Endogenous technological change

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Spring 2017

The dynamics of the Swan-Solow model suggest that that technological progress, or the improvement in total factor productivity, is the probably the key determinant of economic growth, at least when considering sustainable growth on the long term. The Swan-Solow model treats technological progress as exogenous which leaves open the question of what actually determines technological developments. The Romer model tries to answer this question by treating technological change as endogenous; it is determined by the actions of the agents in the model.

Romer accepts the results from the Swan-Solow model but in additioan attempts to explain what determines technological progress.

How is technology determined anyway?

So far we have been pretty vague about what technology term A actually entails. Consider the following aggregate production function

$$\begin{aligned} Y &= L_y^{1-\alpha}(x_1^\alpha + x_2^\alpha + ... + x_A^\alpha) \\ &= L_y^{1-\alpha}\sum_{i=1}^{A} x_i^A \end{aligned}$$

L_Y is the number of workers producing the output, the x_i's are the different types of capital goods, A are the different types of capital inputs.

This production function has two important features

1. Diminishing marginal returns
   - Separately to each individual capital good since $0 < \alpha < 1$
2. A is variable
   - If A is fixed, the pattern of diminishing returns would mean that growth would taper off to zero

Labour allocation

The model includes multiple capital goods. These capital goods are the result of inventions due to a share of labour force allocated to R&D tasks. Let's say that there are L_A workers active in R&D, than the production function for the change in the number of capital goods can be described by

$$\Delta A = \gamma L_A^\lambda A^\phi$$

λ is and index of how slowly diminishing marginal productivity sets in for researchers. ϕ accounts for the fact that researchers can build upon the existing research stock.

The change in the number of capital good depends positively on the

1. Number of researchers
2. Prevailing value of A

Important to note is that the positive level effect of A comes from a standing-on-the-shoulders-of-giants effect: inventions often rely on previous inventions. People familiar with the computer game Civilization will be all too aware of this.
Note the absence of a need for marketing.

s_A is taken as given here.

The allocation of labour is split between producing output and working in the research sector. If s_A is the fraction of the labour force working in the research sector

$$L = L_A + L_Y$$
$$L_A = s_A L$$

And also assume that the total number of workers grows at an exogenous rate n

$$\frac{\Delta L}{L} = n$$

Simplifying the production function

We can now simplify the production function if we define capital stock as

$$K = \sum_{i=1}^{A} x_i$$

Again, the savings rate is treated as exogenous and we assume that

$$\Delta K = s_K Y - \delta K$$

In the production process, all capital goods will play an identical role meaning that the demand from the producers for each good is the same. This means that for all i we have $x_i = \bar{x}$

$$K = A\bar{x}$$
$$\bar{x} = \frac{K}{A}$$

The production function can than be written as

$$\begin{aligned} Y &= AL_Y^{1-\alpha}\bar{x}^{\alpha} \\ &= AL_Y^{1-\alpha}\left(\frac{K}{A}\right)^{\alpha} \\ &= (AL_Y)^{1-\alpha}K^{\alpha} \end{aligned}$$

The production function looks very similar to the one used in the Swan-Solow model. The main difference of course is that the technology term A is written as $A^{1-\alpha}$. In this case output per worker can still be expressed in terms of TFP and the capital/output ratio

$$\frac{Y}{L_Y} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A$$
$$\frac{Y}{L} = (1 - s_A)\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A$$

$L_y = (1 - s_A)L$

The capital/output ratio will converge to

The technology term is A as opposed to $A^{\frac{\alpha}{1-\alpha}}$

$$\left(\frac{K}{Y}\right)^* = \frac{s_K}{n + g + \delta}$$

g takes that place of $\frac{g}{1-\alpha}$ because the TFP term grows at rate $(1-\alpha)g$ instead of g.

