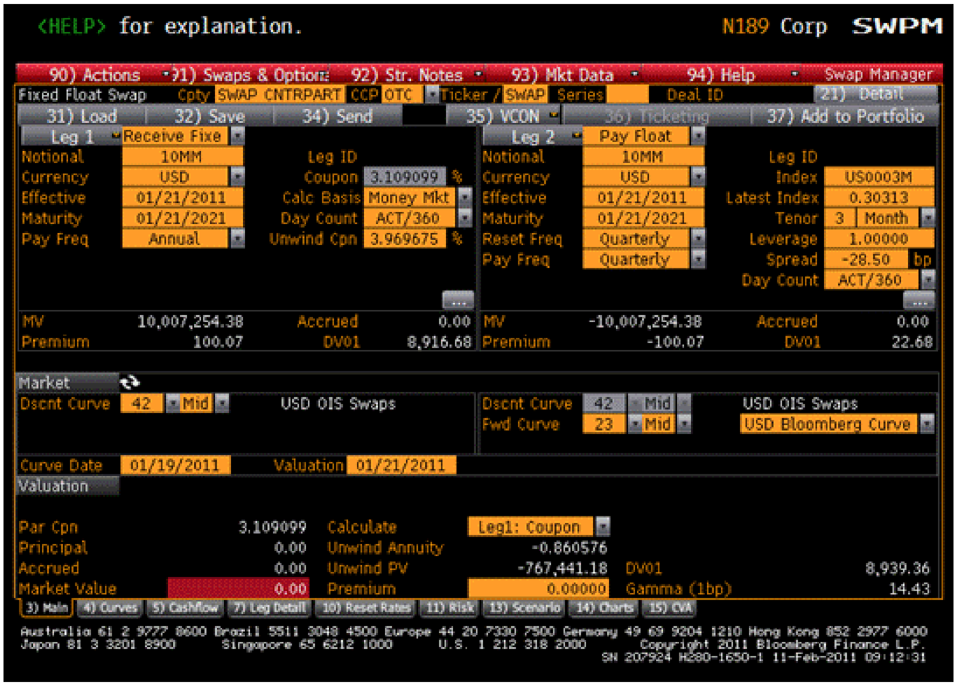


Figure 3 Rearranging FF swap for OIS calculation

Method 2 takes an alternative approach. Under the no-arbitrage assumption, OIS coupons can be calculated directly using FEDL01 projections from the FF curve. The problem with this procedure is that stripping FF curve is computationally expensive due to weighted daily average compounding in FF swaps. However, a quick but accurate approximate conversion formula can be derived based on this idea. Given its efficiency and simplicity, method 2 is the current implementation of choice in Bloomberg.

Note that if OIS and FF swaps were to have the same compounding conventions, OIS swap rates could be calculated from the LIBOR curve with FF basis spread and fixed-leg convention adjustment as shown in the custom SWPM screen in Figure 3. Unfortunately, there is no easy way to account for the compounding differences without the actual daily forward projection from the FF curve, and this compounding difference accounts for error of about 1.5bps or more in OIS swap rates under the current market conditions.

When minor discrepancies such as business day adjustment are ignored, the OIS coupon can be approximated by an analytic formula without the need to actually carry out the swap evaluations. Let s_N^L, s_N^O and b_N^{FF} denote the N-year LIBOR swap rate, OIS rate and FF basis spread, respectively. The implied OIS rate, $\hat{s}_N^O$, can be approximated as:

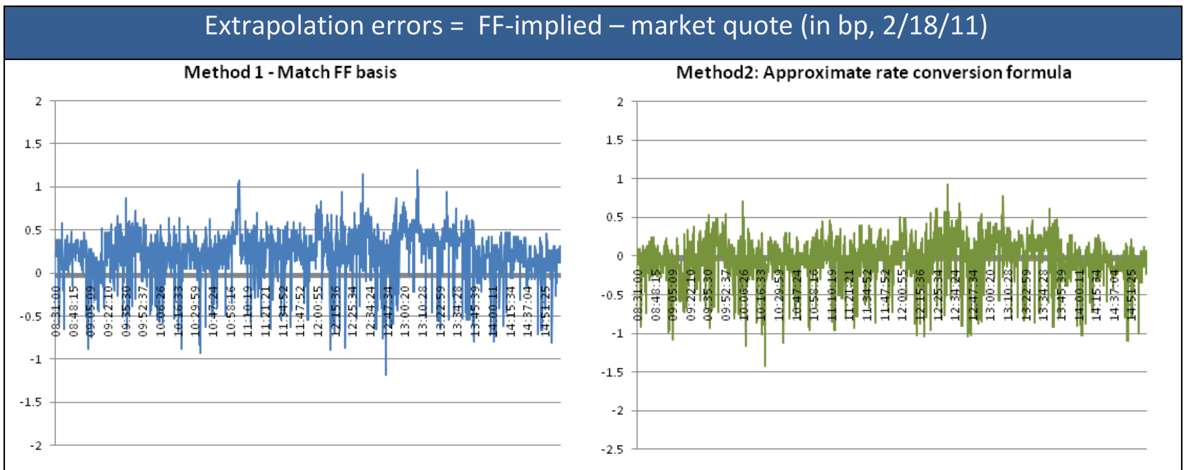
$$\hat{s}_N^O = \left(1 + \frac{(r_Q - b_N^{FF})}{4}\right)^4 - 1$$

