

# Credit Default Swaps with R

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# 1 Introduction

Over the past three decades, **credit derivatives** have become increasingly common tools for financial institutions that wish to pass on **credit risk** to investors who accept that risk in exchange for interest. To put it simply, credit risk is the possibility that a borrower will **default** on her loan, or not pay it back. There are many types of credit derivatives, including collateralized debt obligations (CDOs) and collateralized loan obligations (CLOs). In this paper, we will focus on **credit default swaps** (CDS), a type of credit derivative that has become quite popular among pension funds, hedge funds, and investment banks, among other financial institutions. We will explain the mechanics of this financial product and show how this **CDS** package can be used to manipulate and calculate relevant CDS information.

CDS originated in the late 1980s and were popularized by a team at J.P. Morgan that included Blythe Masters (Lenzner, 2009; Lanchester, 2009). Many different types of CDS have since emerged including basket default swaps (BDSs), index CDS, credit-linked notes, etc. (Kallianiotis, 2013). In this **CDS** package, we focus on calculations related to single-name, or corporate, credit default swaps.

## 2 Fixed Income Securities and Credit Risk

CDS are used for the purpose of hedging against the **credit risk** associated with **bonds** or **fixed income**. Before we approach understanding how CDS work, we must understand the mechanics of a standard coupon bond and its associated risks.

### 2.1 Bond Basics

Consider the following example: let's say that there exists a portfolio manager who has to allocate her clients' money (a total sum of \$10 million). This particular manager is a risk-averse investor who seeks to minimize her losses, even if that ultimately creates lower returns on investments. She also wants to invest in assets that are **liquid** i.e. assets that are easily retrievable or can be easily converted into cash. In such a scenario, coupon bonds seem like an attractive investment.

A **coupon bond** is an agreement in which one party lends money to another party in exchange for that same sum of money, plus periodic **coupon payments**, at a future date. A major advantage of holding bonds over shares (which are also known as equity) is that bondholders are compensated before shareholders in the case of liquidation (such as a default). If a company or other reference entity defaults on its debt and goes bankrupt, the remaining assets associated with the company are gathered and distributed among bondholders and shareholders according to the issuer's capital structure (another term for the hierarchy of investors). Bonds can be further categorized as "senior" and "junior," so although bondholders will be compensated first, those with a greater seniority ("senior" bonds) will be compensated before those with a lesser seniority ("junior" bonds). The amount of assets remaining after default and bankruptcy is known as the **recovery rate**.

To understand the different variables used to determine cash flows and bond pricing, let's look at actual data for an Alcoa bond that will mature, or be paid out, five years from now (since the data describes a bond that was initiated seven years ago).

DES			
ALCOA INC		AA 5.72 02/23/19	112.063/112.063 (2.928/2.928) TRAC
AA 5.72 02/23/19 Corp		Page 1/11	Description: Bond
		94 Notes	95 Buy
		96 Sell	97 Settings
21 Bond Description		22 Issuer Description	
Pages		Identifiers	
1) Bond Info		ID Number EG3379369	
2) Addtl. Info		CUSIP 013817AP6	
3) Covenants		ISIN US013817AP64	
4) Guarantors		Bond Ratings	
5) Bond Ratings		Moody's Ba1	
6) Identifiers		S&P BBB-	
7) Exchanges		Fitch BB+	
8) Inv Parties		DBRS BBB	
9) Fees, Restrict		Issuance & Trading	
10) Schedules		Amt Issued/Outstanding	
11) Coupons		USD 749,500.00 (M) /	
Quick Links		USD 749,500.00 (M)	
32) ALLQ Pricing		Min Piece/Increment	
33) QRD Quote Reca		100,000.00 / 1,000.00	
34) TDH Trade Hist		Par Amount 1,000.00	
35) CAC Corp Action		Book Runner	
36) CF Prospectus		Reporting	
37) CN Sec News		TRACE	
38) HDS Holders			
39) VPR Underly Info			
66) Send Bond			
Issuer Information			
Name ALCOA INC			
Industry Metals & Mining			
Security Information			
Mkt Iss Global			
Country US		Currency USD	
Rank Sr Unsecured		Series	
Coupon 5.72		Type Fixed	
Cpn Freq S/A			
Day Cnt 30/360		Iss Price	
Maturity 02/23/2019			
MAKE WHOLE @15 until 02/23/19/BULLET			
Iss Sprd			
Calc Type (1)STREET CONVENTION			
Announcement Date 04/02/2007			
Interest Accrual Date 02/23/2007			
1st Settle Date 05/02/2007			
1st Coupon Date 08/23/2007			
ISSUED IN EXCH 1440/REGS: 013817AM3/USU01347AA84. CALL @ MAKE WHOLE +15BP. POISON PUT @ 101% SUBJ TO			
RATINGS TRIGGER.			

Figure 1: Bloomberg data taken on June 24, 2014 for senior Alcoa bonds that will mature on February 23, 2019. These bonds have a duration of twelve years, since the bonds were initiated on April 2, 2007 (see "Announcement Date"). This figure displays essential information regarding the bonds that a potential investor would need to consider, such as the company ("name"), industry, seniority ("rank"), the value of the coupon payments ("coupon"), maturity, coupon payment dates and credit ratings. (Many of these attributes and concepts will be further discussed in Sections 2.2 and 2.4.)

## 2.2 Credit Risk Associated with Bonds

A bond has many kinds of risks associated with it such as interest rate risk, inflation risk or liquidity risk. For our purposes, we will concentrate on credit risk.

If we look to the right of Figure 1, under "Bond Ratings," we can see certain symbols representing **credit ratings** that are provided by rating agencies such as Moody's, S&P and Fitch. These symbols are indicators of a company's credit risk. When a company like Alcoa issues bonds, there is always a possibility that it may not be able to meet its debt obligations. That possibility, or risk, is known as the credit risk. Naturally, if a company's credit risk goes up, investors would demand a higher yield and consequently, a lower price. If a company is very likely to meet its debt obligations—or if it has consistently done so in the past—the company can be known as **credit worthy**.

A risk-averse investor, like our pension fund portfolio manager, would naturally want to purchase senior bonds from a company that has a low credit risk—a company like Alcoa, which has a rating of BBB- from S&P, BB+ from Fitch and Ba1 from Moody's. We will not go into the details of how these ratings are determined, since those details are beyond the scope of this vignette. What we should note is that companies that have credit ratings of BBB- or higher from S&P or Fitch or Baa3 or higher from Moody's can be classified as **Investment Grade (IG)** bonds. Such companies, at least from the

viewpoint of the rating agencies, have low credit risks. Bonds that are rated below the IG benchmark are termed as **speculative grade** bonds or **junk** bonds, and have a higher yield than IG bonds. We can see a complete list of classifications from Moody's and S&P in Figure 2:

Investment Grade					Speculative Grade				
S&P	Fitch	Moody's	Interpretation	Numeric Scale	S&P	Fitch	Moody's	Interpretation	Numeric Scale
AAA	AAA	Aaa	Very high credit quality	1	BB+	BB+	Ba1	It has speculative elements and it is subject to substantial credit risk	11
					BB	BB	Ba2		12
					BB-	BB-	Ba3		13
AA+	AA+	Aa1	High credit quality	2	B+	B+	B1	It is considered speculative and it has high credit risk	14
AA	AA	Aa2		3	B	B	B2		15
AA-	AA-	Aa3		4	B-	B-	B3		16
A+	A+	A1	Medium-high grade, with low credit risk	5	CCC+	CCC+	Caa1	Bad credit conditions and it is subject to high credit risk	17
A	A	A2		6	CCC	CCC	Caa2		18
A-	A-	A3		7	CCC-	CCC-	Caa3		19
					CC	CC	Ca		20
BBB+	BBB+	Baa1	Moderate Credit Risk	8					
BBB	BBB	Baa2		9	C	C	C		22
BBB-	BBB-	Baa3		10	SD	DDD	-	Very close to default or in default	22
					D	DD			22
					-	D			22

Figure 2: Credit rating classifications from S&P, Fitch and Moody's for long-term debt obligations. Even though S&P and Fitch have the same credit rating scale, note how the rating determined by S&P and the rating determined by Fitch for Alcoa bonds in Figure 1 are *different*, which implies that the two rating agencies use different measurements to determine a credit rating. Herein lies the value of having three different ratings agencies that can provide three different ratings: if, for example, a company can be classified as IG across *all three* ratings—which are all not determined by the exact same combination of metrics—then one can be more confident in the fact that the bond or the company actually is of IG quality.

## 2.3 Risk-free Bonds

Bonds issued by the U.S. Government (known as treasury bonds) are generally considered to be **risk-free** since the probability of the U.S. Government failing to meet its debt obligations is almost negligible. Bonds issued by Japan (Japanese Government Bonds or JGBs) and Britain (Gilts) are also considered to be risk-free. Therefore, the interest paid by sovereign bonds is an important benchmark for the pricing of corporate bonds or even the bonds of other governments.