Steady-state growth

Looking at the steady-state growth rate, the production function can be re-written as

$s_Y = 1 - s_A$

$$Y = (A s_Y L)^{1-\alpha} K^\alpha$$

Using the usual method for calculating growth rates for Cobb-Douglas items we get[1]

[1] Note that we can use the fact that the steady-state growth rates of capital and output will be the same.

$$\frac{\Delta Y}{Y} = (1-\alpha)\left(\frac{\Delta A}{A} + \frac{\Delta s_Y}{s_y} + \frac{\Delta L}{L}\right) + \alpha\frac{\Delta K}{K}$$
$$\left(\frac{\Delta Y}{Y}\right)^* = (1-\alpha)\left(\frac{\Delta A}{A} + \frac{\Delta s_Y}{s_y} + \frac{\Delta L}{L}\right) + \alpha\left(\frac{\Delta Y}{Y}\right)^*$$

The share of labour allocated to the non-research sector (s_Y) cannot be changing along the steady-state path. If it could, the fraction of researchers would eventually go to zero or become greater than one, which would not be feasible. So we get

$$\left(\frac{\Delta Y}{Y} - \frac{\Delta L}{L}\right) = \frac{\Delta A}{A}$$

The main difference between the Swan-Solow model and the Romer model is that A is determined within the model as opposed to evolving at a fixed exogenous rate. The growth rate of capital goods is

Recall that

$$\Delta A = \gamma L_A^\lambda A^\phi$$

$$\frac{\Delta A}{A} = \gamma (s_A L)^\lambda A^{\phi - 1}$$

In the steady-state of the economy A is growing at a constant rate, calculating the growth rate of the Cobb-Douglas items we get

In the steady-state, the growth rate of the fraction of researchers ($\frac{\Delta s_A}{s_A}$) must be zero.

$$\lambda \left(\frac{\Delta s_A}{s_A} + \frac{\Delta L}{L} \right) - (1-\phi) \frac{\Delta A}{A} = 0$$
$$\left(\frac{\Delta A}{A} \right)^* = \frac{\lambda n}{1-\phi}$$

The long-run growth rate of output per worker depends on three factors

1. n; growth rate of the number of workers
2. λ; describes extent to which diminishing marginal productivity sets in as researchers are added
3. ϕ; more past inventions help to boost the rate of current inventions, increasing the growth rate

A and the steady-state path

Of interest is how fast the TFP terms (A) grows along the steady-state path. Along the path we have

$$\frac{\Delta A}{A} = \gamma (s_A L)^\lambda A^{\phi - 1} = \frac{\lambda n}{1-\phi}$$

This means that the steady-state level A^* is determined by

$$A^{\phi-1} = \frac{\lambda n}{1-\phi} \left(\gamma (s_A L)^\lambda \right)^{-1}$$
$$A^* = \left(\frac{\gamma(1-\phi)}{\lambda n} \right)^{\frac{1}{1-\phi}} (s_A L)^{\frac{1}{1-\phi}}$$

Now in terms of the convergence dynamics of A, we can show that it will always revert back to its steady-state path. Let the growth rate of A be

$$g_A = \frac{\Delta A}{A} = \gamma (s_A L)^\lambda A^{\phi-1}$$

The growth rate of g_A can be calculated as

$$\frac{\Delta A}{A} = \lambda \left(\frac{\Delta s_A}{s_A} + n \right) - (1-\phi) g_A$$

Here g_A will be falling whenever

$$g_A > \frac{\lambda n}{1-\phi} + \frac{\lambda}{1-\phi} \frac{\Delta s_A}{s_A}$$

In this case the growth rate of A will always be declining whenever it is greater that its steady state value of

$$\frac{\lambda n}{1-\phi}$$

and increasing when it is below this rate.

Except for periods when the share of researchers is changing.

Output per worker steady-state level

Output per worker can be decomposed into a capital/output ratio component and a TFP component.

$$\frac{Y}{L_y} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A$$
$$\frac{Y}{L} = (1 - s_A)\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A$$

Can use the fact that $L_y = (1 - s_A)L$.

We know the determinants of the steady-state growth rate g, which we can substitute in the formula for the steady-state capital-output ratio to arrive at

$$\left(\frac{K}{Y}\right)^* = \frac{s_K}{n + \frac{\lambda n}{1-\phi} + \gamma}$$

This can be combined with the formula for the steady-state level of A

$$A^* = \left(\frac{\gamma(1-\phi)}{\lambda n}\right)^{\frac{1}{1-\phi}} (s_A L)^{\frac{\lambda}{1-\phi}}$$

Output per worker in the steady-state path can be given by

$$\left(\frac{Y}{L}\right)^* = (1 - s_A)\left(\frac{s_K}{n + \frac{\lambda n}{1-\phi} + \gamma}\right)^{\frac{1}{1-\alpha}} \left(\frac{\gamma(1-\phi)}{\lambda n}\right)^{\frac{1}{1-\phi}} (s_A L)^{\frac{\lambda}{1-\phi}}$$

Optimal R&D levels

Although A is determined endogenously in the model due to the allocation of labour to the R&D section, we haven't really discussed how this share of labour is determined. Surely there is some optimal level that determines s_A. Although allocating more labour to $R\&D$ will increase the creation of new capital goods, we can't increase the labour share indefinitely. Increasing s_A has two separate offsetting effects on output.

