Growth accounting

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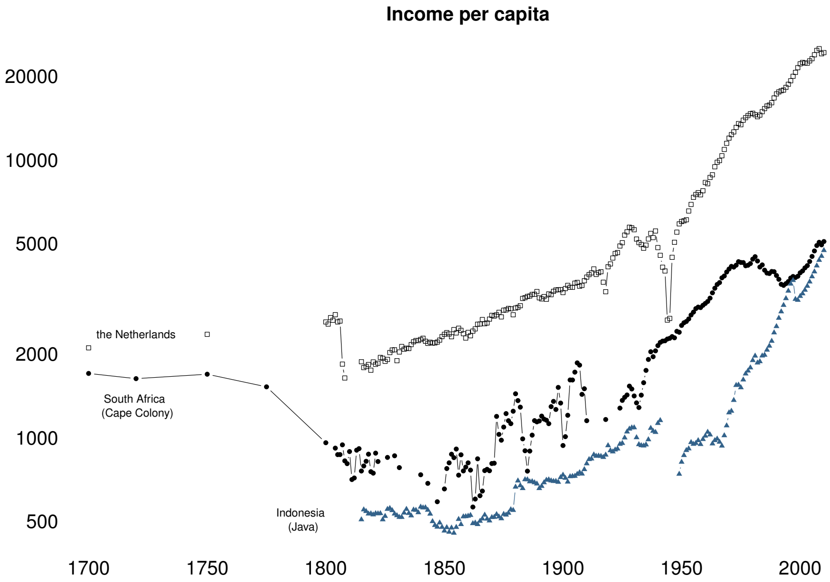

Income per capita
20000
10000
5000
2000
1000
500
the Netherlands
South Africa
(Cape Colony)
Indonesia
(Java)
1700
1750
1800
1850
1900
1950
2000

A stylised fact from economic history: standards of living have been roughly similar in Assyria (1500 BCE), Roman Egypt, and late 18th century England (Clark, 2007)

- Strong link between income per capita and income growth
- Any increases in aggregate outcome offset by increase in population size

In this Malthusian world income per capita remains at constant level, despite technological progress

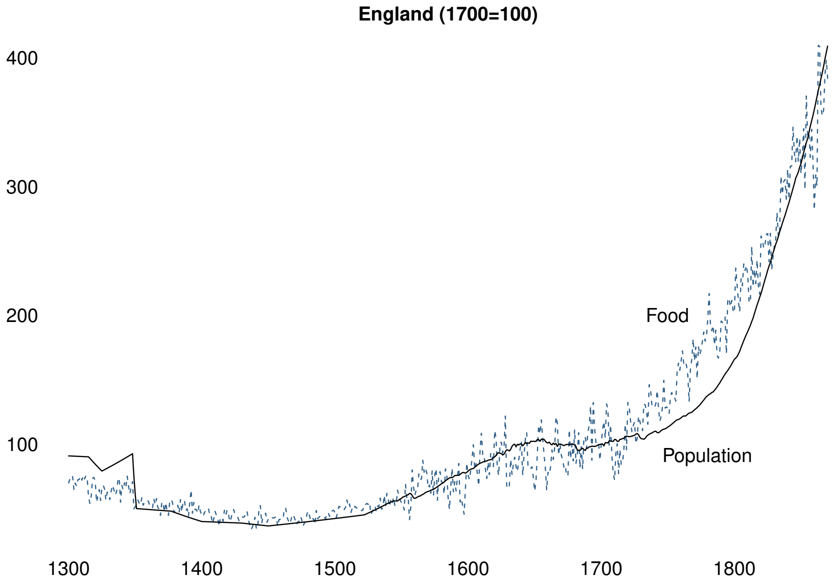
1. Long-run stagnation
2. Population growth will outpace agricultural productivity

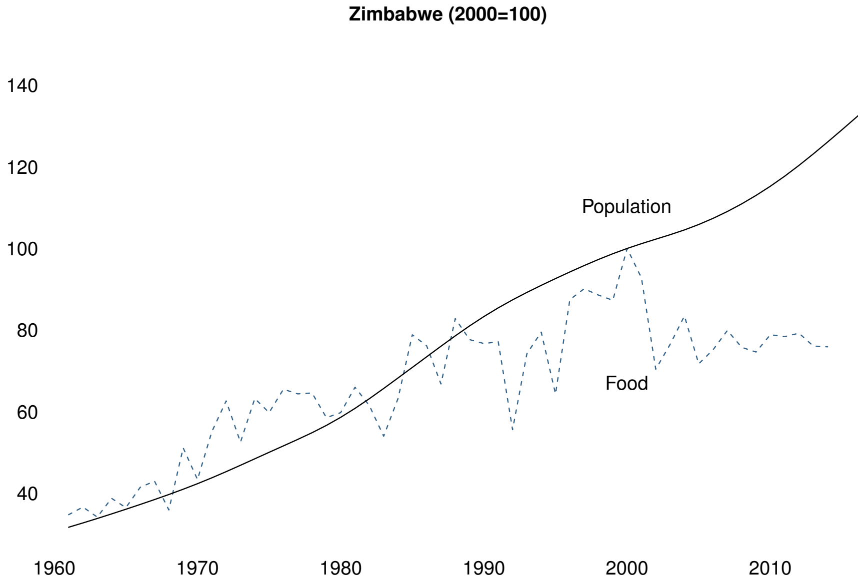
Empirical predictions of the Malthusian model are not that impressive

- Specifically that mortality should rise as living standards fall (Kelly & O'Grada, 2014)

Nonetheless, the Malthusian has had great influence on economic thinking

- Research on environment and conflict
- 'Shithole countries'


England (1700=100)
400
300
200
100
Food
Population
1300
1400
1500
1600
1700
1800


Zimbabwe (2000=100)
140
120
100
80
60
40
Population
Food
1960
1970
1980
1990
2000
2010

Let's consider a Malthusian model where output is determined by two factors

1. Land X
2. Labour L

For simplicity we assume that

- Amount of land is fixed
- Labour force $=$ total population

Aggregate production function can be written as

$$Y = AX^{\alpha}L^{1-\alpha} \tag{1}$$

Y is total aggregate output

A is technology parameter
α is output elasticity

Output Y will increase when total population L increases

$$\frac{\partial Y}{\partial L} = (1-\alpha)AX^{\alpha}L^{-\alpha} \tag{2}$$

$$= (1-\alpha)A\left(\frac{X}{L}\right)^{\alpha} > 0 \tag{3}$$

The marginal product is positive but it will decrease given that L is in the denominator

$$\frac{\delta^2 Y}{\delta L^2} = -\alpha(1-\alpha)AX^{\alpha}L^{-\alpha-1} < 0 \tag{4}$$

To measure living standards we can use output per capita y. Take

$$Y = AX^{\alpha}L^{1-\alpha} \tag{5}$$

and divide by L

$$\frac{Y}{L} = \frac{AX^{\alpha}L^{1-\alpha}}{L} \tag{6}$$

$$= A\left(\frac{X}{L}\right)^{\alpha} = Ax^{\alpha} = y \tag{7}$$

x is land per capita

Increase in population size will have two effects on living standards

1. Production increase, resulting in increase in living standards
2. More people to share production with, meaning a decrease in living standards

In the Malthusian framework effect 2 will dominate, therefore population growth will lead to a fall in living standards.

