Vector auto regression

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Monetary policy

James Stock and Mark Watson (2001), Vector Autoregressions

The paper by Stock & Watson provides a very useful introduction into VAR methods. This particular paper focuses on an important application; the effects of monetary policy shocks. As discussed in the paper you can think of using VAR models in two useful ways

1. From a scientific perspective
   - Monetary policy co-moves with lots of other macro variables
   - Only by identifying the structural or exogenous shocks to policy can we discover its true effects
2. From a policy perspective
   - Can help answer the question "if I choose to raise interest rates by an extra quarter point today, what is likely to happen over the next year to inflation and output relative to the case where I keep rates unchanged?"
   - This is basically a question about impulse responses

The model in the paper uses quarterly data and includes three variables

1. inflation π_t
2. unemployment rate u_t
3. federal funds rate i_t

Recall from last lecture that estimating VARs often includes creating a causal chain. This paper posits a lower-triangular causal chain of the form

$$AZ_t = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \pi_t \\ u_t \\ i_t \end{pmatrix} = BZ_{t-1} + \epsilon_t$$

The identifying assumptions of the model are the following

1. Inflation depends only on lagged values of the other variables[1]
2. Unemployment depends on contemporaneous inflation but not the funds rate

[1] Perhaps motivated by the idea of sticky prices.

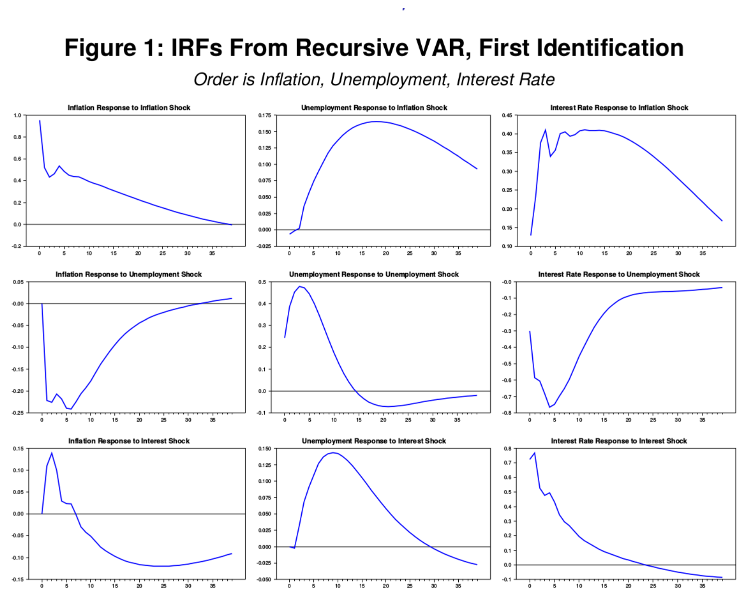

Figure 1: Impulse Response Functions from recursive VAR following Stock and Watson identification. Estimates made by Whelan.

3. The funds rate depends on both contemporaneous inflation and unemployment[2]

[2] Fed using its knowledge about the current state of the economy when it is setting interest rates.

Figure 1 summarises the results. What do the results of this paper tell us?

1. A positive interest rate shock raises the unemployment rate, and over time it reduces the inflation rate

2. Short-run response of the inflation rate is a bit puzzling: interest rate increase seems to raise inflation rate for a few periods

   - This price puzzle result has been obtained in a number of VAR studies; provides a good illustration of the potential limitations of VAR analysis.
   - Some think the explanation is that the Fed is acting on information not captured in the VAR and that this information may provide signals of future inflationary pressures
   - Thus, interest rate increases can tend to occur just before an increase in inflation. The VAR may be capturing this pattern and confusing causation and correlation

For example, information about commodity prices. Indeed, adding commodity prices to the VAR has often been found to eliminate the price puzzle. However, improving the specification this way to get the result we want does smell a bit of data mining.

In general the results from the Stock-Watson identification make intuitive sense, although you could imagine other possible identifications. For instance we could alter the ordering in which the VAR is run

1. interest rate
2. unemployment
3. inflation

You could argue this on the grounds that the Fed can only respond to the economy with a lag because it takes a while to receive data about the current state of the economy but that inflation should be able to respond immediately to economic events. Although this might sound reasonable, the results from this identification strategy don't make much sense (figure 2)

So they are reacting to lagged information.