1. positive effect due to the effect of researchers on technology level
2. negative effect caused by the fact that researchers don't produce actual output

So what would be the right level of labour allocation in this case? Using the formula for output-per-worker, we can re-write this the following way

X are all terms that don't contain s_A bundled together and $Z = \frac{\lambda}{1-\phi}$.

$$\left(\frac{Y}{L}\right)^* = X(1 - s_A)(s_A)^Z$$

Differentiate with respect to s_A, set equal to zero, and solve to obtain the optimizing share of researchers.

$$X(1-s_A)(s_A)^Z$$
$$Xs_A^Z - Xs_A s_A^Z$$
$$Zs_A^{Z-1} - (Z+1)s_A^Z = 0$$
$$\frac{Zs_A}{s_A^Z} = Z+1$$
$$\frac{s_A^{Z-1}}{s_A^Z} = \frac{Z+1}{Z}$$
$$s_A^{Z-1-Z} = \frac{Z+1}{Z}$$
$$s_A^{-1} = \frac{Z+1}{Z}$$
$$s_A = \frac{Z}{Z+1}$$

$$s_A^{**} = \frac{Z}{1+Z} = \frac{\frac{\lambda}{1-\phi}}{1+\frac{\lambda}{1-\phi}} = \frac{\lambda}{1-\phi+\lambda}$$

When s_A is determined endogenously the optimal level is generally not reached. The reason for this is that research generates externalities which affect the output level per worker but which are not taken into account by private individuals or firms when they make a decision to do research.

- Positive externalities arise due to the compound nature of research
  - Researchers don't account for fact that their work will benefit future productivity of other researchers
  - At high ϕ the R&D share is likely to be too low
- Negative externality arises due to the fact that $\lambda < 1$
  - Diminishing marginal productivity applies to the number of researchers

Whether there is too much or too little research in the economy depends on the strength of the externalities.

Romer model trade-offs

Although the Romer model provides some useful insights into how technological progress is determined, there are a number of trade-offs in the model.

1. Present versus the future
   - The government could stimulate people going into research in the hope of future increases in productivity
   - This comes at the cost of having lower output levels today since they are not producing
2. Competition versus growth
   - People who invent stuff can influence future inventions but likely don't profit from those inventions
   - Laws to strengthen patent protection might increase research incentives
   - This points to a potential conflict between policies aimed at raising macroeconomic growth and microeconomic policies aimed at reducing the inefficiencies due to monopoly power

Romer model and economic history

Looking at the various phases of historical invention actually seem to provide some empirical evidence that supports the Romer model. Let's look at the main waves of industrial progress

1. First industrial revolution (1750-1830)
   - Improvement of steam engine, advent of railroads and steamships.
   - Took 150 years to have full impact
2. Second industrial revolution (1870-1900)
   - Electric light, internal combustion engine, sewer systems
   - Telephone, radio, movies, machinery
   - Later inventions build on this progress
   - Lot of this progress applied during the World Wars
3. Third industrial revolution (1960-present)
   - Computers

Cross-country technology diffusion

The Romer model illustrates how the invention of new technologies can promote economic growth. However, in real life it seems that only very few countries seem to be at the technological frontier. Hall & Jones (1999) looked at this issue by examining the TFP levels for different countries. The basis of their study is a level accounting exercise where they examine the effect of education on labour productivity using the following production function

$$Y_i = K_i^{\alpha}(h_i A_i L_i)^{1-\alpha}$$

In this function h_i is a variable that measures human capital, based on the returns to education. The production function can be re-formulated as

$$\frac{Y_i}{L_i} = \left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} h_i A_i$$