More formally;

$$\frac{\partial y}{\partial L} = -\alpha A X^{\alpha} L^{-\alpha-1} < 0 \tag{8}$$

$$\frac{\partial^2 y}{\partial L^2} = \alpha(1-\alpha) A X^{\alpha} L^{-\alpha-2} > 0 \tag{9}$$

Let's have a closer look at the population dynamics: In the model population size L_t equals last year's population size plus the number of births B_t and minus the number of deaths D_t during the same year

$$L_t = L_{t-1} + B_t(y_{t-1}) - D_t(y_{t-1}) \tag{10}$$

Key feature here is that the number of births and deaths is a function of living standards or output per capita, lagged one year: y_{t-1}

$$B'(y_{t-1}) > 0 \tag{11}$$
$$D'(y_{t-1}) < 0 \tag{12}$$

For simplicity the birth and death rate are assumed to be linear functions of output.

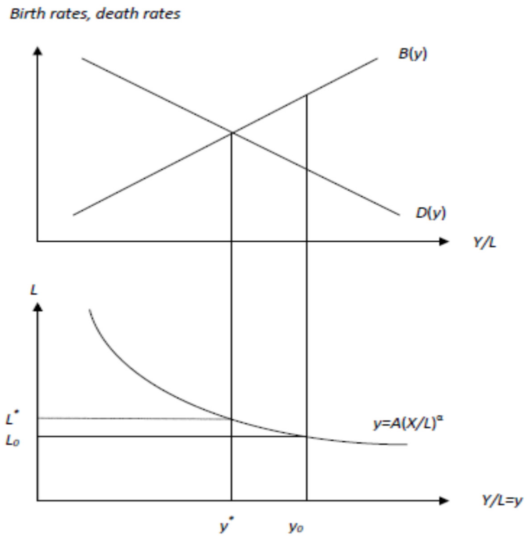


Figure 2.1: The Malthusian trap

When output per capita increases, the food supply will increase which allows families to expand and reduce mortality due to better nourishment. Output per capita will tend to converge towards an equilibrium indicated by y^*, at which point the population ceases to grow.

- This is known as the subsistence level

$$L_t - L_{t-1} = B_t(y^*) - D_t(y^*) = 0 \tag{13}$$

Consider the case where we start at a relatively high output level, y^0. Given the relatively high output per capita there will be (relatively)

- Few deaths
- Many births

This will increase the population, decreasing living standards shifting the equilibrium to the left.

- One can imagine a scenario starting at the left of the equilibrium where many people die and few children are born
- In this case the shrinking population will increase output per capita

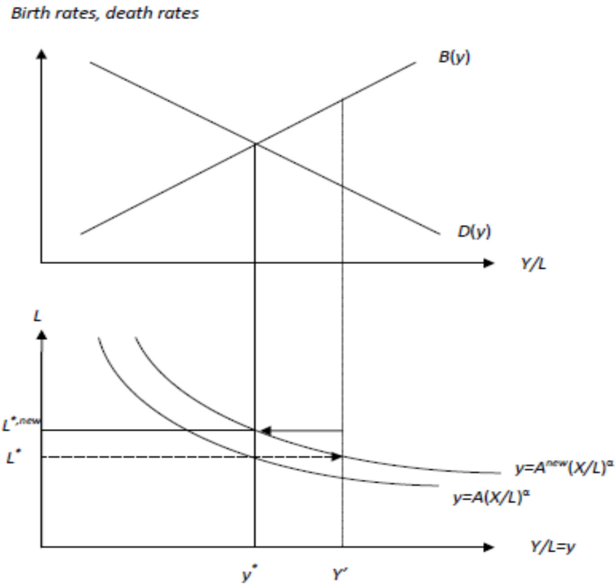


Figure 2.2: Technological progress in the Malthusian model

Following a positive technology shocks there will be an increase in productivity: Productivity increase will shift equilibrium punt outward on y-curve: temporary increase in output per capita

- This will increase birth rate and decrease death rate
- Population will increase reducing the living standards

As such, the only lasting thing of a technology shock is a larger population.

History has shown that there are a number of factors contributing to a break with the Malthusian trap:

1. Fall in birth rate ending link with income per capita
2. Increase in education level
3. Increase in growth of technological knowledge
4. Increase in output per capita far beyond subsistence level

Moving on to the Cobb-Douglas model

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} \tag{14}$$

Assume that output is determined by an aggregate production function technology depending on the total amount of labour (L) and capital (K).

In the Cobb-Douglas model A_t accounts for technology, a measure for productive efficiency

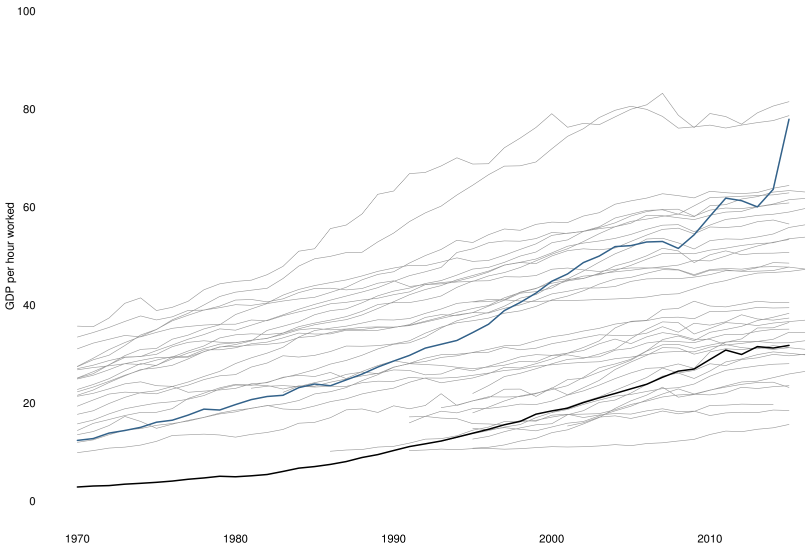
- Increase in A_t results in higher output without having to raise inputs
- Fluctuates for various reasons, e.g. new technology, government regulation, better management

Since A_t increases productiveness of other factors, it is also known as Total Factor Productivity (TFP)

Again, we are interested in output per worker (productivity) which is given by

$$\frac{Y_t}{L_t} = A_t K^{\alpha} L_t^{\beta-1} = A_t \left(\frac{K_t}{L_t}\right)^{\alpha} L_t^{\alpha+\beta-1} \tag{15}$$

Here increases in output per worker is productivity growth.



