Credit Default Swaps with R

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Abstract

This paper explains the basics of Credit Defaults Swaps (CDS) by beginning with a simplified example of property insurance and progressing to increasingly complicated examples of a one-period CDS, two-period CDS, and n-period CDS. These examples consider cases in which two parties have agreed to a particular CDS; based on the agreement, we, as outside observers, are able to infer different attributes about that CDS. Following these examples, the paper introduces Markit and Bloomberg as two sources of financial data that take the CDS attributes discussed in the previous section as inputs to determine CDS pricing. In other words, the paper discusses fundamental CDS concepts and then explores how to apply these concepts to pricing in the CDS market. After the reader can understand basic CDS pricing calculations and concepts, the paper explores the CDS package—a program written in R that allows users to calculate information regarding a specific CDS.

1 Introduction

This paper aims to explain the mechanics of Credit Default Swaps (CDS), a type of **credit derivative** that allows finanical institutions to pass on **credit risk** to investors who accept that risk in exchange for interest. In other words, a CDS allows one party to purchase protection on a certain investment from another party. The paper introduces simplied CDS examples in which a portfolio manager passes on risk to J.P. Morgan in a one-period CDS, a two-period CDS, and an n-period CDS. With each example, we gradually introduce complications that alter pricing calculations. We make several assumptions, some of which we account for in later calculations, and some of which we maintain throughout the paper. A reader with minimal previous knowledge of CDS should be able to understand these fundamental CDS concepts.

Once the reader understands how CDS work and the basic concepts of CDS pricing, we look at data from Bloomberg and Markit that applies these concepts to price CDS. Bloomberg and Markit are two sources that investors can use to determine what kinds of CDS agreements they may want to make.

Finally, once the reader has seen how to apply these concepts to pricing in the CDS market, we introduce the CDS package, which allows users to calculate information regarding a particular CDS.

2 CDS Basics

2.1 An Example: Property Insurance

Consider a simpler form of purchasing protection: property insurance.

Suppose that a homeowner wants to purchase \$100,000 worth of property insurance on her house from January 1 to December 31. For one year of coverage, an insurance company charges a fee of \$1,000. Call this \$1,000 the premium. In exchange for the premium, the insurance company agrees to pay \$100,000 to the homeowner if there is any property damage during that year. If damage does not occur, then the insurance company pockets the \$1,000 premium and doesn't pay anything to the homeowner.

In this simplified insurance agreement, the homeowner pays the premium on January 1, the beginning of the coverage period. If the property is damaged, the insurance company pays the $100,000 on December 31, regardless of when the damage occured. The interest rate (discussed in Section 2.2) is 0%.

The **expected cash flows** for the agreement depend on the probability of property damage. Since the homeowner will pay the $1,000 premium and will potentially receive $100,000—if the house gets damaged—the homeowner's expected cash flows are as follows. ("P" refers to the homeowner's prediction of probability of the property damage.)

$$\text{Homeowner's expected cash flows} = -\ \$1,000 + (\mathrm{P} * \$100,000) \tag{1}$$

Since the insurance company will receive the $1,000 premium and will potentially pay $100,000—if the house gets damaged—the insurance company's expected cash flows are as follows. ("P" refers to the insurance company's prediction of the probability of property damage.)

$$\text{Insurance company's expected cash flows} = \ \$1,000 - (\mathrm{P} * \$100,000) \tag{2}$$

Note that the values for "P" in Equations 1 and 2 do *not* have to be the same. The homeowner and the insurance company may have two different predictions for the probability that the house will get damaged during that year. The homeowner doesn't know the insurance company's prediction for the probability of damage, and the insurance company doesn't know the homeowner's prediction for the probability of damage. In fact, an outside third party would not be able to determine the two parties' predictions, either.

Assume that both the homeowner and the insurance company are **risk-neutral**, meaning that they do not account for risk. For example, a risk-neutral investor would be willing to pay $1 for a 1% chance of a $100 payment and would view the two sides—the $1 and the 1% chance of $100—as equal.

Assume also that the homeowner and the insurance company can only agree on deals in which the expected cash flows are equal; otherwise, the party with the lesser expected cash flow will refuse to agree to the deal. By looking at Equations 1 and 2, we can deduce that "P" must represent the same probability in both expected cash flows if those cash flows are equal.

Consider that both parties agree to the above insurance agreement. What does this imply about "P", the probability of damage that equates the expected cash flows?

Even though the expected cash flows must be equal in order for the parties to strike a deal, this does not mean that the homeowner and the insurance company conveniently hold the same prediction for the probability of damage. Even if they have reached a deal, the homeowner does not even necessarily know the insurance company's prediction for the probability of damage, nor does the insurance company know that of the homeowner. Instead, since both parties conceded to the deal, we can infer that the parties were willing to assume or compromise on a certain value of "P"—the **risk-neutral probability of property damage**—even if that value was not the same as their own prediction for the probability of damage. We calculate the risk-neutral probability of damage by equating the expected cash flows in Equations 1 and 2, since that is the condition upon which the parties will agree:

$$- \$1,000 + (P * \$100,000) = \$1,000 - (P * \$100,000) \tag{3}$$

$$(2 * P * \$100,000) = (2 * \$1,000) \tag{4}$$

$$P * \$100,000 = \$1,000 \tag{5}$$

Therefore, the risk-neutral probability of property damage is 1%. Based on this calculation, we can determine that the homeowner's prediction for the probability of damage is at *least* 1%, and the insurance company's prediction for the probability of damage is at *most* 1%. For example, say the homeowner believes that there is a 1.5% chance that the house will be damaged. She would want to make a deal that assumes a damage probability of 1% because, in that case, she would be paying the same coupon for a stronger likelihood that she will receive the $100,000. If her prediction was at or above 1%, she would be willing to make a deal that assumed a damage probability of 1%.

Conversely, say that the insurance company believes that there is a 0.5% chance that the house will get damaged. The company would want to make a deal that assumed a damage probability of 1% because, in that case, it would be receiving the same coupon payment for a smaller likelihood that it'll have to pay $100,000. If its prediction was at or below 1%, it would be willing to make a deal that assumes a damage probability of 1%.

Clearly, 1%—the risk neutral probability of property damage—is a very important value for this agreement. We need to note, however, that this value is not necessarily the *true* probability of property damage, nor are the parties' predictions for the probability of damage. Consider an extreme example: what if, as the two parties initiate the agreement, an asteriod sets on a course to crash into the house in six months. In such a case, the probability of damage is 100%. However, neither party has forseen this occurence, so the risk-neutral value of "P" (1%) and the parties' predictions of "P" (1.5% and 0.5%) are certainly not equal to the true value of "P" (100%).

Property Insurance Complications

Unfortunately, the above insurance purchase—although relatively simple—made several assumptions that excluded real aspects of the insurance market. Some of these aspects are parallel to aspects in the CDS market that we will assume and exclude from calculations in all sections of this paper. Below we list such assumptions:

1. We assumed that the homeowner was risk-neutral when, in reality, homeowners who purchase property insurance are generally risk-adverse. Since they want to avoid risk, they view the process of purchasing insurance as more of a necessity than an option. They are willing to pay more (i.e. a larger coupon payment) than required for the expected cash flows to be equal. Taking our example, if the homeowner is risk-adverse and believes the probability of damage to be 1.5%, she would be willing to pay a coupon of more than 1.5%.

2. We assumed that both parties would only concede to the insurance agreement if the expected cash flows were equal. As the above assumption demonstrated, an insurance buyer is generally willing to have a smaller expected cash flow than the insurance company. Conversely, insurance companies

need to have higher expected cash flows than their clients (the insurance buyers) because they aim to *profit* from selling insurance; if they charge as much as they expect to pay, then they can't expect to make any money as a company.

3. We discussed the insurance agreement in a way that assumed we knew if there had been damage. How does the insurance company determine that there has been, for example, a house fire when the homeowner claims that one occured? The insurance company would probably want some form of proof, and the company might even have someone visit the house and confirm the damage.

While the above assumptions represent assumptions that we will maintain throughout the paper, the following assumptions are one that this paper will, in fact, account for in parallel situations regarding CDS. We list them below:

1. The insurance agreement assumed that any damage would merit the insurance payment of $100,000. However, what if, for example, the toaster caught on fire and damaged only the toaster? What types of **events** does the insurance company define as "damage"? These questions highlight the importance of a detailed insurance contract that specifies these terms.

2. The agreement considered only the following two cases: either the entire house was damaged (and the insurance company paid $100,000), or the house was not damaged (and the insurance company paid $0). However, there are other likely outcomes we need to account for. For example, what if only half of the house was damaged? In that case, it would not make sense for the insurance company to pay $100,000; instead, the company should compensate only for the damaged section of the house by paying $50,000.

3. We stated in the beginning of the property insurance example that the interest rate was 0%. Clearly, this is not the case in the insurance market, so we would need to account for that change in our calculations (which we'll discuss in Section 2.2).

4. We largely simplified the insurance agreement by creating a one-year agreement from January 1 to December 31. This, too, does not have to be the case in the insurance market. The agreement could be longer or shorter than a year and does not need to fit neatly within a calendar year.

5. The insurance agreement included one premium payment of $1,000. However, there are cases in the insurance market where there are multiple premium payments on different dates. We did not account for that possibility in our example, but we will account for parallel situations in the CDS market.

2.2 Simple One-Period Case

Instead of buying insurance against property damage, consider buying insurance against a company's inability to meet its obligations. For example, a portfolio manager from the hedge fund, Citadel, believes that the company, Alcoa, isn't likely to return the money that its bondholders have lent it. In other words, she believes that Alcoa will **default** on its bonds—not pay them back. She enters a one-year CDS agreement with an investment bank—say, J.P. Morgan (JPM)[1]—in which she purchases protection of $100,000, an amount known as the **notional amount** of the CDS. Note that bondholders

[1]For ease of reading, we desginate the buyers of protection in our examples as female and the sellers of protection as male.

can purchase protection on their bonds (i.e. an Alcoa bondholder can enter a CDS in which she purchases protection against Alcoa's default), but this does not have to be the case. In our example, the portfolio manager does not own Alcoa bonds.

Keep the same timeline (January 1 to December 31) and numerical figures from our property insurance example. The interest rate is still 0%. In exchange for protection on the $100,000, the portfolio manager agrees to pay a **coupon** (equivalent to the premium in our property insurance example) of 1% of the notional amount. The coupon payment of $1,000 is paid on January 1, the beginning of the coverage period. This side of the CDS is called the **premium leg**, and the portfolio manager is known as the **protection buyer**, since she is purchasing protection.

The other side of the CDS agreement is known as the **protection leg**, since it involves protection payment in case of default, and involves JPM, called the **protection seller**. If Alcoa defaults, JPM pays the notional amount of $100,000 to the portfolio manager on December 31, the end of the coverage period. Alcoa is known as the **reference entity** in this CDS since the protection buyer desires protection for Alcoa.

Note that we are using the terms "premium" and "coupon" synonymously. Premium is commonly used to describe the periodic fee paid by the protection buyer, which is why this side of a CDS is called the premium leg. However, even though the coupon is commonly used to describe the periodic fee paid to a bondholder, coupon can also be used to describe the periodic fee in a CDS context. For clarity, we will just use coupon in this paper when referring to the periodic fee paid by the protection buyer.

Since the portfolio manager makes money if Alcoa deteriorates, she can be said to be **short** credit. Shorting is generally a method of profiting from the deterioration of a security, such as a bond. Alternatively, since JPM loses money if Alcoa collapses, JPM can be said to be **long** credit. In other words, JPM's returns are similar to those of a person who owns Alcoa bonds. If Alcoa does well, JPM does well, and if Alcoa doesn't, then JPM doesn't either.

The expected cash flows for the agreement depend on the probability of default. Since the portfolio manager will pay the $1,000 premium and will potentially receive $100,000 if Alcoa defaults, the portfolio manager's expected cash flows—which represent the premium leg—are as follows. ("P" refers to the portfolio manager's prediction of probability of default.)

$$\text{Portfolio manager's expected cash flows} = -\ \$1,000 + (\mathrm{P} * \$100,000) \tag{6}$$

Since JPM will receive the $1,000 premium and will potentially pay $100,000 if Alcoa defaults, JPM's expected cash flows—which represent the protection leg—are as follows. ("P" refers to JPM's prediction of probability of default.)

$$\text{JPM's expected cash flows} = \ \$1,000 - (\mathrm{P} * \$100,000) \tag{7}$$

Note that the portfolio manager's expected cash flows are the same as that of the homeowner in the property insurance example, and JPM's expected cash flows are the same as that of the insurance company. This shouldn't be too surprising, though, since the numerical figures in both examples are the same.

We will again assume that both parties in the CDS are risk-neutral and that both parties will only enter deals in which the expected cash flows are equal.