$\alpha = \frac{1}{3}$

In their study h_i is estimated using data on educational levels, which allows them to express cross-country differences in output per worker in three multiplicative terms. Their results suggested that the output per worker in the five richest countries was 31.7 times that of the poorest five countries

1. Capital intensity

- Contributed to differences by a factor of 1.8

2. Human capital per worker
   - Contributed to differences by a factor of 2.2
3. Total factor productivity
   - Contributed to differences by a factor of 8.3

Technological diffusion

The Romer model should not be thought of as a model that explains growth in just a single country. Most countries in the world use technologies that were not invented in that particular country, as technologies end up being used all around the world. As such, the Romer model is probably best thought of as a long-run model for the world. So how does technology spread across the globe? Let's describe a model where there is a lead country with technology level A_t that grows every period at rate g.

An interesting example is the industrial development in the recently unified German state in the late 1800s which could benefit from British inventions to catalyst the second industrial revolution.

$$\frac{\Delta A_t}{A_t} = g$$

All other j countries in the world have a growth rate that is determined by

$$\frac{\Delta A_{jt}}{A_{jt}} = \lambda_j + \sigma_j \frac{(A_t - A_{jt})}{A_{jt}}$$

We assume $\sigma_j > 0$ because countries can learn from superior technologies in leader countries (see Japan).

We also assume $\lambda_j < g$ so country j can't grow faster than the lead country without the learning that comes from having lower technology than the frontier.

The growth in the lead country is such that

$$\frac{\delta A_t}{dt} = \Delta A_t = g A_t$$

Which can be characterised by what is known as exponential growth

e is a mathematical constant which has the property that $\frac{de^x}{dx} = e^x$

$$A_t = A_o e^{gt}$$

The equation for the dynamics of A_{jt} can be re-written as

$$\Delta A_{jt} = \lambda_j A_{jt} + \sigma_j (A_t - A_{jt})$$

We have seen this type of equation before as it is a first-order linear differential equation. This equation can be solved to illustrate how A_j changes over time. Drawing terms together we can re-write the equation as

Recall that it is a differential equation because it involves a derivative; first-order because it only involves a first derivative; linear because it doesn't involve any terms taken to powers than are not one.

$$\Delta A_{jt} + (\sigma_j - \lambda_j) A_{jt} = \sigma_j A_t$$
$$\Delta A_{jt} + (\sigma_j - \lambda_j) A_{jt} = \sigma_j A_0 e^{gt}$$

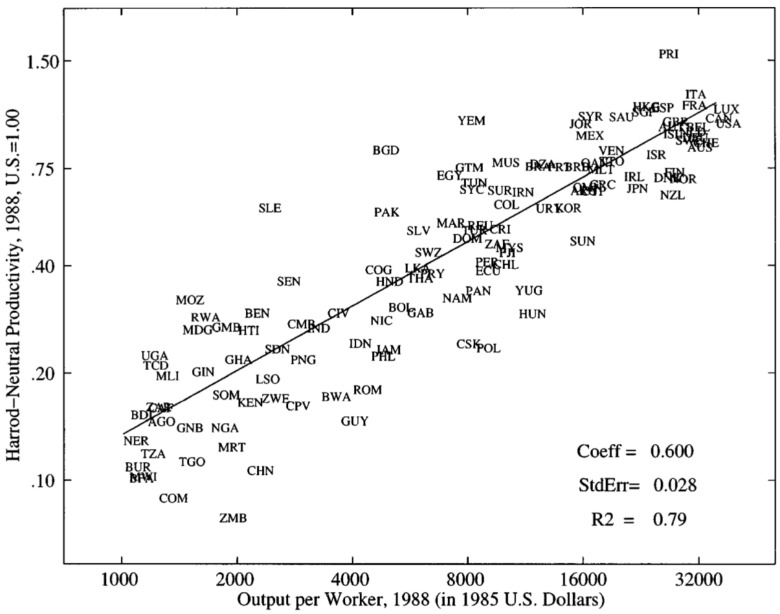

FIGURE I
Productivity and Output per Worker

TABLE I
PRODUCTIVITY CALCULATIONS: RATIOS TO U. S. VALUES

Country	Y/L	Contribution from $(K/Y)^{\alpha/(1-\alpha)}$	H/L	A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 countries:	0.296	0.853	0.565	0.516
Standard deviation:	0.268	0.234	0.168	0.325
Correlation with Y/L (logs)	1.000	0.624	0.798	0.889
Correlation with A (logs)	0.889	0.248	0.522	1.000

The elements of this table are the empirical counterparts to the components of equation (3), all measured as ratios to the U. S. values. That is, the first column of data is the product of the other three columns.

