An Empirical Comparison of Convertible Bond Valuation Models

This paper empirically compares three convertible bond valuation models. We use an innovative approach where all model parameters are estimated by the Marquardt (1963) algorithm using a subsample of convertible bond prices. The model parameters are then used for out-of-sample forecasts of convertible bond prices. The mean absolute deviation is 1.86% for the Ayache-Forsyth-Vetzal (2003) model, 1.94% for the Tsiveriotis-Fernandes (1998) model, and 3.73% for the Brennan-Schwartz (1980) model. For this and other measures of fit, the Ayache-Forsyth-Vetzal (2003) and Tsiveriotis-Fernandes (1998) models outperform the Brennan-Schwartz (1980) model.

Yuriy Zabolotnyuk is an Assistant Professor in the Sprott School of Business, Carleton University, Ottawa, ON, Canada.

Robert Jones is a Professor of Economics at the Department of Economics, Simon Fraser University, Burnaby, BC, Canada.

Chris Veld is a Professor of Finance in the Department of Accounting and Finance, University of Stirling, United Kingdom and Adjunct Professor of Finance, Faculty of Business Administration, Simon Fraser University.

The authors thank Christina Atanasova, Alex Paseka, Christophe Perignon, Amir Rubin, Yulia Veld-Merkoulova, seminar participants at Simon Fraser University and the University of Stirling, and participants at the Financial Management Conference in Orlando (October 2007), the Scottish BAA in Glasgow (September 2008), and the Northern Finance Association in Calgary (September 2008) for helpful comments and suggestions. Special thanks go to an anonymous referee and to the editor for their very useful comments. In addition, Chris Veld gratefully recognizes the financial support of the Social Sciences and Humanities Research Council of Canada. The usual disclaimer applies.

I. Introduction

Exchange-listed companies frequently attract capital by issuing convertible bonds. During the period from 1990-2003, there were globally more than seven thousand issues of convertible bonds.[1] An important issue with convertible bonds is that they are difficult to value. This is due to the fact that the exercise of the conversion right requires the bond to be redeemed in order to acquire the shares. For this reason, a conversion right is, in fact, a call option with a stochastic exercise price. In addition, most convertible bonds are callable in practice.[2] This means that the issuing firm has the right to pay a specific amount, the call price, to redeem the bond before the maturity date. In some convertible bond contracts the call notice period is specified thereby requiring the firm to announce the calling date well before the redemption can be performed. Often, the call notice period is combined with a soft call feature where the bond can only be called if the underlying stock price stays above a certain pre-specified level for a pre-specified period. All these features complicate the valuation process for convertible bonds.

Despite the importance of convertible bond valuation for both academic and practical purposes, there is not much empirical literature on this topic. This paper aims to fill this gap by empirically comparing three different convertible bond valuation models for a large sample of Canadian convertible bonds.

Convertible bonds are issued by corporate issuers and, as such, are subject to the possibility of default. There are two main approaches for valuing securities with default risk. The

---

[1] See Loncarski, ter Horst, and Veld (2006).

[2] Surveys amongst managers that have issued convertible bonds generally find that the ability to call or force conversion of convertible debt, if and when the company needs it, is one the most important reasons to issue these securities. Graham and Harvey (2001) and Bancel and Mittoo (2004) find that this ability is important or very important, respectively, for 48% of the U.S. managers and 55% of the European managers that they survey. Brounen, De Jong, and Koedijk (2006) ask the same question and find important differences between the UK (80%) on one side and the Netherlands (16.67%), France (12.50%), and Germany (20%) on the other side.

first approach, called structural approach, assumes that default is an endogenous event and bankruptcy happens when the value of the firm's assets reaches some low threshold level. This approach was pioneered by Merton (1974) who assumes that the firm value follows a stochastic diffusion process and default happens as soon as the firm value falls below the face value of the debt. However, as pointed out by Longstaff and Schwartz (1995), a default usually happens well before the firm depletes all of its assets. Valuation of the multiple debt issues in Merton (1974) is subject to strict absolute priority where any senior debt has to be valued before any subordinated debt is considered. This creates additional computational difficulties for valuing defaultable debt of firms with multiple debt issues. Moreover, the credit spreads implied by the approach of Merton (1974) are much smaller than those observed in financial markets.

In contrast, a default in the Longstaff and Schwartz (1995) model happens before the firm exhausts all of its assets and as soon as the firm value reaches some predefined level common for all issues of debt. The values of the credit spreads predicted by their model are comparable to the market observed spreads. The common default threshold for all securities allows valuation of multiple debt issues. To obtain more realistic credit spreads, particularly for short maturity issues, Zhou (2001) develops a structural approach model where both diffusion and jumps are allowed in the asset value process. The addition of a jump process allows for the possibility of instantaneous default caused by a sudden drop in firm value.

In the structural approach, debt is viewed as an option on the value of the assets of the firm, and an option embedded in the convertible bond can be viewed as a compound option on the value of firm assets. Therefore, the Black and Scholes (1973) methodology can be used for valuing convertible bonds. The models of Brennan and Schwartz (1977) and Ingersoll (1977) apply the structural approach to the valuation of convertible bonds. In these models, the interest rates are assumed to be non-stochastic. Brennan and Schwartz (1980) correct this by incorporating stochastic interest rates. However, they conclude that for a reasonable range of interest rates, the errors from the non-stochastic interest rate model are small. For practical purposes, it is preferable to use the simpler model with non-stochastic rates.

Nyborg (1996) argues that one of the main problems inherent in the implementation of structural form models is that the convertible bond value is assumed to be a function of the firm value, a variable not directly observable. To circumvent this problem, some authors model the price of convertible bonds as a function of the stock price, a variable directly observable in the market. The model of McConnell and Schwartz (1986) is such an example. In this model, they price Liquid Yield Option Notes (LYONs), which are zero coupon convertible bonds callable by the issuer and putable by the bondholder. However, one of the main drawbacks of their model is the absence of a bankruptcy feature.

The second approach used for the valuation of defaultable corporate obligations is the reduced form approach. In contrast to the structural approach where default is an endogenous event tied to the firm's value and capital structure, in the reduced form models default is an exogenous event. In the reduced form approach, the default risk of a firm and its value are not explicitly related; at any point in time, the probability of default is defined by a Poisson arrival process and is described by a hazard function. The application of this approach for valuing defaultable non-convertible bonds can be found in the models of Jarrow and Turnbull (1995), Duffie and Singleton (1999), and Madan and Unal (2000).[3] The attractiveness of this approach is that the convertible bond value can be modeled as a function of the stock price. The models of

[3] See Andersen and Buffum (2004) for an excellent discussion of problems associated with the calibration and numerical implementation of reduced form convertible bond valuation models.

Tsiveriotis and Fernandes (1998), Takahashi, Kobayashi, and Nakagawa (2001), and Ayache et al. (2003) use the reduced form approach for valuing convertible bonds.[4]

In contrast to the extensive theoretical literature on convertible bond pricing, there is very little empirical literature on this topic. Some researchers use market data to verify the degree of accuracy of their own models. Cheung and Nelken (1994) and Hung and Wang (2002) use market data on single convertible bonds to verify their models. Ho and Pfeffer (1996) use market data on seven convertible bonds to perform a sensitivity analysis of their two-factor multinomial model. King (1986) uses a sample of 103 U.S. convertible bonds and finds that the average predicted prices of the Brennan-Schwartz (1980) model with non-stochastic interest rates are not significantly different from the mean market prices. Carayannopoulos (1996) uses the stochastic interest rate variant of the Brennan-Schwartz (1980) model. For a sample of 30 U.S. convertible bonds, he finds a significant overpricing of deep-in-the-money convertible bonds. Takahashi et al. (2001) use data on four Japanese convertible bonds to compare their model to the models of Tsiveriotis and Fernandes (1998), Cheung and Nelken (1994), and Goldman-Sachs (1994). On the basis of the mean absolute deviation, which is calculated as the difference between the model and the market price expressed as a percentage of the market price, they conclude that their model produces the best predictions of convertible bond prices. Ammann, Kind, and Wilde (2003) use 18 months of daily French market data and the Tsiveriotis-Fernandes (1998) model to find that, on average, market prices of French convertibles are 3% lower than the model predicted prices.

[4] Other examples are the models of Cheung and Nelken (1994), Ho and Pfeffer (1996), and Hung and Wang (2002). The Cheung and Nelken (1994) and Ho and Pfeffer (1996) models are two-factor models, where the state variables are the stock price and the credit spread-adjusted interest rate. Cheung and Nelken (1994) assume no correlation between the interest rates and stock price changes while Ho and Pfeffer (1996) assume that the correlation is constant across periods. Hung and Wang (2002) add one more state variable, the default event, in addition to the stock price and interest rate.

Most companies that issue convertible bonds don't have straight bonds outstanding. For this reason, we cannot use straight bond parameters, such as the credit rating, when calculating model prices for convertible bonds. Moreover, other model parameters, such as the underlying state variable volatility, the dividend yield, and the default rate, are often not directly observable. Therefore, we use an innovative technique that allows for the calculation of model prices even when the values of the parameters are not observable. We divide our sample into two parts: 1) a historical sample and 2) a forecasting sample. Instead of using the values of the parameters inferred from the plain debt data or underlying stock market data, we use the information contained in the historical convertible bond prices to estimate the necessary parameters. This approach allows for forecasting the convertible bond prices using the convertible bond price series only. The data from the historical sample are used to calibrate model parameters. We then calculate model prices for the forecasting period and compare these to market prices.

The estimation procedure becomes very complicated if all the features of convertible bond contracts are taken into account. Lau and Kwok (2004) demonstrate that the dimension of the valuation procedure increases rapidly if the soft-call feature is accommodated. They also determine that the calling period essentially increases the optimal call price at which the issuers should call the bond. This effective call price can be viewed as the original call price multiplied by a call price adjustment, $(1+\pi)$, in which π is the excess calling cost defined as the difference between the critical call price and the published call price. In our study, we only account for call and call notice features and we do not price the soft-call feature. The call notice period is featured by including and calibrating the excess calling cost parameter. Therefore, the results of this study are subject to the fact that some convertible bond contract features are ignored.[5]

[5] Notice that none of the bonds in our sample are putable; therefore, this is not a problem for our estimations.

First, we estimate the Brennan-Schwartz (1980) model. This seminal model for the valuation of convertible bonds has a very sound theoretical background as it explains the economic mechanisms behind the default event connecting the bankruptcy with the capital structure of the firm. However, since it is a structural form model, it requires a simultaneous estimation of all the other defaultable assets. This fact seriously complicates the estimation of the model. To eliminate this complication, we estimate this model using a subsample of firms with a simple capital structure defined as a capital structure that only consists of equity, risk-free straight debt, and convertible debt. The assumption of a simple capital structure substantially simplifies the estimation process. However, it also reduces the domain of applicability for this model.

In order to be able to value the convertible bonds of firms with a non-simple capital structure, we need to rely on the reduced form approach as it is not dependent on the capital structure of a firm. For this reason, we use two other convertible bond valuation models. The first is the Tsiveriotis-Fernandes (1998) model, now referred to as the TF (1998) model. The second is the model of Ayache et al. (2003), now referred to as the AFV (2003) model.

