Growth accounting

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So far the course has focused mainly on the short terms dynamics of the economy. However, an important branch of macroeconomics concerns the study of what happens in the long run, focusing on factors that can help explain the growth rate of the economy, and subsequently how these can be addressed by policy measures. This field contributes to answering the question of why some countries and rich and others are poor, and overlaps with other disciplines such as history. In this first lecture focusing on long-term economic processes we will cover growth accounting, which is a technique explaining the factors that determine economic growth.

Malthusian world

Before we dive into the theoretical models that set out to explain growth we first have a look at one of the earlier economic idea linking output to population. For a long time, and still in many parts of the world, income is mainly driven by population growth, even in the presence of rapid technological progress. Thomas Malthus famously discussed this in *An Essay on the Principle of Population*.[1] The Malthusian model hinges on the principle of diminishing returns to the factors of production, and is well known for two important features

[1] Some of Malthus' insights were further developed by David Ricardo.

1. Long-run stagnation
2. The idea that population growth will outpace agricultural productivity

Let's consider a Malthusian society where output is determined by just two factors: land X and labour L. For simplicity we assume that the labour force is equal to the size of the population and that the total available area of land is fixed. The aggregate production function can be described as

$$Y = AX^{\alpha}L^{1-\alpha}$$

Y is total output, A is a technology parameter, X the fixed quantity of land, L is the size of the population/labour force.

Although output will increase when the population size increases, the marginal product of labour will decrease as it is defined by

$$\begin{aligned}\frac{\delta Y}{\delta L} &= (1-\alpha)AX^{\alpha}L^{-\alpha} \\ &= (1-\alpha)A\left(\frac{X}{L}\right)^{\alpha} > 0\end{aligned}$$

The marginal product is positive, so one extra worker will produce some extra output. But since L is in the denominator the marginal product will decrease. To measure living standard economists and policy makers often rely on output per capita as an indicator We can express output per capita y as

$$\frac{\delta^2 Y}{\delta L^2} = -\alpha(1-\alpha)AX^{\alpha}L^{-\alpha-1} < 0$$

$$\begin{aligned}\frac{Y}{L} = y &= \frac{AX^{\alpha}L^{1-\alpha}}{L} \\ &= A\left(\frac{X}{L}\right)^{\alpha} = Ax^{\alpha}\end{aligned}$$

x is land per capita.

An important implication that follows from the Malthusian model is that living standards will fall with greater population size. Although an additional person means an increase in production, it also entails another person to share the production with. The latter negative effect will dominate.

$$\begin{aligned}\frac{\delta y}{\delta L} &= -\alpha AX^{\alpha}L^{-\alpha-1} < 0 \\ \frac{\delta^2 y}{\delta L^2} &= \alpha(1-\alpha)AX^{\alpha}L^{-\alpha-2} > 0\end{aligned}$$

The Malthusian trap

The key feature of the model is the strong link between output and population growth. Let's consider a simple set up for population where the population size L_t is equal to last year's population size plus the number of births B_t and minus the number of deaths D_t during the same year.

$$L_t = L_{t-1} + B_t(y_{t-1}) - D_t(y_{t-1})$$

A key feature here is that the number of births and deaths is a function of living standards or output per capita, lagged one year: y_{t-1}.

$B'(y_{t-1}) > 0$, $D'(y_{t-1}) < 0$

When output per capita increases, the food supply will increase which allows families to expand and reduce mortality due to better nourishment. The upper panel of figure 1 shows the dynamics between the population and output.[2] The figure illustrates that output per capita will tend to converge towards an equilibrium indicated by y^*, at which point the population ceases to grow[3]

[2] For simplicity the birth and death rate are assumed to be linear functions of output.

[3] This is known as the subsistence level.

$$L_t - Lt - 1 = B_t(y^*) - D_t(y^*) = 0$$

Let's say that we would start at a relatively high output level at y^0. In this case, due to a high output per capita there will be relatively few deaths and many births increasing the population, leading to a reduction in living standards shifting to the equilibrium point on the left. One could imagine a scenario where one would start at the left of the equilibrium point were few children are born and many people