100
80
60
40
20
0
GDP per hour worked
1970
1980
1990
2000
2010

The Cobb-Douglas model shows that there are three potential ways through which to increase productivity

1. L: Increase in labour force
2. K: Increasing the amount of capital per worker (capital deepening)
3. A: Improving the efficiency of the economy (technological progress)

Increasing the labour will add to growth if

$$\alpha + \beta > 1 \tag{16}$$

i.e. under increasing returns to scale; most growth theories assume constant returns to scale (CRS)

$$\alpha + \beta = 1 \tag{17}$$

$$\frac{Y_t}{L_t} = A_t \left(\frac{K_t}{L_t} \right)^{\alpha} \tag{18}$$

Let's examine what determines growth under the CRS assumptions, given

$$\alpha + \beta = 1 \tag{19}$$

We get

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{20}$$

Time is assumed to be continuous

- t evolves smoothly instead of taking integer values like $t = 1, t = 2, ...$

The growth rate of Y_t can be denoted by G_t^Y and defined as

$$G_t^Y = \frac{1}{Y_t} \frac{dY_t}{dt} \tag{21}$$

i.e growth equals the change in output divided by output level.

Can differentiate production function with respect to time; expressing it as a function of G_t^Y. E.g. for capital

$$\frac{\partial K_t^\alpha}{\partial t} = \frac{\partial K_t^\alpha}{\partial K_t}\frac{\partial K_t}{\partial t} = \alpha K_t^{\alpha-1}\frac{\partial K_t}{\partial t} \tag{22}$$

We can apply this for the whole function using the product rule

$$\frac{\partial ABC}{\partial x} = BC\frac{\partial A}{\partial x} + AC\frac{\partial B}{\partial x} + AB\frac{\partial C}{\partial x} \tag{23}$$

Can calculate the terms involving the impact of changes in capital and labour inputs as

$$\begin{aligned}
\frac{\partial Y_t}{\partial t} &= \frac{\partial A_t K_t^{\alpha} L_t^{1-\alpha}}{\partial t} \\
&= K_t^{\alpha} L_t^{1-\alpha} \frac{\partial A_t}{\partial t} + A_t L_t^{1-\alpha} \frac{\partial K_t^{\alpha}}{\partial t} + A_t K_t^{\alpha} \frac{\partial L_t^{1-\alpha}}{\partial t} \\
&= K_t^{\alpha} L_t^{1-\alpha} \frac{\partial A_t}{\partial t} + \alpha A_t K_t^{\alpha-1} L_1^{1-\alpha} \frac{\partial K_t}{\partial t} + (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} \frac{\partial Lt}{\partial t}
\end{aligned} \tag{24}$$

Calculate growth rate of output by dividing both sides by Y_t

$$\frac{1}{Y_t}\frac{\partial Y_t}{\partial t} = \frac{K_t^{\alpha}L_t^{1-\alpha}}{A_t K_t^{\alpha} L_t^{1-\alpha}}\frac{\partial A_t}{\partial t}+ \tag{25}$$

$$\alpha\frac{A_t K_t^{\alpha-1}L_1^{1-\alpha}}{A_t K_t^{\alpha} L_t^{1-\alpha}}\frac{\partial K_t}{\partial t}+(1-\alpha)\frac{A_t K_t^{\alpha} L_t^{-\alpha}}{A_t K_t^{\alpha} L_t^{1-\alpha}}\frac{\partial Lt}{\partial t}$$

$$\frac{1}{Y_t}\frac{\partial Y_t}{\partial t} = \frac{1}{A_t}\frac{\partial A_t}{\partial t}+\alpha\frac{1}{Kt}\frac{\partial K_t}{\partial t}+(1-\alpha)\frac{1}{L_t}\frac{\partial L_t}{\partial t}$$

$$G_t^Y = G_t^A + \alpha G_t^K + (1-\alpha)G_t^L \tag{26}$$

NB- This is the same as dividing by $A_t K_t^{\alpha} L_t^{1-\alpha}$

$$G_t^Y = G_t^A + \alpha G_t^K + (1-\alpha) G_t^L \tag{27}$$

i.e. output growth equals technology growth plus a weighted average of the growth rates of capital and labour

- Weight determined by α

This is the key equation in growth accounting studies.

Growth accounting studies provide estimates of how much GDP growth over a certain time span is determined by

1. Growth in the number of workers
2. Growth in capital stock
3. Improvement in Total Factor Productivity

Calculating the growth equation

$$G_t^Y = G_t^A + \alpha G_t^K + (1-\alpha) G_t^L \tag{28}$$

requires

- A measure for output i.e. GDP
- Number of workers
- Capital stock estimate
- Value for Total Factor Productivity

One empirical issue is that the value for TFP (A_t) can't be directly observed. However, if we know the value of parameter α we can get an estimate for growth.

$$G_t^A = G_t^Y - \alpha G_t^K - (1-\alpha) G_t^L \tag{29}$$

Solow (1957) pointed out that an α estimate could be obtained by looking at the shares of GDP paid to workers and to capital. To illustrate, consider a perfectly competitive firm that is seeking to maximise profits and the firm

- Sells product at price P_t
- Pays wages W_t
- Rents its capital at a rate of R_t

The firm's profits are given by

$$\begin{aligned}\Pi_t &= P_t Y_t - R_t K_t - W_t L_t \\ &= P_t A_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - W_t L_t\end{aligned} \tag{30}$$

$P_t Y_t$ is total nominal GDP
$R_t K_T$ is the total amount of income paid to capital
$W_t L_t$ is the total amount of income paid to wages

The firm only needs to decide how much labour and capital to use It will therefore maximise profits by differentiating the function with respect to capital and labour and set the derivatives equal to zero which provides two conditions

$$\begin{aligned}
\frac{\partial \Pi_t}{\partial K_t} &= \alpha P_t A_t K_t^{\alpha-1} L_t^{1-\alpha} - R_t && (31)\\
&= \alpha \frac{P_t Y_t}{K_t} - R_t = 0\\
\frac{\partial \Pi_t}{\partial L_t} &= (1-\alpha) P_t A_t K_t^{\alpha} L_t^{-\alpha} - W_t && (32)\\
&= (1-\alpha) \frac{P_t Y_t}{L_t} - W_t = 0
\end{aligned}$$

Rearranging we get

$$\alpha = \frac{R_t K_t}{P_t Y_t} \tag{33}$$

$$1 - \alpha = \frac{W_t L_t}{P_t Y_t} \tag{34}$$

α will be the total amount of income paid to capital relative to total income, at the aggregate level nominal GDP

- $1 - \alpha$ can be calculated as the fraction of income paid to workers instead of compensating capital.

