New Keynesian model

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Spring 2018

Kydland & Prescott (1982) "Time to Build and Aggregate Fluctuations" showed strength of DSGE modeling

- Small coherent model of economy
- Optimising agents, rational expectations, market clearing
- Model generated data that resembled observed data

Some shortcomings

- Volatility of hours, persistence of output

But model did remarkably well excluding a lot of presumed *sine qua nons*

- Money, nominal rigidities (i.e. stickiness), non-market clearing

Q: Does money matter?

Short-term fluctuations linked to money increase through price stickiness

- Hume, Keynes, Friedman

Strong empirical case supporting notion that money matters

- "A Monetary History of the U.S.", Friedman & Schwartz (1971)
- Empirical evidence from VAR models

Can include money into DSGE model, requires

1. Monopolistic competition
2. Role to justify existence of money (e.g. in utility)
3. Monetary authority inducing nominal shocks to economy

Model fit improves by

1. Delay/extend response economy to shock (e.g. habit persistence)
2. Adding extra shocks (e.g. preferences)

New Keynesian model addresses some of critiques on Keynesian model

- Rational expectations
- People behave optimally

Room for systematic effects of monetary policy

- Central mechanism for monetary policy is **sticky prices**
- If prices don't move in line with money; central bank can't control real money supply or interest rate

Basic features of NK model

- General equilibrium model
- Two stages of production: firms are monopolistically competitive
- Firms cannot reoptimise prices each period
- Due to price stickiness monetary policy has real effects: needs to be described by model

Basically NK model is RBC model with

1. Sticky prices
2. Monetary authority operating via interest rate feedback rule
3. Additional simplifications
    - No capital accumulation (only labour); no trend productivity growth, only stationary shocks

Why are prices sticky?

- Imperfect information
- Costs of changing prices
- Agents trust in price stability (in stable economic environment)

To describe optimal behaviour, use **Dixit-Stiglitz model**

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \tag{1}$$

Consumers maximise utility function $U(Y_t)$ over an aggregate of a continuum of differentiated goods

- θ denotes constant elasticity of substitution

Model only includes consumption goods, no capital

For each differentiated good, demand function has form

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} \tag{2}$$

P_t is the aggregate price index which is defined by

$$P_t = \left(\int_o^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \tag{3}$$

To describe price rigidity use **Calvo model**

$$P_t = \left[(1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{4}$$

$$P_t^{1-\theta} = (1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \tag{5}$$

Random fraction of firms is able to reset prices

$$1 - \alpha \tag{6}$$

Firms reset price to

$$X_t \tag{7}$$

1. All other firms keep prices unchaged
2. All firms setting new prices today set the same price.

Firms are completely symmetric: except for timing of price-setting

Prices may be fixed for many periods: Firms pick a price to maximise

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k (Y_{t+k} P_{t+k}^{\theta-1} X_t^{1-\theta} - P_{t+k}^{-1} C(Y_{t+k} P_{t+k}^{\theta} X_t^{-\theta})) \right] \quad (8)$$

$C(.)$ is the cost function
Differentiate with respect to X_t to get solution of maximisation problem

$$X_t = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)}{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} \right)} \quad (9)$$

$$X_t = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)}{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} \right)}$$

Without pricing frictions, firm sets

$$X_t = \frac{\theta}{\theta - 1} MC_t \tag{10}$$

i.e. price is markup over marginal costs

Price likely to be fixed for number of periods

- Optimal price is markup over weighted average of future marginal costs

$$(\alpha\beta)^k \tag{11}$$

Less weight on future MC because of

i Discounting

ii Lower probability for price set at t to be around at k as k increases

$$Y_{t+k} P_{t+k}^{\theta-1} \tag{12}$$

Represents aggregate factors affecting future firm demand

i Y_{t+k} increases firm will sell more; as P_{t+k} goes up, firm's relative price down and demand increases

ii Will offset discounting term (somewhat)

