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4 March 2009

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Convertible bond pricing

1 Introduction

Convertible bonds can be thought of as normal corporate bonds with embedded options, which enable the holder to exchange the bond asset for the issuer's stock. Having properties of both stocks and bonds, convertibles can be an attractive choice for investors. Studies suggest that the historical average return of convertibles has been roughly the same as that for the equity market, but convertibles have tended to have lower risk.

When the holder converts a convertible bond, they exchange the bond for a fixed number of shares. Thus the conversion decision on a convertible bond is to give up remaining bond cash flows, and get the shares (and subsequent dividends), or to leave unexercised, and continue receiving bond cash flows.

For low values of the stock price, the convertible acts like a straight bond (although the credit risk is increasing as the stock value decreases), while for high values of the stock price, the convertible acts like equity. The intermediate cases are the most interesting.

2 A common bad error

It is often stated that a convertible bond is simply a normal corporate bond and a warrant, and priced accordingly. This is grossly erroneous. A warrant is detachable from the bond. A warrant has a fixed strike price. The exercise decision for a warrant is to pay the strike, and get the shares, or to leave unexercised. When you exercise a warrant you still have the bond, and carry on enjoying the bond repayments, as well as having the equity.

By contrast, when you convert a convertible bond, you exchange the bond for a fixed number of shares. Thus the conversion decision on a convertible bond is to give up remaining bond cash-flows, and get the shares, or to leave unexercised, and continue receiving bond cash flows. Thus, it would be more precise to say that the convertible bond is a corporate bond and an exchange option: the option is to exchange the bond for shares.

3 Typical features

Usually convertibles may be converted by the holder at any time (American) but on occasion only during specified periods (Bermudan). Often there are additional options, of types that are both in favour of the holder and the issuer:

- Often these bonds can be sold back (or "put") to the issuer at specific dates for a guaranteed price. This is a sweetener for the purchaser, because if the stock performs badly, the bond behaves like an ordinary bond (with low coupons). The holder would like to be rid of the bond, perhaps also fearing bankruptcy of the issuer, in which case the stock is worth nothing and the bond is only worth the recovery value of the put.

- Often the issuer may have the right to redeem or call back the convertible at a specified call price. If the issuer does so, the investor can choose between receiving the call price, or converting into shares. Typically, the call is invoked when the share price is high, to force the investor to convert into shares. If the holder elects to convert, the issuer can create new stock (much as in the case of an option issued by the underlying corporation) and so the factor of stock dilution comes into play. This makes convertible bonds attractive vehicles for corporations to raise finance - if the company is successful, it can issue new shares seamlessly, which may be a more attractive option than the continual payment of interest. Of course, the holder of the convertible bond is typically happy with the conversion, as the shares they receive are trading at a high price in the market, even after the possible effects of dilution.

- Often there are associated soft call features, which means that if the stock trades above a certain level for a certain prescribed period of time (or, above that level for a certain number of trade days in a prescribed period) then the issuer can call back the bond at par. In this case, the holder will certainly elect to convert into shares.

In addition the issuer usually has to give notice some period in advance (e.g. 1 month) of calling the issue. There are many other variations, such as refix clauses (where the number of stock that a bond can be converted into changes).

4 Pricing models

The pricing of convertible bonds has an interesting history. There is an extensive academic literature on this topic. Many of the approaches use binomial trees to value the convertible bond. However, such an approach is problematic, as trees cannot cope adequately with discrete events, such as stock dividends or reset clauses.

Given the predefined variables, such as put and call provisions and conversion ratios, the price of a convertible bond is subject to the following four stochastic factors

- stock price,
- varying probabilities of default on the convertible bond,
- the volatility of the stock price,
- the stochastic nature of interest rates,

however it is known that the last factor is (perhaps surprisingly) of fairly minor significance compared to the other factors. Hence a common approach, which is the one we take, is to save on computational time by fixing the risk free rates, either as a single input rate, or as an input yield curve.

