Real Business Cycle

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Business cycles

Economies fluctuate over time: some things that need explaining

1. Volatility
2. Comovements
3. Persistence (autocorrelation)
4. Effect of expectations on current decisions

Two theoretical approached for explaining this

1. Market clearing
2. Non-market clearing

Two competing models

1. Real Business Cycle model (RBC)
2. New-Keynesian model (NKM)

Similarities

- Dynamic general equilibrium
- Stochastic shocks
- Forward looking expectations

Main difference concerns information and prices

- RBC: Complete and flexible
- NKM: Incomplete and sticky

Real Business Cycle model

1. Take Swan-Solow growth model
2. Insert (1) into dynamics optimisation framework
   - No more constant savings rate
3. Add shocks to total factor productivity (A)
   - Include uncertainty about shocks
4. Add leisure to account for changes in hours of work

Why real?

Equilibrium is about

- Household preferences
- Technology used by firms
- Government policy decisions

These are **real** factors

Recap: **Swan-Solow model**

$$
\begin{aligned}
Y_t &= K_t^{\alpha}(A_t N_t)^{1-\alpha} && (1)\\
K_t &= (1+\gamma)K_{t-1} + I_t && (2)\\
I_t &= sY_t && (3)\\
N_t &= (1+n)N_{t-1} && (4)\\
A_t &= (1+m)A_{t-1} && (5)\\
Y_t &\equiv C_t + I_t && (6)
\end{aligned}
$$

$$0 < \alpha < 1, \gamma > 0, 0 < s < 1$$

Fundamental mechanism of model is shocks to Total Factor Productivity (TFP)

- Recall positive correlation between GDP and productivity

Major result: Fluctuations as an equilibrium outcome

- Work harder when productivity is high: wages increase as labour becomes more productive
- Save more when productivity is high: interest rates increase as capital becomes more productive

Fluctuations in economy are not that bad

Real Business Cycle model: Assumes

1. Perfectly functioning competitive markets
2. Rational expectations

Outcomes generated by decentralized decisions of firms and households: can be replicated as solution to a social planner problem who want so maximise

$$E_t \left[\sum_{i=0}^{\infty} \beta^i (U(C_{t+i}) - V(N_{t+i})) \right] \tag{7}$$

C_t is consumption
N_t hours worked
β is the household's rate of time preference

$$U(C_t) - V(N_t) = \frac{C_t^{1-\eta}}{1-\eta} - \nu N_t \tag{8}$$

Economic constraints

$$Y_t = C_t + I_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha} \tag{9}$$
$$K_t = I_t + (1-\gamma) K_{t-1} \tag{10}$$

Technology process A_t is usually a log-linear AR(1) process

- For simplicity assume that A_t does not trend over time: economy has average growth rate of zero.

$$lnA_t = (1-\rho) lnA^* + \rho lnA_{t-1} + \epsilon_t \tag{11}$$

A^* indicates the steady-state for technology.

Solving the model

1. Formulating the Lagrangean
2. Finding the first order conditions (FOCs)
3. Log-linearisation of the FOCs
4. Finding the steady-state

Constraints

$$
\begin{aligned}
Y_t &= C_t + I_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha} \qquad (12)\\
K_t &= I_t + (1-\gamma) K_{t-1}
\end{aligned}
$$

Can combine in single equation

$$
A_t K_{t-1}^{\alpha} N_t^{1-\alpha} = C_t + K_t - (1-\gamma) K_{t-1} \qquad (13)
$$

Can formulate this as a Langrangian problem

$$
\begin{aligned}
\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i [U(C_{t+i}) - V(N_{t+i})] + \\
E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} [A_t K_{t+i-1}^{\alpha} N_t^{1-\alpha} + (1-\gamma) K_{t+i-1} - C_{t+i} - K_{t+i}]
\end{aligned} \tag{14}
$$

The Langrangian involves picking a series of values for consumption and labour, subject to satisfying a series of constraints.