Have two non-linear equations for price

$$P_t^{1-\theta} = (1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \tag{13}$$

$$X_t = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} Pt + k^{\theta-1} MC_{t+k} \right)}{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} Pt + k^{\theta-1} \right)} \tag{14}$$

Not easy to solve or simulate price equation

- Use log-linear approximations taken around constant growth, zero inflation path

Zero-inflation steady-state is such that

$$X_t^* = P_t^* = P_{t-1}^* = P^* \tag{15}$$

$$\tag{16}$$

$$P_t^{1-\theta} = (1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \tag{17}$$

Becomes

$$\begin{aligned}(P^*)^{1-\theta}(1+(1-\theta)p_t) =& (1-\alpha)(P^*)^{1-\theta}(1+(1-\theta)x_t) + \\ & \alpha(P^*)^{1-\theta}(1+(1-\theta)p_{t-1})\end{aligned} \tag{18}$$

Simplifies to

$$p_t = (1-\alpha)x_t + \alpha p_{t-1} \tag{19}$$

FOC for optimal pricing is given by

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\alpha\beta)^k\left((1-\theta)Y_{t+k}P_{t+k}^{\theta-1}X_t^{-\theta}+\theta MC_{t+k}Y_{t+k}P_{t+k}^{\theta-1}X_t^{-\theta-1}\right)\right]=0 \tag{20}$$

Around steady-state we get

$$(1-\theta)Y_{t+k}P_{t+k}^{\theta-1}X_t^{-\theta} \approx (1-\theta)Y^*(P^*)^{\theta-1}(X^*)^{-\theta} \tag{21}$$
$$(1+y_{t+k}+(\theta-1)p_{t+k}-\theta x_t)$$
$$\theta MC_{t+k}Y_{t+k}P_{t+k}^{\theta-1}X_t^{-\theta-1} \approx \tag{22}$$
$$\theta MC^*Y^*(P^*)^{\theta-1}(X^*)^{-\theta-1}$$
$$(1+mc_{t+k}+y_{t+k}+(\theta-1)p_{t+k}-(1-\theta)x_t)$$

Can use that in steady-state

$$X^* = \left(\frac{\theta}{\theta - 1}\right) MC^* \tag{23}$$

Simplifies to

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k (x_t - mc_{t+k})\right] = 0 \tag{24}$$

$$x_t = (1 - \alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k \mathbb{E}_t mc_{t+k}$$

Aggregate output and price level drop out: log-price is weighted average of expected future logs of marginal costs

Reverse engineering

$$x_t = (1 - \alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k \mathbb{E}_t mc_{t+k} \tag{25}$$

can write optimal reset price as

$$x_t = (1 - \alpha\beta) mc_t + (\alpha\beta) \mathbb{E}_t x_{t+1} \tag{26}$$

Can combine this with fact that

$$p_t = (1 - \alpha) x_t + \alpha p_{t-1} \tag{27}$$

$$\frac{1}{1-\alpha}(p_t - \alpha p_{t-1}) = x_t$$

Note that

$$\pi_t = p_t - p_{t-1} \tag{28}$$

After bunch of re-arranging you get

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1+\alpha)(1-\alpha\beta)}{\alpha}(mc_t - p_t) \tag{29}$$

π_t is a function

1. Expected inflation in $t+1$
2. Ratio of marginal cost to price (real marginal cost)

This is the **New-Keynesian Phillips curve** (NKPC)

Output

Assume diminishing returns to labour production function

- Higher output reduces marginal productivity and raises marginal cost

Makes real marginal costs a function of output gap

$$mc_t - p_t = \eta x_t \tag{30}$$

Concerning x_t

$$x_t = y_t - y_t^n \tag{31}$$

y_t^n is output path in zero-inflation price friction free economy

NKPC has form

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t \tag{32}$$

Looks like traditional expectations-augmented Phillips curve.

