New Keynesian model

School of Economics, University College Dublin

Spring 2018

Kydland & Prescott (1982) "Time to Build and Aggregate Fluctuations" showed strength of DSGE modeling

- Small coherent model of economy
- Optimising agents, rational expectations, market clearing
- Model generated data that resembled observed data

Some shortcomings

- Volatility of hours, persistence of output

But model did remarkably well excluding a lot of presumed *sine qua nons*

- Money, nominal rigidities (i.e. stickiness), non-market clearing

Q: Does money matter?

Short-term fluctuations linked to money increase through price stickiness

- Hume, Keynes, Friedman

Strong empirical case supporting notion that money matters

- "A Monetary History of the U.S.", Friedman & Schwartz (1971)
- Empirical evidence from VAR models

Can include money into DSGE model, requires

1. Monopolistic competition
2. Role to justify existence of money (e.g. in utility)
3. Monetary authority inducing nominal shocks to economy

Model fit improves by

1. Delay/extend response economy to shock (e.g. habit persistence)
2. Adding extra shocks (e.g. preferences)

New Keynesian model addresses some of critiques on Keynesian model

- Rational expectations
- People behave optimally

Room for systematic effects of monetary policy

- Central mechanism for monetary policy is **sticky prices**
- If prices don't move in line with money; central bank can't control real money supply or interest rate

Basic features of NK model

- General equilibrium model
- Two stages of production: firms are monopolistically competitive
- Firms cannot reoptimise prices each period
- Due to price stickiness monetary policy has real effects: needs to be described by model

Basically NK model is RBC model with

1. Sticky prices
2. Monetary authority operating via interest rate feedback rule
3. Additional simplifications
    - No capital accumulation (only labour); no trend productivity growth, only stationary shocks

Why are prices sticky?

- Imperfect information
- Costs of changing prices
- Agents trust in price stability (in stable economic environment)

To describe optimal behaviour, use **Dixit-Stiglitz model**

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \tag{1}$$

Consumers maximise utility function $U(Y_t)$ over an aggregate of a continuum of differentiated goods

- θ denotes constant elasticity of substitution

Model only includes consumption goods, no capital

For each differentiated good, demand function has form

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} \tag{2}$$

P_t is the aggregate price index which is defined by

$$P_t = \left(\int_o^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \tag{3}$$

To describe price rigidity use **Calvo model**

$$P_t = \left[(1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{4}$$

$$P_t^{1-\theta} = (1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}$$

Random fraction of firms is able to reset prices

$$1 - \alpha \tag{5}$$

Firms reset price to

$$X_t \tag{6}$$

1. All other firms keep prices unchaged
2. All firms setting new prices today set the same price.

Firms are completely symmetric: except for timing of price-setting

Prices may be fixed for many periods: firms pick price to maximise

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\alpha\beta)^k(Y_{t+k}P_{t+k}^{\theta-1}X_t^{1-\theta}-P_{t+k}^{-1}C(Y_{t+k}P_{t+k}^{\theta}X_t^{-\theta}))\right] \quad (7)$$

$C(.)$ is the cost function
Differentiate with respect to X_t to get solution of maximisation problem

$$X_t=\frac{\theta}{\theta-1}\frac{\mathbb{E}_t\left(\sum_{k=0}^{\infty}(\alpha\beta)^kY_{t+k}P_{t+k}^{\theta-1}MC_{t+k}\right)}{\mathbb{E}_t\left(\sum_{k=0}^{\infty}(\alpha\beta)^kY_{t+k}P_{t+k}^{\theta-1}\right)} \quad (8)$$

$$X_t = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)}{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} \right)}$$

Without pricing frictions, firm sets

$$X_t = \frac{\theta}{\theta - 1} MC_t \tag{9}$$

i.e. price is markup over marginal costs

Price likely to be fixed for number of periods

- Optimal price is markup over weighted average of future marginal costs

$$(\alpha\beta)^k \tag{10}$$

Less weight on future MC because of

i Discounting

ii Lower probability for price set at t to be around at k as k increases

$$Y_{t+k} P_{t+k}^{\theta-1} \tag{11}$$

Represents aggregate factors affecting future firm demand

i Y_{t+k} increases firm will sell more; as P_{t+k} goes up, firm's relative price down and demand increases

ii Will offset discounting term (somewhat)

