Separating Probability Elicitation From Utilities

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This article deals with the separation of probability elicitation from utilities. We show that elicited probabilities can be related to utilities not just through the explicit or implicit payoffs related to the elicitation process, but also through other stakes the expert may have in the events of interest. We study three elicitation procedures—lotteries, scoring rules, and promissory notes—and show how the expert's utility function and stakes in the events can influence the resulting probabilities. Particularly extreme results are obtained in an example involving a market at equilibrium. The applicability of a no-stakes condition and some implications for probability elicitation are discussed. Let π represent an expert's probability for an event A, and let p denote the elicited probability from some elicitation procedure. We determine the value of p that maximizes the expert's expected utility. When utility is linear in money, $p = \pi$ for all of the procedures studied here. Under nonlinear utility, the lottery procedure still yields $p = \pi$ as long as the expert has no other stakes involving the occurrence or nonoccurrence of A (the no-stakes condition). With the scoring-rule and promissory-note procedures, the no-stakes condition is no longer sufficient for $p = \pi$ in the presence of nonlinear utility. If the no-stakes condition holds and the elicitation-related payoffs approach 0, then $p = \pi$ in the limit. For all three procedures, the combination of nonlinear utility and other stakes can lead to values of p other than π. Furthermore, an analysis of the promissory-note procedure in a market setting gives a very extreme result: In a complete market at equilibrium for such promissory notes, the elicited probability depends on the market price, not on π. Is the no-stakes condition reasonable? We suggest that it often is not, since experts are likely to have significant stakes already, particularly in important situations. Moreover, it may be difficult to determine exactly what those stakes are (and perhaps to obtain accurate information about the expert's utility function). This creates somewhat of a dilemma for probability elicitation, implying that, at least in theory, it is difficult to separate probability elicitation from utilities.

KEY WORDS: Lotteries; No-stakes condition; Promissory notes; Scoring rules; Subjective probability.

1. INTRODUCTION

The elicitation of experts' judgments in probabilistic form is often important in Bayesian inference and decision analysis, and a key question is whether or not elicited probabilities are influenced by experts' utility functions. Some elicitation methods involve real or hypothetical payoffs and thus may be subject to utility-related effects. Furthermore, even in the absence of specific elicitation-related payoffs, implicit rewards may cause utility-related effects. We find that the existence and extent of these effects depends on other stakes that the expert may have in the events of interest. The effects are eliminated or tend to be reduced under a no-stakes condition stating that, ignoring elicitation-related payoffs, the fortune of the expert is independent of the events for which probabilities are being elicited.

The purpose of this article, then, is to study conditions under which probability elicitation can be separated from utilities and to study the impact of utilities when such separation is not possible. In Section 2 we consider the elicitation of probabilities via lotteries and show that either linear utility or the no-stakes condition is sufficient to provide assessed probabilities consistent with an expert's judgments. Violations of the no-stakes condition, however, may cause systematic shifts in elicited probabilities if the expert's utility function is nonlinear. Alternative elicitation devices, using scoring rules and promissory notes, are studied in light of the no-stakes condition and the expert's utility function in Sections 3 and 4. Section 5 shows how elicitation in a market at equilibrium can lead to extreme violations of the no-stakes condition and thus to startling and unreasonable results in terms of elicited probabilities. In Section 6 we ask when the no-stakes condition might be reasonable and find that it seems of doubtful validity in many circumstances. Some implications for probability elicitation and concluding comments are presented in Section 7.

2. LOTTERIES

When lotteries are used in probability elicitation, the general idea is for the expert to choose between two lotteries with identical payoffs. We consider the elicitation of a probability by an expert for a single event A. In the first lottery, the expert receives a reward r if A occurs and receives a "non-reward" n otherwise. In the second lottery, the expert receives r with probability p and n with probability $1 - p$, where p is an "accepted" probability from a standard device (e.g., a probability wheel). We assume without loss of generality that $n = 0$ and $r > 0$, although all that is required is $r \neq n$. The expert is asked: What value of p makes you indifferent between the two lotteries? This indifference value of p is then taken as the expert's assessed probability for A.