where

$$r_Q = \left(\left(1 + \frac{s_N^L \times \frac{360}{365}}{2} \right)^{\frac{2}{4}} - 1 \right) \times 4$$

To further improve accuracy, the OIS rate $\hat{s}_N^O$ is adjusted to compensate for the compounding difference between OIS and FF based on the assumption of a flat curve. The compounding adjusted OIS rate, $\hat{s}_N'^O$, is therefore calculated as follows:

$$\hat{s}_N'^O = \hat{s}_N^O + \left(\left(1 + \frac{\hat{s}_N^O}{360} \right)^{90} - 1 - \frac{\hat{s}_N^O}{4} \right) \times 4 = \left(\left(1 + \frac{\hat{s}_N^O}{360} \right)^{90} - 1 \right) \times 4$$



(a) Method 1: match FF basis (b) Method 2: approximate rate conversion

Figure 4 Cross-validation results for 10Y OIS rates based on real-time quotes on 2/18/11

In an effort to predict how well these two methods perform, cross-validation is used to evaluate them by comparing implied 10Y OIS rates against market quotes. As shown in Figure 4, there is a remarkable agreement between quoted against implied rates for both methods using real-time LIBOR, OIS and FF rates on 2/18/11. A small bias less than ¼ bp is observed, which is likely the result of not making convexity adjustment in FF basis swap valuation. Similar error characteristics are observed when they are applied to market data of various other days.

Constant LIBOR-OIS spread is applied to extend the OIS curve to maturity beyond 30 years.

OIS Discounting and Cross-Currency CSA's

Many OTC derivative transactions follow multi-currency credit support annexes (CSA's). Thus collateral may be posted in a currency other than the currency of the transaction's cash flows. Dealers and other participants may in fact choose collateral for delivery to "optimize" their transaction valuation where there exists a "cheapest to fund" option. In this case, the cash flows should be discounted using the cross-currency swaps (CCS) curve against the currency of collateral instead of the OIS curve.

This CCS curve is closely related to the OIS curve via the OIS FX-Basis. Given a float-float OIS FX-basis swap that is collateralized in currency 1, the present values (PVs) of its two legs, PV_1 and PV_2, are calculated as follows[1]:

$$\mathrm{PV}_{1,N} = \sum_{i=1}^{N} f_{1,i}^{O} \cdot t_i \cdot D_{1,i}^{O} + D_{1,N}^{O} \tag{3}$$

$$\mathrm{PV}_{2,N} = \sum_{i=1}^{N} (f_{2,i}^{O} + b_{2,N}^{FX,O}) \cdot t_i \cdot D_{2,i}^{FX} + D_{2,N}^{FX} \tag{4}$$

where

T_i	=	time of i-th cash flow payment time
t_i	=	day count fraction between times T_{i-1} and T_i
$f_{k,i}^{O}$	=	$\frac{D_{k,i-1}^{O} - D_{k,i}^{O}}{D_{k,i}^{O}}$ = OIS forward rate in $(T_{i-1}, T_i]$ for k-th leg
$D_{k,i}^{O}$, $D_{k,i}^{FX}$	=	OIS or FX discount factor from T_i to T_0 for k-th leg
$b_{2,N}^{FX,O}$	=	OIS FX-basis spread of currency 2 to currency 1

Recognizing that PV_1 is at par, the discount curve $\{D_{2,i}^{FX}\}$ stripped from $\{f_{2,i}^{O} + b_{2,N}^{FX,O}\}$ will also make PV_2 par. As a rule-of-thumb, this resulting discount curve can be very accurately approximated by a swap curve that is stripped from the implied OIS rates shifted by the basis vector, namely $\{ s_{2,N}^{O} + b_{2,N}^{FX,O} \}$.