We find that, using the full sample of 64 bonds, the mean absolute deviation (MAD) of the model price from the market price, expressed as a percentage of the market price, is 1.86% for the AFV (2003) model. This deviation is 1.94% for the TF (1998) model. For the subsample of 17 firms that have a simple capital structure, the Brennan-Schwartz (referred to as the BS) (1980) model has a MAD of 3.73%. The corresponding results of the AFV (2003) and TF (1998) models for the subsample used by the BS (1980) model are 2.16% and 2.17%, respectively. The BS (1980) model illustrates the smallest range of pricing errors. For the TF (1998) and AFV (2003) models, we find a negative correlation between moneyness and the absolute values of the pricing errors while this relationship is positive for the BS (1980) model. This means that the AFV (2003) and TF (1998) models mis-price convertibles that have in-the-money conversion options less than convertibles with conversion options that are at-the-money or out-of-the-money. We find a positive association for the reduced form models between the absolute values of pricing errors and the volatility of the returns of the underlying stocks. The effect of volatility on the absolute deviations in the BS (1980) model is statistically insignificant.

The remainder of this paper is structured as follows. In Section II, we present different convertible bond valuation models. Section III includes the data description. Section IV is devoted to the estimation of the parameters. The results of the estimation are presented in Section V. The paper wraps up with Section VI where the summary and conclusions are presented.

II. Convertible Bond Valuation Models

A. Model Selection

A comparison of valuation models is possible if all the model input variables are either directly observable or can be estimated. Structural models that use non-directly observable variables, such as firm value, are very difficult to estimate. Their estimation becomes easier if a simple capital structure of the firm is assumed. Alternatively, reduced form models use directly observable market variables and are much easier to estimate. This explains their popularity among practitioners. The selection of models that are used for the comparison in our study is based on popularity with practitioners as well as their sound theoretical underpinnings. In this paper, we compare the models of Brennan and Schwartz (BS) (1980), Tsiveriotis and Fernandes (TF) (1998), and Ayache et al. (AFV) (2003). [6]

[6] In their overview of convertible bond valuation models, Grimwood and Hodges (2002) argue that the approach of Tsiveriotis and Fernandes (1998) is the most popular among practitioners.

B. Convertible bond valuation models

1. The BS (1980) Model

Brennan and Schwartz (1977, 1980) develop a structural type approach for valuing convertibles where the convertible bond value is modeled in terms of the firm value. The main assumptions of their approach are: 1) the firm value, W, is the central state variable, the risk-adjusted return on which is the risk-free rate at each instant; 2) the dilution effect resulting from conversion must be handled consistently; 3) the effect of all cash flows on the evolution of the firm value must be accounted for; 4) assets must be sufficient to fund all assumed recoveries in default; and 5) the share price process is endogenously determined by all of this.

The firm value, W, is assumed to be governed by the stochastic process $dW = (rW - D(W) - rB_s - cB_c)dt + \sigma W dW$ in which r is the instantaneous risk-free interest rate, B_s is the par value of senior straight bonds outstanding, B_c is the par value of convertible subordinated bonds outstanding, c is the annualized continuous coupon rate on convertible bonds, $D(W) = d\max\{0, W - B_s - B_c\}$ is the total continuous dividend payout on shares, d is the constant dividend yield on the book value of equity, and σ is the constant proportional volatility of the asset value. The stochastic process above is applied when the firm is not in default. Following Brennan and Schwartz (1980), we further assume a constant default boundary prior to convertible debt maturity at the firm asset level $\underline{W} \equiv B_s + \rho B_c$, where ρ denotes the convertible bond early recovery rate as a fraction of par. [7] This early default boundary implies W is just

[7] The original Brennan and Schwartz (1977) paper does not permit early default, while the 1980 paper has an early default boundary.

sufficient to fund full recovery on the senior straight debt, recovery on the convertibles, and zero recovery on equity at the time of default.[8]

The assumptions described earlier imply that due to standard arbitrage arguments, the value of the entire convertible bond issue, *V*, has to follow the partial differential equation

(PDE) $\frac{1}{2}V_{WW}\sigma^2W^2 + (rW - D(W) - rB_S - rB_C)V_W + cB_C + V_t - rV = 0$ where the subscripts indicate partial differentiation.

Boundary conditions characterize the convertible bond value at maturity, at early default point $\underline{W}$, at the rational early conversion level W^*, and at the rational early call-level *W* if applicable. These conditions are as follows:

$$V(W,T) = \begin{cases} \max\{B_c, C(W)\} \text{ for } W \geq B_c + B_s \\ \max\{0, W\text{-}B_s\} \text{ for } W < B_c + B_s \end{cases} \quad \text{(maturity)}$$

$$V(W,t) \geq C(W) \text{ for all } W, t \quad \text{(voluntary conversion)}$$

$$V(\underline{W},t) = \rho B_c \quad \text{(early default)}$$

$$V(W,t) = \max\{P_c, C(W)\} \text{ for } V \geq (1+\pi)P_c \text{ for all } t \geq T_c \text{ (early call)}$$

In the above, *T* is the maturity date of the convertible bond, P_c is the early call price of the bond, T_c is the first call date of the bond, *C(W)* is the conversion value of the bond given *W*, and π is

[8] Note that we treat coupons as being continuously paid for purposes of the evolution of *W*. If the coupons are periodic, this is equivalent to saying that the accruing interest is continuously and irrevocably paid into a segregated escrow account which pays full accrued interest to bondholders in the event of default. In the actual computation, coupons are paid at discrete intervals and (linearly) accrued interest is assumed to be paid to the bondholder at the instant of conversion, call, or default.

the excess value required for an early call.[9] The excess value π required for early call is introduced to accommodate the presence of a call notice period. Note that upon notice of early call, bondholders exercise the conversion right if that produces a higher value. The BS (1980) model is the only model that allows for the possibility of share dilution after the conversion of convertible bonds. The other models in our study ignore this feature by assuming no dilution after conversion. Solving the above PDE subject to the boundary conditions gives the theoretical value of the entire outstanding convertible bond issue.

There is little guidance regarding the empirical implementation of the original Brennan-Schwartz (1977, 1980) models. In our study, we have filled in the missing elements of the BS (1980) model as simply as possible by: 1) postulating fixed, but unobservable senior claims B_s (bonds, bank loans, amounts due to government and suppliers, etc.); 2) specifying dividend flows in a way that they are non-negative, yet embody likely covenants in senior and subordinated debt; 3) selecting an early default barrier consistent with the assumed risk-free nature of senior debt and assumed partial recovery rate on subordinated debt; 4) assuming a floating coupon rate equal to r on senior debt so that its market value is constant over time; and 5) assuming agents expect that the risk-free rate $r(t)$ will follow a deterministic path implied by the term structure of Treasury rates at each time t. Note that the share price process implicit in all of this cannot exhibit constant proportional volatility as typically assumed in competing models. Therefore, the stock price volatility will increase as assets fall closer to the default point. Similarly, the proportional dividend yield on the shares varies with the level of assets.

[9] The bond conversion value $C(W)$ is defined as follows. Let K denote the exercise share price for the convertible bond, N_0 the number of shares outstanding prior to exercise, and z as the fraction of firm assets and net of senior debt, owned by bondholders after conversion. $q \equiv \frac{B_c / K}{N_0}$ $z \equiv \frac{q}{1+q}$. Then, $C(W) = (W - B_s)z$

Determining the convertible bond value is a problematic task if the firm has a complex capital structure. The value of the convertible bonds has to be determined simultaneously with the values of all senior claims. In our study, we only estimate values for the BS (1980) model for companies that have a simple capital structure. This is defined as a capital structure that only consists of equity, straight debt, and convertible debt. This means that we have to exclude companies that have preferred equity, warrants, and/or different types of subordinated debt in their capital structure. In this approach, the straight debt is assumed to be risk-free. The value of the firm is simply a sum of the values of the equity, convertible debt, and straight debt. This assumption eliminates the necessity of simultaneous valuation of convertibles and senior claims.

The list of observable constant parameters for the model is: c, T, T_c, P_c, K, and N_0. We further observe at each time t, the then current risk-free forward rate structure $r(\tau)$, $\tau \geq t$, and the combined market value of shares plus convertibles (identical to firm assets net of senior claims) $W(t)-B_s$. The list of unobserved constant parameters, to be either specified or estimated, is: d, σ, ρ, π, and B_s. These parameter estimates are chosen via an extended Marquardt (1963) algorithm to minimize the sum of squared deviations of theoretical quotes from market quotes for the convertibles. This estimation procedure is, in effect, non-linear least squares since the predicted quotes are non-linear functions of the parameters being estimated.

2. The TF (1998) Model

The TF (1998) model, which is based on the methodology of Jarrow and Turnbull (1995), discriminates between two parts of the convertible bond: 1) the bond-like or cash only part (COCB) and 2) the equity-like part. The COCB is entitled to all cash payments and no equity flows that an optimally behaving owner of a convertible bond would receive. Therefore, the value of the convertible bond, denoted as *V*, is the sum of the COCB value, denoted as Σ, and the equity value, (V-Σ). The stock price is assumed to follow the continuous time process $dS = rSdt + \sigma Sdw$, where *r* is the risk-free interest rate, *σ* is the standard deviation of stock returns, and *w* is a Wiener process. Since the bond-like part is subject to default, the authors propose to discount it at a risky rate. The equity-like part is default-free and is discounted at the risk-free rate. Additionally we assume that in the event of a default, convertible bondholders recover a proportion *ρ* of the bond face value. Convertible bond valuation then becomes a system of two coupled PDEs:

For V: $$\frac{1}{2}V_{SS}\sigma^2V^2 + (r-d)SV_S + V_t - r(V-\Sigma) - (r+r_C)\Sigma = 0$$

For Σ: $$\frac{1}{2}\Sigma_{SS}\sigma^2V^2 + (r-d)S\Sigma_S + \Sigma_t - (r+r_C)\Sigma = 0$$

S is the underlying stock price, r_c is the credit spread reflecting the pay-off default risk, and *d* is the underlying stock dividend yield.

In order to find the value of the convertible bond, it is necessary to solve the system of PDEs. At each point in time, the convertible bond prices should satisfy boundary conditions. At the maturity date, the following conditions should hold: $\mathrm{V(S,T) = max(aS, F + Coupon)}$, $\Sigma(\mathrm{S,T}) = \max(\mathrm{F}, 0)$ where *a* is the conversion ratio, and *F* is the face value of the bond. At the conversion points,the constraints are: $\mathrm{V(S, t)} \geq \mathrm{aS}$; $\Sigma = 0$ if $\mathrm{V(S, t)} \leq \mathrm{aS}$. Callability constraints are: $\mathrm{V} \leq \max\text{(Call Price, aS)}$; $\Sigma = 0$ if $V \geq \text{Call Price}$.

The prices of the convertible bond are first calculated for different stock prices at the maturity date. In the equity-like region of underlying stock prices, where the value of the bond if converted is higher than the face value plus accrued coupons, the convertible bond price is equal to the conversion value. In this range, the price of the convertible bond is discounted one period back at the risk-free rate, *r*. In the stock price range, where the total of face value and accrued coupon is higher than the conversion value, the convertible bond prices are discounted at the risky rate, $r+r_c$. Working one period back, the convertible prices are calculated and the points are found where the issuer can call the bond. The iterations continue until the initial date is reached.

3. The AFV (2003) Model

The AFV (2003) model is a modified, reduced form model that assumes a Poisson default process. The authors of this model argue that the TF (1998) model does not properly treat stock prices at default as it does not stipulate what happens to the stock price of a distressed firm in the case of bankruptcy.