- interest rate shock raises inflation for almost four years
- unemployment drops for a while after the increase in interest rates

The identification a researcher uses may depend on how sensible they believe the results to be. Needless to say, this may cause issues.

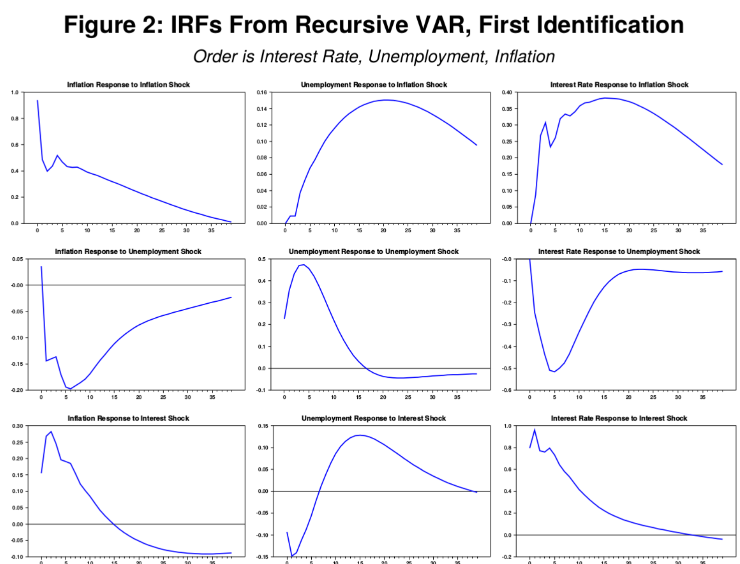

Figure 2: Impulse Response Functions from recursive VAR following alternative identification. Estimates made by Whelan.

Oil shocks

Lutz Kilian (2009).Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market

Oil shocks are large run-ups and subsequent declines in the price of oil, and receive a lot of attention. Interestingly, many recession have followed after oil price increases. Why this has happened is not really obvious since oil consumption is a relatively small input compared to the size of GDP. Most empirical work on this topic has focused on what the effects of oil price shocks are, but Killian focuses on two other things

1. what is an oil price shock?

2. are there different type of shocks?

To answer these questions Kilian uses a VAR model that distinguishes between three types of shock, and uses monthly data

1. oil supply, measured by oil production growth rate $\Delta prod_t$

2. global demand, measured by real global economic activity rea_t

See figure 3

3. speculation in oil price market, measured by real oil price rpo_t

$z_t = (\Delta prod_t, rea_t, rpo_t)'$

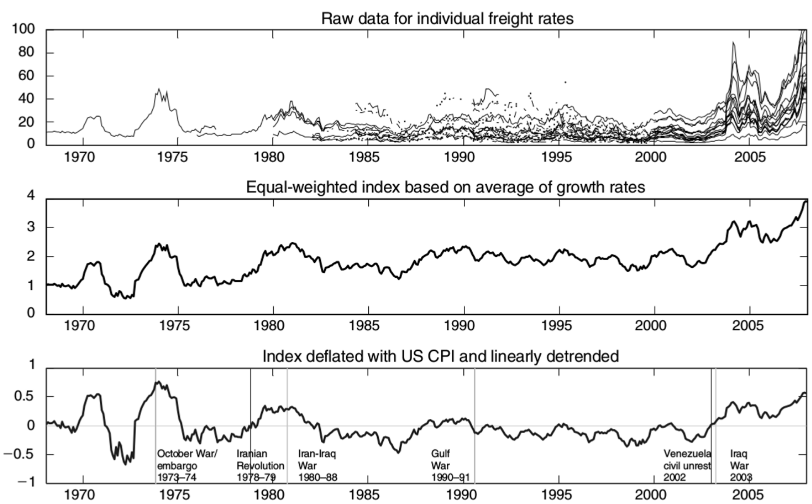

FIGURE 1. MONTHLY INDEX OF GLOBAL REAL ECONOMIC ACTIVITY BASED ON DRY CARGO BULK FREIGHT RATES (1968:1–2007:12)

Figure 3: Monthly index of global real economic activity based on dry cargo bulk freight rates for 1968:1-2007:12. Figure taken from Kilian (2009)