Consider that a deal is reached: both parties agree to the CDS. As in the property insurance example, we can equate the cash flows to find the **risk-neutral probability of default**, which is the value of "P" implied by the deal. Note that an alternate definition of this probability is the value of "P" at which the premium leg equals the protection leg:

$$\text{Portfolio manager's expected cash flows} = \text{JPM's expected cash flows} \tag{8}$$

$$\text{Premium leg} = \text{Protection leg} \tag{9}$$

$$-\mathrm{C} + (\mathrm{P} * \mathrm{V}) = \mathrm{C} - (\mathrm{P} * \mathrm{V}) \tag{10}$$

The left side of Equation 10 indicates that the portfolio manager has to pay the coupon, "C", but could receive the protection leg of "V", depending on "P". The right indicates the converse: JPM will receive the coupon payment, "C", but could potentially have to pay the protection leg of "V", depending on "P". Plugging in known values ("C" = $1,000 and "V" = $100,000), we get:

$$-\$1,000 + (\mathrm{P} * \$100,000) = \$1,000 - (\mathrm{P} * \$100,000) \tag{11}$$

Since this is the same equation and the same numerical figures from the property insurance example, we know that P = 1%. Note that 1% only represents the risk-neutral value of "P"; it does not indicate the values of "P" that the two parties believe to be true, nor does it indicate the true value of "P". Parallel to the property insurance example, we can infer (given both parties have agreed to the CDS) that the portfolio manager's prediction of "P" must be at least 1%, and JPM's prediction of "P" must be at most 1%. In fact, from here on out, when we mention the risk-neutral probability of default for a CDS example, we can take this to mean that the protection buyer's prediction of "P" is at least the risk-neutral value, and the protection seller's prediction of "P" is at most that risk-neutral value.

We will call the above CDS example the simple one-period CDS.

One-Period Case Over Time

In order to fully understand the nature of a CDS, we should look at what happens to the simple one-period CDS over the life of its contract—specifically, how the risk-neutral probability of default (or risk-neutral "P"), the **mark-to-market** value, the **spread**, and the **profits and losses** (PNL) of the Citadel portfolio manager and JPM change from January 1 to December 31.

We can think of the mark-to-market value as the price at which the CDS would sell at any given time over the life of the contract. For example, on June 30, midway through the year, a prospective buyer would pay $500 to the portfolio manager in order to replace her as the protection buyer. Since the portfolio manager pays the $1,000 coupon payment on Janaury 1 and would recieve $500 from the prospective buyer on June 30, both the portfolio manager and prospective buyer pay a net amount of $500—which makes sense since one year of coverage merits a coupon payment of $1,000 and thus six months of coverage merits $500.

We can consider the spread to be equal to the coupon in this particular CDS (we will discuss the spread in more detail in Section sec:nonStandardCoupon). A different is that we refer to coupons in percentages—in this case, 1%—and we refer to spreads in basis points (bps). 1% is equal to 100 bps.

PNL represents the change in market value of the CDS contract on, in this case, a day to day basis. A good way of thinking about the PNL of, for example, JPM in this CDS is to consider the fraction of the coupon payment that JPM *earns* each day. We know that JPM recieves a coupon payment of $1,000 on January 1, but JPM's PNL measures what JPM earns each day that it provides protection during that year.

Here is a table that considers what happens to the above-mentioned variables as the contract matures from January 1 to December 31. Note that we are considering the case in which Alcoa does not default during the one-year contract.

Date	Cash flows (Citadel)	Cash flows (JPM)	Risk-neutral "P"	Mark-to-market	Spread	PNL (Citadel)	PNL (JPM)
Jan 1	-$1,000	$1,000	1%	$1,000	100 bps	$0	$0
Mar 31	$0	$0	.75%	$750	75 bps	-$250	$250
Jun 30	$0	$0	.50%	$500	50 bps	-$500	$500
Dec 31	$0	$0	0%	$0	0 bps	-$1,000	$1,000

Table 1: This table measures how several variables—the cash flows for each party, the risk-neutral value of "P", the mark-to-market value of the contract, the spread, and the PNL for each party—change as the simple one-period CDS matures. Since this table considers the case in which the reference entity—Alcoa—does not default, the only cash flow is the $1,000 coupon payment from the Citadel portfolio manager to JPM on January 1. The risk-neutral value of "P" decreases from 1% to 0% as the contract matures, a process we will discuss later in this section. The mark-to-market value decreases from $1,000 to $0 from January 1 to December 31 and the spread decreases from 100 bps to 0 bps. Note that as JPM profits from providing protection coverage (gains $1,000 by Dec 31), the portfolio manager loses. On January 1, the PNL of both parties is 0.

As we can see in Table 1, the risk-neutral "P" decreases from 1% on January 1 to 0% on December 31. On June 30, the risk-neutral "P" has dropped to 0.5% because only half of a year remains for Alcoa to default during the contract—and thus the risk-neutral "P" is half of its initial value. This demonstrates the direct relationship between the risk-neutral "P" and the contract duration.

In our simple one-period case, we say that the coupon is 1% and the spread is 100 bps. We can observe in Table 1 that the spread decreases from 100 bps to 0 bps over the year—a reflection of the fact that, at the end of the contract, JPM has earned and been paid the full $1,000 coupon payment by providing a full year of coverage. This process—the spread drop over the duration of the contract—is known as **rolling down the curve**.

Simple One-Period Case Complications

Similar to the property insurance example, we made several assumptions in our simple one-period CDS that are not consistent with the CDS market.

1. Similar to the property insurance example, we assumed that the protection buyer was risk-neutral, when she could very well be risk-adverse. She may be willing to pay a coupon larger than 1% even if she predicts the probability of default to be 1% because she strongly desires to have protection on Alcoa.

2. Also like the property insurance example, we assumed that both parties would only agree to the deal if the expected cash flows were equal. This could very well not be the case if, for example, JPM is selling protection in many CDS agreements and needs or wants to make a profit.

3. In the simple one-period CDS, we assumed that the protection leg would only be paid out in the case of default and bankruptcy. However, depending on the particular CDS, the protection leg could be paid out even if the reference entity defaults and doesn't go bankrupt. In fact, there are several scenarios that can be considered **credit events**—occurences that merit the payout of the protection leg in a CDS. Besides default and bankruptcy, common credit events include failure to return money to bondholders within a certain amount of time, a credit rating downgrade (explained in Section 3.2) and the confiscation of assets, among other events.

Because we made many more assumptions and failed to address many aspects of a CDS, we have split up the following simple one-period CDs complications into five separate sections that address interest rate, recovery rate, accrued coupon, non-standard coupon and upfront payment.

Interest Rate

Up until now, we have assumed that the interest rate is 0% in the simple one-period CDS. The **interest rate** is a benchmark rate that participants in the CDS market use to discount cash flows. How might the cash flows in the CDS agreement change if the interest rate was *not* 0%?

First, look at our CDS agreement: the portfolio manager pays the premium on January 1 and JPM pays the $100,000 (if Alcoa defaults) on December 31. JPM receives the $1,000 coupon a full *year* before the portfolio manager would receive the $100,000 payment if Alcoa defaults. In an environment where the interest rate is 10%, JPM could theoretically invest the $1,000 sum starting January 1 and earn interest for that year:

$$\text{Coupon with Interest} = \frac{\$1,000}{(1+.10)} = \$1,100 \tag{12}$$

By December 31, the $1,000 would grow to $1,100, which gives JPM an extra $100 that renders the expected cash flows unequal. In order to equate them, we need to discount the payment made by the protection buyer on January 1 by the interest rate—known as the **discount rate** in this context. In other words, the payment made by the protection buyer on January 1 should be an amount that, if invested at an interest rate of 10% for one year, would equal the coupon payment of $1,000. This payment, which we will call the discounted coupon, can be calculated as follows. ("C" is the coupon payment and "i" is the interest rate.)

$$\text{Discounted Coupon} = \frac{\text{C}}{(1+\text{i})} \tag{13}$$

Here are the expected cash flows for a generic case. ("P" refers to the risk-neutral probability of default; "V" represents the notional value; "i" stands for the interest rate; and "C" is the coupon payment).

$$-\frac{C}{(1+i)}+(P*V)=\frac{C}{(1+i)}-(P*V) \tag{14}$$

How might a discounted coupon affect "P", the risk-neutral probability of default which, by definition, equates the expected cash flows? Plugging in known values:

$$-\frac{\$1,000}{(1+.10)}+(P*\$100,000)=\frac{\$1,000}{(1+.10)}-(P*\$100,000) \tag{15}$$

Eventually, we get:

$$(P*\$100,000)=\$909.09 \tag{16}$$

Solving for "P", we get .91% (rounded to the nearest hundreth). Since both parties agreed to this CDS in an environment where the interest rate is 10%, we can say that the portfolio manager's prediction for the probability of default is at least .91%, and JPM's prediction for the probability of default is at most .91%.

Recovery Rate

Recall that in our property insurance example, we mentioned that some cases—such as a house fire that burns half of a house—may not merit a full payment equal to the notional amount (in that case, $100,000). Similarly, when a company defaults, an **auction** occurs in which some of the company's bondholders come to sell their bonds, and prospective buyers come to buy the bonds at, usually, lower prices. We note this because the price at which bonds can be sold after default affects the protection leg payment in a CDS contract.

Suppose that Alcoa defaults during the simple-one period CDS, and a bond that had a face value of $100 can now be sold at the auction for a price of $55. (Note for each complication in the simple one-period CDS, we are ignoring the effects of the other complications. For example, we assume that that there is an interest rate of 0% in this example.)

In this case, the **recovery rate**—the rate representing the amount of value a bond retains after default—is 55%. As such, JPM would only have to pay 45% of the notional amount—$45,000—instead of the notional amount of $100,000 because bondholders who have purchased protection are able to retain 55% of their bonds' worth.

So, we need to factor the recovery rate into our calculation of expected cash flows because it changes the value of the protection leg in the simple one-period CDS. Since the portfolio manager and JPM will only both agree to this CDS if the expected cash flows are equal, we set the portfolio manager's expected cash flows and JPM's expected cash flows equal to each other. ("RR" refers to the recovery rate and, as before, "P" is the risk-neutral probability of default, "C" is the coupon payment, and "V" is the notional value.)

$$\text{Portfolio manager's expected cash flows} = \text{JPM's expected cash flows} \tag{17}$$

$$-\text{C} + (\text{P} * \text{V} * (1 - \text{RR})) = \text{C} - (\text{P} * \text{V} * (1 - \text{RR})) \tag{18}$$

Note that the above equation is an extension of Equation 10, the only difference being that we mupltipled "V" by (1 - RR) to account for the change in the protection leg. The protection seller only has to pay the fraction of the notional amount that the recovery rate does *not* account for (hence (1 - RR) instead of just "RR"). Another term for this amount is the loss given default (LGD):

$$(1 - \text{RR}) = \text{LGD} \tag{19}$$

We'll stick to using "RR" instead of the LGD for now.

To solve for the risk-neutral probability of default, we plug in known values ("C" = $1,000, "V" = $100,000, and "RR" = 0.55)

$$-\$1,000 + (\text{P} * \$100,000 * (1 - 0.55)) = \$1,000 - (\text{P} * \$100,000 * (1 - 0.55)) \tag{20}$$

After combining like terms, dividing both sides by 2 and simplifying, we get:

$$\text{P} * \$55,000 = \$1,000 \tag{21}$$

Therefore, "P" is 1.8% (rounded to the nearest tenth) in this scenario where the recovery rate is 55%.

Accrued Coupon

Back to our simple one-period CDS example—0% interest rate, 0% recovery rate. For this scenario, both the premium leg and the protection leg are paid on December 31. We have so far assumed that our CDS agreement between the portfolio manager and JPM begins on January 1 and ends on December 31 of the same year. However, what if this agreement wasn't made exactly on January 1? What if it was made on March 31, one quarter through the year? Using our simple one-period CDS example, this implies that the portfolio manager would have to pay the same coupon of 1% ($1,000) on December 31 for only receiving nine months (instead of one year) of protection coverage.

Assume that both parties agreed to a deal under these conditions.

Since the risk-neutral probability of default decreases as the contract duration decreases (see 1), the risk-neutral probability that Alcoa will default in nine months is less than the risk-neutral probability that it will default in twelve. As such, the expected cash flows are no longer equal and therefore the two parties must have accounted for this discrepancy—otherwise the portfolio manager would not have agreed to the above CDS. In other words, given that the agreement was made, JPM must have paid an additional sum to the portfolio manager to compensate for the fact that she is paying the coupon for one-year coverage and is only receiving nine months.

In order to determine this sum, we need to calculate the fraction of the coupon she is unfairly paying, or the fraction of the coupon that accounts for the first three months of the year (January 1 to March 31). This fraction of the coupon payment is known as the **accrued coupon**.

$$\text{Accrued} = \frac{90}{360} * \frac{1.0}{100} * \$100,000 = \$250 \tag{22}$$

In order to determine the accrued coupon, we multiply the coupon (1%) by the notional amount ($100,000) to get the coupon payment ($1,000), and then we multiply the coupon payment by the fraction of the year that the portfolio manager did not receive coverage for—in this case, 90/360 or one fourth.