A broad family of models much in use are those where the convertible bond is split into two components: a bond only part, which is subject to credit risk, and an equity part, which is not. In the case of binomial trees, this approach is prevalent, where it is often encapsulated in the dictum 'grow risk free, discount risky'. Essentially, equity performance is seen as riskless (as a company can always issue more stock) and the appropriate pricing world is the Black-Scholes-Merton world. On the other hand, it is argued, the bond component is part of the corporate bond world. Thus it should be valued by discounting at the relevant spread to risk free. Typically, the split of value at any point in the tree is performed via the delta of the convertible, see [Connolly, 1998, §6.4.4] or Goldman Sachs [1994], for example.

In the model of Tsiveriotis and Fernandes [1998] this approach leads to a pair of coupled partial differential equations that can be solved to value convertibles. A simple description of this model in the binomial context may be found in Hull [2002].

In the papers of Ayache et al. [2002] and Ayache et al. [2003] the approach of Tsiveriotis and Fernandes [1998] is brought fundamentally into question, in that their approach is claimed to be internally inconsistent. Fundamentally, the entire suite of approaches mentioned above are criticised because the split into equity and bond is arbitrary (model dependent) and the credit spread is then only applied to the bond component. Fundamentally, this is self-referential, and hence is not really a linear problem. The approach of Ayache, Forsyth and Vetzal is to treat the entire convertible bond as the contingent claim, and to derive a Black-Scholes-Merton type PDE which inter alia models the (residual) value of the convertible bond in the event of default. In particular, default occurs with an intensity (inverse to the ruling stock price). This intensity and the inverse proportionality coefficients are direct inputs to the B-S-M type differential equations that are solved.

They show that the Tsiveriotis and Fernandes [1998] model is a special case of theirs, where the stock price does not jump in case of default on the bond, and in the case of default, there is no recovery on the bond. In reality, neither of these conditions is realistic: when the company announces default on the bond it affects the perceived value of the company as an entity and hence the stock price will jump down. Furthermore, the bond-holder will

typically be entitled to some partial recompense in the courts. By analysing some special extreme cases, they show that the model of Tsiveriotis and Fernandes [1998] predicts sub-optimal behaviour by the bond holder.

In the 'hedge model' of Ayache et al. [2003], which we have implemented, the inputs to pricing are

- the current price of the stock;
- the annual volatility of the stock;
- the risk free rate or risk free yield curve, and the associated compounding convention;
- the maturity date of the bond;
- declared or predicted cash dividends or discrete dividend yields, and their timing, on the common stock, for the life of the bond;
- conversion ratio(s), being the number of shares into which the bond can be converted at any particular time. This number may change through time, but some conversion is always permitted. The usual reason for the conversion ratio changing, is because of pre-specified events, such as dividends.
- hard put and call provisions, their strike prices, and timing;
- soft call provisions, their strike prices, and timing;
- the annual coupon paid on the bond and its annual frequency, more generally, the sizes of coupons and their payment dates, for the life of the bond;
- an additional parameter needed for a parsimonious model of the hazard rate as increasing as the stock price declines;
- the proportional loss in the price of the share when default on the convertible bond occurs. This model does not assume that the firm as a whole becomes insolvent if default on the bond occurs, although by setting this parameter to 100% one recovers this model;
- the current market implied intensity of default (hazard rate) on the debt of the underlying (such intensities can be found using standard credit software, such as KMV KMV [2008]);
- a recovery percentage of the bond value embedded in the convertible bond, on default of the issuer on the convertible bond;
- the credit spread associated with similar straight bonds issued by the same or similar institution.

Of the last three parameters, only two are needed. These three parameters are related, so given any two, the third, if required, is derived internally.

As already discussed, setting both the percentage loss in the share price and the recovery percentage to 0 one recovers the model from Tsiveriotis and Fernandes [1998].

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