Solow's results implied that $\alpha < 0.5$

- For most countries the national income accounts show that wage income explain most of GDP
- $\alpha = \frac{1}{3}$ is used, based on the estimates for the US economy

Solow's paper also concluded that capital deepening had not been that important for US growth

- TFP growth accounted for 87.5% of growth in productivity over the period

TFP is sometimes called the Solow residual because it is a backed out calculation that makes things add up

In the standard Swan-Solow model the production functions links output to capital and labour inputs as well as a technological efficiency parameter.

$$Y_t = AF(K_t, L_t) \tag{35}$$

A key feature of the model is that, with a constant labour supply,

there are diminishing marginal returns to capital accumulation meaning that each increase in capital will give a progressively smaller increase in output

$$\frac{\delta^2 Y_t}{\delta K_t} < 0 \tag{36}$$

Some additional assumptions of the model

$$Y_t = C_t + I_t \tag{37}$$

$$S_t = Y_t - C_t = I_t \tag{38}$$

$$\frac{\partial K_t}{\partial t} = I_t - \delta K_t \tag{39}$$

$$S_t = sY_t \tag{40}$$

i.e.

- All output takes the form of consumption or investment
- Savings will equal investment
- Capital will depreciate
- Consumers save constant share of income

These assumptions tell us something about the model's capital dynamics since the amount of savings equals the amount of investment, this means that investment is also a constant fraction of output

$$I_t = sY_t \tag{41}$$

This entails that the capital stock changes over time according to

$$\frac{\partial K_t}{\partial t} = sY_t - \delta K_t \tag{42}$$

Capital stock development over time depends on whether investments are greater, equal to, or less than the depreciation rate

$$\partial K_t < sY_t \Rightarrow \frac{\partial K_t}{\partial t} > 0 \tag{43}$$

$$\partial K_t = sY_t \Rightarrow \frac{\partial K_t}{\partial t} = 0 \tag{44}$$

$$\partial K_t > sY_t \Rightarrow \frac{\partial K_t}{\partial t} < 0 \tag{45}$$

The stock of capital will stay constant if the capital/output ratio is

$$\frac{K_t}{Y_t} = \frac{s}{\delta} \tag{46}$$

The level of investments is given by

$$I_t = sY_t = sAF(K_t, L_t) \tag{47}$$

This means that an one-off increase in technology level A has the same effect as a one off increase in s

- Capital and output gradually increase to a new level

The model implies a very important difference between these two determinants of growth

- Savings rate s is subject to a limit, whereas as A does not face such constraints

Therefore, in order to have long-term sustainable growth increases in TFP matter

- Specifically, growth through capital accumulation will taper off over time producing a one-off increase in output per worker whereas TFP growth can lead to sustained higher growth rates of output per worker

We can define the capital-output ratio as

$$\frac{K_t}{Y_t} = K_t Y_t^{-1} = x_t \tag{48}$$

and the growth rate can be written as

$$\frac{\Delta x_t}{x_t} = \frac{\Delta K_t}{K_t} - \frac{\Delta Y_t}{Y_t} \tag{49}$$

We can now define the growth equation

$$G_t^Y = G_t^A + \alpha G_t^K + (1-\alpha) G_t^L \tag{50}$$

as

$$\frac{\Delta Y_t}{Y_t} = g + \alpha \frac{\Delta K_t}{K_t} + (1-\alpha) n \tag{51}$$

Capital growth is given as

$$\frac{\Delta K_t}{K_t} = s \frac{Y_t}{K_t} - \delta = \frac{s}{x_t} - \delta \tag{52}$$

Meaning that the growth of the capital-output ratio is given by

$$\begin{aligned} \frac{\Delta x_t}{x_t} &= (1-\alpha) \frac{\Delta K_t}{K_t} - g - (1-\alpha) n \\ &= (1-\alpha) \left(\frac{s}{x_t} - \frac{g}{1-\alpha} - n - \delta \right) \end{aligned} \tag{53}$$

The growth rate of x_t depends negatively on the value of x_t

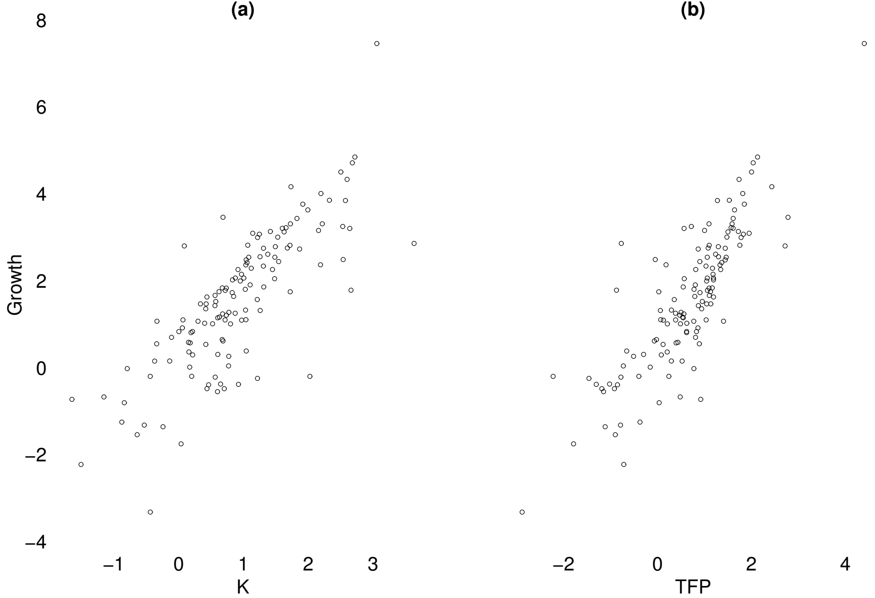
- When it is above a certain x_t value the growth rate will decline and it will increase under said x_t value

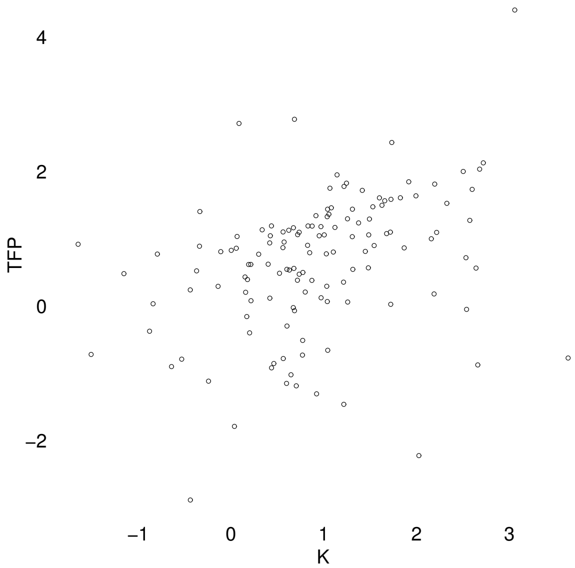
The capital-output ratio therefore exhibits convergent dynamics leading to a particular long-run steady state value. In equilibrium the capital-output ratio equals 0, this is when

$$x^* = \frac{s}{\frac{g}{1-\alpha} + n + \delta} \tag{54}$$

NB- Results from growth accounting studies can potentially be misleading as they misidentify the source of growth