Infinity

Equations sums to infinity

- Entails infinite number of first-order conditions for current and expected values of C_t, K_t, N_t.
- Can be simplified by looking at when exactly the time t and $t + n$ variables appear

Example; Capital

$$\frac{\partial \mathcal{L}}{\partial K_t} \tag{15}$$

Find when K_t appears.

$$U(C_t) - V(N_t) + \lambda_t(A_t K_{t-1}^{\alpha} N_t^{1-\alpha} - C_t - K_t + (1-\gamma)K_{t-1}) \tag{16}$$
$$+\beta E_t[\lambda_{t+1}(A_{t+1} K_t^{\alpha} N_{t+1}^{1-\alpha} + (1-\gamma)K_t)]$$

t variables only appear once

- FOCs consist of differentiating the model end setting the derivatives equal to zero

$t + n$ appear exactly as the t variables: only in expectation form and multiplied by discount β^n

- FOCs are identical to the t variables

$$Y_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha} \tag{17}$$

Differentiating we get the following FOCs

$$\frac{\partial \mathcal{L}}{\partial C_t} : U'(C_t) - \lambda_t = 0 \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial K_t} : -\lambda_t + \beta E_t \left[\lambda_{t+1} \left(\alpha \frac{Y_{t+1}}{K_t} + 1 - \gamma \right) \right] = 0 \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} : -V'(N_t) + (1-\alpha)\lambda_t \frac{Y_t}{N_t} = 0 \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : A_t K_{t-1}^{\alpha} N_t^{1-\alpha} - C_t - K_t + (1-\gamma) K_{t-1} = 0 \tag{21}$$

Keynes-Ramsey condition Now in order to make the system a bit easier to understand, it helps to define the marginal value of an additional unit of capital next year as

$$R_{t+1} = \alpha \frac{Y_{t+1}}{K_t} + 1 - \gamma \tag{22}$$

$$FOC : \lambda_t = \beta E_t(\lambda_{t+1} R_{t+1}) \tag{23}$$

This can then be combined with the FOC for consumption to give

$$U'(C_t) = \beta E_t[U'(C_{t+1}) R_{t+1}] \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = U'(C_t) - \lambda_t = 0 \tag{25}$$

$$\lambda_t = U'(C_t) \tag{26}$$

This means that

- The marginal utility of consumption must equal the marginal utility of capital
- And the marginal utility of capital must equal the expected value of capital at $t+1$ times the return of capital times a discount factor

Interpretation Keynes-Ramsey condition:
Δ decrease in C_t will lead to utility losse

$$U'(C_t)\Delta$$

Invest to get $R_{t+1}\Delta$ next period which will be worth

$$\beta E_t[U'(C_{t+1})R_{t+1}]$$

in current period's utiity

- Along an optimal path, the household must be indifferent

CCRA Consumption and Separable Consumption-Leisure

The model uses the utility function

$$U(C_t) - V(N_t) = \frac{C_t^{1-\eta}}{1-\eta} - \nu N_t \tag{27}$$

This formulation of the Constant Relative Risk Aversion (CRRA) utility from consumption and separate disutility from labour turns out to be necessary for the model to have a stable growth path solution.

The Keynes-Ramsey condition becomes

$$C_t^{-\eta} = \beta E_t(C_{t+1}^{-\eta} R_{t+1}) \tag{28}$$

And the condition for optimal worked hours becomes

$$-\nu + (1-\alpha) C_t^{-\eta} \frac{Y_t}{N_t} = 0 \tag{29}$$

$$\frac{Y_t}{N_t} = \frac{v}{1-\alpha} C_t^{\eta} \tag{30}$$

The RBC model can be defined by six equations

1. three identities describing resource constraints
2. one definition
3. and two FOCs describing optimal behaviour

Process for the technology variable is

$$lnA_t = (1-\rho)lnA^* + \rho lnA_{t-1} + \epsilon_t \tag{31}$$

$$
\begin{aligned}
Y_t &= C_t + I_t && (32)\\
Y_t &= A_t K_{t-1}^{\alpha} N_t^{1-\alpha} && (33)\\
K_t &= I_t + (1-\gamma) K_{t-1} && (34)\\
R_t &= \alpha \frac{Y_t}{K_{t-1}} + 1 - \gamma && (35)\\
C_t^{-\eta} &= \beta E_t (C_{t+1}^{-\eta} R_{t+1}) && (36)\\
\frac{Y_t}{N_t} &= \frac{v}{1-\alpha} C_t^{\eta} && (37)
\end{aligned}
$$