The structure of the VAR is as follows

$$A_0 z_t = \alpha \sum_{i=1}^{24} A_i z_{t-1} + \epsilon_t$$

$$A_0 = \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix}$$

And the identifying assumptions are as follows

1. Oil production does not respond within the month to world demand and oil prices

2. World demand is affected within the month by oil production, but not by oil prices

3. Oil prices respond immediately to oil production and world demand

Now in the model A_0 is lower-triangular matrix, which means that A_0^{-1} is as well. The reduced-form VAR therefore becomes

$$z_t = A_0^{-1}\alpha + A_0^{-1}\sum_{i=1}^{24} A_i z_{t-1} + A_0^{-1}\epsilon_t$$

The reduced-form shocks e_t are related to structural shocks $A_0^{-1}\epsilon_t$

$$\begin{pmatrix} e_t^{\Delta prod} \\ e_t^{rea} \\ e_t^{rpo} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \epsilon_t^{\Delta prod} \\ \epsilon_t^{rea} \\ \epsilon_t^{rpo} \end{pmatrix}$$

Practically this entails that

1. Oil production reduced form shock is a structural shock

2. Economic activity reduced form shock is combination of structural oil shock and structural activity shock

3. Reduced form oil price shock is combination of all three structural shocks

Let's check the identification restrictions. We know that the general model has the form

$$AY_t = BY_{t-1} + C\epsilon_t$$

which means that the model needs 18 identifying restrictions ($2n^2 = 18$). In this case we have that

- $C = I$ assuming contemporaneous interactions between variables: 9
- Lower diagonal assumption on A_0: 3
- Unit coefficient normalisation on A_0 diagonal: 3
- Orthogonal structural shocks: 3 off-diagonal elements of $\sum$ are 0

In terms of the results of the estimation, Kilian shows, in addition to the standard impulse response analysis, how the real price of oil can be decomposed into components related to the three shocks (figure 4). Recall the Vector Moving Average representation

$$Y_t = e_t + Ae_{t-1} + A^2 e_{t-2} + A^3 e_{t-3} + ... + A^t e_0$$

One can repeat this calculation three times, each time with only one type of shock turned on and the other set to zero. Adding these up, one will get the realized values of Y_t. Alternatively, one can do a dynamic simulation of the model

$$Y = AY_{t-1} + \epsilon_t$$

Here we let ϵ_t represent one of the realised shocks, setting the others to zero

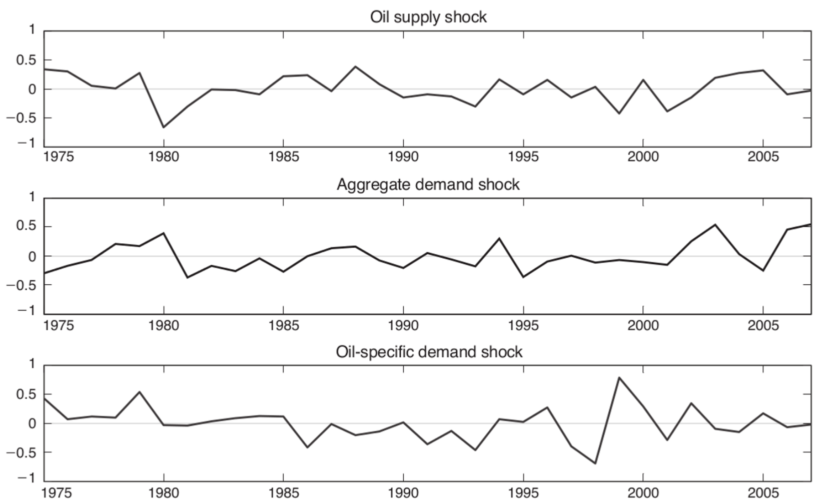

FIGURE 2. HISTORICAL EVOLUTION OF THE STRUCTURAL SHOCKS, 1975–2007

Figure 4: Historical evolution of the structural shock 1975:2007. Figure taken from Kilian (2009)

DESPITE GETTING A LOT OF ATTENTION, SHOCKS TO OIL SUPPLY HAVE LIMITED EFFECTS ON OIL PRICES AND HAVE BEEN OF NEGLIGIBLE IMPORTANCE IN DRIVING OIL PRICES OVER TIME.

