Vector Autoregression: Examples

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Stock & Watson (2001) Effect of monetary policy shocks.
VAR model can be useful from two perspectives:

1. Scientific
   - Monetary policy co-moves with lots of other macro variables
   - Only by identifying the structural or exogenous shocks to policy can we discover its true effects

2. Policy
   - Can help answer the question "if I choose to raise interest rates by an extra quarter point today, what is likely to happen over the next year to inflation and output relative to the case where I keep rates unchanged?"
   - This is basically a question about impulse responses

Quarterly data, three variables

1. inflation π_t
2. unemployment rate u_t
3. federal funds rate i_t

Lower-triangular causal chain

$$AZ_t = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \pi_t \\ u_t \\ i_t \end{pmatrix} = BZ_{t-1} + \epsilon_t \tag{1}$$

Identifying assumptions

1. Inflation depends only on lagged values of the other variables (sticky prices)
2. Unemployment depends on contemporaneous inflation but not the funds rate
3. Funds rate depends on both contemporaneous inflation and unemployment

Table 1

VAR Descriptive Statistics for (π, u, R)

A. Granger-Causality Tests

Regressor	*Dependent Variable in Regression* π	u	R
π	0.00	0.31	0.00
u	0.02	0.00	0.00
R	0.27	0.01	0.00

B. Variance Decompositions from the Recursive VAR Ordered as π, u, R

B.i. Variance Decomposition of π

Forecast Horizon	*Forecast Standard Error*	*Variance Decomposition (Percentage Points)* π	u	R
1	0.96	100	0	0
4	1.34	88	10	2
8	1.75	82	17	1
12	1.97	82	16	2

B.ii. Variance Decomposition of u

Forecast Horizon	*Forecast Standard Error*	*Variance Decomposition (Percentage Points)* π	u	R
1	0.23	1	99	0
4	0.64	0	98	2
8	0.79	7	82	11
12	0.92	16	66	18

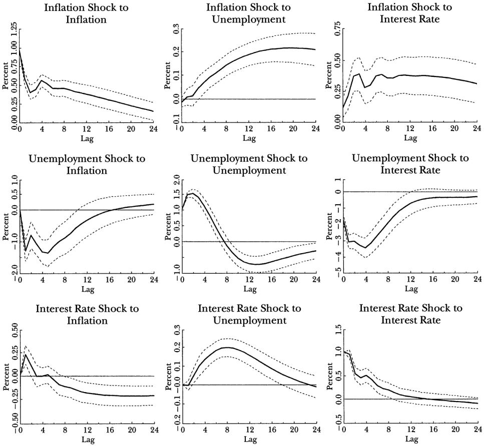
B.iii. Variance Decomposition of R

Forecast Horizon	*Forecast Standard Error*	*Variance Decomposition (Percentage Points)* π	u	R
1	0.85	2	19	79
4	1.84	9	50	41
8	2.44	12	60	28
12	2.63	16	59	25

Notes: π denotes the rate of price inflation, u denotes the unemployment rate and R denotes the Federal Funds interest rate. The entries in Panel A show the p-values for F-tests that lags of the variable in the row labeled *Regressor* do not enter the reduced form equation for the column variable labeled *Dependent Variable*. The results were computed from a VAR with four lags and a constant term over the 1960:I–2000:IV sample period.

Figure 1

Impulse Responses in the Inflation-Unemployment-Interest Rate Recursive VAR



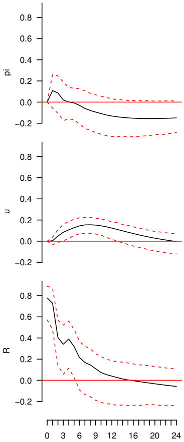
Prize puzzle: A shock to the interest rate seems to actually increase the inflation rate for a couple of periods.