Note that we are dividing 90 by 360 instead of 365 in the above calculation. This particular CDS contract can be said to have a 30/360 **day count convention**, which means that the accrued coupon calculations are based on the assumptions that there are 30 days in a month and 360 days in a year. For example, if we are trying to find the number of days between date one, $M_1/D_1/Y_1$, and date two, $M_2/D_2/Y_2$, using the 30/360 convention, we use the following formula with exceptions listed below:

$$\text{Number of days} = 360 * (\mathrm{Y_2} - \mathrm{Y_1}) + 30 * (\mathrm{M_2} - \mathrm{M_1}) + (\mathrm{D_2} - \mathrm{D_1}) \tag{23}$$

Exceptions:

1. If D_1 is 31, assume that D_1 is 30.
2. If D_2 is 31 and D_1 is 30 or 31, assume that D_2 is 30.
3. If M_1 is 2, and D_1 is 28 (not in a leap year) or 29, assume that D_1 is 30.

For example, according to this convention, if the simple one-period CDS agreement was initiated on February 28 (during a non-leap year), then we would calculate the number of days between $01/01/Y_a$ and $02/28/Y_a$ (where Y_a is a specific year) to be 59, since we consider February 28 to be the last day in a 30-day month (according to exception 3 above):

$$\text{Number of days} = 360 * (0) + 30 * (2 - 1) + (30 - 1) = 59 \tag{24}$$

If a CDS agreement was initiated on October 31, then we would calculate the number of days in between $01/01/Y_a$ and $10/31/Y_a$ (where Y_a is some year) to be 300. Note that it does not matter for this calculation if Y_a was a leap year or not.

$$\text{Number of days} = 360 * (0) + 30 * (10 - 1) + (31 - 1) = 300 \tag{25}$$

The purpose of this day count convention is to make calculations easier.

Since a $250 cash flow from JPM to the portfolio manager compensates for the amount of the coupon that the portfolio manager is unfairly paying, we can model the expected cash flows for the agreement:

$$-\$1,000 + (\mathrm{P} * \$100,000) + \$250 = \$1,000 - (\mathrm{P} * \$100,000) - \$250 \tag{26}$$

What do these expected cash flows imply about "P", the risk-neutral probability of default? Solving for "P", we get:

$$\mathrm{P} * \$100,000 = \$750 \tag{27}$$

Therefore, the risk-neutral probability of default in this CDS is .75%. Note that this is different than that of the simple one-period CDS (1%). Note that in Table 1, we observed that the risk-neutral probability of default decreases as the contract matures.

In Section 2.2, we measured different aspects of a CDS over the life of the contract. Recall that we defined the mark-to-market value as the coupon that a prospective buyer would pay at a given time to own the protection for the remainder of the CDS. We could similarly measure the value of the accrued coupon over the life of the contract—or the accrued coupon value that JPM would have to pay a prospective buyer if she bought the protection from the portfolio manager at a given time in the contract. Consider the table below:

Date	Mark-to-market	PNL (Citadel)	PNL (JPM)	Accrued Coupon
Jan 1	$1,000	$0	$0	$0
Mar 31	$750	-$250	$250	$250
Jun 30	$500	-$500	$500	$500
Dec 31	$0	-$1,000	$1,000	$1,000

Table 2: This table measures the mark-to-market value, the PNL for both parties, and the accrued coupon value over the life of the contract. Note that the mark-to-market values and accrued coupon values are the same for each date. In other words, the accured coupon that accounts for the time passed is equal to the price that a prospective buyer would have to pay the portfolio manager for protection for the remainder of the CDS. The absolute value of each party's PNL is also equal to the accrued coupon value; the accrued coupon that accounts for the time passed is equal to the money the JPM has earned by providing protection for the time passed.

As we can see, the accrued coupon increases as the contract matures—or, in other words, the coupon that accounts for the time in the contract that has passed *increases* as the contract matures.

Note that accrued coupon only becomes relevant when the contract does not initiate on a coupon payment date. In the real-world CDS market, there are four dates each year—instead of, in our example, the one date of January 1—when coupon payments can be made. These dates are known as **roll dates** and comprise the following: March 20, June 20, September 20, and December 20. As we proceed with our simple CDS example, however, we'll continue with the assumption that January 1 is the one and only "roll date."

Non-standard Coupon

In all of our CDS examples, we have assumed a coupon of 1%. We have done so because we have been working with the same basic numerical figures: a risk-neutral probability of default of 1% and a notional amount of $100,000:

$$-\$1,000 + ((P = 1\%) * \$100,000) = \$1,000 - ((P = 1\%) * \$100,000) \tag{28}$$

In other words, we have been considering cases in which the portfolio manager believes the probability of default to be at least 1%, and JPM believes the probability of default to be at most 1%.

What if, however, the portfolio manager actually thought that the probability of default was 3%, and JPM thought it was 4%? The portfolio manager would want to make CDS agreements with risk-neutral probabilities of default of 3% or greater because if it's greater then she is paying the same coupon for a higher probability that she'll receive $100,000. JPM would want to make CDS agreements with risk-neutral probabilities of default of 4% or smaller because if it's smaller then JPM is receiving the same coupon for a smaller probability that it'll have to pay $100,000. The range of possible risk-neutral probabilities at which they would agree is 3% to 4%. Say they decide on 3%.

In such a case, the expected cash flows would only be equal if the coupon payment was $3,000: 3% of the notional amount of $100,000. As such, here is a case in which the coupon would not be 1%:

$$-\$3,000 + ((P = 3\%) * \$100,000) = \$3,000 - ((P = 3\%) * \$100,000) \tag{29}$$

In fact, that is a simplified example of how protection buyers and sellers determined coupon payments before April, 2009. Traders could theoretically enter CDS agreements with any coupon—say, .72% or 1.63%—because there were not any market conventions for coupon values. Each CDS had a specific **spread**—in this context, a specific coupon—that equated the two expected cash flows in the CDS. If a person was interested in buying CDS protection, she and the protection seller would refer to different CDS agreements by simply quoting the corresponding coupons.

One issue with this system was that traders could not easily trade CDS, since each CDS had a specific coupon. For example, investment banks like JPM often buy protection for one CDS and sell protection for another so as to offset each deal and **hedge** its risk. However, this became difficult to do when the CDS agreements had different coupons and thus different expected cash flows.

So, in 2009, a regulatory organization known as the International Swaps and Derivates Association (ISDA) introduced new CDS market conventions in North America which required buyers and sellers to trade at coupons of 100 bps or 500 bps. Since then, the spread of a CDS no longer signifies the coupon at which the CDS is traded; instead, the spread is the coupon that the CDS *would* be traded at in a market without standardized coupons—or the coupon that the CDS would have been traded at before 2009. In order to account for the discrepancy between the spread and the standardized coupon at which a CDS must trade, protection buyers and sellers use **upfront payments**, discussed in the next section.

Upfront payment

An **upfront payment**—a payment made from one party to the other at the beginning of a contract (in this case, January 1)—compensates for the difference between the spread that both parties *want* to trade at and the standardized coupon that they *have* to trade at. Suppose that, like in the previous example, both parties want to trade at a coupon of 3%. Clearly, they have to trade at a coupon of 1% or 5%—assume they choose 1%. They will only make a deal that acts *as if* they traded at a coupon of 3%.

Consider that both parties agree to the deal. How do we determine the value of the upfront payment?

First, review the expected cash flows we had in the previous section with a coupon of 3%:

$$-\$3,000 + ((P = 3\%) * \$100,000) = \$3,000 - ((P = 3\%) * \$100,000) \tag{30}$$

Since the difference between the protection buyer's desired coupon payment ($3,000 at 3%) and the payment she has to make due to CDS market convention ($1,000 at 1%) is $2,000, the portfolio manager's upfront payment to JPM should be $2,000. The best way of thinking about an upfront payment is to consider the difference in expected cash flows between the two parties. Since the portfolio manager's expected cash flow is larger than that of JPM, the upfront payment ("U") is negative on the left side of the equation (the portfolio manager's expected cash flow) and positive on the other side (JPM's expected cash flow) so as to equate the two sides. (As before, "C" designates the coupon payment, "P" is the risk-neutral probability of default, and "V" is the notional value.)

$$\text{Premium leg} = \text{Protection leg} \tag{31}$$

$$-U - C + (P * V) = U + C - (P * V) \tag{32}$$

Plugging in known values:

$$-U - \$1,000 + (.03 * \$100,000) = U + \$1,000 - (.03 * \$100,000) \tag{33}$$

Simplifying:

$$(2 * \$3,000) = (2 * \$1,000) + (2 * U) \tag{34}$$

$$\$3,000 - \$1,000 = U \tag{35}$$

This verifies that the correct upfront payment in this scenario is $2,000.

That way, with an upfront payment, the two parties can trade with a coupon of 1% and a risk-neutral probability of default of 3%. On January 1, the portfolio manager would pay $2,000 of the upfront payment to JPM and the $1,000 coupon payment to JPM.

In Section 2.2, we measured different aspects of a CDS such as the risk-neutral "P" and the mark-to-market value over the life of the contract. Recall that we defined the mark-to-market value as the coupon that a prospective buyer would pay at a given time to own the protection for the remainder of the CDS. We could similarly measure the value of the upfront payment over the life of the contract—or the upfront payment that a prospective buyer would have to pay at a given time to own the protection for the remainder of the CDS. Consider the table below:

Date	Risk-neutral "P"	Mark-to-market	Upfront Payment
Jan 1	3%	$1,000	$2,000
Mar 31	2.25%	$750	$1,500
Jun 30	1.5%	$500	$1,000
Dec 31	0%	$0	$0

Table 3: As the contract matures from January 1 to December 31, the value of the upfront payment (in a CDS where the coupon is 1%, the notional amount is $100,000, and the risk-neutral probability of default is 3%) decreases from $2,000 to $0. Note that the true mark-to-market value of the CDS on, say June 30, would actually be the value in the mark-to-market column *plus* the value in the upfront payment column—becasue that is the total amount that the prospective protection buyer would have to pay to own the protection. However, for clarity, in this table we will view the values in the mark-to-market column as values that represent the coupon the prospective buyer would have to pay.

Earlier, we calculated the upfront payment using the following equation:

$$\text{Premium leg} = \text{Protection leg} \tag{36}$$

$$-\mathrm{U} - \mathrm{C} + (\mathrm{P} * \mathrm{V}) = \mathrm{U} + \mathrm{C} - (\mathrm{P} * \mathrm{V}) \tag{37}$$

We can verify that the upfront payment values in Table 3 are correct by inputting the risk-neutral probability of default as "P" and by inputting the mark-to-market value as "C". For example, take June 30 (where, according to Table 3, the risk-neutral "P" is 1.5% and the mark-to-market value is $500.)

$$-\mathrm{U} - \$500 + (.0015 * \$100,000) = \ \mathrm{U} + \$500 - (.0015 * \$100,000) \tag{38}$$

$$(2 * \$1,500) = (2 * \$500) + (2 * \mathrm{U}) \tag{39}$$

$$\$1,500 - \$500 = \mathrm{U} \tag{40}$$

As such, on June 30, the value of the upfront payment is $1,000—which is what Table 3 indicates, as well.

Note that in Equation 37, "C" and "V" are known or pre-determined values, since "C" is standardized by the ISDA and "V" is agreed upon by the protection buyer and seller. As such, "P" and "U" can be considered the unknown variables; one needs to know the value of "P" to determine the value of "U", and vice versa.

One-Period Summary

We have introduced a simple one-period CDS between a portfolio manager and JPM, and we have explored five complications that affect the expected cash flows: interest rate, recovery rate, accrued coupon, non-standard coupon and upfront payment.

In a real-world scenario, we'd have to look at *all* these factors at the same time, and come up with a way to model cash flows that take all factors into account.

Combining Equations 14, 19 and 37, we get a master equation that accounts for interest rate ("i"), recovery rate ("RR"), and upfront payment ("U"). Accrued coupon can be calculated separately based on the date the CDS was initiated. (As before, "C" stands for the coupon payment, "P" for the probability of default, and "V" for the notional value.)

$$\text{Portfolio manager's expected cash flows} = \text{JPM's expected cash flows} \tag{41}$$

$$\text{Premium leg} = \text{Protection leg} \tag{42}$$

$$\mathrm{U} - \frac{\mathrm{C}}{(1+\mathrm{i})} + (\mathrm{P} * \mathrm{V} * (1-\mathrm{RR})) = -\,\mathrm{U} + \frac{\mathrm{C}}{(1+\mathrm{i})} - (\mathrm{P} * \mathrm{V} * (1-\mathrm{RR})) \tag{43}$$

Note that for any standard CDS contract, the two unknown variables are "P" and "U", since "C" and "RR" are fixed by the International Swaps and Derivatives Association (ISDA)[2] (discussed in the next section), and "V" is agreed upon by the two parties (essentially fixed). As such, if we know "P", we can calculate "U" and vice versa.