Have two non-linear equations for price

$$P_t^{1-\theta} = (1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \tag{12}$$

$$X_t = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} Pt + k^{\theta-1} MC_{t+k} \right)}{\mathbb{E}_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} Pt + k^{\theta-1} \right)} \tag{13}$$

Not easy to solve or simulate price equation

- Use log-linear approximations taken around constant growth, zero inflation path

Zero-inflation steady-state is such that

$$X_t^* = P_t^* = P_{t-1}^* = P^* \tag{14}$$

$$\tag{15}$$

$$P_t^{1-\theta} = (1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \tag{16}$$

Becomes

$$\begin{aligned}(P^*)^{1-\theta}(1+(1-\theta)p_t) =& (1-\alpha)(P^*)^{1-\theta}(1+(1-\theta)x_t)+ \\ & \alpha(P^*)^{1-\theta}(1+(1-\theta)p_{t-1})\end{aligned} \tag{17}$$

Simplifies to

$$p_t = (1-\alpha)x_t + \alpha p_{t-1} \tag{18}$$

FOC for optimal pricing is given by

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\alpha\beta)^k\left((1-\theta)Y_{t+k}P_{t+k}^{\theta-1}X_t^{-\theta}+\theta MC_{t+k}Y_{t+k}P_{t+k}^{\theta-1}X_t^{-\theta-1}\right)\right]=0 \tag{19}$$

Around steady-state we get

$$(1-\theta)Y_{t+k}P_{t+k}^{\theta-1}X_t^{-\theta}\approx(1-\theta)Y^*(P^*)^{\theta-1}(X^*)^{-\theta} \tag{20}$$
$$(1+y_{t+k}+(\theta-1)p_{t+k}-\theta x_t)$$
$$\theta MC_{t+k}Y_{t+k}P_{t+k}^{\theta-1}X_t^{-\theta-1}\approx \tag{21}$$
$$\theta MC^*Y^*(P^*)^{\theta-1}(X^*)^{-\theta-1}$$
$$(1+mc_{t+k}+y_{t+k}+(\theta-1)p_{t+k}-(1-\theta)x_t)$$

Can use that in steady-state

$$X^* = \left(\frac{\theta}{\theta - 1}\right) MC^* \tag{22}$$

Simplifies to

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k (x_t - mc_{t+k})\right] = 0 \tag{23}$$

$$x_t = (1 - \alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k \mathbb{E}_t mc_{t+k}$$

Aggregate output and price level drop out: log-price is weighted average of expected future logs of marginal costs

Calvo model pricing dynamics can be summarised by

$$
\begin{aligned}
p_t &= (1-\alpha)x_t + \alpha p_{t-1} \\
x_t &= (1-\alpha\beta)\sum_{k=0}^{\infty}(\alpha\beta)^k \mathbb{E}_t mc_{t+k}
\end{aligned}
$$

- p_t can be solved using Binder-Pesaran method
- x_t has infinite sum: describes standard solution to first-order stochastic difference equation

Reverse engineering

$$x_t = (1 - \alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k \mathbb{E}_t mc_{t+k} \tag{24}$$

can write optimal reset price as

$$x_t = (1 - \alpha\beta) mc_t + (\alpha\beta) \mathbb{E}_t x_{t+1} \tag{25}$$

Can combine this with fact that

$$p_t = (1 - \alpha) x_t + \alpha p_{t-1} \tag{26}$$

$$\frac{1}{1 - \alpha} (p_t - \alpha p_{t-1}) = x_t$$

Inflation rate given by

$$\pi_t = p_t - p_{t-1} \tag{27}$$

After bunch of re-arranging you get

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1+\alpha)(1-\alpha\beta)}{\alpha}(mc_t - p_t) \tag{28}$$

π_t is a function

1. Expected inflation in $t+1$
2. Real marginal cost

This is the **New-Keynesian Phillips curve** (NKPC)

Output

Assume diminishing returns to labour production function

- Higher output reduces marginal productivity and raises marginal cost

Makes real marginal costs a function of output gap

$$mc_t - p_t = \eta x_t \tag{29}$$

Concerning x_t

$$x_t = y_t - y_t^n \tag{30}$$

y_t^n is output path in zero-inflation price friction free economy

NKPC has form

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t \tag{31}$$

Looks like traditional expectations-augmented Phillips curve.