- Both global demand and speculative oil price shocks can have significant effects on oil prices, but speculative oil price shocks have limited effects on global economic activity
- Speculative oil-market shocks have accounted for most of the month-to-month movements in oil prices
- But the steady increase in oil prices from 2000 onwards was almost solely due to strong global demand

The main lesson of this study is that how the economy reacts to an oil price shock will depend on the origins of that shock.

This helps to explain why the world economy survived a long period of increasing oil prices in the 2000s without going into recession. And when it did go into recession, it had little to do with oil prices.

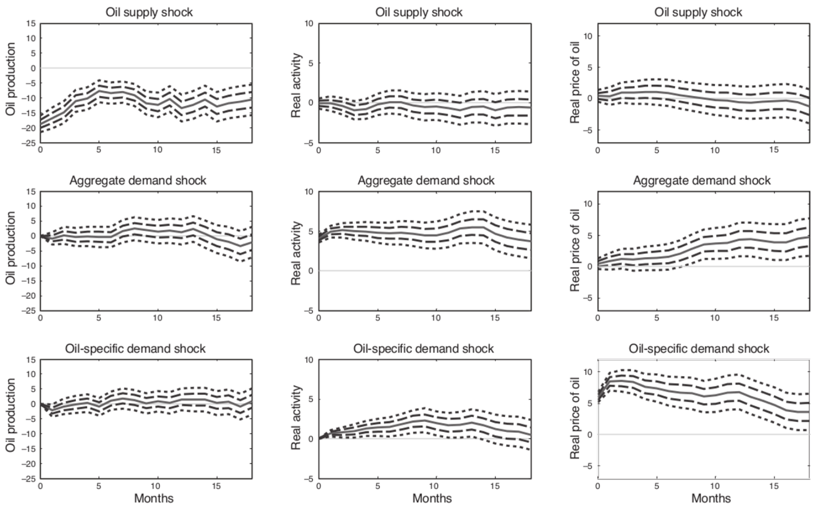

FIGURE 3. RESPONSES TO ONE-STANDARD-DEVIATION STRUCTURAL SHOCKS

(Point estimates with one- and two-standard error bands)

Figure 5: Response to one-standard deviation structural shock. Figure taken from Kilian (2009)

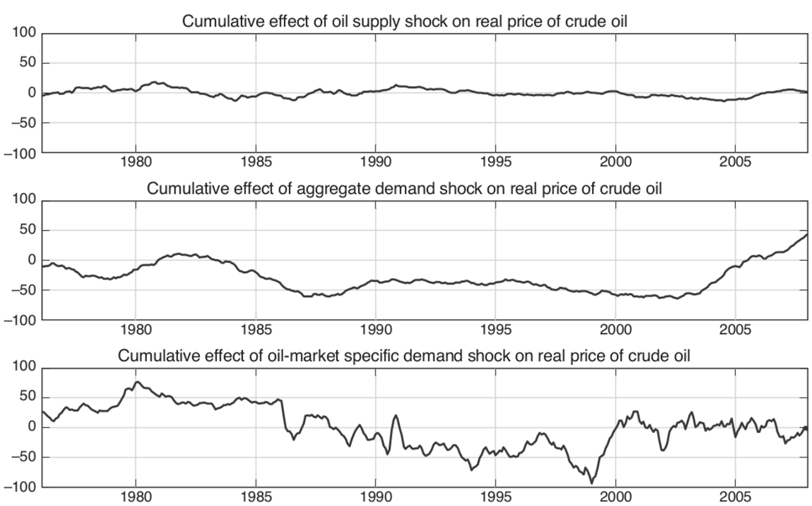

FIGURE 4. HISTORICAL DECOMPOSITION OF REAL PRICE OF OIL

(1976:1–2007:12)

Figure 6: Historical decomposition of real oil prices 1976:1-2007:12. Figure taken from Kilian (2009)

