High-Frequency Market Making in a Limit Order Book: The effect of inventory and adverse selection on order book dynamics

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Abstract

Contents

1 Introduction

The *Limit Order Book* (LOB) lies at the heart of modern market microstructure. This exchange mechanism provides the means for any market participant to act as either a liquidity *provider* or *taker*.

It is conventional in market making models to consider agents who can post limit orders or market orders, where a market order is simply one which crosses the bid/ask spread in order to take liquidity. We will stick with this convention, while noting that actual market orders are simply an order type with a specific set of rules: to fill an order regardless of the resulting price. While a limit order specifies a price.

The bid/ask spread is primarily modelled as a function of three terms: *order processing costs*, *inventory risk*, and *adverse selection*.

2 Background

Starting with Garman (1976), there is a long history of market microstructure theory.

Garman innovates on the standard supply/demand function by having the supply/demand price relationship be a function of the arrival rate of orders.

2.1 Ho and Stoll (1981)

Ho and Stoll (1981 [**?**]) "Optimal dealer pricing under transactions and return uncertainty"

2.2 Avellaneda and Stoikov (2008)

Avellaneda and Stoikov use an arithmetic brownian motion mid-point price process S_t

$$S_t = s + \sigma W_t \tag{1}$$

The market maker seeks to maximize the expected utility through the value function:

$$v(x, s, q, t) = E_t[-e^{(-\gamma(x+qS_T))}] \tag{2}$$

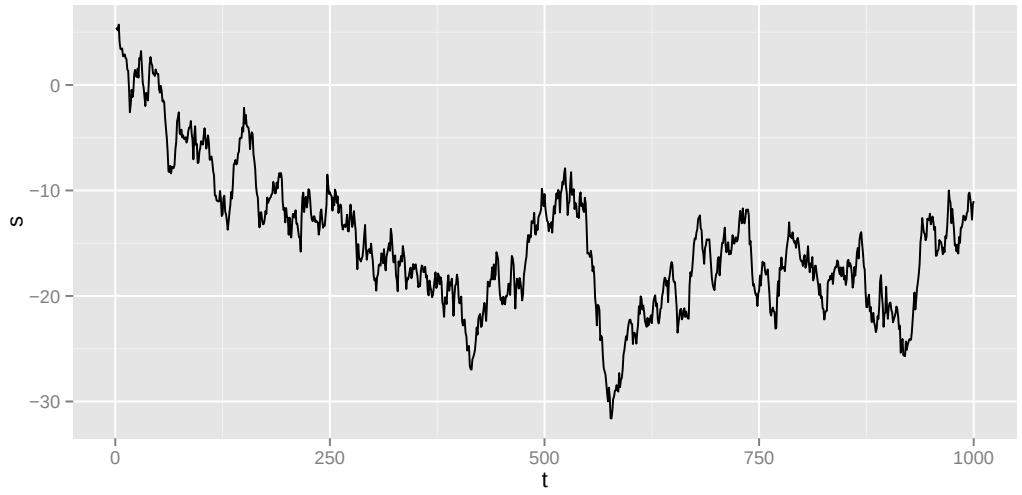

Figure 1: Brownian motion S_t mid-point process.

2.3 Gueant, Lehalle, and Fernandez-Tapia (2012)

The position at time t is represented by q_t is a function of the number of fills at the bid N_t^b and ask N_t^a respectively:

$$q_t = N_t^b - N_t^a \tag{3}$$

2.4 Guilbaud and Pham (2013a)

2.5 Guilbaud and Pham (2013b)

2.6 Overview

We follow in the tradition of Avellaneda/Stoikov (2006), but extend the model in some fundamental ways.

- We add a term for *adverse selection.*

3 The Model

3.1 The mid-price of the stock

$$S_t = s + \sigma W_t \tag{4}$$

4 Further Reading

I provide references throughout the text to the canonical papers and books relevant to each topic. As a general topic, I recommend the following as more detailed texts on the subject:

- Joel Hasbrouck "Empirical Market Microstructure". An early draft of the book can be found online at ...

References

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