First-order stochastic difference equation; has solution in form

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t x_{t+k} \tag{32}$$

No backward-looking element in

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t x_{t+k} \tag{33}$$

- No intrinsic inertia in inflation
- Lagged inflation effects statistical artifacts in conventional models

Original NKPC formulation has no shock/error term

- Maybe price movements not consistent with this formulation.

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{34}$$

Can add **cost-push shock** NKPC

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{35}$$

u_t accounts for misc. shocks:

- π no longer results of just expected inflation and output gap

Central bank can no longer implement a stabilisation policy by only addressing the output gap.

NKPC links inflation to output

- Need to consider how to link output to monetary policy

NK model uses interest rates: recall exclusion of capital in model

$$Y = C \tag{36}$$

Relation between C and i comes from standard intertemporal optimization problem; consumer wants to maximise

$$\sum_{k=0}^{\infty} \left(\frac{1}{1+\beta} \right)^{k} U(C_{t+k}) \tag{37}$$

Intertemporal budget constraint given by

$$\sum_{k=0}^{\infty} \frac{\mathbb{E}_t C_{t+k}}{\left(\prod_{m=1}^{k+t} R_{t+m}\right)} = A_t + \sum_{k=0}^{\infty} \frac{\mathbb{E}_t Y_{t+k}}{\left(\prod_{m=1}^{k+t} R_{t+m}\right)} \tag{38}$$

R_t is the interest rate
Can write Lagrangian as

$$\begin{aligned}\mathcal{L} = & \sum_{k=0}^{\infty} \left(\frac{1}{1+\beta}\right)^k U(C_{t+k}) \\ & + \lambda \left[A_t + \sum_{k=0}^{\infty} \frac{\mathbb{E}_t Y_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)} - \sum_{k=0}^{\infty} \frac{\mathbb{E}_t C_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)}\right]\end{aligned} \tag{39}$$

Derive **Euler equation** by combining FOCs for C_t and C_{t+1}

$$U'(C_t) = \mathbb{E}_t \left[\left(\frac{R_{t+1}}{1+\beta} \right) U'(C_{t+1}) \right] \tag{40}$$

Can set

$$U(C_t) = U(Y_t) = \frac{Y_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{41}$$

$$\mathbb{E}_t \left[\left(\frac{R_{t+1}}{1+\beta} \right) \left(\frac{Y_t}{Y_{t+1}} \right)^{\frac{1}{\sigma}} \right] = 1 \tag{42}$$

Similar to the Real Business Cycle model this is a Constant Relative Risk Aversion (CRRA) utility from consumption

Set

$$\rho = -log \beta \tag{43}$$

Log-linearised version of the Euler equation is

$$y_t = \mathbb{E}_t y_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - \rho) \tag{44}$$

i.e. today's output depends negatively on the real interest rate

Recall that inflation equation featured output gap

$$x_t = y_t - y_t^n \tag{45}$$

Can rewrite Euler equation as

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - \rho) + \mathbb{E}_t y_{t+1}^n - y_t^n \tag{46}$$

$$\tag{47}$$

More naturally this becomes

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{48}$$

$$r_t^n = \sigma^{-1} \mathbb{E}_t \Delta y_{t+1}^n - log \beta \tag{49}$$

This defines a 'natural' real interest rate r_t which is consistent with

$$x_t = \mathbb{E}_t x_{t+1} \tag{50}$$

and a function of

$$\mathbb{E}_t \Delta y^n_{t+1} \tag{51}$$

Meaning that it is determined by

1. Technology
2. Preferences

Output gap x_t follows a first-order stochastic difference equation which has a solution of the form

$$x_t = \sigma \sum_{k=0}^{\infty} (i_{t+k} - \mathbb{E}_t \pi_{t+k+1} - r^n_{t+k}) \tag{52}$$

Policy implications

No backward-looking elements in x_t

- Output has no intrinsic persistence

Implication for monetary policy is that what matters for today's output is

1. Current policy
2. All future interest rates

Central bankers should therefore take care in managing expectations about future policy

- Future interest rates are their key tool

Interpreting i_t as the short-term interest rate, and assuming that the expectations theory of the term structure holds, this model states that it is the long-term interest rates that matter for spending.