Taking log differences (Δ *logs*)

$$
\begin{aligned}
Y_t = 2X_t &\Leftrightarrow y = x \\
Y_t = 2X_t Z_t &\Leftrightarrow y = x + z \\
Y_t = 2X_t Z_t^{-3} &\Leftrightarrow y = x - 3z \\
Y_{t+1} = X_{t+1} + Z_{t+1} &\Leftrightarrow y = x\frac{X_t}{Y_t} + z\frac{Z_t}{Y_t} \\
Y_{t+1} = X_{t+1} + a &\Leftrightarrow y = x\frac{X_t}{Y_t}
\end{aligned}
$$

Log-linearisation

Nonlinear systems can generally not be solved analytically

- Solution can be approximated using corresponding set of linear equations: Use Taylor series

Non-linear function $F(x_t, y_t)$ can be approximated around any point x_t^*, y_t^* using

$$\begin{aligned}
F(x_t, y_t) = {} & F(x_t^*, y_t^*) \\
& + F_x(x_t^*, y_t^*)(x_t - x_t^*) \\
& + F_y(x_t^*, y_t^*)(y_t - y_t^*) \\
& + F_{xx}(x_t^*, y_t^*)(x_t - x_t^*)^2 \\
& + F_{xy}(x_t^*, y_t^*)(x_t x_t^*)(y_t - y_t^*) \\
& + F_{yy}(x_t^*, y_t^*)(y_t - y_t^*) + \ldots
\end{aligned} \tag{38}$$

If the gap between (x_t, y_t) and (x_t^*, y_t^*) is small, then terms in second and higher order powers and cross-terms will all be very small and can be ignored leaving something like

$$F(x_t, y_t) \approx \alpha + \beta_1 x_t + \beta_2 y_t \tag{39}$$

If we linearise around point that is far away from (x_t, y_t), then the approximation will not be accurate.

Steady-state path

DSGE models take log-linearise variables around steady-state path

- Around this path all real variables grow at same rate

Stochastic economy will on average fluctuate around values given by steady state path

- Can get therefore an accurate approximation
- Provides set of linear equations in log-deviations of variables from steady-state values

$$x_t = ln\, X_t - ln\, X^*$$

Recall: log-differences are approximately percentage deviations

$$ln\, X - ln\, Y \approx \frac{X - Y}{Y}$$

Approach provides

- System of variables expressed in percentage deviations from steady-state path
- System that can be thought of as business-cycle component of model
- Coefficients are elasticities (also easy with IRF)
- Easy to implement

Write variable as

$$X_t = X^* \frac{X_t}{X^*} = X^* e^{x_t} \tag{40}$$

First-order Taylor approximation for e^{x_t} given by

$$e^{x_t} \approx 1 + x_t \tag{41}$$

Can write variable as

$$X_t \approx X^*(1 + x_t) \tag{42}$$

Can set

$$x_t y_t = 0 \tag{43}$$

Multiplying small deviations from steady-state will produce term close to zero anyway.

$$X_t Y_t \approx X^* Y^* (1 + x_t)(1 + y_t) \tag{44}$$
$$\approx X^* Y^* (1 + x_t + y_t) \tag{45}$$

