Smets-Wouters model

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Smets & Wouters (2007) "Shocks and Frictions in US Business Cycles : A Bayesian DSGE Approach"

Using a Bayesian likelihood approach, we estimate a dynamic stochastic general equilibrium model for the US economy using seven macroeconomic time series. The model incorporates many types of real and nominal frictions and seven types of structural shocks. We show that this model is able to compete with Bayesian Vector Autoregression models in out-of-sample prediction. We investigate the relative empirical importance of the various frictions. Finally, using the estimated model, we address a number of key issues in business cycle analysis: What are the sources of business cycle fluctuations? Can the model explain the cross correlation between output and inflation? What are the effects of productivity on hours worked? What are the sources of the Great Moderation?

Supply side

$$y_t = \phi_p(\alpha k_t^s + (1-\alpha) l_t + \epsilon_t^a) \tag{1}$$

y_t is GDP
k_t^s is capital in use: determined by lagged level of capital and a capacity utilisation variable

$$k_t^s = k_{t-1} + z_t \tag{2}$$

l_t is labour input
ϵ_t^a is total factor productivity

z_t linked to marginal productivity of capital; function of capital to labour ratio and the real wage

$$r_t^k = -(k_t - l_t) + w_t \tag{3}$$

Total factor productivity evolves over time according to

$$\epsilon_t^a = \rho\epsilon_{t-1}^a + \eta_t^a \tag{4}$$

Demand side

$$y_t = c_y c_t + i_y i_t + z_y z_t + \epsilon_t^g \quad (5)$$

y_t is GDP
c_t is consumption
i_t is investment
ϵ_t^g is exogenous spending

Variables with subscript y are steady-state shares
z_t is included in the resource constraint because of the assumption that there are costs associated with having high rates of capital utilisation.

Exogenous spending is assumed to develop over time according to

$$\epsilon_t^g = \rho \epsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a \tag{6}$$

Exogenous spending is assumed to have two components

1. Government spending
2. Element related to productivity

Consumption

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2(l_t - E_t l_{t+1}) - c_3(r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{7}$$

c_1, c_2, c_3 are constant parameters (functions of deeper structural parameters)

r_t is the interest rate on a one-period safe bond (quarterly)

ϵ_t^b is a risk premium shock determining the willingness of a household to hold the one-period bond

- Preference shock that influence short-term consumption-saving decisions

$$e_t^b = \rho_b \epsilon_{t-1} + \eta_t^b \tag{8}$$

Two other important things concerning consumption equation

- Backward looking consumption term represent habit forming
- Equation allows for substitution of consumption with labour input

Investment

$$i_t = i_t i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \epsilon_t^i \tag{9}$$

Main driver of investment: q_t

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) r_{t+1}^k - (r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{10}$$

Prices

$$\mu_t^p = \alpha(k_t - l_t) + \epsilon_t^a - w_t \tag{11}$$

Equation accounts for

1. Diminishing marginal productivity of capital
2. Productivity shocks effect on costs
3. Real wage

Inflation

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \epsilon_t^p \tag{12}$$

New Keynesian Philips curve

- Adjusted to account for lagged inflation
- Most firms will index their prices based on past inflation levels and can only set an optimal price occasionally

ϵ_t^p is a price mark-up disturbance which is described by

$$\epsilon_t^p = \rho^p \epsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p \tag{13}$$

Shock affects both current and lagged inflation in order to get a temporary price level shock.

Wages

$$w_t = w_1 w_{t-1} + (1 - w_1)E_t(w_{t+1} + \pi_{t+1}) - \\ w_2\pi_t + w_3\pi_{t-1} - w_t\mu_t^w + \epsilon_t^w \tag{14}$$

$$\epsilon_t^w = \rho^w \epsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w \tag{15}$$

μ_t^w is the wage mark-up

- Gap between real wage and marginal rate of substitution between working and consuming

$$
\begin{aligned}
\mu_t^w &= w_t - mrs_t && (16)\\
&= w_t - \left(\sigma l_t - \frac{1}{1-\lambda/\gamma}(c_t - \lambda c_{t-1})\right) && (17)
\end{aligned}
$$

Sort of sticky: wages adjust gradually to equate the marginal costs and benefits of working