Bonds from Alcoa, or any corporation, are generally riskier than bonds issued by the U.S. Government. A rational investor would want a riskier bond to pay a higher coupon rate than a U.S. treasury bond since otherwise she could get the same interest payment at no risk at all. In other words, she would want the expected return of a corporate bond to be the same as the expected return of a risk-free bond. For this to be the case, the riskier bond would have to pay more interest than the risk-free asset in order to compensate for the added risk. The amount to which the riskier bond's interest exceeds that

of a risk-free bond is known as the **risk premium**. So, the interest that an investor would want from a bond would change as the risk-free coupon rate changes. It is important to understand the intuition behind this before learning how bonds are priced.

### Swap Curve or Risk-free Curve

Bond yields are directly proportional to bond maturities. This relationship results from the fact that the risk of default for any entity is higher over a long period than over a short one. This should make sense intuitively since a longer maturity is more likely to capture the time at which an entity may default; the risk that a government may default on its debt over 30 years is higher than the risk that it defaults in six months. In rare cases, the short-term yields are higher than long term yields; this can often be a sign of a recession. If, for a given date, we plot the maturities of different bonds, such as treasury bonds, on the x-axis and their corresponding yields on the y-axis, we get a **Swap Curve**.

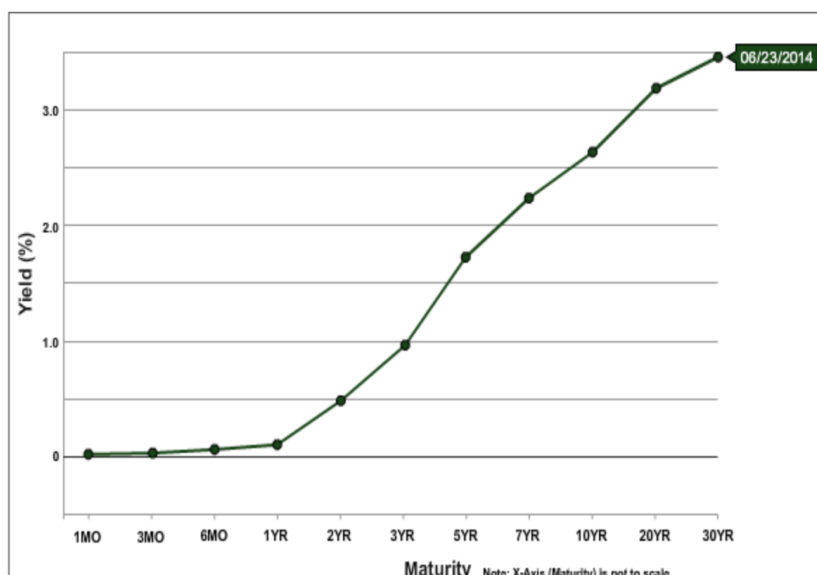


Figure 3: Swap Curve for treasury bonds on June 23, 2014. Clearly, yields are higher for debt obligations with longer maturities. This makes sense since investors would want a higher overall yield if they were to take on more risk by buying a bond with a longer maturity. Note that the x-axis is not to scale (the points are not at equal intervals), so it is difficult to make any conclusions or observations about slope or the change in yield for a certain amount of time.

## 2.4 Bond Pricing

### Present Value (PV)

**Present value** refers to the current value of a future payment. For example, if an investor has \$1 million, that asset (if invested) it is worth more today than \$1 million ten years from now. If this person invests his \$1 million in, say, a risk-free asset earning a coupon of 5% a year (compounded semi-annually), that initial amount will become \$1.63 million in ten years. So, the present value of that \$1.63 million

(that will accumulate over ten years) is \$1 million. The 5% coupon rate that we are using to discount the future value is known as the **discount rate**.

Applying this concept to bond pricing, we can say that a bond's price is essentially the sum of the present values of its cash flows i.e. the sum of the present values of coupon payments and principal. Basically, we have to pretend as though we are investing the coupon payments and principal at different time periods. So for our \$1 million bond with a discount rate of 5% paid semi-annually (2.5% every six months), the present value of the first coupon of \$25,000 is  $\$25,000/1.025$ , that of the second coupon is  $\$25,000/1.025^2$  and so on. The present value (and price) of a bond can be calculated using the equation below:

$$\text{Price} = C * \frac{1 - \frac{1}{(1+i)^n}}{i} + \frac{M}{(1+i)^n} \quad (1)$$

"M" represents the principal payment and "C" refers to the coupon payment of \$25,000. The variable, "i", is the discount rate and is also called the **yield to maturity (YTM)** or just **yield**. It is the required interest or discount rate for the present value of a future payment to be equal to the bond price.

If we look at the top of Figure 1, we see that a bond with a face value of \$100 for Alcoa on June 24, 2014 has a coupon rate of 5.72%, a price of \$112.063 and a yield of 2.928%. As we can see, the bond price is greater than the bond's face value—the amount that Alcoa will return to the bondholder when the contract ends—because the bondholder is receiving 5.72% interest, which is larger than the coupon rate of a treasury bond or other risk-free bond. The **Cpn Freq** (Coupon Frequency) is quoted as "S/A" which stands for "semi-annually." So, if our portfolio manager were to purchase Alcoa bonds that have a principal of \$10 million and that pay a coupon of \$286,000 semi-annually, it would cost her \$11,206,300. If we substitute "i" in the above equation with 0.014514 (the discount rate for a five-year bond), "C" with \$2.86, "M" with \$100 and "n" (the number of coupon payments) with 10, we would get the bond price quoted in the Bloomberg screenshot in Figure 1: \$112.063.

When our portfolio manager has to purchase a bond, she has to determine what a fair yield, or discount rate, for that specific bond would look like. The factors affecting the yield include the risk-free rate, as explained earlier, and the credit health of the company, represented partly by the credit rating. If we look at the equation above or even think of these concepts intuitively, we can see that the bond yield has an inverse relationship with its price. If there is a fixed coupon rate, and an investor wants a higher bond yield, the only way to keep the same coupon rate and increase the yield is the lower the bond price—and decrease what the investor has to pay—accordingly. Moreover, the difference between the yield of a bond and the yield of a risk-free bond of the same maturity is a common measurement of the company's credit risk.



## 3 CDS Basics

### 3.1 Why CDS?

Let's assume that our portfolio manager buys bonds of Alcoa with a face value of \$10 million and with a maturity of five years. However, six months after she makes this trade, her confidence in the creditworthiness of Alcoa begins to waiver. Of course, she could simply sell the bonds. However, that may be premature or even unprofitable. Instead, she could **hedge** her position by purchasing a financial product whose value rises with a drop in the bonds' value. In other words, she could take a position that allows her to *profit* from increased credit risk. Hedging is a process that allows someone to profit from both a rise and a drop in an asset's value. When an investors hedges, she takes two opposite positions on an asset such that in the case of a rise or drop, her downside is limited, and one position offsets the losses from the other.

When the asset in question is a bond, one option is to be **long**, a position in which you simply own the bond and profit when its price increases (or yield drops). The opposite position is to be **short**. Shorting is the process of borrowing security (i.e. purchasing it), immediately selling the security to other investors, and then buying it back from the investors and returning it to the lender. Clearly, this is profitable if the value of the security drops by the time the lender repurchases the security in the future. It is a conventional technique of hedging against credit risk. However, there are several issues associated with this method. It is difficult to short bonds that are not liquid. The portfolio manager may not be able to find enough bonds that will mature at the same time. Moreover, she would have to spend \$10 million to hedge her entire portfolio of \$10 million, which is a substantial opportunity cost. Therefore, it makes sense for her to seek out an alternate method of reducing her credit risk.

### 3.2 CDS Mechanics

Instead of shorting a security, our portfolio manager can enter a credit default swap agreement to buy protection on her credit risk. In order to understand this type of exchange, let's first talk about a simpler form of purchasing protection: life insurance.

Let's say that an insurance company charges \$1,000 for one year of life insurance coverage. This \$1,000 is known as the **premium**. If a person buys life insurance for that one year, and if he lives through that year, then the only cash flow present is the \$1,000 fee, paid by the insurance buyer to the insurance company. If, however, the buyer passes away during that year, then the insurance company has agreed to pay \$100,000 to the buyer's spouse or other family members. This \$100,000 amount is considered the value of the policy. Let's assume that the premium is paid at the beginning of each calendar year, on January 1, and pays for insurance until December 31 of that year.

Is this a fair deal? In other words, is the **expected payment value** of the insurance agreement for both parties—the insurance company and the buyer's family—equal? This would be the case if the buyer's expected payment (i.e. the premium amount of \$1,000) is equal to the insurance company's expected payment: (the probability of the buyer's death)\*(the payment in case of death: \$100,000). So,

in answering this question, we must determine a value for the probability of the portfolio manager's death. In this example, if both parties believe that the probability of the buyer's death is 1%, then this is a fair agreement; the expected payments for each party are equal.