Example; Income

$$Y_t = C_t + I_t \tag{46}$$

Rewrite

$$Y^* e^{y_t} = C^* e^{c_t} + I^* e^{i_t} \tag{47}$$

Use first-order approximation

$$Y^*(1 + y_t) = C^*(1 + c_t) + I^*(1 + i_t) \tag{48}$$

Steady-state terms must obey identities

$$Y^* = C^* + I^* \tag{49}$$

Canceling terms on both sides

$$Y^* y_t = C^* c_t + I^* i_t \tag{50}$$

Which we can write

$$y_t = \frac{C^*}{Y^*} c_t + \frac{I^*}{Y^*} i_t \tag{51}$$

Example: Production function

$$Y_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha} \tag{52}$$

Re-write in terms of steady-state and log deviations

$$Y^* e^{y_t} = (A^* e^{a_t})(K^*)^{\alpha} e^{\alpha k_{t-1}} (N^*)^{1-\alpha} e^{(1-\alpha) n_t} \tag{53}$$

Steady-state must obey identities

$$Y^* = A^* (K^*)^{\alpha} (N^*)^{1-\alpha} \tag{54}$$

Canceling terms we get

$$e^{y_t} = e^{a_t} e^{\alpha k_{t-1}} e^{(1-\alpha) n_t} \tag{55}$$

Use first-order Taylor approximation

$$(1 + y_t) = (1 + \alpha_t)(1 + \alpha k_{t-1})(1 + (1 - \alpha) n_t) \tag{56}$$

Ignore cross-products of log-deviations: simplifies to

$$y_t = a_t + \alpha k_{t-1} + (1 - \alpha) n_t \tag{57}$$

Log-linearised system

$$
\begin{aligned}
y_t &= \frac{C^*}{Y^*} c_t + \frac{I^*}{Y^*} i_t \\
y_t &= a_t + \alpha k_{t-1} + (1-\alpha) n_t \\
k_t &= \frac{I^*}{K^*} i_t + (1-\gamma) k_{t-1} \\
n_t &= y_t - \eta c_t \\
c_t &= E_t c_{t+1} - \frac{1}{\eta} E_t r_{t+1} \\
r_t &= \left(\frac{\alpha}{R^*} \frac{Y^*}{K^*} \right) (y_t - k_{t-1}) \\
a_t &= \rho a_{t-1} + \epsilon_t
\end{aligned}
$$

Technology is assumed to be given by

$$a_t = \rho a_{t-1} + \epsilon_t \tag{58}$$

Entail no trend growth in economy

- Implies that steady-state variables are constants

Calculating steady-state

Three steady-state variables that need to be calculated; involves terms

$$\frac{C^*}{Y^*}, \frac{I^*}{Y^*}, \frac{I^*}{K^*}, \frac{\alpha}{R^*}\frac{Y^*}{K^*} \tag{59}$$

Take original non-linearised RBC system: figure out what it looks like along zero growth path

$$y_t = y_{t+1} = y^* \tag{60}$$

Therefore

$$\frac{y_t}{y_{t+1}} = 1 \tag{61}$$

Start with interest rate

- Linked to consumption behaviour via Keynes-Ramsey condition

$$C_t^{-\eta} = \beta E_t(C_{t+1}^{-\eta} R_{t+1}) \tag{62}$$

$$1 = \beta E_t \left(\left(\frac{C_t}{C_{t+1}} \right)^{\eta} R_{t+1} \right) \tag{63}$$

Have no trend growth in technology

- Constant values for steady-state consumption, investment, and output

In steady-state we have

$$C_t^* = C_{t+1}^* = C^* \tag{64}$$

$$R^* = \beta^{-1} \tag{65}$$

In a no-growth economy, the rate of return on capital is determined by the rate of time preference.

Let's look at the rate of return on capital

$$R_t = \alpha \frac{Y_t}{K_{t-1}} + 1 - \gamma \tag{66}$$

In steady-state we have

$$R^* = \beta^{-1} = \alpha \frac{Y^*}{K^*} + 1 - \gamma \tag{67}$$

So we get

$$\frac{Y^*}{K^*} = \frac{\beta^{-1} + \gamma - 1}{\alpha} \tag{68}$$

Together with the steady-state interest equation this tells us that

$$\frac{\alpha}{R^*} \frac{Y^*}{K^*} = \alpha\beta \left(\frac{\beta^{-1} + \gamma - 1}{\alpha} \right) \tag{69}$$

