

The Modern New-Keynesian Model

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New-Keynesian Model

The New-Keynesian model (NKM) addresses some of the critiques that proponents of the rational expectations school had on the Keynesian model. The NKM is a model in which people have rational expectations and behave optimally, but there is also room for monetary policy to have systematic effects. The central mechanism for monetary policy to have an effect on output is the concept of sticky prices. If prices don't move in line with money, then the central bank can't control the real money supply or the real interest rate.

There is actually some evidence that prices tend to be sticky.

The Dixit-Stiglitz model

The standard NKM uses the Dixit-Stiglitz model as vantage point to describe optimal behaviour. This model does not include capital but only consumption goods. Consumers will maximise their utility function $U(Y_t)$ over an aggregate of a continuum of differentiated goods

θ denotes constant elasticity of substitution.

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

This model implies that the demand function for each differentiated good is of the form

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta}$$

Where P_t is the aggregate price index which is defined by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$

The Calvo model

To describe price rigidity, the NKM relies on the Calvo model. In this model, for each period only a random fraction of firms, $1 - \alpha$, are able to reset their price. All other firms keep their prices unchanged. Apart from this difference in timing, of when they set prices, the firms are completely symmetric. Therefore, the price level can be

For further technical details on this, see last year's [notes](#).

i.e. all firms setting new prices today set the same price.

defined as

$$P_t = \left[(1 - \alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$P_t^{1-\theta} = (1 - \alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}$$

When firms reset their prices they must take into account that the price may be fixed for many periods. They pick a price to maximise

$$E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k (Y_{t+k} P_{t+k}^{\theta-1} X_t^{1-\theta} - P_{t+k}^{-1} C(Y_{t+k} P_{t+k}^{\theta} X_t^{-\theta})) \right]$$

The solution for this maximisation problem is, differentiating with regard to X_t

$$X_t = \frac{\theta}{\theta - 1} \frac{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)}{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} \right)}$$

This entails that changed price X_t is a markup over a weighted average of future marginal costs.

We now have two non-linear equations for price

$$P_t^{1-\theta} = (1 - \alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}$$

$$X_t = \frac{\theta}{\theta - 1} \frac{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)}{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} \right)}$$

These are not easy to solve or simulate so instead we use log-linearised approximations.

$$p_t = (1 - \alpha)x_t + \alpha p_{t-1}$$

$$x_t = (1 - \alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k E_t mc_{t+k}$$

X_t is the price that the firms resetting today have chosen.

$C(\cdot)$ is the cost function

Without frictions the firm would set $X_t = \frac{\theta}{\theta-1} MC_t$, i.e. the price equals $\frac{\theta}{\theta-1}$ times the marginal costs.

These are taken around a constant growth, zero inflation path. We use the fact that in the steady state we have

$$X_t^* = P^* = P_{t-1}^* = P^*$$

$$X^* = \left(\frac{\theta}{1 - \theta} \right) MC^*$$

This is the solution to a first-order stochastic difference equation, reverse engineering we can see that the optimal reset price can also be written as

$$x_t = (1 - \alpha\beta)mc_t + (\alpha\beta)E_t x_{t+1}$$

This equation can be combined with the fact that

$$p_t = (1 - \alpha)x_t + \alpha p_{t-1}$$

$$x_t = \frac{1}{1 - \alpha}(p_t - \alpha p_{t-1})$$

and after a bunch of re-arranging you get

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 + \alpha)(1 - \alpha\beta)}{\alpha}(mc_t - p_t)$$

In this definition inflation (π_t) is a function of the expected inflation in the next period and the ratio of marginal cost to price. This relationship is known as the New-Keynesian Phillips curve (NKPC).

Basically write it out for t .

$$\pi_t = p_t - p_{t-1}$$

i.e. real marginal cost.

Output and the NKPC

Let's have a closer look at the relation between output and the NKPC. We will assume that there are standard diminishing returns to labour production function. This makes that the real marginal costs are a function of the output gap.

$$mc_t - p_t = \eta x_t$$

Which implies a NKPC of the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

This NKPC looks a lot like the traditional expectations-augmented Phillips curve. There are some very different implications however. It is a first-order stochastic difference equation, which entails a solution in the form

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t x_{t+k}$$

As you can see, there is no backward-looking element. This means that there is no intrinsic inertia in inflation, implying that the lagged inflation effects in the conventional models are actually a statistical artifact. Note also that in the original formulation the NKPC does not have an error or shock term. Nonetheless, there maybe price movements not consistent with this formulation. For this reason the

Higher output reduces marginal productivity and raises marginal cost.