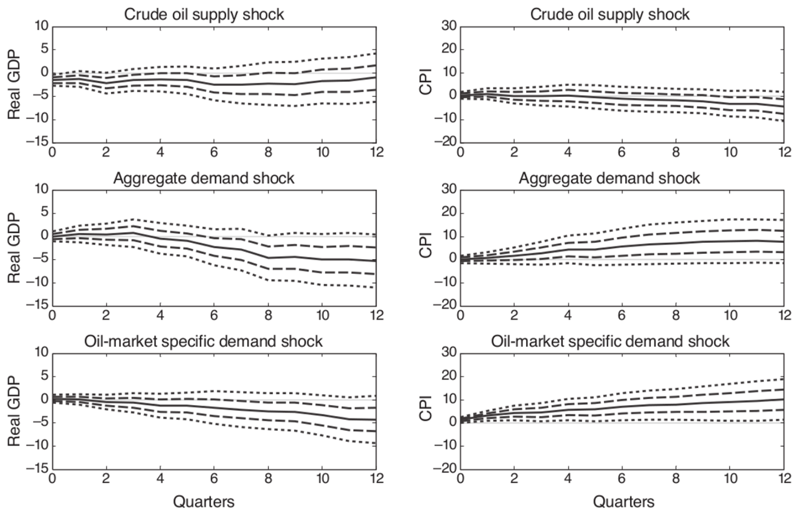

FIGURE 5. RESPONSES OF US REAL GDP AND CPI LEVEL TO EACH STRUCTURAL SHOCK
(Point estimates with one- and two-standard error bands)

Figure 7: Responses of US real GDP and CPI level to each structural shock. Figure taken from Kilian (2009)

Fiscal policy

Blanchard & Perotti examined a three-variable quarterly VAR using US data on

Olivier Blanchard and Roberto Perotti (2002). An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output

1. Federal tax revenues
2. Federal government spending
3. GDP

The specification of their VAR model is

Quarterly data is used as this is essential to identify the fiscal shocks. Such as the presence of seasonal patterns in the response of some of the taxes to economic activity.

$$Y_t = A(L,q)Y_{t-1} + U_t$$

- $Y_t = (T_t, G_t, X_t)'$, outcome, logarithm measured in real per capita terms
- $U_t = (t_t, g_t, x_t)'$, reduced-form residual shocks, assumed to have nonzero cross-correlations
- $A(L,q)$, four-quarter distributed lag polynomial

  The reduced-form residual shocks are defined as

e_t^t, e_t^g, e_t^x are mutually uncorrelated structural shocks

$$\begin{aligned} t_t &= a_1 x_t + a_2 e_t^g + e_t^t \\ g_t &= b_1 x_t + b_2 e_t^t + e_t^g \\ x_t &= c_1 t_t + c_2 g_t + e_t^x \end{aligned}$$

The equation for t_t states that the unexpected movements for taxes within a quarter are the result of the response to

This is similar for the unexpected movements in spending g_t

1. unexpected movements in GDP $a_1 x_t$
2. structural shocks to spending $a_2 e_t^g$
3. structural shocks to taxes e_t^t

In this system the unexpected movements to spending x_t can be due to unexpected movements in taxing , spending, or other unexpected shocks e_t^x

Similar to the other studies discussed so far, they used prior information to set three different contemporaneous effect coefficients

1. Assume that there is no within-quarter effect of GDP on government spending[3]
2. Use separate information on tax elasticities to set a specific positive value for the contemporaneous effect of GDP on tax revenues.
3. They swap between setting
   - the contemporaneous effect of taxes on spending equal to zero
   - setting the contemporaneous effect of spending on taxes equal to zero

   and report that results are similar when either is used

[3] "Direct evidence on the conduct of fiscal policy suggests that it takes policymakers ... more than a quarter to learn about a GDP shock, decide what fiscal measures, if any, to take in response, pass these measures through the legislature, and actually implement them."

The authors assume that taxes and spending can both affect GDP within the same quarter, i.e. they place GDP last in the ordering as described. Figure 8 shows the impulse response function following a unit tax shock, assuming that taxes are ordered first ($a_2 = 0$), using both a deterministic (top) and stochastic (bottom) trend. The impulse response functions show that

- Under a deterministic trend
  - Output falls by 70%, reverts back to trend after 5 quarters
  - Effect on government spending is small; largest after 12 quarters
- Under a stochastic trend
  - Output response is stronger and more persistent; takes 7 quarters to revert back to trend
  - Effect on spending is similar

Figure 9 shows the impulse response to a spending shock.

Spending is ordered first: $b_2 = 0$.