Instead of buying insurance against death, let's say that our portfolio manager wants to buy insurance against the bankruptcy of a company—say, Alcoa. She owns \$100,000 worth of Alcoa bonds, and she is concerned that the company may go bankrupt and default on her bonds in the next year. So, she enters a CDS agreement with an investment bank—say, J.P. Morgan. If Alcoa goes bankrupt and defaults on its bonds, J.P. Morgan agrees to pay the buyer the value of her Alcoa bonds (\$100,000)—an amount known as the **notional amount**. In exchange for this protection, the buyer agrees to pay a **coupon** (similar to the premium in the example above) of 1%, or 100 basis points (bps), of the notional amount (so, \$1,000) to J.P. Morgan. This agreement is valid for one year. In summary, as shown in Figure 4, a CDS is an agreement between two **counterparties** in which the buyer (in this case, the portfolio manager) pays a **fixed periodic coupon** to the seller in exchange for protection against a credit event associated with the buyer's bond.

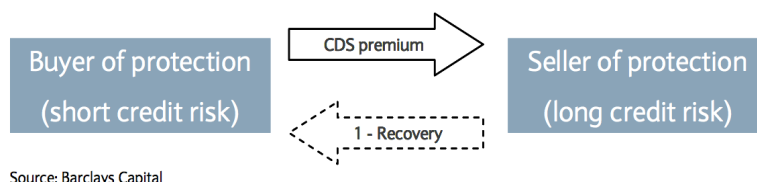


Figure 4: Cash flows in a single-name CDS contract.

Again, for this CDS agreement to be considered a fair deal, the expected payments from each party must be equal. Since this scenario involves all the same figures as the life insurance example above (coupon of 1%, notional amount of \$100,000), then the probability of default must be 1% for the expected payments to be equal.

### Upfront payment

However, what if the chances of Alcoa's default is *not* 1%—what if it's 0.5%? Clearly, this is no longer a fair deal. The expected payment of the portfolio manager—known as the **protection buyer** in the CDS agreement—at the end of the year will be greater than J.P. Morgan's expected payment since the probability of default (and thus the probability that J.P. Morgan will have to pay out any money at all) is less than the example above. A simple solution to this issue would be to lower the protection buyer's coupon to 0.5% (or \$500).

However, let's say that the CDS market is regulated—that is, regulations require that CDS agreements only trade at a standard coupon of 1%. If the probability of Alcoa's default is less 1%, how do we modify the CDS agreement to once again make it a fair trade for both parties?

The answer lies in an **upfront payment**—a payment made from one party to the other at the beginning of a contract (in this case, January 1). If the probability of Alcoa's default is 0.5%, then J.P. Morgan has to compensate the protection buyer for paying a coupon of 1% instead of a coupon of 0.5%. Since

the difference between the protection buyer's fair payment (\$500 at 0.5%) and the payment she has to make due to regulations (\$1,000 at 1%) is \$500, J.P. Morgan's upfront payment to the protection buyer should be \$500. That way, the two parties can trade according to CDS regulations and still create a fair deal in which the expected payments from each party are equal. Let's assume that both the upfront payment and the coupon payment are made at the same time: on January 1, the first day of the one-year contract. In this case, since the portfolio manager would have to pay J.P. Morgan \$1,000, and J.P. Morgan would have to pay the buyer \$500, the two parties can just agree to have one equivalent cash flow: the protection buyer paying \$500 to J.P. Morgan.

### Discount Rate

Although the above example simplifies cash flow calculations, these simplifications render the scenario unrealistic. For example, coupon payments are often (always? check.) paid at the *end* of a time period—in this case, at the end of one year. To make our example more realistic, let's consider our above example, with the exception that coupon payments are now made at the *end* of one year.

This begins to make our CDS trade a bit more complex. Since the upfront payment would be made on January 1, and since the coupon payment would be made one year later (on December 31), the \$500 upfront payment would have more value *per dollar* than the \$1,000 coupon payment. The assumption is that, theoretically, the portfolio manager could invest the upfront payment of \$500 starting January 1 and could earn interest on the \$500 for that year. At the bare minimum, the protection buyer could invest that money in a treasury bond and earn the **interest rate** on her investment with essentially no risk. Since this affects the fairness of this agreement, we should account for this in the pricing of the upfront payment. Otherwise, J.P. Morgan's expected payment is larger than that of the portfolio manager—and why would J.P. Morgan want to enter this agreement if it were unfair on its own end?

Let's say that interest rates are currently at 2%, and that the portfolio manager will invest the upfront payment at 2% compounded semi-annually (compounded 1% every six months). In order to account for the upfront payment's greater value per dollar compared to that of the coupon payment *at the inception of the trade*, we must discount our previously calculated upfront payment by the interest rate. The rate by which we discount the upfront payment is known as the **discount rate**. In other words, the upfront payment should be an amount that, if invested at the current interest rate of 2% for one year, would equal \$500. So,  $\$500 / (1 + .01)^2 = \$490$  (rounded to the nearest dollar). Thus, the upfront payment that J.P. Morgan should make to the protection buyer is approximately \$490.

### Accrued Interest

Up until now, we have assumed that this CDS agreement was initiated on January 1 and will last until the end of that calendar year. However, what if this agreement wasn't made exactly on January 1? What if it was made halfway through the year, on Jun 30? Let's assume the probability of default for those six months is still 0.5%. (This may very well not be the case in the real-world CDS market—see later section discussing relationship between maturity date and probability of default—but we'll make this assumption for now for simplicity's sake.) Under our current description of a CDS agreement, this

implies that the portfolio manager would have to pay the same coupon of 1% (\$1,000) on December 31 for only receiving six months (instead of one year) of protection coverage. This, too, renders the agreement unfair.

To account for this, first let's calculate the amount of the coupon that the portfolio manager is unfairly paying—that is, the fraction of the coupon payment that the portfolio manager is making for the first six months of the year (January 1 - June 30), which he does not receive coverage for. Note that in this example, the protection buyer is only paying \$500 of the required \$1,000 coupon to begin with (and J.P. Morgan is accounting for the other half in the upfront payment) because the actual probability of default is 0.5%, not 1%. So, we will input the probability of default in the following equation as so:

$$\text{Accrued} = \frac{180}{360} * \frac{0.5}{100} * \$100,000 = \$250 \quad (2)$$

Note that we just divided the coupon payment that the portfolio manager actually pays in half, since we are looking for the six month equivalent. We are also dividing 180 by 360 instead of 365 in the above calculation. This has to do with the **day count convention** of the contract (something we will discuss in a later section).

The portfolio manager does not want (and understandably does not have the obligation) to pay for the six months he did not receive coverage for. Instead, J.P. Morgan will pay the portfolio manager the accrued interest so that the expected payments from each party are equal. Thus, if the CDS agreement was initiated on June 30, then the total upfront payment made by J.P. Morgan to the portfolio manager would be \$495 (which is equal to \$500/(1+ 1%) rounded to nearest dollar) + \$250 = \$745. This value is known as the **dirty upfront**, since it specifies the actual upfront payment that (in this case) the protection seller will pay the protection buyer. The **clean upfront**, on the other hand, does not take accrued interest into account, and (in this case) is equal to \$495. Note that the portion of the upfront payment that accounts for a lower probability of default than 1% is discounted by the discount rate, and the portion that accounts for the accrued interest is not. This is because the accrued interest should be worth \$250 *now* (June 30), and the other portion of the upfront payment should be worth \$500 on December 31.

## Two-Period Case

Up until now, we have described various aspects of a CDS agreement that has one coupon payment on December 31. However, most CDS include more than one coupon payment. Let's explore an agreement in which there are two coupon payments in during a two-year contract: one on December 31 of the first year and one on December 31 of the second year. For this case, let's assume that the interest rate (and thus the discount rate) is 0% and the probability of default is constant for both years of the contract. If the standardized coupon is still 1% (so the buyer will make two \$1,000 coupon payments) and the notional amount is still \$100,000, the probability of default would have to be 2% for this to be a fair deal:  $(.02) * (100,000) = (1,000 + 1,000)$ .

In this example, however, let's say that the probability default is actually 2.5%, and the standardized coupon must remain at 1%. What would be an appropriate upfront payment for this type of contract?

In this case, it is the protection buyer that has to pay the upfront payment. The amount of money that J.P. Morgan expects to pay increases as the probability of default increases, whereas the total payment for two coupon payments of \$1,000 is set at \$2,000 and does not change. Thus, the portfolio manager will have to pay the difference between the total coupon payment that would merit a "fair deal" with a %2.5 default probability (\$2,500) and the standardized total coupon payment (\$2,000). The upfront payment would then be \$500. Note, again, that this is assuming an interest rate of 0%, so discounting is not necessary.

For this scenario, there are three basic ways in which the CDS agreement may unfold. Alcoa could default during the first year, Alcoa could default during the second year, or Alcoa could just not default during the two-year contract.

If the probability of default changes from year one to year two, then this can make our calculation of the upfront payment a bit more complex. If, for example, the probability of default during year one is 2%, and the probability of default during year two is 2.5%, then the expected payment from J.P. Morgan to the portfolio manager would be the expected payments for year one  $(.02 * 100,000)$  plus the expected payment for year two *assuming Alcoa doesn't default during year one*  $(.98) * (.025 * 100,000)$ : \$4,450. Since there is a 2% probability of default for year one and thus a 98% chance that Alcoa does not default during year one, we have to multiply .98 by year two's expected payment if the contract survives until year two.