In most basic form NK model has three equations

1. New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{53}$$

2. Euler equation for output

$$x_t = \mathbb{E}_t x_{t+1} - \sigma\big(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\big) \tag{54}$$

Only need one final equation: description of how interest rate policy is set.

Output-inflation dynamics

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{55}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{56}$$

Can rewrite as

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \mathbb{E}_t x_{t+1} - \kappa\sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) + u_t \tag{57}$$

Put in vector form

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa\sigma \end{pmatrix} \begin{pmatrix} \mathbb{E}_t x_{t+1} \\ \mathbb{E}_t \pi t + 1 \end{pmatrix} + \begin{pmatrix} \sigma(r_t^n - i_t) \\ \kappa\sigma(r_t^n - i_t) + u_t \end{pmatrix} \tag{58}$$

Have model in form

$$Z_t = A\mathbb{E}_t Z_{t+1} + BV_t \tag{59}$$

Unique stable solution requires eigenvalues A to be less than 1

$$A = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa\sigma \end{pmatrix} \tag{60}$$

Recall that there is an eigenvector that when multiplied by $A - \lambda I$ equals a vector of zeroes, meaning that the determinants of the matrix equal zero

$$A - \lambda I = \begin{pmatrix} 1-\lambda & \sigma \\ \kappa & \beta + \kappa\sigma - \lambda \end{pmatrix} \tag{61}$$

Eigenvalues satisfy

$$\begin{aligned} P(\lambda) &= (1-\lambda)(\beta + \kappa\sigma - \lambda) - \kappa\sigma = 0 \\ P(\lambda) &= \lambda^2 - (1 + \beta + \kappa\sigma)\lambda + \beta = 0 \end{aligned} \tag{62}$$

$P(\lambda)$ is a U-shaped polynomial, we have that

$$P(0) = \beta > 0 \tag{63}$$

$$P(1) = -\kappa\sigma < 0 \tag{64}$$

$P(\lambda) > 0$ when $\lambda > 1$; this implies

1. one eigenvalue between zero and one
2. one eigenvalue greater than 1

Serious problem for the model: no unique stable solution; model has multiple equilibria

Two options to deal with λ issue

1. Accept multiple equilibria: analyse impact interest rate changes on output and inflation across range of different possible equilibria
2. Specify monetary policy following a particular rule; rule designed to produce a unique stable equilibrium

Taylor rule

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_x x_t \tag{65}$$

Interest rate based on inflation and output gap

- Increase in π, x will increase i

Note inclusion of natural interest rate: set interest rate moves with the natural interest rate

- Rule here allows i to move with natural rate: Taylor's rule has a constant intercept

Rule can be substituted in the equation for x_t to give

$$x_t = \mathbb{E}_t x_{t+1} + \sigma \mathbb{E}_t \pi_{t+1} - \sigma \phi_\pi \pi_t - \sigma \phi_x x_t \tag{66}$$

To look at dynamics rewrite equations in matrix form

$$Z_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}; V_t = \begin{pmatrix} 0 \\ u_t \end{pmatrix} \tag{67}$$

$$Z_t = A \mathbb{E}_t Z_{t+1} + B V_t \tag{68}$$

In standard model

$$Z_t = A\mathbb{E}_t Z_{t+1} + BV_t$$

We have

$$A = \frac{1}{1 + \sigma\pi_x + \kappa\sigma\phi_\pi} \begin{pmatrix} 1 & \sigma(1 - \beta\phi_\pi) \\ \kappa & \beta + \sigma\kappa + \beta(1 + \sigma\phi_x) \end{pmatrix} \tag{69}$$

$$B = \frac{1}{1 + \sigma\phi_x + \kappa\sigma\phi_\pi} \begin{pmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1 + \sigma\phi_x \end{pmatrix} \tag{70}$$