Let $g(f|A)$ and $g(f|\overline{A})$ represent the probability distributions of the expert's fortune given A and its complement $\overline{A}$, respectively, *not* including any payoffs from the lotteries used in the elicitation of the expert's probability for A. In addition, let U denote the expert's utility function

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for his or her fortune, including elicitation-related payoffs. Then the expert's expected utility if the first lottery is selected is

$$\mathrm{E}[U(f)|\text{Lottery 1}] = \pi \int U(f + r)g(f|A)\, df + (1 - \pi) \int U(f)g(f|\bar{A})\, df, \quad (1)$$

where π represents the expert's probability that A will occur. If the second lottery is chosen, the expected utility is

$$\begin{aligned}\mathrm{E}[U(f)&|\text{Lottery 2}] \\ &= p\left[\pi \int U(f + r)g(f|A)\, df + (1 - \pi) \int U(f + r)g(f|\bar{A})\, df\right] \\ &\quad + (1 - p)\left[\pi \int U(f)g(f|A)\, df + (1 - \pi) \int U(f)g(f|\bar{A})\, df\right]. \quad (2)\end{aligned}$$

The expert's assessed probability is the value of p for which

$$\mathrm{E}[U(f)|\text{Lottery 1}] = \mathrm{E}[U(f)|\text{Lottery 2}]. \quad (3)$$

Equating (1) with (2) and solving for p yields the following expression for the assessed odds ratio:

$$p/(1 - p) = c\pi/(1 - \pi), \quad (4)$$

where

$$c = \frac{\int [U(f + r) - U(f)]g(f|A)\, df}{\int [U(f + r) - U(f)]g(f|\bar{A})\, df}. \quad (5)$$

From (4), we see that the assessed odds ratio $p/(1 - p)$ accurately reflects the expert's judgments, as represented by $\pi/(1 - \pi)$, iff $c = 1$.

If U is linear (i.e., the expert is risk neutral), then $c = 1$; however, if we allow the possibility of U being nonlinear, then c clearly depends on $g(f|A)$ and $g(f|\bar{A})$ as well as U. For example, suppose that $g(f|A)$ and $g(f|\bar{A})$ are degenerate, placing probability 1 at f_A and $f_{\bar{A}}$, respectively. From (5), we have

$$c = \frac{U(f_A + r) - U(f_A)}{U(f_{\bar{A}} + r) - U(f_{\bar{A}})}.$$

For U increasing and strictly concave, indicating a risk-averse expert, $c < 1$ if $f_A > f_{\bar{A}}$ and $c > 1$ if $f_A < f_{\bar{A}}$. Generalizing this result beyond the degenerate case, we can say that for risk-averse experts, $c < 1$ if $g(f|A)$ stochastically dominates $g(f|\bar{A})$ and $c > 1$ if $g(f|\bar{A})$ stochastically dominates $g(f|A)$. For risk-taking experts (i.e., experts with increasing, strictly convex U), the inequalities on c are reversed.

The foregoing discussion shows that an expert's elicited probabilities and utilities can be intertwined. Under what conditions can we separate the elicitation process from utilities and say something about c without making any assumptions about the expert's utility function? A sufficient condition for $c = 1$ is

$$g(f|A) = g(f|\bar{A}) \quad \text{for all } f. \quad (6)$$

This condition implies that, apart from payoffs involving the elicitation-related lotteries, no part of the expert's fortune is contingent on whether or not A occurs. Independence of f and the events or variables for which probabilities are being elicited can be thought of in terms of the expert having no stakes in these events or variables other than stakes that might be created as part of the elicitation process through lotteries or other devices. Thus, for convenience, we refer to (6) as the *no-stakes condition.* The no-stakes condition is close in spirit to Ramsey's (1931) notion of ethical neutrality.

Discussions of lottery-based probability-elicitation methods seem to focus solely on elicitation-related payoffs and thus to assume implicitly that the no-stakes condition holds. For example, LaValle (1978, p. 79) stated that the expert's probability "should *not* depend on the choice of reference consequences" (r and n). From (5), c may well depend on r (and the choice of $n = 0$) if the no-stakes condition is not satisfied.

