```
            If AmeEurFlag = "e" Then
                OptionValue(i) = (p * OptionValue(i + 1) + (1 - p) _
        * OptionValue(i)) * Df
            ElseIf AmeEurFlag = "a" Then
                OptionValue(i) = Max((z * (S * u^i * d^(j - i) - X)), _
                (p * OptionValue(i + 1) + (1 - p) * OptionValue(i)) * Df)
            End If

        Next
        If j = 2 Then
                ReturnValue(2) = ((OptionValue(2) - OptionValue(1)) _
        / (S * u^2 - S * u * d) _
                - (OptionValue(1) - OptionValue(0)) _
        / (S * u * d - S * d^2)) / (0.5 * (S * u^2 - S * d^2))
                ReturnValue(3) = OptionValue(1)
            End If
         If j = 1 Then
                ReturnValue(1) = (OptionValue(1) - OptionValue(0)) _
        / (S * u - S * d)
            End If
    Next
    ReturnValue(0) = OptionValue(0)
    If OutputFlag = "p" Then 'Option value
        LeisenReimerBinomial = ReturnValue(0)
    ElseIf OutputFlag = "d" Then 'Delta
        LeisenReimerBinomial = ReturnValue(1)
    ElseIf OutputFlag = "g" Then 'Gamma
        LeisenReimerBinomial = ReturnValue(2)
   ElseIf OutputFlag = "a" Then
        LeisenReimerBinomial = Application.Transpose(ReturnValue())
    End If

End Function
```

7.1.5 Convertible Bonds in Binomial Trees

A convertible bond can be seen as a combination of a plain bond and a stock option. If the stock price is far below the strike (conversion price), the convertible behaves like a straight bond. If the stock price is far above the strike, the convertible behaves like a stock. This should also affect the discounting of the cash flows. When the convertible is deep-out-of-the-money, the future cash flows should be discounted by a rate that takes into account the credit spread k above the treasury rate of the particular bond. If the convertible is deep-in-the-money, it is almost certain to be converted, and the cash flows should be discounted at the risk-free rate.

Bardhan, Bergier, Derman, Dosembet, and Kani (1994) have incorporated these effects by using a discounting rate that is a function of a variable conversion probability. The model starts out with a standard binomial stock price tree. The convertible bond price is then found by starting at the end of the stock price tree. At each end node, the convertible value must be equal to the maximum of the value of converting the bond into stocks or the face value plus the final coupon.

One next rolls backward through the tree, using backward induction. If it is optimal to convert the bond, the value is set equal to the conversion value at that node, or else the convertible bond value $P_{n,i}$ is set equal to

$$P_{n,i} = \max[mS,\ pP_{n+1,i+1}e^{-r_{n+1,i+1}\Delta t} + (1-p)P_{n+1,i}e^{-r_{n+1,i}\Delta t}], \tag{7.12}$$

where m is the conversion ratio. Some convertible bonds have an initial lockout period during which the investor is not allowed to convert the bond. The convertible bond value at these nodes can be simplified to

$$P_{n,i} = pP_{n+1,i+1}e^{-r_{n+1,i+1}\Delta t} + (1-p)P_{n+1,i}e^{-r_{n+1,i}\Delta t}$$

Instead of using a constant discount rate r, the discount rate $r_{n,i}$ is set to fluctuate with the conversion probability $q_{n,i}$ at each node.

The conversion probabilities $q_{n,i}$, where n is the time step and i the number of up moves (the state), are calculated by starting at the end of the stock price tree. If it is optimal to convert the bond, the conversion probability is set to 1; otherwise, the conversion probability is set to 0. For time steps before the end of the tree, the conversion probability is set equal to 1 if it is optimal to convert at that node; otherwise,

$$q_{n,i} = pq_{n+1,i+1} + (1-p)q_{n+1,i} \tag{7.13}$$

The credit-adjusted discount rate is set equal to a conversion probability weighted mixture of the risk-free rate and the credit-adjusted rate. This gives a discount rate for up moves equal to

$$r_{n,i} = q_{n,i}r + (1-q_{n,i})(r+k) \tag{7.14}$$

The discount rate is thus set equal to the constant risk-free rate r when the conversion probability is 1, and set equal to the risk-free rate plus the credit spread $r+k$ when the conversion probability is 0. The discount rate moves smoothly between the risk-free rate and the credit-adjusted rate for conversion probabilities between 0 and 1.

Example

Consider a convertible corporate bond with five years to maturity. The continuously compounding yield on a five-year treasury bond is 7%, the credit spread on the corporate bond is 3% above treasury, the face value is 100, the annual coupon is 6, the conversion ratio is 1, the current stock price is 75, and the volatility of the stock is 20%. What is the value of the convertible bond? $S = 75$, $T = 5$, $r = b = 0.07$, $k = 0.03$, $m = 1$, and $\sigma = 0.2$.