System is a matrix version of the first-order stochastic difference equation: can be solved in a similar fashion to give

$$Z_t = \sum_{k=0}^{\infty} A^k B \mathbb{E}_t V_{t+k} \tag{71}$$

For unique stable equilibrium absolute values of both eigenvalues of A need to be less than 1, which will be the case when

$$\phi_\pi + \frac{(1-\beta)\phi_x}{\kappa} > 1 \tag{72}$$

$\beta \approx 1$ so the condition is approximately $\phi_\pi > 1$

If the policy rule satisfies this requirement, known as the **Taylor principle**, there is a unique stable equilibrium

- Nominal interest rates must rise by more than inflation so that real rates rise in response to an increase in inflation
- Needed for stability because otherwise inflationary shocks reduces real interest rates which stimulates the economy which will further stimulate inflation

A big question for central banks of course is what is optimal to do? In general we know that central banks

- Don't like inflation
- Like to keep output on a steady path close to potential output

Central bank behaviour can be modeled using **loss function**

$$L_t = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\mathbb{E}_t(\pi_{t+k}^2 + \gamma x_{t+k}^2) \tag{73}$$

x_t is the output gap
γ weight put on output stabilisation relative to inflation stabilisation

Research has shown that

$$\gamma = \frac{\kappa}{\theta} \tag{74}$$

κ is coefficient on output gap in NKPC
θ is elasticity of demand for firms

$$x_t^2$$

Risk-averse consumers prefer smooth consumption paths which keeps output close to its natural rate to achieve this.

$$\pi_t^2$$

Consumers don't just care about the level of consumption but also its allocation.

- With inflation, sticky prices imply different prices for symmetric goods and thus different consumption levels
- Optimality requires equal consumption of all items in the bundle; rationale for welfare effect of inflation, independent of its effect on output

Central bank has two options in setting policy

1. Under commitment: solves problem at beginning of time
2. Under discretion: solves problem each period

Optimal policy under commitment

Suppose that the central bank can commit today to a strategy it can adopt now and in the future.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_t \left[-\frac{1}{2}(\pi_{t+k}^2 + \gamma x_{t+k}^2) + \lambda_{t+k}(\pi_{t+k} - \beta\pi_{t+k+1} - \kappa x_{t+k}) \right] \tag{75}$$

FOCs are

$$\frac{\partial \mathcal{L}}{\partial x_t} = \gamma \mathbb{E}_t x_{t+k} - \kappa \mathbb{E}_t \lambda_{t+k} = 0 \tag{76}$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = \mathbb{E}_t \pi_{t+k} + \mathbb{E}_t \lambda_{t+k} - \mathbb{E}_t \lambda_{t+k-1} = 0 \tag{77}$$

for $t = 0, 1, 2, \ldots$ where $\lambda_{-1} = 0$

- There is no constraint on time $t = -1$

Can re-arrange terms

$$\mathbb{E}_t x_{t+k} = \frac{\kappa}{\gamma}\mathbb{E}_t \lambda_{t+k} = \theta \mathbb{E}_t \lambda_{t+k} \tag{78}$$

$$\mathbb{E}_t \pi_{t+k} = \mathbb{E}_t \lambda_{t+k-1} - \mathbb{E}_t \lambda_{t+k} = -\frac{1}{\theta}\mathbb{E}_t \Delta x_{t+k} \tag{79}$$

$$\Delta \mathbb{E}_t x_{t+k} = -\theta \mathbb{E}_t \pi_{t+k} \tag{80}$$

Optimal policy **under commitment** is therefore characterised by

$$x_t = -\theta \pi_t = \theta(p_{t-1} - p_t) \tag{81}$$

$$\mathbb{E}_t \Delta x_{t+1} = -\theta \mathbb{E}_t \pi_{t+k} = \theta(p_{t+k-1} - p_{t+k}) \tag{82}$$

Considering initial price level p_{-1} we get

$$\mathbb{E}_t x_{t+k} = \theta(p_{-1} - \mathbb{E}_t p_{t+k}) \tag{83}$$

Since

$$\pi_t = p_t - p_{t-1} \tag{84}$$

Price level targeting rule

- Implies that price level always returns to trend
- Inflation will be zero, on average

Optimal policy under discretion

Consider scenario where a central bank cannot commit to taking a particular course of action in the future

