# Testing the one-sided Pairs Trading Investment Strategy

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**ABSTRACT**
The study investigates the performance of one-sided pairs trading investment strategy. This is variation of the pairs trading strategy, which aims to be adapted for the use by individual investors, who are usually constrained from short selling. The one-sided strategy prescribes to invest into relatively underpriced stock from the pair, and then close the position when the two prices get close to each other.
The strategy performance showed that it can be used to generate return in excess that of the market index. At the same time the strategy is prone to high variability of returns and also to the risk that co-integrating relation between paired stocks weakens.

**KEYWORDS:**
Pair trading, statistical arbitrage, back-testing

## 1 INTRODUCTION

The goal of the study is to investigate the performance of one-sided Pairs Trading strategy. The mentioned investment strategy is based on the widely known pairs trading strategy, which belongs to the category of statistical arbitrage investment strategies. The pairs trading strategy requires selection of one or several pairs of stocks, which move similarly, or are co-integrated. Then the long and short position is taken simultaneously on relatively under-priced and relatively over-priced stocks, respectively. As the stocks are co-integrated, so that they co-move in the long run, the short-term deviations are sure to be reversed. Pairs trading therefore dwells firmly on the mean reversion property of stock returns. This strategy was proven to have strong investment performance by many authors (Gatev, 2006; Schmidt, 2004).
The contribution of this study is its focus on 'one-sided' version of pairs trading. Such strategy implies only taking long position for relatively under-priced stock,

therefore it is suitable for implementation by individual investors, whereas 'traditional' pairs trading is aimed for institutional investors.

The following research questions are considered. Does 'one-sided' version of pairs trading generate abnormal returns (e.g. returns that are higher than risk-adjusted expected returns)? Which investment rules of a pair of stock (pairs of stocks) provides better results? Such rules in relation to identified paired stocks involve the decision when to buy a stock (stocks), and when to sell the stock. Does profitability of pair trading change over time?

The remaining part of the study has the following structure. Section 2 discusses the prior related work. The used data-set is described in section 3. Then the method for analysis is provided in section 4. The results and their discussion are provided in section 5. Conclusions, review of the implications for investor and further research suggestions complete the study.

## 2 RELATED WORK

The pair trading strategy was invented in 1980s in Morgan Stanley by a group of quantitative traders. The strategy was successful and in 1987 it earned over $87 million in profits for the company (Schmigt, 2004).

Vidyamurthy (2004) discusses the nature of pairs trading method of investing as one of statistical arbitrage methods. Some versions of pair trading are investigated - including the risk-neutral pair trading based on capital asset pricing model, as well as pair trading based on co-integrated stocks. The book indicates several different methods for selection paired stocks, and investing approaches.

More focused studies were performed by Gatev (2006) and Schmigt (2004) among others. These researchers tested pair tradings strategy and confirmed its high performance. For example, Gatev (2006) tested the standard pairs trading strategy and concluded that it, on average, delivered abnormal return of 11% during 1962 - 2002.

Efficient selection of paired stocks is another direction of research. McSharry (2015) indicates that there are over 50,000 stocks in the world that would require testing and ranking of over 1.2 billion unique pairs. Therefore, for international institutional investors the methods of resource-efficient selection of pairs is of relevance.

## 3 DATASET

The task is based on the use of daily stock returns of S&P 500 index constituent firms for the period from January 2012 till November 2017. The sample is broke into three sub-samples, namely 2012-2013, 2014-2015, and 2016-2017. Each sub-sample, is then divided into 'training' and 'testing' parts, whereas 'training' part is used to identify paired stocks, and 'testing' one is used to implement investing with paired stocks.

The following is the summary of mean annualized returns and annualized standard deviations of returns for a sample of stocks, which are constituents of S&P500 index. The table shows that stocks are significantly different by their risk and return profiles, even these are stocks of the largest and most reputable US corporations.