One possible solution is where the A_{jt} is something in the form of $B_j e^{gt}$ where B_j is an unknown coefficient, which must satisfy

$$gB_j e^{gt} + (\sigma_j - \lambda_j)B_j e^{gt} = \sigma_j A_0 e^{gt}$$

Canceling the e^{gt} terms we get

$$B_j = \frac{\sigma_j A_0}{\sigma_j + g - \lambda_j}$$

Which takes the form

$$A_{jt}^p = B_j e^{gt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) A_0 e^{gt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) A_t$$

By adding an additional term we can get a more general solution. Assume that solution takes the form

$$A_{jt} = B_j e^{gt} + D_{jt}$$

This solution would obey

$$gBe^{gt} + D_{jt} + (\sigma_j - \lambda_j)(Be^{gt} + D_{jt}) = \sigma_j A_0 e^{gt}$$

The e^{gt} terms cancel out as we've seen by construction of B_j so

$$\begin{aligned} D_{jt} + (\sigma_j - \lambda_j)D_{jt} &= 0 \\ D_{jt} &= D_{j0} e^{-(\sigma_j - \lambda_j)t} \end{aligned}$$

All possible solutions for technology in country j must take the form

$$A_{jt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) A_t + D_{j0} e^{(-\sigma_j - \lambda_j)t}$$

Technology convergence over time

We can express A_{jt} as a ratio of the frontier level of technology

$$\frac{A_{jt}}{A_t} = \frac{\sigma_j}{\sigma_j + g - \lambda_j} + \frac{D_{j0}}{A_0} e^{-(\sigma_j + g - \lambda_j)t}$$

Recall that

- $\lambda_j < g$
  - Without catch-up growth, the follower's technology grows slower than the lead
- $\sigma_j > 0$
  - Some learning takes place

Taken together this mean that we get

$$e^{-(\sigma_j+g-\lambda_j)t} \to 0 \; as \; t \to \infty$$

This entails that $e^{-(\sigma_j+g-\lambda_j)t}$ tends towards zero, so over time, as this terms disappears, the country converges towards a constant level of technology that is a constant ratio on the frontier level

$$\frac{\sigma_j}{\sigma_j + g - \lambda_j}$$

While its growth rate tends towards g.

Steady-state properties

Given that $g - \lambda_j > 0$ we know that

$$0 < \frac{\sigma_j}{\sigma_j + g - \lambda_j} < 1$$

Basically this tells us that each country actually never catches up with the leader; it converges to some fraction of the lead country's technology level.

It also implies the following

$$\frac{d}{d\sigma_j}\left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) > 0$$

Meaning that the equilibrium ratio of a country's technology to the leader's depends positively on learning parameter σ_j.

We also have that

$$\frac{d}{d\lambda_j}\left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) > 0$$

Meaning that the more growth a country can generate in each period, independent of learning from the leader, the higher it's equilibrium ratio will be.

Transition dynamics

The equation for A_{jt} can be expressed as a ratio of the frontier level of technology

$$\frac{A_{jt}}{A_t} = \frac{\sigma_j}{\sigma_j + g - \lambda_j} + \frac{D_{j0}}{A_0}e^{-(\sigma_j+g-\lambda_j)t}$$

The second term in the equation tends to go to zero over time. Nonetheless, it is still important given that the country's development along the transition path depends on the value of of parameter D_{j0}.