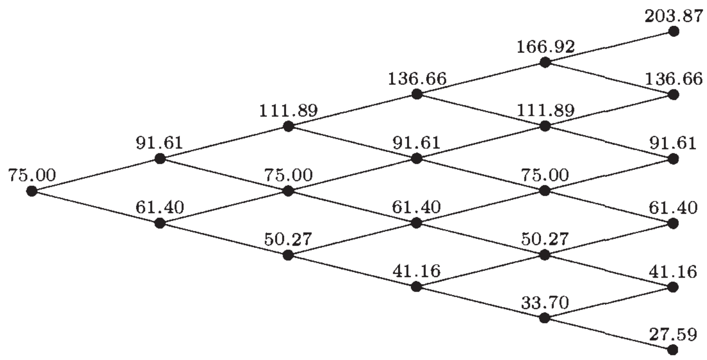
To price the convertible bond, we need to build a standard binomial stock price tree. With the number of time steps $n = 5$, we get $\Delta t = 1$ and up and down factors

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2\sqrt{1}} = 1.2214, \qquad d = \frac{1}{u} = 0.8187$$

The probability of an increase in price is thus given by

$$p = \frac{e^{b\Delta t} - d}{u - d} = \frac{e^{0.07\times 1} - 0.8187}{1.2214 - 0.8187} = 0.6302,$$

and we obtain the following binomial stock price tree:



The next step is to find the convertible bond values and the conversion probabilities at each node in the tree. To see how this works, let's look at the calculation of several nodes.

At the end node with stock price 203.87, it is better to convert the bond into one stock and receive the stock price 203.87 than to get the notional plus the coupon $100 + 6$. The probability of conversion at this node, $q_{5,5}$, is 100%, which we write as 1.00 in the conversion probability tree.

At the end node, with a stock price of 91.61, it is better not to convert the bond and receive the face value plus the coupon of 106. The probability of conversion is $q_{5,3} = 0$.

For the node at year four ($n = 4$) with stock price 111.89, the convertible bond value of 121.77 is found by using equation (7.12):

$$P_{4,4} = \max\left[1 \times 111.89,\ 0.6302 \times 136.66e^{-r_{n+1,i+1}\times 1} + (1 - 0.6302)106.00e^{-r_{n+1,i}\times 1}\right]$$

The credit-adjusted discount rates are found by using equation (7.14):

$$r_{n+1,i+1} = 1 \times 0.07 + (1 - 1)(0.07 + 0.03) = 0.07$$

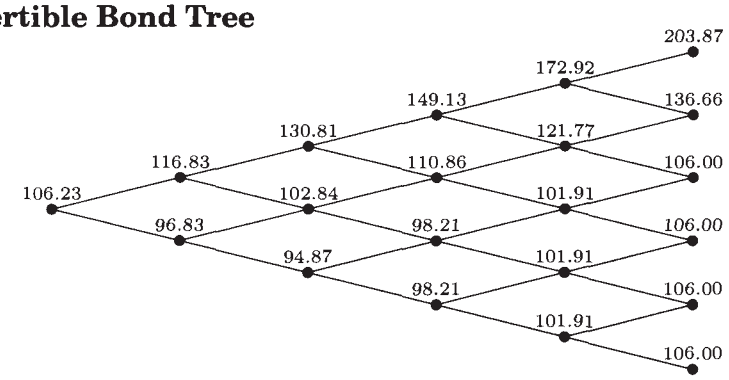
$$r_{n+1,i} = 0 \times 0.07 + (1 - 0)(0.07 + 0.03) = 0.1$$

The conversion probability of 0.63 at this node is given by equation (7.13):

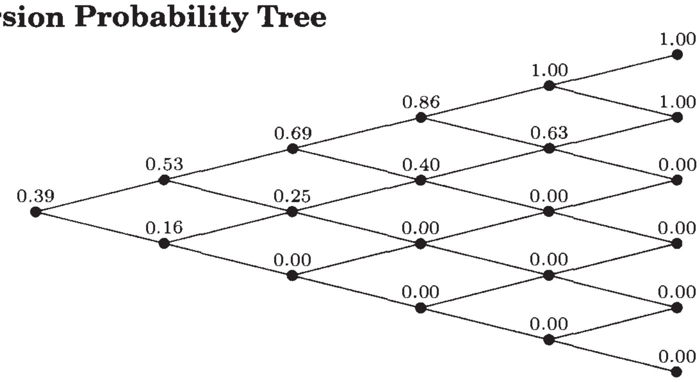
$$q_{4,4} = 0.6302 \times 1 + (1 - 0.6302) \times 0$$

The same procedure can be used to find any convertible bond value and conversion probability.

Convertible Bond Tree



Conversion Probability Tree



In the above section the main principles of how to incorporate a convertible bond model were outlined. In practice, there are many additional issues to take into account. Some convertible bonds allow the issuer to force investors to convert the bond if the stock price reaches a certain prespecified level (barrier). To include a barrier in the convertible binomial model, the number of time steps should be chosen to make the barrier fall exactly *on* the nodes. The conversion probability is then set to 1 if the stock price is larger than or equal to the barrier. The issuer of the convertible bond also often has the right to call the bond, and the investor has the right to put the bond. The paper of Bardhan, Bergier, Derman, Dosembet, and Kani (1994) is a good start to look into such practicalities.