High-Frequency Market Making in a Limit Order Book: The effect of inventory and adverse selection on order book dynamics

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June 10, 2012

Abstract

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1   Introduction

The *Limit Order Book* (LOB) lies at the heart of modern market microstructure. This exchange mechanism provides the means for any market participant to act as either a liquidity *provider* or *taker*.

It is conventional in market making models to consider agents who can post limit orders or market orders, where a market order is simply one which crosses the bid/ask spread in order to take liquidity. We will stick with this convention, while noting that actual market orders are simply an order type with a specific set of rules: to fill an order regardless of the resulting price. While a limit order specifies a price.

The bid/ask spread is primarily modelled as a function of three terms: *order processing costs*, *inventory risk*, and *adverse selection*.

2   Background

Starting with Garman (1976), there is a long history of market microstructure theory.

Garman innovates on the standard supply/demand function by having the supply/demand price relationship be a function of the arrival rate of orders.

2.1   Stoll (1978)

Stoll (1978) "The Supply of Dealer Services in Securities Markets" introduced an important set of innovations over the Garman (1976) model for dealer prices.

2.2   Ho and Stoll (1981)

Ho and Stoll (1981 [**?**]) "Optimal dealer pricing under transactions and return uncertainty" expands on the model from Ho (1978) by considering a multistage problem. They seek to define the behavior of a single dealer in a security who seeks to maximize his expected utility (terminal wealth) while considering his inventory risk.

2.3   Avellaneda and Stoikov (2008)

Avellaneda and Stoikov use an arithmetic brownian motion mid-point price process S_t
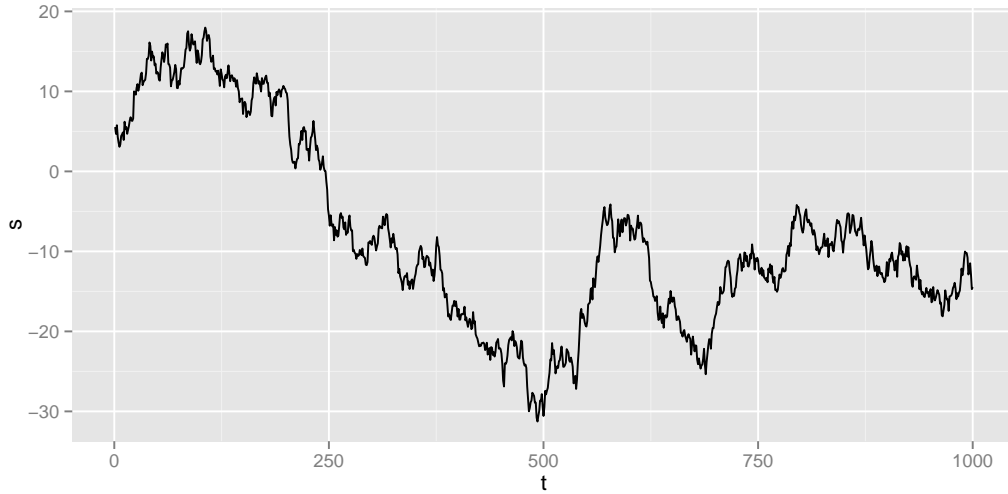
$$S_t = s + \sigma W_t \tag{1}$$

Figure 1: Brownian motion S_t mid-point process.

The market maker seeks to maximize the expected utility through the value function:

$$v(x, s, q, t) = E_t[-e^{(-\gamma(x+qS_T))}] \tag{2}$$

2.4 Gueant, Lehalle, and Fernandez-Tapia (2012)

The position at time t is represented by q_t is a function of the number of fills at the bid N_t^b and ask N_t^a respectively:

$$q_t = N_t^b - N_t^a \tag{3}$$

2.5 Guilbaud and Pham (2013a)

2.6 Guilbaud and Pham (2013b)

2.7 Overview

We follow in the tradition of Avellaneda/Stoikov (2006), but extend the model in some fundamental ways.

- We add a term for *adverse selection*.

3 The Model

3.1 The mid-price of the stock

$$S_t = s + \sigma W_t \qquad (4)$$

4 Further Reading

I provide references throughout the text to the canonical papers and books relevant to each topic. As a general topic, I recommend the following as more detailed texts on the subject:

- Joel Hasbrouck "Empirical Market Microstructure". An early draft of the book can be found online at ...

References

author = Ho, Thomas and Stoll, Hans R., doi = 10.1016/0304-405X(81)90020-9, file = :Users/smc77/papers/1-s2.0-0304405X81900209-main.pdf:pdf, issn = 0304405X, journal = Journal of Financial Economics, mendeley-groups = Market Microstructure,Market Making, month = mar, number = 1, pages = 47–73, title = Optimal dealer pricing under transactions and return uncertainty, url = http://linkinghub.elsevier.com/retrieve/pii/0304405X81900209, volume = 9, year = 1981Kyle, A.S. (1985). Continuous auctions and insider trading. *Econometrica*, 53: 1315-1336 . Roll, R., 1984. A simple implicit measure of the e!ective bid-ask spread. Journal of Finance 39, 1127-1139. O'Hara, M. (1995). Market Microstructure Theory. Cambridge, MA: Blackwell 1995 [1] B. Biais, L. Glosten and C. Spatt, TheMicrostructure of StockMarkets,Working paper (2004). [2] J.-P. Bouchaud, M.Mezard andM. Potters, Statistical Properties of Stock Order Books: Empirical Results and Models, Quantitative Finance, 2 (2002) 251-256. [3] X. Gabaix, P. Gopikrishnan, V. Plerou, H.E. Stanley, A Theory of Large Fluctu- ations in Stock Market Activity, MIT Department of Economics Working Paper No. 03-30. [4] P. Gopikrishnan, V. Plerou, X. Gabaix and H.E. Stanley, Statistical Properties of Share Volume Traded in Financial Markets, Physical Review E, 62 (2000) R4493-R4496. [5] T. Ho and R. Macris, Dealer Bid-Ask Quotes and Transaction Prices: An Em- pirical Study of Some AMEX Options, Journal of Finance, 39, (1984) 23-45. [6] T. Ho and H. Stoll, Optimal Dealer Pricing

under Transactions and Return Uncertainty, Journal of Financial Economics, 9 (1981) 47-73.