$x_t = y_t - y_t^n$, where y_t^n is the path of output that would have been obtained in a zero inflation price friction free economy.

As an example there maybe firm-specific shocks to marginal costs where firms differ randomly in the markup they charge.

literature often adds a so called cost-push shock to the curve, so the NKPC becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

Note that due to the inclusion of the error term to account for miscellaneous shocks the effect of monetary policy changes. No longer is inflation just a result of expected inflation and the output gap. As a result, a central bank can no longer implement a stabilisation policy by only addressing the output gap.

Optimal consumption problem

The NKPC, which links inflation to output, is the first of three equations that make up the New Keynesian model. Now we must consider how we link output to monetary policy. In the NKM this is done using interest rates. The relation between consumption and the interest rate comes from a standard intertemporal optimization problem where the consumer wants to maximise

Remember that the basic model does not include capital, meaning that output equals consumption.

$$\sum_{k=0}^{\infty} \left(\frac{1}{1+\beta} \right)^k U(C_{t+k})$$

Which is subject to the intertemporal budget constraint

R_t is the interest rate.

$$\sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{\left(\prod_{m=1}^{k+t} R_{t+m} \right)} = A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{\left(\prod_{m=1}^{k+t} R_{t+m} \right)}$$

The Lagrangian for which is

$$L = \sum_{k=0}^{\infty} \left(\frac{1}{1+\beta} \right)^k U(C_{t+k}) + \lambda \left[A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m} \right)} - \sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m} \right)} \right]$$

Combining the first-order conditions for C_t and C_{t+1} results in the Euler equation

$$U'(C_t) = E_t \left[\left(\frac{R_{t+1}}{1+\beta} \right) U'(C_{t+1}) \right]$$

Here we can set $U(C_t) = U(Y_t) = \frac{Y_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$ and this becomes

Similar to the Real Business Cycle model this is a Constant Relative Risk Aversion (CRRA) utility from consumption.

$$E_t \left[\left(\frac{R_{t+1}}{1+\beta} \right) \left(\frac{Y_t}{Y_{t+1}} \right)^{\frac{1}{\sigma}} \right] = 1$$

The log-linearised version of the Euler equation is

$$\rho = -\log \beta$$

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - \rho)$$

The interpretation of this function is that today's output depends negatively on the real interest rate.

The natural rate of interest

Recall that the equation for inflation featured the output gap $x_t = y_t - y_t^n$. This can be substituted into the Euler equation to produce

$E_t y_{t+1}$ with $E_t x_{t+1} + E_t y_{t+1}^n$

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - \rho) + E_t y_{t+1}^n - y_t^n \\ x_t &= E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \end{aligned}$$

Where the natural interest rate is given by

$$r_t^n = \sigma^{-1} E_t \Delta y_{t+1}^n - \log \beta$$

This natural interest rate, being a function of $E_t \Delta y_{t+1}^n$ is determined by the technology and preferences. Note that output gap x_t also follows a first-order stochastic difference equation which has a solution of the form

$$x_t = \sigma \sum_{k=0}^{\infty} (i_{t+k} - E_t \pi_{t+k+1} - r_{t+k}^n)$$

Here again we see that there is no backward-looking element, indicating that output has no intrinsic persistence. The policy implication of this is that in terms of monetary policy what matters for today's output is not only current policy but all future interest rates. In practice this means that central bankers should take care in managing expectations about future policy. Interpreting i_t as the short-term interest rate, and assuming that the expectations theory of the term structure holds, this model states that it is the long-term interest rates that matter for spending.

In fact, future interest rates are their key tool.

Basic New-Keynesian model

In its most basic form the New Keynesian model has three equations. We have already derived

1. The New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

2. The Euler equation for output

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n)$$

Now we only have to find an equation describing how interest rate policy is set. This is usually described as an explicit interest rate rule.