Given that \$4,450 is the expected payment for J.P. Morgan, that of our portfolio manager should be equivalent. Since the coupon payment is set 1%, and since there are two coupon payments, the upfront payment (made by the portfolio manager) is equal to  $\$4,450 - \$2,000 = \$2,450$ .

Let's add another layer of complexity. What if we have the same exact scenario as described above, but the interest rate is not 0%? What if the interest rate *changes* from year one to year two?

First let's assume that the interest rate remains constant over the two years at 2%, compounded annually. So, if J.P. Morgan were to invest its upfront payment in a treasury bond earning the interest rate, their \$2,450 would become  $(\$2,450) * (1.02) = \$2,499$  by the end of year one and  $(\$2,499) * (1.02) = \$2,549$  (rounded to the nearest dollar) by the end of year two. Overall, J.P. Morgan could gain an extra \$99 just by receiving the upfront payment at the beginning of the two years instead of at the end. In order to discount for this advantage, we need to think of the upfront payment as two parts: one \$1,225 part that accounts for the discrepancy in year one, and another \$1,225 part that accounts for the discrepancy in year two. By discounting each part by the interest rate for that year and adding the amounts back together, we get the discounted upfront payment:  $\$1,225/1.02 + \$1,225/1.02 = \$2,401$  (rounded to the nearest dollar).

Using the equation above, we can now see that calculating the discounted upfront payments where there is one fixed interest rate for year one, and a different fixed interest rate for year two would not be so different. Because we are viewing these at a year-by-year case (viewing the upfront payment as two parts, each accounting for one year), we can just substitute the interest rate for one part.

## 4 CDS Terminology and Cash Flows

Although the cash flows involved in a CDS seem pretty simple in Figure 4, the underlying mechanics of a CDS are a bit more complex. To understand this, let's look at actual CDS data from Bloomberg for Alcoa. Figure 5 below displays many different variables such as RED Pair Code, REF Entity, Trade Date, Debt Type etc. for the CDS of Alcoa for June 24, 2014. These are standard terms of CDS contracts that were set by the **International Swaps and Derivatives Association (ISDA)**, the organization that regulates over-the-counter derivatives such as CDS. Let's understand what these variables are and how they are important for understanding the cash flow<sup>1</sup>.



Figure 5: CDS figures from Bloomberg for Alcoa Note that this **REF Obligation** (Reference Obligation) matches the ISIN in Figure 1.

### 4.1 Entity-Specific Variables

Some of these variables seem self-explanatory. **REF Entity** (Reference Entity) refers to the name of the company (Alcoa, in this case) that the buyer wants protection against. The **ticker** is an abbreviated reference symbol for the Reference Entity, which is "AA" for Alcoa. Moreover, the **Trade Date** is the date on which we are making the trade: June 24, 2014.

The **RED Pair Code** is a Markit product that stands for Reference Entity Database. Each entity/seniority pair has a unique six-digit RED Pair Code that matches the first six digits of the nine-digit RED Pair Code. Each entity also has a "preferred reference obligation," which is the default reference

<sup>1</sup>Some of the definitions come from *Credit Derivatives Glossary* (Markit, 2009), *Standard Corporate CDS Handbook* (Leeming et al., 2010), *Credit Derivatives* (Green and Witschen, 2012), and *The Pricing and Risk Management of Credit Default Swaps, with a Focus on the ISDA Model* (White, 2013).

obligation for CDS trades. A user can input either the six-digit RED Pair Code or the nine-digit RED Pair Code. The input "014B98" is the six-digit RED Pair Code for "Alcoa"

We can also note the label **Debt type**, marked as "Senior." Clearly, this refers to the seniority of the debt. The **notional amount** is printed as "10" or \$10 million USD (US Dollars). The **REF Obligation** (Reference Obligation) refers to the bond involved in the CDS. Since our portfolio manager is purchasing protection on the bonds she previously purchased, this **REF Obligation** matches the ISIN in Figure 1.

**Maturity** refers to the **tenor**, or length, of the contract. The most commonly traded contracts have maturities of five years. The Bloomberg screenshot displays the length of the contract, as well as the implied maturity date. Interestingly, the maturity date for the CDS of Alcoa on June 24, 2014 is September 20, 2019. This might seem odd since we would expect it to be on June 24, 2019, exactly five years from the Trade Date. However, the maturity date always falls on one of the four **roll dates**: March 20, June 20, September 20 and December 20. Therefore, the maturity date for this contract will be on the roll date after June 24, 2019, which is September 20, 2019. Also note that this contract is of the type **SNAC**, or Standard North American Contract, which is a convention that specifies how North American single-name CDS are supposed to trade. In European markets, CDS belong to the **STEC** category, or Standard European Contract.

## 4.2 Premium Leg

The stream of cash flows from the protection buyer to the seller, when there is no default, is known as the **premium leg**. To understand the premium leg, we must look at the **Trade Spread** (marked as **Trd Sprd** in Figure 5) and **Coupon** in Figure 5. In section 3.2, we stated that the protection buyer pays the protection seller a fixed coupon for purchasing protection. Until 2009, the two counterparties in a CDS contract would agree on the coupon level before the trade. Then, as the market moved, this tradable coupon would vary. So if our portfolio manager was purchasing a CDS before 2009, she would have had to negotiate a fixed coupon—which would then vary as the company's credit risk varied—with the hedge fund.

In April 2009 in North America, the ISDA introduced a series of mandatory modifications to the CDS contract known as the "Big Bang" protocol. Under the new rules, coupon rates were standardized in North America and Europe starting June 2009. Dealers now had to quote **standard coupons** of 100bps or 500bps in North America, or 25bps, 100bps, 500bps or 1000bps in Europe, and all coupons were paid quarterly on one of the four roll dates. The coupon printed in the figure above is the fixed coupon for Alcoa, which is 100bps, or 1% of the notional amount. However, the dealers may not feel that this is the fair premium for protection. For instance, the hedge fund selling protection to the portfolio manager may feel that the fair premium should be 160bps, and not 100bps. This fair premium rate is known as **trade spread** (or **par spread**, or just **spread**), labeled as 160 in the figure above.

Naturally, if the hedge fund believes that the fair premium should be 160bps, it would like to be compensated for receiving just 100bps. As a result, the portfolio manager would have to make what is known as an **upfront payment** at the trade inception. Therefore, in the absence of a credit event, the

cash flow between the two parties, over the life of the contract, would look like Figure 6.

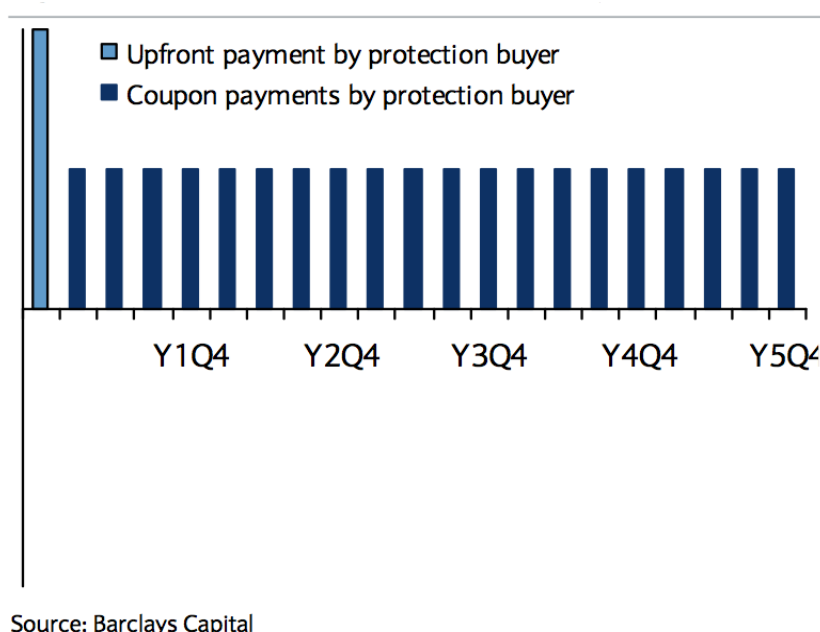


Figure 6: CDS cash flows when there is no credit event.

The ISDA protocol, since April 2009, specifies that all premium payments, by default, start on the roll date before the Trade Date. So if the Trade Date is June 24, 2014, the **Accrual Begin Date** is, by default, June 20, 2014. Now if the Accrual Begin Date is 4 days before the Trade Date, the portfolio manager would not want (and is not obligated) to pay interest for the 5 days she has not received protection for. The **accrued interest** can be calculated using the equation below:

$$\text{Accrued} = \frac{5}{360} * \frac{1}{100} * \$10000000 = \$1,389 \quad (3)$$

We must note that we are dividing 5 by 360 instead of 365 in the above calculation. This has to do with the **day count convention** (**Day Cnt** in Figure 5) of the contract, which specifies that the accrual factor between two dates is ACT/360, or Actual/360.

