Vector Autoregression: Examples

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Stock & Watson (2001) Effect of monetary policy shocks.
VAR model can be useful from two perspectives

1. Scientific
   - Monetary policy co-moves with lots of other macro variables
   - Only by identifying the structural or exogenous shocks to policy can we discover its true effects

2. Policy
   - Can help answer the question "if I choose to raise interest rates by an extra quarter point today, what is likely to happen over the next year to inflation and output relative to the case where I keep rates unchanged?"
   - This is basically a question about impulse responses

Quarterly data, three variables

1. inflation π_t
2. unemployment rate u_t
3. federal funds rate i_t

Lower-triangular causal chain

$$AZ_t = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \pi_t \\ u_t \\ i_t \end{pmatrix} = BZ_{t-1} + \epsilon_t \tag{1}$$

Identifying assumptions

1. Inflation depends only on lagged values of the other variables (sticky prices?)
2. Unemployment depends on contemporaneous inflation but not the funds rate
3. The funds rate depends on both contemporaneous inflation and unemployment

Kilian (2009) Oil price shocks

1. What is an oil price shock?
2. Are there different type of shocks?

Kilian identifies three types of shock

1. Supply: oil production growth rate $\Delta prod_t$
2. Demand: global demand measured by real global economic activity rea_t
3. Speculation: in oil price market, measured by real oil price rpo_t

$$z_t = (\Delta prod_t, rea_t, rpo_t)' \tag{2}$$

$$A_0 z_t = \alpha + \sum_{i=1}^{24} A_i z_{t-1} + \epsilon_t$$

$$A_0 = \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix}$$

Identifying assumptions

1. Oil production does not respond within the month to world demand and oil prices
2. World demand is affected within the month by oil production, but not by oil prices
3. Oil prices respond immediately to oil production and world demand

Reduced-form model

$$z_t = A_0^{-1}\alpha + A_0^{-1}\sum_{i=1}^{24} A_i z_{t-1} + A_0^{-1}\epsilon_t$$

A_0 is lower-triangular matrix, so is A_0^{-1} as well.
Relation between reduced-form shocks e_t and structural shocks $A_0^{-1}\epsilon_t$

$$\begin{pmatrix} e_t^{\Delta prod} \\ e_t^{rea} \\ e_t^{rpo} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \epsilon_t^{\Delta prod} \\ \epsilon_t^{rea} \\ \epsilon_t^{rpo} \end{pmatrix}$$

Practically this entails that

1. Oil production reduced form shock is a structural shock
2. Economic activity reduced form shock is combination of structural oil shock and structural activity shock
3. Reduced form oil price shock is combination of all three structural shocks

Structural model

$$AY_t = BY_{t-1} + C\epsilon_t$$

Needs 18 identifying restrictions: $2n^2 = 18$

1. Assuming contemporaneous interaction between variables $C = I$ (9)
2. Zero restrictions/lower diagonal assumtpion on A_0 (3)
3. Unit coefficient normalisation on diagonal A_0 (3)
4. Orthogonal structural shocks: off-diagonal elements of $\sum$ are 0 (3)

Recall Vector Moving Average representation

$$Y_t = e_t + Ae_{t-1} + A^2 e_{t-2} + A^3 e_{t-3} + ... + A^t e_0$$

One can repeat this calculation three times, each time with only one type of shock turned on and the other set to zero. Adding these up, one will get the realized values of Y_t. Alternatively, one can do a dynamic simulation of the model

$$Y = AY_{t-1} + \epsilon_t$$

Here we let ϵ_t represent one of the realised shocks, setting the others to zero

Brunner (2002) El Niño and world commodity prices

$$
\begin{aligned}
ENSO_t &= \mu_s + A_{11}(L)ENSO_{t-1} + \epsilon_t && (3)\\
X_t &= \phi_s + A_{21}(L)ENSO_t + A_{22}(L)X_{t-1} + \eta_t \\
\begin{bmatrix} \epsilon_t \\ \eta_t \end{bmatrix} &\sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sum_\eta \end{bmatrix} \right) && (4)
\end{aligned}
$$

$$
X_t = [\pi_t^{cp} - \pi_t^g \pi_t^g \Delta y_t] \tag{5}
$$

Identifying assumptions

1. ENSO events are not influences by contemporaneously economic events
2. Assumption that ENSO events are exogenous can be tested
3. $\sum_\eta$ is expecte to be nondiagonal