The AFV (2003) model boils down to solving the following equation $MV - p\max(aS(1-\eta), \rho X) = 0$, where MV is defined as:

$$MV \equiv -\frac{1}{2}V_{SS}\sigma^2 V^2 - (r + p\eta - d)SV_s - V_t + (r + p)V\ ,$$

subject to the boundary condition $\mathrm{V} \leq \max\ (\mathrm{Call\ Price,\ aS})$, where *S* is the stock price, *p* is the probability of default, η is the proportional fall in the underlying stock value after a default occurs, *r* is the interest rate, *a* is the conversion ratio,and *d* is the underlying stock dividend yield.

In their original paper (Ayache et al., 2003), the authors argue that *X* can take many forms, be it the face value of the bond or the pre-default market value of the bond. In our study, we use the version of the AFV (2003) model where we assume *X* to be equal to the bond's face value. Thus, ρ is the proportion of the bond face value that is recovered immediately after a default, and d is the continuous dividend yield on the underlying stock.

The AFV (2003) model assumes the probability of default to be a decreasing function of stock price: $p(S) = p_0^{'}(\frac{S}{S_0})^{\alpha}$ The symbols S_0, $p_0^{'}$, and α represent constants for a given firm; $p_0^{'}$ is the probability of default when the stock price is S_0. We can also group S_0 and $p_0^{'}$ together, introduce $\gamma = \frac{p_0^{'}}{S_0^{\alpha}}$, and rewrite the hazard function as $p(S) = \gamma S^{\alpha}$.

III. Data Description

In our study, we use a sample of 97 actively traded Canadian convertible and exchangeable bonds listed on the Toronto Stock Exchange as of November 1, 2005.[10] These bonds have different issue dates going back from April 1997-October 2005. The maturity dates range from March 2007-October 2015. From the original sample of 97 bonds, we exclude all exchangeable bonds as well as bonds traded in currencies other than the Canadian dollar. None of the issues in our sample is putable. After screening for the issues that have price series and prospectuses available, as well as information on underlying stocks and financial statements with dividend information, the sample reduces to 64 issues. Fifty-seven bonds of the 64 were issued by income trusts. This number is surprising since convertible bonds tend to be issued by young and growing firms while the income trust structure is more suited to stable, mature firms (Halpern, 2004). Forty-three of these 57 convertible bonds were issued by income trusts operating in the oil and gas and real estate industries.

[10] The pricing data for the convertible bonds in our study comes from the Toronto Stock Exchange. All historical price series reflect the actual prices and are not derived by extrapolation. This allows us to avoid the common problems associated with the pricing of privately traded bonds.

Seventeen firms from this sample have a simple capital structure consisting of equity, straight debt, and convertible debt only. These bonds are used for estimating the BS (1980) model. Detailed information concerning the issues used in the study can be found in Appendix A.[11]

Even though there should theoretically be no difference in the pricing of convertible bonds issued by income trusts and ordinary corporations, we briefly outline the essence of income trusts as there are no corresponding securities in U.S. market. Income trusts raise funds by issuing units of securities to the public; they purchase most of the equity and debt of successful businesses with the acquired funds. Operating businesses act as subsidiaries of income trusts which, in turn, distribute 70%-95% of their cash flow to unit holders as cash distributions (Department of Finance, Canada, 2005). Since most of the earnings are distributed to unit holders, little or no funds are left for research and development and/or capital expenditures. Therefore, the stable and mature types of businesses are deemed most suitable for an income trust structure. The preferential tax treatment of publicly traded investment vehicles made income trusts a widespread business structure in Canada in the late 1990s and early 2000s. However, in October 2006, the Canadian Government introduced changes in the tax legislation that effectively eliminated the preferential tax treatment of the income trusts.[12]

For the purpose of determining which convertible bond valuation model predicts the prices that are the closest to the market prices, the data is divided into two subsamples: 1) historical and 2) forecasting. The pricing data that we use in both the historical and forecasting samples is weekly data with prices being observed every Wednesday (to ensure a high trading

[11] The assumption of a simple capital structure reduces the sample to 17 firms. This subsample is only used only for the evaluation of the Brennan-Schwartz (1980) model. All other models are evaluated using the complete sample of 64 bonds.

[12] For a more detailed discussion regarding the changes in the income trust taxation rules, see the Department of Finance news release at http://www.fin.gc.ca/n06/06-061-eng.asp

activity on the market). The models' parameters are calibrated using the data from the historical subsample. Then, weekly model prices are calculated for each convertible bond for the forecasting period using the calibrated parameters. The best model is selected based on the distance between the actual forecasting period market prices and the model predicted prices for the forecast period.

The historical subsample for each bond starts with its issue date and ends at the start of the forecast period. The first date of the forecast period for each bond varies with the bond's issue date. The historical sub-period is defined in a way that it is not shorter than one year. If the bond was issued before January 1, 2004, then the forecast period begins on January 1, 2005. If the bond was issued from January 1, 2004-July 1, 2004, then the starting date is set to July 1, 2005. If the bond was issued after July 1, 2004, the starting date of the forecast period is set to January 1, 2006. Thus, the bonds with the earlier issue dates have longer forecast subsamples. This choice of starting date is stipulated by the need for a large enough historical subsample for the estimation of the model parameters.[13] The end of the forecasting period is fixed on April 28, 2006. We equally weigh the errors from observations in the historical subsample as the approach we use assumes that parameters stay constant over time, thus enabling us to use the calibrated values of parameters for out-of-sample convertible bond price predictions.

The Globe Investor Gold database provides data regarding historical bond prices. We take detailed information on each issue including coupon rates, maturity dates, and conversion conditions from the prospectuses available at the SEDAR (System of Electronic Document and Archive Retrieval) and the Bloomberg databases. We use the Dominion Bond Rating Service

[13] Loncarski, ter Horst, and Veld (2009) find that during the first six months after their issue, convertible bonds are underpriced. This provides a possibility for convertible arbitrage. As a robustness check, we perform an alternative pricing procedure using reduced historical samples where the first six months of data are dropped. The results of the reduced sample estimation are similar to those using the original data samples. These results are available from the authors on request.

data on existing debt and issuer ratings. The information on underlying stocks' dividends comes from companies' websites and from the Toronto Stock Exchange. The information on the number of stocks and convertible bonds outstanding is taken from the Canadian Financial Markets Research Centre database. The descriptive statistics of the convertible bond characteristics are presented in Table I.

Insert Table I about here.

As can be seen from Panel A of Table I, the shortest time to maturity for the bonds in the sample is 1.25 years while the longest time to maturity is almost 10 years; the average time to maturity is around five years. The degree of moneyness of the bonds for the sample period, S/K, ranges from 0.20-4.38. The average bond is slightly in-the-money with a ratio of the underlying stock price to the exercise price of 1.13. The least volatile underling stock has an annual standard deviation of 13%, the most volatile, 60%. The average volatility of the underlying stocks is 25%. The average coupon rate for the convertible bonds in the sample is 7.18%. Panel B of Table I indicates that the average time to maturity for the subsample of bonds used for estimating the BS (1980) model is approximately 4.5 years. The average degree of moneyness for these bonds is 1.08 and the average volatility is 27%.

Since in the evaluation of the models risk-free interest rates are used, we use the forward interest rates, derived from the Bank of Canada zero coupon bond curves, as a proxy. The forward rates used are three-month forward rates for horizons from 3 months-30 years. Zero-coupon bonds data is taken from the Bank of Canada.

Andersen and Buffum (2004) develop a method for calibrating time varying convertible bond valuation model parameters. They confirm, in their study, that the naïve assumption of constant non-time varying model parameters may cause significant convertible bond price estimation errors. Their estimation approach relies on the presence and abundance of data on the underlying firm's equity options and straight debt pricing. In contrast to the estimation techniques that require use of the firms' straight corporate bonds and/or equity options for estimating model parameters, we employ the method that uses information inherent in the convertible bond prices for calibrating the parameters of the models. Many of the firms in our sample issue convertible debt instead of straight bonds in order to save the costs of interest in the absence of a high credit rating. These young and growing firms offer investors convertible bonds with lower coupons. In exchange for these lower coupons, the conversion feature is added. The majority of these firms do not have other publicly traded corporate bonds in their capital structure. Therefore, using a method for convertible valuation that does not hinge on the presence of the firm's straight corporate debt promises to be valuable.

Unlike the approach of Andersen and Buffum (2004), our approach relies solely on the presence of convertible bond pricing data. Assuming constancy of model parameters allows us to calibrate all parameters simultaneously without referring to the pricing data on other financial instruments of the firm (many of which may be non-existent). Our assumption of market efficiency makes the use of large historical series of pricing data justifiable since a wealth of information about model parameters is contained in these prices. The information contained in the prices of the convertible bonds may be helpful to calibrate parameters of the models in the absence of other types of bonds for the firm. Moreover, by using information contained in the historical convertible price series, it is possible to estimate all other convertible bond parameters such as the underlying state variable (stock price or firm value) volatility, dividend yield, and diffusion processes parameters.

Using historical convertible prices, we employ the Marquardt algorithm (Marquardt, 1963) to search for the model parameters that minimize the squared sum of residuals between model predicted prices and market prices. Later, we use these parameters for forecasting the convertible prices for our forecasting subsample. Initial values and boundaries for parameters are provided based on the assumption of the corresponding models.

Given the convertible valuation model, the Marquardt (1963) algorithm finds the theoretical convertible bond prices given the initial values for model parameters. In the next step, the algorithm changes the model parameters until the values that return the minimum squared deviations of the model prices from the observed market prices are found.[14] The data needed for the estimation of the parameters and prediction of the out-of-sample theoretical convertible bond prices consist of the convertibles' market prices, conversion prices, issue, settlement, and maturity dates, coupon rates, number of coupons per year, market prices of the underlying stock, call schedules and call prices, and numbers of outstanding convertible bonds and stocks for the BS (1980) model.

We limit the parameters' values to ensure that the Marquardt (1963) algorithm does not assume unrealistic or non-plausible parameter values:

- The volatility σ has a lower floor of 0.05 and an upper floor of 1 (representing a range for the standard deviation from 5%-100% of stock returns or firm value gains);

[14] This technique allows estimation not just of the parameters of the hazard function, but also of the characteristics of the convertible bonds such as volatility and dividend yields that are implied by the convertible prices.

- The bond recovery fraction ρ is assumed to be between 0-1;
- The dividend yield *d* is assumed to be between 0-30%;
- The excess calling cost π to be between 0-30% of the call price;
- The debt value for the BS (1980) model to be between zero and ten times the face value of convertible debt;
- The credit spread to have a lower floor of zero.

To ensure that the calibrated parameters are robust to the starting values, we repeat the calibration for each firm several times using different starting values until we find a parameter set that represents the lowest value of squared residuals between the algorithm predicted prices and the market prices.

Note that together with the convertible bond data, data on straight bonds can be used to calibrate the parameters. In this case, the "conversion price" of straight debt has to be specified as some unrealistically large number and "call dates" have to be set after the maturity date. The parameter calibration approach where both convertible and straight bonds data is used may yield superior results as compared to the exploitation of only convertible bond data. This is because the former approach uses a wider set of market information. However, since most of the firms in our sample do not have straight corporate debt, we only use convertible bond prices for calibration of parameters.

IV. Estimation

To be able to predict the theoretical convertible bond prices, parameters such as the underlying state variable volatility, dividend yield, and the credit spread are needed. Many of these parameters are not directly observable. We take the approach using historical convertible bond prices for estimating all necessary parameters of the models.