To investigate the behavior of c for small r, we expand c from (5) in a Taylor series in r around 0, as follows:

$$c = c_0 + rc_1 + O(r^2), \quad (7)$$

where

$$c_0 = \int U'(f)g(f|A)\, df \Big/ \int U'(f)g(f|\bar{A})\, df \quad (8)$$

is the limiting value of c as r approaches 0 and

$$c_1 = \frac{c_0}{2}\left[\frac{\int U''(f)g(f|A)\, df}{\int U'(f)g(f|A)\, df} - \frac{\int U''(f)g(f|\bar{A})\, df}{\int U'(f)g(f|\bar{A})\, df}\right].$$

We can rewrite c_1 as

$$c_1 = (c_0/2)[E_{h_{\bar{A}}}(w) - E_{h_A}(w)], \quad (9)$$

with

$$w(f) = -U''(f)/U'(f) \quad (10)$$

representing the Pratt–Arrow risk-aversion function (Pratt 1964),

$$h_A(f) = U'(f)g(f|A) \Big/ \int U'(f)g(f|A)\, df, \quad (11)$$

and

$$h_{\bar{A}}(f) = U'(f)g(f|\bar{A}) \Big/ \int U'(f)g(f|\bar{A})\, df. \quad (12)$$

From (7)–(9),

$$c = c_0\{1 + (r/2)[E_{h_{\bar{A}}}(w) - E_{h_A}(w)]\} + O(r^2). \quad (13)$$

Whether $c < 1$, $c = 1$, or $c > 1$ for r close to 0 depends on U, $g(f|A)$, and $g(f|\bar{A})$. For example, if the expert exhibits constant risk aversion through an exponential utility function, then the term in square brackets in (13) is 0 and c is approximately c_0 for small r. On the other hand, if $w(f)$ is not constant, then the sign of the term in square brackets in (13) depends on the relationship between $g(f|A)$ and $g(f|\bar{A})$. Finally, making r small does not mean that we are only concerned about the shape of U over a very limited domain; the entire set of values of f and $f + r$ implied by $g(f|A)$ and $g(f|\bar{A})$ is relevant. The size of this set reflects the various uncertainties related to the expert's fortune.

With lottery methods, the case for small stakes is not as strong as it is for the scoring rules and promissory notes discussed in Sections 3 and 4. In fact, LaValle (1978) claimed that "as a matter of convenience, [we] should select reference outcomes [r and n] which are easily conceptualizable and *sufficiently distinct* [emphasis ours] to encourage [the expert] to think *seriously* about the [lotteries]" (p. 80). Whatever the difference between r and n, the no-stakes condition is important to guarantee that $c = 1$ and thus $p = \pi$ in the presence of nonlinearities in the expert's utility function.

3. SCORING RULES

An alternative operational procedure for eliciting an expert's probability involves scoring rules. The expert receives a payoff equal to a score S from a scoring rule that is strictly proper in the sense that the expected score is maximized iff the reported value is exactly the expert's probability π. A frequently used strictly proper scoring rule is quadratic, with

$$S = -r(x - p)^2, \tag{14}$$

where p is the expert's stated probability of A, r is a positive constant, and x is an indicator variable corresponding to the occurrence ($x = 1$) or nonoccurrence ($x = 0$) of A.

The expert's expected utility as a function of p under the scoring scheme given by (14) is

$$EU(p) = \pi \int U[f - r(1 - p)^2]g(f|A)\,df + (1 - \pi) \int U(f - rp^2)g(f|\bar{A})\,df. \tag{15}$$

To maximize expected utility, the expert sets the derivative of $EU(p)$ with respect to p equal to 0, as follows:

$$\pi(1 - p) \int U'[f - r(1 - p)^2]g(f|A)\,df - (1 - \pi)p \int U'(f - rp^2)g(f|\bar{A})\,df = 0,$$

which simplifies to

$$p/(1 - p) = c\pi/(1 - \pi) \tag{16}$$

with

$$c = \frac{\int U'[f - r(1 - p)^2]g(f|A)\,df}{\int U'(f - rp^2)g(f|\bar{A})\,df}. \tag{17}$$