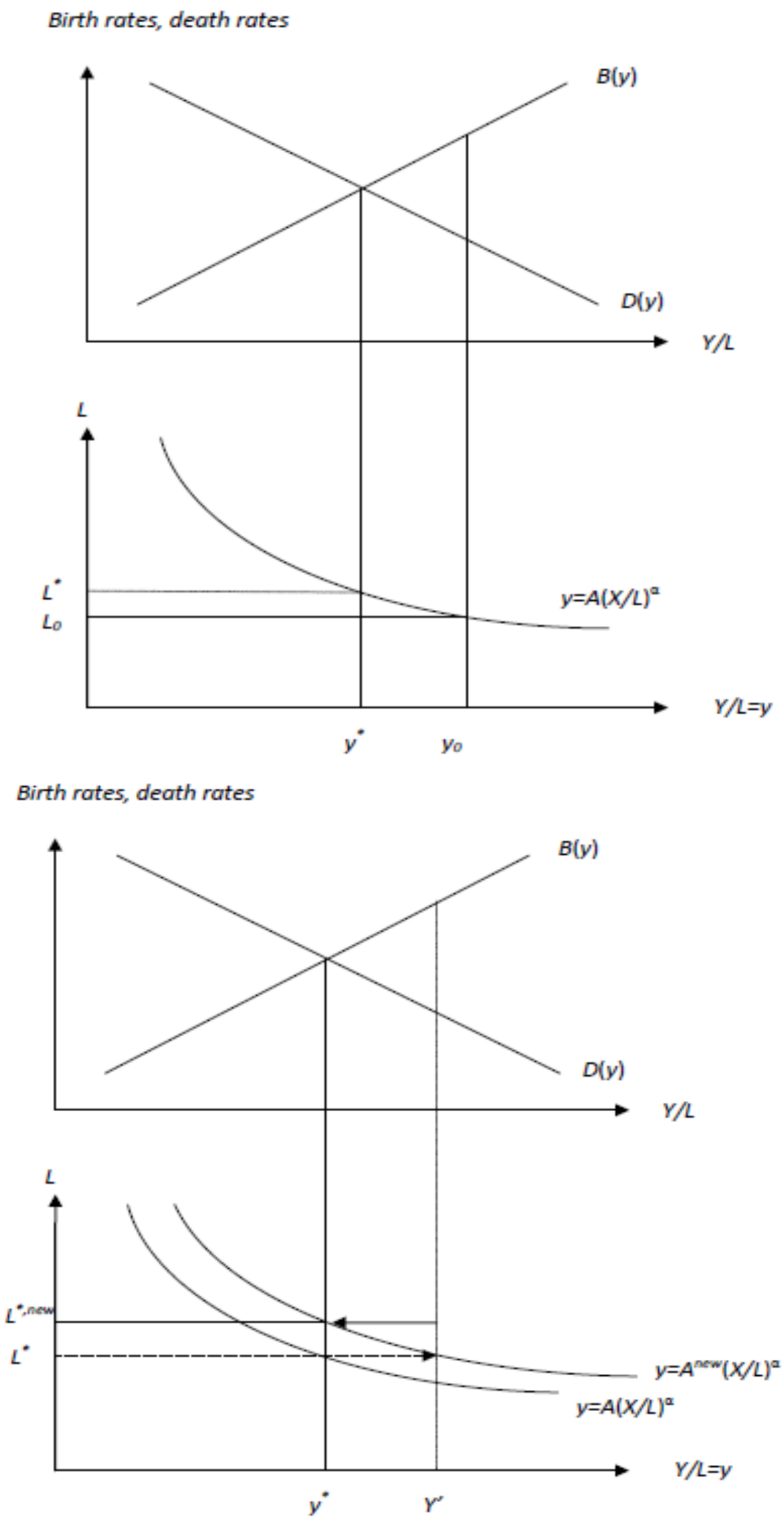


Figure 1: The Malthusian trap illustrated. The dynamics between population size and output (upper panel) and the effect of a technology shock (lower panel). Figures taken from Olsson Essential of Advanced Macroeconomic Theory (2010).

die. In this case the shrinking of the population leads to a gradual increase in output per capita.

One can think here about the situation in Europe during the Black Plague.

Consider a positive technology shock increasing productivity. The increase in productivity will lead to an outward shift in the y-curve and a temporary increase in output per capita. But again, this will correspond with an increase in the birth rate and decrease in the death rate, causing the population to rise, leading to a reduction in living standards. The only lasting result of such a technology shock is a larger population. History has shown however that there are a number of factors that can contribute to a break with the Malthusian trap.