Since the upfront payment is calculated as if the trade began on the roll date before the date of trade inception, the payment that the protection buyer *actually* makes—known as the **dirty upfront** or **cash settlement amount**—is the calculated upfront payment minus the accrued interest. As we can see in Figure 5, the dirty upfront value for this contract is \$286,069. The **clean upfront** payment or **principal**, on the other hand, is simply another term for the initial, calculated upfront payment. In this case, it is \$287,458. Moreover, we can also note a variable called **Pts Upf** in Figure 5. This variable, known as **points upfront**, or simply **points**, is the clean upfront expressed as a percentage of the notional amount.

As we can see, the upfront and points upfront values are positive, since the spread of the CDS is higher than the fixed coupon rate. If instead, the portfolio manager and hedge fund agree that the fair premium should be 60bps, the upfront value would be negative i.e. the hedge fund would pay



the portfolio manager a compensation for receiving a coupon higher than the fair premium or spread. The cash flows would then look like the values in Figure 7. The clean and dirty values are now both negative.


Calculation Results		
Market Value	-117,057	
Cash Settlement	-117,061	
Accrued Days	75	
Accrued Amt	20,833.33	
Currency	USD	
Details		Show Cash Flows 
	Transformed	User
Market Value	-117,057	-117,059
Clean Price	100.96 %	100.96 %
Cash Settlement	-117,058	-117,061
Accrued Days	75	75
Accrued Amt	20,833.33	20,833.33
Credit DV01	4,849	4,849
IR DV01	25	25

Figure 7: CDS cash flows for Alcoa when spread is equal to 60bps and the coupon is equal to 100bps. These have been calculated using the calculator provided by Markit.

### 4.3 Protection Leg

So far we have only discussed the stream of payments for a contract in which a credit event does not occur. The stream of payments that would have to be made in the case of a credit event is known as the **protection leg** or **contingent leg**. The resulting cash flow would look like that in Figure 8.

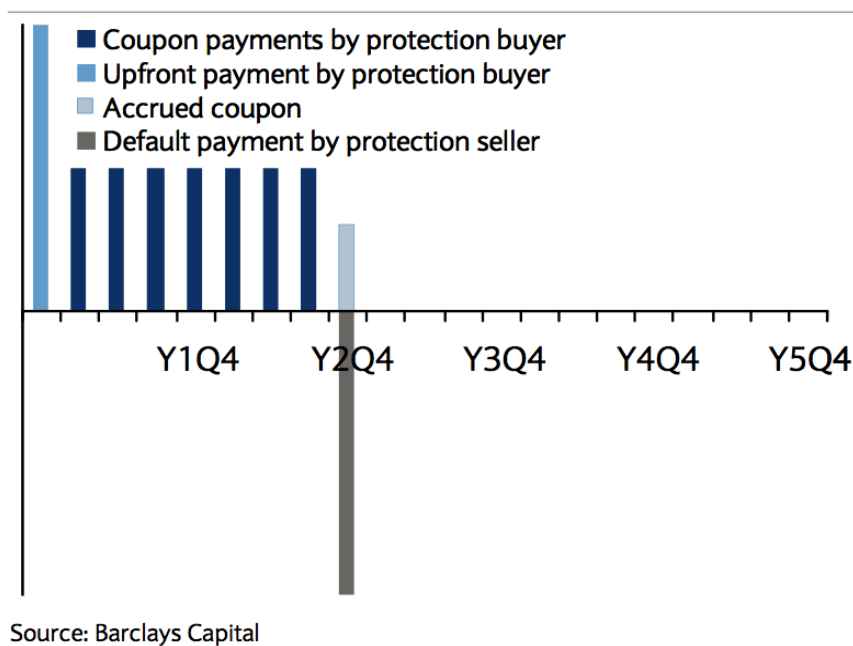


Figure 8: CDS cash flows when there is a credit event.

When a default does occur, the protection seller would owe the protection buyer the notional amount minus any money recovered from the company. It must be noted that the protection is effective for the credit events that have taken place since 60 days *before* the trade date; this date is known as the **backstop date**. Before the "Big Bang" protocol in April 2009, this date used to be one day *after* the trade date. If there is a delay in awareness about a credit event, the new system allows sufficient time for the two dealers to discover and process the information.

### Types of Settlements in the Case of a Default

Should a bond (that is the reference obligation in a CDS agreement) default, the counterparties can compensate accordingly in two ways. The first is a **physical settlement** in which the buyer will actually deliver the defaulted bonds to seller, and the seller will then pay the face value of those bonds. The disadvantage to this particular transaction is that the buyer(s) of protection will have to find and deliver those bonds to the seller even if they don't own the bonds themselves. This may artificially drive up the price of the bonds. This is more likely to happen when there is a large number of outstanding CDS contracts.

The alternative to a physical settlement is a **cash settlement**, in which the seller simply pays the following to the buyer:  $(\text{notional amount}) \times (1 - \text{recovery rate})$ . Unfortunately, determining a recovery rate can often prove to be an issue. One approach the ISDA has been using lately is an **auction** style process in which major dealers submit their bids for the value they place on a company's debt. CDS contracts for corporate bonds generally assume a 40% recovery rate for valuation purposes.

## What Constitutes a Default?

Another issue that we have to face when creating CDS is the definition of "default." A default does not always have to be outright bankruptcy. Often companies **restructure** their debt—or renegotiate the terms of their debt—instead of declaring for bankruptcy, and in some CDS contracts, restructuring does not constitute a default. In Figure 5, if we look between the notional amount and the currency, we will see a text box printed as "MM." This stands for "Modified Modified" restructuring and implies that debt restructuring does in fact constitute a default. So, if Alcoa decides to restructure this specific debt, the hedge will have to pay the protection leg to the portfolio manager.

## Counterparty Risk

When a counterparty such as the hedge fund sells protection on a bond, there is always the possibility that, in case of a default, the counterparty may not be able to pay the protection payment to the protection buyer. Or, conversely, perhaps our portfolio manager is unable to make her periodic payments. The risk of either of these eventualities taking place is known as **counterparty risk**.

## 4.4 CDS Indices

Holding a disproportionate number of Alcoa bonds in a portfolio is naturally a risky idea. It is akin to putting all your eggs in one basket, as the saying goes. In the real world, most successful portfolio managers prefer to distribute their risk by holding a portfolio with hundreds, if not thousands, of different positions.

Instead of holding Alcoa bonds with a face value of \$10 million, let's say our portfolio manager decides to take on less risk by holding bonds of 100 different IG companies. This is known as **diversification**. Although this strategy reduces her exposure to the credit risk of any particular company, she hasn't reduced her exposure to factors that might affect a wide variety of assets simultaneously, such as interest rate risk or political risk. Fortunately, in the modern financial world, she has the option of purchasing multi-name CDS, or CDS indices, which contain a basket of CDS. The two most common indices are CDX and iTraxx, which represent North American CDS and European CDS, respectively.

Moreover, if our portfolio manager has a high-risk appetite and wants to earn a higher return for her investors, she may invest a portion of her portfolio in bonds that have a lower credit rating but provide a higher yield. She may hedge this position by investing in a tranche of CDS indices such as the North American High Yield CDS index, which specifically offers protection on high yield bonds of 100 different companies. CDX and iTraxx are broken into smaller CDS based on sectors such as automobiles or consumer goods.

These index products trade in high volumes and are very liquid. In this vignette, we only discuss the pricing mechanisms of single-name CDS and do not delve into the more complex world of multi-name CDS. However, it is important to understand their relevance as most CDS contracts involve companies in these indices.

## 5 CDS Pricing

The ISDA has created the "Standard Model," which allows market participants to calculate cash settlement from conventional spread quotations, convert between conventional spread and upfront payments, and build the spread curve of a CDS. In this section we will lay out the assumptions made by the Standard Model, and explain the methods and formulas used to calculate certain information related to CDS.

There are several ways of calculating the price of a CDS such as hedge-based valuation or bond-yield-based pricing. We will apply the **discounted-cash flow pricing** or **risk-neutral valuation**, where the present value of the CDS is equal to the expected value of its discounted future cash flows. This model assumes that the probability of default is risk-neutral.

### 5.1 Discounted-Cash Flow Pricing or Risk-Neutral Valuation

Let's say our portfolio manager wants to sell her CDS contract for whatever reason. Finding the market value of a CDS contract for the protection buyer is known as **Marking-to-market (MTM)** and the value is known as the **mark-to-market** value. Theoretically, the value of this CDS (of Alcoa) for the portfolio manager is:

$$[\text{Money she expects to receive}] - [\text{Money she expects to pay}]$$

Clearly, if she expects to receive more than she expects to pay, the value of the contract will be positive. So, if the fair spread for this CDS contract rises and Alcoa's credit risk increases, the market value of her CDS will increase and she will be in a **mark-to-market profit**. If, on the other hand, the credit risk and fair spread drop, she would have a **mark-to-market loss**.

The Standard Model assumes that the mark-to-market value is computed by discounting the expected protection leg and premium leg cashflows from the trade date until the maturity date. In Section 2.4 we explained the concept of **present value** and how a bond's price is the present value of its future cash flows. This concept applies to the pricing of CDS as well. If we are looking at the value of a CDS from the protection buyer's perspective, the premium leg has a negative value and the protection leg has a positive value. So it is equivalent to saying that,

$$PV(\text{CDS}) = PV(\text{Protection}) - PV(\text{Premium})$$

Using the above equation, we can intuitively understand how the upfront value is calculated. We know that one of the central components in determining the price of a CDS is the spread. When the spread is different from the standard coupon of the contract, an upfront payment has to be made as a compensation. This upfront value is the expected present value of the extra cash flows that will be made hypothetically, on top of the standard coupon payments.

