The Price of Immediacy

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Abstract

I provide a general overview of Market Microstructure Theory. I start with a basic review of market structure and rules. I then introduce the standard microstructure theories of price discovery: inventory models, sequential trade models, and strategic trade models. I discuss the transaction cost literature, including trade cost optimization models. Finally, I briefly discuss some of the issues raised by recent topics on high-frequency trading and quoting.

Contents

1 Introduction

The *Law of One Price* is a central feature of many theories in modern finance and economics. It states that ...

This concept ignores some common sense observations. It seems obvious that market structure matters. It is readily available that we can go from one grocery store to another and buy the exact same products for different prices. Why does this happen? The simple answer is market inefficiency. It is not readily known to everyone what prices are in both markets in advance. And there are other factors that might contribute to one's decision.

This also ignores one of the major philosophical revolutions in the twentieth century: pluralism. The law of one price harkens back to classical platonism, and inhabits an abstract world of ideals which is only a faint connection to the real world. In modern financial markets there are always *many prices.*

Markets are human institutions. Even when governed by computer algorithms, these are still following rules designed by human rules and intuitions. There is no true abstract mathematical reality in finance. We have known this for a long time. Most especially established by Benoit Mandlebrot in ...

Tail events are the signature of humanity.

The functioning of individual asset exchange is one of the most fundamental aspects of microeconomics and finance. Marketplace exchange has been transformed over the last two decades due to the introduction of electronic exchanges along with the related regulations (Order Handling Rules, Reg ATS, and Reg NMS). I aim to provide a concise literature review related to the different aspects of this field. The literature might be loosely divided into Market Microstruture Theory (modeling the behavior of market makers) and Optimal Execution (transaction cost analysis). There is also a burgeoning literature on high-frequency trading, which is the primary source of liquidity in modern electronic markets.

"As good a starting point as any is the coinage of the term *market microstructure* in the paper of the same title by Garman (1976):

> We depart from the usual approaches of the theory of exchange by (1) making the assumption of asynchonous, temporally discreate market activities on the part of the market agents and (2) adopting a viewpoint which treats the temporal microstructure, i.e. moment-to-moment aggregate exchagne behavior, as an important descriptive aspect of such markets. (p.257)

1.1 Overview

"Confessions of a Stock Operator" is one of the most widely read texts in financial literature.

Markets are how we are able to get what we want, when we want it. In order to achieve this level of immediacy, a customer needs to be willing to pay a premium over the cost of production.

There are several key themes throughout:

- *Equilibrium* is a state when all market participants are satisfied with the current exchange.
- *Liquidity* denotes the idea of being about to buy or sell quickly and at a given price.
- *Information* covers any knowledge that can impact the buy and sell decisions of market participants. Prices are ultimately decided by the exchange between individuals and firms; there is no abstract "true" price. Some traders are "informed" and others are "uninformed".
- *Risk* is the uncertainty associated with a transaction.
- *Middlemen* enable the exchange between disparate places and times.
- *Transaction costs* cover the difference between the desired price and the realized price.
- *Algorithms* are specific descriptions for how a trade should take place.
- *Market Clearing* denotes the arrival at an equilibrium price.

Similarly, I also use some important structural divisions to help clarify ideas:

- *Theoretical vs. empirical* analyses: theoretical models aim to provide an
- *Ex-ante vs. ex-post*: ex-ante includes any analysis that is based on information available "at the time", while ex-post can use future information "after the fact".

I aim to provide a general introduction to this field so that it can be understood by the general reader. I assume no prior knowledge of the subject, although exposure to statistics, time series analysis, and optimization will be helpful.

I will be using R to explore model dynamics. However, I assume throughout that the reader is familiar with R and with basic time series analysis and econometrics. For a more thorough introduction to R, I recommend reading ... X provides a nice introduction to econometrics with R. I also like "Mostly Harmless Econometrics" by ...

For simplicity, the code is implemented in the *microstructure* package on CRAN. This can be installed in R by running:

```
> install.package("microstructure")
```

This imports a few packages that are generally useful for this kind of analysis.
http://journal.r-project.org/archive/2011-1/RJournal$_2$011 $- 1_K$*ane et al.pdf*

1.2 General Equilibrium, Rational Expectations, and the Law of One Price

Price formation is generally viewed as the equilibrium between supply and demand. "The equilibrium price is the price at which quantity demanded equals quantity supplied." (O'Hara 14)

General Equilibrium theory dates to the 1870s, particularly NA

Traditional economics is focused on assumptions of *equilibrium*. Under these assumptions, the trading mechanism is largely irrelevant. As, for instance, in Hicks (1939) and Radner (1982).

1.3 Market Efficiency

Market efficiency describes the predictability of asset prices based on information.

1.4 Digression on Academics, Practicioners, and Performativity

Sociologists have taken an increased interest in the interaction between financial theory and real world markets over the last few decades in the field of "Social Studies of Finance". David MacCauley developed the idea of *performativity* in his excellent study "An Engine Not a Camera" (2003).

An alternative social science viewpoint is being provided from Anthropologists, as for instance, by ... His recent book "Debt" ...