Monetary policy in the New-Keynesian model

Before diving into the monetary policy rules, let's first examine the joint dynamics of output and inflation in the model. The output and inflation equations are given as

$$\begin{aligned}x_t &= E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_t\end{aligned}$$

These can be rewritten as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa E_t x_{t+1} - \kappa \sigma(i_t - E_t \pi_{t+1} - r_t^n) + u_t$$

and put into vector form

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa \sigma \end{pmatrix} \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} + \begin{pmatrix} \sigma(r_t^n - i_t) \\ \kappa \sigma(r_t^n - i_t) + u_t \end{pmatrix}$$

Here we have a model in the form $Z_t = AE_t Z_{t+1} + BV_t$. We know that for this model to have a unique stable solution the eigenvalues of A need to be less than 1. In this case we have that

$$A = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa \sigma \end{pmatrix}$$

Considering the eigenvalues we have

$$A - \lambda I = \begin{pmatrix} 1 - \lambda & \sigma \\ \kappa & \beta + \kappa \sigma - \lambda \end{pmatrix}$$

Therefore the eigenvalues satisfy

$$\begin{aligned}P(\lambda) &= (1 - \lambda)(\beta + \kappa \sigma - \lambda) - \kappa \sigma = 0 \\ P(\lambda) &= \lambda^2 - (1 + \beta + \kappa \sigma)\lambda + \beta = 0\end{aligned}$$

$P(\lambda)$ is a U-shaped polynomial. If $\lambda = 0$ we get that $P(0) = \beta > 0$ and for $P(1) = -\kappa \sigma < 0$. $P(\lambda)$ will be greater than 0 when λ rises above one. This implies that one eigenvalue is between zero and one, and the other eigenvalue is greater than 1, which is a serious problem for the model. In general there is no unique stable solution and the model has multiple equilibria. There are two ways to deal with this

1. Accept that there are multiple equilibria and analyse the impact of interest rate changes on output and inflation across a range of different possible equilibria
2. Specify that monetary policy follows a particular rule and this rule is designed to produce a unique stable equilibrium.

Recall that there is an eigenvector that when multiplied by $A - \lambda I$ equals a vector of zeroes, meaning that the determinants of the matrix equal zero.

The second option is the most popular option in the New Keynesian literature.

A Taylor-type rule

To consider monetary policy let's start with a rule similar to the one proposed by John Taylor.

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_x x_t$$

In this case the monetary policy sets the interest rate on the basis of inflation and the output gap. When these increase interest rates will be raised. Also note that the natural interest rate is included here, so the set interest rate moves with the natural interest rate. This rule can be substituted in the equation for x_t to give

$$x_t = E_t x_{t+1} + \sigma E_t \pi_{t+1} - \sigma \phi_\pi \pi_t - \sigma \phi_x x_t$$

What do the dynamics look like in this rule? We can rewrite the equations in matrix form where we have that $Z_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}$ and $V_t = \begin{pmatrix} 0 \\ u_t \end{pmatrix}$. Again we can describe the economy in a standard model

$$Z_t = A E_t Z_{t+1} + B V_t$$

This system is a matrix version of the first-order stochastic difference equations and it can be solved in a similar fashion to give

$$Z_t = \sum_{k=0}^{\infty} A^k B E_t V_{t+k}$$

For this model to have a unique stable equilibrium the absolute values of both eigenvalues of A need to be less than 1. This will be the case when

$$\phi_\pi + \frac{(1-\beta)\phi_x}{\kappa} > 1$$

If the policy rule satisfies this requirement, known as the Taylor principle, there is a unique stable equilibrium. Ergo, the nominal interest rates must rise by more than inflation so that real rates rise in response to an increase in inflation. This is needed for stability because otherwise inflationary shocks reduces real interest rates which stimulates the economy which will further stimulate inflation.

Optimal monetary policy

A big question for central banks of course is what is optimal to do? In general we know that central banks

The rule is similar not identical to the Taylor rule because it allows the interest rate to move with the natural rate whereas Taylor's rule has a constant intercept.

This can be combined with the NKPC to produce a system of first-order stochastic difference equations.

Where

$$A = \frac{1}{1 + \sigma \phi_x + \kappa \sigma \phi_\pi} \begin{pmatrix} 1 & \sigma(1 - \beta \phi_\pi) \\ \kappa & \beta + \sigma \kappa + \beta(1 + \sigma \phi_x) \end{pmatrix}$$

$$B = \frac{1}{1 + \sigma \phi_x + \kappa \sigma \phi_\pi} \begin{pmatrix} 1 & -\sigma \phi_\pi \\ \kappa & 1 + \sigma \phi_x \end{pmatrix}$$

$\beta \approx 1$ so the condition is approximately $\phi_\pi > 1$.

- Don't like inflation¹
- Like to keep output on a steady path close to potential

¹ Ask the Germans

The behaviour of central banks has been modeled using something that is called a loss function, which takes the form of

$$L_t = \frac{1}{2} \sum_{k=0}^{\infty} \beta^k E_t (\pi_{t+k}^2 + \gamma x_{t+k}^2)$$

x_t is the output gap and γ indicates the weight put on stabilisation relative to inflation stabilisation.