- Consider a country that allocates a fixed share of GDP to investments but is experiencing a steady growth in TFP
- The Swan-Solow model predicts in this case a steady increase in output per worker and an increase in capital stock
- Observing this increase a growth accounting study might conclude that a certain percentage of growth is caused by capital accumulation, whereas all growth is caused by the TFP


(a)
(b)
Growth
8
6
4
2
0
−2
−4
−1
0
1
2
3
K
−2
0
2
4
TFP

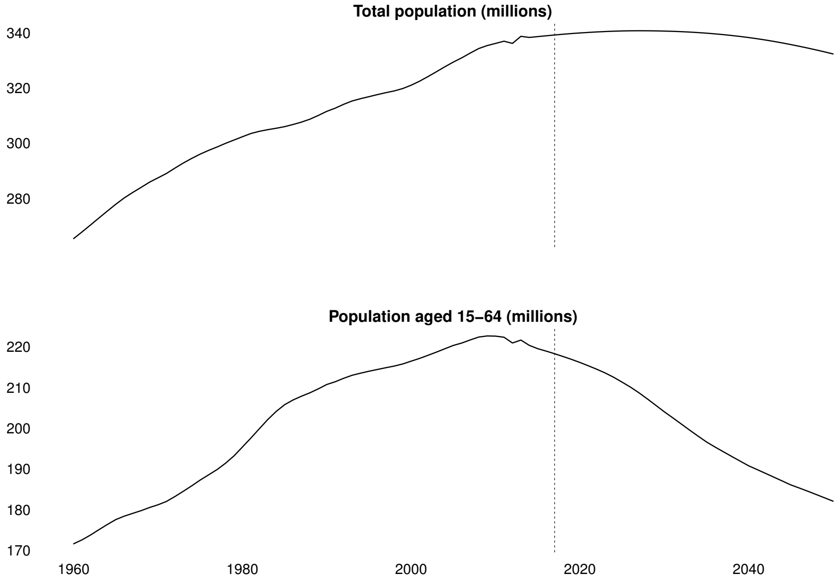

4
2
0
−2
TFP
−1
0
1
2
3
K

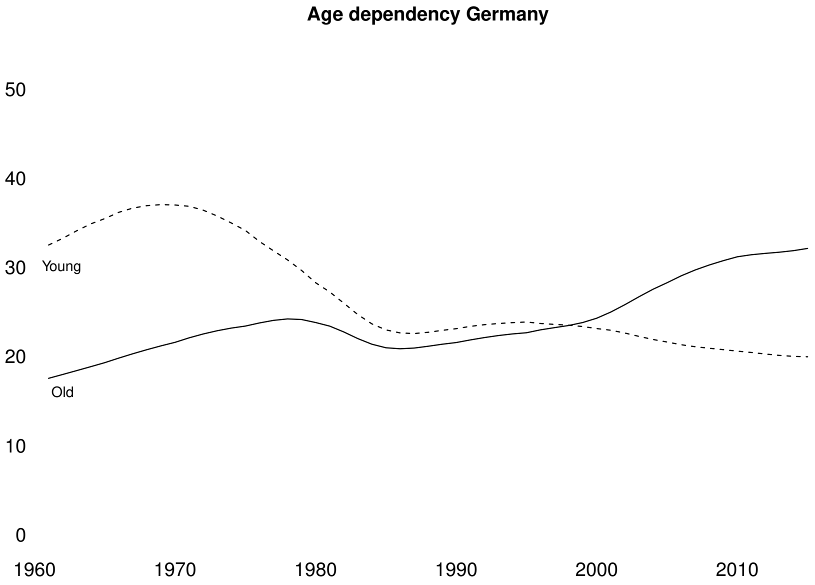
One main challenge for future economic growth in Europe is dealing with demographic changes

- Specifically the aging of the population
- Population growth is slowing and will peak in the middle of the century
- Worryingly, the working population (15-64 year) has already peaked and will decline

Macroeconomic problems facing Europe are twofold

1. Short term: weak aggregate demand and high level of debt
2. Long term: Changing demography


Total population (millions)
340
320
300
280
Population aged 15−64 (millions)
220
210
200
190
180
170
1960
1980
2000
2020
2040

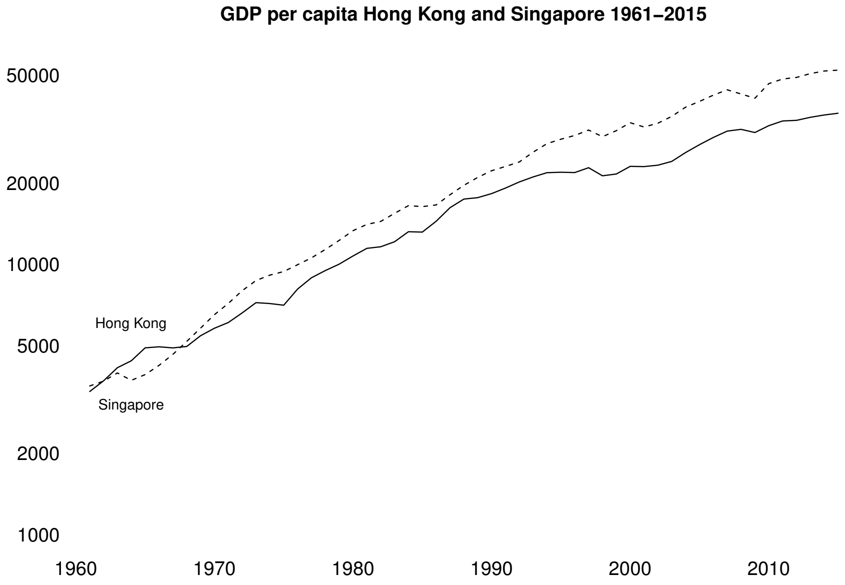

Age dependency Germany
50
40
30
20
10
0
1960
1970
1980
1990
2000
2010
Young
Old

Young (1992) provides a growth accounting study focusing on Hong Kong and Singapore

- Both cities have been very successful in restructuring their economy; Hong Kong experienced an economic growth of 147% between the early 1970s and 1990; Singapore 154%

Young focuses on these two cities as they have a similar background yet are different on a number of issues emphasized by growth theory. Some similarities in the prewar period include

- Both British colonies
- Entrepôt trading ports
- Little domestic manufacturing


GDP per capita Hong Kong and Singapore 1961−2015
50000
20000
10000
5000
2000
1000
1960
1970
1980
1990
2000
2010
Hong Kong
Singapore

During the postwar period, both city-states developed export-dependent manufacturing industries, going from producing textiles to clothing, plastics, electronics, and since the 1980s shifting to banking and financial services. Two important differences between the cities are

