Banking and risk

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Recall interest rate equation from NK model

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_x x_t \tag{1}$$

i_t is a specific interest rate

- Short-term risk-free overnight rate that banks charge each other
- Quite different from consumer interest rates

To understand consumer interest rates need to understand risk involved with consumer lending.

Suppose investor can choose between following two assets

1. Risk-free bond with interest rate r
2. Loan with interest rate R and probability of default p
    - Return of R with probability $1 - p$
    - Return of -1 (losing all your money) with probability p

Expected return on loan

$$\begin{aligned} R(1-p) - p \\ R - Rp - p \\ R - p \end{aligned} \tag{2}$$

For same expected return, interest r needs to be

$$\begin{aligned} r &= R - p \\ r + p &= R \end{aligned} \tag{3}$$

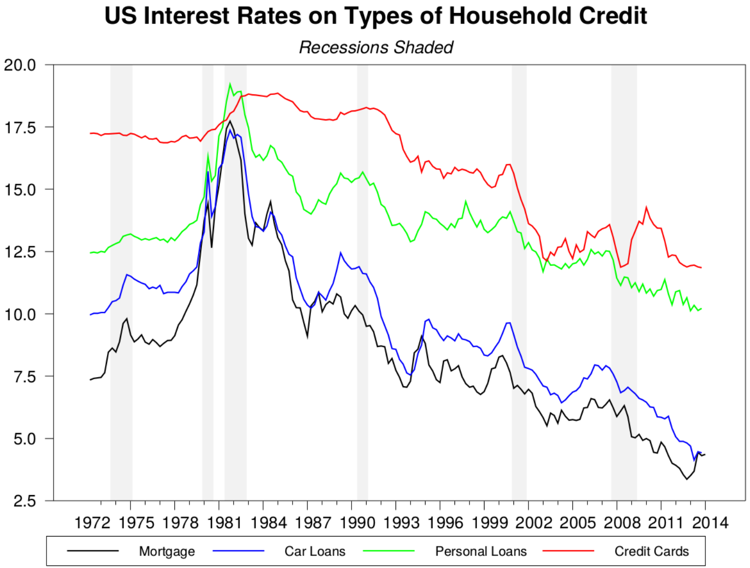
For loan with collateral, default return:

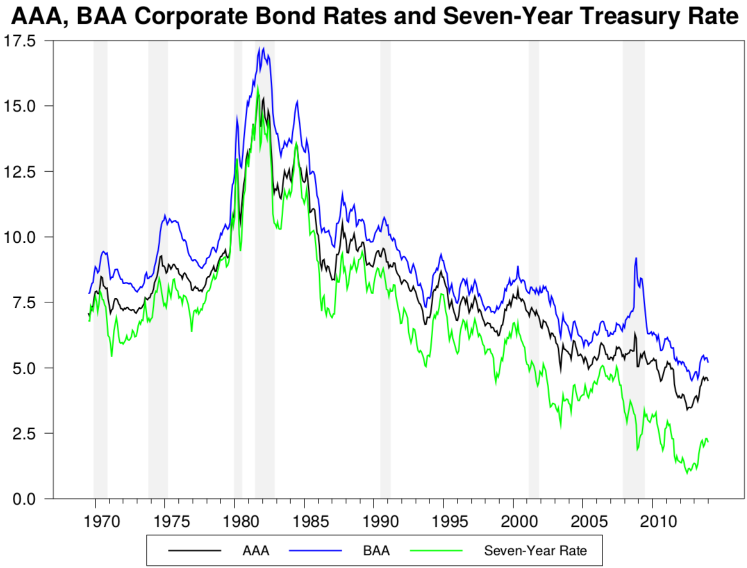
$$c - 1 < 0 \tag{4}$$

Eq.3 becomes

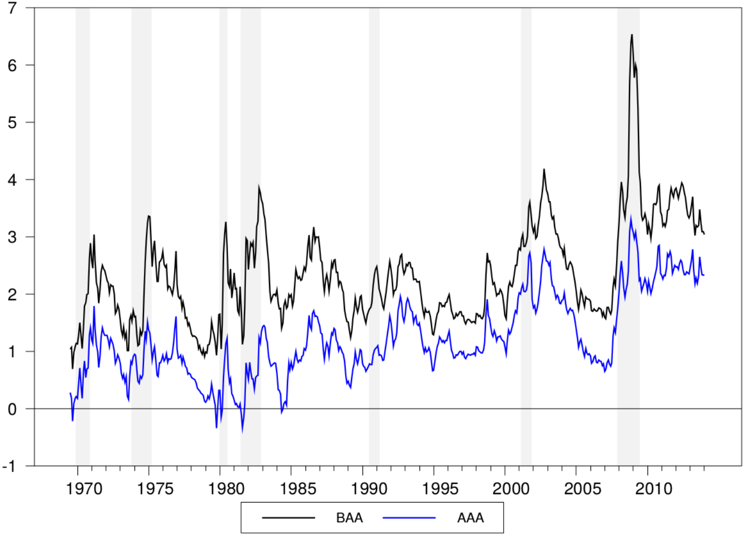
$$R = r + (1 - c)p \tag{5}$$

1. Collateralised loans have lower interest rates
2. Interest rate depends on type of collateral


US Interest Rates on Types of Household Credit
Recessions Shaded
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
1972
1975
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
Mortgage
Car Loans
Personal Loans
Credit Cards


AAA, BAA Corporate Bond Rates and Seven-Year Treasury Rate
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
1970
1975
1980
1985
1990
1995
2000
2005
2010
AAA
BAA
Seven-Year Rate

Spreads of AAA and BAA over 7-Year Treasury (Recessions Shaded)



Default risk and collateral

1. Interest rate affected by value of collateral
2. Value of assets fluctuate with state of economy

Financial accelerator: mechanism by which financial sector can propagate business cycle shocks

- Recession-causing shock will increase interest rate spreads which will worsen the recession

Bernanke, Gertler & Gilchrist (1999)