Monetary policy

$$r_t = \rho r_{t-1} + (1-\rho)(r_\pi \pi_t + r_y(y_t - y_t^p)) + \\ r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \epsilon_t^r \tag{18}$$

$$\epsilon_t^r = \rho^r \epsilon_{t-1}^r + \eta_t^r \tag{19}$$

Central bank sets short-term interest rate according to

1. Last period's interest rate
2. Gradual adjustment towards target interest rate
   - Depends on inflation and output gap
3. Output gap growth rate

Potential output defined as the level of output that would prevail if prices and wages were fully flexible

- Model effectively needs to be expanded to add shadow flexible-price economy

VAR system

$$Y_t = \begin{pmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \end{pmatrix} = \begin{pmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \bar{l} \\ \overline{\pi} \\ \bar{r} \end{pmatrix} + \begin{pmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{pmatrix} \tag{20}$$

Additional features compared to RBC or NK model:

- Adjustment costs for investment
- Capacity utilisation cost
- Habit persistence
- Price indexation
- Wage indexation
- All kinds of autocorrelated shock terms

Fixes included to overcome shortcomings previous model: slow things down

- Give random shocks longer lasting effects
- Make development of variables more sluggish

Velocity major shortcoming of RBC: wage/price indexation addresses NK shortcoming

- Failed to deal with inflation persistence

Adjustment are largely ad hoc: no clear theoretical grounding.

TABLE 1A—PRIOR AND POSTERIOR DISTRIBUTION OF STRUCTURAL PARAMETERS

	Prior distribution			Posterior distribution			
	Distr.	Mean	St. Dev.	Mode	Mean	5 percent	95 percent
φ	Normal	4.00	1.50	5.48	5.74	3.97	7.42
σ_c	Normal	1.50	0.37	1.39	1.38	1.16	1.59
h	Beta	0.70	0.10	0.71	0.71	0.64	0.78
ξ_w	Beta	0.50	0.10	0.73	0.70	0.60	0.81
σ_l	Normal	2.00	0.75	1.92	1.83	0.91	2.78
ξ_p	Beta	0.50	0.10	0.65	0.66	0.56	0.74
ι_w	Beta	0.50	0.15	0.59	0.58	0.38	0.78
ι_p	Beta	0.50	0.15	0.22	0.24	0.10	0.38
ψ	Beta	0.50	0.15	0.54	0.54	0.36	0.72
Φ	Normal	1.25	0.12	1.61	1.60	1.48	1.73
r_π	Normal	1.50	0.25	2.03	2.04	1.74	2.33
ρ	Beta	0.75	0.10	0.81	0.81	0.77	0.85
r_y	Normal	0.12	0.05	0.08	0.08	0.05	0.12
$r_{\Delta y}$	Normal	0.12	0.05	0.22	0.22	0.18	0.27
$\bar{\pi}$	Gamma	0.62	0.10	0.81	0.78	0.61	0.96
$100(\beta^{-1} - 1)$	Gamma	0.25	0.10	0.16	0.16	0.07	0.26
$\bar{l}$	Normal	0.00	2.00	−0.1	0.53	−1.3	2.32
$\bar{\gamma}$	Normal	0.40	0.10	0.43	0.43	0.40	0.45
α	Normal	0.30	0.05	0.19	0.19	0.16	0.21

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 1B—Prior and Posterior Distribution of Shock Processes

