

New Keynesian model

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New Keynesian model addresses some of the critiques on the Keynesian model

- ▶ Rational expectations
- ▶ People behave optimally

Also room for monetary policy to have systematic effects

- ▶ Central mechanism for monetary policy is **sticky prices**
- ▶ If prices don't move in line with money; central bank can't control real money supply or interest rate

Dixit-Stiglitz model

Used as vantage point to describe optimal behaviour

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \quad (1)$$

Consumers will maximise their utility function $U(Y_t)$ over an aggregate of a continuum of differentiated goods

- ▶ θ denotes constant elasticity of substitution

This model does not include capital but only consumption goods.

Demand function for each differentiated good is of the form

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} \quad (2)$$

P_t is the aggregate price index which is defined by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \quad (3)$$

Calvo model

Used to describe price rigidity, or sticky prices

$$P_t = \left[(1 - \alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (4)$$

$$P_t^{1-\theta} = (1 - \alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \quad (5)$$

$1 - \alpha$ is random fraction of firms able to reset their price

X_t is the price that the firms resetting today have chosen

1. All other firms keep prices unchanged
2. All firms setting new prices today set the same price.

Apart from this difference in timing, of when they set prices, the firms are completely symmetric

Prices may be fixed for many periods: Firms pick a price to maximise

$$E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k (Y_{t+k} P_{t+k}^{\theta-1} X_t^{1-\theta} - P_{t+k}^{-1} C(Y_{t+k} P_{t+k}^{\theta} X_t^{-\theta})) \right] \quad (6)$$

$C(.)$ is the cost function

Solution for maximisation problem is (differentiating with regard to X_t)

$$X_t = \frac{\theta}{\theta - 1} \frac{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)}{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} \right)} \quad (7)$$

This entails that changed price X_t is a markup over a weighted average of future marginal costs: without frictions the firm would set

$$X_t = \frac{\theta}{\theta - 1} MC_t \quad (8)$$

i.e. the price equals $\frac{\theta}{\theta-1}$ times the marginal costs
 Got two non-linear equations for price

$$P_t^{1-\theta} = (1 - \alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \quad (9)$$

$$X_t = \frac{\theta}{\theta - 1} \frac{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k} + k^{\theta-1} MC_{t+k} \right)}{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k} + k^{\theta-1} \right)} \quad (10)$$

Solving or simulating price equations is not easy: use log-linear approximations taken around constant growth, zero inflation path

$$X_t^* = P^* = P_{t-1}^* = P^* \quad (11)$$

$$X^* = \left(\frac{\theta}{1 - \theta} \right) MC^* \quad (12)$$

$$p_t = (1 - \alpha)x_t + \alpha p_{t-1} \quad (13)$$

$$x_t = (1 - \alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k E_t mc_{t+k} \quad (14)$$

Reverse engineering can see that optimal reset price can be written as

$$x_t = (1 - \alpha\beta)mc_t + (\alpha\beta)E_tx_{t+1} \quad (15)$$

Can combine this with fact that

$$p_t = (1 - \alpha)x_t + \alpha p_{t-1} \quad (16)$$

$$x_t = \frac{1}{1 - \alpha}(p_t - \alpha p_{t-1}) \quad (17)$$

Note that

$$\pi_t = p_t - p_{t-1} \quad (18)$$

After bunch of re-arranging you get

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 + \alpha)(1 - \alpha\beta)}{\alpha} (mc_t - p_t) \quad (19)$$

π_t is a function

1. Expected inflation in $t + 1$
2. Ratio of marginal cost to price (real marginal cost)

This is the **New-Keynesian Phillips curve** (NKPC)

Output

Assume that there are standard diminishing returns to labour production function

- Higher output reduces marginal productivity and raises marginal cost

This means that real marginal costs are a function of the output gap

$$mc_t - p_t = \eta x_t \quad (20)$$

Concerning x_t

$$x_t = y_t - y_t^n \quad (21)$$

y_t^n is the path of output that would have been obtained in a zero inflation price friction free economy

We get NKPC of the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \quad (22)$$

Looks a lot like traditional expectations-augmented Phillips curve

It is a first-order stochastic difference equation, which entails a solution in the form

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t x_{t+k} \quad (23)$$

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t x_{t+k} \quad (24)$$

Has no backward-looking element

- ▶ No intrinsic inertia in inflation
- ▶ Lagged inflation effects, in conventional models, are actually a statistical artifact

Note that original formulation of NKPC does not have error/shock term

- ▶ Maybe price movements not consistent with this formulation.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \quad (25)$$

Cost-push shock added to NKPC

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \quad (26)$$

u_t accounts for misc. shocks:

- ▶ π no longer results of just expected inflation and output gap

Central bank can no longer implement a stabilisation policy by only addressing the output gap.