Demand

$$
\begin{aligned}
y_t &= \frac{C}{Y} c_t + \frac{I}{Y} inv_t + \frac{G}{Y} g_t + \frac{C^e}{Y} c_t^e + \ldots && (6) \\
c_t &= -\sigma r_{t+1} + E_t c_{t+1} && (7) \\
c_t^e &= \frac{1-\phi}{\phi} n_{t+1} && (8) \\
q_t &= \phi(i_t - k_t) && (9) \\
E_t r_{kt+1} &= (1-\rho) E_t (p_{wt+1} - p_{t-1} + y_{t-1} && (10) \\
&\quad - k_{t+1}) + \rho E_t q_{t+1} - q_t \\
E_t r_{kt+1} - r_{t+1} &= -v(n_t - q_t - k_{t+1}) && (11)
\end{aligned}
$$

Supply

$$y_t = a_t + \alpha k_t + (1-\alpha) l_t \tag{12}$$

$$y_t - l_t = \mu_t + \gamma_l l_t + c_t \tag{13}$$

$$\pi_t = \kappa(p_{wt} - p_t) + \beta E_t \pi_{t+1} \tag{14}$$

The evolution of the state variables

$$k_{t+1} = \delta inv_t + (1-\delta)k_t \tag{15}$$

$$n_t = \frac{\theta RK}{N}[r_t^k - r_t] + \theta R(r_t + n_{t-1}) \tag{16}$$

$$r_t = i_{t-1} - \pi_{t-1} \tag{17}$$

Monetary policy rule is given by

$$i_t = \rho i_{t-1} + (1-\rho)[\gamma_\pi \pi_t + \gamma_y (y_t - y_t^n)] + \epsilon_t^{rn} \tag{18}$$

$$i_t = r_{t+1} - E_t \pi_{t+1} \tag{19}$$

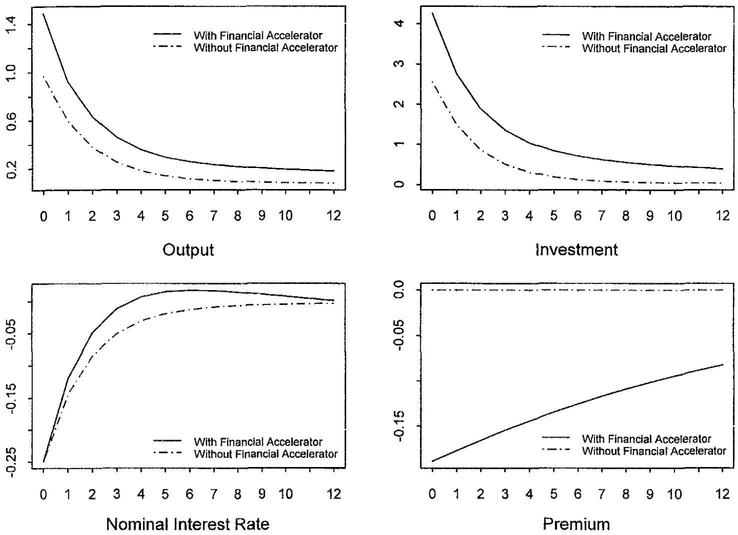


Fig. 3. Monetary shock – no investment delay. All panels: time horizon in quarters.

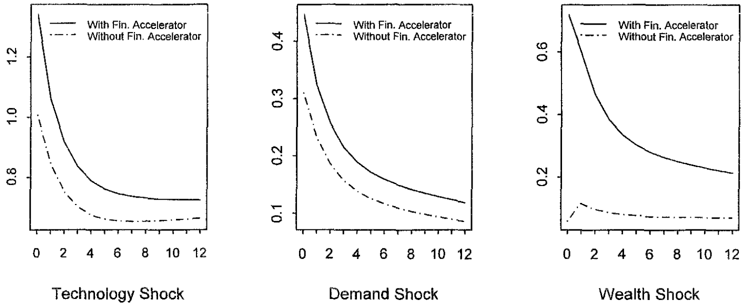


Fig. 4. Output response – alternative shocks. All panels: time horizon in quarters.

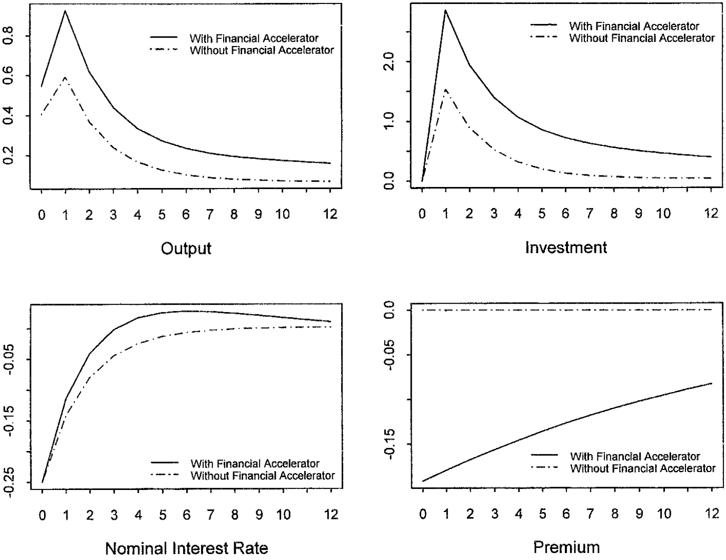


Fig. 5. Monetary shock – one period investment delay. All panels: time horizon in quarters.

Borrowers with higher risk pay higher interest rates.

- Assumption: for each risk level, someone willing to lend at high enough rates

Credit suppliers can refuse to make a loan, rather than trying to balance the loss by raising the interest rate: **credit rationing**

> *Lenders provide a smaller amount of loans than is demanded at the market interest rate.*

Asymmetric information in credit market:

- Banks can't always tell good borrowers from bad
- From bank's point of view borrowers worsen as interest rates increase

Credit rationing can be quite severe, turning down credit-worthy borrowers

Stiglitz & Weiss (1981); Suppose number of borrowers each with a project to undertake

1. All borrowers look to borrow B and put up collateral C
2. Projects deliver a sum of R
3. Interest rate on bank loan r is determined endogenously