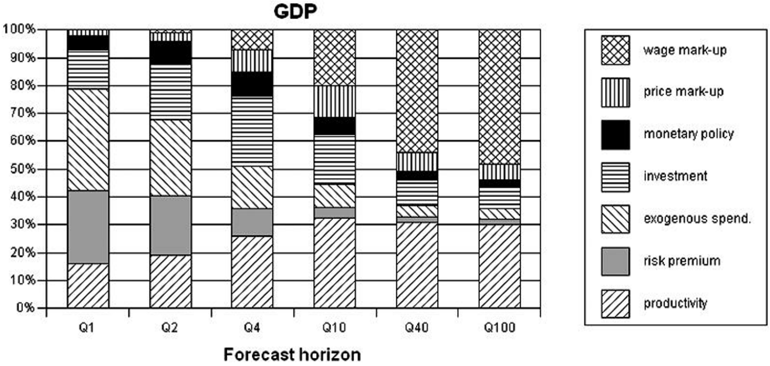
	Prior distribution			Posterior distribution			
	Distr.	Mean	St. Dev.	Mode	Mean	95 percent	5 percent
σ_a	Invgamma	0.10	2.00	0.45	0.45	0.41	0.50
σ_b	Invgamma	0.10	2.00	0.24	0.23	0.19	0.27
σ_g	Invgamma	0.10	2.00	0.52	0.53	0.48	0.58
σ_I	Invgamma	0.10	2.00	0.45	0.45	0.37	0.53
σ_r	Invgamma	0.10	2.00	0.24	0.24	0.22	0.27
σ_p	Invgamma	0.10	2.00	0.14	0.14	0.11	0.16
σ_w	Invgamma	0.10	2.00	0.24	0.24	0.20	0.28
ρ_a	Beta	0.50	0.20	0.95	0.95	0.94	0.97
ρ_b	Beta	0.50	0.20	0.18	0.22	0.07	0.36
ρ_g	Beta	0.50	0.20	0.97	0.97	0.96	0.99
ρ_I	Beta	0.50	0.20	0.71	0.71	0.61	0.80
ρ_r	Beta	0.50	0.20	0.12	0.15	0.04	0.24
ρ_p	Beta	0.50	0.20	0.90	0.89	0.80	0.96
ρ_w	Beta	0.50	0.20	0.97	0.96	0.94	0.99
μ_p	Beta	0.50	0.20	0.74	0.69	0.54	0.85
μ_w	Beta	0.50	0.20	0.88	0.84	0.75	0.93
ρ_{ga}	Beta	0.50	0.20	0.52	0.52	0.37	0.66

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 3—Out-of-Sample Prediction Performance

	GDP	dP	Fedfunds	Hours	Wage	CONS	INV	Overall
VAR(1)	*RMSE-statistic for different forecast horizons*							
1q	0.60	0.25	0.10	0.46	0.64	0.60	1.62	−12.87
2q	0.94	0.27	0.18	0.78	1.02	0.95	2.96	−8.19
4q	1.64	0.34	0.36	1.45	1.67	1.54	5.67	−3.25
8q	2.40	0.53	0.64	2.13	2.88	2.27	8.91	1.47
12q	2.78	0.63	0.79	2.41	4.09	2.74	10.97	2.36
BVAR(4)	*Percentage gains (+) or losses (−) relative to VAR(1) model*							
1q	2.05	14.14	−1.37	−3.43	2.69	12.12	2.54	3.25
2q	−2.12	15.15	−16.38	−7.32	−0.29	10.07	2.42	0.17
4q	−7.21	31.42	−12.61	−8.58	−3.82	1.42	0.43	0.51
8q	−15.82	33.36	−13.26	−13.94	−8.98	−8.19	−11.58	−4.10
12q	−15.55	37.59	−13.56	−4.66	−15.87	−3.10	−23.49	−9.84
DSG	*Percentage gains (+) or losses (−) relative to VAR(1) model*							
1q	5.68	2.05	−8.24	0.68	5.99	20.16	9.22	3.06
2q	14.93	10.62	−17.22	10.34	6.20	25.85	16.79	2.82
4q	20.17	46.21	1.59	19.52	9.21	26.18	21.42	6.82
8q	22.55	68.15	28.33	22.34	15.72	21.82	25.95	11.50
12q	32.17	74.15	40.32	27.05	21.88	23.28	41.61	13.51

Notes: All models are estimated starting in 1966:1. The forecast period is 1990:1–2004:4. VAR(1) and BVAR(4) models are reestimated each quarter, the DSGE model each year. The overall measure of forecast performance is the log determinant of the uncentered forecast error covariance matrix. Gains and losses in the overall measure are expressed as the difference in the overall measure divided by the number of variables and by two to convert the variance to standard errors (times 100).