1. Hong Kong had a better educated population in the early postwar years
2. Hong Kong has pursuit laissez-faire policies, whereas Singapore implemented forced national savings and attracted a lot of foreign direct investment

Table 5 CRUDE ESTIMATE OF TOTAL FACTOR PRODUCTIVITY GROWTH

Time period	*Growth of*			*Average capital share*	*Percentage contribution of*		
	Output	*Labor*	*Capital*		*Labor*	*Capital*	*TFP Δ*
Hong Kong							
71–76	0.406	0.165	0.447	0.330	0.27	0.36	0.36
76–81	0.512	0.253	0.527	0.386	0.30	0.40	0.30
81–86	0.294	0.095	0.388	0.421	0.19	0.55	0.26
86–90	0.260	0.036	0.237	0.414	0.08	0.38	0.54
71–90	1.472	0.549	1.599	0.384	0.23	0.42	0.35
Singapore							
70–75	0.454	0.247	1.005	0.553	0.24	1.22	−0.47
75–80	0.408	0.256	0.503	0.548	0.28	0.68	0.04
80–85	0.300	0.069	0.620	0.491	0.12	1.01	−0.13
85–90	0.383	0.252	0.273	0.468	0.35	0.33	0.31
70–90	1.545	0.825	2.402	0.533	0.25	0.83	−0.08

Table 6 TOTAL FACTOR PRODUCTIVITY GROWTH

Time period	*Growth of*			*Average capital share*	*Percentage contribution of*		
	Output	*Labor*	*Capital*		*Labor*	*Capital*	*TFP Δ*
Hong Kong							
61–66	0.577	0.130	0.694	0.393	0.14	0.47	0.39
66–71	0.322	0.126	0.377	0.355	0.25	0.42	0.33
71–76	0.406	0.098	0.361	0.330	0.16	0.29	0.54
76–81	0.512	0.350	0.527	0.386	0.42	0.40	0.18
81–86	0.294	0.108	0.374	0.421	0.21	0.54	0.25
Singapore							
66–70	0.507	0.157	0.576	0.562	0.14	0.64	0.23
70–75	0.454	0.317	0.860	0.553	0.31	1.05	−0.36
75–80	0.408	0.289	0.466	0.548	0.32	0.63	0.05
80–85	0.300	0.249	0.474	0.491	0.42	0.78	−0.20

Young finds that both cities experienced an economic transformation going roughly through the same industries

- Singapore seemed to have done this in a more compressed period
- Rate of structural transformation is 0.209 for Singapore, 0.082 for Hong Kong
- Structural transformation is here measured by the allocation of labour in across two-digit ISIC manufacturing sectors

Important is the source of growth: Capital deepening or TFP growth

- TFP growth played important role in Hong Kong, contributing about 30 to 50% to output growth, with an average of 35% between 1971-1990
- In Singapore capital was key, contributing to 83% of output growth (did not contribute to TFP growth)

One advantage of TFP growth over growth based on capital accumulation is that it is more sustainable over the long run.

Young provides a theoretical explanation for the different patterns across the two cities focusing on

1. Innovations (N)
2. Bounded learning by doing (T)

This is to account for the fact that new technologies do not achieve their full productivity potential directly upon implementation but that there are gains made by continuous adjustments.

Figure 2 INVENTION AND BOUNDED LEARNING BY DOING

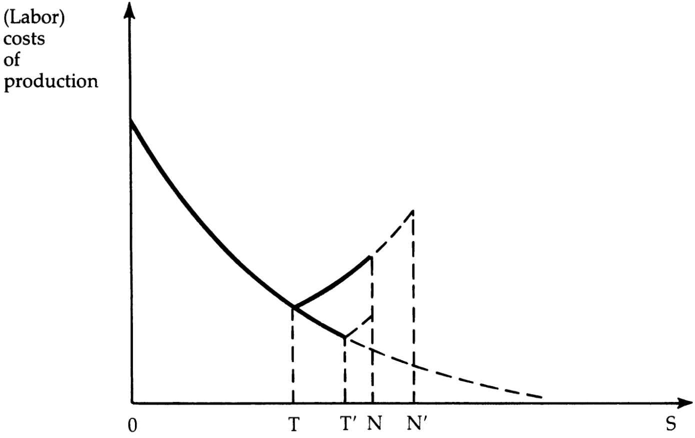
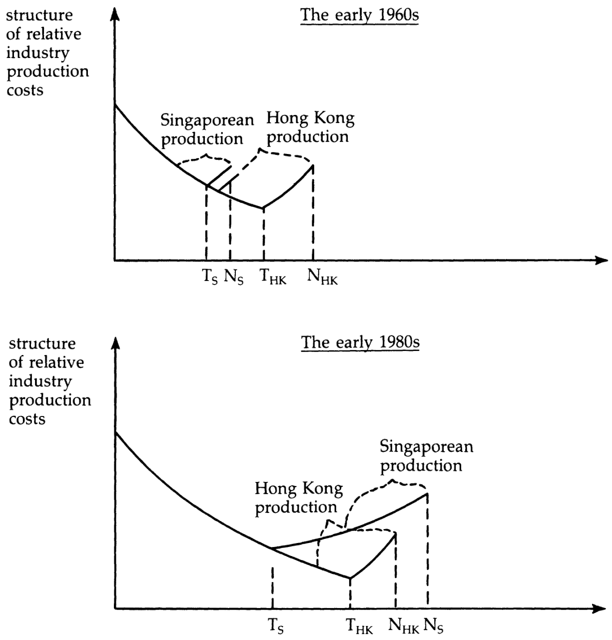
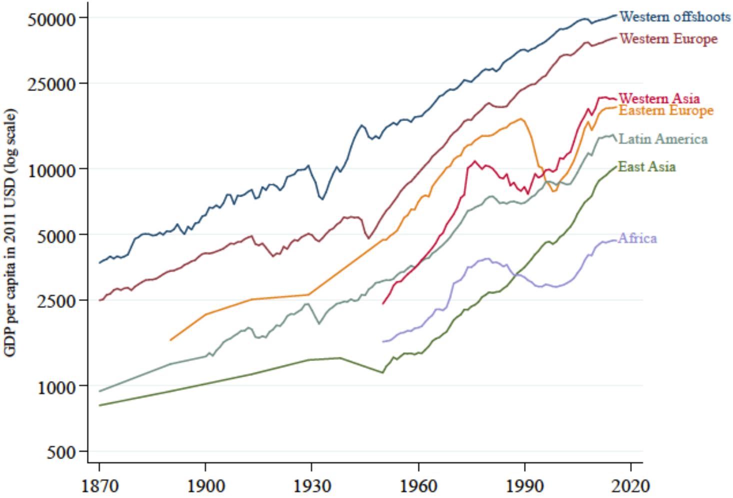


Figure 3 THE DEVELOPMENT OF HONG KONG AND SINGAPORE



Young argues that

- In the early 1960s Hong Kong learning maturity was greater than that of Singapore: $T_{HK} > T_S$
- Hong Kong found it easier to copy technologies and enter new sectors: length of $[T_{HK}, N_{HK}]$ relative to $[T_S, N_S]$
- By the early 1980s Singapore had caught up, and both economies experienced substantial learning by doing: Rightward movement of T_{HK}, T_S



Source: Maddison Project Database 2018, Bolt et al. (2018)

What matters for long-term growth?