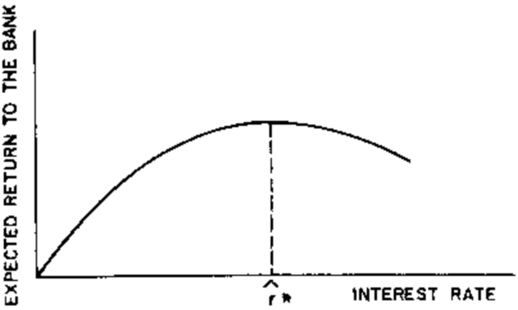


FIGURE 1. THERE EXISTS AN INTEREST RATE WHICH MAXIMIZES THE EXPECTED RETURN TO THE BANK

Return R is uncertain

- Outcome distribution varies across borrowers

Return distribution type θ borrowers

$$PDF : f(R, \theta) \tag{20}$$

Distribution mean identical across borrowers, but greater θ values correspond to greater riskiness

- High values of θ induce a mean-preserving spread in the distribution of projected payoffs

$$PDF : g(\theta) \tag{21}$$
$$CDF : G(\theta) \tag{22}$$

Borrowers are observably identical to banks: don't observe an individual's value of θ

Mechanism: bank sets r which might affect risk of loan through

1. **Adverse selection**: sorting potential borrowers
2. **Moral hazard**: affecting the actions of the borrowers

Default

$$C + R \leq B(1 + r) \tag{23}$$

B, amount borrowed

C, collateral

Returns

For borrower

$$\pi(R, r) = max(R - (1 + r)B; -C) \tag{24}$$

For bank

$$\rho(R, r) = min(R + C; B(1 + r)) \tag{25}$$

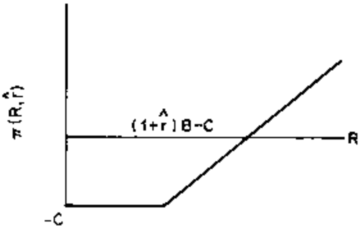


FIGURE 2a. FIRM PROFITS ARE A CONVEX FUNCTION OF THE RETURN ON THE PROJECT

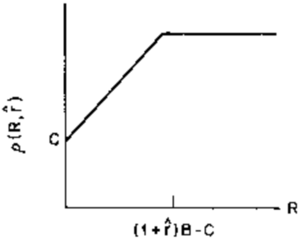


FIGURE 2b. THE RETURN TO THE BANK IS A CONCAVE FUNCTION OF THE RETURN ON THE PROJECT

The worst a firm can do is default on the loan and lose the collateral when the project has a bad return

- After that the return increases one for one with outcome R

Return R depends negatively on borrowing rate r

- Not all firms decide to go ahead and borrow as not all firms have a positive expected value.

Borrow if

$$\mathbb{E}[\pi(R,r)] = \int_0^\infty \pi(R,r) f(R,\theta) dR > 0 \tag{26}$$

Main question is how $\mathbb{E}[\pi(R,r)]$ varies with type θ

Types only borrow if

$$\theta > \hat{\theta}(r) \tag{27}$$

From utility theory;

- Concave utility function: mean-preserving spread in distribution reduces expected utility because people are risk averse
- In this case outcome is a convex function of R, so more uncertainty increases the expected return

Bad case: outcome still $-C$, but increased risk raises the chance of a really good outcome.

Effect of increase in r on loan demand

1. R depends negatively on r: increase r reduces expected project returns
2. R depends positively on θ: some θ have $\mathbb{E}[\pi(R, r)] > 0$

Increase in r raises the cut-off $\hat{\theta}(r)$ for potential borrowers

1. Adverse selection: borrower pool changes consisting of more risky projects
2. Moral hazard problem: risk-neutral investors prefer project with higher bankruptcy probability

Bank pay off

$$Min(R + C; (1 + r)B) \tag{28}$$

If bank knows it is lending to type θ; expected return

$$\rho(\theta, r) = \int_0^\infty [Min(R + C; (1 + r)B)] f(R, \theta) dR \tag{29}$$

Increase in θ reduces bank's expected return.

- Bank's pay off concave function of R

1. Best case: bank gets principal and interest
2. Worst case: only get collateral

More risk is bad, but bank cannot tell if borrowers is risky

- Expected payoff can be calculated averaging across all types that look for loans at interest rate r

$$\mathbb{E}[\rho(\theta, r)] = \frac{\int_{\hat{\theta}(r)}^{\infty} \rho(\theta, r) dG(\theta)}{1 - G(\hat{\theta})} \tag{30}$$

r increase has two effects on bank's pay off

1. +ve: higher interest revenues from each project that pays off
2. -ve: adverse selection.

At some point second effect dominates: bank profits rise as the interest rate goes up, reach a maximum and then decline

Borrower type

1. Low risk
2. High risk

Profits drop at point where low-risk types drop out

- Continuous number of types: interest rate r^*, which is consistent with maximum profit level
- Assuming that loan supply depends positively on the expected pay off

Problem: banks cannot simply choose r^*

- No equilibrium if there is not sufficient demand
- Banks would chase consumers offering them r^* and lots of people would be turning them down

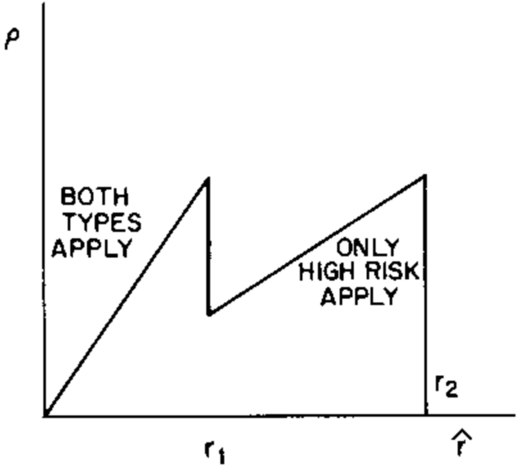
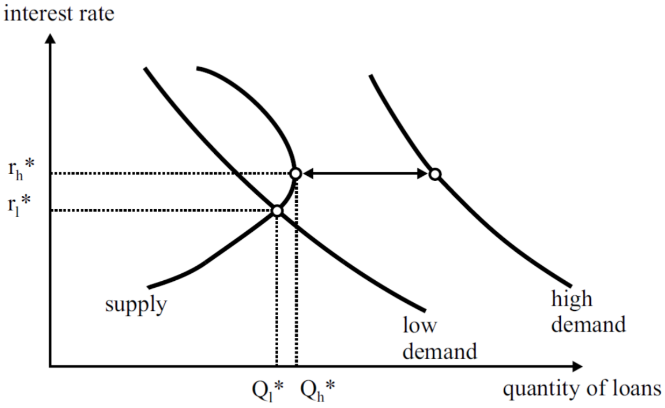