So if we have a five-year contract for the CDS of Alcoa, in which the standard coupon is 100bps and the fair spread is 160bps, the upfront value paid by the portfolio manager to the hedge fund is the *net expected* present value of the extra 60bps payments made quarterly (15 bps per quarter) over a period

of five years. When we say *expected* present value, it implies that there is a chance that this hypothetical payment will never have to be made, since the company might default, and then the protection seller may have to make the contingent payment. We have to account for that probability in calculating the present value of those payments.

When an upfront payment is made, the transaction simply implies that the expected present value of the protection and premium legs are not equal. In the all-running contracts before the "Big Bang Protocol," where the coupon could be set at any level that the two parties agreed to, the premium leg would naturally equal the protection leg and the present value of the contract would be zero. But when the coupon is fixed and when the fair spread is not equal to the coupon, the expected present values of the two legs would not be the same and an upfront payment would have to be made as compensation. The upfront value essentially equates the premium leg and the protection leg. Let us see how the expected present values of these legs are calculated.

### Present Value of the Net Expected Cash Flows

As mentioned before, the premium leg is the present value of the stream of payments made by the protection buyer to the protection seller. Here we have to account for the **survival probability**—the conditional probability that the reference entity survives until a given time period. We will use a term structure of survival probabilities to weight each of the contingent premium payments.

Moreover we also have to discount each of the contingent payments with a discount factor, which is the interest rate curve explained earlier. Therefore, the present value of the premium leg can be calculated using the formula:

$$PV(Premium) = C * \sum_t Df(t_i)S(t_i)B(t_i),$$

The protection leg is the present value of the expected contingent protection payment of (1-recovery rate) from the protection seller to the protection buyer. We will weight this contingent protection payment in each time period with the **default probability**, which is essentially the difference in the survival probabilities between two time periods. Therefore, the present value of the protection leg is:

$$PV(Protection) = (1 - R) * \int_0^T Df(t)dQ(t)$$

### Survival Probability Curve and Risk Neutral Probability

The survival probability is used to determine the expected cash flow of a CDS contract. The survival probability function can be equated with the **hazard rates** function or the distribution of the company's risk-neutral probability of default given below,

$$S_t = 1 - Q_t = 1 - \int_0^t q_t dt$$

There are several assumptions made here:

- The risk-free rate and credit risk are not correlated

- Counterparty risk is not considered
- 

This **Hazard Rate function** is piecewise constant. What this means is that the function varies across a scale but remains constant between two points.

To calculate the hazard rates, we need a **CDS curve** or the **term structure** of CDS rates. A CDS curve is similar to the interest rate curve in that it is a curve with the fair spreads of CDS contracts for different maturities. For reference names that are liquid, these rates are available quite easily; however, if the CDS is not liquid we can use the underlying bond's price to determine the hazard rates. Moreover, if there are only a couple of maturity dates available, we can assume a flat CDS curve. We will not go into the details of how these curves are derived but simply note that they are used in calculating the present value of the two legs and the upfront payment.

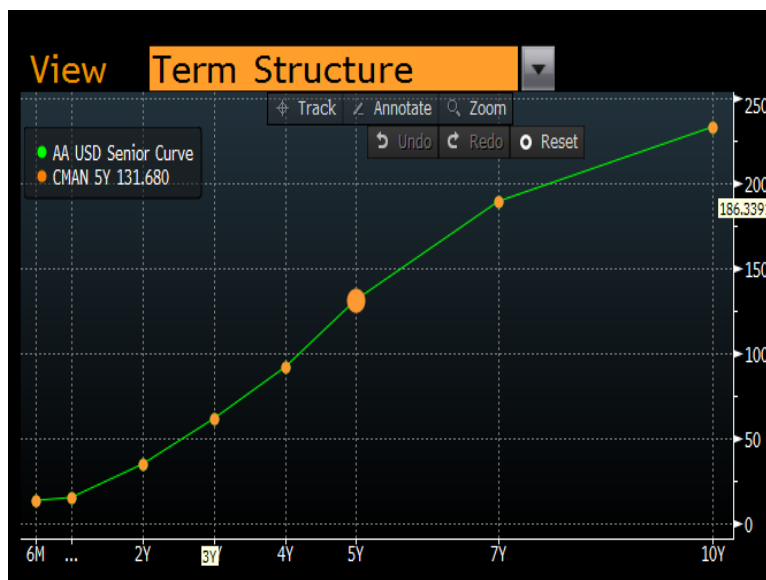


Figure 9: Term-structure of CDS rates for Alcoa on June 24, 2014

### Discount factor: Using the Swap Curve or Risk-Free Curve

In the risk-neutral valuation framework, the risk free curve is the discount factor used in the calculation of the present value of CDS. In calculating the price of a CDS, we use the swap curve of the *previous day*, and use the rates for the corresponding currency. So for CDS denominated in USD, we will use the US treasury rates, and for those in GBP we will use UK Government bond yields, and so on. If we look at Figure 3, we can see the yields for different treasury bonds at 4pm EST on June 23, 2014, which will be used for pricing the CDS of Alcoa

The Standard Model allows market participants to convert between the par spread and the upfront payment, and compute the cash settlement amount for a standard contract.

Yield Curve		
Date	23-Jun-2014	
Snap	1600	TimeZone: New York
Recalculate		
Period	Type	Rate
1M	Deposit	0.1520 %
2M	Deposit	0.1963 %
3M	Deposit	0.2326 %
6M	Deposit	0.3253 %
1Y	Deposit	0.5471 %
2Y	Swap	0.6190 %
3Y	Swap	1.0630 %
4Y	Swap	1.4665 %
5Y	Swap	1.7930 %
6Y	Swap	2.0570 %
7Y	Swap	2.2725 %
8Y	Swap	2.4450 %
9Y	Swap	2.5880 %
10Y	Swap	2.7110 %
12Y	Swap	2.9085 %
15Y	Swap	3.1110 %
20Y	Swap	3.2915 %
25Y	Swap	3.3710 %
30Y	Swap	3.4100 %
Interest Rate Conventions		
Spot Date: 26-Jun-2014		Swap DCC: 30/360
MM DCC: A/360		Swap Interval: 6M
Floating DCC: A/360		Holidays: none
Floating Interval: 3M		Bad Day Conv: MF

Figure 10: Risk-free rates with their day count conventions on June 23, 2014 at 4pm EST. These are the rates that will be used to price the CDS of Alcoa on June 24, 2014. **MM DCC**, **Floating DCC** and **Swap DCC** refer to the Day Count Conventions being used to calculate the different kinds of rates. A **Swap Interval** of '6M' implies that interest is paid every six months or semi-annually. These conventions are different for each currency

The ISDA also standardizes the interest rates used by the Standard Model in valuing a CDS contract. There are two types of rates used in valuing a USD-denominated CDS contract: cash rates and swap rates. Cash rates are of the following maturities: one, two, three, and six month(s), and one year. These refer to the yields of zero-coupon bonds that have a maturity of less than one year; US treasuries with a maturity of one year or less are zero-coupon bonds. They are provided by the British Bankers' Association (BBA). Swap rates, or yields of coupon bond with a maturity of over one year, are of maturity 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 25, and 30 years, and are provided by ICAP (Markit, 2013). The Standard Model follows the conventions below for interpolation of the entire USD yield curve:

- The day count convention (DCC) for money market instruments and the floating legs of the swaps is **ACT/360**.
- DCC for floating legs of the swaps is **30/360**.

- Payment frequency for fixed legs of the swaps is 6 months.
- Payment frequency for floating legs of the swaps is 3 months.<sup>2</sup>
- A business day calendar of weekdays (Monday to Friday) is assumed. Saturdays and Sundays will be the only non-business days.
- If a date falls on a non-business day, the convention used for adjusting coupon payment dates is **M** (Modified Following).
- Recovery Rate is the estimated percentage of par value that bondholders will receive after a credit event. It is commonly reported in percentage of notional value. CDS contracts for corporate bonds assume a 40% recovery rate for valuation purposes.

### Credit Triangle

The probability of default, recovery rate and spread form a credit triangle such that if we have two of them, we can use it to calculate the third:

$$\text{Hazard Rate} = \text{Spread} / (1 - \text{Recovery Rate})$$

For a constant premium, higher recovery rates imply a high default probability.