1. Fall in birth rate ending link with income per capita
2. Increase in education level
3. Increase in growth of technological knowledge
4. Increase in output per capita far beyond subsistence level

Productivity

A more modern production function is the Cobb-Douglas model. Here we assume that output is determined by an aggregate production function technology depending on the total amount of labour (L) and capital (K).

$$Y_t = A_t K_t^{\alpha} L_t^{\beta}$$

In this function A_t accounts for technology

- Increase in A_t results in higher output without having to raise inputs
- Measure of productive efficiency
  - Fluctuates for various reasons, e.g. new technology, government regulation, better management
- Since A_t increases productiveness of other factors, it is also known as Total Factor Productivity (TFP)

We are often interested in the output per worker or productivity. In the model output per worker is given by

Increases in output per worker is productivity growth.

$$\frac{Y_t}{L_t} = A_t K^{\alpha} L_t^{\beta-1} = A_t \left(\frac{K_t}{L_t}\right)^{\alpha} L_t^{\alpha+\beta-1}$$

This function shows that there are three potential ways to increase productivity

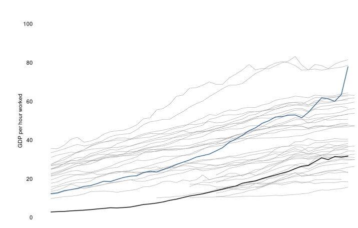


Figure 2: Productivity, measured in GDP per hour worked, for OECD member states and some additional economies. Blue indicates Ireland, black line South Korea. Data: OECD

1. Increase in labour force
   - Increasing the number of workers will only add to growth if $\alpha + \beta > 1$
   - i.e. increasing returns to scale, whereas most growth theories assume constant returns to scales (CRS)
   - Under CRS $\alpha + \beta = 1$ and productivity is

$$\frac{Y_t}{L_t} = A_t \left(\frac{K_t}{L_t}\right)^{\alpha}$$

2. Technological progress
   - Improving the efficiency of the economy
3. Capital deepening
   - Increasing the amount of capital per worker

Growth determinants

Most growth theories assume constant returns to scale (CRS), let's examine what determines growth under these assumptions, using the standard Cobb-Douglas production function.

$\beta = 1 - \alpha$

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Time is assumed to be continuous; t evolves smoothly instead of taking integer values like $t = 1, t = 2, \ldots$.

The growth rate of Y_t can be denoted by G_t^Y and defined as[4]

[4] i.e growth equals the change in output divided by output level.

$$G_t^Y = \frac{1}{Y_t}\frac{dY_t}{dt}$$

This production function can be differentiated with respect to time, and expressing it as a function of G_t^Y we can define the growth rates of labour, capital, and technology.

For instance, for capital and labour the differentiated model is

Use the product rule here, which implies

$$\frac{dABC}{dx} = BC\frac{dA}{dx} + AC\frac{dB}{dx} + AB\frac{dC}{dx}$$

$$\frac{dK_t^{\alpha}}{dt} = \frac{dK_t^{\alpha}}{dK_t}\frac{dK_t}{dt} = \alpha K_t^{\alpha-1}\frac{dK_t}{dt}$$
$$\frac{dL_t^{1-\alpha}}{dt} = \frac{dL_t^{1-\alpha}}{dL_t}\frac{dL_t}{dt} = (1-\alpha)L_t^{-\alpha}\frac{dL_t}{dt}$$

For the whole production function we get

Can use the chain rule to calculate the terms involving the impact of changes in capital and labour inputs

$$\begin{aligned}\frac{dY_t}{dt} &= \frac{dA_t K_t^{\alpha} L_t^{1-\alpha}}{dt} \\ &= K_t^{\alpha} L_t^{1-\alpha}\frac{dA_t}{dt} + A_t L_t^{1-\alpha}\frac{dK_t^{\alpha}}{dt} + A_t K_t^{\alpha}\frac{dL_t^{1-\alpha}}{dt} \\ &= K_t^{\alpha} L_t^{1-\alpha}\frac{dA_t}{dt} + \alpha A_t K_t^{\alpha-1} L_1^{1-\alpha}\frac{dK_t}{dt} + (1-\alpha)A_t K_t^{\alpha} L_t^{-\alpha}\frac{dLt}{dt}\end{aligned}$$

The growth rate of output is calculated by dividing both sides by Y_t which is the same as dividing by $A_t K_t^{\alpha} L_t^{1-\alpha}$ this becomes

$$\frac{1}{Y_t}\frac{dY_t}{dt} = \frac{K_t^{\alpha} L_t^{1-\alpha}}{A_t K_t^{\alpha} L_t^{1-\alpha}}\frac{dA_t}{dt} + \alpha \frac{A_t K_t^{\alpha-1} L_1^{1-\alpha}}{A_t K_t^{\alpha} L_t^{1-\alpha}}\frac{dK_t}{dt} + (1-\alpha)\frac{A_t K_t^{\alpha} L_t^{-\alpha}}{A_t K_t^{\alpha} L_t^{1-\alpha}}\frac{dLt}{dt}$$
$$\frac{1}{Y_t}\frac{dY_t}{dt} = \frac{1}{A_t}\frac{dA_t}{dt} + \alpha \frac{1}{Kt}\frac{dK_t}{dt} + (1-\alpha)\frac{1}{L_t}\frac{dL_t}{dt}$$
$$G_t^Y = G_t^A + \alpha G_t^K + (1-\alpha) G_t^L$$