Quadratic loss functions like these are popular since differentiating things to the power 2 produces linear relationships. This function was purely ad hoc though. Now a quadratic loss function can also be used as an approximation to consumer utility in the NKM. Research has shown that the correct value for $\gamma = \frac{\kappa}{\theta}$. The intuition behind the function is the following

κ is the coefficient on the output gap in the NKPC and θ is the elasticity of demand for firms.

1. x_t^2

- Risk-averse consumers prefer smooth consumption paths which keeps output close to its natural rate to achieve this.

2. π_t^2

- Consumers don't just care about the level of consumption but also its allocation. With inflation, sticky prices imply different prices for the symmetric goods and thus different consumption levels. Optimality requires equal consumption of all items in the bundle. Rationale for welfare effect of inflation, independent of its effect on output.

Optimal policy under commitment

Suppose that the central bank can commit today to a strategy it can adopt now and in the future. The Lagrangian is

$$L = \sum_{t=0}^{\infty} \beta^t E_t \left[\frac{1}{2} (\pi_{t+k}^2 + \gamma x_{t+k}^2) + \lambda_{t+k} (\pi_{t+k} - \beta \pi_{t+k+1} - \kappa x_{t+k}) \right]$$

The constraint here is the NKPC.

The first order conditions of which are

$$\gamma E_t x_{t+k} - \kappa E_t \lambda_{t+k} = 0$$

$$E_t \pi_{t+k} + E_t \lambda_{t+k} - E_t \lambda_{t+k-1} = 0$$

for $t = 0, 1, 2, \dots$ where $\lambda_{-1} = 0$.

There is no constraint on time $t = -1$.

From this we get that

$$E_t x_{t+k} = \frac{\kappa}{\gamma} E_t \lambda_{t+k} = \theta E_t \lambda_{t+k}$$

$$E_t \pi_{t+k} = E_t \lambda_{t+k-1} - E_t \lambda_{t+k} = -\frac{1}{\theta} E_t \Delta x_{t+k}$$

$$\Delta E_t x_{t+k} = -\theta E_t \pi_{t+k}$$

Therefore the optimal policy under commitment will be characterised by

$$\begin{aligned}x_t &= -\theta\pi_t = \theta(p_{t-1} - p_t) \\E_t\Delta x_{t+1} &= -\theta E_t\pi_{t+k} = \theta(p_{t+k-1} - p_{t+k})\end{aligned}$$

If we consider some initial price level p_{-1} we get

$$E_t x_{t+k} = \theta(p_{-1} - E_t p_{t+k})$$

In this case the optimal policy is set against the price level. Importantly, shocks will only temporarily affect price level but have no cumulative effect. On average inflation will be zero.

Optimal policy under discretion

Let's consider the scenario where a central bank cannot commit to taking a particular course of action in the future. Instead, all they can do is adopt the optimal strategy for what to do today. How different would the impact be? Recall that the optimality conditions for period t and $t + 1$ are

$$\begin{aligned}x_t &= -\theta\pi_t \\E_t x_t - E_t x_{t+1} &= -\theta\pi_{t+1}\end{aligned}$$

Under discretion the policy maker always sets $x_t = -\theta\pi_t$. So in this case the policy is set against inflation, where inflation can be characterised by

$$\pi_t = \beta E_t \pi_{t+1} - \kappa\theta\pi_t + u_t$$

With new first-order different equation

$$\pi_t = \left(\frac{1}{1 + \theta\kappa}\right) (\beta E_t \pi_{t+1} + u_t)$$

and a repeated iteration solution

$$\pi_t = \left(\frac{1}{1 + \theta\kappa}\right) \sum_{k=0}^{\infty} \left(\frac{\beta}{1 + \theta\kappa}\right)^k E_t u_{t+k}$$

It is often assumed that the cost-push shocks follow an $AR(1)$ process which implies that $E_t u_{t+k} = \rho^k u_t$ and inflation now becomes

$$\begin{aligned}\pi_t &= \left(\frac{1}{1 - \theta\kappa}\right) \left[\sum_{k=0}^{\infty} \left(\frac{\beta\rho}{1 + \theta\kappa}\right)^k \right] u_t \\&= \left(\frac{1}{1 - \theta\kappa}\right) \left(\frac{1}{1 - \frac{\beta\rho}{1 + \theta\kappa}}\right) u_t \\&= \frac{u_t}{1 + \theta\kappa - \beta\rho}\end{aligned}$$

Since $\pi_t = p_t - p_{t-1}$.