### Price

**Price** refers to the clean dollar price of the contract. The price of this Alcoa CDS is 97.13, less than 100. A CDS will have a price less than 100 if the points upfront are positive; that is, the CDS buyer needs to pay money to obtain protection because he promises to pay a coupon of, say, 100 even if the spread is 160. This is analogous to a bond investor paying less than the face value of a bond because current interest rates are higher than the coupon rate on the bond. It can be calculated by

$$\begin{aligned} \text{Price} &= (1 - \text{Principal/Notional}) * 100 \\ &\& = 100 - \text{Points Upfront.} \end{aligned}$$

### Present Value 01 (PV01)

The PV01 is the present value of a stream of 1 basis points. It can also be used to calculate the cash flows and risk measures of a CDS. It is sometimes referred to as the **CDS duration** or **risky duration**. Analytically, PV01 can be calculated by

$$PV01 = \sum_i Df(t_i)S(t_i)B(t_i),$$

- $i$  = coupon index,

---

<sup>2</sup>See <http://www.fincad.com/derivatives-resources/wiki/swap-pricing.aspx> for details on floating and fixed legs calculation.



- $t_i$  = coupon date,
- $B(t_i)$  = day count fraction at  $t_i$ .
- $Df(f_i)$  = discount factor until  $t_i$ ,
- $S(t_i)$  = survival probability until  $t_i$ ,

As we can see in the equation above, we need the coupon index, coupon dates, the day count fraction, discount factor for each date, and survival probability until the date of the respective coupon payment. Before go into the details of how PV01 is used in determining the market price or the risk measures, it is essential to understand the different components used to calculate it. Coupon dates refer to the dates on which coupon payments are made. Day count fraction is the fraction of the day on which the coupon payment is made upon 360 or 365, depending on the day count convention of that CDS. While the first three components of the above equation seem straightforward, the other two - survival probability and discount factor require more explanation.

We can also use the PV01 to calculate the premium leg as premium leg is simply the present value of future coupon payments. It is simply the coupon payment multiplied by the PV01

We can use the formula above for PV01 to calculate the principal amount (clean upfront payment) paid from the protection buyer to the seller using the following formula:

$$\text{Principal Amount} = \text{PV(Protection)} - \text{PV(Premium)} = (\text{Par Spread} - \text{Coupon}) \times \text{PV01}$$

## 5.2 Risk Measures of a Standard CDS Contract

The PV01 can be used to compute certain risk measures related to interest rates, spread and the default probability.

### Spread DV01

Using the concept of PV01, we show the calculation of the main risks (exposures) of a CDS position, **Spread DV01**. Spread DV01 reflects the risk duration of a CDS trade, also known as **Sprd DV01**, **Credit DV01**, **Spread Delta**, and just **DV01**.

It measures the sensitivity of a CDS contract mark-to-market to a parallel shift in the term structure of the par spread. DV01 should always be positive for a protection buyer since he or she is short credit, and a rising spread is a sign of credit deterioration. Starting with PV01 and taking the derivative with respect to the spread gives us:

$$\begin{aligned} PV &= (S - C) * PV01 \\ DV01 &= \frac{\partial PV}{\partial S} \\ &= PV01 + (S - C) \frac{\partial PV01}{\partial S}, \end{aligned}$$

where  $S$  is the spread of the contract and  $C$  is the coupon. Both DV01 and PV01 are measured in dollars and are equal if the spread equals the coupon.

### **IRDV01 or Interest Rate Dollar Value 01**

The **IR DV01** is the change in value of a CDS contract for a 1 bp parallel increase in the interest rate curve. IR DV01 is, typically, a much smaller dollar value than Spread DV01 because moves in overall interest rates have a much smaller effect on the value of a CDS contract than does a move in the CDS spread itself.

### **Default Probability or Probability of Default (PD)**

**Default Probability** refers to the default probability which is the estimated probability of default for each maturity by a given time. It can be approximated by

$$\text{Default Prob} \approx \left[ 1 - \exp\left(\frac{rt}{1-R}\right) \right],$$

where  $r$  is the spread,  $t$  is the time to maturity, and  $R$  is the recovery rate.

### **Default Exposure**

"Default Expo" refers to the exposure to the default of a CDS contract based on the formula below.

$$\text{Default Exposure} = (1 - \text{Recovery Rate}) * \text{Notional} - \text{Principal}.$$

## **5.3 Risk-Neutral Pricing**

## **6 Pricing Sources**

Bloomberg and Markit are the most widely used sources for data related to pricing of CDS. We compared the results from our package with results from Bloomberg and Markit. Interestingly, the results from Markit and Bloomberg were not always identical.

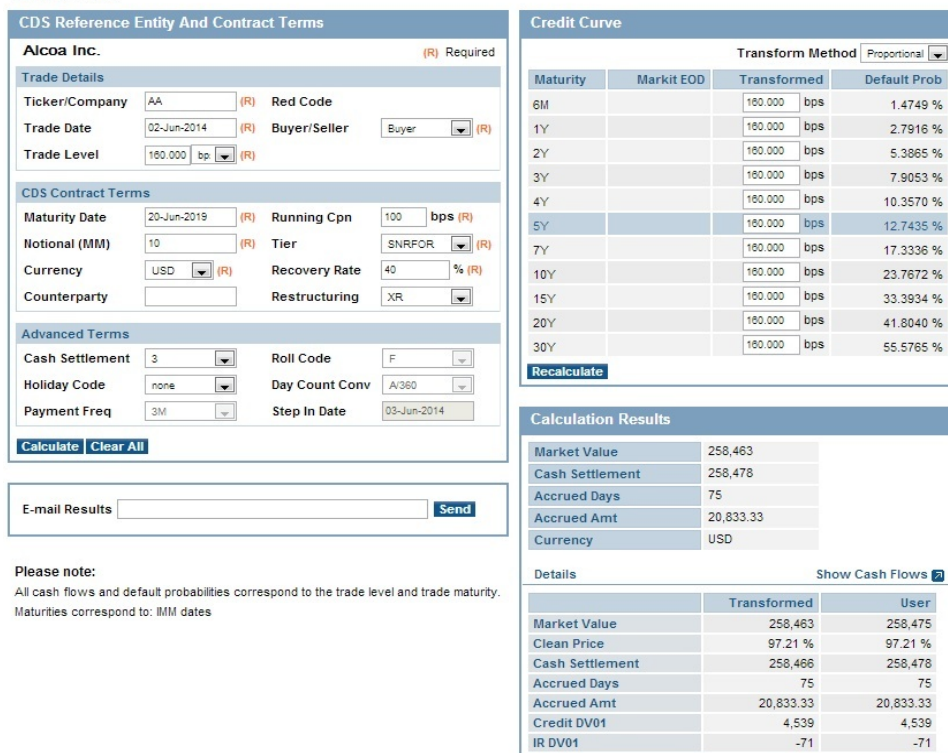


Figure 11: CDS figures from Markit.com



Figure 12: CDS figures from Bloomberg

The interest rate results were identical, at least until the fourth decimal place.

Yield Curve		
Date	01-Jun-2014	
Snap	1600	TimeZone: New York
Recalculate		
Period	Type	Rate
1M	Deposit	0.1510 %
2M	Deposit	0.1908 %
3M	Deposit	0.2274 %
6M	Deposit	0.3219 %
1Y	Deposit	0.5344 %
2Y	Swap	0.5105 %
3Y	Swap	0.9095 %
4Y	Swap	1.2895 %
5Y	Swap	1.6175 %
6Y	Swap	1.8915 %
7Y	Swap	2.1165 %
8Y	Swap	2.2995 %
9Y	Swap	2.4525 %
10Y	Swap	2.5810 %
12Y	Swap	2.7885 %
15Y	Swap	2.9970 %
20Y	Swap	3.1825 %
25Y	Swap	3.2670 %
30Y	Swap	3.3055 %
Interest Rate Conventions		
Spot Date:	04-Jun-2014	Swap DCC: 30/360
MM DCC:	A/360	Swap Interval: 6M
Floating DCC:	A/360	Holidays: none
Floating Interval:	3M	Bad Day Conv: MF

Figure 13: Interest Rate figures from Markit.com

Benchmark		S260 (USD ISDA Standard Curve - Mid)	
Date		06/02/2014	
Term	Rate	Term	Rate
1 Month	0.1510	12 Year	2.7885
2 Month	0.1908	15 Year	2.9970
3 Month	0.2274	20 Year	3.1825
6 Month	0.3219	25 Year	3.2670
12 Month	0.5344	30 Year	3.3055
2 Year	0.5105		
3 Year	0.9095		
4 Year	1.2895		
5 Year	1.6175		
6 Year	1.8915		
7 Year	2.1165		
8 Year	2.2995		
9 Year	2.4525		
10 Year	2.5810		

Figure 14: Interest Rate figures from Bloomberg

## 7 Using the CDS package

In this section, we will demonstrate the use of the **CDS** package in detail and provide a series of examples. The **CDS** package implements the Standard Model, allowing users to value credit default swaps and calculate various risk measures associated with these instruments. Currently, a market participant can conduct CDS-related calculations by using the **CDSW Calculator** on a Bloomberg Terminal or the Markit CDS Calculator<sup>3</sup>. The **CDS** package provides tools for valuing a single-name CDS contract. The default setting allows a user to value a USD-denominated CDS contract following the Standard Model as mentioned before. She can also specify her own set of parameters to customize the calculation.