- Result has been showing up consistently in VAR studies

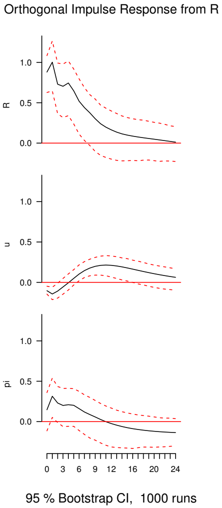
Fed could be acting on information that is not captured by the VAR model which may provide information/signals on future inflationary pressure:

- i.e. interest rate increase occurs just before inflation increase: VAR confuses correlation/causation
- Including commodity prices often eliminates prize puzzle

Orthogonal Impulse Response from R



95 % Bootstrap CI, 1000 runs



Orthogonal Impulse Response from R
R
u
pi
1.0
0.5
0.0
0 3 6 9 12 16 20 24
95 % Bootstrap CI, 1000 runs

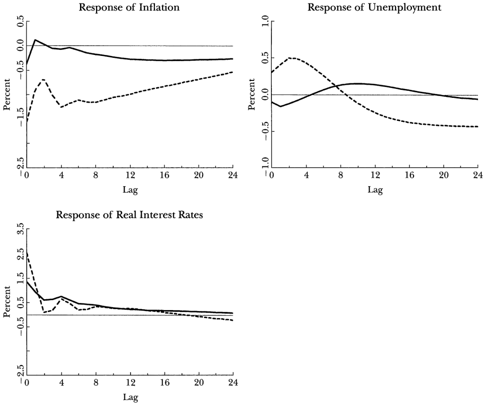
Table 2

Root Mean Squared Errors of Simulated Out-Of-Sample Forecasts, 1985:1–2000:IV

Forecast Horizon	*Inflation Rate*			*Unemployment Rate*			*Interest Rate*		
	RW	*AR*	*VAR*	*RW*	*AR*	*VAR*	*RW*	*AR*	*VAR*
2 quarters	0.82	0.70	0.68	0.34	0.28	0.29	0.79	0.77	0.68
4 quarters	0.73	0.65	0.63	0.62	0.52	0.53	1.36	1.25	1.07
8 quarters	0.75	0.75	0.75	1.12	0.95	0.78	2.18	1.92	1.70

Notes: Entries are the root mean squared error of forecasts computed recursively for univariate and vector autoregressions (each with four lags) and a random walk ("no change") model. Results for the random walk and univariate autoregressions are shown in columns labeled RW and AR, respectively. Each model was estimated using data from 1960:I through the beginning of the forecast period. Forecasts for the inflation rate are for the average value of inflation over the period. Forecasts for the unemployment rate and interest rate are for the final quarter of the forecast period.

Figure 2
Impulse Responses of Monetary Policy Shocks for Different Taylor Rule Identifying Assumptions



Notes: The solid line is computed with the backward-looking Taylor rule; the dashed line, with the forward-looking Taylor rule.

Kilian (2009): Oil price shocks

1. What is an oil price shock?
2. Are there different type of shocks?

Kilian identifies three types of shock

1. Supply: oil production growth rate $\Delta prod_t$
2. Demand: global demand measured by real global economic activity rea_t
3. Speculation: in oil price market, measured by real oil price rpo_t

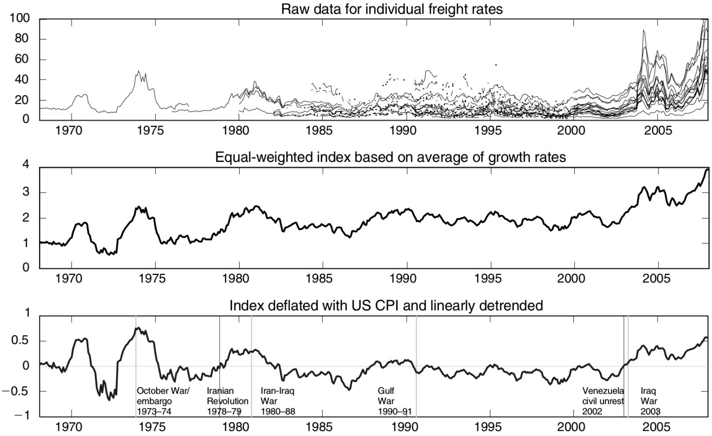


Figure 1. Monthly Index of Global Real Economic Activity Based on Dry Cargo Bulk Freight Rates
(1968:1–2007:12)

Notes: The monthly raw data were manually collected from *Drewry's Shipping Monthly*, various issues since 1970. The two oldest series in the first panel are indices of iron ore, coal, and grain shipping rates compiled by Drewry's. The remaining series are differentiated by cargo, route, and ship size and may include, in addition, shipping rates for oilseeds, fertilizer, and scrap metal. In the 1980s, there are about 15 different rates for each month; by 2000 that number rises to about 25; more recently that number has dropped to about 15.