Table 1: Mean returns for selected stocks

| Ticker | Mean annualized return, % | Annualized standard deviation, % |
|---|---|---|
| ACN | 18.99 | 18.29 |
| ADBE | 27.91 | 24.62 |
| ADS | -0.05 | 28.32 |
| AET | 26.37 | 23.88 |
| AKAM | 5.58 | 33.83 |
| AMZN | 26.98 | 29.42 |
| BAC | 17.06 | 25.62 |
| BSX | 19.62 | 24.13 |
| CAT | 16.21 | 23.28 |
| CSCO | 17.98 | 19.58 |
| MMM | 7.26 | 15.27 |
| T | 18.99 | 18.30 |

## 4 METHOD

The implementation steps include the co-integration testing with Engle-Granger methodology, designing investment rules, measuring the performance, and comparison of the results against a benchmark - S&P500 stock index. Investment performance is assessed based on cumulative wealth index, mean annualized return, standard deviation, Sharpe ratio, and portfolio Alpha.

### *4.1 Pairs Selection*

The universe for the stock selection are the stocks that were S&P500 index constituents as of November 2017. Given *N* number of stocks in the 'universe', the number of possible unique pairs is equal to ($N^2$-$N$)/2 (Vidyamurthy, 2004). Therefore, out of 500 stocks there are 124,750 unique pairs.

Compare performance of the strategy when selection of paired stocks is based on two methods - co-integration analysis and measure of Euclidean distance.

Using Engle and Granger (1987) methodology, co-integration test involves two steps. On step one log stock prices of stock *i* are regressed against those for stock *j* and residuals are obtained. Then residuals are tested for null hypothesis that they involve unit root (e.g. are non-stationary), which is performed by using Dickey-Fuller test. When the null is rejected, the conclusion is that the pair of stocks is co-integrated. In case when no co-integrated pair of stock is identified, then the 'most co-integrated' one (the one with the highest value of the test statistic) can be picked.

The Euclidean distance is compared between the logs of stock prices within each possible pair of stocks (eq. 1). The pair with the minimum Euclidean distances (e.g. '*Pair**') is selected as the best pair of considered stocks (Gatev, 2006).

$$Pair^* = \arg\min_{\{Pair_k\}} \left( \sum_t (\log P_{k,1,t} - \log P_{k,2,t})^2 \right)^{0.5} \tag{1}$$

### *4.2 Investing Algorithm*

The investing is implemented based on the following algorithmic rule. The first step is that for the selected pair of stocks, in the investing period (back-testing period) their prices are normalized (eq. 2).

$$P_{i,t}^n = \frac{P_{i,t} - \overline{P_i}}{\sigma_{P_i}} \tag{2}$$

Where, $\overline{P_i}$ and $\sigma_{P_i}$ are the mean price and standard deviation of stock prices.

Then every period normalized prices of both stocks are compared, and distance ($D$) between the two is measured. The one-sided pairs trading strategy is then implemented as follows.

First, when the distance between normalized prices of the two stocks is higher than the trigger (e.g. $D > D_1$=0.5[1]), need to open long position on the relatively undervalued stock, which is the stock with a lower normalized price .

Second, when the distance between the normalized prices of the two stocks comes close, so that the distance gets less than the trigger (e.g. $D < D_2 = 0.1$).

Several alternatives for the trigger parameters can be considered. For example, the investing trigger $D_1$ can take on values in the interval from 0.5 to 3.0 with a step of 0.1. And the trigger for closing investment position $D_2$ can take on values in the interval from 0.5 to 0 with a step of 0.1.

### *4.3 Evaluation*

Evaluation is performed based on cumulative return, as well as the risk adjusted return. Performance is compared against the S&P500 stock index, based on the following indicators - cumulative wealth index, annualized return, Jensen's Alpha, and Sharpe ratio. The returns for stock $i$ ($R_{i,t}$) were computed using the log-approach (eq. 2).

$$R_{i,t} = \ln(P_{i,t}) - \ln(P_{i,t-1}) \qquad (2)$$

Cumulative wealth index shows the evolution of a $1,000 investment over the investment period. Mean annualized return is obtained from mean daily return via multiplying it by a factor of 251, being the number of trading days in a year. Sharpe ratio, known as the reward to variability ratio (Bodie et al., 2011), is the ratio of excess return to standard deviation of returns (eq. 3).

$$Sharpe_p = \frac{\overline{R}_p - R_f}{\sigma_{R_i}} \qquad (3)$$

Where, $\overline{R}_p$ and $\sigma_{R_p}$ are, respectively, the mean annualized return and annualized standard deviation of the returns on portfolio $p$. Also, $R_f$ is the risk-free rate of return, proxied with yield to maturity on 10-year T-bills, it is assumed 1.23% for the purpose of this analysis.