### 7.1 CDS Class

In the **CDS** package, we call the function `CDS` to construct an object of a class `CDS`. Below we show an example of how to construct a CDS contract in the package.

```
> library(CDS)
> cds1 <- CDS(entityName = "Alcoa",
+             RED = "49EB20",
+             TDate = "2014-06-24",
+             tenor = "5Y",
+             notional = 1e7,
+             coupon = 100,
+             parSpread = 160)
```

Here the user enters the CDS contract with "Alcoa" as the underlying entity and sets the spread at 160 bps and the coupon at 100 bps. However, the valuation of a CDS contract requires neither the Reference Entity or the RED Code. She does not have to know that information to use the **CDS** package. As shown below, as long as she inputs the same Trade Date, parSpread, and maturity information, the valuation of the contract will be the same.

Besides parSpread, a market participant can choose to specify either `ptsUpfront` or `upfront` to construct a CDS class object.<sup>4</sup> One of the three arguments has to be specified in order to construct the CDS class object.

#### Default Settings of the CDS Contract

The default settings of valuing a CDS contract in the **CDS** package follow the Standard North American Corporate (SNAC) CDS Contract specifications.<sup>5</sup> Below we list the ISDA specifications implemented in the **CDS** package. Additional default settings in the package which are not specified by the Standard Model, such as the default notional amount, are also listed.

- Currency: USD.

---

<sup>3</sup>The Markit CDS Calculator is available at <http://www.markit.com/markit.jsp?jspage=pv.jsp>.

<sup>4</sup>See Section 4.2 for definitions on both terms.

<sup>5</sup>See <http://www.cdsmodel.com/assets/cds-model/docs/Standard%20CDS%20Contract%20Specification.pdf> for details.

- Trade Date (T): the current business day.
- CDS Date: Mar/Jun/Sep/Dec 20th of a year.
- Maturity: five years.
- Maturity Date (End Date): It falls on a CDS date without adjustment.
- Coupon Rate: 100 bps.
- Notional Amount (MM): 10MM.
- Recovery Rate (%): 40% for senior debts.
- Premium Leg:
  - Payment Frequency: quarterly
  - DCC: ACT/360
  - Pay Accrued On Default: It determines whether accrued interest is paid on a default. If a company defaults between payment dates, there is a certain amount of accrued payment that is owed to the protection seller. "True" means that this accrued will need to be paid by the protection buyer, "False" otherwise. The default is "True,"
  - Adjusted CDS Dates: "F." It means that it assumes the next available business day when a CDS date falls on a non-business day except the maturity date.
  - First Coupon Payment: It is the earliest Adjusted CDS Date after T + 1.
  - Accrual Begin Date (Start Date): It is the latest Adjusted CDS Date on or before T + 1.
  - Accrual Period: It is from previous accrual date (inclusive) to the next accrual date (exclusive), except for the last accrual period where the accrual end date (Maturity Date) is included.
- Protection Leg:
  - Protection Effective Date (Backdrop Date): T - 60 calendar days for credit events.
  - Protection Maturity Date: Maturity Date.
  - Protection Payoff: Par minus Recovery.

## 7.2 Generic Methods

### summary Method

A user can call `summary` on a `cds1` to view essential information on the contract.

```
> summary(cds1)
```

Contract Type:	SNAC	TDate:	2014-06-24
Entity Name:	Alcoa	RED:	49EB20
Currency:	USD	End Date:	2019-09-20
Spread:	160	Coupon:	100
Upfront:	286,048	Spread DV01:	4,667
IR DV01:	-75.63	Rec Risk (1 pct):	-330.15

In the summary output, it shows that the type of the CDS contract is "SNAC". Trade Date refers to the trade date and is April 15, 2014. Reference Entity (called `entityName` in the package) refers to the entity name of the CDS contract and is "Alcoa". The RED code is "48EB20" as specified by the user. spread shows that the quoted spread for the contract is 50 bps and the coupon is 100 bps as shown in the coupon field. upfront indicates the dirty upfront payment in dollars or the cash settlement amount.

The remaining three items from the summary output are Spread DV01, IR DV01, and Rec Risk (1 pct). In `cds1`, the IR DV01 is \$66.14. **Recovery Risk 01** or Rec Risk (1 pct) as shown in the summary output, is the dollar value change in market value if the recovery rate used in the CDS valuation were increased by 1%. It is \$90.03 in `cds1`.

Besides calling the `summary` method, one can type in the name of the CDS class object in the current R Session and obtain a full description of the CDS contract.

### update method

A market participant can also update the CDS class objects she has constructed by calling the `update` method. It updates a CDS class object with a new spread and points upfront by specifying the relevant input.

```
> cds3 <- update(cds1, spread = 55)
```

`cds3` is a new CDS class object with a spread of 55 bps; all other specifications of the contract are the same as those in `cds1` since it is updated from `cds1`. One can also specify upfront (in dollar amount) or `ptsUpfront` (in bps) in the `update` method.

There are three parts of the output. The first part "CDS Contract" provides basic information on the contract including "Contract Type", "Currency", "Reference Name" (called Entity Name in the package), "RED", "Trade Date", various dates related to the contract, and the day count conventions for `cds3`. The last part of the output reports the interest rates used in the calculation. The second part of the output contains relevant risks measures of `cds3`. Calling the function `getRates` also produces the rates used in building a curve for CDS valuation.

### show method

```
> cds1
```

CDS Contract

Contract Type:	SNAC	Currency:	USD
----------------	------	-----------	-----

Entity Name:	Alcoa	RED:	49EB20
TDate:	2014-06-24	End Date:	2019-09-20
Start Date:	2014-06-20	Backstop Date:	2014-04-25
1st Coupon:	2014-09-20	Pen Coupon:	2019-06-20
Day Cnt:	ACT/360	Freq:	Q

#### Calculation

Value Date:	2014-06-27	Price:	97.13
Spread:	160	Pts Upfront:	0.0287
Principal:	287,436	Spread DV01:	4,667
Accrual:	-1,389	IR DV01:	-75.63
Upfront:	286,048	Rec Risk (1 pct):	-330.15
Default Prob:	0.1322	Default Expo:	5,712,564

Credit curve effective of 2014-06-24

Term	Rate	Term	Rate
1M	0.001520	7Y	0.022725
2M	0.001963	8Y	0.024450
3M	0.002326	9Y	0.025880
6M	0.003253	10Y	0.027110
1Y	0.005471	12Y	0.029085
2Y	0.006190	15Y	0.031110
3Y	0.010630	20Y	0.032915
4Y	0.014665	25Y	0.033710
5Y	0.017930	30Y	0.034100
6Y	0.020570		

## 7.3 CDS Pricing Related Functions

### CS 10

CS10 is a method which calculates the change in value of the CDS contract when the spread of the contract increases by 10%. CS10 takes in a CDS class object formed by calling the CDS function. The CS10 of cds1 is \$25385.2.

```
> cds1.CS10 <- CS10(cds1)
> cds1.CS10
[1] 74191.79
```

### getRates function

```
> cds3Rates <- getRates(date = "2014-06-24")
```



The output from the `getRates` function below consists of two list objects. The first list contains rates of various maturities. They are directly obtained from the Markit website based on the specifications (Markit, 2013).

	expiry	matureDate	rate	type
1	1M	2014-07-28	0.00152	M
2	2M	2014-08-26	0.001963	M
3	3M	2014-09-26	0.002326	M
4	6M	2014-12-26	0.003253	M
5	1Y	2015-06-26	0.005471	M
6	2Y	2016-06-26	0.00619	S
7	3Y	2017-06-26	0.01063	S
8	4Y	2018-06-26	0.014665	S
9	5Y	2019-06-26	0.01793	S
10	6Y	2020-06-26	0.02057	S
11	7Y	2021-06-26	0.022725	S
12	8Y	2022-06-26	0.02445	S
13	9Y	2023-06-26	0.02588	S
14	10Y	2024-06-26	0.02711	S
15	12Y	2026-06-26	0.029085	S
16	15Y	2029-06-26	0.03111	S
17	20Y	2034-06-26	0.032915	S
18	25Y	2039-06-26	0.03371	S
19	30Y	2044-06-26	0.0341	S

The second list reports the specific day count conventions and payment frequencies regarding the interest rate curve used.

	text
effectiveDate	"2014-06-24"
badDayConvention	"M"
mmDCC	"ACT/360"
mmCalendars	"none"
fixedDCC	"30/360"
floatDCC	"ACT/360"
fixedFreq	"6M"
floatFreq	"3M"
swapCalendars	"none"

## 7.4 Risk-related Functions

`recRisk01`

[1] -330.1512

**IRDV01**

[1] -73.71414

**spreadDV01**

[1] 4609.709

**defaultExpo**

[1] 5720689

**defaultProb**

[1] 0.1248267

## 8 Conclusion

In this paper, we describe the basics of a CDS contract and the ISDA Standard Model. We also provide a simple collection of tools to implement the Standard Model in **R** with the `CDS` package. Moreover, the flexibility of **R** itself allows users to extend and modify this package to suit their own needs and/or create their preferred models for valuing CDS contracts. An **R** package, `backtest` Campbell et al. (2007), provides facilities to explore portfolio-based conjectures about credit default swaps. It is possible to use the `backtest` package based on the output from the `CDS` package. Before reaching that level of complexity, however, `CDS` provides a good starting point for valuing credit default swaps.

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