The key scholars in this subject are often directly involved in the markets, and hence have incentives ...

- *Andrew Lo* is a professor at MIT and a pivotal figure in the foundations of modern finance.

- *Neil Chriss* was previously the head of quantitative strategies at S.A.C. Capital Partners, and now runs his own firm called Hutchin Hill.

- *Robert Almgren* is a professor at NYU and founded Quantitative Brokers in 2009.

1.5 Outline

In essence, this paper is

We start by defining the rules.

Then we consider market makers and liquidity provision, the *sell side*.

The we look at algorithmic trading, market impact, and optimal executions, the *buy side*.

Lastly, we consider high frequency trading, which sits somewhere in between these two parties.

2 Data Analysis

In this section, I aim to provide a short summary of all the basic tools required for analysis of financial market data, especially given some of the complexities inherent in high-frequency market data.

2.1 Value, Price, Quantity, Uncertainty, and Time

The time value of money is the central idea of finance. This might simply be thought of in terms of interest rates. In this sense, the bid-ask spread is the instantaneous interest rate that the market maker charges for temporal risk.

In an abstract sense we are most concerned with the *value* of an asset, although the simplest proxy for value is *price* and I will consider concepts to be interchangable. Price is generally easier to process.

The interaction between the price and time lies at the heart of market microstructure and transaction costs. In one of the foundational works, Demsetz (1968)... "Demsetz argued that this lack of equilibrium could be overcome by paying a price for immediacy." (O'Hara 5)

Another important aspect of exchange is quantity: namely, at different prices, individuals would be will to exchange varying amounts of things.

2.1.1 Stochastic Processes

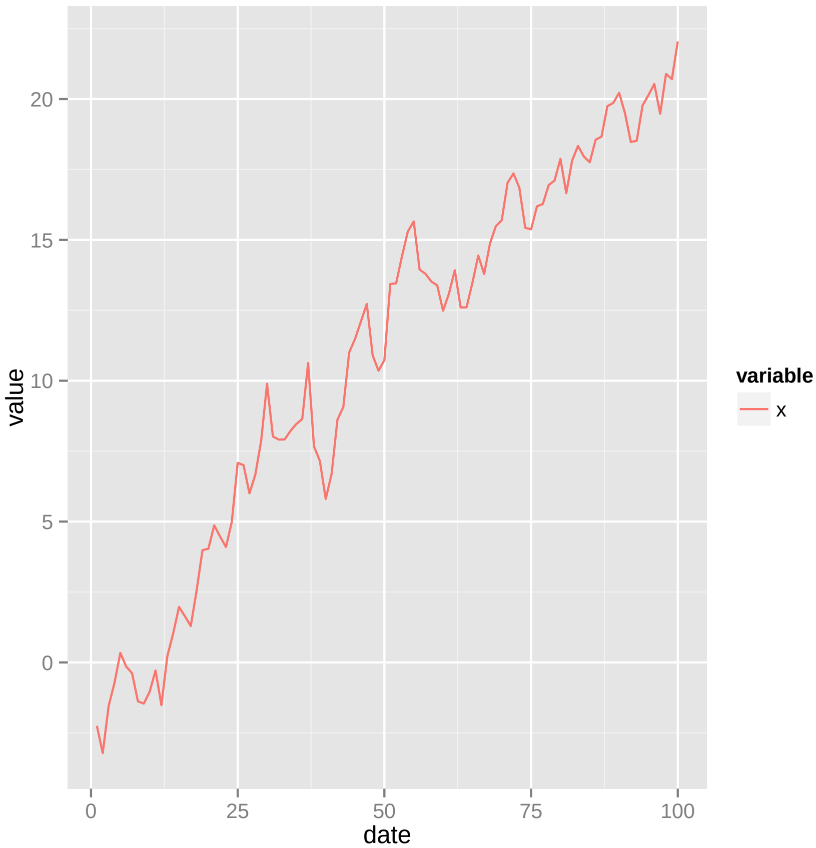
The random walk (or brownian motion) is the most basic model for price dynamics, first proposed by Bachelier (1895). This is generally one of the first things being considered in *stochastic processes*.

The basic basic model can be defined as a sequential addition of the prior value plus a random variable ϵ (typically gaussian).

$$p_t = p_{t-1} + \epsilon \tag{1}$$

It is simple to test these models in R with the following command:

```
> time = 1:100
> epsilon = rnorm(time)
> prices = cumsum(epsilon)
> prices_ts = zoo(prices)
```



When dealing with arrival times, it is common to use a *Poisson process*, which models time as ...

2.2 Convex Optimization

Another important feature of many market microstructure models is their dynamics over time. In particular, not only are many of these models stochastic (changing randomly over time), but they often have an element of dependence.

In general, when trying to solve a mathematical problem in finance, we can either find explicit solutions or optimal solutions. Explicit solutions require us to solve the given problem directly, which is not possible under certain common conditions.

In optimization problems, we aim to *learn the underlying relationship by fitting a model to the data*.

I like to think about optimization problems in terms of their domain space. Consider the following simple model:

$$Score_{SAT} = \beta_0 + \beta_1 Income + \epsilon \tag{2}$$

This states that there is a relationship between SAT scores and income.