- Can only set optimal strategy for what to do today

Optimality conditions for period t is

$$x_t = -\theta \pi_t \tag{85}$$

"Lean against the wind" policy

- $x_t > 0$: pursue policy that lowers inflation

Under optimal discretionary policy, policy is set against inflation

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} - \kappa \theta \pi_t + u_t \tag{86}$$

First-order difference equation

$$\pi_t = \left(\frac{1}{1+\theta\kappa} \right) \left(\beta \mathbb{E}_t \pi_{t+1} + u_t \right) \tag{87}$$

Repeated iteration solution

$$\pi_t = \left(\frac{1}{1+\theta\kappa} \right) \sum_{k=0}^{\infty} \left(\frac{\beta}{1+\theta\kappa} \right)^k \mathbb{E}_t u_{t+k} \tag{88}$$

Cost-push shocks assumed to follow $AR(1)$ process, implying

$$\mathbb{E}_t u_{t+k} = \rho^k u_t \tag{89}$$

With

$$\begin{aligned} u_t &= \rho u_{t-1} + v_t \\ v_t &\sim N(0, \sigma^2) \end{aligned} \tag{90}$$

Using

$$\sum_{k=0}^{\infty} c^k = \frac{1}{1-c} \tag{91}$$

for $|c| < 1$, inflation becomes

$$\begin{aligned}
\pi_t &= \left(\frac{1}{1-\theta\kappa}\right)\left[\sum_{k=0}^{\infty}\left(\frac{\beta\rho}{1+\theta\kappa}\right)^k\right] u_t &(92)\\
&= \left(\frac{1}{1-\theta\kappa}\right)\left(\frac{1}{1-\frac{\beta\rho}{1+\theta\kappa}}\right) u_t &(93)\\
&= \frac{u_t}{1+\theta\kappa-\beta\rho} &(94)
\end{aligned}$$

$AR(1)$ cost-push shock implies that

$$\mathbb{E}_t x_{t+1} = \rho x_t \tag{95}$$

$$\mathbb{E}_t \pi_{t+1} = \rho \pi_t \tag{96}$$

Substitute into Euler equation

$$x_t = \mathbb{E}_t x_{t+1} - \sigma\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right) \tag{97}$$

along with

$$x_t = -\theta \pi_t \tag{98}$$

to back out what the optimal interest rate looks like

$$-\theta\pi_t = \rho x_t - \sigma(i_t - \rho\pi_t - r_t^n) \tag{99}$$

$$i_t = r_t^n + \left(\rho + \frac{(1-\rho)\theta}{\sigma}\right)\pi_t \tag{100}$$

This will be greater than 1 if

$$\frac{\theta}{\sigma} > 1 \tag{101}$$

which will hold for all reasonable parameterisations

- Satifies Taylor principle
- Inflation and interest rates do not depend at all on what happened in the past.

Welfare gains: Consider transitory cost-push shock u_t

- Assume that expectations about shock won't affect future policy

Short-run trade-off between inflation and output gap at t, vertical shift

$$u_t \tag{102}$$

Requires the central bank to choose

1. Increase inflation
2. Have negative output gap
3. Combination of both

In contrast, if central bank is expected to pursue tighter policy from $t + 1$ onwards; short-run trade-off will be shifted by total **change**

$$u_t + \mathbb{E}_t \pi_{t+1} \tag{103}$$

Shift will actually be smaller and thus possibly increase

stabilisation.

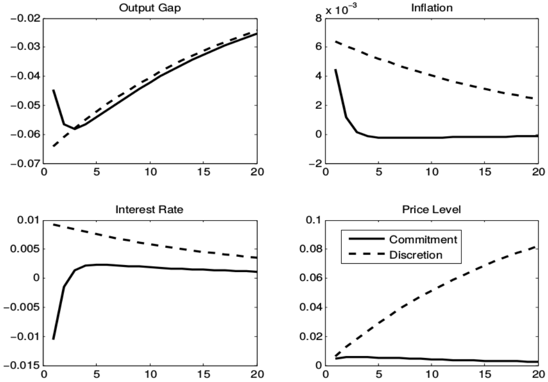
Difference between commitment and discretion boils down to implicit price versus inflation target

- Price target better at anchoring $\mathbb{E}_t \pi_{t+1}$
- Agents know central bank will enact policy to return price level to its target

Less volatile expected inflation improves available tradeoff between current inflation and output gap

- More movement in $\mathbb{E}_t \pi_{t+1}$ means more movement in either inflation or output gap: reduces welfare

Main issue is that it might not be practically feasible to pick a policy and stick to it.