[1] For normalized prices, SD=1.0

Jensen's Alpha ($\alpha_p$ below) is the abnormal return on the portfolio, compared to the risk-adjusted expected return on the given stock. Jensen's alpha is also the intercept of the regression of portfolio returns on the index returns (eq. 4).

$$R_{p,t} = \alpha_p + \beta_p \times R_{m,t} + \varepsilon_t \quad (4)$$

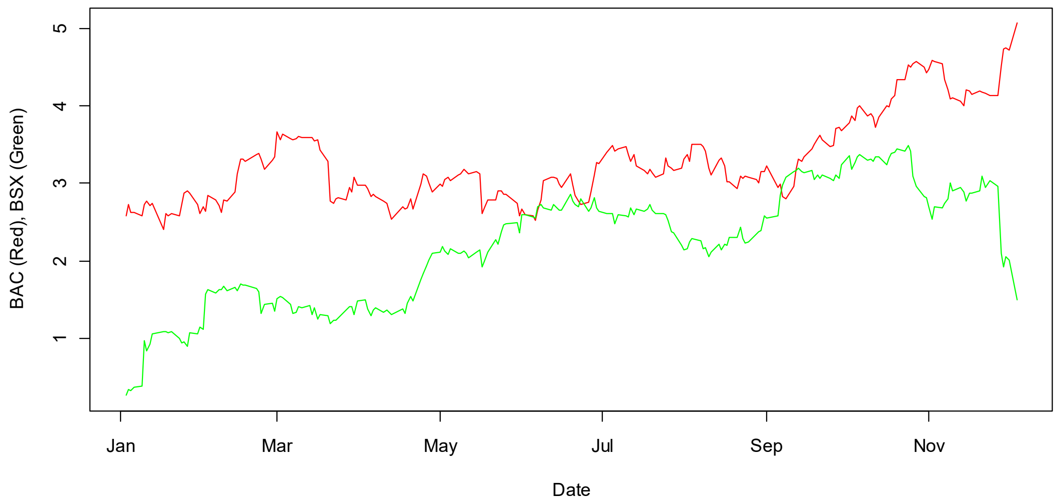
For a portfolio of superior performance, Jensen's alpha is positive and statistically significant.

## 5 EVALUATION OF RESULTS

The section presents the obtained results for the pair selection in year 2016 and using the pair for subsequent trading in year 2017. The presented results involve the dynamics of the normalized prices of the selected stocks, and the summary of the appraisal metrics - cumulative wealth index, annualized return, Sharpe ratio and Jensen's alpha.