For the BS (1980) model, the dividend payout is assumed to be a fixed proportion of the amount by which the firm value exceeds the principal owed on the debt (this is the sum of straight debt and convertible debt).[15] For the other models, the dividend yield is simply estimated as a constant proportion of the price of the underlying stocks. To account for the call notice period feature, we introduce the call price adjustment parameter π; multiplying the original call price specified in the bond prospectus by $1 + \pi$ gives us the effective call price as in Lau and Kwok (2004). This parameter is unknown and is calibrated for all the models together with the dividend yield (d), stock/firm value volatility (σ), credit spread for the TF (1998) model (r_c), default bond value recovery fraction (ρ), and hazard process parameters (α, γ) for the AFV (2003) model from the historical convertible bond prices. The dividend yield is assumed to be a constant proportion of the book value of the shares. The other unobservable variable necessary for the estimation of the BS (1980) model is the value of the firm. We calculate the firm value as the sum of the values of its common shares (market price times the amount of shares outstanding), convertible bonds (market value of the bonds times the number of convertibles outstanding), and unobserved value of senior straight debt. The value of the senior straight debt is assumed to be equal to a constant amount over time and is calibrated by the Marquardt (1963) algorithm to provide the smallest possible deviation of the model price from the market price in the historical subsample.[16] In the original BS (1980) model, the authors assume that convertible

[15] This assumption is consistent with the numerical illustration of Brennan and Schwartz (1980). In addition, it has an advantage when compared to their approach since their dividend specification can lead to negative dividends, while ours does not. Furthermore, it forces dividend payouts to stop while the firm value is still sufficient to repay senior debt completely, justifying our treatment of straight debt as risk-free.
[16] We also estimate the version of the BS (1980) model where we assume senior straight debt to be zero. In that case, the model pricing errors tend to be larger than those where we assume a constant positive value of the senior debt.

bondholders recover two-thirds of the face value in case of bankruptcy. This implies that bankruptcy of a firm occurs as soon as the firm value net of senior debt drops to two-thirds of the value of the outstanding convertible bonds. In our study, we allow this recovery proportion to be calibrated for all models.

We use the Crank and Nicolson (1947) finite difference algorithm to solve the corresponding partial differential equations and the Marquardt (1963) iterative procedure for finding the values of parameters that produce the smallest deviations of model prices from the market prices. The Crank-Nicolson (1947) algorithm assumes a time dimension step of one month (0.0833 years). The upper boundary of the spatial grid is set at a level 15 times higher than the current value of the state variable (the stock price in the TF (1998) and AFV (2003) models and the firm value in the BS (1980) model). The space between the lower and upper bounds in the spatial grid has 200 nodes. The descriptive statistics of the model parameters calibrated are included in Table II.

Insert Table II about here.

The parameters' values calibrated by the models are in a range consistent with the real market observations. The implied underlying return stock volatility (annualized standard deviation) is 29% for the AFV (2003) model and 36% for the TF (1998) model and the implied volatility of firm value is 75% for the BS (1980) model. The calibrated dividend yield is typically equal to 21% for the AFV (2003) model, 22% for the TF (1998) model, and 29% for the BS (1980) model. These numbers are reasonable considering that the majority of the bonds in the sample are issued by income trusts that historically have high cash distribution yields. The price implied recovery rates are, on average, 1% for the AFV (2003) model, 18% for the TF (1998) model, and 2% for the BS (1980) model.

The excess cost values range from 0%-30% of the original call price. The average values for the excess calling cost are 1%-15% depending on the model and fall in the range reported by Lau and Kwok (2004) for a 30-60 day call notice period.

To calculate the convertible bonds values with the TF (1998) model, the following data are needed: 1) bond issue date, 2) trading date, 3) risk-free rate, 4) price of the underlying stock at the settlement date, 5) maturity date, 6) coupon rate, 7) conversion ratio, 8) call schedule, and 9) the credit spread that reflects the credit rating of the issuer. The only input needed for calculating prices with the TF (1998) model that cannot be directly observed from the market is the credit spread. We use the average value of the credit spread for bonds that have the same credit ranking. Many of the bonds in our sample are issued by small firms, and, as such, don't have credit ratings assigned. For companies that do not have credit ratings assigned, we assume a BBB rating. The average credit spreads are taken from the Canadian Corporate Bond Spread Charts published by RBC Capital Markets.

Prices are also calculated for the AFV (2003) model. This model allows for a different behavior of stock prices in case the firm defaults on its corporate debt. The partial default version assumes that the price of the underlying stock is partially affected by the firm's default on its bonds. The total default model assumes that the stock price jumps to zero when default takes place. We also assume that in case of a default, convertible bondholders recover a fraction ρ of the bond face value. In this study, we use the total default version of the AFV (2003) model.[17]

[17] The results of the estimation of the partial default AFV (2003) model with the assumed value of parameter $\rho=0$ (no change in the stock price at default) are very similar to the total default ($\rho=1$) AFV (2003) model. These results are available on request from the authors.

It should be mentioned that we cannot capture all the conditions of convertible bond contracts in our estimation. Some of the bonds in our sample have "soft call" provisions. In a typical case, the bond can be called only if the underlying trades above the barrier for 20 out of the previous 30 trading days resulting in a very high dimensional valuation problem. In addition, there are occasionally other special conditions. For example, with regard to the first bond mentioned in Appendix A, the issuer has the option to pay (either on the redemption date or at maturity and subject to a prior notice period) either in terms of cash or in terms of additional shares, the number of which is determined by "95% of the weighted average trading price…for the 20 consecutive trading days ending on the fifth trading day preceding the date fixed for redemption on the maturity date" (see page 3 of the prospectus). Such special conditions are not captured in our valuation and our results should be read against this background.

V. Results

Our comparison of convertible bond pricing models is based on the scores that demonstrate the ability of the models to generate prices that are close to market prices. We will base our decision on several scores. The first and the most important score is the "Mean Absolute Deviation" (MAD). It is calculated as:

$$\text{MAD} = \text{average}\left[\text{abs}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]\right]$$

The MAD measures best the pricing ability since it takes into account deviations from market prices from both sides. The second indicator is the "Mean Deviation" (MD). This is calculated as the average deviation of the model price from the market price as a percentage of the convertible bond market price:

$$\text{MD} = \text{average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]$$

The MD gives us an idea as to the average model over or under pricing of the convertible bonds. Another indicator of model fit is the "Root of Mean Squared Error" (RMSE), which is calculated as:

$$\text{RMSE} = \sqrt{\text{average}\left[\frac{\text{Market Price - Model Price}}{\text{Market Price}}\right]^2}$$

The MAD and MD scores assign the same weight to all errors. There is no additional penalty for the instances when the model price is far from the market price. The RMSE score gives larger weight to large deviations.

We also calculate the percentage of forecasted model prices that fall within specific intervals around the market price as one of the measures of model pricing precision. We use 10%, 5%, and 1% intervals around market prices for this purpose. Table III provides rankings of the models based on the indicators mentioned above.

Insert Table III about here.

Based on the results in Table III, we conclude that the AFV (2003) model and the TF (1998) model best demonstrate the predictive power for both the full sample of 64 convertibles and the subsample of 17 convertibles issued by firms with a simple capital structure consisting of equity, straight debt, and convertible debt only (BS (1980) subsample). For the full sample of 64 bonds, the AFV (2003) model illustrates the lowest values of mean absolute deviations. The value of the MAD is 1.86%. The AFV (2003) model reports a slight overpricing of convertibles; the value of the MD is -0.35%. Almost 99% of the model errors are lower than 10% of bond market prices; 45% of the errors are smaller than 1% of the bond market value. For the subsample used for estimating the BS (1980) model, the AFV (2003) model has a MAD of 2.16% and a MD of -0.52%.

For the full sample of bonds, the TF (1998) model has an average MAD of 1.94%. On average, the TF (1998) model overprices convertibles by 0.19%. This result is in line with that of Ammann et al. (2003) who find that the TF (1998) model typically overprices French convertible bonds by 3%. Slightly more than 99% of the model predictions fall within 10% of market prices; 43% of the predicted pricing errors are less than 1% of the market price as compared to 45% for the AFV (2003) model. For the BS (1980) subsample, the TF (1998) model has a MAD of 2.17% and a MD of 0.29%.

The BS (1980) model presents the largest MAD score: 3.73%. The BS (1980) model, on average, overprices the convertibles relative to the market by 2.78%; 96% and 22% of pricing errors are less than 10% and 1% of the market price, respectively. The average MD is significantly different from zero for all three models. Therefore, we can reject the null hypothesis that the models have a mean zero pricing error.

Based on the RMSE, the most accurate models are the AFV (2003) model and the TF (1998) model. The RMSE for the AFV (2003) model and the TF (1998) model using the full sample is 2.93 and 2.95, respectively. The corresponding figures for the BS (1980) subsample are 2.94 and 3.19. The BS (1980) model has the largest RMSE score of 4.94. Even though the RMSE score weights large pricing errors more heavily, the ranking of the models by the means of RMSE is the same as by means of MD.

To see whether there exist any differences in the way the models price convertibles of income trusts versus convertibles of ordinary corporations, we separate our sample into two parts: 1) the first part contains only income trusts and 2) the second part only contains ordinary corporations. The results of these two subsamples are reported in Panel A of Table IV.

Insert Table IV about here.

The average MD score for the TF (1998) model is -1.49% and -0.45% for trusts and non-trusts, respectively. The AFV (2003) model predicts errors of -0.57% and -2.47% and the BS (1980) model elicits errors of -3.95% and -0.16% for trusts and non-trusts, respectively. The sample means tests reject the null hypothesis of equal means at the 1% significance level. Therefore, we can conclude that the models price convertibles of income trusts and of ordinary corporations differently.

Loncarski, ter Horst, and Veld (2008), who study motives for the issuance of convertible bonds in Canada, find that convertible bonds issued by income trusts are more debt-like and that convertible bonds issued by ordinary corporations are more equity-like. This difference may explain the fact that convertibles issued by income trusts are priced differently from convertibles issued by ordinary corporations. In order to test whether this is really the case, we follow the approach of Loncarski et al. (2008) and divide the convertible bonds in our study into equity-like and debt-like convertibles. Their distinction is based on the Black-Scholes (1973) delta measure.[18] Convertible bonds with a delta higher than 0.5 are defined as equity-like while convertibles with a delta lower than 0.5 are defined as debt-like.[19] In Panel B of Table IV, we follow their approach and come to the same conclusion as Loncarski, et al. (2008). Convertibles of ordinary companies are more equity-like and convertibles of income trusts are more debt-like.

Using the Black-Scholes (1973) delta, we find that 51 of 57 bonds issued by income trusts are debt-like and 6 are equity-like; three of seven bonds issued by corporations are debt-like, and four are equity-like. Of the 17 issues used in the BS (1980) model, 15 are debt-like and two are equity-like. Using differences in the mean deviations (MDs), we find significant differences in the pricing of equity- and debt-like convertibles for all three models. The difference is significant at the 1% level for the BS (1980) and AFV (2003) models and significant at the 10% level for the TF (1998) model.

However, one issue with the approach of Loncarski et al. (2008) is that they assume that conversion rights can be valued using a standard option pricing model. In practice, this may cause important problems because, as we mentioned earlier, the conversion right is "paid" by redeeming the accompanying bond making it an option with a stochastic exercise price. Additionally, the convertible bonds in our sample are largely callable and many have call notice

[18] Their approach follows earlier studies of e.g. Lewis, Rogalski, and Seward (1999), Burlacu (2000), and Dutordoir and Van de Gucht (2007).