2.3 Two-Period Case

Thus far, we have described a simple one-period CDS with one coupon payment on January 1. However, as mentioned in Section 2.2, CDS have four coupon payments each year. Before we get into the exact mechanics of a four-period or more case, consider a simple two-period CDS that has a duration of two years and two coupon payments: one on January 1 of year one and one on January 1 of year two. The interest rate is 0%, and the probability of default is constant for both years of the contract. Like the simple one-period CDS, the notional amount is $100,000 and the coupon is 1%. Note that the term "coupon" refers to the protection buyer's *annual* payment. So, the buyer will make two $1,000 coupon payments, thus paying a total of $2,000 over the two years. The protection buyer is still the portfolio manager and the protection seller is still JPM.

Consider that both parties agree to this deal.

The risk-neutral probability of default—the default that the deal implies both parties were willing to compromise on—can be calculated by equating the expected cash flows. Consider the expected cash flows for year one of the CDS contract:

$$\text{Portfolio manager's expected cash flows} = -\,\$1,000 + (\mathrm{P} * \$100,000) \tag{44}$$

$$\text{JPM's expected cash flows} = \ \$1,000 - (\mathrm{P} * \$100,000) \tag{45}$$

The expected cash flows for the portfolio manager and for JPM are the same for year two.

In order to find the risk-neutral probability of default for each year, we must equate the expected cash flows:

$$-\,\mathrm{C} + (\mathrm{P} * \mathrm{V}) = \mathrm{C} - (\mathrm{P} * \mathrm{V}) \tag{46}$$

$$-\$1,000 + (\mathrm{P} * \$100,000) = \$1,000 - (\mathrm{P} * \$100,000) \tag{47}$$

[2] Note, ISDA does not set standard recovery rates for all reference entities.

$$P * \$100,000 = \$1,000 \tag{48}$$

Thus, the risk-neutral probability of default for year one (or for year two, since the expected cash flows are the same) is 1%.

Now that we have considered the expected cash flows and risk-neutral probability of default for each year, we may want consider what those values for both years combined. What are the overall expected cash flows and the overall risk-neutral "P"?

$$\text{Portfolio manager's expected cash flows} = -(\ \$1,000 + \$1,000) + (P * \$100,000) \tag{49}$$

$$\text{JPM's expected cash flows} = \ (\$1,000 + \$1,000) - (P * \$100,000) \tag{50}$$

In order to find the risk-neutral probability of default for both years combined, we need to equate the two expected cash flows:

$$-(\$1,000 + \$1,000) + (P * \$100,000) = (\$1,000 + \$1,000) - (P * \$100,000) \tag{51}$$

$$P * \$100,000 = \$2,000 \tag{52}$$

The risk-neutral probability of default for the two-year contract is 2%. Note that the risk-neutral probability of default in the simple one-period CDS was 1%. The only reason that this probability is different is because the portfolio manager's coupon payment in the two-period case is double that of the one-period case.

Recall how we discussed in Section 2.2 that the probability of default within the context of a CDS decreases to 0% as the contract matures. Based on our calculations above, we can verify this relationship; sfter one year of the two-year contract, the probability of default has decreased from 2% at the beginning of the contract to 1% midway through the contract.

We will call the above CDS example the simple two-period CDS.

Simple Two-Period Case Complications

Since this is a simplified example, we have excluded aspects of the CDS market that are worth noting.

1. We again assumed that both parties are risk-neutral when, in fact, this may not be the case.

2. When calculating the probability of default during each year of the simple two-period CDS, we essentially assumed that we can just add probabilities: a probability of default of 1% during year one and a probability of default of 1% during year two equals a probability of default of 2% over the two years. This is clearly not exactly the case.

Non-constant Probability of Default

In the simple two-period CDS, the overall probability of default is 2%. Since we conveniently assumed that the probability of default was constant over the two-year contract, each year had a probability of default of 1%. What if the overall probability of default was still 2%, but the probability of default is not constant—what if it's 2% the first year and 0% the second year?

In such a case, the expected cash flows for year one (where "P" = 2%) are as follows:

$$\text{Portfolio manager's expected cash flows} = -\ \$1,000 + ((\text{P} = 2\%) * \$100,000) \tag{53}$$

$$\text{JPM's expected cash flows} = \ \$1,000 - ((\text{P} = 2\%) * \$100,000) \tag{54}$$

Note that these expected cash flows are *not* be equal. We will address this fact soon.

Now consider the expected cash flows for year two (where "P" = 0%):

$$\text{Portfolio manager's expected cash flows} = -\ \$1,000 + ((\text{P} = 0\%) * \$100,000) \tag{55}$$

$$\text{JPM's expected cash flows} = \ \$1,000 - ((\text{P} = 0\%) * \$100,000) \tag{56}$$

Clearly, the expected cash flows for the portfolio manager and JPM during year one are not equal, and the expected cash flows for the portfolio manager and JPM during year two are not equal, either.

However, note that in year one, JPM's expected cash flows are $1,000 greater than that of JPM, and in year two, JPM's expected cash flows are $1,000 less than that of JPM. As such, the overall expected cash flows (which are the same as Equation 51 because the overall risk-neutral probability of default is the same) are equal.

What it's reversed: what if the risk-neutral probability of default for year one is 0% and that of year two is 2%?

Non-constant interest rate

Two-Period Case Over Time

Consider what happens to variables such as the cash flows of both parties, the risk-neutral "P", the mark-to-market value, the spread, and the PNL of both parties as the two-year contract matures.

Date	Cash flows (Citadel)	Cash flows (JPM)	Risk-neutral "P"	Mark-to-market	Spread	PNL (Citadel)	PNL (JPM)
Jan 1, Y_a	-$1,000	$1,000	2%	$1,000	200 bps	$0	$0
Mar 31, Y_a	$0	$0	1.75%	$750	175 bps	-$250	$250
Jun 30, Y_a	$0	$0	1.50%	$500	150 bps	-$500	$500
Jan 1, Y_b	-$1,000	$1,000	1%	$1,000	100 bps	-$1,000	$1,000
Mar 31, Y_b	$0	$0	.75%	$750	75 bps	-$1,250	$1,250
Jun 30, Y_b	$0	$0	.50%	$500	50 bps	-$1,500	$1,500
Dec 31, Y_b	$0	$0	0%	$0	0 bps	-$2,000	$1,000

Table 4: This table displays how several variables—the cash flows of both parties, the risk-neutral "P", the mark-to-market value, the spread, and the PNL of both parties—change as the two year contract matures. Y_a stands for year one and Y_b stands for year two. Note that this table shows these changes in the case that the reference entity—Alcoa–does not default. As such, the only cash flows are the two $1,000 coupon payments on January 1 of each year. The risk-neutral probability of default decreases from 2% to 0% over the life of the contract, and the spread decreases from 200 bps to 0 bps. Note that the mark-to-market value, however, goes from $1,000 to $0 over the first year, and $1,000 to $0 over the second year.

Suppose that the spread (a.k.a. the coupon) is 160 bps (equivalent to 1.60%), and the portfolio manager and JPM have to trade at 100 bps because of CDS market conventions. On January 1, the portfolio manager pays an upfront payment to JPM that takes into account the extra 60 bps she doesn't have to pay as part of the coupon. Even though this payment happens once during the year, we can think about the value of this payment by considering what JPM earns every day. In return for providing protection each day, JPM earns a fraction of the total upfront payment. In other words, as the year progresses, the protection seller slowly makes money and the protection buyer slowly loses money. By the end of the year, both the upfront and the coupon payments have been made, effectively canceling each other out to produce a one cash flow: a coupon payment of 1% from the protection buyer to seller. Thus, by the end of the year, the value of the upfront payment goes to zero since by the end of the year, since the cash flows cancel out as if there was never any upfront payment at all.

The way in which the value of the upfront payment drops over the life of the CDS contract can be called the **pull to par**.

3 Fixed Income Securities and Credit Risk

CDS are used for the purpose of hedging against the **credit risk** associated with **bonds** or **fixed income**. Before we approach understanding how CDS work, we must understand the mechanics of a standard coupon bond and its associated risks.

3.1 Bond Basics

Consider the following example: say that there exists a portfolio manager who has to allocate her clients' money (a total sum of $10 million). This particular manager is a risk-averse investor who seeks to minimize her losses, even if that ultimately creates lower returns on investments. She also wants to invest in assets that are **liquid** i.e. assets that are easily retrievable or can be easily converted into cash. In such a scenario, coupon bonds seem like an attractive investment.

A **coupon bond** is an agreement in which one party lends money to another party in exchange for that same sum of money, plus periodic **coupon payments**, at a future date. A major advantage of holding bonds over shares (which are also known as equity) is that bondholders are compensated before shareholders in the case of liquidiation (such as a default). If a company or other reference entity defaults on its debt and goes bankrupt, the remaining assets associated with the company are gathered and distrubited among bondholders and shareholders according to the issuer's capital structure (another terms for the hierarchy of investors). Bonds can be further categorized as "senior" and "junior," so although bondholders will be compensated first, those with a greater seniority ("senior" bonds) will be compensated before those with a lesser seniority ("junior" bonds). The amount of assets remaining after default and bankruptcy is known as the **recovery rate**.

To understand the different variables used to determine cash flows and bond pricing, look at actual data for an Alcoa bond that will mature, or be paid out, five years from now (since the data descibes a bond that was initiated seven years ago).

Figure 1: Bloomberg data taken on June 24, 2014 for senior Alcoa bonds that will mature on February 23, 2019. These bonds have a duration of twelve years, since the bonds were initiated on April 2, 2007 (see "Announcement Date"). This figure displays essential information regarding the bonds that a potential investor would need to consider, such as the company ("name"), industry, seniority ("rank"), the value of the coupon payments ("coupon"), maturity, coupon payment dates and credit ratings. (Many of these attributes and concepts will be further discussed in Sections 3.2 and 3.4.)

3.2 Credit Risk Associated with Bonds

A bond has many kinds of risks associated with it such as interest rate risk, inflation risk or liquidity risk. For our purposes, we will concentrate on credit risk.

If we look to the right of Figure 1, under "Bond Ratings," we can see certain symbols representing **credit ratings** that are provided by rating agencies such as Moody's, S&P and Fitch. These symbols are indicators of a company's credit risk. When a company like Alcoa issues bonds, there is always a possibility that it may not be able to meet its debt obligations. That possibility, or risk, is known as the credit risk. Naturally, if a company's credit risk goes up, investors would demand a higher yield and consequently, a lower price. If a company is very likely to meet its debt obligations—or if it has consistently done so in the past—the company can be known as **credit worthy**.

A risk-averse investor, like our pension fund portfolio manager, would naturally want to purchase senior bonds from a company that has a low credit risk—a company like Alcoa, which has a rating of BBB- from S&P, BB+ from Fitch and Ba1 from Moody's. We will not go into the details of how these ratings are determined, since those details are beyond the scope of this vignette. What we should note is that companies that have credit ratings of BBB- or higher from S&P or Fitch or Baa3 or higher from Moody's can be classified as **Investment Grade** (IG) bonds. Such companies, at least from the viewpoint of the rating agencies, have low credit risks. Bonds that are rated below the IG benchmark are termed as **speculative grade** bonds or **junk** bonds, and have a higher yield than IG bonds. We can see a complete list of classifications from Moody's and S&P in Figure 2:

Investment Grade					Speculative Grade				
S&P	Fitch	Moody's	Interpretation	Numeric Scale	S&P	Fitch	Moody's	Interpretation	Numeric Scale
					BB+	BB+	Ba1		11
AAA	AAA	Aaa	Very high credit quality	1	BB	BB	Ba2	It has speculative elements and it is subject to substantial credit risk	12
					BB-	BB-	Ba3		13
AA+	AA+	Aa1		2	B+	B+	B1		14
AA	AA	Aa2	High credit quality	3	B	B	B2	It is considered speculative and it has high credit risk	15
AA-	AA-	Aa3		4	B-	B-	B3		16
A+	A+	A1		5	CCC+	CCC+	Caa1		17
A	A	A2	Medium-high grade, with low credit risk	6	CCC	CCC	Caa2	Bad credit conditions and it is subject to high credit risk	18
A-	A-	A3		7	CCC-	CCC-	Caa3		19
					CC	CC	Ca		20
BBB+	BBB+	Baa1		8					
BBB	BBB	Baa2	Moderate Credit Risk	9	C	C	C		22
BBB-	BBB-	Baa3		10	SD	DDD	-	Very close to default or in default	22
					D	DD			22
					-	D			22

Figure 2: Credit rating classifications from S&P, Fitch and Moody's for long-term debt obligations. Even though S&P and Fitch have the same credit rating scale, note how the rating determined by S&P and the rating determined by Fitch for Alcoa bonds in Figure 1 are *different*, which implies that the two rating agencies use different measurements to determine a credit rating. Herein lies the value of having three different ratings agencies that can provide three different ratings: if, for example, a company can be classified as IG across *all three* ratings—which are all not determined by the exact same combination of metrics—then one can be more confident in the fact that the bond or the company actually is of IG quality.