Note that the policy is history dependent: policy today depends on the whole past sequence of shocks that have determined today's price level.

The conditions for the first period are different from the rest. At $t, t - 1$ is gone and doesn't matter. But we do have to take into account the effect that time t decisions will have at time $t + 1$.

$$u_t = \rho u_{t-1} + v_t, \quad v_t \sim N(0, \sigma^2)$$

Use $\sum_{k=0}^{\infty} c^k = \frac{1}{1-c}$ for $|c| < 1$.

The AR(1) cost-push shock implies that

$$E_t x_{t+1} = \rho x_t$$

$$E_t \pi_{t+1} = \rho \pi_t$$

which can be substituted in the Euler equation along with $x_t = -\theta \pi_t$ to back out what the optimal interest rate looks like

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n)$$

$$i_t = r_t^n + \left(\rho + \frac{(1-\rho)\theta}{\sigma} \right) \pi_t$$

This will be greater than 1 if $\frac{\theta}{\sigma} > 1$ which will hold for all reasonable parameterisations. Note also that inflation and thus interest rates do not depend at all on what happened in the past.

Comparing policy under commitment and discretion

In a 2003 paper, Woodford argues that policy under commitment produces superior welfare outcomes. He notes that an optimal policy is history dependent because the private sector will anticipate that future policies will be different because of the conditions at time t have the potential to improve stabilisation outcomes at time t . This holds even if these conditions will actually no longer matter at a later time. Now we also have to consider the transitory cost-push shock u_t . Here Woodford notes that in the case that the expectations about the shock are that it won't affect future policy, the short-run trade-off between inflation and the output gap will be shifted vertically by u_t . In this case the central bank has to choose whether to increase inflation, have a negative output gap, or possibly a bit of both. On the other hand, if due to the shocks people expect the central bank to pursue tighter monetary policy from $t+1$ onward, the short-run trade-off will be shifted by the change $u_t + E_t \pi_{t+1}$. This shift will actually be smaller and thus possibly increase stabilisation. Now the main issue here of course is that it might not be practically feasible to pick a policy and stick to it.

Empirical problems for the model

In contemporary monetary policy analysis there is a central role for the NKPC. There are however some empirical problems with the NKPC as a model for inflation. Given that it relies on the output gap (x_t), a central issue is of course how to measure this gap. One approach to tackle this problem is to assume that on average output tend to return to its natural rate. This means that this natural rate can

be proxied by a simple trend. Let's say that to proxy $x = y_t - y_t^n$ we use

$$\tilde{y}_t = y_t - y_t^{tr}$$

The NKPC can be estimated with data using

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$

However, a problem that arises using this approach is that the sign for κ often comes out negative. Although this might seem counter intuitive, we already know that

- $\Delta \pi_t$ is negatively correlated with the unemployment rate
- Therefore positively correlated with the output gap

Given that $\beta \approx 1$, we can proxy

$$\pi_t - \beta E_t \pi_{t+1}$$

with

$$\pi_t - \pi_{t+1} = -\Delta \pi_{t+1}$$

Which shows that the negative sign on κ might not be that surprising. There are two possible reasons for this failure

1. The model is wrong
2. The output gap is measured with error

Gali and Gertler (1999) argue the latter suggesting that deterministic trends do a bad job in capturing movements in the natural rate of output and suggest an alternative approach. They argue for proxying marginal cost with unit labour costs so that the proxy for real marginal costs is the labour share of income. Estimating $\pi_t = \beta E_t \pi_{t+1} + \gamma s_t$ they find a positive value for γ which puts the NKPC on solid empirical ground. However, Rudd and Whelan (2007) have shown that these results are not so robust as initially thought. For instance, in many countries there has been a downward trend in the labour share. An additional issue is the persistence problem.

If the KNPC would work well with either the labour share or another measure of real marginal cost, this would imply that it is completely forward looking.

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t s_{t+k}$$

For instance using the HP filter.

We can't observe $E_t \pi_{t+1}$ so we substitute realised π_{t+1} and use an instrumental variable to deal with the fact this is a noisy estimator of what we really want.



Figure 1: Labour share of income in the U.S.A.

A VAR model could be used to forecast the levels of s_{t+k} and give a fitted value for the equations above. However, research has shown (Rudd & Whelan, 2006) that the fits are not really good. In contrast, adding lagged inflation to the model

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t s_{t+k} + \rho \pi_{t-1}$$

improves the fit of the model considerably. So a main problem with the New-Keynesian Phillips Curve is that it doesn't account properly for inflation's dependence on its own lags.