This shows that the output growth rate equals the technology growth rate plus a weighted average of the growth rates of capital and labour, with the weight determined by α. This is the key equation in growth accounting studies. These studies provide estimates of how much of GDP growth over a certain time span is determined by

- Growth in the number of workers
- Growth in capital stock
- Improvement in Total Factor Productivity

Estimating productivity

Calculating the growth equation

$$G_t^Y = G_t^A + \alpha G_t^K + (1-\alpha) G_t^L$$

requires

- A measure for output[5]
- Number of workers
- Capital stock estimate
- Value for Total Factor Productivity

[5] i.e. GDP

One empirical issue is that the value for TFP (A_t) can't be directly observed. However, if we know the value of parameter α we can get an estimate for growth.

$$G_t^A = G_t^Y - \alpha G_t^K - (1-\alpha) G_t^L$$

In a seminal paper, Solow (1957) pointed out that an estimate of α could be obtained by looking at the shares of GDP paid to workers and to capital. Consider a perfectly competitive firm that is seeking to maximise profits and the firm

- Sells product at price P_t
- Pays wages W_t

- Rents its capital at a rate of R_t

The firm's profits are given by

$$\begin{aligned}\Pi_t &= P_t Y_t - R_t K_t - W_t L_t \\ &= P_t A_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - W_t L_t\end{aligned}$$

The only thing the firm need to do is decide on how much labour and capital to use. It will therefore maximise profits by differentiating the function with respect to capital and labour and the the derivatives equal to zero which provides two conditions

$$\frac{\delta \Pi_t}{\delta K_t} = \alpha P_t A_t K_t^{\alpha-1} L_t^{1-\alpha} - R_t = \alpha \frac{P_t Y_t}{K_t} - R_t = 0$$
$$\frac{\delta \Pi_t}{\delta L_t} = (1-\alpha) P_t A_t K_t^{\alpha} L_t^{-\alpha} - W_t = (1-\alpha) \frac{P_t Y_t}{L_t} - W_t = 0$$

Rearranging we get

$$\begin{aligned}\alpha &= \frac{R_t K_t}{P_t Y_t} \\ 1-\alpha &= \frac{W_t L_t}{P_t Y_t}\end{aligned}$$

We have that

- $P_t Y_t$ is total nominal GDP
- $W_t L_t$ is the total amount of income paid to wages
- $R_t K_T$ is the total amount of income paid to capital

α will be the total amount of income paid to capital relative to total income, at the aggregate level nominal GDP. And $1-\alpha$ can be calculated as the fraction of income paid to workers instead of compensating capital.

Solow's results

Solow found that for most countries the national income accounts show that wage income explains most of GDP, corresponding to $\alpha < 0.5$. Based on the estimates for the US economy, commonly $\alpha = \frac{1}{3}$ is used. However, studies assume that firms are imperfectly competitive, and if that assumption holds, that means that the share if income earned by labour and capital depend on the degree of monopoly power. Solow's paper also concluded that capital deepening had not been that important for US growth; TFP growth accounted for 87.5% of growth in productivity over the period.

TFP is sometimes called the Solow residual because it is a backed out calculation that makes things add up.

The Swan-Solow model

In the standard Swan-Solow model the production functions links output to capital and labour inputs as well as a technological efficiency parameter.

$$Y_t = AF(K_t, L_t)$$

A key feature of the model is that, with a constant labour supply, there are diminishing marginal returns to capital accumulation meaning that each increase in capital will give a progressively smaller increase in output

$$\frac{\delta^2 Y_t}{\delta K_t} < 0$$

Additional assumptions include

1. Closed economy; no government sector or international trade
   - All output takes the form of consumption or investment

$$Y_t = C_t + I_t$$

2. Savings equal investment

$$S_t = Y_t - C_t = I_t$$

3. Capital depreciates
   - Capital stock depends positively on investments and negative on depreciation at rate δ

$$\frac{dK_t}{dt} = I_t - \delta K_t$$

4. Consumers save constant share of income

$$S_t = sY_t$$

These assumptions tell us something about the capital dynamics in the model. Since the amount of savings equals the amount of investment, this means that investment is also a constant fraction of output

$$I_t = sY_t$$

This entails that the capital stock changes over time according to

$$\frac{dK_t}{dt} = sY_t - \delta K_t$$

How the capital stock develops thus depends on whether investments are greater, equal to, or less than the depreciation rate.