2.3 Dynamic Programming

Finance is a social science, and thus bares more similarity to fields such as economics, political science, sociology, and psychology than to physical sciences like Physics and Biology. One way to think about this is that the data that we study, primarily prices, are *social* variables: people not only set prices based on a large, complex set of reasons, but they also are inherently *dependent*, that is to say that futures prices always depend on prior prices. We look at what happens and consciously react.

Having dependent time series variables poses a serious challenge to many basic statistical tools.

2.3.1 Discrete Time, Discrete State Dynamic Programming

3 Trading and Exchanges

The exchange mechanism plays an important role in the establishment of prices.

Harris (2003) is the . DMA (2010) provides a more recent treatment.

On some level, these previous discussions are outdated as exchanges have been dramatically transformed over the last decade by electronic trading.

3.1 Market Participants

The literature typically refers to four different types of market participants: ...

Brokers Dealers Investors

3.2 A Brief History

The story of modern exchanges is primarily an American story. The innovation and regulations that took place in the US in the last 20 years have defined the global standard.

Buttonwood Tree Agreement (Harris, 1598)

Futures exchanges (Bloomberg book)

Auction markets ECN's

"Geek's on Wall Street" provides a very entertaining look at market evolution.

3.3 Auctions and Order Books

There are two primary structures of modern market exchange: auctions and order books. Of these, the order book model provides the most popular model for continuous trading in liquid assets, and hence is the primary focus for market microstructure analysis.

4 Microstructure Theory

http://www.acsu.buffalo.edu/ keechung/MGF743/Readings/Thehttp://www.acsu.buffalo.edu/ keec

http://www.cambridge.org/resources/0521687276/8763$_d$*ejongrindislidesChapter*2345*.pdf Slide*
http : //www.ems.bbk.ac.uk/faculty/sciubba/microstructure$_s$*lides*1*.pdf http : //www.ems.bbk.ac.u*

Surveys:

http://web.cenet.org.cn/upfile/5056.pdf

Hans R. Stoll: "Market Microstructure", Handbook of The Economics of Finance, Volume 1A: Corporate Finance, Chapter 9, pp. 555-600. http://www.eco.sdu.edu.cn/jrtzx../u

David Easley and Maureen O'Hara: "Microstructure and Asset Pricing", Handbook of The Economics of Finance, Volume 1B: Financial Markets and Asset Pricing, Chapter 17, pp. 1021-1047.

NA http://studenttheses.cbs.dk/bitstream/handle/10417/746/jens$_v$*alloe*$_c$*hristiansen.pdf*

Mauricio Labadie and Charles-Albert Lehalle "Optimal algorithmic trading and market microstructure" October, 2010 http://hal.archives-ouvertes.fr/docs/00/59/84/28/PDF/mem

Demsetz http://people.stern.nyu.edu/lpederse/courses/LAP/papers/Overview/Demsetz68.pdf

"Market microstructure refers to the study of the process and outcomes of exchanging assets under a specific set of rules." (O'Hara http://www.bis.org/publ/bppdf/bispap02a.pdf

Working (1953) and Houthakker (1957)

The first real focus on the interaction between trading mechanisms and price discovery was in Demsetz (1968).

There are two standard reference frameworks in the literature. The first is called the ”continuous auction framework” first developed by Kyle (1985). The second is the ”sequential trade framework” proposed by Glosten and Milgrom (1985). A large amount of research has been done involving the application of these two frameworks. Both frameworks are suffciently simple and well behaved that they easily lend themselves to analysis of policy issues and empirical testing (see Madhavan (2000) and Biais et al. (2005) for extensive surveys of the literature).

This follows along with the analysis set forth first by O'Hara (1995). Hasbrouck (2007) took a different approach, focusing on a simple econometric model (the Roll model), then the assymmetric information models, and then the inventory models. Hasbrouck places far more emphasis on econometrics than O'Hara, and as such could be used by an advanced undergraduate more than O'Hara (which is mostly a graduate text). O'Hara includes very little discussion of one of the most prevalent market structures now: the limit order book (LOB).

My treatment is shorter, with less emphasis on econometrics.

The key models are:

- *Inventory models* focus on the market maker's risk aversion to having inventory risk. The simplest model considers the dealer's ruin if he runs out of inventory or cash.
- *Sequantial trade models* consider the risk that the counterparty possesses valuable information.
- *Strategic trade models* allows for multiple trades by one informed trader, in order to limit market impact by splitting up orders and hiding his orders.

These latter two models are also considered in light of *adverse selection*, which occurs whenever a passive trader (limit order) is hit by an informed trader (market order).

I am here following what I would consider to be a logical order, following that set out by O'Hara (1985). Hasbrouck moved Inventory models to the end, which makes more sense given his econometric focus.

We will consider each of these in turn, focusing the foundational ideas and then giving a brief review of the current perspectives. In each case, there are various simplifying assumptions which make each model unrealistic of actual market exchange. But these also lead to interesting results that can be applied to real-world problems. There is also an entire body of literature that considers the empiric

4.1 Walrasian Auctioneer

One of the most basic and long-established models for price formation was the *Walrasian auctioneer*, following on ...