Structural VAR

$$z_t = (\Delta prod_t, rea_t, rpo_t)' \tag{2}$$

$$A_0 z_t = \alpha + \sum_{i=1}^{24} A_i z_{t-i} + \epsilon_t \tag{3}$$

A_0^{-1} has recursive structure; reduced form errors e_t can be decomposed as

$$\begin{aligned} e_t &= A_0^{-1} \epsilon_t \\ &\equiv \begin{pmatrix} e_t^{\delta prod} \\ e_t^{rea} \\ e_t^{rpo} \end{pmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{pmatrix} \epsilon_t^{oil\ supply\ shock} \\ \epsilon_t^{aggregate\ demand\ shock} \\ \epsilon_t^{oil\ specific-demand\ shock} \end{pmatrix} \end{aligned} \tag{4}$$

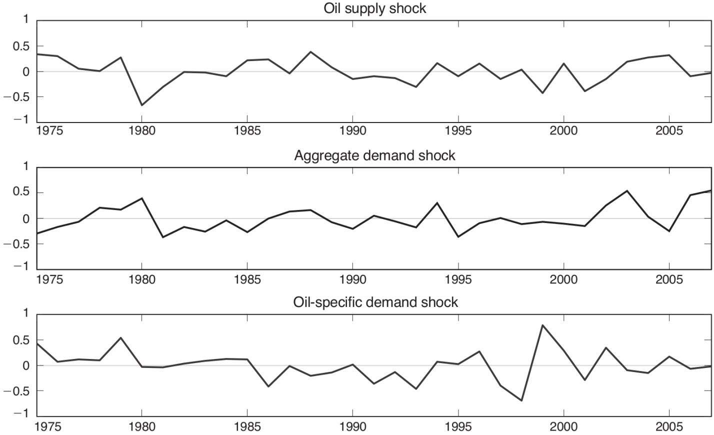


Figure 2. Historical Evolution of the Structural Shocks, 1975–2007

Note: Structural residuals implied by model (1), averaged to annual frequency.

Exclusion restrictions

1. Crude oil supply does not respond within same month to changes in oil demand/price
2. Global demand is affected within the month by oil production, but not prices
3. Oil prices respond immediately to oil production and global demand

For the shocks this means

1. Oil production reduced form shock is a structural shock
2. Economic activity reduced form shock is combination of structural oil shock and structural activity shock
3. Reduced form oil price shock is combination of all three structural shocks

Identifying restrictions

$$AY_t = BY_{t-1} + C\epsilon_t$$

Needs 18 identifying restrictions: $2n^2 = 18$

1. Assuming contemporaneous interaction between variables $C = I$ (9)
2. Zero restrictions/lower diagonal assumption on A_0 (3)
3. Unit coefficient normalisation on diagonal A_0 (3)
4. Orthogonal structural shocks: off-diagonal elements of $\sum$ are 0 (3)

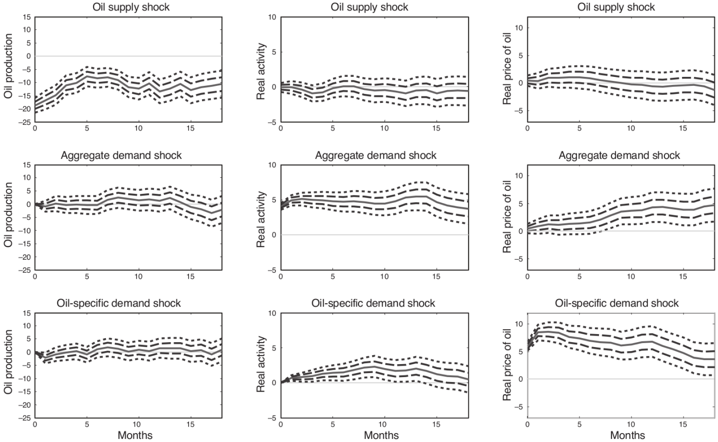


Figure 3. Responses to One-Standard-Deviation Structural Shocks

(Point estimates with one- and two-standard error bands)

Notes: Estimates based on model (1). The confidence intervals were constructed using a recursive-design wild bootstrap.