From (16), the expert's stated probability p equals π iff $c = 1$. As in Section 2, $c = 1$ if U is linear. In the absence of linearity, however, the no-stakes condition is not sufficient for $c = 1$ with scoring rules; large values of r and/or probabilities near 0 or 1 can cause discrepancies between $U'[f - r(1 - p)^2]$ and $U'(f - rp^2)$. For a strictly risk-averse individual, U' is strictly decreasing. Therefore, from (16) and (17), a risk-averse expert with (6) satisfied will have $c < 1$ when $\pi > \frac{1}{2}$ and $c > 1$ when $\pi < \frac{1}{2}$. As a result, risk aversion causes p to move away from π toward $\frac{1}{2}$. For a risk taker satisfying (6), the movement is in the opposite direction, toward 0 when $\pi < \frac{1}{2}$ and toward 1 when $\pi > \frac{1}{2}$. If the risk taking is extreme enough, the second-order condition is violated and the expert moves all the way to $p = 0$ or $p = 1$. As in Section 2, potential nonlinearities in U must be considered not just over the range of elicitation-related payoffs [in this case, possible values of S from (14)], but over the entire range of possible fortunes $f + S$. Furthermore, violations of the no-stakes condition can cause c in (17) to deviate considerably from 1 just as they can cause c in (5) to differ from 1.

When r approaches 0, implying that the payoff from the scoring rule is of little consequence, then the no-stakes condition is sufficient for $c = 1$ to hold approximately. In the limit, we have

$$\lim_{r \to 0} c = c_0 = \frac{\int U'(f)g(f|A)\,df}{\int U'(f)g(f|\bar{A})\,df}, \tag{18}$$

which equals 1 if the no-stakes condition holds. Expanding c in a Taylor series in r around 0, we find that

$$c = c_0 + rc_1 + O(r^2), \tag{19}$$

where

$$c_1 = \left[c_0 - (1 - p)^2 \frac{\int U''(f)g(f|A)\,df}{\int U'(f)g(f|A)\,df} + p^2 \frac{\int U''(f)g(f|\bar{A})\,df}{\int U'(f)g(f|\bar{A})\,df} \right]. \tag{20}$$

Thus from (19) and (20),

$$c = c_0\{1 + r[(1 - p)^2 E_{h_A}(w) - p^2 E_{h_{\bar{A}}}(w)]\} + O(r^2), \tag{21}$$

with w, h_A, and $h_{\bar{A}}$ given by (10)–(12). This expression for c is similar to that in (13) for the lottery method, with changes in the weights given to the two expectations in the term in square brackets. When U is exponential with $w(f) = w > 0$,

$$c = c_0[1 + r(1 - 2p)w] + O(r^2).$$

For small r, $c > c_0$ if $\pi < \frac{1}{2}$ and $c < c_0$ if $\pi > \frac{1}{2}$.

The limiting value of c, c_0, is the same for lotteries and scoring rules. Insofar as the separation of probability elicitation from utilities is concerned, a major difference between lotteries and scoring rules is that the no-stakes condition by itself is sufficient for $c = 1$ with lotteries but not with scoring rules. In that sense, stronger assumptions are needed to obtain $c = 1$ when scoring rules are needed. Of course, with either method, c can differ considerably from 1 if the no-stakes condition is not satisfied.

4. PROMISSORY NOTES

De Finetti (1974) used promissory notes in the elicitation of probabilities. Consider a promissory note that pays r if A occurs and nothing otherwise, where $r > 0$. This promissory note is identical to the lottery involving A in Section 2, but we will not compare it with another lottery. Instead, we ask the expert: What is the largest price you will pay for the promissory note (i.e., the price that makes you indifferent between buying the note and not buying it)? Denote the indifference price by q. Then the expert's elicited probability for A is taken to be $p = q/r$.