"As generations of economists learned, the price formation process ould be captured by the general represetation of a Walrasian auctioneers, who aggregates traders' demands and supplies to fina d market-clearing price." (O'Hara, p.4)

http://beyondmicrofoundations.blogspot.com/2011/01/calculating-walrasian-auctioneer.html (with python code)

NA The tatonnement process is a model for investigating stability of equilibria. Prices are announced (perhaps by an "auctioneer"), and agents state how much of each good they would like to offer (supply) or purchase (demand). No transactions and no production take place at disequilibrium prices. Instead, prices are lowered for goods with positive prices and excess supply. Prices are raised for goods with excess demand. The question for the mathematician is under what conditions such a process will terminate in equilibrium where demand equates to supply for goods with positive prices and demand does not exceed supply for goods with a price of zero. Walras was not able to provide a definitive answer to this question (see Unresolved Problems in General Equilibrium below)." http://en.wikipedia.org/wiki/General$_equilibrium_theory$

4.2 Order Processing Costs

The first basic driver of the bid/ask spread is order processing costs, although this has become increasingly unimportant.

4.2.1 Roll model (1984)

> This paper presents a method for inferring the effective bid-ask spread directly from a time series of market prices. The method requires no data other than the prices themselves...

Richard Roll (faculty page) developed the Roll model in "A simple implicit measure of the e!ective bid-ask spread" (1984).

Hasbrouck (2007) starts his microstructure overview with a simple econometric model – the Roll model – as introduced in Roll (1984). The Roll model is based on standard time series modelling, and Hasbrouck subsequently extends this to include multivariate techniques in what he calls the "generalized Roll model".

We take these into account by adjusting the random walk in the following way:

$$m_t = m_{t-1} + u_t \tag{3}$$

We make the following assumptions:

- All trading in the market is conducted through dealers. These dealers will post bid/ask prices at each point in time as a_t and b_t.

- Dealers are competing with each other so that the spread between the bid/ask prices is equivalent to the dealers' cost c per trade.

Then the bid and ask are given by:

$$a_t = m_t + c \tag{4}$$
$$b_t = m_t - c \tag{5}$$

We can infer the trade price p_t as a function of the the mid-quote price m_t, half of the bid/ask cost c, and the sign of the trade q_t (+1 for buy and -1 for sell):

$$p_t = m_t + cq_t \tag{6}$$

We can model this simply in R. First let's look at bid/ask prices assuming a constant cost c:

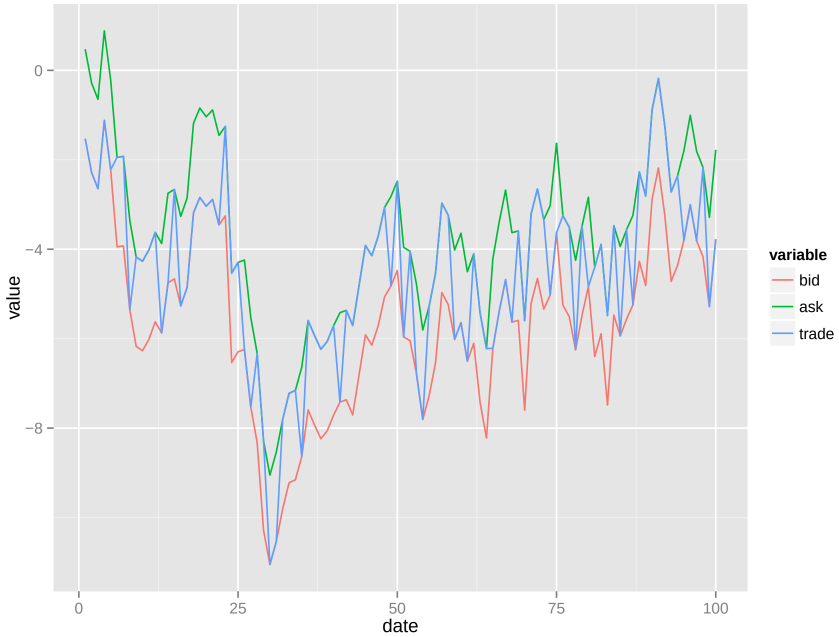
```
> c = 1
> time = 1:100
> epsilon = rnorm(time)
> prices = cumsum(epsilon)
> m_t = zoo(prices)
> a_t = m_t + c
> b_t = m_t - c
```

We can simulated trade prices by adding a normally distributed trade sign variable:

```
> q_t = sign(rnorm(time))
> p_t = m_t + (c * q_t)
```

And plot this, so we can see the trades bouncing back and forth between the bid and ask:

```
> x = merge(b_t, a_t, p_t)
> colnames(x) <- c("bid", "ask", "trade")
> print(qplot.zoo(x))
```



We can also use this model in the opposite direction, to infer transaction costs from actual prices. The roll model has two parameters that can be estimated, c and σ^2. This is very useful as it implies that we can estimate transaction costs by only using transaction data (i.e. without knowing quoted spreads). Here, transaction costs are inferred from serial covariance of daily returns.