[19] The convertible bond can be equity-like either because the issuer's equity value is very high (in which case conversion is very attractive for the holder) or because it is very low (so that having the bond effectively is giving ownership of the firm's assets). Therefore, we inspected the equity-like issues and found that all of them except one had stock prices at levels higher than the conversion price most of the time. This means that high deltas for these issues were caused by high equity values. Additionally, we checked the financial statements of the only exception for the research period to find that the company was in a relatively good standing with the ability to repay all of its debt obligations.

periods. Therefore, we also calculate deltas directly from the Crank-Nicolson (1947) algorithm used for calculating convertible bond prices using the AFV (2003) model.[20] This methodology takes the specific convertible bond characteristics into account.[21]

The "numerical" delta is calculated as the change in the convertible bond price caused by a unit change in the underlying stock price at any given date. Since a convertible bond can be converted into multiple shares, we normalize the numerical delta by dividing the calculated delta by the bond's conversion ratio. Again, we define convertible bonds with a delta higher than 0.5 as equity-like and convertibles with a delta lower than 0.5 as debt-like. According the numerical deltas, of the 57 issues distributed by the income trusts in our sample, 25 issues were debt-like and 32 issues were equity-like. The percentage of equity-like issues among the convertibles issued by ordinary corporations was similar. Four of seven convertibles issued by these corporations were equity-like. Thus, in contrast to the results in Panel B, we find that the proportion of debt-like or equity-like convertible bonds issued is similar for income trusts and ordinary corporations.

Further, we re-evaluate the mispricing of the debt-like and the equity-like convertibles and we test whether there is a difference in the mispricing. In Panel C, we can see that the MD for the debt-like convertibles is -0.90% for the TF (1998) model and the score for the equity-like issues is -0.03% for the same model. The sample means tests reject the null hypothesis that the errors for the equity and debt-like issues come from the same distribution at the 1% significance level for this model. Similarly, at the 5% level of significance, we cannot reject the null hypothesis of equal sample means for the BS (1980) and AFV (2003) models. The analysis in Panels B and C suggests that the different results for income trusts versus non-trusts can partly

[20] The classification into debt-like and equity-like issues does not change regardless as to whether the numerical deltas were calculated from the AFV (2003) model or the TF (1998) model numerical routines.
[21] We thank an anonymous referee for this suggestion.

be explained by the difference in equity- and debt-likeness of the convertibles. Probably, a more important lesson from the differences between Panels B and C is that the delta measure from the original Black-Scholes (1973) model is unsuitable to distinguish between equity- and debt-like convertibles. If the specific characteristics of convertibles are taken into account, the delta measures strongly differ from the Black-Scholes (1973) deltas.

Table V provides the descriptive statistics for the mean deviations of the models' prices from the market prices as the percentage of the market prices.

Insert Table V about here.

The BS (1980) model's pricing ranges from underpricing the convertible securities by 22.44% of the market price to an overpricing by 14.38% creating a range of 36.82%. For the full sample of 64 bonds, the pricing errors for the TF (1998) model range from a negative 23.17% to a positive 15.72% and for the AFV (2003) model, the errors span a range of 41.69% (from -24.58% to 17.11%). For the BS (1980) subsample of bonds, the pricing error ranges are 25.35% and 21.99% for the TF (1998) and AFV (2003) models, respectively.

We also examine whether there are convertible bond characteristics that affect the mispricing in a systematic way. In order to check for any such regularities, we perform a regression analysis where the pricing errors are regressed on characteristics of the convertible securities. These characteristics include the moneyness measured as the ratio of the current market price to the conversion price (S/K), the annual historically observed volatility of the underlying stock (VOLAT), the convertible bond coupon rate (COUPON), and the remaining time to maturity of the convertible security (TMAT). We assume that the pricing errors are identically and independently distributed with a normal distribution that has an expected value of zero and a finite variance. Table VI presents the results of the regressions where the dependent variable is the MAD. The results in this table aid in locating the variables that explain the precision of the models.

Insert Table VI about here.

As can be seen from Table VI, the MAD statistically depends on the degree of the convertible bond moneyness for the reduced form models. On average, convertible securities that are deep in-the-money have smaller MADs than convertible securities that are at-the-money or out-of-the-money in the TF (1998) and AFV (2003) models. The BS (1980) model coefficient for moneyness is not statistically significant.

The underlying stock volatility has a positive effect on the MADs for the TF (1998) and AFV (2003) models. This implies that convertible bonds with highly volatile underlying stocks are mispriced more heavily by all models except the BS (1980) model. This may happen because in the BS (1980) model, the firm volatility instead of the stock volatility is used for our calculations.

The TF (1998) model tends to have smaller absolute deviations for the bonds with longer times to maturity as the coefficient for time to maturity is negative and statistically significant at the 1% level. The coefficients for the AFV (2003) and BS (1980) models are not statistically significant. This result contrasts with the results of King (1986) who finds heavier mispricing for the convertibles close to maturity in the BS (1980) model.

The convertible bond coupon rates have a statistically significant positive effect on the size of the absolute values of the pricing errors for BS (1980) model. The BS (1980) model predicts larger absolute deviations for the bonds with higher coupon rates.

Table VII reports the regression results where the dependent variable is the MD. The regression of the actual values of the pricing errors helps to find the variables that explain the direction of mispricing (i.e., whether the convertibles are under or over priced).

Insert Table VII about here.

From Table VII, we note that the BS (1980) model tends to overprice the bonds with higher coupon rates. The effect of the coupon rate is opposite for the AFV (2003) and TF (1998) models. Moneyness has no statistically significant effect on the size of MDs for all three models. This result is in contrast to the results of King (1986), Carayannopoulos (1996), and Ammann et al. (2003) who report a positive correlation between overpricing and moneyness.

Volatility and time to maturity do not have any effect on the direction of pricing for the models in our study per the results in Table VII. This result is dissimilar to the results of Ammann et al. (2003) who find a larger underpricing for bonds with longer terms to maturity.

VI. Summary and Conclusions

We compare the price prediction ability of one structural and two reduced form convertible bond pricing models using actual market data on convertible bonds traded on the Toronto Stock Exchange. To set us apart from other studies, we estimate all model parameters from the convertible bond price series. This approach allows for the calculation of theoretical convertible bond prices even when the issuing firms have no straight debt outstanding or when parameters such as the credit spread or the dividend yield are not observable from market data. The Ayache, Forsyth, and Vetzal (2003) model and the Tsiveriotis and Fernandes (1998) models perform similarly while outperforming the Brennan and Schwartz (1980) model based on the magnitude of the pricing errors. On average, using the full sample of 64 bonds, the mean absolute deviation (calculated as the absolute difference between the market and the model price expressed as a percentage of the market price) is 1.86% for the AFV (2003) model and 1.94% for the TF (1998) model. The BS (1980) model reports a MAD of 3.73% for the subsample of 17 convertible bonds issued by firms with simple capital structure. For the same subsample of bonds used by the BS (1980) model, the AFV (2003) and TF (1998) models have MADs of 2.16% and 2.17%, respectively. In addition to the lower pricing errors, the AFV (2003) and TF (1998) models have an advantage over the BS (1980) model since the former models can be used for pricing convertibles issued by a broader range of firms including bonds of companies with complex capital structures.

References

Ammann, M., A. Kind, and C. Wilde, 2003, "Are Convertible Bonds Underpriced? An Analysis of the French Market," *Journal of Banking and Finance* 27, 635-653.

Andersen, L. and D. Buffum, 2004, "Calibration and Implementation of Convertible Bond Models," *Journal of Computational Finance* 7(2), 1-34.

Ayache, E., P.A. Forsyth, and K.R. Vetzal, 2003, "Valuation of Convertible Bonds with Credit Risk," *Journal of Derivatives* 11(Fall), 9-29.

Bancel, F. and U. Mittoo, 2004, "Cross-Country Determinants of Capital Structure Choice: A Survey of European Firms," *Financial Management* 33(Winter), 103-132.

Black, F. and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, 637-654.

Brennan, M.J. and E.S. Schwartz, 1977, "Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion," *Journal of Finance* 32, 1,699-1,715.

Brennan, M.J. and E.S. Schwartz, 1980, "Analyzing Convertible Bonds," *Journal of Financial and Quantitative Analysis* 15, 907-929.

Brounen, D., A. de Jong, and K. Koedijk, 2006, "Capital Structure Policies in Europe: Survey Evidence," *Journal of Banking and Finance* 30, 1,409-1,442.

Burlacu, R., 2000, "New Evidence on the Pecking Order Hypothesis: The Case of French Convertible Bonds," *Journal of Multinational Financial Management* 10, 439-459.

Carayannopoulos, P., 1996, "Valuing Convertible Bonds under the Assumption of Stochastic Interest Rates: An Empirical Investigation," *Quarterly Journal of Business and Economics* 35, 17-31.

Cheung, W. and I. Nelken, 1994, "Costing the Converts," *RISK* 7, July, 47-49.

Crank J. and P. Nicolson, 1947, "A Practical Method for Numerical Evaluation of Solutions of Partial Differential Equations of the Heat-Conduction Type," *Proceedings of the Cambridge Philosophical Society* 43, 50-67.

Department of Finance, Government of Canada, "Tax and Other Issues Related to Publicly Listed Flow-Through Entities (Income Trusts and Limited Partnerships)," *Consultation Paper*, September 2005.

Duffie, D. and K.J. Singleton, 1999, "Modeling Term Structures of Defaultable Bonds," *Review of Financial Studies* 12, 687-720.

Dutordoir, M. and Van de Gucht, L. (2007), "Are There Windows of Opportunity for Convertible Debt Issuance? Evidence for Western Europe," *Journal of Banking and Finance* 31, 2,828-2,846.

Goldman-Sachs, 1994, "Valuing Convertible Bonds as Derivatives," *Goldman-Sachs Quantitative Strategies Research Note*, November.

Graham, J.R. and C.R. Harvey, 2001, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics* 60, 187-243.

Grimwood, R. and S. Hodges, 2002, "The Valuation of Convertible Bonds: A Study of Alternative Pricing Models," Warwick University Working Paper.

Halpern, P., 2004, "Is the Trust in Trusts Misplaced? A Study of Business Income Trusts and Their Role in Canadian Capital Markets," Rotman School of Management, University of Toronto Working Paper.

Ho, T.S.Y. and D.M. Pfeffer, 1996, "Convertible Bonds: Model, Value Attribution, and Analytics," *Financial Analysts Journal* 52(5), 35-44.

Hung, M.W. and J.Y. Wang, 2002, "Pricing Convertible Bonds Subject to Default Risk," *Journal of Derivatives* 10(Winter), 75-87.

Ingersoll, J.E., 1977, “A Contingent-Claims Valuation of Convertible Securities,” *Journal of Financial Economics* 4, 289-321.

Jarrow, R. and S. Turnbull, 1995, “Pricing Derivatives on Financial Securities Subject to Credit Risk,” *Journal of Finance* 50, 53-85.

King, R., 1986, “Convertible Bond Valuation: An Empirical Test,” *Journal of Financial Research* 9, 53-69.

Lau, K.W. and Y.K. Kwok, 2004, “Anatomy of Option Features in Convertible Bonds,” *Journal of Futures Markets* 24, 513-532.

Lewis, C.M., R.J. Rogalski, and J.K. Seward, 1999, “Is Convertible Debt a Substitute for Straight Debt or for Common Equity?” *Financial Management* 28(Autumn), 5-27.