The analysis of the use of promissory notes is summarized briefly here; for more details, see Kadane and Winkler (1987). The expert's odds ratio $\pi/(1 - \pi)$ is related to $p/(1 - p)$, the odds ratio implied by the indifference price q, by

$$p/(1 - p) = c\pi/(1 - \pi), \tag{22}$$

as in (4) and (16). In this case,

$$c = \frac{\displaystyle\int \left[\frac{U(f - q + r) - U(f)}{r - q}\right] g(f|A)\, df}{\displaystyle\int \left[\frac{U(f) - U(f - q)}{q}\right] g(f|\bar{A})\, df}, \tag{23}$$

and the limiting value of c is the same as that provided by lotteries and scoring rules:

$$\lim_{r\to 0} c = c_0 = \frac{\displaystyle\int U'(f)g(f|A)\, df}{\displaystyle\int U'(f)g(f|\bar{A})\, df}. \tag{24}$$

An expansion of c yields

$$c = c_0\{1 - (r/2)[s\, E_{h_{\bar{A}}}(w) + (1 - s)E_{h_A}(w)]\} + O(r^2), \tag{25}$$

where s is the limit, as $r \to 0$, of q/r.

If the no-stakes condition is satisfied, $c_0 = 1$ but c may differ from 1, as in the case of scoring rules. For instance, from (23), $c < 1$ for risk-averse experts satisfying (6). Thus the promissory-note approach will understate the odds in favor of A for a risk-averse expert with no other stakes involving A. Similarly, the odds in favor of A will be overstated by a risk-taking expert without other stakes involving A. In the presence of other stakes, c might move toward 1 or further away from 1, depending on the nature of $g(f|A)$ and $g(f|\bar{A})$. Clearly, violations of the no-stakes condition can have an important impact on p for any of the three methods we have considered in Sections 2–4.

5. ELICITATION IN A MARKET SETTING

The importance of the no-stakes condition in the elicitation of probabilities can be especially notable in a market setting. In this section we consider a particular type of market with risk-averse participants. If the market is at equilibrium and the promissory-note approach is used to elicit the probabilities of all of the participants, then they would all appear to have the same probability. This startling result provides another illustration of how crucial non-elicitation-related uncertainties concerning an expert's fortune can be.

Suppose that a market exists of the following type: An auctioneer announces a price for promissory notes of the type considered in Section 4. Each person indicates how much he or she wishes to buy or sell at that price. If desired purchases exceed desired sales, the auctioneer raises the price, and conversely. This continues until the auctioneer finds a price at which desired sales and desired purchases are equal. Then all transactions take place in those amounts at that price, q^*. Furthermore, suppose that the participants in the market are risk averse in the sense that they put infinite negative utility on the outcome of being completely broke. For example, the logarithmic utility $U(f) = \log f$ satisfies this condition. Then all individuals whose probabilities of A are less than q^*/r will sell some promissory notes, but not to the extent of their entire fortunes, and all individuals for whom $\pi > q^*/r$ will buy some notes, but again, will not spend their entire fortunes doing so. Those with $\pi = q^*/r$ will be indifferent, and we can suppose that they neither buy nor sell.

Now suppose that after such an auction we enter the room and attempt to use the promissory-note technique to discover the personal probability for A of a participant in the market. We ask what price would leave that person indifferent between buying a note and not buying it. Only the price $q = q^*$ can be given, since if any other price, say $q > q^*$, is given, the person would have bought more promissory notes at the price q^* in the auction. Similarly, if $q < q^*$, the person would have sold more. Thus q^* is given by everyone in the market. Since promissory notes on $\bar{A}$ are valued at $r - q^*$ by everyone (otherwise arbitrage is possible), everyone has the same price ratio of A to $\bar{A}$, $q^*/(r - q^*)$. Following the standard procedure with promissory-note elicitation (i.e., ignoring other stakes in A), we would, therefore, infer that everyone has the same probability q^*/r. Thus, in a market at equilibrium, contrary to the subjective view of probability, objective (i.e., interpersonal) probability appears to exist! This appearance, of course, is due to the stakes that the individuals in the market already have in the event of interest.