NKPC links inflation to output: Consider how we link output to monetary policy.

- ▶ First of three equations that make up the New Keynesian model

NKM uses interest rates: recall that model does not include capital

$$Y = C \quad (27)$$

Relation between C and i comes from standard intertemporal optimization problem; consumer wants to maximise

$$\sum_{k=0}^{\infty} \left(\frac{1}{1 + \beta} \right)^k U(C_{t+k}) \quad (28)$$

Intertemporal budget constraint given by

$$\sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{\left(\prod_{m=1}^{k+t} R_{t+m}\right)} = A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{\left(\prod_{m=1}^{k+t} R_{t+m}\right)} \quad (29)$$

R_t is the interest rate

Can write Lagrangian as

$$L = \sum_{k=0}^{\infty} \left(\frac{1}{1+\beta}\right)^k U(C_{t+k}) \quad (30)$$
$$+ \lambda \left[A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)} - \sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)} \right]$$

Euler equation can be derived by combining FOCs for C_t and C_{t+1}

$$U'(C_t) = E_t \left[\left(\frac{R_{t+1}}{1 + \beta} \right) U'(C_{t+1}) \right] \quad (31)$$

Can set

$$U(C_t) = U(Y_t) = \frac{Y_t^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \quad (32)$$

$$E_t \left[\left(\frac{R_{t+1}}{1 + \beta} \right) \left(\frac{Y_t}{Y_{t+1}} \right)^{\frac{1}{\sigma}} \right] = 1 \quad (33)$$

NB-Similar to the Real Business Cycle model this is a Constant Relative Risk Aversion (CRRA) utility from consumption

Set

$$\rho = -\log \beta \quad (34)$$

Log-linearised version of the Euler equation is

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - \rho) \quad (35)$$

i.e. today's output depends negatively on the real interest rate

Recall that the equation for inflation featured the output gap

$$x_t = y_t - y_t^n \quad (36)$$

Substitute into Euler equation¹

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - \rho) + E_t y_{t+1}^n - y_t^n \quad (37)$$

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \quad (38)$$

Natural interest rate is given by

$$r_t^n = \sigma^{-1} E_t \Delta y_{t+1}^n - \log \beta \quad (39)$$

¹ $E_t y_{t+1}$ with $E_t x_{t+1} + E_t y_{t+1}^n$

r_t is a function of

$$E_t \Delta y_{t+1}^n \quad (40)$$

Meaning that it is determined by

1. Technology
2. Preferences

Output gap x_t follows a first-order stochastic difference equation which has a solution of the form

$$x_t = \sigma \sum_{k=0}^{\infty} (i_{t+k} - E_t \pi_{t+k+1} - r_{t+k}^n) \quad (41)$$

Policy implications

x_t has no backward-looking element; output has no intrinsic persistence

For monetary policy this means that what matters for today's output is

1. Current policy
2. All future interest rates

Central bankers should therefore take care in managing expectations about future policy

- Future interest rates are their key tool

Interpreting i_t as the short-term interest rate, and assuming that the expectations theory of the term structure holds, this model states that it is the long-term interest rates that matter for spending.

In its most basic form the New Keynesian model has three equations. We have already derived

1. The New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \quad (42)$$

2. The Euler equation for output

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \quad (43)$$

Now we only have to find an equation describing how interest rate policy is set. This is usually described as an explicit interest rate rule.

Output-inflation dynamics

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \quad (44)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \quad (45)$$

Can be rewritten as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa E_t x_{t+1} - \kappa \sigma(i_t - E_t \pi_{t+1} - r_t^n) + u_t \quad (46)$$

Put in vector form

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa \sigma \end{pmatrix} \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} + \begin{pmatrix} \sigma(r_t^n - i_t) \\ \kappa \sigma(r_t^n - i_t) + u_t \end{pmatrix} \quad (47)$$

Have model in form

$$Z_t = AE_t Z_{t+1} + BV_t \quad (48)$$

For unique stable solution the eigenvalues of A need to be less than 1

$$A = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa\sigma \end{pmatrix} \quad (49)$$