GDP
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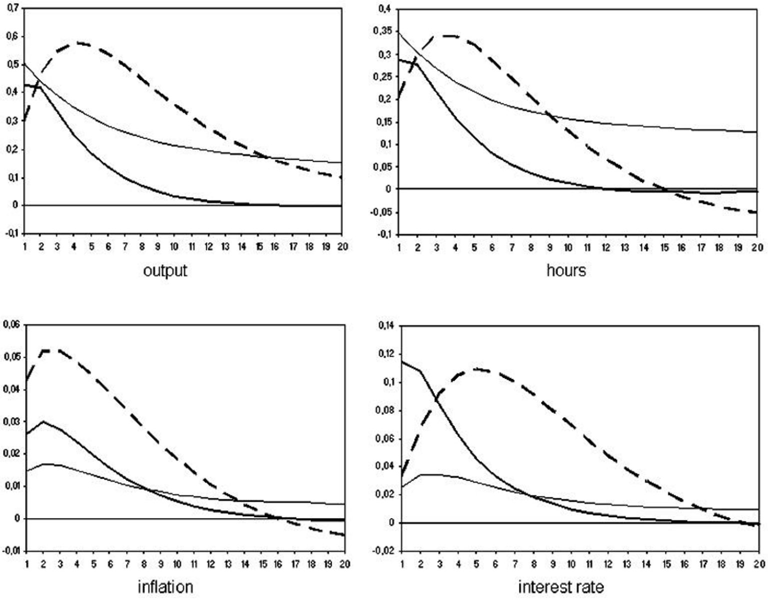
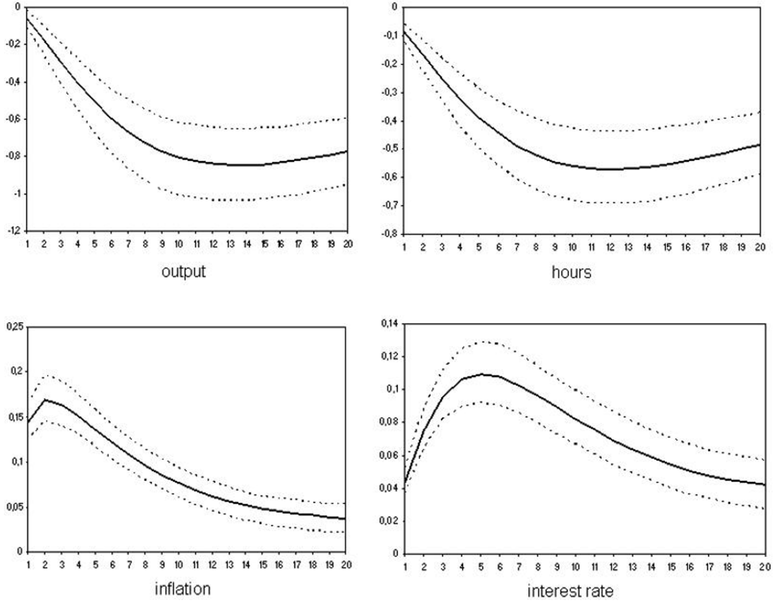
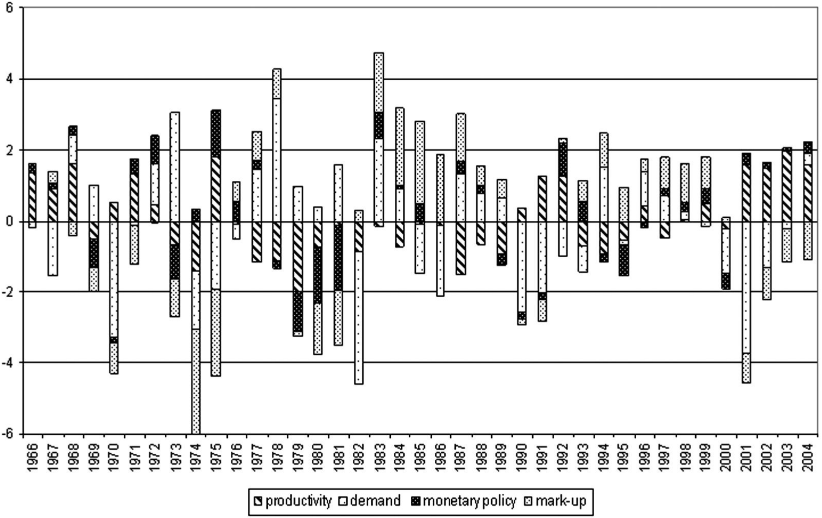
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Q1
Q2
Q4
Q10
Q40
Q100
Forecast horizon
wage mark-up
price mark-up
monetary policy
investment
exogenous spend.
risk premium
productivity