The aforementioned market is complete, which means that the events under consideration, A and $\overline{A}$, exhaust the sure event. Thus the sure event is one of the "securities" in the market. An incomplete market may not yield the result that everyone has the same price, and hence the same inferred probability, if the promissory-note scheme is used. As Leamer (1986) noted, however, in market settings "there is . . . little information in the market price about differences in . . . individuals' pre-exchange valuations" (p. 218). Focusing on market-related and game-theoretic considerations, Leamer claimed that probabilities cannot be elicited economically and accurately.

6. IS THE NO-STAKES CONDITION REASONABLE?

We would like to feel that elicited probabilities accurately reflect an expert's judgments. The analysis in this article suggests that experts may have some incentive to give responses that imply probabilities differing from their judgments. Violations of the no-stakes condition can play an important role in these differences. Yet, in eliciting probabilities, we seldom ask whether, or to what extent, the expert is already making bets on the very stochastic events for which the expert's probabilities are to be elicited.

Are experts likely to already have significant stakes relating to the events of interest? Elicitations of probabilities of experts/stakeholders are done all the time in risk analysis and decision analysis. Experts are elicited about their probabilities concerning health effects associated with pollutants such as carbon monoxide (Keeney, Sarin, and Winkler 1984), even though these experts may have staked varying proportions of their scientific reputations on the outcome. Other experts are asked about their probabilities for nuclear war (Press 1985), in which we all have a large stake (What is the meaning of "You win \$10 if a nuclear war occurs"?), as pointed out in an example given by DeGroot (1970, pp. 74–75). When probabilities are elicited in applications of decision analysis, experts may have vested interests in the decision that will be made as a result of the analysis. When scientists assess the chances of success of research proposals, their own investments in certain research directions and strategies may influence the assessments despite good intentions to remain neutral.

In financial circles, opinions about future prospects for individual investments, for the stock market as a whole, and about the economy are available from numerous "experts," and these opinions are sometimes expressed in probabilistic form. Surely most of these experts have significant stakes in how the stock market and the economy perform, both through their own personal investments and through the income they may derive from their predictions. Of course, finance is an area in which dependence among different investments has been studied in detail and allowed for in the determination of diversified portfolios through portfolio analysis (e.g., Markowitz 1959). Miller (1978) pointed out, however, that standard portfolio theory takes into consideration dependence among investments but generally ignores dependence between these investments and an investor's earned income. For example, those investors whose earned incomes may be closely related to the economy or the stock market (e.g., workers who might be laid off in a recession or stockbrokers who may find their commissions dwindling when the economy and the market are down) might be better off to diversify with non-stock-market investments even if they feel that the prospects for the market are reasonably good. If they agree with this line of reasoning, probabilities that might be inferred from their investment behavior could differ considerably from their actual judgments. But this is precisely the issue that we are concerned with when we ask if an expert's fortune is independent of events for which probabilities are being elicited!

When scientific and practical matters are at stake, we all have various previous "investments" in the outcomes. These may be difficult to untangle; nonetheless, they bear an important weight in the future bets we might make. Without studying these stakes, with no conflict-of-interest statement, elicitations could make serious errors. Moreover, the problem may be most serious precisely when the most important inferences and decisions are being made.

7. ELICITATION WITHOUT ASSUMING THE NO-STAKES CONDITION

The purpose of this section is constructive, to consider ways of eliciting probabilities without the no-stakes condition. The spirit of the section is similar to that of the following quotation:

> A selling point for DMUU (decision making under uncertainty) modeling is that it permits the decomposition of the problem so that, for example, the modeling of uncertainty can be separated from the modeling of preferences. When a probability assessor has a vested interest in the eventual outcomes, this separation may be hard to maintain. How serious is this problem, and how can assessment procedures be designed to minimize the problem? (Winkler 1982, p. 523)

It is natural to ask questions such as these: What are the implications and proposed prescriptions for practice in terms of how probability elicitation should be designed so as to minimize the interaction with utility? What should be done under the worst circumstances with vested interests? We have no definitive answers, and we regard these as valid questions for further research. The discussion in this section is therefore brief; rather than go into great detail, we touch upon points of interest and offer some suggestions regarding elicitation.