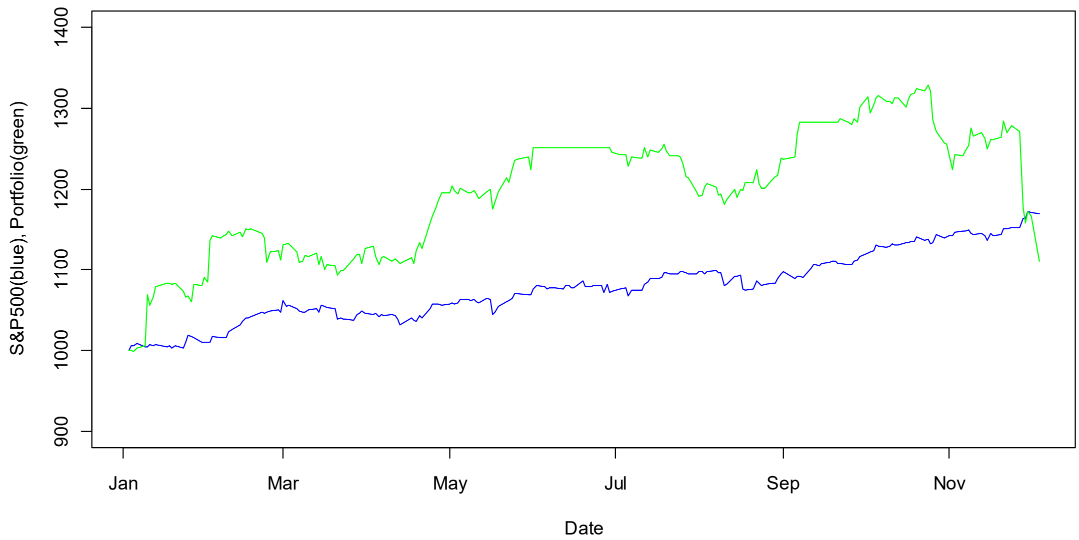
Based on the selection of pair in 2016, the selected pair was Bank of America (ticker: *BAC*) and Boston Scientific Corporation (*BSX*). The best pair *BAC*-*BSX* was selected based on the Euclidean distance measure. Among the considered pairs stocks, the minimum root sum of squared differences between log-prices was 45.55 (pair *BAC*-*BSX*), and the maximum one was 93.95 (pair *ADS*-*AMZN*). Plot of normalized prices for BAC and BSX in January - November 2017 is shown below. There are several instances during the period when investment position could be taken (when difference between the two normalized prices exceeds $D_1$=0.5) and when it should be subsequently closed (when difference between the two normalized prices gets less than $D_2$=0.1).

Figure 1: Normalized prices for the selected pair, Jan-Nov 2017




The investment portfolio during year 2017 was managed based on the mentioned strategy, involving the selected pair of stocks. The results for this portfolio performance were assessed using the metrics of cumulative wealth index, annualized return, Sharpe ratio and Jensen's alpha (table 2).

Figure 2: Cumulative wealth index for the portfolio and SP500 in 2017




During most of the considered period, the cumulative wealth index for the portfolio was higher than for S&P500, but also it was more volatile. At the end of the back-testing period the portfolio value plummeted below that for the S&P index (figure 2).

The summary of performance evaluation of the one-sided paired strategy is provided in table 2. The portfolio showed inferior performance in comparison to the benchmark, in terms of each considered measure. Also, the systematic risk exposure (Beta) is higher for the one-sided paired trading portfolio in 2017. The poor performance of the strategy is to a significant extend attributed to the decline in the portfolio value in the last month over the back-testing period (figure 2).

Table 2: Performance appraisal summary

| Measure | 2016 - 2017 | |
|---|---|---|
| | Portfolio | S&P500 |
| Cumulative wealth index | 1.111 | 1.169 |
| Annualized return | 11.35% | 16.9% |
| Sharpe ratio | 0.56 | 2.33 |
| Jensen's $\alpha$ | -380 | 0.0 |
| Beta | 1.46 | 1.00 |

The analysis showed that the one-sided strategy performance showed higher return in comparison to the S&P500 index over the most beck-testing period, with the exception of the last month. Another features is that one-sided pairs trading strategy does not attain reduction of the volatility of portfolio, unlike the standard pairs trading strategy does (Gatev, 2006).

The reason for the poor performance is that the normalized prices of the paired stocks diverged far from each other over the prolonged time. It is possible that the co-integration relation between the stocks weakened. This calls for a better investigation of the nature (e.g. strength and duration) of the co-integration between paired stocks. One possible suggestion is to re-evaluate possible pairs of stocks more frequently.

## 7 CONCLUSIONS

The goal of this study was to investigate the performance of the one-sided pairs trading strategy. The study was motivated by the need to adapt the pairs trading strategy for the use by individual investors, whereas the standard pairs trading strategy is basically limited for the use by institutional investors.

The findings show that the one-sided pairs trading strategy can generate return in excess of the market return. Besides it typically has higher dispersion of returns, compared to the market.

There are several investment implications. First, strategy can generate return in excess of the market return. Second, the strategy is prone to high dispersion of returns. Third, the strategy is prone to the risk that paired stocks deviate far from each other over a prolonged time.

The suggestions for the future research involve the following ones: optimization of the strategy by selection of optimal algorithm parameters - length of training and back-testing periods, the values for trigger distances, ways to pick paired stocks.