Recall that there is an eigenvector that when multiplied by $A - \lambda I$ equals a vector of zeroes, meaning that the determinants of the matrix equal zero

$$A - \lambda I = \begin{pmatrix} 1 - \lambda & \sigma \\ \kappa & \beta + \kappa\sigma - \lambda \end{pmatrix} \quad (50)$$

Eigenvalues satisfy

$$P(\lambda) = (1 - \lambda)(\beta + \kappa\sigma - \lambda) - \kappa\sigma = 0 \quad (51)$$

$$P(\lambda) = \lambda^2 - (1 + \beta + \kappa\sigma)\lambda + \beta = 0 \quad (52)$$

$P(\lambda)$ is a U-shaped polynomial: if $\lambda = 0$ we get

$$P(0) = \beta > 0 \quad (53)$$

$$P(1) = -\kappa\sigma < 0 \quad (54)$$

$P(\lambda)$ will be greater than 0 when λ rises above one, this implies that

1. one eigenvalue between zero and one
2. one eigenvalue greater than 1

This is a serious problem for the model: no unique stable solution; model has multiple equilibria

Two ways to deal with λ issue

1. Accept that there are multiple equilibria: analyse the impact of interest rate changes on output and inflation across a range of different possible equilibria
2. Specify that monetary policy follows a particular rule and this rule is designed to produce a unique stable equilibrium

Taylor rule

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_x x_t \quad (55)$$

Monetary policy sets interest rate based on inflation and output gap

- ▶ Increase in π, x will increase i

Note inclusion of natural interest rate: set interest rate moves with the natural interest rate

- ▶ Rule here allows i to move with natural rate: Taylor's rule has a constant intercept

Rule can be substituted in the equation for x_t to give

$$x_t = E_t x_{t+1} + \sigma E_t \pi_{t+1} - \sigma \phi_\pi \pi_t - \sigma \pi_x x_t \quad (56)$$

To look at dynamics rewrite equations in matrix form

$$Z_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}; V_t = \begin{pmatrix} 0 \\ u_t \end{pmatrix} \quad (57)$$

$$Z_t = A E_t Z_{t+1} + B V_t \quad (58)$$

In standard model

$$Z_t = AE_t Z_{t+1} + BV_t$$

We have

$$A = \frac{1}{1 + \sigma\pi_x + \kappa\sigma\phi_\pi} \begin{pmatrix} 1 & \sigma(1 - \beta\phi_\pi) \\ \kappa & \beta + \sigma\kappa + \beta(1 + \sigma\phi_x) \end{pmatrix} \quad (59)$$

$$B = \frac{1}{1 + \sigma\phi_x + \kappa\sigma\phi_\pi} \begin{pmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1 + \sigma\phi_x \end{pmatrix} \quad (60)$$

System is a matrix version of the first-order stochastic difference equations: can be solved in a similar fashion to give

$$Z_t = \sum_{k=0}^{\infty} A^k B E_t V_{t+k} \quad (61)$$

To have unique stable equilibrium the absolute values of both eigenvalues of A need to be less than 1, which will be the case when This will be the case when

$$\phi_{\pi} + \frac{(1 - \beta)\phi_x}{\kappa} > 1 \quad (62)$$

$\beta \approx 1$ so the condition is approximately $\phi_{\pi} > 1$

If the policy rule satisfies this requirement, known as the **Taylor principle**, there is a unique stable equilibrium

- ▶ Nominal interest rates must rise by more than inflation so that real rates rise in response to an increase in inflation
- ▶ Needed for stability because otherwise inflationary shocks reduces real interest rates which stimulates the economy which will further stimulate inflation

A big question for central banks of course is what is optimal to do?
In general we know that central banks

- ▶ Don't like inflation²
- ▶ Like to keep output on a steady path close to potential

²Ask the Germans

Loss function

Central bank behaviour can be modeled using loss function

$$L_t = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t E_t (\pi_{t+k}^2 + \gamma x_{t+k}^2) \quad (63)$$

x_t is the output gap

γ indicates the weight put on stabilisation relative to inflation stabilisation

κ is the coefficient on the output gap in the NKPC

θ is the elasticity of demand for firms

Quadratic loss functions like these are popular since differentiating things to the power 2 produces linear relationships

- ▶ Quadratic loss function can also be used as an approximation to consumer utility in the NKM

Research has shown that

$$\gamma = \frac{\kappa}{\theta} \tag{64}$$

$$x_t^2 \quad (65)$$

Risk-averse consumers prefer smooth consumption paths which keeps output close to its natural rate to achieve this.

$$\pi_t^2 \quad (66)$$

Consumers don't just care about the level of consumption but also its allocation.