Decomposing variables
Recall Vector Moving Average representation

$$Y_t = e_t + Ae_{t-1} + A^2 e_{t-2} + A^3 e_{t-3} + ... + A^t e_0$$

One can repeat this calculation three times, each time with only

one type of shock turned on and the other set to zero. Adding these up, one will get the realized values of Y_t. Alternatively, one can do a dynamic simulation of the model

$$Y = AY_{t-1} + \epsilon_t$$

Here we let ϵ_t represent one of the realised shocks, setting the others to zero

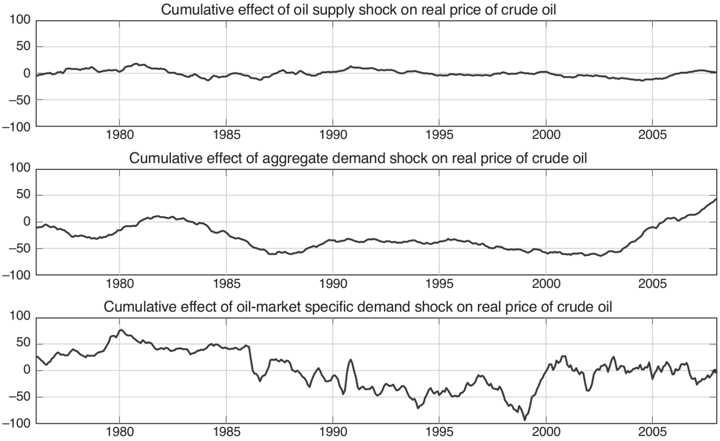


FIGURE 4. HISTORICAL DECOMPOSITION OF REAL PRICE OF OIL

(1976:1–2007:12)

Note: Estimates derived from model (1).

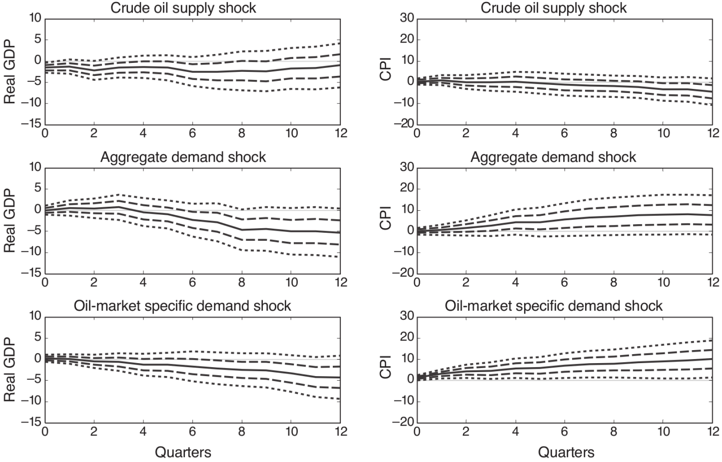


FIGURE 5. RESPONSES OF US REAL GDP AND CPI LEVEL TO EACH STRUCTURAL SHOCK

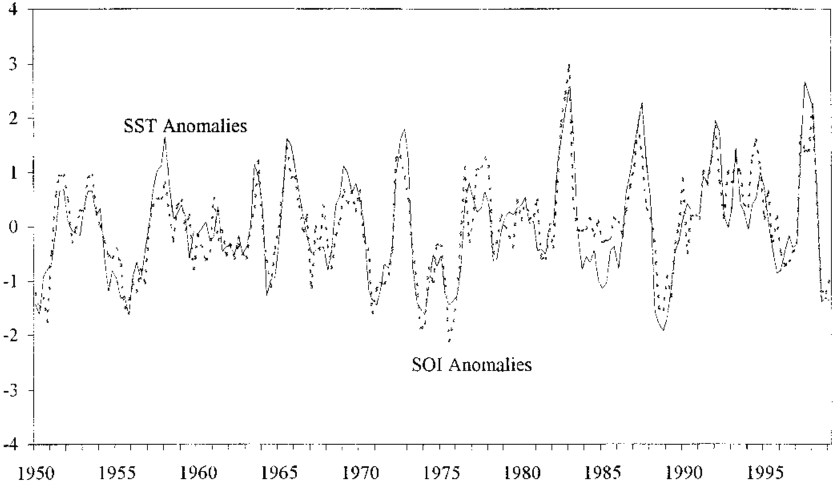
(Point estimates with one- and two-standard error bands)