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APPENDIX: R-CODE (from the next page)

```r
# ---------------------------------------------------------------
#
# Project title: Testing the One-Sided Pairs trading Investment Strategy
#
# By: Yuriy podvysotskiy
# Email: yap7@pitt.edu
#
# Version dated: 12/12/2017
# -----------------------------------------------------------------

rm(list=ls())
setwd('C:\\Users\\Yuriy\\Documents\\Pitt\\Data Mining (INFSCI 2160)\\FinalCourseProject\\Data')


# (1) Get the data:
# ======================================================================
acn<-read.csv2('acn.csv', sep=',', dec='.')
adbe<-read.csv2('adbe.csv', sep=',', dec='.')
ads<-read.csv2('ads.csv', sep=',', dec='.')
aet<-read.csv2('aet.csv', sep=',', dec='.')
akam<-read.csv2('akam.csv', sep=',', dec='.')
amzn<-read.csv2('amzn.csv', sep=',', dec='.')
bac<-read.csv2('bac.csv', sep=',', dec='.')
bsx<-read.csv2('bsx.csv', sep=',', dec='.')
cat<-read.csv2('cat.csv', sep=',', dec='.')
csco<-read.csv2('csco.csv', sep=',', dec='.')
mmm<-read.csv2('mmm.csv', sep=',', dec='.')
t<-read.csv2('t.csv', sep=',', dec='.')

spy<- read.csv2('spy.csv', sep=',', dec='.')

prices<-data.frame(acn$Date, acn$Adj.Close, adbe$Adj.Close, ads$Adj.Close, aet$Adj.Close, akam$Adj.Close,
           amzn$Adj.Close, bac$Adj.Close, bsx$Adj.Close, cat$Adj.Close, csco$Adj.Close,
           t$Adj.Close, acn$Adj.Close)

colnamess<-c('date', 'acn', 'adbe', 'ads', 'aet', 'akam', 'amzn', 'bac', 'bsx', 'cat', 'csco', 'mmm', 't')
stoxx<-c('acn', 'adbe', 'ads', 'aet', 'akam', 'amzn', 'bac', 'bsx', 'cat', 'csco', 'mmm', 't')
colnames(prices)<-colnamess
prices[1:3,]

logprices<-prices
logprices[,2:13]<-log(prices[,2:13])

logprices[1:3,]

# (2) Compute returns:
# ======================================================================
returns<-prices
returns<-returns[2:1007,]
for(i in stoxx){
  returns[[i]]<-diff(logprices[[i]])
}
returns[1:3,]
returns$date2<-as.Date(as.character(returns$date))

mean(returns$acn)*251*100
mean(returns$adbe)*251*100
mean(returns$ads)*251*100
mean(returns$aet)*251*100
mean(returns$akam)*251*100
mean(returns$amzn)*251*100
mean(returns$bac)*251*100
mean(returns$bsx)*251*100
mean(returns$cat)*251*100
mean(returns$csco)*251*100
```

```
mean(returns$mmm)*251*100
mean(returns$t)*251*100

SD=NULL
SD[1]=(sd(returns$acn)^2*251)^0.5*100
SD[2]=(sd(returns$adbe)^2*251)^0.5*100
SD[3]=(sd(returns$ads)^2*251)^0.5*100
SD[4]=(sd(returns$aet)^2*251)^0.5*100
SD[5]=(sd(returns$akam)^2*251)^0.5*100
SD[6]=(sd(returns$amzn)^2*251)^0.5*100
SD[7]=(sd(returns$bac)^2*251)^0.5*100
SD[8]=(sd(returns$bsx)^2*251)^0.5*100
SD[9]=(sd(returns$cat)^2*251)^0.5*100
SD[10]=(sd(returns$csco)^2*251)^0.5*100
SD[11]=(sd(returns$mmm)^2*251)^0.5*100
SD[12]=(sd(returns$t)^2*251)^0.5*100
SD

## (3) Evaluate pairs, get top pair(s):
# ================================================================================
# Need: table with columns: 1-pair, 2-stock1, 3-stock2, 4-measure, 5-rank

TestStart<-"2016-01-01"
TestEnd<-"2016-12-31"

prices$date2<-as.Date(as.character(prices$date))
logprices$date2<-as.Date(as.character(logprices$date))

prices_test<-subset(prices, date2>=as.Date(TestStart)& date2<=as.Date(TestEnd))
prices_test[1:3,]

logprices_test<-subset(logprices, date2>=as.Date(TestStart)& date2<=as.Date(TestEnd))
logprices_test[1:3,]


nn <- length(stoxx)

pairs <- data.frame(matrix(ncol=5, nrow=(nn*nn-nn)/2))
colnames(pairs)<- c('pair', 'stock1', 'stock2', 'distance', 'rank')

k = 0
for(i in 1:12){
  for(j in 1:12){
    if(i>=j) next
    k = k+1
    a=stoxx[i]
    b=stoxx[j]
    pairs[[1]][k]=paste(a,b, sep=" ")
    pairs[[2]][k]=a
    pairs[[3]][k]=b
    pairs[[4]][k]=(sum(logprices_test[[a]]*logprices_test[[b]]))^0.5
  }
}

pairs$rank<-rank(pairs$distance, ties.method = "first")
pairs[1:3,]
pairs[65:66,]

min(pairs$distance)
max(pairs$distance)

toppairs<-pairs[order(pairs$rank),]
toppairs[1:3,]
toppairs[65:66,]

thepair<-as.matrix(toppairs[1,2:3])
thepair[1]
thepair[2]