If we know the expert's utility function U and we also know $g(f|A)$ and $g(f|\overline{A})$, then we can use the results in Sections 2–4 to make the appropriate adjustments in the values provided by the expert. That is, we do not assume that the odds ratio equals the price ratio; instead, we use our information about U, $g(f|A)$, and $g(f|\overline{A})$ to calculate c from (5), (17), or (23). The observed $p/(1 - p)$ can then be divided by c to arrive at the odds ratio.

Unfortunately, we generally do not know the expert's utility function and all of the many uncertainties related to the expert's fortune. We can assess the expert's utility function via standard procedures (e.g., Keeney and Raiffa 1976) that can be used when the relevant consequences are multiattribute in nature as well as in the basic case when only utility for money is being considered. Obtaining information about the expert's non-elicitation-related stakes in the event of interest, however, is a much more difficult task. The expert may be understandably reluctant to reveal these stakes, and some of the connections between the events and the expert's fortune may be so complicated that even the expert cannot completely unravel them. A full Bayesian approach would involve the consideration of a distribution representing our uncertainty about the stakes. This distribution could be used, together with the expert's responses, to generate a distribution for the expert's probability π. In this approach we are admitting that we are uncertain about the expert's non-elicitation-related stakes (and possibly about the expert's U as well) and that, as a result, we cannot be certain about π. But eliciting our own distribution for the expert's stakes and utilities is by no means an easy task.

The elicitation problem of interest here is related to the more general problem of designing appropriate incentives. The study of scoring rules is within the framework of incentives, and elicitation procedures such as lotteries and promissory notes can be thought of as attempts to provide the expert with inducements intended to encourage the expert to reveal probabilistic judgments. When the no-stakes condition is not reasonable, we must adjust the incentive plan to allow for the non-elicitation-related stakes. For example, consider the elicitation of a sales manager's probabilities for next year's sales. We want to get an accurate picture of the prospects for next year, but we also want the incentive plan to inspire the manager to work hard to maximize sales. For some work along these lines, see Sarin and Winkler (1980). Also relevant are the notion of state-dependent preferences (Drèze 1985) and the growing body of literature on "agency theory" in finance and economics (e.g., Pratt and Zeckhauser 1985). Finally, questions of incentives and "small worlds" versus "large worlds" are relevant to the design of organizations. What do various designs and incentives imply for how decisions made at various levels of the organization (small worlds) relate to the goals of the organization as a whole (large world)? Some thoughts of Hogarth (1982) are relevant in this direction.

Multiple elicitations may help to reduce the problem of systematic shifts in the assessed probabilities. Such shifts may cause incoherence, and adjustments in the elicited probabilities might be made in consultation with the expert (reconciling inconsistencies) or might be made by the analyst. In using multiple elicitations, we should keep in mind potential dependence among the payoffs related to the various elicitations.

The discussion in this section has assumed that we are using elicitation techniques such as lotteries, scoring rules, and promissory notes. Another approach, of course, is to abandon such techniques in favor of alternative approaches that attempt to avoid or at least reduce the utility-related difficulties we have encountered with these methods. A common approach (e.g., DeGroot 1970) is to use "is more likely than" as a primitive relation between events. The elicitation process can consist of comparisons of events to see which one is more likely, using reference events and not necessarily using payoffs.

The foundational work of Savage (1954) avoids the criticism here by use of the device of "consequences," which are not random variables and over which individuals have preferences. This device proves awkward, however, when one wants to use Savage's theory in "small worlds," restricted just to the events of immediate interest, avoiding references to the "grand world" that anticipates all future uncertainties and possible actions (see Savage 1954, pp. 82–91). As Seidenfeld and Schervish (1983) pointed out, consequences are a crucial device for permitting finitely additive probabilities to be implied by a theory of preference.

Our focus in this article is on the elicitation of probabilities; however, similar issues could affect the elicitation of utilities. For example, Fukuba and Ito (1984) discussed some difficulties that may arise in utility elicitation when the decision maker's initial wealth is stochastic. Fundamental problems of elicitation remain to be solved.

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