FIGURE 3. OPTIMAL INTEREST RATE r_1

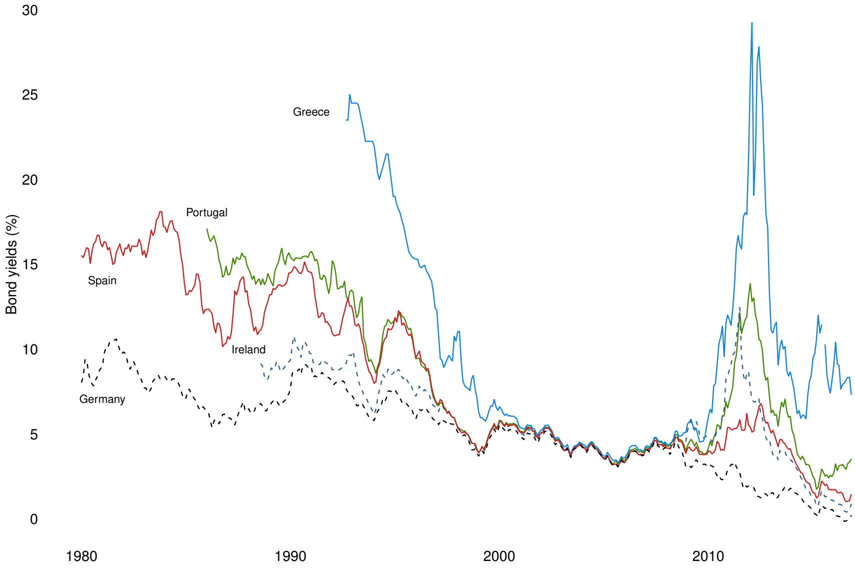
Equilibrium outcome determined by supply/demand interaction

1. Low loan demand
   - Loan demand curve intersects loan supply curve below r^*.
   - Market functions normally: all who request a loan receive one
2. High loan demand
   - Loan demand and supply curves do not intersect: bank picks optimal interest rate r^*
   - Credit rationing: More demand than banks are willing to supply


interest rate
r_h^*
r_l^*
supply
low
demand
high
demand
Q_l^*
Q_h^*
quantity of loans

Examples sovereign defaults:

Early 1800s: number of countries after the Napoleonic Wars, e.g. Denmark, France, the Netherlands, and Sweden
1875: Ottoman Empire
1932: Germany
1982: Mexico
1998: Russia
2006: Zimbabwe
1982, 1989, 2001: Argentina
1826, 1843, 1860, 1894, 1932, 2015: Greece



30
25
20
15
10
5
0
Bond yields (%)
Greece
Portugal
Spain
Ireland
Germany
1980
1990
2000
2010

Consider country has $P(\mathit{default}) = 0.1$ over next year; leading to 50% default on outstanding debt

- Country needs to pay 5% premium on debt relative to safe assets

Premium imposes additional burden on government

- Interest costs rise above the funds that country can access to pay off the interest payments
- Alternatively the country's GDP could expand in order to keep debt stable

Market for government bonds might cease to operate as the country is deemed not credit-worthy: risk goes from unlikely to likely

- Closing of a bond market is an rare and abrupt events: People often don't see it coming

After a default a country needs to restructure it debt which often involves writing off part of it, in order to restore the debt level to a more sustainable level.

Why are financial intermediaries useful?

1. Pooling savings
2. Risk diversification
3. Maturity transformation
4. Information processing

Two types of banking relevant to today's financial system

1. Clearing house banks
2. Fractional reserve banking

Suppose that one business day the following transactions occur

- Bank A: accounts credited with EUR 10M from Bank B depositors
- Bank B: credited with EUR 9M from Bank A depositors

Total transfers: EUR 19M

1. Have couriers transfer money back and forth
2. Settle account at end of the business day

Clearing house bank

- Will order transfer of EUR 1M from Bank B to Bank A
- More efficiently: deduct EUR 1 M from ledger entry for Bank B's account; add it to Bank A's
- All deposits still fully backed by the cash in the vaults

Forerunner of today's central banks

Most of time only small fraction of bank's total deposits will be demanded at any given time

- "most" being important qualifier here

i.e. not all cash has to be in the vault in order to back up the deposits

- Some of it can be used for loans while keeping some cash reserves to deal with day-today demand.

This practice is called **fractional reserve banking**

Some advantages of fractional reserve banking

1. Saves depository money: banks can charge interest on loans
2. Banks serve as an intermediary

Disadvantage: risk of bank run

- Assets $\leq$ liabilities (e.g. due to loan defaults)
- Suspicion of insolvency could lead depositors to claim their money

Main issue is a maturity mismatch

- People who supply funds want to have it available for return at shorter terms than the people who the bank lends the money to

Understanding bank run: bank's balance sheet

- Liabilities: sources of the bank's funds
- Assets: uses of said funds

Table: Stylised bank balance sheet

Assets (use of funds)	Liabilities (source of funds)
Loans	Deposits
Securities	Other borrowings
Cash and reserves	Equity capital

Risky nature of assets

- Borrowers don't pay back loans
- Bad investments are sometimes made in stocks and bonds
- Other assets invested in decline in value

Result: negative equity capital

- Assets go below what is owed to depositors and bond-holders
- Might trigger bank run when bank is suspected to be insolvent

Examples recent bank runs:

2001: Bank run in Argentina during economic crisis (1999-2002)
2007: Northern Rock, UK
2009: DSB Bank, the Netherlands
2015: Bank runs in Greece and Cyprus

What happens during a bank run?