First-order stochastic difference equation; has solution in form

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t x_{t+k} \tag{33}$$

No backward-looking element in

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t x_{t+k} \tag{34}$$

- No intrinsic inertia in inflation
- In conventional model, lagged inflation effects are statistical artifact

Original NKPC formulation has no shock/error term

- Maybe price movements not consistent with this formulation.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{35}$$

Cost-push shock added to NKPC

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{36}$$

u_t accounts for misc. shocks:

- π no longer results of just expected inflation and output gap

Central bank can no longer implement a stabilisation policy by only addressing the output gap.

NKPC links inflation to output

- Need to consider how to link output to monetary policy

NK model uses interest rates: recall exclusion of capital in model

$$Y = C \tag{37}$$

Relation between C and i comes from standard intertemporal optimization problem; consumer wants to maximise

$$\sum_{k=0}^{\infty} \left(\frac{1}{1+\beta} \right)^{k} U(C_{t+k}) \tag{38}$$

Intertemporal budget constraint given by

$$\sum_{k=0}^{\infty} \frac{\mathbb{E}_t C_{t+k}}{\left(\prod_{m=1}^{k+t} R_{t+m}\right)} = A_t + \sum_{k=0}^{\infty} \frac{\mathbb{E}_t Y_{t+k}}{\left(\prod_{m=1}^{k+t} R_{t+m}\right)} \tag{39}$$

R_t is the interest rate
Can write Lagrangian as

$$\begin{aligned} L = & \sum_{k=0}^{\infty} \left(\frac{1}{1+\beta}\right)^k U(C_{t+k}) \\ & + \lambda \left[A_t + \sum_{k=0}^{\infty} \frac{\mathbb{E}_t Y_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)} - \sum_{k=0}^{\infty} \frac{\mathbb{E}_t C_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)} \right] \end{aligned} \tag{40}$$

Derive **Euler equation** by combining FOCs for C_t and C_{t+1}

$$U'(C_t) = \mathbb{E}_t \left[\left(\frac{R_{t+1}}{1+\beta} \right) U'(C_{t+1}) \right] \tag{41}$$

Can set

$$U(C_t) = U(Y_t) = \frac{Y_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{42}$$

$$\mathbb{E}_t \left[\left(\frac{R_{t+1}}{1+\beta} \right) \left(\frac{Y_t}{Y_{t+1}} \right)^{\frac{1}{\sigma}} \right] = 1 \tag{43}$$

NB-Similar to the Real Business Cycle model this is a Constant Relative Risk Aversion (CRRA) utility from consumption

Set

$$\rho = -log\beta \tag{44}$$

Log-linearised version of the Euler equation is

$$y_t = \mathbb{E}_t y_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - \rho) \tag{45}$$

i.e. today's output depends negatively on the real interest rate

Recall that inflation equation featured output gap

$$x_t = y_t - y_t^n \tag{46}$$

Substitute into Euler equation[1]

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - \rho) + \mathbb{E}_t y_{t+1}^n - y_t^n \tag{47}$$
$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{48}$$

Natural interest rate is given by

$$r_t^n = \sigma^{-1} \mathbb{E}_t \Delta y_{t+1}^n - log\beta \tag{49}$$

[1] $\mathbb{E}_t y_{t+1}$ with $\mathbb{E}_t x_{t+1} + \mathbb{E}_t y_{t+1}^n$

r_t is a function of

$$\mathbb{E}_t \Delta y^n_{t+1} \tag{50}$$

Meaning that it is determined by

1. Technology
2. Preferences

Output gap x_t follows a first-order stochastic difference equation which has a solution of the form

$$x_t = \sigma \sum_{k=0}^{\infty} (i_{t+k} - \mathbb{E}_t \pi_{t+k+1} - r^n_{t+k}) \tag{51}$$

Policy implications

x_t has no backward-looking element; output has no intrinsic persistence

For monetary policy this means that what matters for today's output is

1. Current policy
2. All future interest rates

Central bankers should therefore take care in managing expectations about future policy

- Future interest rates are their key tool

Interpreting i_t as the short-term interest rate, and assuming that the expectations theory of the term structure holds, this model states that it is the long-term interest rates that matter for spending.