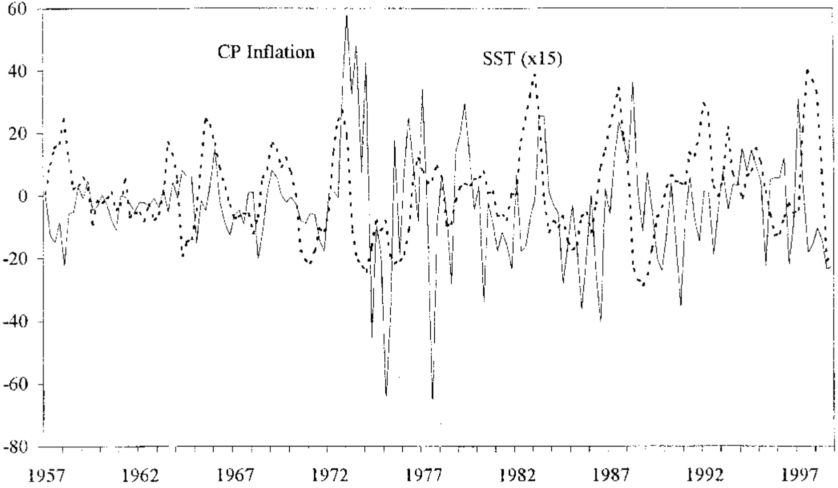
Note: The plots show the cumulated responses estimated from models (2) and (3).

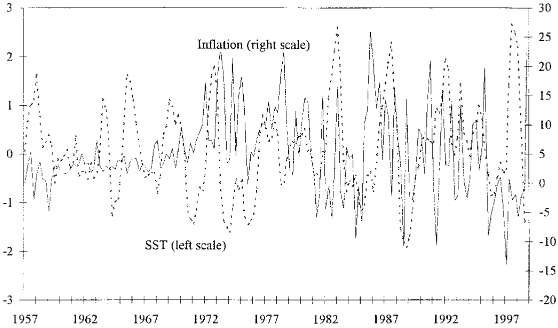
Brunner (2002) Effect of El Niño Southern Oscillation (ENSO) on world primary commodity prices.

1. Examines global economic consequences of the ENSO cycle
2. Uses continuous ENSO measures, rather than dummy variables
3. Constructs uncertainty intervals on estimates

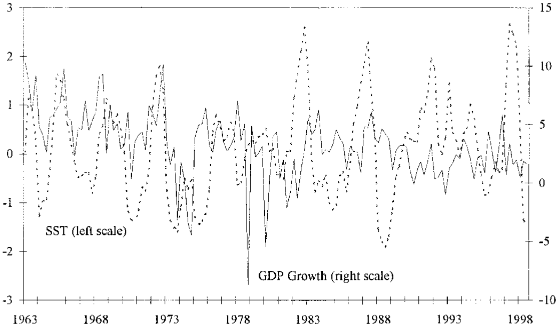
Finds that ENSO accounts for 20% of commodity price inflation movements.


4
3
2
1
0
-1
-2
-3
-4
SST Anomalies
SOI Anomalies
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995


60
40
20
0
-20
-40
-60
-80
CP Inflation
SST (x15)
1957
1962
1967
1972
1977
1982
1987
1992
1997



Inflation (right scale)
SST (left scale)
3
2
1
0
-1
-2
-3
30
25
20
15
10
5
0
-5
-10
-15
-20
1957
1962
1967
1972
1977
1982
1987
1992
1997



SST (left scale)
GDP Growth (right scale)
3
2
1
0
-1
-2
-3
15
10
5
0
-5
-10
1963
1968
1973
1978
1983
1988
1993
1998

Structural VAR

$$
\begin{aligned}
ENSO_t &= \mu_s + A_{11}(L)ENSO_{t-1} + \epsilon_t && (5) \\
X_t &= \phi_s + A_{21}(L)ENSO_t + A_{22}(L)X_{t-1} + \eta_t \\
\begin{bmatrix} \epsilon_t \\ \eta_t \end{bmatrix} &\sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sum_\eta \end{bmatrix} \right) && (6)
\end{aligned}
$$

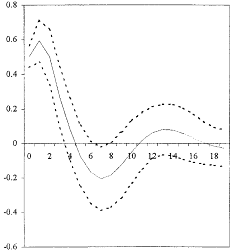
$$X_t = [\pi_t^{cp} - \pi_t^g \; \pi_t^g \Delta y_t] \tag{7}$$

$\pi_t^{cp} - \pi_t^g$ is price inflation
π_t^g is inflation rate
Δy_t is average GDP growth rate for G7 countries.