output
hours
inflation
interest rate


output
hours
inflation
interest rate


6
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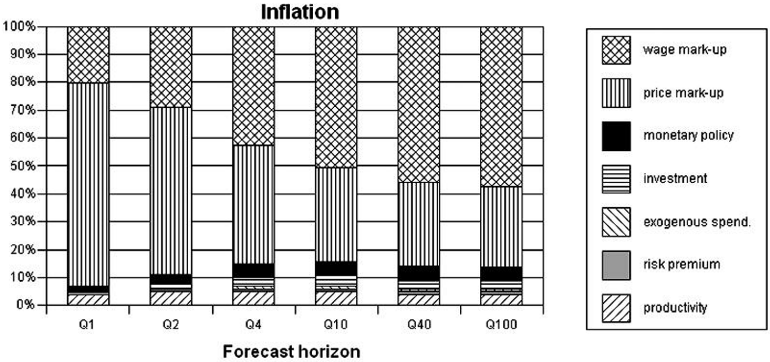
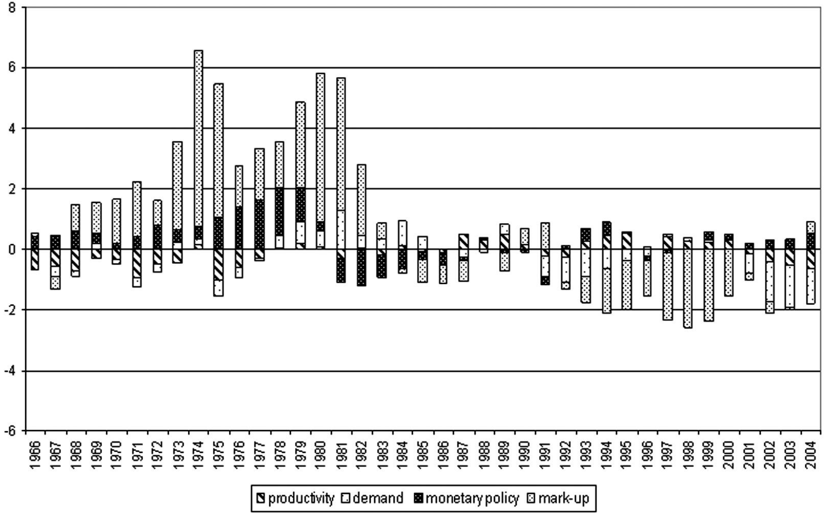
-6
1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004
productivity
demand
monetary policy
mark-up


Inflation
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Q1
Q2
Q4
Q10
Q40
Q100
Forecast horizon
wage mark-up
price mark-up
monetary policy
investment
exogenous spend.
risk premium
productivity


8
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1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004
productivity
demand
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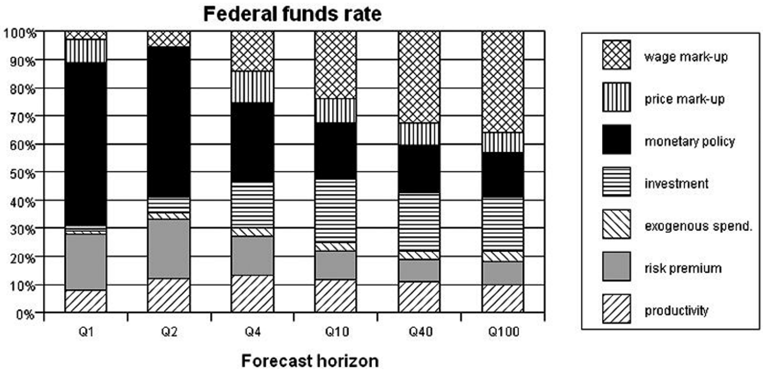
Federal funds rate
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90%
80%
70%
60%
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Q1
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Forecast horizon
wage mark-up
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monetary policy
investment
exogenous spend.
risk premium
productivity