- Demography
- Geography
- Institutions
- Natural resources
- War and conflict
- Trade
- Education
- Colonial history
- Macroeconomic policy

Cross-country evidence on growth determinants is often discounted

- Major issue is that there are many candidate models and choice in variable selection leaves room for data mining

Ciccone & Jarocinski (2010) examine the sensitivity of results to changes in data and variable selection using Bayesian Model Averaging

- They find that the results are very sensitive to measurement error in income estimates

Consider a dataset with N countries, where N is relatively large to the number of possible variables K

- Could regress growth rate on all K variables; likely find some statistically significant results
- If N is close to K, the estimates will be imprecise (not feasible when $N > K$)

Bayesian methods aims to identify the determinants in terms of uncertainty about the true set of explanatory variables.

All K variables are collected in vector x; can denote the 2^K subsets of x by x_j and regress model

$$y_n = \alpha + x_{jn}\beta_j + \epsilon_{jn} \tag{55}$$

y_n is the growth rate of per capita GDP in country n.
For the BMA all we further need is

1. Priors for models p_j
2. Priors for parameters α, β and variance of ϵ
3. Likelihood function for each model j

Important is the likelihood of model j integrated with respect to the parameters using their prior distribution. The marginal likelihood of model j can be written as

$$l_y(M_j) \tag{56}$$

This is the density of the data conditional on the model, and in the Bayesian framework this can be translated into a posterior probability of the model conditional on the observed data

$$p(M_j|y) \propto l_y(M_j)p_j \tag{57}$$

Finally, one can get the posterior inclusion probability for variable k

by summing the posterior probabilities of all models including the variable.

Table 4—Posterior Inclusion Probabilities of Groups of Variables in the Sala-i-Martin, Doppelhofer, and Miller (2004) Approach
(Using income data from PWT 6.2, 6.1, 6.0)

	PWT 6.2	PWT 6.1	PWT 6.0
Panel A. Proxy groups			
Education	**1.00**	**0.99**	**0.80**
Population growth	**0.92**	0.15	0.05
Natural resources	0.35	0.44	0.20
Size of government	0.34	0.14	0.23
Openness to trade	0.25	0.17	0.26
Market access	0.17	**0.81**	0.46
Climate zones	0.14	**0.84**	**0.64**
Inflation	0.09	0.03	0.04
Age structure	0.08	0.07	0.06
War and conflict	0.07	0.05	0.06
Health	0.05	0.26	0.40
Size of the economy	0.05	0.04	0.04
Rights	0.04	0.27	0.09
Panel B. Theory groups			
Neoclassical	**1.00**	**1.00**	**0.92**
Demography	**0.94**	0.39	0.29
Regional heterogeneity	**0.93**	**0.89**	**0.94**
Religion	**0.90**	0.38	0.39
Macroeconomic policy	**0.54**	**0.99**	**0.90**
Natural resources	0.35	0.44	0.20
Geography and trade (within countries and international)	0.27	**0.89**	**0.54**
Colonial history	0.20	0.19	0.19
Absolute geography	0.16	**0.84**	**0.84**
Institutions	0.08	0.30	0.12
War and conflict	0.07	0.05	0.06
Terms of trade	0.03	0.04	0.04

Notes: Posterior probabilities higher than 0.5 are in bold. The prior probability of each group is different and depends on its size. The prior probability of a group of j variables is $1 - (1 - p)^j$, where p is the prior inclusion probability of an individual variable (here: $p = 7/67$). This formula can be obtained by noting that the prior probability of a group being represented equals one minus the prior probability that none of the variables from the group is included. As the inclusion of variables is a priori independent, the prior probability that none of j variables is included in the model equals $(1 - p)^j$.

Growth accounting studies rely on estimate of GDP, however GDP is often badly measured

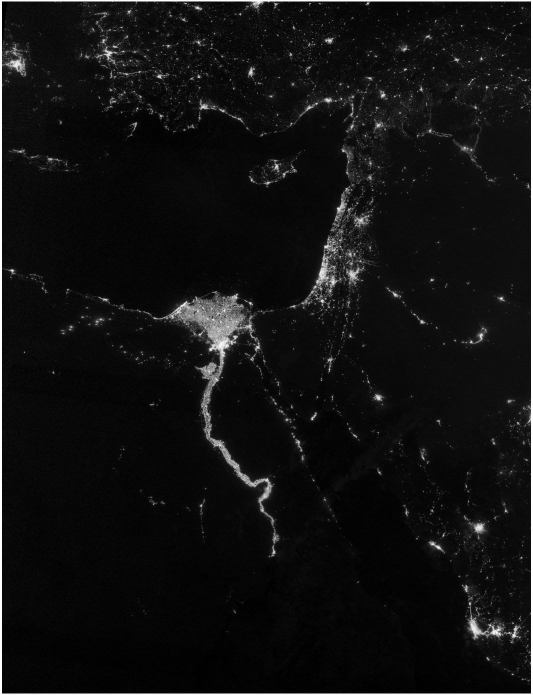
- Need to measure nominal GDP as well as reliable price indices

Particularly a problem in developing countries due to number of factors

1. Smaller fraction of economic activity is formal
2. Economic integration within-country is lower
3. Statistical infrastructure is weak

Henderson et al. (2012) provide a solution using night-light emissions measured by satellites

- Lights at night are conspicuous consumptions
- Can serve as proxy for economic activity; both short and long term



Henderson et al. (2012) use the following framework to relate lights to GDP growth.
First, there is classical measurement error in GDP data

$$z_j = y_j + \epsilon_{z,j} \tag{58}$$

z the growth of real GDP as measured in national income accounts
y is growth in true real GDP
j indexes country, ϵ_z is denoted σ_z^2

Second, relation between growth of lights and true income is given by

$$x_j = \beta y_j + \epsilon_{x,j} \tag{59}$$

i.e. there is a simple constant elasticity between total observable lights X and total income Y

$$X_j = Y_j^{\beta} \tag{60}$$

For predictive purpose we are interested in

$$z_j = \hat{\psi} x_j + e_j \tag{61}$$

OLS parameter $\hat{\psi}$ is simply $cov(x, z)/cov(x)$; relationship between $\hat{\psi}$ and structural parameter β is

$$\hat{\psi} = \frac{1}{\beta}\left(\frac{\beta^2\sigma_y^2}{\beta^2\sigma_y^2 + \sigma_x^2}\right) \tag{62}$$