1. Banks starts paying off depositors; selling off most liquid assets: e.g. cash, excess reserves at central bank, etc.
2. Bank sells non-liquid assets: long-term customer loans, property assets: fire sale

Bank runs often triggered by - assumed- insolvency: makes bank make more insolvent

- Bank run can be triggered by just rumours
- Banks and governments are always quick to declare that the banks are fully solvent
- Main concern of bank run is contagion risk

Banking crisis likely leads to credit squeeze

$$\begin{aligned} \textit{Loans} = \textit{Deposits} + \textit{Other Borrowings} + \textit{Equity Capital} \\ - \textit{Cash and reserves} - \textit{Securities} \end{aligned}$$

1. Loans
    - Hard to call in
    - When paid-off, funds kept as cash, reserves, or invested in securities
    - Pay off deposit outflows or maturing bond liabilities
    - Don't make new loans!
2. Deposits
    - Customers prefer cash at home: banks will have less funds to loan
3. Other borrowings
    - Bond markets/other fund providers likely reluctant to lend to banks, worrying they might fail
4. Cash and reserves
    - Will be kept on balance sheet: needed to survive potential bank run
5. Securities
    - Preferred: can be quickly sold to raise cash

Credit crunch result of behaviour of bank and customers

- Bank no longer in position to lend: financial intermediation breaks down

Banking crisis can lead to severe recession

Modern banking system has number of features that make crisis difficult to deal with

1. Interbank linkages
2. Financial assets and negative feedbacks
3. Non-deposit funding

Also, incentive problem

- Bankers moved from risk-averse moneylenders to risk-loving gamblers

1. High leverage (little equity capital relative to assets)
2. Many risky investments
3. Too much short-term non-deposit funding
4. Too big

Imagine investment group starts a bank with starting capital of EUR 10M

- EUR 1M spend on retail branch network
- Offer 1% interest rate on deposits: attracts EUR 50M
- EUR 50M to make loans: interest rate of 5%
- EUR 9M in cash and reserves

Table: Balance sheet

Assets (use of funds)		Liabilities (source of funds)	
Loans	50	Deposits	50
Branch network building	1	Equity capital	10
Cash and reserves	9		
Total	60		60

1. Revenues
    - Loan interest: EUR 2.5M
    - Fees: EUR 1M
2. Costs
    - Deposit interest: EUR 0.5M
    - Running costs: EUR 1.5M

Table: Income statement

Revenues		Costs	
Interest income	2.5	Interest paid	0.5
Fees	1	Running costs	1.5
Total	3.5		2

Investment: EUR 10M
Profit: EUR 1.5M
Return on Equity

$$RoE = \frac{1.5}{10} = 15\% \tag{31}$$

Time to expand!

- EUR 0.5M paid in dividends
- EUR 1M for more loans
- EUR 20M in debt securities to raise funds to make additional loans

Table: Balance sheet after expanding the business

Assets (use of funds)		Liabilities (source of funds)	
Loans	71	Deposits	50
Branch network building	1	Equity capital	11
Cash and reserves	9	Debt securities	20
Total	81		81

Goal of bank is to expand business; through making more loans

- Means attracting more risky borrowers: increase in probability that people don't pay back loans

Suppose that of the 71M in loans, 5M went to a real estate developer who went bankrupt

Table: Balance sheet after expanding the business

Assets (use of funds)		Liabilities (source of funds)	
Loans	66	Deposits	50
Branch network building	1	Equity capital	6
Cash and reserves	9	Debt securities	20
Total	76		76

1. **Equity capital** is risky; one bad loan removes a fair chunk
2. **Investors** will get paid dividends when there is a profit, but they are the first to lose money when there is a bad loan
3. **Depositors and debt-holders** have first claim to getting their money back

Bank needs to be cautious in assessing credit risk of loan.

Does size matter?

Start bank with equity capital of EUR 10M

- Pay 2% on deposits
- Charge 3% on loans
- 10% of deposits reserve requirements

Consider two approaches to raising funds

1. Conservative
   - 90M raised in deposits
2. Aggressive
   - 90M raised in deposits
   - 100M borrowed from international money markets (2% interest rate)

Table: Balance sheet starting with EUR 100 million

Assets (use of funds)		Liabilities (source of funds)	
Loans	91	Deposits	90
Cash and reserves	9	Equity capital	10
Total	100		100

Table: Balance sheet starting with EUR 200 million

Assets (use of funds)		Liabilities (source of funds)	
Loans	191	Deposits	90
Cash and reserves	9	Equity capital	10
		Borrowings	100
Total	200		200

Profits and RoE for conservative approach

$$\begin{aligned} \Pi &= 3\% * 91 - 2\% * 90 = 2.73 - 1.8 = 0.93 \\ RoE &= 9.3\% \end{aligned}$$

Profits and RoE for aggressive approach

$$\begin{aligned} \Pi &= 3\% * 191 - 2\% * 190 = 5.73 - 3.82 = 1.91 \\ RoE &= 19.1\% \end{aligned}$$

Larger bank has

- Lower capital-to-assets ratio
- Higher profits
- Higher RoE

Highly-leverage banks make larger profits but also take more risks

- More credit risk since loans could go bad,
- More liquidity risk as funds from international money market could dry up

Not in banker's self-interest to maintain sufficient capital levels

Leverage ratio is the assets-capital ratio

1. Small bank: 10: equity capital was 10% of total assets
2. Large bank: 20 equity capital was 5% of total assets

Two sets of incentives

1. Investors
   - Shareholders of highly-leveraged banks willing to lose all their money with prospect of high returns most of the time
   - When things go pear-shaped, may have made a decent enough return from all the dividends
2. Bank management
   - Even when investors are risk adverse there are strong incentives for high leverage
   - E.g. profit-linked bonuses, which means that they want to maximise profit today
   - In case of bankruptcy don't have to pay back bonuses
   - Government bailout: privatised profits, socialised losses

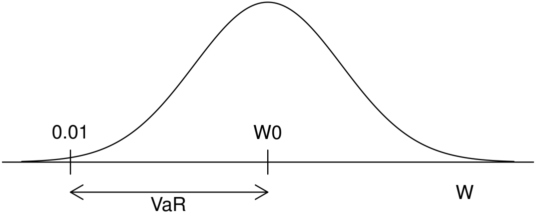
Value at Risk, measure for risk level of investments of a bank, motivated by question

> *What is, realistically, the worst that could happen over one day, one week, or one year?*

VaR at confidence level α relative to base level W_0 is smallest non-negative number such that

$$Pr(W < W_0 - VaR) \leq 1 - \alpha \tag{32}$$

$$VaR \equiv inf\{V | F(W_0 - V) \leq 1 - \alpha\} \tag{33}$$


0.01
W0
VaR
W

VaR estimates possible loss under normal market conditions, using statistical distribution of the bank's credit losses

- Expected loss (distribution average)
  - Banks should deal with these by writing down part of their loans each year as loan loss provision
  - i.e. valuing assets at less than their current book value in anticipation of future losses

- Stress loss (distribution extreme tail)
  - 1% tail is commonly used
  - e.g. at a weekly VaR of EUR 50 million, there is 1% chance that your portfolio will lose more than EUR 50 million over the course of a week

A bank is owed money by two firms

- 1M by firm A, 1M by firm B

Both firms are credit worthy: pay back loan with $Pr = 0.995$

- When A defaults, bank can recover 0.5M
- When B defaults, nothing can be recovered

For $\alpha = 0.99$, what is the bank's VaR for each firm?