$$\Delta p_t = -cq_{t-1} + cq_t + u_t \tag{7}$$

We define the variance and covariance as $\gamma_0 = Var(\Delta p_t)$ and $\gamma_1 = Cov(\Delta p_t, \Delta p_{t-1})$. Following on Hasbrouck (2007), we have that:

$$\gamma_0 = 2c^2 + \sigma_u^2 \tag{8}$$
$$\gamma_1 = -c^2 \tag{9}$$

So we can now solve for c and σ^2 as:

$$c = \sqrt{-\gamma_1} \tag{10}$$
$$\sigma_u^2 = \gamma_0 + 2\gamma_1 \tag{11}$$

(derivation: http://waxworksmath.com/Authors/G$_M/Hasbrouck/WriteUp/weatherwax_hasbro$

So we just need to compute the variance and covariance of returns in order to estimate the expected cost. An example of estimating these values in R using daily data.

```
> returns <- function(x) (x/lag(x))-1
> rets <- na.omit(returns(m_t))
> gamma_0 <- var(rets)
> gamma_1 <- cov(rets[1:length(rets)-1], lag(rets, na.pad=FALSE))
> sigma_2 <- gamma_0 + 2 * gamma_1
> c_est <- sqrt(2 * gamma_1)
```

In our simple example, the actual cost was 1 while the estimated cost is NaN.

There are some existing *empirical* studies that attempt to test the Roll model on real data and verify the validity of its estimates.

As an example, see Hasbrouck (2009) Trading Costs and Returns for U.S. Equities: Estimating Effective Costs from Daily Data which uses a generalized version of the Roll model to estimate transaction costs. Another interesting application is Hasbrouck (2004) "Liquidity in the Futures Pit: Inferring Market Dynamics from Incomplete Data".

Interesting: Thomas J. George, Gautam Kaul, M. Nimalendran "Estimation of the Bid-Ask Spread and its Components: A New Approach" The Review of Financial Studies, Vol. 4, No. 4 (1991), pp. 623-656

There have been many empirical tests of the Roll model, and many subsequent extensions. I want to highlight Harris (1990) "Statistical Properties of the Roll Serial Covariance Bid/Ask Spread Estimator", which provides a nice summary of the basic flaws.

> Unfortunately, the serial covariance estimator yields poor empirical results when used to estimate individual security spreads from a year of daily or weekly data. Estimated first-order serial covariances are positive for about half of all securities so that the square root in the estimator is not properly defined. Further, estimated spreads depend on the observation interval. In particular, daily estimates are smaller than weekly estimates. Both problems appear in Roll's empirical results.

There are some obvious aspects of this model that don't match reality.

- Ordinarily bid and ask prices stay at the same level for different periods of time. This model doesn't include any sense of size of orders, and hence doesn't capture this temporal aspect.
- The bid/ask spread is not a constant in actual trading. It also follows clear intraday patterns where it varies between the open, midday, and close times.
- There is a strong relationship between trades and changes in quotes, this dynamic is completely ignored in the Roll model. A quote level may change

when market makers change their prices or someone trades and moves from one level of an order book to another.

- Trades do not occur with every quote. There can be extended periods of time where markets are being quoted without any trading taking place.

There have been extensions to the basic Roll model, such as Zhang (2012) which we may consider later.

4.3 Inventory models

If we think of market makers as the intermediaries between buyers and sellers, then they

Inventory models have subsequently largely been replaced by information based models.

Covered in O'Hara Chapter 2 or Hasbrouck Chpater n.

Models of how market maker inventories affect pricing and liquidity in financial markets all predict that a market maker will set his ask price above his bid price for reasons including bankruptcy risk (Garman [6]), market power (Amihud Mendelson 6 [1]), and risk aversion (Stoll [19], Ho Stoll [10, 11]).

4.3.1 Garman (1976)

This starts with an important paper by Garman (1976). Garman's focus was on the temporal aspects of exchange, given that imbalances between supply and demand would temporarily arise:

> What mattered was that orders would be submitted to the market and imbalances between supply and demand could temporarally arise. This imbalance gave an imporance to the "temporal microstructure," or how the exchagne between buyer and seller actually occurred at any point in time.

"In Garman's model, there is a single, monopolistic market maker who sets prices, receives all orders, and clears trades. The dealers objective is to maximize expected profit per unit of time, subject to the avoidance of bankruptcy or failure. Failure arises in this model whenever the delaer runs out of either inventory of cash. The market maker's only decision is to set an ask price, p_a, at which he will fill orders wishing to buy the stock, and a bid price, p_b, at which he will fill orders wishing to sell the stock." (OH 16)

Uncertainty is a result of the different arrival times of buy and sell orders, and thus we can model the uncertainty of the arrival times as a Poisson process.

Garman innovates on the standard supply/demand function by having the supply/demand price relationship be a function of the arrival rate of orders.

A key component of the Garman model is that the dealer's inventory is directly related to his viability.

Garman

4.3.2 Amihud and Mendelson (1980)

http://pages.stern.nyu.edu/ lpederse/courses/LAP/papers/InventoryRisk/AmihudMendelson80.pdf

Amihud and Mendelson extend Garman (1976) by considering the relationship between the level of inventory and the price set by the dealer.