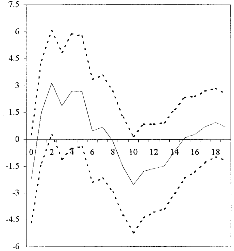
Identifying assumptions

1. ENSO events are not influenced by contemporaneously economic events: ϵ_t orthogonal
2. $\sum_\eta$ is expected to be non-diagonal: shocks in η are correlated
   - Not required since focus is on ϵ_t, which is uncorrelated with η_t

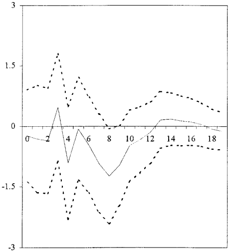
Effects on SST



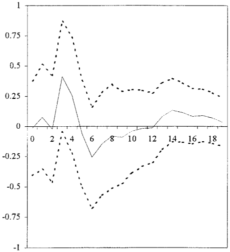
Effects on Real Commodity Price Inflation



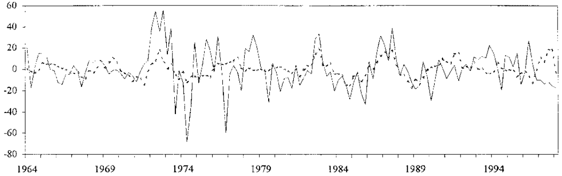
Effects on CPI Inflation



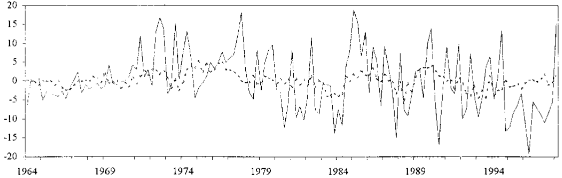
Effects on GDP Growth



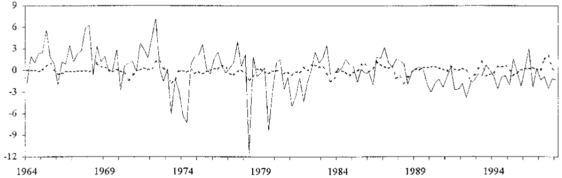
a) Contributions to Real Commodity Price Inflation



b) Contributions to G7 CPI Inflation



c) Contributions to G7 GDP Growth



Rudebusch (1998) provided some criticisms on the use of VAR, specifically in relation to monetary policy

1. Changes over time in monetary policy formation are ignored
2. Relies on final published data, rather than preliminary estimates
3. Underestimates information available to policy makers
4. Models incorporate long lags, but unlikely that policy makers look that far back
5. Monetary shocks don't resemble surprise elements of monetary policy decisions
6. Similar IRF reported by models with different monetary policy shocks (data mining)

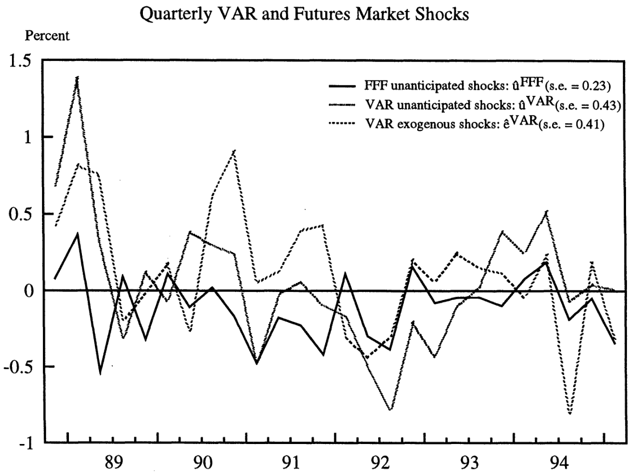

Quarterly VAR and Futures Market Shocks
Percent
1.5
1
0.5
0
-0.5
-1
FFF unanticipated shocks: û FFF (s.e. = 0.23)
VAR unanticipated shocks: û VAR (s.e. = 0.43)
VAR exogenous shocks: ê VAR (s.e. = 0.41)
89
90
91
92
93
94

FIGURE 6