In both cases $VaR = 0$ at $\alpha = 0.99$

$$Pr(\mathit{repayment} < 1M - x) \leq 0.01 \tag{34}$$

Both firms repay with $Pr = 0.995$; can set $x = 0$

$$Pr(\mathit{repayment} < 1M) = 0.005 \leq 0.01 \tag{35}$$

However, banks has more to lose when firm B defaults: need better measure.

$$\mathit{Tail\ loss} = \mathbb{E}(W|W < q) = \frac{\int_{-\infty}^{q} Wf(W)dW}{\int_{-\infty}^{q} f(W)dW} \tag{36}$$

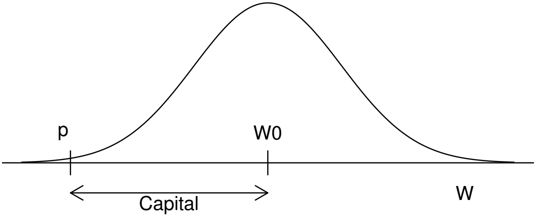
Or conditional expected loss

Suppose

- W is value of assets at some future data
- W_0 is value of assets today

Bank will remain solvent as long as

$$W \geq W_0 - Capital \tag{37}$$



Bank will go bankrupt with probability p

- Failure probability can be decreased by holding more capital
- VaR indicates capital amount required to stave off default with high probability

VaR of interest to creditors and tax payers

- However, these do not control the bank
- Equity holders do: no concern for tail loss

Time horizon in VaR should be long enough so that

- Corrective measures can be taken to rectify problem
- Horizon is appropriate for degree of asset illiquidity
- Uncertainty level chosen to account for ease of recapitalisation

VaR bedrock of capital regulations by Basel regulations

- Minimum level regulatory capital equal to some multiple of unexpected losses

$$Capital\ Required = 3 \cdot VaR$$

Basel regulations also require risk-weighted assets of $\geq$8%

- Tier 1 and 2 capital (equity capital, subordinated bonds)

$$RWA = \frac{3 * VaR}{0.08}$$

Final RWA figure determined by some adjustments

1. Market risk

   ...pertaining to interest rate related instruments, equities, foreign exchange risk and commodities risk.

2. Operational risk

   ...inadequate or failed internal processes, people and systems or from external events.

VaR shortcoming: uses distribution of past asset returns

1. Estimation sample
   - True distribution not known; can only be estimated from historical data
   - Banks rely mainly on using returns from recent years
2. Tail risk
   - Fat tails not accounted for
   - Financial markets generate extreme losses more often than predicted by normal distribution

Interbank markets can help banks coping with reserve requirements

- Lending and borrowing short-term funds
- Allowing banks with lots of deposits but without good loan opportunities to lend to banks with good loan opportunities but limited deposits

Interbank lending can make system unstable: **systemic risk**

Consider three banks (A, B, C), each with a equity capital of EUR 10M[1]

1. A borrows EUR 25M from B
2. B borrows EUR 15M from C

A loses EUR 35M in loans: wipes out equity capital

1. Bank A loses
2. A becomes insolvent $\rightarrow$ B loses EUR 25M
3. B becomes insolvent $\rightarrow$ cannot pay C
4. C becomes insolvent and has no equity capital left

Insolvency of one bank can bring down whole system

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[1] Note that this example is not entirely realistic as the amount of capital lost by the first bank is greater than the total amount of capital in the system.

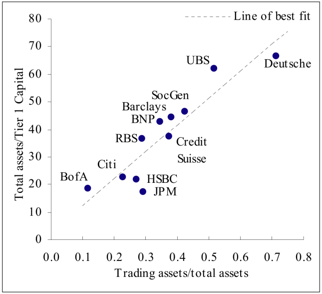
Main mechanisms of systemic risk

1. Contagion (interbank lending)
2. Spillovers (asset sales)
    - Troubled bank sells liquid assets
    - Fire sale puts downward pressure on asset price
    - Due to regulation asset value other banks marked down
    - Fire sale reduces equity capital $\rightarrow$ increasing risk other banks

Alessandri & Haldane(2009) "Banking on the state"

- The banking sector has grown in size relative to the economy
- Banks have become more leveraged and less liquid
- Have engaged in more risky trading activities

Chart 8: LCFIs' trading portfolios and financial leverage – 2007



Sources: Published accounts and Bank calculations

Prudential regulation can lead to financial instability

1. Asset prices increase during boom, loans paid back: increase in equity
2. Can expand operation: acquire new assets; liquidity not issue with large demand: boom continues
3. Boom turns into bust: business cycle plays out; recession arrives
4. Capital requirement will lead to asset sell off: price decrease will erode equity

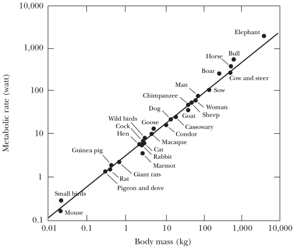
Power law

$$f(x) = Ax^{-\alpha} \tag{38}$$

α, scaling parameter

A, constant

Figure 7
Metabolic Rate as a Function of Mass across Animals



Source: West, Brown, and Enquist (2000).
Notes: "Metabolic rate" is energy requirement per day. The slope of this log-log graph is 3/4: the metabolic rate of an animal of mass M is proportional to $M^{3/4}$ (Kleiber's law).