The stock of capital will stay constant if the capital/output ratio is

$$\frac{K_t}{Y_t} = \frac{s}{\delta}$$

$$\delta K_t < sY_t \Rightarrow \frac{dK_t}{dt} > 0$$
$$\delta K_t = sY_t \Rightarrow \frac{dK_t}{dt} = 0$$
$$\delta K_t > sY_t \Rightarrow \frac{dK_t}{dt} < 0$$

The level of investments is given by

$$I_t = sY_t = sAF(K_t, L_t)$$

which means that a one off increase in technology level A has the same effect as a one off increase in s: capital and output gradually increase to a new level. Nonetheless, the model implies a very important difference between these two determinants of growth. Note that the savings rate s is subject to a limit, whereas as A does not face such constraints. Therefore, in order to have long-term sustainable growth increases in TFP matter.[6]

[6] Specifically, growth through capital accumulation will taper off over time producing a one-off increase in output per worker whereas TFP growth can lead to sustained higher growth rates of output per worker.

Capital-output dynamics in Swan-Solow model

We can define the capital-output ration as

$$\frac{K_t}{Y_t} = K_t Y_t^{-1} = x_t$$

and the growth rate can be written as

$$\frac{\Delta x_t}{x_t} = \frac{\Delta K_t}{K_t} - \frac{\Delta Y_t}{Y_t}$$

We can now define the growth equation as

$$\begin{aligned} G_t^Y &= G_t^A + \alpha G_t^K + (1-\alpha)G_t^L \\ \frac{\Delta Y_t}{Y_t} &= g + \alpha \frac{\Delta K_t}{K_t} + (1-\alpha)n \end{aligned}$$

Capital growth is given as

$$\frac{\Delta K_t}{K_t} = s\frac{Y_t}{K_t} - \delta = \frac{s}{x_t} - \delta$$

Meaning that the growth of the capital-output ratio is given by

$$\begin{aligned} \frac{\Delta x_t}{x_t} &= (1-\alpha)\frac{\Delta K_t}{K_t} - g - (1-\alpha)n \\ &= (1-\alpha)(\frac{s}{x_t} - \frac{g}{1-\alpha} - n - \delta) \end{aligned}$$

The growth rate of x_t depends negatively on the value of x_t; when it is above a certain x_t value the growth rate will decline and it will increase under said x_t value. The capital-output ratio therefore exhibits convergent dynamics leading to a particular long-run steady state value. In equilibrium the capital-output ratio equals 0, this is when

$$x^* = \frac{s}{\frac{g}{1-\alpha} + n + \delta}$$

Caveat in growth accounting

Results from growth accounting studies can potentially be misleading as they misidentify the source of growth. Consider a country that allocates a fixed share of GDP to investments but is experiencing a steady growth in TFP. The Swan-Solow model predicts in this case a steady increase in output per worker and an increase in capital stock. Observing this increase a growth accounting study might conclude that a certain percentage of growth is caused by capital accumulation, whereas all growth is caused by the TFP.

Example: multifactor productivity figure

The U.S. Bureau of Labor Statistics (BLS) produces growth accounting calculations under the name of multifactor productivity calculations. Figure 3 summarises the results for the period 1987-2013. The BLS adds some additional factors to the growth equation to account for improvements in the quality of labour force q_t, such as education and experience

$$Y_t = A_t K_t^{\alpha} (q_t L_t)^{1-\alpha}$$

Figure 3 shows that productivity growth in the US is weakening.

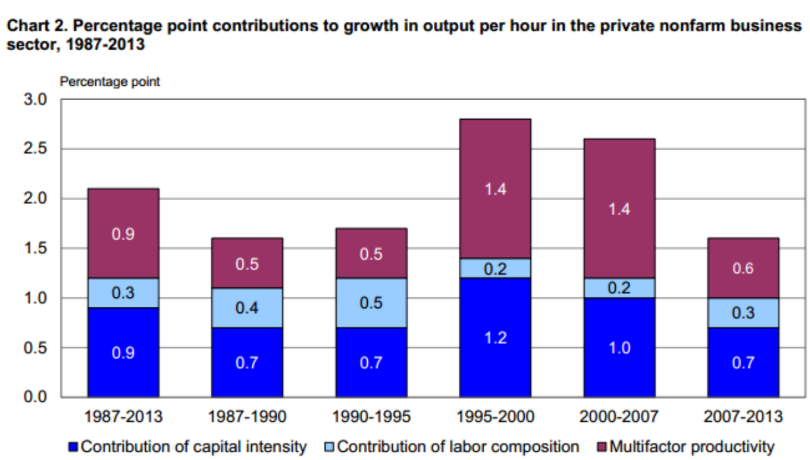


Figure 3: Productivity in the US 1987-2013. Note that they exclude activities in the agricultural sector. For the US data you can check the BLS website. Similarly, the European Union also provides growth accounting calculations up until 2007. More recent data is available at the EU KLEMS webpage.