From "*Optimal Monetary Policy in the New Keynesian Model*", Sims (2014)

Empirical issues

Central role for NKPC in NK model: relies on output gap x_t

- How to measure this gap?

Assume that average output tends to return to natural rate

- Use simple trend as proxy for natural rate (e.g. HP-filter)

Proxy

$$x = y_t - y_t^n \tag{104}$$

with

$$\tilde{x}_t = y_t - \hat{y}_t \tag{105}$$

Can estimate NKPC using

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{x}_t \tag{106}$$

Cannot observe $\mathbb{E}_t \pi_{t+1}$: substitute realised π_{t+1}

- Use an instrumental variable to deal with the fact this this is a noisy estimator of what we really want.

For model

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{x}_t$$

results often show

$$\kappa < 0$$

$\kappa < 0$ seems counterintuitive; however $\Delta\pi_t$ is

1. Negatively correlated with the unemployment rate (accelerationist)
2. Therefore positively correlated with the output gap

Given that $\beta \approx 1$, we can proxy

$$\pi_t - \beta \mathbb{E}_t \pi_{t+1} \tag{107}$$

with

$$\pi_t - \pi_{t+1} = -\Delta\pi_{t+1} \tag{108}$$

Negative sign on κ might not be that surprising
Two possible reasons for failure

1. Model is wrong
2. Output gap measured with error

Gali & Gertler (1999) argue output gap is measured with error

- Deterministic trends do a bad job in capturing movements in the natural rate of output

Suggest using unit labour costs as proxy for marginal costs

- Proxy for real marginal costs is the labour share of income.

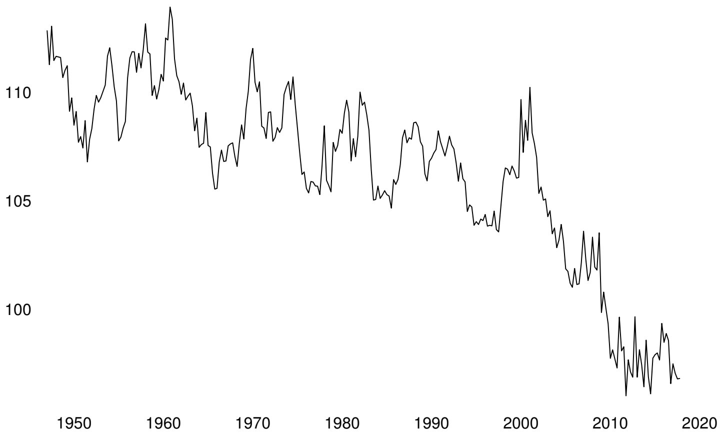
Estimate

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \gamma s_t \tag{109}$$

Find

$$\gamma > 0 \tag{110}$$

**Nonfarm business sector
Labour share**



Some issues

- Real marginal costs should be procyclical: increase when output is above potential
- Labour share is countercyclical
- Downward trend in labour share across countries

Rudd & Whelan (2007) argue that if NKPC works well with labour share this implies that it is completely forward looking

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k {}_t s_{t+k} \tag{111}$$

Use VAR to forecast s_{t+k} and provide fitted value for (111).

Fit of model not really good (Rudd & Whelan, 2006); better when including lagged inflation

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t s_{t+k} + \rho \pi_{t-1} \tag{112}$$

NKPC's main problem: does not account properly for inflation's dependence on own lags

Can use a hybrid version

$$\pi_t = \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1} + \kappa x_t \tag{113}$$

x_t measures inflationary pressure
Some theoretical weaknesses

1. Rule-of-thumb price-setters: some people set backward-looking prices, other don't (Gali-Gertler, 1999)
2. Indexation: each period some set optimal prices, others don't; non-optimising price-setters index to past inflation (Christiano et al, 2005)