- Under a deterministic trend
  - Spending shocks last longer than tax shocks: 95% of the shock is still there after two years
  - GDP decreases and then increases to reach a peak of 1.29 after 4 years

- Taxes respond similarly

- Under a stochastic trend
  - Output peak is smaller at 0.90, and reached at impact
  - Estimate comes with wide uncertainty intervals

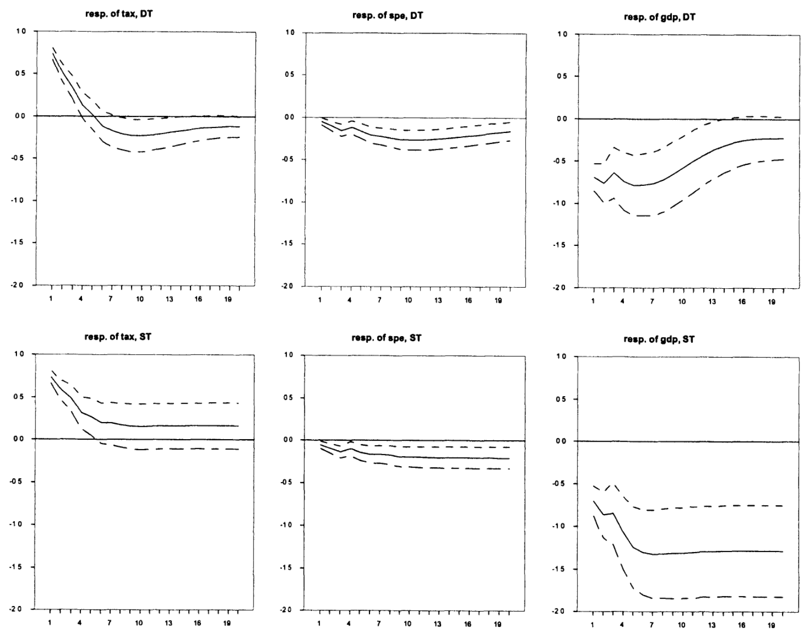

FIGURE III
Response to a Tax Shock

Figure 8: Response to a tax shock. Figure taken from Blanchard & Perotti (2002)

Does VAR make sense?

Glenn Rudebusch (1998). Do Measures of Monetary Policy in a VAR Make Sense?

Christopher Sims (1998). Comment on Glenn Rudebusch's Do Measures of Monetary Policy in a VAR Make Sense?

The paper by Rudebusch contains a strong critique of VARs used to assess monetary policy. The main points of critique are that VARs

1. ignore changes over time in the formulation of monetary policy
2. use final published data instead of the preliminary estimates the Fed has available when it makes decisions
3. greatly underestimate the information available to the Fed when it takes decisions
4. incorporate long lags but he argues it is not credible that the Fed responds to information from over a year prior to taking a decision

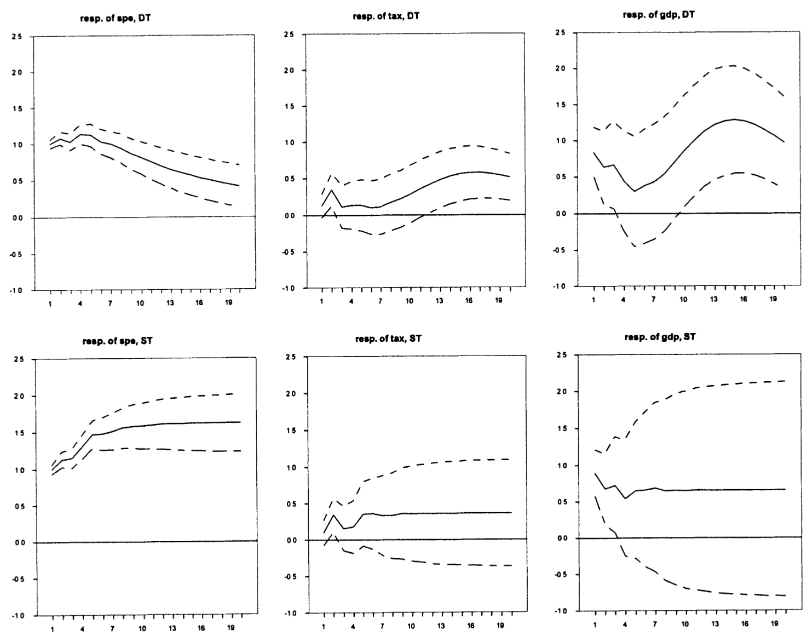

FIGURE V
Response to a Spending Shock

Figure 9: Response to a spending shock. Figure taken from Blanchard & Perotti (2002)

5. the monetary policy shocks donâĂŹt look anything like the surprise element of monetary policy decisions obtained from looking at financial contracts like fed funds futures

Figure 10 and 11

6. Models with very different monetary policy shocks report similar IRFs, suggesting that perhaps the models have been data-mined to give these answers

Sims responded in detail; some of his points were as follows

1. VAR models may differ in their shocks but agree on their effects.

   - For example, one model may include more variables in a supply equation than another so its supply shocks are, by construction, smaller in size but both models could still capture roughly the same effect of a shock to supply.