Demography and productivity

One important factor with regard to the reduction in productivity growth is the slow growth rate of the labour force. In recent years the US labour force has flattened out after years of increasing number of people available for work due to

1. Population growth
2. Female labour participation
3. Immigration

An important long-term demographic trend, that also affects other developed countries is the ageing of the baby boom generation. As more and more people from this generation start to retire, the dependency ratio starts to increase significantly.

The dependency ratio is the ratio of non-working to working people.

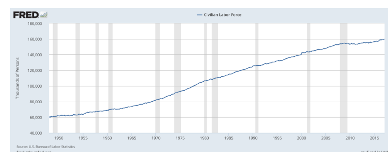
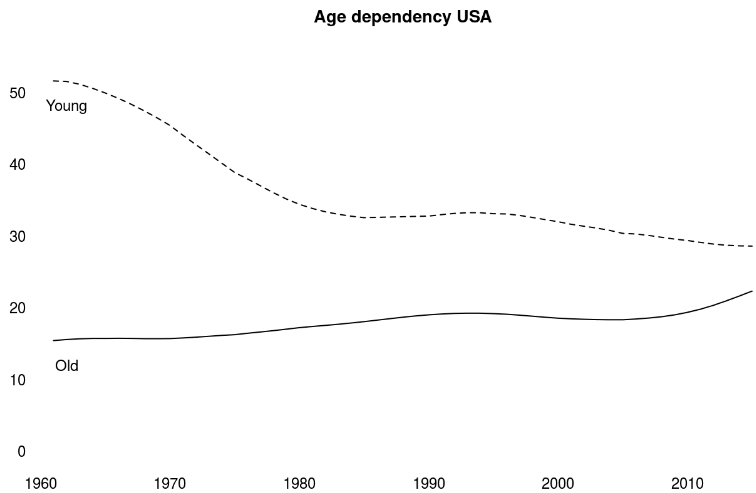


Figure 4: US civilian labour force. Data: St. Louis FED.



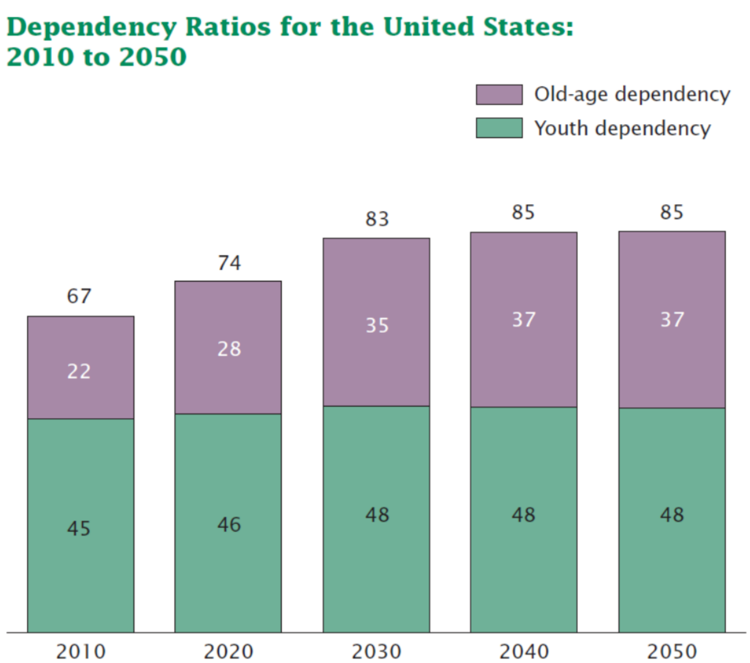


Figure 5: *Top:* Age dependency for old (>64) and young (< 15) for the USA. Data: World Bank World *Bottom:* Projected age dependency ratios for the USA.

Example: Growth accounting in the Euro area

Figure 7 shows growth accounting calculations for the euro area over time and illustrates that output per worker has declined over time, specifically TFP growth. One serious challenge for future economic growth in Europe is dealing with the demographic changes, specifically the effects of an ageing population. Population growth is slowing and to peak in the middle of the century (figure 8). Worryingly, the working population, those aged between 15-64 years, has peaked already and is set to decline. The macroeconomic problems that Europe faces are twofold

- Short term
  - Weak aggregate demand
  - High levels of public and private debt
- Long term
  - Demographic challenges
  - Requiring productivity increase, immigration, higher ages for retirement

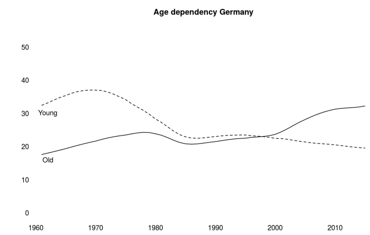


Figure 6: Age dependency for old (>64) and young (< 15) for Germany. Data: World Bank.