1. $D_{j0} < 0$
   - Country will start below equilibrium technology ratio
   - Technology growth will be faster than the leader country for some periods of time
2. $D_{j0} > 0$
   - Country will start above equilibrium technology ratio
   - Technology growth will be slower than the leader country for some periods of time

The transition dynamics are illustrated using a simulation where the leader economy has a growth rate of $g = 0.02$ and the follower economy has parameter values $\lambda_j = 0.01, \sigma_j = 0.04$. These values mean that the follower economy will converge at a technology level 20% below the leader.

$$\frac{\sigma_j}{\sigma_j + g - \lambda_j} = \frac{0.04}{0.04 + 0.02 - 0.01} = 0.8$$

The charts on the next pages show two situations

1. $D_{j0} = -.5$
   - Economy starts with a technology level 30% of leader economy
2. $D_{j0} = 0.5$
   - Economy starts with a technology level 30% above leader economy

Growth miracles

Can also examine the effect of growth miracles, which are sometimes observed when countries start to suddenly experience rapid growth. Well-known examples of this are China, various South-East Asian countries like South Korea and Taiwan, Chile, and Ireland. A country can possibly increase the σ_j value by improving education or via science-related policies. As a result its position in the steady-state distribution of income may move upwards substantially, the economy experiencing a phase of rapid growth. In the following figures the σ_j values changes from 0.005 to 0.04 at $t = 21$. This means that the equilibrium technology ratio changes from $\frac{1}{3}$ to 0.8, where the economy sees a long transitional period of rapid growth. The main message from this is that most countries don't have to re-invent the wheel in order to achieve higher living standard but that they can learn from other, more technologically advanced countries. Japan is a prime example of this.