In most basic form NK model has three equations

1. New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{52}$$

2. Euler equation for output

$$x_t = \mathbb{E}_t x_{t+1} - \sigma\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right) \tag{53}$$

Only need one final equation: description of how interest rate policy is set.

Output-inflation dynamics

$$x_t = \mathbb{E}_t x_{t+1} - \sigma\big(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\big) \tag{54}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{55}$$

Can rewrite as

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \mathbb{E}_t x_{t+1} - \kappa\sigma\big(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\big) + u_t \tag{56}$$

Put in vector form

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa\sigma \end{pmatrix} \begin{pmatrix} \mathbb{E}_t x_{t+1} \\ \mathbb{E}_t \pi t + 1 \end{pmatrix} + \begin{pmatrix} \sigma(r_t^n - i_t) \\ \kappa\sigma(r_t^n - i_t) + u_t \end{pmatrix} \tag{57}$$

Have model in form

$$Z_t = A\mathbb{E}_t Z_{t+1} + BV_t \tag{58}$$

For unique stable solution the eigenvalues of A need to be less than 1

$$A = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa\sigma \end{pmatrix} \tag{59}$$

Recall that there is an eigenvector that when multiplied by $A - \lambda I$ equals a vector of zeroes, meaning that the determinants of the matrix equal zero

$$A - \lambda I = \begin{pmatrix} 1-\lambda & \sigma \\ \kappa & \beta + \kappa\sigma - \lambda \end{pmatrix} \tag{60}$$

Eigenvalues satisfy

$$P(\lambda) = (1-\lambda)(\beta + \kappa\sigma - \lambda) - \kappa\sigma = 0 \tag{61}$$

$$P(\lambda) = \lambda^2 - (1 + \beta + \kappa\sigma)\lambda + \beta = 0 \tag{62}$$

$P(\lambda)$ is a U-shaped polynomial: if $\lambda = 0$ we get

$$P(0) = \beta > 0 \tag{63}$$

$$P(1) = -\kappa\sigma < 0 \tag{64}$$

$P(\lambda)$ will be greater than 0 when λ rises above one, this implies that

1. one eigenvalue between zero and one
2. one eigenvalue greater than 1

This is a serious problem for the model: no unique stable solution; model has multiple equilibria

Two options to deal with λ issue

1. Accept multiple equilibria: analyse impact interest rate changes on output and inflation across range of different possible equilibria
2. Specify monetary policy following a particular rule; rule designed to produce a unique stable equilibrium

Taylor rule

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_x x_t \tag{65}$$

Monetary policy sets interest rate based on inflation and output gap

- Increase in π, x will increase i

Note inclusion of natural interest rate: set interest rate moves with the natural interest rate

- Rule here allows i to move with natural rate: Taylor's rule has a constant intercept

Rule can be substituted in the equation for x_t to give

$$x_t = \mathbb{E}_t x_{t+1} + \sigma \mathbb{E}_t \pi_{t+1} - \sigma \phi_\pi \pi_t - \sigma \pi_x x_t \tag{66}$$

To look at dynamics rewrite equations in matrix form

$$Z_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}; V_t = \begin{pmatrix} 0 \\ u_t \end{pmatrix} \tag{67}$$

$$Z_t = A \mathbb{E}_t Z_{t+1} + B V_t \tag{68}$$

In standard model

$$Z_t = A\mathbb{E}_t Z_{t+1} + BV_t$$

We have

$$A = \frac{1}{1+\sigma\pi_x+\kappa\sigma\phi_\pi}\begin{pmatrix}1 & \sigma(1-\beta\phi_\pi) \\ \kappa & \beta+\sigma\kappa+\beta(1+\sigma\phi_x)\end{pmatrix} \tag{69}$$

$$B = \frac{1}{1+\sigma\phi_x+\kappa\sigma\phi_\pi}\begin{pmatrix}1 & -\sigma\phi_\pi \\ \kappa & 1+\sigma\phi_x\end{pmatrix} \tag{70}$$