Loncarski, I., J. ter Horst, and C. Veld, 2006, “Why Do Companies Issue Convertible Bonds? A Review of Theory and Empirical Evidence,” in L. Renneboog, Ed., *Advances in Corporate Finance and Asset Pricing* (p. 311-339), Amsterdam, Elsevier.

Loncarski, I., J. ter Horst, and C. Veld, 2008, “Why Do Companies Issue Convertible Bonds: An Empirical Analysis for the Canadian Market,” *Canadian Journal of Administrative Sciences* 25, 214-236.

Loncarski, I., J. ter Horst, and C. Veld, 2009, “The Rise and Demise of the Convertible Arbitrage Strategy,” *Financial Analysts Journal* 65 (September/October), 35-50.

Longstaff, F.A. and E.S. Schwartz, 1995, “A Simple Approach to Valuing Risky Fixed and Floating Rate Debt,” *Journal of Finance* 50, 789-819.

Madan, D. and H. Unal, 2000, “A Two-Factor Hazard Rate Model for Pricing Risky Debt and the Term Structure of Credit Spreads,” *Journal of Financial and Quantitative Analysis* 35, 43-65.

Marquardt, D.W., 1963, “An Algorithm for Least-Squares Estimation of Nonlinear Parameters,” *Journal of the Society for Industrial and Applied Mathematics* 11, 431-441.

McConnell, J.J. and E.S. Schwartz, 1986, “LYON Taming,” *Journal of Finance* 41, 561-576.

Merton, R.C., 1974, “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *Journal of Finance* 29, 449-470.

Nyborg, K.G., 1996, “The Use and Pricing of Convertible Bonds,” *Applied Mathematical Finance* 3, 167-190.

Takahashi, A., T. Kobayashi, and N. Nakagawa, 2001, “Pricing Convertible Bonds with Default Risk,” *Journal of Fixed Income* 11, 20-29.

Tsiveriotis, K. and C. Fernandes, 1998, “Valuing Convertible Bonds with Credit Risk,” *Journal of Fixed Income* 8, 95-102.

Zhou, C., 2001, “The Term Structure of Credit Spreads with Jump Risk,” *Journal of Banking and Finance* 25, 2,015-2,040.

Table I. Descriptive Statistics of the Convertible Bonds Sample Used

Panel A of this table presents the descriptive statistics of the sample of 64 convertible bonds used in our study on the comparison of convertible bond pricing models. All the bonds in the sample are traded on the Toronto Stock Exchange. VOLAT refers to the annualized historical standard deviation of the returns on the underlying stock; TMAT refers to the remaining time to maturity (in years) of the bonds as of December 1, 2005. COUPON refers to the convertible bond coupon rates and is expressed in percent. S/K refers to the ratio of the average stock price during the forecast period to the conversion price. Panel B of this table provides the descriptive statistics for the subsample of convertible bonds (17 convertibles) issued by firms with a capital structure that only consists of equity, straight debt, and convertible debt.

	VOLAT	COUPON	TMAT	S/K
Panel A. Full Sample				
Minimum	0.13	5	1.25	0.2
Maximum	0.6	10	9.92	4.38
Average	0.25	7.18	4.92	1.13
Median	0.23	6.75	4.72	1.1
Standard Deviation	0.08	1.18	2.4	0.46
Panel B: Simple Capital Structure Firms Subsample				
Minimum	0.14	5	1.25	0.2
Maximum	0.6	10	9	4.38
Average	0.27	7.06	4.49	1.08
Median	0.23	6.65	4.12	0.95
Standard Deviation	0.12	1.31	2.53	0.72

Table II. Calibrated Model Parameters' Descriptive Statistics

This table reports the descriptive statistics of the model parameters calibrated using the Marquardt (1963) algorithm. Parameter *d* is the implied dividend yield of an underlying stock; σ is the implied volatility of underlying stock, π is the excess calling cost as the proportion of the original call price used to proxy the presence of the early call notice period; *ρ* indicates the price implied proportion of the bond value recovered in case of default. The model specific parameters are: for the AFV (2003) model, α and γ are parameters of hazard function *p(S)*, where $p(S) = p_0^{'}(\frac{S}{S_0})^{\alpha}$, $\gamma = \frac{p_0^{'}}{S_0^{\alpha}}$; for the TF (1998) model, r_c is an implied credit spread (in decimal form); for the BS (1980) model, *debt* denotes the calibrated value of the senior debt in the capital structure of a firm (expressed as times the total face value of the convertible bonds issued).

Ayache-Forsyth-Vetzal (2003)						
	α	γ	***σ***	***d***	**π**	***ρ***
Mean	-0.32	0.13	0.29	0.21	0.13	0.01
Median	-0.18	0.07	0.19	0.23	0.10	0.00
Min	-5.91	0.00	0.05	0.00	0.00	0.00
Max	4.81	0.88	0.97	0.30	0.30	0.45

Tsiveriotis-Fernandes (1998)					
	r_c	***σ***	***d***	**π**	***ρ***
Mean	0.11	0.36	0.22	0.01	0.18
Median	0.10	0.30	0.22	0.00	0.00
Min	0.00	0.05	0.02	0.00	0.00
Max	1.00	1.00	0.30	0.30	1.00

Brennan-Schwartz (1980)					
	Debt	***σ***	***d***	**π**	***ρ***
Mean	3.64	0.75	0.29	0.15	0.02
Median	0	0.77	0.30	0.14	0.00
Min	0	0.44	0.04	0.00	0.00
Max	9.99	1.00	0.30	0.30	0.18

Table III. Models' Mispricing Scores

This table reports the mispricing scores for all three models. The sample for the Ayache, Forsyth, and Vetzal (AFV) (2003) and Tsiveriotis and Fernandes (TF) (1998) models consists of 64 Canadian convertible bonds traded at the Toronto Stock Exchange. The subsample for the Brennan and Schwartz (BS) (1980) model consists of 17 firms, which have simple capital structure that consists of equity, straight debt, and convertible debt. For comparison, the results for the AFV (2003) and TF (1998) models are presented using both the full sample and the subsample of convertibles issued by firms with a simple capital structure and used for estimating the BS (1980) model. Errors are calculated as the convertible bond market prices minus the corresponding model predicted prices. The mean deviation (MD) expressed in dollars refers to the average pricing error (in dollars) for the entire sample. The MD in the percentage form refers to the average error as a percentage of the convertible market price and is calculated as: $\text{MD=average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]$

The Mean absolute deviation (MAD) refers to the average absolute error as a percentage of the convertible market price and is calculated as: $\text{MAD=average}\left[abs\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]\right]$

The Root of Mean Squared Error (RMSE) is calculated as the square root of mean squared error: $\text{MRSE=}\sqrt{\text{average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]^2}$

The last three rows report the percentage of all predictions that fall within the defined range (i.e., "within 10%" means that the pricing error was less than 10% of the market value).

	TF		AFV		BS
	Full Sample	BS Subsample	Full Sample	BS Subsample	
MD, $	-0.14	0.41	-0.38	-0.68	-2.95
t-statistics	(-2.11)**	(2.86)**	(-5.92)**	(-5.34)**	(-17.21)**
MD, %	-0.19	0.29	-0.35	-0.52	-2.78
t-statistics	(-3.18)**	(2.31)**	(-6.13)**	(-4.78)**	(-18.54)**
MAD, %	1.94	2.17	1.86	2.16	3.73
RMSE	2.95	3.19	2.93	2.94	4.94
Percentage of Errors Within 10% of Market Price	99.14	98.87	98.80	99.29	95.70
Percentage of Errors Within 5% of Market Price	91.97	89.79	93.65	89.59	70.20
Percentage of Errors Within 1% of Market Price	43.07	40.84	44.93	38.52	21.74

* and ** denote significance at the 5% and 1% levels, respectively

Table IV. Models' Mispricing Scores for Subsamples of Income Trusts and Non-Trusts and Debt-like and Equity-like Convertible Bond Issues

This table reports separate mispricing scores for income trusts and non-trusts (Panel A) and debt-like and equity-like convertible bond issues for all three models (Panels B and C). In Panel B, the debt- or equity-likeness is calculated using the Black-Scholes (1973) delta (Δ): $\Delta = e^{-dT}\left[\frac{\ln(\frac{S}{X})+(r-d+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right]$ where S is the current price of the underlying stock, K is the conversion price, d is the continuously compounded dividend yield, r is the continuously compounded yield on a selected risk-free bond, σ is the annualized stock return volatility, T is the maturity of the bond, and $N(.)$ is the cumulative standard normal probability distribution. In Panel C, delta is calculated as the sensitivity of convertible bond price to the change in the underlying stock price. In this panel, the deltas are obtained from the numerical routine used to calculate convertible bond prices using the Ayache, Forsyth, and Vetzal (AFV) (2003) model. In both Panels B and C, the debt-like convertibles are those with average Δ less than 0.5 and the equity-like convertibles are those with average Δ greater or equal to 0.5.

The sample for the Ayache, Forsyth, and Vetzal (AFV) (2003) and Tsiveriotis and Fernandes (TF) (1998) models consists of 64 Canadian convertible bonds traded at the Toronto Stock Exchange. Fifty-seven of these bonds are issued by income trusts and the remaining seven are issued by non-income trusts (ordinary corporations). The subsample for the Brennan and Schwartz (BS) (1980) model consists of 17 firms (4 of which are ordinary corporations and 13 are income trusts) that have a capital structure that consists of equity, straight debt, and convertible debt. Errors are calculated as convertible bond market prices minus the corresponding model predicted prices. The mean deviation (MD) expressed in dollars refers to the average pricing error (in dollars) for the entire sample. The MD in the percentage form refers to the average error as a percentage of the convertible market price and is calculated as:

$$\text{MD=average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]$$

The Mean absolute deviation (MAD) refers to the average absolute error as a percentage of the convertible market price and is calculated as:

$$\text{MAD=average}\left[abs\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]\right]$$

The Root of Mean Squared Error (RMSE) is calculated as the square root of mean squared error: $\text{MRSE=}\sqrt{\text{average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]^2}$

The last three rows report the percentage of all predictions that fall within the defined range (i.e., "within 10%" means that the pricing error was less than 10% of the market value. The last three columns of this table illustrate the p-values of t-tests testing the equality of the sample means. Panel A reports the p-values for equal sample means tests for convertible bonds issued by income trusts versus ordinary corporations. Panels B and C report the p-values for equal sample mean tests for equity-like and debt-like convertible bonds. The number of observations in brackets refers to the number of weekly observations for the AFV (2003) and TF (1998) models. The number of observations for the BS (1980) model differs since for this model, we use only the issues from the firms with simple capital structure. For the BS (1980) model, we have 470 weekly observations of bonds issued by income trusts and 275 observations of the convertibles issued by ordinary corporations. Based on the Black-Scholes (1973) deltas in Panel B, 632 weekly observations are for debt-like bonds and 113 weekly observations are for equity-like bonds. Based on the deltas calculated from the numerical routine in Panel C, 460 weekly observations are for the debt-like bonds and 285 observations are for equity-like bonds.