3.3 Risk-free Bonds

Bonds issues by the U.S. Government (known as treasury bonds) are generally considered to be **risk-free** since the probability of the U.S. Government failing to meet its debt obligations is almost negligible. Bonds issued by Japan (Japanese Government Bonds or JGBs) and Britain (Gilts) are also considered to be risk-free. Therefore, the interest paid by sovereign bonds is an important benchmark for the pricing of corporate bonds or even the bonds of other governments.

Bonds from Alcoa, or any corporation, are generally riskier than bonds issued by the U.S. Government. A rational investor would want a riskier bond to pay a higher coupon rate than a U.S. treasury bond since otherwise she could get the same interest payment at no risk at all. In other words, she would want the expected return of a corporate bond to be the same as the expected return of a risk-free bond. For this to be the case, the riskier bond would have to pay more interest than the risk-free asset in order to compensate for the added risk. The amount to which the riskier bond's interest exceeds that of a risk-free bond is known as the **risk premium**. So, the interest that an investor would want from a bond would change as the risk-free coupon rate changes. It is important to understand the intuition behind this before learning how bonds are priced.

Swap Curve or Risk-free Curve

Bond yields are directly proportional to bond maturities. This relationship results from the fact that the risk of default for any entity is higher over a long period than over a short one. This should make sense intuitively since a longer maturity is more likely to capture the time at which an entity may default; the risk that a government may default on its debt over 30 years is higher than the risk that it defaults in six months. In rare cases, the short-term yields are higher than long term yields; this can often be a sign of a recession. If, for a given date, we plot the maturities of different bonds, such as treasury bonds, on the x-axis and their corresponding yields on the y-axis, we get a **Swap Curve**.

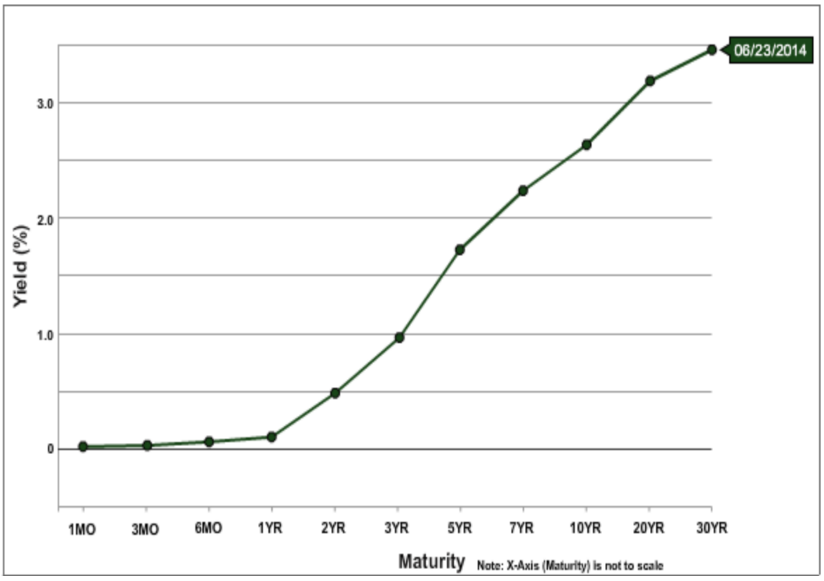


Figure 3: Swap Curve for treasury bonds on June 23, 2014. Clearly, yields are higher for debt obligations with longer maturities. This makes sense since investors would want a higher overall yield if they were to take on more risk by buying a bond with a longer maturity. Note that the x-axis is not to scale (the points are not at equal intervals), so it is difficult to make any conclusions or observations about slope or the change in yield for a certain amount of time.

3.4 Bond Pricing

Present Value (PV)

Present value refers to the current value of a future payment. For example, if an investor has $1 million, that asset (if invested) it is worth more today than $1 million ten years from now. If this person invests his $1 million in, say, a risk-free asset earning a coupon of 5% a year (compounded semi-annually), that initial amount will become $1.63 million in ten years. So, the present value of that $1.63 million (that will accumulate over ten years) is \$1 million. The 5% coupon rate that we are using to discount the future value is known as the **discount rate**.

Applying this concept to bond pricing, we can say that a bond's price is essentially the sum of the present values of its cash flows i.e. the sum of the present values of coupon payments and principal. Basically, we have to pretend as though we are investing the coupon payments and principal at different time periods. So for our \$1 million bond with a discount rate of 5% paid semi-annually (2.5% every six months), the present value of the first coupon of \$25,000 is \$25,000/1.025, that of the second coupon is $\$25{,}000/1.025^2$ and so on. The present value (and price) of a bond can be calculated using the equation below:

$$\text{Price} = C * \frac{1 - \frac{1}{(1+i)^n}}{i} + \frac{M}{(1+i)^n} \tag{57}$$

"M" represents the principal payment and "C" refers to the coupon payment of \$25,000. The variable, "i", is the discount rate and is also called the **yield to maturity (YTM)** or just **yield**. It is the required interest or discount rate for the present value of a future payment to be equal to the bond price.

If we look at the top of Figure 1, we see that a bond with a face value of \$100 for Alcoa on June 24, 2014 has a coupon rate of 5.72%, a price of \$112.063 and a yield of 2.928%. As we can see, the bond price is greater than the bond's face value—the amount that Alcoa will return to the bondholder when the contract ends—because the bondholder is receiving 5.72% interest, which is larger than the coupon rate of a treasury bond or other risk-free bond. The **Cpn Freq** (Coupon Frequency) is quoted as "S/A" which stands for "semi-annually." So, if our portfolio manager were to purchase Alcoa bonds that have a principal of \$10 million and that pay a coupon of \$286,000 semi-annually, it would cost her \$11,206,300. If we substitute "i" in the above equation with 0.014514 (the discount rate for a five-year bond), "C" with \$2.86, "M" with \$100 and "n" (the number of coupon payments) with 10, we would get the bond price quoted in the Bloomberg screenshot in Figure 1: \$112.063.

When our portfolio manager has to purchase a bond, she has to determine what a fair yield, or discount rate, for that specific bond would look like. The factors affecting the yield include the risk-free rate, as explained earlier, and the credit health of the company, represented partly by the credit rating. If we look at the equation above or even think of these concepts intuitively, we can see that the bond yield has an inverse relationship with its price. If there is a fixed coupon rate, and an investor wants a higher bond yield, the only way to keep the same coupon rate and increase the yield is the lower the bond price—and decrease what the investor has to pay—accordingly. Moreover, the difference between the yield of a bond and the yield of a risk-free bond of the same maturity is a common measurement of the company's credit risk.

4 CDS Terminology and Cash Flows

Although the cash flows involved in a CDS seem pretty simple in Figure **??**, the underlying mechanics of a CDS are a bit more complex. To understand this, look at actual CDS data from Bloomberg for Alcoa. Figure 4 below displays many different variables such as RED Pair Code, REF Entity, Trade

Date, Debt Type etc. for the CDS of Alcoa for June 24, 2014. These are standard terms of CDS contracts that were set by the **International Swaps and Derivatives Association (ISDA)**, the organization that regulates over-the-counter derivatives such as CDS. These variables are important for understanding the cash flow[3].

Figure 4: CDS figures from Bloomberg for Alcoa Note that this **REF Obligation** (Reference Obligation) matches the ISIN in Figure 1.

4.1 Entity-Specific Variables

Some of these variables seem self-explanatory. **REF Entity** (Reference Entity) refers to the name of the company (Alcoa, in this case) that the buyer wants protection against. The **ticker** is an abbreviated reference symbol for the Reference Entity, which is "AA" for Alcoa Moreover, the **Trade Date** is the date on which we are making the trade: June 24, 2014.

The **RED Pair Code** is a Markit product that stands for Reference Entity Database. Each entity/seniority pair has a unique six-digit RED Pair Code that matches the first six digits of the nine-digit RED Pair Code. Each entity also has a "preferred reference obligation," which is the default reference obligation for CDS trades. A user can input either the six-digit RED Pair Code or the nine-digit RED Pair Code. The input "014B98" is the six-digit RED Pair Code for "Alcoa"

We can also note the label **Debt type**, marked as "Senior." Clearly, this refers to the seniority of the debt. The **notional amount** is printed as "10" or \$10 million USD (US Dollars). The **REF Obligation**

[3]Some of the definitions come from *Credit Derivatives Glossary* (Markit, 2009), *Standard Corporate CDS Handbook* (Leeming et al., 2010), *Credit Derivatives* (Green and Witschen, 2012), and *The Pricing and Risk Management of Credit Default Swaps, with a Focus on the ISDA Model* (White, 2013).

(Reference Obligation) refers to the bond involved in the CDS. Since our portfolio manager is purchasing protection on the bonds she previously purchased, this **REF Obligation** matches the ISIN in Figure 1.

Maturity refers to the **tenor**, or length, of the contract. The most commonly traded contracts have maturities of five years. The Bloomberg screenshot displays the length of the contract, as well as the implied maturity date. Interestingly, the maturity date for the CDS of Alcoa on June 24, 2014 is September 20, 2019. This might seem odd since we would expect it to be on June 24, 2019, exactly five years from the Trade Date. However, the maturity date always falls on one of the four **roll dates**: March 20, June 20, September 20 and December 20. Therefore, the maturity date for this contract will be on the roll date after June 24, 2019, which is September 20, 2019. Also note that this contract is of the type **SNAC**, or Standard North American Contract, which is a convention that specifies how North American single-name CDS are supposed to trade. In European markets, CDS belong to the **STEC** category, or Standard European Contract.

4.2 Premium Leg

The stream of cash flows from the protection buyer to the seller, when there is no default, is known as the **premium leg**. To understand the premium leg, we must look at the **Trade Spread** (marked as **Trd Sprd** in Figure 4) and **Coupon** in Figure 4. In section **??**, we stated that the protection buyer pays the protection seller a fixed coupon for purchasing protection. Until 2009, the two counterparties in a CDS contract would agree on the coupon level before the trade. Then, as the market moved, this tradable coupon would vary. So if our portfolio manager was purchasing a CDS before 2009, she would have had to negotiate a fixed coupon—which would then vary as the company's credit risk varied—with the hedge fund.

In April 2009 in North America, the ISDA introduced a series of mandatory modifications to the CDS contract known as the "Big Bang" protocol. Under the new rules, coupon rates were standardized in North America and Europe starting June 2009. Dealers now had to quote **standard coupons** of 100bps or 500bps in North America, or 25bps, 100bps, 500bps or 1000bps in Europe, and all coupons were paid quarterly on one of the four roll dates. The coupon printed in the figure above is the fixed coupon for Alcoa, which is 100bps, or 1% of the notional amount. However, the dealers may not feel that this is the fair premium for protection. For instance, the hedge fund selling protection to the portfolio manager may feel that the fair premium should be 160bps, and not 100bps. This fair premium rate is known as **trade spread** (or **par spread**, or just **spread**), labeled as 160 in the figure above.

Naturally, if the hedge fund believes that the fair premium should be 160bps, it would like to be compensated for receiving just 100bps. As a result, the portfolio manager would have to make what is known as an **upfront payment** at the trade inception. Therefore, in the absence of a credit event, the cash flow between the two parties, over the life of the contract, would look like Figure 5.

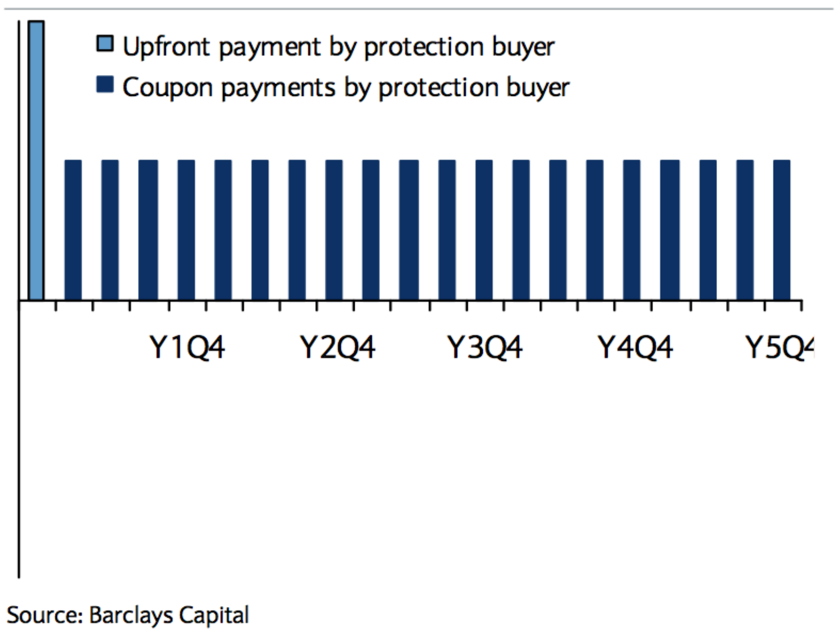


Figure 5: CDS cash flows when there is no credit event.