2. Financial market surprises are not necessarily the best measures of the exogenous element of monetary policy.

   - A Fed governor could give a speech the week before an FOMC meeting indicating that the Fed is going to raise rates (even if this isn't predicted by inflation or GDP or other standard VAR variables.) When this rate increase happens, we'd like to know what effect it has even though, on the day, it is not a surprise for financial markets.

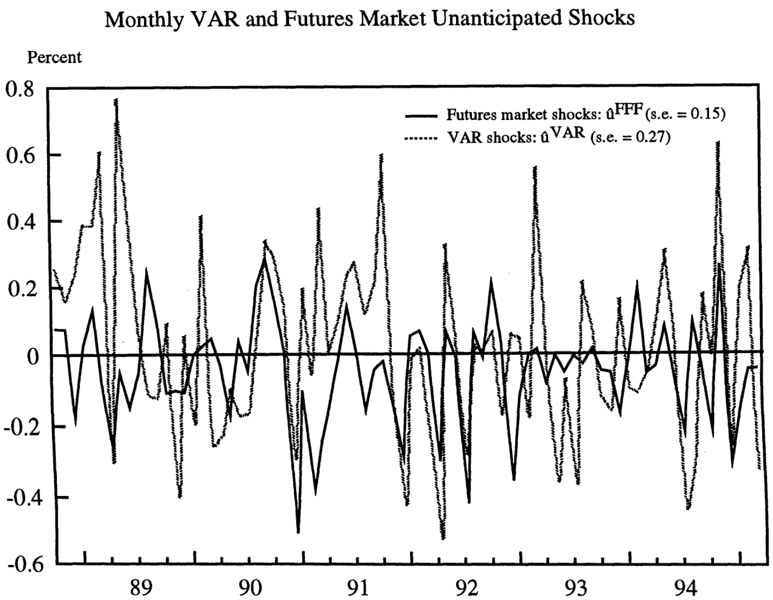

FIGURE 4

Figure 10: Monthly VAR and futures market unanticipated shocks. Figure taken from Rudebusch (1998)

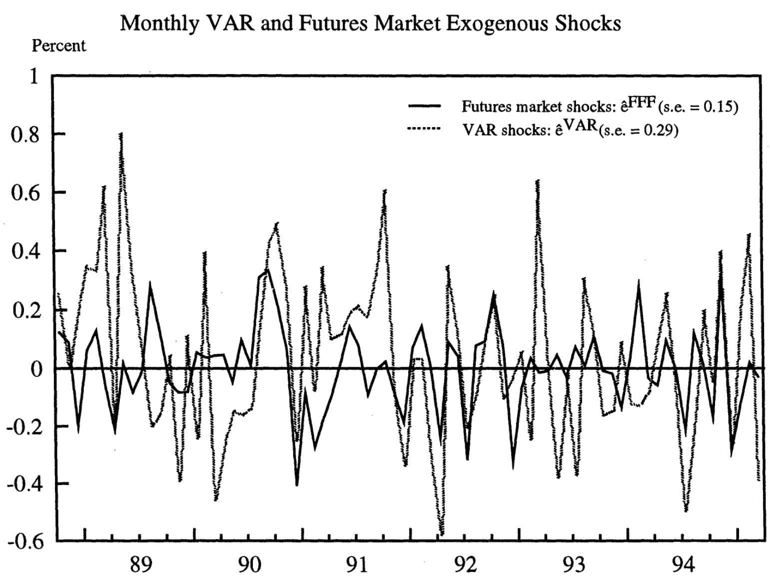

FIGURE 5

Figure 11: Monthly VAR and futures markets exogenous shocks. Figure taken from Rudebusch (1998)

3. Issues like time invariance, linearity, and variable selection are universal in macroeconomic modeling and are not special to VARs.