NA

AM 1986: (http://pages.stern.nyu.edu/ lpederse/courses/LAP/papers/TransactionCosts/Amihu

http://pages.stern.nyu.edu/ lpederse/courses/LAP/papers/InventoryRisk/GrossmanMiller.pdf
http://pages.stern.nyu.edu/ lpederse/courses/LAP/papers/InventoryRisk/GrossmanMillerDiscussio

4.3.3 Stoll (1978)

Stoll (1978) extends the Garman model to account for the *Market Makers' Ruin Problem.*

4.3.4 Ho and Stoll (1981)

Stoll (1978) extends the Garman model to account for the *Market Makers' Ruin Problem.*

4.4 Sequential trade models

The sequential trade model was first proposed in Glosten and Milgrom (1985). This involves the study of *assymetric information*, wherein there is an interaction between *informed* and *uninformed* traders. Informed traders are trading on the basis of some information; a simple assumption is that these traders know the actual future evolution of the trade prices. Uninformed traders are trading without any explicit knowledge for future prices, and might be trading on the basis of a simple rebalance (e.g. an index which rebalances due to an inflow/outflow of assets). Each trader is allow to trade once in this model, and a market maker tries to set the bid/ask prices on the basis of the informed trader flow.

Why would a market maker care if orders are placed by informed traders? The short answer is *adverse selection.*

The simple way to build this model is to consider all the potential scenarios involved in trading.

4.4.1 Glosten and Milgrom (1985)

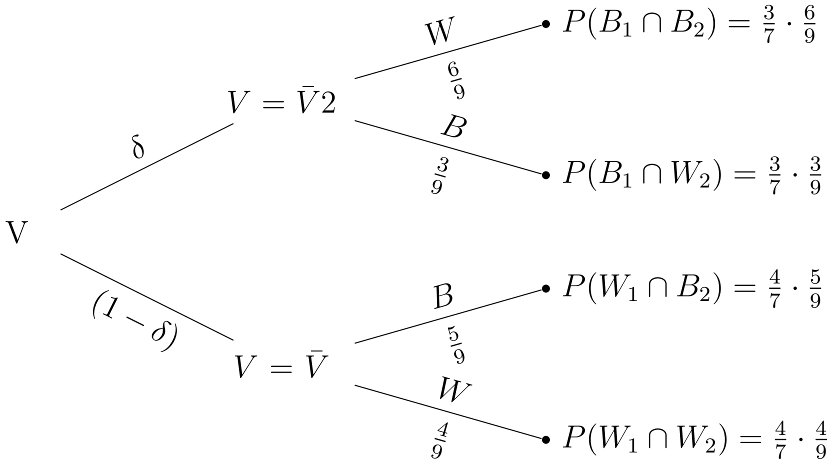
The Glosten and Milgron model is one of the most well known mwas introduced in ””

https://raw.github.com/alexchinco/Glosten-and-Milgrom–1985-/master/glosten$_a nd_m ilgrom_1 98$ $//github.com/alexchinco/Glosten-and-Milgrom--1985-http : //www.alexchinco.com/?p =$ 3691

4.4.2 Probability of Informed Trading (PIN)

Easley et al. (1996) build on the model of Glosten and Milgrom (1987) to estimate the probability that a counterpart in the trading process enjoys superior ”asymmetric” information.

http://scholar.lib.vt.edu/theses/available/etd-05022007-153028/unrestricted/Marius$_E TD.pdf$



4.5 Strategic trade (continuous auction) models

The *strategic trade* model was first set forth in Kyle (1984, 1985).

4.5.1 Kyle (1985)

A. S. Kyle. Continuous Auctions and Insider Trading. Econometrica, 53(6):1315-1335, 1985

Kyle’s model aims to answer how a single informed trader would choose to trade in order to maximize the value of private information.

D. Easley and M. O’Hara. Price, Trade Size, and Information in Securities Markets. Journal of Financial Economics, 19(1), 69-90, 1987. http://www.fsa.ulaval.ca/personnel/PHGI

L. R. Glosten and L. E. Harris. Estimating the Components of the Bid/Ask Spread. Journal of Financial Economics, 21(1), 123-142, 1988.

5 Limit Order Book

5.1 Limit Orders as Option Prices

5.1.1 Copeland and Galai (1983)

Copeland and Galai (1983) provided the foundational study on how market making is equivalent to option writing.

> An individual who chooses to serve as a market-maker is assumed to optimize his position by setting a bid-ask spread which maximizes the difference between expected revenues received from liquidity-motivated traders and expected losses to information motivated traders. By characterizing the cost of supplying quotes, as writing a put and a call option to an information-motivated trader, it is shown that the bid-ask spread is a positive function of the price level and return variance, a negative function of measures of market activity, depth, and continuity, and negatively correlated with the degree of competition. Thus, the theory of information effects on the bid-ask spread proposed in this paper is consistent with the empirical literature.