The ISDA protocol, since April 2009, specifies that all premium payments, by default, start on the roll date before the Trade Date. So if the Trade Date is June 24, 2014, the **Accrual Begin Date** is, by default, June 20, 2014. Now if the Accrual Begin Date is 4 days before the Trade Date, the portfolio manager would not want (and is not obligated) to pay interest for the 5 days she has not received protection for. The **accrued interest** can be calculated using the equation below:

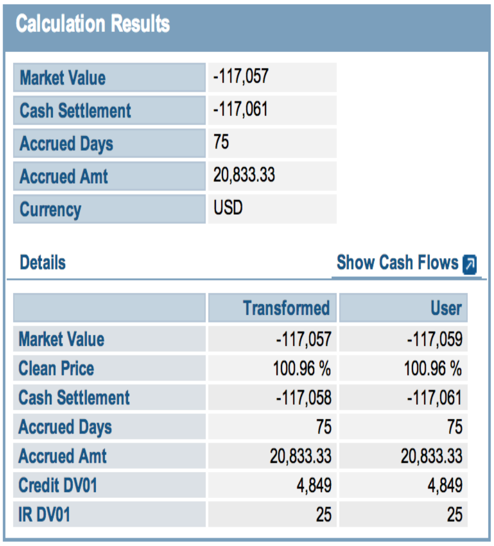
$$\text{Accrued} = \frac{5}{360} * \frac{1}{100} * \$10000000 = \$1,389 \tag{58}$$

We must note that we are dividing 5 by 360 instead of 365 in the above calculation. This has to do with the **day count convention** (**Day Cnt** in Figure 4) of the contract, which specifies that the accrual factor between two dates is ACT/360, or Actual/360.

Since the upfront payment is calculated as if the trade began on the roll date before the date of trade inception, the payment that the protection buyer *actually* makes—known as the **dirty upfront** or **cash settlement amount**—is the calculated upfront payment minus the accrued interest. As we can see in Figure 4, the dirty upfront value for this contract is $286,069. The **clean upfront** payment or **principal**, on the other hand, is simply another term for the initial, calculated upfront payment. In this case, it is $287,458. Moreover, we can also note a variable called **Pts Upf** in Figure 4. This variable, known as **points upfront**, or simply **points**, is the clean upfront expressed as a percentage of the notional amount.

As we can see, the upfront and points upfront values are positive, since the spread of the CDS is higher than the fixed coupon rate. If instead, the portfolio manager and hedge fund agree that the fair premium should be 60bps, the upfront value would be negative i.e. the hedge fund would pay the portfolio manager a compensation for receiving a coupon higher than the fair premium or spread.

The cash flows would then look like the values in Figure 6. The clean and dirty values are now both negative.



Calculation Results

Market Value	-117,057
Cash Settlement	-117,061
Accrued Days	75
Accrued Amt	20,833.33
Currency	USD

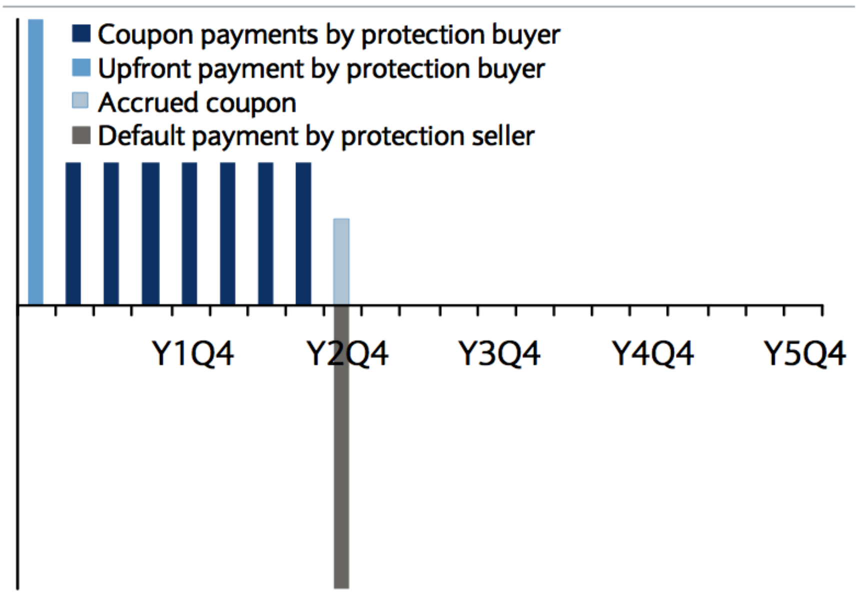
Details — Show Cash Flows

	Transformed	User
Market Value	-117,057	-117,059
Clean Price	100.96 %	100.96 %
Cash Settlement	-117,058	-117,061
Accrued Days	75	75
Accrued Amt	20,833.33	20,833.33
Credit DV01	4,849	4,849
IR DV01	25	25

Figure 6: CDS cash flows for Alcoa when spread is equal to 60bps and the coupon is equal to 100bps. These have been calculated using the calculator provided by Markit.

4.3 Protection Leg

So far we have only discussed the stream of payments for a contract in which a credit event does not occur. The stream of payments that would be have to be made in the case of a credit event is known as the **protection leg** or **contingent leg**. The resulting cash flow would like that in Figure 7.



Source: Barclays Capital

Figure 7: CDS cash flows when there is a credit event.

When a default does occur, the protection seller would owe the protection buyer the notional amount minus any money recovered from the company. It must be noted that the protection is effective for the credit events that have taken place since 60 days *before* the trade date; this date is known as the **backstop date**. Before the "Big Bang" protocol in April 2009, this date used to be one day *after* the trade date. If there is a delay in awareness about a credit event, the new system allows sufficient time for the two dealers to discover and process the information.

Types of Settlements in the Case of a Default

Should a bond (that is the reference obligation in a CDS agreement) default, the counterparties can compensate accordingly in two ways. The first is a **physical settlement** in which the buyer will actually deliver the defaulted bonds to seller, and the seller will then pay the face value of those bonds. The disadvantage to this particular transcation is that the buyer(s) of protection will have to find and deliver those bonds to the seller even if they don't own the bonds themselves. This may artificially drive up the price of the bonds. This is more likely to happen when there is a large number of outstanding CDS contracts.

The alternative to a physical settlement is a **cash settlement**, in which the seller simply pays the following to the buyer: (notional amount)*(1-recovery rate). Unfortunately, determining a recovery rate can often prove to be an issue. One approach the ISDA has been using lately is an **auction** style process in which major dealers submit their bids for the value they place on a company's debt. CDS contracts for corporate bonds generally assume a 40% recovery rate for valuation purposes.

What Constitutes a Default?

Another issue that we have to face when creating CDS is the definition of "default." A default does not always have to be outright bankruptcy. Often companies **restructure** their debt—or renegotiate the terms of their debt—instead of declaring for bankruptcy, and in some CDS contracts, restructuring does not constitute a default. In Figure 4, if we look between the notional amount and the currency, we will see a text box printed as "MM." This stands for "Modified Modified" restructuring and implies that debt restructuring does in fact constitute a default. So, if Alcoa decides to restructure this specific debt, the hedge will have to pay the protection leg to the portfolio manager.

Counterparty Risk

When a counterparty such as the hedge fund sells protection on a bond, there is always the possibility that, in case of a default, the counterparty may not be able to pay the protection payment to the protection buyer. Or, conversely, perhaps our portfolio manager is unable to make her periodic payments. The risk of either of these eventualities taking place is known as **counterparty risk**.

4.4 CDS Indices

Holding a disproportionate number of Alcoa bonds in a portfolio is naturally a risky idea. It is akin to putting all your eggs in one basket, as the saying goes. In the real world, most successful portfolio managers prefer to distribute their risk by holding a portfolio with hundreds, if not thousands, of different positions.

Instead of holding Alcoa bonds with a face value of $10 million, say our portfolio manager decides to take on less risk by holding bonds of 100 different IG companies. This is known as **diversification**. Although this strategy reduces her exposure to the credit risk of any particular company, she hasn't reduced her exposure to factors that might affect a wide variety of assets simulatenously, such as interest rate risk or political risk. Fortunately, in the modern financial world, she has the option of purchasing multi-name CDS, or CDS indices, which contain a basket of CDS. The two most common indices are CDX and iTraxx, which represent North American CDS and European CDS, respectively.

Moreover, if our portfolio manager has a high-risk appetite and wants to earn a higher return for her investors, she may invest a portion of her portfolio in bonds that have a lower credit rating but provide a higher yield. She may hedge this position by investing in a tranche of CDS indices such as the North American High Yield CDS index, which specifially offers protection on high yield bonds of 100 different companies. CDX and iTraxx are broken into smaller CDS based on sectors such as automobiles or consumer goods.

These index products trade in high volumes and are very liquid. In this vignette, we only discuss the pricing mechanisms of single-name CDS and do not delve into the more complex world of multi-name CDS. However, it is important to understand their relevance as most CDS contracts involve companies in these indices.

5 CDS Pricing

The ISDA has created the "Standard Model," which allows market participants to calculate cash settlement from conventional spread quotations, convert between conventional spread and upfront payments, and build the spread curve of a CDS. In this section we will lay out the assumptions made by the Standard Model, and explain the methods and formulas used to calculate certain information related to CDS.

There are several ways of calculating the price of a CDS such as hedge-based valuation or bond-yield-based pricing. We will apply the **discounted-cash flow pricing** or **risk-neutral valuation**, where the present value of the CDS is equal to the expected value of its discounted future cash flows. This model assumes that the probability of default is risk-neutral.

5.1 Discounted-Cash Flow Pricing or Risk-Neutral Valuation

Our portfolio manager wants to sell her CDS contract for whatever reason. Finding the market value of a CDS contract for the protection buyer is known as **Marking-to-market (MTM)** and the value is known as the **mark-to-market** value. Theoretically, the value of this CDS (of Alcoa) for the portfolio manager is:

[Money she expects to receive] — [Money she expects to pay]

Clearly, if she expects to receive more than she expects to pay, the value of the contract will be positive. So, if the fair spread for this CDS contract rises and Alcoa's credit risk increases, the market value of her CDS will increase and she will be in a **mark-to-market profit**. If, on the other hand, the credit risk and fair spread drop, she would have a **mark-to-market loss**.

The Standard Model assumes that the mark-to-market value is computed by discounting the expected protection leg and premium leg cashflows from the trade date until the maturity date. In Section 3.4 we explained the concept of **present value** and how a bond's price is the present value of its future cash flows. This concept applies to the pricing of CDS as well. If we are looking at the value of a CDS from the protection buyer's perspective, the premium leg has a negative value and the protection leg has a positive value. So it is equivalent to saying that,

$$PV(CDS) = PV(Protection) - PV(Premium)$$

Using the above equation, we can intuitively understand how the upfront value is calculated. We know that one of the central components in determining the price of a CDS is the spread. When the spread is different from the standard coupon of the contract, an upfront payment has to be made as a compensation. This upfront value is the expected present value of the extra cash flows that will be made hypothetically, on top of the standard coupon payments.

So if we have a five-year contract for the CDS of Alcoa, in which the standard coupon is 100bps and the fair spread is 160bps, the upfront value paid by the portfolio manager to the hedge fund is the *net expected* present value of the extra 60bps payments made quarterly (15 bps per quarter) over a period of five years. When we say *expected* present value, it implies that there is a chance that this hypothetical payment will never have to be made, since the company might default, and then the protection seller may have to make the contingent payment. We have to account for that probability in calculating the present value of those payments.

When an upfront payment is made, the transaction simply implies that the expected present value of the protection and premium legs are not equal. In the all-running contracts before the "Big Bang Protocol," where the coupon could be set at any level that the two parties agreed to, the premium leg would naturally equal the protection leg and the present value of the contract would be zero. But when the coupon is fixed and when the fair spread is not equal to the coupon, the expected present values of the two legs would not be the same and an upfront payment would have to be made as compensation. The upfront value essentially equates the premium leg and the protection leg.

Present Value of the Net Expected Cash Flows

As mentioned before, the premium leg is the present value of the stream of payments made by by the protection buyer to the protection seller. Here we have to account for the **survival probability**—the conditional probability that the reference entity survives until a given time period. We will use a term structure of survival probabilities to weight each of the contingent premium payments.

Moreover we also have to discount each of the contingent payments with a discount factor, which is the interest rate curve explained earlier. Therefore, the present value of the premium leg can be calculated using the formula:

The protection leg is the present value of the expected contingent protection payment of (1-recovery rate) from the protection seller to the protection buyer. We will weight this contingent protection payment in each time period with the **default probability**, which is essentially the difference in the survival probabities between two time periods. Therefore, the present value of the protection leg is:

Survival Probability Curve and Risk Neutral Probability

The survival probability is used to determine the expected cash flow of a CDS contract. The survival probability function can be equated with the **hazard rates** function or the distribution of the company's risk-neutral probability of default given below,

There are several assumptions made here:

- The risk-free rate and credit risk are not correlated
- Counterparty risk is not considered
-

This **Hazard Rate function** is piecewise constant. What this means is that the function varies across a scale but remains constant between two points.