System is a matrix version of the first-order stochastic difference equation: can be solved in a similar fashion to give

$$Z_t = \sum_{k=0}^{\infty} A^k B \mathbb{E}_t V_{t+k} \tag{71}$$

For unique stable equilibrium absolute values of both eigenvalues of A need to be less than 1, which will be the case when

$$\phi_\pi + \frac{(1-\beta)\phi_x}{\kappa} > 1 \tag{72}$$

$\beta \approx 1$ so the condition is approximately $\phi_\pi > 1$

If the policy rule satisfies this requirement, known as the **Taylor principle**, there is a unique stable equilibrium

- Nominal interest rates must rise by more than inflation so that real rates rise in response to an increase in inflation
- Needed for stability because otherwise inflationary shocks reduces real interest rates which stimulates the economy which will further stimulate inflation

A big question for central banks of course is what is optimal to do? In general we know that central banks

- Don't like inflation
- Like to keep output on a steady path close to potential

Loss function

Central bank behaviour can be modeled using loss function

$$L_t = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t \mathbb{E}_t(\pi_{t+k}^2 + \gamma x_{t+k}^2) \tag{73}$$

x_t is the output gap
γ indicates the weight put on stabilisation relative to inflation stabilisation
κ is the coefficient on the output gap in the NKPC
θ is the elasticity of demand for firms

Quadratic loss functions are popular because differentiating them will produce linear relationships

- Quadratic loss function can also be used as an approximation to consumer utility in the NK model

Research has shown that

$$\gamma = \frac{\kappa}{\theta} \tag{74}$$

$$x_t^2 \tag{75}$$

Risk-averse consumers prefer smooth consumption paths which keeps output close to its natural rate to achieve this.

$$\pi_t^2 \tag{76}$$

Consumers don't just care about the level of consumption but also its allocation.

- With inflation, sticky prices imply different prices for symmetric goods and thus different consumption levels
- Optimality requires equal consumption of all items in the bundle; rationale for welfare effect of inflation, independent of its effect on output

Optimal policy under commitment

Suppose that the central bank can commit today to a strategy it can adopt now and in the future.

$$L = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_t \left[\frac{1}{2}(\pi_{t+k}^2 + \gamma x_{t+k}^2) + \lambda_{t+k}(\pi_{t+k} - \beta\pi_{t+k+1} - \kappa x_{t+k}) \right] \tag{77}$$

FOCs are

$$\gamma \mathbb{E}_t x_{t+k} - \kappa \mathbb{E}_t \lambda_{t+k} = 0 \tag{78}$$

$$\mathbb{E}_t \pi_{t+k} + \mathbb{E}_t \lambda_{t+k} - \mathbb{E}_t \lambda_{t+k-1} = 0 \tag{79}$$

for $t = 0, 1, 2, ...$ where $\lambda_{-1} = 0$

- There is no constraint on time $t = -1$

We get

$$\mathbb{E}_t x_{t+k} = \frac{\kappa}{\gamma}\mathbb{E}_t\lambda_{t+k} = \theta\mathbb{E}_t\lambda_{t+k} \tag{80}$$

$$\mathbb{E}_t\pi_{t+k} = \mathbb{E}_t\lambda_{t+k-1} - \mathbb{E}_t\lambda_{t+k} = -\frac{1}{\theta}\mathbb{E}_t\Delta x_{t+k} \tag{81}$$

$$\Delta\mathbb{E}_t x_{t+k} = -\theta\mathbb{E}_t\pi_{t+k} \tag{82}$$

Optimal policy **under commitment** is therefore characterised by

$$x_t = -\theta\pi_t = \theta(p_{t-1} - p_t) \tag{83}$$

$$\mathbb{E}_t\Delta x_{t+1} = -\theta\mathbb{E}_t\pi_{t+k} = \theta(p_{t+k-1} - p_{t+k}) \tag{84}$$