- ▶ With inflation, sticky prices imply different prices for the symmetric goods and thus different consumption levels
- ▶ Optimality requires equal consumption of all items in the bundle. Rationale for welfare effect of inflation, independent of its effect on output

Optimal policy under commitment

Suppose that the central bank can commit today to a strategy it can adopt now and in the future.

$$L = \sum_{t=0}^{\infty} \beta^t E_t \left[\frac{1}{2} (\pi_{t+k}^2 + \gamma x_{t+k}^2) + \lambda_{t+k} (\pi_{t+k} - \beta \pi_{t+k+1} - \kappa x_{t+k}) \right] \quad (67)$$

FOCs are

$$\gamma E_t x_{t+k} - \kappa E_t \lambda_{t+k} = 0 \quad (68)$$

$$E_t \pi_{t+k} + E_t \lambda_{t+k} - E_t \lambda_{t+k-1} = 0 \quad (69)$$

for $t = 0, 1, 2, \dots$ where $\lambda_{-1} = 0$

- There is no constraint on time $t = -1$

From this we get that

$$E_t x_{t+k} = \frac{\kappa}{\gamma} E_t \lambda_{t+k} = \theta E_t \lambda_{t+k} \quad (70)$$

$$E_t \pi_{t+k} = E_t \lambda_{t+k-1} - E_t \lambda_{t+k} = -\frac{1}{\theta} E_t \Delta x_{t+k} \quad (71)$$

$$\Delta E_t x_{t+k} = -\theta E_t \pi_{t+k} \quad (72)$$

Therefore the optimal policy under commitment will be characterised by

$$x_t = -\theta \pi_t = \theta(p_{t-1} - p_t) \quad (73)$$

$$E_t \Delta x_{t+1} = -\theta E_t \pi_{t+k} = \theta(p_{t+k-1} - p_{t+k}) \quad (74)$$

If we consider some initial price level p_{-1} we get

$$E_t x_{t+k} = \theta(p_{-1} - E_t p_{t+k}) \quad (75)$$

Since

$$\pi_t = p_t - p_{t-1} \quad (76)$$

$$E_t x_{t+k} = \theta(p_{-1} - E_t p_{t+k}) \quad (77)$$

Optimal policy is set against the price level

- ▶ Shocks will only temporarily affect price level but have no cumulative effect
- ▶ On average inflation will be zero

NB- Policy is history dependent: policy today depends on the whole past sequence of shocks that have determined today's price level

Optimal policy under discretion

Consider scenario where a central bank cannot commit to taking a particular course of action in the future

- ▶ All they can do is adopt the optimal strategy for what to do today.

Recall that the optimality conditions for period t and $t + 1$ are

$$x_t = -\theta\pi_t \quad (78)$$

$$E_t x_t - E_t x_{t+1} = -\theta\pi_{t+1} \quad (79)$$

The conditions for the first period are different from the rest

- ▶ At t , $t - 1$ is gone and doesn't matter
- ▶ Have to take into account the effect that time t decisions will have at time $t + 1$

Under discretion the policy maker always sets

$$x_t = -\theta\pi_t \quad (80)$$

Policy is set against inflation, where inflation can be characterised by

$$\pi_t = \beta E_t \pi_{t+1} - \kappa\theta\pi_t + u_t \quad (81)$$

With new first-order difference equation

$$\pi_t = \left(\frac{1}{1 + \theta\kappa} \right) (\beta E_t \pi_{t+1} + u_t) \quad (82)$$

and a repeated iteration solution

$$\pi_t = \left(\frac{1}{1 + \theta\kappa} \right) \sum_{k=0}^{\infty} \left(\frac{\beta}{1 + \theta\kappa} \right)^k E_t u_{t+k} \quad (83)$$