Barro & Jin (2011) "On the size distribution of macroeconomic disasters"

$$log(Y_{t+1}) = log(Y_t) + g + u_{t+1} + v_{t+1} \tag{39}$$

$$Y_t = C_t; g \geq 0 \tag{40}$$

Low-probability disasters v_{t+1}: output contracts by fraction b

$$0 < b \leq 1$$

$$Pr(1-p) : v_{t+1} = 0 \tag{41}$$

$$Pr(p) : v_{t+1} = log(1-b)$$

$$g^* = g + \frac{1}{2} \cdot \sigma^2 - p \cdot Eb \tag{42}$$

Disaster size

$$z \equiv 1/(1-b) \tag{43}$$

Single power law

$$f(z) = Az^{-(\alpha+1)} \tag{44}$$

$$A = \alpha z_0^{\alpha} \tag{45}$$

Double power law

$$f(z) = \begin{Bmatrix} 0, & \text{if } z < z_0, \\ Bz^{-(\beta+1)}, & \text{if } z_0 \leq z \leq \delta, \\ Az^{-(\alpha+1)}, & \text{if } \delta \leq z, \end{Bmatrix} \tag{46}$$

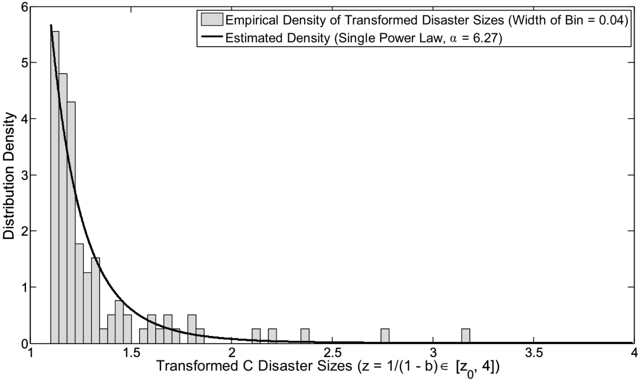


FIGURE 1.—Histogram of the empirical density of transformed C disasters and the density estimated by the single power law. The threshold is $z_0 = 1.105$, corresponding to $b = 0.095$. For the histogram, multiplication of the height on the vertical axis by the bin width of 0.04 gives the fraction of the total observations (99) that fall into the indicated bin. The results for the single power law for C, shown by the curve, correspond to Table I.

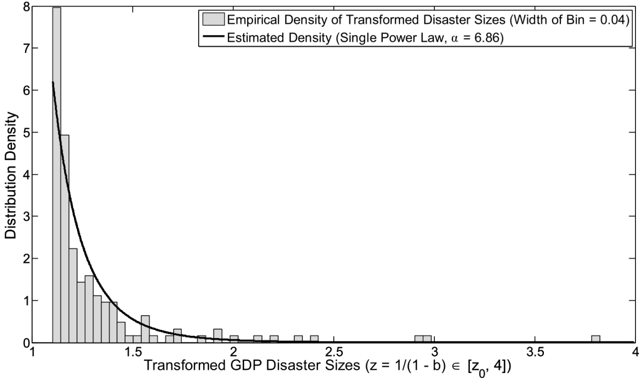


FIGURE 2.—Histogram of the empirical density of transformed GDP disasters and the density estimated by the single power law. The threshold is $z_0 = 1.105$, corresponding to $b = 0.095$. For the histogram, multiplication of the height on the vertical axis by the bin width of 0.04 gives the fraction of the total observations (157) that fall into the indicated bin. The results for the single power law for GDP, shown by the curve, correspond to Table I.

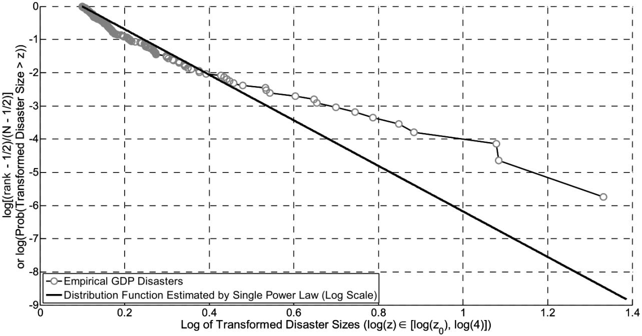


FIGURE 4.—Estimated log-scale tail distribution and log of transformed ranks of GDP disaster sizes versus log of transformed GDP disaster sizes. The straight line corresponding to the log-scale tail distribution comes from the estimated single power law for GDP in Table I. See note to Figure 3 for further information.

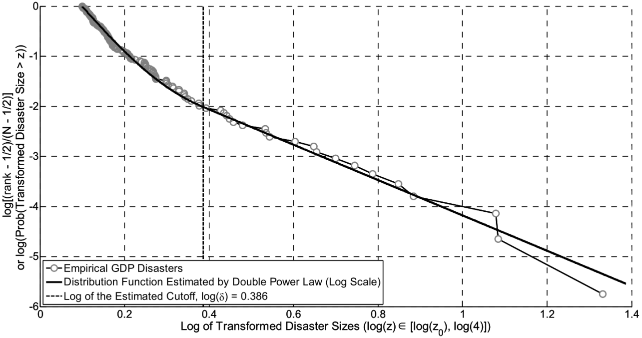


FIGURE 7.—Estimated log-scale tail distribution and log of transformed ranks of GDP disaster sizes versus log of transformed GDP disaster sizes. The line with two segments corresponding to the log-scale tail distribution comes from the estimated double power law for GDP in Table I. See note to Figure 3 for further information.

Gourio (2012) "Disaster risk and business cycles"

$$log\ z_{p,t} = log\ z_{p,t-1} + \mu + \epsilon_t + x_t\theta_t, \tag{47}$$

$$log\ z_{r,t} = \rho_z log\ z_{r,t-1} + (\varphi_t - \theta_t)x_t, \tag{48}$$

$$K_{t+1} = \left((1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t\right)e^{x_{t+1}\zeta_{t+1}}, \tag{49}$$

$$Pr(x_{t+1} = 1 | x_t = 1) = max(q, p_t);\ Pr(x_{t+1} = 1 | x_t = 0) = p_t \tag{50}$$

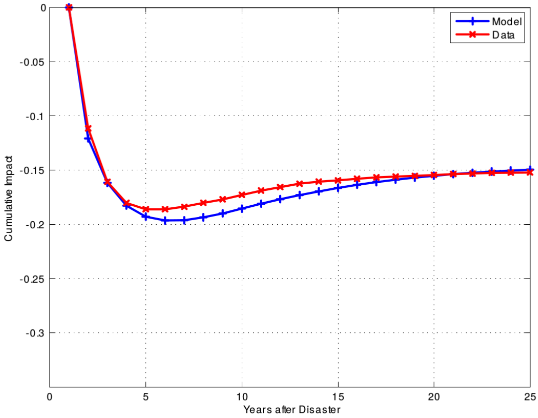


FIGURE 1. IMPULSE RESPONSE FUNCTION OF CONSUMPTION TO A DISASTER REALIZATION: MODEL VS. DATA (ESTIMATE OF BARRO ET AL. (2011)).