$\hat{\psi}$ is an estimate of the inverse of elasticity of lights with respect to income, but it is a biased estimate

- Due to inversion of production relationship and measurement error in x

Eq. 59 can be used as proxy for best-fit relationship

$$\hat{z}_j = \hat{\psi}x_j \tag{63}$$

$\hat{z}$ is a separate estimate of income growth which can be combine with a national account measure to create a composite measure $\hat{y}$

$$\hat{y}_j = \lambda z_j + (1 - \lambda)\hat{z}_j \tag{64}$$

The variance of the composite estimate is given by

$$var(\hat{y} - y) = \lambda^2 \sigma_z^2 + (1 - \lambda)^2 \frac{\sigma_y^2 \sigma_x^2}{\beta^2 \sigma_y^2 + \sigma_x^2} \tag{65}$$

This is solved for the weight λ^* which minimises the variance

$$\lambda^* = \frac{\sigma_x^2 \sigma_y^2}{\sigma_z^2(\beta^2 \sigma_y^2 + \sigma_x^2) + \sigma_x^2 \sigma_y^2} \tag{66}$$

λ^* is a function of four unknown parameters but there are only three known sample moments

$$var(z) = \sigma_y^2 + \sigma_z^2 \tag{67}$$

$$var(x) = \beta^2\sigma_y^2 + \sigma_x^2 \tag{68}$$

$$cov(x, z) = \beta\sigma_y^2 \tag{69}$$

i.e. for the last moment $cov(y, x) = cov(x, z)$ one more equation is needed, which is based on the assumption about the signal-to-noise ratio in measured GDP growth z

$$\phi = \frac{\sigma_y^2}{\sigma_y^2\sigma_z^2} \tag{70}$$

Assuming a specific value for ϕ, optimal λ will be given by

$$\begin{aligned} \lambda^* &= \frac{\phi var(z)var(x) - cov(z, x)^2}{var(z)var(x) - cov(z, x)^2} \\ &= \frac{\phi - \rho^2}{1 - \rho^2} \end{aligned} \tag{71}$$

with ρ being the correlation between z and x

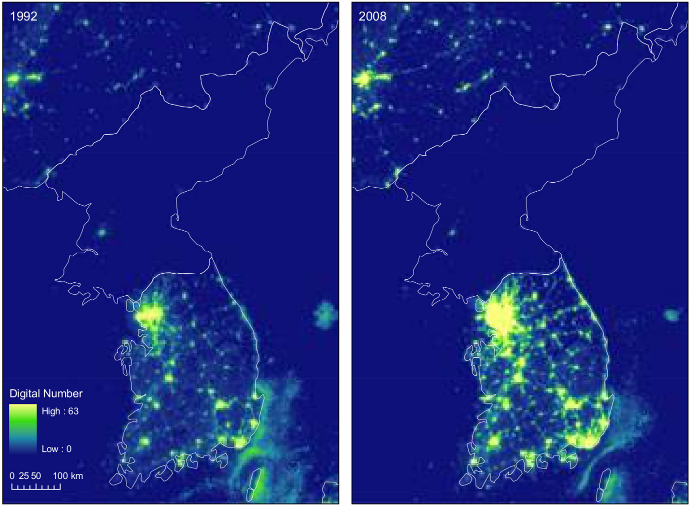
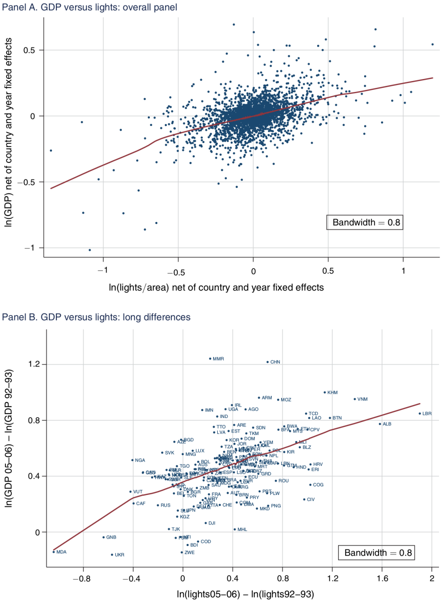


FIGURE 2. LONG-TERM GROWTH: KOREAN PENINSULA



Panel A. GDP versus lights: overall panel
ln(GDP) net of country and year fixed effects
ln(lights/area) net of country and year fixed effects
Bandwidth = 0.8
Panel B. GDP versus lights: long differences
ln(GDP 05–06) – ln(GDP 92–93)
ln(lights05–06) – ln(lights92–93)
Bandwidth = 0.8

FIGURE 6

Table 6—Average Annual Growth Rates in True Income for Bad Data Countries (*percent*) 1992/93–2005/06

Country	ISO code	WDI (LCU)	Fitted lights	Optimal combination of WDI and fitted lights	Difference
Myanmar	MMR	10.02	3.26	6.48	−3.22
Angola	AGO	6.99	3.88	5.37	−1.51
Nigeria	NGA	4.04	1.92	2.94	−1.06
Sudan	SDN	5.92	4.01	4.93	−0.94
Vietnam	VNM	7.60	5.80	6.67	−0.87
Burkina Faso	BFA	5.80	4.45	5.10	−0.66
Benin	BEN	4.52	3.49	3.99	−0.51
Ghana	GHA	4.60	3.71	4.14	−0.44
Rwanda	RWA	3.06	2.25	2.64	−0.40
Oman	OMN	4.28	3.83	4.05	−0.22
Algeria	DZA	3.29	2.85	3.06	−0.22
Mali	MLI	5.08	4.76	4.92	−0.16
Iran, Islamic Rep.	IRN	4.03	3.74	3.88	−0.15
Cameroon	CMR	3.29	3.00	3.14	−0.14
Niger	NER	3.48	3.21	3.34	−0.14
Sierra Leone	SLE	3.04	2.78	2.91	−0.13
Gambia, The	GMB	3.80	3.73	3.76	−0.03
Liberia	LBR	6.75	7.03	6.89	0.14
Central African Republic	CAF	1.59	1.94	1.77	0.18
Mauritania	MRT	3.68	4.04	3.86	0.18
Swaziland	SWZ	3.42	3.93	3.68	0.26
Lebanon	LBN	3.85	4.43	4.15	0.29
Madagascar	MDG	2.74	3.38	3.07	0.32
Eritrea	ERI	3.51	4.97	4.26	0.73
Guinea-Bissau	GNB	−0.29	1.40	0.58	0.87
Congo, Rep.	COG	2.63	5.03	3.86	1.20
Haiti	HTI	−0.28	2.73	1.27	1.55
Côte d'Ivoire	CIV	1.82	4.91	3.40	1.56
Congo, Dem. Rep.	COD	−0.52	3.05	1.30	1.84
Burundi	BDI	−0.71	2.89	1.13	1.85