Period	Euro Area			
	$\triangle y$	$\triangle a$	$\triangle k$	$\triangle l$
1970-1976	3.6	2.7	1.5	-0.5
1977-1986	2.1	1.6	0.8	-0.4
1987-1996	2.3	1.5	0.8	0.0
1997-2006	2.2	0.7	0.8	0.7
2007-2013	-0.3	-0.2	0.5	-0.6
2000-2013	0.9	0.2	0.7	0.0
2010-2013	0.1	0.3	0.3	-0.5

Figure 7: Growth accounting in the Eurozone. Based on work by McQuinn and Whelan [Prospects for Growth in the Euro Area](#).

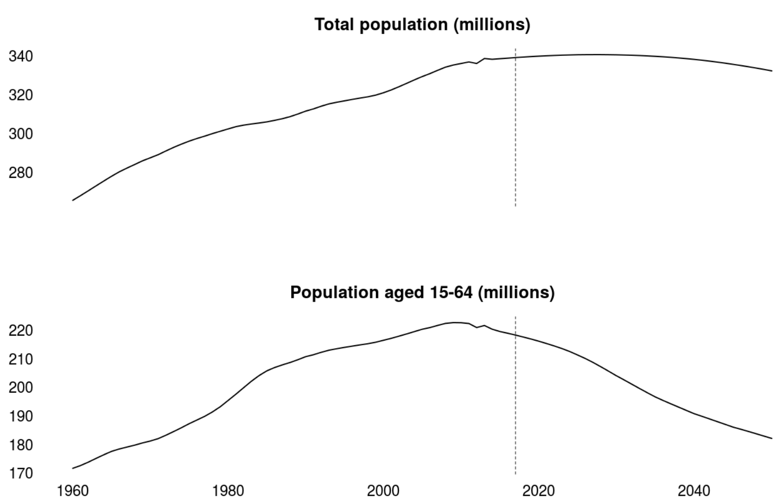


Figure 8: Demographic projections for the Euro area. Data: Knoema

Example: Singapore and Hong Kong

"A Tale of Two Cities" by Alwyn Young (1992) uses two city-states in the Orient, Hong Kong and Singapore, as a case study to examine the various factors that contributed to their growth experience. Both cities have been very successful in restructuring their economy; Hong Kong experienced an economic growth of 147% between the early 1970s and 1990, and Singapore 154%.

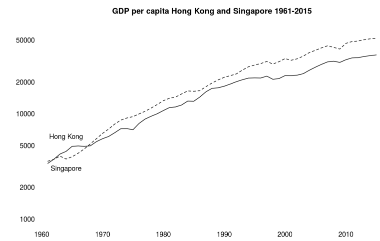


Figure 9: GDP over time for Hong Kong and Singapore. Data: World Bank

Young focuses on these two cities as they have a similar background yet are different on a number of issues emphasised by growth theory. Some similarities in the prewar period include

- Both British colonies
- Entrepôt trading ports[7]
- Little domestic manufacturing

[7] Hong Kong processed trade with mainland China whereas Singapore processed trade with Malaysia and Indonesia.

For the postwar period both cities developed export-dependent manufacturing industries, going from producing textiles to clothing, plastics, electronics, and since the 1980s shifting to banking and financial services.[8] Two important differences between the cities are

1. Hong Kong had a better educated population in the early postwar years[9]
2. Hong Kong has pursuit laissez faire policies, whereas Singapore implemented forced national savings and attracted a lot of foreign direct investment[10]

[8] Each industry experienced a period of fast growth, then moderate growth, and eventually decline.

[9] Capitalists from Shanghai.

[10] Singapore has heavily taxed labour income.

Hong Kong and Singapore both experienced an economic transformation going roughly through the same industries, only Singapore seemed to have done this in a more compressed period. Young therefore finds that the rate of structural transformation is higher in Singapore (0.209) compared to that of Hong Kong (0.082).[11] Focusing on total factor productivity Young finds that TFP has played a significant role in economic growth in Honk Kong were it contributed about 30 to 50% to output growth, with an average of 35% between 1971-1990. In contrast, in Singapore TFP is actually negative during some periods while positive during others, which can be contributed to fluctuations in the business cycle. For Singapore, capital has been a more important factor contributing on average 83% of output growth. Interestingly, the capital capital deepening, and forces saving, in Singapore did not contribute to TFP growth. One advantage of TFP growth over growth based on capital accumulation is that it is more sustainable over the long run. Young provides a theoretical explanation for the different patterns across the two cities. Specifically he focuses on

[11] Structural transformation is here measured by the allocation of labour in across two-digit ISIC manufacturing sectors.