Table IV. Models' Mispricing Scores for Subsamples of Income Trusts and Non-Trusts and Debt-like and Equity-like Convertible Bond Issues (Continued)

	TF	BS	AFV	TF	BS	AFV	TF	BS	AFV
	Panel A. Income Trusts and Corporations						P-values of Tests of Equal Means for Income Trust and Non-income Trusts		
	Income Trusts (2230 obs)			**Non-income Trusts (352 obs)**					
MD, $	-1.58	-4.35	-0.74	-0.32	0.05	-2.85	0.00	0.00	0.00
t-statistics	(-7.33)**	(-14.64)**	(-3.39)**	(-1.32)	(0.17)	(-12.70)**			
MD, %	-1.49	-3.95	-0.57	-0.45	-0.16	-2.47	0.00	0.00	0.00
t-statistics	(-7.31)**	(-15.77)**	(-3.12)**	(-2.25)**	(-0.58)	(-11.97)**			
MAD, %	2.83	4.42	2.31	2.29	2.45	3.34			
RMSE	3.80	5.41	3.53	3.51	4.18	4.63			
Percentage of Errors Within:									
10% of Market Price	99.66	94.93	98.34	98.3	97.70	96.4			
5% of Market Price	77.55	60.37	91.97	88.1	87.1	79.5			
1% of Market Price	29.93	12.44	31.86	40.14	41.94	26.04			
	Panel B. Debt-like and Equity-like Issues (Black-Scholes (1973) delta)								
	Debt-like (2067 obs)			**Equity-like (515 obs)**					
MD, $	-0.24	-3.36	-0.15	-0.53	0.11	-1.42	0.178	0.00	0.00
t-statistics	(-3.22)**	(-19.15)**	(-2.49)**	(-2.60)**	(0.29)	(-6.49)**			
MD, %	-0.24	-3.09	-0.20	-0.56	-0.32	-1.05	0.055	0.00	0.00
t-statistics	(-3.65)**	(-19.37)**	(-3.38)**	(-3.73)**	(-0.99)	(-6.05)**			
MAD, %	2.30	4.02	1.78	1.90	2.54	2.24			
RMSE	3.09	5.17	2.66	3.31	4.33	3.91			
Percentage of Errors Within									
10% of Market Price	99.11	95.27	99.24	98.33	97.80	96.79			
5% of Market Price	91.01	67.16	94.61	91.42	86.26	89.32			
1% of Market Price	39.79	17.01	44.89	50.42	41.21	45.09			

Table IV. Models' Mispricing Scores for Subsamples of Income Trusts and Non-Trusts and Debt-like and Equity-like Convertible Bond Issues (Continued)

	TF	BS	AFV	TF	BS	AFV	TF	BS	AFV
	Panel C. Debt-like and Equity-like Issues (numerical delta)								
	Debt-like (866 obs)			**Equity-like (1716 obs)**					
MD, $	-0.87	-3.16	-0.51	0.01	0.12	-0.32	0.00	0.148	0.145
t-statistics	(-7.15)**	(-19.16)**	(-8.41)**	(0.08)	(0.34)	(-4.78)**			
MD, %	-0.90	-3.00	-0.54	-0.03	-0.17	-0.25	0.00	0.024	0.047
t-statistics	(-7.22)**	(-18.94)**	(-8.72)**	(-0.48)	(-0.97)	(-4.67)**			
MAD, %	2.74	4.08	2.18	1.69	1.22	1.7			
RMSE	3.80	5.26	3.2	2.74	1.83	2.78			
Percentage of Errors Within:									
10% of Market Price	98.60	95.43	98.73	99.13	100	98.83			
5% of Market Price	84.70	64.78	92.84	94.11	97.12	94.06			
1% of Market Price	25.47	18.04	35.68	49.86	57.69	49.59			

* and ** denote significance at the 5% and 1% levels, respectively

Table V. Descriptive Statistics of Models' Over/Underpricing Errors

This table provides the descriptive statistics for the deviations of the observed market prices from the model prices expressed as a percentage of market prices. The MD refers to the average error as a percentage of the convertible market price and is calculated as:

$$MD = \text{average}\left[\frac{\text{Market Price-Model Price}}{\text{Market Price}}\right]$$

The full sample for the Ayache, Forsyth, and Vetzal (2003) and Tsiveriotis and Fernandes (1998) models consists of 64 Canadian convertible bonds traded on the Toronto Stock Exchange. For the estimation of the Brennan-Schwartz (1980) model, the subsample of 17 firms with a capital structure consisting of equity, straight debt, and convertible debt is used. For comparison, the results for AFV (2003) and TF (1998) models are presented using both the full sample and the subsample of convertibles used for the estimation of the Brennan-Schwartz (1980) model. The start of the sample period depends on the issue date of the bond and starts either on January 1, 2005, July 1, 2005, or January 1, 2006. The sample period ends on April 28, 2006. The TF (1998) model refers to the Tsiveriotis-Fernandes (1998) model. The AFV (2003) model refers to the model of Ayache, Forsyth, and Vetzal (2003). The BS (1980) model refers to Brennan-Schwartz (1980) model. The *t*-statistics are for the test of the average error being equal to zero.

	TF		AFV		BS
	Full Sample	**BS Subsample**	**Full Sample**	**BS Subsample**	
Mean	-0.19	0.29	-0.35	-0.52	-2.78
Median	-0.12	0.25	-0.22	-0.42	-2.61
Minimum	-23.17	-9.63	-24.58	-10.88	-14.38
Maximum	15.72	15.72	17.11	11.11	22.44
Range	38.89	25.35	41.69	21.99	36.82
t-statistics	-3.18	2.31	-6.13	-4.78	-18.54

Table VI. Regression Results for the Mean Absolute Deviations (MADs)

This table presents regression results of the models' Mean Absolute Deviations (MADs) on the ratio of the stock market price to the conversion price (S/K), the time to maturity of the convertible security (TMAT), the historically observed volatility of the underlying stock (VOLAT), and the convertible bond coupon rate. MADs are defined as the absolute difference between market the price and the model price divided by the market price. The sample consists of 64 Canadian convertible bonds traded on the Toronto Stock Exchange. The start of the sample period depends on the issue date of the bond and starts either on January 1, 2005, July 1, 2005, or January 1, 2006. The sample period ends on April 28, 2006. The TF (1998) model refers to the Tsiveriotis-Fernandes model. The AFV (2003) model refers to Ayache-Vetzal-Forsyth (2003) model. The BS (1980) model refers to the Brennan-Schwartz (1980) model. The sample for the TF (1998) and AFV (2003) models consists of 64 firms traded on the Toronto Stock Exchange. For the estimation of the BS (1980) model, a subsample of 17 firms with the capital structure consisting of equity, straight debt, and convertible debt is used. The *t*-values of the coefficient estimates are reported in parentheses.

Model	Intercept	S/K	VOLAT	COUPON	TMAT	R^2
TF	2.82	-1.20	3.90	-0.35	-0.10	0.10
	(5.86)**	(-4.56)**	(2.82)**	(-0.43)	(-2.24)**	
AFV	1.76	-1.05	6.59	-0.71	-0.05	0.10
	(4.41)**	(-4.08)**	(4.12)**	(-0.87)	(-1.52)	
BS	-5.48	0.57	0.12	1.21	0.12	0.16
	(-2.49)**	(1.55)	(0.06)	(4.91)**	(0.67)	

* and ** denote significance at the 5% and 1% levels, respectively

Table VII: Regression Results for the Mean Deviations

This table reports regression results of the Mean Deviations (MDs) on the ratio of the stock market price to the conversion price (S/K), the time to maturity of the convertible security (TMAT), the historically observed volatility of the underlying stock (VOLAT), and the convertible bond coupon rate (COUPON). MDs are defined as the market price minus the model price divided by the market price. The sample consists of 64 Canadian convertible bonds traded on the Toronto Stock Exchange. The start of the sample period depends on the issue date of the bond and starts either on January 1, 2005, July 1, 2005, or January 1, 2006. The sample period ends on April 28, 2006. The TF (1998) model refers to the Tsiveriotis-Fernandes model. The AFV (2003) model refers to Ayache-Vetzal-Forsyth (2003) model. The BS (1980) model refers to the Brennan-Schwartz (1980) model. The sample for the TF (1998) and AFV (2003) models consists of 64 firms traded on the Toronto Stock Exchange. For the estimation of the BS (1980) model, a subsample of 17 firms with the capital structure consisting of equity, straight debt, and convertible debt is used. The *t*-values of the coefficient estimates are reported in parentheses.

Model	Intercept	S/K	VOLAT	COUPON	TMAT	R^2
TF	-1.02	0.56	-0.96	3.07	0.04	0.01
	(-1.58)	(1.80)	(-0.49)	(2.41)**	(0.62)	
AFV	-0.32	0.54	-4.74	3.49	0.05	0.03
	(-0.51)	(1.65)	(-1.88)	(2.63)**	(0.95)	
BS	6.77	0.82	4.30	-1.47	-0.31	0.18
	(2.23)*	(1.60)	(1.33)	(-4.66)**	(-1.43)	

* and ** denote significance at the 5% and 1% levels, respectively.

Appendix A: Characteristics of the Convertible Bonds Used in the Study

This table reports the main characteristics of the convertible securities used in our study. The sample consists of 64 Canadian convertible bonds traded on the Toronto Stock Exchange. The “Conversion ratio” indicates the number of stocks that can be obtained in the case of conversion for each 100 dollars of bond face value. The underlying stock volatility is expressed as the annualized standard deviation. In the call schedule, the first number refers to the call price per 100 dollars of face value, while the second number refers to the starting date of calling at this price. Calling continues until the next call date (if any) or until the maturity date if not specified otherwise. Credit spreads are derived from the corporate credit rating using the 2005 Royal Bank of Canada relative value curves for Canadian corporate bonds. The “Income Trust” column reports whether the issuing entity was an income trust. The “Call Notice Period” provides the minimum period between the calling announcement and actual call date. “Industry” specifies the area of specialization of the issuing firm. "Soft Call" lists conditions to be met before the bond can be called. For example, “20 days cumulative (consecutive) above 125% of conversion” means that the volume weighted average stock price has to be above 125% of its conversion price for at least 20 (consecutive) days in any given 30-day period before the issuer can call the bond; “N” refers to the absence of the soft call condition for a given bond. Asterisks (*) denote the firms with a simple capital structure consisting of equity, straight debt, and convertible debt. These firms are used in the estimation of the Brennan-Schwartz model.