$$= 1 - \beta(1 - \gamma) \tag{70}$$

Now we only have to find the ratios for

- investment-capital
- investment-output

Here we can use the identity

$$K_t = I_t + (1-\gamma)K_{t-1} \tag{71}$$

$$K^* = I^* + (1-\gamma)K^* \tag{72}$$

$$K^* = I^* + K^* - \gamma K^* \tag{73}$$

$$I^* = \gamma K^* \tag{74}$$

$$\frac{I^*}{K^*} = \gamma \tag{75}$$

This identity is in steady-state and combined with the fact that $K_t^* = K_{t-1}^* = K^*$ we get

$$\frac{I^*}{K^*} = \gamma \tag{76}$$

This can be combined with the previous steady-state ratio to give $\frac{Y^*}{K^*} = \frac{\beta^{-1}+\gamma-1}{\alpha}$

$$\frac{I^*}{Y^*} = \frac{\frac{I^*}{K^*}}{\frac{Y^*}{K^*}} = \frac{\alpha\gamma}{\beta^{-1}+\gamma-1} \tag{77}$$

From this it follows that the consumption-output ratio must be

$$\frac{C^*}{Y^*} = 1 - \frac{\alpha\gamma}{\beta^{-1}+\gamma-1} \tag{78}$$

Final system

$$y_t = \left(1 - \frac{\alpha\gamma}{\beta^{-1} + \gamma - 1}\right) c_t + \left(\frac{\alpha\gamma}{\beta^{-1} + \gamma - 1}\right) i_t \tag{79}$$

$$y_t = a_t + \alpha k_{t-1} + (1-\alpha) n_t \tag{80}$$

$$k_t = \gamma i_t + (1-\gamma) k_{t-1} \tag{81}$$

$$n_t = y_t - \eta c_t \tag{82}$$

$$c_t = E_t c_{t+1} - \frac{1}{\eta} E_t r_{t+1} \tag{83}$$

$$r_t = (1 - \beta(1-\gamma))(y_t - k_{t-1}) \tag{84}$$

$$a_t = \rho a_{t-1} + \epsilon_t \tag{85}$$

Estimation

1. Make assumption about underlying parameter values
2. Use Binder-Pesarant algorith to get reduced-form solution
3. Simulate model

1. Perfect markets and rational expectations
    - Markets are not always competitive and people are not always rational (in their economic decisions)
    - RBC model should be seen as a benchmark against which more complicated models can be assessed.
    - Separate modeling of the decisions of firms and households to account for imperfect competition can be done
2. Monetary and fiscal policy
    - RBC models exhibit complete monetary neutrality, so there is no role at all for monetary policy, something which many people think is crucial to understanding the macroeconomy
    - Most models build on the RBC approach introducing mechanisms that are allowed to have Keynesian effects, such as sticky prices and wages
3. Skepticism about technology shocks
    - RBC models give primacy to technology shocks as the source of economic fluctuations (all variables apart from A_t are deterministic). But what are these shocks?
    - Link between long-term growth and TFP

Can check the parameterizing of the the model and simulate and check the impluse response functions. The following graphs are based on a model with parameter values intended for the analysis of quarterly time series

$$\alpha = \frac{1}{3} \tag{86}$$

$$\beta = 0.99 \tag{87}$$

$$\gamma = 0.015 \tag{88}$$

$$\rho = 0.95 \tag{89}$$

$$\eta = 1 \tag{90}$$

Figure shows a 200-period simulation of the model and illustrates the main feature of the RBC model, namely that it can generate business cycles that don't look too far-fetched. We can notice two things here

1. The model roughly matches the observed fluctuations in output
2. The model reflects the fact that investment cycles are more volatile than consumption

Part of the early hype surrounding RBC models stemmed from the idea that the model contained important propagation mechanisms turning technology shocks into business cycles. The idea behind this is that increases in technology would lead to extra output through higher capital accumulation and by inducing people to work more. This entails, as suggested in early research, that in a world with identical technology level one would expect the RBC model still to generate business cycles. However, these propagation mechanisms are quite weak as shown in figure which illustrates that fluctuations in output follow fluctuations in technology quite closely.