5.1.2 Brenner and Subrahmanyam (1988)

5.1.3 Jarnecic and McInish (1997)

"An Empirical Investigation of the Option Value of the Limit Order Book on the Australian Stock Exchange"

5.1.4 Harris (2003)

5.1.5 Chacko, Jurek, and Stafford (2008)

In "The Price of Immediacy", Chacko, Jurek, and Stafford

> This paper models transaction costs as the rents that a monopolistic market maker extracts from impatient investors who trade via limit orders. We show that limit orders are American options. The limit prices inducing immediate execution of the order are functionally equivalent to bid and ask prices and can be solved for various transaction sizes to characterize the market maker's entire supply curve. We find considerable empirical support for the model's predictions in the cross-section of NYSE firms. The model produces unbiased, out-of-sample forecasts of abnormal returns for firms added to the SP 500 index.

5.2 Spread and Depth: Lee, Mucklow, and Ready (1993)

Lee, Mucklow, and Ready (1993) [**?**] showed that "wide spreads are accompanied by low depths, and that spreads widen and depths fall in response to higher volume."

5.3 Adverse Selection

5.3.1 Glosten and Harris (1988)

"This paper develops and implements a technique for estimating a model of the bid/ask spread. The spread is decomposed into two components, one due to asymmetric information and one due to inventory costs, specialist monopoly power, and clearing costs. The model is estimated using NYSE common stock transaction prices in the period 1981-1983. Cross-sectionaI regrzsion analysis is then used to relate time-series estimated spread components to other stock $_{t}ics.Theresultscannotrejectthehypothes$

6 Algorithmic Trading and Transaction Cost Analysis

The primary theoretical papers on the subject are Lo and Habermas (1997) and Almgren and Chriss (1998). The Almgren/Chriss model

"Optimal Trading Algorithms: Portfolio Transactions, Multiperiod Portfolio Selection, and Competitive Online Search" http://e-collection.library.ethz.ch/eserv/eth:30852/eth-30852-02.pdf

Algorithmic trading covers the electronic execution of trades, typically aimed at transaction cost reduction.

Transaction cost reduction.

Transaction Cost Analysis (TCA) is one of the most widely adopted applications of microstructure theory.

1. Direct costs such as commissions, custody fees, taxes, and infra-structure costs
   - primarily determined by quantity of trading
   - easy to measure
2. Indirect costs such as market impact and opportunity costs
   - primarily determined by the trading strategy at micro-level
   - hard to measure

DAVID J. LEINWEBER "Using Information From Trading: In Trading And Portfolio Management: Ten Years Later" http://www.hss.caltech.edu/SSPapers/wp1135.pdf

6.1 Transaction Costs

6.1.1 Implementation Shortfall: Treynor (1981) and Perold (1988)

Jack L. Treynor "What Does It Take to Win the Trading Game?" Financial Analysts Journal, Vol. 37, No. 1 (Jan. - Feb., 1981), pp. 55-60, http://www.jstor.org/stable/4478421

NA

Hitesh Mittal: "Implementation Shortfall - One Objective, Many Algorithms" http://www.cis.upenn.edu/ mkearns/finread/impshort.pdf

The most basic model for transaction costs is the *Implementation Shortfall* (Teynor 1981, Perold 1988):

$$implementationshortfall = tradeprice - (decisionprice + commissions) \quad (12)$$

From Kolm:

Desirable aspects for a t-cost model:

Unbiasedness: Low error: High explanatory power:

6.1.2 The Square-Root Formula

http://www.math.nyu.edu/financial$_mathematics/content/02_financial/2008-6.pdf$

Discussed in Chapter 16 of: Grinold, Richard C., and Ronald N. Kahn, 1995, Active Portfolio Management (New York: The McGraw-Hill Companies, Inc.)

$$Cost = Spreadterm + c\sigma\sqrt{\frac{n}{V}} \quad (13)$$

6.1.3 Market Impact

Transaction costs are generally broken into explicit and implicit. Explicit includes anything that is visible, such as commissions and taxes. Implicit covers the difference between the desired and realized execution prices, and covers the bid/ask spread and market impact.

Logically, we know that the more we trade, the larger the market impact. And that the type of asset matters, so trying to trade in an illiquid asset should cost more. Lastly, time should be a factor: trying to execute quickly should cost more. We can think of this in terms of the time variation of supply/demand.

In the implementation-shortfall framwork of Perold (JPM, 1988), MI is defined as the difference between pre-trade or paper price and the realized execution price.

MI can be decomposed into temporary and permanent impact components.

http://www.dbquant.com/Presentations/Berlin200812.pdf http://www.cmap.polytechnique.fr/f $//www.cmap.polytechnique.fr/\ euroschoolmathfi09/ParisSummerschool0908_schied.pdf http:$ $//www.northinfo.com/documents/257.pdf http: //www.haas.berkeley.edu/groups/finance/hidd$

Recent: http://arxiv.org/abs/1205.4008

Anomalous price impact and the critical nature of liquidity in

nancial markets B. Toth a , Y. Lemperiere a , C. Deremble a , J. de Lataillade a , J. Kockelkoren a , J.-P. Bouchaud a http://arxiv.org/pdf/1105.1694.pdf

Stoll (1989): Temporary and Permanent

http://www.acsu.buffalo.edu/ keechung/MGF743/Readings/Stoll

Almgren, Thum, Hauptmann, and Li (2005)