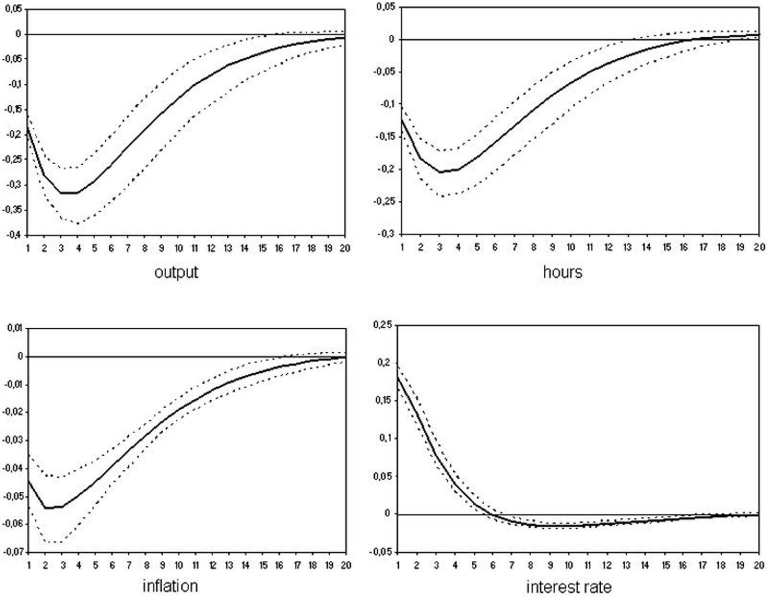
output
hours
inflation
interest rate

Table 5—Subsample Estimates

	Structural parameters					Shock processes			
	1966:1–1979:2		1984:1–2004:4			1966:1–1979:2		1984:1–2004:4	
	Mode	SD	Mode	SD		Mode	SD	Mode	SD
φ	3.61	1.03	6.23	1.12	σ_a	0.58	0.05	0.35	0.02
σ_c	1.39	0.22	1.47	0.13	σ_b	0.22	0.04	0.18	0.02
h	0.63	0.07	0.68	0.04	σ_g	0.54	0.05	0.41	0.03
ξ_w	0.65	0.07	0.74	0.13	σ_I	0.52	0.09	0.39	0.05
σ_l	1.52	0.65	2.30	0.67	σ_r	0.20	0.02	0.12	0.01
ξ_p	0.55	0.08	0.73	0.04	σ_p	0.22	0.03	0.11	0.01
ι_w	0.58	0.13	0.46	0.16	σ_w	0.20	0.02	0.21	0.03
ι_p	0.45	0.18	0.21	0.09	ρ_a	0.97	0.01	0.94	0.02
ψ	0.34	0.13	0.69	0.11	ρ_b	0.39	0.17	0.14	0.08
Φ	1.43	0.09	1.54	0.09	ρ_g	0.91	0.03	0.96	0.01
r_π	1.65	0.19	1.77	0.29	ρ_I	0.60	0.10	0.64	0.07
ρ	0.81	0.03	0.84	0.02	ρ_r	0.22	0.10	0.29	0.10
r_y	0.17	0.03	0.08	0.05	ρ_p	0.51	0.24	0.74	0.13
$r_{\Delta y}$	0.20	0.03	0.16	0.02	ρ_w	0.96	0.02	0.82	0.15
$\bar{\pi}$	0.72	0.11	0.67	0.10	μ_p	0.46	0.20	0.59	0.18
$\beta^{-1} - 1$	0.14	0.06	0.12	0.05	μ_w	0.84	0.07	0.62	0.17
$\bar{l}$	0.03	0.62	−0.55	1.21	ρ_{ga}	0.58	0.11	0.39	0.11
$\bar{\gamma}$	0.33	0.04	0.44	0.02					
α	0.19	0.02	0.21	0.02					

Note: SD stands for standard deviation.

Table 6—Actual, Model-Based, and Counterfactual Standard Deviations of GDP Growth and Inflation

	1966:1–2004:4		1966:1–1979:2		1984:1–2004:4		Counterfactual 1984:1–2004:4		
	Actual	Model	Actual	Model	Actual	Model	Shocks	Policy	Structure
Growth	0.86	0.94	1.01	1.13	0.59	0.73	1.21	0.70	0.75
Inflation	0.62	0.57	0.55	0.81	0.25	0.34	1.30	0.39	0.32

Notes: "Actual" refers to the data-based standard deviations over the indicated sample; "model" refers to the standard deviations generated by the DSGE model estimated over the indicated sample. The counterfactual standard deviations for the period 1984:1–2004:4 refer to the standard deviations that would have occurred in this period if the shock processes ("shocks"), the monetary policy rule ("policy"), or the structural parameters ("structure") would have been the same as the ones estimated in the 1966:1–1979:2 sample.