```

```r
## (4) Traiding:
# ==============================================================================
# Input: period
# Step1: 1-normalized prices, 2-distances, 3-Whether hold stock, 4-Whether hold cash

TradeStart<-"2017-01-01"
TradeEnd<-"2017-12-05"
Amount<-1000.0

prices$date2<-as.Date(as.character(prices$date))
prices_trade<-subset(prices, date2>=as.Date(TradeStart)& date2<=as.Date(TradeEnd))
prices_trade$amount<- Amount
prices_trade[1:3,]

keeps<-c('date2', thepair[1], thepair[2])
prices_trade<-prices_trade[keeps]
prices_trade[1:3,]

prices_trade$bac_n <-(prices_trade$bac-mean(prices_test$bac))/sd(prices_test$bac)
prices_trade$bsx_n <-(prices_trade$bsx-mean(prices_test$bsx))/sd(prices_test$bsx)
prices_trade[1:3,]

minn=min(c(min(prices_trade$bac_n),min(prices_trade$bsx_n)))
maxx=max(c(max(prices_trade$bac_n),max(prices_trade$bsx_n)))

plot(prices_trade$date2, prices_trade$bac_n, ylim=range(c(minn, maxx)), ylab="BAC
(Red), BSX (Green)", xlab="Date", type="l", col="red")
lines(prices_trade$date2, prices_trade$bsx_n, type="l", col="green")


# ---------TRADES tracking --------------------------------

prices_trade$bac_inv <- 0
prices_trade$bsx_inv <- 0

prices_trade$cash <- 0
prices_trade$cash[[1]] <- 1

prices_trade$bac_num <-0
prices_trade$bsx_num <-0

prices_trade$bac_val <-0
prices_trade$bsx_val <-0

prices_trade$cash_val <-0
prices_trade$cash_val[[1]] = 1000

# Think about sequencing: e.g. today and yesterdays (!!!)

for(i in 2:length(prices_trade[,1])){

  if(prices_trade$cash[[i-1]]==1){
    if(prices_trade$bac_n[[i]] - prices_trade$bsx_n[[i]] < -0.5){
      prices_trade$bac_inv[[i]] = 1
      prices_trade$cash[[i]] = 0
      prices_trade$cash_val[[i]] = 0

      prices_trade$bac_num[[i]] = prices_trade$cash_val[[i-1]]/prices_trade$bac[[i]]
      prices_trade$bac_val[[i]] = prices_trade$bac_num[[i]]*prices_trade$bac[[i]]


    } else if(prices_trade$bsx_n[[i]] - prices_trade$bac_n[[i]]  < -0.5){
      prices_trade$bsx_inv[[i]] = 1
      prices_trade$cash[[i]] = 0
      prices_trade$cash_val[[i]] = 0

      prices_trade$bsx_num[[i]] = prices_trade$cash_val[[i-1]]/prices_trade$bsx[[i]]
      prices_trade$bsx_val[[i]] = prices_trade$bsx_num[[i]]*prices_trade$bsx[[i]]
    } else {
```

```r
   prices_trade$cash[[i]] = 1
   prices_trade$cash_val[[i]] = prices_trade$cash_val[[i-1]]
  }


 } else if(prices_trade$bac_inv[[i-1]] == 1){
   if(prices_trade$bac_n[[i]] - prices_trade$bsx_n[[i]] >=-0.1){
    prices_trade$bac_inv[[i]] = 0
    prices_trade$cash[[i]] = 1