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	Conversion Ratio, per 100$ of Par Value	Underlying Stock Volatility	Credit Spread, Basis Points	Call Schedule	Income Trust	Call Notice Period, Days	Industry	Soft Call
Advantage Energy AVN.DB.A	Jul-03	Aug-08	9	5.9	0.21	45	105 - 08/01/06, 102.5 -08/01/07	Y	30-60	Oil and gas	N
Advantage Energy AVN.DB.B	Dec-03	Feb-09	8.25	6.1	0.21	65	105 - 02/01/07, 102.5 -02/01/08	Y	30-60	Oil and gas	N
Advantage Energy AVN.DB.C	Jul-03	Oct-09	7.5	4.9	0.21	65	105 - 10/01/07, 102.5 -10/01/08	Y	30-60	Oil and gas	N
Advantage Energy AVN.DB.D	Jan-05	Dec-11	7.75	4.8	0.21	75	105 - 12/01/07, 102.5 -12/01/08	Y	30-60	Oil and gas	N
Agricore United AU.DB	Nov-02	Nov-07	9	13.3	0.30	65	100 - 12/01/05	N	30-60	Agriculture	N
Alamos Gold AGI.DB	Jan-05	Feb-10	5.5	18.9	0.48	65	100 - 02/15/08	N	30-60	Metals and mining	20 days cumulative above 125%
Alexis Nihon AN.DB*	Aug-04	Jun-14	6.2	7.3	0.14	128	100 - 06/30/08	Y	30-60	Real estate	20 days cumulative above 125%
Algonquin Power APF.DB*	Jul-04	Jul-11	6.65	9.4	0.17	69	100 - 07/31/07	Y	30-60	Utilities	20 days cumulative above 125%
Baytex Energy BTE.DB	Jun-05	Dec-10	6.4	6.8	0.26	115	105 - 12/31/08, 102.5 -12/31/09	Y	30-60	Oil and gas	N
Bonavista Energy BNP.DB	Jan-04	Jun-09	7.5	4.4	0.26	83	105 - 02/01/07, 102.5 -02/01/08	Y	30-60	Oil and gas	N
Bonavista Energy BNP.DB.A	Dec-04	Jul-10	6.75	3.5	0.26	105	105 - 12/31/07, 102.5 -12/31/08	Y	30-60	Oil and gas	N
Boyd Group BYD.DB*	Sep-03	Sep-08	8	11.6	0.60	45	105 - 09/30/04, 102.5 -09/30/05	Y	N/A		

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	Conversion Ratio, per 100$ of Par Value	Underlying Stock Volatility	Credit Spread, Basis Points	Call Schedule	Income Trust	Call Notice Period, Days	Industry	Soft Call
Calloway REIT CWT.DB	Apr-04	Jun-14	6	5.9	0.22	88	100 - 06/30/08	Y	30-60	Real estate	Previous day price above 125%
Cameco Corp CCO.DB*	Sep-03	Oct-13	5	4.6	0.40	82	100 - 10/01/08	N	30-60	Metals and mining	N
Can Hotel Inc. HOT.DB	Feb-02	Sep-07	8.5	10.4	0.15	65	100 - 03/01/05	Y	30-60	Real estate	20 days consecutive above 115%
Can Hotel Inc. HOT.DB.A	Nov-04	Nov-14	6	8.5	0.15	128	100 - 11/30/08	Y	30-60	Real estate	20 days consecutive above 125%
Chemtrade CHE.DB*	Dec-02	Dec-07	10	6.9	0.23	45	105 - 12/31/05, 102.5 -12/31/06	Y	30-60	Chemicals	Previous day price above 125%
Cineplex Galaxy CGX.DB*	Jul-05	Dec-12	6	5.3	0.27	75	100 - 12/31/08	Y	30-60	Media	20 days consecutive above 125%
Clean Power CLE.DB	Jun-04	Dec-10	6.75	9.8	0.34	69	100 - 06/30/07	Y	30-60	Utilities	Previous day price above 125%
Clublink LNK.DB*	Apr-98	May-08	6	5	0.18	65	100 - 03/15/03	N	30-60	Leisure	N
Cominar CUF.DB	Sep-04	May-14	6.3	5.8	0.15	88	100 - 06/30/08	Y	30-60	Real estate	20 days consecutive above 125%
Creststreet Power CRS.DB	Jan-05	Mar-10	7	10	0.19	65	100 - 03/15/08	Y	30-60	Utilities	Current price above 125%
Daylight Energy DAY.DB*	Oct-04	Dec-09	8.5	10.5	0.21	65	105 - 12/01/07, 102.5 -12/01/08	Y	N/A	Oil and gas	N
Dundee REIT D.DB	May-04	Jun-14	6.5	4	0.17	117	100 - 06/30/08	Y	30-60	Real estate	20 days consecutive above 125%
Dundee REIT D.DB.A	Apr-05	Mar-15	5.7	3.3	0.17	117	100 - 03/31/09	Y	30-60	Real estate	20 days consecutive above 125%
Esprit Energy EEE.DB*	Jul-05	Dec-10	6.5	7.2	0.23	65	105 - 12/31/08, 102.5 -12/31/09	Y	30-60	Oil and gas	N
Fort Chicago Energy FCE.DB.A	Jan-03	Jun-08	7.5	11.1	0.22	45	100 - 01/31/06	Y	30-60	Oil and gas	20 days consecutive above 125%
Fort Chicago Energy FCE.DB.B	Oct-03	Dec-10	6.75	9.4	0.22	55	100 - 12/31/06	Y	30-60	Oil and gas	20 days consecutive above 125%
Gerdau AmeriSteel Corp. GNA.DB*	Apr-97	Apr-07	6.5	3.8	0.44	65	100 - 04/30/02	N	30-60	Metals and mining	N

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	Conversion Ratio, per 100$ of Par Value	Underlying Stock Volatility	Credit Spread, Basis Points	Call Schedule	Income Trust	Call Notice Period, Days	Industry	Soft Call
Harvest Energy HTE.DB	Jan-04	May-09	9	7.1	0.31	90	105 - 05/31/07, 102.5 -05/31/08	Y	30-60	Oil and gas	N
Harvest Energy HTE.DB.B	Jul-05	Dec-10	6.5	3.2	0.31	90	105 - 12/31/08, 102.5 -12/31/09	Y	40-60	Oil and gas	N
InnVest INN.DB.A	Mar-04	Apr-11	6.25	8	0.19	105	100 - 04/15/08	Y	30-60	Real estate	20 days consecutive above 125%
Inter Pipeline IPL.DB	Nov-02	Dec-07	10	16.7	0.22	45	100 - 12/31/05	Y	30-60	Oil and gas	20 days consecutive above 125%
IPC US REIT IUR.DB.U	Nov-04	Nov-14	6	10.5	0.21	90	100 - 11/30/08	Y	30-60	Real estate	20 days consecutive above 125%
IPC US REIT IUR.DB.V	Sep-05	Sep-10	5.75	9.1	0.21	65	100 - 09/30/08	Y	30-60	Real estate	20 days consecutive above 125%
Keyera KEY.DB*	Jun-04	Jun-11	6.75	8.3	0.24	65	100 - 06/30/07	Y	30-60	Oil and gas	20 days consecutive above 125%
Legacy Hotels LGY.DB*	Feb-02	Apr-07	7.75	11.4	0.20	65	100 - 04/01/04	Y	30	Hotels	20 days consecutive above 115%
Magellan Aerospace MAL.DB*	Dec-02	Jan-08	8.5	22.2	0.44	45	100 - 01/31/06	N	40-60	Aerospace	20 days consecutive above 125%
MDC Partners MDZ.DB	Jun-05	Jun-10	8	7.1	0.34	83	100 - 06/30/08	N	30-60	Marketing services	at least 20 days in 30 consecutive day period above 125%
Morguard Real Estate MRT.DB.A	Jul-02	Nov-07	8.25	10	0.13	45	100 - 11/01/05	Y	30-60	Real estate	20 days consecutive above 125%
NAV Energy NVG.DB	May-04	Jun-09	8.75	9.1	0.27	65	105 - 06/30/07, 102.5 -06/30/08	Y	30-60	Oil and gas	N
Northland Power NPI.DB*	Aug-04	Jun-11	6.5	8	0.25	69	100 - 06/30/07	Y	30-60	Utilities	20 days consecutive above 125%
Paramount Energy PMT.DB	Aug-04	Sep-09	8	7	0.22	65	105 - 09/30/07, 102.5 -09/30/08	Y	40-60	Oil and gas	N
Paramount Energy PMT.DB.A	Apr-05	Jun-10	6.25	5.2	0.22	65	105 - 06/30/08, 102.5 -06/30/09	Y	30-60	Oil and gas	N
Pembina PIF.DB.A	Dec-01	Jun-07	7.5	9.5	0.24	45	100 - 06/30/05	Y	30-60	Oil and gas	20 days consecutive above 125%
Pembina PIF.DB.B	Jun-03	Dec-10	7.35	8	0.24	55	100 - 06/30/06	Y	30-60	Oil and gas	20 days consecutive above 125%

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	Conversion Ratio, per 100$ of Par Value	Underlying Stock Volatility	Credit Spread, Basis Points	Call Schedule	Income Trust	Call Notice Period, Days	Industry	Soft Call
Primaris REIT PMZ.DB*	Jun-04	Jun-14	6.75	8.2	0.20	117	100 - 06/30/08	Y	40-60	Real estate	20 days consecutive above 125%
Primewest Energy PWI.DB.A	Aug-04	Sep-09	7.5	3.8	0.24	83	105 - 09/30/07, 102.5 -09/30/08	Y	30-60	Oil and gas	N
Primewest Energy PWI.DB.B	Aug-04	Dec-11	7.75	3.8	0.24	105	105 - 12/31/07, 102.5 -12/31/08	Y	30-60	Oil and gas	N
Progress Energy PGX.DB	Jan-05	May-10	6.75	6.7	0.26	65	105 - 12/31/07, 102.5 -12/31/08	Y	30-60	Oil and gas	N
Provident Energy PVE.DB.A	Sep-03	Dec-08	8.75	9.1	0.21	65	100 - 01/01/07	Y	30-60	Oil and gas	N
Provident Energy PVE.DB.B	Jul-04	Jul-09	8	8.3	0.21	83	100 - 07/31/07	Y	30-60	Oil and gas	N
Provident Energy PVE.DB.C	Feb-05	Aug-12	6.5	7.3	0.21	105	100 - 08/31/08	Y	30-60	Oil and gas	N
Retirement Res REIT RRR.DB.B	Jul-03	Jan-11	8.25	8.1	0.25	95	100 - 07/31/07	Y	30-60	Real estate	20 days consecutive above 125%
Retirement Res REIT RRR.DB.C	Apr-05	Mar-15	5.5	8.8	0.25	95	100 - 03/31/09	Y	30-60	Real estate	20 days consecutive above 125%
Retrocom Mid-Market RMM.DB*	Jul-05	Jul-12	7.5	12.1	0.29	75	100 - 08/31/09	Y	30-60	Real estate	20 days consecutive above 125%
Rogers Sugar RSI.DB.A	Mar-05	Jun-12	6	18.9	0.24	75	100 - 06/29/08	Y	30-60	Food	20 days consecutive above 125%
Royal Host Real Estate RYL.DB	Feb-02	Mar-07	9.25	14.3	0.19	65	100 - 03/01/05	Y	30	Real estate	20 days consecutive above 125%
Summit Real Estate SMU.DB	Feb-04	Mar-14	6.25	4.7	0.21	88	100 - 03/31/08	Y	30-60	Real estate	20 days consecutive above 125%
Superior Propane SPF.DB	Jan-01	Jul-07	8	6.3	0.27	45	100 - 02/01/04	Y	30-60	Diverse	20 days consecutive above 125%
Superior Propane SPF.DB.A	Dec-02	Nov-08	8	5	0.27	45	100 - 11/01/05	Y	30-60	Diverse	20 days consecutive above 125%
Superior Propane SPF.DB.B	Jun-05	Dec-12	5.75	2.8	0.27	69	100 - 07/01/08	Y	30-60	Diverse	20 days consecutive above 125%

Issuer (Symbol)	Issue Date	Maturity Date	Coupon, %	Conversion Ratio, per 100$ of Par Value	Underlying Stock Volatility	Credit Spread, Basis Points	Call Schedule	Income Trust	Call Notice Period, Days	Industry	Soft Call
Superior Propane SPF.DB.C	Oct-05	Oct-15	5.85	3.2	0.27	88	100 - 10/31/08	Y	30-60	Diverse	20 days consecutive above 125%
Taylor NGL TAY.DB*	Mar-05	Sep-10	5.85	9.7	0.27	75	100 - 09/10/08	Y	30-60	Oil and gas	20 days consecutive above 125%