It is often assumed that the cost-push shocks follow an $AR(1)$ process which implies

$$E_t u_{t+k} = \rho^k u_t \quad (84)$$

With

$$u_t = \rho u_{t-1} + v_t, \quad v_t \sim N(0, \sigma^2) \quad (85)$$

Using

$$\sum_{k=0}^{\infty} c^k = \frac{1}{1-c} \quad (86)$$

for $|c| < 1$, inflation becomes

$$\pi_t = \left(\frac{1}{1-\theta\kappa} \right) \left[\sum_{k=0}^{\infty} \left(\frac{\beta\rho}{1+\theta\kappa} \right)^k \right] u_t \quad (87)$$

$$= \left(\frac{1}{1-\theta\kappa} \right) \left(\frac{1}{1-\frac{\beta\rho}{1+\theta\kappa}} \right) u_t \quad (88)$$

$$= \frac{u_t}{1+\theta\kappa-\beta\rho} \quad (89)$$

The $AR(1)$ cost-push shock implies that

$$E_t x_{t+1} = \rho x_t \quad (90)$$

$$E_t \pi_{t+1} = \rho \pi_t \quad (91)$$

which can be substituted in the Euler equation along with

$$x_t = -\theta \pi_t \quad (92)$$

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n)$$

to back out what the optimal interest rate looks like

$$i_t = r_t^n + \left(\rho + \frac{(1 - \rho)\theta}{\sigma} \right) \pi_t \quad (93)$$

This will be greater than 1 if

$$\frac{\theta}{\sigma} > 1 \quad (94)$$

which will hold for all reasonable parameterisations

- Inflation and thus interest rates do not depend at all on what happened in the past.

Woodford (2003) argues that policy under commitment produces superior welfare outcomes

- ▶ Private sector will anticipate future policies will be different
- ▶ Conditions at time t have the potential to improve stabilisation outcomes at time $t + 1$
- ▶ Holds even if these conditions will actually no longer matter at a later time.

Have to consider the transitory cost-push shock u_t ; Woodford argues expectations about shock won't affect future policy

- ▶ Short-run trade-off between inflation and the output gap; shift vertically by

$$u_t \quad (95)$$

- ▶ Central bank has to choose whether to increase inflation, have a negative output gap, or possibly a bit of both

Due to shocks people can expect central bank to pursue tighter policy from $t + 1$ onwards; short-run trade-off will be shifted by the change

$$u_t + E_t \pi_{t+1} \quad (96)$$

Shift will actually be smaller and thus possibly increase stabilisation.

Now the main issue here of course is that it might not be practically feasible to pick a policy and stick to it.

Empirical issues

Central role for NKPC in NKM: relies on output gap x_t

- ▶ How to measure this gap?

Can assume that on average output tend to return to its natural rate

- ▶ Use simple trend as proxy for natural rate (e.g. HP-filter)

Proxy

$$x = y_t - y_t^n \quad (97)$$

with

$$\tilde{y}_t = y_t - y_t^{tr} \quad (98)$$

NKPC can be estimated with data using

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t \quad (99)$$

We can't observe $E_t \pi_{t+1}$ so we substitute realised π_{t+1} and use an instrumental variable to deal with the fact this is a noisy estimator of what we really want.

One issue arising here is that often we find that

$$\kappa < 0 \quad (100)$$

Seems counterintuitive but we know that

1. $\Delta\pi_t$ is negatively correlated with the unemployment rate
2. Therefore positively correlated with the output gap

Given that $\beta \approx 1$, we can proxy

$$\pi_t - \beta E_t \pi_{t+1} \quad (101)$$

with

$$\pi_t - \pi_{t+1} = -\Delta \pi_{t+1} \quad (102)$$

Negative sign on κ might not be that surprising: two possible reasons for failure

1. The model is wrong
2. The output gap is measured with error

Gali & Gertler (1999) argue that the output gap is measured with error

- ▶ Deterministic trends do a bad job in capturing movements in the natural rate of output

Suggest using unit labour costs as proxy for marginal costs

- ▶ Proxy for real marginal costs is the labour share of income.

Estimate

$$\pi_t = \beta E_t \pi_{t+1} + \gamma s_t \quad (103)$$

Find

$$\beta > 0 \quad (104)$$

figure

Rudd & Whelan shown that there is actually downward trend in labour share across countries

If the KNPC would work well with either the labour share or another measure of real marginal cost, this would imply that it is completely forward looking.

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} \quad (105)$$

A VAR model could be used to forecast the levels of s_{t+k} and give a fitted value for the equations above. However, research has shown (Rudd & Whelan, 2006) that the fits are not really good. In contrast, adding lagged inflation to the model

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} + \rho \pi_{t-1} \quad (106)$$

improves the fit of the model considerably. So a main problem with the New-Keynesian Phillips Curve is that it doesn't account properly for inflation's dependence on its own lags.