    prices_trade$cash_val[[i]] = prices_trade$bac_num[[i-1]]*prices_trade$bac[[i]]
    prices_trade$bac_val[[i]] = 0
    prices_trade$bac_num[[i]] = 0
   } else {
     prices_trade$bac_num[[i]] = prices_trade$bac_num[[i-1]]
     prices_trade$bac_val[[i]] = prices_trade$bac_num[[i]]*prices_trade$bac[[i]]
     prices_trade$bac_inv[[i]] = 1
   }
  } else if(prices_trade$bsx_inv[[i-1]] == 1){
  if(prices_trade$bsx_n[[i]] - prices_trade$bac_n[[i]] >=-0.1){
   prices_trade$bsx_inv[[i]] = 0
   prices_trade$cash[[i]] = 1
   prices_trade$cash_val[[i]] = prices_trade$bsx_num[[i-1]]*prices_trade$bsx[[i]]
   prices_trade$bsx_val[[i]] = 0
   prices_trade$bsx_num[[i]] = 0
  } else {
    prices_trade$bsx_num[[i]] = prices_trade$bsx_num[[i-1]]
    prices_trade$bsx_val[[i]] = prices_trade$bsx_num[[i]]*prices_trade$bsx[[i]]
    prices_trade$bsx_inv[[i]] = 1
  }
 }
}


sum(prices_trade$bac_inv)
sum(prices_trade$bsx_inv)
prices_trade$portfolio = prices_trade$cash_val + prices_trade$bac_val + prices_trade$bsx_val
# ---------TRADES tracking  end --------------------------------

prices_trade[1:20,]


# (5) Evaluation:
# ==========================================================================
# Getting the benchmark:
spy$date2<-as.Date(as.character(spy$Date))
spy_trade <-subset(spy, date2>=as.Date(TradeStart)& date2<=as.Date(TradeEnd))
spy_trade$Ths <-spy_trade$Adj.Close/spy_trade$Adj.Close[1]*1000

(spy_trade$Ths[length(spy_trade$Ths)]/spy_trade$Ths[1]-1)*100

prices_trade$portfolio[length(prices_trade$portfolio)]

# Mean annualized return, SD, Sharpe ratio:
Rf <- 1.23
R1<- mean(diff(log(spy_trade$Adj.Close)))*251*100
R2<- mean(diff(log(prices_trade$portfolio)))*251*100

SD1 <- sd(diff(log(spy_trade$Adj.Close)))*(251^0.5)*100
SD2 <- sd(diff(log(prices_trade$portfolio)))*(251^0.5)*100

Sharpe1 <- (R-Rf)/SD
Sharpe2 <- (R2-Rf)/SD2

# Cumulative wealth index plot:
plot(spy_trade$date2, spy_trade$Ths, ylab='SP500 Index', xlab='Date', type="l", col="blue")
plot(prices_trade$date2, prices_trade$portfolio, ylab='Portfolio', xlab='Date', type="l", col="green")
```

```
plot(spy_trade$date2, spy_trade$Ths, ylab = "S&P500(blue), Portfolio(green)", xlab = "Date",
     ylim=range(c(900, 1400)), type="l", col="blue")
lines(prices_trade$date2, prices_trade$portfolio, col="green")

plot(prices_trade$date2, prices_trade$bac_n, ylim=range(c(minn, maxx)),type="l", col="red")
lines(prices_trade$date2, prices_trade$bsx_n, type="l", col="green")

# Jensen's Alpha:
linmod <-lm(prices_trade$portfolio ~ spy_trade$Ths)
summary(linmod)
```