However OIS FX-basis spreads are not as widely quoted as LIBOR FX-basis. In this case, they can be closely approximated using the LIBOR, OIS and LIBOR FX-basis rates as shown in [7]:

$$b_{2,N}^{FX,O} \approx b_{2,N}^{FX,L} - \left(s_{1,N}^{L} - s_{1,N}^{O}\right) + \left(s_{2,N}^{L} - s_{2,N}^{O}\right) \tag{5}$$

where $\{s_{k,N}^{L}\}$ and $\{s_{k,N}^{O}\}$ are the implied LIBOR and OIS rates based on the float leg conventions of currency 2. Note that the implied LIBOR and OIS rate are often slightly different than the quoted rates due to convention differences, and convention corrections from quoted rate are often needed for better approximation accuracy.

The validity of the above approximation is backed by a very strong agreement between the quoted OIS FX-basis spreads and their calculated values using Equation (5). Examples of overlaying calculated OIS FX basis spreads on the quoted USD-EUR and USD-JPY basis curves (quotes from ICPL on 9/19/11) are shown in Figure 5 and Figure 6, respectively.

[1] Any applicable FX spot rate has been absorbed into the notional.

Given Equation (5), the effects of OIS/DC on CCS curve can be easily quantified. Let $\{s_{2,N}^{FX,L}\}$ and $\{s_{2,N}^{FX,O}\}$ denote the implied swap rates the LIBOR- and OIS-calibrated CCS discount curves, respectively. Following the same reasoning as for OIS FX-basis curve, we can show that

$$S_{2,N}^{FX,L} \approx S_{2,N}^{L} + b_{2,N}^{FX,L} \tag{6}$$

$$S_{2,N}^{FX,O} \approx S_{2,N}^{O} + b_{2,N}^{FX,O} \approx S_{2,N}^{L} + b_{2,N}^{FX,L} - \left(s_{1,N}^{L} - s_{1,N}^{O}\right) \tag{7}$$

Equations (6) and (7) imply that the LIBOR-calibrated CCS curve can be interpreted as shifting the LIBOR swap curve by the associated LIBOR FX-basis spread vector, and adoption of OIS/DC further shifts the CCS curve by the spread vector between LIBOR and OIS rates in the currency of collateralization.