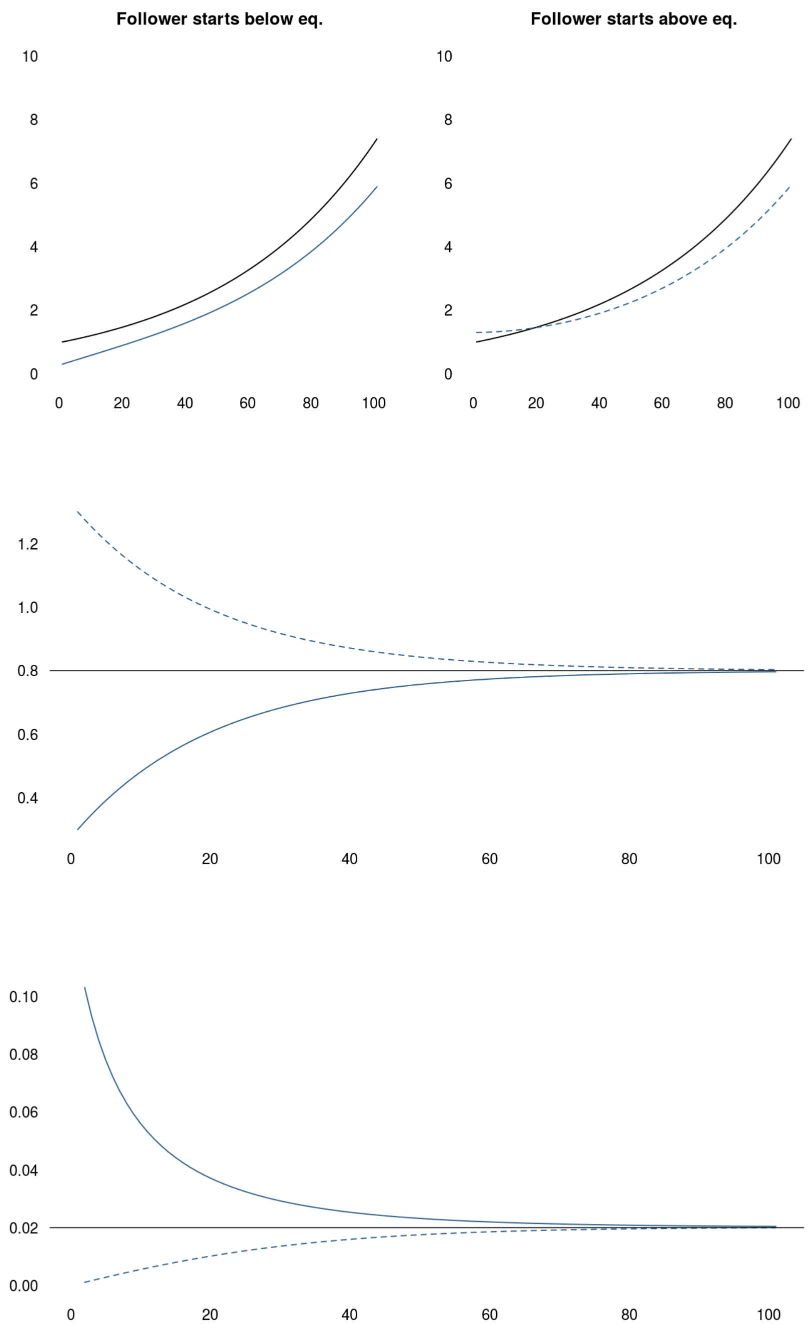

Figure 1: Simulating technology convergence for one economy ahead (dashed blue line) of the technology leader (black line) and one economy behind (blue line). *Top:* Technology level over time for country behind (left) and ahead (right) of technology leader. *Middle:* Technology ratio compared to leader economy. *Bottom:* Technology growth rate.

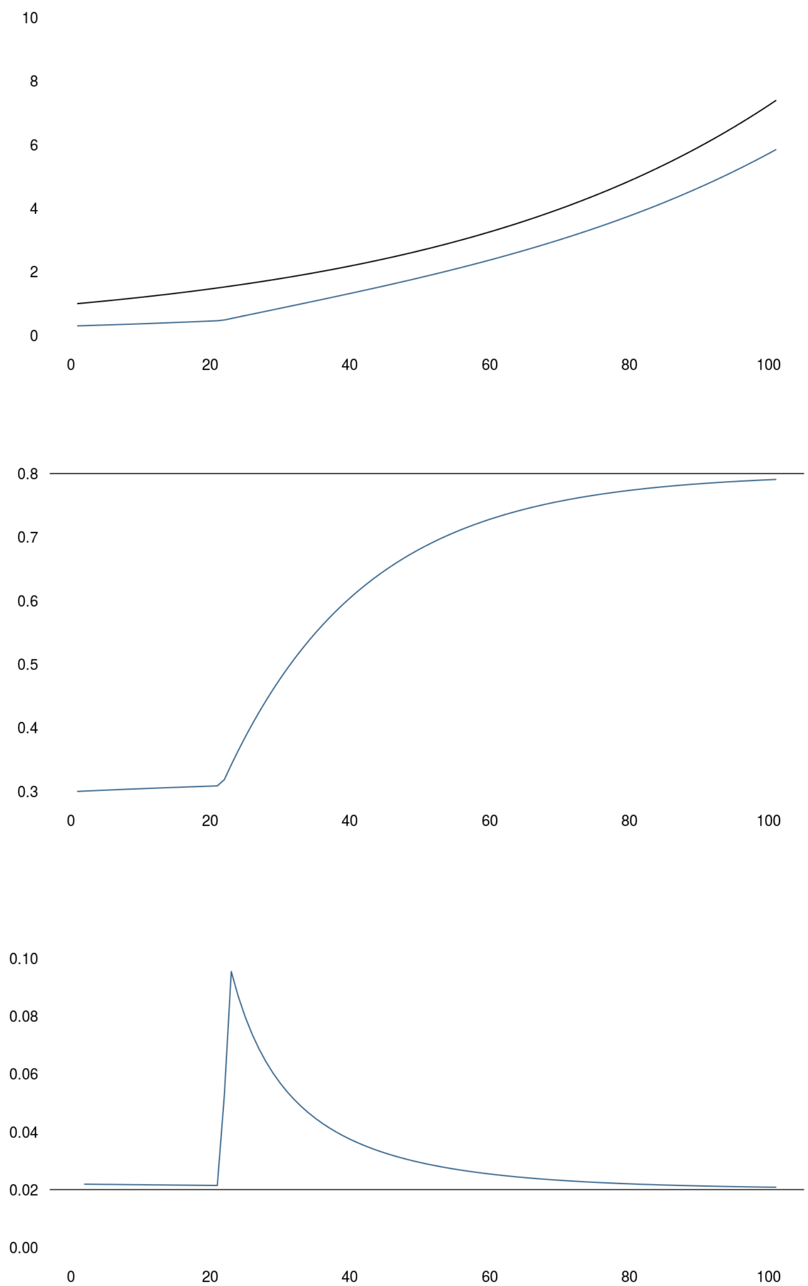

Figure 2: Simulating a growth miracle (blue line). *Top:* Technology level over time. *Middle:* Technology ratio compared to leader economy. *Bottom:* Technology growth rate.