To calculate the hazard rates, we need a **CDS curve** or the **term structure** of CDS rates. A CDS curve is similar to the interest rate curve in that it is a curve with the fair spreads of CDS contracts for different maturities. For reference names that are liquid, these rates are available quite easily; however, if the CDS is not liquid we can use the underlying bond's price to determine the hazard rates. Moreover, if there are only a couple of maturity dates available, we can assume a flat CDS curve. We will not go into the details of how these curves are derived but simply note that they are used in calculating the present value of the two legs and the upfront payment.

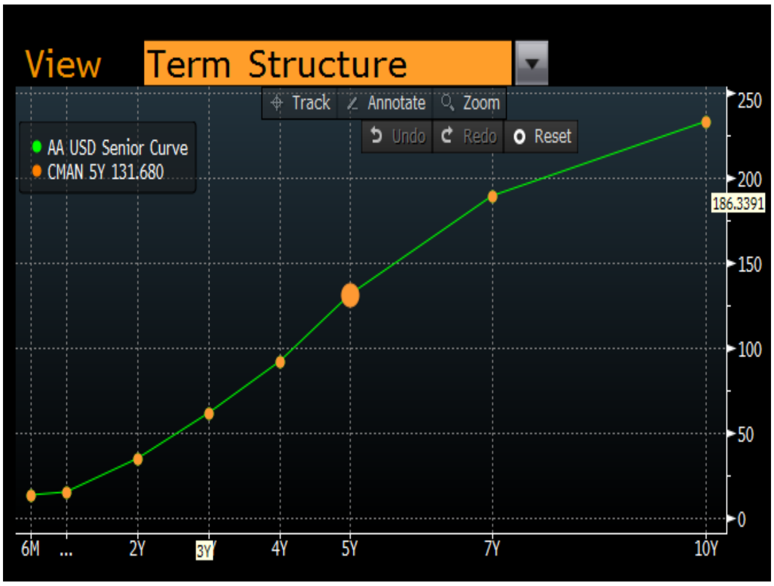


Figure 8: Term-structure of CDS rates for Alcoa on June 24, 2014

Discount factor: Using the Swap Curve or Risk-Free Curve

In the risk-neutral valuation framework, the risk free curve is the discount factor used in the calculation of the present value of CDS. In calculating the price of a CDS, we use the swap curve of the *previous day*, and use the rates for the corresponding currency. So for CDS denominated in USD, we will use the US treasury rates, and for those in GBP we will use UK Government bond yields, and so on. If we look at Figure 3, we can see the yields for different treasury bonds at 4pm EST on June 23, 2014, which will be used for pricing the CDS of Alcoa

The Standard Model allows market participants to convert between the par spread and the upfront payment, and compute the cash settlement amount for a standard contract.

Yield Curve

Date 23-Jun-2014

Snap 1600 **TimeZone:** New York

Recalculate

Period	Type	Rate
1M	Deposit	0.1520 %
2M	Deposit	0.1963 %
3M	Deposit	0.2326 %
6M	Deposit	0.3253 %
1Y	Deposit	0.5471 %
2Y	Swap	0.6190 %
3Y	Swap	1.0630 %
4Y	Swap	1.4665 %
5Y	Swap	1.7930 %
6Y	Swap	2.0570 %
7Y	Swap	2.2725 %
8Y	Swap	2.4450 %
9Y	Swap	2.5880 %
10Y	Swap	2.7110 %
12Y	Swap	2.9085 %
15Y	Swap	3.1110 %
20Y	Swap	3.2915 %
25Y	Swap	3.3710 %
30Y	Swap	3.4100 %

Interest Rate Conventions

Spot Date: 26-Jun-2014	**Swap DCC:** 30/360
MM DCC: A/360	**Swap Interval:** 6M
Floating DCC: A/360	**Holidays:** none
Floating Interval: 3M	**Bad Day Conv:** MF

Figure 9: Risk-free rates with their day count conventions on June 23, 2014 at 4pm EST. These are the rates that will be used to price the CDS of Alcoa on June 24, 2014. **MM DCC**, **Floating DCC** and **Swap DCC** refer to the Day Count Conventions being used to calculate the different kinds of rates. A **Swap Interval** of '6M' implies that interest is paid every six months or semi-annually. These conventions are different for each currency

The ISDA also standardizes the interest rates used by the Standard Model in valuing a CDS contract. There are two types of rates used in valuing a USD-denominated CDS contract: cash rates and swap rates. Cash rates are of the following maturities: one, two, three, and six month(s), and one year. These refer to the yields of zero-coupon bonds that have a maturity of less than one year; US treasuries with a maturity of one year or less are zero-coupon bonds. They are provided by the British Bankers' Association (BBA). Swap rates, or yields of coupon bond siwht a maturity of over one year, are of maturity 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 25, and 30 years, and are provided by ICAP (Markit, 2013). The Standard Model follows the conventions below for interpolation of the entire USD yield curve:

- The day count convention (DCC) for money market instruments and the floating legs of the swaps is **ACT/360**.
- DCC for floating legs of the swaps is **30/360**.

- Payment frequency for fixed legs of the swaps is 6 months.
- Payment frequency for floating legs of the swaps is 3 months.[4]
- A business day calendar of weekdays (Monday to Friday) is assumed. Saturdays and Sundays will be the only non-business days.
- If a date falls on a non-business day, the convention used for adjusting coupon payment dates is **M** (Modified Following).
- Recovery Rate is the estimated percentage of par value that bondholders will receive after a credit event. It is commonly reported in percentage of notional value. CDS contracts for corporate bonds assume a 40% recovery rate for valuation purposes.

Credit Triangle

The probability of default, recovery rate and spread form a credit triangle such that if we have two of them, we can use it to calulate the third:

Hazard Rate = Spread/(1 - Recovery Rate)

For a constant premium, higher recovery rates imply a high default probability.

Price

Price refers to the clean dollar price of the contract. The price of this Alcoa CDS is 97.13, less than 100. A CDS will have a price less than 100 if the points upfront are positive; that is, the CDS buyer needs to pay money to obtain protection because he promises to pay a coupon of, say, 100 even if the spread is 160. This is analogous to a bond investor paying less than the face value of a bond because current interest rates are higher than the coupon rate on the bond. It can be calculated by

$$\text{Price} = (1 - \text{Principal}/\text{Notional}) * 100$$
$$\& = 100 - \text{Points Upfront}.$$

Present Value 01 (PV01)

The PV01 is the present value of a stream of 1 basis points. It can also be used to calculate the cash flows and risk measures of a CDS. It is sometimes referred to as the **CDS duration** or **risky duration**. Analytically, PV01 can be calculated by

- i = coupon index,

[4]See `http://www.fincad.com/derivatives-resources/wiki/swap-pricing.aspx` for details on floating and fixed legs calculation.

- t_i = coupon date,
- $B(t_i)$ = day count fraction at t_i.
- $Df(f_i)$ = discount factor until t_i,
- $S(t_i)$ = survival probability until t_i,

As we can see in the equation above, we need the coupon index, coupon dates, the day count fraction, discount factor for each date, and survival probability until the date of the respective coupon payment.Before go into the details of how PV01 is used in determining the market price or the risk measures, it is essential to understand the different components used to calculate it. Coupon dates refer to the dates on which coupon payments are made. Day count fraction is the fraction of the day on which the coupon payment is made upon 360 or 365, depending on the day count convention of that CDS. While the first three components of the above equation seem straightforward, the other two - survivial probability and discount factor require more explanation.

We can also use the PV01 to calculate the premoum leg as premium leg is simply the present value of future coupon payments. It is simply the coupon payment mutliplied by the PV01

We can use the formula above for PV01 to calculate the principal amount (clean upfront payment) paid from the protection buyer to the seller using the following formula:

Principal Amount = PV(Protection) - PV(Premium) = (Par Spread - Coupon) $\times$ PV01

5.2 Risk Measures of a Standard CDS Contract

The PV01 can be used to compute certain risk measures related to interest rates, spread and the default probability.

Spread DV01

Using the concept of PV01, we show the calculation of the main risks (exposures) of a CDS position, **Spread DV01**. Spread DV01 reflects the risk duration of a CDS trade, also known as **Sprd DV01**, **Credit DV01**, **Spread Delta**, and just **DV01**.

It measures the sensitivity of a CDS contract mark-to-market to a parallel shift in the term structure of the par spread. DV01 should always be positive for a protection buyer since he or she is short credit, and a rising spread is a sign of credit deterioration. Starting with PV01 and taking the derivative with respect to the spread gives us:

$$\begin{aligned} PV &= (S - C) * PV01 \\ DV01 &= \frac{\partial PV}{\partial S} \\ &= PV01 + (S - C)\frac{\partial PV01}{\partial S}, \end{aligned}$$

where S is the spread of the contract and C is the coupon. Both DV01 and PV01 are measured in dollars and are equal if the spread equals the coupon.

IRDV01 or Interest Rate Dollar Value 01

The **IR DV01** is the change in value of a CDS contract for a 1 bp parallel increase in the interest rate curve. IR DV01 is, typically, a much smaller dollar value than Spread DV01 because moves in overall interest rates have a much smaller effect on the value of a CDS contract than does a move in the CDS spread itself.

Default Probability or Probability of Default (PD)

Default Probability refers to the default probability which is the estimated probability of default for each maturity by a given time. It can be approximated by

$$\text{Default Prob} \approx \left[1 - exp\left(\frac{rt}{1-R}\right)\right],$$

where r is the spread, t is the time to maturity, and R is the recovery rate.

Default Exposure

"Default Expo" refers to the expoure to the default of a CDS contract based on the formula below.

$$\text{Default Exposure} = (1 - \text{Recovery Rate}) * \text{Notional} - \text{Principal}.$$

5.3 Risk-Neutral Pricing

6 Pricing Sources

Bloomberg and Markit are the most widely used sources for data related to pricing of CDS. We compared the results from our package with results from Bloomberg and Markit. Interestingly, the results from Markit and Bloomberg were not always identical.

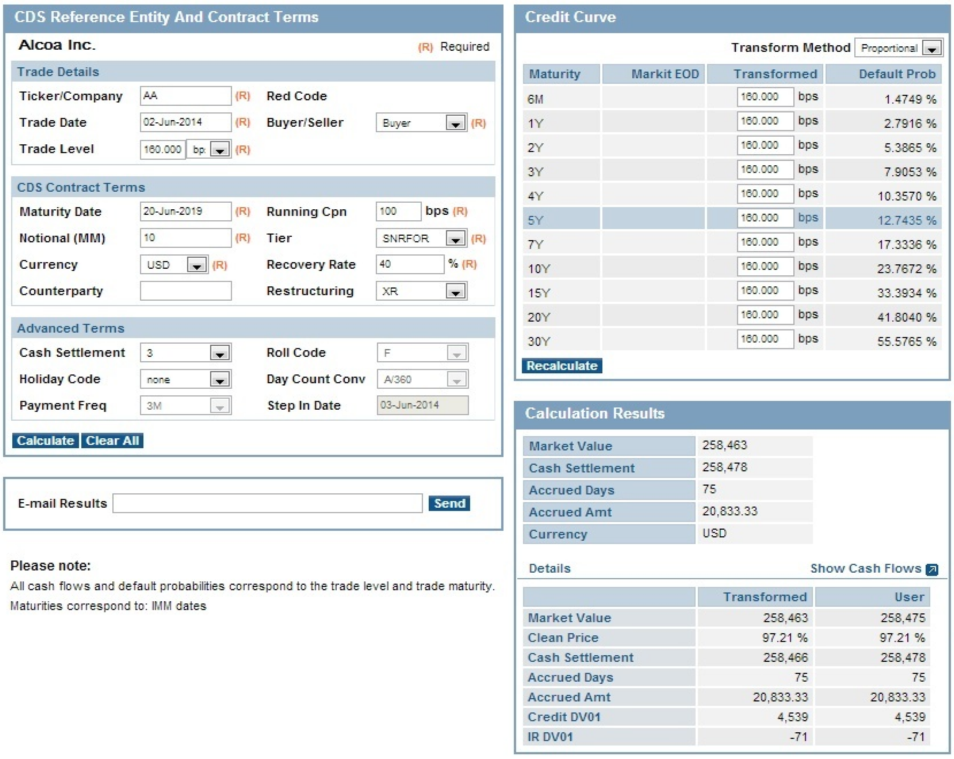


Figure 10: CDS figures from Markit.com

Figure 11: CDS figures from Bloomberg

The interest rate results were identical, at least until the fourth decimal place.