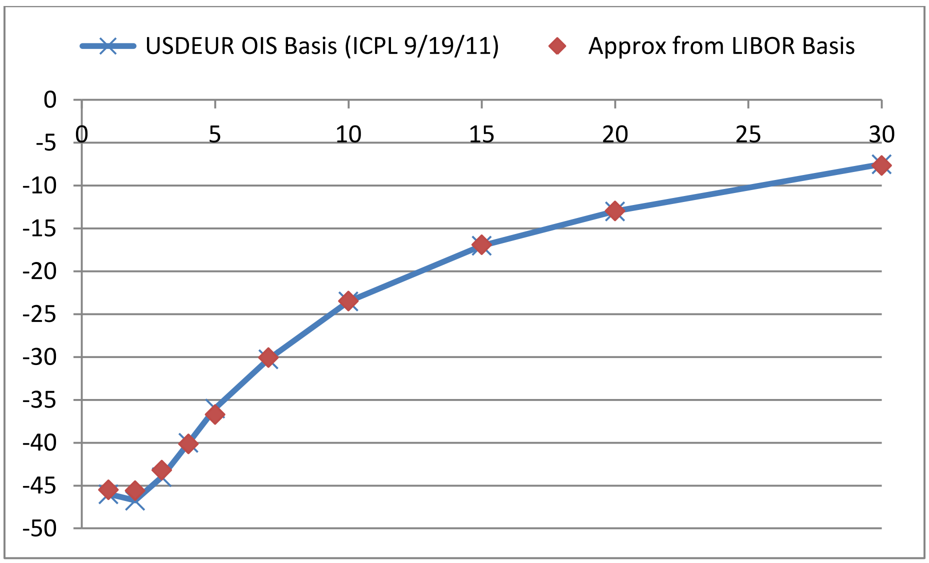


Figure 5 USD-EUR OIS basis spreads: quoted vs. approximation from LIBOR basis

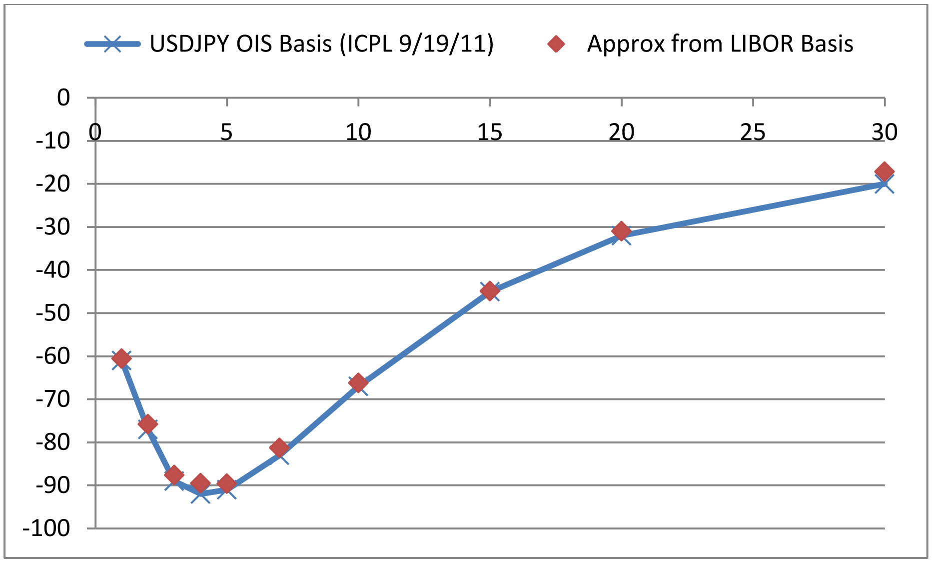


Figure 6 -JPY OIS basis spreads: quoted vs. approximation from LIBOR basis

Conclusion

This paper presents an overview of the OIS discounting and dual-curve stripping methodology currently implemented in Bloomberg. In addition to the new swap math for OIS/DC stripping and methods for extending OIS curves beyond the maturity of available market quotes, it also shows impacts of OIS/DC on forward projection and provides intuitive interpretations of CCS curves and the ramifications of adopting OIS/DC.

Appendix A: Bootstrapping OIS Curve using FF Basis Swaps

Let s_N^L, s_N^O and b_N^{FF} denote the LIBOR swap rate, OIS rate and FF basis spread of the maturity T_N, respectively. We seek to bootstrap the OIS rate, s_N^O, using s_N^L and b_N^{FF}, assuming that the OIS curve has been constructed up to the maturity T_{N-1}, pursuant to the procedure described in [6]. Constant-forward interpolation is used for the OIS curve during bootstrapping for efficiency reasons. Let

$t_{k,i}$	=	i-th cash-flow payment date in k-th leg (k=1,2)
$\Delta t_{k,i}$	=	year fraction in i-th payment period $(t_{k,i-1}, t_{k,i}]$ in k-th leg
d_t	=	discount factor at time t on the OIS curve
r_N^{df}	=	constant OIS daily-forward rate in the interval $(T_{N-1}, T_N]$
PV_k	=	PV of k-th leg

Then the PVs in a FF basis swap are evaluated as follows:

$$\mathrm{PV}_1 = s_N^L \sum_i \Delta t_{1,i}\, d_{t_{1,i}} + d_{T_N}$$

$$\mathrm{PV}_2 = \sum_j (\bar{r}_j + b_N^{FF})\, \Delta t_{2,j}\, d_{t_{2,j}} + d_{T_N}$$

where

$\bar{r}_j$ = weighted average daily forward rate in *j*-th payment period in leg 2

$$= \frac{n_{t_j,1} \cdot \left(r_N^{df} \cdot \delta \right) + \sum_{k=2}^{\cdots} n_{t_j,k} \cdot \left(\left(1 + r_N^{df} \cdot \delta \right)^k - 1 \right)}{\Delta t_{2,j}}$$

δ = year fraction of one day

$n_{t_j,k}$ = number of days in $(t_{2,j-1}, t_{2,j}]$ that accrue k days of interest

For example, $n_{t_j,3}$ is the number of Fridays in j-th period that are not part of a long weekend. Note that both PVs are function of r_N^{df}, which can be solved from $\mathrm{PV}_1 = \mathrm{PV}_2$. The bootstrapping procedure can be carried out efficiently using pre-computed $\{n_{t_j,k}\}$.

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