Almgren, Robert, Chee Thum, Emmanuel Hauptmann, and Hong Li, 2005, Equity market impact, Risk July, 576 http://www.math.nyu.edu/ almgren/papers/costestim.pdf

6.2 Optimal Execution (Trade Scheduling)

Almgren, R. (2001): ”Optimal Execution of Portfolio Transactions”, Journal of Risk, 3, 5-39. Almgren, R. (2003): ”Optimal Execution with Nonlinear Impact Functions and Trading-enhanced risk,” Applied Mathematical Finance, 10, 1-18

http://www.courant.nyu.edu/ almgren/papers/arrival1.pdf

Bertsimas, Andrew Lo ”Optimal control of execution costs”, Journal of Financial Markets, 1, 1-50, 1998. http://web.mit.edu/dbertsim/www/papers/Finance/Optimal(with Paul Hummel and Andrew Lo) ”Optimal control of execution costs for portfolios”, Computing in Science and Engineering, 40-53, 1999. http://www.mit.edu/ dbertsim/papers/Finance/Optimal

http://hal.archives-ouvertes.fr/docs/00/59/84/28/PDF/memoire$_m$*aster.pdf*

Huberman and Stanzl ”Optimal Liquidity Trading”, (2005)

6.2.1 Bertsimas and Lo (1998)

Dimitris Bertsimas and Andrew Lo (BL) wrote ”Optimal Control of Execution Costs” in 1998 and provided the first exposition of trade scheduling based on a t-cost minimization. ”We derive dynamic optimal trading strategies that minimize the expected cost of trading a large block of equity over a fixed time horizon.” (BL 1)

6.2.2 Almgren and Chriss (2000)

[From Walia, p.15]

In Almgren and Chriss (2000), the underlying stock movement is modelled as arithmetic Brownian motion in discrete time.

$$S_k = S_{k+1} + \sigma\tau^{1/2}\gamma_k - \tau g(n_k/\tau) \tag{14}$$

Following from Stoll (1989), Almgren and Chriss also include a temporary

$$S_k = S_{k+1} - h(n_k/\tau) \tag{15}$$

6.2.3 Dynamic Trading Schedules: Extensions to Almgren/Chriss

Obizhaeva, Anna, and Jiang Wang, 2005, Optimal trading strategy and supply/demand dynamics, Discussion paper MIT Sloan School of Management. Alfonsi, Aurelien, Alexander Schied, and Antje Schulz, 2007, Optimal execution strategies in limit order books with general shape functions, Discussion paper CERMICS, projet MATHFI

Malamut, R. (2002), "Multi-Period Optimization Techniques for Trade Scheduling," QWAFAFEW, April 2002

6.3 Optimal Execution (Order Type)

A separate branch of literature is concerned with the type of order that is used, and where it should be placed within the limit order book.

6.4 Algorithmic Trading

http://iafe.org/html/documents/KolmIAFEMay272009.pdf http://www.cims.nyu.edu/ kolm/Kolm

7 High Frequency Quoting and Trading

With the arrival of electronic markets, market makers were primarily replaced by high frequency traders.

Scalping and market making.

There are several books on high frequency trading, including

http://dspace.mit.edu/bitstream/handle/1721.1/59122/658860705.pdf http://papers.ssrn.com/s
568363$http : //www.iaeng.org/publication/WCE2010/WCE2010_pp352-357.pdfhttp :$
$//www.londonstockexchange.com/about-the-exchange/regulatory/responsetocesrscallforevide$
$structuralissuesoftheeuropeanequitymarkets.pdf$

http://papers.ssrn.com/sol3/papers.cfm?abstract$_id = 1722924rec = 1srcabs =$ 1695596

http://www.tinbergen.nl/discussionpapers/11076.pdf

7.1 NBBO

http://people.stern.nyu.edu/jhasbrou/Research/Working

7.2 Flow Toxicity and VPIN

Optimal Execution Horizon (OEH): http://economics.cornell.edu/deasley/OptimalFlow toxicity:

http://papers.ssrn.com/sol3/papers.cfm?abstract$_i d = 1695596 rec = 1 srcabs = 568363$

8 Conclusion

Our understanding of how markets behave at the individual trade and quote level is still developing, although it has evolved dramatically over the last 30 years.

9 Further Reading

I provide references throughout the text to the canonical papers and books relevant to each topic. As a general topic, I recommend the following as more detailed texts on the subject:

- Joel Hasbrouck "Empirical Market Microstructure". An early draft of the book can be found online at ...

References

[Figueredo and Wolf, 2009] Figueredo, A. J. and Wolf, P. S. A. (2009). Assortative pairing and life history strategy - a cross-cultural study. *Human Nature*, 20:317–330.

[Kyle, 1985] Kyle, A.S. (1985). Continuous auctions and insider trading. *Econometrica*, 53: 1315-1336 .

Roll, R., 1984. A simple implicit measure of the e!ective bid-ask spread. Journal of Finance 39, 1127-1139.

O'Hara, M. (1995). Market Microstructure Theory. Cambridge, MA: Blackwell 1995