Yield Curve

Date 01-Jun-2014

Snap 1600 TimeZone: New York

Recalculate

Period	Type	Rate
1M	Deposit	0.1510 %
2M	Deposit	0.1908 %
3M	Deposit	0.2274 %
6M	Deposit	0.3219 %
1Y	Deposit	0.5344 %
2Y	Swap	0.5105 %
3Y	Swap	0.9095 %
4Y	Swap	1.2895 %
5Y	Swap	1.6175 %
6Y	Swap	1.8915 %
7Y	Swap	2.1165 %
8Y	Swap	2.2995 %
9Y	Swap	2.4525 %
10Y	Swap	2.5810 %
12Y	Swap	2.7885 %
15Y	Swap	2.9970 %
20Y	Swap	3.1825 %
25Y	Swap	3.2670 %
30Y	Swap	3.3055 %

Interest Rate Conventions

Spot Date: 04-Jun-2014	Swap DCC: 30/360
MM DCC: A/360	Swap Interval: 6M
Floating DCC: A/360	Holidays: none
Floating Interval: 3M	Bad Day Conv: MF

Figure 12: Interest Rate figures from Markit.com

Benchmark S260 (USD ISDA Standard Curve - Mid)

Date 06/02/2014

Term	Rate	Term	Rate
1 Month	0.1510	12 Year	2.7885
2 Month	0.1908	15 Year	2.9970
3 Month	0.2274	20 Year	3.1825
6 Month	0.3219	25 Year	3.2670
12 Month	0.5344	30 Year	3.3055
2 Year	0.5105		
3 Year	0.9095		
4 Year	1.2895		
5 Year	1.6175		
6 Year	1.8915		
7 Year	2.1165		
8 Year	2.2995		
9 Year	2.4525		
10 Year	2.5810		

Figure 13: Interest Rate figures from Bloomberg

7 Using the CDS package

In this section, we will demonstrate the use of the **CDS** package in detail and provide a series of examples. The **CDS** package implements the Standard Model, allowing users to value credit default swaps and calculate various risk measures associated with these instruments. Currently, a market participant can conduct CDS-related calculations by using the **CDSW Calculator** on a Bloomberg Terminal or the Markit CDS Calculator[5]. The **CDS** package provides tools for valuing a single-name CDS contract. The default setting allows a user to value a USD-denominated CDS contract following the Standard Model as mentioned before. She can also specify her own set of parameters to customize the calculation.

7.1 CDS Class

In the **CDS** package, we call the function `CDS` to construct an object of a class `CDS`. Below we show an example of how to construct a CDS contract in the package.

```
> library(CDS)
> cds1 <- CDS(entityName = "Alcoa",
+             RED = "49EB20",
+             TDate = "2014-06-24",
+             tenor = "5Y",
+             notional = 1e7,
+             coupon = 100,
+             parSpread = 160)
```

Here the user enters the CDS contract with "Alcoa" as the underlying entity and sets the spread at 160 bps and the coupon at 100 bps. However, the valuation of a CDS contract requires neither the Reference Entity or the RED Code. She does not have to know that information to use the **CDS** package. As shown below, as long as she inputs the same `Trade Date`, `parSpread`, and `maturity` information, the valuation of the contract will be the same.

Besides `parSpread`, a market paricipant can choose to specify either `ptsUpfront` or `upfront` to construct a `CDS` class object.[6] One of the three arguments has to be specified in order to construct the `CDS` class object.

Default Settings of the CDS Contract

The default settings of valuing a CDS contract in the **CDS** package follow the Standard North American Corporate (SNAC) CDS Contract specifications.[7] Below we list the ISDA specifications implemented in the **CDS** package. Additional default settings in the package which are not specified by the Standard Model, such as the default notional amount, are also listed.

- Currency: USD.

[5]The Markit CDS Calculator is available at `http://www.markit.com/markit.jsp?jsppage=pv.jsp`.

[6]See Section 4.2 for definitions on both terms.

[7]See `http://www.cdsmodel.com/assets/cds-model/docs/Standard%20CDS%20Contract%20Specification.pdf` for details.

- Trade Date (T): the current business day.
- CDS Date: Mar/Jun/Sep/Dec 20th of a year.
- Maturity: five years.
- Maturity Date (End Date): It falls on a CDS date without adjustment.
- Coupon Rate: 100 bps.
- Notional Amount (MM): 10MM.
- Recovery Rate (%): 40% for senior debts.
- Premium Leg:
  - Payment Frequency: quarterly
  - DCC: ACT/360
  - Pay Accrued On Default: It determines whether accrued interest is paid on a default. If a company defaults between payment dates, there is a certain amount of accrued payment that is owed to the protection seller. "True" means that this accrued will need to be paid by the protection buyer, "False" otherwise. The defalt is "True,"
  - Adjusted CDS Dates: "F." It means that it assumes the next available business day when a CDS date falls on a non-business day except the maturity date.
  - First Coupon Payment: It is the earliest Adjusted CDS Date after T + 1.
  - Accrual Begin Date (Start Date): It is the latest Adjusted CDS Date on or before T + 1.
  - Accrual Period: It is from previous accrual date (inclusive) to the next accrual date (exclusive), except for the last accrual period where the accrual end date (Maturity Date) is included.
- Protection Leg:
  - Protection Effective Date (Backdrop Date): T - 60 calendar days for credit events.
  - Protection Maturity Date: Maturity Date.
  - Protection Payoff: Par minus Recovery.

7.2 Generic Methods

summary Method

A user can call `summary` on a `cds1` to view essential information on the contract.

```
> summary(cds1)
```

Contract Type:	SNAC	TDate:	2014-06-24
Entity Name:	Alcoa	RED:	49EB20
Currency:	USD	End Date:	2019-09-20
Spread:	160	Coupon:	100
Upfront:	286,069	Spread DV01:	4,667
IR DV01:	-75.64	Rec Risk (1 pct):	-330.19

In the summary output, it shows that the type of the CDS contract is "SNAC". `Trade Date` refers to the trade date and is April 15, 2014. Reference Entity (called `entityName` in the package) refers to the entity name of the CDS contract and is "Alcoa". The `RED` code is "48EB20" as specified by the user. `spread` shows that the quoted spread for the contract is 50 bps and the coupon is 100 bps as shown in the `coupon` field. `upfront` indicates the dirty upfront payment in dollars or the cash settlement amount.

The remaining three items from the `summary` output are `Spread DV01`, `IR DV01`, and `Rec Risk (1 pct)`. In `cds1`,the IR DV01 is $66.14. **Recovery Risk 01** or `Rec Risk (1 pct)` as shown in the `summary` output, is the dollar value change in market value if the recovery rate used in the CDS valuation were increased by 1%. It is $90.03 in `cds1`.

Besides calling the `summary` method, one can type in the name of the `CDS` class object in the current **R** Session and obtain a full description of the CDS contract.

update method

A market participant can also update the `CDS` class objects she has constructed by calling the `update` method. It updates a `CDS` class object with a new spread and points upfront by specifiying the relevant input.

```
> cds3 <- update(cds1, spread = 55)
```

`cds3` is a new `CDS` class object with a spread of 55 bps; all other specifications of the contract are the same as those in `cds1` since it is updated from `cds1`. One can also specify `upfront` (in dollar amount) or `ptsUpfront` (in bps) in the `update` method.

There are three parts of the output. The first part "CDS Contract" provides basic information on the contract including "Contract Type", "Currency", "Reference Name" (called Entity Name in the package), "RED", "Trade Date", various dates related to the contract, and the day count conventions for `cds3`. The last part of the output reports the interest rates used in the calculation. The second part of the output contains relevant risks measures of `cds3`. Calling the function `getRates` also produces the rates used in building a curve for CDS valuation.

show method

```
> cds1
```

```
CDS Contract
Contract Type:                      SNAC   Currency:                           USD
```

```
Entity Name:                  Alcoa   RED:                    49EB20
TDate:                   2014-06-24   End Date:           2019-09-20
Start Date:              2014-06-20   Backstop Date:      2014-04-25
1st Coupon:              2014-09-20   Pen Coupon:         2019-06-20
Day Cnt:                    ACT/360   Freq:                        Q

Calculation
Value Date:              2014-06-27   Price:                   97.13
Spread:                         160   Pts Upfront:            0.0287
Principal:                  287,458   Spread DV01:             4,667
Accrual:                     -1,389   IR DV01:                -75.64
Upfront:                    286,069   Rec Risk (1 pct):      -330.19
Default Prob:                0.1322   Default Expo:        5,712,542

Credit curve effective of 2014-06-24
 Term     Rate Term     Rate
   1M 0.001520   7Y 0.022725
   2M 0.001963   8Y 0.024450
   3M 0.002326   9Y 0.025880
   6M 0.003253  10Y 0.027110
   1Y 0.005471  12Y 0.029085
   2Y 0.006190  15Y 0.031110
   3Y 0.010630  20Y 0.032915
   4Y 0.014665  25Y 0.033710
   5Y 0.017930  30Y 0.034100
   6Y 0.020570
```

7.3 CDS Pricing Related Functions

CS 10

`CS10` is a method which calculates the change in value of the CDS contract when the spread of the contract increases by 10%. `CS10` takes in a `CDS` class object formed by calling the `CDS` function. The CS10 of `cds1` is $25385.2.

```
> cds1.CS10 <- CS10(cds1)
> cds1.CS10
[1] 74197.34
```

getRates function

This is a function used to obtain the interest rate curve for a given currency on a given date, along with the day count convention for that currency.

```
> cds3Rates <- getRates(date = "2014-06-24")
```

The output from the getRates function below consists of two list objects. The first list contains rates of various maturities. They are directly obtained from the Markit website based on the specifications (Markit, 2013).

```
   expiry matureDate     rate type
1      1M 2014-07-28  0.00152    M
2      2M 2014-08-26 0.001963    M
3      3M 2014-09-26 0.002326    M
4      6M 2014-12-26 0.003253    M
5      1Y 2015-06-26 0.005471    M
6      2Y 2016-06-26  0.00619    S
7      3Y 2017-06-26  0.01063    S
8      4Y 2018-06-26 0.014665    S
9      5Y 2019-06-26  0.01793    S
10     6Y 2020-06-26  0.02057    S
11     7Y 2021-06-26 0.022725    S
12     8Y 2022-06-26  0.02445    S
13     9Y 2023-06-26  0.02588    S
14    10Y 2024-06-26  0.02711    S
15    12Y 2026-06-26 0.029085    S
16    15Y 2029-06-26  0.03111    S
17    20Y 2034-06-26 0.032915    S
18    25Y 2039-06-26  0.03371    S
19    30Y 2044-06-26   0.0341    S
```

The second list reports the specific day count conventions and payment frequencies regarding the interest rate curve used.

```
                 text
effectiveDate    "2014-06-24"
badDayConvention "M"
mmDCC            "ACT/360"
mmCalendars      "none"
fixedDCC         "30/360"
floatDCC         "ACT/360"
fixedFreq        "6M"
floatFreq        "3M"
swapCalendars    "none"
```

7.4 Risk-related Functions

recRisk01

Function to calculate the increase in the upfront value when the recovery rate increase by one percentage point.

```
[1] -330.1858
```

IRDV01

Function to calculate the increase in the upfront value when the interest rate on all maturities of the interest curve increase by a single basis point.

```
[1] -73.72284
```

spreadDV01

Function to calculate the increase in the upfront value when the spread increase by one basis point.

```
[1] 4609.709
```

defaultExpo

Function to calculate the default exposure.

```
[1] 5720689
```

defaultProb

Probability of default calculated using the function provided in the Bloomberg manual.

```
[1] 0.1248267
```

8 Conclusion

In this paper, we describe the basics of a CDS contract and the ISDA Standard Model. We also provide a simple collection of tools to implement the Standard Model in **R** with the `CDS` package. Moreover, the flexibility of **R** itself allows users to extend and modify this package to suit their own needs and/or create their preferred models for valuing CDS contracts. An **R** package, `backtest` Campbell et al. (2007), provides facilities to explore portfolio-based conjectures about credit default swaps. It is possible to use the `backtest` package based on the output from the `CDS` package. Before reaching that level of complexity, however, `CDS` provides a good starting point for valuing credit default swaps.

Bibliography

K. Campbell, J. Enos, D. Gerlanc, and D. Kane. Backtests. *R News*, 7(1):36–41, April 2007. URL `http://cran.r-project.org/doc/Rnews/Rnews_2007-1.pdf`.

S. Green and E. Witschen. Credit Derivatives. *Bloomberg*, January 2012.

M. Leeming, S. Willemann, A. Ghosh, and R. Hagemans. Standard Corporate CDS Handbook, Ongoing Evolution of the CDS Market. *Barclays Capital*, February 2010.

Markit. Credit Derivatives Glossary. *Markit*, March 2009.

Markit. Markit Interest Rate Curve XML Specifications Version 1.15. *Markit*, May 2013.

R. White. The Pricing and Risk Management of Credit Default Swaps, with a Focus on the ISDA Model. *OpenGamma*, October 2013.