1. Innovations (N)

2. Bounded learning by doing (T)

This is to account for the fact that new technologies do not achieve their full productivity potential directly upon implementation but that there are gains made by continuous adjustments.[12] Young argues that

- In the early 1960s Hong Kong learning maturity was greater than that of Singapore: $T_{HK} > T_S$
- Hong Kong found it easier to copy technologies and enter new sectors: length of $[T_{HK}, N_{HK}]$ relative to $[T_S, N_S]$
- By the early 1980s Singapore had caught up, and both economies experienced substantial learning by doing: Rightward movement of T_{HK}, T_S

[12] In the diagram an upward sloping cost curve is smaller for less developed countries since costs are lower for adopting existing technologies rather than exploring the knowledge frontier.

Table 5 CRUDE ESTIMATE OF TOTAL FACTOR PRODUCTIVITY GROWTH

Time period	Growth of Output	Growth of Labor	Growth of Capital	Average capital share	Percentage contribution of Labor	Percentage contribution of Capital	Percentage contribution of TFP Δ
Hong Kong							
71–76	0.406	0.165	0.447	0.330	0.27	0.36	0.36
76–81	0.512	0.253	0.527	0.386	0.30	0.40	0.30
81–86	0.294	0.095	0.388	0.421	0.19	0.55	0.26
86–90	0.260	0.036	0.237	0.414	0.08	0.38	0.54
71–90	1.472	0.549	1.599	0.384	0.23	0.42	0.35
Singapore							
70–75	0.454	0.247	1.005	0.553	0.24	1.22	−0.47
75–80	0.408	0.256	0.503	0.548	0.28	0.68	0.04
80–85	0.300	0.069	0.620	0.491	0.12	1.01	−0.13
85–90	0.383	0.252	0.273	0.468	0.35	0.33	0.31
70–90	1.545	0.825	2.402	0.533	0.25	0.83	−0.08

Figure 3 THE DEVELOPMENT OF HONG KONG AND SINGAPORE

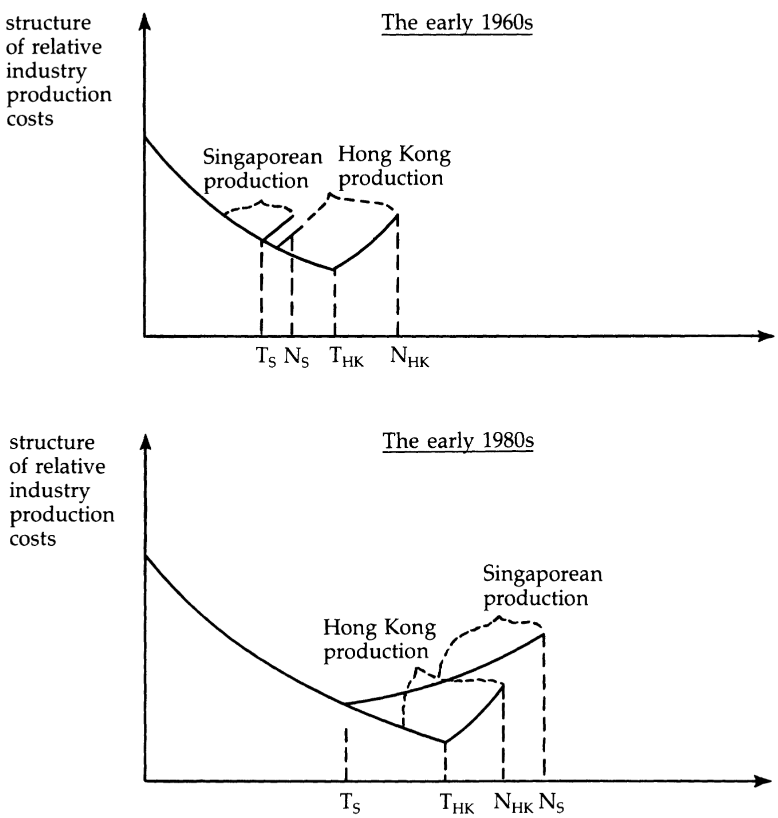


Figure 10: Estimates of total factor productivity (top) and theoretical model of invention and learning by doing (bottom) from Young (1992).