Considering initial price level p_{-1} we get

$$\mathbb{E}_t x_{t+k} = \theta(p_{-1} - \mathbb{E}_t p_{t+k}) \tag{85}$$

Since

$$\pi_t = p_t - p_{t-1} \tag{86}$$

$$\mathbb{E}_t x_{t+k} = \theta(p_{-1} - \mathbb{E}_t p_{t+k}) \tag{87}$$

Optimal policy is set against the price level

- Shocks will temporarily affect price level: no cumulative effect
- Inflation will be zero, on average

Policy is history dependent: policy today depends on the whole past sequence of shocks that have determined today's price level

Optimal policy under discretion

Consider scenario where a central bank cannot commit to taking a particular course of action in the future

- Can only set optimal strategy for what to do today

Optimality conditions for period t and $t+1$ are

$$x_t = -\theta \pi_t \tag{88}$$

$$\mathbb{E}_t x_t - \mathbb{E}_t x_{t+1} = -\theta \pi_{t+1} \tag{89}$$

First period conditions differ from rest

- At t, $t-1$ is gone and doesn't matter
- Account for fact that time t affect $t+1$

Under discretion policy maker always sets

$$x_t = -\theta\pi_t \tag{90}$$

Policy is set against inflation; inflation characterised by

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} - \kappa\theta\pi_t + u_t \tag{91}$$

First-order difference equation

$$\pi_t = \left(\frac{1}{1+\theta\kappa}\right)(\beta\mathbb{E}_t\pi_{t+1} + u_t) \tag{92}$$

Repeated iteration solution

$$\pi_t = \left(\frac{1}{1+\theta\kappa}\right)\sum_{k=0}^{\infty}\left(\frac{\beta}{1+\theta\kappa}\right)^k \mathbb{E}_t u_{t+k} \tag{93}$$

Cost-push shocks assumed to follow $AR(1)$ process, implying

$$\mathbb{E}_t u_{t+k} = \rho^k u_t \tag{94}$$

With

$$\begin{aligned} u_t &= \rho u_{t-1} + v_t \\ v_t &\sim N(0, \sigma^2) \end{aligned} \tag{95}$$

Using

$$\sum_{k=0}^{\infty} c^k = \frac{1}{1-c} \tag{96}$$

for $|c| < 1$, inflation becomes

$$\begin{aligned}
\pi_t &= \left(\frac{1}{1-\theta\kappa}\right)\left[\sum_{k=0}^{\infty}\left(\frac{\beta\rho}{1+\theta\kappa}\right)^k\right] u_t &(97)\\
&= \left(\frac{1}{1-\theta\kappa}\right)\left(\frac{1}{1-\frac{\beta\rho}{1+\theta\kappa}}\right) u_t &(98)\\
&= \frac{u_t}{1+\theta\kappa-\beta\rho} &(99)
\end{aligned}$$

$AR(1)$ cost-push shock implies that

$$\mathbb{E}_t x_{t+1} = \rho x_t \tag{100}$$

$$\mathbb{E}_t \pi_{t+1} = \rho \pi_t \tag{101}$$

Substitute into Euler equation along with

$$x_t = -\theta \pi_t \tag{102}$$

$$x_t = \mathbb{E}_t x_{t+1} - \sigma\big(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\big)$$

to back out what the optimal interest rate looks like

$$i_t = r_t^n + \left(\rho + \frac{(1-\rho)\theta}{\sigma}\right)\pi_t \tag{103}$$

This will be greater than 1 if

$$\frac{\theta}{\sigma} > 1 \tag{104}$$

which will hold for all reasonable parameterisations

- Inflation and thus interest rates do not depend at all on what happened in the past.

Woodford (2003) argues that policy under commitment produces superior welfare outcomes

- Private sector will anticipate future policies will be different
- Conditions at time t have the potential to improve stabilisation outcomes at time $t+1$
- Holds even if these conditions will actually no longer matter at a later time.