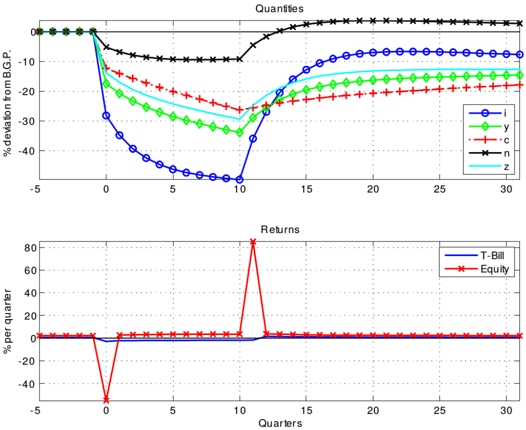


FIGURE 2. RESPONSE OF MACROECONOMIC QUANTITIES AND ASSET RETURNS TO A TYPICAL DISASTER IN THE MODEL.

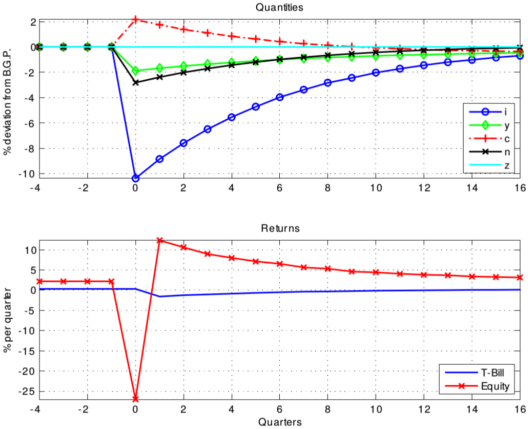


FIGURE 3. IMPULSE RESPONSE FUNCTION TO A TEMPORARY INCREASE IN DISASTER PROBABILITY FROM 0.72% TO 4%. TOP PANEL: MACROECONOMIC QUANTITIES; BOTTOM PANEL: ASSET RETURNS.

Boissay et al. (2016) "Booms and Banking Crises"

Financial recessions are sui generis

1. They are rare events
2. Are deeper and last longer compared to other recessions
3. Follow credit booms (i.e. not random)

Model: bank heterogeneity leads to interbank market

- Moral hazard and asymmetric information may lead eventually to financial recession

Illustrate that these recession are not triggered by adverse exogenous shocks

$$P(z_{t+1} < \overline{z}_{t+1} | z_t, A_t) = \Phi(log\overline{z}_{t+1} - \rho_z logz_t) \qquad (51)$$

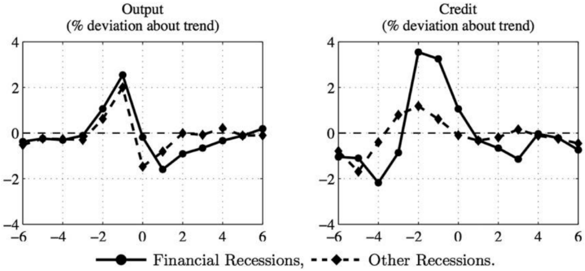


FIG. 1.—Dynamics of GDP and credit gaps around recessions. Average dynamics of the Hodrick-Prescott ($\lambda = 6.25$) cyclical component of (log) output and credit 6 years before and after the beginning of a recession (period 0) with and without a banking crisis. The data are from Jordà et al. (2011, 2013) and Schularick and Taylor (2012).

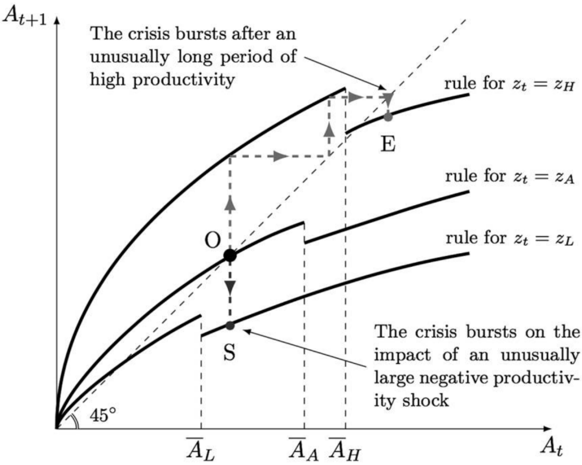


FIG. 3.—Stylized representation of the optimal decision rules

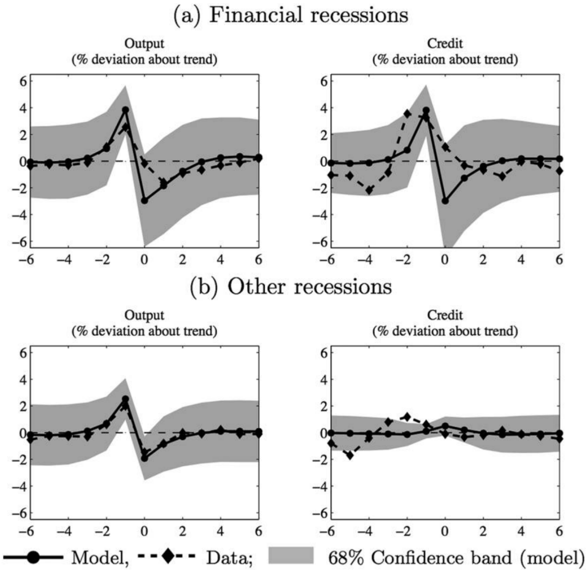


FIG. 9.—Dynamics of output and credit gaps around recessions. Average dynamics of the Hodrick-Prescott ($\lambda = 6.25$) cyclical component of (log) output and credit six periods before and after the beginning of a recession (period 0) with and without a banking crisis. To be consistent, we treat the simulated series of output and credit as we treat the actual data.