Have to consider the transitory cost-push shock u_t; Woodford argues expectations about shock won't affect future policy

- Short-run trade-off between inflation and the output gap; shift vertically by

$$u_t \tag{105}$$

- Central bank has to choose whether to increase inflation, have a negative output gap, or possibly a bit of both

Due to shocks people can expect central bank to pursue tighter policy from $t+1$ onwards;short-run trade-off will be shifted by the change

$$u_t + \mathbb{E}_t \pi_{t+1} \tag{106}$$

Shift will actually be smaller and thus possibly increase stabilisation: main issue is that it might not be practically feasible to pick a policy and stick to it.

Empirical issues

Central role for NKPC in NK model: relies on output gap x_t

- How to measure this gap?

Can assume that on average output tend to return to its natural rate

- Use simple trend as proxy for natural rate (e.g. HP-filter)

Proxy

$$x = y_t - y_t^n \tag{107}$$

with

$$\tilde{y}_t = y_t - y_t^{tr} \tag{108}$$

Can estimate NKPS using

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t \tag{109}$$

Cannot observe $\mathbb{E}_t \pi_{t+1}$: substitute realised π_{t+1}

- Use an instrumental variable to deal with the fact this this is a noisy estimator of what we really want.

Results often show

$$\kappa < 0 \tag{110}$$

Seems counterintuitive but we know that

1. $\Delta\pi_t$ is negatively correlated with the unemployment rate
2. Therefore positively correlated with the output gap

Given that $\beta \approx 1$, we can proxy

$$\pi_t - \beta \mathbb{E}_t \pi_{t+1} \tag{111}$$

with

$$\pi_t - \pi_{t+1} = -\Delta \pi_{t+1} \tag{112}$$

Negative sign on κ might not be that surprising: two possible reasons for failure

1. Model is wrong
2. Output gap measured with error

Gali & Gertler (1999) argue that the output gap is measured with error

- Deterministic trends do a bad job in capturing movements in the natural rate of output

Suggest using unit labour costs as proxy for marginal costs

- Proxy for real marginal costs is the labour share of income.

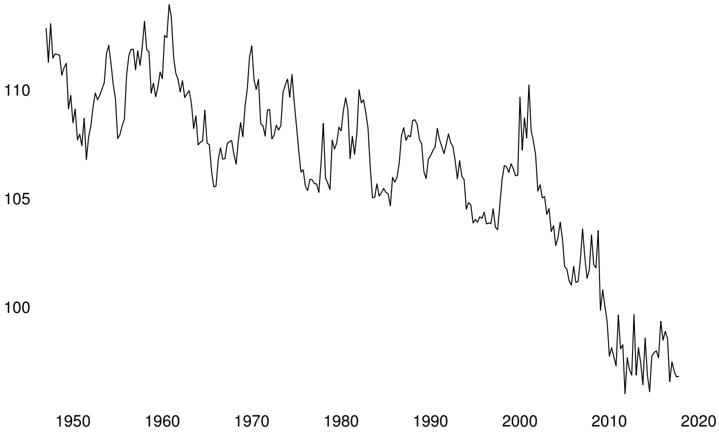
Estimate

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \gamma s_t \tag{113}$$

Find

$$\beta > 0 \tag{114}$$

Nonfarm business sector
Labour share



Rudd & Whelan (2007) show downward trend in labour share across countries

- If NKPS works well with labour share, or other measure for real marginal costs, implies that it is completely forward looking

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k{}_t s_{t+k} \tag{115}$$

Can use VAR model to forecast level of s_{t+k} and give a fitted value for the equations above.

Fit of model not really good (Rudd & Whelan, 2006); better when including lagged inflation

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} + \rho \pi_{t-1} \tag{116}$$

NKPC's main problem: does not account properly for inflation's dependence on own lags

Can use a hybrid version

$$\pi_t = \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1} + \kappa x_t \tag{117}$$

x_t measures inflationary pressure
Some theoretical weaknesses

1. Rule-of-thumb price-setters: some people set backward-looking prices, other don't (Gali-Gertler, 1999)
2. Indexation: each period some set optimal prices, others don't; non-optimising price-setters index to past inflation (Christiano et al, 2005)